

Image Restoration and Reconstruction

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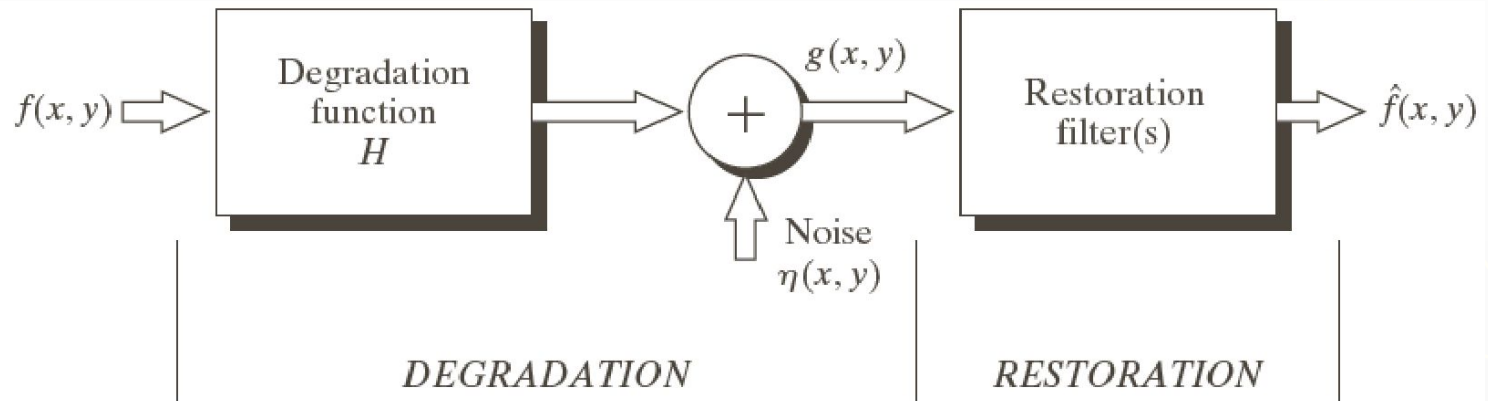
Image Restoration

- ▶ Image restoration: recover an image that has been degraded by using a prior knowledge of the degradation phenomenon.
- ▶ Model the degradation and applying the inverse process in order to recover the original image.

A Model of Image Degradation/Restoration Process

FIGURE 5.1

A model of the image degradation/restoration process.



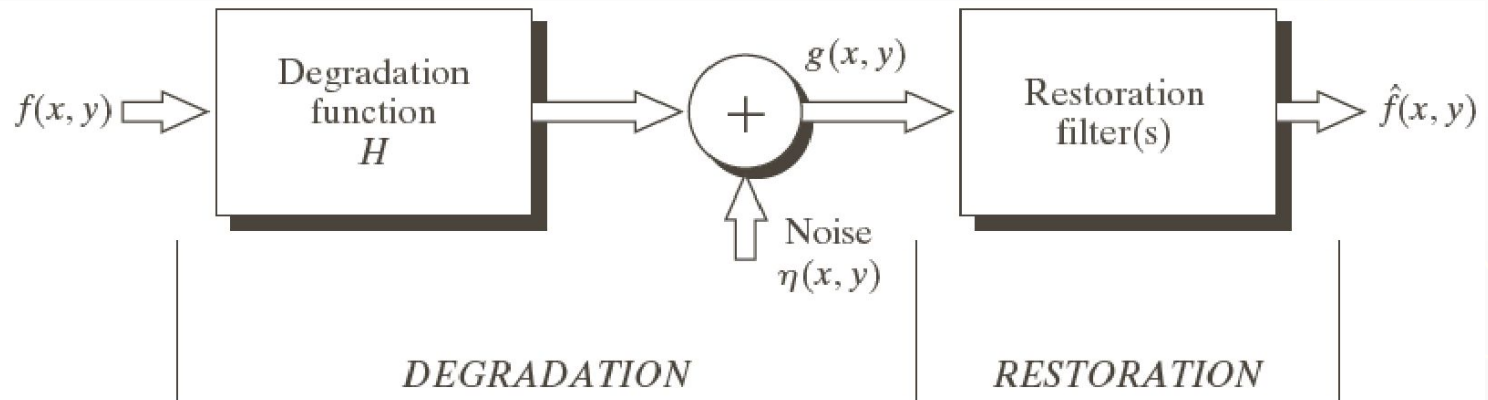
► Degradation

- Degradation function H
- Additive noise $\eta(x, y)$

A Model of Image Degradation/Restoration Process

FIGURE 5.1

A model of the image degradation/restoration process.



If H is a linear, position-invariant process, then the degraded image is given in the spatial domain by

$$g(x, y) = h(x, y) \star f(x, y) + \eta(x, y)$$

Noise Sources

- ▶ The principal sources of noise in digital images arise during **image acquisition and/or transmission**
- ✓ Image acquisition
e.g., light levels, sensor temperature, etc.
- ✓ Transmission
e.g., lightning or other atmospheric disturbance in wireless network

Noise Models

- ▶ White noise
 - The Fourier spectrum of noise is constant
- ▶ With the exception of spatially periodic noise, we assume
 - Noise is independent of spatial coordinates
 - Noise is uncorrelated with respect to the image itself

Noise Models

□ **Gaussian noise**

Electronic circuit noise, sensor noise due to poor illumination and/or high temperature

□ **Rayleigh noise**

Range imaging: a collection of techniques that are used to produce a 2D image showing the distance to points in a scene from a specific point

□ **Erlang (gamma) noise:** Laser imaging

□ **Exponential noise:** Laser imaging

□ **Uniform noise:** Least descriptive; Basis for numerous random number generators

□ **Impulse noise:** quick transients, such as faulty switching

Gaussian Noise

The PDF of Gaussian random variable, z , is given by

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(z-\bar{z})^2/2\sigma^2}$$

- 70% of its values will be in the range

$$[(\mu - \sigma), (\mu + \sigma)]$$

- 95% of its values will be in the range

$$[(\mu - 2\sigma), (\mu + 2\sigma)]$$

Rayleigh Noise

The PDF of Rayleigh noise is given by

$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{-(z-a)^2/b} & \text{for } z \geq a \\ 0 & \text{for } z < a \end{cases}$$

The mean and variance of this density are given by

$$\bar{z} = a + \sqrt{\pi b / 4}$$
$$\sigma^2 = \frac{b(4 - \pi)}{4}$$

Erlang (Gamma) Noise

The PDF of Erlang noise is given by

$$p(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!} e^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases}$$

The mean and variance of this density are given by

$$\bar{z} = b / a$$

$$\sigma^2 = b / a^2$$

Exponential Noise

The PDF of exponential noise is given by

$$p(z) = \begin{cases} ae^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases}$$

