

Image Processing in Frequency Domain



Outline

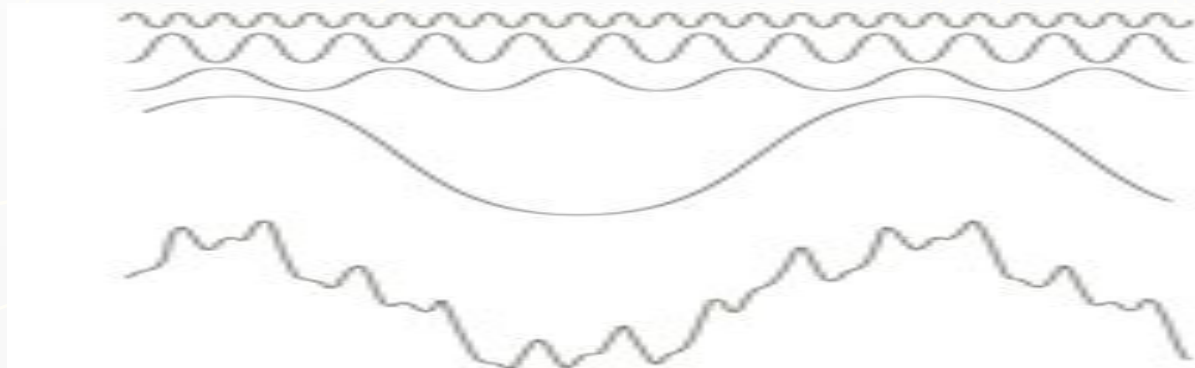
- ▶ Fourier Series , Fourier Transform
- ▶ Convolution
- ▶ 2-D discrete Fourier transform
- ▶ Filtering in Fourier Transform Domain



Fourier Series and Fourier Transform

► Fourier Series

Any periodic function can be expressed as the sum of sines and /or cosines of different frequencies, each multiplied by a different coefficients



► Fourier Transform

Any function that is not periodic can be expressed as the integral of sines and /or cosines multiplied by a weighing function

Preliminary Concepts

$j = \sqrt{-1}$, a complex number

$$C = R + jI$$

the conjugate

$$C^* = R - jI$$

$$|C| = \sqrt{R^2 + I^2} \text{ and } \theta = \arctan(I / R)$$

$$C = |C| (\cos \theta + j \sin \theta)$$

Using Euler's formula,

$$C = |C| e^{j\theta}$$

Fourier Series

$$\theta = \omega \tau = (2\pi n/T) \tau$$

A function $f(t)$ of a continuous variable t that is periodic with period, T , can be expressed as the sum of sines and cosines multiplied by appropriate coefficients

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{j \frac{2\pi n}{T} t}$$

where

$$c_n = \frac{1}{T} \int_{-T/2}^{T/2} f(t) e^{-j \frac{2\pi n}{T} t} dt \quad \text{for } n = 0, \pm 1, \pm 2, \dots$$

Impulses and the Sifting Property (1)

A *unit impulse* of a continuous variable t located at $t=0$, denoted $\delta(t)$, defined as

$$\delta(t) = \begin{cases} \infty & \text{if } t = 0 \\ 0 & \text{if } t \neq 0 \end{cases}$$

and is constrained also to satisfy the identity

$$\int_{-\infty}^{\infty} \delta(t) dt = 1$$

The *sifting property*

$$\int_{-\infty}^{\infty} f(t) \delta(t - t_0) dt = f(t_0)$$

$$\int_{-\infty}^{\infty} f(t) \delta(t) dt = f(0)$$

Impulses (Discrete) and the Sifting Property (2)

A *unit impulse* of a discrete variable x located at $x=0$, denoted $\delta(x)$, defined as

$$\delta(x) = \begin{cases} 1 & \text{if } x = 0 \\ 0 & \text{if } x \neq 0 \end{cases}$$

and is constrained also to satisfy the identity

$$\sum_{x=-\infty}^{\infty} \delta(x) = 1$$

The *sifting property*

$$\sum_{x=-\infty}^{\infty} f(x) \delta(x - x_0) = f(x_0)$$

$$\sum_{x=-\infty}^{\infty} f(x) \delta(x) = f(0)$$

Impulses and the Sifting Property (3)

impulse train $s_{\Delta T}(t)$,

$$s_{\Delta T}(t) = \sum_{n=-\infty}^{\infty} \delta(t - n\Delta T)$$

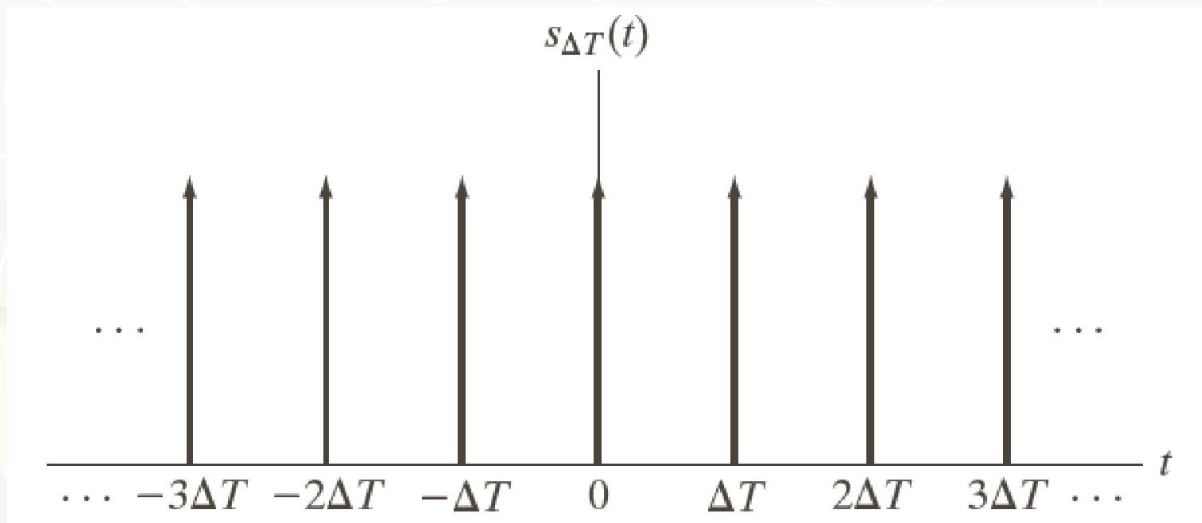


FIGURE 4.3 An impulse train.

Fourier Transform: Continuous Variable

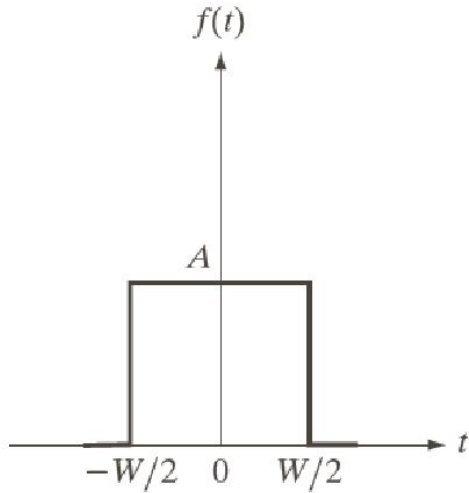
The *Fourier Transform* of a continuous function $f(t)$

$$F(\mu) = \mathfrak{F}\{f(t)\} = \int_{-\infty}^{\infty} f(t)e^{-j2\pi\mu t} dt$$

The *Inverse Fourier Transform* of $F(\mu)$

$$f(t) = \mathfrak{F}^{-1}\{F(\mu)\} = \int_{-\infty}^{\infty} F(\mu)e^{j2\pi\mu t} d\mu$$

Fourier Transform: One Continuous Variable



a b c

FIGURE 4.4 (a) A simple function that is finite in both directions.

