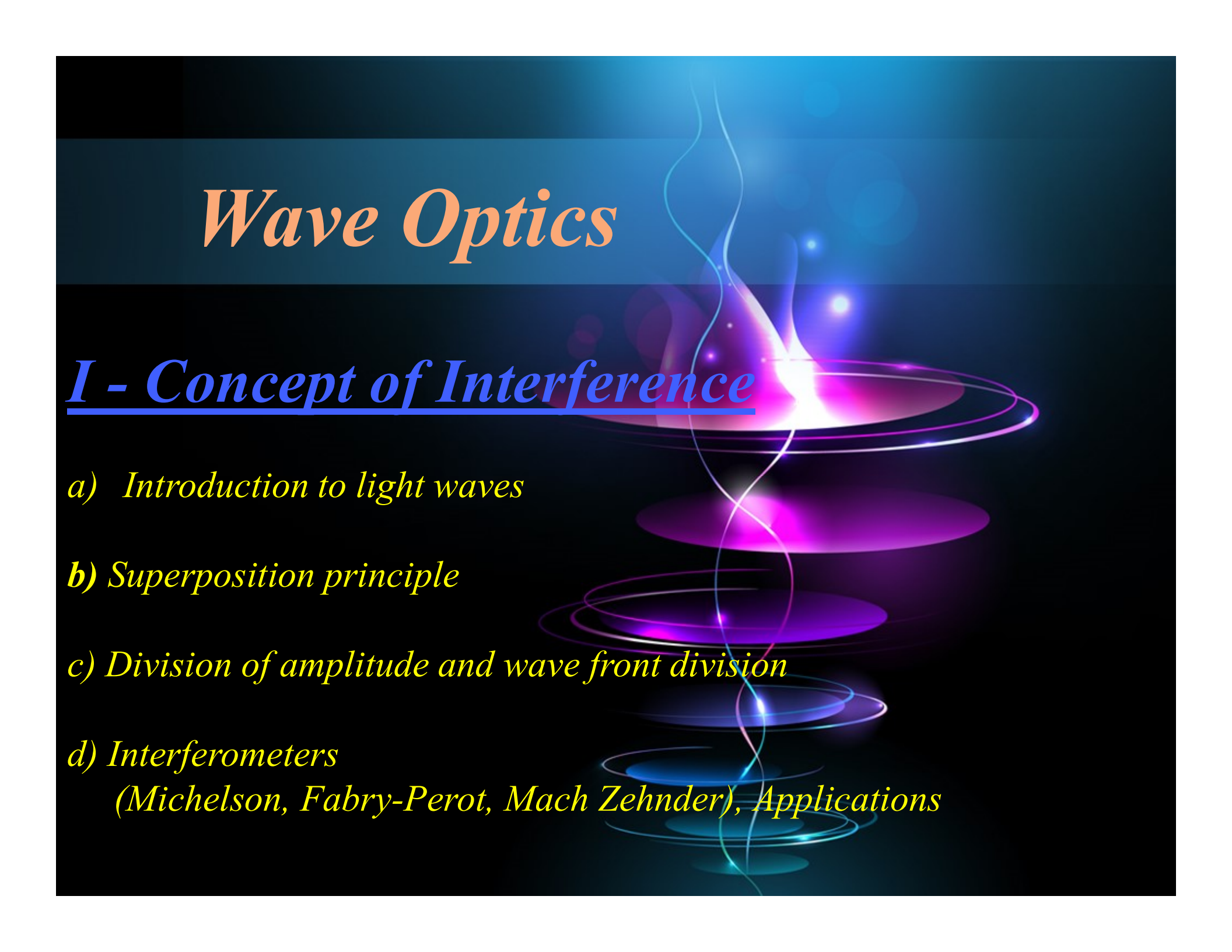


Wave Optics



I - Concept of Interference

- a) Introduction to light waves*
- b) Superposition principle*
- c) Division of amplitude and wave front division*
- d) Interferometers
(Michelson, Fabry-Perot, Mach Zehnder), Applications*

Wave equation in stretching strings

Evidently, small disturbances on the string satisfy

$$\boxed{\frac{\partial^2 f}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2}}, \quad (9.2)$$

where v (which, as we'll soon see, represents the speed of propagation) is

$$v = \sqrt{\frac{T}{\mu}}. \quad (9.3)$$

Equation 9.2 is known as the (classical) **wave equation**, because it admits as solutions all functions of the form

$$f(z, t) = g(z - vt), \quad (9.4)$$

Maxwell's Equations

Describe EVERYTHING in Electromagnetism

$$\oint \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0}$$

$$\oint \vec{B} \cdot d\vec{A} = 0$$

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$



Electromagnetic Waves

- From Maxwell's equations applied to empty space, the following relationships can be found:

$$\frac{\partial^2 E}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} \qquad \frac{\partial^2 B}{\partial x^2} = \mu_0 \epsilon_0 \frac{\partial^2 B}{\partial t^2}$$

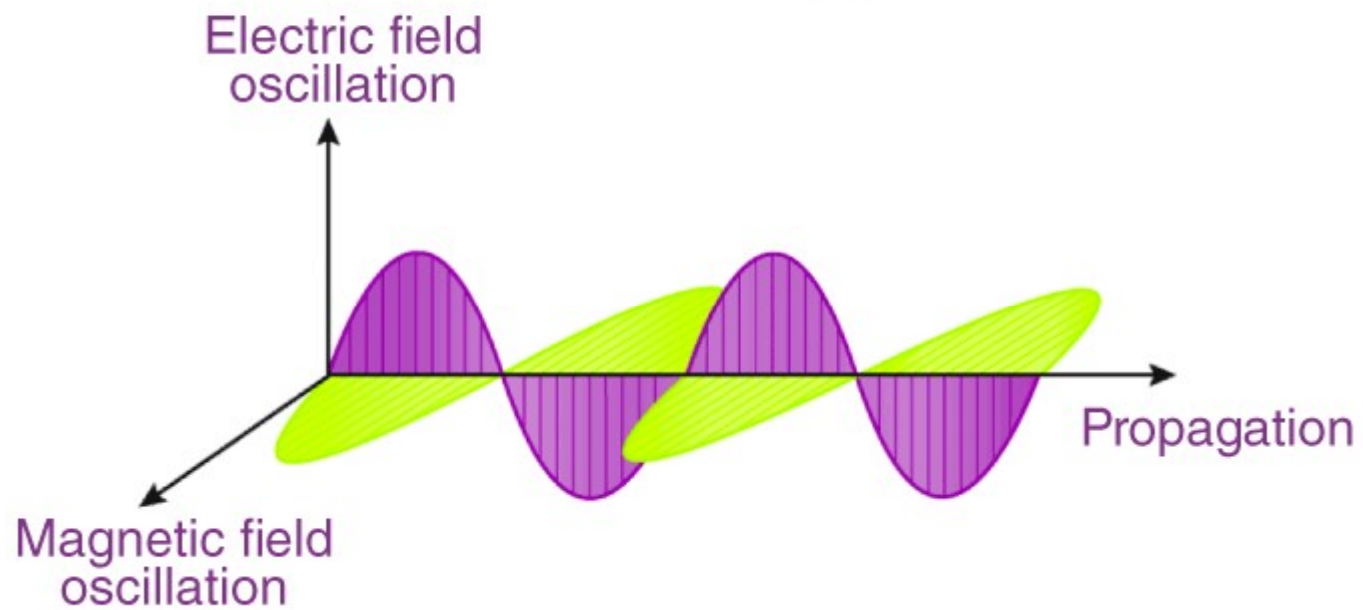
- The simplest solutions to these partial differential equations are sinusoidal waves – **electromagnetic waves**:

$$E = E_{\max} \cos(kx - \omega t); B = B_{\max} \cos(kx - \omega t)$$

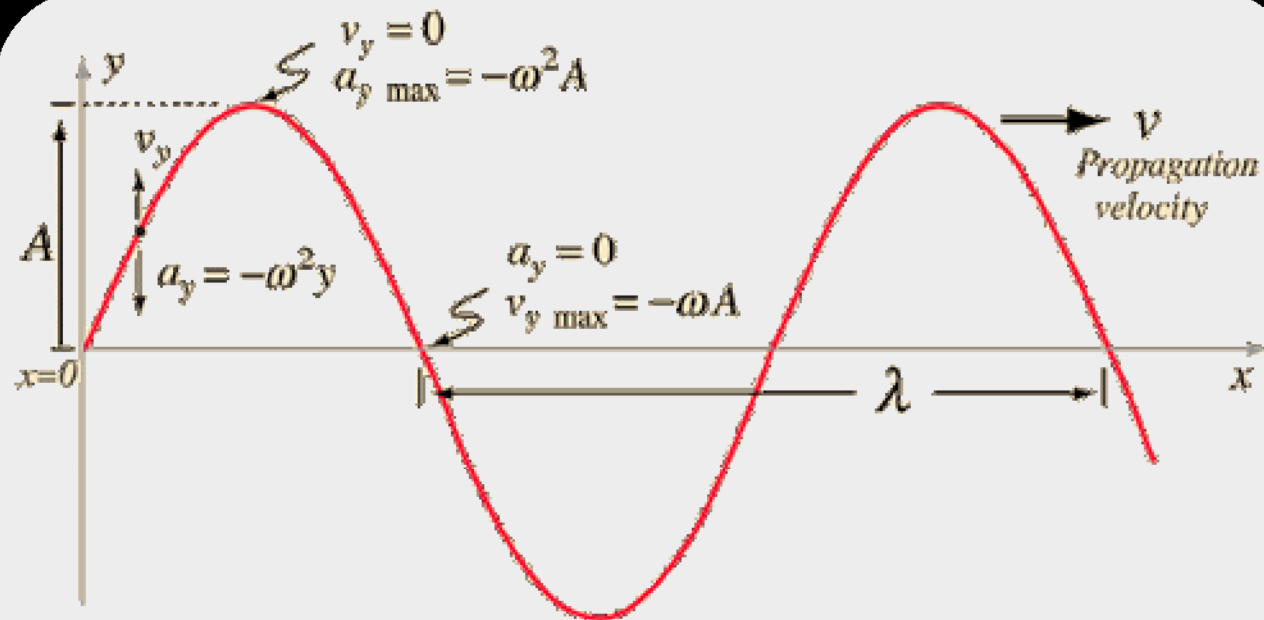
- The speed of the electromagnetic wave is:

$$v = \frac{\omega}{k} = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \equiv c = 2.99792 \times 10^8 \text{ m/s} = \frac{E}{B} = \frac{E_{\max}}{B_{\max}}$$

ELECTROMAGNETIC WAVES



The Wave?



*Description of
the transverse
motion.*

$$\frac{2\pi v}{\lambda} = 2\pi f = \omega$$

$$v = f\lambda$$

$$y(x,t) = A \sin \frac{2\pi}{\lambda}(x - vt)$$

$$v_y(x,t) = \frac{dy}{dt} = \omega A \cos \frac{2\pi}{\lambda}(x - vt)$$

$$a_y(x,t) = \frac{d^2y}{dt^2} = -\omega^2 y = -\omega^2 A \sin \frac{2\pi}{\lambda}(x - vt)$$

Light Wave

$$C = \lambda \times f$$

Where,

C = velocity of light (3×10^8 m/s)

λ = wavelength of light

f = frequency of light

$$E = h \times f$$

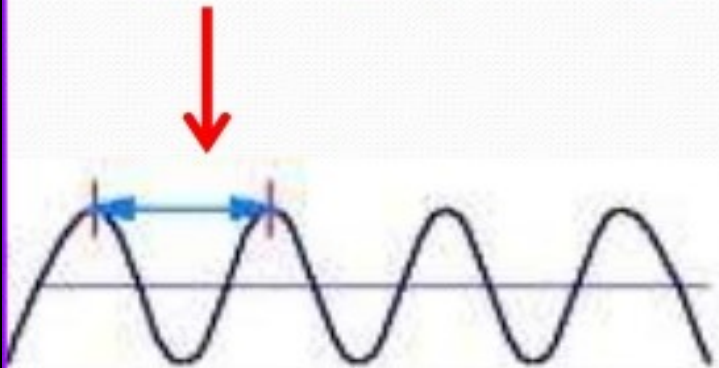
Where,

E = light energy

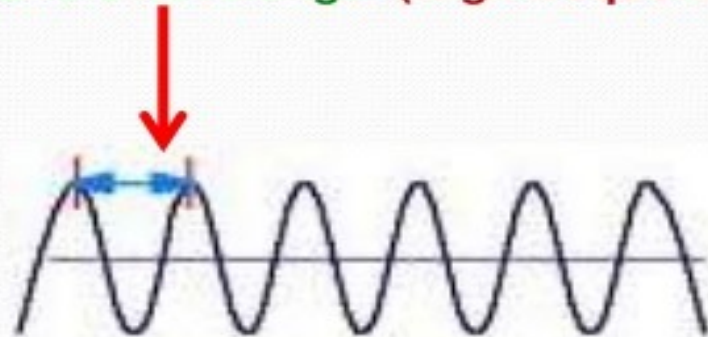
h = Planck's constant = 6.626×10^{-34} J.s

f = frequency of light

Long wavelength (Low frequency)

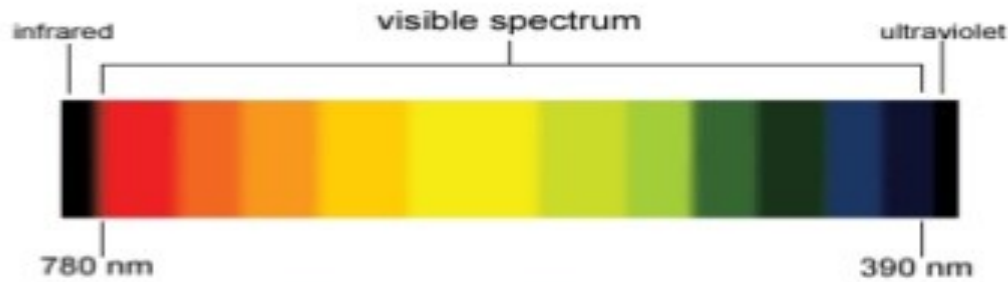


Short wavelength (High frequency)

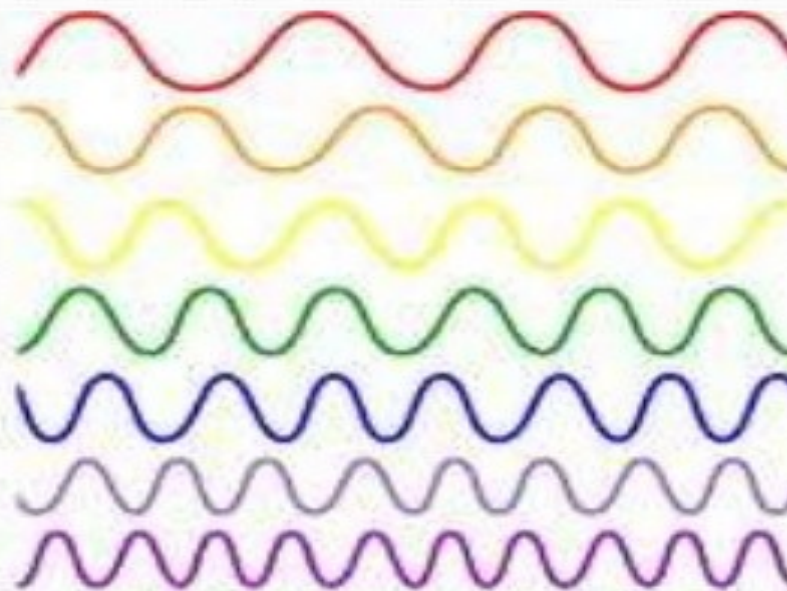


Lights of different color

The Visible Light Spectrum



RED
ORANGE
YELLOW
GREEN
BLUE
INDIGO
VIOLET



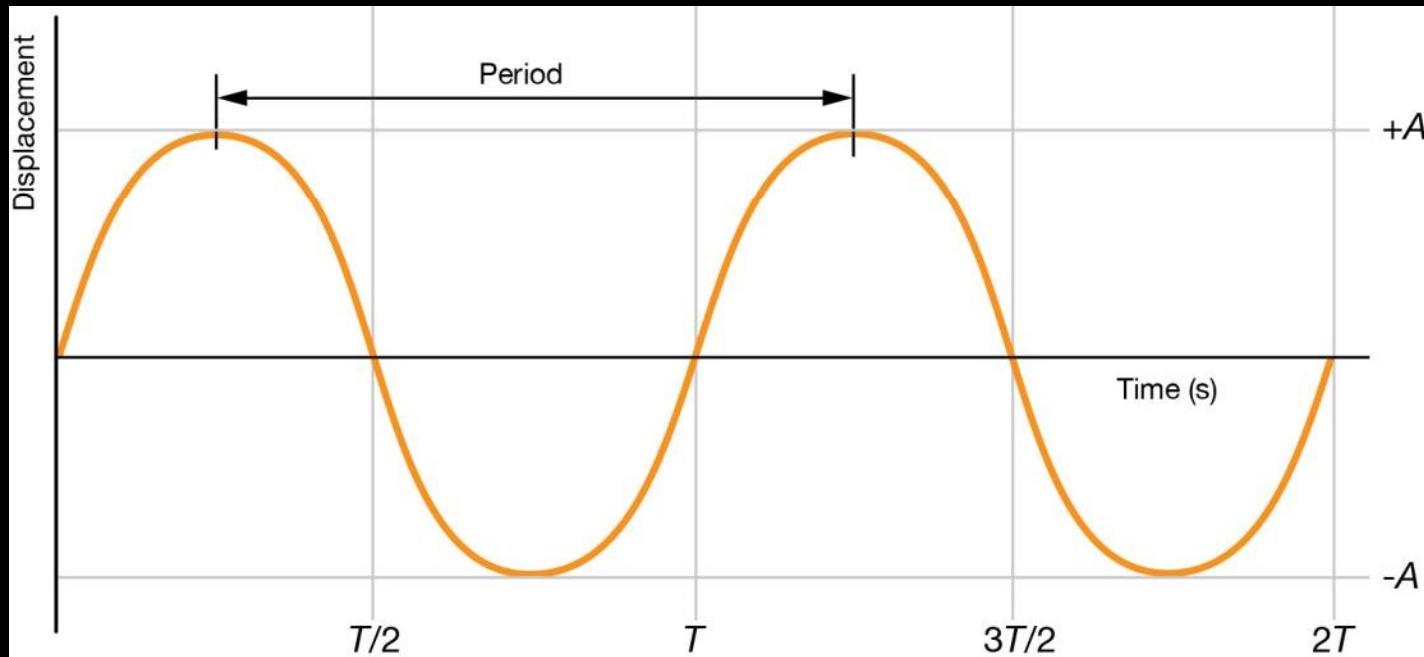
Long wavelength
(Low frequency)

Short wavelength
(High frequency)

Violet light has higher energy

Displacement vs time

at a specific point in space (for a specific particle).



Superposition Principle

When two or more waves overlap, the resultant wave is the algebraic sum of the individual waves.

PRINCIPLE OF SUPERPOSITION

Terminology

- *Phase Difference:*

The fractional part of a period which represents the offset in peak positions of waves.

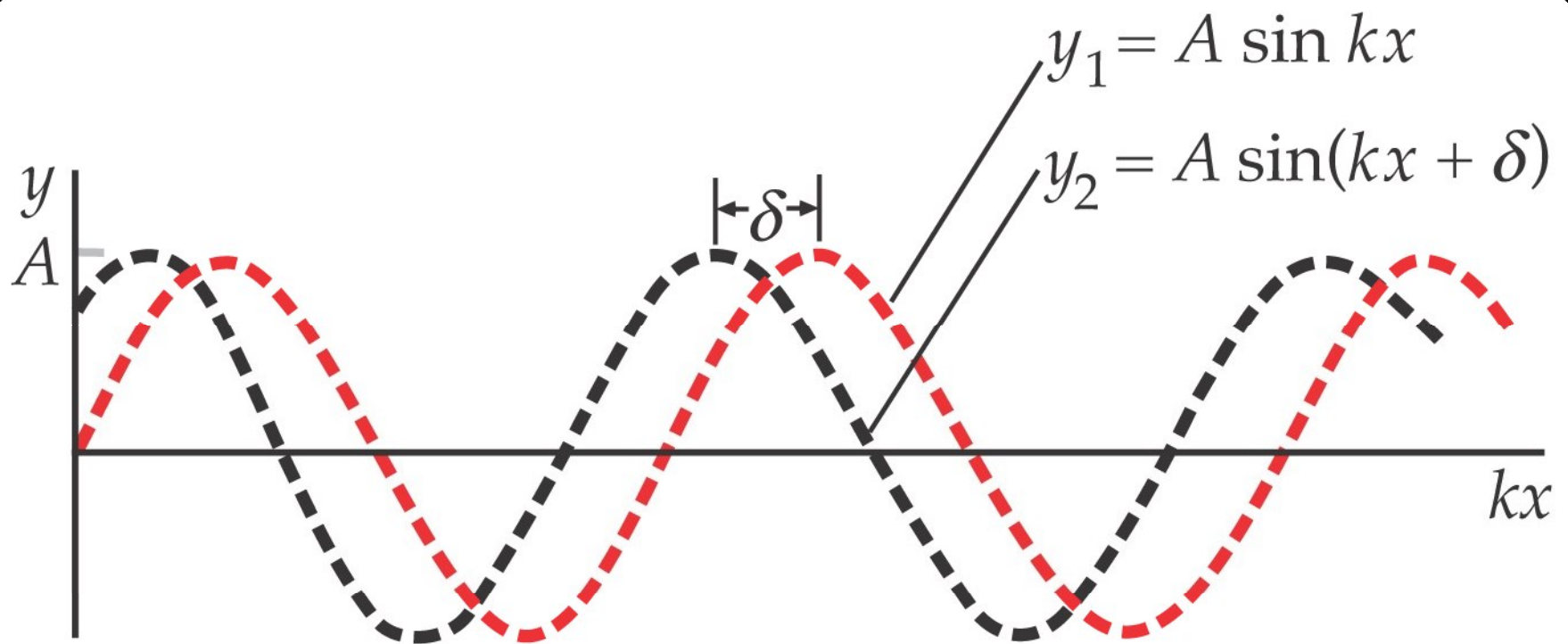
- *Coherence:*

The existence of a correlation between the phases of two or more waves such that interference effects can be observed.

- *Interference:*

The variation with distance or time of the amplitude of a wave which results from the superposition of two or more waves.

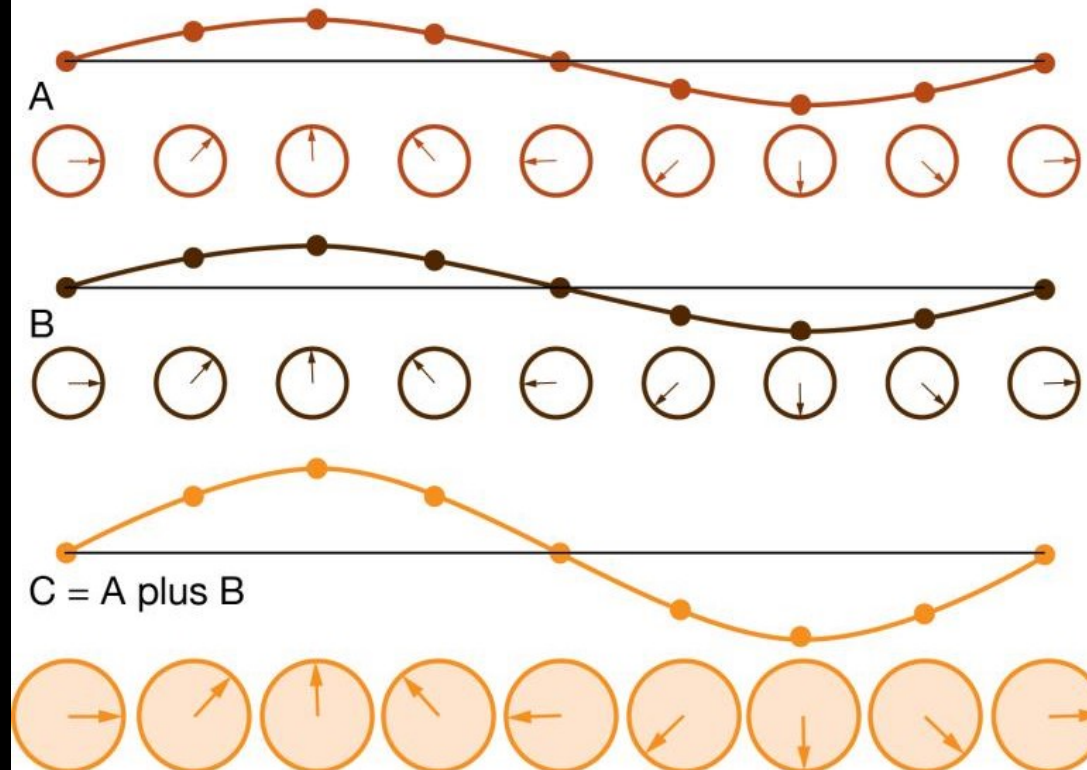
Phase Difference



In phase of two waves

Superposition and phase difference

Oscillations in phase



Rotating arrows add up:



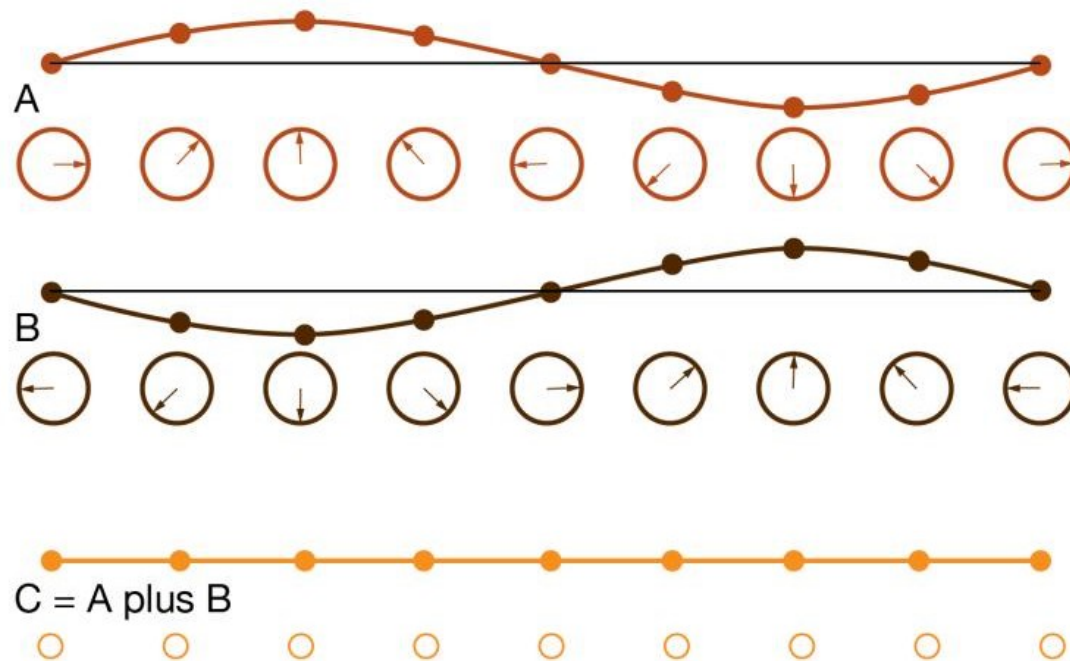
arrows add tip to tail 

If phase difference = 0
then amplitude of resultant
= sum of amplitudes
of components

Out of Phase

Superposition and phase difference

Oscillations in antiphase



Rotating arrows add up:

$$\text{Red arrow} + \text{Brown arrow} = \text{Orange circle}$$

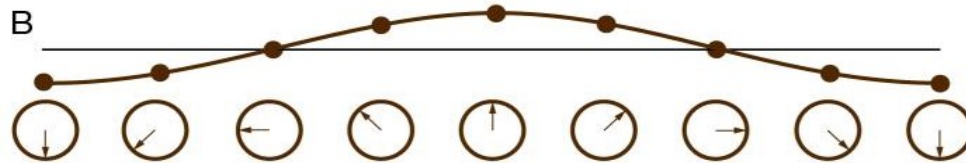
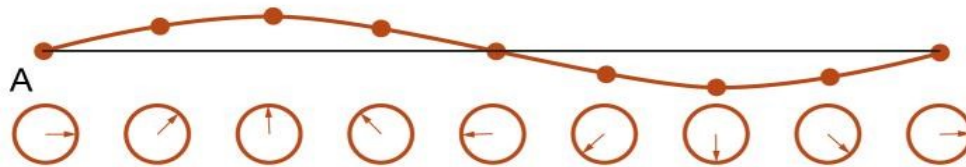
arrows add tip to tail $\text{Orange circle} \rightleftharpoons$

If phase difference = π rad ($=180^\circ$),
then amplitude of resultant
= difference in amplitudes
of components

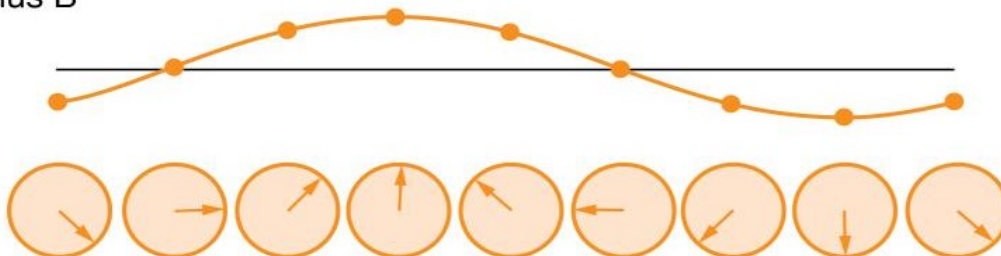
Phase difference $-\pi/2$

Superposition and phase difference

Oscillations in phase



C = A plus B



Rotating arrows add up:



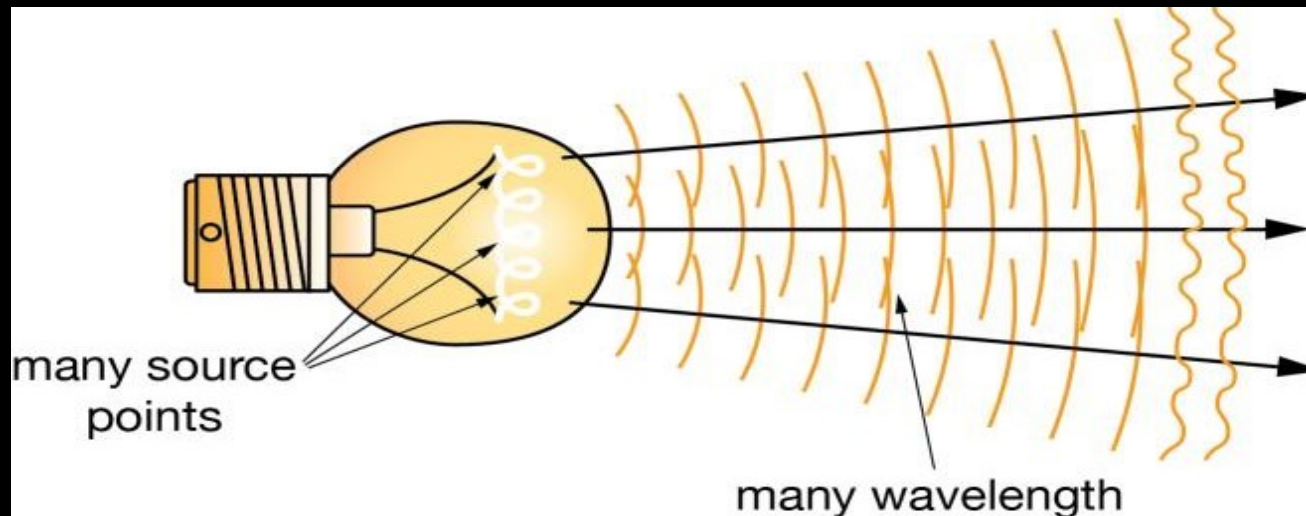
arrows add tip to tail



For any phase difference,
amplitude of resultant
= arrow sum of components

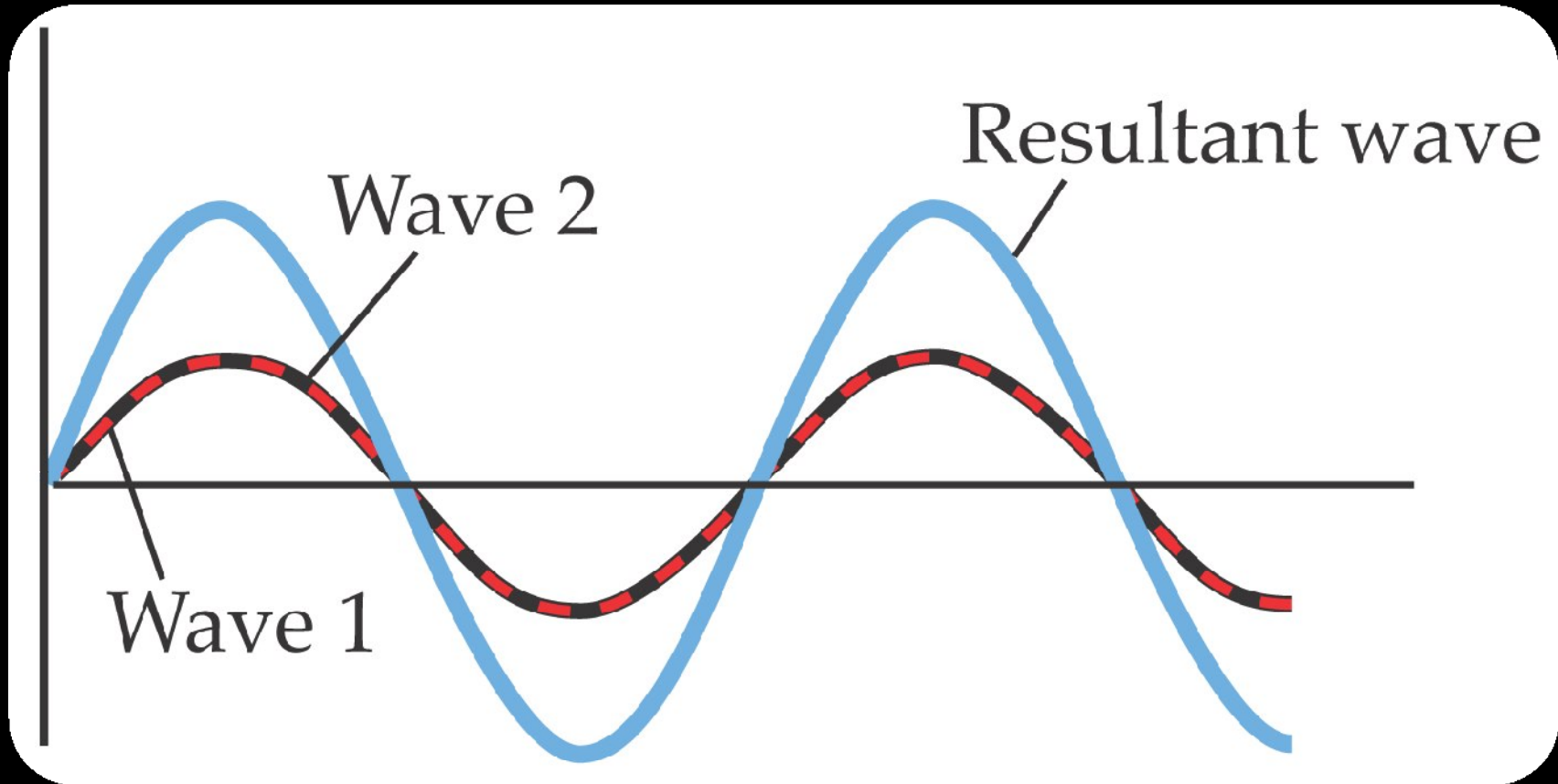
Coherence of light sources

Two waves will only produce a stable interference pattern if their phase relationship remains constant.



Young created coherent light sources by dividing the wavefronts from a single source into two, using slits.

Constructive Interference

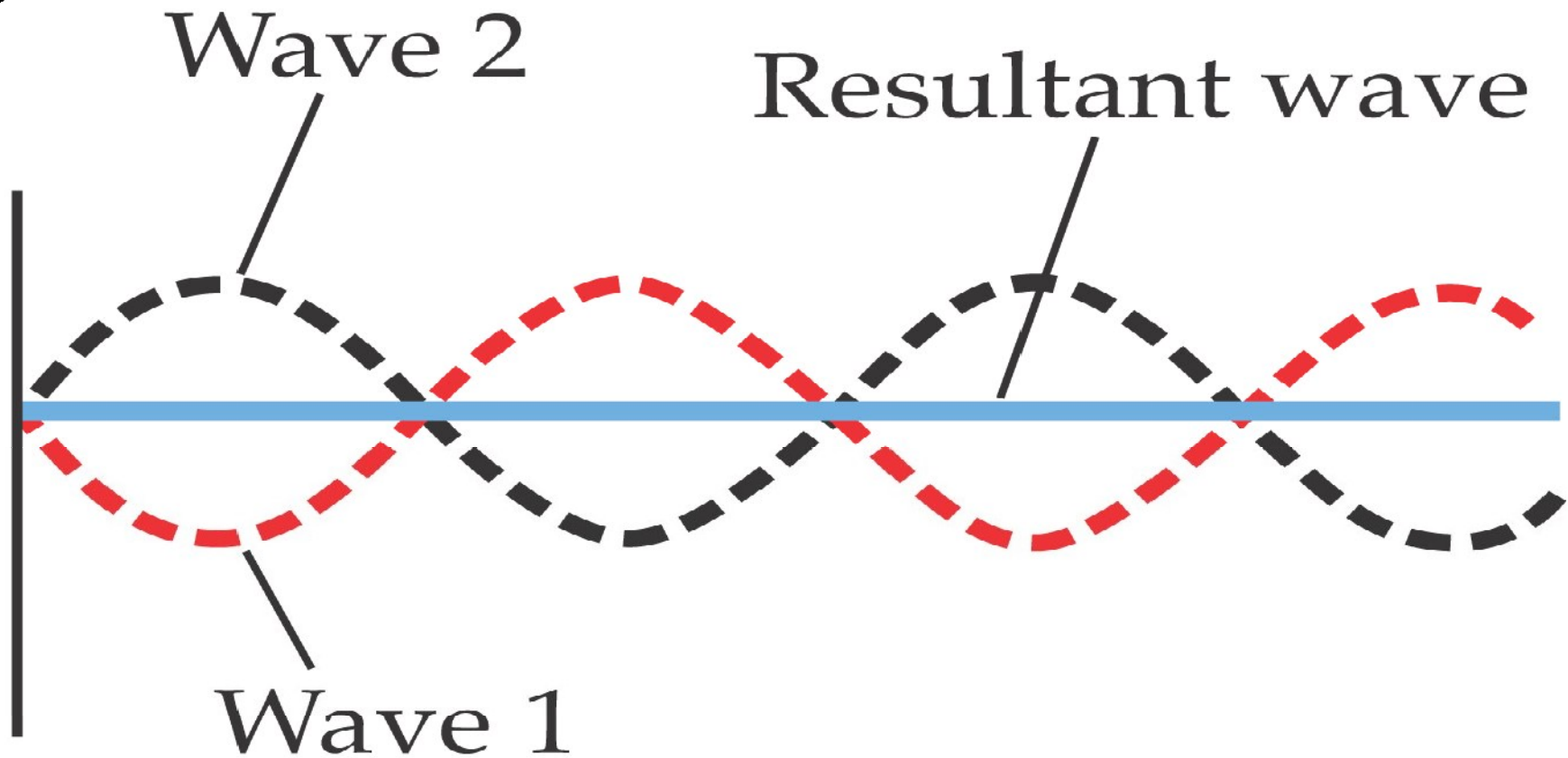


$$y_1 + y_2 = [2A \cos \frac{1}{2}\delta] \sin(kx - \omega t + \frac{1}{2}\delta)$$

16-6

SUPERPOSITION OF TWO WAVES OF THE SAME AMPLITUDE AND FREQUENCY

Destructive Interference



$$y_1 + y_2 = [2A \cos \frac{1}{2}\delta] \sin(kx - \omega t + \frac{1}{2}\delta)$$

16-6

SUPERPOSITION OF TWO WAVES OF THE SAME AMPLITUDE AND FREQUENCY

Interference of light

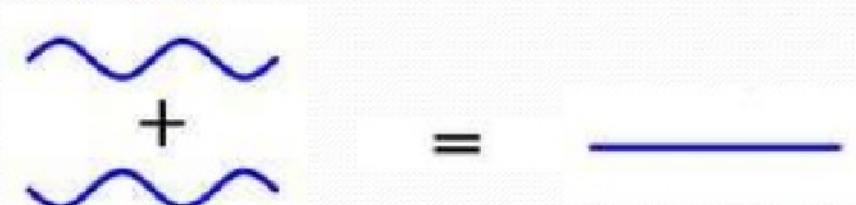
Interference is a phenomenon in which two coherent light waves superpose to form a resultant wave of greater or lower amplitude.

Types of Interference

(a) Constructive interference: If a crest of one wave meets a crest of another wave, the resultant intensity increases.



(b) Destructive interference: If a crest of one wave meets a trough of another wave, the resultant intensity decreases.



Conditions for Interference

- [1] The two interfering waves should be coherent, i.e. the phase difference between them must remain constant with time.
- [2] The two waves should have same frequency.
- [3] If Interfering waves are polarized, they must be in same state of polarization.
- [4] The separation between the light sources should be as small as possible.
- [5] The distance of the screen from the sources should be quite large.
- [6] The amplitude of the interfering waves should be equal or at least very nearly equal.
- [7] The two sources should be narrow
- [8] The two sources should give monochromatic or very nearly monochromatic, or else the path difference should be very small.

Techniques for Producing Interference

- * The **phase** relation between the waves emitted by **two independent** light sources **rapidly changes with time** and therefore they can never be coherent, though the sources are identical in all respects.
- * If **two** sources are derived from a **single source** by **some device**, then any phase change occurring in one source is simultaneously accompanied by the same phase change in the other source. Therefore, the phase difference between the waves emerging from the two sources remains constant and the sources are **coherent**.

The techniques used for creating coherent sources of light are

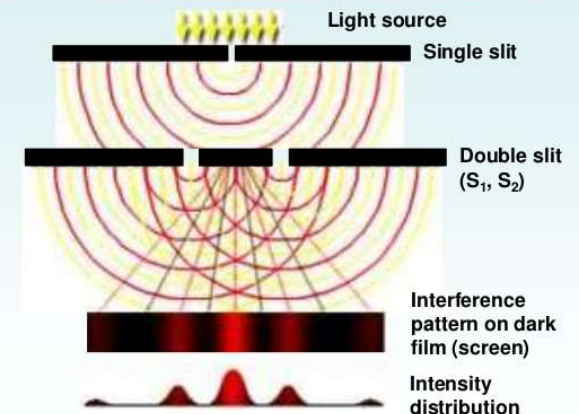
- (a) **Wavefront splitting**
- (b) **Amplitude splitting**

Continued....

(a) Wavefront splitting:

- * One of the methods consists in dividing a light wavefront, emerging from a narrow slit, by passing it through two slits closely spaced side by side.
- * The two parts of the same wavefront travel through different paths and reunite on a screen to produce fringe pattern. This is known as interference due to the division of wavefront.
- * This method is useful only with narrow sources.
- * **Examples:** Young's double slit, Fresnel's double mirror, Fresnel's biprism, Lloyd's mirror etc.

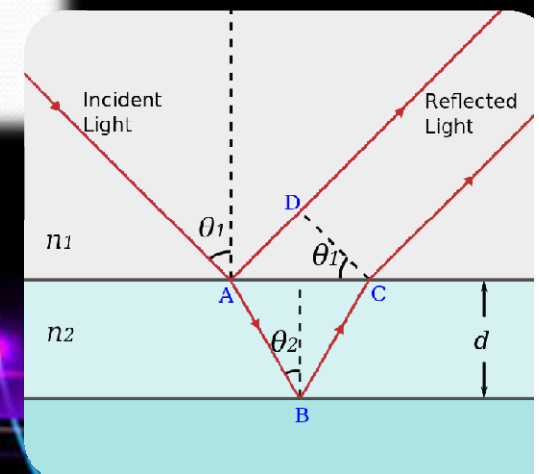
Interference of Light by Division of Wavefront



Continued....

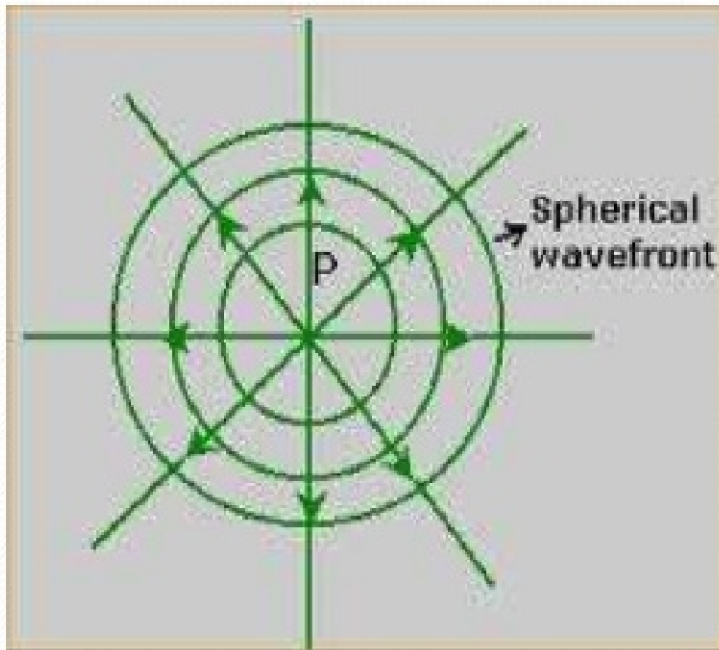
(b) Amplitude splitting:

- * The amplitude (intensity) of a light wave is divided into two parts, namely reflected and transmitted components, by partial reflection at a surface.
- * The two parts travel through different paths and reunite to produce interference fringes. This is known as interference due to division of amplitude.
- * Optical elements such as beam splitters, mirrors are used for achieving amplitude division.
- * **Examples:** Newton's ring experiment, Michelson's interferometer, etc.

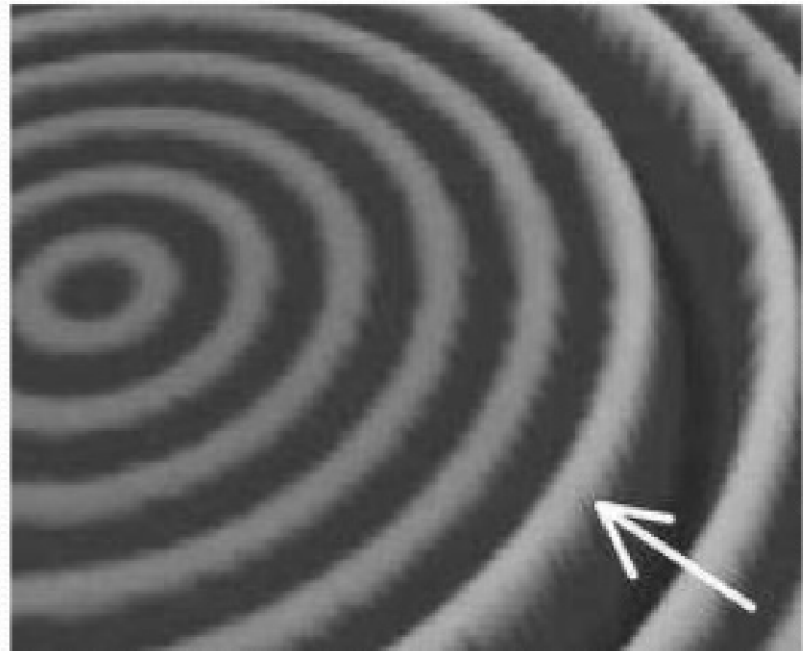


Wavefront

In physics, a **wavefront** is the locus of points having the same phase (line or curve in 2D, or a surface for a wave propagating in 3D)



2-Dimensional



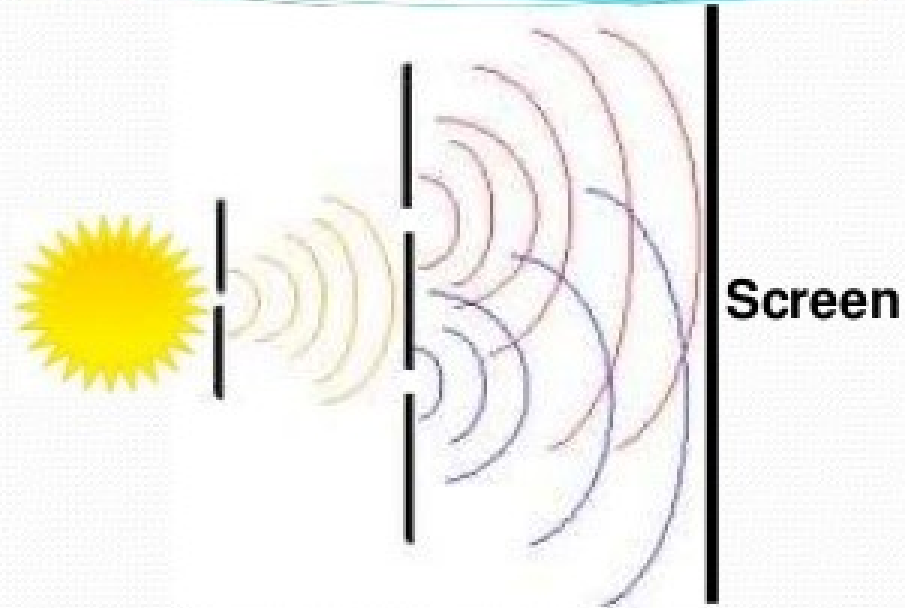
3-Dimensional

Thomas Young



Thomas Young
(1773-1829)

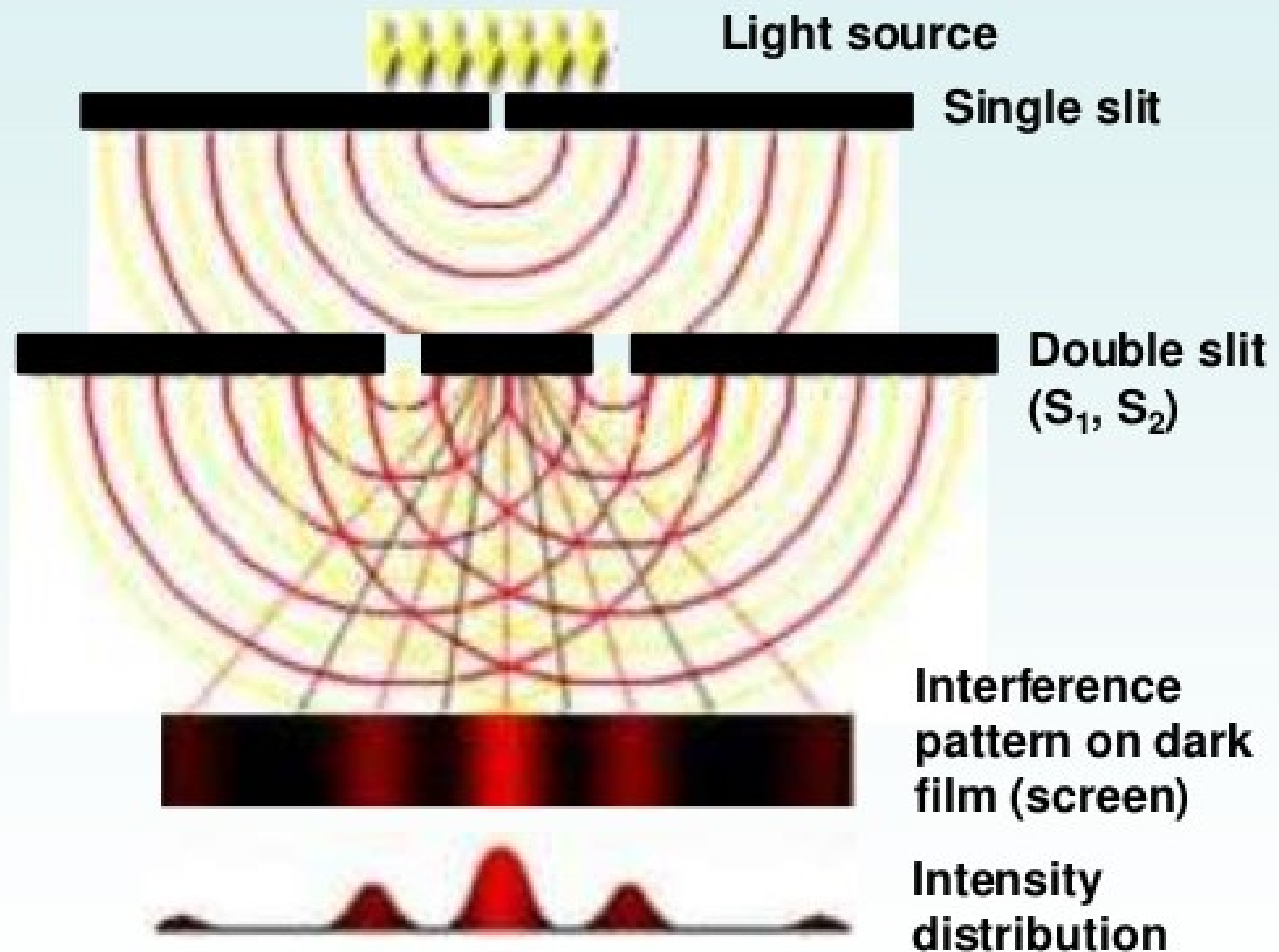
Thomas Young



Interference of light by division of wavefront

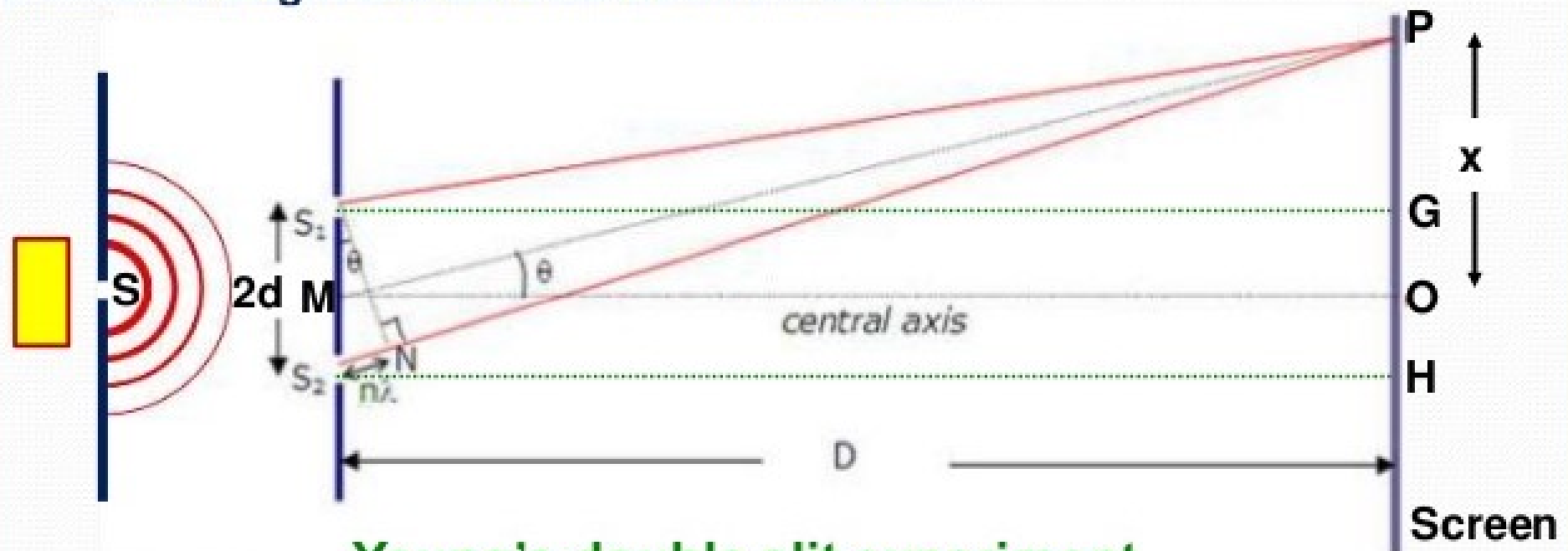
- * In 1801, Young admitted the sunlight through a single pinhole and then directed the emerging light onto two pinholes.
- * The spherical waves emerging from the pinholes interfered with each other and a few colored fringes were observed on the screen.
- * The pinholes were latter replaced with narrow slits and the sunlight was replaced by monochromatic light (Ex: Sodium lamp, Yellow, $5893 \text{ \AA}$)

Interference of Light by Division of Wavefront



Calculation of optical path difference between two waves

As slits (S_1 and S_2) are **equidistant** from source (S), the **phase** of the wave at S_1 will be **same** as the **phase** of the wave at S_2 and therefore, S_1, S_2 act as **coherent** sources. The waves leaving from S_1 and S_2 **interfere** and produce alternative **bright** and **dark** bands on the screen.



Young's double slit experiment

Let P is an arbitrary point on screen, which is at a distance D from the double slits.

$2d$ is the distance between S_1 and S_2

θ is the angle between MO and MP

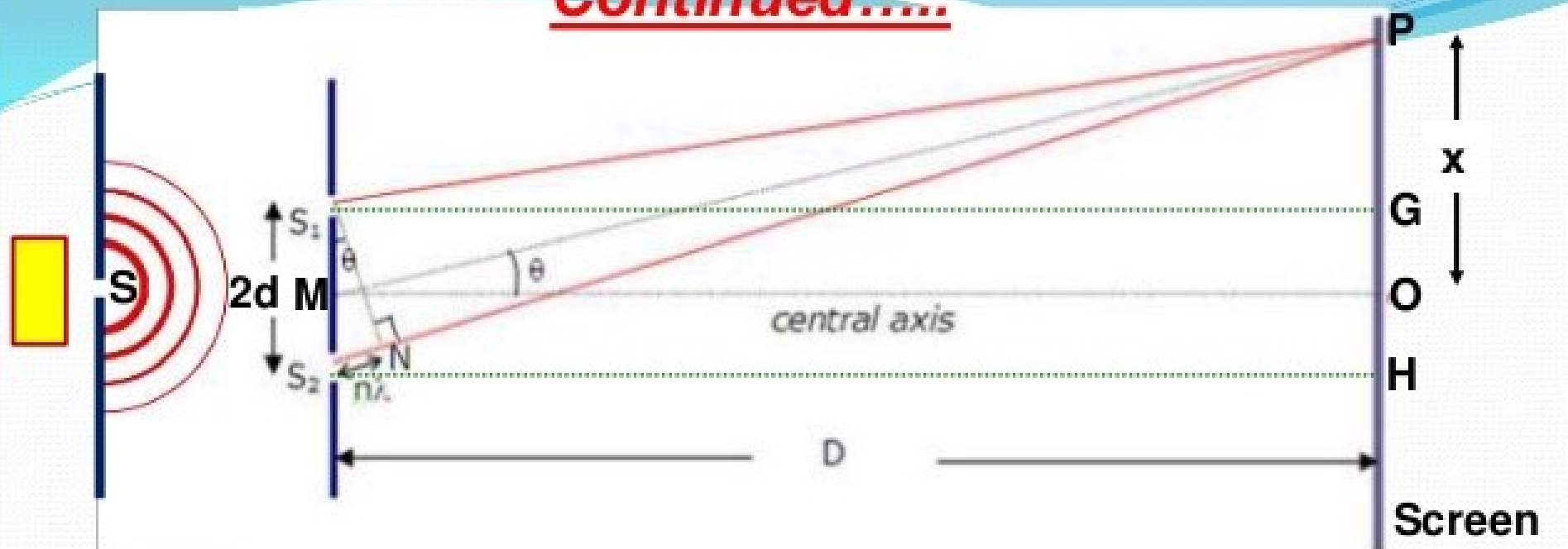
x is the distance between O and P

S_1N is the normal on to the line S_2P

MO bisects S_1S_2 and GH (i.e. $S_1M = S_2M = GO =$

$HO = d$)

Continued.....



