

Digital Image Processing

Lecture 1 Introduction & Fundamentals

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Introduction to the course

▶ Article Reading and Project

- Develop face Recognition-
- Gender Classification
- Code correction
- Obstacle detection

Journals & Conferences in Image Processing

▶ Journals:

- IEEE T IMAGE PROCESSING
- IEEE T MEDICAL IMAGING
- INTL J COMP. VISION
- IEEE T PATTERN ANALYSIS MACHINE INTELLIGENCE
- PATTERN RECOGNITION
- COMP. VISION AND IMAGE UNDERSTANDING
- IMAGE AND VISION COMPUTING

... ..

▶ Conferences:

- CVPR: Comp. Vision and Pattern Recognition
- ICCV: Intl Conf on Computer Vision
- ACM Multimedia
- ICIP
- SPIE
- ECCV: European Conf on Computer Vision
- CAIP: Intl Conf on Comp. Analysis of Images and Patterns

... ..

Introduction

- ▶ What is Digital Image Processing?
- ▶ Digital image processing deals with manipulation of digital images through a digital computer. It is a subfield of signals and systems but focus particularly on images.

Digital Image

— a two-dimensional function , x and y are spatial coordinates , The amplitude f is called intensity or gray level at the point (x, y)

Digital Image Processing

— process digital images by means of computer, it covers low-, mid-, and high-level processes

low-level: inputs and outputs are images

mid-level: outputs are attributes extracted from input images

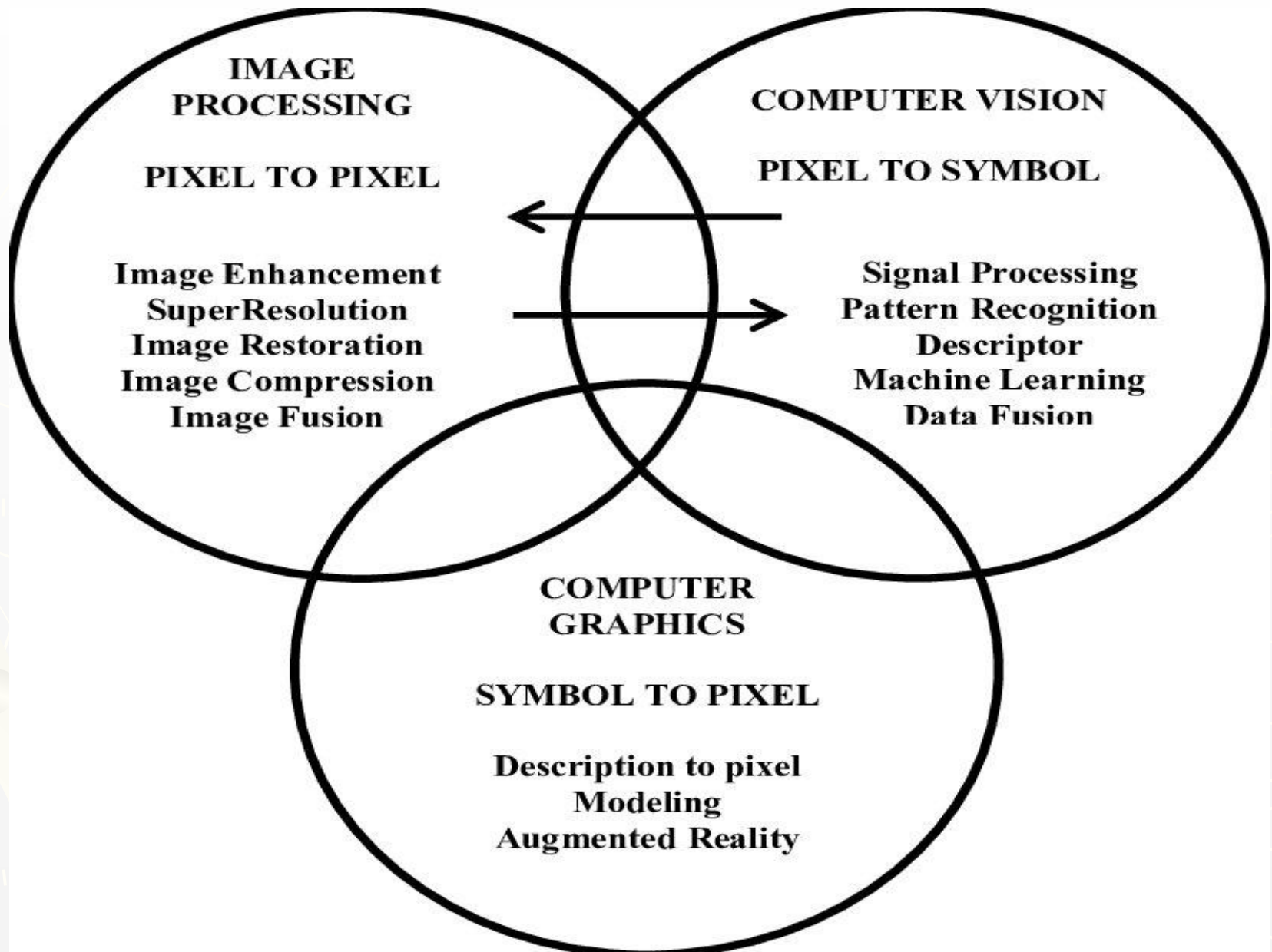
high-level: recognition of individual objects

Pixel — the elements of a digital image

Relationship between a digital image and a signal

- ▶ **If the image is a two dimensional array then what does it have to do with a signal?**
- ▶ Since anything that conveys information or broadcast a message in physical world between two observers is a signal.
- ▶ **How a digital image is formed**
- ▶ Since capturing an image from a camera is a physical process. The sunlight is used as a source of energy.
- ▶ A sensor array is used for the acquisition of the image.
- ▶ So when the sunlight falls upon the object, then the amount of light reflected by that object is sensed by the sensors, and a continuous voltage signal is generated by the amount of sensed data.

Overlapping fields

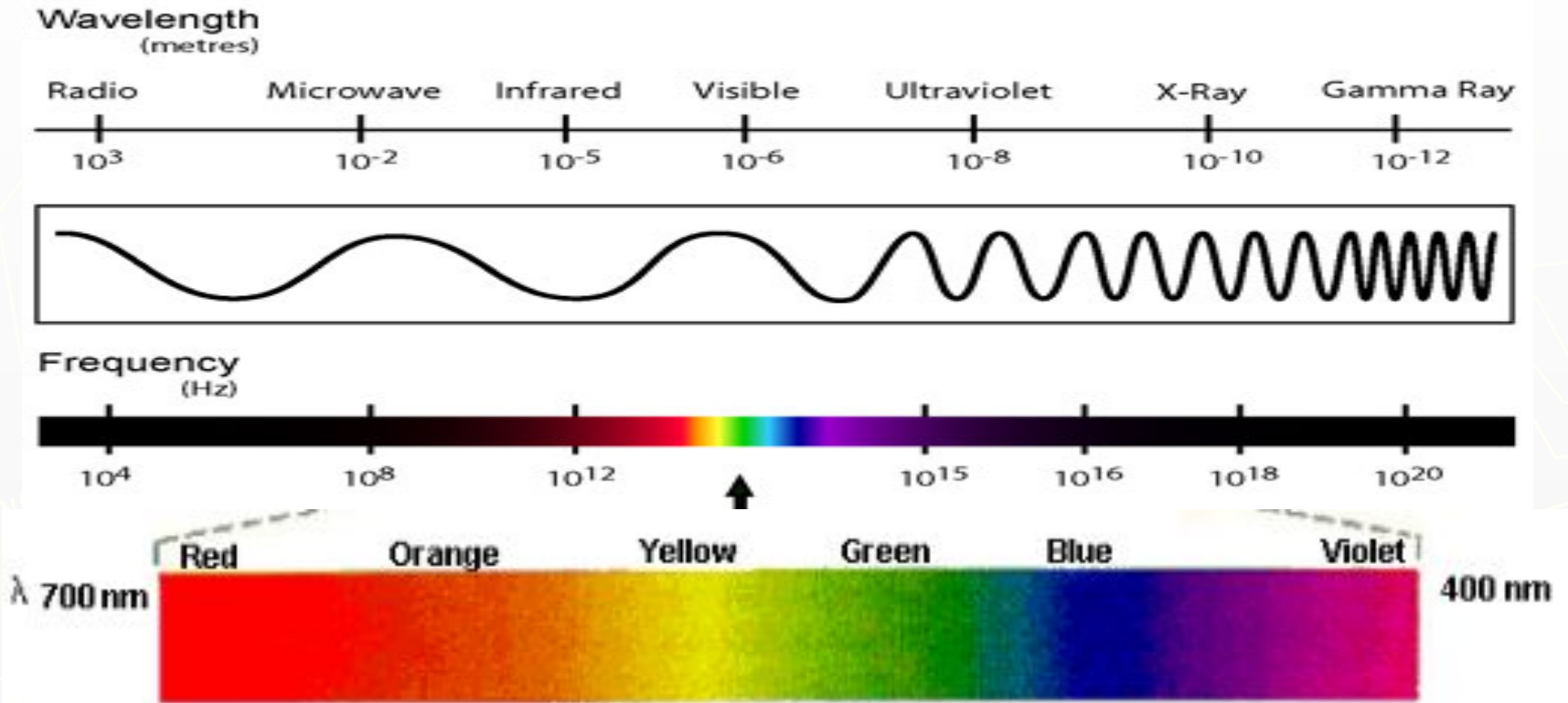


Q &A

- ▶ How can you tell that an image is a form of signal?
- ▶ Difference among image processing, computer vision, and computer graphics.

Light and EM Spectrum

THE ELECTRO MAGNETIC SPECTRUM



$$c = \lambda \nu$$

$$E = h\nu, \quad h: \text{Planck's constant.}$$

In physics, electromagnetic radiation refers to the waves of the electromagnetic field, propagating through space, carrying electromagnetic radiant energy. It includes radio waves, microwaves, infrared, light, ultraviolet, X-rays, and gamma rays.

Electromagnetic (EM) energy spectrum

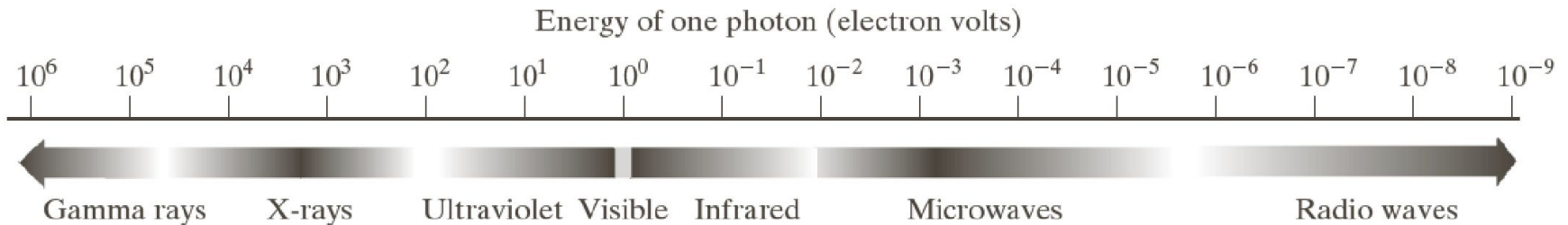


FIGURE 1.5 The electromagnetic spectrum arranged according to energy per photon.

