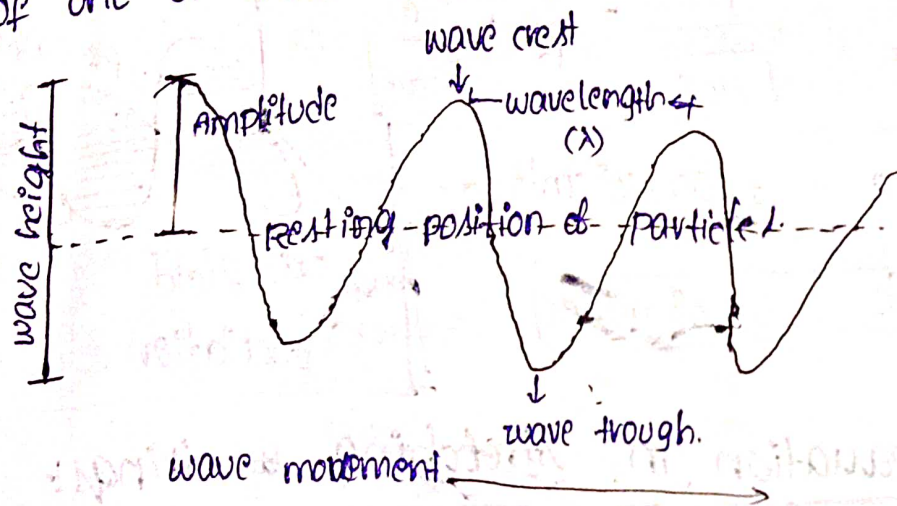


* WAVE OPTICS *

1. CONCEPT OF INTERFERENCE

a) introduction of light waves:

Wave:- wave is a propagating dynamic disturbance of one or more quantities.



electromagnetic waves:-

From maxwell's equations $\oint \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0}$

$$\oint \vec{B} \cdot d\vec{A} = 0$$

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\phi_B}{dt}$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i + \mu_0 \epsilon_0 \frac{d\phi_E}{dt}$$

$$\phi_E = \int \vec{E} \cdot d\vec{A} \quad , \quad \phi_B = \int \vec{B} \cdot d\vec{A}$$

$$\therefore \frac{\partial^2 E}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}$$

$$\frac{\partial^2 B}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 B}{\partial t^2}$$

→ The simplest solutions to these partial differential equations are sinusoidal waves - electromagnetic waves.

$$E = E_{\text{max}} \cos(kx - \omega t)$$

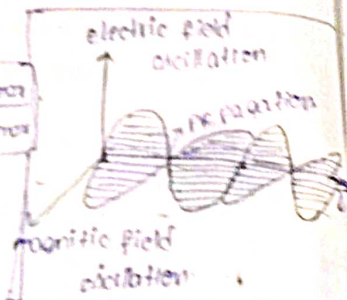
$$B = B_{\text{max}} \cos(kx - \omega t)$$

the speed of electromagnetic wave is:

$$v = \frac{\omega}{k} = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \frac{E}{B} = \frac{E_{\text{max}}}{B_{\text{max}}}$$

$$c = 3.00 \times 10^8 \text{ m/sec}$$

$$c = 3 \times 10^8 \text{ m/sec}$$



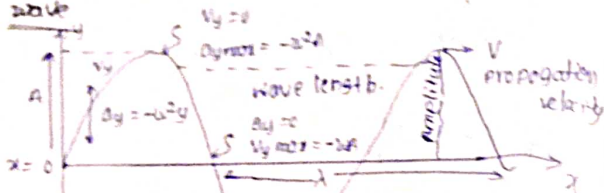
wave equation in stretching string:

$$\frac{d^2 y}{dx^2} = \frac{1}{v^2} \frac{d^2 y}{dt^2}$$

where $v = \sqrt{\frac{T}{\mu}}$

$$y(x, t) = g(z - vt)$$

the wave



description of the transverse motion

$$\frac{2\pi v}{\lambda} = 2\pi f = \omega$$

$$v = f\lambda$$

$$y(x, t) = A \sin \frac{2\pi}{\lambda} (x - vt)$$

$$v_y(x, t) = \frac{dy}{dt} = \omega A \cos \frac{2\pi}{\lambda} (x - vt)$$

$$a_y(x, t) = \frac{d^2 y}{dt^2} = -\omega^2 y$$

Light wave:

$$c = \lambda \times f \quad \text{or} \quad E = hf$$

where,

c = velocity of light

$$= 3 \times 10^8 \text{ m/sec}$$

λ = wavelength of light

f = frequency of light

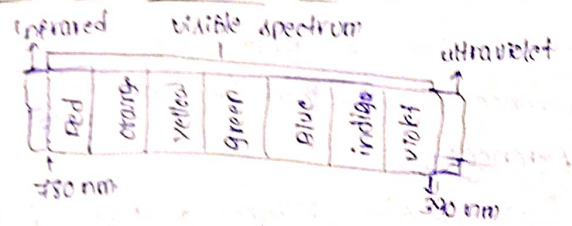
where,

E = light energy

h = plank's constant = $6.626 \times 10^{-34} \text{ J.s}$

f = frequency of light

wavelength is ^{inversely} proportional to frequency. (wavelength of different color)

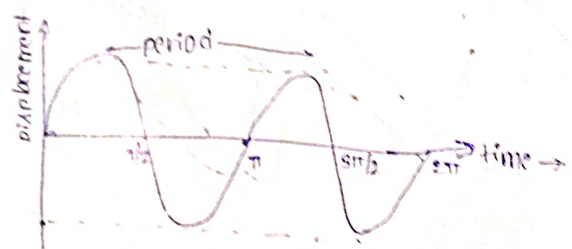


red has long wavelength (low frequency)

violet has short wavelength (high frequency)

$\therefore$ violet has higher energy

displacement vs time:



Principle of Superposition:-

When two or more waves overlap, the resultant wave is the algebraic sum of the individual waves.

Phase difference:-

The fractional part of a period which represents the offset in peak positions of waves.

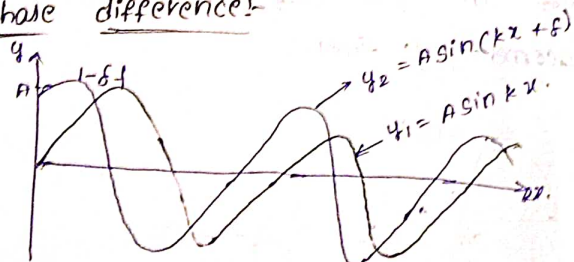
Coherence:-

The existence of a correlation between the phases of two or more waves such that interference effects can be observed.

Interference:-

The variation with distance or time of the amplitude of a wave which results from the superposition of two or more waves.

Phase difference:-



In phase of two waves:-

Superposition and phase difference:-

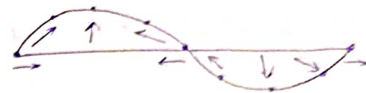
Oscillations in phase:-



$$\theta + \theta = \theta$$

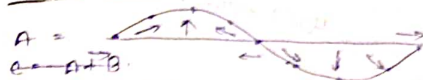


$$C = A + B$$



if phase difference is equal to zero then amplitude of resultant = amplitude of component.

Out of phase:-



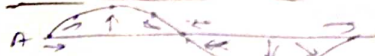
$$C = A + B$$



$$\theta + \theta = 0$$

if phase difference = π then amplitude of resultant = difference in amplitude of component.

