BASIC ELECTRICAL AND ELECTRONICS ENGINEERING

(Computer *Science Engineering*)

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UNIT-2

AC CIRCUITS

Introduction to AC Circuits

- Introduction : A circuit in which currents and voltages vary sinusoidally i.e vary with time is called alternating current or A.C circuit
- All AC circuits are made up of combination of R, L, C
- The circuit elements R,L & C are called circuit parameters
- Alternating Quantity: an alternating quantity is that which acts in alternate directions and whose magnitude undergoes a definite cycle of changes in definite interval of time
- Cycle: A cycle may be defined as one complete set of positive and negative values of alternating quantity repeating at equal intervals, each complete cycle spread over 360 degrees electrical
- Time period: The time taken by an alternating quantity in seconds to trace one complete cycle is called periodic time or time period, it is usually denoted by symbol “T”

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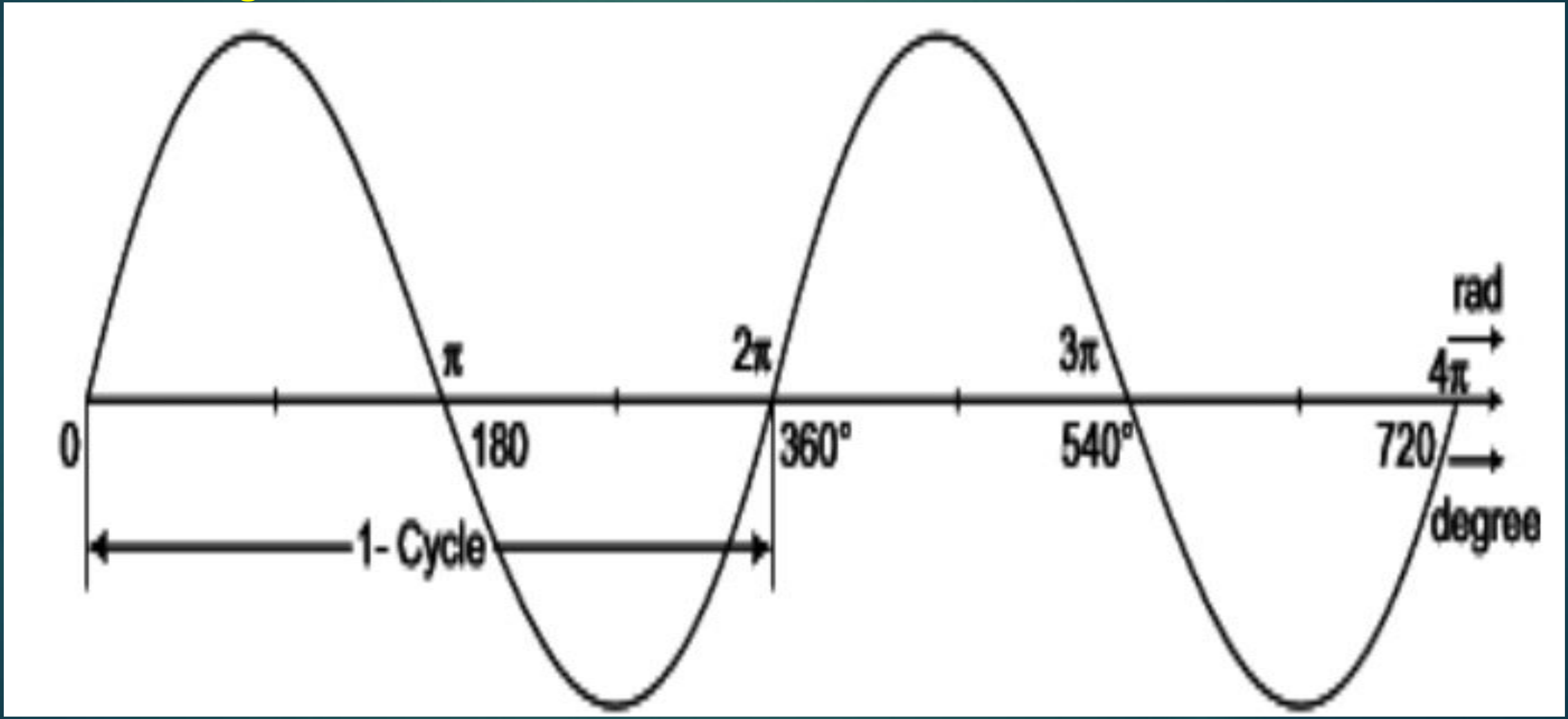
- Sinusoidal signal :



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- Frequency : The number of cycles per second is called frequency and it is denoted by symbol "f"

$$f = 1/T$$

if the angular velocity is "w" is expressed in radians per seconds then

$$w = 2\Pi f = 2\Pi/T$$

- Peak factor: Ratio of maximum value to the RMS value is known as peak factor or crest factor or amplitude factor

Peak factor = maximum value / RMS value

- Form factor: Ratio of RMS value to Average Value is known as form factor

Form factor = RMS value / Average Value

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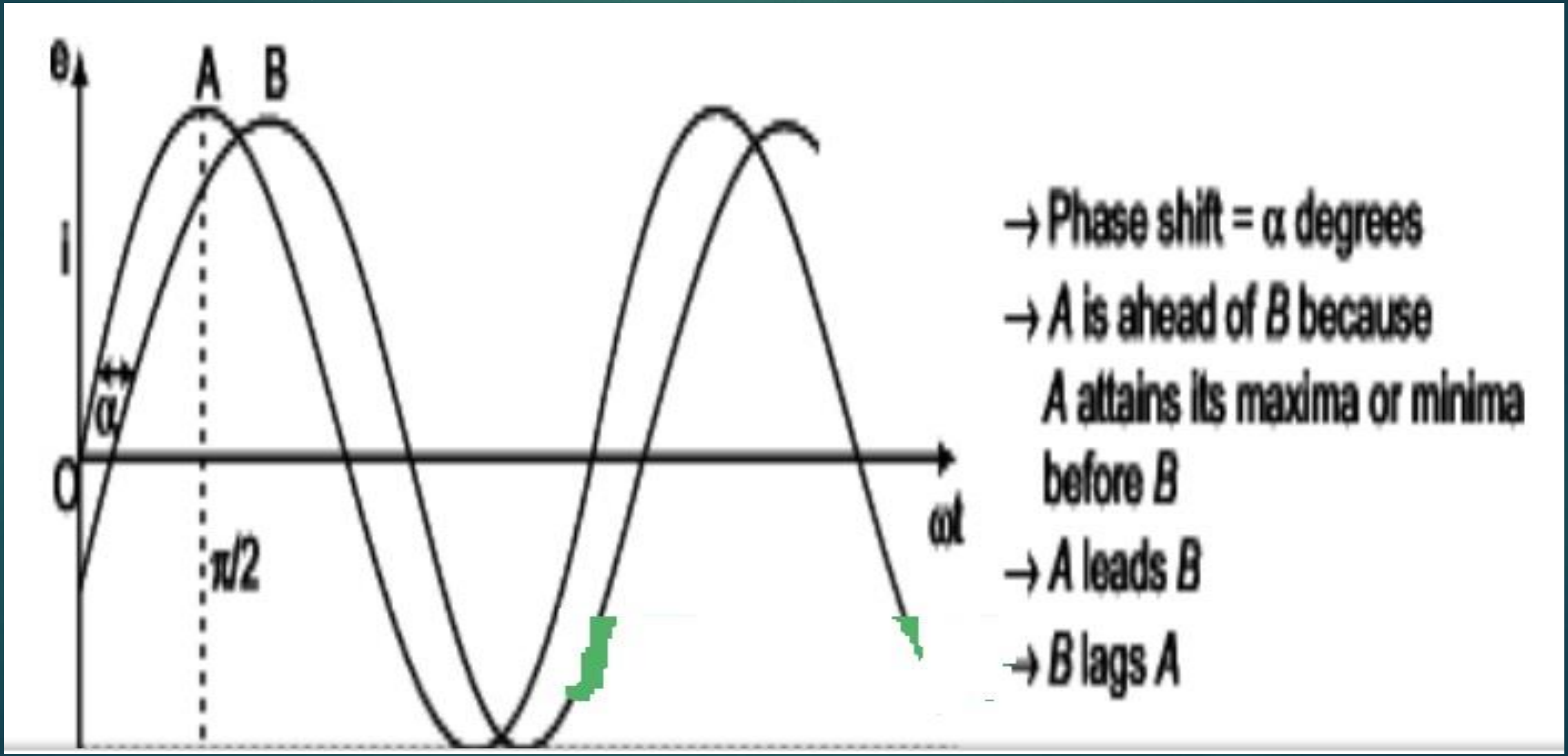
- Phase difference :



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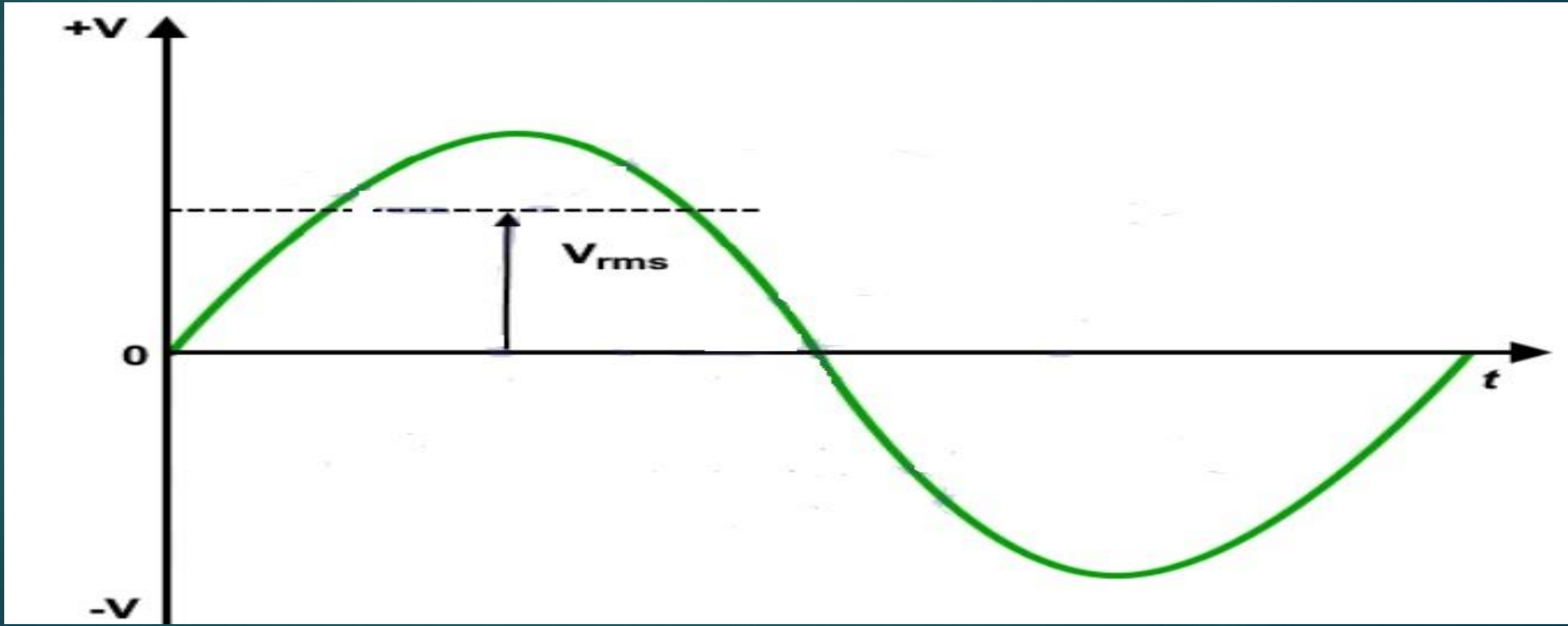
- Root mean square value (RMS): It is also known as effective value or virtual value of an alternating current is the value of direct current(DC) when flowing through a circuit or resistor for the specific time period and produces same amount of heat which produced by the alternating current (AC)when flowing through a circuit or resistor for a specific time



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- Formula for RMS value: Vrms = Squrt(1/2Π ($\int_0^{2\Pi}$ $V^2(t)$ dwt))

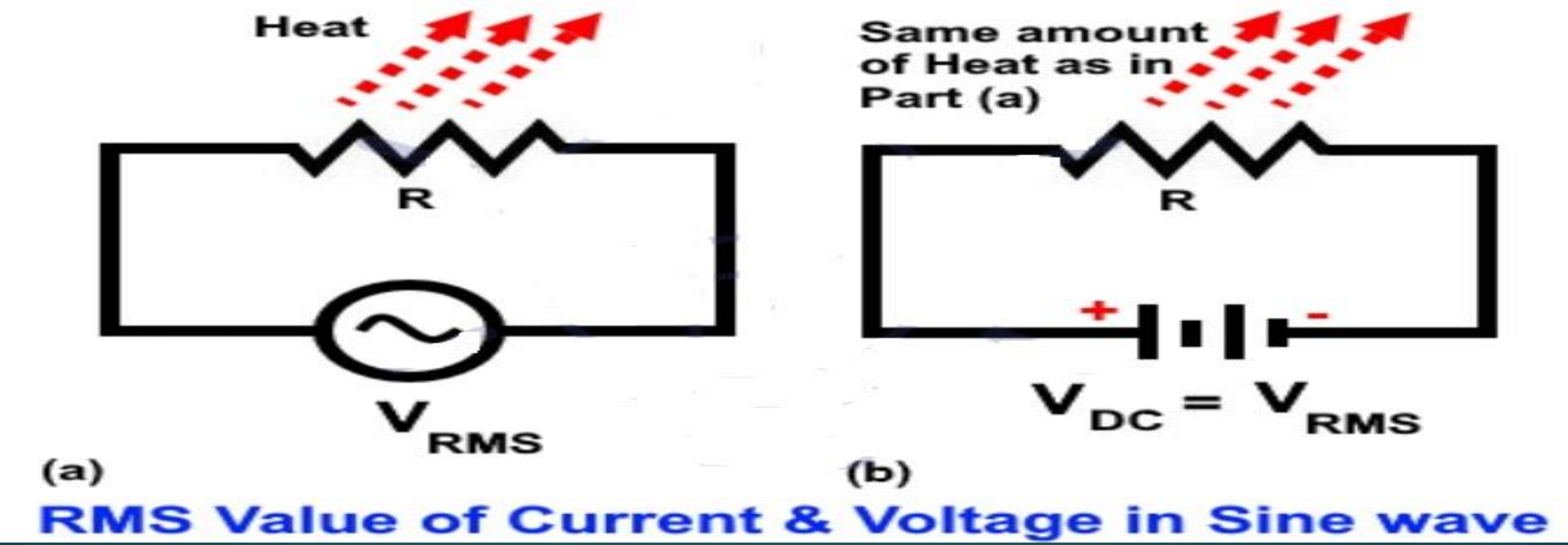
  where V(t) = Vm Sinwt =10 sin 2pi100 t=Vm sin 2 pi f t

  Vm = maximum voltage (volts)

  w = angular frequency (rad/sec)

  t = time period of sin wave (sec)

  for sinusoidal signal Vrms = Vm/√2 , Irms =Im/√2



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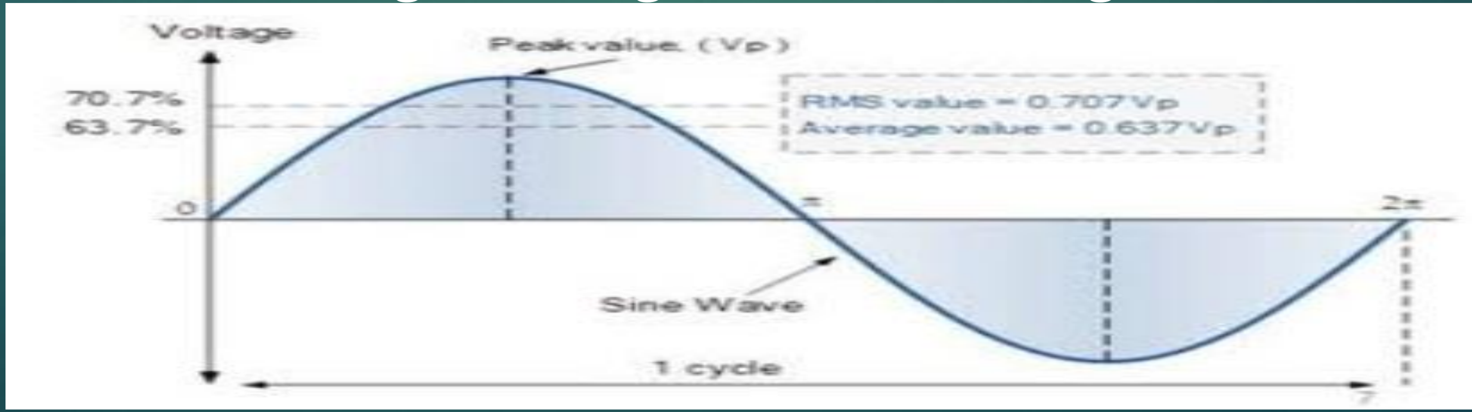
- Average value : Average value is defined based on charge transfer in the circuit ,the voltage at which charge transfer in an AC circuit is equal to the charge transfer in a DC circuit is called as Average value
- Average value of one complete cycle or symmetrical wave form is equal to zero
- For finding average value of symmetrical wave form only positive half cycle is considered
- For symmetrical wave form: $V_{avg} = 1/\Pi \int_0^{\Pi} V(t)\, dwt$

For unsymmetrical wave form: $V_{avg} = 1/2\Pi \left(\int_0^{\Pi} V(t)\, dwt + \int_{\Pi}^{2\Pi} V(t)\, dwt \right)$

Average value of Sinusoidal signal $V_{avg} = 2\, V_m/\Pi$, $I_{avg} = 2\, I_m/\Pi$



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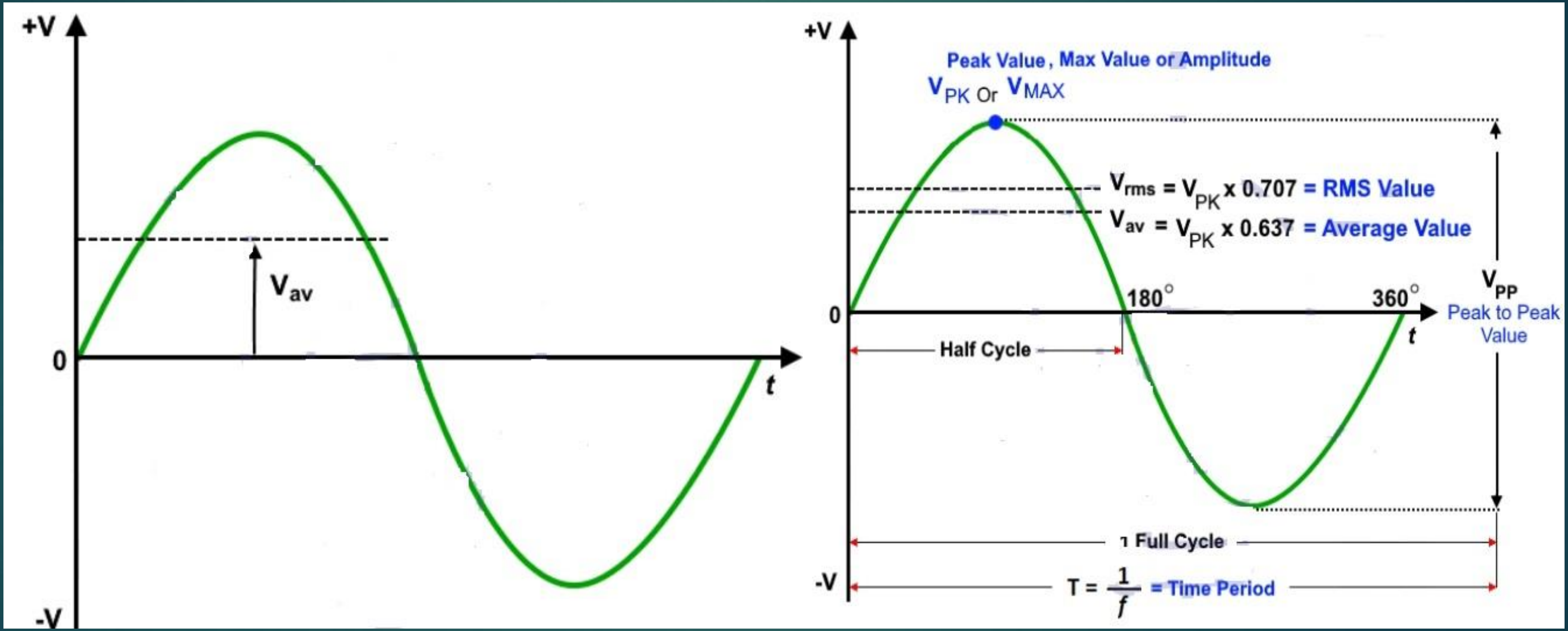
- Sinusoidal wave average value :



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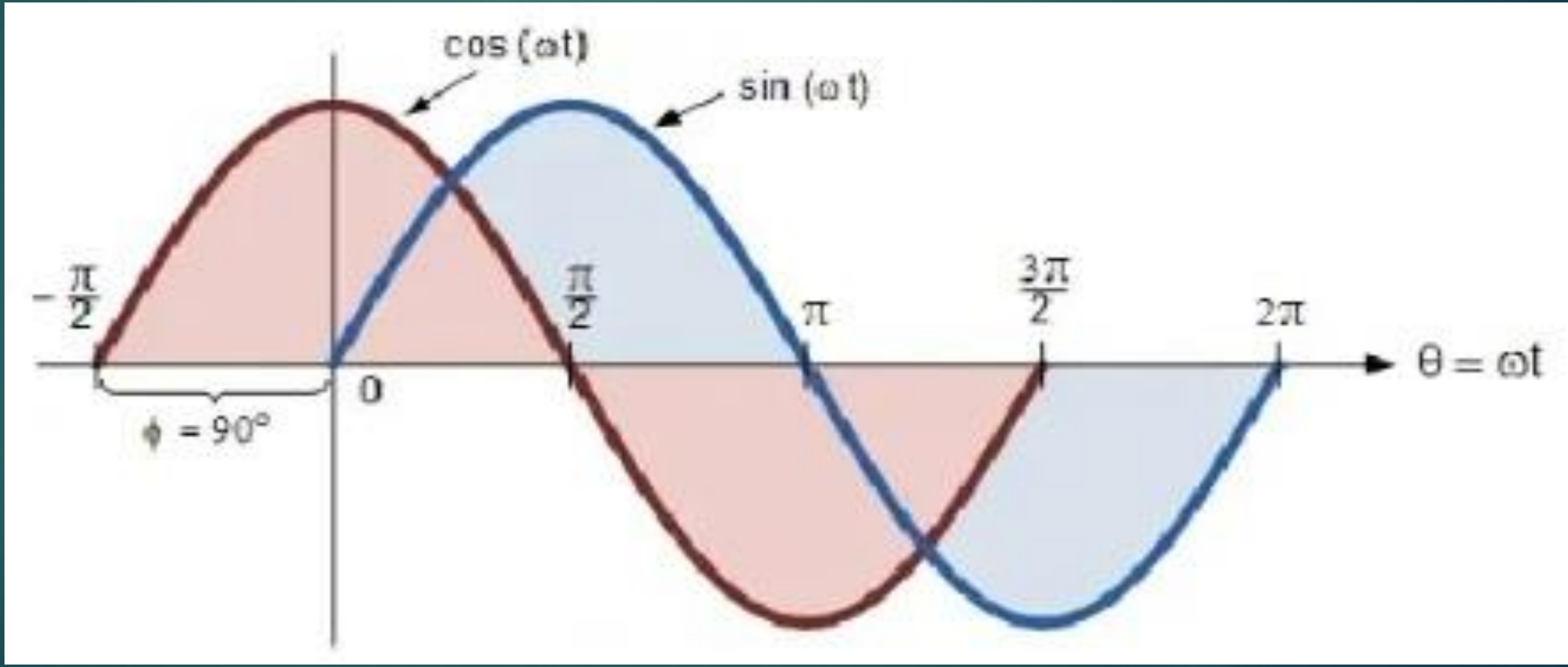
- Leading wave form: let us considered a sinusoidal signal input wave V(t) = Vm Sinwt
- Input signal lead by with an angle of θ, V(t) = Vm Sin(wt + θ) = Vm Sin(wt – (- θ))
- From figure $\theta = 90^0$,i.e V(t) = Vm sin(wt + 90)