Major uses

Gamma-ray imaging: nuclear medicine and astronomical observations

X-rays: medical diagnostics, industry, and astronomy, etc.

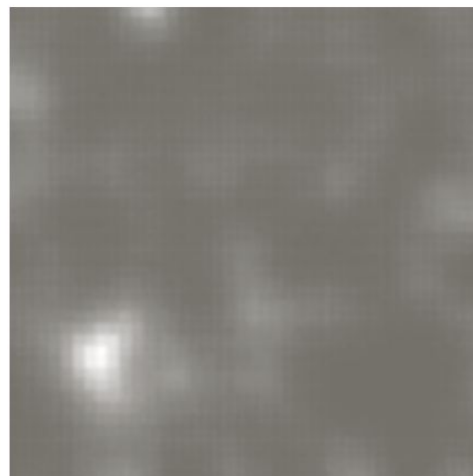
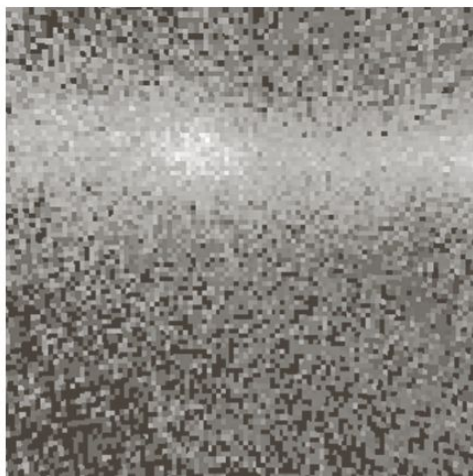
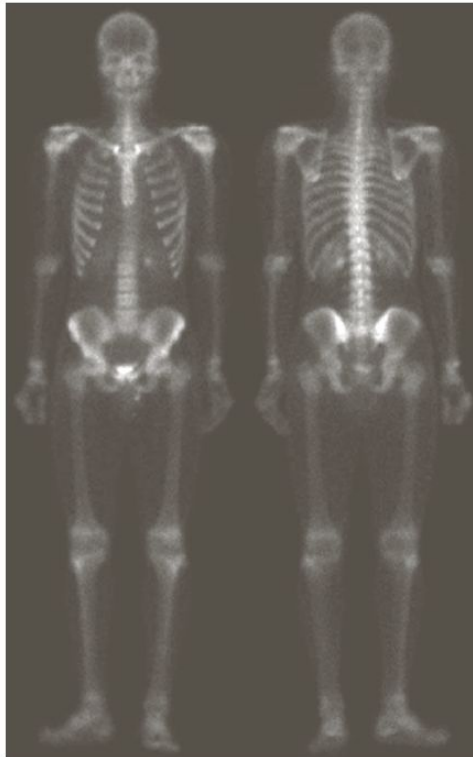
Ultraviolet: lithography, industrial inspection, microscopy, lasers, biological imaging, and astronomical observations

Visible and infrared bands: light microscopy, astronomy, remote sensing, industry, and law enforcement

Microwave band: radar

Radio band: medicine (such as MRI) and astronomy

Examples: Gama-Ray Imaging



a	b
c	d

FIGURE 1.6

Examples of gamma-ray imaging. (a) Bone scan. (b) PET image. (c) Cygnus Loop. (d) Gamma radiation (bright spot) from a reactor valve.

(Images courtesy of (a) G.E.

Medical Systems,

(b) Dr. Michael

E. Casey, CTI

PET Systems,

(c) NASA,

(d) Professors

Zhong He and

David K. Wehe,

University of

Michigan.)

PET(positron
emission
tomography),
Cygnus loop(
stationary gas
cloud generated
around 15000
years ago when
a star was
exploded

Examples: X-Ray Imaging

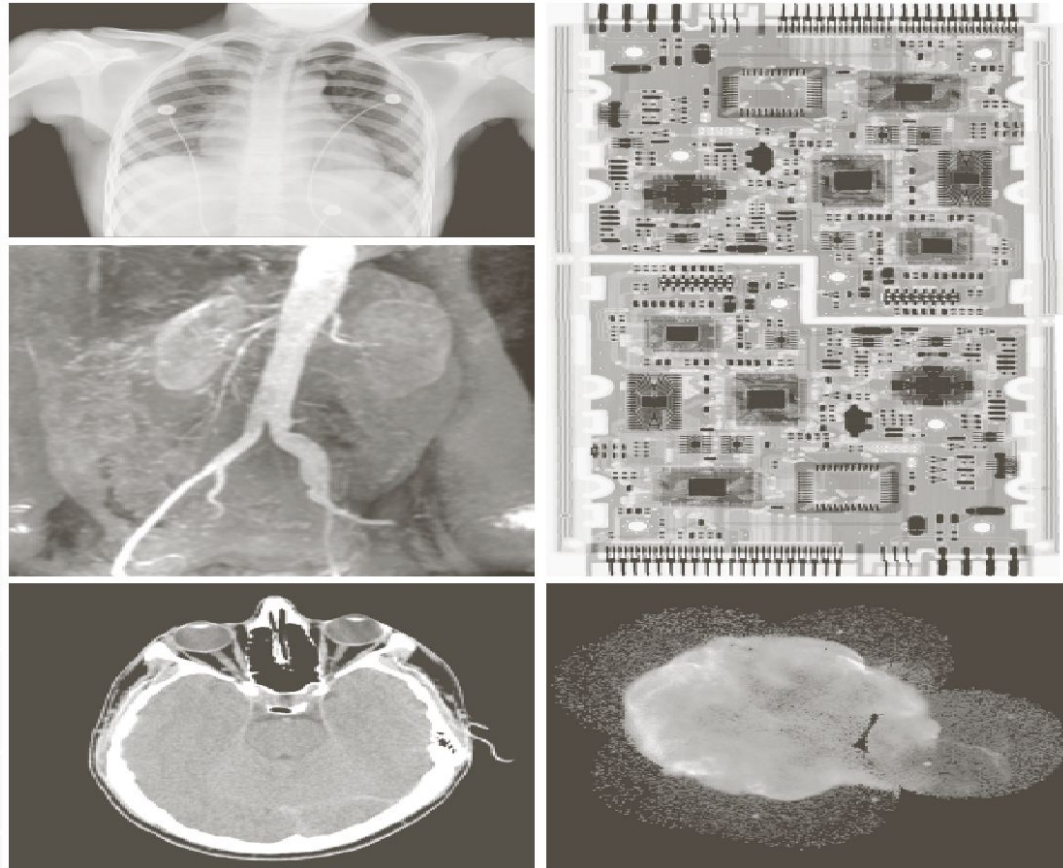
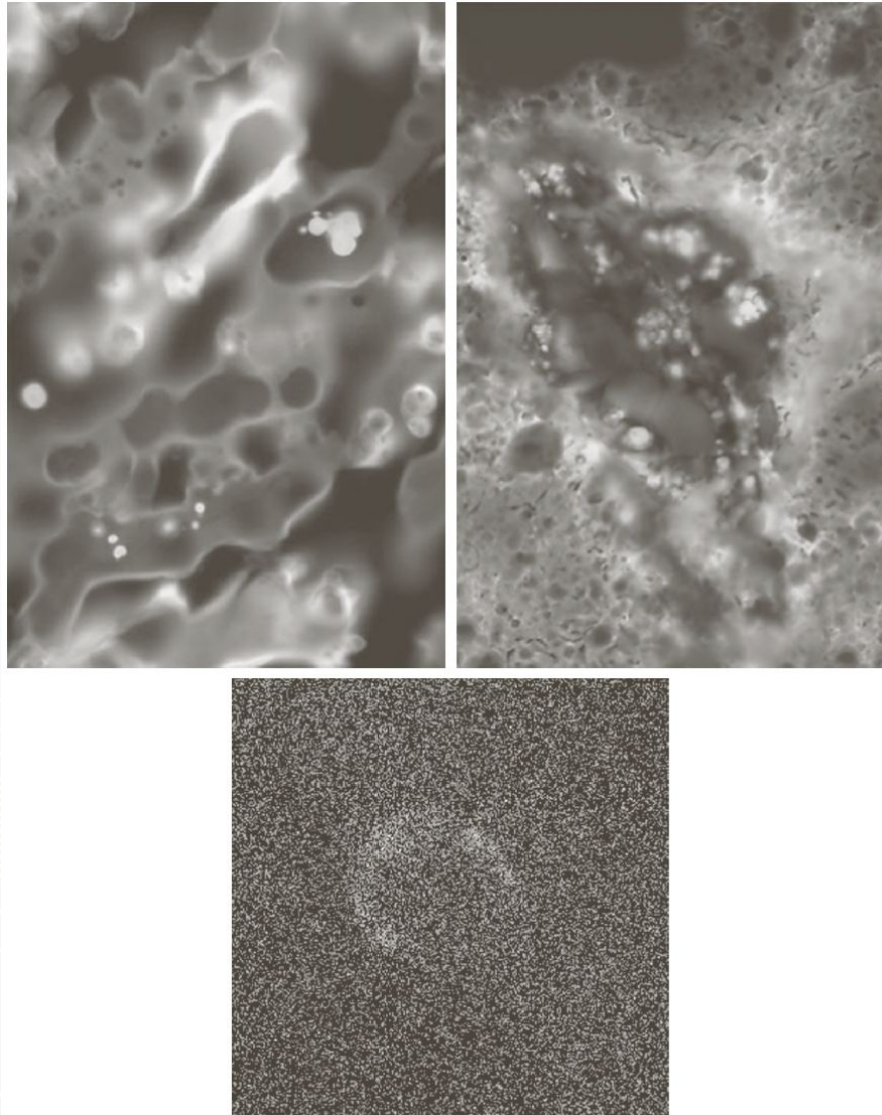


FIGURE 1.7 Examples of X-ray imaging. (a) Chest X-ray. (b) Aortic angiogram. (c) Head CT. (d) Circuit boards. (e) Cygnus Loop. (Images courtesy of (a) and (c) Dr. David R. Pickens, Dept. of Radiology & Radiological Sciences, Vanderbilt University Medical Center; (b) Dr. Thomas R. Gest, Division of Anatomical Sciences, University of Michigan Medical School; (d) Mr. Joseph E. Pascente, Lixi, Inc.; and (e) NASA.)

Examples: Ultraviolet Imaging



a b
c

FIGURE 1.8

Examples of ultraviolet imaging.

(a) Normal corn.
(b) Smut corn.
(c) Cygnus Loop.
(Images courtesy of (a) and (b) Dr. Michael W. Davidson, Florida State University, (c) NASA.)

Examples: Visual and Infrared Imaging

TABLE 1.1

Thematic bands
in NASA's
LANDSAT
satellite.