Phase difference $-\pi/2$:-



$$C = A + B$$

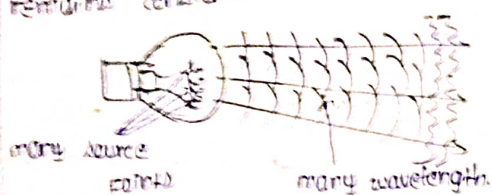


$$\rightarrow + \rightarrow = 2\rightarrow$$

For any phase difference amplitude of resultant = arrow sum of component.

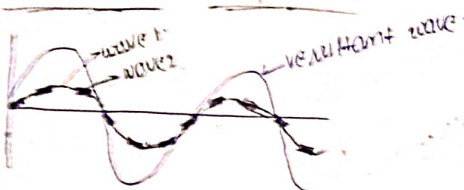
Coherence of light sources:-

Two waves will only produce a stable interference pattern if their phase relationship remains constant.



YOUNG created coherent light sources by dividing the wavefronts from a single source into two using a slit.

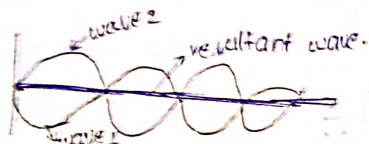
Constructive Interference:-



$$y_1 + y_2 = [2A \cos \frac{1}{2} \delta] \sin (kx - \omega t + \frac{1}{2} \delta)$$

* Superposition of two waves of the same amplitude and frequency.

Destructive Interference:-



$$y_1 + y_2 = [2A \cos \frac{1}{2} \delta] \sin (kx - \omega t + \frac{1}{2} \delta)$$

Interference of light:-

Interference is a phenomenon in which two coherent light waves superpose to form a resultant wave of greater or lower amplitude.

Types of interference:-

1) Constructive interference:- if a crest of one wave meets a crest of another wave, the resultant intensity increases (bright fringes)



2) Destructive interference:- if a crest of one wave meets a trough of another wave, the resultant intensity decreases (dark fringes)



Conditions for interference:-

- 1) the two interfering waves should be coherent, i.e. the phase difference between them must remain constant with time.
- 2) the two waves should have same frequency.
- 3) if interfering waves are polarized, they be in same state of polarization.
- 4) The separation between the light sources should be as small as possible.

- The distance of the screen from the source should be quite large.
- c) The amplitude of the interfering waves should be equal or atleast very nearly equal.
- e) The two sources should be narrow.
- f) The two sources should give monochromatic or very nearly monochromatic or else the path difference should be very small.

Techniques for producing interference:

- The phase relation between the waves emitted by two independent light sources rapidly change with time and then they can never be called coherent, though the sources are identical in all respects.
- If two sources are derived from a single source by same device, then any phase change occurring in one source is simultaneously accompanied by the same phase change in the other source. So the phase difference between the waves emerging from the two sources remains constant and the sources are coherent.

The techniques used for creating coherent sources of light are:

- a) wavefront splitting.
- b) Amplitude splitting.

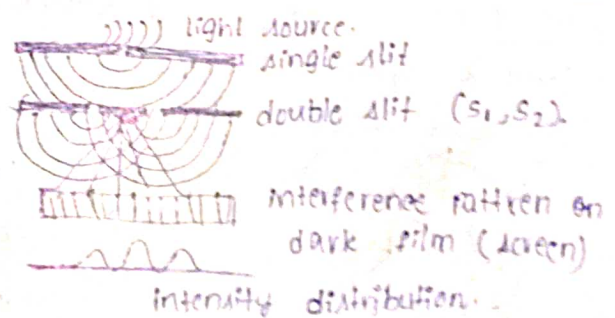
a) wavefront splitting:-

One of the methods consists in dividing a light wavefront, emerging from a narrow slit by passing it through two slits closely spaced side by side.

→ The two parts of the same wavefront travel through different paths and reunite on a screen to produce fringe pattern. This is known as interference due to the division of wavefront.

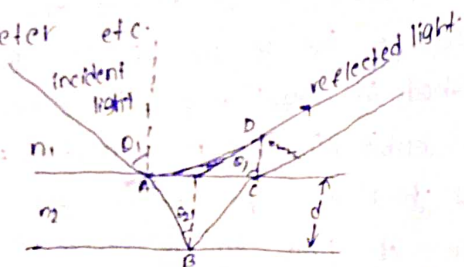
→ This method is useful only with narrow sources, eg: Young's double slit, Fresnel's double mirror, Fresnel's biprism, Lloyd's mirror etc.

Interference of light by division of wavefront:

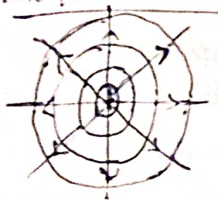


b) Amplitude Splitting:-

- the amplitude (intensity) of a light wave is divided into two parts, namely reflected and transmitted components, by partial reflection at a surface.
 - the two parts travel through different paths and reunite to produce interference fringes. This is known as interference due to division of amplitude.
 - optical elements such as beam splitters, mirrors are used for achieving amplitude division.
- eg) Newton's ring experiment, Michelson's interferometer etc.

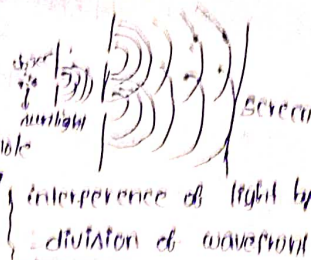


wavefront:- in physics a wavefront is the locus of points having the same phase.
line / curve in 2D surface for wave propagating



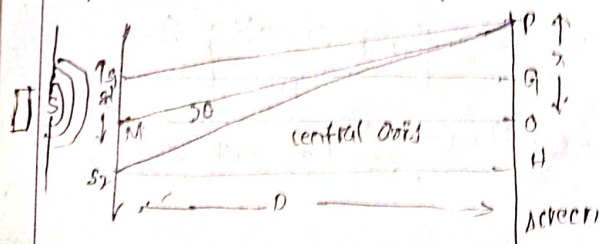
Thomas Young:-

- in 1801, Young admitted the sunlight through a single pinhole and then directed the emerging light onto two pinholes. interference of light by division of wavefront
- the spherical waves emerging from the pinholes interfered with each other and a few colored fringes were observed on the screen.
- the pinholes were later replaced with narrow slits and the sunlight was replaced by monochromatic light (e.g. sodium lamp, yellow, $\lambda = 589.3 \text{ nm}$).



calculation of optical path difference between two waves:-

As slits S_1 and S_2 are equidistant from source (S), the phase of the wave at S_1 will be same as the phase of the wave at S_2 and therefore S_1, S_2 act as coherent sources. The waves leaving from S_1 and S_2 interfere and produce alternative bright and dark bands on screen.



Young's double slit experiment:

Let P be an arbitrary point on screen.

which is at a distance D from the double slit

and is the distance between S_1 and S_2

is the distance between O and P

S_1N is the normal to the line S_2P
and bisects S_1S_2 and GH (i.e. $S_1M = S_2M =$
 $GO = HO = d$).

The optical paths are identical with the geometrical paths if the experiment is carried out in air.