The optical paths are identical with the geometrical paths, if the experiment is carried out in air.

The path difference between two waves is $S_2P - S_1P = S_2N$

Let S_1G and S_2H are perpendiculars on the screen and from S_2HP triangle

$$\begin{aligned}(S_2P)^2 &= (S_2H)^2 + (HP)^2 \\ &= D^2 + (x+d)^2 \\ &= D^2 \left[1 + \frac{(x+d)^2}{D^2} \right]\end{aligned}$$

Continued.....

$$(S_2P)^2 = D^2 \left[1 + \frac{(x+d)^2}{D^2} \right]$$

$$S_2P = D \left[1 + \frac{(x+d)^2}{D^2} \right]^{\frac{1}{2}}$$

Since $D \gg (x+d)$, $(x+d)^2/D^2$ is very small

After expansion,

$$S_2P = D \left[1 + \frac{1}{2} \frac{(x+d)^2}{D^2} \right]$$

$$S_2P = D + \frac{1}{2} \frac{(x+d)^2}{D}$$

Similarly, $S_1P = D + \frac{1}{2} \frac{(x-d)^2}{D}$

$$\text{Path difference} = S_2P - S_1P = \frac{1}{2D} [(x+d)^2 - (x-d)^2] = \frac{2xd}{D}$$



$$[(A+B)^2 - (A-B)^2 = A^2 + B^2 + 2AB - (A^2 + B^2 - 2AB) = 4AB]$$

Continued.....

The nature of the interference of the two waves at P depends simply on how many waves are contained in the length of path difference (S_2N).

- If the path difference (S_2N) contains an integral number of wavelengths, then the two waves interfere constructively producing a maximum in the intensity of light on the screen at P.
- If the path difference (S_2N) contains an odd number of half-wavelengths, then the two waves interfere destructively and produce minimum intensity of light on the screen at P.

Continued.....

Thus for bright fringes (maxima),

$$S_2 N = \frac{2xd}{D} = n\lambda \quad \text{where } n = 0, 1, 2, ..$$

$$x = \frac{D}{2d} n\lambda$$

For dark fringes (minima),

$$S_2 N = \frac{2xd}{D} = (2n + 1) \frac{\lambda}{2} \quad \text{where } n = 0, 1, 2, ..$$

$$x = \frac{D}{2d} (2n + 1) \frac{\lambda}{2}$$

Let x_n and x_{n+1} denote the distances of n^{th} and $(n+1)^{\text{th}}$ bright fringes.

Then,

$$x_n = \frac{D}{2d} n\lambda$$

$$x_{n+1} = \frac{D}{2d} (n + 1)\lambda$$

Interference of light

Interference is a phenomenon in which two coherent light waves superpose to form a resultant wave of greater or lower amplitude.

Types of Interference

(a) **Constructive interference:** If a crest of one wave meets a crest of another wave, the resultant intensity increases.



(b) **Destructive interference:** If a crest of one wave meets a trough of another wave, the resultant intensity decreases.



Continued.....

Spacing between n^{th} and $(n+1)^{\text{th}}$ bright fringe is

$$\begin{aligned}x_{n+1} - x_n &= \frac{D}{2d} (n+1)\lambda - \frac{D}{2d} n\lambda \\ &= \frac{D\lambda}{2d}\end{aligned}$$

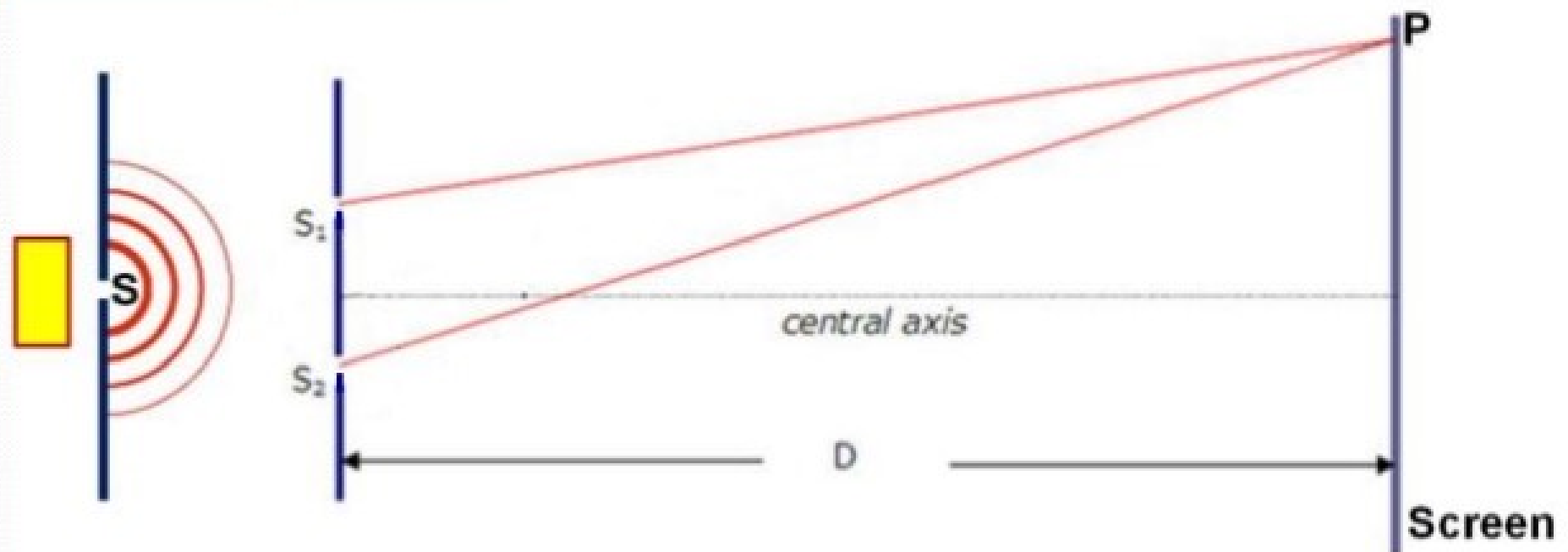
Hence, the spacing between any consecutive bright fringe is the same.
Similarly, the spacing between two dark fringes is $D\lambda/2d$,

The spacing between the fringes is independent of n

The spacing between any two consecutive bright or dark fringe is called the “fringe width” and is denoted by $\bar{X}$

$$\bar{X} = \frac{D\lambda}{2d}$$

Resultant Intensity due to superposition of two interfering waves



Let S be a narrow slit illuminated by a monochromatic (single wavelength) source

S_1 and S_2 are two similar parallel slits (S_1 and S_2 are equidistant from S and very close to each other)

Suppose the waves from S reach S_1 and S_2 in the same phase (coherent)

Beyond S_1 and S_2 , the waves proceed as if they were started from S_1 and S_2 .

Continued....

Let us calculate the resultant intensity of light of wavelength (λ) at point P on a screen placed parallel to S_1 and S_2 .

Let A_1 and A_2 be the amplitude at P due to the waves from S_1 and S_2 , respectively.

The waves arrive at P, having traversed different paths S_1P and S_2P .

Hence, they are superposed at P with a phase differences δ

$$\begin{aligned}\delta &= \frac{2\pi}{\lambda} \times \text{path difference} \\ &= \frac{2\pi}{\lambda} \times (S_2P - S_1P)\end{aligned}$$

The displacement at P due to the simple harmonic waves from S_1 and S_2 can be represented by

$$y_1 = A_1 \sin \omega t$$

$$y_2 = A_2 \sin (\omega t + \delta)$$

Where $\omega/2\pi$ is the common frequency (f) of two waves

Continued....

By the principle of superposition, when two or more waves simultaneously reach at point, the resultant displacement is equal to the sum of the displacements of all the waves.

Hence the resultant displacement is

$$\begin{aligned}y &= y_1 + y_2 \\&= A_1 \sin \omega t + A_2 \sin (\omega t + \delta) \\&= A_1 \sin \omega t + A_2 \sin \omega t \cos \delta + A_2 \cos \omega t \sin \delta\end{aligned}$$

$$\sin (A+B) = \sin A \cos B + \cos A \sin B$$


$$= \sin \omega t (A_1 + A_2 \cos \delta) + \cos \omega t (A_2 \sin \delta)$$

Let us make a change in constants A_1 , A_2 and δ by putting

$$A_1 + A_2 \cos \delta = R \cos \theta$$

$$A_2 \sin \delta = R \sin \theta$$

Where R and θ are new constants

Continued....

The resultant displacement is

$$y = \sin \omega t R \cos \theta + \cos \omega t R \sin \theta$$

$$y = R \sin (\omega t + \theta) \quad \leftarrow \sin A \cos B + \cos A \sin B = \sin (A+B)$$

Hence, the resultant displacement at P is simple harmonic and of amplitude R.

$$A_1 + A_2 \cos \delta = R \cos \theta$$

$$A_2 \sin \delta = R \sin \theta$$

$$R^2 \sin^2 \theta = (A_2 \cos \delta)^2$$

$$R^2 \cos^2 \theta = (A_1 + A_2 \cos \delta)^2$$

Adding the above two equations,

$$R^2 \cos^2 \theta + R^2 \sin^2 \theta = (A_1 + A_2 \cos \delta)^2 + (A_2 \cos \delta)^2$$

$$R^2 = A_1^2 + A_2^2 + 2A_1 A_2 \cos \delta$$

Continued....

$$R^2 = A_1^2 + A_2^2 + 2A_1A_2 \cos \delta$$

The resultant intensity I at P , which is proportional to the square of the resultant amplitude, is given by

$$I = R^2 \quad (\text{by assuming proportionality constant} = 1)$$

$$\text{Thus, } I = A_1^2 + A_2^2 + 2A_1A_2 \cos \delta$$

$$\text{Where, } \delta = \frac{2\pi}{\lambda} \times \text{path difference} = \frac{2\pi}{\lambda} \times (S_2P - S_1P)$$

In case of interference, the resultant intensity at P is not just the sum of the intensities of the individual waves.

Conditions for Maxima and Minima Intensity

The resultant intensity I at P,

$$I = A_1^2 + A_2^2 + 2A_1A_2\cos \delta$$

The intensity I is a maximum, when $\cos \delta = +1$ or $\delta = 2n\pi$, $n=0,1,2,\dots$

Phase difference (δ) = $2n\pi$

Path difference (S_2P-S_1P) = $2n\pi \times \lambda/2\pi = n\lambda$

$$\begin{aligned} I_{max} &= A_1^2 + A_2^2 + 2A_1A_2 = (A_1 + A_2)^2 \\ &= I_1 + I_2 + 2\sqrt{I_1I_2} \end{aligned}$$

The maximum intensity is greater than the sum of two separate intensities

The intensity I is a minimum, when $\cos \delta = -1$ or

$$\delta = (2n+1)\pi, \quad n=0,1,2,\dots$$

Phase difference (δ) = $(2n+1)\pi$

Path difference (S_2P-S_1P) = $(2n+1)\pi \times \lambda/2\pi$
= $(2n+1) \lambda/2$

$$I_{min} = A_1^2 + A_2^2 - 2A_1A_2 = (A_1 - A_2)^2$$

The minimum intensity is less than the sum of two separate intensities

PROBLEM-1

Q: In an interference pattern, at a point we observe the 12th order maximum for $\lambda_1 = 6000 \text{ \AA}$. What order will be visible here if the source is replaced by light of wavelength $\lambda_2 = 4800 \text{ \AA}$?

A: In double slit interference, the distance x of a bright fringe from the centre (zero-order fringe) is

$$x = (D/2d)n\lambda, \text{ where } n = 0, 1, 2, \dots$$

Thus at a given point $n\lambda$ is constant

$$\text{Or, } n_1\lambda_1 = n_2\lambda_2$$

$$n_2 = n_1\lambda_1/\lambda_2 = 12 \times 6000/4800 = 15$$

PROBLEM-2

Q: Two straight narrow parallel slits (2 mm apart) are illuminated with a monochromatic light of wavelength 5896 Å. Fringes are observed at a distance of 60 cm from the slits. Find the width of the fringes?

A: The interference fringe-width for a double slit is given by

$$\bar{X} = \frac{D\lambda}{2d}$$

$$2d = 2 \text{ mm} = 0.2 \text{ cm},$$

$$D = 60 \text{ cm},$$

$$\lambda = 5896 \text{ Å} = 5896 \times 10^{-8} \text{ cm}$$

$$\text{Fringe-width} = (60 \text{ cm} \times 5896 \times 10^{-8} \text{ cm}) / 0.2 \text{ cm} = 1.77 \times 10^{-2} \text{ cm}$$

PROBLEM-3

Q: In a two-slit interference pattern with $\lambda = 6000 \text{ \AA}$ the zero-order and tenth-order maxima fall at 12.34 mm and 14.73 mm, respectively. If λ is changed to 5000 $\text{\AA}$, deduce the positions of the zero-order and twentieth-order fringes, while other arrangements remaining the same.

A: Fringe-width is $\bar{X} = \frac{D\lambda}{2d}$

with $\lambda = 6000 \text{ \AA}$, the distance between zero-order and tenth-order fringe is $14.73 \text{ mm} - 12.34 \text{ mm} = 2.39 \text{ mm}$

Fringe-width = $2.39 \text{ mm} / 10 = 0.239 \text{ mm}$

$(\text{Fringe-width})_{6000} / (\text{Fringe-width})_{5000} = 6000 \text{ \AA} / 5000 \text{ \AA} = 6/5$

$(\text{Fringe-width})_{5000} = (\text{Fringe-width})_{6000} \times 5/6 = 0.239 \text{ mm} \times 5/6 = 0.199 \text{ mm}$

Position of zero-order fringe (for $\lambda = 5000 \text{ \AA}$) = 12.34 mm

Position of twentieth-order fringe (for $\lambda = 5000 \text{ \AA}$)

= $12.34 \text{ mm} + (0.199 \text{ mm} \times 20) = 16.32 \text{ mm}$

PROBLEM-4

Q: Two coherent beams of wavelength 5000 Å reaching a point would individually produce intensities 1.44 and 4.00 units. If they reach there together, the intensity is 0.90 units. Calculate the lowest phase difference with which the beams reach that point.

A: The resultant intensity at a point due to two coherent waves of amplitudes A_1 and A_2 reaching the point with a difference δ is given by

$$I = A_1^2 + A_2^2 + 2A_1A_2\cos\delta$$

$$I = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\delta$$

Here $I_1 = 1.44$, $I_2 = 4.00$ and $I = 0.9$

$$0.9 = 1.44 + 4.00 + 2\sqrt{1.44 \times 4.00} \cos\delta$$

$$\cos\delta = -0.9458,$$

$$\cos(180^\circ - \delta) = 0.9458$$

$$[\text{As } \cos(180^\circ - \theta) = -\cos\theta]$$

$$180^\circ - \delta = \cos^{-1}0.9458$$

$$\delta = 161^\circ$$

PROBLEM-5

Q: Find the ratio of the intensity at the centre of a bright fringe in an interference pattern to the intensity at a point one-quarter of the distance between two fringes from the centre.

A: The resultant intensity at a point due to two coherent waves of amplitudes A_1 and A_2 reaching the point with a difference δ is given by

$$I = A_1^2 + A_2^2 + 2A_1A_2\cos\delta$$

If A be the amplitude of each of the two interfering waves, then the resultant intensity at any point on the screen is given by

$$I = 2A^2 (1 + \cos\delta)$$

Where δ is the phase difference between two waves at that point

At the centre of a bright fringe, $\delta = 0, 2\pi, \dots$

So that $I_0 = 2A^2 (1 + 1) = 4A^2$

The phase difference between successive fringes is 2π . Thus the phase difference at a point distance **one quarter** of the distance between two fringes from the centre will be $\pi/2$.

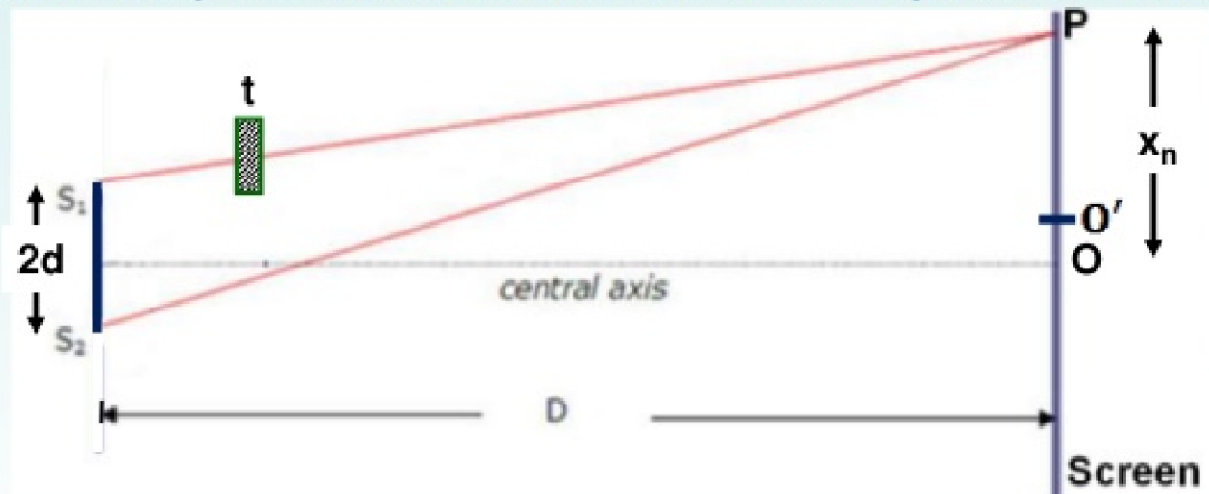
$$I_1 = 2A^2 (1 + \cos 90^\circ) = 2A^2$$

$$I_0/I_1 = 4A^2 / 2A^2 = 2$$

Applications of Young's double slit interference

Displacement of Fringes by Thin Lamina

When a thin transparent plate (glass or mica) is introduced in the path of one of the two interfering beams, the entire fringe-pattern is displaced to a point towards the beam in which the plate is introduced.



Let S_1 and S_2 be two coherent monochromatic sources giving light of wavelength λ .

The thin plate of thickness " t " is introduced in the path of light from S_1 .

For a particular λ , the refractive index of the plate is μ .

Now, light from S_1 travel partly in air and partly in the plate.

For the light path from S_1P , the distance travelled in air is $S_1P - t$

Continued....

If v_o and v_1 are the velocities of light in air and in the plate respectively, then the time taken by light to travel from S_1 to P is

$$= \frac{S_1P - t}{V_o} + \frac{t}{V_1}$$

$$= \frac{S_1P - t}{V_o} + \frac{\mu t}{V_o} \quad \leftarrow \quad \left[\mu = \frac{V_o}{V_1} \right]$$

$$= \frac{S_1P + (\mu - 1)t}{V_o}$$

Hence, the effective path in air from S_1 to P is $[S_1P + (\mu - 1)t]$, i.e. the air path S_1P is increased by an amount $(\mu - 1)t$ due to the introduction of the plate.

Determination of Thickness of the Plate

The introduction of the plate produces shifting in the fringe by an amount x_0 .

$$x_o = \frac{D}{2d} (\mu - 1)t$$

$$t = \frac{x_o(2d)}{D(\mu - 1)}$$

This technique can be used to measure the thickness of a plate.

PROBLEM-8

Q:On introducing a thin sheet of mica (thickness 12×10^{-5} cm) in the path of one of the interfering beams in a biprism arrangement, the central fringe is shifted through a distance equal to the spacing between successive bright fringes. Calculate the refractive index of mica. ($\lambda = 6 \times 10^{-5}$ cm)

A: The shift x_0 of the central fringe is given by

$$x_0 = \text{FW}(\mu - 1)t / \lambda, \quad \text{where FW} = \text{Fringe-width}$$

$$(\mu - 1) = x_0 \lambda / \text{FW} t$$

Hera, $x_0 = \text{FW}$, $\lambda = 6 \times 10^{-5}$ cm and $t = 12 \times 10^{-5}$ cm

$$\mu = 1.5$$

Interference involving multiple reflections

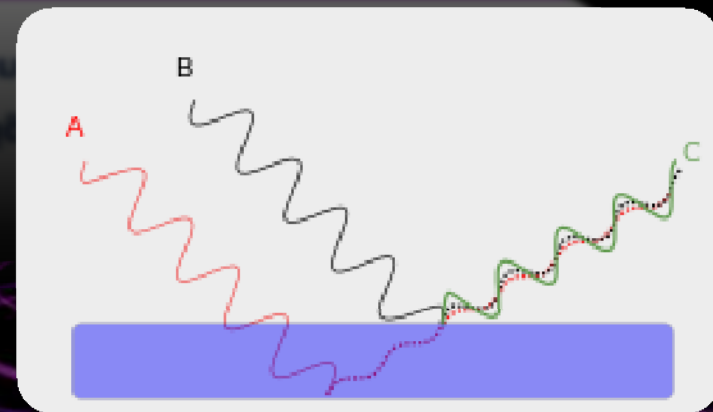
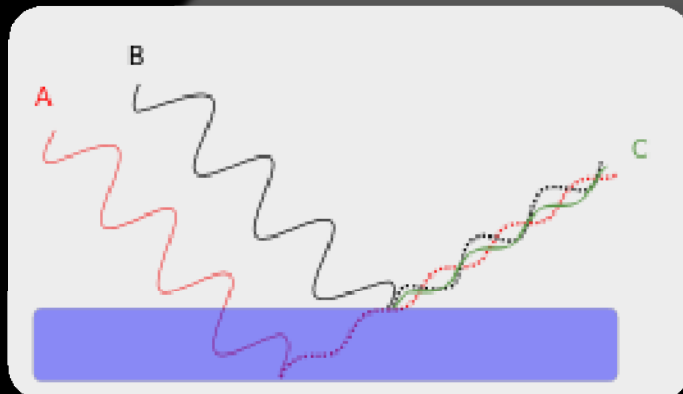


Interference of Light by Thin Film

Stoke's treatment

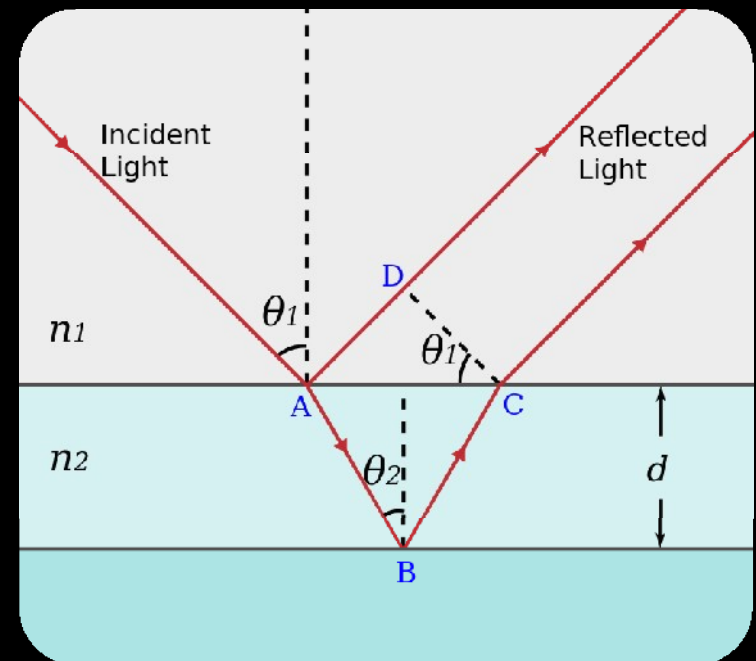
Interference of Light by Thin Film:

- * Interference occurs between the monochromatic light waves (reflected and transmitted), when a film of oil spread over the surface of water, or a thin glass plate.
- * Interference occurs between the light waves reflected from the film and also between the light waves transmitted through the film.



Thin film Interference

- ❑ In optics, a thin film is a layer of material with thickness in the sub-nanometer to micron range.
- ❑ As light strikes the surface of a film it is either transmitted or reflected at the upper surface. Light that is transmitted reaches the bottom surface and may once again be transmitted or reflected.
- ❑ The light reflected from the upper and lower surfaces will interfere.
- ❑ The degree of constructive or destructive interference between the two light waves depends on the difference in their phase.
- ❑ This difference in turn depends on the thickness of the film layer, the refractive index of the film, and the angle of incidence of the original wave on the film.
- ❑ Additionally, a phase shift of 180° or $\{\pi\}$ radians may be introduced upon reflection at a boundary depending on the refractive indices of the materials on either side of the boundary.



❑ This phase shift occurs if the refractive index of the medium the light is travelling through is less than the refractive index of the material it is striking.

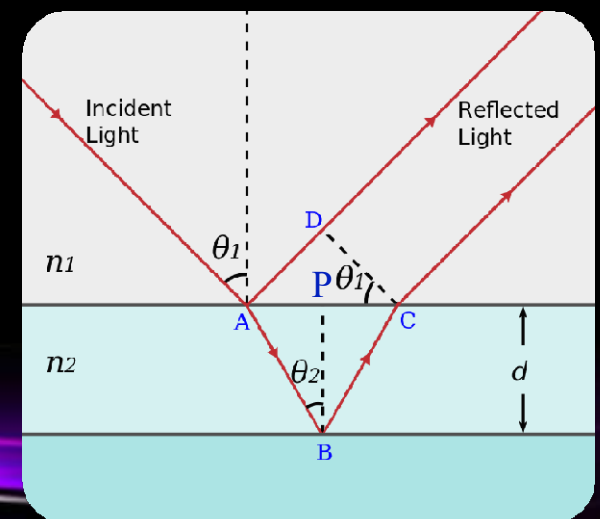
❑ In other words, if $n_1 < n_2$ and the light is travelling from material -1 (n_1) to material -2 (n_2), then a phase shift occurs upon reflection.

❑ The pattern of light that results from this interference can appear either as light and dark bands or as colorful bands depending upon the source of the incident light.

❑ Consider light incident on a thin film and reflected by both the upper and lower boundaries. The optical path difference (OPD) of the reflected light must be calculated in order to determine the condition for interference

$$OPD = n_2(\overline{AB} + \overline{BC}) - n_1(\overline{AD})$$

$$\text{Where, } \overline{AB} = \overline{BC} = \frac{d}{\cos(\theta_2)} \quad (\text{From } \triangle ABP \text{ and } \triangle PBC)$$



$$OPD = n_2(\overline{AB} + \overline{BC}) - n_1(\overline{AD}) \text{-----} (1)$$

$$\text{Where, } \overline{AB} = \overline{BC} = \frac{d}{\cos \theta_2} \text{-----} (2)$$

From

$$\Delta ACD, \sin(\theta_1) = \frac{AD}{AC} \text{-----} (3)$$

$$AC = AP + PC$$

$$AP = PC = BP(\tan \theta_2)$$

$$\therefore AC = 2BP(\tan \theta_2)$$

$$\text{From(3), } AD = AC \sin(\theta_1)$$

$$\Rightarrow AD = 2BP(\tan \theta_2) \sin(\theta_1) = 2d(\tan \theta_2) \sin(\theta_1) \text{-----} (4)$$

from(1),(2)and(4)

$$OPD = n_2\left(\frac{2d}{\cos \theta_2}\right) - n_1(2d(\tan \theta_2) \sin(\theta_1)) \text{-----} (5)$$

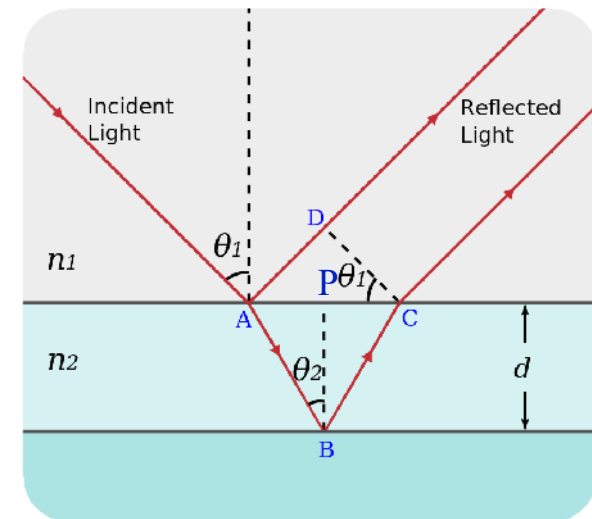
From,snell'sLaw

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

From(5),

$$OPD = n_2\left(\frac{2d}{\cos \theta_2}\right) - n_2\{(2d(\tan \theta_2) \sin(\theta_2))\} = 2n_2d\left(\frac{1 - \sin^2 \theta_2}{\cos(\theta_2)}\right)$$

$$OPD = 2n_2d \cos(\theta_2)$$



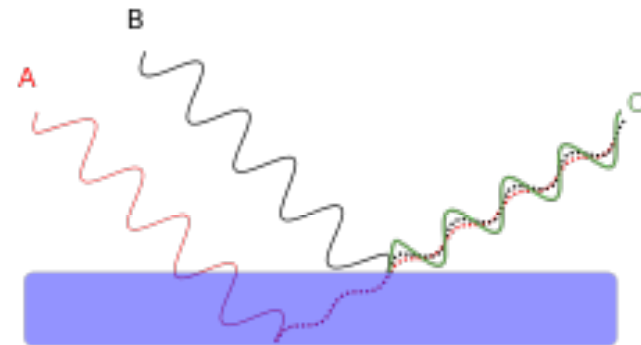


Interference will be constructive if the optical path difference is equal to an integer multiple of the wavelength of light.