The mean and variance of this density are given by

$$\bar{z} = 1/a$$

$$\sigma^2 = 1/a^2$$

Uniform Noise

The PDF of uniform noise is given by

$$p(z) = \begin{cases} \frac{1}{b-a} & \text{for } a \leq z \leq b \\ 0 & \text{otherwise} \end{cases}$$

The mean and variance of this density are given by

$$\bar{z} = (a + b) / 2$$

$$\sigma^2 = (b - a)^2 / 12$$

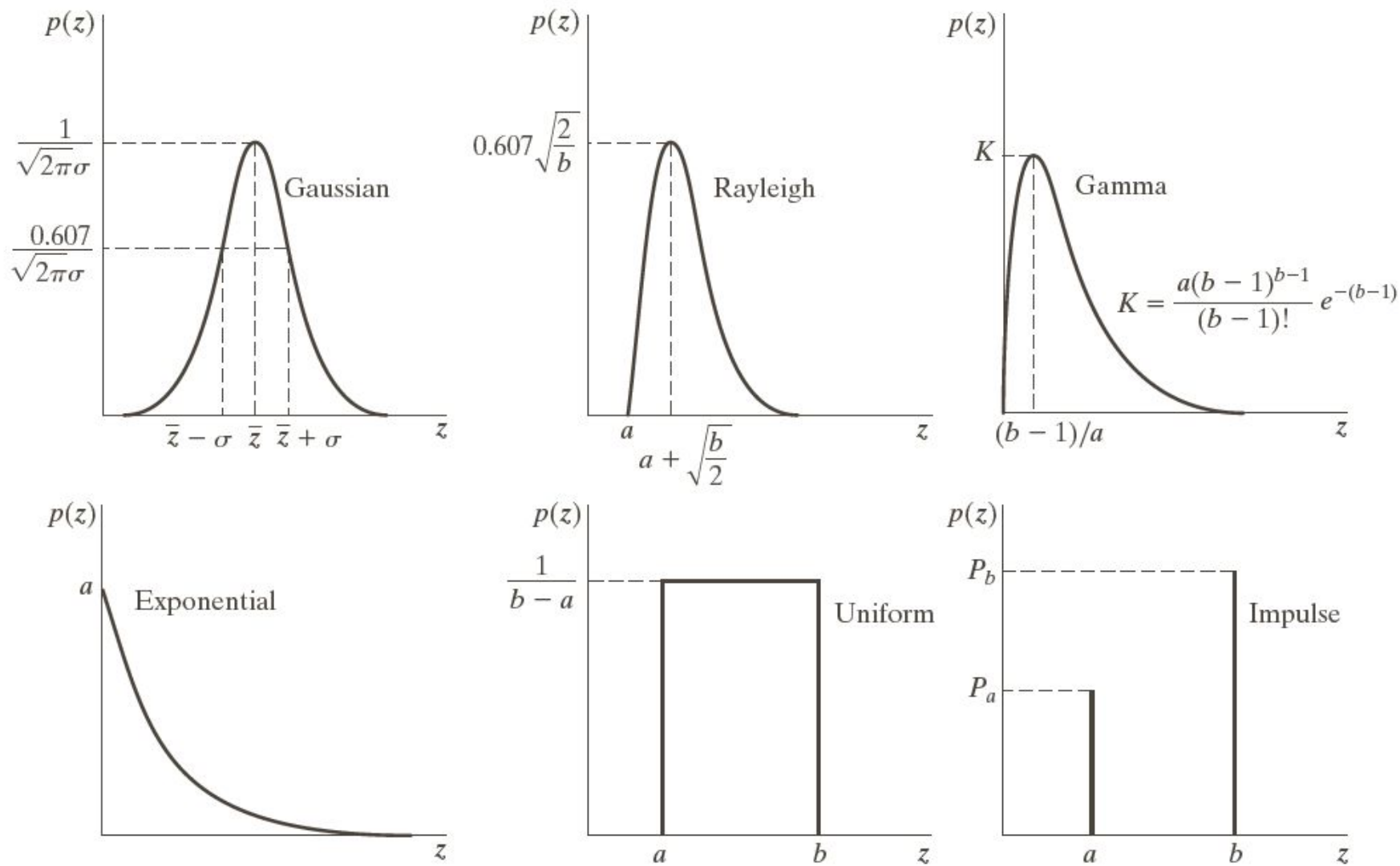
Impulse (Salt-and-Pepper) Noise

The PDF of (bipolar) impulse noise is given by

$$p(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases}$$

if $b > a$, gray-level b will appear as a light dot, while level a will appear like a dark dot.

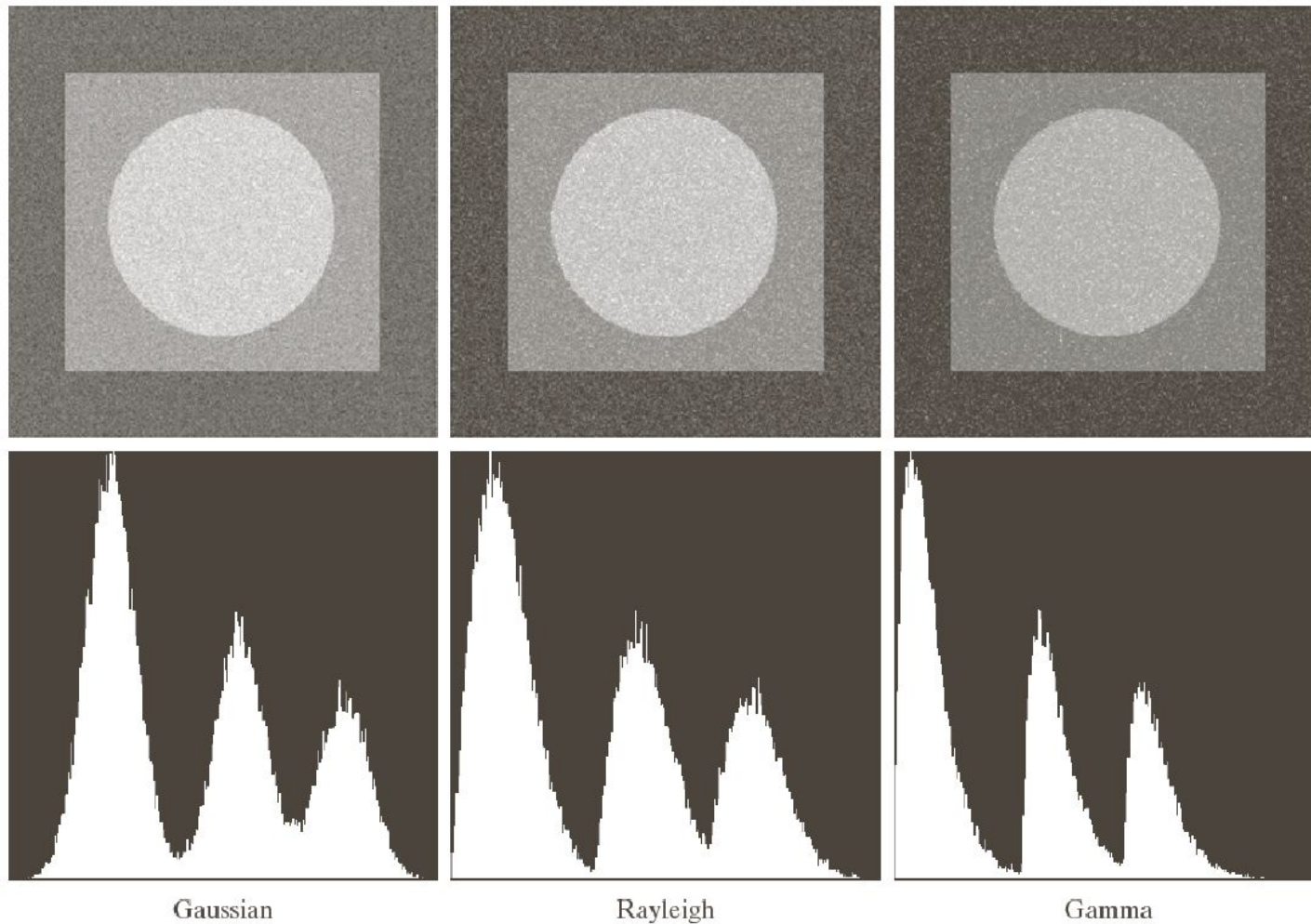
If either P_a or P_b is zero, the impulse noise is called *unipolar*