Fourier Transform: Impulses

The Fourier transform of a unit impulse located at the origin:

$$F(\mu) = \int_{-\infty}^{\infty} \delta(t) e^{-j2\pi\mu t} dt$$

The Fourier transform of a unit impulse located at $t = t_0$:

$$F(\mu) = \int_{-\infty}^{\infty} \delta(t - t_0) e^{-j2\pi\mu t} dt$$

Fourier Transform and Convolution

The convolution of two functions is denoted by the operator $\star$

$$f(t) \star h(t) = \int_{-\infty}^{\infty} f(\tau)h(t - \tau)d\tau$$

Fourier Transform and Convolution

Fourier Transform Pairs

$$f(t) \star h(t) \Leftrightarrow H(\mu)F(\mu)$$

$$f(t)h(t) \Leftrightarrow H(\mu) \star F(\mu)$$

The Discrete Fourier Transform (DFT) of One Variable

$$F(\mu) = \sum_{x=0}^{M-1} f(x) e^{-j2\pi\mu x/M}, \quad \mu = 0, 1, \dots, M-1$$

$$f(x) = \frac{1}{M} \sum_{\mu=0}^{M-1} F(\mu) e^{j2\pi\mu x/M}, \quad x = 0, 1, 2, \dots, M-1$$

2-D Impulse and Sifting Property: Continuous

The impulse $\delta(t, z)$,
$$\delta(t, z) = \begin{cases} \infty & \text{if } t = z = 0 \\ 0 & \text{otherwise} \end{cases}$$

and
$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \delta(t, z) dt dz = 1$$

The sifting property

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(t, z) \delta(t, z) dt dz =$$

2-D Impulse and Sifting Property: Discrete

The impulse $\delta(x, y)$,
$$\delta(x, y) = \begin{cases} 1 & \text{if } x = y = 0 \\ 0 & \text{otherwise} \end{cases}$$

The sifting property

$$\sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} f(x, y) \delta(x, y) = f(0, 0)$$

and

$$\sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} f(x, y) \delta(x - x_0, y - y_0) = f(x_0, y_0)$$

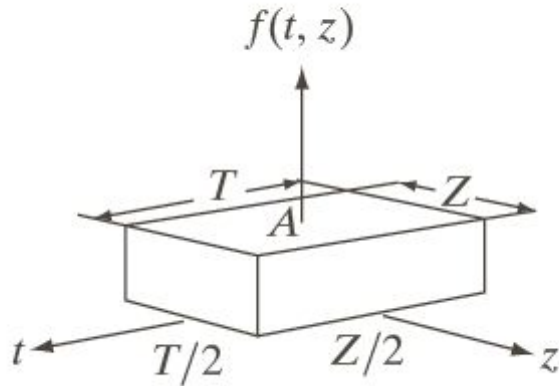
2-D Fourier Transform: Continuous

$$F(\mu, \nu) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(t, z) e^{-j2\pi(\mu t + \nu z)} dt dz$$

and

$$f(t, z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(\mu, \nu) e^{j2\pi(\mu t + \nu z)} d\mu d\nu$$

2-D Fourier Transform: Continuous



$$\begin{aligned}
 F(\mu, \nu) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(t, z) e^{-j2\pi(\mu t + \nu z)} dt dz \\
 &= \int_{-T/2}^{T/2} \int_{-Z/2}^{Z/2} A e^{-j2\pi(\mu t + \nu z)} dt dz \\
 &= ATZ \left[\frac{\sin(\pi\mu T)}{\pi\mu T} \right] \left[\frac{\sin(\pi\nu T)}{\pi\nu T} \right]
 \end{aligned}$$

a b

FIGURE 4.13 (a) A 2-D function, and (b) a section of its spectrum (not to scale). The block is longer along the t -axis, so the spectrum is more “contracted” along the μ -axis. Compare with Fig. 4.4.

2-D Discrete Fourier Transform and Its Inverse

DFT:

$$F(\mu, \nu) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) e^{-j2\pi(\mu x/M + \nu y/N)}$$

$$\mu = 0, 1, 2, \dots, M-1; \nu = 0, 1, 2, \dots, N-1;$$

$f(x, y)$ is a digital image of size $M \times N$.

IDFT:

$$f(x, y) = \frac{1}{MN} \sum_{\mu=0}^{M-1} \sum_{\nu=0}^{N-1} F(\mu, \nu) e^{j2\pi(\mu x/M + \nu y/N)}$$

Properties of the 2-D DFT

relationships between spatial and frequency intervals

Let ΔT and ΔZ denote the separations between samples, then the separations between the corresponding discrete, frequency domain variables are given by

$$\Delta\mu = \frac{1}{M\Delta T}$$

and

$$\Delta\nu = \frac{1}{N\Delta Z}$$

Properties of the 2-D DFT

translation and rotation

$$f(x, y)e^{j2\pi(\mu_0 x/M + \nu_0 y/N)} \Leftrightarrow F(\mu - \mu_0, \nu - \nu_0)$$

and

$$f(x - x_0, y - y_0) \Leftrightarrow F(\mu, \nu)e^{-j2\pi(\mu x_0/M + \nu y_0/N)}$$

Using the polar coordinates

$$x = r \cos \theta \quad y = r \sin \theta \quad \mu = \omega \cos \varphi \quad \nu = \omega \sin \varphi$$

results in the following transform pair:

$$f(r, \theta + \theta_0) \Leftrightarrow F(\omega, \varphi + \theta_0)$$

Properties of the 2-D DFT

periodicity

2-D Fourier transform and its inverse are infinitely periodic

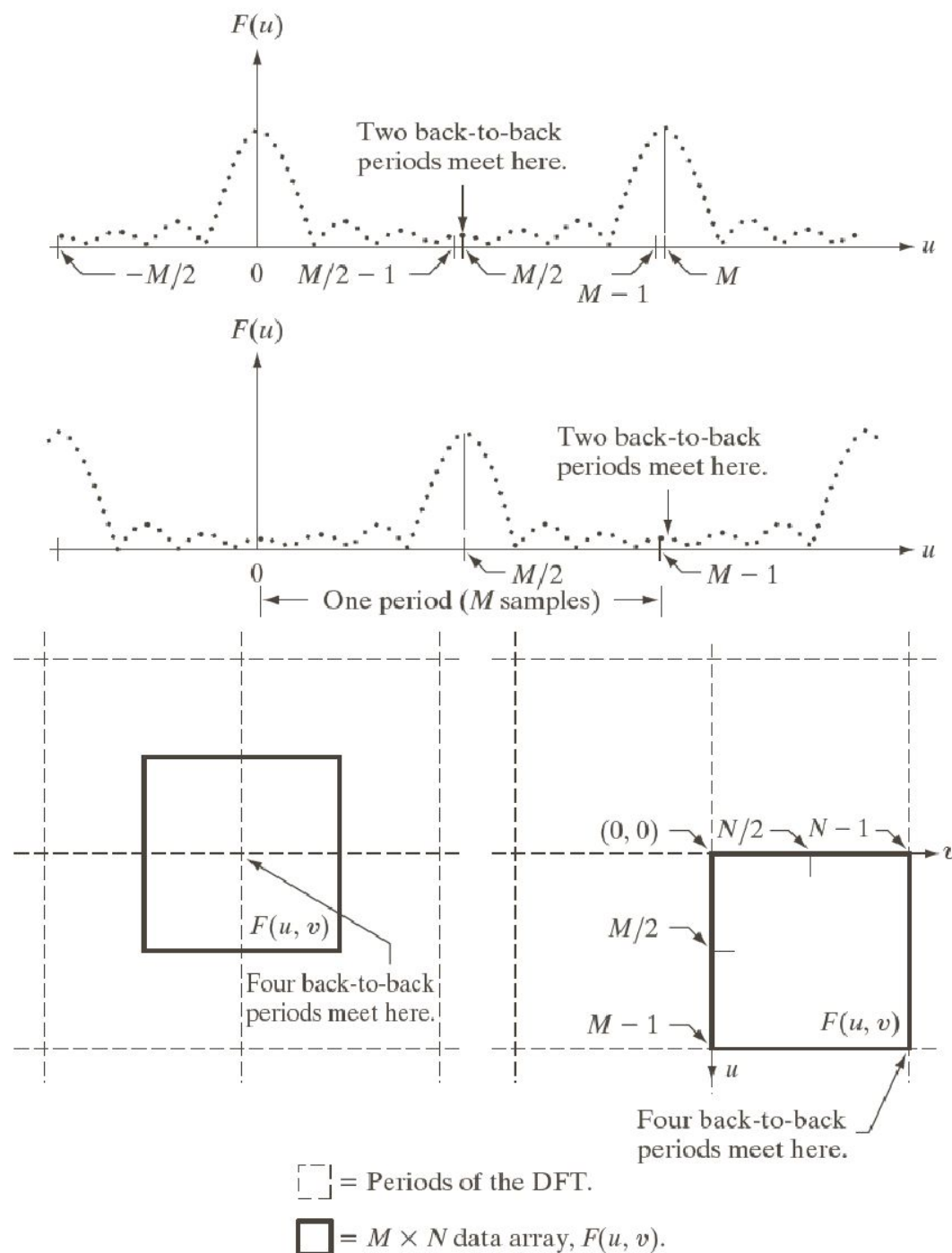
$$F(\mu, \nu) = F(\mu + k_1 M, \nu) = F(\mu, \nu + k_2 N) = F(\mu + k_1 M, \nu + k_2 N)$$

