UNIT-III

Formal Grammar:

* Introduction
* classification of formal Grammar.
    1. chomsky Hirearchy.
    2. Types.

* **Introduction:—**

Mathematically A formal Grammar is a tuple like

$G = (V, T, P, S)$ where,

V = finite and non Empty set of non-Terminal symbols (or) Variables.

variables are represented by upper case letters

T = Finite and non Empty set of Terminal symbols Represented by lower case letters and some special symbols are there.

P = It is a set of production rules are of the form

$$P \rightarrow \alpha \rightarrow \beta$$

$$\alpha \in V$$

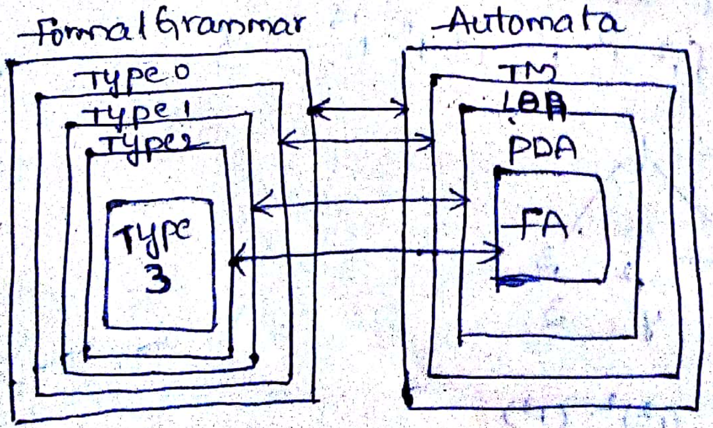
$$\beta \in (V \cup T)^*$$

S → It is the starting symbol of the Grammar is always, a variable which is $S \in V$.

NOTE:— Grammar's are used to describe a language

* **classification of Grammar:—**

- Using chomsky hierarchy.



Type 3 Grammar :—

* It is also called as Regular grammar.
* Type 3 Grammar is defined as $G=(V,T,P,S)$ where,

  V → set of variables.
  T → set of Terminals
  P → set of production rules are of the form

$\boxed{\begin{array}{l} A \to Ba \\ A \to a \end{array}}$ — According to left linear grammar

(or)

$\boxed{\begin{array}{l} A \to aB \\ A \to a \end{array}}$ — According to Right linear grammar.

Ex :— $A \to aB$
$A \to Ba$
$A \to a$
$A \to \epsilon$

where.
$(A,B) \in V$
$a \in T^*$

* Type 3 Grammar is used to generating Regular language
* Regularlanguages are recognised (or) accepted by finite automata. i.e; NFA (or) DFA.

Type 2 Grammar :—

* It is also called as Context-free grammar.
* The context-free grammar is defined as $G=(V,T,P,S)$

  where V → finite set of variables
  T → finite set of Terminals
  P → finite set of production rules are of the form

$$\alpha \to \beta$$

where $\alpha \in V$
$\beta \in (V \cup T)^*$

Ex :— $S \to aSa$
$S \to bSb$
$S \to ab$
$S \to \epsilon$

* Context-free grammars are used to generate context-free language.

* Context-free language recognised (or) Accepted by pushdown Automata.

Type 1 Grammar :—

* It is also called as context - Sensitive Grammar.
* A CSG is defined as $G=(V,T,P,S)$ where
  V = finite set of variables
  T = finite set of Terminals.
  P = set of production rules are of the form $\alpha \to \beta$
  where, $\alpha \in (V \cup T)^+$
  $\beta \in (V \cup T)^*$
  length of $|\alpha| \le$ length of $|\beta|$

Ex:- $S \to aBb$
$bB \to aa$
$B \to b$

* CSG is used to generating Context - Sensitive language
* CSL recognised (or) Accepted by Linear Bounded Automata