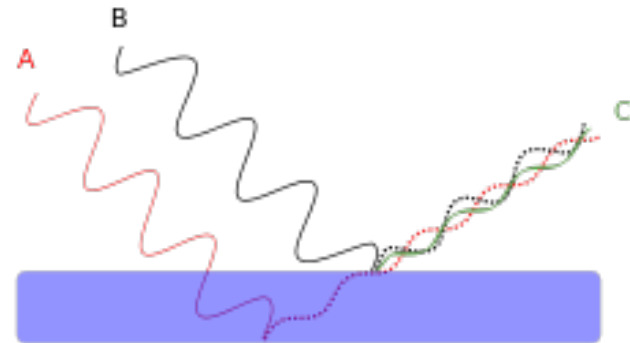
$$OPD = 2n_2d \cos(\theta_2) = m\lambda$$

This condition **may change** after considering possible phase shifts that occur upon reflection.

Constructive Phase
Interaction



Destructive Phase
Interaction



PROBLEM-9

Q: A thin mica sheet ($\mu = 1.6$) of 7 microns thickness introduced in the path of one of the interfering beams in a biprism arrangement shifts the central fringes to a position normally occupied by the 7th bright fringe from the centre. Find the wavelength of light used.

A: The shift x_0 of the central fringe is given by

$$x_0 = \text{FW}(\mu - 1)t / \lambda, \quad \text{where FW} = \text{Fringe-width}$$

$$\lambda = \text{FW}(\mu - 1)t / x_0$$

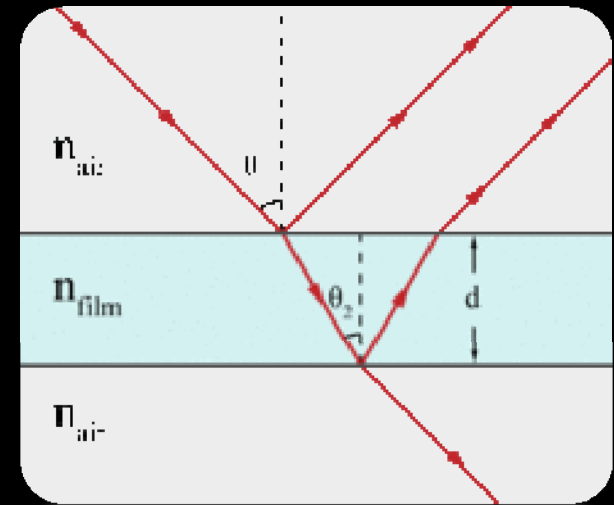
Hera, $x_0 = 7 \text{ FW}$, $\mu = 1.6$ and $t = 7 \text{ } \mu\text{m} = 7 \times 10^{-4} \text{ cm}$

$$\lambda = 6 \times 10^{-5} \text{ cm} = 6000 \text{ } \text{\AA}$$

Examples

1. Soap bubble

- ❖ In the case of a soap bubble, light travels through air and strikes a soap film. The air has a refractive index of 1 ($n_{\text{air}}=1$) and the film has an index that is larger than 1 ($n_{\text{film}}>1$).
- ❖ The reflection that occurs at the upper boundary of the film (the air-film boundary) will introduce a 180° phase shift in the reflected wave because the refractive index of the air is less than the index of the film ($n_{\text{film}}>1$).
- ❖ Light that is transmitted at the upper air-film interface will continue to the lower film-air interface where it can be reflected or transmitted.
- ❖ The reflection that occurs at this boundary will not change the phase of the reflected wave because $\{n_{\text{film}}>1\}$.



The condition for interference for a soap bubble is the following:

$$2n_{\text{film}}d \cos(\theta) = \left(m - \frac{1}{2}\right)\lambda, m = 1, 2, \dots \quad \text{For constructive interference of reflected light}$$

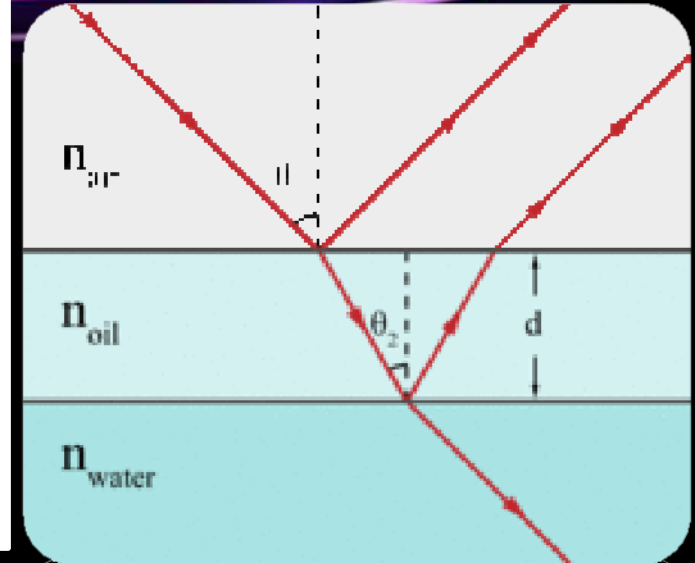
$$2n_{\text{film}}d \cos(\theta) = m\lambda \quad \text{For destructive interference of reflected light}$$

2. Oil film

$$n_{\text{air}} < n_{\text{water}} < n_{\text{oil}}$$

$$2n_{\text{film}}d \cos(\theta) = (m - \frac{1}{2})\lambda, m=1,2,3,\dots \quad \text{For constructive interference of reflected light}$$

$$2n_{\text{film}}d \cos(\theta) = m\lambda \quad \text{For destructive interference of reflected light}$$

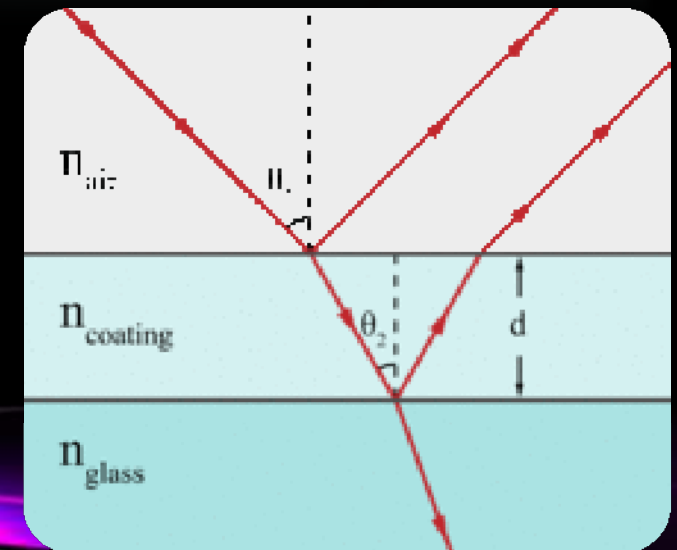


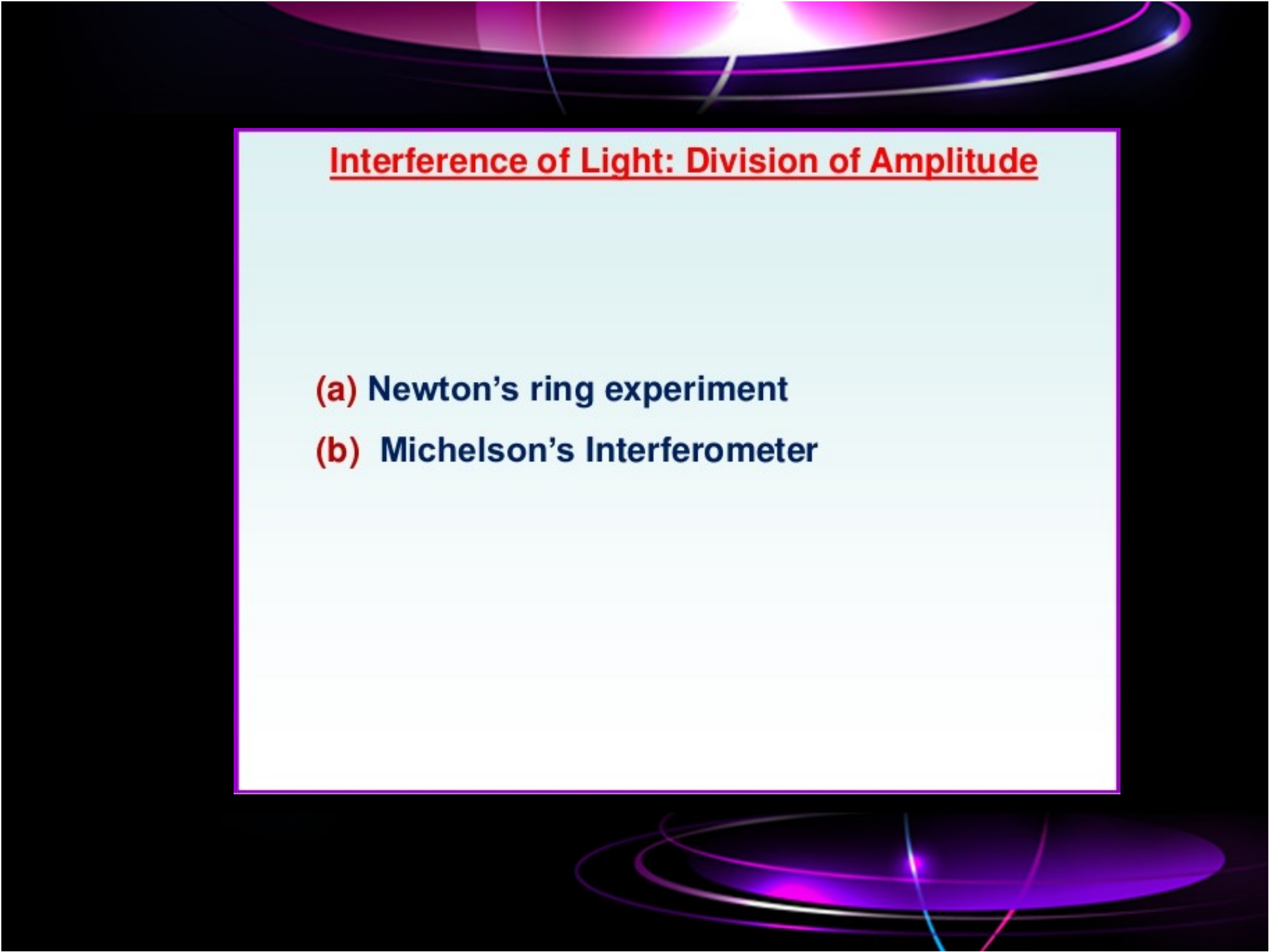
3. Anti-reflection coatings

$$n_{\text{air}} < n_{\text{coating}} < n_{\text{glass}}$$

$$2n_{\text{film}}d \cos(\theta) = (m - \frac{1}{2})\lambda, m=1,2,3,\dots \quad \text{For destructive interference of reflected light}$$

$$2n_{\text{film}}d \cos(\theta) = m\lambda \quad \text{For constructive interference of reflected light}$$





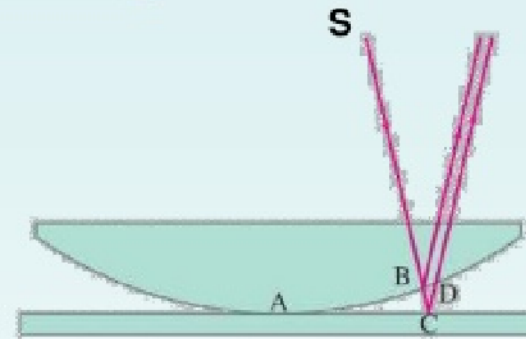
Interference of Light: Division of Amplitude

- (a) Newton's ring experiment
- (b) Michelson's Interferometer

Formation of Newton's Ring

When a plano-convex lens of large radius of curvature is placed with its convex surface in contact with a plane glass plate, an air film is formed between the lower surface of the lens and the upper surface of the glass plate.

- * The thickness of the film gradually increases from the point of contact outwards.
- * If monochromatic light is allowed to fall normally on this film, a system of alternate bright and dark concentric rings with their centre dark is formed in the air film.
- * These are called Newton's rings



Continued....

Newton's rings are formed as a result of interference between the light waves reflected from the upper and lower surfaces of the air film.

The effective path difference between the two rays is

$$p = 2\mu t \cos \theta + \frac{\lambda}{2}$$

Where μ is the refractive index of the film (air), t is the thickness of the film at the point D and θ is the inclination of the ray.

The factor $\lambda/2$ accounts for the phase change of π on reflection at the lower surface of the film.

For air-film ($\mu = 1$) and normal incidence of light ($\theta = 0^\circ$)

$$p = 2t + \frac{\lambda}{2}$$

Continued....

The effective path difference between two rays for air-film ($\mu = 1$) and normal incidence of light ($\theta = 0^\circ$)

$$p = 2t + \frac{\lambda}{2}$$

The condition for maximum intensity (bright-fringe) is

$$p = n\lambda \quad n = 0, 1, 2, \dots$$

$$2t + \frac{\lambda}{2} = n\lambda$$

$$2t = (2n - 1) \frac{\lambda}{2} \quad (\text{Maxima})$$

The condition for minimum intensity (dark fringe) is

$$p = (2n + 1) \frac{\lambda}{2}$$

$$2t + \frac{\lambda}{2} = (2n + 1) \frac{\lambda}{2}$$

$$2t = n\lambda$$

Continued....

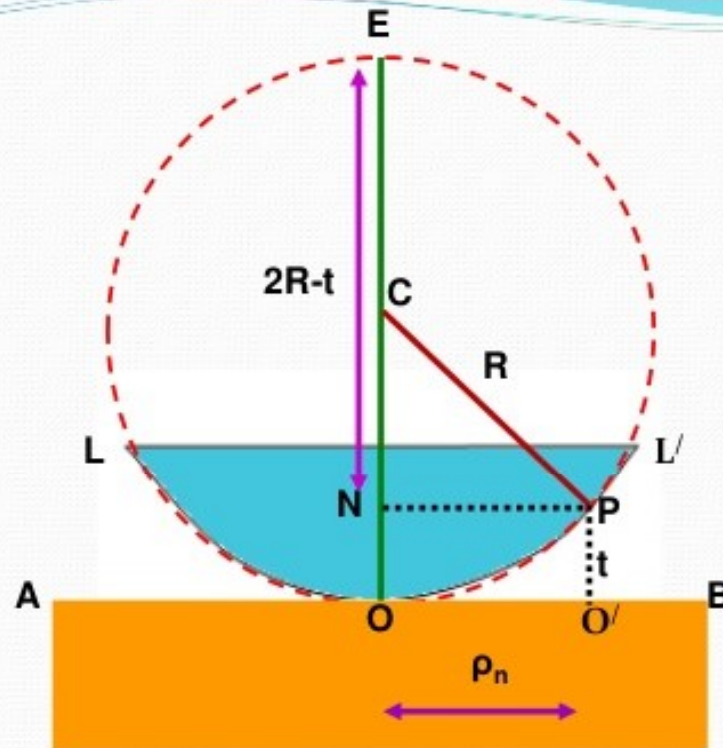
For $t = 0$, at the point of contact of the lens and the plate

$$p = \lambda/2$$

This is the condition for minimum intensity and hence the central spot is dark

- * It is clear that a bright or dark fringe of any particular order n will occur for a constant value of t .
- * Since in the air film “ t ” remains constant along a circle with its centre at the point of contact, the fringes are in the form of concentric circles.
- * Since each fringe is the locus of constant film thickness, these are called fringes of constant thickness.

Diameters of Bright Rings



* Let LOL' be the lens placed on the glass plate AB , the point of contact being O .

* Let R be the radius of curvature of the curved surface of the lens.

Continued....

* Let ρ_n be the radius of a Newton's ring corresponding to a point P, where the film thickness is t .

$$opd = 2\mu t, \mu = \text{air}$$

* Draw a perpendicular PN.

refractive

index

According to the property of the circle,

$$PN^2 = ON \times NE$$

$$\mu = 1$$

Or, $\rho_n^2 = t \times (2R - t) = 2Rt - t^2$

$$opd = 2t$$

Since t is small compared to R , one can neglect t^2

$$\rho_n^2 = t \times (2R - t) = 2Rt$$

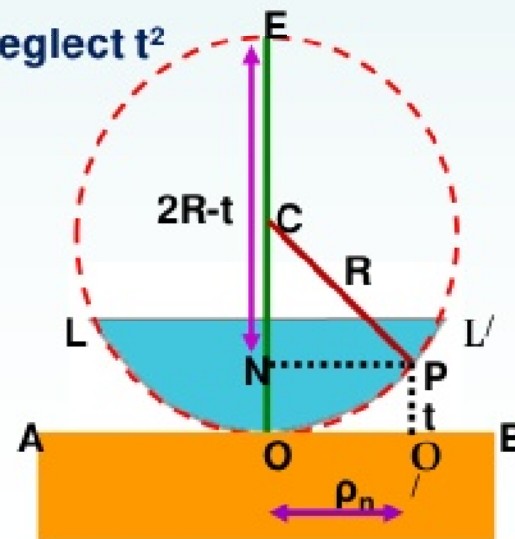
$$2t = \rho_n^2 / R$$

The condition for a bright ring is

$$2t = (2n-1)\lambda/2$$

$$\rho_n^2 / R = (2n-1)\lambda/2$$

$$\rho_n^2 = (2n-1)\lambda R/2$$



Continued....

If D_n be the diameter of the n^{th} bright ring, then $D_n = 2\rho_n$

$$D_n^2 = 2(2n-1)\lambda R$$

$$D_n = (2\lambda R)^{1/2} (2n-1)^{1/2}$$

$$D_n \propto (2n-1)^{1/2}$$

As n is an integer, $(2n-1)$ is an odd number

Thus the diameter of the bright rings are proportional to the square-root of the odd natural number.

The diameters of the first few rings are in the ratio

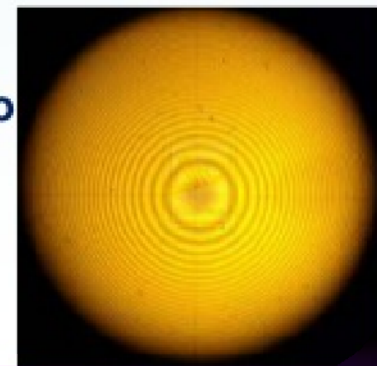
$$1 : \sqrt{3} : \sqrt{5} : \sqrt{7} \dots\dots\dots$$

$$= 1 : 1.732 : 2.236 : 2.446$$

The separation between successive rings are in the ratio

$$= 0.732, 0.504, 0.410, \dots\dots$$

i.e. the separation decreases as the order increases



Diameters of Dark Rings

The condition for a dark ring is

$$2t = n\lambda$$

$$\rho_n^2/R = n\lambda$$

If D_n be the diameter of the n^{th} bright ring, then $\rho_n = D_n/2$

$$D_n^2/4R = n\lambda$$

$$D_n = (4nR\lambda)^{1/2}$$

$$D_n = (4nR\lambda)^{1/2} (n)^{1/2}$$

$$D_n \propto n^{1/2}$$

Thus the diameter of the dark fringes are proportional to the square root of natural numbers.

Applications

You can use them to check the thickness of a surface, and to check whether a surface is uniform

PROBLEM-10

Q: Calculate the thickness of the thinnest film ($\mu=1.4$) in which interference of violet component ($\lambda = 4000 \text{ \AA}$) of incident light can take place by reflection.

A: The condition for constructive interference of light reflected from a film of thickness t is

$$2\mu t \cos\theta = (2n+1)\lambda/2 \quad n = 0,1,2,\dots$$

For normal incident $\theta = 0$ and for minimum thickness n should be 0.

$$2\mu t = \lambda/2$$

$$t = 714.3 \text{ \AA}$$

PROBLEM-11

Q: A thin soap film ($\mu=1.33$) seen by sodium light ($\lambda = 5893 \text{ \AA}$) by normal reflection appears dark. Find the minimum thickness of the film.

A: The condition for destructive interference of light reflected from the a film of thickness t is

$$2\mu t \cos\theta = n\lambda$$

For normal incident $\theta = 0$ and for minimum thickness n should be 1.

$$t = \lambda/2\mu = 2215 \text{ \AA}$$

PROBLEM-12

Q: Newton's rings are observed normally in reflected light of wavelength 5.9×10^{-5} cm. The diameter of the 10th dark ring is 0.5 cm. Find the curvature of the lens and the thickness of the film.

A: The diameter of the nth dark ring is given by

$$D_n^2 = 4n\lambda R$$

After putting the values of $D_n = 0.5$ cm, $n = 10$ and $\lambda = 5.9 \times 10^{-5}$ cm,

$$R = 106 \text{ cm}$$

If t is the thickness of the film corresponding to a ring of diameter D , then we have

$$2t = D^2/4R$$

$$t = 3 \times 10^{-4} \text{ cm}$$

PROBLEM-13

Q: A convex lens of radius 350 cm placed on a flat plate and illuminated by monochromatic light gives the 6th bright ring of diameter 0.68 cm. calculate the wavelength of light used.

A: The diameter of the nth bright fringe is

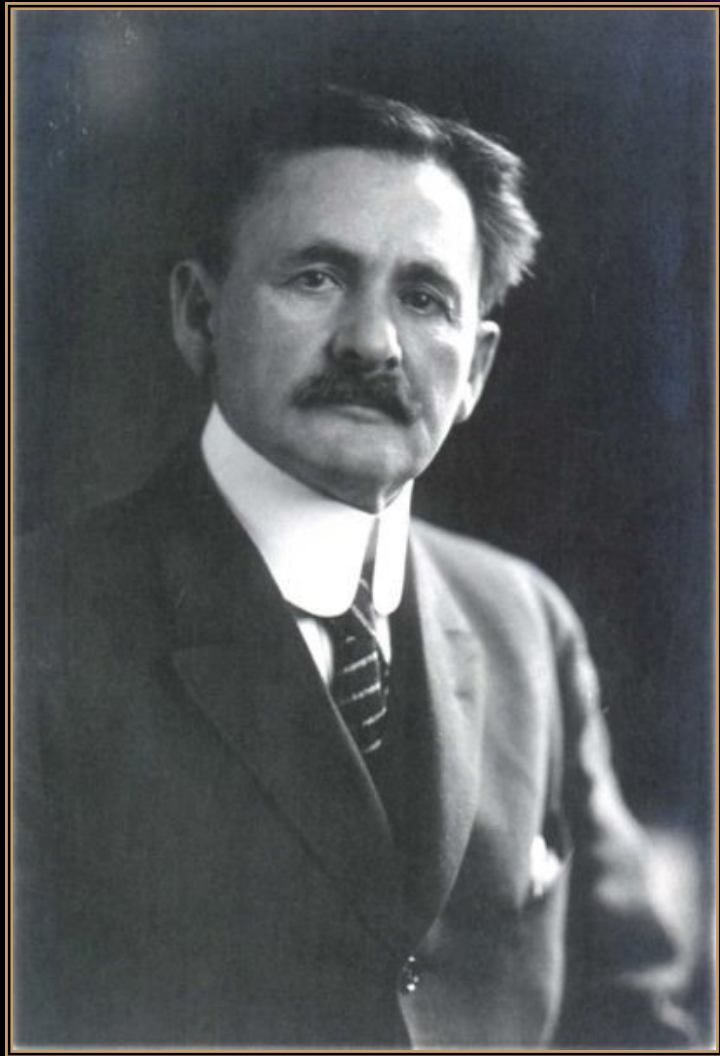
$$D_n^2 = 2(2n-1)\lambda R, \quad \text{where } n = 1, 2, 3, \dots$$

$$R = 350 \text{ cm}, n = 6, D_n = 0.68 \text{ cm}$$

$$\lambda = 6.0 \times 10^{-5} \text{ cm} = 6000 \text{ \AA}$$

INTERFEROMETER





Michelson Interferometer

Albert Abraham Michelson
(1852-1931)

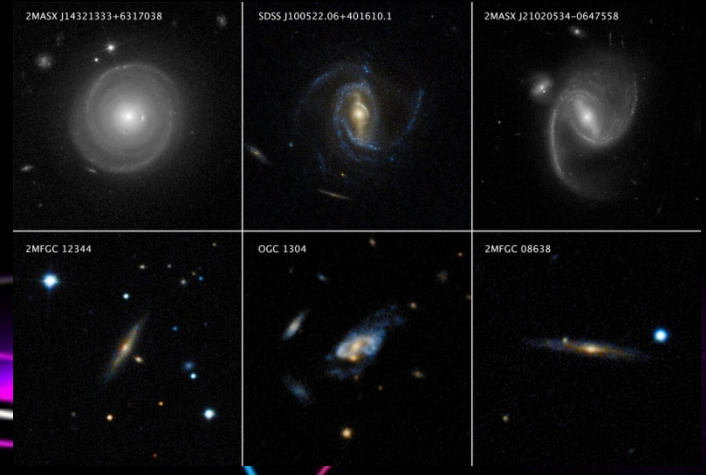
Michelson-Morley Experiment

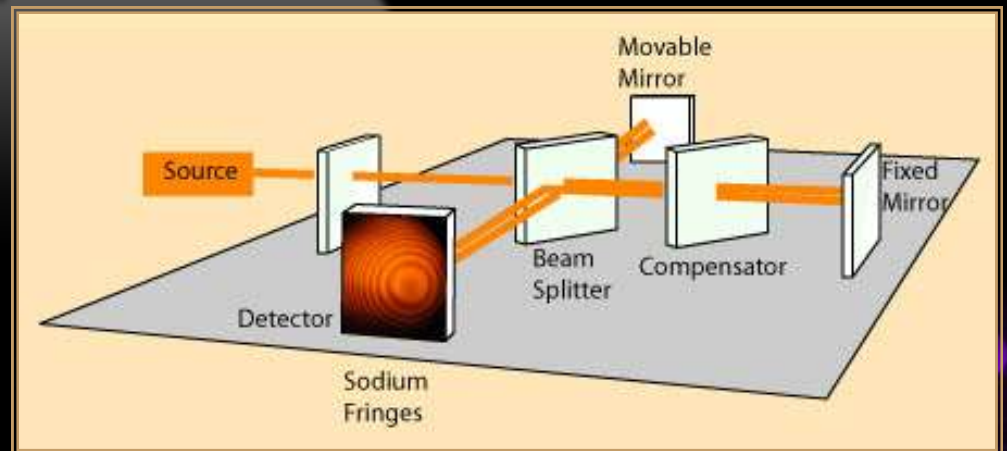
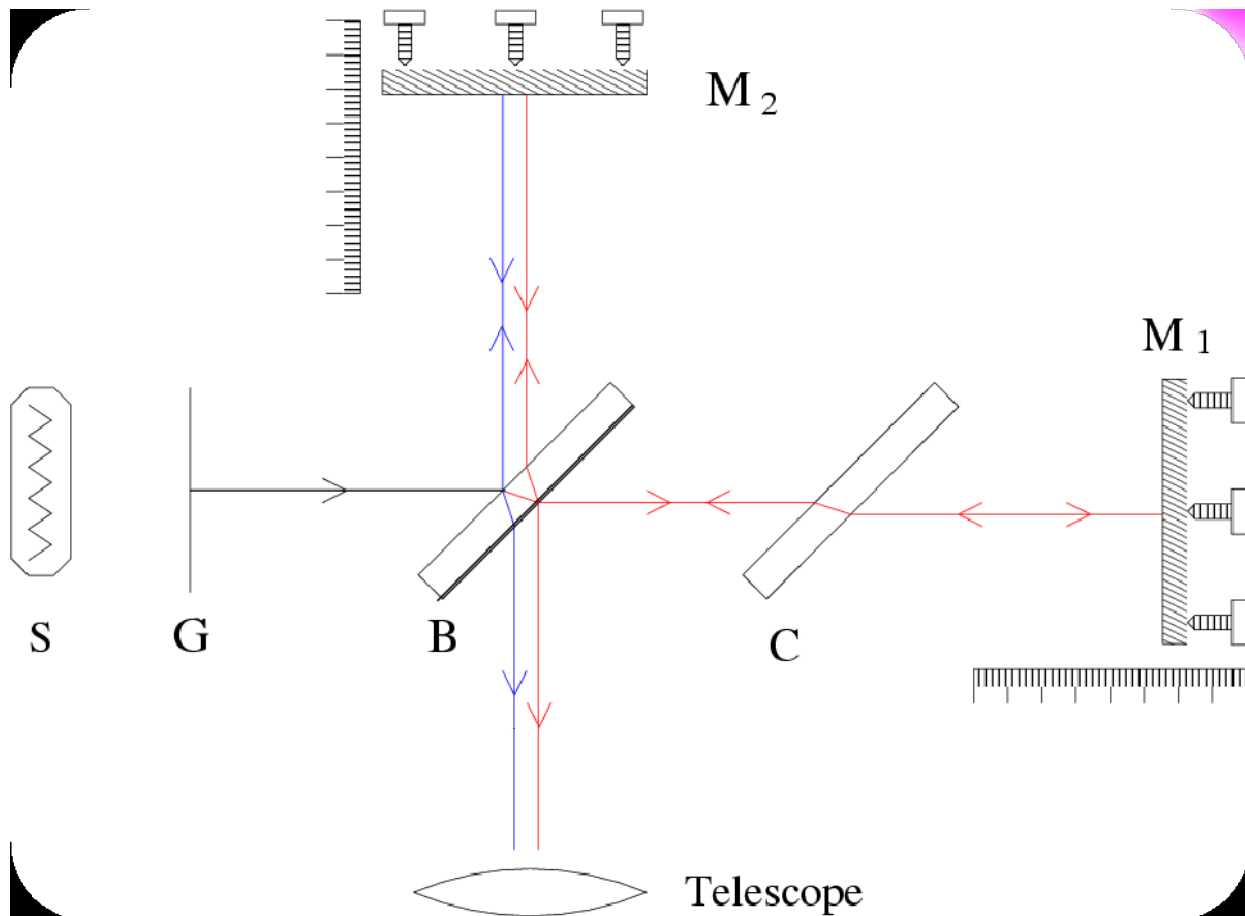
In 1878, Michelson thought the detection of motion through the 'ether' might be measurable.