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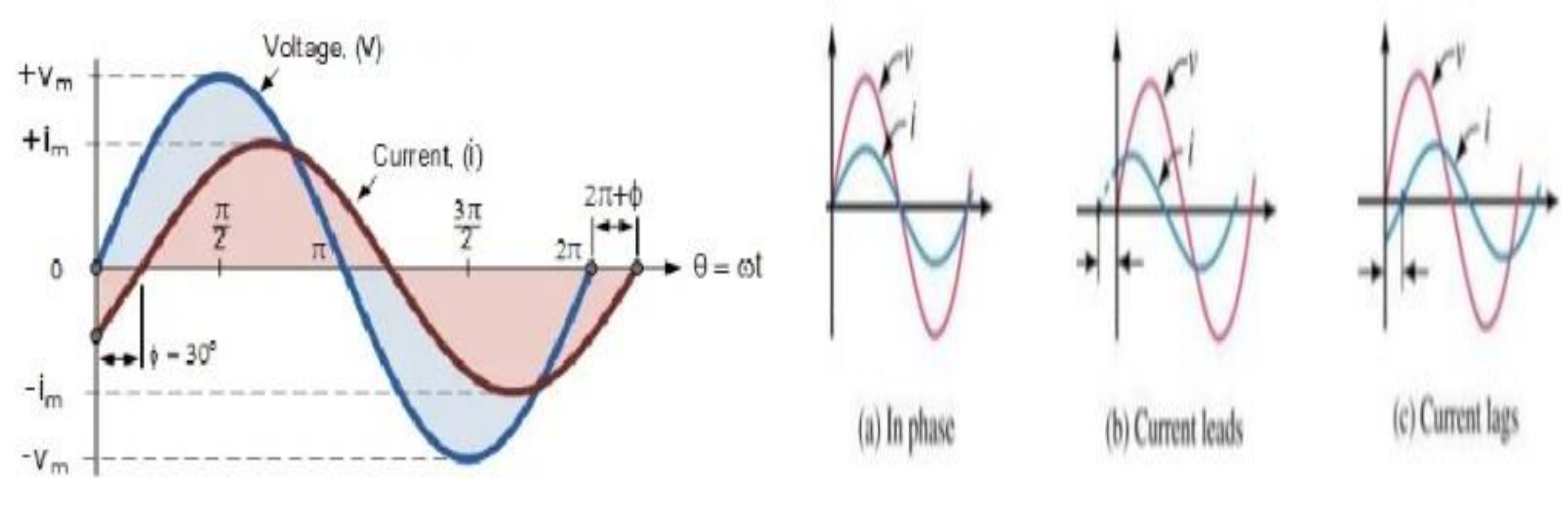
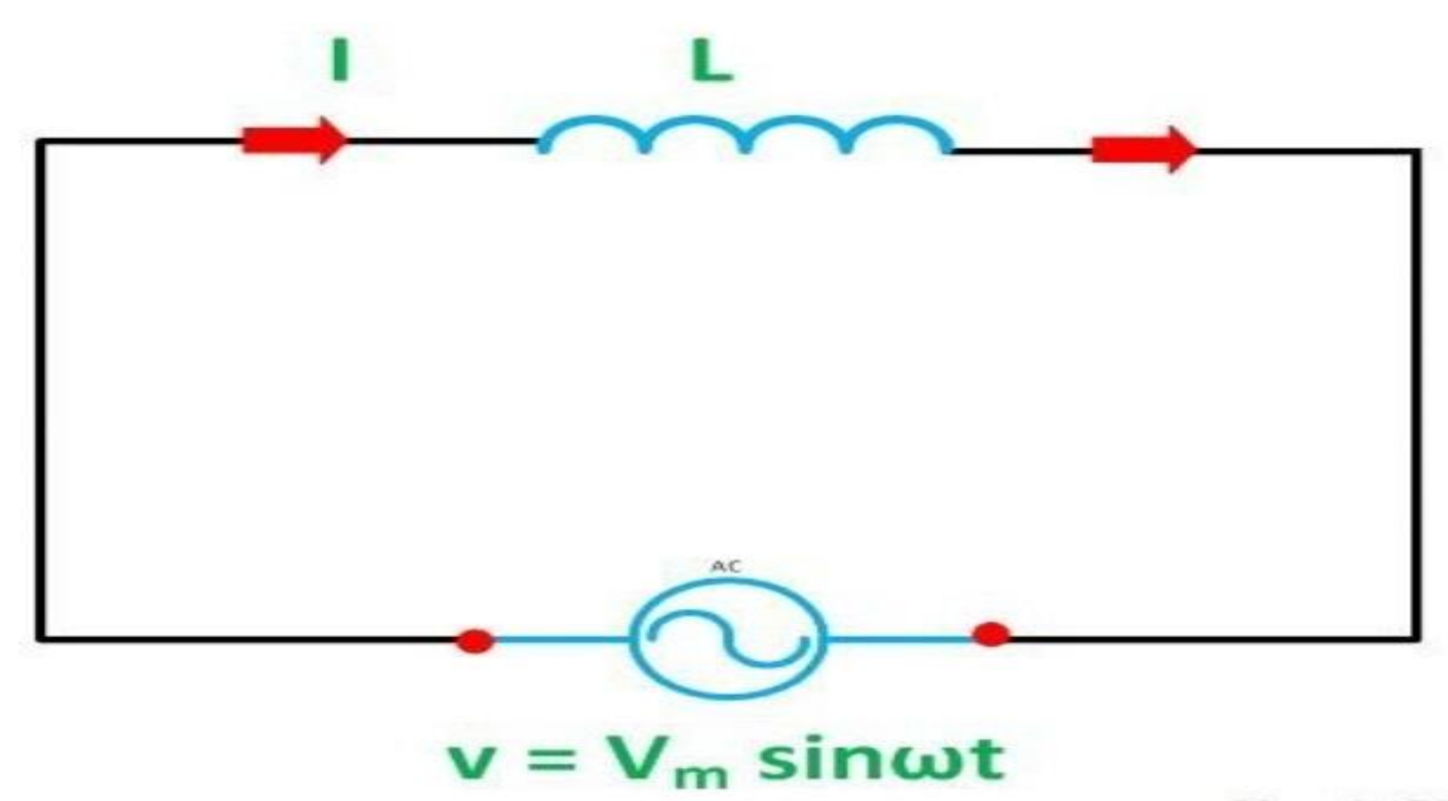
- Lagging wave form: let us considered a sinusoidal signal input wave V(t) = Vm Sinwt
- Input signal lag by with an angle of θ, V(t) = Vm Sin(wt - θ) = Vm Sin(wt + (- θ))
- From figure $\theta = 30^0$,i.e V(t) = Vm sin(wt - 30^0)



Inductive Circuit

- Pure inductive circuit : The circuit which contains only Inductance (L) and not any other quantities like resistor and capacitance in the circuit is called pure Inductive circuit
- In this circuit voltage leads current by an angle of 90^0 or current lags voltage by an angle of 90^0 $Z= R+ jX_L$



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- Derivation: Alternating Current applied to the circuit is given by

$$I = Im\ Sinwt$$

Voltage across inductor is given by

$$V_L = L\ di/dt$$

current flowing through the circuit is given by

$$I = dq/dt$$

$$V_L = L\ d(Im\ siwt)/dt$$

$$V_L = L\ Im\ w\ Coswt = w\ L\ Im\ Sin\ (wt + 90^0) = Im\ X_L\ Sin\ (wt + 90^0)$$

$$Im = V_L / X_L$$

$$X_L = w\ L \quad i.e\ w = 2\ \Pi\ f$$

$$V_L = w\ L\ I$$

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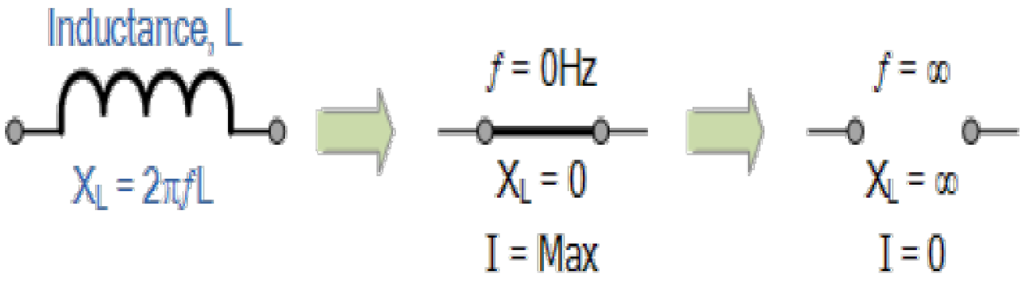

In an AC circuit containing pure inductance the following formula applies:

$$\text{Current, I} = \frac{\text{Voltage}}{\text{Opposition to current flow}} = \frac{V}{X_L}$$

In the phasor domain the voltage across the coil is given as:

$$V_L = j\omega LI$$

where: $j\omega L = jX_L = 2\pi f L = \text{IMPEDANCE, Z}$

and in Polar Form this would be written as: $X_L \angle 90^o$ where:

$$X_L \angle \theta = \frac{V_L \angle +90^o}{I_L \angle 0^o}$$

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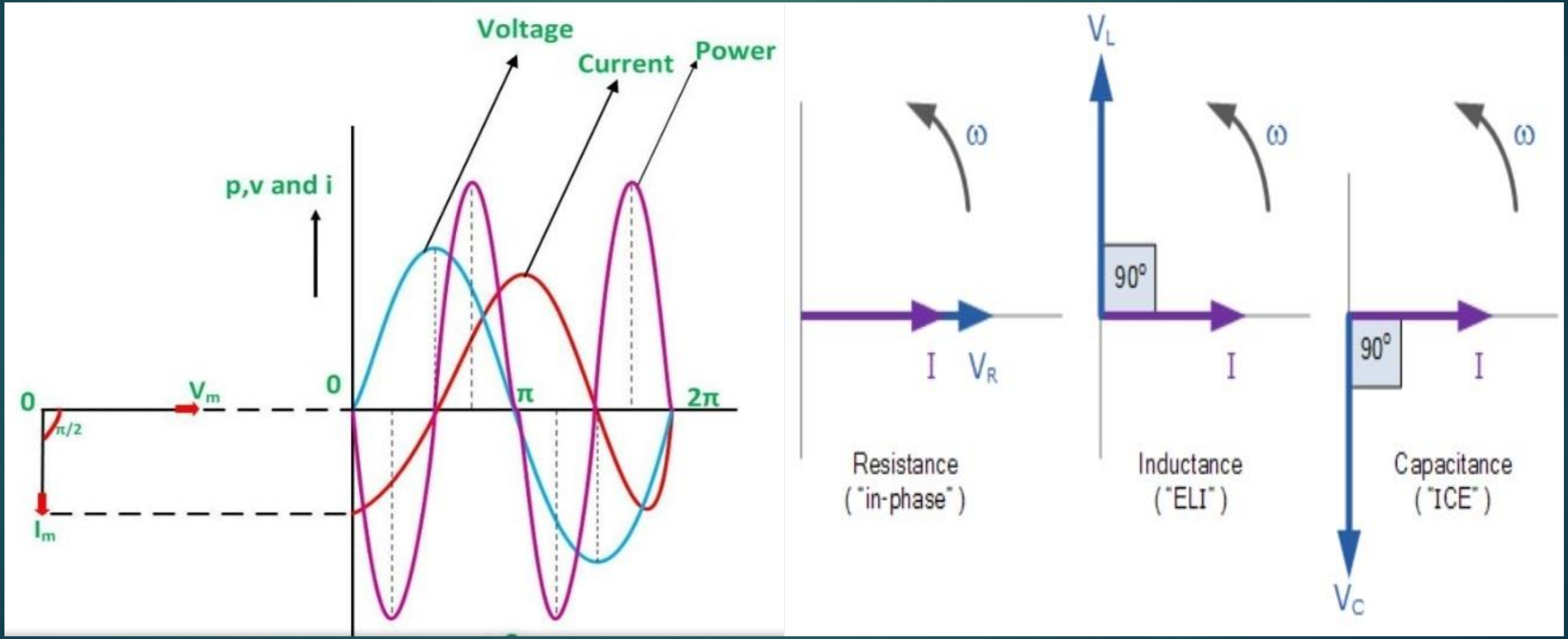
- Phasor diagram & power curve of inductive circuit :The current in the pure inductive AC circuit lags the voltage by 90 degrees



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Power in Pure Inductive Circuit

Instantaneous power in the inductive circuit is given by

$$p = vi$$

$$P = (V_m \sin\omega t)(I_m \sin(\omega t + \pi/2))$$

$$P = V_m I_m \sin\omega t \cos\omega t$$

$$P = \frac{V_m I_m}{2} 2 \sin\omega t \cos\omega t$$

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \sin 2\omega t \text{ or}$$

$$P = 0$$

Hence, the average power consumed in a purely inductive circuit is zero.

Capacitive Circuit

- Pure capacitive circuit :The circuit contains only a pure capacitor of capacitance C faradays is known as pure capacitor circuit
- The capacitors stores electrical power in the form of electric field, their effect is known as the capacitance. It is also known as “Condenser”
- The capacitors consists of two conductive plates which are separated by the dielectric medium
- The dielectric material is made up of glass or paper or insulating material
- It stores energy in electrical form
- The capacitor works as a storage device
- Capacitor gets charged when the supply is “ON” and it is discharged when the supply is “OFF”
- If it is connected to the direct supply, it gets charged equal to the value of applied voltage

Continued…………

- Derivation: Alternating voltage applied to the circuit is given by

$$V = V_m \sin wt$$

charge of capacitor at any instant of time

$$q = C\,V$$

current flowing through the circuit is given by

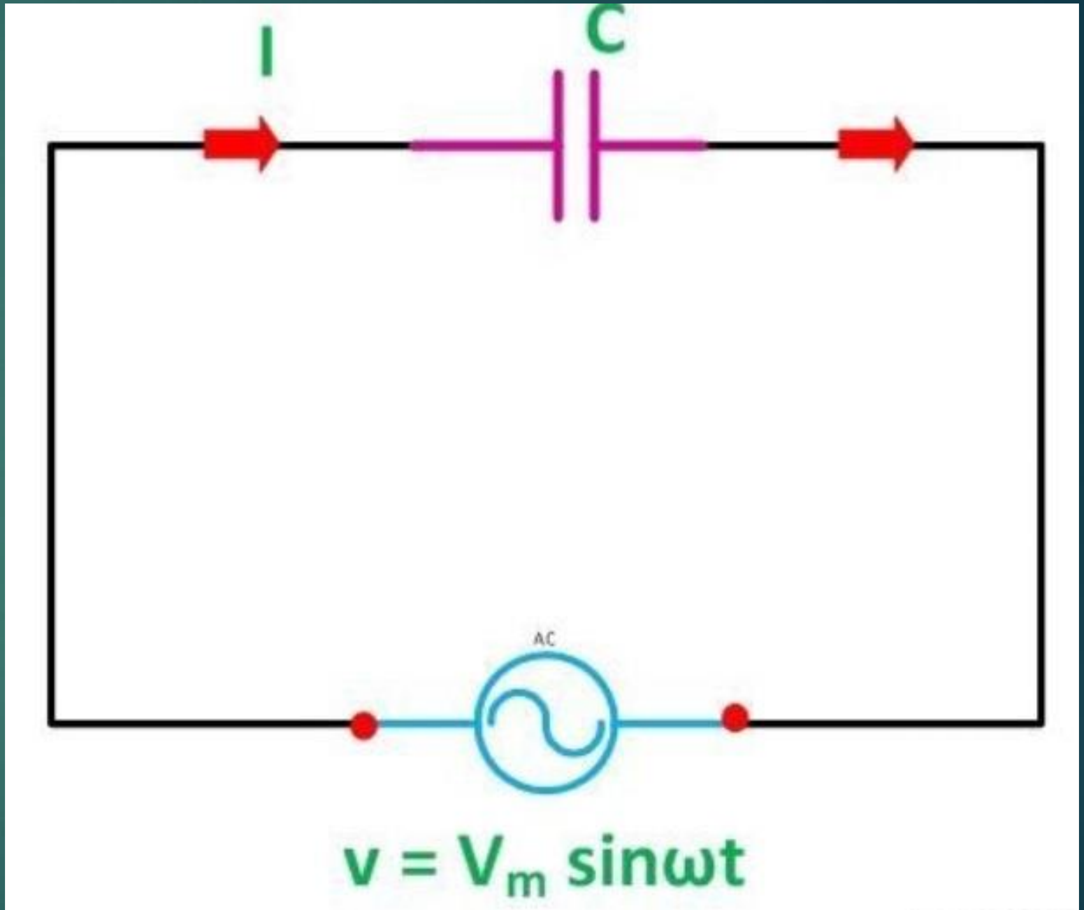
$$I = dq/dt$$

from above expressions

$$I = d(cv)/dt = C\,dv/dt$$

replace $V = V_m \sin wt$ in above equation

$$I = C\,d(V_m \sin wt)/dt$$



$$I = C\,V_m\,w \cos wt = V_m/(1/wC) \sin(wt + 90^0) = V_m/X_c \sin(wt + 90^0)$$

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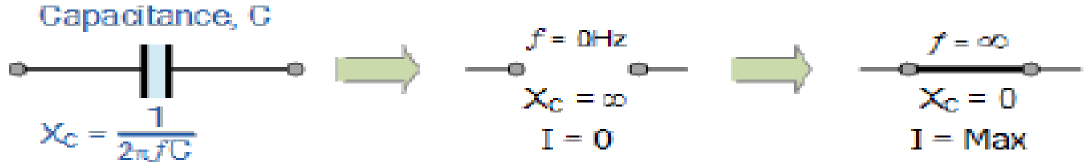
- Phase domain:



In an AC circuit containing pure capacitance the current (electron flow) flowing into the capacitor is given as:

$$I_{C(t)} = \frac{dq}{dt} \quad \text{where: } q = CV_C = CV_{max}\sin(\omega t)$$

$$\therefore I_{C(t)} = \frac{d}{dt}CV_{max}\sin(\omega t) = \omega CV_{max}\cos(\omega t)$$

$$\text{If: } I_{max} = \frac{V_{max}}{Xc} \quad \text{where: } Xc = \frac{1}{2\pi f C} = \frac{1}{\omega C}$$

$$\text{then: } I_{max} = \omega CV_{max}$$

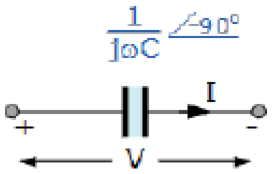
In the phasor domain the voltage across the plates of an AC capacitance will be:

$$V_C = \frac{1}{j\omega C} \times I_C$$

$$\text{where: } \frac{1}{j\omega C} = jX_C = \frac{1}{2\pi f C} = \text{IMPEDANCE, Z}$$

and in Polar Form this would be written as: $X_C\angle{-90^\circ}$ where:

$$X_C\angle\theta = \frac{V_C\angle 0^\circ}{I_C\angle{+90^\circ}}$$



$$X_C\angle\theta = \frac{1}{j\omega C} = 0 - jX_C = \frac{1}{\omega C}\angle{-90^\circ} = Z\angle{-90^\circ}$$

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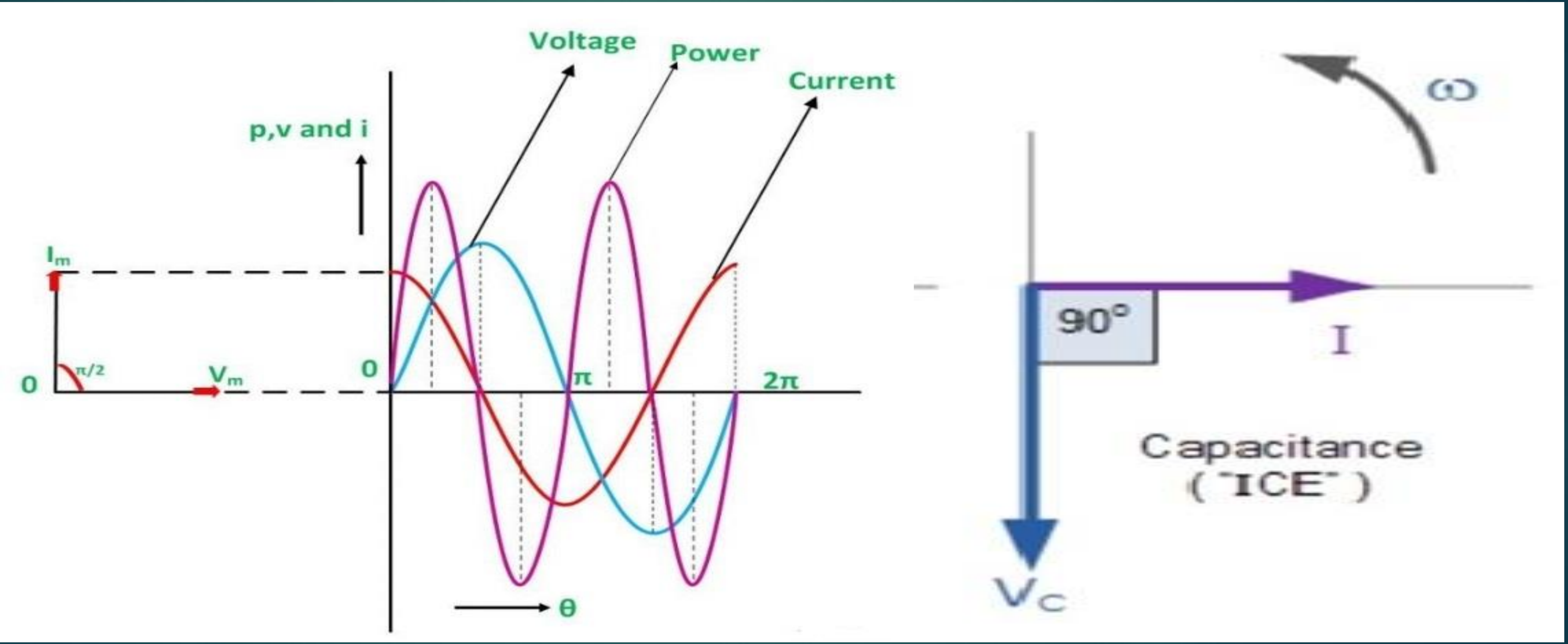
- Phase diagram & power curve :In the pure capacitor circuit, the current flowing through the capacitor leads the voltage by an angle of 90 degrees



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Power in Pure Capacitor Circuit

Instantaneous power is given by p = vi

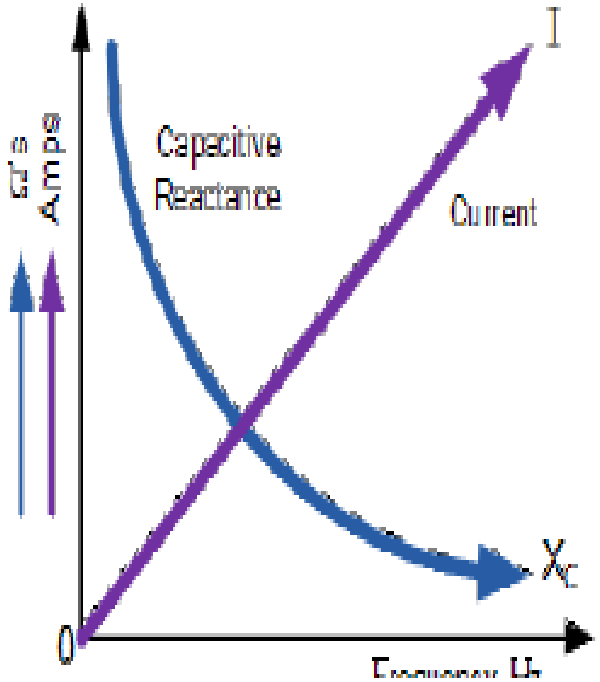
$$P = (V_m \sin\omega t)(I_m \sin(\omega t + \pi/2))$$

$$P = V_m I_m \sin\omega t \cos\omega t$$

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \sin 2\omega t \quad \text{or}$$

$$P = 0$$

Capacitive Reactance against Frequency



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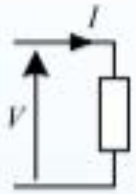
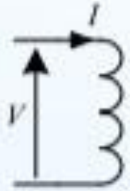
Resistor:

$v(t) = Ri(t) \Rightarrow V = RI \quad \Rightarrow \frac{V}{I} = R$



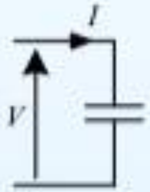
Inductor:

$v(t) = L\frac{di}{dt} \Rightarrow V = j\omega LI \quad \Rightarrow \frac{V}{I} = j\omega L$



Capacitor:

$i(t) = C\frac{dv}{dt} \Rightarrow I = j\omega CV \quad \Rightarrow \frac{V}{I} = \frac{1}{j\omega C}$



For all three components, phasors obey Ohm's law if we use the *complex impedances* $j\omega L$ and $\frac{1}{j\omega C}$ as the "resistance" of an inductor or capacitor.