The path difference b/w $S_2P - S_1P = S_2N$.
Let S_1G and S_2H are perpendicular on the screen and form S_2HP triangle.

$$\begin{aligned}(S_2P)^2 &= (S_2H)^2 + (HP)^2 \\&= D^2 + (x+d)^2 \\&= D^2 \left[1 + \frac{(x+d)^2}{D^2} \right]\end{aligned}$$

$$S_2P = D \left[1 + \frac{(x+d)^2}{D^2} \right]^{1/2}$$

Since $D \gg (x+d)$, $\frac{(x+d)^2}{D^2}$ is very small

after expansion,

$$S_2P = D \left[1 + \frac{1}{2} \frac{(x+d)^2}{D^2} \right]$$

$$S_2P = D + \frac{1}{2} \frac{(x+d)^2}{D}$$

$$\text{Similarly } S_1P = D + \frac{1}{2} \frac{(x-d)^2}{D}$$

$$\text{path difference} = S_2P - S_1P$$

$$= \frac{1}{2D} ((x+d)^2 - (x-d)^2)$$

$$\boxed{\text{path difference} = \frac{2xd}{D}} = x$$

$$\therefore (x+d)^2 - (x-d)^2 = 4xd$$

thus for bright fringes (maxima)

$$S_2N = \frac{2xd}{D} = n\lambda, \quad n=0,1,2,\dots$$

$$\boxed{x = \frac{D}{2d} n\lambda}$$

for dark fringes (minima)

$$S_2N = \frac{2xd}{D} = (2n+1) \frac{\lambda}{2}, \quad n=0,1,2,\dots$$

$$\boxed{x = \frac{D}{2d} (2n+1) \frac{\lambda}{2}}$$

Let x_n and x_{n+1} denote the distance of n th and $(n+1)$ th bright fringes then $x_n = \frac{D}{2d} n\lambda$

$$x_{n+1} = \frac{D}{2d} (n+1)\lambda$$

spacing between n^{th} and $(n+1)^{\text{th}}$ bright fringe

$$x_{n+1} - x_n = \frac{D}{2d} (n+1)\lambda - \frac{D}{2d} (n\lambda)$$

$$= \frac{D}{2d} (\cancel{n\lambda} + \lambda - \cancel{n\lambda})$$

$$x_{n+1} - x_n = \frac{D\lambda}{2d}$$

$\therefore$ The space between any consecutive bright fringe is the same. Similarly,

the spacing between two dark fringe is $\frac{D\lambda}{2d}$.

The spacing between the fringes is independent of n .

fringe width

The spacing between any two consecutive bright or dark fringe is called "fringe width", and it is denoted by x .

$$x = \frac{D\lambda}{2d}$$

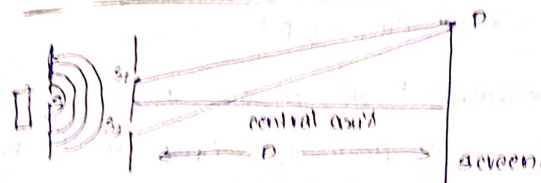
The nature of the interference of the two waves at P depends simply on how many waves are contained in the length of path difference (S_2N).

if the path difference (S_2N) contains an integral number of wavelengths, then two waves

interfere constructively producing a maximum in the intensity of light on the screen at P .

if the path difference (S_2N) contains an odd number of half-wavelengths, then the two waves interfere destructively and produce minimum intensity of light on the screen at P .

resultant intensity due to superposition of two interference waves



Let S be a narrow slit illuminated by a monochromatic (single wavelength) source.

S_1 and S_2 are two similar parallel slits (S_1 and S_2 are equidistant from S and very close to each other).

Suppose the waves from S reach S_1 and S_2 in the same phase (coherent). Beyond S_1 and S_2 the waves proceed as if they were started from S_1 and S_2 .

Let us calculate the resultant intensity of light of wavelength (λ) at point P on screen placed parallel to S_1 and S_2 .

let A_1 and A_2 be the amplitude at P due to the waves from S_1 and S_2 respectively.

→ The waves arrive at P, having traversed different paths S_1P and S_2P .

→ Hence, they are superposed at P with a phase difference δ .

$$\delta = \frac{2\pi}{\lambda} \times \text{path difference}$$

$$= \frac{2\pi}{\lambda} (S_2P - S_1P)$$

The displacement at P due to the simple harmonic waves from S_1 and S_2 can be represented by

$$y_1 = A_1 \sin \omega t$$

$$y_2 = A_2 \sin (\omega t + \delta)$$

where $\frac{\omega}{2\pi}$ is the common frequency (f) of two waves.

→ by the principle of superposition, when two or more waves simultaneously reach at point, the resultant displacement is equal to the sum of the displacements of all the waves.

Hence the resultant displacement is

$$y = y_1 + y_2$$

$$= A_1 \sin \omega t + A_2 \sin (\omega t + \delta)$$

$$= A_1 \sin \omega t + A_2 \sin \omega t \cos \delta + A_2 \cos \omega t \sin \delta$$

$$= \sin \omega t (A_1 + A_2 \cos \delta) + \cos \omega t (A_2 \sin \delta)$$

let us make a change in constants A_1 , A_2 and δ by putting $A_1 + A_2 \cos \delta = R \cos \theta$

$$A_2 \sin \delta = R \sin \theta$$

where R and θ are new constants.

$$R^2 = A_1^2 + A_2^2 + 2A_1A_2 \cos \delta$$

The resultant intensity I and R which is proportional to the square of the resultant amplitude i.e. $I = R^2$

$$I = A_1^2 + A_2^2 + 2A_1A_2 \cos \delta$$

$$\text{where } \delta = \frac{2\pi}{\lambda} \times \text{path difference} = \frac{2\pi}{\lambda} (S_2P - S_1P)$$

→ in case of interference, the resultant intensity at P is not just the sum of the intensities of the individual waves.

Conditions for maxima and minima intensity:

The resultant intensity I at P.

$$I = A_1^2 + A_2^2 + 2A_1A_2 \cos \delta$$

The intensity I is maximum,

$$\text{when } \cos \delta = +1 \text{ or } \delta = 2n\pi, n=0,1,2,\dots$$

$$\text{phase difference } (\delta) = 2n\pi$$

$$\text{path difference } (S_2P - S_1P) = 2n\pi \times \frac{\lambda}{2\pi} = n\lambda$$

$$I_{\max} = A_1^2 + A_2^2 + 2A_1A_2$$

$$= (A_1 + A_2)^2$$

$$I_{\max} = I_1 + I_2 + 2\sqrt{I_1I_2}$$

the maximum intensity is greater than the sum of two separate intensities
the intensity is a minimum,

$$\text{when } \cos \delta = -1$$

$$\delta = (2n+1)\pi, \quad n=0,1,2$$

$$\text{path difference } (\delta) = (n+1)\lambda$$

$$\text{path difference } (S_2P - S_1P) = (2n+1)\lambda \times \frac{\lambda}{2\lambda}$$

$$= \frac{(2n+1)\lambda}{2}$$

$$I_{\min} = A_1^2 + A_2^2 - 2A_1A_2 = (A_1 - A_2)^2$$

the minimum intensity is less than sum of two separate intensities.

Q. in an interference pattern, at a point we observe the 12th order maximum for $\lambda_1 = 6000 \text{ \AA}$. what order will be visible here if the source is replaced by light of wavelength $\lambda_2 = 4800 \text{ \AA}$?

sol. in double slit interference, the distance x of a bright fringe from the center (zero order)

$$x = \frac{d}{2D} n\lambda, \quad \text{where } n=0,1,2$$

at given point $n\lambda$ is constant (8)

$$n_1\lambda_1 = n_2\lambda_2$$

$$n_2 = n_1 \frac{\lambda_1}{\lambda_2} = 12 \times \frac{6000}{4800}$$

$$n_2 = 15$$

Q. two straight narrow parallel slits (2mm) are illuminated with a monochromatic light of wavelength 5896 \text{ \AA}. Fringes are observed at a distance of 60cm from the slits. Find the wavelength width of the fringes?

$$\text{fringe-width } x = \frac{\lambda D}{2d}$$

$$2d = 2\text{mm} = 0.2\text{cm}, \quad D = 60\text{cm},$$

$$\lambda = 5896 \text{ \AA} = 5896 \times 10^{-8}\text{cm}.$$

$$x = \frac{60 \times 5896 \times 10^{-8}}{0.2} = 1.77 \times 10^{-2}\text{cm}.$$

Q. In a two-slit interference pattern with $\lambda = 5000 \text{ \AA}$ the zero order and tenth order maxima fall at 12.34 mm and 14.73 mm, respectively. if λ is changed to 5000 \text{ \AA}, deduce the positions of the zero order and twentieth order fringes, while other arrangements remaining the same.

sol:- Fringe width $\lambda = \frac{\lambda D}{2d}$

with $\lambda = 6000 \text{ \AA}$,

→ distance between 0th order and 10th order fringe

→ is $14.73 \text{ mm} - 12.34 \text{ mm} = 2.39 \text{ mm}$.