In trying to measure the speed of the Earth through the supposed "ether", you could depend upon one component of that velocity being known - the velocity of the Earth around the sun, about 30 km/s. Using a wavelength of about 600 nm, there should be a shift of about 0.04 fringes as the spectrometer was rotated 360°. Though small, this was well within Michelson's capability.

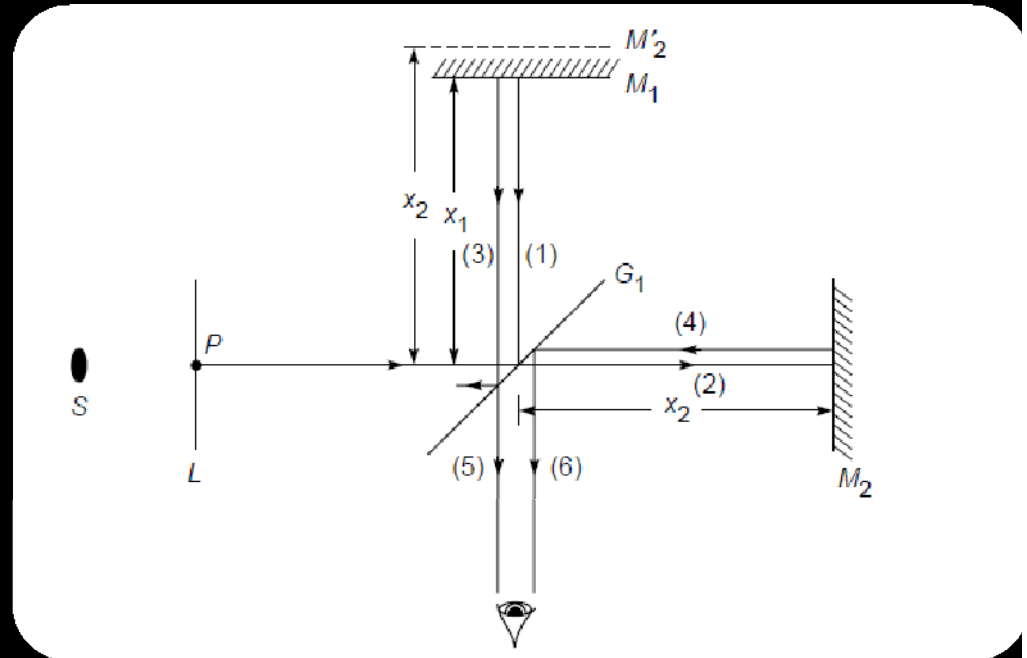
Michelson, and everyone else, was surprised that there was no shift. Michelson's description of the experiment: "The interpretation of these results is that there is no displacement of the interference bands. ... The result of the hypothesis of a stationary ether is thus shown to be incorrect."

(A. A. Michelson, Am. J. Sci, 122, 120 (1881))





1. **S** is the source of almost uniform intensity.
2. **L** represents a ground glass plate to extend uniform intensity.
3. **G1 Beam splitter**: Partially reflect and partially transmit at **45 degrees** to the mirrors.
4. **M1** and **M2** are two perpendicular to each other.
5. Waves from **P** partially get reflected and transmitted through the **G1**.
6. The reflected waves gets further reflection from Mirror **M1**. And this will transmitted through **G1**.
7. The transmitted wave gets reflected by the Mirror **M2**. And reflected from **G1**.
8. Waves (5) and (6) are similar are shown in the figure.



If the beam splitter is glass material the phase accumulated by π , Since the beam is extra path travelled hence the path difference will be,

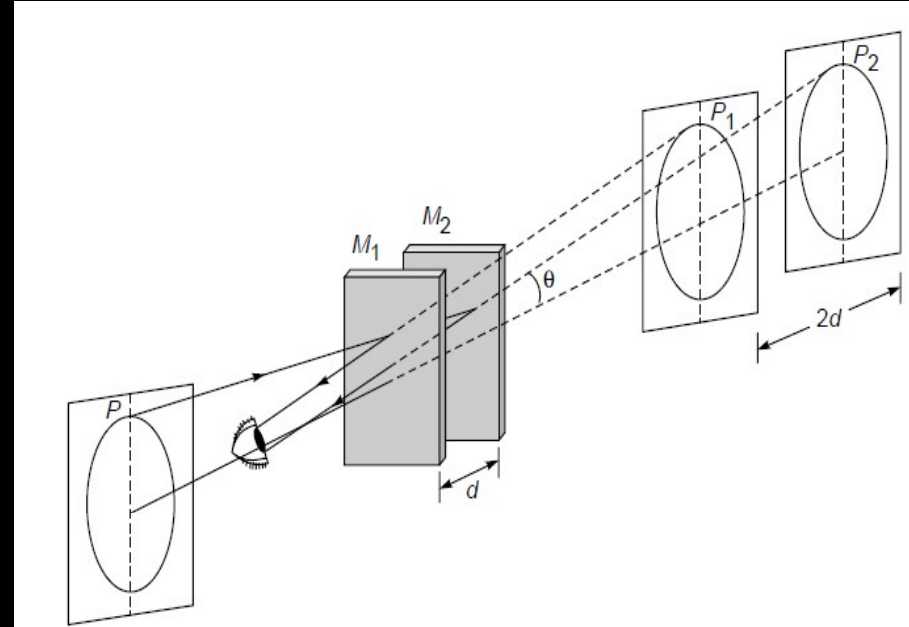
$$2(X_2 \sim X_1) + \lambda/2 = m\lambda \text{ (For constructive interference)}$$

Hence

$$2(X_2 \sim X_1) = (m+1/2) \lambda .$$

For Destructive interference,

$$2(X_2 \sim X_1) = m \lambda$$



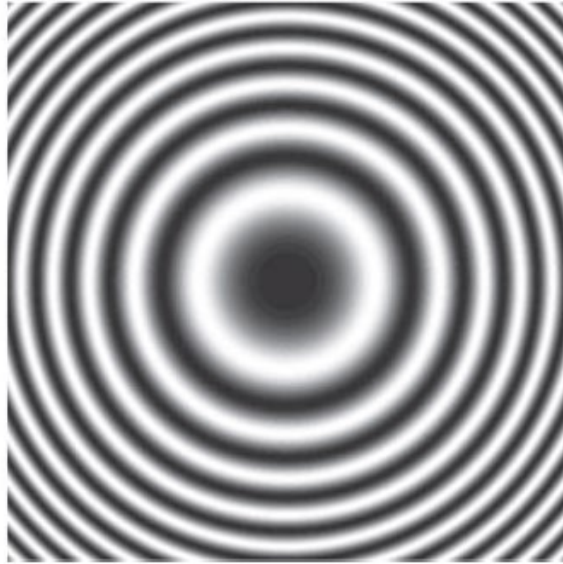
If there is angle (θ) between two rays from two mirrors then,

Path difference

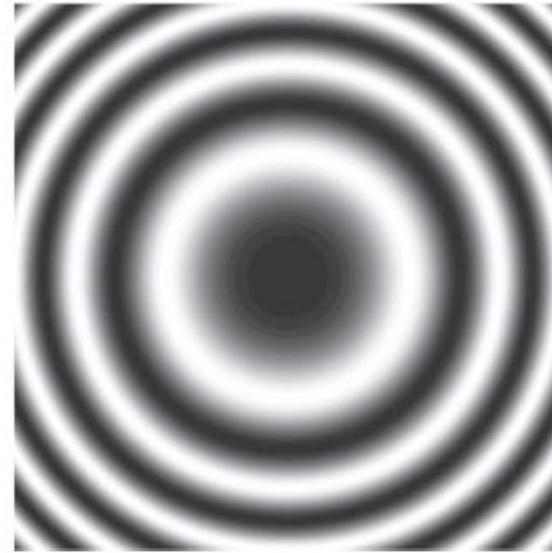
$$2d \cos(\theta) = (m+1/2) \lambda \text{ For constructive interference}$$

$$2d \cos(\theta) = m\lambda \text{ For Destructive interference}$$

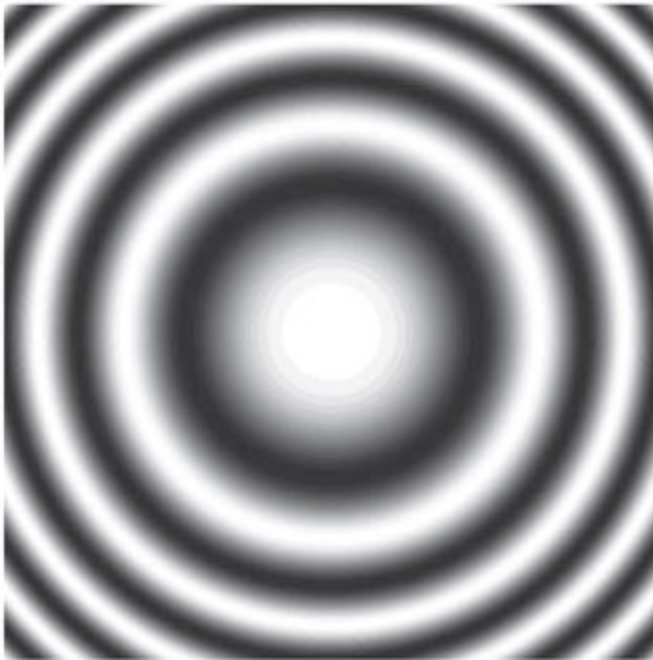
$d = 0.3 \text{ mm}$



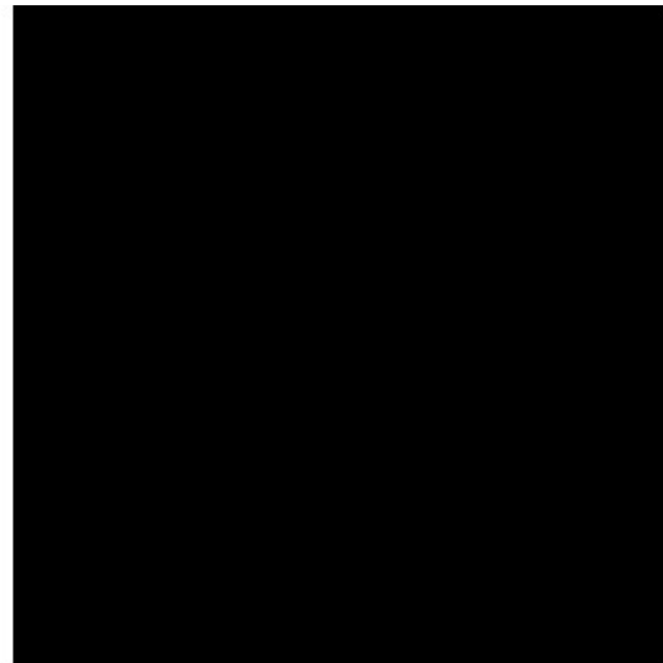
$d = 0.15 \text{ mm}$



$d = 0.14985 \text{ mm}$



$d = 0.0 \text{ mm}$



Order of the fringe:

When the central fringe is dark the order of the fringe is

$$m = \frac{2d}{\lambda}$$

- As d is increased new fringes appear at the center and the existing fringes move outwards, and finally move out of the field of view.
- For any value of d , the central fringe has the largest value of m .

1. Measurement of wavelength of light

$$2d\cos\theta = m\lambda$$

$$2d = m_0\lambda \quad (\theta = 0)$$

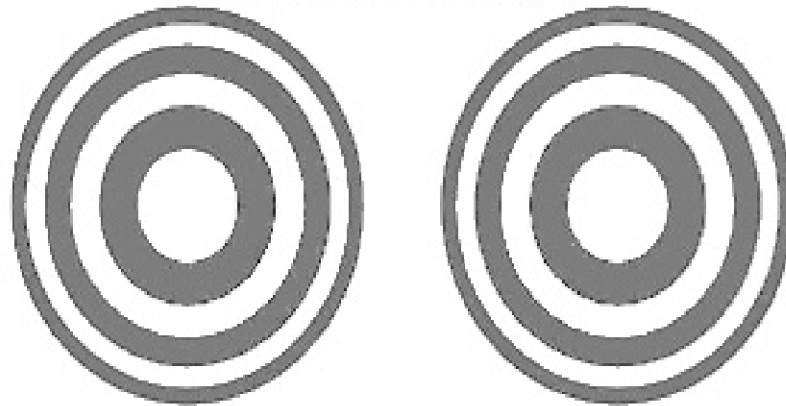
Move one of the mirrors to a new position d' so that the order of the fringe at the centre is changed from m_0 to m .

$$2d' = m\lambda$$

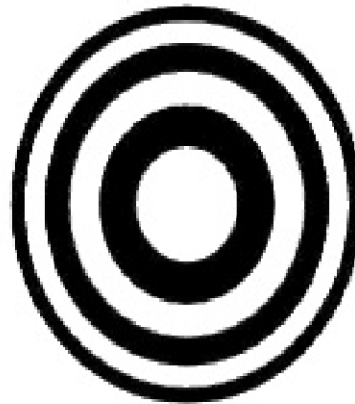
$$2|d' - d| = |m - m_0|\lambda = n\lambda$$

$$\lambda = \frac{2\Delta d}{\Delta m}$$

Concordance



$$2d_1 = p\lambda_1$$
$$= q(\lambda_1 + \Delta\lambda)$$



2. Measurement of wavelength separation of a doublet (λ_1 and $\lambda_1 + \Delta\lambda$)

$$2d_1 = p\lambda_1 = q(\lambda_1 + \Delta\lambda) \quad 2d = m_0\lambda \quad (\theta = 0)$$

As d is increased p and q increase by different amounts, with

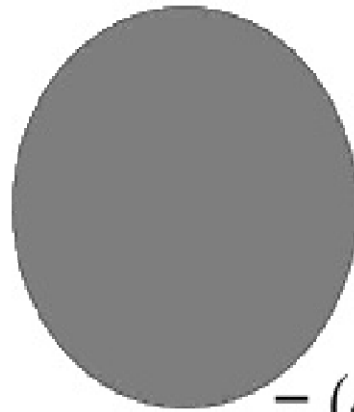
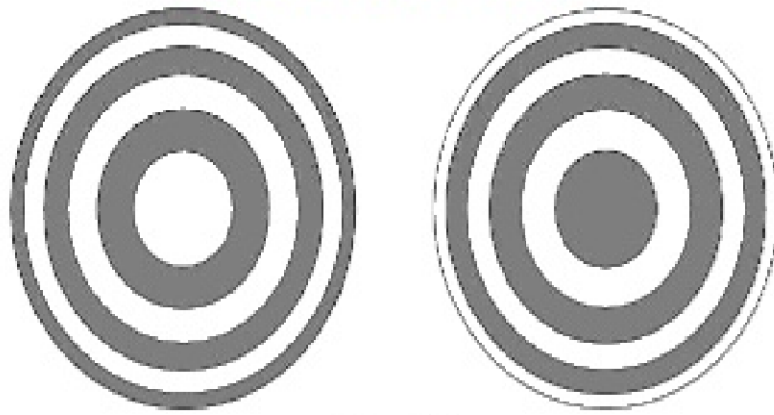
$$\Delta q < \Delta p$$

When

$$q = p - (1/2)$$

the bright fringes of λ_1 coincide with the dark fringes of $\lambda_1 + \Delta\lambda$, and vice-versa and the fringe pattern is washed away (**Discordance**).

Discordance



$$2d_1 = p\lambda_1$$
$$= (q + 1/2)(\lambda_1 + \Delta\lambda)$$

2. Measurement of wavelength separation of a doublet (λ_1 and $\lambda_1 + \Delta\lambda$)

$$2d_1 = p\lambda_1 = q(\lambda_1 + \Delta\lambda) \quad 2d = m\lambda \quad (\theta = 0)$$

- $\Delta\lambda$ can be measured by increasing d_1 to d_2 so that the two sets of fringes, initially concordant, become discordant and are finally concordant again.
- If p changes to $p+n$, and q changes to $q+(n-1)$ we have concordant fringes again.

$$2d_2 = (p+n)\lambda_1 = (q+n-1)(\lambda_1 + \Delta\lambda)$$

$$2(d_2 - d_1) = n\lambda_1 = (n-1)(\lambda_1 + \Delta\lambda)$$

$$\Delta\lambda \approx \frac{\lambda_1^2}{2(d_2 - d_1)}$$

.APPLICATIONS AND USES

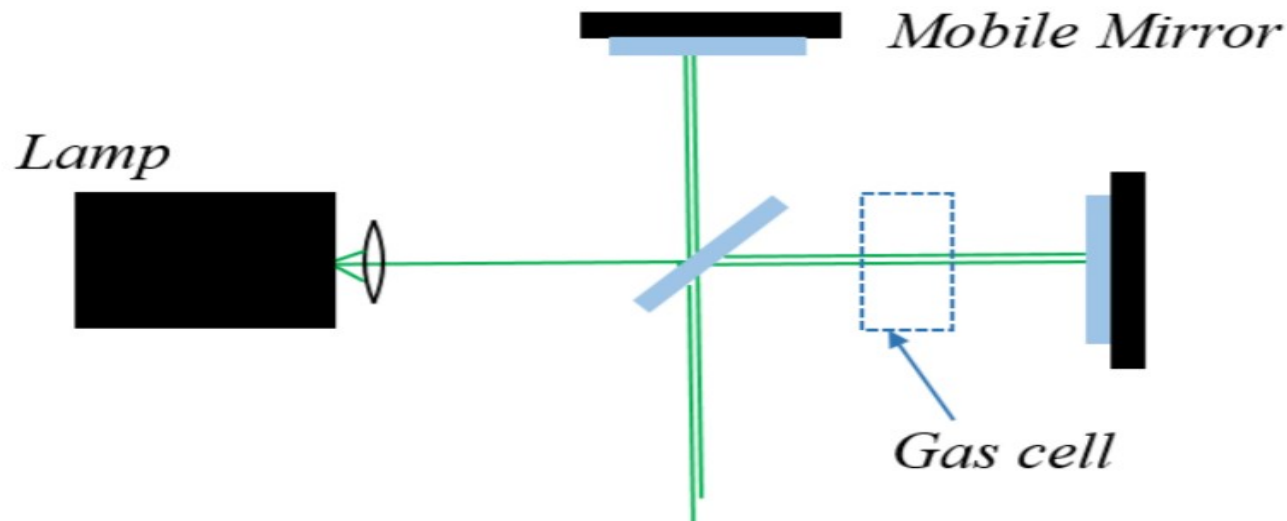
- ▶ The key applications of a Michelson interferometer are as follows:
- ▶ In the Michelson-Morley experiment, which led to the development of the special theory of relativity.
- ▶ In astronomical interferometry.
- ▶ In optical coherence tomography.
- ▶ In analyzing the upper atmosphere, by revealing temperatures and winds, and by measuring the Doppler widths and shifts in the spectra of airglow and aurora.
- ▶ As a component in the helioseismic and magnetic imager to study solar variability, and to illustrate the sun's interior, along with the many aspects of magnetic activity.
- ▶ For the detection of gravitational waves .
- ▶ As a tunable narrow band filter.
- ▶ As the core of Fourier transform spectroscopy.

Question 1.7 (5)

A bulb for a discharge tube lamp is filled with an unknown gas and installed in a lamp. The emission spectrum is dominated by a single wavelength according to a diffraction grating based spectrometer. To more accurately determine the wavelength, and check for multiple closely spaced lines, a Michelson interferometer is used. To determine the wavelength, the movable mirror of the interferometer is shifted by 0.12 mm and 485 fringes are counted shifting past the cross hairs of the viewing telescope mounted on the interferometer. No signs of multiple emission lines are observed.

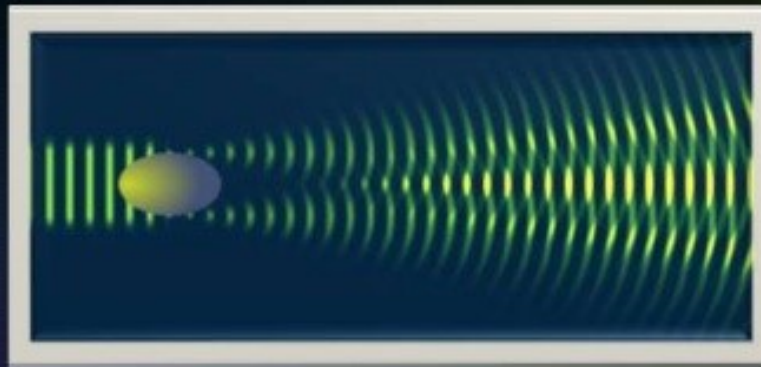
(a) What is the wavelength of the light?

(b) Continuing to use the discharge lamp as the light source, a gas cell that is 3 cm long and is installed along one of the arms of the interferometer is now flooded with the gas under study. A total of 79 fringes are counted passing the cross hairs as the cell shifts from vacuum to 1 atmosphere of pressure. What is the index of refraction of the gas at atmospheric pressure?



Diffraction

A Ray of light bends slightly as it passes around the edge of an object.

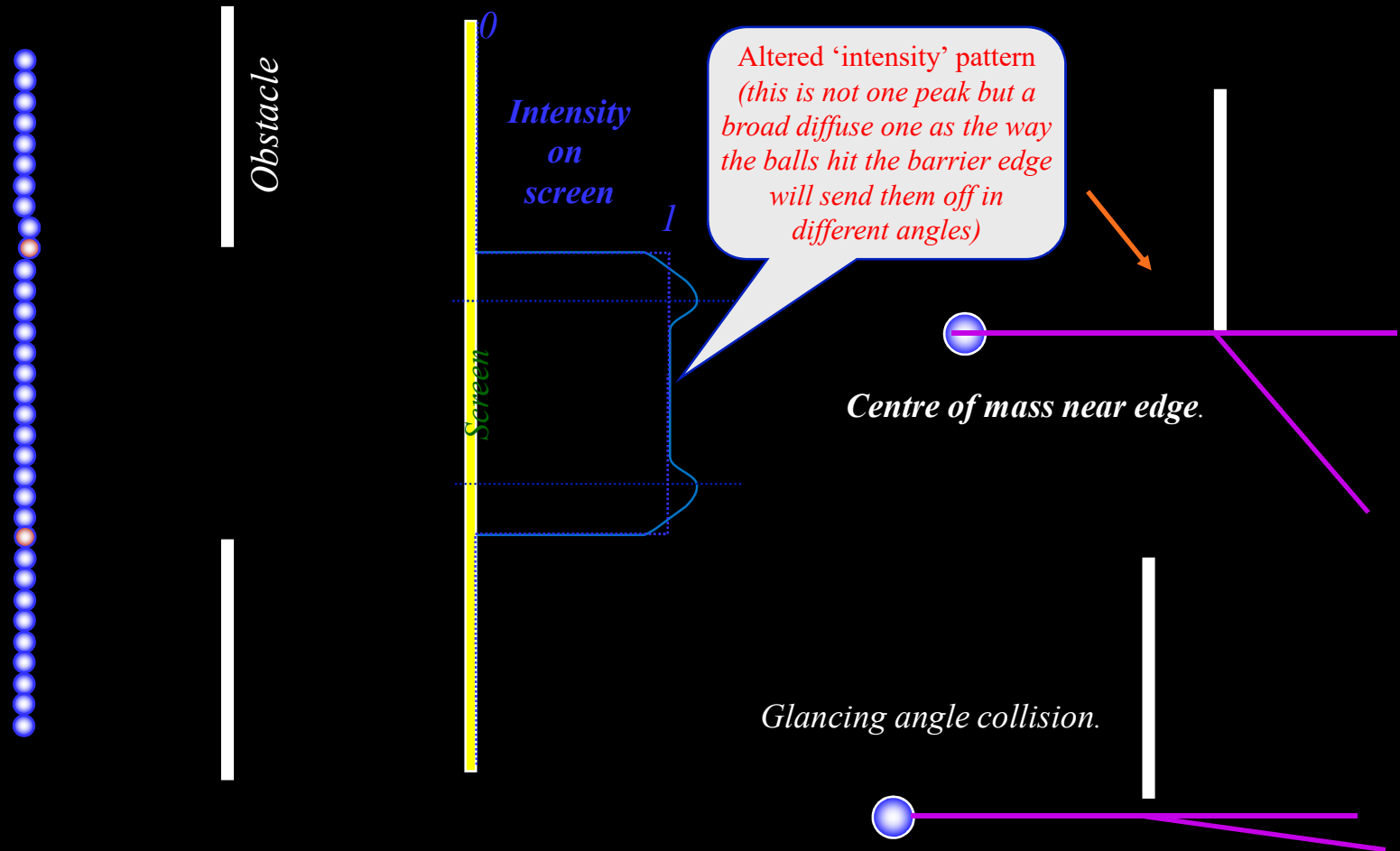


The phenomenon of bending of light from its straight line path around the corners of an obstacle or slit is known as DIFFRACTION

- # German Optician
- # Known for the discovery of dark absorption lines (Fraunhofer lines)
- # 1814- Invented SPECTROSCOPE
- # 1821- developed DIFFRACTION GRATING

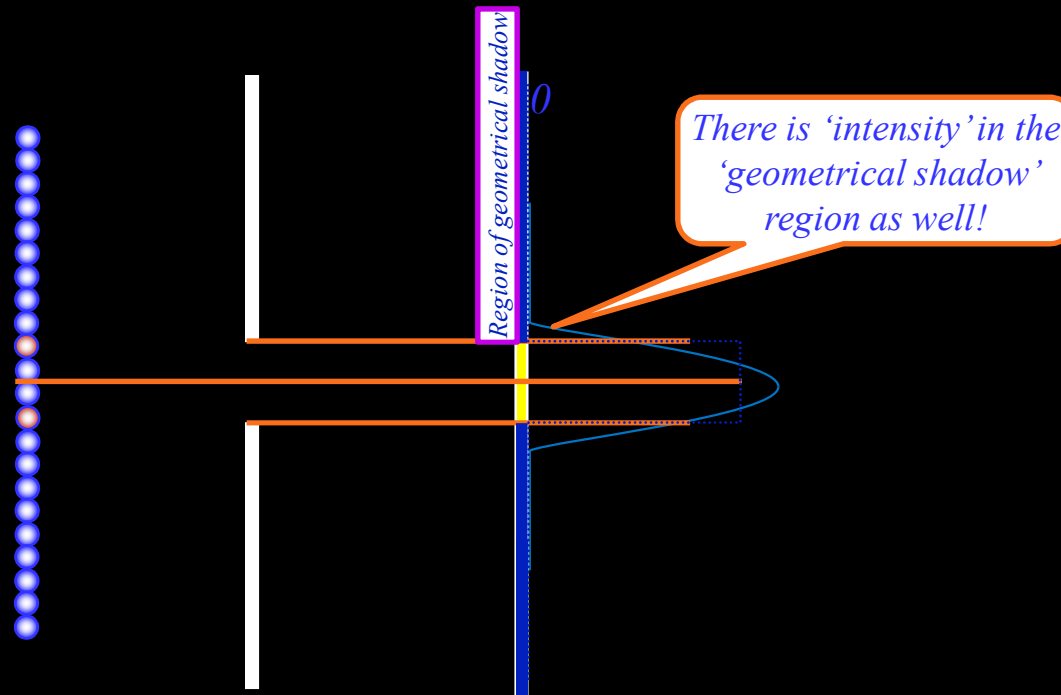


What about the ones hitting the edge?



This is not what happens in diffraction. This is to tell you that 'watch out for sharp corners'!!

What if the slit width is of the order of the 'ball size'?