Circuit Element	Symbol	Resistance or Reactance	Phase of Current	Phase Constant	Amplitude Relation
Resistor	R	R	In phase with v_R	0° (0 rad)	$V_R=I_R R$
Capacitor	C	$X_C=1/W_0C$	Leads v_R by 90°	90°	$V_C=I_C X_C$
Inductor	L	$X_L=W_0L$	Lags v_R by 90°	90°	$V_L=I_L X_L$

R – L Series Circuit

- R L-Series Circuit: A circuit that contains a pure resistance R ohms connected in series with a coil having a pure inductance L is known as R L series circuit.
- When an AC supply voltage V is applied the current I flows in the circuit
- So I_R and I_L will be the current flowing in the resistor and inductor respectively
- But the amount of current flowing through both the elements will be same as they connected in series
- V_R = Voltage across the resistor
- V_L = Voltage across the inductor
- V = Total applied AC voltage

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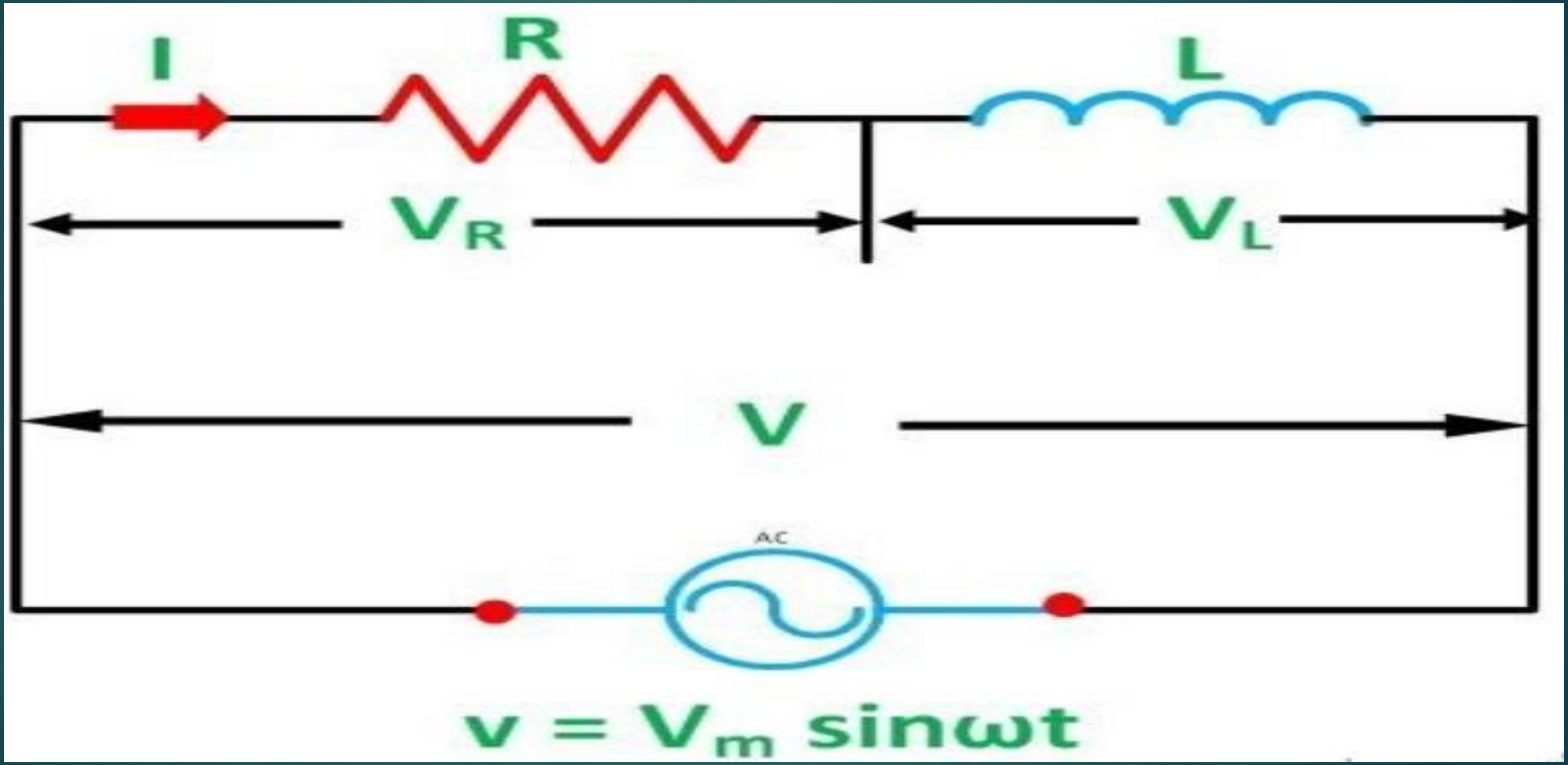
- Circuit Diagram:



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- Steps to draw the phasor diagram of RL series circuit:
- Current I is taken as a reference
- The voltage drop across the resistance $V_R = I_R R$ is drawn in phase with the current I
- The voltage drop across the inductive reactance $V_L = I X_L$ is drawn ahead of the current I, as the current lags voltage by an angle of 90^0 in the pure inductive circuit
- The vector sum of the two voltage drops V_R and V_L is equal to the applied voltage V

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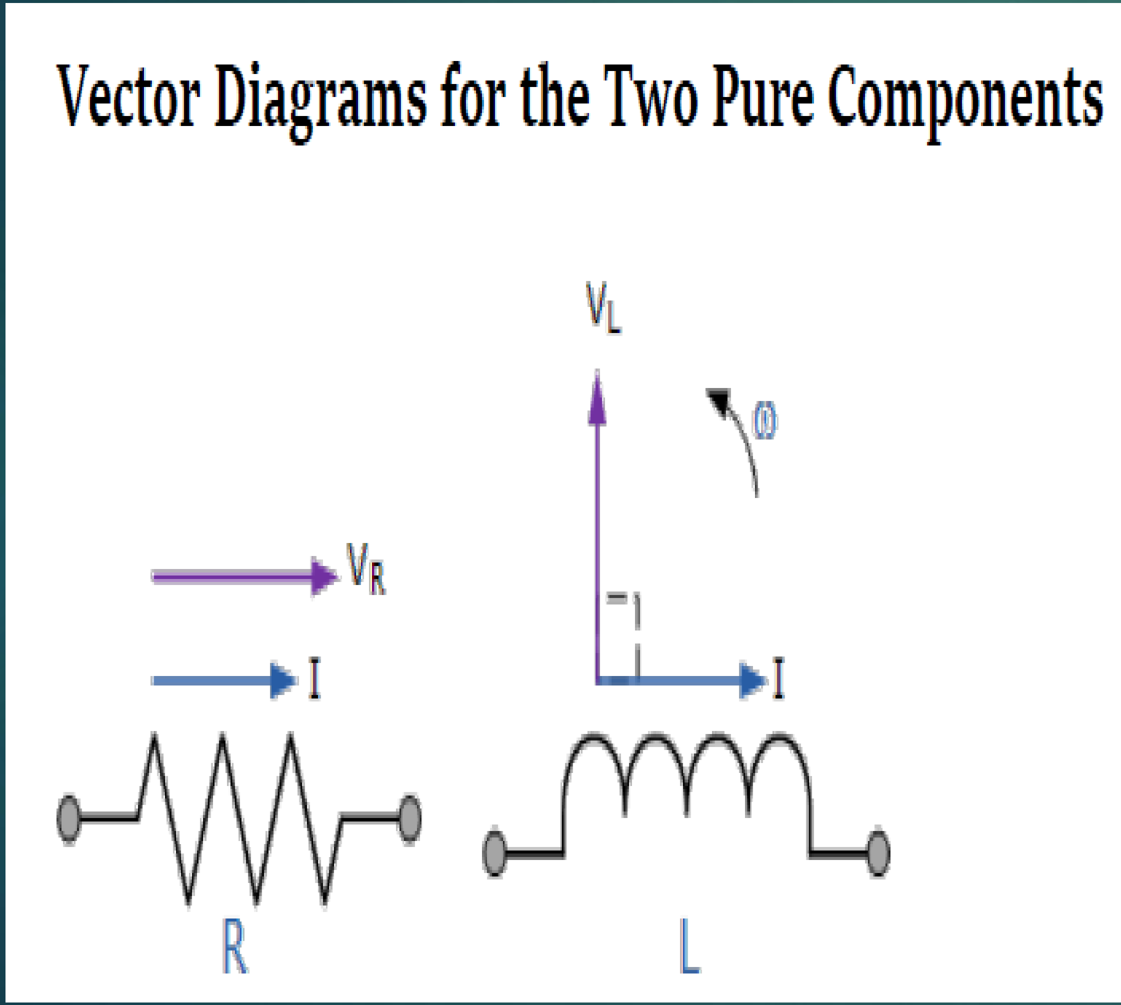
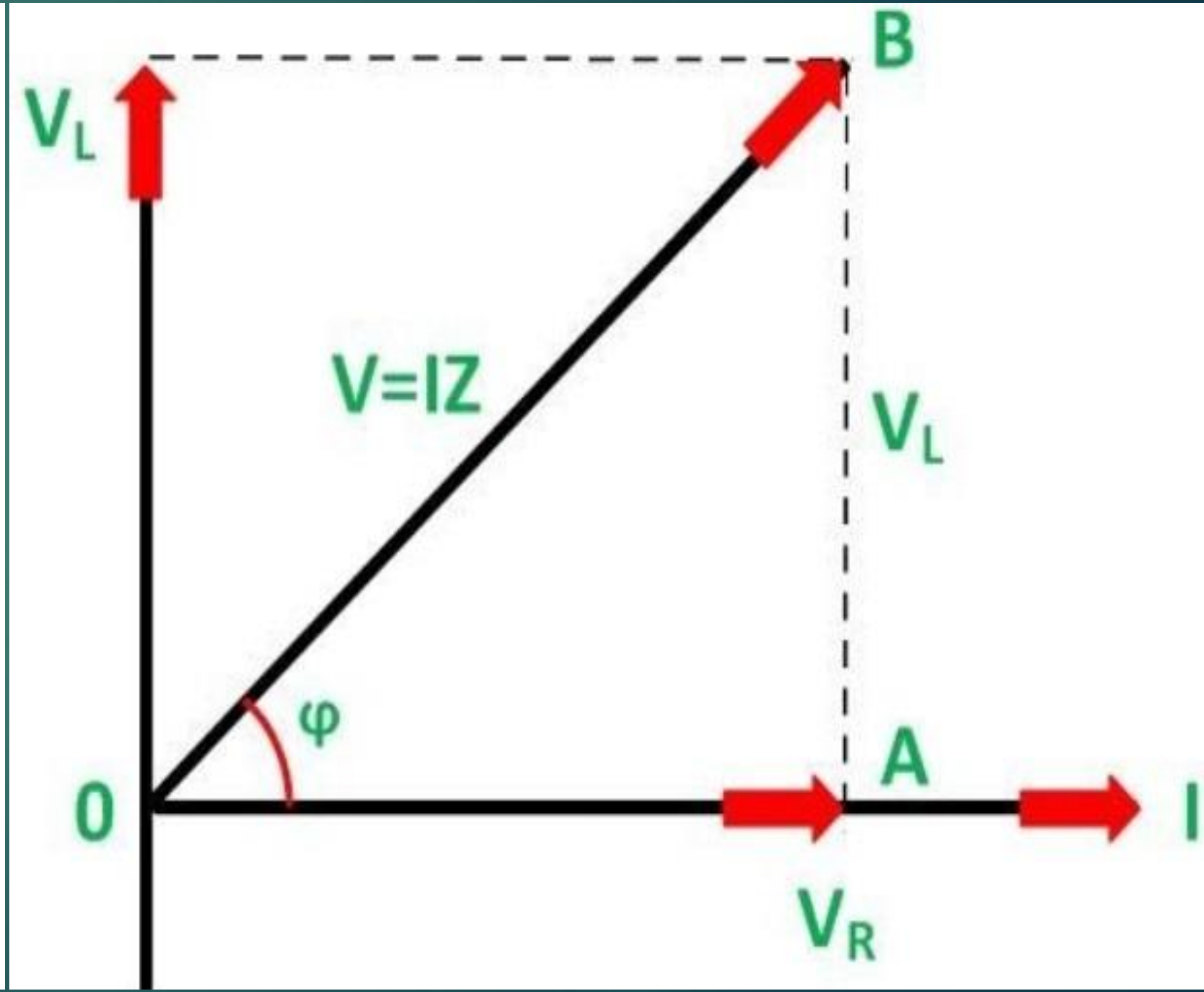
- Phasor diagram:





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- In the right angle triangle OAB :

$V_R = IR$ and $V_L = IX_L$ where $X_L = 2\pi fL$

$$V = \sqrt{(V_R)^2 + (V_L)^2} = \sqrt{(IR)^2 + (IX_L)^2}$$

$$V = I\sqrt{R^2 + X_L^2} \quad \text{or}$$

$$I = \frac{V}{Z}$$

Where,

$$Z = \sqrt{R^2 + X_L^2}$$

Phase Angle

In RL Series circuit the current lags the voltage by 90 degrees angle known as phase angle. It is given by the equation:

$$\tan\varphi = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{X_L}{R} \quad \text{or}$$

$$\varphi = \tan^{-1}\frac{X_L}{R}$$

Continued…………

Power in R L Series Circuit

If the alternating voltage applied across the circuit is given by the equation:

$$v = V_m \mathrm{Sin}\omega t \ldots\ldots\ldots\ldots\ldots(1)$$

The equation of current I is given as:

$$i = I_m \sin(\omega t - \varphi) \ldots\ldots\ldots\ldots(2)$$

Then the instantaneous power is given by the equation:

$$p = v\, i \ldots\ldots\ldots(3)$$

Putting the value of v and i from the equation (1) and (2) in the equation (3) we will get

$$P = (V_m \mathrm{Sin}\omega t) x\, I_m \sin(\omega t - \varphi)$$

$$p = \frac{V_m I_m}{2} 2\sin(\omega t - \varphi)\sin\omega t$$

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} [\cos\varphi - \cos(2\omega t - \varphi)]$$

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\varphi - \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos(2\omega t - \varphi)$$

The average power consumed in the circuit over one complete cycle is given by the equation shown below:

$$P = \text{average of } \frac{V_m}{\sqrt{2}} \frac{V_m}{\sqrt{2}} \cos\varphi - \text{average of } \frac{V_m}{\sqrt{2}} \frac{V_m}{\sqrt{2}} \cos(2\omega t - \varphi) \quad \text{or}$$

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\varphi - \text{Zero} \quad \text{or}$$

$$P = V_{r.m.s} I_{r.m.s} \cos\varphi = V\, I \cos\varphi$$

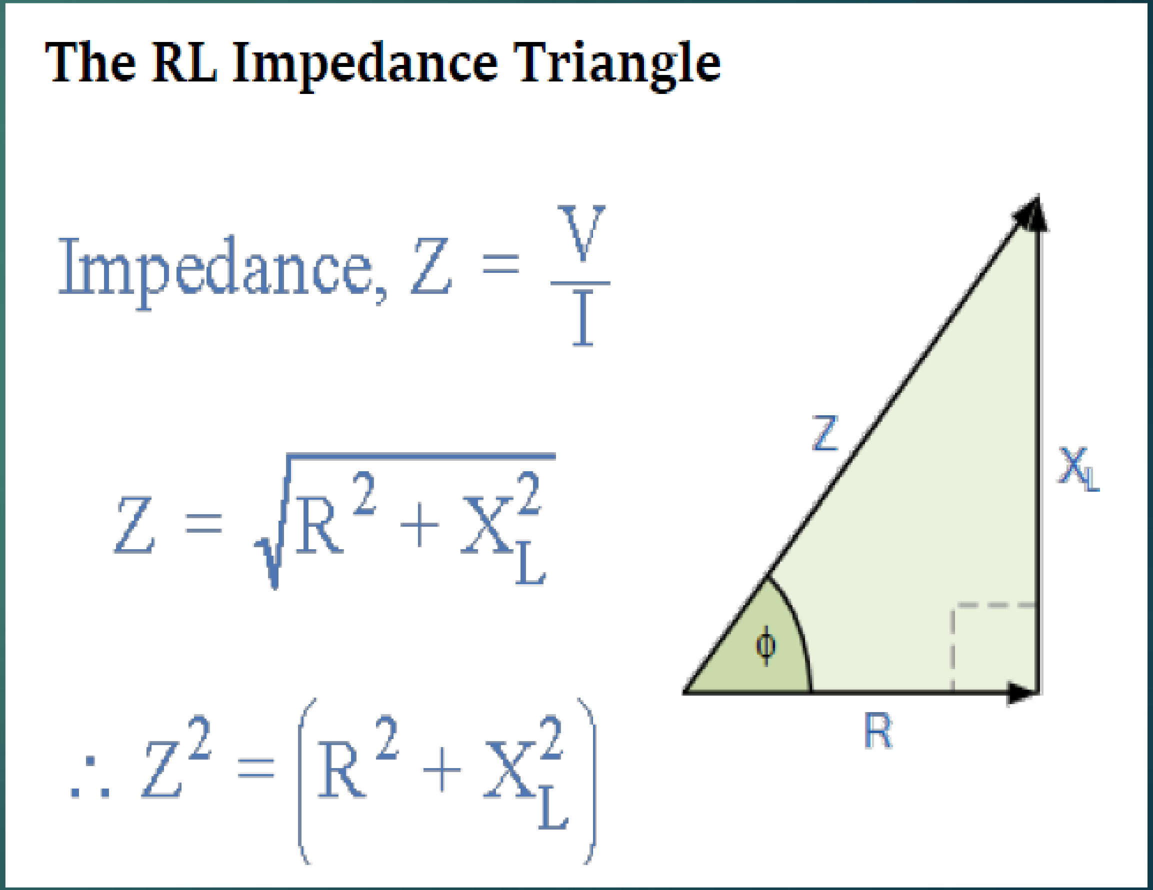
Where cosф is called the power factor of the circuit.

Continued…………

- Power factor : The power factor is defined as the ratio of resistance to the impedance of an AC circuit or voltage across resistor (or) voltage across inductor to total applied AC voltage

$$\text{Cos } \phi = V_R / V = I\,R / I\,Z = R / Z$$

$$\text{Power } P = I^2 R$$



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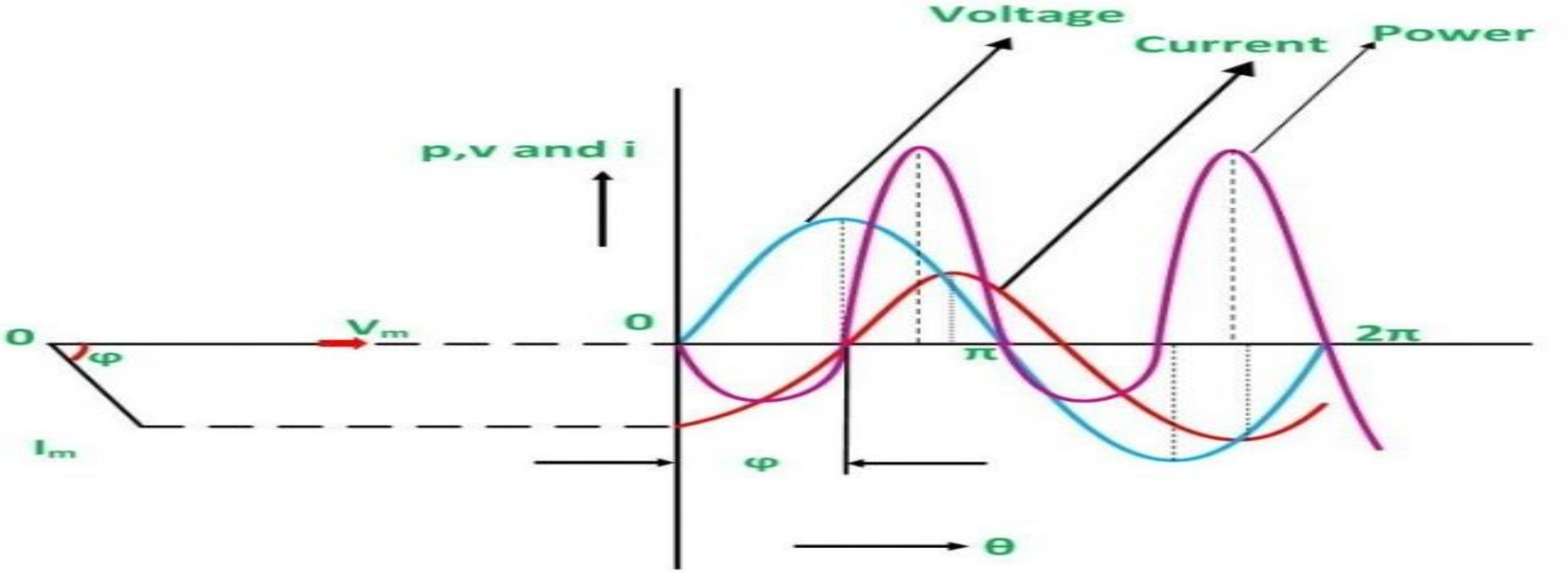
- Wave forms :

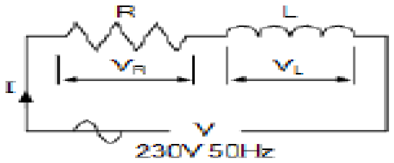
The **waveform** and **power curve** of the RL series circuit is shown below:



The various points on the power curve are obtained by the product of voltage and current.