Fringe width $= \frac{2.39}{10} = 0.239 \text{ mm}$.

→ $\frac{(\text{Fringe width})_{6000}}{(\text{Fringe width})_{5000}} = \frac{6000 \text{ \AA}}{5000 \text{ \AA}} = \frac{6}{5}$.

→ $(\text{Fringe width})_{5000} = (\text{Fringe width})_{6000} \times \frac{5}{6}$
 $= 0.239 \times \frac{5}{6}$
 $= 0.199 \text{ mm}$.

→ position of 0th order fringe ($\lambda = 5000 \text{ \AA}$) $= 12.34 \text{ mm}$.

→ position of 20th order fringe ($\lambda = 5000 \text{ \AA}$).

$= 12.34 \text{ mm} + (0.199 \text{ mm} \times 20) = 16.32 \text{ mm}$.

Part The coherent beams of wavelength $5000 \text{ \AA}$ reaching a point would individually produce intensities 1.44 and 4.00 units. if they reach there together, the intensity is 0.90 units. calculate the lowest phase difference with which the beams reach the point.

$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$.

Here $I_1 = 1.44$, $I_2 = 4.00$ and $I = 0.9$.

$0.9 = 1.44 + 4 + 2\sqrt{1.44 \times 4} \cos \phi$

$\cos \phi = -0.9458$

$\cos(180 - \phi) = 0.9458$

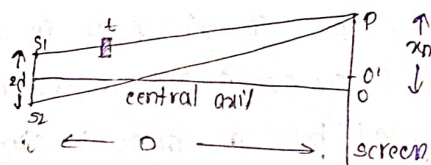
$180 - \phi = \cos^{-1}(0.9458)$

$\phi = 161^\circ$

Applications of Young's double slit interference

Displacement of fringes by thin lamina :-

When a thin transparent plate (glass or mica) is introduced in the path of one of the two interfering beams, the entire fringe-pattern is displaced to a point towards the beam in which the plate is introduced.



Let S_1 and S_2 are two monochromatic sources giving light of wavelength λ .

→ the thin plate of thickness t is introduced in the path of light from S_1 .

For a particular λ , the refractive index of the plate is μ .

Now light from S_1 travel partly in air and partly in plate.

For the light path from S_1P , the distance travelled in air is $S_1P - t$

if v_0 and v_1 are the velocities of light in air and in the plate respectively then the time taken by the light to travel from S_1

$$\begin{aligned} \text{to } P \text{ is } &= \frac{S_1P - t}{v_0} + \frac{t}{v_1} \\ &= \frac{S_1P - t}{v_0} + \frac{\mu t}{v_0} \quad \left[\mu = \frac{v_0}{v_1} \right] \\ &= \frac{S_1P + (\mu - 1)t}{v_0} \end{aligned}$$

Hence, the effective path in air from S_1 to P is $[S_1P + (\mu - 1)t]$, i.e. the air path S_1P is increased by an amount $(\mu - 1)t$ due to the introduction of the plate.

the introduction of the plate produces shifting in the fringe by an amount of x_0 .

$$x_0 = \frac{D}{2d} (\mu - 1)t$$

$$\left(t = \frac{x_0 (2d)}{D(\mu - 1)} \right)$$

The technique can be used to measure the thickness of a plate.

By introducing a thin sheet of mica ($12 \times 10^{-5} \text{ cm}$) in the path of one of the interfering beams in a biprism arrangement, the central fringe is shifted through a distance equal to the spacing between successive bright fringes. calculate the refractive index of mica ($\lambda = 5 \times 10^{-5} \text{ cm}$).

$$x_0 = \frac{FW (\mu - 1)t}{\lambda}, \quad FW = \text{Fringewidth}$$

$$(\mu - 1) = \frac{x_0 \lambda}{FW t}$$

$$x_0 = FW, \quad \lambda = 5 \times 10^{-5} \text{ cm}, \quad t = 12 \times 10^{-5} \text{ cm}$$

$$\mu = \frac{FW \cdot 5 \times 10^{-5}}{FW \cdot 12 \times 10^{-5}} + 1$$

$$\mu = 0.5 + 1 = 1.5$$

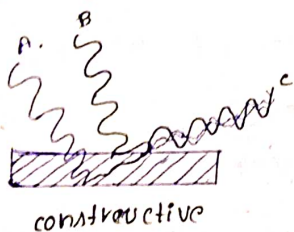
Interference involving multiple reflections:-

- 1) Interference of light by thin film.
- 2) Stroke's treatment.

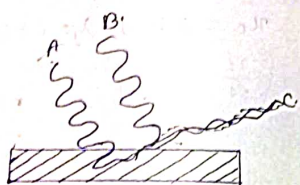
Interference of light by thin film:-

Interference occurs between the monochromatic light waves (reflected and transmitted), when a film of oil spread over the surface of water or a thin glass plate.

Interference occurs between the light waves reflected from the film and also between the light waves transmitted through the film.



constructive



destructive

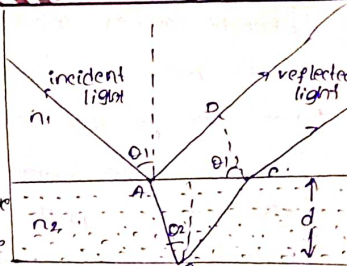
Thin film interference:-

- * In optics a thin film is a layer of material with thickness in the sub-nanometer to micrometer range.
- * As light strikes the surface of a film it is either transmitted or reflected at the upper surface. Light that is transmitted reaches the bottom surface and may once again be

transmitted or reflected.

- * The light reflected from the upper and lower surfaces will interfere

- * The degree of constructive or destructive interference between the two light waves depends on the difference in their phase.



- * This difference in phase turn depends on the thickness of the film layer, the refractive index of the film, and the angle of incidence of the original wave on the film.

- * Additionally, a phase shift of 180° (or π radians) may be introduced upon reflection at a boundary depending on the refractive indices of the materials on either side of the boundary.

- * This phase shift occurs if the refractive index of the medium the light is travelling through is less than the refractive index of the material it is striking.

- * If $n_1 < n_2$ and the light is travelling from n_1 to n_2 then phase shift occurs upon reflection.

- * The pattern of light that results from the interference can appear either as light and dark

bands or as colourful bands depending on the source of incident light.

