

Digital Image Processing

Image Segmentation



Fundamentals

- Let R represent the entire spatial region occupied by an image. Image segmentation is a process that partitions R into n sub-regions, $R_1, R_2, \dots, R_n$, such that

(a) $\bigcup_{i=1}^n R_i = R.$

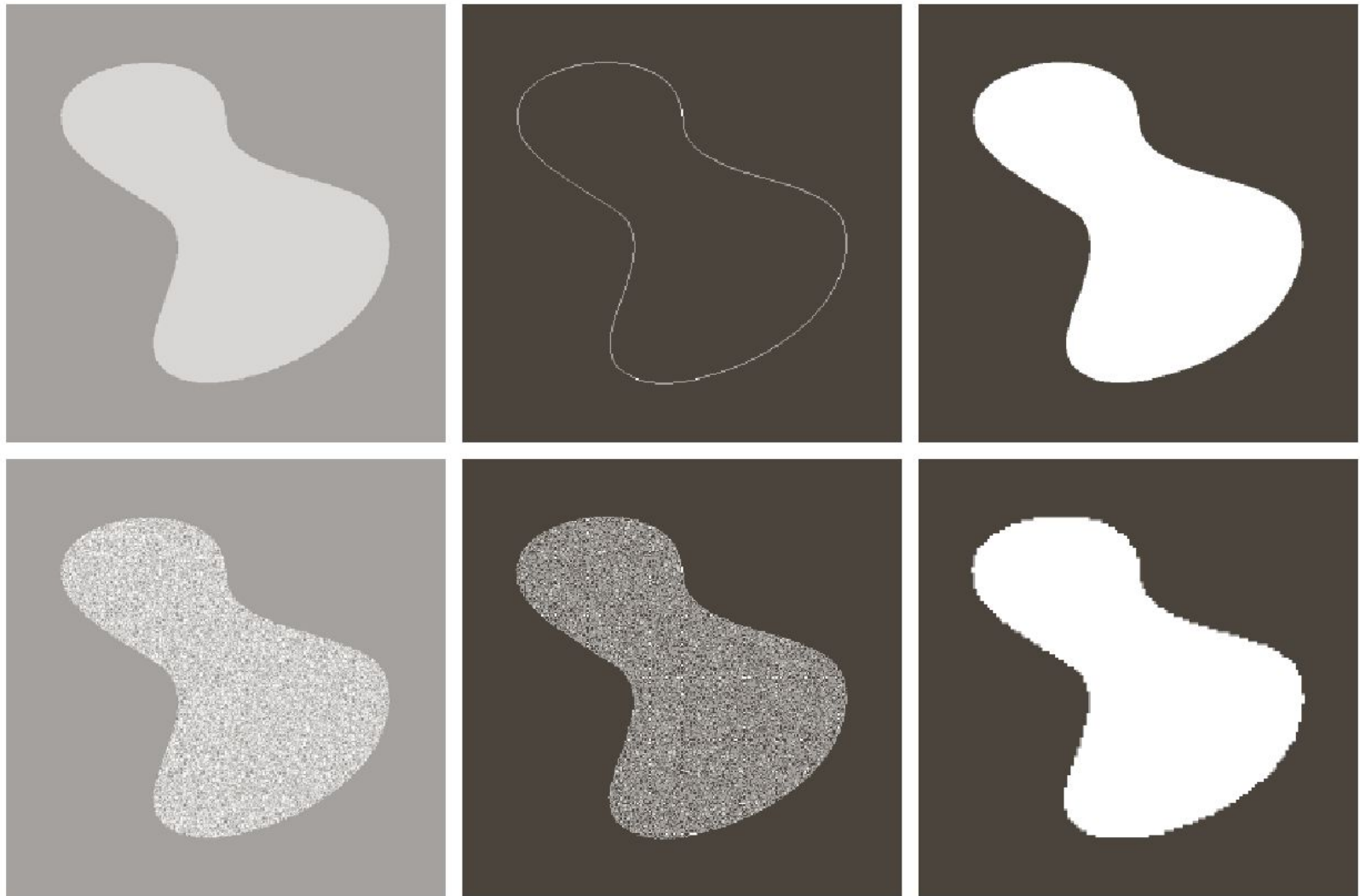
(b) R_i is a connected set. $i = 1, 2, \dots, n.$