$$f(x, y) = f(x + k_1 M, y) = f(x, y + k_2 N) = f(x + k_1 M, y + k_2 N)$$

$$f(x) e^{j2\pi(\mu_0 x/M)} \Leftrightarrow F(\mu - \mu_0)$$

$$\mu_0 = M / 2, \quad f(x)(-1)^x \Leftrightarrow F(\mu - M / 2)$$

$$f(x, y)(-1)^{x+y} \Leftrightarrow F(\mu - M / 2, \nu - N / 2)$$



a
b
c d

FIGURE 4.23

Centering the Fourier transform.

(a) A 1-D DFT showing an infinite number of periods.

(b) Shifted DFT obtained by multiplying $f(x)$ by $(-1)^x$ before computing $F(u)$.

(c) A 2-D DFT showing an infinite number of periods. The solid area is the $M \times N$ data array, $F(u, v)$, obtained with Eq. (4.5-15). This array consists of four quarter periods.

(d) A Shifted DFT obtained by multiplying $f(x, y)$ by $(-1)^{x+y}$ before computing $F(u, v)$. The data now contains one complete, centered period, as in (b).

Properties of the 2-D DFT

Symmetry

Spatial Domain [†]		Frequency Domain [†]
1)	$f(x, y)$ real	$\Leftrightarrow F^*(u, v) = F(-u, -v)$
2)	$f(x, y)$ imaginary	$\Leftrightarrow F^*(-u, -v) = -F(u, v)$
3)	$f(x, y)$ real	$\Leftrightarrow R(u, v)$ even; $I(u, v)$ odd
4)	$f(x, y)$ imaginary	$\Leftrightarrow R(u, v)$ odd; $I(u, v)$ even
5)	$f(-x, -y)$ real	$\Leftrightarrow F^*(u, v)$ complex
6)	$f(-x, -y)$ complex	$\Leftrightarrow F(-u, -v)$ complex
7)	$f^*(x, y)$ complex	$\Leftrightarrow F^*(-u - v)$ complex
8)	$f(x, y)$ real and even	$\Leftrightarrow F(u, v)$ real and even
9)	$f(x, y)$ real and odd	$\Leftrightarrow F(u, v)$ imaginary and odd
10)	$f(x, y)$ imaginary and even	$\Leftrightarrow F(u, v)$ imaginary and even
11)	$f(x, y)$ imaginary and odd	$\Leftrightarrow F(u, v)$ real and odd
12)	$f(x, y)$ complex and even	$\Leftrightarrow F(u, v)$ complex and even
13)	$f(x, y)$ complex and odd	$\Leftrightarrow F(u, v)$ complex and odd

TABLE 4.1 Some symmetry properties of the 2-D DFT and its inverse. $R(u, v)$ and $I(u, v)$ are the real and imaginary parts of $F(u, v)$, respectively. The term *complex* indicates that a function has nonzero real and imaginary parts.

[†]Recall that x, y, u , and v are *discrete* (integer) variables, with x and u in the range $[0, M - 1]$, and y , and v in the range $[0, N - 1]$. To say that a complex function is *even* means that its real *and* imaginary parts are even, and similarly for an odd complex function.

2-D Convolution Theorem

1-D convolution

$$f(x) \star h(x) = \sum_{m=0}^{M-1} f(m)h(x-m)$$

2-D convolution

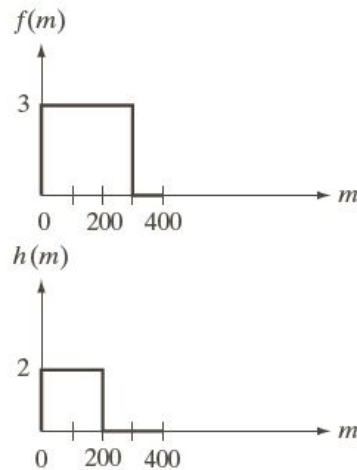
$$f(x, y) \star h(x, y) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f(m, n)h(x-m, y-n)$$

$$x = 0, 1, 2, \dots, M-1; y = 0, 1, 2, \dots, N-1.$$

$$f(x, y) \star h(x, y) \Leftrightarrow F(u, v)H(u, v)$$

$$f(x, y)h(x, y) \Leftrightarrow F(u, v) \star H(u, v)$$

An Example of Convolution



Mirroring h
about the
origin

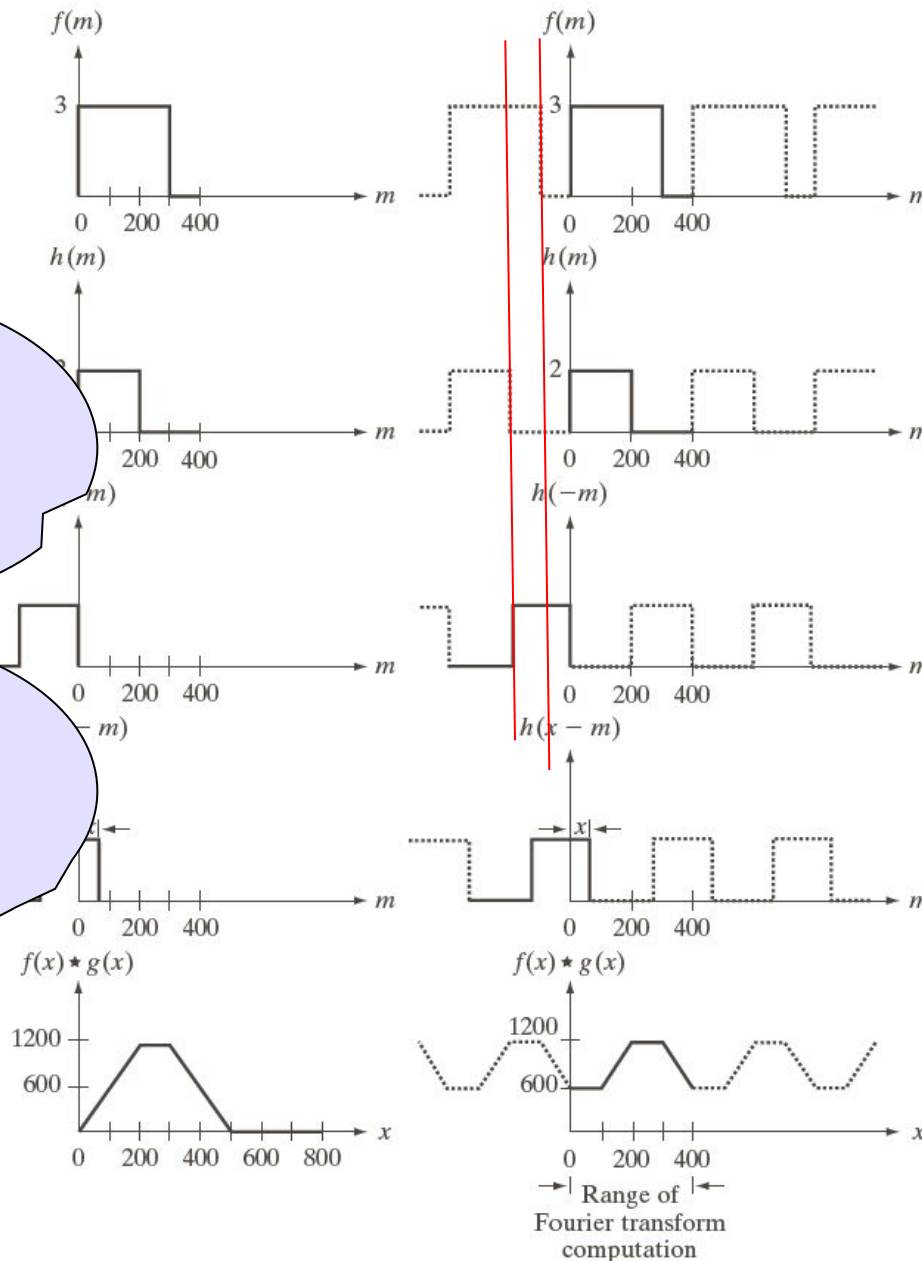
Translating
the mirrored
function by x

Computing the
sum for each x

a	f
b	g
c	h
d	i
e	j

FIGURE 4.28 Left column: convolution of two discrete functions obtained using the approach discussed in Section 3.4.2. The result in (e) is correct. Right column: Convolution of the same functions, but taking into account the periodicity implied by the DFT. Note in (j) how data from adjacent periods produce wraparound error, yielding an incorrect convolution result. To obtain the correct result, function padding must be used.