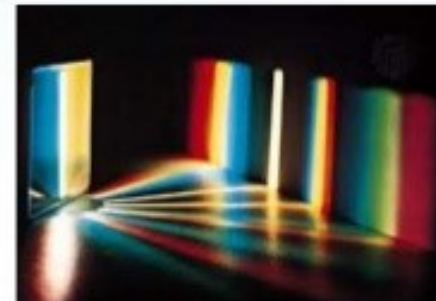


- ❑ So we have seen that even with macroscopic balls it is possible to get **'intensity'** in the region of the geometrical shadow.
- ❑ For this effect to be prominent we have noticed that the slit width has to be of the order of the **'size'** of the ball.

Diffraction of Light

When a narrow opaque obstacle (aperture) be placed between an source of the light and a screen, light bends round the corners of the aperture. This encroachment of light is called “diffraction”.

- * For diffraction, the size of the aperture is small (comparable to the wavelength).
- * As a result of diffraction, the edges of the shadow (or illuminated region) are not sharp, but the intensity is distributed in a certain way depending upon the nature of the aperture.



Difference between Interference and Diffraction

Interference: It occurs between waves starting from two (or more, but finite in numbers) coherent sources.

Diffraction: It occurs between the secondary wavelets starting from the different points (infinite in number) of the same wave.

- Both are superposition effects and often both are present simultaneously (like young's experiment)

Comparison:

- (a) In an interference pattern the minima are usually almost perfectly dark while in a diffraction pattern they are not so.
- (b) In an interference pattern all the maxima are of same intensity but not in the diffraction pattern.
- (c) The interference fringes are usually (although not always) equally spaced. The diffraction fringes are never equally spaced.

Types of Diffraction

(a) Fresnel diffraction

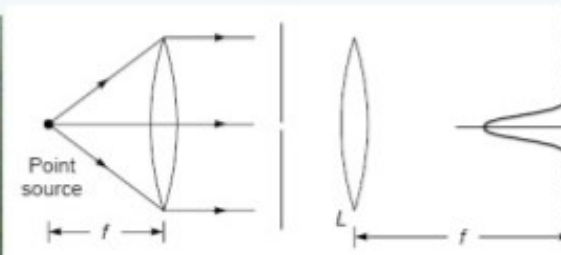
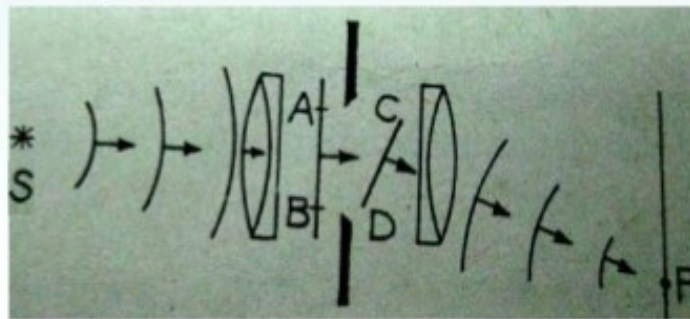
(b) Fraunhofer diffraction

Important aspects of Fraunhofer Diffraction :-

1. Distance of slit from source and screen is infinite.
2. Wavefront incident on slit & screen is plane.
3. Lenses are required to observe diffraction pattern.

Fraunhofer's Diffraction

- * In this case, the source of light and screen are effectively at infinite distance from the diffracting aperture (obstacle).
- * This is achieved by placing the source and screen in the focal planes of two lenses.
- * The incident wavefront is plane.
- * Fraunhofer diffraction is a limiting case of the more general Fresnel's diffraction.



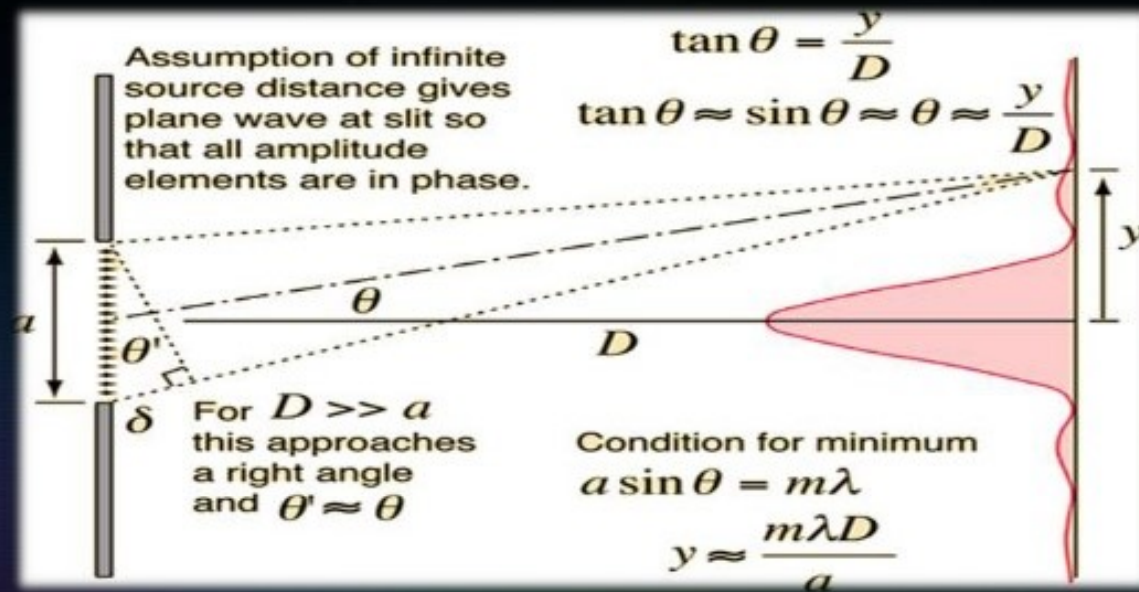
- * Much easier to calculate the intensity distribution of a Fraunhofer diffraction .



FRAUNHOFER DIFFRACTION At SINGLE SLIT

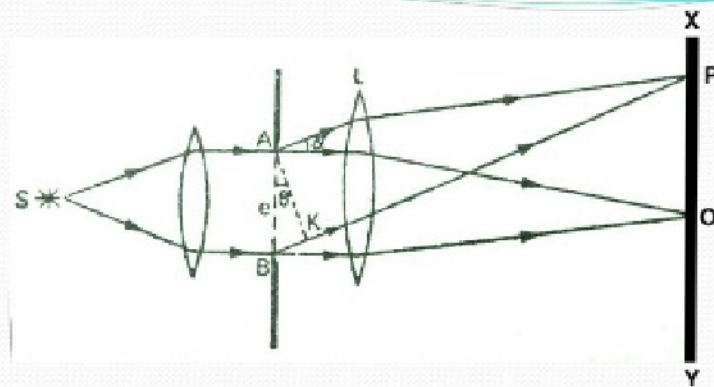
SAIT 01 02 03 04 05 06 07 08 09 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100

Theory :-



Slit of width a (comparable to wavelength of light) is divided into N equal parts of width x . All points on it behaves as point source Emitting secondary waves in all directions and we get a diffraction pattern of alternate maxima's and minima's of decreasing intensities.

Fraunhofer's Diffraction at a Single Slit



- * Let a parallel beam of monochromatic light of wavelength λ be incident normally upon a narrow slit of width $AB = e$
- * Let diffracted light be focused by a convex lens L on a screen XY placed in the focal plane of the lens.
- * The diffraction pattern obtained on the screen consists of a central bright band, having alternate dark and weak bright bands of decreasing intensity on both sides.

Continued.....

In terms of wave theory, a plane wavefront is incident on the slit AB.

According to the Huygen's principle, each point in AB sends out secondary wavelets in all directions.

The rays proceeding in the same direction as the incident rays focussed at O; while those diffracted through an angle θ are focused at P.

Let us find out the resultant intensity at P.

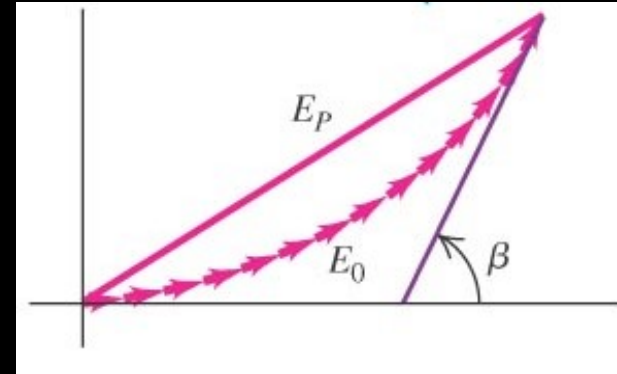
Let AK be perpendicular to BK. As the optical paths from the plane AK to P are equal, the path difference between the wavelets from A to B in the direction $\theta = BK = AB \sin \theta = e \sin \theta$

The corresponding phase difference $= (2\pi/\lambda) e \sin \theta$

Let the width AB of the slit be divided into n equal parts. The amplitude of vibration at P due to the waves from each part will be the same ($= a$)

Quantitative Intensity in the single-slit pattern

- The angle β is the phase angle of the ray from the top of the slit, while the phase angle from the bottom of the slit is 0. The vectors lie along a circle whose center is at C , so E_p is a chord of the circle. The arc length E_0 is subtended by this same angle β , so the radius of the circle is E_0/β .



- From the diagram,

$$E_p = 2 \frac{E_0}{\beta} \sin \frac{\beta}{2} = E_0 \frac{\sin \beta/2}{\beta/2}$$

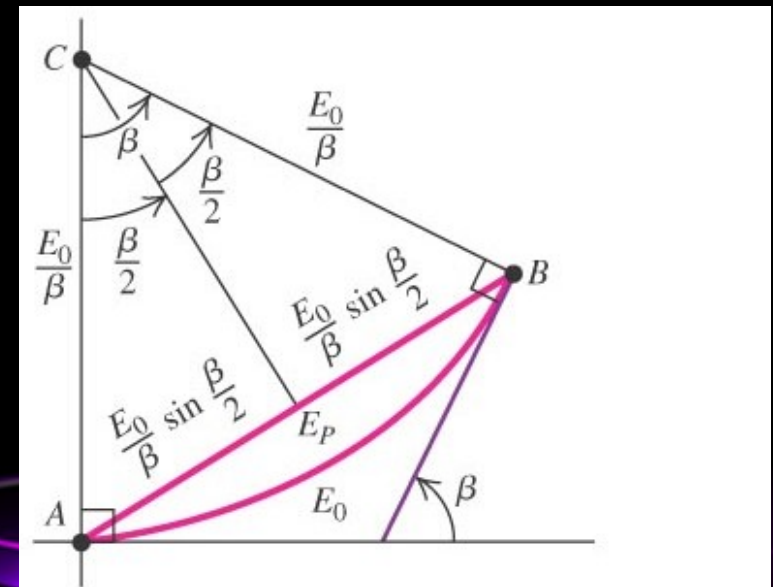
- Since

$$\beta = \frac{2\pi}{\lambda} a \sin \theta$$

- We have

$$I = I_0 \left[\frac{\sin \left[\pi a (\sin \theta) / \lambda \right]}{\pi a (\sin \theta) / \lambda} \right]^2$$

(sinc function)



Continued.....

The phase difference between the waves from any two consecutive parts is

$$\frac{1}{n} \left(\frac{2\pi}{\lambda} e \sin \theta \right) = d$$

Hence the resultant amplitude at P is given by

$$R = \frac{a \sin \left(\frac{nd}{2} \right)}{\sin \left(\frac{d}{2} \right)} = \frac{a \sin \left(\frac{\pi e \sin \theta}{\lambda} \right)}{\sin \left(\frac{\pi e \sin \theta}{n\lambda} \right)}$$

Let $\frac{\pi e \sin \theta}{\lambda} = \alpha$, then

$$R = \frac{a \sin \alpha}{\sin \left(\frac{\alpha}{n} \right)} = \frac{a \sin \alpha}{\frac{\alpha}{n}} \quad \left(\text{as } \frac{\alpha}{n} \text{ is small} \right)$$

$$R = \frac{na \sin \alpha}{\alpha}$$

As $n \rightarrow \infty$, $a \rightarrow 0$,

but the product remains finite

Let the width AB of the slit be divide into n equal parts. The amplitude of vibration at P due to the waves from each part will be same, say a . The phase difference between the waves from any two consecutive parts is

$$\frac{1}{n} \left(\frac{2\pi}{\lambda} e \sin \theta \right) = d$$

Then the resultant amplitude at P is given by

$$R = \frac{a \sin(nd/2)}{\sin(d/2)} = \frac{a \sin \left(\frac{\pi e \sin \theta}{\lambda} \right)}{\sin \left(\frac{\pi e \sin \theta}{n\lambda} \right)}$$

Let us put

$$\left(\frac{\pi}{\lambda} e \sin \theta \right) = \alpha$$

Then
$$R = \frac{a \sin \alpha}{\sin(\alpha/n)} = \frac{a \sin \alpha}{\alpha/n}$$

$$R = \frac{na \sin \alpha}{\alpha}$$

When $n \rightarrow \infty$, $a \rightarrow 0$, but the product na remains finite.

Let $na = A$

The resultant intensity at P, being proportional to the square of the amplitude, is

$$I = R^2 = A^2 \left(\frac{\sin \alpha}{\alpha} \right)^2$$

$$I = R^2 = A^2 \left(\frac{\sin \alpha}{\alpha} \right)^2$$

Condition for Maxima

$$R = \frac{A \sin \alpha}{\alpha} = \frac{A}{\alpha} \left[\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} + \dots \right]$$

$$R = \frac{A \sin \alpha}{\alpha} = A \left[1 - \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} - \frac{\alpha^6}{7!} + \dots \right]$$

$$R = A \text{ for } \alpha = 0$$

This is the intensity of central maximum

$$\alpha = \left(\frac{\pi}{\lambda} e \sin \theta \right) = 0 \quad \text{or} \quad \sin \theta = 0$$

Continued.....

$$\text{Let } na = A, \text{ then } R = \frac{A \sin \alpha}{\alpha}$$

The resultant intensity at P is

$$I = R^2 = A^2 \left(\frac{\sin \alpha}{\alpha} \right)^2$$

Continued.....

Direction of Minima:

The intensity is minimum (zero) when

$$\frac{\sin \alpha}{\alpha} = 0$$

or, $\sin \alpha = 0$ (but $\alpha \neq 0$, because for $\alpha = 0$, then $\sin \alpha / \alpha = 1$)

$\alpha = \pm m\pi$ where m has an integral value 1,2,3 ...except Zero

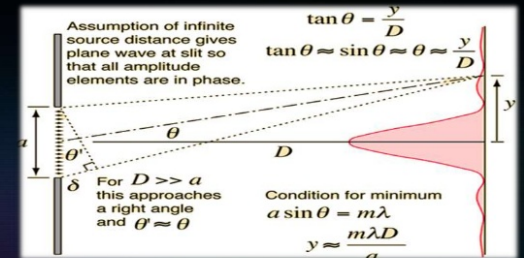
$$\text{But } \alpha = \frac{\pi e \sin \theta}{\lambda}$$

$$\frac{\pi e \sin \theta}{\lambda} = \pm m\pi$$

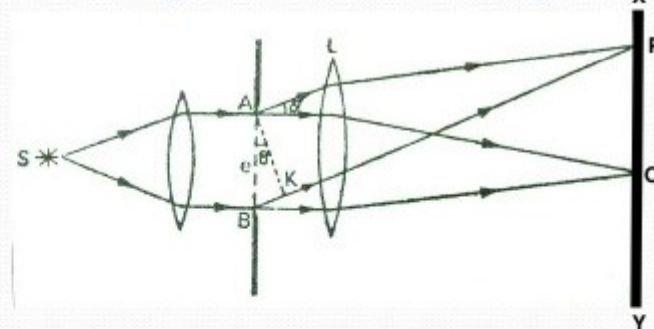
$$e \sin \theta = \pm m\lambda$$

This equation gives the directions of the first, second, third.....minima by putting $m = 1, 2, 3, \dots$

Theory :-



Slit of width a (comparable to wavelength of light) is divided into N equal parts of width x . All points on it behave as point source emitting secondary waves in all directions and we get a diffraction pattern of alternate maxima's and minima's of decreasing intensities.



Continued.....

Direction of Maxima:

To find the direction of maximum intensity, let us differentiate the Intensity with respect to α and equate it to zero

$$dI/d\alpha = 0$$

$$A^2 \left(\frac{2 \sin \alpha}{\alpha} \right) \frac{\alpha \cos \alpha - \sin \alpha}{\alpha^2} = 0$$

$$\frac{\alpha \cos \alpha - \sin \alpha}{\alpha^2} = 0$$

$$\alpha \cos \alpha - \sin \alpha = 0$$

$$\alpha = \tan \alpha$$

This equation is solved graphically by plotting the curves

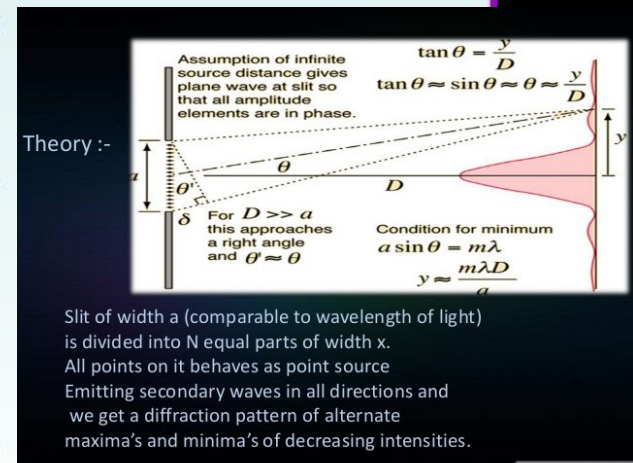
$$y = \alpha$$

$$y = \tan \alpha$$

$$I_p = I_o \left(\frac{\sin \alpha}{\alpha} \right)^2$$

$$\frac{dI}{d\alpha} = I_o 2 \left(\frac{\sin \alpha}{\alpha} \right) * \frac{(\alpha \cos \alpha - 1 \cdot \sin \alpha)}{\alpha^2}$$

$$\text{Formula, } (u/v)^1 = (u^1 v - v^1 u) / v^2$$



Continued.....

The 1st equation is a straight line through origin making an angle of 45°

The 2nd is a discontinuous curve having a number of branches.

The point of intersection of the two curves give the value of α

These values are approximately given by

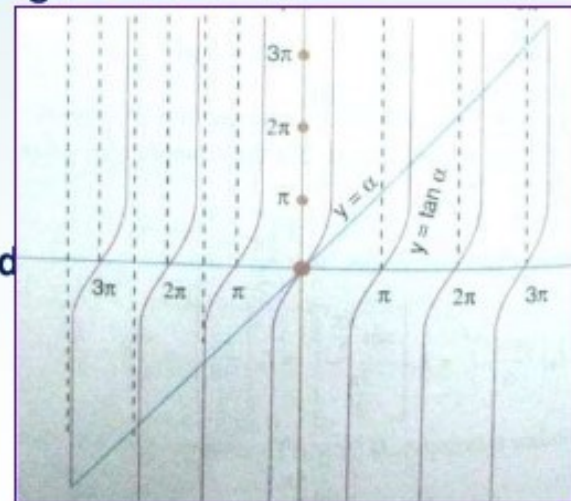
$$\alpha = 0, 3\pi/2, 5\pi/2, 7\pi/2, \dots$$

Substituting these values of α , we can find

$$I_0 = A^2$$

$$I_1 \approx A^2/22$$

$$I_2 \approx A^2/61$$



- The intensity of the 1st maximum is about 4.96% of the central maximum.

$$I = I_0 \left(\frac{\sin \alpha}{\alpha} \right)^2$$

$$\text{Put } \alpha = \frac{3\pi}{2}$$

$$I_1 = \frac{4}{9\pi^2} I_0 \quad I_2 = \frac{4}{25\pi^2} I_0 \quad I_3 = \frac{4}{49\pi^2} I_0$$

The ratio of relative intensities of successive maxima are

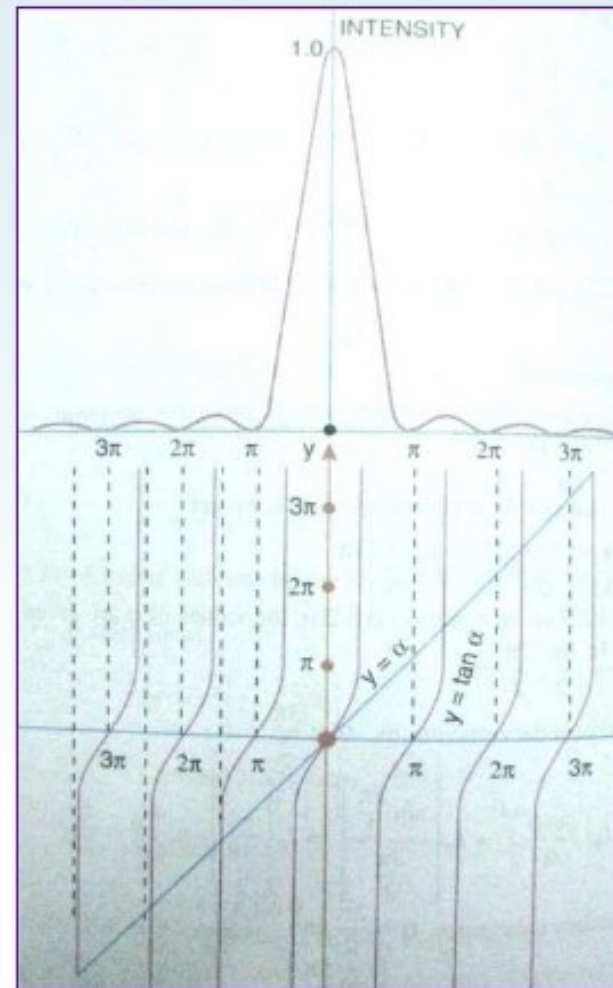
$$1 : \frac{4}{9\pi^2} : \frac{4}{25\pi^2} : \frac{4}{49\pi^2} : \dots\dots\dots$$

Continued.....

The principal maxima occurs at $\alpha = (\pi e \sin \theta)/\lambda = 0$, or $\theta = 0$
i.e. the principal maxima occurs at the same direction of light

The diffraction pattern consists of a bright principal maximum in the direction of incident light, having alternately minima and weak subsidiary maxima of rapidly decreasing intensity on either side of it.

The minima lie at $\alpha = \pm \pi, \pm 2\pi, \dots$



Continued....

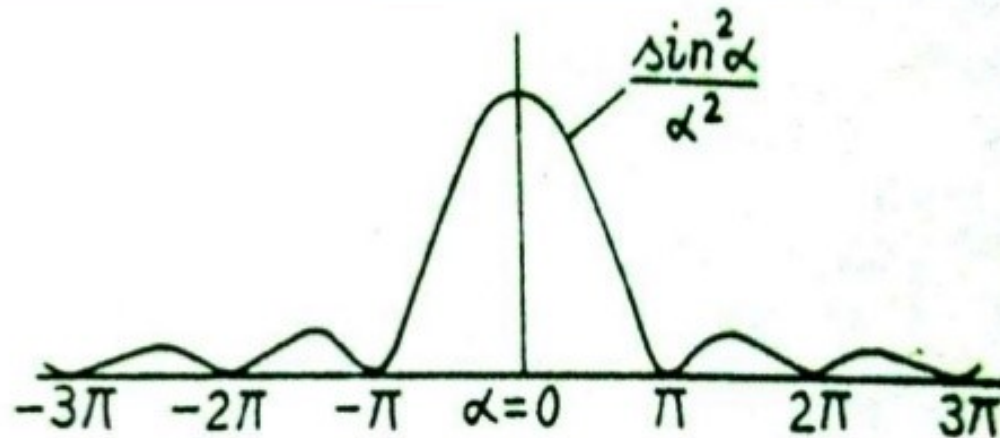
(a) The diffraction term, $(\sin^2 \alpha) / \alpha^2$, gives a central maximum in the direction $\theta = 0$, having alternately minima and subsidiary maxima of decreasing intensity on either side.

The minima are obtained in the directions given by

$$\sin \alpha = 0, \quad \alpha = \pm m\pi$$

$$(\pi e \sin \theta) / \lambda = \pm m\pi$$

$$e \sin \theta = \pm m\lambda \quad (m=1,2,3,\dots, \text{except zero})$$



Continued....

(b) The interference term, $\cos^2 \beta$, gives a set of equidistant dark and bright fringes, as in Young's double-slit interference experiment.

The bright fringes (maxima) are obtained in the directions given by

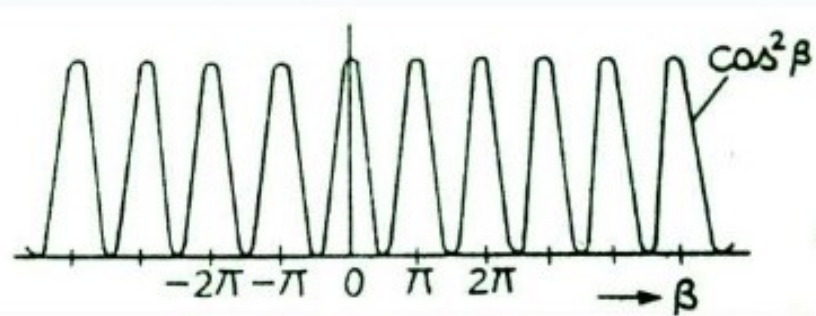
$$\cos^2 \beta = 1$$

$$\beta = \pm n\pi$$

$$(\pi/\lambda) (e+d) \sin \theta = \pm n\pi$$

$$(e+d) \sin \theta = \pm n\lambda, \quad \text{Where } n = 0, 1, 2, \dots$$

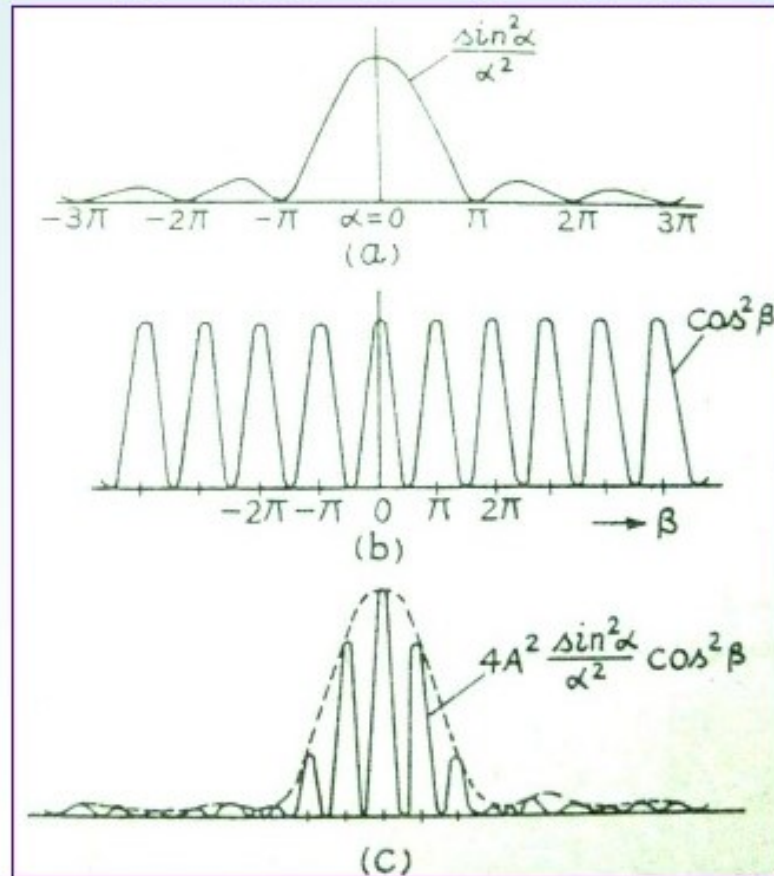
The various maxima corresponding to $n = 0, 1, 2, \dots$ are zero-order, first-order, second-order maxima



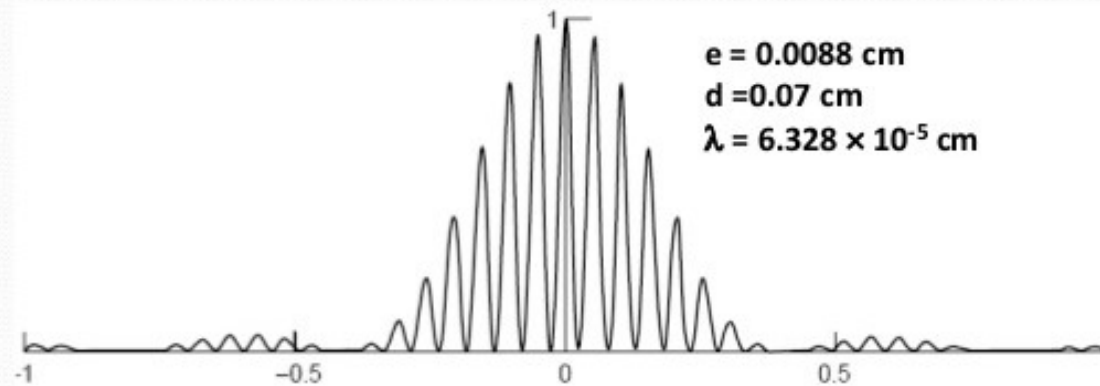
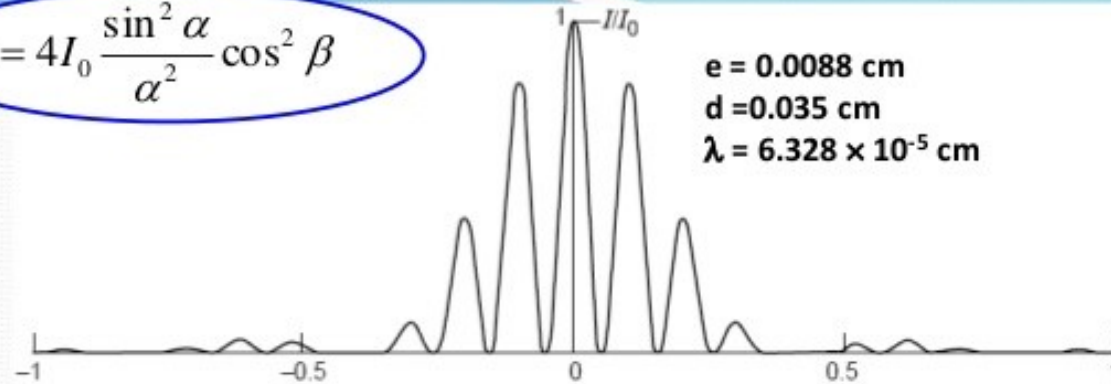
Continued....