Type 0 Grammar :—

* It is also called also Recursive - Grammar (or) Recursive Enumerable grammar (or) phrase structured Grammar.
* mathematically Recursive grammar is defined as
  $G=(V,T,P,S)$ where $V \to$ finite set of variables
  $T \to$ finite set of Terminals
  $P \to$ set of production rules are of the form.
  $\alpha \to \beta$
  $\alpha \in (V \cup T)^{+}$
  $\beta \in (V \cup T)^*$
  $|\alpha| \ge |\beta|$

Ex:- $S \to aAbB$
$aAbB \to aB$
$abB \to aA$
$A \to \epsilon$

* Recursive Grammars are used to generating recursive language (or) Recursive-Enumerable language (or) phrase structured language.

* Recursive languages are recognised are accepted by Turing machine.

Relationship b/w formal grammar and automata :-

1. Type 3 $\subseteq$ Type 2 $\subseteq$ Type 1 $\subseteq$ Type 0
2. FA $\subseteq$ PDA $\subseteq$ LBA $\subseteq$ TM

Context - Free Grammar :

* Introduction
* Design of CFL
* closure properties of CFL.

* Introduction :-

Context-free Grammar is a Grammar which is defined by four tuples like $G=(V,T,P,S)$ where,

V - It is finite and non-empty set of non-terminal symbols (or) variables.

T - finite and non-Empty set of Terminal symbols.

P - finite and non-Empty set of production rules are of the form $\alpha \rightarrow \beta$

$\alpha \in V$

$\beta \in (V \cup T)^*$

Ex:- $S \rightarrow aSa$

$S \rightarrow bSb$

$S \rightarrow aa|bb$

$S \rightarrow \epsilon$

$S \rightarrow$ It is starting symbol.

Context free - language :-

Let $G=(V,T,P,S)$ be a Context-free grammar. The CFG generating a language 'L' is called Context-free language

*It is denoted by L(G)..

*Context - free languages are organized by PDA.

Design of CFL :—

1) construct a CFL for the following set. $\{\epsilon, a, aa, aaa, \ldots a^n\}$

sol:- Given set $\{\epsilon, a, aa, aaa, aaaa, \ldots a^n\}$

minimum string $= \epsilon$

Next minimum string $= a$

Maximum string $= a^n$

$S \rightarrow a^n$

$\downarrow$

$a \cdot a^{n-1} \Rightarrow S \rightarrow aS$

$\downarrow$

$a \cdot a \cdot a^{n-2}$ $\quad S \rightarrow \epsilon$

$\quad S \rightarrow a$

$\downarrow$

$a \cdot a \cdot a \cdot a^{n-3}$

CFG:- S

$S \rightarrow aS$

$S \rightarrow \epsilon$

$S \rightarrow a$

$L = \{a^n \mid n \geq 0\}$ //.

2) construct a CFL for the following set $\{\epsilon, ab, aabb, \ldots\}$

sol:- Minimum String $= \epsilon$

Next minimum string $= ab$

Maximum string $= a^n b^n$

$S \rightarrow a^n b^n$

$S \rightarrow a\, \underline{a^{n-1} \cdot b^{n-1}}\, b$

$S \rightarrow aa\, \underbrace{a^{n-2}\, b^{n-2}}_{(V)}\, bb$

$\therefore S \rightarrow aSb$

$S \rightarrow \epsilon$

$S \rightarrow ab$

$\therefore$ CFG :- $S \rightarrow aSb$

$S \rightarrow \epsilon$

$S \rightarrow ab$

$\therefore L = \{a^n b^n \mid n \geq 0\}$

3)

4)

5)

6)