* consider a thin film of incident and reflected light by both the upper and lower boundaries. the optical path difference (OPD) of the reflected light must be calculated in order to determine the condition of interference.

$$OPD = n_2 (AB + BC) - n_1 (AD)$$

where $AB = BC = \frac{d}{\cos(\theta_2)}$

$$OPD = n_2 (AB + BC) - n_1 (AD) \rightarrow (1)$$

where $AB = BC = \frac{d}{\cos(\theta_2)}$ $\rightarrow (2)$

And, $AD = \frac{AD}{\sin(\theta_1)} \rightarrow (3)$

$$AC = AB + BC$$

$$AC = BC = 2d \tan(\theta_2)$$

$$\therefore AC = 2d \tan(\theta_2)$$

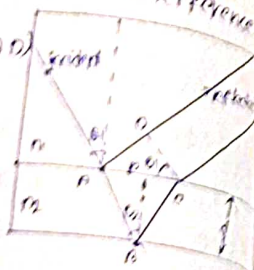
from (3) $\Rightarrow AD = AC \sin(\theta_1)$

$$AD = 2d \tan(\theta_2) \sin(\theta_1) = 2d \tan(\theta_2) \sin(\theta_1) \rightarrow (4)$$

from (1), (2) and (4)

$$OPD = n_2 \left[\frac{2d}{\cos(\theta_2)} \right] - n_1 (2d \tan(\theta_2) \sin(\theta_1)) \rightarrow (5)$$

from small angle $n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$



from (5)

$$OPD = n_2 \left[\frac{2d}{\cos(\theta_2)} \right] - n_1 [2d \tan(\theta_2) \sin(\theta_1)]$$

$$= 2n_2 d \frac{(1 - \sin^2(\theta_2))}{\cos(\theta_2)}$$

$$= 2n_2 d \cos(\theta_2)$$

$$OPD = 2n_2 d \cos(\theta_2)$$

interference will be constructive if the optical path difference is equal to an integer multiple of the wavelength of light.

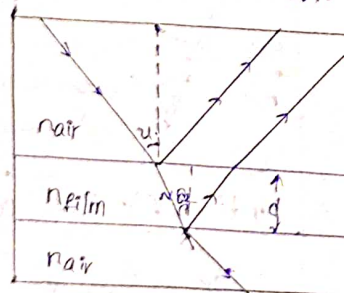
$$OPD = 2n_2 d \cos(\theta_2)$$

$$OPD = m\lambda$$

this condition may change after considering possible phase shifts that occur upon reflection. examples:-

1) soap bubbles:-

light travels through air and strikes a soap film. the air has a refractive index of 1



($n_{air} = 1$) and the film has an index $n_{film} > 1$.

the reflection occurs at the upper boundary of the film (the air film boundary) will introduce a

180° phase shift in the reflected wave because the refractive index of air is less than the index of the film ($n_{\text{film}} > 1$).

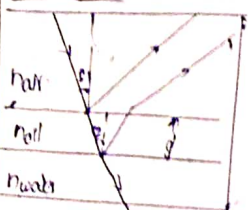
- light that is transmitted at the upper air-film interface will continue to the lower film-air interface where it can be reflected or transmitted.
- the reflection that occurs at this boundary will not change the phase of reflected wave because $n_{\text{film}} > 1$.

condition of interference for soap bubbles:-

$$2n_{\text{film}} d \cos \theta = (m - \frac{1}{2}) \lambda, \quad m = 0, 1, 2 \quad (\text{constructive})$$

$$2n_{\text{film}} d \cos \theta = m \lambda \quad (\text{destructive})$$

2) oil film:-



$$n_{\text{air}} < n_{\text{oil}} < n_{\text{glass}}$$

$$2n_{\text{film}} d \cos \theta = (m - \frac{1}{2}) \lambda \quad (\text{constructive})$$

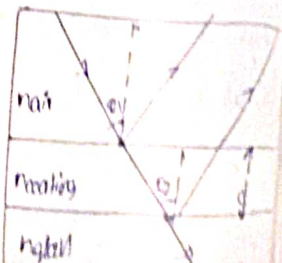
$$2n_{\text{film}} d \cos \theta = m \lambda \quad (\text{destructive})$$

3) anti-reflection coating:-

$$n_{\text{air}} < n_{\text{coating}} < n_{\text{glass}}$$

$$2n_{\text{film}} d \cos \theta = (m - \frac{1}{2}) \lambda \quad (\text{destructive})$$

$$2n_{\text{film}} d \cos \theta = m \lambda \quad (\text{constructive})$$



Formation of Newton's rings:-

When a plano-convex lens of large radius of curvature is placed with its convex surface in contact with a plane glass plate, an air film is formed between the lower surface of the lens and the upper surface of glass plate.



The thickness of the film gradually increases from the point of contact outwards.

If monochromatic light is allowed to fall normally on this film, a system of alternate bright and dark concentric rings with their centre dark is formed in the air film.



These are called Newton rings.

Newton rings are formed as a result of interference between the light waves reflected from the upper and lower surfaces of air film.

∴ The effective path difference between two rays is

$$p = 2\mu t \cos \theta + \frac{\lambda}{2}$$

where μ = refractive index of film (air)

t = thickness of film at point O

θ = inclination of the ray.

Factor $\frac{\lambda}{2}$ accounts for phase change of π on reflection at the lower surface of film.

for air-film ($\mu=1$) and normal incidence of light ($\theta=0$) then $\boxed{p = 2t + \frac{\lambda}{2}}$

→ The effective path difference between two rays of air-film ($\mu=1$) and normal of incidence of light ($\theta=0$) then $p = 2t + \frac{\lambda}{2}$.

* The condition for max intensity (bright fringe);

$$p = n\lambda \quad (n=0, 1, 2, \dots)$$

$$2t + \frac{\lambda}{2} = n\lambda$$

$$2t = (2n-1) \frac{\lambda}{2} \quad (\text{maximum})$$

* The condition for min intensity (dark fringe)

$$p = (2n+1) \frac{\lambda}{2}$$

$$2t + \frac{\lambda}{2} = (2n+1) \frac{\lambda}{2}$$

$$2t = n\lambda$$

→ for $t=0$ at the point of contact of the lens and the plate $\boxed{p = \frac{\lambda}{2}}$

* This is the condition for minimum intensity and hence the central spot is dark.

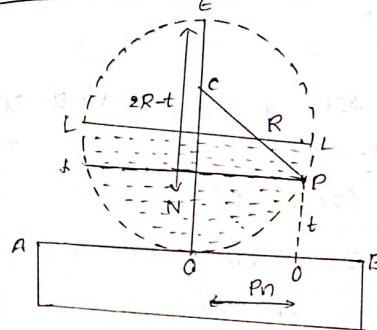
* it is clear that a bright or dark fringe of any particular order n will occur for a

constant value of t .

→ since in the air film " t " remains constant along a circle with its centre at the point of contact, the fringes are in the form of concentric circles.

→ since each fringe is the locus of constant film thickness, these are called "fringes of constant thickness".

Diameters of Bright Rings:



let LOL be the lens placed on the glass plate AB , the point of contact being O .

→ let R be the radius of curvature of the curved surface of the lens.

→ let R_n be the radius of a Newton's ring corresponding to a point P , where film thickness is t .

Draw a perpendicular PN.

According to the property of the circle

$$PN^2 = ON \times NE$$

$$\text{or, } P_n^2 = t \times (2R - t)$$

$$= 2Rt - t^2$$

$$\text{OPD} = 2\mu t$$

$$\mu = \text{air refractive index}$$

$$\mu = 1$$

$$\text{OPD} = 2t$$

Since t is small compared to R , we can neglect t^2 .

$$P_n^2 = t \times (2R - t) = 2Rt$$

$$2t = \frac{P_n^2}{R}$$

The condition for a bright ring is

$$2t = (2n-1)\lambda/2$$

$$P_n^2/R = (2n-1)\lambda/2$$

$$\boxed{P_n^2 = \frac{(2n-1)\lambda R}{2}}$$

if D_n be the diameter of the n^{th} bright ring, then $D_n = 2P_n$.

$$D_n^2 = 2(2n-1)\lambda R$$

$$D_n^2 = (2\lambda R)^{1/2} (2n-1)^{1/2}$$

$$D_n \propto (2n-1)^{1/2}$$

As n is an integer, $(2n-1)$ is odd number. Thus the diameter of the bright rings are proportional to square root of odd natural numbers.