An Example of Convolution



a	f
b	g
c	h
d	i
e	j

FIGURE 4.28 Left column: convolution of two discrete functions obtained using the approach discussed in Section 3.4.2. The result in (e) is correct. Right column: Convolution of the same functions, but taking into account the periodicity implied by the DFT. Note in (j) how data from adjacent periods produce wraparound error, yielding an incorrect convolution result. To obtain the correct result, function padding must be used.

It causes the **wraparound error**

It can be solved by appending zeros

Zero Padding

- ▶ Consider two functions $f(x)$ and $h(x)$ composed of A and B samples, respectively
- ▶ Append zeros to both functions so that they have the same length, denoted by P , then wraparound is avoided by choosing

$$P \geq A+B-1$$

Zero Padding

- ▶ Let $f(x,y)$ and $h(x,y)$ be two image arrays of sizes $A \times B$ and $C \times D$ pixels, respectively. Wraparound error in their convolution can be avoided by padding these functions with zeros

$$f_p(x, y) = \begin{cases} f(x, y) & 0 \leq x \leq A-1 \text{ and } 0 \leq y \leq B-1 \\ 0 & A \leq x \leq P \text{ or } B \leq y \leq Q \end{cases}$$

$$h_p(x, y) = \begin{cases} h(x, y) & 0 \leq x \leq C-1 \text{ and } 0 \leq y \leq D-1 \\ 0 & C \leq x \leq P \text{ or } D \leq y \leq Q \end{cases}$$

Here $P \geq A + C - 1; Q \geq B + D - 1$

Summary

Name	Expression(s)
1) Discrete Fourier transform (DFT) of $f(x, y)$	$F(u, v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) e^{-j2\pi(ux/M + vy/N)}$
2) Inverse discrete Fourier transform (IDFT) of $F(u, v)$	$f(x, y) = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u, v) e^{j2\pi(ux/M + vy/N)}$
3) Polar representation	$F(u, v) = F(u, v) e^{j\phi(u, v)}$
4) Spectrum	$ F(u, v) = [R^2(u, v) + I^2(u, v)]^{1/2}$ $R = \text{Real}(F); \quad I = \text{Imag}(F)$
5) Phase angle	$\phi(u, v) = \tan^{-1} \left[\frac{I(u, v)}{R(u, v)} \right]$
6) Power spectrum	$P(u, v) = F(u, v) ^2$
7) Average value	$\bar{f}(x, y) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x, y) = \frac{1}{MN} F(0, 0)$

(Continued)

Summary

Name	DFT Pairs
1) Symmetry properties	See Table 4.1
2) Linearity	$af_1(x, y) + bf_2(x, y) \Leftrightarrow aF_1(u, v) + bF_2(u, v)$
3) Translation (general)	$f(x, y)e^{j2\pi(u_0x/M + v_0y/N)} \Leftrightarrow F(u - u_0, v - v_0)$ $f(x - x_0, y - y_0) \Leftrightarrow F(u, v)e^{-j2\pi(ux_0/M + vy_0/N)}$
4) Translation to center of the frequency rectangle, $(M/2, N/2)$	$f(x, y)(-1)^{x+y} \Leftrightarrow F(u - M/2, v - N/2)$ $f(x - M/2, y - N/2) \Leftrightarrow F(u, v)(-1)^{u+v}$
5) Rotation	$f(r, \theta + \theta_0) \Leftrightarrow F(\omega, \varphi + \theta_0)$ $x = r \cos \theta \quad y = r \sin \theta \quad u = \omega \cos \varphi \quad v = \omega \sin \varphi$
6) Convolution theorem [†]	$f(x, y) \star h(x, y) \Leftrightarrow F(u, v)H(u, v)$ $f(x, y)h(x, y) \Leftrightarrow F(u, v) \star H(u, v)$

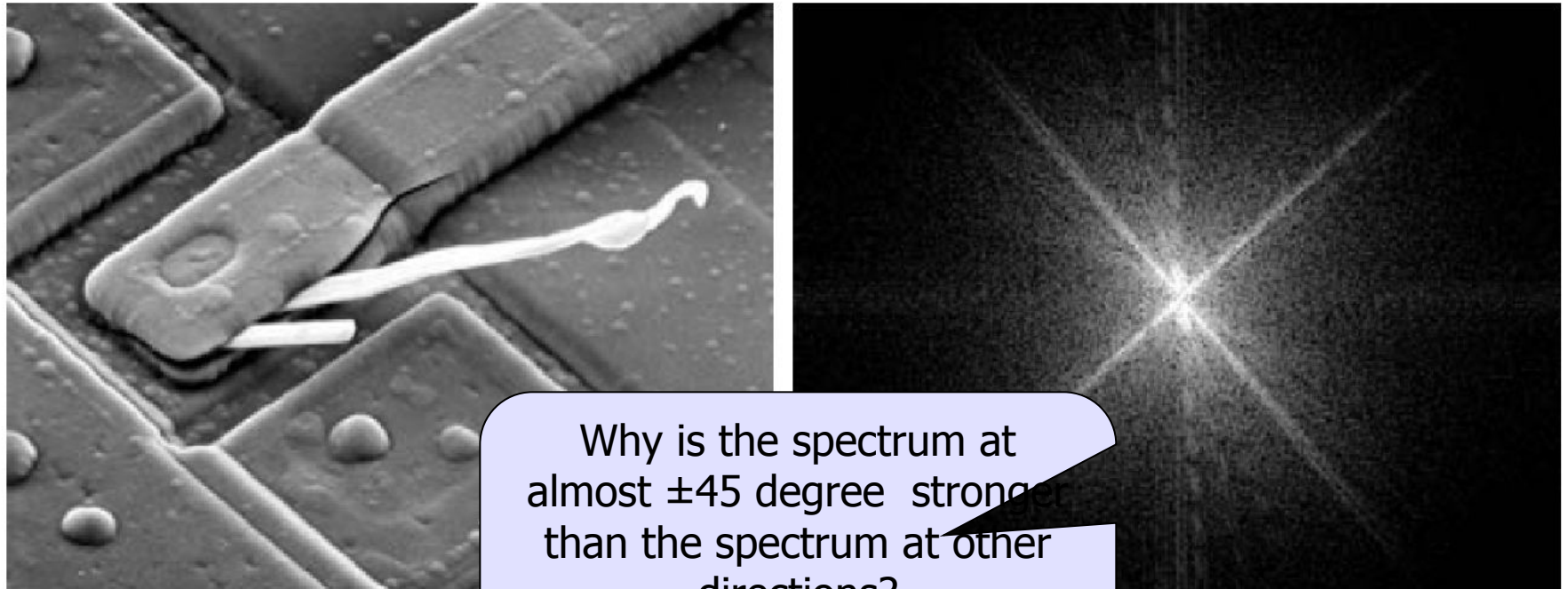
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Summary