Band No.	Name	Wavelength (μm)	Characteristics and Uses
1	Visible blue	0.45–0.52	Maximum water penetration
2	Visible green	0.52–0.60	Good for measuring plant vigor
3	Visible red	0.63–0.69	Vegetation discrimination
4	Near infrared	0.76–0.90	Biomass and shoreline mapping
5	Middle infrared	1.55–1.75	Moisture content of soil and vegetation
6	Thermal infrared	10.4–12.5	Soil moisture; thermal mapping
7	Middle infrared	2.08–2.35	Mineral mapping

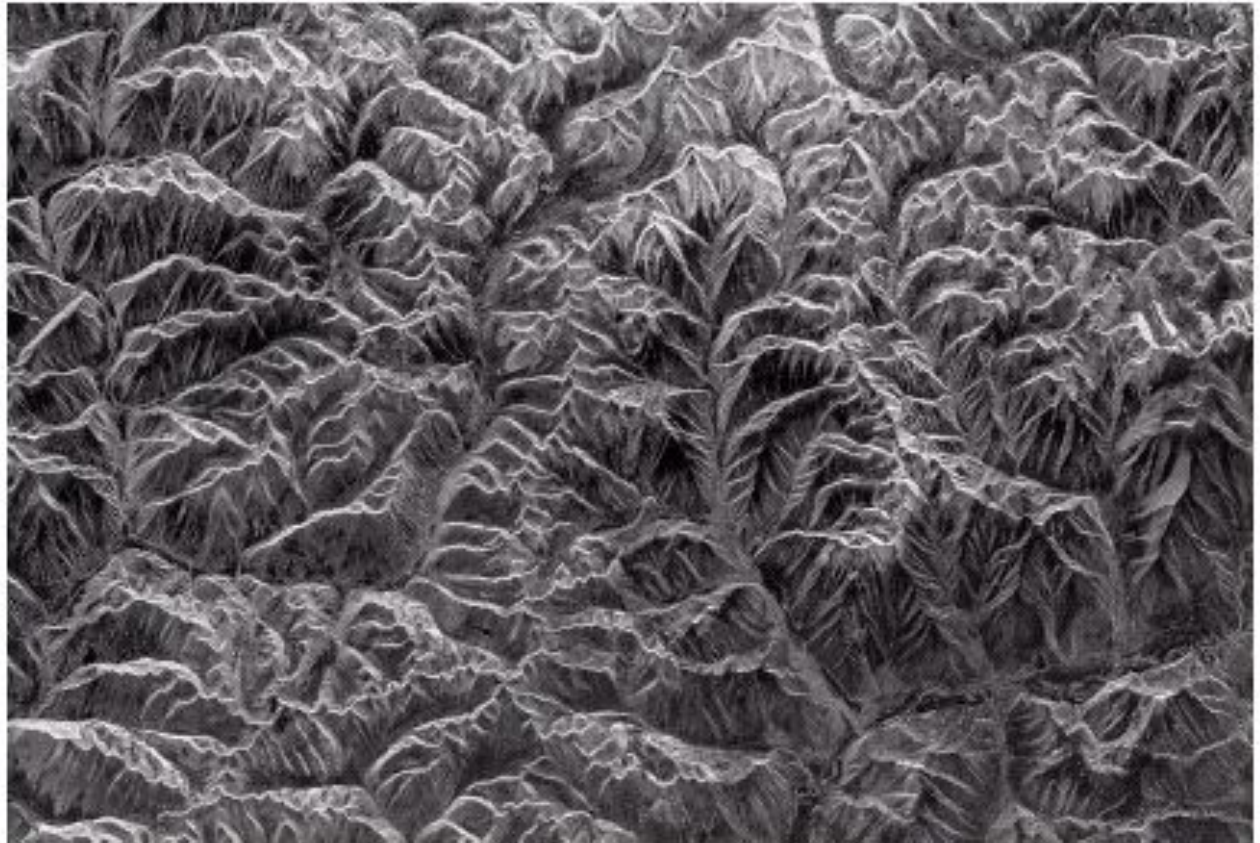
Examples: Infrared Satellite Imaging



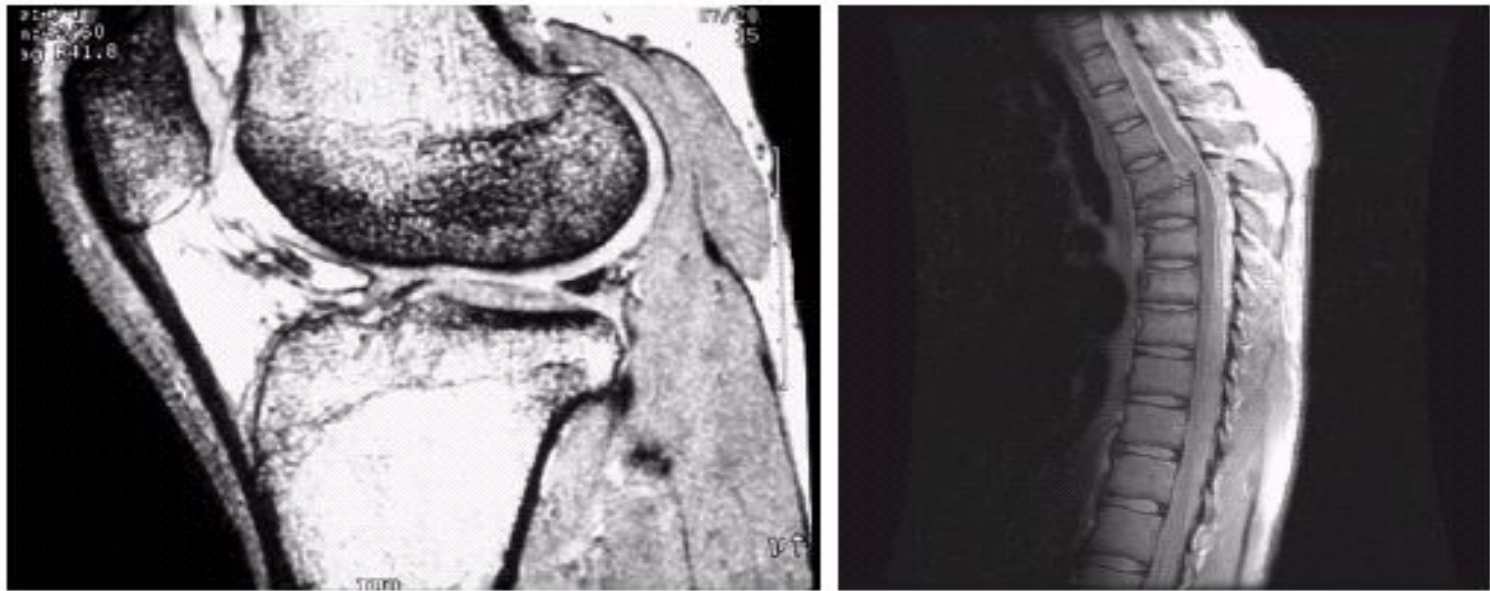
Example of Radar Image (Microwave)

FIGURE 1.16

Spaceborne radar image of mountains in southeast Tibet. (Courtesy of NASA.)



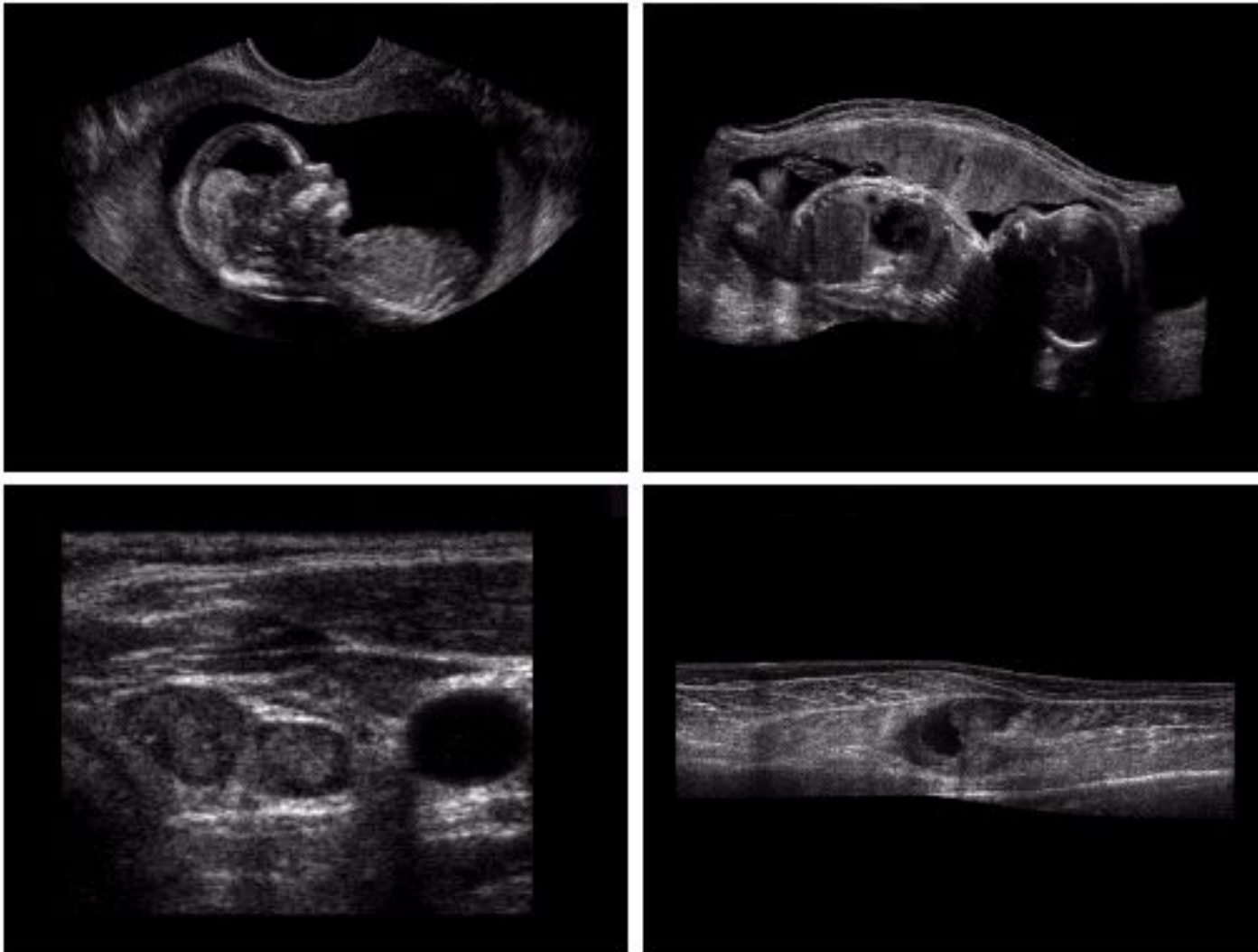
Examples: MRI(Magnetic Resonance Imaging) (Radio Band)



a b

FIGURE 1.17 MRI images of a human (a) knee, and (b) spine. (Image (a) courtesy of Dr. Thomas R. Gest, Division of Anatomical Sciences, University of Michigan Medical School, and (b) Dr. David R. Pickens, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)

Examples: Ultrasound Imaging



a	b
c	d

FIGURE 1.20
Examples of
ultrasound
imaging. (a) Baby.
(2) Another view
of baby.
(c) Thyroids.
(d) Muscle layers
showing lesion.
(Courtesy of
Siemens Medical
Systems, Inc.,
Ultrasound
Group.)