The diameter of the n^{th} bright ring is

$$D_n = \sqrt{2\lambda R (2n-1)}$$

The separation b/w successive rings are

i.e. the separation decreases as the order is increased.

Diameter of dark rings:-

The condition for a dark ring is

$$2t = n\lambda$$

$$P_n^2/R = n\lambda$$

if D_n be the diameter of the n^{th} bright ring, then $P_n = D_n/2$.

$$D_n^2/4R = n\lambda$$

$$D_n = (4n\lambda R)^{1/2}$$

$$D_n = (4\lambda R)^{1/2} (n)^{1/2}$$

$$D_n \propto n^{1/2}$$

Thus the diameter of the dark fringes are proportional to the square root of natural numbers.

Applications:-

You can use them to check the thickness of surface, and to check whether a surface is uniform.

Q. calculate the thickness of film ($\mu = 1.4$) in which interference of violet component ($\lambda = 4000 \text{ \AA}$) of incident light can take place by reflection.

The condition for constructive interference of light reflected from a film of thickness t ,

$$2\mu t \cos \theta = (2n+1) \lambda / 2 \quad n=0, 1, 2, \dots$$

For normal incident $\theta=0$ and for minimum thickness n should be 0.

$$2\mu t = \lambda / 2$$

$$t = \frac{\lambda}{2} \times \frac{1}{2\mu}$$

$$t = \frac{4000}{2} \times \frac{1}{2 \times 1.4}$$

$$t = 714.28 \text{ \AA}$$

Q. A thin soap film ($\mu = 1.33$) seen by sodium light ($\lambda = 5893 \text{ \AA}$) by normal reflection appears dark. Find the minimum thickness of film.

For the condition for destructive interference of light reflected from the a film of thickness t is $2\mu t \cos \theta = n\lambda$.

For normal incident $\theta=0$ and for minimum thickness n should be 1.

$$t = \frac{\lambda}{2\mu}$$

$$t = \frac{5893}{2 \times 1.33}$$

$$t = 2215.41 \text{ \AA}$$

Newton's rings are observed normally in reflection light of wavelength $5.9 \times 10^{-5} \text{ cm}$. The diameter of the 10th dark ring is 0.5 cm . Find the curvature of the lens and the thickness of the film.

The diameter of the n^{th} dark ring is

$$D_n^2 = 4n\lambda R$$

after putting the values $D_n = 0.5 \text{ cm}$, $n = 10$ and $\lambda = 5.9 \times 10^{-5} \text{ cm}$, $R = 106 \text{ cm}$

if t is the thickness of the film corresponding to a ring of diameter D , then

$$2t = D^2 / 4R$$

$$t = \frac{D^2}{4R} \times \frac{1}{2}$$

$$t = \frac{(0.5)^2}{8(106)}$$

$$t = 3 \times 10^{-4} \text{ cm}$$

Q. A convex lens of radius 350 cm placed on flat plate and illuminated by monochromatic light. The 10th bright ring is diameter

0.58 cm. calculate the wavelength of light.

The diameter of the n^{th} bright fringe is

$$D_n^2 = 2(2n-1)\lambda R, \quad n = 1, 2, 3, \dots$$

$$R = 350 \text{ cm}, \quad n = 6, \quad D_n = 0.58 \text{ cm.}$$

$$\lambda = 6.0 \times 10^{-5} \text{ cm.}$$

$$\lambda = 6000 \text{ \AA.}$$

Find the ratio of the intensity at the centre of a bright fringe in an interference pattern to the intensity at a point one-quarter of the distance between two fringes from the centre.

The resultant intensity at a point due to two coherent waves of amplitude A_1 and A_2 reaching the point with a difference ϕ is given by.

$$I = A_1^2 + A_2^2 + 2A_1A_2 \cos \phi$$

If A be the amplitude of each of two interfering waves, then $I = 2A^2(1 + \cos \phi)$.

At the centre of a bright fringe, $\phi = 0, 2\pi, \dots$

$$\text{So that } I_0 = 2A^2(1+1) = 4A^2.$$

The phase difference b/w successive fringes is 2π . The phase difference at a point distance one quarter of the distance b/w two fringes from centre will be $\frac{\pi}{2}$.

$$I_1 = 2A^2(1 + \cos 90^\circ) = 2A^2$$

$$\frac{I_0}{I_1} = \frac{4A^2}{2A^2} = 2$$

Standard Inference:-

order of the fringes:-

when the central fringe is dark the order of fringe is $m = \frac{2d}{\lambda}$.

→ As d increased new fringes appear at the centre and existing fringes move outwards and finally move out of the field of view.

→ For any value of d , the central fringe has the largest value of m .

DIFFRACTION

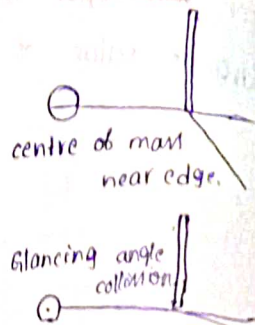
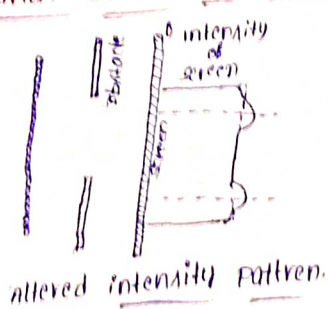
The phenomenon of bending of light from its straight line path around the corners of an obstacle or slit is known as DIFFRACTION.

→ Joseph von Fraunhofer - German optician.

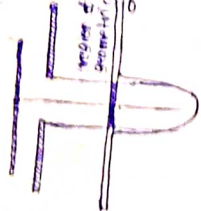
1814 - invented SPECTROSCOPE

1821 - developed DIFFRACTION GRATING.

What about the ones hitting the edge:



→ This is not one peak but a broad diffuse one as the way the balls hit the barrier edge will send them off in different angles.



There is "intensity" in the geometrical shadow region as well.

→ so we have seen that even with macroscopic balls, it is possible to get "intensity" in the region of geometrical shadow.

→ For this effect to be prominent we have noted that the slit width has to be of the order of the size of the ball.

diffraction of light:

When a narrow opaque obstacle (aperture) is placed between a source of the light and a screen, light bends around the corners of the aperture. This encroachment of light is called "diffraction".

→ For diffraction, the size of aperture is important (comparable to the wavelength).

→ As a result of diffraction, the edges of the shadow are not sharp, but the intensity is distributed in a certain way depending upon the nature of the aperture.

difference between interference and diffraction:

Interference:

It occurs between waves starting from two (or more, but finite in number) coherent sources.

It occurs between the secondary wavelets starting from the different points (infinite in number) of the same wave.

* both are superposition effects and often are present simultaneously (like young's experiment comparison)

- in an interference pattern the minima are usually almost perfectly dark while in a diffraction pattern they are not so.
- in an interference pattern all the maxima are of same intensity but not in the diffraction pattern.
- the interference fringes are usually (although not always) equally spaced. The diffraction never equally spaced.