(c) $R_i \cap R_j = \Phi.$

(d) $\mathbb{Q}(R_i) = \text{TRUE}$ for $i = 1, 2, \dots, n.$

(e) $\mathbb{Q}(R_i \cup R_j) = \text{FALSE}$ for any adjacent regions
 R_i and $R_j.$

Q is a compound stmt like "average intensity of the region is less than m and standard deviation of their intensity is Greater than s ".



a	b	c
d	e	f

FIGURE 10.1 (a) Image containing a region of constant intensity. (b) Image showing the boundary of the inner region, obtained from intensity discontinuities. (c) Result of segmenting the image into two regions. (d) Image containing a textured region. (e) Result of edge computations. Note the large number of small edges that are connected to the original boundary, making it difficult to find a unique boundary using only edge information. (f) Result of segmentation based on region properties.

Background

- ▶ First-order derivative

$$\frac{\partial f}{\partial x} = f'(x) = f(x+1) - f(x)$$

- ▶ Second-order derivative

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$

Characteristics of First and Second Order Derivatives

- ▶ First-order derivatives generally produce thicker edges in image
- ▶ Second-order derivatives have a stronger response to fine detail, such as thin lines, isolated points, and noise
- ▶ The sign of the second derivative can be used to determine whether a transition into an edge is from light to dark or dark to light.
- ▶ This isotropic (rotation invariant) filter

Detection of Points in 2-D space

► The Laplacian

$$\nabla^2 f(x, y) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

0	1	0
1	-4	1
0	1	0

1	1	1
1	-8	1
1	1	1

$$= f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)$$

$$g(x, y) = \begin{cases} 1 & \text{if } |R(x, y)| \geq T \\ 0 & \text{otherwise} \end{cases} \quad R = \sum_{k=1}^9 w_k z_k$$

Line Detection :Second derivatives to result in a stronger response and to produce thinner lines than first derivatives

Detecting Line in Specified Directions

-1	-1	-1	2	-1	-1	-1	2	-1	-1	-1	2
2	2	2	-1	2	-1	-1	2	-1	-1	2	-1
-1	-1	-1	-1	-1	2	-1	2	-1	2	-1	-1
Horizontal			+45°			Vertical			-45°		

FIGURE 10.6 Line detection masks. Angles are with respect to the axis system in Fig. 2.18(b).

- ▶ Let R_1 , R_2 , R_3 , and R_4 denote the responses of the masks. If, at a given point in the image, $|R_k| > |R_j|$, for all $j \neq k$, that point is said to be more likely associated with a line in the direction of mask k .

Edge Detection

- ▶ Edges are pixels where the brightness function changes abruptly
- ▶ Edge models (Step edge, Ramp edge, roof edge model)

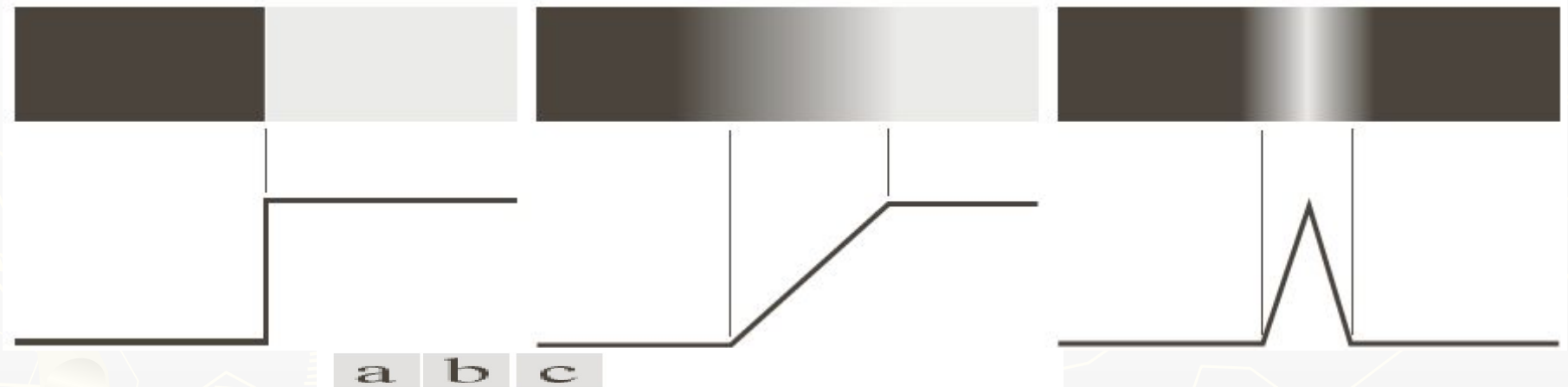


FIGURE 10.8
From left to right,
models (ideal
representations) of
a step, a ramp, and
a roof edge, and
their corresponding
intensity profiles.