Q and A

- ▶ What is order of wavelength of visible light?
- ▶ Which EM radiation is used in RADAR?



Image Acquisition Process

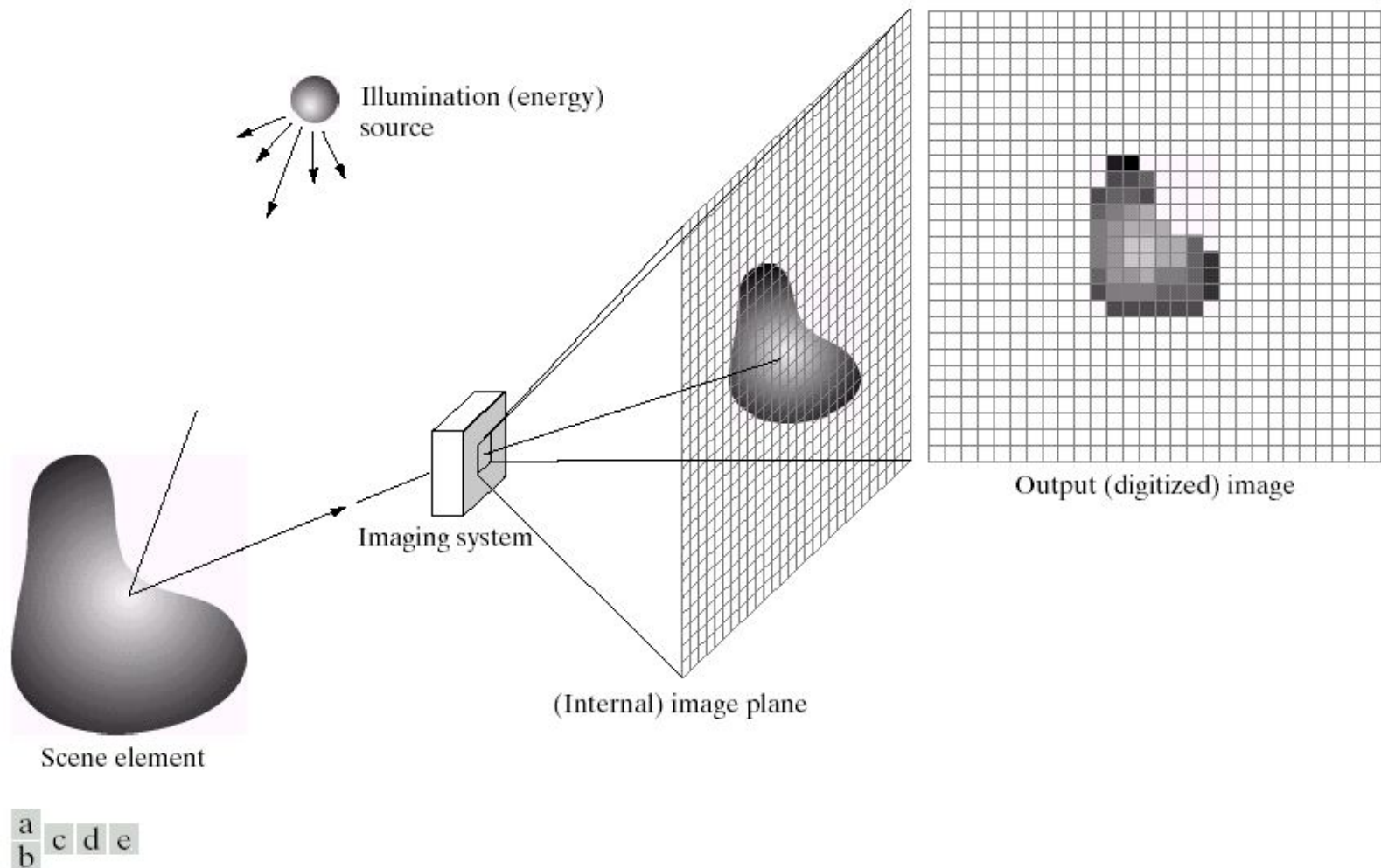


FIGURE 2.15 An example of the digital image acquisition process. (a) Energy (“illumination”) source. (b) An element of a scene. (c) Imaging system. (d) Projection of the scene onto the image plane. (e) Digitized image.

A Simple Image Formation Model

$$f(x, y) = i(x, y)r(x, y)$$

$f(x, y)$: intensity at the point (x, y)

$i(x, y)$: illumination at the point (x, y)

(the amount of source illumination incident on the scene)

$r(x, y)$: reflectance/transmissivity at the point (x, y)

(the amount of illumination reflected/transmitted by the object)

where $0 < i(x, y) < \infty$ and $0 < r(x, y) < 1$

Important Terminologies

- ▶ In an image, **intensity** of a pixel is defined as the value of the pixel. For example in an 8 bit grayscale image there are 256 gray levels.
- ▶ **brightness**, is a relative term. Suppose A, B and C are three pixels having intensities 1, 30 and 250, then C is brighter and A & B are darker with respect to C. In general you can say the higher the intensity the brighter is the pixel.
- ▶ **Luminance** is the luminous intensity, projected on a given area and direction. Luminance is an objectively measurable attribute.
- ▶ The unit is 'Candela per Square Meter' (cd/m²)..
- ▶ It measures of the amount of light emitting, passing through or reflected from a particular surface from a solid angle.
- ▶ **contrast**. It is the difference between maximum and minimum pixel intensities in an image.
- ▶ Consider two images A having pixel intensities between 30 to 200 and B having pixel intensities 70 to 130. Then A has more contrast than B.

Some Typical Ranges of illumination

► Illumination

Lumen — A unit of light flow or luminous flux

Lumen per square meter (lm/m^2) — Illuminance is a term that describes the measurement of the **amount of light** falling onto (illuminating) and spreading over a given surface area.

- On a clear day, the sun may produce in excess of 90,000 lm/m^2 of illumination on the surface of the Earth
- On a cloudy day, the sun may produce less than 10,000 lm/m^2 of illumination on the surface of the Earth
- On a clear evening, the moon yields about 0.1 lm/m^2 of illumination
- The typical illumination level in a commercial office is about 1000 lm/m^2

Some Typical Ranges of Reflectance

► Reflectance

- 0.01 for black velvet
- 0.65 for stainless steel
- 0.80 for flat-white wall paint
- 0.90 for silver-plated metal
- 0.93 for snow

Image formation on analog cameras

- ▶ In analog cameras , the image formation is due to the chemical reaction that takes place on the strip that is used for image formation.
- ▶ It is denoted in the figure by 35mm film cartridge. This strip is coated with silver halide (a chemical substance).
- ▶ So when light photon particles are passed through the camera, it reacts with the silver halide particles on the strip and it results in the silver which is the negative of the image.
- ▶ although image formation involves many other concepts like shutter and shutter speed and aperture and its opening.



▶ **Aperture**

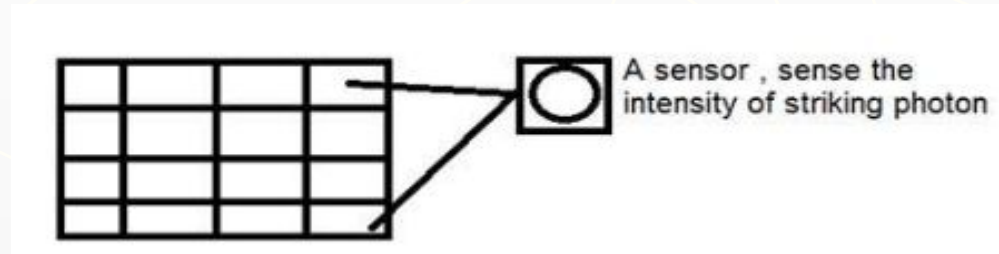
- ▶ Aperture is a small opening which allows the light to travel inside into camera. Here is the picture of aperture
- ▶ If the aperture opening is wide, it would allow more light to pass into the camera. More light would result in more photons, which ultimately result in a brighter image.

▶ **Shutter:**

- ▶ After the aperture, there comes the shutter just before the CCD sensor. Shutter has further two main concepts: Shutter Speed & Shutter time
- ▶ More shutter speed = less shutter time.
- ▶ To capture image of a fast moving object: increase shutter speed & Decrease shutter time.

Image formation on digital cameras

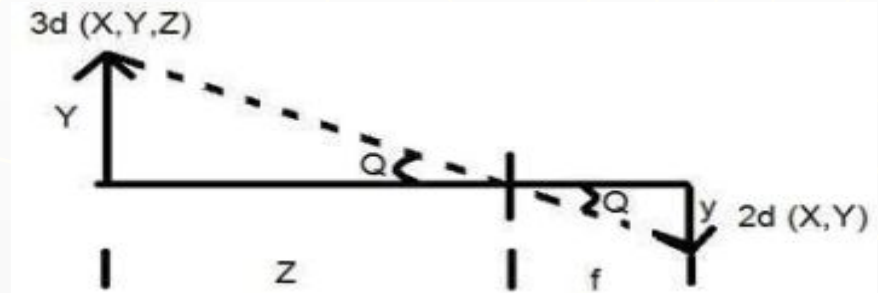
- ▶ In the digital camera , a CCD array of sensors is used for the image formation.
- ▶ CCD stands for charge-coupled device.
- ▶ This CCD is actually in the shape of array or a rectangular grid.



- Each sensor of the CCD array itself is an analog sensor. When photons of light strike on the chip , **it is held as a small electrical charge in each photo sensor.** The response of each sensor is directly equal to the amount of light or (photon) energy strike on the surface of the sensor.
- The value of each sensor of the CCD array refers to each the value of the individual pixel.
- The number of sensors = number of pixels.

Calculating the size of image formed

- ▶ Suppose a person 5m tall, and standing at a distance of 50m from the camera, and we have to tell that what is the size of the image of the person, with a camera of focal length is 50mm.
- ▶ Solution:
- ▶ $Y = 5000$ mm.
- ▶ $f = 50$ mm.
- ▶ $Z = 50000$ mm.
- ▶ Putting the values in the formula, we get



$$Y = -f \frac{Y}{Z} = -50 \times 5000 / 50000$$

$$= -5 \text{ mm.}$$

Q & A

- ▶ What is intensity in an image?
- ▶ Full form of CCD
- ▶ What is difference between optical zoom and digital zoom.