Types of diffraction:

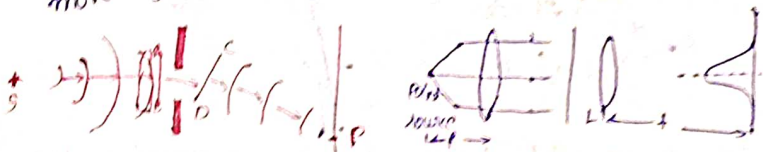
- Fresnel diffraction.
- Fraunhofer diffraction.

Important aspects of Fraunhofer diffraction:

- Distance of slit from source and screen is infinite.
- Wavefront incident on slit and screen is plane.
- Lenses are required to observe diffraction pattern.

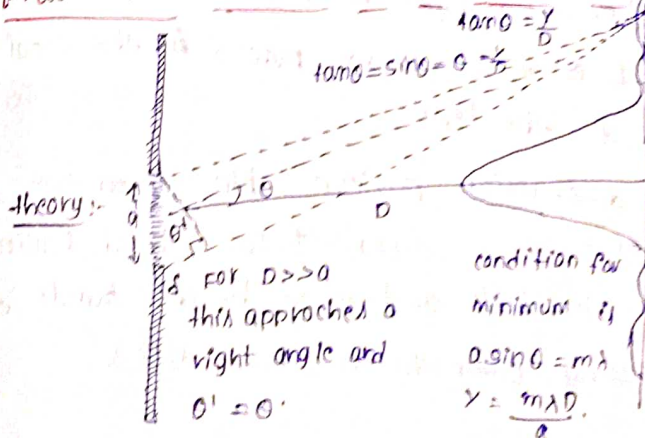
Fraunhofer's diffraction:

- In this case, the source of light and screen are effectively at infinite distance from the diffracting aperture (obstacle).
- This is achieved by placing the source and screen in the focal planes of two lenses.
- The incident wavefront is plane.
- Fraunhofer diffraction is a limiting case of the more general Fresnel's diffraction.

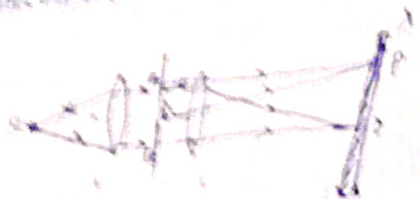


- Much easier to calculate the intensity distribution of a Fraunhofer diffraction.

Fraunhofer diffraction at a single slit:



slit of width a (comparable to wavelength λ)
 is divided into n equal parts of width $\frac{a}{n}$.
 all points on it behave as point source emitting secondary waves in all directions and we get a diffraction pattern of alternate maxima and minima of decreasing intensities.



Let a parallel beam of monochromatic light of wavelength λ be incident normally upon a narrow slit of width $AB = a$.
 Let diffracted light be focused by a convex lens L on a screen XX' placed in the focal plane of the lens.

The diffraction pattern obtained on the screen consists of a central bright band having alternate dark and weak bright bands of decreasing intensity on both sides.

A plane wavefront is incident on the slit AB according to the Huygens principle,

each point in AB sends out secondary wavelets in all directions.

The rays proceeding in the same direction as the incident rays focused at O , while those diffracted through an angle θ focused at P .

Let AK be perpendicular to BK . As the optical paths from the plane AK to P are equal

$\therefore$ path difference between the wavelets from A to B in the direction at the angle

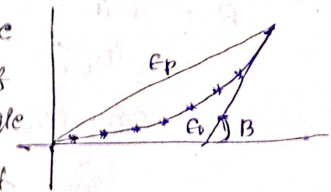
$$D = BK = AB \sin \theta = a \sin \theta.$$

$$\text{phase difference} = \left(\frac{2\pi}{\lambda}\right) a \sin \theta.$$

Let the width AB of the slit be divided into n equal parts. The amplitude of vibration at P due to the waves from each other part will be the same ($=a$).

quantitative intensity in single slit pattern

The θ is the phase angle of the ray from the top of the slit, while the phase angle from the bottom of the slit

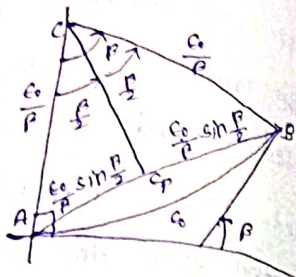


the vectors lie along whole center is at C, so E_p is chord of a circle. The arc length E_0 is subtended by this same angle β , so the radius of the circle is E_0/β .

From the diagram,

$$E_p = 2 \frac{E_0}{\beta} \sin \frac{\beta}{2}$$

$$E_p = E_0 \frac{\sin \beta/2}{\beta/2}$$



$$\therefore \beta = \frac{2\pi}{\lambda} a \sin \theta, \quad I = I_0 \left[\frac{\sin [\pi a (\sin \theta) / \lambda]}{\pi a (\sin \theta) / \lambda} \right]^2$$

The phase difference between the waves from any two consecutive parts is $\frac{1}{n} \left[\frac{2\pi}{\lambda} a \sin \theta \right]$

Hence the resultant amplitude at P is given by

$$R = \frac{a \sin \left(\frac{n\delta}{2} \right)}{\sin \left(\frac{\delta}{2} \right)} = \frac{a \sin \left(\frac{\pi a \sin \theta}{\lambda} \right)}{\sin \left(\frac{\pi a \sin \theta}{n\lambda} \right)}$$

let $\frac{\pi a \sin \theta}{\lambda} = \alpha$, then

$$R = \frac{a \sin \alpha}{\sin \left(\frac{\alpha}{n} \right)} = \frac{a \sin \alpha}{\alpha/n} \quad (\text{as } \frac{\alpha}{n} \text{ is small})$$

$$R = \frac{n a \sin \alpha}{\alpha}$$

As $n \rightarrow \infty$, $a \rightarrow 0$, but product is finite.

$$\text{let } na = A, \text{ then } R = \frac{A \sin \alpha}{\alpha}$$

the resultant intensity at P is

$$I = R^2 = A^2 \left[\frac{\sin \alpha}{\alpha} \right]^2$$

direction of minima:-

the intensity is minimum (zero) when $\frac{\sin \alpha}{\alpha} = 0$

(or) $\sin \alpha = 0$ (but $\alpha \neq 0$, i.e. for $\alpha = 0$, then $\frac{\sin \alpha}{\alpha} = 1$)

$\alpha = \pm m\pi$ where m is an integral value $m = 1, 2, 3, \dots$

$$\text{But } \alpha = \frac{\pi a \sin \theta}{\lambda}$$

$$\frac{\pi a \sin \theta}{\lambda} = \pm m\pi$$

$$[a \sin \theta = \pm m\lambda] \quad m = 1, 2, 3, \dots$$

direction of maxima:-

let us differentiate the intensity with respect to α and equate it to zero.

$$\frac{dI}{d\alpha} = 0$$

$$A^2 \left(\frac{2 \sin \alpha}{\alpha} \right) \frac{a \cos \alpha - \sin \alpha}{\alpha^2} = 0$$

$$a \cos \alpha - \sin \alpha = 0$$

$$[a = \tan \alpha]$$

The equation is graphically by plotting the curve

$$y = \alpha$$

$$I_p = I_0 \left(\frac{\sin \alpha}{\alpha} \right)^2$$

at $\alpha = 0$, $I_p = I_0$

- * The 1st equation is a straight line through origin making an angle of 45° .
- * The 2nd is a discontinuous curve having a number of branches.