a	b	c
d	e	f

FIGURE 5.2 Some important probability density functions.

Examples of Noise: Noisy Images(1)



a b c
d e f

FIGURE 5.4 Images and histograms resulting from adding Gaussian, Rayleigh, and gamma noise to the image in Fig. 5.3.

Examples of Noise: Noisy Images(2)

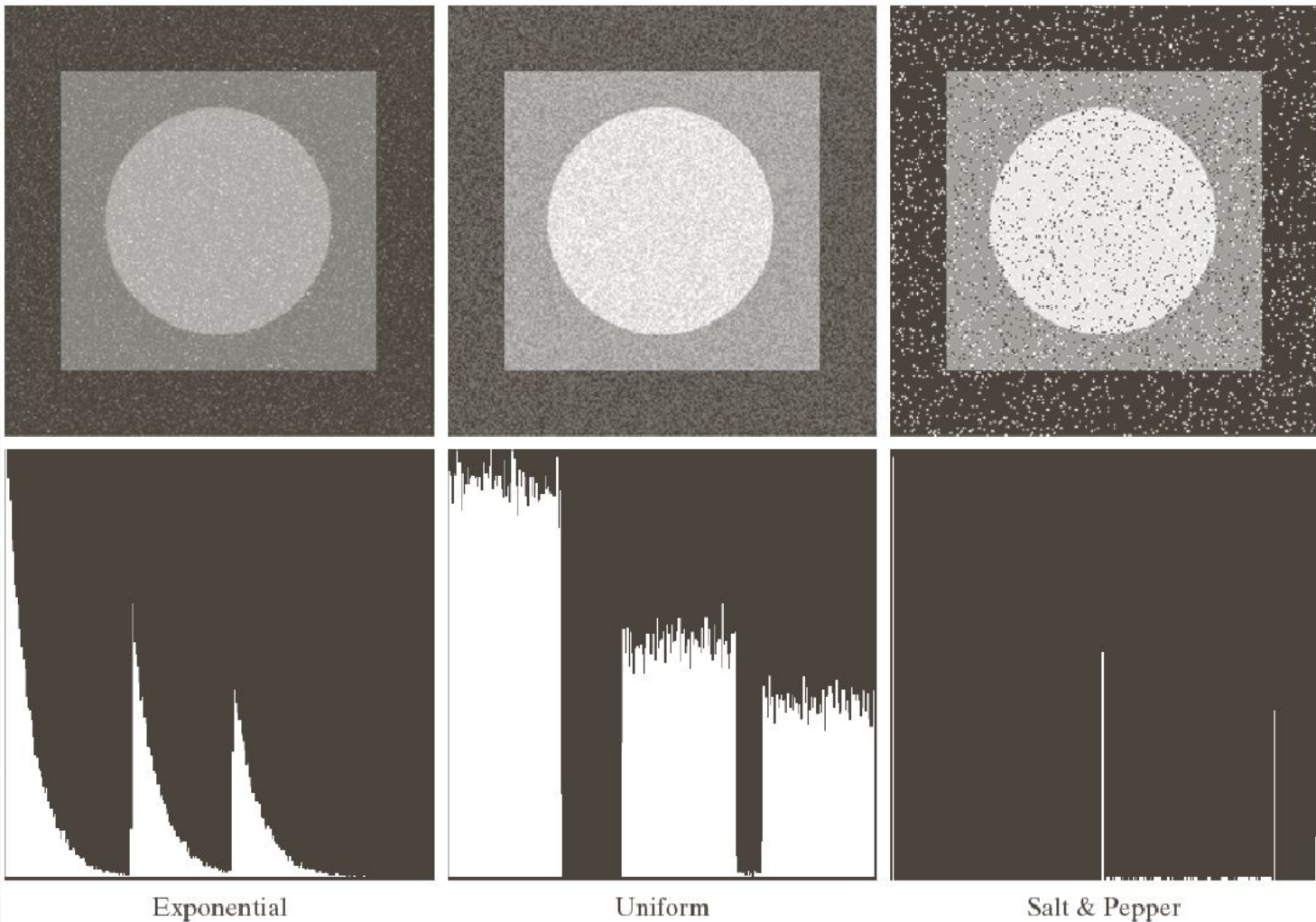
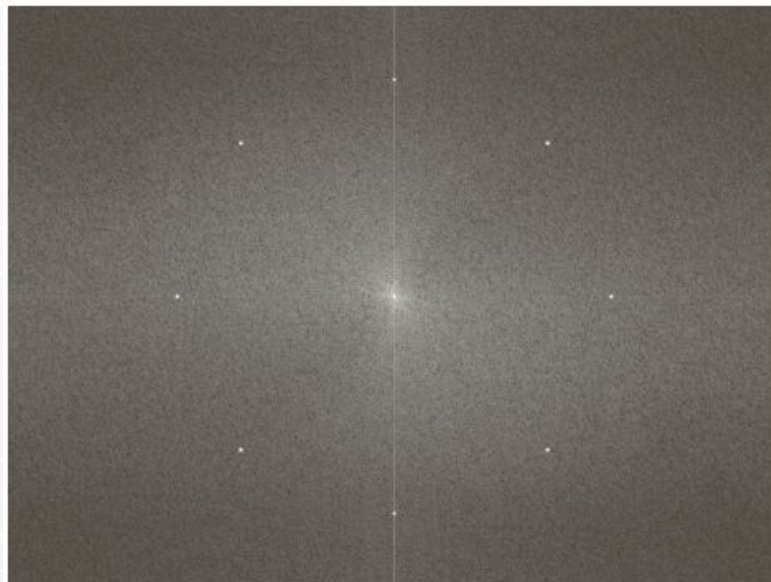
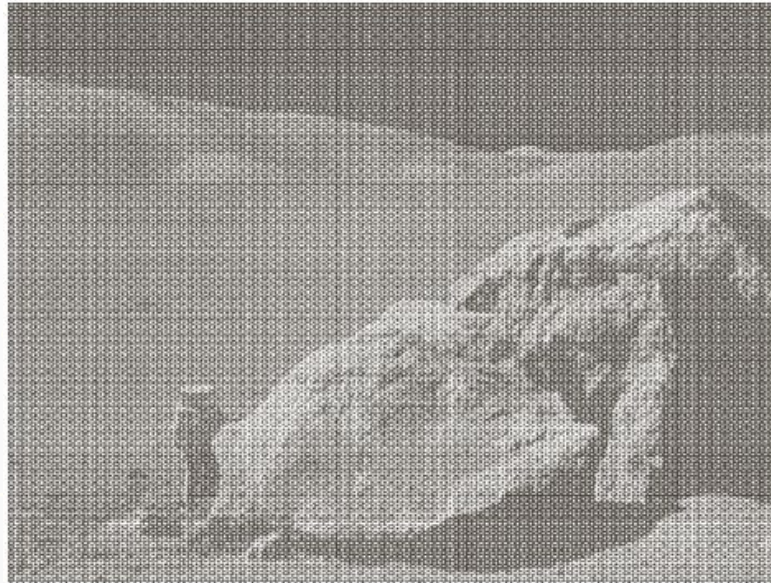