Digitization

- Real world is continuous , The computer world is discrete
- Mapping a continuous function to a discrete one is called sampling
- Mapping a continuous variable to a discrete one is called quantizaion
- To represent or render an image using a computer, we must both sample and quantize

- ▶ Two stages in the digitization process:

- **Spatial sampling:** Spatial domain
- **Quantization:** Gray level

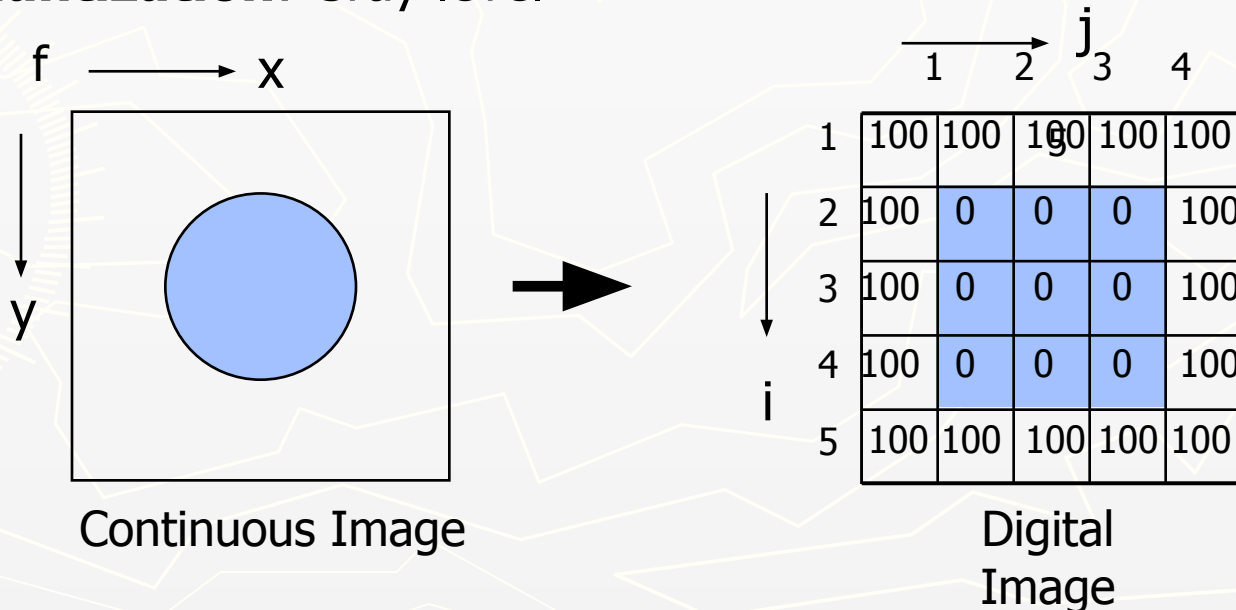


Image Sampling and Quantization

You can see in this image, that the signal has been quantified into four different levels.

And the vertically ranging values have been quantized into 5 different levels or partitions.

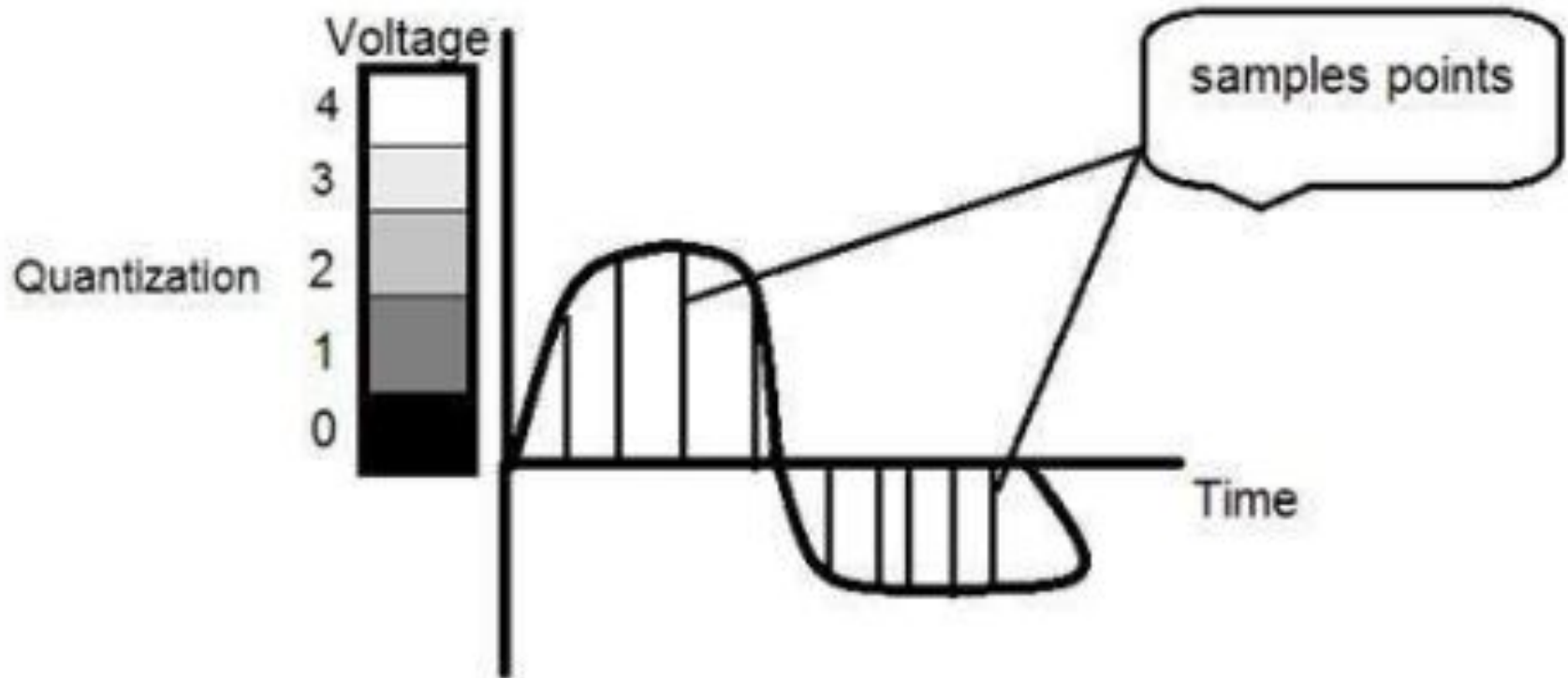
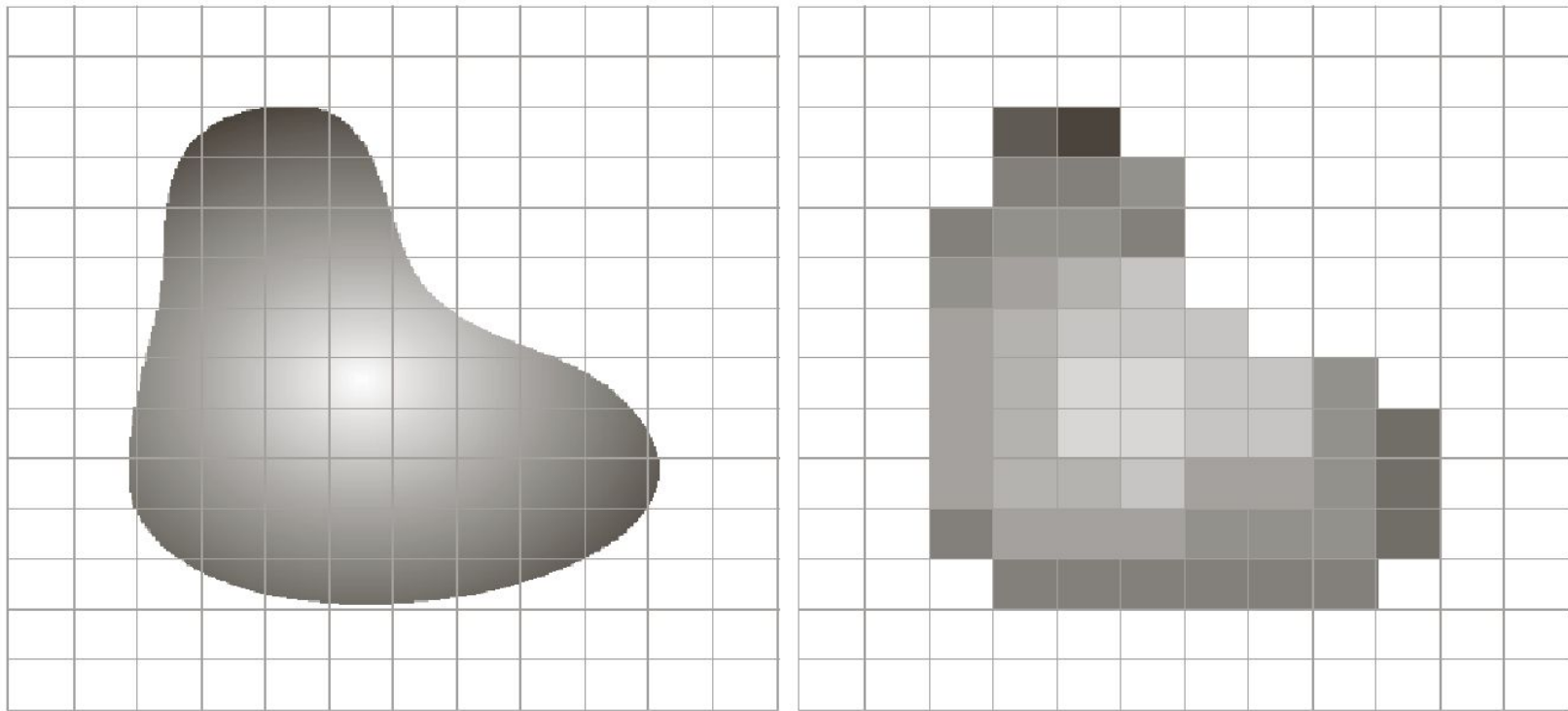


Image Sampling and Quantization



a b

FIGURE 2.17 (a) Continuous image projected onto a sensor array. (b) Result of image sampling and quantization.

The sampling rate determines the spatial resolution of the digitized image, while the quantization level determines the number of grey level resolution in the digitized image

Quantization

256



64



16



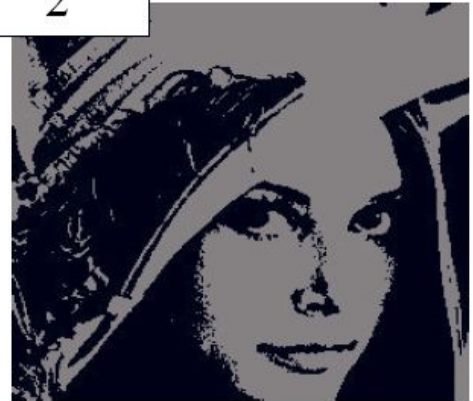
8



4



2



Representing Digital Images

- The representation of an $M \times N$ numerical array as

$$f(x, y) = \begin{bmatrix} f(0,0) & f(0,1) & \dots & f(0,N-1) \\ f(1,0) & f(1,1) & \dots & f(1,N-1) \\ \dots & \dots & \dots & \dots \\ f(M-1,0) & f(M-1,1) & \dots & f(M-1,N-1) \end{bmatrix}$$

Discrete intensity interval $[0, L-1]$, $L=2^k$

The number b of bits required to store a $M \times N$ digitized image

$$b = M \times N \times k$$

Representing Digital Images

TABLE 2.1

Number of storage bits for various values of N and k .