Name	DFT Pairs
7) Correlation theorem [†]	$f(x, y) \star h(x, y) \Leftrightarrow F^*(u, v) H(u, v)$ $f^*(x, y) h(x, y) \Leftrightarrow F(u, v) \star H(u, v)$
8) Discrete unit impulse	$\delta(x, y) \Leftrightarrow 1$
9) Rectangle	$\text{rect}[a, b] \Leftrightarrow ab \frac{\sin(\pi ua)}{(\pi ua)} \frac{\sin(\pi vb)}{(\pi vb)} e^{-j\pi(ua+vb)}$
10) Sine	$\sin(2\pi u_0 x + 2\pi v_0 y) \Leftrightarrow$ $j \frac{1}{2} [\delta(u + Mu_0, v + Nv_0) - \delta(u - Mu_0, v - Nv_0)]$
11) Cosine	$\cos(2\pi u_0 x + 2\pi v_0 y) \Leftrightarrow$ $\frac{1}{2} [\delta(u + Mu_0, v + Nv_0) + \delta(u - Mu_0, v - Nv_0)]$
The following Fourier transform pairs are derivable only for continuous variables, denoted as before by t and z for spatial variables and by μ and ν for frequency variables. These results can be used for DFT work by sampling the continuous forms.	
12) <i>Differentiation</i> (The expressions on the right assume that $f(\pm\infty, \pm\infty) = 0$.)	$\left(\frac{\partial}{\partial t}\right)^m \left(\frac{\partial}{\partial z}\right)^n f(t, z) \Leftrightarrow (j2\pi\mu)^m (j2\pi\nu)^n F(\mu, \nu)$ $\frac{\partial^m f(t, z)}{\partial t^m} \Leftrightarrow (j2\pi\mu)^m F(\mu, \nu); \frac{\partial^n f(t, z)}{\partial z^n} \Leftrightarrow (j2\pi\nu)^n F(\mu, \nu)$
13) <i>Gaussian</i>	$A 2\pi\sigma^2 e^{-2\pi^2\sigma^2(t^2+z^2)} \Leftrightarrow A e^{-(\mu^2+\nu^2)/2\sigma^2} \quad (A \text{ is a constant})$

[†] Assumes that the functions have been extended by zero padding. Convolution and correlation are associative, commutative, and distributive.

The Basic Filtering in the Frequency Domain



a b

FIGURE 4.29 (a) SEM image of a damaged integrated circuit. (b) Fourier spectrum of (a). (Original image courtesy of Dr. J. M. Hudak, Brockhouse Institute for Materials Research, McMaster University, Hamilton, Ontario, Canada.)

The Basic Filtering in the Frequency Domain

- ▶ Modifying the Fourier transform of an image
- ▶ Computing the inverse transform to obtain the processed result

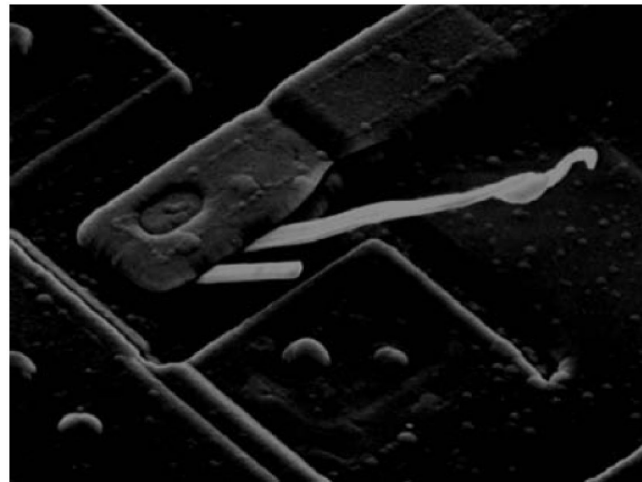
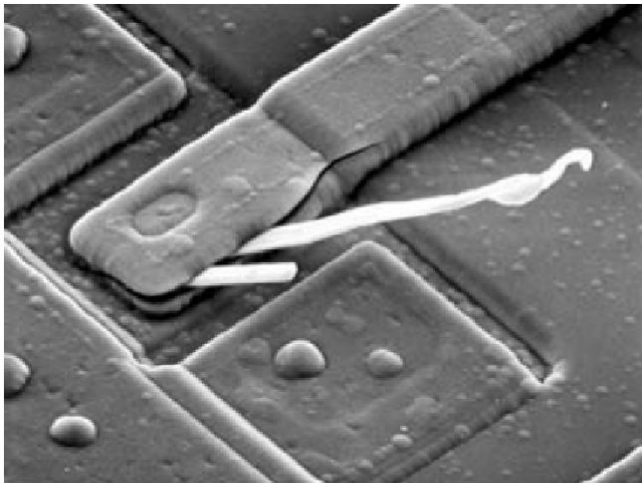
$$g(x, y) = \mathfrak{F}^{-1} \{H(u, v)F(u, v)\}$$

$F(u, v)$ is the DFT of the input image

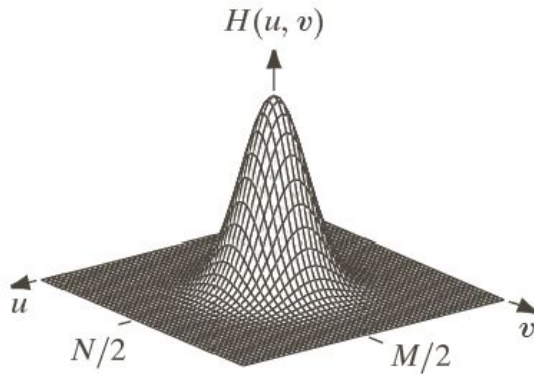
$H(u, v)$ is a filter function.

The Basic Filtering in the Frequency Domain

- ▶ In a filter $H(u,v)$ that is 0 at the center of the transform and 1 elsewhere, what's the output image?



The Basic Filtering in the Frequency Domain



a	b	c
d	e	f

FIGURE 4.31 Top row: frequency domain filters. Bottom row: corresponding filtered images obtained using Eq. (4.7-1). We used $a = 0.85$ in (c) to obtain (f) (the height of the filter itself is 1). Compare (f) with Fig. 4.29(a).

Correspondence Between Filtering in the Spatial and Frequency Domains (1)