3) Construct a CFL for the following set {a, b, ab, aabb, aaabbb, ...}

Sol:- Minimum string = a / b

Maximum string = $a^n b^n$

$S \to a^n b^n$

$\to a a^{n-1} b^{n-1} b \Rightarrow S \to aSb$

$\to a a a^{n-2} b^{n-2} bb$ $S \to a$

$S \to b.$

$\therefore$ CFG $S \to aSb$

$S \to a$

$S \to b$

$\therefore L = \{a^n b^n \mid n \geq 1\}$

4) Construct a CFG to generate the language $L = \{a^n b^{2n} / n \geq 1\}$

Sol:- Minimum string = abb

Maximum string = $a^n b^{2n}$

$S \to a^n b^{2n}$

$S \to a a^{n-1} b^{2n-2} bb \Rightarrow S \to aSbb$

$S \to abb$

$\therefore$ CFG = $S \to aSbb$

$S \to abb$.

5) Construct CFG for the following CFL

$L = \{0^i 1^{i+1} \mid i \geq 0\}$

Sol: $L = \{0^i 1^{i+1} \mid i \geq 0\}$

$= 0^i 1^i . 1$

$A \to 0^i 1^i$

$\to 0 0^{i-1} 1^{i-1} 1$

$S \to A1$

$\to 0A1$

$A \to 0A1$

$A \to \epsilon$

$A \to 01$

CFG: $S \to A1$

$A \to 0A1$

$A \to \epsilon$

$A \to 01$

6) Construct a CFL from the following Language

$L = \{a^m b^m c^n \mid m, n \geq 0\}$

Sol:- $\underbrace{a^m b^m}_{A} \underbrace{c^n}_{B}$

$A \to a^m b^m$

$\to a\, a^{m-1} b^{m-1} b$

$A \to aAb$

$A \to \epsilon$

$A \to ab$

$B \to c^n$

$B \to c\, c^{n-1}$

$B \to cB$

$B \to \epsilon$

$B \to c$

CFG:- $S \to AB$

$A \to aAb$

$A \to \epsilon$

$A \to ab$

$B \to cB$

$B \to \epsilon$

$B \to c$

Closure properties of CFL:-

- content free languages are closed under union
- " " " " " " concatenation.
- " " " " " " Kleene closure
- " " " " " " Reversal

- content free languages are not closed under Complement
- " " " " " " " Intersection
- " " " " " " " difference.

Derivation:-

* Introduction * Types of Derivation * Derivation tree

Derivation is a process of generating a string from a given grammar.

Derivation process can be represented graphically is called Derivation Tree (or)

* left most derivation * Rightmost derivation.

Left most derivation:- with example

In this, we can replace a left most Variable to obtain the given input string.

Right most derivation:-

In this, each step we can replace a Right most variable to obtain the given input string.

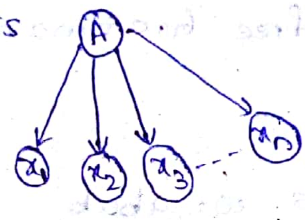
Derivation tree :–

Let $G = (V, T, P, S)$ be a CFG. Then there is a derivation tree for G. If and only if.

* The root node of the Tree is labelled with start symbol of G
* All leaf nodes of Tree are labelled by Terminals (or) special symbols of G.
* The interior nodes are labelled by variables of G.
* If any production rule in G. is are the form.

$A \rightarrow x_1 x_2 x_3 \cdots x_n$ then the

derivation tree is



find the i) left most derivation
ii) Right most derivation
iii) parse tree for the i/p string id+id*id

from the following grammar. $E \rightarrow E+E$
$E \rightarrow E*E$
$E \rightarrow id$.

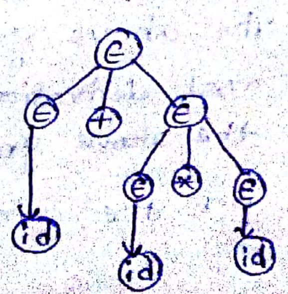
Sol:- The given grammar is
$E \rightarrow E+E$
$E \rightarrow E*E$
$E \rightarrow id$.

Input string. id+id*id.