N/k	1 ($L = 2$)	2 ($L = 4$)	3 ($L = 8$)	4 ($L = 16$)	5 ($L = 32$)	6 ($L = 64$)	7 ($L = 128$)	8 ($L = 256$)
32	1,024	2,048	3,072	4,096	5,120	6,144	7,168	8,192
64	4,096	8,192	12,288	16,384	20,480	24,576	28,672	32,768
128	16,384	32,768	49,152	65,536	81,920	98,304	114,688	131,072
256	65,536	131,072	196,608	262,144	327,680	393,216	458,752	524,288
512	262,144	524,288	786,432	1,048,576	1,310,720	1,572,864	1,835,008	2,097,152
1024	1,048,576	2,097,152	3,145,728	4,194,304	5,242,880	6,291,456	7,340,032	8,388,608
2048	4,194,304	8,388,608	12,582,912	16,777,216	20,971,520	25,165,824	29,369,128	33,554,432
4096	16,777,216	33,554,432	50,331,648	67,108,864	83,886,080	100,663,296	117,440,512	134,217,728
8192	67,108,864	134,217,728	201,326,592	268,435,456	335,544,320	402,653,184	469,762,048	536,870,912

Resolution

- ▶ The resolution can be defined in many ways. Such as **pixel resolution, spatial resolution (sampling), Intensity (Gray level) resolution, temporal resolution.**
- ▶ **pixel resolution:** You have probably seen that in your own computer settings, you have monitor resolution of 800 x 600, 640 x 480 e.t.c
- ▶ In pixel resolution, the term resolution refers to the total number of count of pixels in an digital image.
- ▶ For example. If an image has M rows and N columns, then its resolution can be defined as $M \times N$.

Resolution (Cont..)

▶ **Megapixels**

- ▶ We can calculate mega pixels of a camera using pixel resolution.
- ▶ Column pixels (width) X row pixels (height) / 1 Million.
- ▶ The size of an image can be defined by its pixel resolution.
- ▶ Size = pixel resolution X bpp (bits per pixel)

▶ **Calculating the mega pixels of the camera**

- ▶ Lets say we have an image of dimension: 2500 X 3192.
- ▶ Its pixel resolution = $2500 * 3192 = 7982350$ bytes.
- ▶ Dividing it by 1 million = $7.9 = 8$ mega pixel (approximately).

▶ **Aspect ratio**

- ▶ Another important concept with the pixel resolution is aspect ratio.
- ▶ Aspect ratio is the ratio between width of an image and the height of an image. 1.33:1, 1.37:1, 1.50:1, 1.56:1, 1.66:1, 1.75:1, e.t.c

Resolution(Cont..)

► If you are given an image with aspect ratio of 6:2 of an image of pixel resolution of 480000 pixels given the image is an gray scale image.

► calculate the dimensions of image

► Given:

► Aspect ratio: $c:r = 6:2$

► Pixel resolution:

$$c * r = 480000$$

► Bits per pixel:

grayscale image = 8bpp

$$\text{Size} = \text{rows} * \text{cols} * \text{bpp}$$

Size of image in bits

$$= 400 * 1200 * 8$$

$$= 3840000 \text{ bits}$$

$$= 48 \text{ kb (approx).}$$

$$\text{Equation 1.} \quad c:r = 6:2 \rightarrow c = 6r / 2$$

$$\text{Equation 2.} \quad c = 480000/r$$

$$\text{Comparing both equations} \rightarrow \frac{6r}{2} = \frac{480000}{r}$$

$$r^2 = \sqrt{\frac{480000 * 2}{6}}$$

$$\text{That gives} \quad r = 400.$$

$$\text{Put } r \text{ in equation 1, we get} \rightarrow c = 1200.$$

$$\text{So rows} = 400 \text{ cols} = 1200.$$

Spatial and Intensity Resolution

► Spatial resolution

- A measure of the smallest discernible detail in an image
- stated with *line pairs per unit distance, dots (pixels) per unit distance, dots per inch (dpi)*

► Intensity resolution

- The smallest discernible change in intensity level
- stated with *8 bits, 12 bits, 16 bits, etc.*

Q & A

- ▶ What is difference between spatial resolution and pixel resolution?
- ▶ If you are given an image with aspect ratio of 5:3 of an image of pixel resolution of 450000 pixels given the image is an gray scale image. calculate the dimensions of image.
- ▶ What is Intensity resolution for binary image?

Basic Relationships Between Pixels

- ▶ Neighborhood
- ▶ Adjacency
- ▶ Connectivity
- ▶ Paths
- ▶ Regions and boundaries

Basic Relationships Between Pixels

► **Neighbors** of a pixel p at coordinates (x,y)

- **4-neighbors of p** , denoted by $\mathbf{N}_4(p)$:
 $(x-1, y)$, $(x+1, y)$, $(x, y-1)$, and $(x, y+1)$.
- **4 diagonal neighbors of p** , denoted by $\mathbf{N}_D(p)$:
 $(x-1, y-1)$, $(x+1, y+1)$, $(x+1, y-1)$, and $(x-1, y+1)$.
- **8 neighbors of p** , denoted $\mathbf{N}_8(p)$
$$\mathbf{N}_8(p) = \mathbf{N}_4(p) \cup \mathbf{N}_D(p)$$

Basic Relationships Between Pixels

► Adjacency

Let V be the set of intensity values

- **4-adjacency:** Two pixels p and q with values from V are 4-adjacent if q is in the set $N_4(p)$.
- **8-adjacency:** Two pixels p and q with values from V are 8-adjacent if q is in the set $N_8(p)$.

Basic Relationships Between Pixels

► Adjacency

Let V be the set of intensity values

□ **m-adjacency:** Two pixels p and q with values from V are m-adjacent if

(i) q is in the set $N_4(p)$, or

(ii) q is in the set $N_D(p)$ and the set $N_4(p) \cap N_4(q)$ has no pixels whose values are from V .

Basic Relationships Between Pixels

► Path

- A (digital) path (or curve) from pixel p with coordinates (x_0, y_0) to pixel q with coordinates (x_n, y_n) is a sequence of distinct pixels with coordinates

$$(x_0, y_0), (x_1, y_1), \dots, (x_n, y_n)$$

Where (x_i, y_i) and (x_{i-1}, y_{i-1}) are adjacent for $1 \leq i \leq n$.

- Here n is the *length* of the path.
- If $(x_0, y_0) = (x_n, y_n)$, the path is **closed** path.
- We can define 4-, 8-, and m-paths based on the type of adjacency used.

Examples: Adjacency and Path

$V = \{1, 2\}$

0 1 1

0 2 0

0 0 1

0 1 1

0 2 0

0 0 1

0 1 1

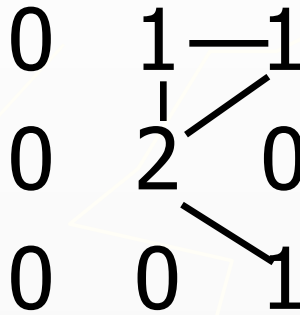
0 2 0

0 0 1

Examples: Adjacency and Path

$$V = \{1, 2\}$$

0	1	1
0	2	0
0	0	1



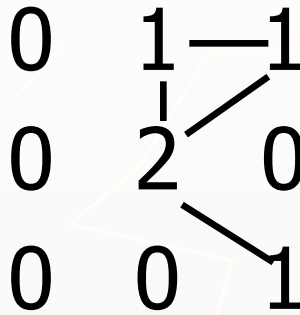
0	1	1
0	2	0
0	0	1

8-adjacent

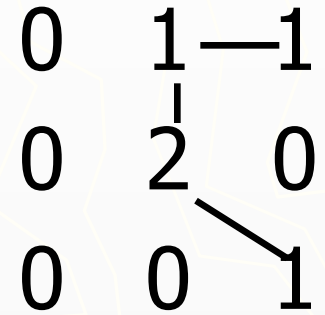
Examples: Adjacency and Path

$$V = \{1, 2\}$$

0	1	1
0	2	0
0	0	1



8-adjacent

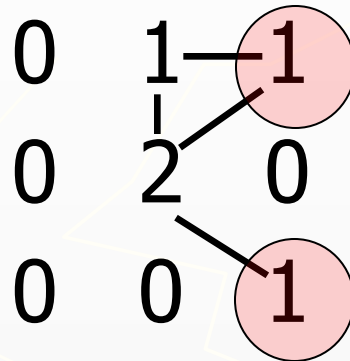


m-adjacent

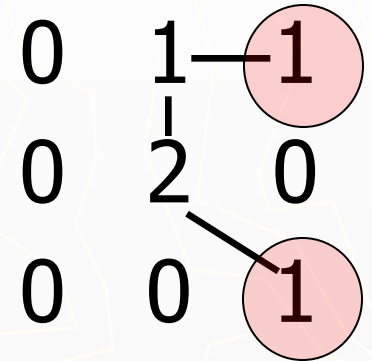
Examples: Adjacency and Path

$$V = \{1, 2\}$$

$0_{1,1}$	$1_{1,2}$	$1_{1,3}$
$0_{2,1}$	$2_{2,2}$	$0_{2,3}$
$0_{3,1}$	$0_{3,2}$	$1_{3,3}$



8-adjacent



m-adjacent

The 8-path from (1,3) to (3,3):

- (i) (1,3), (1,2), (2,2), (3,3)
- (ii) (1,3), (2,2), (3,3)

The m-path from (1,3) to (3,3):

(1,3), (1,2), (2,2), (3,3)

Basic Relationships Between Pixels

► Connected in S

Let S represent a subset of pixels in an image. Two pixels p with coordinates (x_0, y_0) and q with coordinates (x_n, y_n) are said to be **connected in S** if there exists a path

$$(x_0, y_0), (x_1, y_1), \dots, (x_n, y_n)$$

Where $\forall i, 0 \leq i \leq n, (x_i, y_i) \in S$

Basic Relationships Between Pixels

Let S represent a subset of pixels in an image

- ▶ For every pixel p in S , the set of pixels in S that are connected to p is called a ***connected component*** of S .
- ▶ If S has only one connected component, then S is called ***Connected Set***.
- ▶ We call R a **region** of the image if R is a connected set
- ▶ Two regions, R_i and R_j are said to be ***adjacent*** if their union forms a connected set.
- ▶ Regions that are not to be adjacent are said to be ***disjoint***.

Basic Relationships Between Pixels

► **Boundary (or border)**

- The ***boundary*** of the region R is the set of pixels in the region that have one or more neighbors that are not in R .
- If R happens to be an entire image, then its boundary is defined as the set of pixels in the first and last rows and columns of the image.

► **Foreground and background**

- An image contains K disjoint regions, R_k , $k = 1, 2, \dots, K$. Let R_u denote the union of all the K regions, and let $(R_u)^c$ denote its complement.
All the points in R_u is called **foreground**;
All the points in $(R_u)^c$ is called **background**.