R L Series circuit Problem

- Problem :



A solenoid coil with a resistance of 30 ohms and an inductance of 200mH is connected to a 230VAC, 50Hz supply. Calculate: (a) the solenoids impedance, (b) the current consumed by the solenoid, (c) the phase angle between the current and the applied voltage, and (d) the average power consumed by the solenoid.

Data given: R = 30Ω, L = 200mH, V = 230V and f = 50Hz.

(a) Impedance (Z) of the solenoid coil:

$$R = 30\Omega$$

$$X_L = 2\pi f L = 2\pi \times 50 \times 200 \times 10^{-3} = 62.8\Omega$$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{30^2 + 62.8^2} = 69.6\Omega$$

(b) Current (I) consumed by the solenoid coil:

$$V = I \times Z$$

$$\therefore I = \frac{V}{Z} = \frac{230}{69.6} = 3.3\,A_{(rms)}$$

$$\cos\theta = \frac{R}{Z}, \quad \text{or} \quad \sin\theta = \frac{X_L}{Z}, \quad \text{or} \quad \tan\theta = \frac{X_L}{R}$$

$$\therefore \cos\theta = \frac{R}{Z} = \frac{30}{69.6} = 0.431$$

$$\cos^{-1}(0.431) = 64.5^\circ \text{ lagging}$$

(d) Average AC power consumed by the solenoid coil:

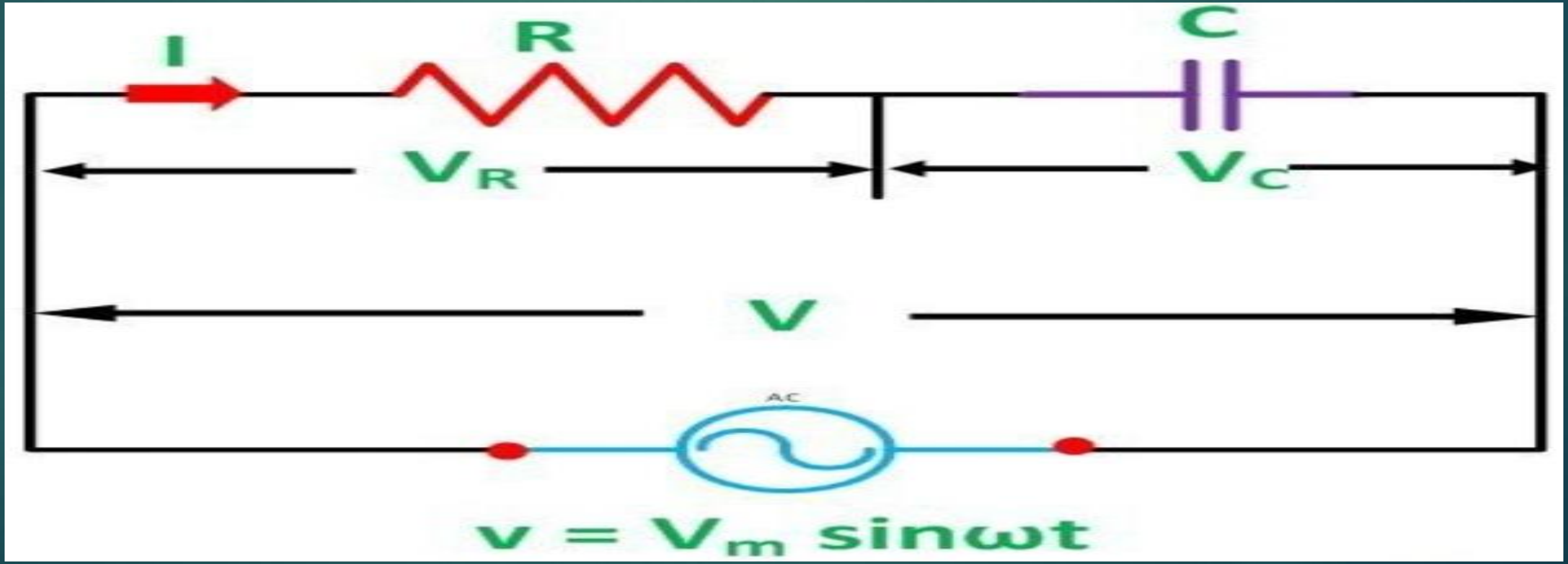
$$P = V \times I \times \cos\theta$$

$$P = 230 \times 3.3 \times \cos(64.5^\circ)$$

$$\therefore P = 327 \text{ watts}$$

R C –Series Circuit

- R C Series Circuit : The current flowing into a pure AC Capacitance leads the voltage by 90^0.But in the real world it is impossible to have a pure AC capacitance ,as well as capacitors will have a certain amount of internal resistance across their plates giving rise to a leakage current
- Series RC circuit consists of one series resistance connected in series with capacitor



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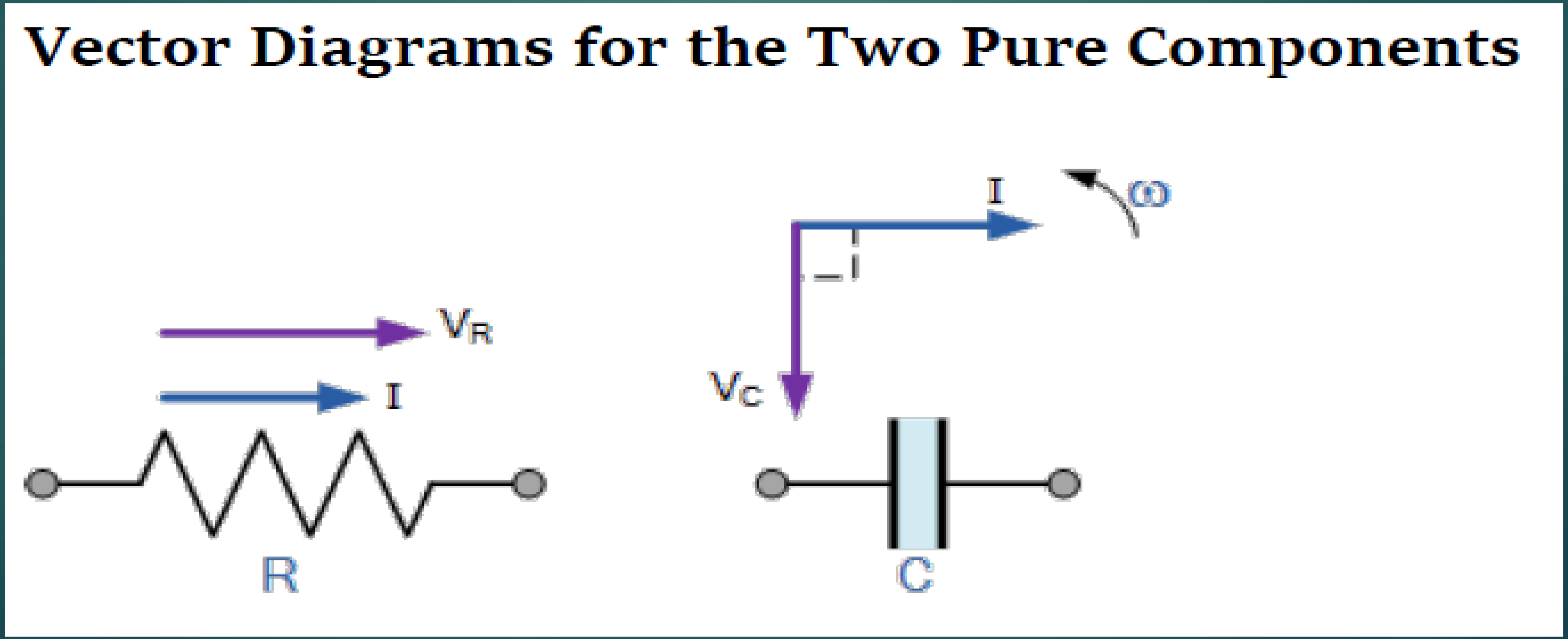
- V_R = Voltage across the Resistor 'R'
- V_C = Voltage across the capacitor
- V = Total voltage across Series RC circuit
- A sinusoidal voltage is applied current I flows through the resistance (R) and capacitance (C) of the circuit



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- Steps to draw the phasor diagram of RC series circuit:
- Current I is taken as a reference
- The voltage drop across the resistance $V_R = I_R R$ is drawn in phase with the current I
- The voltage drop across the capacitive reactance $V_C = I X_C$ is drawn behind of the current I, as the current leads voltage by an angle of 90^0 in the pure inductive circuit
- The vector sum of the two voltage drops V_R and V_C is equal to the applied voltage V
- $X_C = 1/2\Pi f C$

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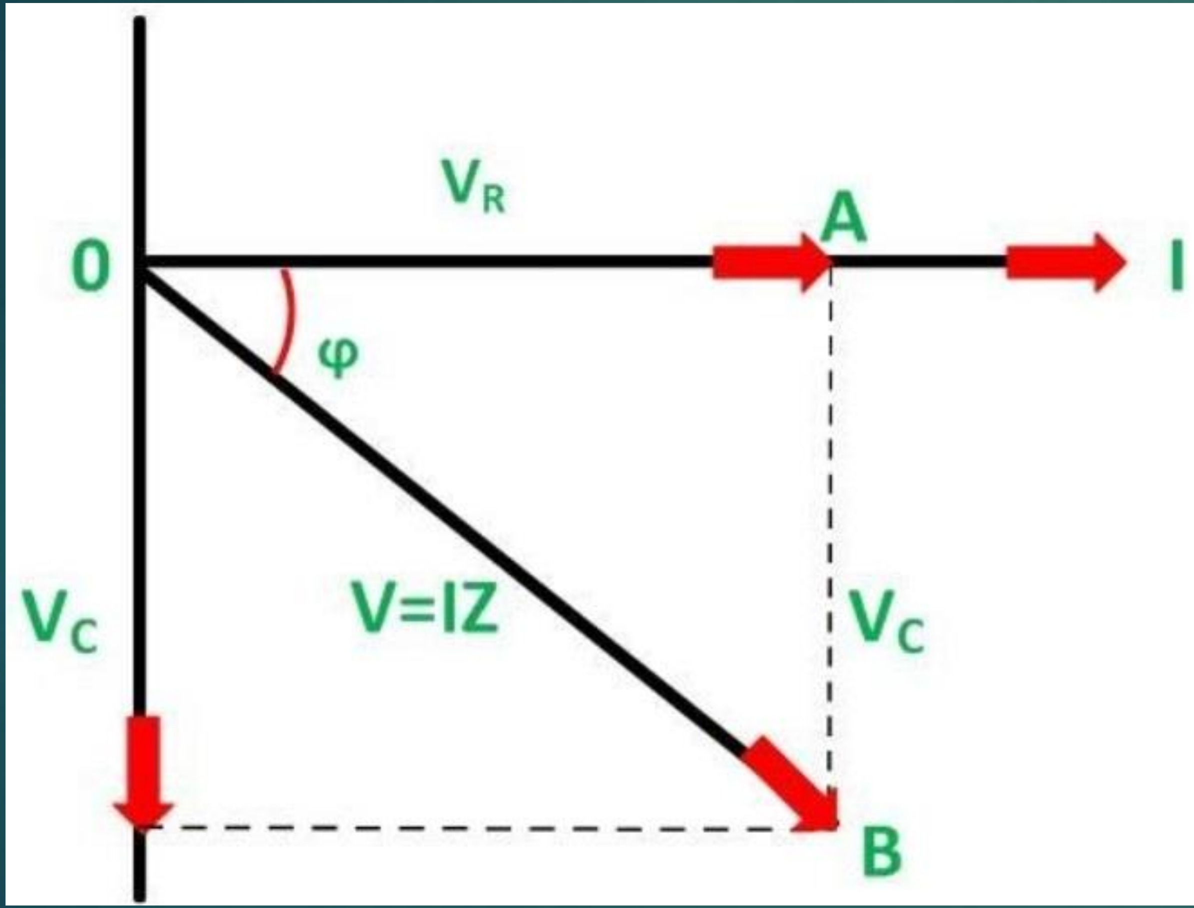
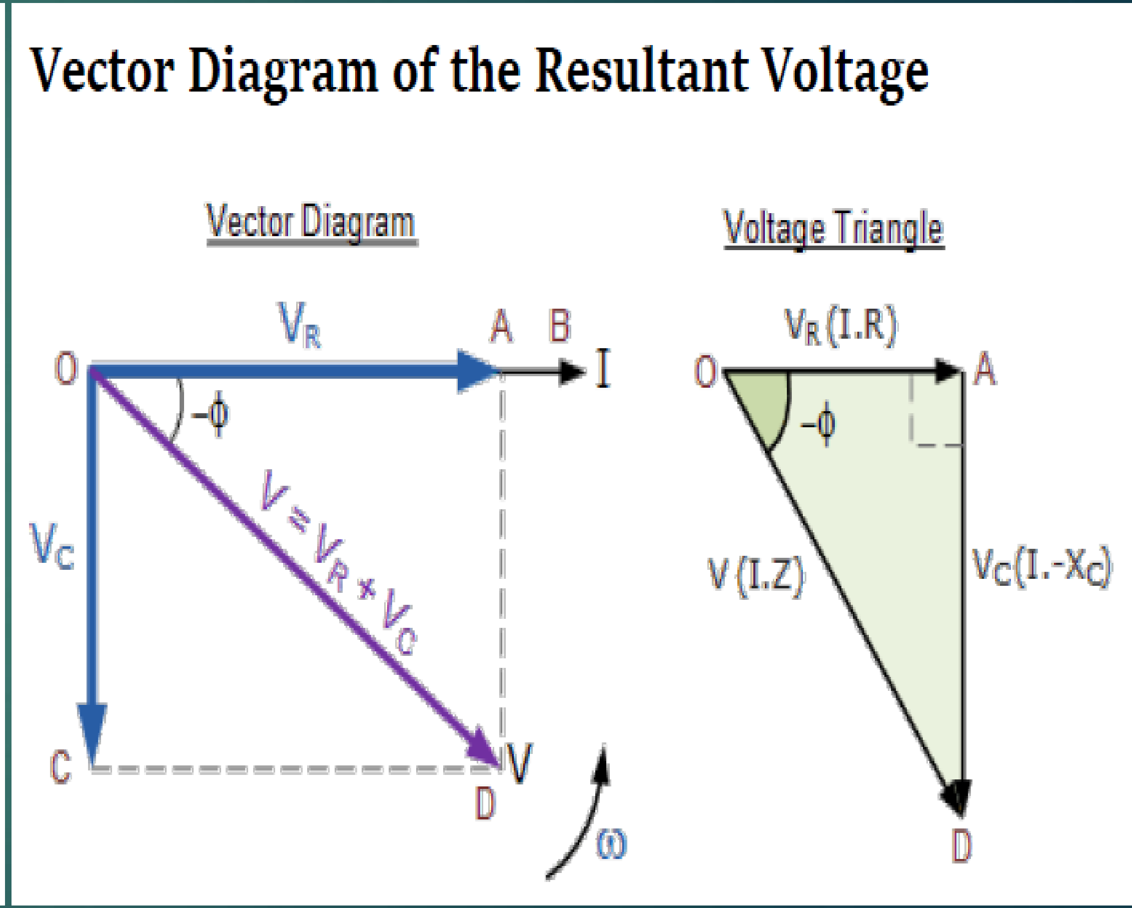
- Phasor diagram :





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- In right triangle OAB :

$$V = \sqrt{(V_R)^2 + (V_C)^2} = \sqrt{(IR)^2 + (IX_C)^2}$$

$$V = I\sqrt{R^2 + X_C^2} \quad \text{or}$$

$$I = \frac{V}{\sqrt{R^2 + X_C^2}} = \frac{V}{Z}$$

Where,

$$Z = \sqrt{R^2 + X_C^2}$$

Phase angle

From the phasor diagram shown above, it is clear that the current in the circuit leads the applied voltage by an angle ϕ and this angle is called the **phase angle**.

$$\tan\varphi = \frac{V_C}{V_R} = \frac{IX_C}{IR} = \frac{X_C}{R} \quad \text{or}$$

$$\varphi = \tan^{-1}\frac{X_C}{R}$$

Continued…………

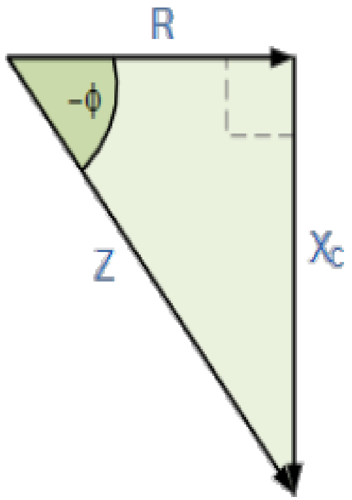
- Impedance Triangle :

The RC Impedance Triangle

$$\text{Impedance, } Z = \frac{V}{I}$$

$$Z = \sqrt{R^2 + X_C^2}$$

$$\therefore Z^2 = R^2 + X_C^2$$



Then: (Impedance)^2 = (Resistance)^2 + (j Reactance)^2 where j represents the 90° phase shift.

Power:

$$P = (V_m \mathrm{Sin}\omega t) x\, I_m \sin(\omega t + \varphi)$$

$$p = \frac{V_m I_m}{2} 2\sin(\omega t + \varphi) \sin\omega t$$

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} [\cos\varphi - \cos(2\omega t + \varphi)]$$

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\varphi - \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos(2\omega t + \varphi)$$

The average power consumed in the circuit over a complete cycle is given by:

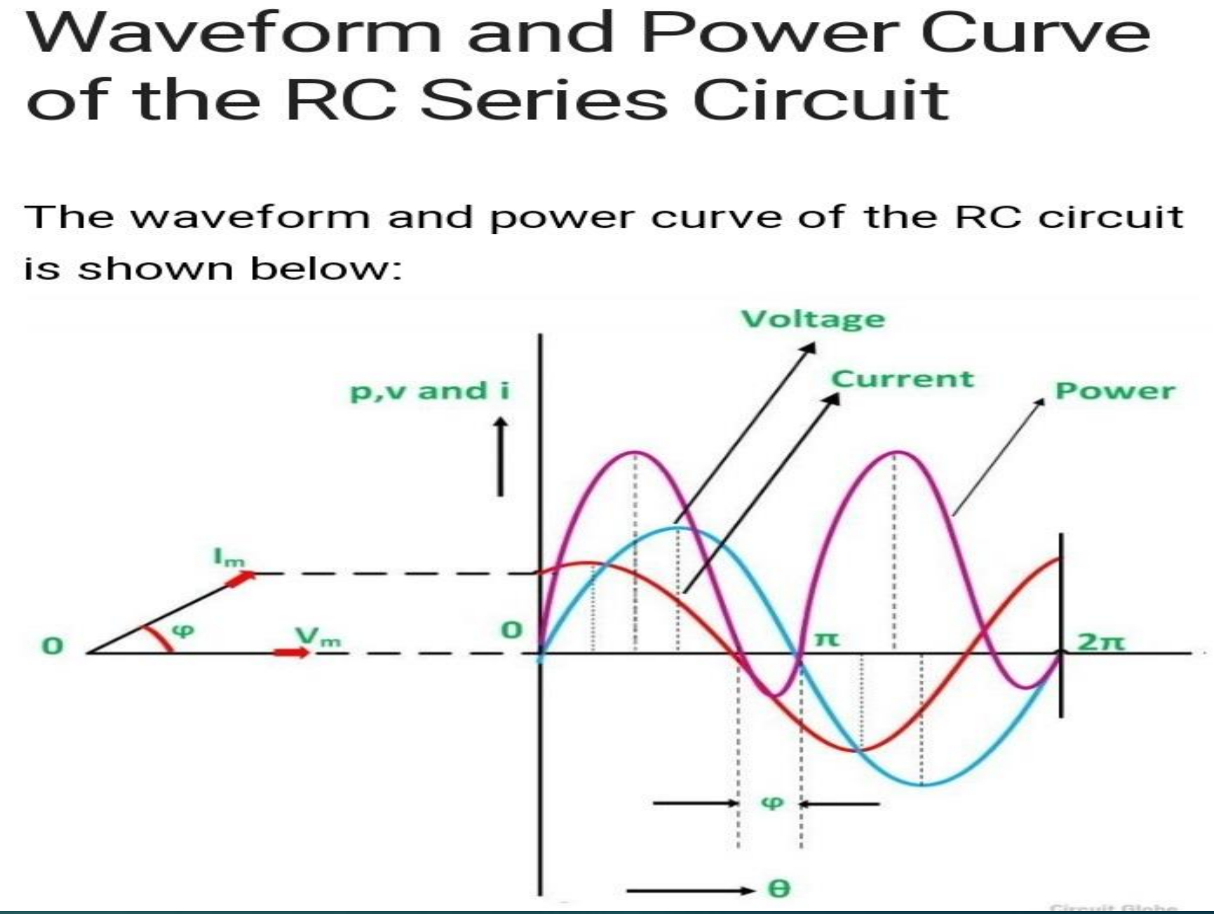
$$P = \text{average of } \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\varphi - \text{average of } \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos(2\omega t + \varphi) \quad \text{or}$$

Continued…………

- Power Factor :The power factor is defined as the ratio of resistance to the impedance of an AC circuit or voltage across resistor (or) voltage across inductor to total applied AC voltage

$\text{Cos } \phi = V_R / V = I R / I Z = R / Z = V_C / V$

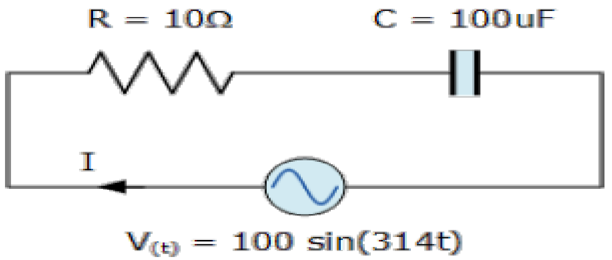
$\text{Power } P = I^2 R$



problems on RC series circuit

- Problem :

A capacitor which has an internal resistance of 10Ω and a capacitance value of 100uF is connected to a supply voltage given as $V_{(t)} = 100 \sin(314t)$. Calculate the current flowing into the capacitor. Also construct a voltage triangle showing the individual voltage drops.