Basic Edge Detection by Using First-Order Derivative

$$\nabla f \equiv \text{grad}(f) = \begin{bmatrix} g_x \\ g_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

The magnitude of ∇f

$$M(x, y) = \text{mag}(\nabla f) = \sqrt{g_x^2 + g_y^2}$$

The direction of ∇f

$$\alpha(x, y) = \tan^{-1} \left[\frac{g_x}{g_y} \right]$$

The direction of the edge

$$\phi = \alpha - 90^\circ$$

Basic Edge Detection by Using First-Order Derivative

$$\text{Edge normal: } \nabla f \equiv \text{grad}(f) = \begin{bmatrix} g_x \\ g_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

$$\text{Edge unit normal: } \nabla f / \text{mag}(\nabla f)$$

In practice, sometimes the magnitude is approximated by

$$\text{mag}(\nabla f) = \left| \frac{\partial f}{\partial x} \right| + \left| \frac{\partial f}{\partial y} \right| \text{ or } \text{mag}(\nabla f) = \max \left(\left| \frac{\partial f}{\partial x} \right|, \left| \frac{\partial f}{\partial y} \right| \right)$$

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

-1	0	0	-1
0	1	1	0

Roberts

-1	-1	-1	-1	0	1
0	0	0	-1	0	1
1	1	1	-1	0	1

Prewitt

-1	-2	-1	-1	0	1
0	0	0	-2	0	2
1	2	1	-1	0	1

Sobel

a	
b	c
d	e
f	g

FIGURE 10.14

A 3×3 region of an image (the z 's are intensity values) and various masks used to compute the gradient at the point labeled z_5 .

0	1	1	-1	-1	0
-1	0	1	-1	0	1
-1	-1	0	0	1	1

Prewitt

0	1	2	-2	-1	0
-1	0	1	-1	0	1
-2	-1	0	0	1	2

Sobel

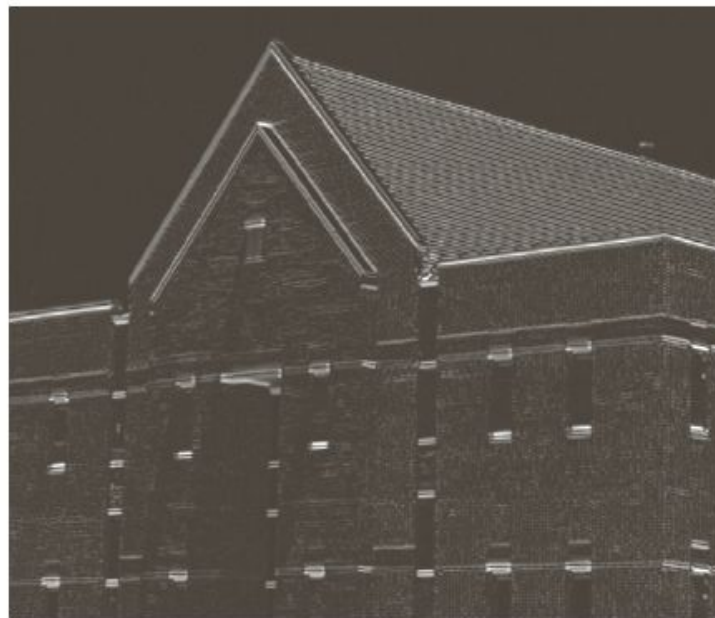


FIGURE 10.15
Prewitt and Sobel
masks for
detecting diagonal
edges.



a	b
c	d

FIGURE 10.16
 (a) Original image of size 834×1114 pixels, with intensity values scaled to the range $[0, 1]$.
 (b) $|g_x|$, the component of the gradient in the x -direction, obtained using the Sobel mask in Fig. 10.14(f) to filter the image.
 (c) $|g_y|$, obtained using the mask in Fig. 10.14(g).
 (d) The gradient image, $|g_x| + |g_y|$.



a	b
c	d

FIGURE 10.18
 Same sequence as in Fig. 10.16, but with the original image smoothed using a 5×5 averaging filter prior to edge detection.