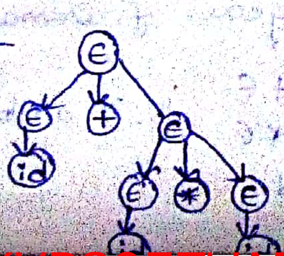
LMD:- $E \rightarrow E+E$
~~$\rightarrow E+E*E$~~
$\rightarrow id+E$
$\rightarrow id+E*E$
$\rightarrow id+id*E$
$\rightarrow id+id*id$.

RMD:- $E \rightarrow E+E$
$\rightarrow$ ~~id~~ $E+E*E$
$\rightarrow E+E*id$
$\rightarrow E+id*id$
$\rightarrow id+id*id$

Parse tree:–



Parse tree:–



Ambiguious Grammar:-

* A CFG $G=(V,T,P,S)$ which generates two (or) more parse trees for given i/p string is called Ambiguious grammar.

* That means an Ambegeous grammar has two or more left most derivations (or) right most derivation (or) parse tree.

Ex:- Prove that $S \to aSbS$ is ambegeous for the i/p
$S \to bSaS$
$S \to \epsilon$

string abab

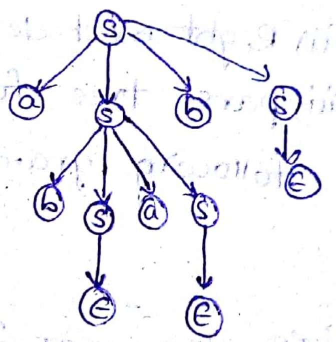
sol:- The given context free Grammar is $S \to aSbS$
$S \to bSaS$
$S \to \epsilon$.

the input string is $w = abab$

① LMD:

$S \to aSbS$
$\to abSaSbS$
$\to ab\epsilon aSbS$
$\to abaSbS$
$\to aba\epsilon bS$
$\to ababS$
$\to abab\epsilon$
$\to abab$

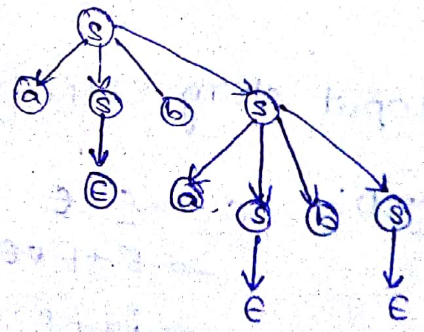
parsetree



② LMD:

$S \to aSbS$
$\to a\epsilon bS$
$\to abS$
$\to abaSbS$
$\to aba\epsilon bS$
$\to ababS$
$\to abab\epsilon$
$\to abab$

Parsetree



∴ The above grammar generates two parse trees (or) two left most derivation for the same i/p string $w=abab$. Hence the above grammar is ambegeous grammar.

2) P.T the grammar $E \to E+E$
$E \to E*E$ to is ambegeous for i/p
$E \to id$
string $id+id*id$.

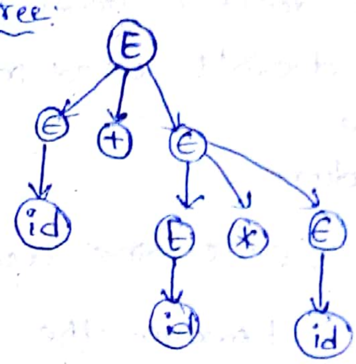
sol:- The given context free grammar is $E \to E+E$
$E \to E*E$
$E \to id$

The input string is $w = id+id*id$.