The capacitive reactance and circuit impedance is calculated as:

$$X_C = \frac{1}{\omega C} = \frac{1}{314 \times 100uF} = 31.85\Omega$$

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{10^2 + 31.85^2} = 33.4\Omega$$

Then the current flowing into the capacitor and the circuit is given as:

$$I = \frac{V_C}{Z} = \frac{100}{33.4} = 3\text{Amps}$$

$$\phi = \tan^{-1}\left(\frac{X_C}{R}\right) = \frac{31.85}{10} = 72.6^\circ \text{ leading}$$

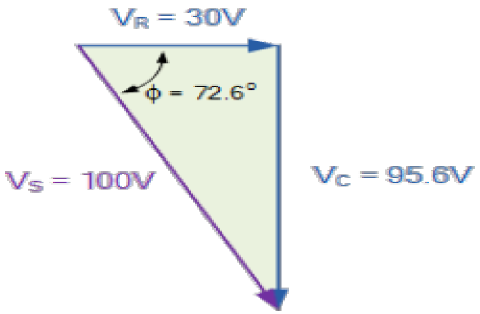
Then the individual voltage drops around the circuit are calculated as:

$$V_R = I \times R = 3 \times 10 = 30V$$

$$V_C = I \times X_C = 3 \times 31.85 = 95.6V$$

$$V_S = \sqrt{V_R^2 + V_C^2} = \sqrt{30^2 + 95.6^2} = 100V$$

Then the resultant voltage triangle for the calculated peak values will be:



RLC Series Circuit

- RLC Series Circuit: When a pure resistance (R) ,pure inductance of (L) and pure capacitance of (C) are connected together in series combination with each other then RLC circuit is formed.
- As all the three elements are connected in series, so the current flowing through each element of the circuit will be the same as the total current I flowing in the circuit
- In the series RLC circuit :$X_L = 2 \Pi f L$ & $X_C = 1/ 2 \Pi f C$
- When the AC voltage is applied through the RLC series circuit ,the resulting current I flows through the circuit and the voltage across the each element will be

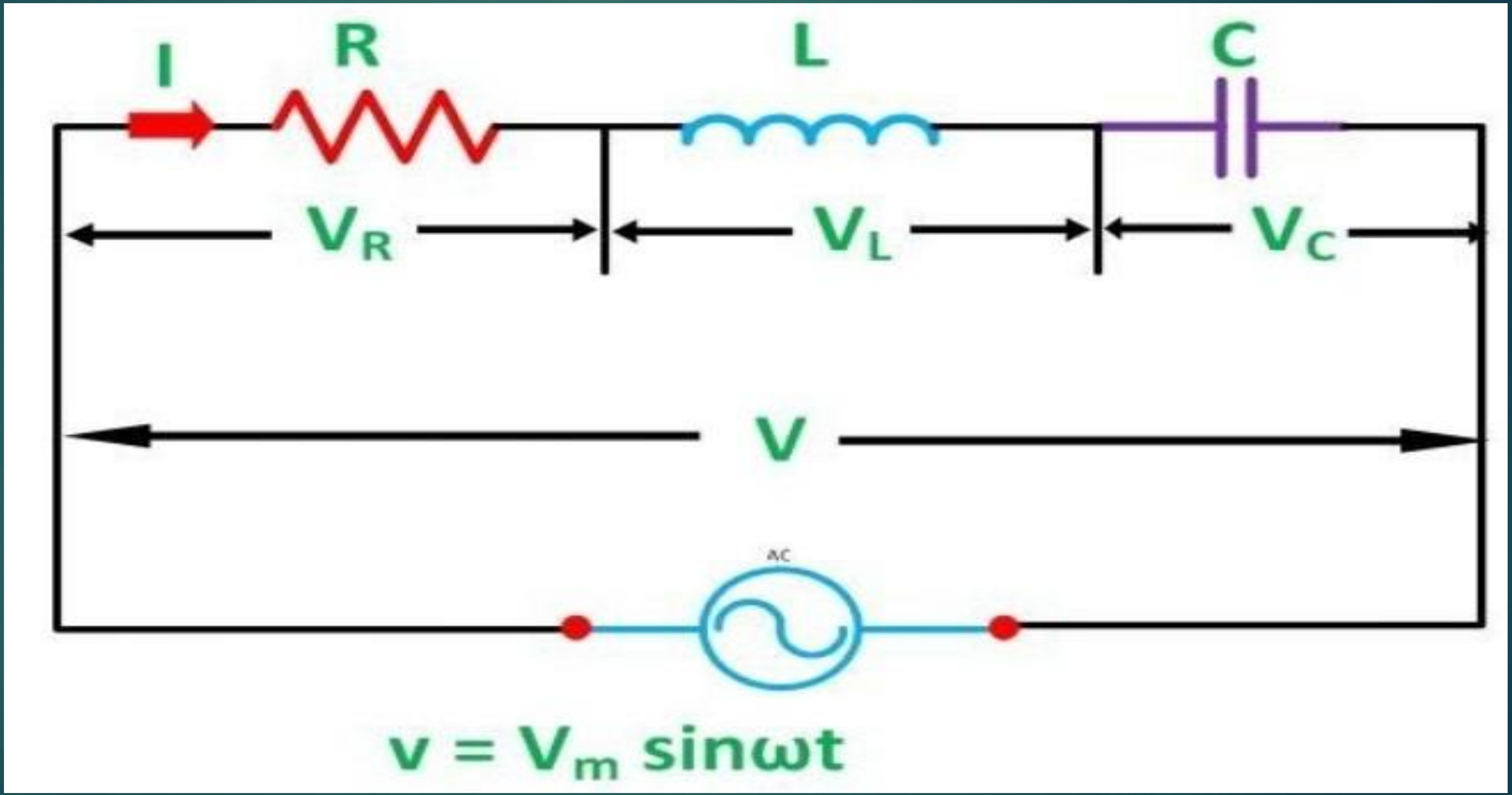
$V_R = I R$ that is the voltage across the resistance R and is in phase with current I

$V_L = I X_L$ that is the voltage across the inductor L and it leads current I by an angle of 90^0

$V_C = I X_C$ that is the voltage across the capacitor C and it lags current I by an angle of 90^0

Continued…………

- Circuit diagram :Z= R + jwL + 1/jwC =R + jwL- j/wC

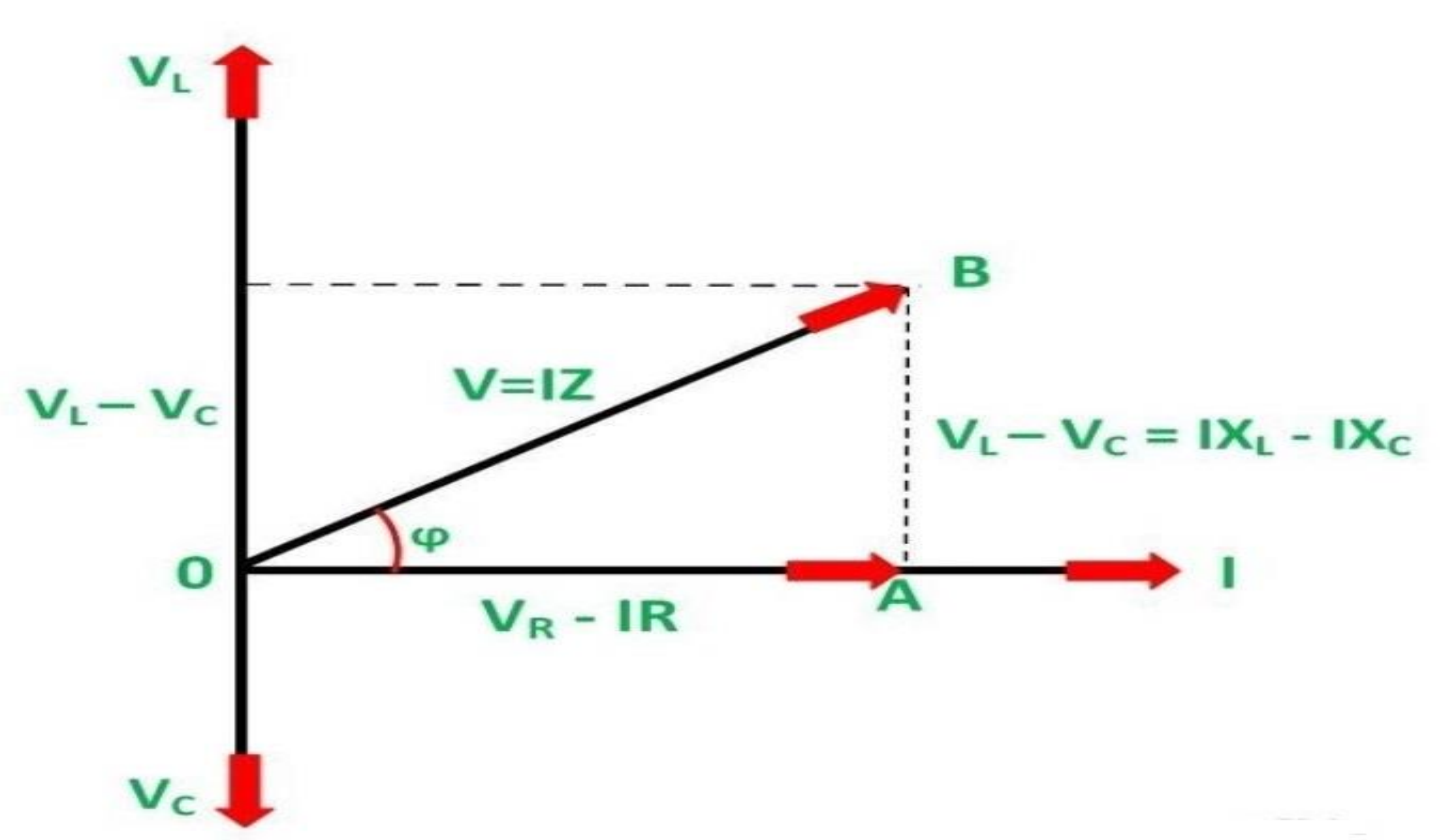


Continued…………

- Steps to draw the phasor diagram of RLC series circuit:
- Current I is taken as a reference
- The voltage drop across the resistance $V_R = I_R R$ is drawn in phase with the current I
- The voltage drop across the inductive reactance $V_L = I X_L$ is drawn ahead of the current I, as the current lags voltage by an angle of 90^0 in the pure inductive circuit
- The voltage drop across the capacitive reactance $V_C = I X_C$ is drawn behind of the current I, as the current leads voltage by an angle of 90^0 in the pure inductive circuit
- The two vectors V_L and V_C are opposite to each other

Continued…………

- Phasor diagram :The phasor diagram of the RLC series circuit ,when the circuit is acting as an inductive circuit that means ($V_L > V_C$) and if ($V_L < V_C$) the circuit will be behaves as a capacitive circuit



Continued………….

- Total Voltage :

$$V = \sqrt{(V_R)^2 + (V_L - V_C)^2} = \sqrt{(IR)^2 + (IX_L - IX_C)^2} \quad \text{or}$$

$$V = I\sqrt{R^2 + (X_L - X_C)^2} \quad \text{or}$$

$$I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{Z}$$

Where,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Phase Angle

From the phasor diagram, the value of phase angle will be

$$\tan\varphi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R} \quad \text{or}$$

$$\varphi = \tan^{-1}\frac{X_L - X_C}{R}$$

Power in RLC Series Circuit

The product of voltage and current is defined as power.

$$P = VI\cos\varphi = I^2R$$

Where cosφ is the power factor of the circuit and is expressed as

$$\cos\varphi = \frac{V_R}{V} = \frac{R}{Z}$$

Continued…………

- Conditions of RLC Series circuit :

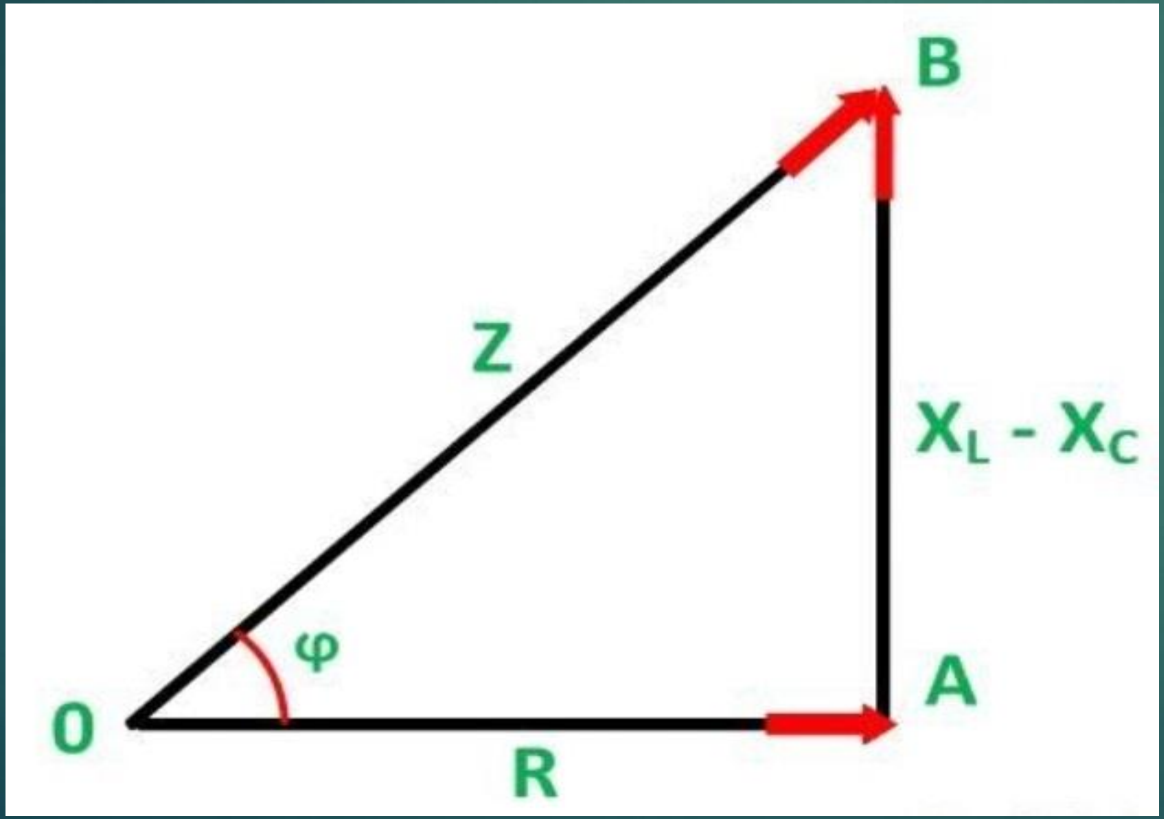
- When $X_L > X_C$, the phase angle ϕ is positive. The circuit behaves as a RL series circuit in which the current lags behind the applied voltage and the power factor is lagging.
- When $X_L < X_C$, the phase angle ϕ is negative, and the circuit acts as a series RC circuit in which the current leads the voltage by 90 degrees.
- When $X_L = X_C$, the phase angle ϕ is zero, as a result, the circuit behaves like a purely resistive circuit. In this type of circuit, the current and voltage are in phase with each other. The value of power factor is unity.

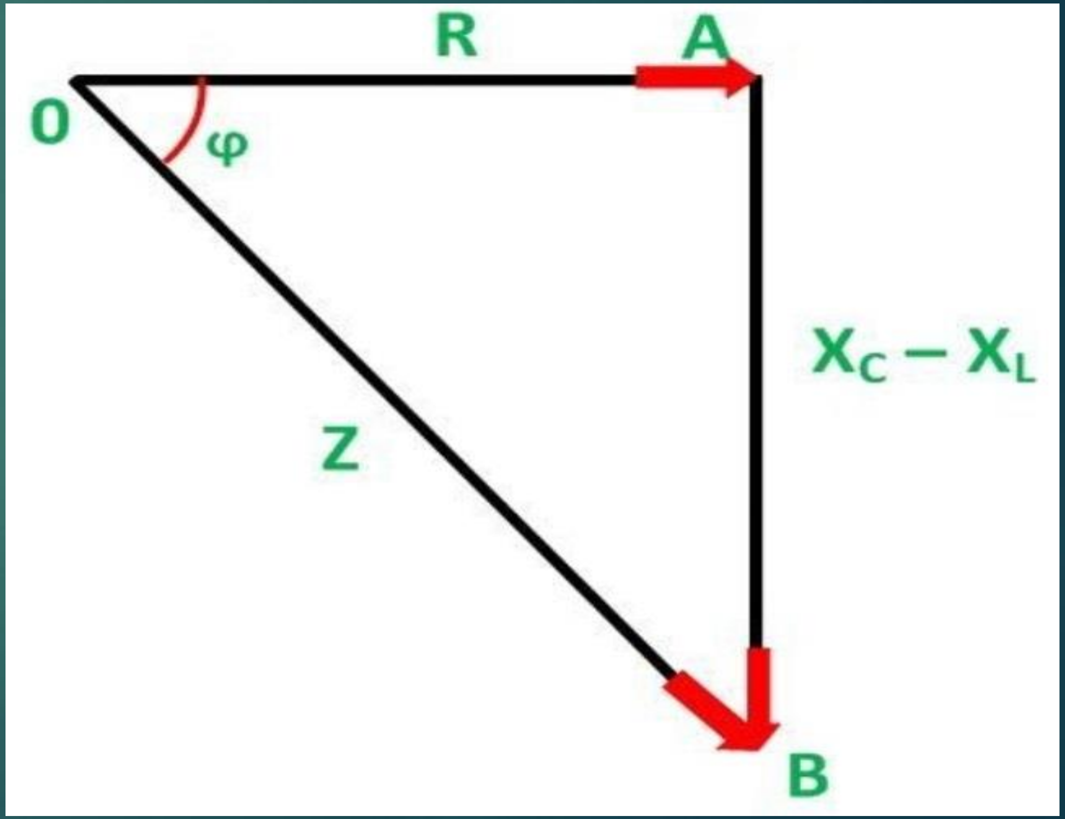
Continued…………

- Impedance Triangle of Series RLC Circuit : Z= R+jwL+1/jwC =R +j(WL-1/WC)

When XL > XC so it acts as a Inductive Circuit

When XL < XC so it acts as a Capacitive Circuit





Continued…………

- Apparent Power: The product of rms value of current & voltage ,V I is called the apparent power and is measured volt-amp or Kilo-Volt-Amp(KVA)
- True Power: True power in an AC circuit is obtained by multiplying the apparent power by the power factor and is expressed in watts or Kilo watts (KW)
- Reactive Power: The product of apparent power (VI) and sine of the angle between voltage and current (Sinψ) is called the reactive power,Kilo –Volt -Amp Reactive (KVAR)

i.e. Apparent power, $S = VI$ volt-amperes or $\dfrac{V\,I}{1{,}000}$ kVA

$$\text{True power, } P = VI\cos\Phi \text{ watts or } \frac{V\,I\cos\Phi}{1{,}000} \text{ kW}$$

$$\text{Reactive power, } Q = VI\sin\Phi \text{ VAR or } \frac{V\,I\sin\Phi}{1{,}000} \text{ kVAR}$$

$$\text{and kVA} = \sqrt{(\text{kW})^2 + (\text{kVAR})^2}$$

Continued…………

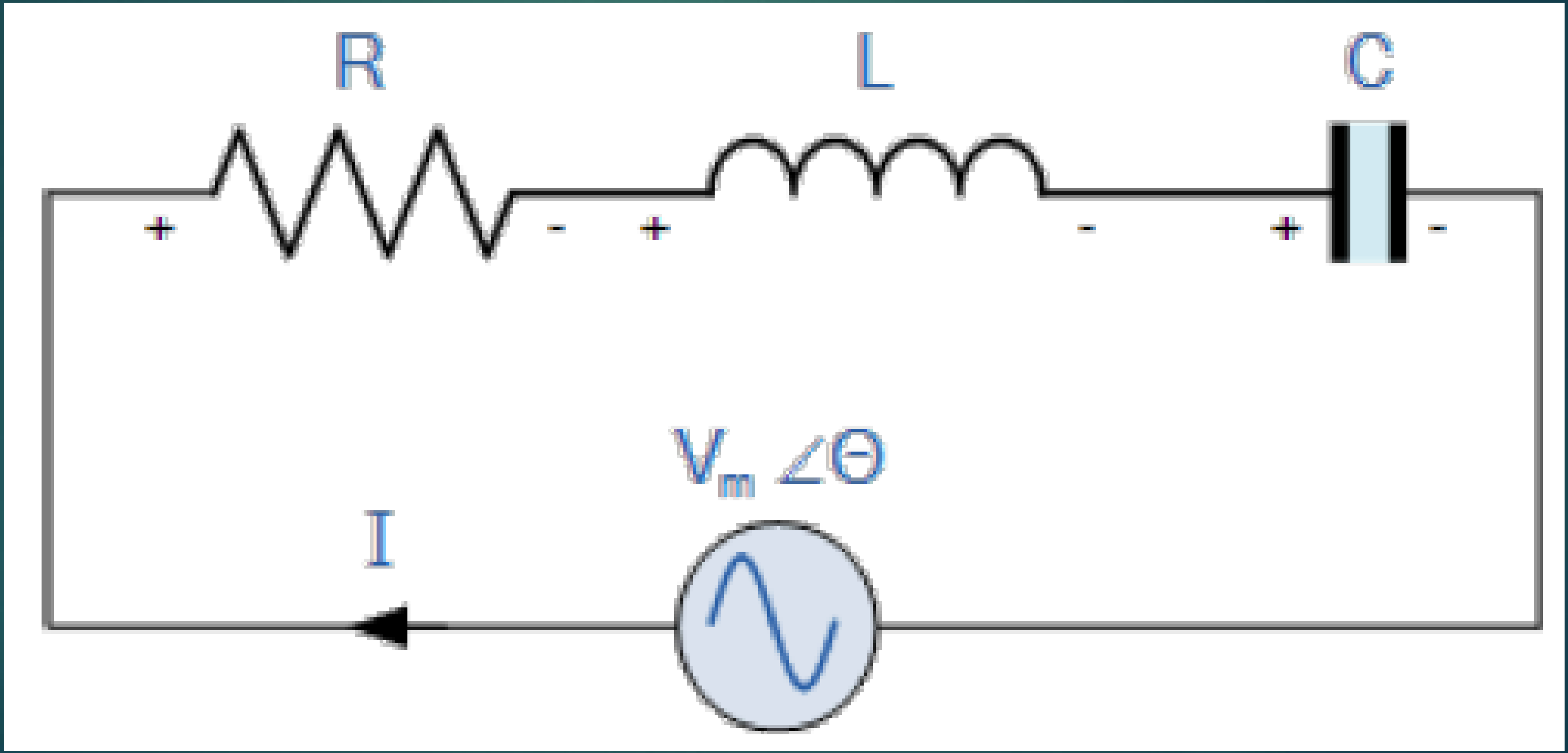
- Applications:
- It acts as a variable tuned circuit
- It acts as a low pass, high pass, band pass & band reject filters depending up on the type of frequency
- The circuit also works as an oscillator
- Voltage multiplier and pulse discharge circuit

Series Resonance Circuits

- Resonance : Resonance occurs in a series circuit when the supply frequency causes the voltage across the L and C to be equal and opposite in phase
- Resonance is a condition in a series RLC circuit in which the capacitive and inductive reactances are equal in magnitude
- Resonant Frequency :The frequency at which inductive reactance is equal to capacitive reactance is called Resonant Frequency i.e $X_L = X_C$
- The result is a purely resistive impedance
- as we are analysing a series RLC circuit this resonance frequency produces a Series Resonance.
- Types of Resonance Circuits:
- Series Resonance Circuit
- Parallel Resonance Circuit

Continued…………

- Series RLC Circuit :