Let $H(u)$ denote the 1-D frequency domain Gaussian filter

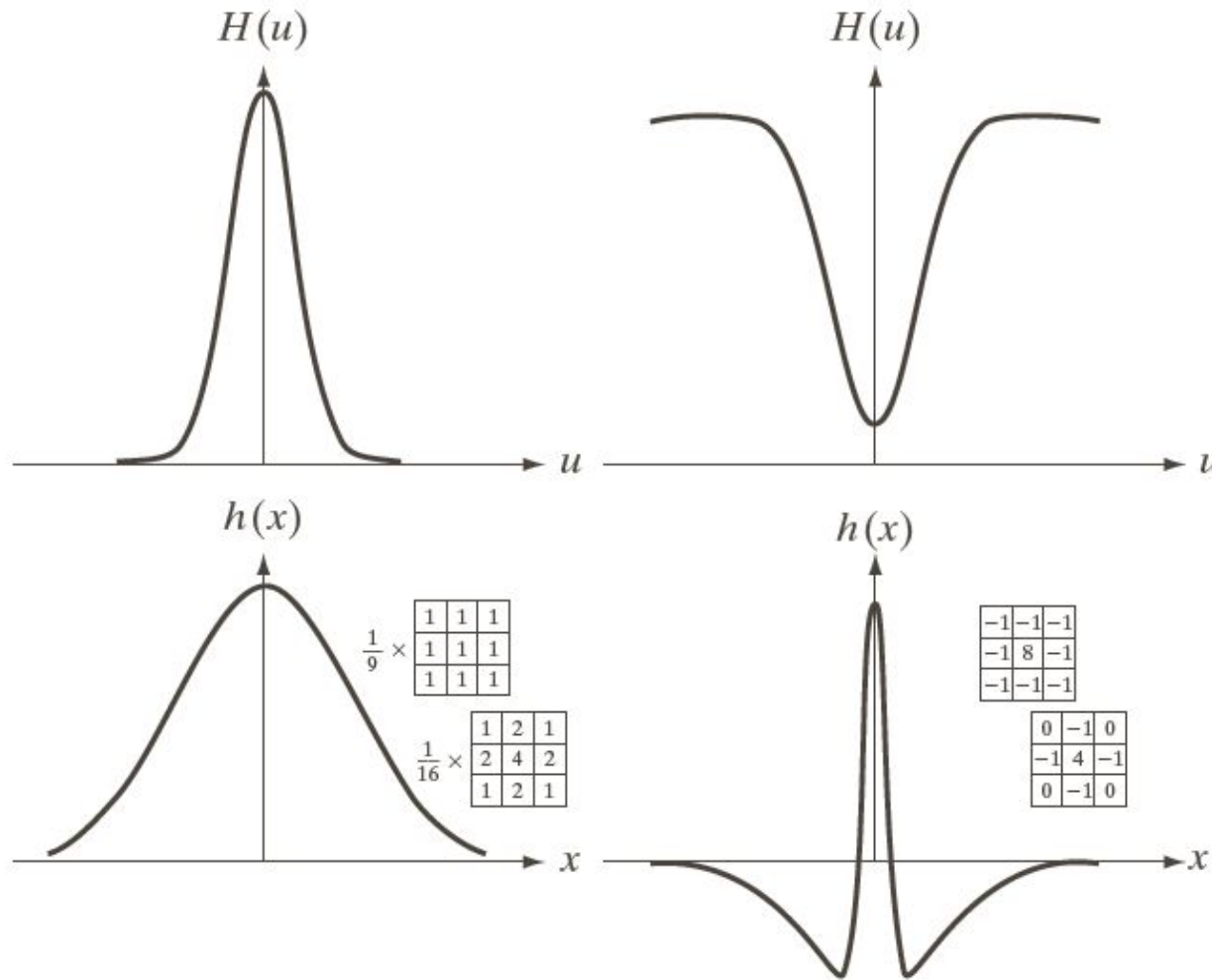
$$H(u) = Ae^{-u^2/2\sigma^2}$$

The corresponding filter in the spatial domain

$$h(x) = \sqrt{2\pi}\sigma Ae^{-2\pi^2\sigma^2x^2}$$

1. Both components are Gaussian and real
2. The functions behave reciprocally

Correspondence Between Filtering in the Spatial and Frequency Domains (3)



a	c
b	d

FIGURE 4.37

(a) A 1-D Gaussian lowpass filter in the frequency domain.

(b) Spatial lowpass filter corresponding to (a).

(c) Gaussian highpass filter in the frequency domain.

(d) Spatial highpass filter corresponding to (c).

The small 2-D masks shown are spatial filters we used in Chapter 3.

Generate $H(u,v)$

1. Multiply $h_p(x, y)$ by $(-1)^{x+y}$ to center the frequency domain filter
2. Compute the forward DFT of the result in (1)
3. Set the real part of the resulting DFT to 0 to account for parasitic real parts
4. Multiply the result by $(-1)^{u+v}$, which is implicit when $h(x, y)$ was moved to the center of $h_p(x, y)$.

Image Smoothing Using Filter Domain Filters: ILPF

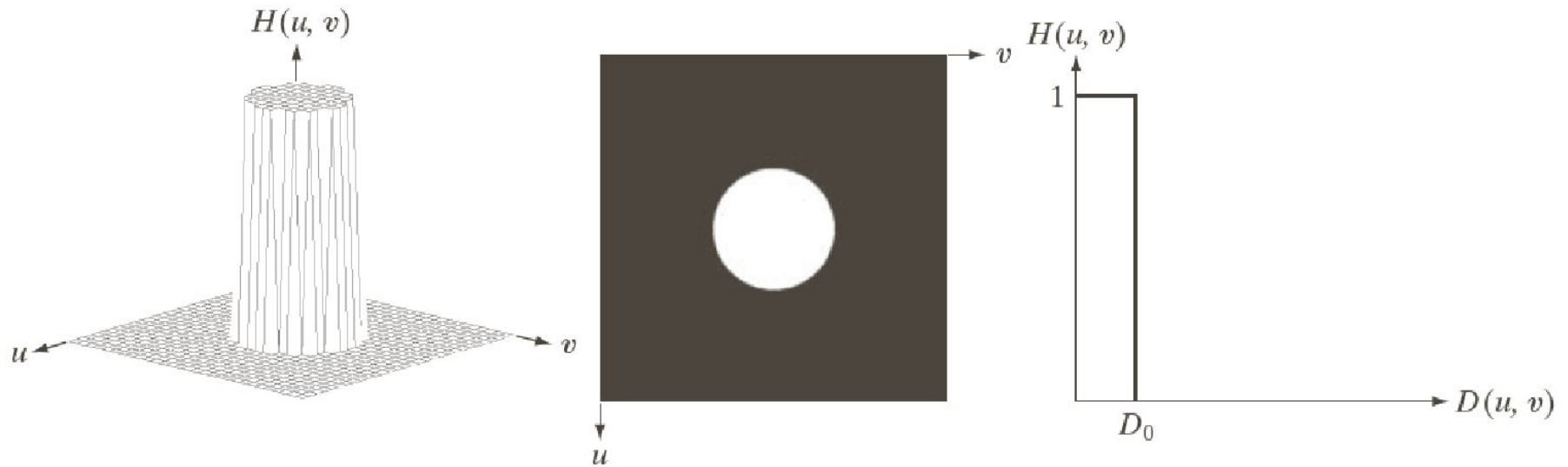
Ideal Lowpass Filters (ILPF)

$$H(u, v) = \begin{cases} 1 & \text{if } D(u, v) \leq D_0 \\ 0 & \text{if } D(u, v) > D_0 \end{cases}$$

D_0 is a positive constant and $D(u, v)$ is the distance between a point (u, v) in the frequency domain and the center of the frequency rectangle

$$D(u, v) = \left[(u - P / 2)^2 + (v - Q / 2)^2 \right]^{1/2}$$

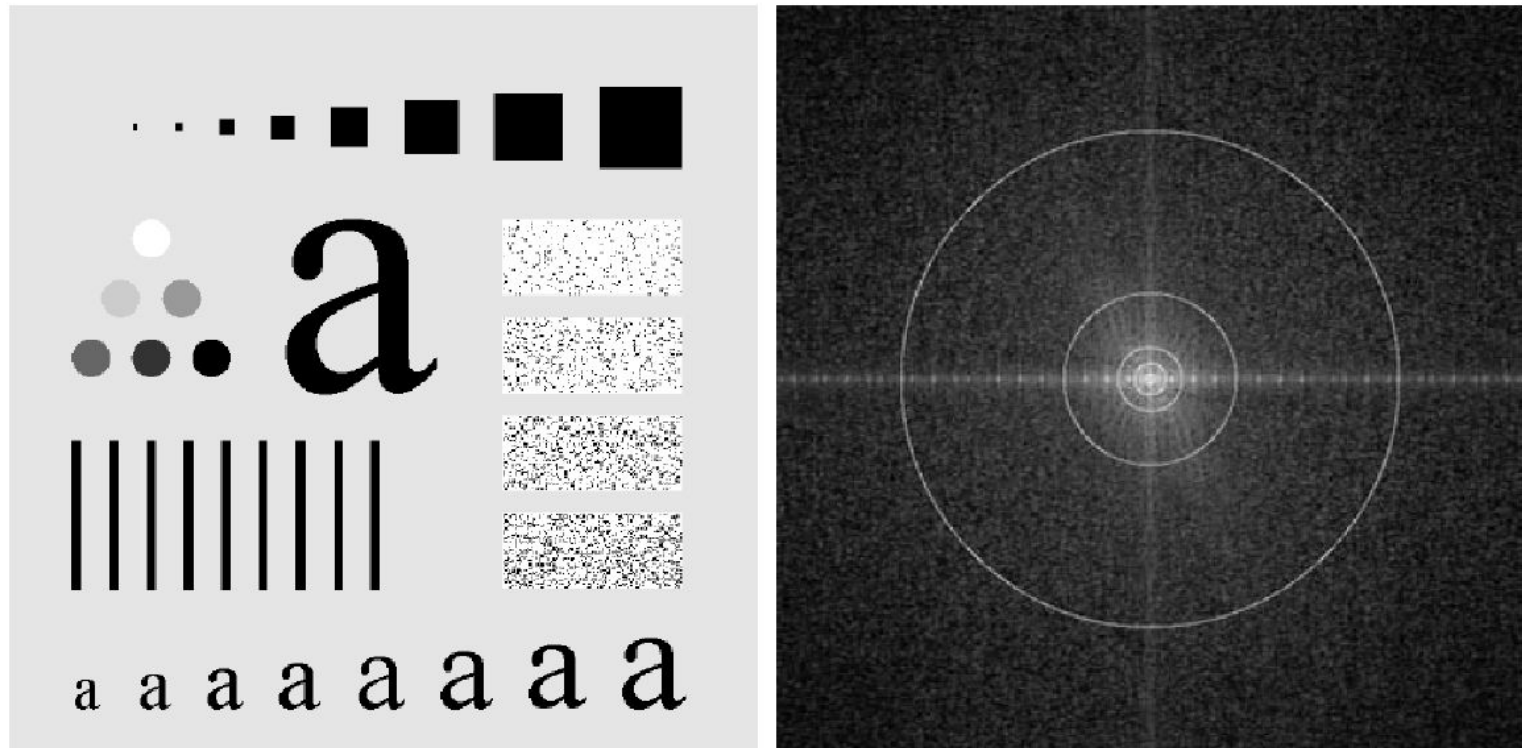
Image Smoothing Using Filter Domain Filters: ILPF



a b c

FIGURE 4.40 (a) Perspective plot of an ideal lowpass-filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross section.