(c) The intensity distribution in the resultant diffraction pattern is a plot of the product of the constant term $4 A^2$, diffraction term $(\sin^2 \alpha) / \alpha^2$, and the interference term $\cos^2 \beta$.

The entire pattern may be considered as consisting interference fringes due to light from both slits, the intensities of these fringes being governed by the diffraction occurring at the individual slits.



$$I = 4I_0 \frac{\sin^2 \alpha}{\alpha^2} \cos^2 \beta$$

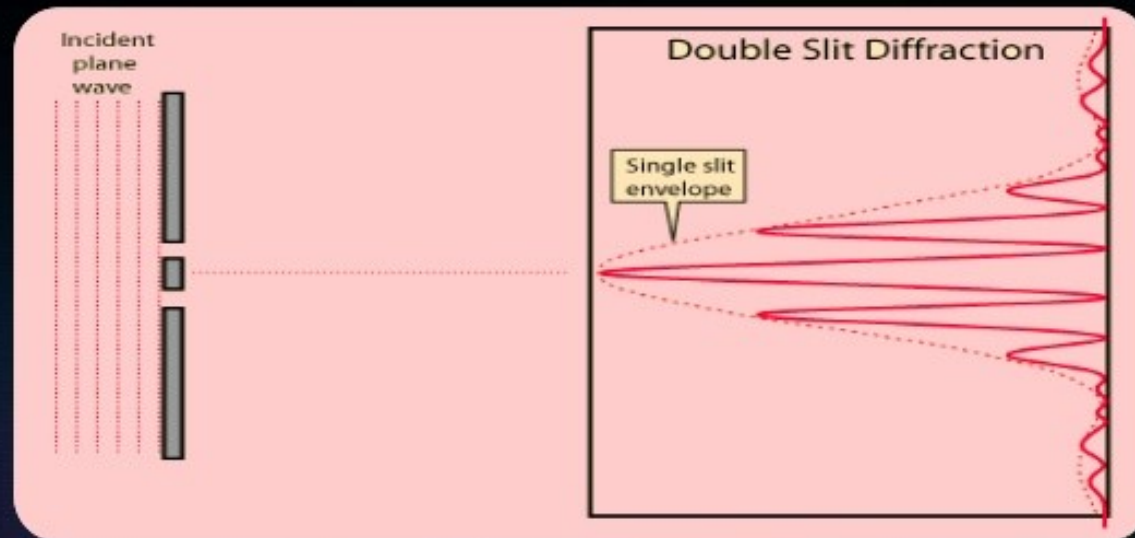


The double-slit intensity distribution corresponding to $e = 0.0088 \text{ cm}$, $\lambda = 6.328 \times 10^{-5} \text{ cm}$, and $d = 0.035 \text{ cm}$ and $d = 0.07 \text{ cm}$ respectively.



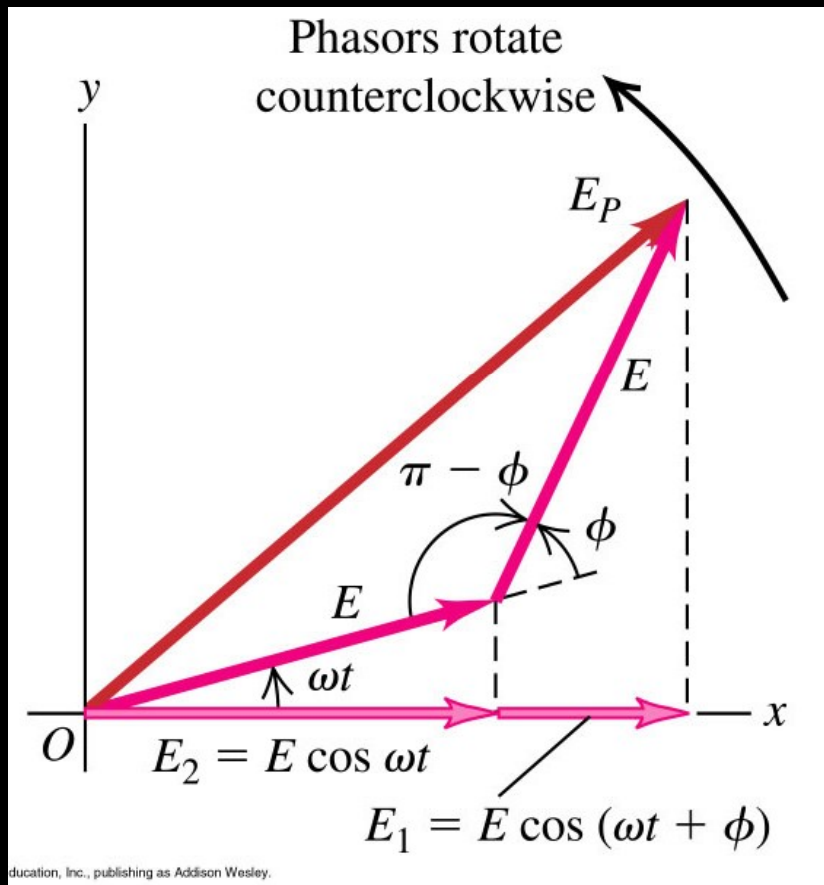
FRAUNHOFER DOUBLE SLIT DIFFRACTION

SAFETY: Do not look directly at the laser beam. Do not touch the laser or the equipment.



1. Two slits of width a are used separated by distance b .
2. It is combination of diffraction and interference pattern.
3. Central Maxima is formed 4 times more intense than in single slit.

Intensity Distribution



Use phasor diagram to do the addition $E_1 + E_2$

$$E_1 = E_0 \cos(\omega t)$$

$$E_2 = E_0 \cos(\omega t + \phi)$$

Using cosine rule:

$$E_{TOT}^2 = E_0^2 + E_0^2 - 2E_0^2 \cos(\pi - \phi)$$

$$= E_0^2 + E_0^2 + 2E_0^2 \cos(\phi)$$

$$= 2E_0^2 (1 + \cos(\phi))$$

$$E_{TOT}^2 = 4E_0^2 \left(\cos^2 \left(\frac{\phi}{2} \right) \right)$$


$$E_1 = A \frac{\sin \beta}{\beta} \cos(\omega t - \beta)$$

Similarly, the second slit will produce a field

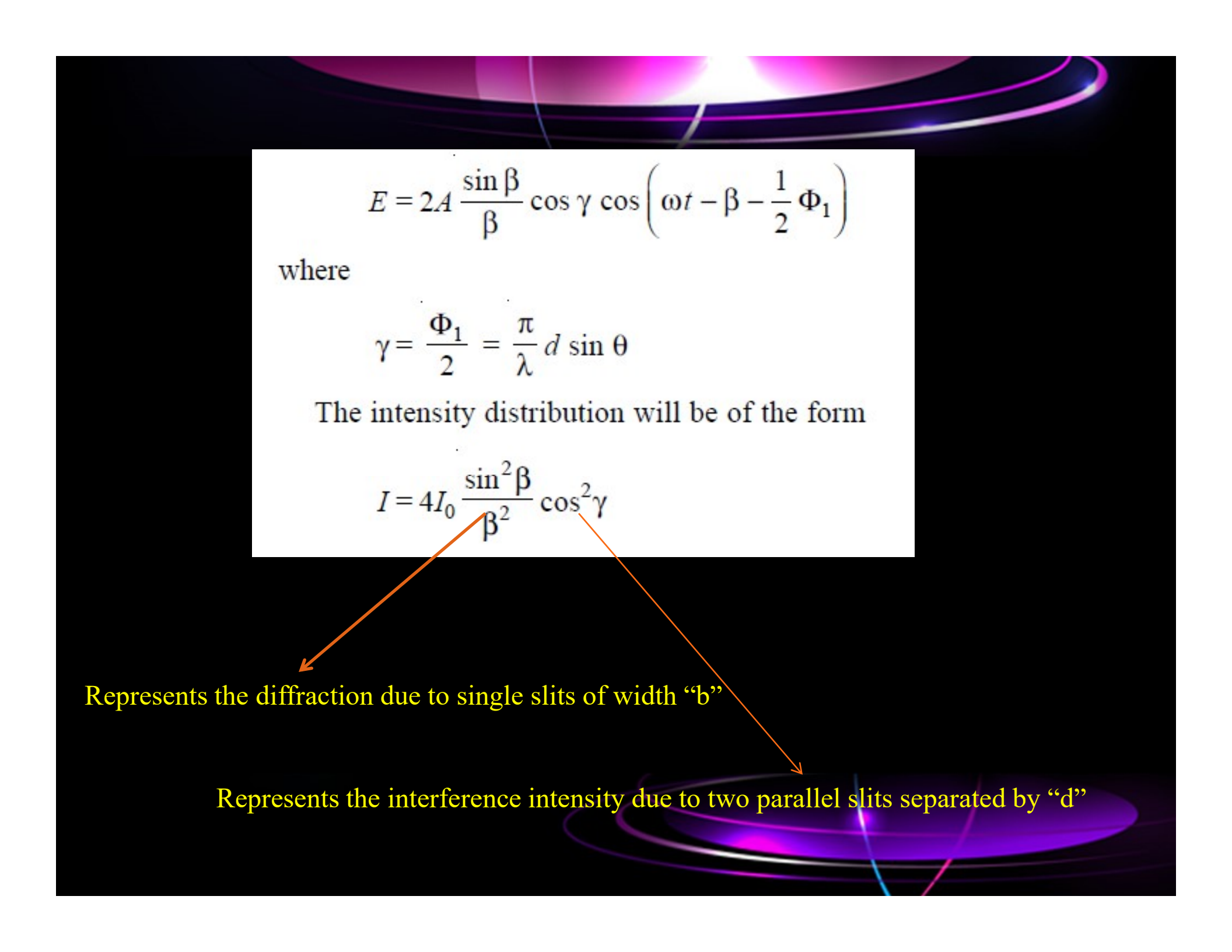
$$E_2 = A \frac{\sin \beta}{\beta} \cos(\omega t - \beta - \Phi_1)$$

at point P , where

$$\Phi_1 = \frac{2\pi}{\lambda} d \sin \theta$$

$$E = E_1 + E_2$$

$$= A \frac{\sin \beta}{\beta} [\cos(\omega t - \beta) + \cos(\omega t - \beta - \Phi_1)]$$



$$E = 2A \frac{\sin \beta}{\beta} \cos \gamma \cos \left(\omega t - \beta - \frac{1}{2} \Phi_1 \right)$$

where

$$\gamma = \frac{\Phi_1}{2} = \frac{\pi}{\lambda} d \sin \theta$$

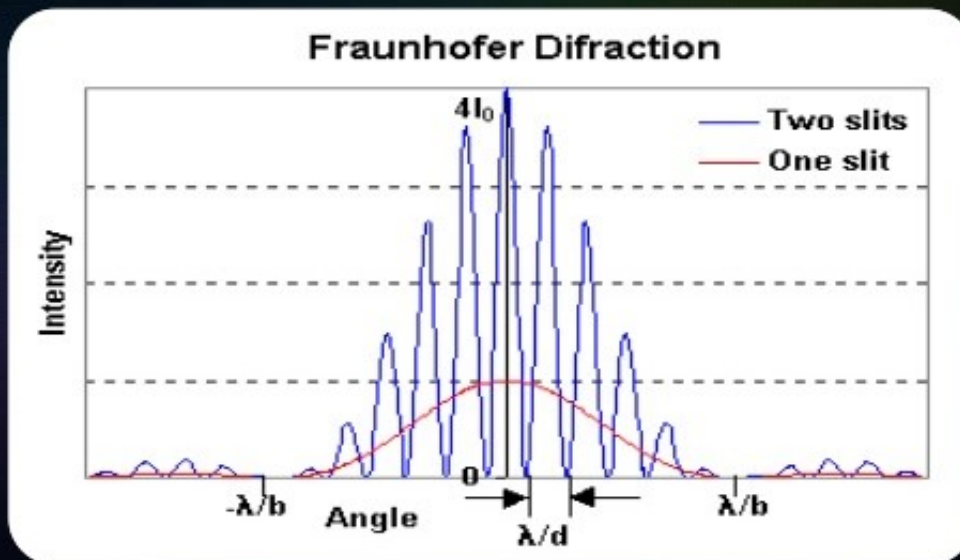
The intensity distribution will be of the form

$$I = 4I_0 \frac{\sin^2 \beta}{\beta^2} \cos^2 \gamma$$

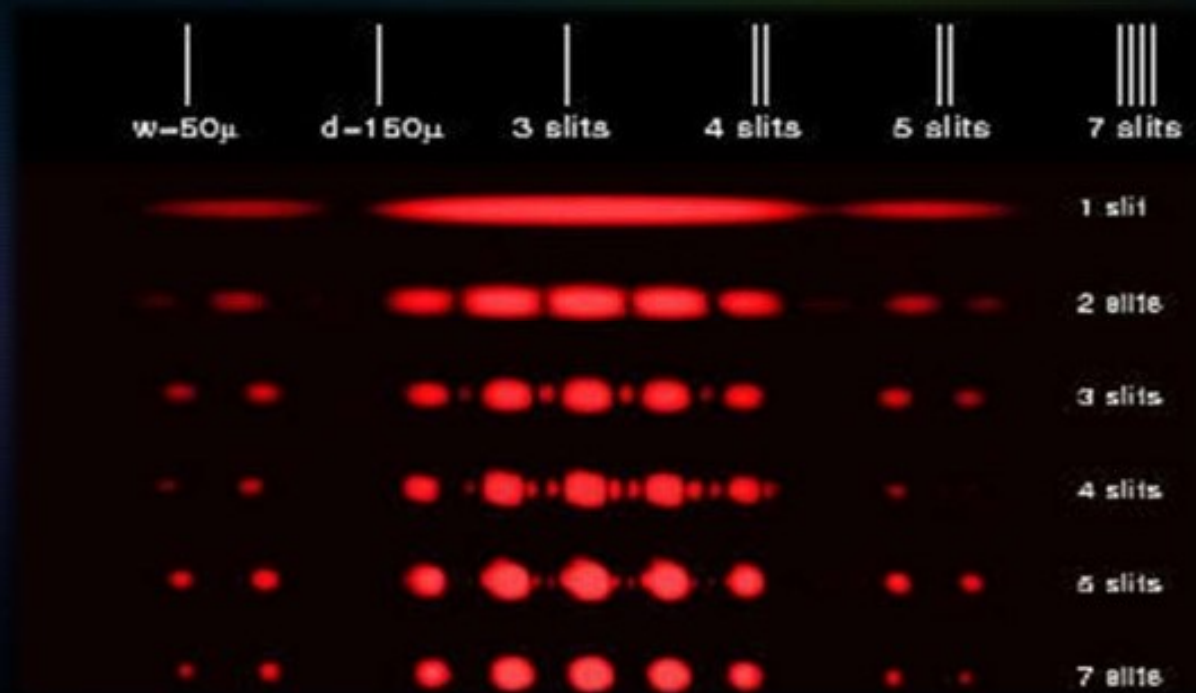
Represents the diffraction due to single slits of width “b”

Represents the interference intensity due to two parallel slits separated by “d”

Comparison between Single & Double slit :-



A plane diffraction grating is obtained when large number of parallel slits of equal width are separated by opaque spaces



Application :-

1. Diffraction plays a major role in light- and electron microscopy.
2. Diffraction is used in X-ray crystallography, called *Bragg diffraction* to deduce the structure of a crystal from the angles at which X-rays are diffracted from it.

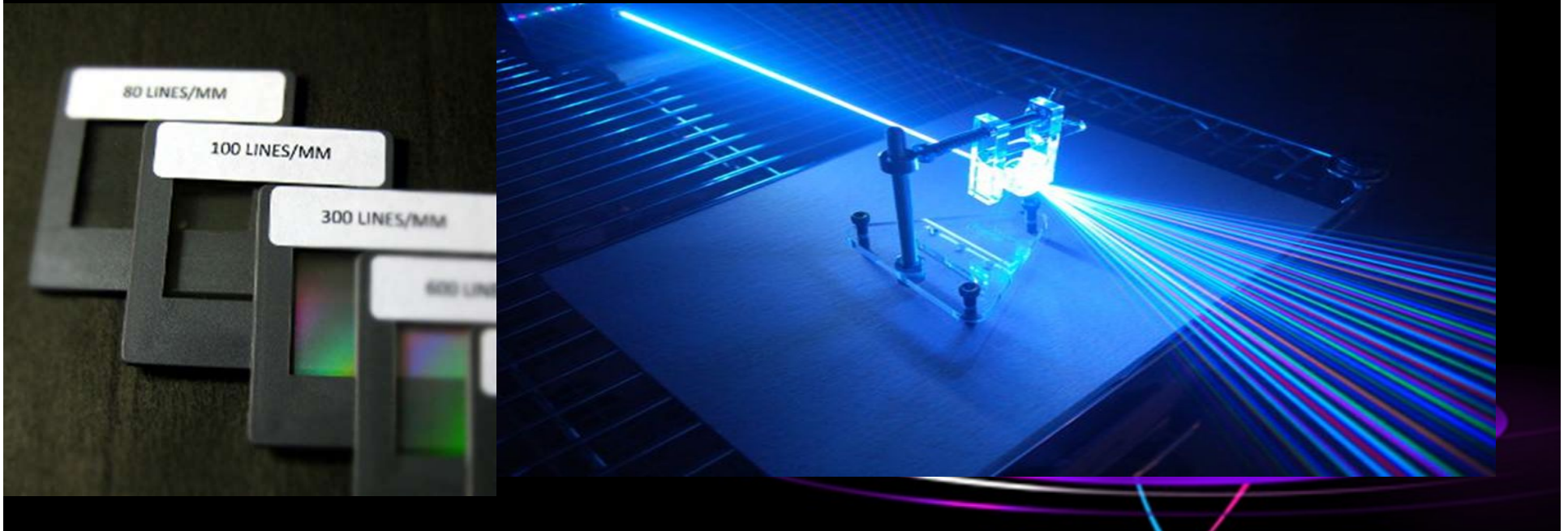
Grating

An arrangement consisting of large number of parallel slits of the same width and separated by equal opaque spaces is known as Diffraction grating. Gratings are constructed by ruling equidistant parallel lines on a transparent material such as glass, with a diamond point.

The ruled lines are opaque to light while the space between any two lines is transparent to light and acts as a slit. This is known as Gratings are constructed by ruling equidistant parallel lines on a transparent material such as glass, with a fine diamond point.

The ruled lines are opaque to light while the space between any two lines is transparent to light and acts as a slit. This is known as plane transmission grating.

When the spacing between the lines is of the order of the wavelength of light, then an appreciable deviation of the light is produced.



Diffraction Grating

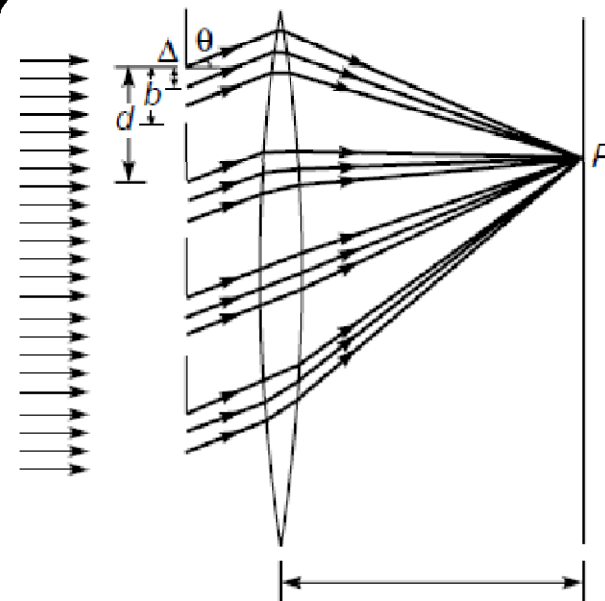
□ We assume that each slit consists of n equally spaced point sources with spacing (Δ). Thus the field at an arbitrary point P will essentially be a sum of N terms.

□ Where the first term represents the amplitude produced by the first slit, the second term by the second slit, etc. and the various symbols have the same meaning

$$E = A \frac{\sin \beta}{\beta} \cos (\omega t - \beta) + A \frac{\sin \beta}{\beta} \cos (\omega t - \beta - \Phi_1) + \cdots + A \frac{\sin \beta}{\beta} \cos [\omega t - \beta - (N-1)\Phi_1] \quad (4)$$

$$E = \frac{A \sin \beta}{\beta} \{ \cos (\omega t - \beta) + \cos (\omega t - \beta + \Phi_1) + \cdots + \cos [\omega t - \beta - (N-1)\Phi_1] \}$$

$$= \frac{A \sin \beta}{\beta} \frac{\sin N\gamma}{\sin \gamma} \cos \left[\omega t - \beta - \frac{1}{2} (N-1)\Phi_1 \right]$$



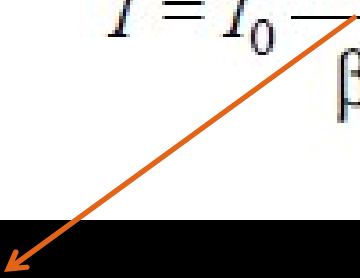
Fraunhofer diffraction of a plane wave incident normally on a multiple slit.

where

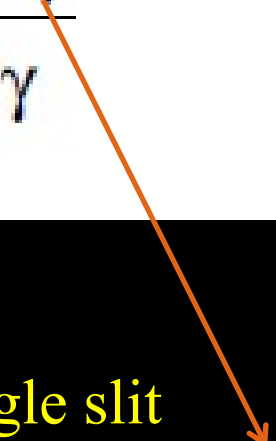
$$\gamma = \frac{\Phi_1}{2} = \frac{\pi}{\lambda} d \sin \theta$$

The corresponding intensity distribution will be

$$I = I_0 \frac{\sin^2 \beta}{\beta^2} \frac{\sin^2 N\gamma}{\sin^2 \gamma}$$


$$I_0 (\sin^2 \beta) / \beta^2$$

Represents the intensity due to single slit


$$(\sin^2 N\gamma) / \sin^2 \gamma$$

Represents the interference intensity due to N equispaced parallel slits.



For $N = 1$, the above Eq. reduces to the single-slit diffraction pattern and for $N = 2$, to the double-slit diffraction pattern.

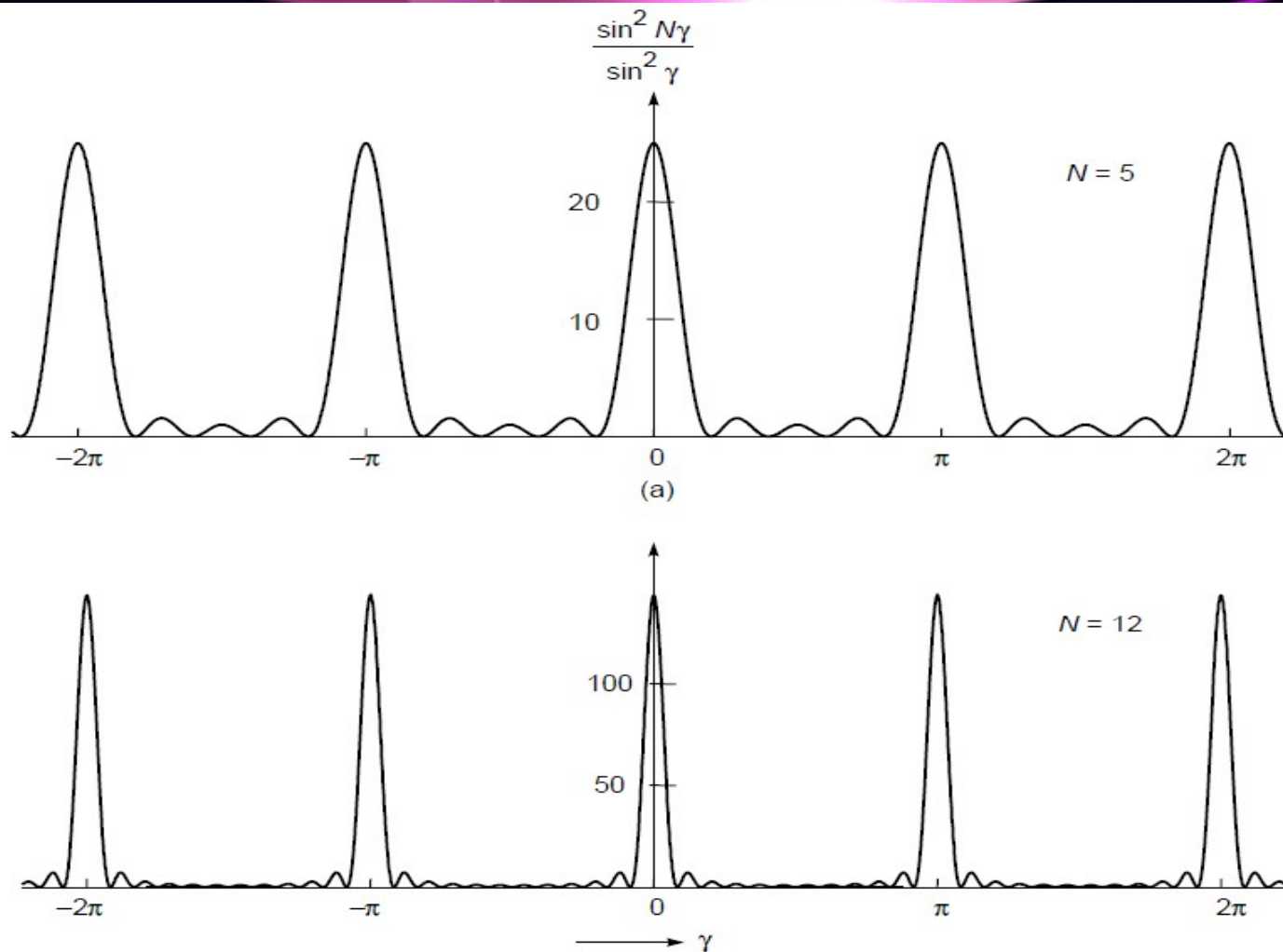
In we have given a plot of the function, As a function of g for $N = 5$ and $N = 11$. One can immediately see that as the value of N becomes very large, the above function becomes very sharply peaked at $\gamma = 0, \pi, 2\pi, \dots$

Between the two peaks, the function vanishes when,

$$\gamma = \frac{p\pi}{N} \quad p = \pm 1, \pm 2, \dots \quad \text{but} \quad p \neq 0, \pm N, \pm 2N$$

which are referred to as secondary minima.





The variation of the function $(\sin^2 N\gamma)/\sin^2 \gamma$ with γ for $N = 5$ and 12. As N becomes larger, the function becomes more and more sharply peaked at $\gamma = 0, \pm\pi, \pm2\pi, \pm3\pi, \dots$

When the value of N is very large, one obtains intense maxima at $\gamma \sim m\pi$, i.e., when

$$d \sin \theta = m\lambda \quad m = 0, 1, 2, \dots$$

This can be easily seen by noting that,

$$\lim_{\gamma \rightarrow m\pi} \frac{\sin N\gamma}{\sin \gamma} = \lim_{\gamma \rightarrow m\pi} \frac{N \cos N\gamma}{\cos \gamma} = \pm N$$

thus, the resultant amplitude and the corresponding intensity distributions are given by,

$$E = N \frac{A \sin \beta}{\beta}$$

$$I = N^2 I_0 \frac{\sin^2 \beta}{\beta^2}$$

$$\beta = \frac{\pi b \sin \theta}{\lambda} = \frac{\pi b}{\lambda} \frac{m\lambda}{d} = \frac{\pi b m}{d}$$

Such maxima are known as principal maxima. Physically, at these maxima the fields produced by each of the slits are in phase, and therefore, they add and the resultant field is N times the field produced by each of the slits.