* The point of intersection of the two curves gives

$$\alpha = 0, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \dots$$

- * The intensity of the 1st maximum is about 4.96% of the central maximum.

$$I = I_0 \left[\frac{\sin \alpha}{\alpha} \right]^2$$

$$\text{Put } \alpha = \frac{3\pi}{2}$$

$$I_1 = \frac{4}{9\pi^2} I_0, \quad I_2 = \frac{4}{25\pi^2} I_0, \quad I_3 = \frac{4}{49\pi^2} I_0$$

ratio of relative intensities of successive maxima

$$\text{are } I : \frac{4}{9\pi^2} : \frac{4}{25\pi^2} : \frac{4}{49\pi^2} : \dots$$



Fresnel diffraction:-

In this diffraction the source and screen are separated at finite distance.

→ To study this diffraction lenses are not used because the source and screen separated at finite distance.

→ This diffraction can be studied in the direction of propagation of light.

→ In this diffraction the incidence wave front must be spherical or cylindrical.

Diffraction effect:-

The diffraction pattern consists of central principal maxima for $\alpha = 0$ value. The secondary maxima of decreasing intensity are present on either side of central maxima for $\alpha = \pm \frac{3\pi}{2}, \pm \frac{5\pi}{2}, \pm \frac{7\pi}{2}, \dots$ values.

→ Between the secondary maxima and minima values are present for $\alpha = \pm \pi, \pm 2\pi, \pm \dots$ values.

Interference effect:-

In interference pattern the variation of $\cos^2 \beta$ with respect to β .

Interference maximum will occur for $\cos^2 \beta = 1$.

$$\beta = \pm m\pi \quad \text{where } m=0, 1, 2, 3, 4, \dots$$

$$\beta = \pm \pi, \pm 2\pi, \pm 3\pi, \pm 4\pi, \dots$$

$$= \frac{\pi(e+d) \sin \theta}{\lambda} = \pm m\pi$$

$$(e+d) \sin \theta = m\lambda$$

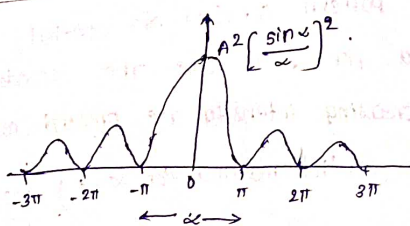
Interference minima will occur for $\cos^2 \beta = 0$.

$$\beta = \pm (2m+1) \frac{\pi}{2}$$

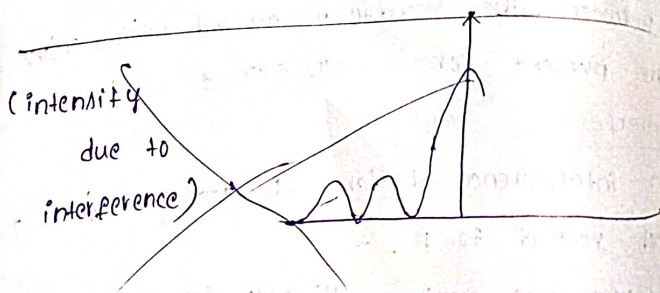
$$\frac{\pi(e+d) \sin \theta}{\lambda} = \pm (2m+1) \frac{\pi}{2}$$

$$(e+d) \sin \theta = \pm (2m+1) \frac{\lambda}{2}$$

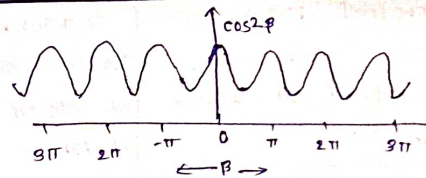
Intensity distribution:-



(Intensity due to diffraction)



Intensity due to interference).



(Intensity of both effects)

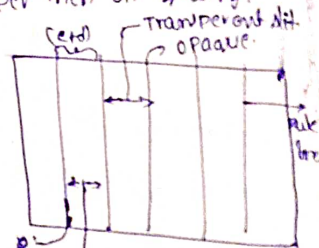


Diffraction grating:-

A set of large number of parallel slits of same width and separated by opaque spaces is known as "diffraction grating". Fraunhofer used the first grating consisting of a large number of parallel wires placed side by side very closely at regular separation.

Let 'e' be width of line and 'd' be the width of the slit. Then (e+d) is known as grating element. N is number of lines per inch on grating. then $N(e+d) = 1 \text{ inch} = 2.54 \text{ cm}$

$$(e+d) = \frac{2.54 \text{ cm}}{N}$$



Commercial gratings are produced by taking the coat of actual grating on translucent film like that of cellulose acetate.

Fresnel diffraction

1) Source and screen are placed at finite distances.

2) No lenses are used

3) The incident wavefront is spherical or cylindrical

4) it can be studied in the direction of propagation of light

Fraunhofer Diffraction

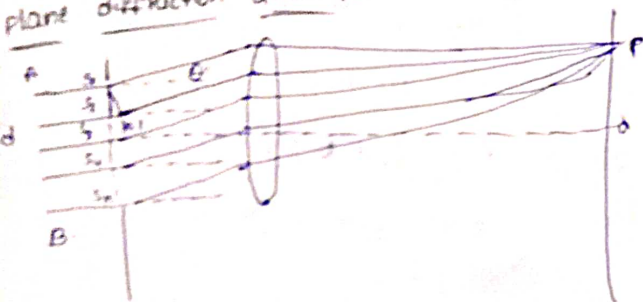
1) source and source are at placed at infinite distances.

lenses are used

The incident wave front is plane wave front.

it can be studied in any of propagation of light.

plane diffraction Grating:-



The amplitude from each slit in the direction θ is $R_0 = \frac{A \sin \alpha}{\alpha}$ where $\alpha = \frac{\pi}{\lambda} \sin \theta$

The path difference between the wavelets from S_1 and S_2 the direction θ is $S_2 k_1 = (e+d) \sin \theta$.

Hence the phase difference between them $\frac{2\pi}{\lambda} (e+d) \sin \theta = 2\beta$

if N be the total number of slits in grating, then by the method of vector addition of amplitude the resultant amplitude in the direction of θ will be.

$$R = R_0 \frac{\sin N\beta}{\sin \beta} = \frac{A \sin \alpha}{\alpha} \frac{\sin N\beta}{\sin \beta}$$

hence the resultant intensity at point P is

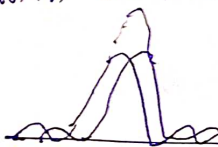
$$I = R^2 = A^2 \left[\frac{\sin \alpha}{\alpha} \right]^2 \left[\frac{\sin N\beta}{\sin \beta} \right]^2$$

$\left[\frac{\sin \alpha}{\alpha} \right]^2$ gives the intensity distribution due to single slit. While $\left[\frac{\sin N\beta}{\sin \beta} \right]^2$ gives the distribution of intensity in the diffraction pattern due to the interference in the wave due to N slits.

Rayleigh criterion for resolving power:-

Resolving power:-

An optical instrument used to resolve the two closely spaced objects is called Resolving Power. According to Rayleigh, two closely spaced points are said to be just resolved if the principal maxima of one coincides with the first minima of other or vice versa.



Resolution example

- 1) Pluto and its moon, Charon
- 2) earth-based telescope is blurred
- 3) Hubble space telescope clearly resolves the two objects.

Dispersive power:

Dispersive power is the property of a material, irrespective of its shape. It is defined as the ratio of angular deviation to the mean deviation of the material.

$$\omega = \frac{n_F - n_C}{n_D - 1}$$

C - line of hydrogen with $\lambda = 656 \text{ nm}$

D - line of sodium with $\lambda = 589 \text{ nm}$

F - line of hydrogen with $\lambda = 486 \text{ nm}$

C, D, and F lines as red, yellow, blue,