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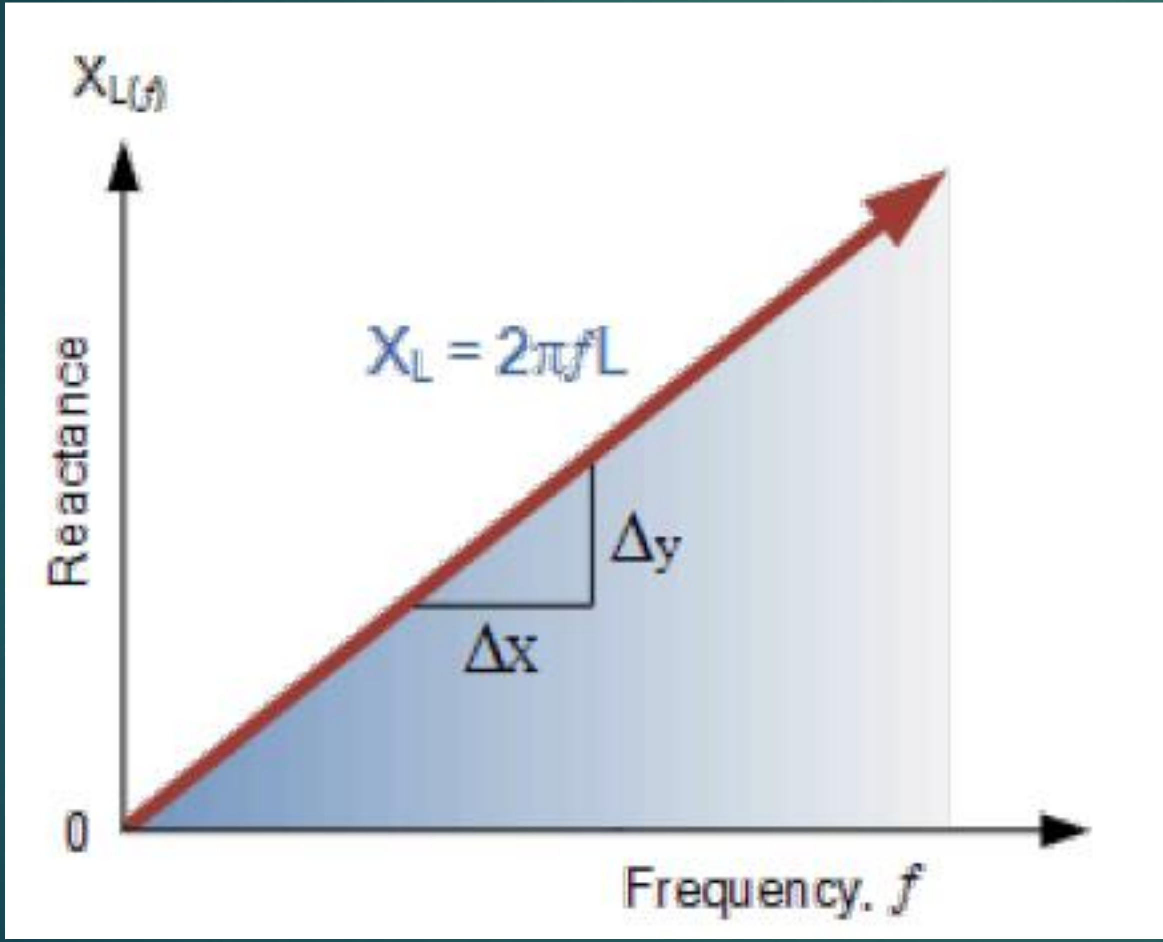
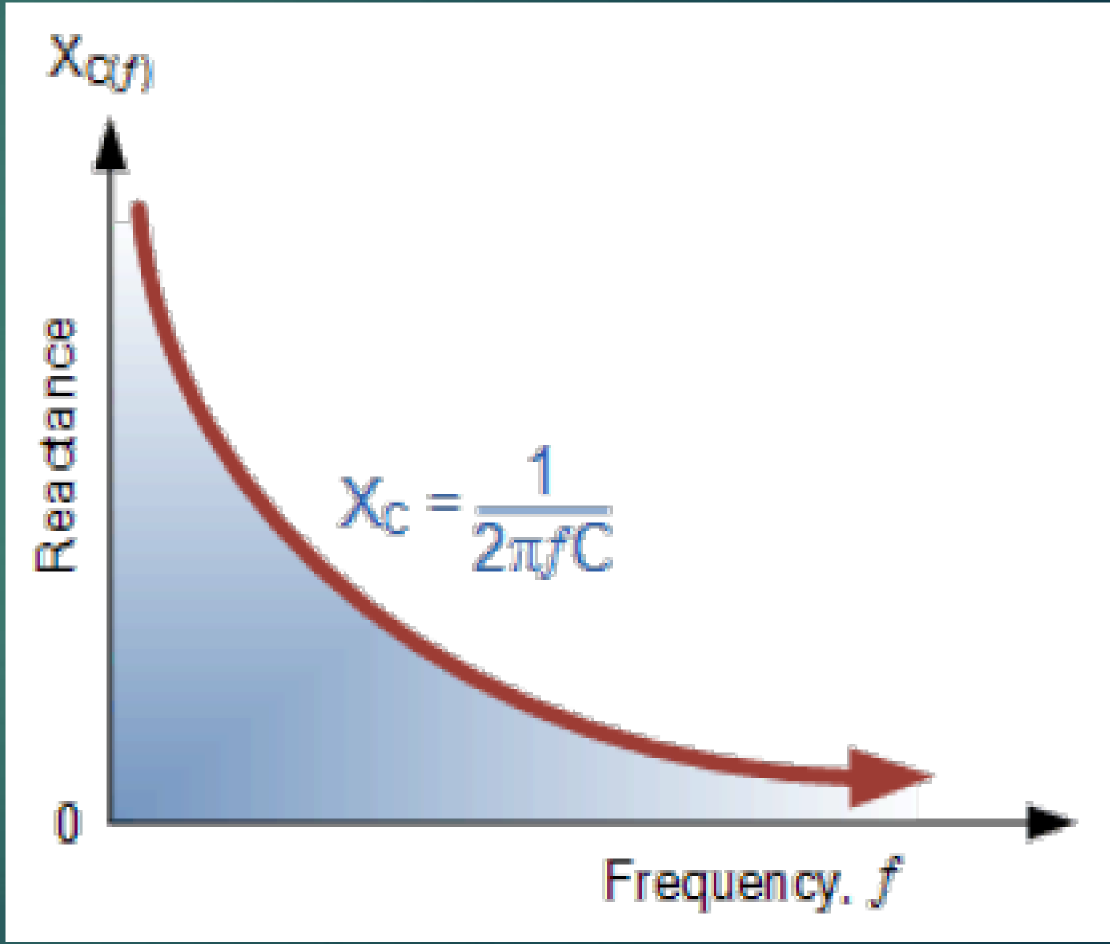
Firstly, let us define what we already know about series RLC circuits.

- Inductive reactance : $X_L = 2\pi f L = \omega L$
- Capacitive reactance : $X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{\omega C}$
- When $X_L > X_C$ the circuit is Inductive
- When $X_C > X_L$ the circuit is Capacitive
- Total circuit reactance = $X_T = X_L - X_C$ or $X_C - X_L$
- Total circuit impedance = $Z = \sqrt{R^2 + X_T^2} = R + jX$

Continued…………

- Inductive Reactance against Frequency (X_L ά f) : Capacitive Reactance against Frequency (X_C ά 1/f)





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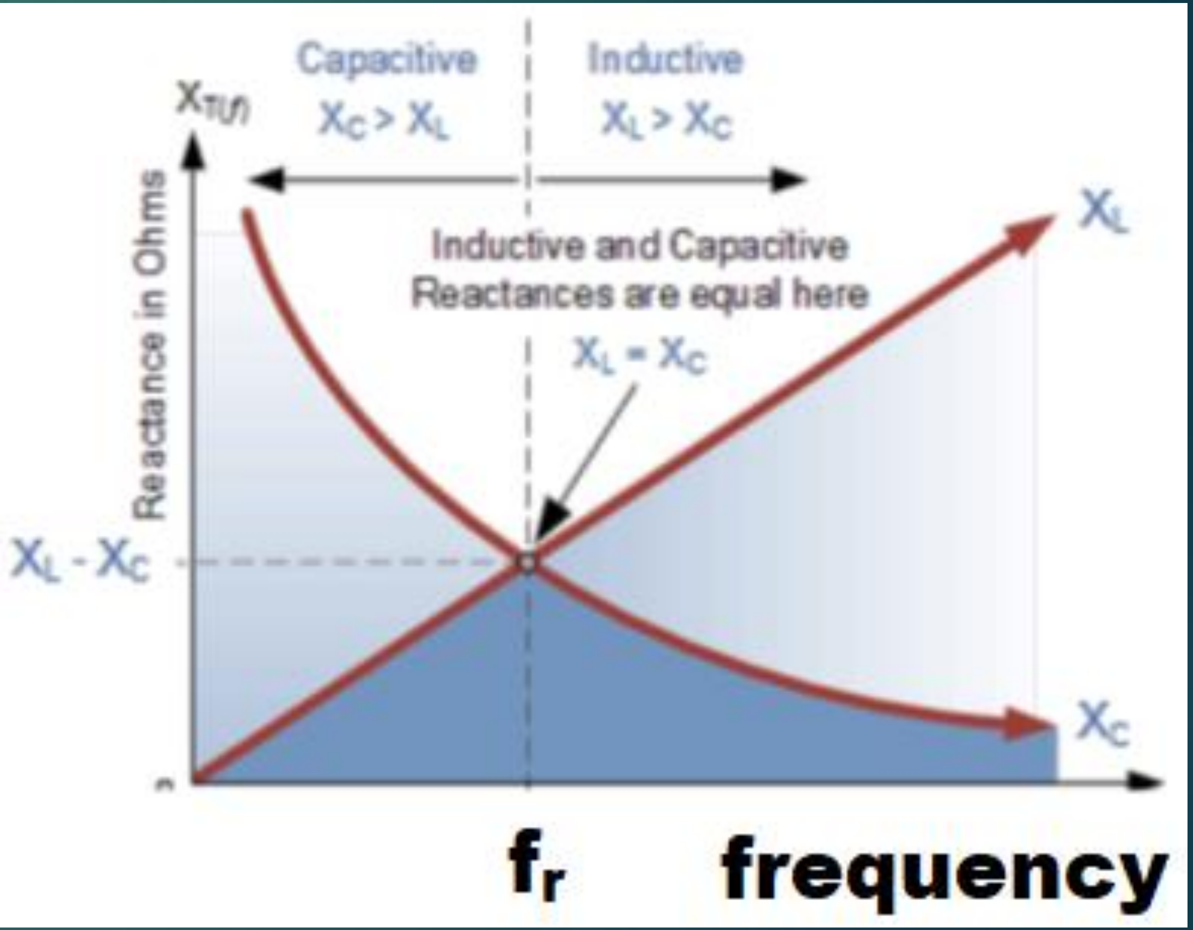
- Series Resonance Frequency :

$$X_L = X_C \quad \Rightarrow \quad 2\pi f L = \frac{1}{2\pi f C}$$

$$f^2 = \frac{1}{2\pi L \times 2\pi C} = \frac{1}{4\pi^2 LC}$$

$$f = \sqrt{\frac{1}{4\pi^2 LC}}$$

$$\therefore \; f_r = \frac{1}{2\pi\sqrt{LC}}\,(\mathrm{Hz}) \quad \text{or} \quad \omega_r = \frac{1}{\sqrt{LC}}\,(\mathrm{rads})$$



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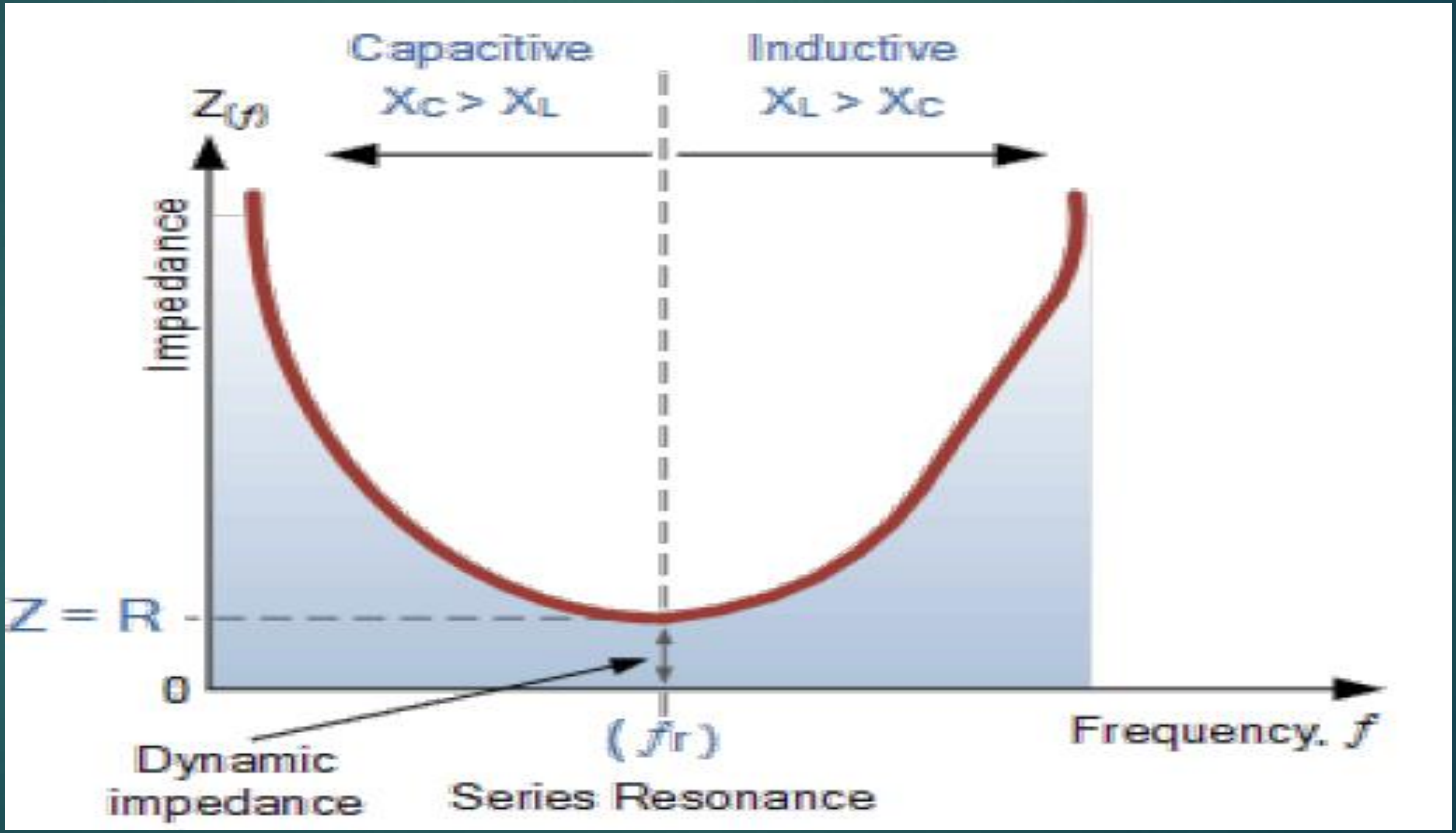
- Impedance in a series resonance circuit (Z = R):



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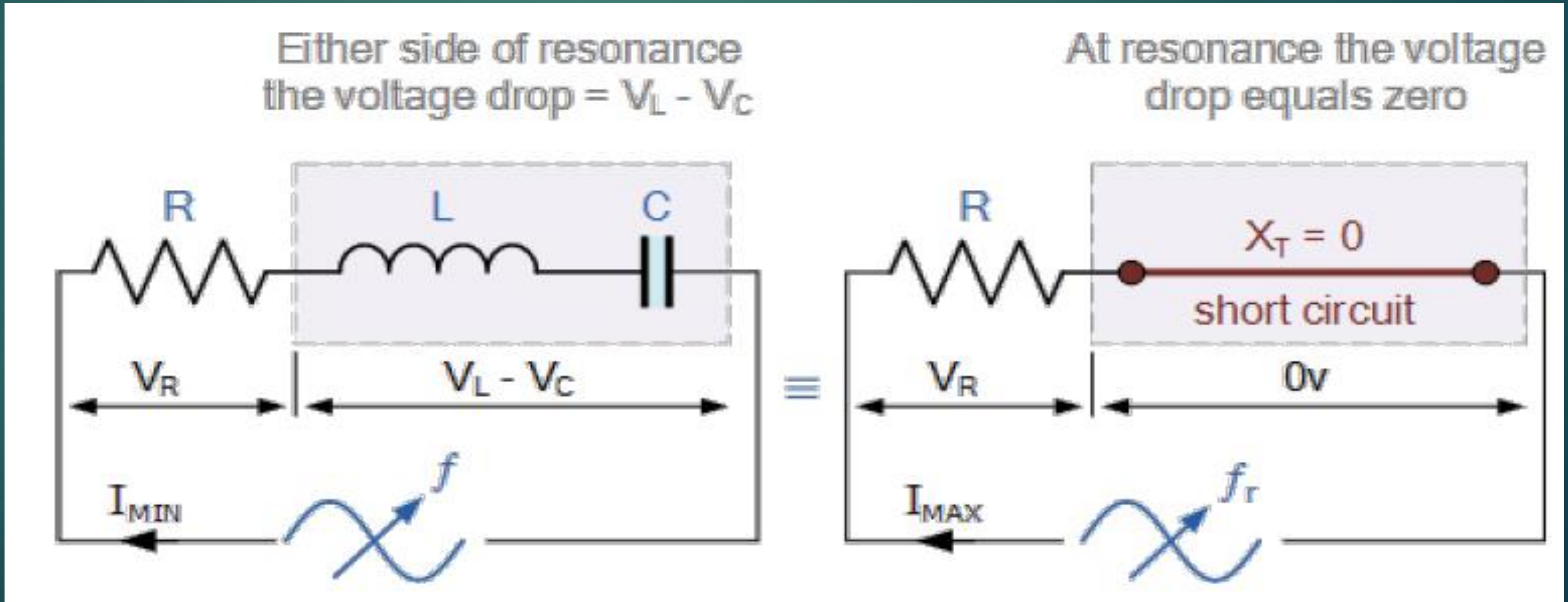
- Series RLC Circuit at Resonance : In a series Resonance circuit as $V_L = - V_C$ the resulting reactive voltages are zero and all the supply voltage dropped across the resistor. Therefore

  $V_R = V_{SUPPLY}$

- So for this reason the series resonance circuit is also known as "Voltage Magnification circuit" or voltage resonance circuit



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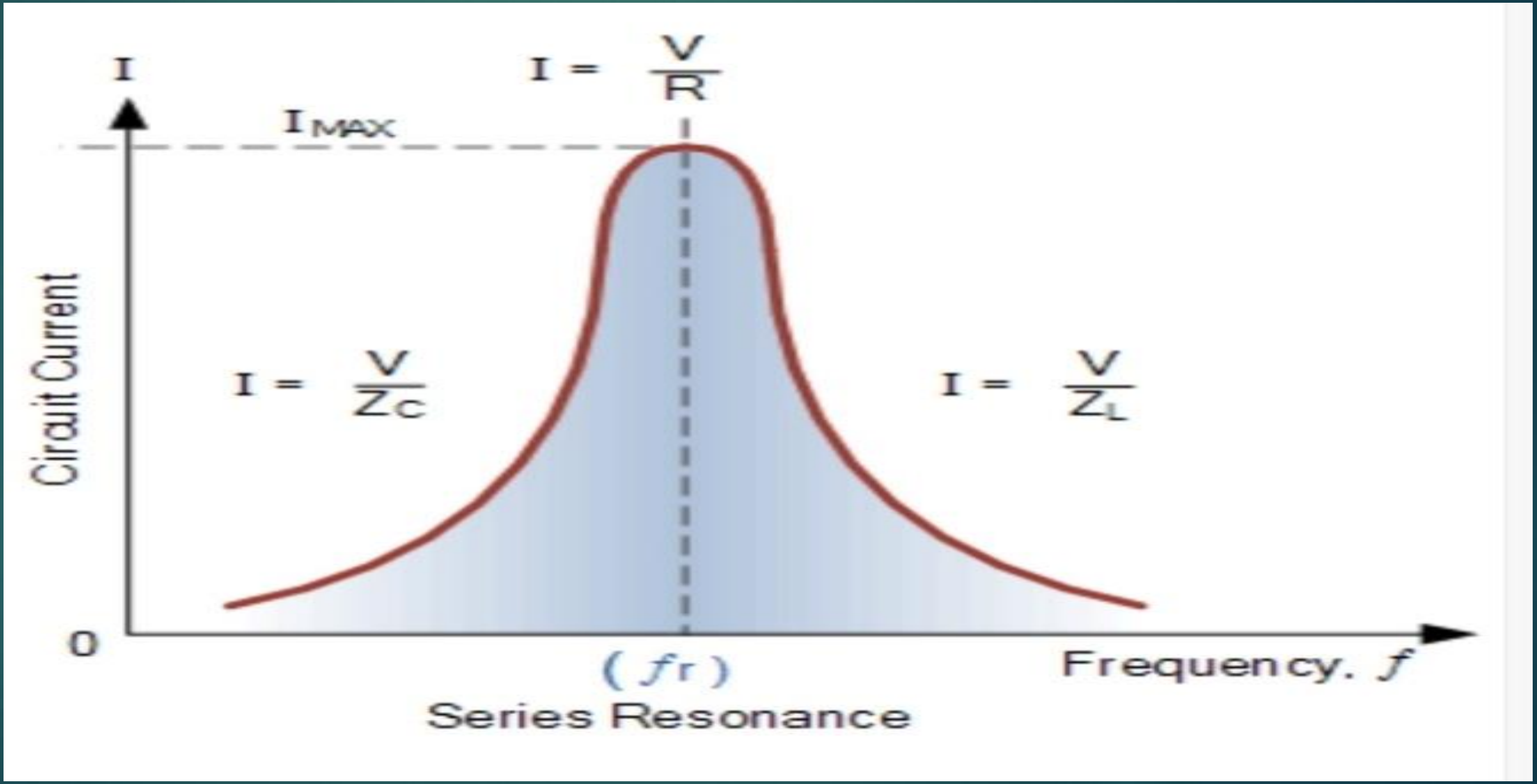
- Series Circuit Current at Resonance : $I_{Max} = V/(R=Z)_{Min}$