Edge Linking and Boundary Detection

- ▶ Edge detection typically is followed by linking algorithms designed to assemble edge pixels into meaningful edges and/or region boundaries
- ▶ Three approaches to edge linking
 - Local processing**
 - Regional processing
 - Global processing

Local Processing

- ▶ Analyze the characteristics of pixels in a small neighborhood about every point (x,y) that has been declared an edge point
- ▶ All points that similar according to predefined criteria are linked, forming an edge of pixels.
Establishing similarity: (1) the strength (magnitude) and (2) the direction of the gradient vector.
A pixel with coordinates (s,t) in S_{xy} is linked to the pixel at (x,y) if both magnitude and direction criteria are satisfied.

Local Processing

Let S_{xy} denote the set of coordinates of a neighborhood centered at point (x, y) in an image. An edge pixel with coordinate (s, t) in S_{xy} is similar in *magnitude* to the pixel at (x, y) if

$$|M(s, t) - M(x, y)| \leq E$$

An edge pixel with coordinate (s, t) in S_{xy} is similar in *angle* to the pixel at (x, y) if

$$|\alpha(s, t) - \alpha(x, y)| \leq A$$

Local Processing: Steps (1)

1. Compute the gradient magnitude and angle arrays, $M(x,y)$ and $\alpha(x,y)$, of the input image $f(x,y)$
2. Form a binary image, g , whose value at any pair of coordinates (x,y) is given by

$$g(x,y) = \begin{cases} 1 & \text{if } M(x,y) > T_M \text{ and } \alpha(x,y) = A \pm T_A \\ 0 & \text{otherwise} \end{cases}$$

T_M : threshold A : specified angle direction

T_A : a "band" of acceptable directions about A

Local Processing: Steps (2)

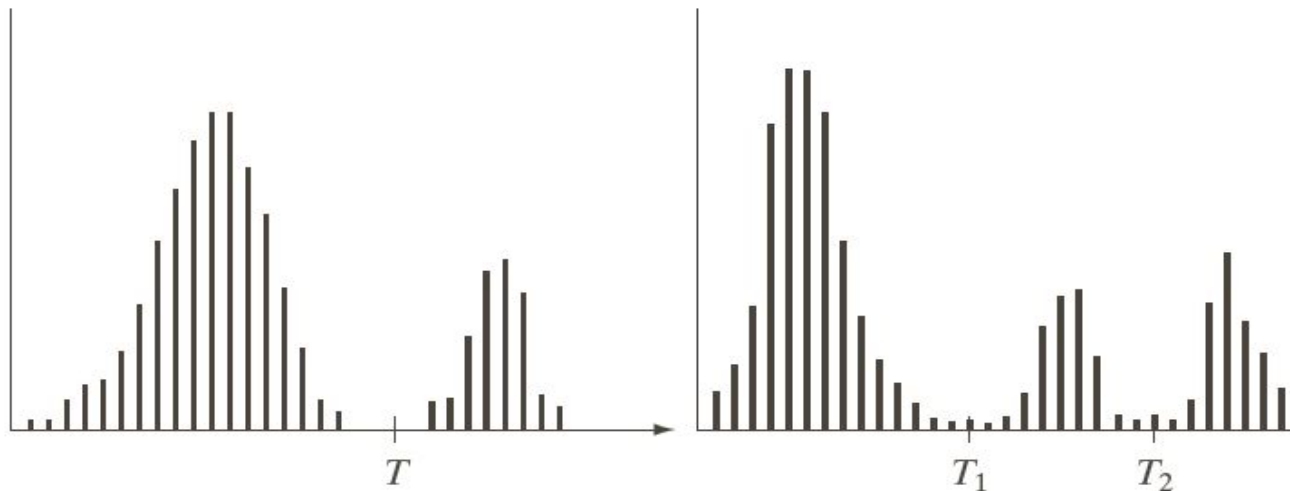
3. Scan the rows of g and fill (set to 1) all gaps (sets of 0s) in each row that do not exceed a specified length, K .
4. To detect gaps in any other direction, rotate g by this angle and apply the horizontal scanning procedure in step 3.



Thresholding

$$g(x, y) = \begin{cases} 1 & \text{if } f(x, y) > T \quad (\text{object point}) \\ 0 & \text{if } f(x, y) \leq T \quad (\text{background point}) \end{cases}$$

T : global thresholding



a b

FIGURE 10.35
Intensity histograms that can be partitioned (a) by a single threshold, and (b) by dual thresholds.