FIGURE 5.4 (Continued) Images and histograms resulting from adding exponential, uniform, and salt and pepper noise to the image in Fig. 5.3.

Periodic Noise

- ▶ Periodic noise in an image arises typically from electrical or electromechanical interference during image acquisition.
- ▶ It is a type of spatially dependent noise
- ▶ Periodic noise can be reduced significantly via frequency domain filtering

An Example of Periodic Noise



a
b

FIGURE 5.5

(a) Image corrupted by sinusoidal noise.
(b) Spectrum (each pair of conjugate impulses corresponds to one sine wave). (Original image courtesy of NASA.)

Estimation of Noise Parameters (1)

The shape of the histogram identifies the closest PDF match from the histogram plot of a small background strip of the image

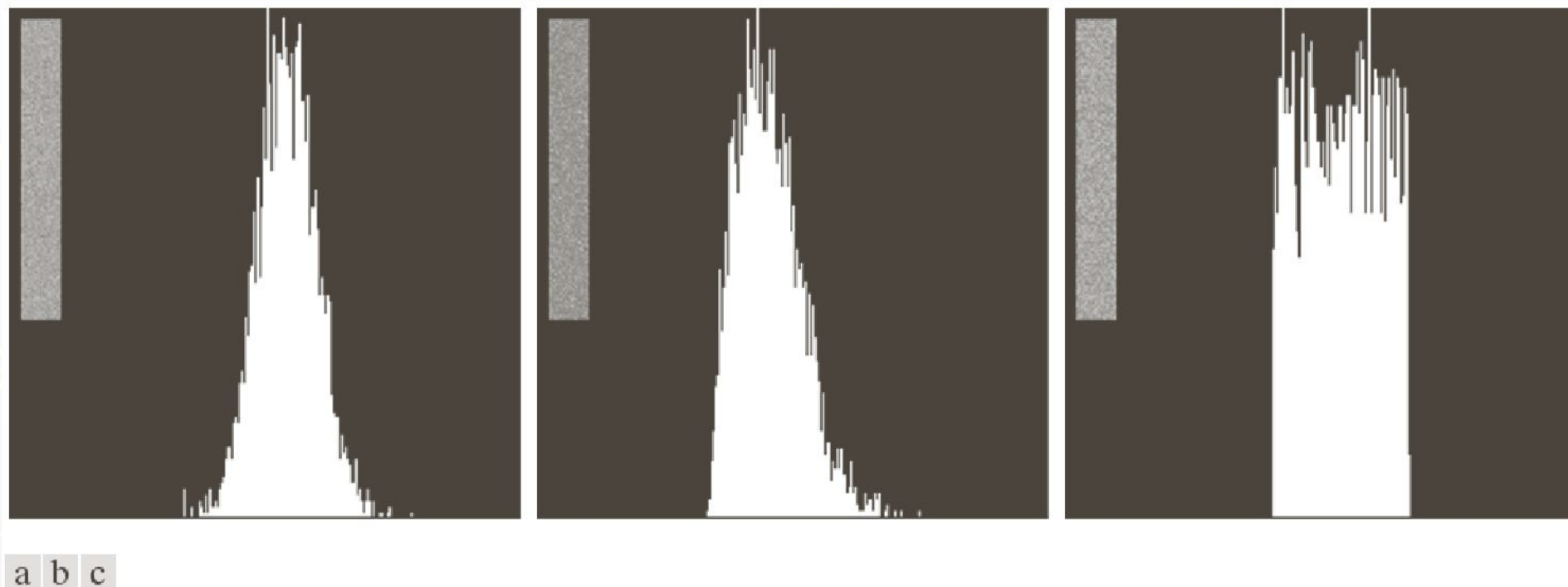


FIGURE 5.6 Histograms computed using small strips (shown as inserts) from (a) the Gaussian, (b) the Rayleigh, and (c) the uniform noisy images in Fig. 5.4.

Estimation of Noise Parameters (2)

Consider a subimage denoted by S , and let $p_s(z_i)$, $i = 0, 1, \dots, L-1$, denote the probability estimates of the intensities of the pixels in S . The mean and variance of the pixels in S :

$$\bar{z} = \sum_{i=0}^{L-1} z_i p_s(z_i)$$

and

$$\sigma^2 = \sum_{i=0}^{L-1} (z_i - \bar{z})^2 p_s(z_i)$$

For Gaussian, we need mean and variance, and for other distributions we calculate a and b.

Restoration in the Presence of Noise Only and the noise is not periodic is to use

– Spatial Filtering

Noise model without degradation

$$g(x, y) = f(x, y) + \eta(x, y)$$

and

$$G(u, v) = F(u, v) + N(u, v)$$

Spatial Filtering: Mean Filters (1)

Let S_{xy} represent the set of coordinates in a rectangle subimage window of size $m \times n$, centered at (x, y) .