① LMD:

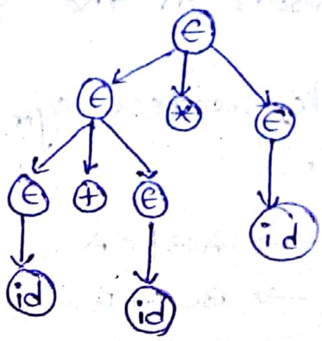
$E \to \underline{E}+E$
$\to id+\underline{E}$
$\to id+\underline{E}*E$
$\to id+id*\underline{E}$
$\to id+id*id.$

Parse tree:



② RMD:-

$E \to \underline{E}*E$
$\to \underline{E}+E*E$
$\to id+\underline{E}*E$
$\to id+id*\underline{E}$
$\to id+id*id.$



** (7m) Simplification of CFG:

* Introduction
* Methods
  1. Elimination of useless symbols.
  2. Elimination of ε-productions
  3. Elimination of unit productions.

Introduction:-

It's means minimizing the no. of productions in the given CFG. that is reducing size of CFG. size of CFG is equal to no. of productions.

Methods:- $S \to AB$
$A \to a$
$A \to aA$
$B \to SB$

Elimination of useless symbols:-

useful symbol:- A variable is said to be useful if and only if

* It generates a terminal string.
* It is used in derivation of a string at least one time

useless symbol :-

* A variable is said to be useless if and only if.
  * It doesn't generates a terminal string.
  * It doesn't used in derivation of a string at least one time.

Procedure :-

step 1 :- Determine useless symbols in the grammar.

step 2 :- Remove the productions which contains useless symbols in the grammar.

Ex :- Eliminate useless symbols from the following grammar.

$S \to AB \mid CA$
$B \to BC \mid AB$
$A \to a$
$C \to aB \mid b$

sol :- The given CFG is

$S \to AB$
$S \to CA$
$B \to BC$
$B \to AB$
$A \to a$
$C \to aB$
$C \to b$

In the given grammar 'B' doesn't generating a terminal string.

$S \to \underline{A}B$
$S \to a\underline{B}$
$\to a\underline{B}C$
$\to a\underline{A}BC$
$\to aaB\underline{C}$
$\to aaBb$
$\to aa\underline{A}Bb$
$\to aaaBb$
⋮

so, ∴ 'B' is useless symbol.

so, we can eliminate the productions which contains 'B'.

∴ The reduced CFG is

$S \to CA$ | $C \to b$.
$A \to a$

2) Elimination of ϵ-production :-

ϵ-production :- A production is of the form $A \to \epsilon$ is called ϵ-production (or) NULL production.

procedure :-

step 1 :- If the grammar contains $A \to \epsilon$ then replace 'A' with ϵ in the remaining productions.

step 2 :- Remove $A \to \epsilon$ from the grammar.

Ex :- Remove ϵ-productions from the following grammar

$A \to 0B1 \mid 1B1$

$B \to 0B \mid 1B \mid \epsilon$

sol :- the given CFG is $A \to 0B1$

$A \to 1B1$

$B \to 0B$

$B \to 1B$

$B \to \epsilon$.

$A \to 0B1$
$\to 0\epsilon 1$
$\to 01$

$\therefore A \to 0B1$
$A \to 01$

$B \to 0B$
$\to 0\epsilon$
$\to 0$

$\therefore B \to 0B$
$B \to 0$

$A \to 1B1$
$\to 1\epsilon 1$
$\to 11$

$\therefore A \to 1B1$
$A \to 11$

$B \to 1B$
$\to 1\epsilon$
$\to 1$

$\therefore B \to 1B$
$B \to 1$

After eliminating $B \to \epsilon$ the resultant CFG is

$A \to 0B1$ $\quad$ $B \to 1B$

$A \to 01$ $\quad$ $B \to 1$

$A \to 1B1$

$A \to 11$

$B \to 0B$

$B \to 0$

14M
** Normal forms :-

*Introduction

*Types of Normal forms

1. chomsky Normal Form (CNF)
2. Greiback Normal Form (GNF).