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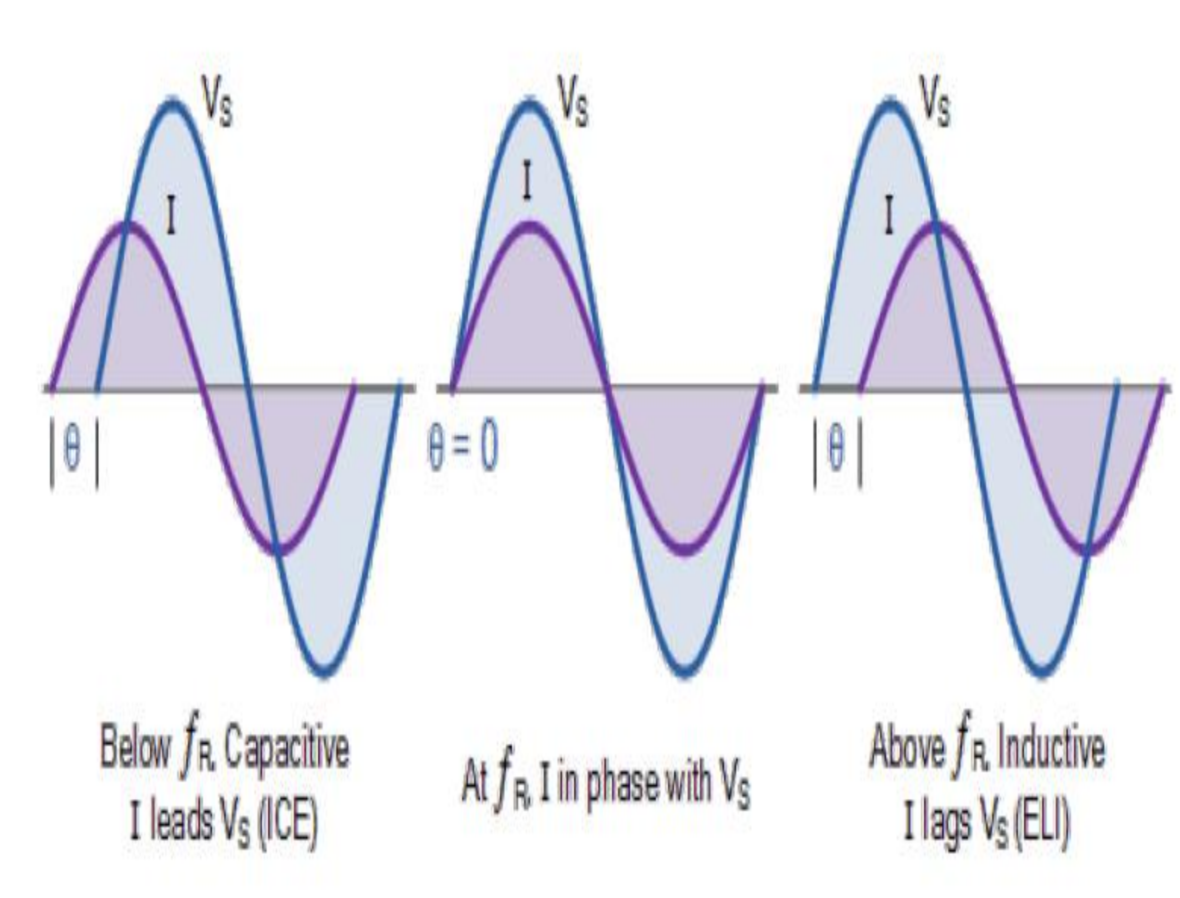
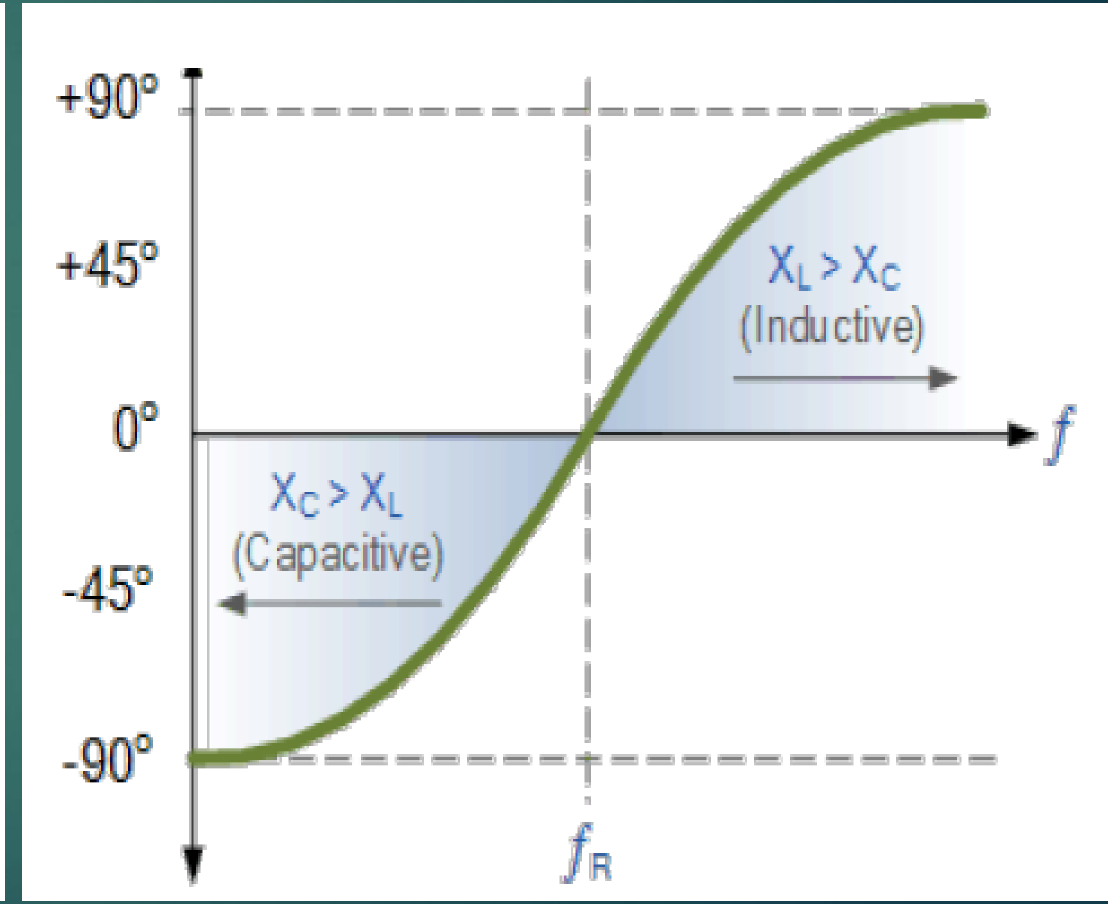
- Phase Angle of a Series Resonance Circuit : arctan = $X_L - X_C$ / R = 0^0 all real





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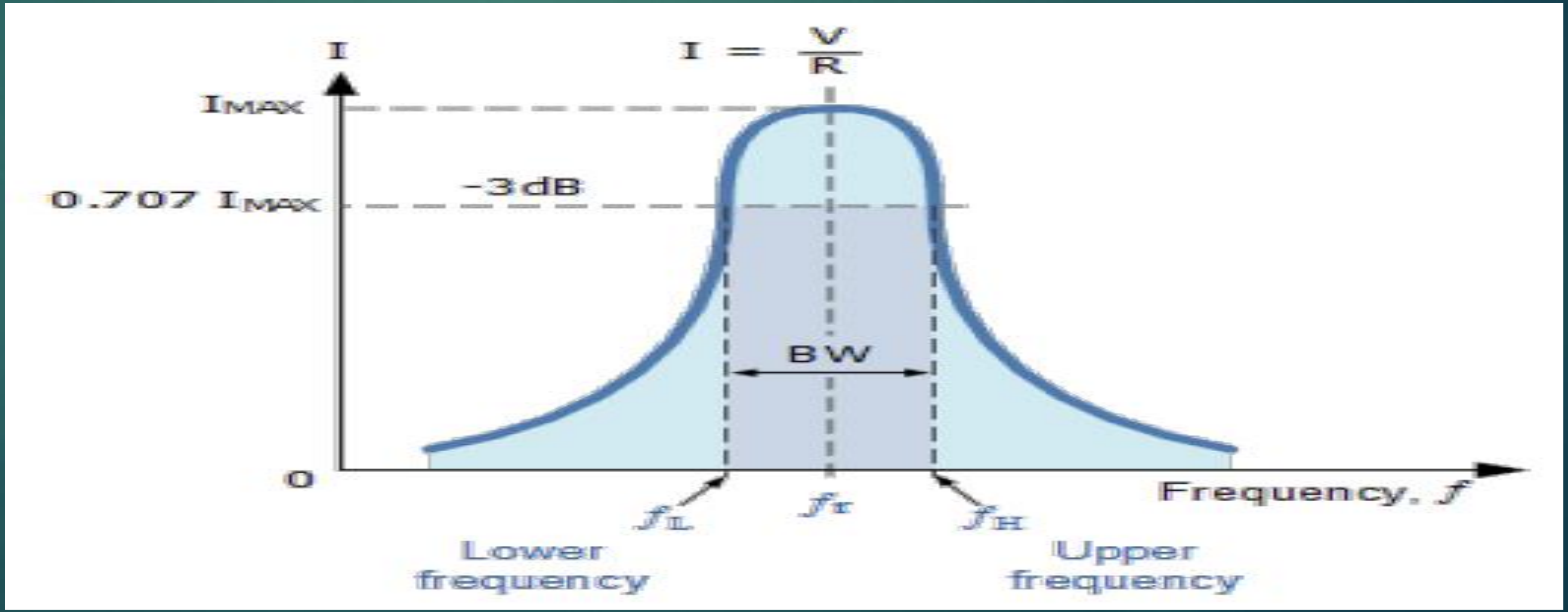
- Bandwidth of a Series Resonance Circuit :
- In the series RLC circuit is driven by the variable frequency at a constant voltage ,then the magnitude of the current I is proportional to impedance Z, therefore at resonance power absorbed by the circuit must be at its maximum value as $P = I^2 R$
- If we now reduce or increase the frequency until the average power absorbed by the resistor in the series resonance circuit is half that of its maximum value at resonance, we produce two frequency points called the half-power points which are -3 dB down from maximum, taking 0 dB as the maximum current reference.
- These -3dB points give us a current value that is 70.7% of its maximum resonant value which is defined as: $0.5(I^2 R) = (0.707 \times I)^2 R$. Then the point corresponding to the lower frequency at half the power is called the "lower cut-off frequency", labelled f_L with the point corresponding to the upper frequency at half power being called the "upper cut-off frequency", labelled f_H. The distance between these two points, i.e. $(f_H – f_L)$ is called the Bandwidth, (BW)

Continued…………

- The frequency response of the circuits current magnitude below, relates to the sharpness of the resonance in a series resonance circuit. “The sharpness of the peak is measured quantitatively and is called the Quality factor,” Q of the circuit. The quality factor relates the maximum or peak energy stored in the circuit (the reactance) to the energy dissipated (the resistance) during each cycle of oscillation. meaning that it is a ratio of resonant frequency to bandwidth and the higher the circuit Q, the smaller the bandwidth,

$Q = f_r / BW$.



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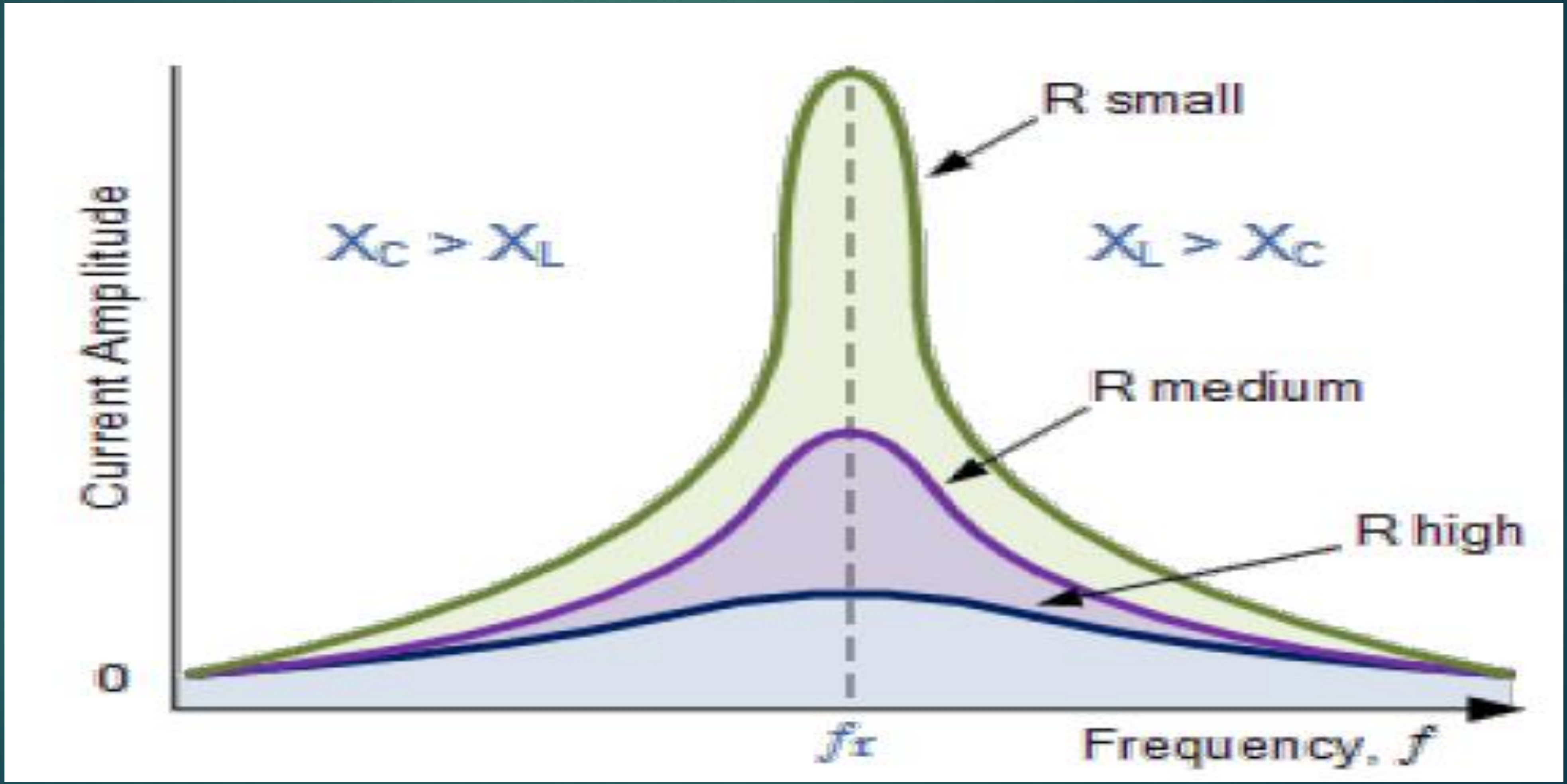
- Bandwidth of a Series RLC Resonance Circuit:



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- The relationship between Resonance, Bandwidth , Selectivity and Quality factor for a Series Resonance Circuit :

1). Resonant Frequency, (f_r)

$$X_L = X_C \quad \Rightarrow \quad \omega_r L - \frac{1}{\omega_r C} = 0$$

$$\omega_r^2 = \frac{1}{LC} \quad \therefore \quad \omega_r = \frac{1}{\sqrt{LC}}$$

2). Current, (I)

at ω_r $\quad Z_T = \text{min}, \quad I_S = \text{max}$

$$I_{max} = \frac{V_{max}}{Z} = \frac{V_{max}}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V_{max}}{\sqrt{R^2 + \left(\omega_r L - \frac{1}{\omega_r C}\right)^2}}$$

3). Lower cut-off frequency, (f_L)

At half power, $\frac{P_m}{2}, \quad I = \frac{I_m}{\sqrt{2}} = 0.707 I_m$

$$Z = \sqrt{2}R, \quad X = -R \ (\text{capacitive})$$

$$\omega_L = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

4). Upper cut-off frequency, (f_H)

At half power, $\frac{P_m}{2}, \quad I = \frac{I_m}{\sqrt{2}} = 0.707 I_m$

$$Z = \sqrt{2}R, \quad X = +R \ (\text{inductive})$$

$$\omega_H = +\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

5). Bandwidth, (BW)

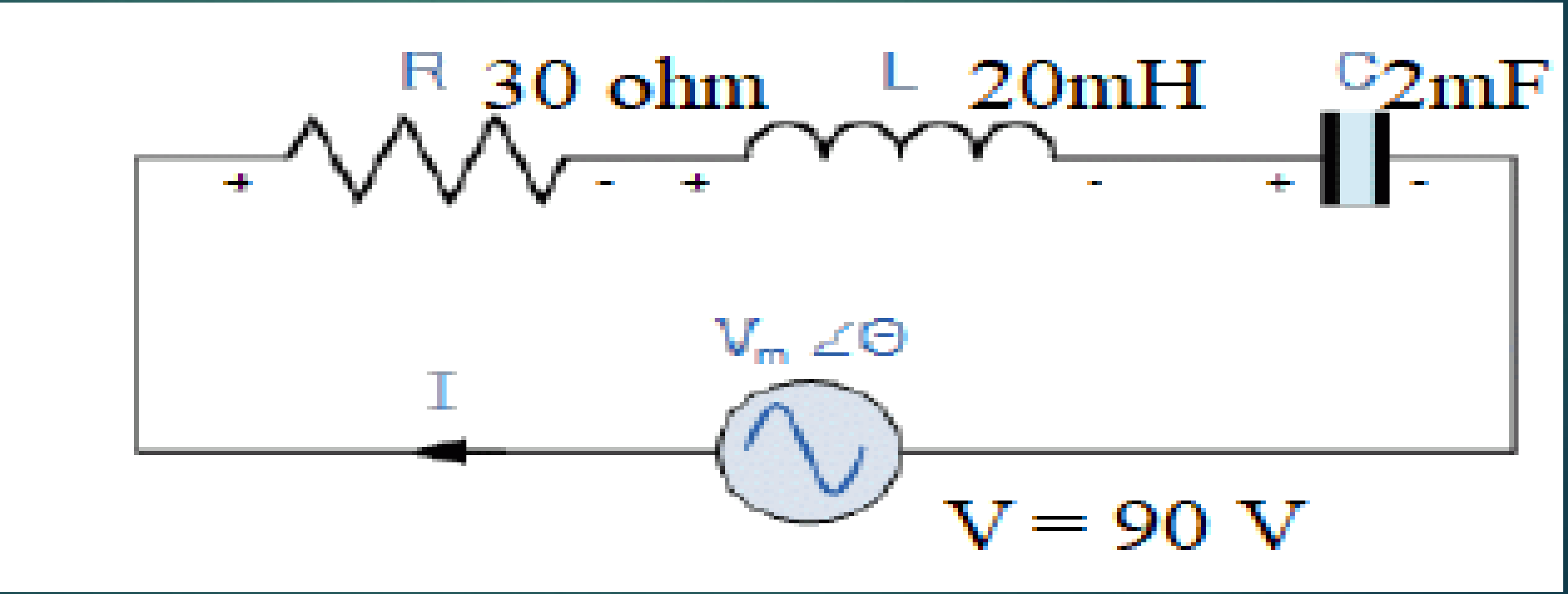
$$BW = \frac{f_r}{Q}, \quad f_H - f_L, \quad \frac{R}{L} \ (\text{rads}) \quad \text{or} \quad \frac{R}{2\pi L} \ (\text{Hz})$$

6). Quality Factor, (Q)

$$Q = \frac{\omega_r L}{R} = \frac{X_L}{R} = \frac{1}{\omega_r CR} = \frac{X_C}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Problems on Series RLC Resonance Circuit

- **Problem** : A series resonance network consisting of a resistor of 30Ω, a capacitor of 2uF and an inductor of 20mH is connected across a sinusoidal supply voltage which has a constant output of 9 volts at all frequencies. Calculate, the resonant frequency, the current at resonance, the voltage across the inductor and capacitor at resonance, the quality factor and the bandwidth of the circuit. Also sketch the corresponding current waveform for all frequencies.



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- Solution :

1. Resonant Frequency, f_r

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.02\times2\times10^{-6}}} = 796\text{Hz}$$

2. Circuit Current at Resonance, I_m

$$I = \frac{V}{R} = \frac{9}{30} = 0.3\text{A} \text{ or } 300\text{mA}$$

3. Inductive Reactance at Resonance, X_L

$$X_L = 2\pi f L = 2\pi\times796\times0.02 = 100\Omega$$

4. Voltages across the inductor and the capacitor, V_L, V_C

$$V_L = V_C$$
$$V_L = I\times X_L = 300\text{mA}\times100\Omega$$
$$V_L = 30\text{volts}$$

Note: the supply voltage may be only 9 volts, but at resonance, the reactive voltages across the capacitor, V_C and the inductor, V_L are 30 volts peak!

5. Quality factor, Q

$$Q = \frac{X_L}{R} = \frac{100}{30} = 3.33$$

6. Bandwidth, BW

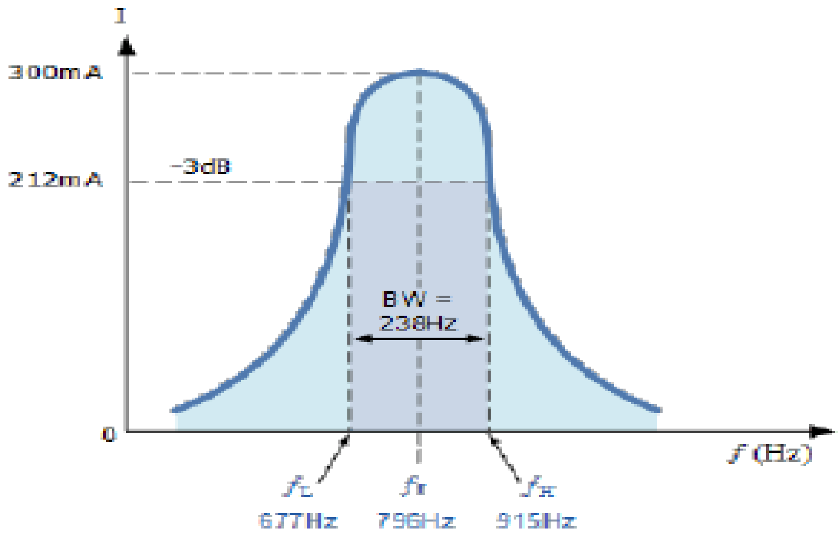
$$BW = \frac{f_r}{Q} = \frac{796}{3.33} = 238\text{Hz}$$

7. The upper and lower -3dB frequency points, f_H and f_L

$$f_L = f_r - \frac{1}{2}BW = 796 - \frac{1}{2}(238) = 677\text{Hz}$$

$$f_H = f_r + \frac{1}{2}BW = 796 + \frac{1}{2}(238) = 915\text{Hz}$$

8. Current Waveform



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- Problem :

EXAMPLE 5.3 *What is the resonance frequency of a series RLC circuit where $R = 10\,\Omega$, $L = 25$ mH and $C = 100\,\mu F$? Evaluate the Q factor also.*

SOLUTION. $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$

$$= \frac{1}{2\pi\sqrt{25 \times 10^{-3} \times 100 \times 10^{-6}}} \text{ Hz}$$

$\simeq$ **100.71 Hz.**

We know, Q factor of a series LRC circuit can be expressed as any of the two following relations

$$Q = \frac{\omega_0 L}{R} \quad \text{or} \quad \frac{1}{\omega_0 RC}$$

Using $\quad Q = \dfrac{\omega_0 L}{R}$,

$$Q = \frac{2\pi \times 100.71 \times 25 \times 10^{-3}}{10} = 1.58$$

[Using $\quad Q = \dfrac{1}{\omega_0 RC}$

$$Q = \frac{1}{2\pi \times 100.71 \times 10 \times 100 \times 10^{-6}} = 1.58]$$

Continued…………

- Problem :

EXAMPLE 5.6 *A coil having an inductance and resistance of 50 mH and 100 ohms is connected in series with a capacitor and a 100 V, 1 kHz source. Obtain the value of capacitance that will cause resonance in the circuit. Find the circuit current at resonance frequency.*

SOLUTION.