Arithmetic mean filter

$$\hat{f}(x, y) = \frac{1}{mn} \sum_{(s,t) \in S_{xy}} g(s, t)$$

Spatial Filtering: Mean Filters (2)

Geometric mean filter

$$\hat{f}(x, y) = \left[\prod_{(s, t) \in S_{xy}} g(s, t) \right]^{\frac{1}{mn}}$$

Generally, a geometric mean filter achieves smoothing comparable to the arithmetic mean filter, but it tends to lose less image detail in the process

Spatial Filtering: Mean Filters (3)

Harmonic mean filter

$$\hat{f}(x, y) = \frac{mn}{\sum_{(s,t) \in S_{xy}} \frac{1}{g(s,t)}}$$

It works well for salt noise, but fails for pepper noise.
It does well also with other types of noise like Gaussian noise.

Spatial Filtering: Mean Filters (4)

Contraharmonic mean filter

$$\hat{f}(x, y) = \frac{\sum_{(s,t) \in S_{xy}} g(s, t)^{Q+1}}{\sum_{(s,t) \in S_{xy}} g(s, t)^Q}$$

Q is the order of the filter.

It is well suited for reducing the effects of salt-and-pepper noise. $Q > 0$ for pepper noise and $Q < 0$ for salt noise.

If $Q = 0$, it is arithmetic mean, If $Q = -1$, it is harmonic mean

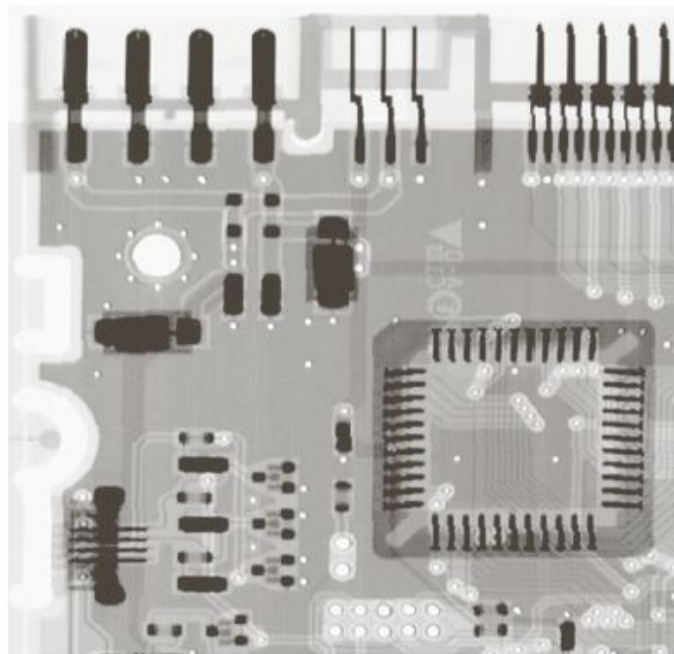
Spatial Filtering: Example (1)

a	b
c	d

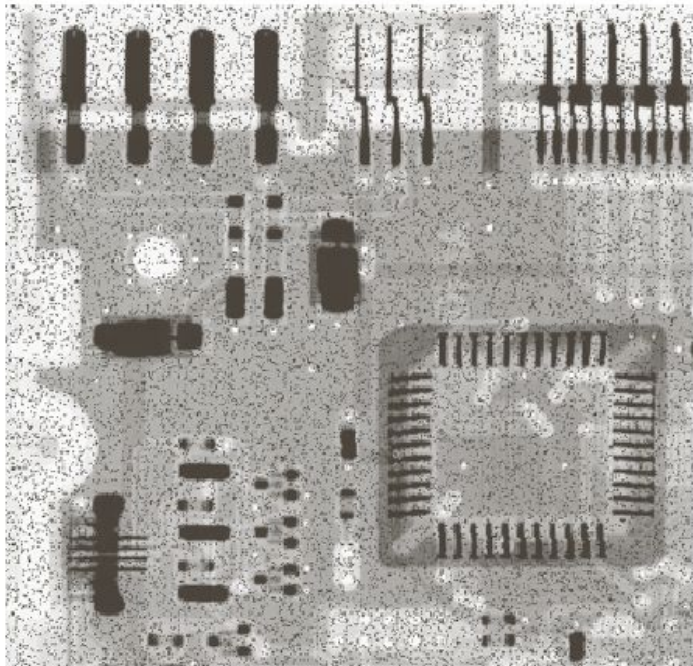
FIGURE 5.7

(a) X-ray image.
(b) Image corrupted by additive Gaussian noise.
(c) Result of filtering with an arithmetic mean filter of size 3×3 .
(d) Result of filtering with a geometric mean filter of the same size.

(Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)



Spatial Filtering: Example (2)



a	b
c	d

FIGURE 5.8

(a) Image corrupted by pepper noise with a probability of 0.1. (b) Image corrupted by salt noise with the same probability. (c) Result of filtering (a) with a 3×3 contra-harmonic filter of order 1.5. (d) Result of filtering (b) with $Q = -1.5$.

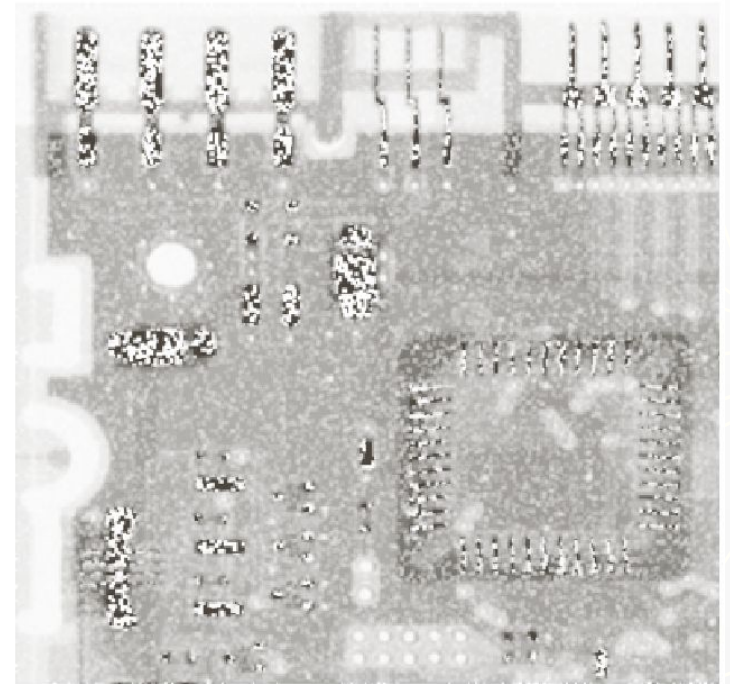
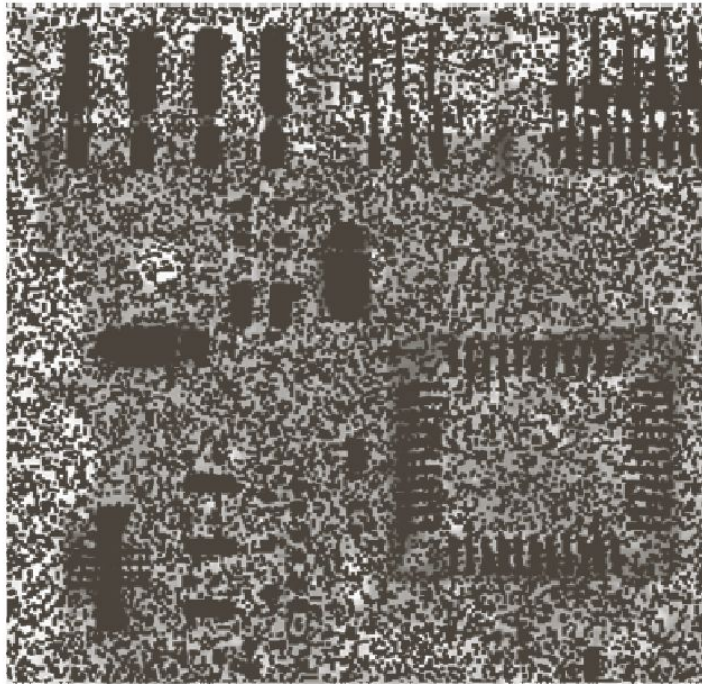
Spatial Filtering: Example (3)