Introduction :-

In CFG each production of the form $\alpha \rightarrow \beta$ where $\alpha \in V$ that means β contains any no. of non-terminal symbols $\beta \in (V \cup T$ and any no. of terminal symbols. But, we need to have a grammar in specific form i.e; we can decide the no. of non-terminals and terminals on R.H.S of the grammar. This can be implemented by using "normalization of CFG".

Normalization :-

The process of Arranging the grammar with fixed no. of non-terminals and terminals on R.H.S of CFG is called normalization.

normal-forms are classified into two types

i) chomsky normal form.

ii) Greiback normal form

chomsky normal form :—

It is defined as $\alpha \to \beta$

non-terminal $\to$ Non-terminal. Non-terminal.
(or)
Non-terminal $\to$ Terminal.

conversion of CFG to CNF :—

procedure :-

step 1 :- simplify the CFG

step 2 :- convert the simplified CFG to CNF.

Ex:- convert the following CFG into chomsky normal form.

$S \to aaaaS$

$S \to aaaa$

Sol:- The given grammar is $S \to aaaaS$

$S \to aaaa$

Consider a non-terminal A that derives terminal a.

$\therefore$ The production rule is $A \to a$. is in CNF

$S \to aaaaS$

$S \to A\boxed{AAAS}$ can be replaced by P_1

$S \to AP_1$ is in CNF.

$P_1 \to A\boxed{AAS}$ can be replaced by P_2

$P_1 \to AP_2$ is in CNF

$P_2 \to A\boxed{AS}$ can be replaced by P_3

$P_2 \to AP_3$ is in CNF

$P_3 \to AS$ is in CNF.

$S \to aaaa$

$S \to A\boxed{AAA}$ can be replaced by P_4

$S \to AP_4$ is in CNF

$P_4 \to A\boxed{AA}$ can be replaced by P_5.

$P_4 \to AP_5$ is in CNF

$P_5 \to AA$ is in CNF.

The resultant Grammar CNF is

$S \to AP_1$

$S \to AP_4$

$P_1 \to AP_2$

$P_2 \to AP_3$

$P_3 \to AS$

$P_4 \to AP_5$

$P_5 \to AA$

$A \to a$

2) Convert the given CFG to CNF. $S \to aSa$
$S \to bSb$
$S \to a$
$S \to b$.

sol:- The given grammar is $S \to aSa$
$S \to bSb$.
$S \to a$
$S \to b$

It is already in simplified form.

Consider a non-terminal A that derives a terminal a and the non-Terminal B. that derives the terminal b.

∴ The production rules is $A \to a$ are in CNF.
$B \to b$.

(i) $S \to aSa$

$S \to A\boxed{SA}$ can be replaced by P_1

$S \to AP_1$ is in CNF.

$P_1 \to SA$ is in CNF.

(ii) $S \to bSb$

$S \to B\boxed{SB}$ can be Replaced by P_2

$S \to BP_2$ is in CNF

$P_2 \to SB$ is in CNF

(iii) $S \to a$ is in CNF

$S \to b$ is in CNF.

∴ The resultant grammar in CNF is

$S \to AP_1$

$S \to BP_2$

$S \rightarrow a$

$S \rightarrow b$

$P_1 \rightarrow SA$

$P_2 \rightarrow SB$

$A \rightarrow a$

$B \rightarrow b.$

Greibach Normal Form (GNF) :-

GNF is defined as

Non-terminal $\rightarrow$ Terminal. any no. of nonterminals

Non-terminal $\rightarrow$ Terminal.

Lemma 1:

Let CFG be $G=(V,T,P,S)$ and there is a production rule $A \rightarrow aB$ and $B \rightarrow \beta_1|\beta_2|\beta_3|\cdots|\beta_n$ then add the new production rule to GNF $A \rightarrow a\beta_1|a\beta_2|a\beta_3|\cdots|a\beta_n$ to GNF.