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

or,

$$1000 = \frac{1}{2\pi\sqrt{50 \times 10^{-3} \times C}}$$

$$\therefore \quad C = \frac{1}{4\pi^2 \times 50 \times 10^{-3} \times (1000)^2} = 0.5\ \mu F$$

$$|I_0| = \frac{|V|}{|Z|}$$

[Z_0 being the circuit impedance at resonance and is equal to the value of resistance]

$$= \frac{100}{10} = 10\ A$$

Problems on Series RLC Resonance Circuit

- Problem : In a series RLC circuit, instantaneous voltage and currents are $v = 100 \sin(314\,t + 30^0)$ V and $i = 4 \sin(314\,t + 30^0)$ A, Find the values of R and C provided L = 0.5 H
- Solution: we observe that in the series RLC circuit ,the instantaneous voltage and current are in phase .This indicates that circuit is at resonance ,the net circuit impedance being R

$$R = Vm/Im = 100/4 = 25 \text{ ohm}$$

$$X_L = X_C \text{ , } wL = 1/wC = C = 1/w^2L = 20.2 \text{ micro F}$$

3-Phase Star-Delta Connection

- Types of Phase System: There are two types of phase systems are available in electrical circuits

1. Single Phase System (1-ϕ) = Phase – Neutral
2. 3-Phase System (3-ϕ) = R –Y – B – N (Red – Yellow –Blue - Neutral)

Single Phase System: In single phase circuit there will be only one phase i.e . current will flow through one wire(conductor) and there will be one return path is called neutral line to complete the circuit

- Three Phase System: In three phase circuit there will be three phases i.e . current will flow through three wire (conductors) and there will be one return path is called neutral line to complete the circuit ,Phase difference of each (wire or R-Y-B) is 120^0

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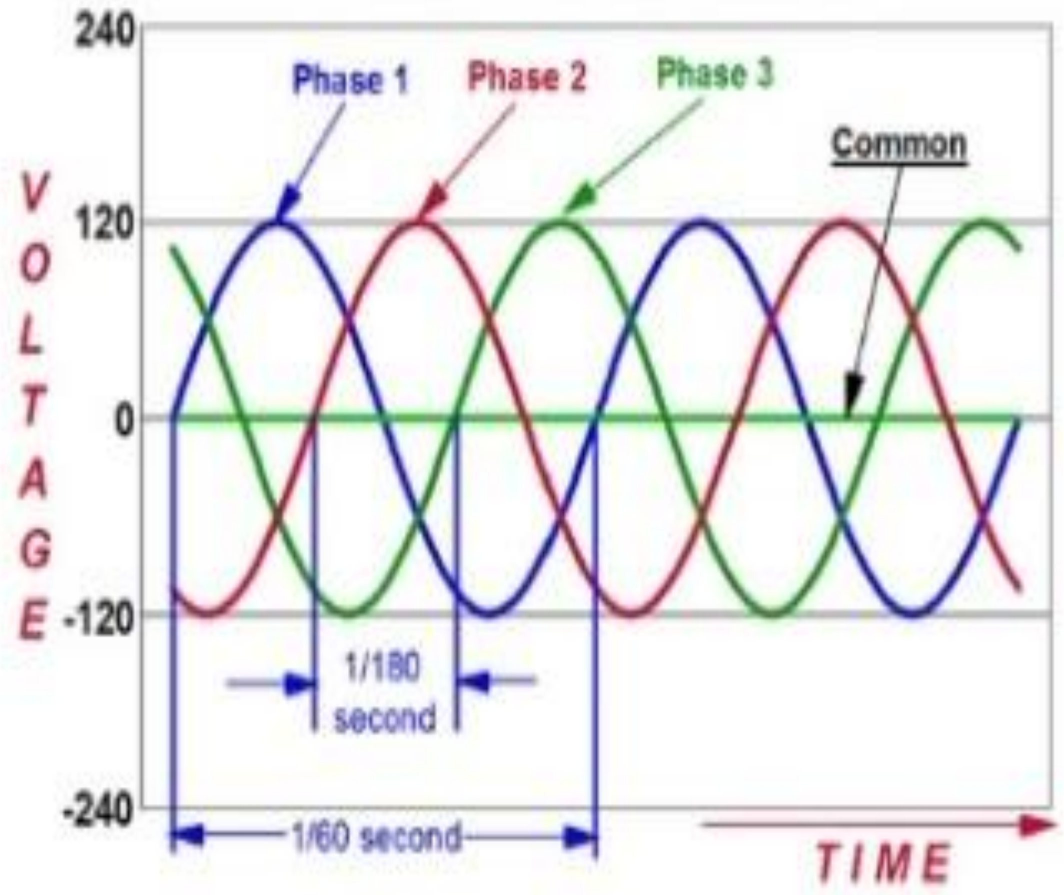
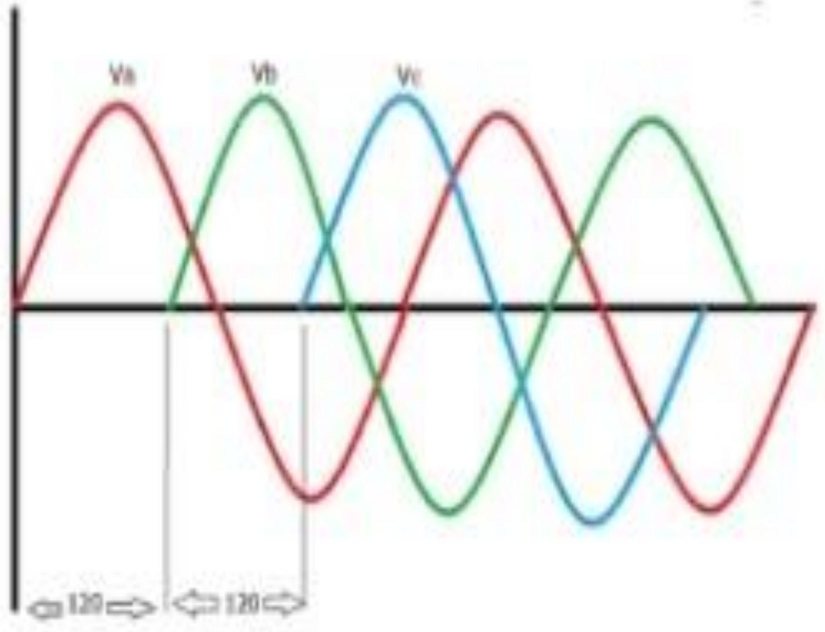
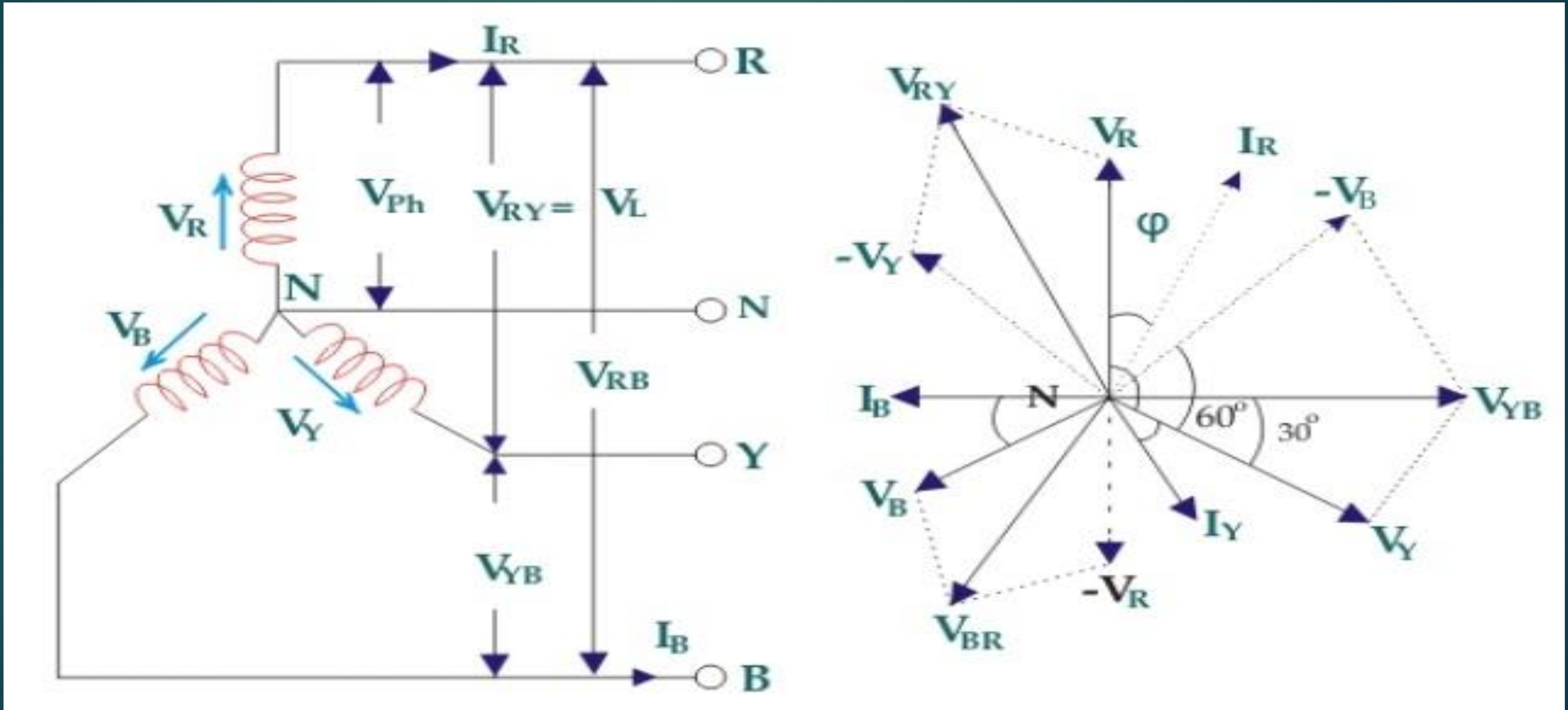
- 3-Phase System :





3- Phase Balanced Star connection

- Star connection :



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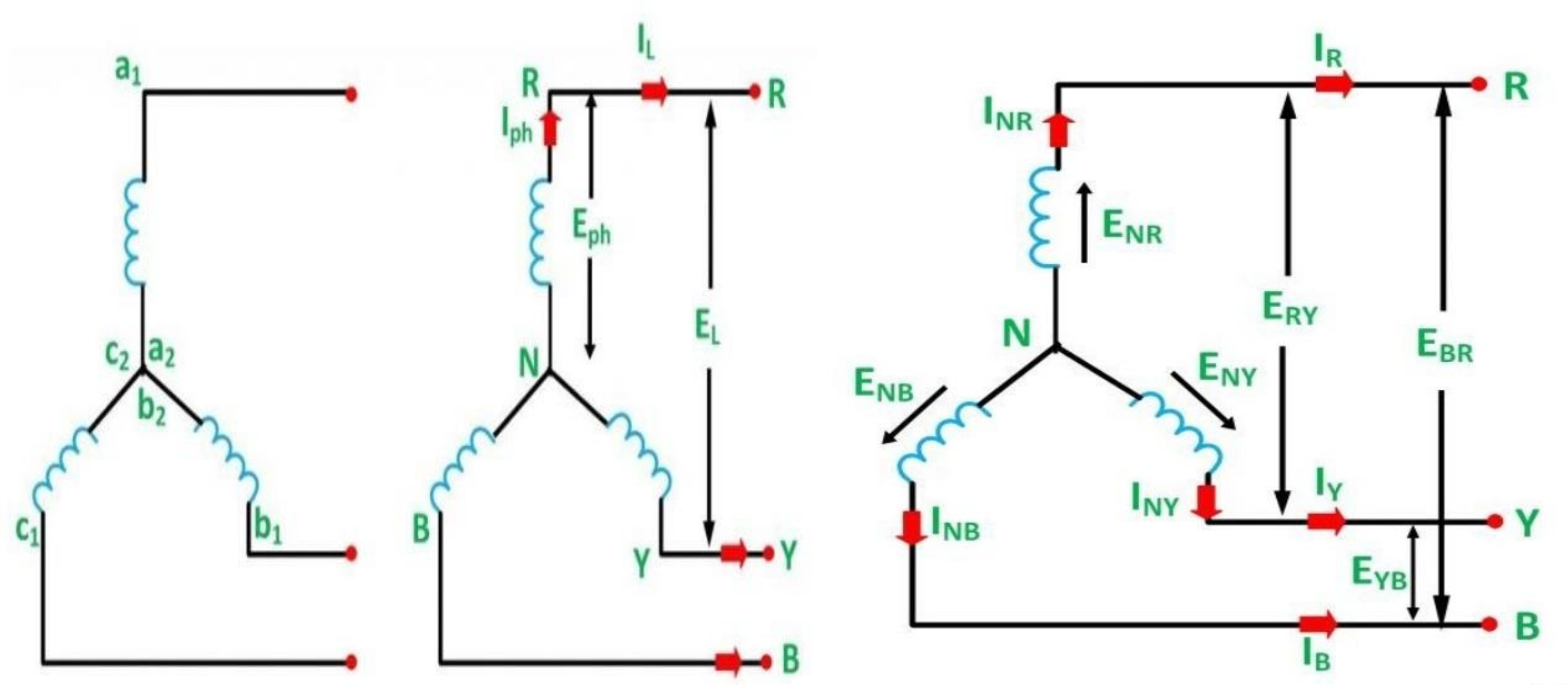
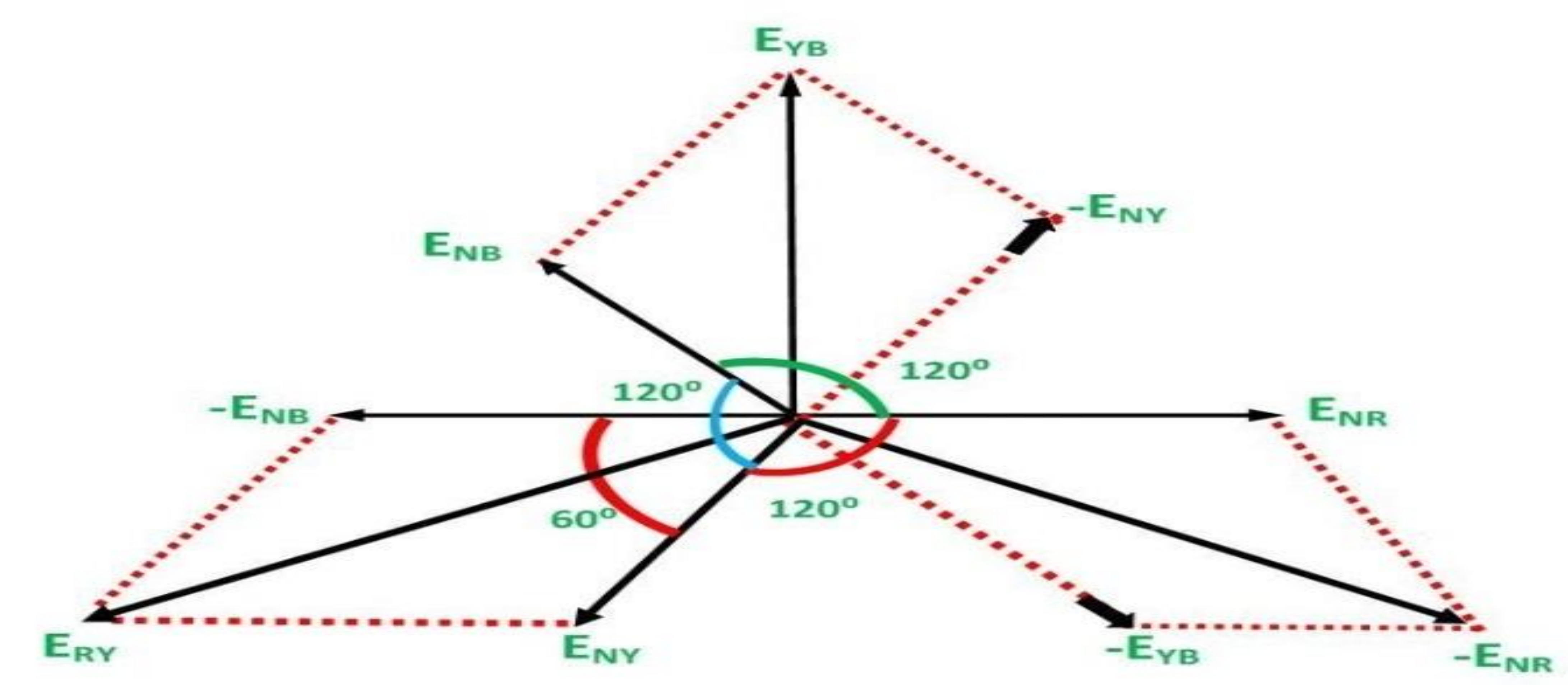


a_1
c_2 a_2
b_2
c_1
b_1
I_L
R
I_{ph}
R
E_{ph}
E_L
N
B
Y
Y
B
I_R
R
I_{NR}
E_{NR}
E_{RY}
E_{BR}
N
E_{NB}
E_{NY}
I_{NY}
I_Y
Y
I_{NB}
E_{YB}
B
I_B

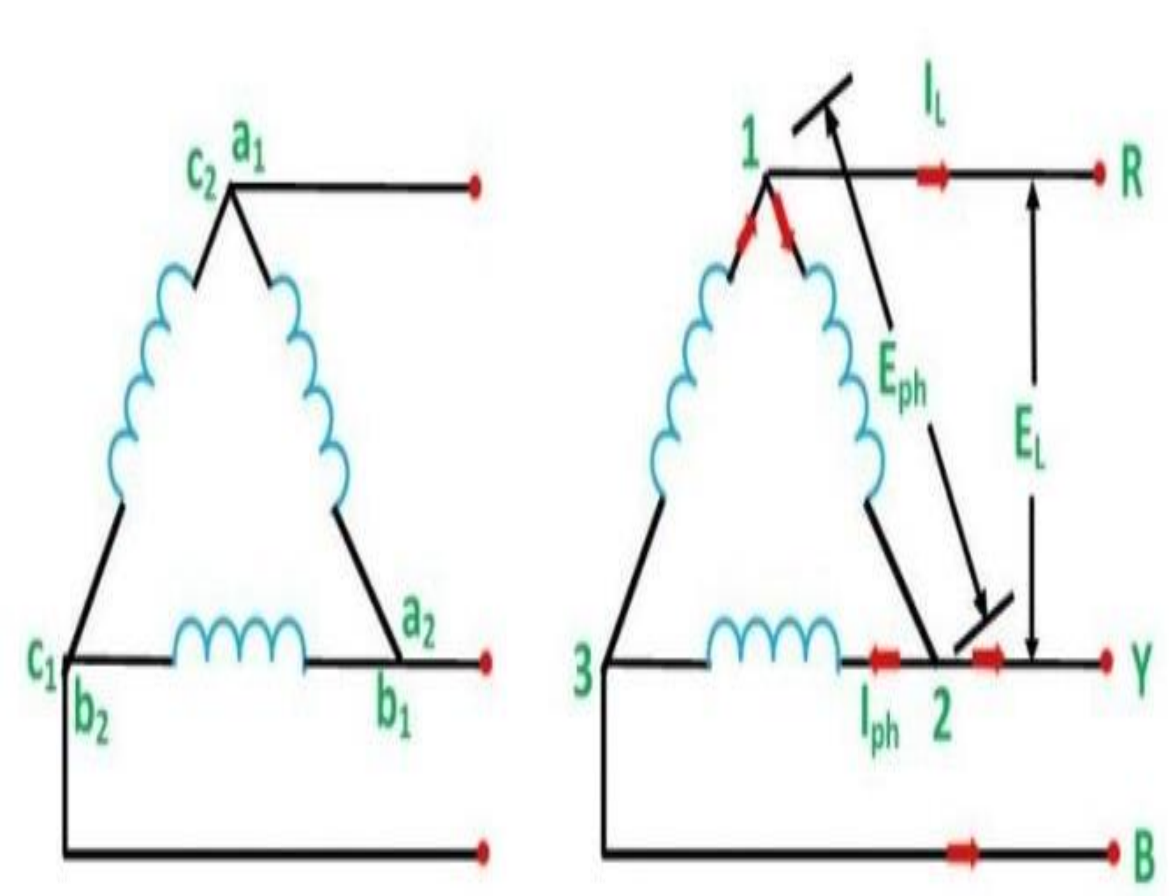
Continued…………

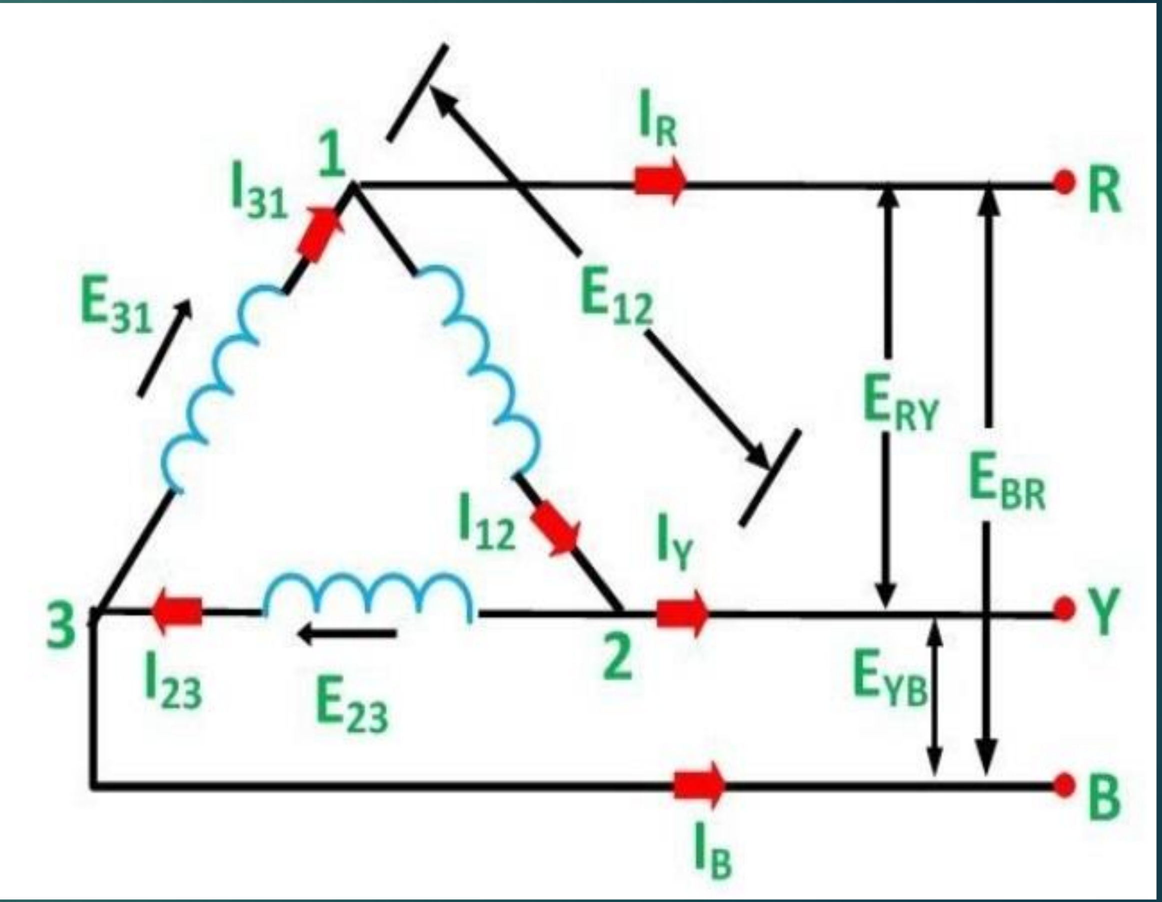
- Phasor diagram :



3- Phase Balanced Delta connection

- Delta Connection :





Continued…………

- Phasor diagram :