Q & A

- ▶ **In the following arrangement of pixels, are the two regions (of 1s) adjacent? (if 8-adjacency is used)**

1	1	1
1	0	1
0	1	0
0	0	1
1	1	1
1	1	1

Region 1

Region 2

Question 3

- ▶ **In the following arrangement of pixels, the two regions (of 1s) are disjoint (if 4-adjacency is used)**

1	1	1
1	0	1
0	1	0

Region 1

0	0	1
1	1	1
1	1	1

Region 2

Question 4

- ▶ **In the following arrangement of pixels, the two regions (of 1s) are disjoint (if 4-adjacency is used)**

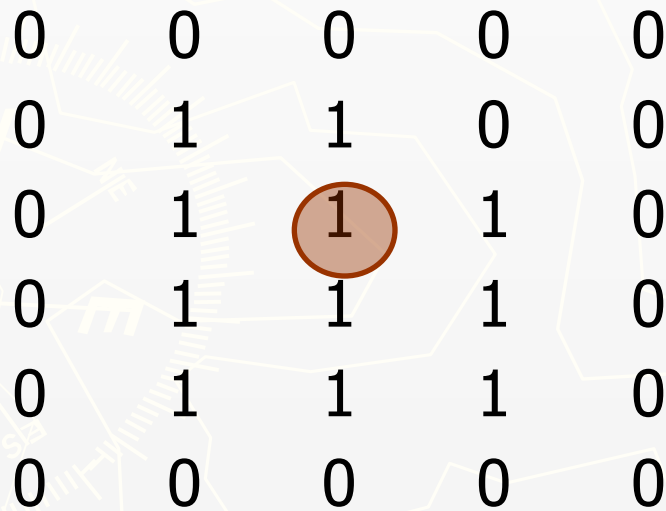
1	1	1
1	0	1
0	1	0
0	0	1
1	1	1
1	1	1

foreground

background

Question 5

- ▶ In the following arrangement of pixels, the circled point is part of the boundary of the 1-valued pixels
 1. if **8-adjacency** is used,
 2. if **4-adjacency** is used, **true or false?**



A 5x5 grid of pixels. The values are as follows:

0	0	0	0	0
0	1	1	0	0
0	1	1	1	0
0	1	1	1	0
0	1	1	1	0
0	0	0	0	0

The pixel at row 3, column 3 (the value 1) is circled in brown.

Distance Measures

► Given pixels p , q and z with coordinates (x, y) , (s, t) , (u, v) respectively, the distance function D has following **properties**:

a. $D(p, q) \geq 0$ $[D(p, q) = 0, \text{ iff } p = q]$

b. $D(p, q) = D(q, p)$

c. $D(p, z) \leq D(p, q) + D(q, z)$

Distance Measures

The following are the different Distance measures:

a. Euclidean Distance :

$$D_e(p, q) = [(x-s)^2 + (y-t)^2]^{1/2}$$

b. City Block Distance:

$$D_4(p, q) = |x-s| + |y-t|$$

c. Chess Board Distance:

$$D_8(p, q) = \max(|x-s|, |y-t|)$$

		2		
	2	1	2	
2	1	0	1	2
	2	1	2	
		2		

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

Introduction to Mathematical Operations in DIP

► Array vs. Matrix Operation

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

Array
product
operator

$$A .* B = \begin{bmatrix} a_{11}b_{11} & a_{12}b_{12} \\ a_{21}b_{21} & a_{22}b_{22} \end{bmatrix}$$

Array product

Matrix
product
operator

$$A * B = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

Matrix product

Introduction to Mathematical Operations in DIP

► Linear vs. Nonlinear Operation

$$H[f(x, y)] = g(x, y)$$

$$H[a_i f_i(x, y) + a_j f_j(x, y)]$$

$$= H[a_i f_i(x, y)] + H[a_j f_j(x, y)]$$

$$= a_i H[f_i(x, y)] + a_j H[f_j(x, y)]$$

$$= a_i g_i(x, y) + a_j g_j(x, y)$$

Additivity

Homogeneity

H is said to be a **linear operator (sum, average, subtraction)**;

H is said to be a **nonlinear operator (mult, div)** if it does not meet the above qualification.

Arithmetic Operations

- ▶ Arithmetic operations between images are array operations. The four arithmetic operations are denoted as

$$s(x,y) = f(x,y) + g(x,y)$$

$$d(x,y) = f(x,y) - g(x,y)$$

$$p(x,y) = f(x,y) \times g(x,y)$$

$$v(x,y) = f(x,y) \div g(x,y)$$

Example: Addition of Noisy Images for Noise Reduction

Noiseless image: $f(x,y)$

Noise: $n(x,y)$ (at every pair of coordinates (x,y) , the noise is uncorrelated and has zero average value)

Corrupted image: $g(x,y)$

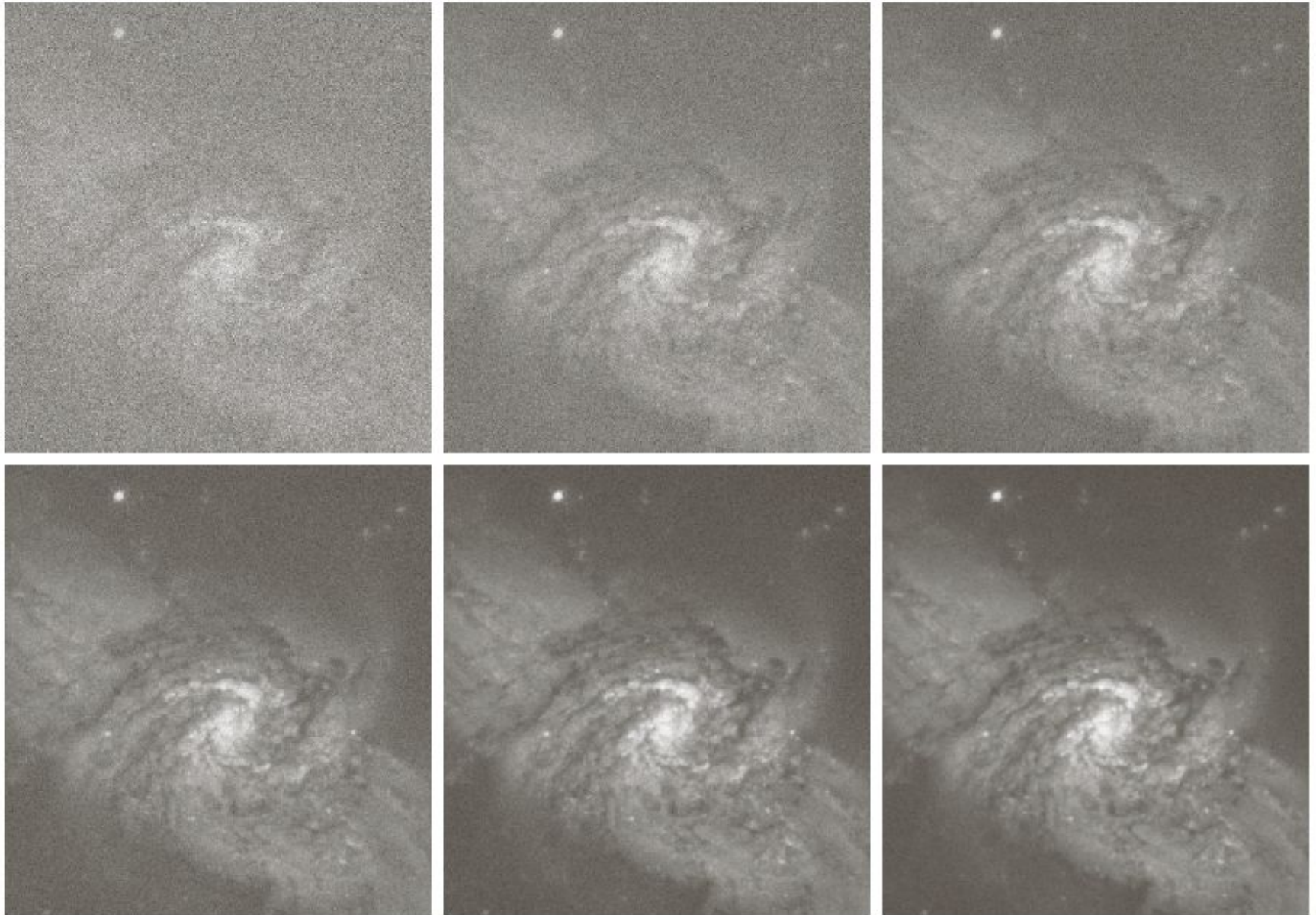
$$g(x,y) = f(x,y) + n(x,y)$$

Reducing the noise by adding a set of noisy images, $\{g_i(x,y)\}$

$$\bar{g}(x,y) = \frac{1}{K} \sum_{i=1}^K g_i(x,y)$$

Application: Addition of Noisy Images for Noise Reduction

- ▶ In astronomy, imaging under very low light levels frequently causes sensor noise to render single image virtually useless for analysis.
- ▶ In astronomical observations, similar sensors for noise reduction by observing the same scene over long periods of time. Image averaging is then used to reduce the noise.



a	b	c
d	e	f

FIGURE 2.26 (a) Image of Galaxy Pair NGC 3314 corrupted by additive Gaussian noise. (b)–(f) Results of averaging 5, 10, 20, 50, and 100 noisy images, respectively. (Original image courtesy of NASA.)

An Example of Image Subtraction: Mask Mode Radiography

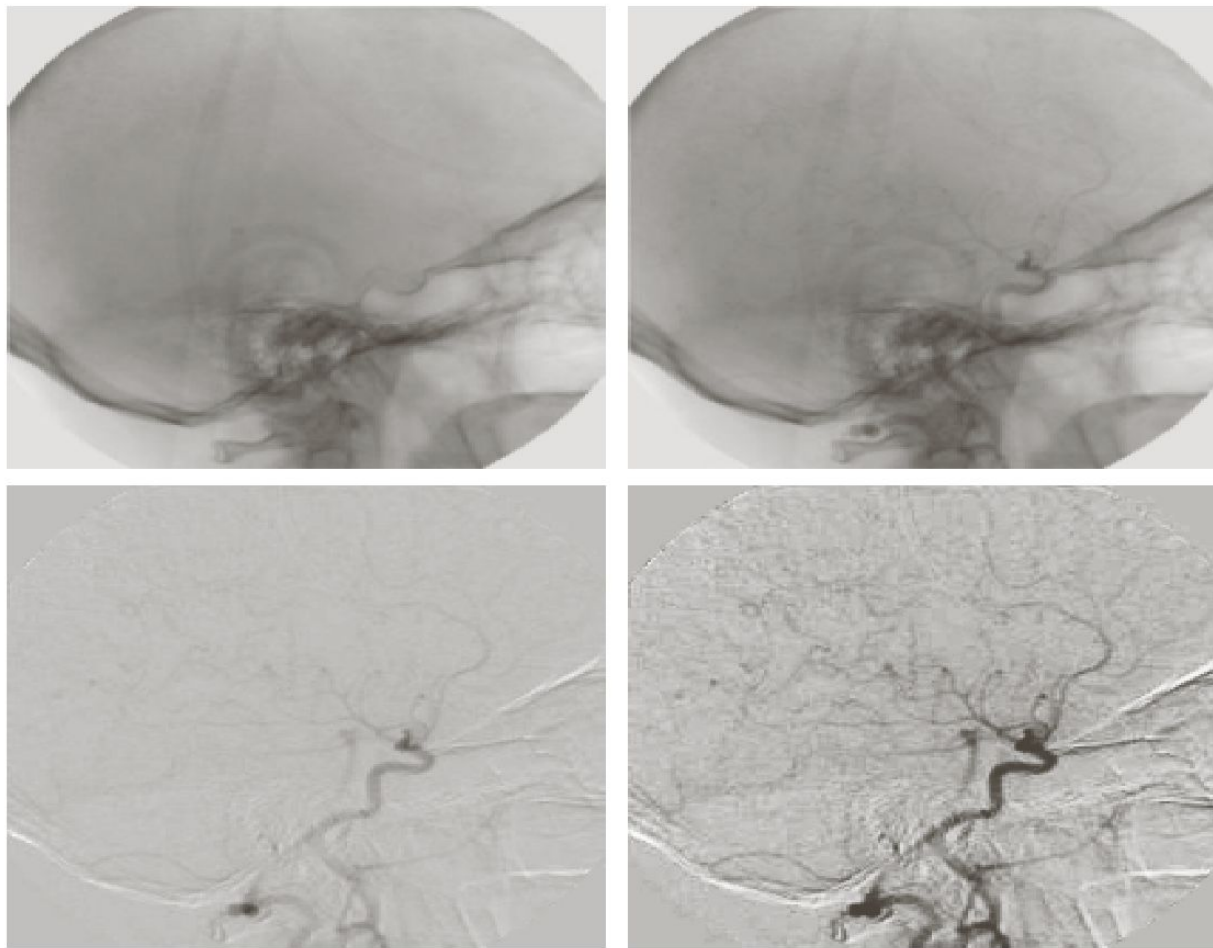
Mask $h(x,y)$: an X-ray image of a region of a patient's body

Live images $f(x,y)$: X-ray images captured at TV rates after injection of the contrast medium

Enhanced detail $g(x,y)$

$$g(x,y) = f(x,y) - h(x,y)$$

The procedure gives a movie showing how the contrast medium propagates through the various arteries in the area being observed.