Multiple thresholding

$$g(x, y) = \begin{cases} a & \text{if } f(x, y) > T_2 \\ b & \text{if } T_1 < f(x, y) \leq T_2 \\ c & \text{if } f(x, y) \leq T_1 \end{cases}$$

The Role of Noise in Image Thresholding

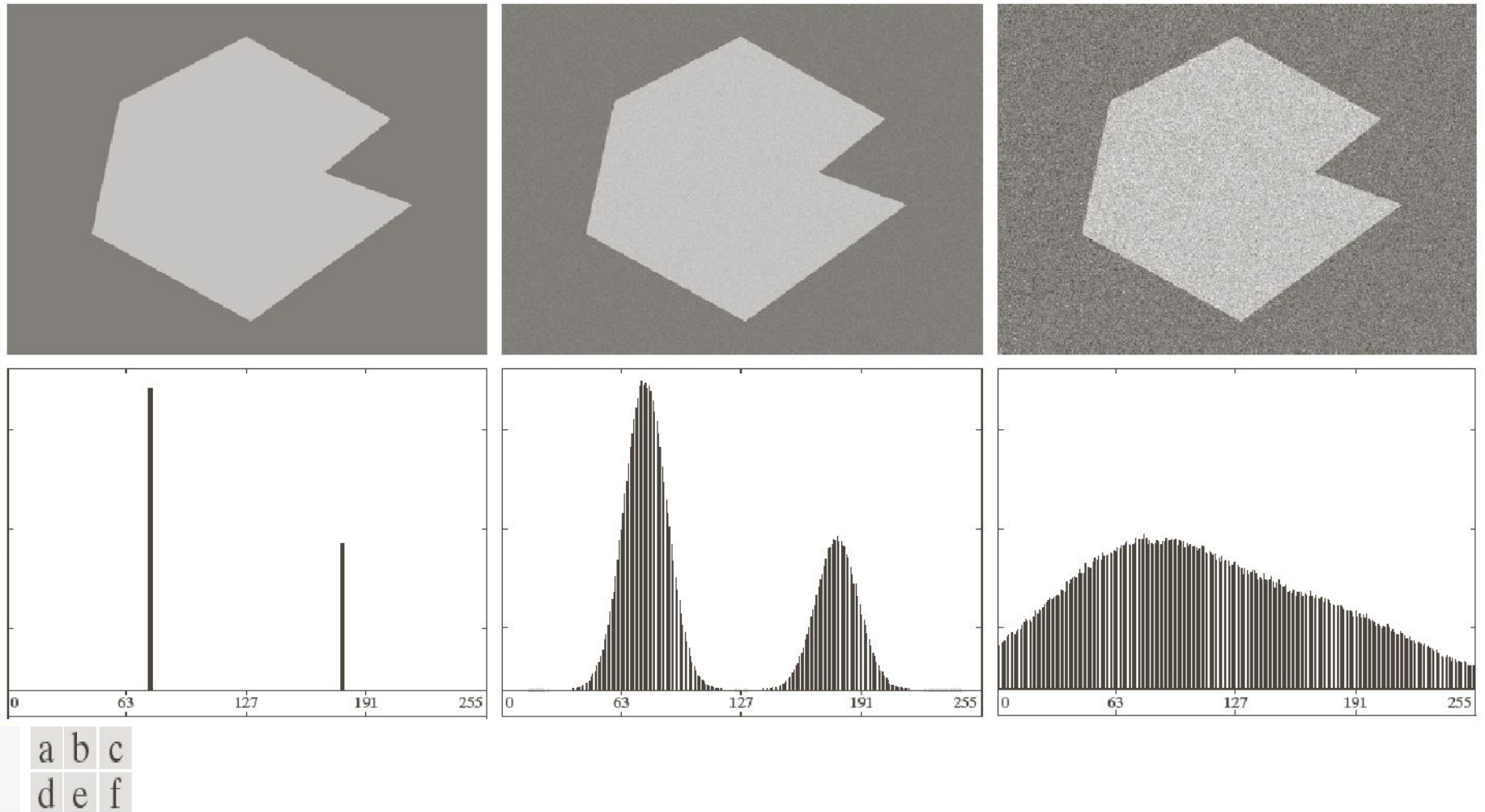


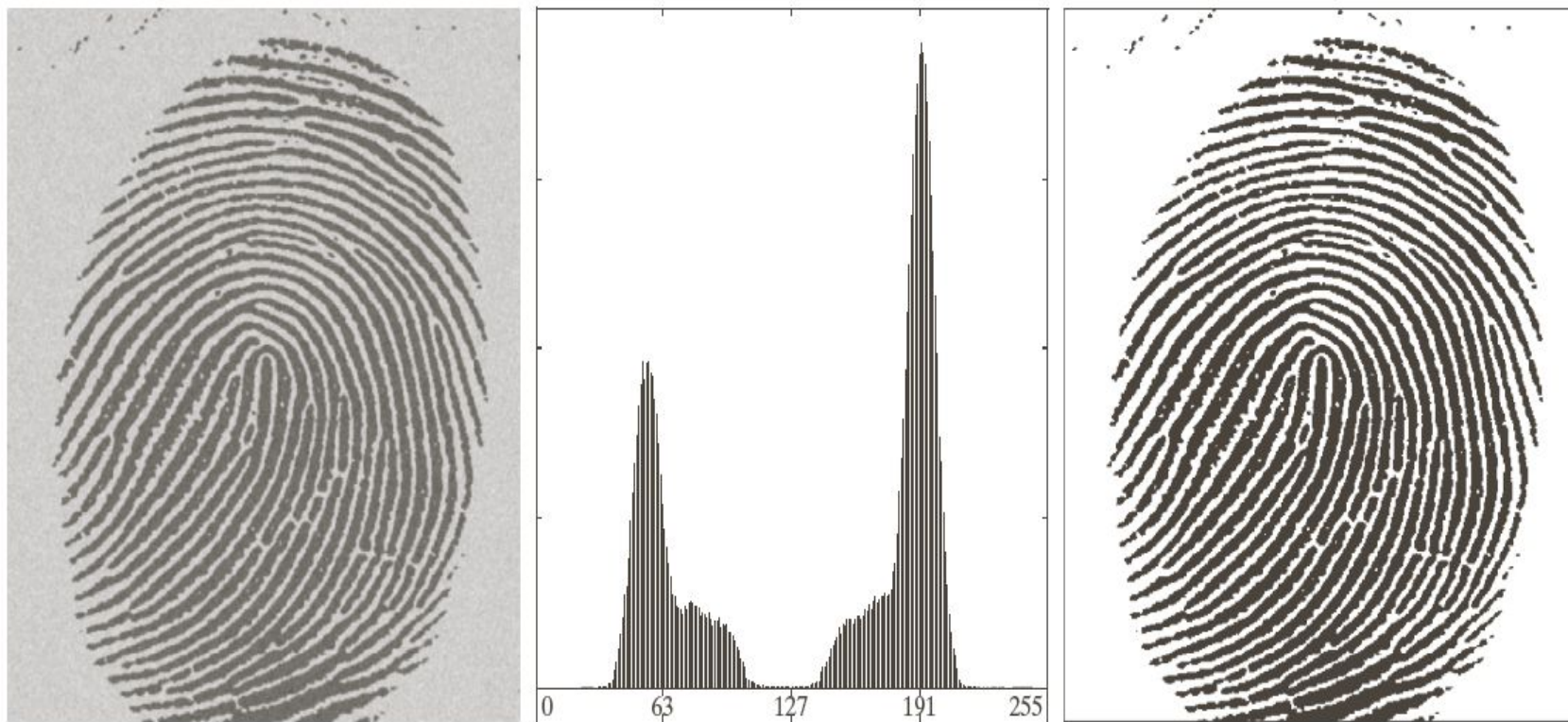
FIGURE 10.36 (a) Noiseless 8-bit image. (b) Image with additive Gaussian noise of mean 0 and standard deviation of 10 intensity levels. (c) Image with additive Gaussian noise of mean 0 and standard deviation of 50 intensity levels. (d)–(f) Corresponding histograms.