Consequently, the intensity has a large value unless

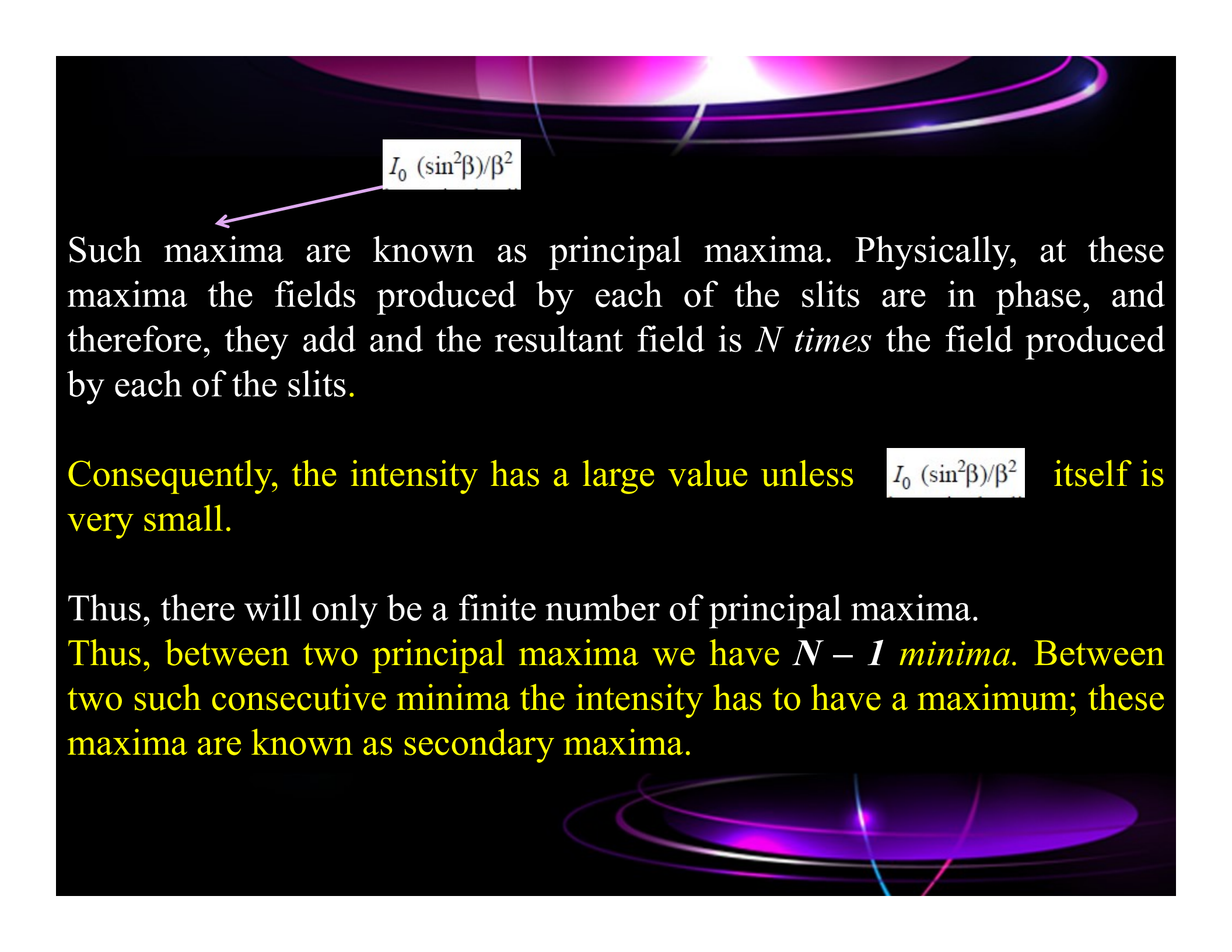
$$I_0 (\sin^2 \beta) / \beta^2 \quad \text{itself is very small.}$$

(Minima corresponding to the single slit diffraction)

$$b \sin \theta = n\lambda \quad n = 1, 2, 3, \dots$$

$$N\gamma = p\pi \quad p \neq N, 2N, \dots$$

$$d \sin \theta = \frac{\lambda}{N}, \frac{2\lambda}{N}, \dots, \frac{(N-1)\lambda}{N}, \frac{(N+1)\lambda}{N}, \frac{(N+2)\lambda}{N}, \dots, \frac{(2N-1)\lambda}{N}, \frac{(2N+1)\lambda}{N}, \frac{(2N+2)\lambda}{N}, \dots$$


$$I_0 (\sin^2\beta)/\beta^2$$

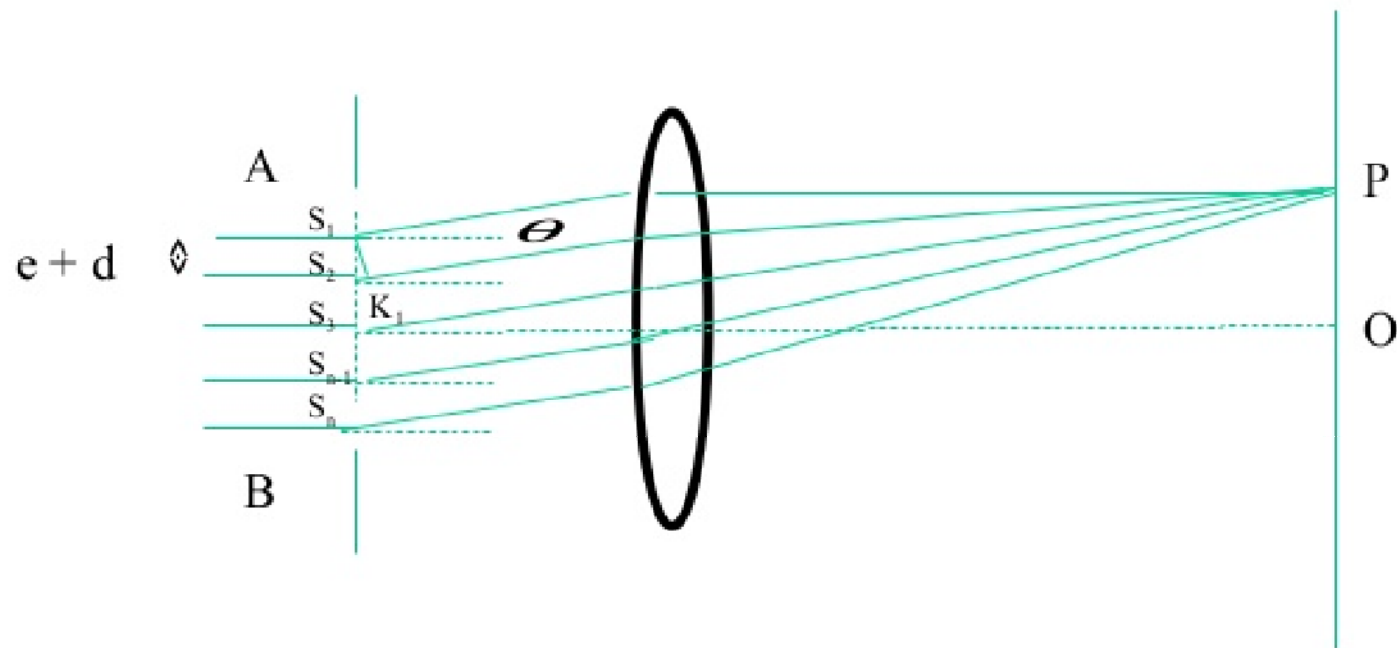

Such maxima are known as principal maxima. Physically, at these maxima the fields produced by each of the slits are in phase, and therefore, they add and the resultant field is N times the field produced by each of the slits.

Consequently, the intensity has a large value unless $I_0 (\sin^2\beta)/\beta^2$ itself is very small.

Thus, there will only be a finite number of principal maxima.

Thus, between two principal maxima we have $N - 1$ minima. Between two such consecutive minima the intensity has to have a maximum; these maxima are known as secondary maxima.

Diffraction due to grating



The amplitude from each slit in the direction θ is

$$R_0 = \frac{A \sin \alpha}{\alpha} \quad \text{where } \alpha = \frac{\pi e}{\lambda} \sin \theta$$

The path difference between the wavelets from S_1 and S_2 in the direction θ is

$$S_2K_1 = (e + d) \sin \theta$$

Hence the phase difference between them

$$\frac{2\pi}{\lambda} (e + d) \sin \theta = 2\beta$$

If N be the total number of slits in the grating, then by the method of vector addition of amplitudes, the resultant amplitude in the direction of θ will be

$$R = R_0 \frac{\sin N\beta}{\sin \beta} = \frac{A \sin \alpha}{\alpha} \frac{\sin N\beta}{\sin \beta}$$

Thus the resultant intensity at point P is

$$I = R^2 = A^2 \left(\frac{\sin \alpha}{\alpha} \right)^2 \left(\frac{\sin N\beta}{\sin \beta} \right)^2$$

The factor $A^2 \left(\frac{\sin \alpha}{\alpha} \right)^2$ gives the intensity distribution due to single slit,

while $\left(\frac{\sin N\beta}{\sin \beta} \right)^2$ gives the distribution of intensity in the diffraction pattern due to the interference in the waves due to N slits.

Principal Maxima

$$I = R^2 = A^2 \left(\frac{\sin \alpha}{\alpha} \right)^2 \left(\frac{\sin N\beta}{\sin \beta} \right)^2$$

The intensity will be maximum when

$$\sin \beta = 0 \quad \Rightarrow \quad \beta = \pm n\pi \quad n = 0, 1, 2, 3, \dots$$

This results in

$$\frac{\sin N\beta}{\sin \beta} = \frac{0}{0} \quad (\text{Indeterminate})$$

Apply L' Hospital rule

$$\lim_{\beta \rightarrow \pm n\pi} \frac{\sin N\beta}{\sin \beta} = \lim_{\beta \rightarrow \pm n\pi} \frac{\frac{d}{d\beta}(\sin N\beta)}{\frac{d}{d\beta}(\sin \beta)}$$

$$\lim_{\beta \rightarrow \pm n\pi} \frac{N \cos N\beta}{\cos \beta} \Rightarrow \pm N$$

This results in $I = A^2 \left(\frac{\sin \alpha}{\alpha} \right)^2 N^2$

The condition for principal maxima is

$$\sin \beta = 0 \quad \text{or} \quad \beta = \pm n\pi$$

$$\frac{\pi}{\lambda}(e + d) \sin \theta = \pm n\pi$$

$$(e + d) \sin \theta = \pm n\lambda$$

For $n = 0$, we get $\theta = 0$ and this gives the direction of zero order principal maxima. The value of $n = 1, 2, 3, \dots$ gives the direction of first, second, third order principal maxima.

Minima

$$I = R^2 = A^2 \left(\frac{\sin \alpha}{\alpha} \right)^2 \left(\frac{\sin N\beta}{\sin \beta} \right)^2$$

The intensity will be minimum when

$$\sin N\beta = 0 \quad \text{but} \quad \sin \beta \neq 0$$

Therefore $N\beta = \pm m\pi$

$$\frac{N\pi}{\lambda} (e + d) \sin \theta = \pm m\pi \quad \Rightarrow \quad N(e + d) \sin \theta = \pm m\lambda$$

Here m can have all integral values except 0, N, 2N, 3N.....

Here m can have values $1, 2, 3, \dots, (N-1)$. Thus there are $(N-1)$ equispaced minima between two consecutive principal maxima.

Secondary Maxima

In order to differentiate between the two consecutive minima there should be a maximum between them. Therefore there are $(N-2)$ maxima between $(N-1)$ minima. These maxima are known as secondary maxima.

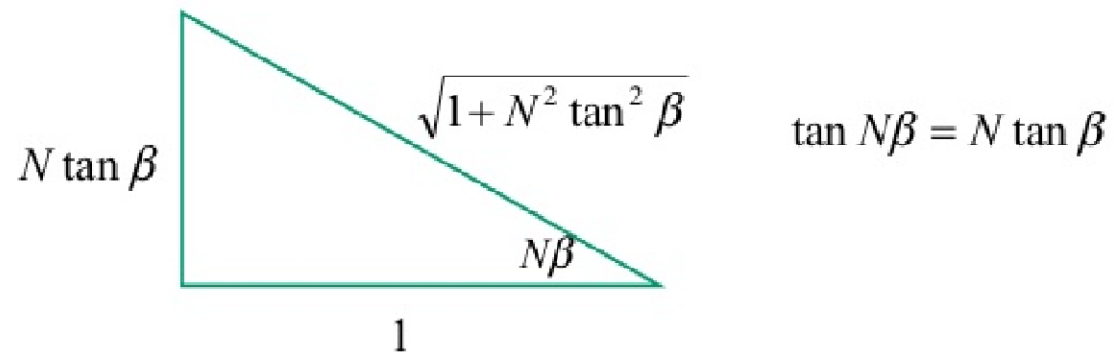
$$I = A^2 \left(\frac{\sin \alpha}{\alpha} \right)^2 \left(\frac{\sin N\beta}{\sin \beta} \right)^2$$

$$\frac{dI}{d\beta} = 0$$

$$\frac{A^2 \sin^2 \alpha}{\alpha^2} 2 \left(\frac{\sin N\beta}{\sin \beta} \right) \left[\frac{N \cos N\beta \sin \beta - \sin N\beta \cos \beta}{\sin^2 \beta} \right] = 0$$

$$N \cos N\beta \sin \beta - \sin N\beta \cos \beta = 0$$

$$\tan N\beta = N \tan \beta$$



$$\sin N\beta = \frac{N \tan \beta}{\sqrt{1 + N^2 \tan^2 \beta}}$$

$$\frac{\sin^2 N\beta}{\sin^2 \beta} = \frac{N^2 \tan^2 \beta / (\sqrt{1 + N^2 \tan^2 \beta})^2}{\sin^2 \beta}$$

$$= \frac{N^2}{\cos^2 \beta (1 + N^2 \tan^2 \beta)}$$

$$= \frac{N^2}{\cos^2 \beta + N^2 \sin^2 \beta}$$

$$= \frac{N^2}{1 - \sin^2 \beta + N^2 \sin^2 \beta}$$

$$\frac{\sin^2 N\beta}{\sin^2 \beta} = \frac{N^2}{1 + (N^2 - 1) \sin^2 \beta}$$

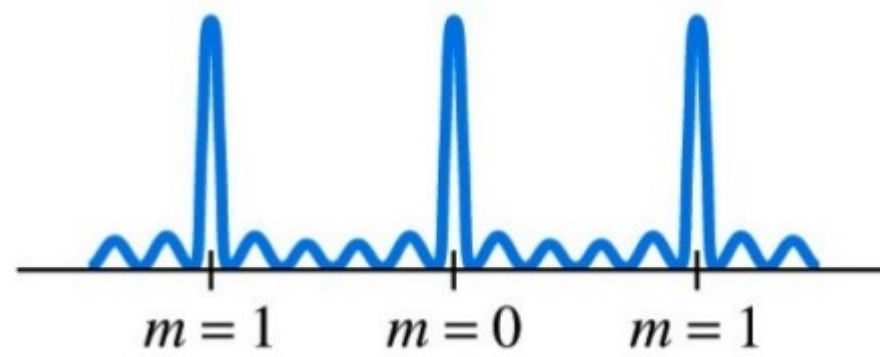
$$I = A^2 \left(\frac{\sin \alpha}{\alpha} \right)^2 \left(\frac{\sin N\beta}{\sin \beta} \right)^2$$

$$I = A^2 \frac{\sin^2 \alpha}{\alpha^2} \frac{N^2}{1 + (N^2 - 1) \sin^2 \beta}$$

The intensity of secondary maxima is proportional to

$$\frac{N^2}{1 + (N^2 - 1) \sin^2 \beta}$$

While the intensity of primary maxima is proportional to N^2



$$\frac{\text{Intensity of secondary maxima}}{\text{Intensity of principal maxima}} = \frac{1}{1 + (N^2 - 1) \sin^2 \beta}$$

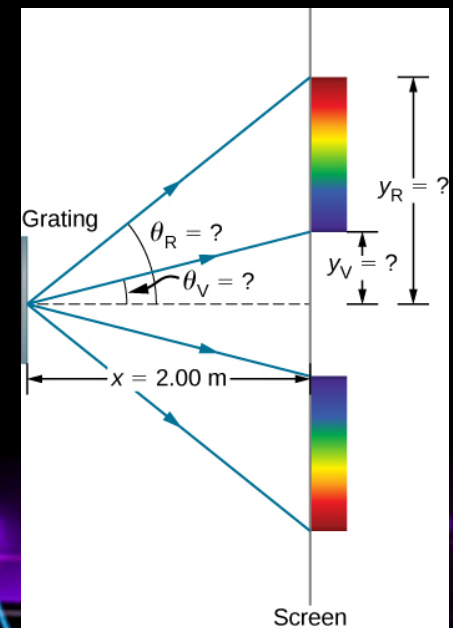
As N increases the intensity of secondary maxima decreases.

In case of diffraction grating N is very large.

Therefore the secondary maxima are not visible in the spectrum and there is complete darkness between two successive principal Maxima.

Problem 1

Diffraction gratings with 10,000 lines per centimeter are readily available. Suppose you have one, and you send a beam of white light through it to a screen 2.00 m away. (a) Find the angles for the first-order diffraction of the shortest and longest wavelengths of visible light (380 and 760 nm, respectively). (b) What is the distance between the ends of the rainbow of visible light produced on the screen for first-order interference?



Solution

The distance between slits is

$$d = (1\text{cm})/10,000$$

$$d = 1.00 \times 10^{-4}\text{cm} \text{ or } 1.00 \times 10^{-6}\text{m}.$$

$$d = (1\text{cm})/10,000 = 1.00 \times 10^{-4}\text{cm} \text{ or } 1.00 \times 10^{-6}\text{m}.$$

$$d \sin\theta = m\lambda, m = 1, 2, 3, \dots$$

For Violet of $m=1$ order

$$d \sin\theta_v = \lambda_v$$

$$1.00 \times 10^{-6}\text{m} \sin\theta_v = 380 \times 10^{-9}\text{m}$$

$$\theta_v = 22.33^\circ$$

Similarly for red

$$\theta_r = 49.46^\circ$$

The distances on the screen are labeled y_v and y_r

Notice that $\tan\theta = y/x$.

We can solve for y_v and y_r That is,

$$y_v = x \tan \theta_v \text{ and } y_r = x \tan \theta_r$$

$$y_v - y_r = 1.523 \text{ m}$$

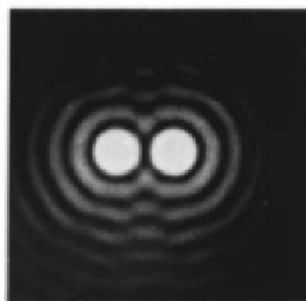
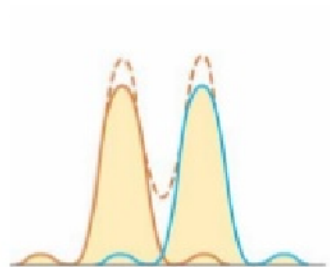


Rayleigh Criteria For Resolution

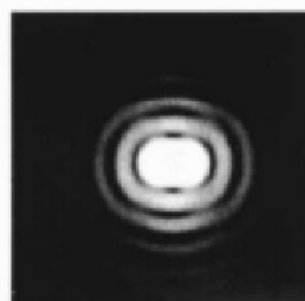
Resolving Power: An optical instrument used to resolve the two closely spaced objects is called Resolving power.

According to Raleigh, two closely spaced points are said to be just resolved if the principal maxima of one coincides with the first minima of the other or vice versa.

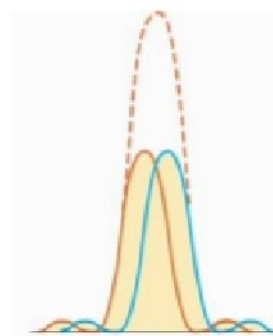




(a)

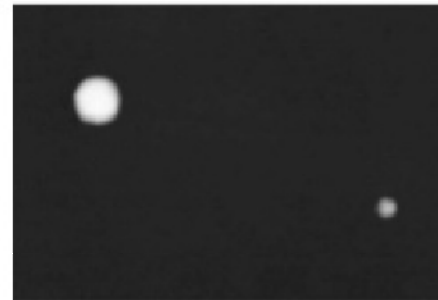
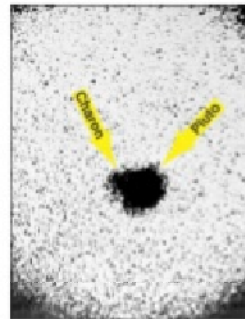


(b)



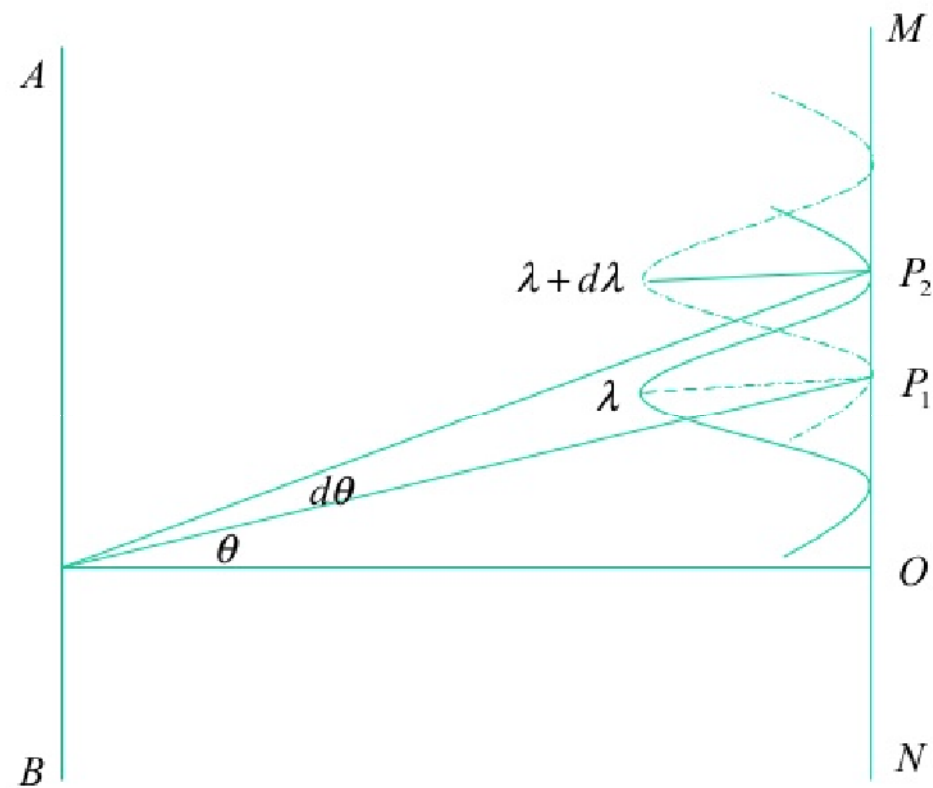
(c)

Resolution, Example



- Pluto and its moon, Charon
- Left: Earth-based telescope is blurred
- Right: Hubble Space Telescope clearly resolves the two objects

Resolving Power of Plane Diffraction grating



Resolving Power of Plane Diffraction grating

The resolving power of the grating is defined as the ratio of wavelength (λ) to the difference $d\lambda$ of the wavelength

The direction of n^{th} principal maxima for wavelength λ is given by

$$(e + d) \sin \theta = n\lambda$$

The direction of n^{th} principal maxima for wavelength $\lambda + d\lambda$ is given by

$$(e + d) \sin(\theta + d\theta) = n(\lambda + d\lambda) \quad (1)$$

The minima for wavelength λ is given by

$$N(e+d)\sin\theta = m\lambda$$

Here m can have all integral values except $0, N, 2N, 3N, \dots$ because for these values of m the condition of maxima is satisfied.

The first minimum adjacent to n^{th} principal maxima in the direction $(\theta+d\theta)$ can be obtained by putting m as $(nN+1)$

$$N(e+d)\sin(\theta+d\theta) = (nN+1)\lambda$$

$$(e+d)\sin(\theta+d\theta) = \frac{(nN+1)\lambda}{N} \quad (2)$$

Comparing equation (1) and (2)

$$n(\lambda + d\lambda) = \frac{(nN + 1)\lambda}{N}$$

$$n\lambda + nd\lambda = n\lambda + \frac{\lambda}{N}$$

$$nd\lambda = \frac{\lambda}{N} \Rightarrow \frac{\lambda}{d\lambda} = nN$$

Since resolving power is [], it means that larger will be the number of lines per cm of a grating greater will be the resolving power.

DISPERSIVE POWER OF A GRATING

Dispersive power of a grating is defined as the ratio of the difference in the angle of diffraction of any two neighbouring spectral lines to the difference in wavelength between the two spectral lines.

It can also be defined as the difference in the angle of diffraction per unit change in wavelength.

The diffraction of the n^{th} order principal maximum for a wavelength λ is given by the equation,

$$(a+b) \sin \theta = n\lambda$$

Differentiating this equation with respect to θ and λ

[$a+b$ is constant and n is constant in a given order]

$$\begin{aligned}(a+b) \cos \theta \, d\theta &= n \, d\lambda \\ \frac{d\theta}{d\lambda} &= \frac{n}{(a+b) \cos \theta} \\ \frac{d\theta}{d\lambda} &= \frac{n N'}{\cos \theta}\end{aligned}$$

$\frac{d\theta}{d\lambda}$ Is the dispersive power

n Is the order of the spectrum

N' Is the number of lines per cm in the grating.

θ Is the angle of diffraction for the n^{th} order principal maximum of wavelength λ



Dispersive power of the Grating depends on:

1. Directly proportional to the order of n
2. Inversely proportional to grating element $(a+b)$
3. Inversely proportional to $\cos\theta$.

If the value of $\cos\theta$ is large, then $\cos\theta$ will be small and Dispersive power $d\theta/d\lambda$ is high

As $\theta_r > \theta_v$ in a given spectrum, therefore dispersive power in the red Region is greater than in violet region.

If θ is small, $\cos\theta = 1$, hence $d\theta \propto d\lambda$.

Such spectrum is known as normal spectrum



Resolving power of Microscope

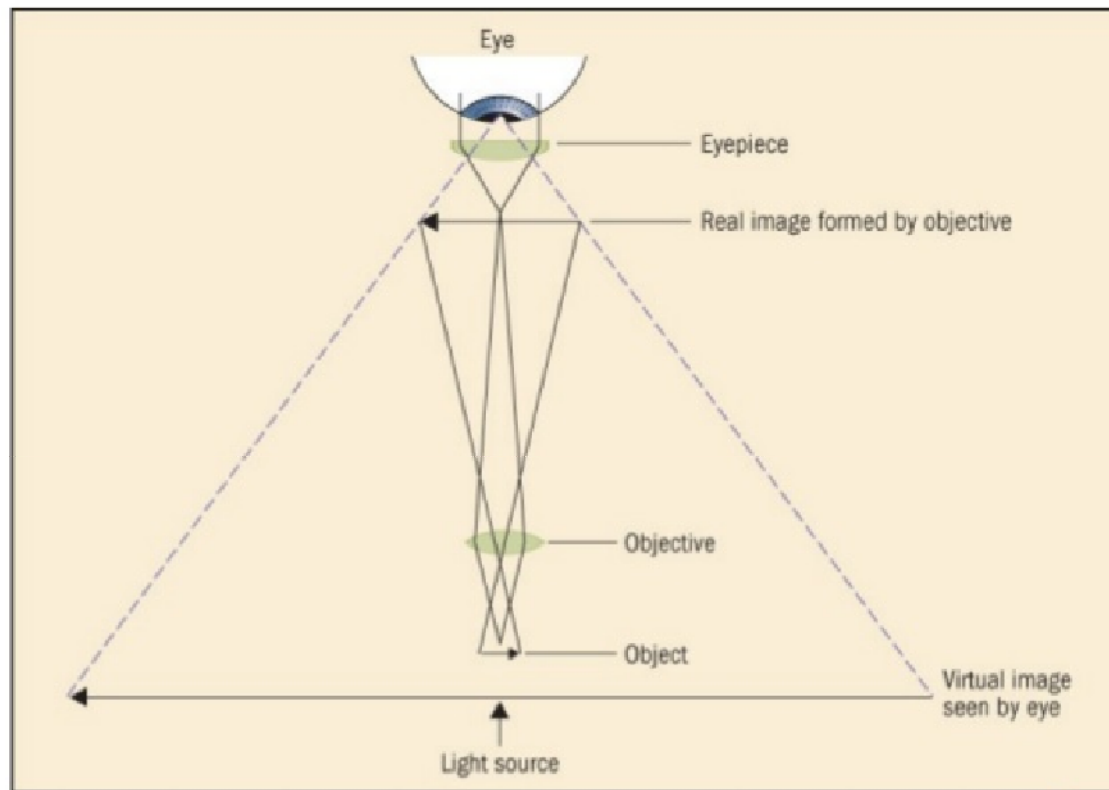
INTRODUCTION

- Micro: small and Scope: to view
- A microscope is an optical instrument that uses a lens or a combination of lenses to magnify and resolve the fine details of an object.
- The earliest methods for examining physical evidence relied solely on the microscope.
- The magnified image seen by looking through a lens is known as a *virtual image*, whereas an image viewed directly is known as a *real image*.
- The object to be magnified is placed under the lower lens, called the objective and viewed through the upper lens, called the eyepiece.

COMPOUND MICROSCOPE

○ Principle

A real, inverted and magnified image of the object is first produced by the objective lens. This collects the light from the specimen and forms the primary image. The eyepiece enlarges the primary image thus an ***erect, virtual and magnified image*** is formed. This image again acts as virtual object for the eye (lens of eye) finally an image is formed on retina thus enabling to see magnified image.



The principle of the compound microscope. The passage of light through two lenses forms the virtual image of the object seen by the eye.

RESOLVING POWER

- **Resolving Power** – the ability of an optical system to distinguish or separate two small, distinct objects or points from one another
Resolving power is a function of the wavelength of light being used (λ), in nm, and another factor called “the numerical aperture” (NA) of the objective lens
- **Numerical Aperture** – a mathematical constant that describes the relative efficiency of a lens in bending light rays
- In general, the shorter the wavelength of light being used (λ), the better the resolving power

$$\text{Resolving power (R.P.)} = \frac{\lambda \text{ in nm}}{2 \times \text{N.A.}}$$

Depth of Focus – The range in distance between the lens and the specimen that can be changed without upsetting the sharpness or resolution of the image. *Depth of focus decreases as magnification increases.*

- **Diameter of Field** – The diameter of the total field of view through a microscope at a given magnification. *Diameter of field decreases as magnification increases.*