$\therefore$ B is replaced by $B \rightarrow \beta_1|\beta_2|\cdots|\beta_n$

Lemma 2:

Let CFG be $G=(V,T,P,S)$ and there is production rule $A \rightarrow A\alpha_1|A\alpha_2|\cdots|A\alpha_n|\beta_1|\beta_2|\cdots|\beta_n$ then the production rules are added to GNF.

$A \rightarrow \beta_1|\beta_2|\beta_3|\cdots|\beta_n$

$A \rightarrow \beta_1 Z|\beta_2 Z|\beta_3 Z|\cdots|\beta_n Z$

$Z \rightarrow \alpha_1|\alpha_2|\alpha_3|\cdots|\alpha_n$

$Z \rightarrow \alpha_1 Z|\alpha_2 Z|\alpha_3 Z|\cdots|\alpha_n Z$

Converting ~~CNG~~ CFG into GNF :-

Procedure:-

step 1 :- Simplify the CFG.

step 2 :- Converting simplified CFG into GNF.

Ex:- Convert the given CFG to GNF $S \rightarrow ABA$

$A \rightarrow aA|\epsilon$

$B \rightarrow bB|\epsilon$.

Sol:- The Given CFG is $S \rightarrow ABA$

$A \rightarrow aA$

$A \to \epsilon$

$B \to bB$

$B \to \epsilon$

Simplified of given CFG :-

ⓐ elimination of ϵ-productions :-

$A \to \epsilon \quad B \to \epsilon$

① $S \to ABA$
$S \to \epsilon BA$
$S \to BA$

② $S \to ABA$
$S \to AB\epsilon$
$S \to AB$

③ $S \to ABA$
$S \to A\epsilon A$
$S \to AA$

④ $S \to ABA$
$S \to \epsilon\epsilon A$
$S \to A$

⑤ $S \to ABA$
$S \to \epsilon B \epsilon$
$S \to B$

$A \to aA$
$A \to a\epsilon$
$A \to a$

$B \to bB$
$B \to b\epsilon$
$B \to b$

$\therefore$ After eliminating $A \to \epsilon$, $B \to \epsilon$ from the grammar the resultant grammar is

$S \to ABA \qquad A \to aA$
$S \to BA \qquad A \to a$
$S \to AB \qquad B \to bB$
$S \to AA \qquad B \to b$
$S \to A$ ✓
$S \to B$ ✓

Elimination of unit productions :-

The above grammar has two unit productions like

$S \to A$ × $\qquad S \to B$ ×

$S \to aA \qquad S \to bB$

$S \to a \qquad S \to b$

$\left[\because \text{since } A \to aA,\ A \to a \quad B \to bB,\ B \to b\right]$

$\therefore$ After elimination unit productions $S \to A$, $S \to B$ from the grammar. The resultant grammar is

$S \to ABA \qquad A \to aA$
$S \to BA \qquad A \to a$
$S \to AB \qquad B \to bB$
$S \to AA \qquad B \to b$
$S \to aA$
$S \to a$
$S \to bB$
$S \to b$

there is no useless production.

∴ The simplified CFG is

$S \to ABA$ $\quad A \to aA$
$S \to BA$ $\quad A \to a$
$S \to AB$ $\quad B \to bB$
$S \to AA$ $\quad B \to b$
$S \to aA$
$S \to a$
$S \to bB$
$S \to b$

Converting simplified CFG to GNF:

i) $S \to ABA$ $\quad A \to aA$ ✓
$S \to aABA$ ✓ $\quad A \to a$ ✓
$S \to aBA$ ✓ $\quad B \to bB$ ✓

ii) $S \to BA$ $\quad B \to b$ ✓
$S \to bBA$ ✓
$S \to bA$ ✓

iii) $S \to AB$
$S \to aAB$
$S \to aB$ ✓

iv) $S \to AA$
$S \to aAA$
$S \to aA$

v) $S \to aA$ ✓
$S \to a$ ✓

vi) $S \to bB$ ✓
$S \to b$ ✓

∴ The resultant grammar is in GNF. is

$S \to aABA \mid aBA \mid bBA \mid bA \mid aAB \mid aB \mid aAA \mid aA \mid bB \mid a \mid b$
$A \to aA \mid a$
$B \to bB \mid b.$