a b

FIGURE 5.9

Results of selecting the wrong sign in contraharmonic filtering.

(a) Result of filtering Fig. 5.8(a) with a contraharmonic filter of size 3×3 and $Q = -1.5$.
(b) Result of filtering 5.8(b) with $Q = 1.5$.



Spatial Filtering: Order-Statistic Filters (1)

Median filter

$$\hat{f}(x, y) = \underset{(s, t) \in S_{xy}}{\text{median}} \{g(s, t)\}$$

Suitable for
Impulse noise

Max filter

$$\hat{f}(x, y) = \max_{(s, t) \in S_{xy}} \{g(s, t)\}$$

Suitable for Pepper
noise

Min filter

$$\hat{f}(x, y) = \min_{(s, t) \in S_{xy}} \{g(s, t)\}$$

Suitable for salt
noise

Midpoint filter

$$\hat{f}(x, y) = \frac{1}{2} \left[\max_{(s, t) \in S_{xy}} \{g(s, t)\} + \min_{(s, t) \in S_{xy}} \{g(s, t)\} \right]$$

Suitable for
Gaussian
noise

a	b
c	d

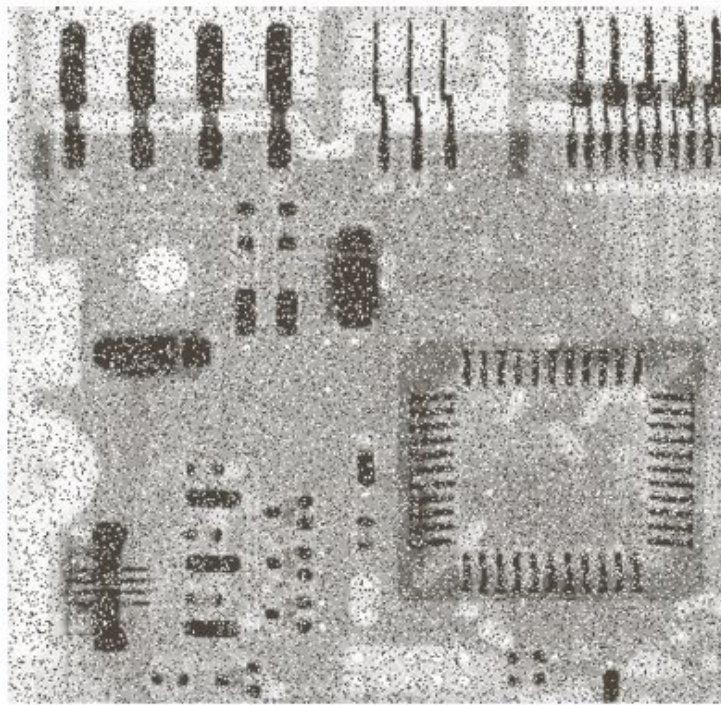
FIGURE 5.10

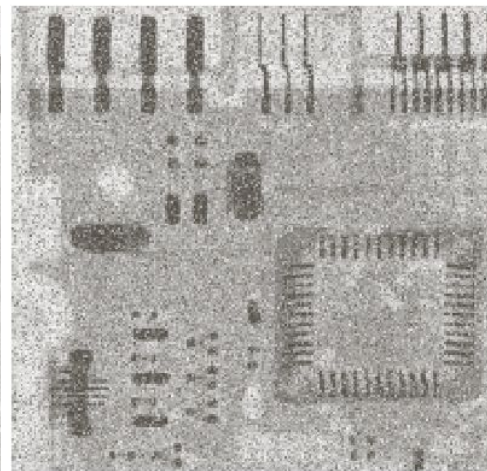
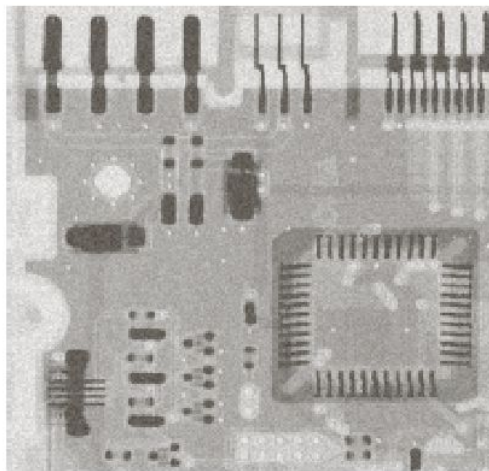
(a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.1$.

(b) Result of one pass with a median filter of size 3×3 .

(c) Result of processing (b) with this filter.

(d) Result of processing (c) with the same filter.





a	b
c	d
e	f

FIGURE 5.12

(a) Image corrupted by additive uniform noise.

(b) Image additionally corrupted by additive salt-and-pepper noise.