ILPF Filtering Example



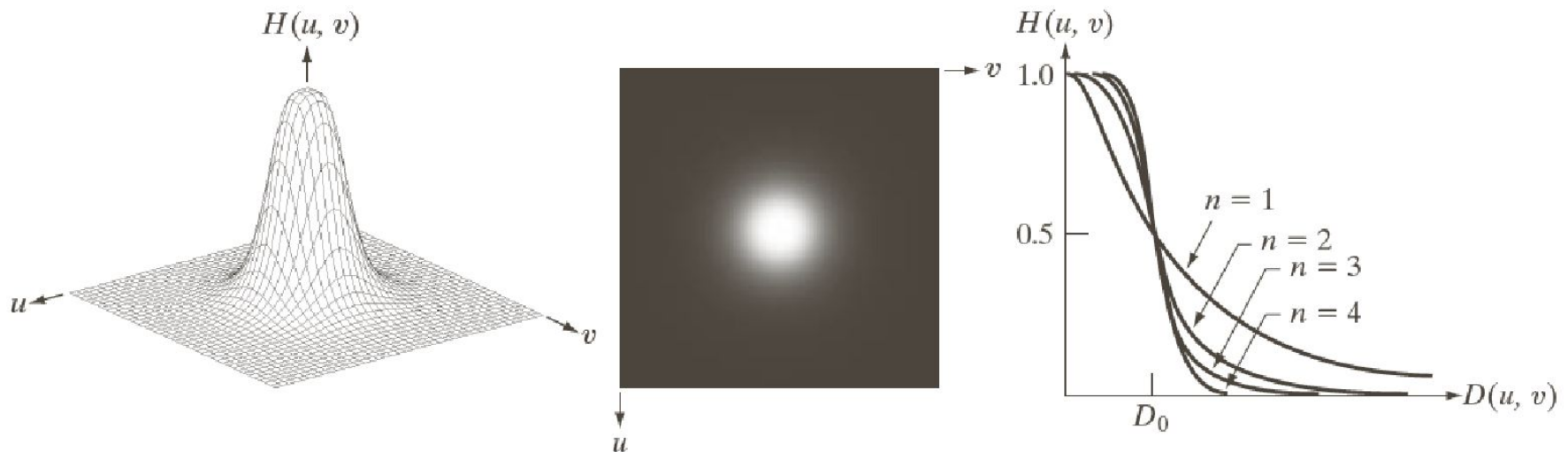
a b

FIGURE 4.41 (a) Test pattern of size 688×688 pixels, and (b) its Fourier spectrum. The spectrum is double the image size due to padding but is shown in half size so that it fits in the page. The superimposed circles have radii equal to 10, 30, 60, 160, and 460 with respect to the full-size spectrum image. These radii enclose 87.0, 93.1, 95.7, 97.8, and 99.2% of the padded image power, respectively.

Image Smoothing Using Filter Domain Filters: BLPF

Butterworth Lowpass Filters (BLPF) of order n and with cutoff frequency D_0

$$H(u, v) = \frac{1}{1 + [D(u, v) / D_0]^{2n}}$$



a b c

FIGURE 4.44 (a) Perspective plot of a Butterworth lowpass-filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.



a b
c d
e f

FIGURE 4.42 (a) Original image. (b)–(f) Results of filtering using ILPFs with cutoff frequencies set at radii values 10, 30, 60, 160, and 460, as shown in Fig. 4.41(b). The power removed by these filters was 13, 6.9, 4.3, 2.2, and 0.8% of the total, respectively.

a b
c d
e f

FIGURE 4.45 (a) Original image. (b)–(f) Results of filtering using BLPFs of order 2, with cutoff frequencies at the radii shown in Fig. 4.41. Compare with Fig. 4.42.

Image Smoothing Using Filter Domain Filters: GLPF

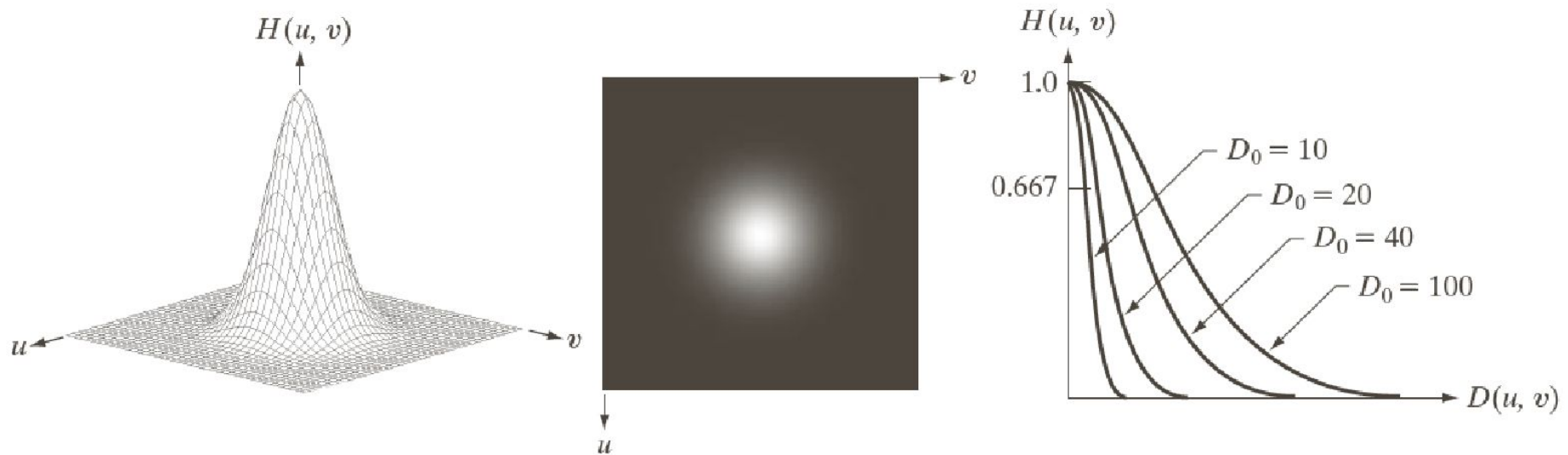
Gaussian Lowpass Filters (GLPF) in two dimensions is given

$$H(u, v) = e^{-D^2(u, v)/2\sigma^2}$$

By letting $\sigma = D_0$

$$H(u, v) = e^{-D^2(u, v)/2D_0^2}$$

Image Smoothing Using Filter Domain Filters: GLPF



a b c

FIGURE 4.47 (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of D_0 .



FIGURE 4.42 (a) Original image. (b)–(f) Results of filtering using ILPFs with cutoff frequencies set at radii values 10, 30, 60, 160, and 460, as shown in Fig. 4.41(b). The power removed by these filters was 13, 6.9, 4.3, 2.2, and 0.8% of the total, respectively.

FIGURE 4.48 (a) Original image. (b)–(f) Results of filtering using GLPFs with cutoff frequencies at the radii shown in Fig. 4.41. Compare with Figs. 4.42 and 4.45.