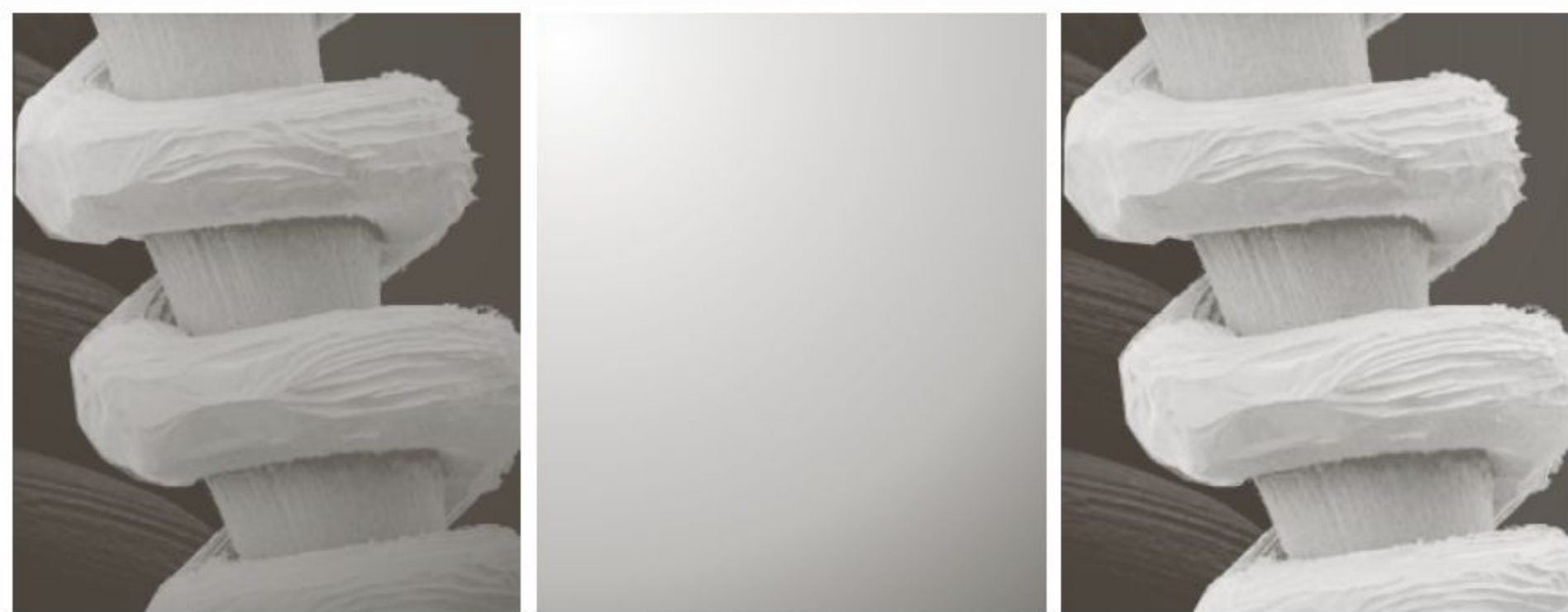
a	b
c	d

FIGURE 2.28

Digital subtraction angiography. (a) Mask image. (b) A live image. (c) Difference between (a) and (b). (d) Enhanced difference image. (Figures (a) and (b) courtesy of The Image Sciences Institute, University Medical Center, Utrecht, The Netherlands.)

- ❑ A mask X-ray (using intensified TV camera) image of a patient's head prior to injection of an iodine (X-ray contrast) medium into bloodstream, (b) a sample of live image taken after the injection (c) difference between two (d) enhanced image
- ❑ Angiography or arteriography is a medical imaging technique used to visualize the inside, or lumen, of blood vessels and organs of the body, with particular interest in the arteries, veins, and the heart chambers.

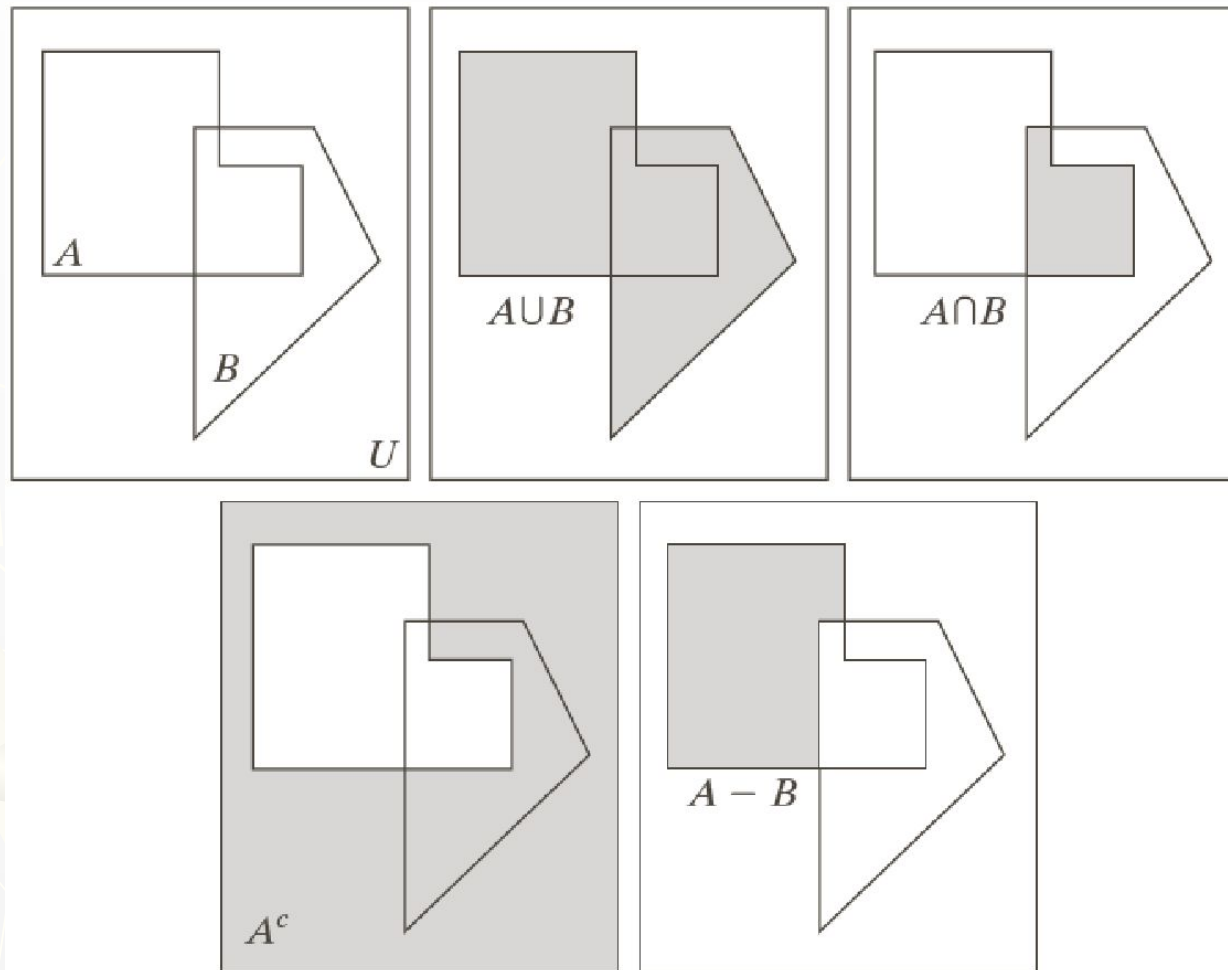
An Example of Image Multiplication (shading correction & masking)



a b c

FIGURE 2.29 Shading correction. (a) Shaded SEM image of a tungsten filament and support, magnified approximately 130 times. (b) The shading pattern. (c) Product of (a) by the reciprocal of (b). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

Set and Logical Operations



a	b	c
d	e	

FIGURE 2.31

(a) Two sets of coordinates, A and B , in 2-D space. (b) The union of A and B . (c) The intersection of A and B . (d) The complement of A . (e) The difference between A and B . In (b)–(e) the shaded areas represent the member of the set operation indicated.

Set and Logical Operations

- ▶ Let A be the elements of a gray-scale image
The elements of A are triplets of the form (x, y, z) , where x and y are spatial coordinates and z denotes the intensity at the point (x, y) .

$$A = \{(x, y, z) \mid z = f(x, y)\}$$

- ▶ The complement of A is denoted A^c

$$A^c = \{(x, y, K - z) \mid (x, y, z) \in A\}$$

$K = 2^k - 1$; k is the number of intensity bits used to represent z

Set and Logical Operations

The union of two gray-scale images (sets) A and B is defined as the set

$$A \cup B = \{\max(a, b) \mid a \in A, b \in B\}$$

a b c

FIGURE 2.32 Set operations involving gray-scale images. (a) Original image. (b) Image negative obtained using set complementation. (c) The union of (a) and a constant image. (Original image courtesy of G.E. Medical Systems.)

Question 1

- In the following arrangement of pixels, what's the value of the chessboard distance between the circled two points?

0	0	0	0	0
0	0	1	1	0
0	1	1	0	0
0	1	0	0	0
0	0	0	0	0
0	0	0	0	0

Question 2

- In the following arrangement of pixels, what's the value of the city-block distance between the circled two points?

0	0	0	0	0
0	0	1	1	0
0	1	1	0	0
0	1	0	0	0
0	0	0	0	0
0	0	0	0	0

Question 3

- ▶ Write Python code to add, subtract, and multiply two images.



Thank You