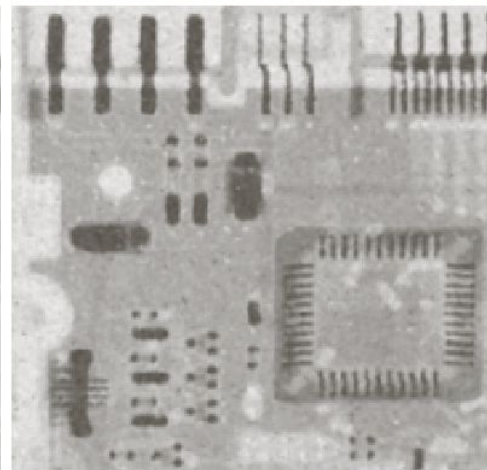
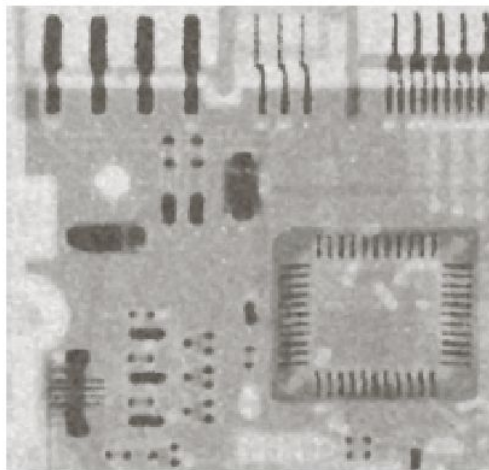
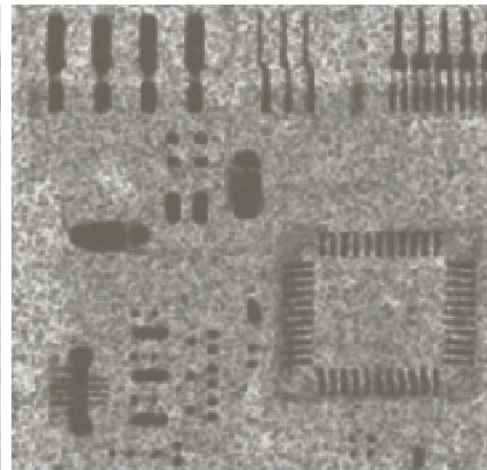
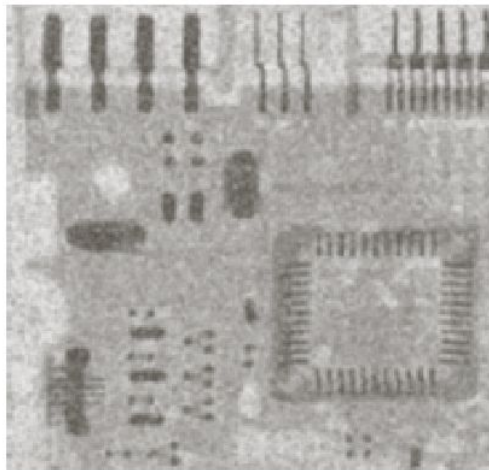
Image (b) filtered with a 5×5 ;

(c) arithmetic mean filter;

(d) geometric mean filter;

(e) median filter;

and (f) alpha-trimmed mean filter with $d = 5$.



Linear, Position-Invariant Degradations

H is linear

$$H[af_1(x, y) + bf_2(x, y)] = aH[f_1(x, y)] + bH[f_2(x, y)]$$

f_1 and f_2 are any two input images.

An operator having the input-output relationship
 $g(x, y) = H[f(x, y)]$ is said to be position invariant
if

$$H[f(x - \alpha, y - \beta)] = g(x - \alpha, y - \beta)$$

for any $f(x, y)$ and any α and β .

Linear, Position-Invariant Degradations

$$f(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\alpha, \beta) \delta(x - \alpha, y - \beta) d\alpha d\beta$$

Assume for a moment that $\eta(x, y) = 0$

if H is a linear operator,

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Mathematical Modeling (1)

- ▶ Environmental conditions cause degradation

A model about atmospheric turbulence

$$H(u, v) = e^{-k(u^2 + v^2)^{5/6}}$$

k : a constant that depends on
the nature of the turbulence

a	b
c	d

FIGURE 5.25

Illustration of the
atmospheric
turbulence model.

(a) Negligible
turbulence.

(b) Severe
turbulence,
 $k = 0.0025$.

(c) Mild
turbulence,
 $k = 0.001$.

(d) Low
turbulence,
 $k = 0.00025$.

(Original image
courtesy of
NASA.)



Mathematical Modeling (2)

- ▶ Derive a mathematical model from basic principles

E.g., An image blurred by uniform linear motion between the image and the sensor during image acquisition



Mathematical Modeling (3)

Suppose that an image $f(x, y)$ undergoes planar motion, $x_0(t)$ and $y_0(t)$ are the time-varying components of motion in the x - and y -directions, respectively.

The optical imaging process is perfect. T is the duration of the exposure. The blurred image $g(x, y)$

$$g(x, y) = \int_0^T f[x - x_0(t), y - y_0(t)] dt$$

Mathematical Modeling (4)

$$g(x, y) = \int_0^T f[x - x_0(t), y - y_0(t)] dt$$

$$G(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x, y) e^{-j2\pi(ux+vy)} dx dy$$

Mathematical Modeling (4)

$$H(u, v) = \int_0^T e^{-j2\pi[ux_0(t) + vy_0(t)]} dt$$

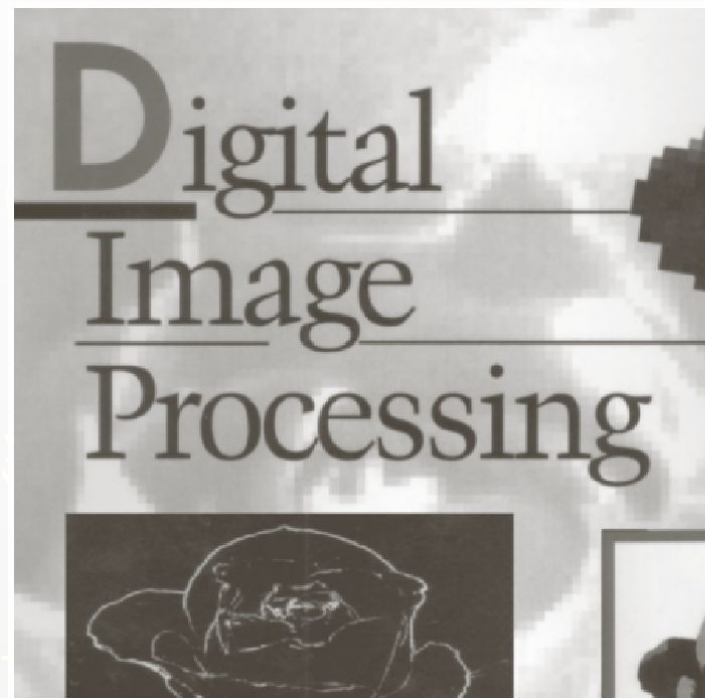


Mathematical Modeling (5)

Suppose that the image undergoes uniform linear motion in the x -direction and y -direction, at a rate given by

$$x_0(t) = at / T \text{ and } y_0(t) = bt / T$$

$$\begin{aligned} H(u, v) &= \int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]} dt \\ &= \int_0^T e^{-j2\pi[ua+vb]t/T} dt \end{aligned}$$



a b

FIGURE 5.26

(a) Original image.
(b) Result of blurring using the function in Eq. (5.6-11) with $a = b = 0.1$ and $T = 1$.