(2) Convert the following CFG into GNF $S \to AA \mid 0$
$A \to SS \mid 1$

Sol: Given Grammar $S \to AA$
$S \to 0$
$A \to SS$
$A \to 1$

The simplified CFG is. $S \to AA$
$S \to 0$
$A \to SS$
$A \to 1$

① S → AA | 0
S → SSA | 0
S → 0
S → 0Z
Z → SA
Z → SAZ
Z → SA
Z → 0A
Z → 0ZA
Z → 1AA
Z → 0A

② S → AA | 0
S → 1A | 0
S → 1A
S → 0
Z → SAZ
Z → 0AZ
Z → 0ZAZ
Z → 1AAZ
Z → 0AZ

③ A → S1
A → 0S
A → 0ZS
A → 1AS
A → 0S

∴ The resultant grammar is

S → 0 | 0Z | 1A

Z → 0A | 0ZA | 1AA | 0A | 0AZ | 0ZAZ | 1AAZ |

A → 0S | 0ZS | 1AS | 1

③ convert the given CFG to GNF S → CA
A → a
C → aB | b

⇒ Given CFG is not a simplified Grammar
After eliminating the useless symbols the resultant CFG is. S → CA
A → a
C → b

By Applying Lemma 1 S → CA
S → bA

∴ The resultant GNF is S → bA
A → a
C → b

④ Convert the given CFG to GNF S → SS
S → 0S1 | 01

The given CFG is a simplified CFG
The resultant grammar is S → SS
S → 0S1
S → 01

Replaced 0 by A, 1 by B
Then productions are A → 0
B → 1

$S \to SS$

$S \to ASB$

$S \to AB$

Applying Lemma ①

① $S \to SS$

$S \to ASBS$

$S \to 0SBS$

② $S \to SS$

$S \to ABS$

$S \to 0BS$

② $S \to ASB$

$S \to 0SB$

$S \to AB$

$S \to 0B$

The resultant grammar GNF is

$S \to 0SBS \mid 0BS \mid 0SB \mid 0B$

$A \to 0$

$B \to 1$

Pumping Lemma for CFL:—

pumping lemma is used for proving the given language is CFL (or) not.

Lemma :- let 'L' be any CFL, then there is a constant 'n' which depends only a part 'L' such that there exist a string. $w = uvxyz$ such that
1. $|vy| \geq 1$
2. $|vxy| \leq n$
3. for $\forall\, i > 0$ uv^ixy^iz is in L

Then 'L' is said to be CFL. otherwise it is not a CFL.

① Prove that $L = \{a^n b^n c^n \mid n \geq 0\}$ is not a CFL.

The given language $L = \{a^n b^n c^n \mid n \geq 0\}$.

$L = \{\epsilon, abc, aabbcc, ----\}$

consider a constant n and the string $w = a^n b^n c^n$

consider a string $w \in L$

$w = abc$ for $n = 1$

$|w| = 3n$

for $i = 1$ $w = abc$

for $i = 2$

$w = uv^ixy^iz$

$w = uv^2xy^2z$

$w = a^{n-1} a^2 b^{n-1} b^2 c^n$

$w = a^{n+1} b^{n+1} c^n \notin L$

$a^n \quad b^n \quad c^n$

$a^{n-1} \quad a \quad b^{n-1} \quad b \quad c^n$

$u \quad v \quad x \quad y \quad z$

$\therefore$ The given language is not a CFL.

② show that the language $L = \{ ss^T / s \in \{a,b\}^* \}$

Given language $L = \{ ss^T / s \in \{a,b\}^* \}$

$L = \{ \epsilon,$