Image Sharpening Using Frequency Domain Filters

A highpass filter is obtained from a given lowpass filter using

$$H_{HP}(u, v) = 1 - H_{LP}(u, v)$$

A 2-D ideal highpass filter (IHPL) is defined as

$$H(u, v) = \begin{cases} 0 & \text{if } D(u, v) \leq D_0 \\ 1 & \text{if } D(u, v) > D_0 \end{cases}$$

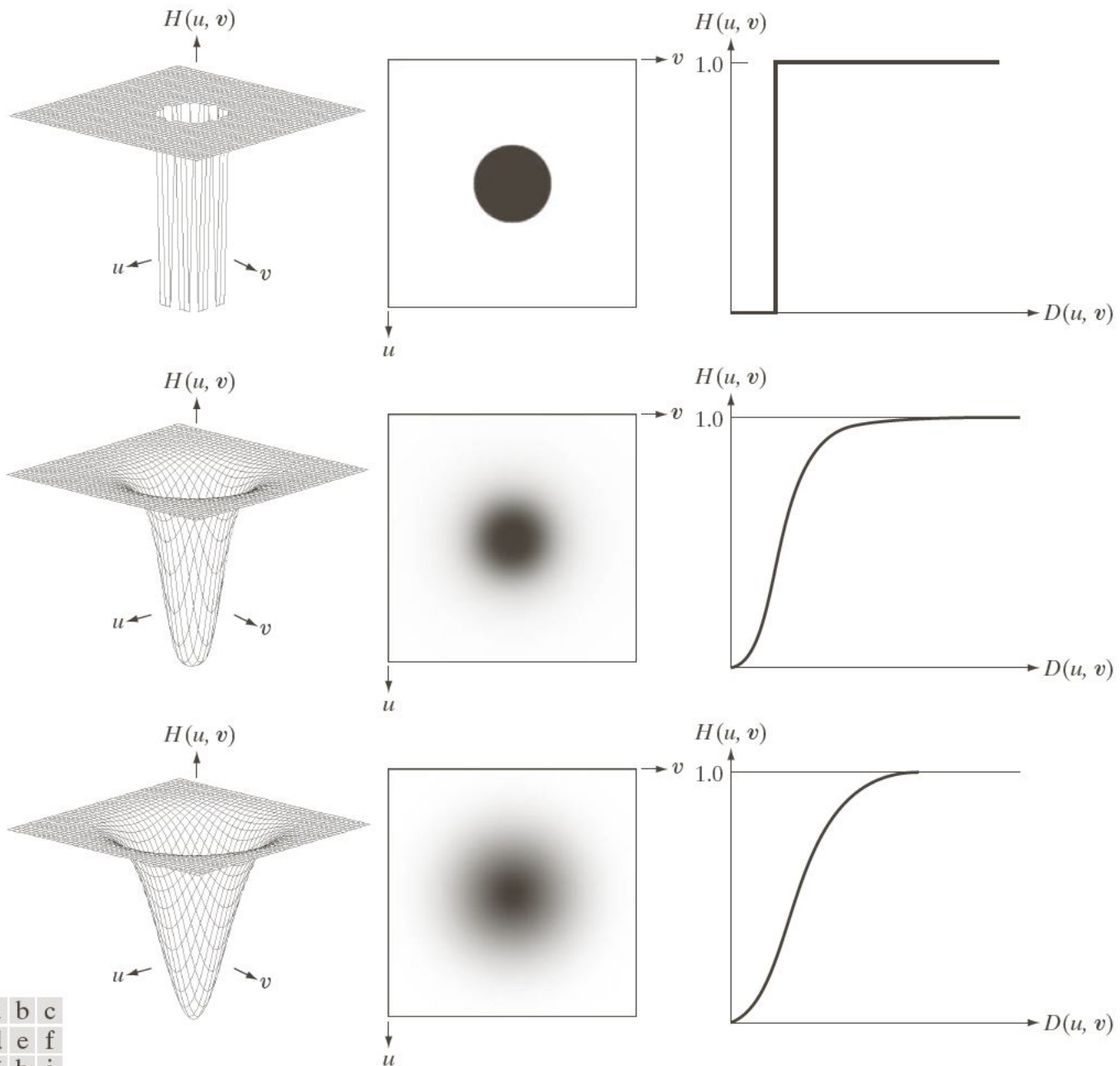
Image Sharpening Using Frequency Domain Filters

A 2-D Butterworth highpass filter (BHPL) is defined as

$$H(u, v) = \frac{1}{1 + [D_0 / D(u, v)]^{2n}}$$

A 2-D Gaussian highpass filter (GHPL) is defined as

$$H(u, v) = 1 - e^{-D^2(u, v) / 2D_0^2}$$



a	b	c
d	e	f
g	h	i

FIGURE 4.52 Top row: Perspective plot, image representation, and cross section of a typical ideal highpass filter. Middle and bottom rows: The same sequence for typical Butterworth and Gaussian highpass filters.

Selective Filtering: Bandreject and Bandpass Filters

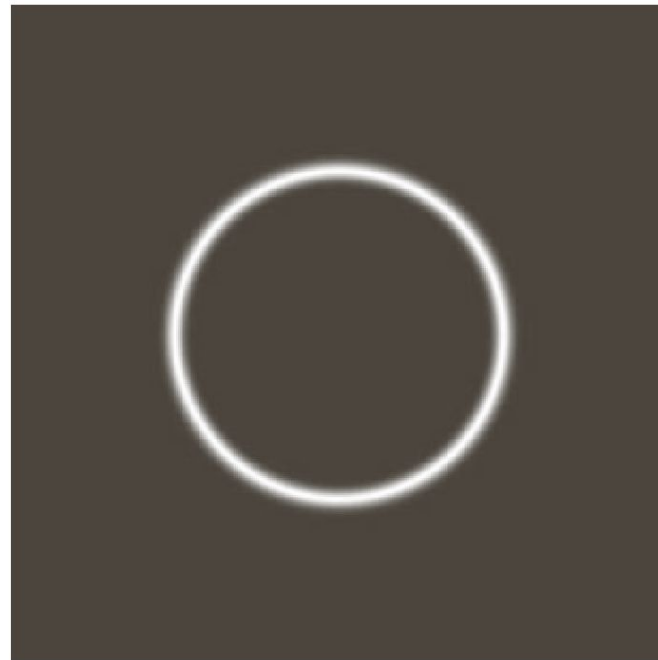
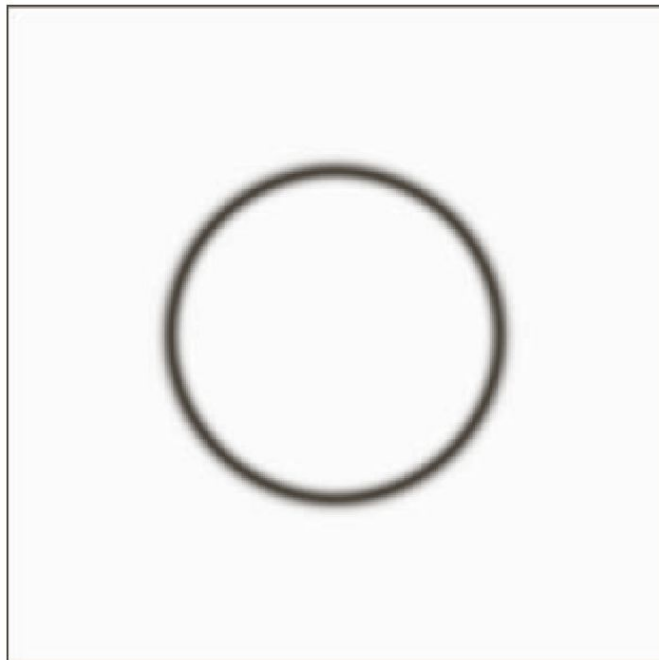
TABLE 4.6

Bandreject filters. W is the width of the band, D is the distance $D(u, v)$ from the center of the filter, D_0 is the cutoff frequency, and n is the order of the Butterworth filter. We show D instead of $D(u, v)$ to simplify the notation in the table.

Ideal	Butterworth	Gaussian
$H(u, v) = \begin{cases} 0 & \text{if } D_0 - \frac{W}{2} \leq D \leq D_0 + \frac{W}{2} \\ 1 & \text{otherwise} \end{cases}$	$H(u, v) = \frac{1}{1 + \left[\frac{DW}{D^2 - D_0^2} \right]^{2n}}$	$H(u, v) = 1 - e^{-\left[\frac{D^2 - D_0^2}{DW} \right]^2}$

$$H_{BP}(u, v) = 1 - H_{BR}(u, v)$$

Selective Filtering: Bandreject and Bandpass Filters



a b

FIGURE 4.63

(a) Bandreject Gaussian filter.
(b) Corresponding bandpass filter.
The thin black border in (a) was added for clarity; it is not part of the data.