Minimum Mean Square Error (Wiener) Filtering

□ **N. Wiener (1942)**

□ **Objective**

Find an estimate of the uncorrupted image such that the mean square error between them is minimized

$$e^2 = E \left\{ (f - \hat{f})^2 \right\}$$

Minimum Mean Square Error (Wiener) Filtering

The minimum of the error function is given in the frequency domain by the expression

$$\begin{aligned} \hat{F}(u, v) &= \left[\frac{H^*(u, v) S_f(u, v)}{S_f(u, v) |H(u, v)|^2 + S_\eta(u, v)} \right] G(u, v) \\ &= \left[\frac{H^*(u, v)}{|H(u, v)|^2 + S_\eta(u, v) / S_f(u, v)} \right] G(u, v) \\ &= \left[\frac{1}{H(u, v)} \frac{|H(u, v)|^2}{|H(u, v)|^2 + S_\eta(u, v) / S_f(u, v)} \right] G(u, v) \end{aligned}$$

Minimum Mean Square Error (Wiener) Filtering

$$\hat{F}(u, v) = \left[\frac{1}{H(u, v) \frac{|H(u, v)|^2}{|H(u, v)|^2 + S_\eta(u, v) / S_f(u, v)}} \right] G(u, v)$$

$H(u, v)$: degradation function

$H^*(u, v)$: complex conjugate of $H(u, v)$

$$|H(u, v)|^2 = H^*(u, v)H(u, v)$$

$S_\eta(u, v) = |N(u, v)|^2$ = power spectrum of the noise

$S_f(u, v) = |F(u, v)|^2$ = power spectrum of the undegraded image

Minimum Mean Square Error (Wiener) Filtering

$$\hat{F}(u, v) = \left[\frac{1}{H(u, v)} \frac{|H(u, v)|^2}{|H(u, v)|^2 + K} \right] G(u, v)$$

K is a specified constant. Generally, the value of K is chosen interactively to yield the best visual results.

Minimum Mean Square Error (Wiener) Filtering



FIGURE 5.28 Comparison of inverse and Wiener filtering. (a) Result of full inverse filtering of Fig. 5.25(b). (b) Radially limited inverse filter result. (c) Wiener filter result.

Some Measures (1)

Singal-to-Noise Ratio (SNR)

$$SNR = \frac{\sum_{u=0}^{M-1} \sum_{v=0}^{N-1} |F(u, v)|^2}{\sum_{u=0}^{M-1} \sum_{v=0}^{N-1} |N(u, v)|^2}$$

This ratio gives a measure of the level of information bearing singal power to the level of noise power.

Some Measures (2)

Mean Square Error (MSE)

$$\text{MSE} = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \left[f(x, y) - \hat{f}(x, y) \right]^2$$

Root-Mean-Square-Error (RMSE)

$$\text{RMSE} = \frac{\sum_{u=0}^{M-1} \sum_{v=0}^{N-1} \hat{f}(x, y)^2}{\sum_{u=0}^{M-1} \sum_{v=0}^{N-1} |f(x, y) - \hat{f}(x, y)|^2}$$

Constrained Least Squares Filtering

- ▶ In Wiener filter, the power spectra of the undegraded image and noise must be known. Although a constant estimate is sometimes useful, it is not always suitable.
- ▶ Constrained least squares filtering just requires the mean and variance of the noise.

Constrained Least Squares Filtering

$$F(u, v) = \left[\frac{H^*(u, v)}{|H(u, v)|^2 + \gamma |P(u, v)|^2} \right] G(u, v)$$

$P(u, v)$ is the Fourier transform of the function

$$p(x, y) = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

γ is a parameter

Examples

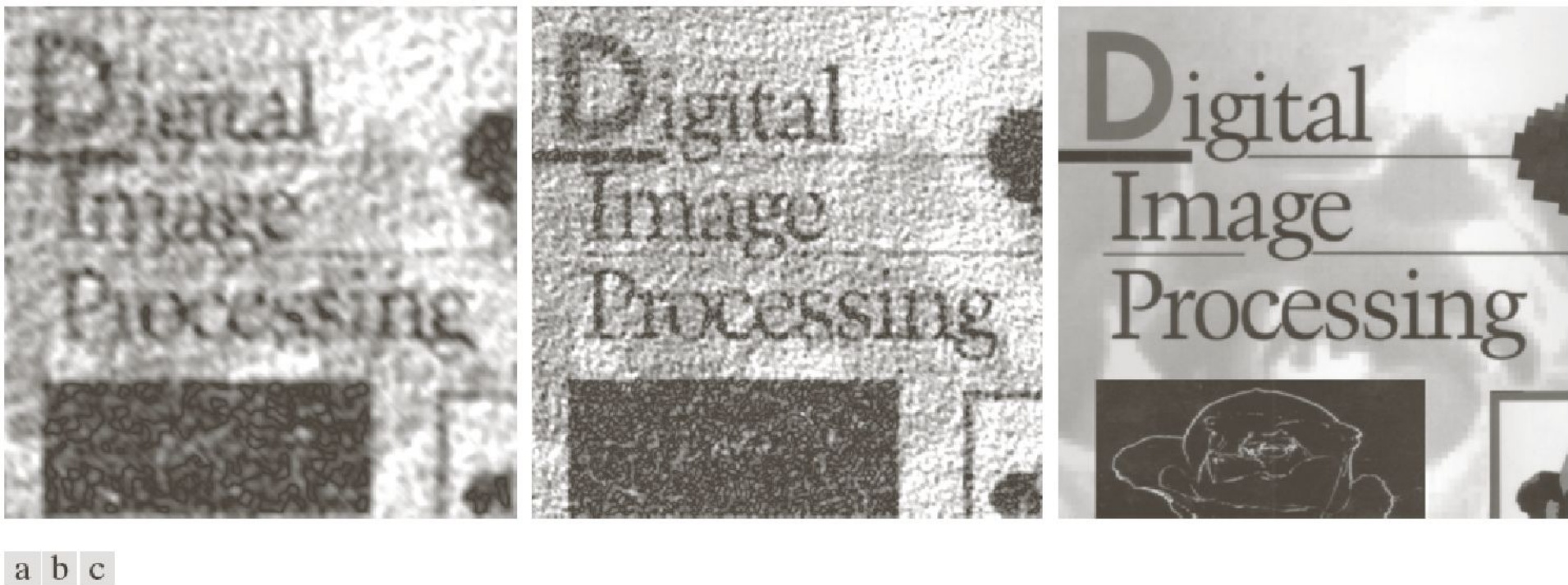


FIGURE 5.30 Results of constrained least squares filtering. Compare (a), (b), and (c) with the Wiener filtering results in Figs. 5.29(c), (f), and (i), respectively.