Basic Global Thresholding

1. Select an initial estimate for the global threshold, T .
2. Segment the image using T . It will produce two groups of pixels: $G1$ consisting of all pixels with intensity values $> T$ and $G2$ consisting of pixels with values $\leq T$.
3. Compute the average intensity values $m1$ and $m2$ for the pixels in $G1$ and $G2$, respectively.
4. Compute a new threshold value.

$$T = \frac{1}{2}(m1 + m2)$$

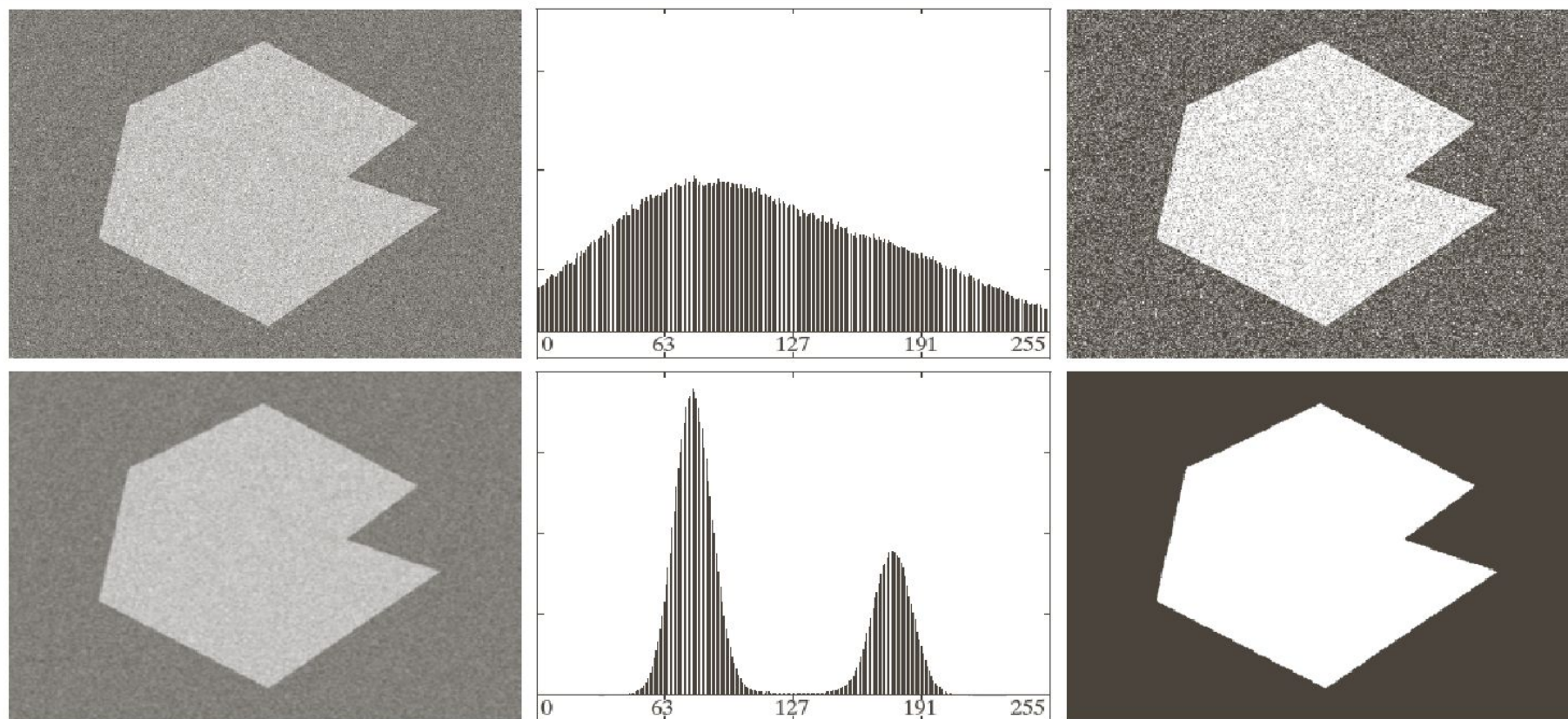
5. Repeat Steps 2 through 4 until the difference between values of T in successive iterations is smaller than a predefined parameter ΔT .



a b c

FIGURE 10.38 (a) Noisy fingerprint. (b) Histogram. (c) Segmented result using a global threshold (the border was added for clarity). (Original courtesy of the National Institute of Standards and Technology.)

Using Image Smoothing to Improve Global Thresholding



a	b	c
d	e	f

FIGURE 10.40 (a) Noisy image from Fig. 10.36 and (b) its histogram. (c) Result obtained using Otsu's method. (d) Noisy image smoothed using a 5×5 averaging mask and (e) its histogram. (f) Result of thresholding using Otsu's method.

Variable Thresholding Based on Local Image Properties

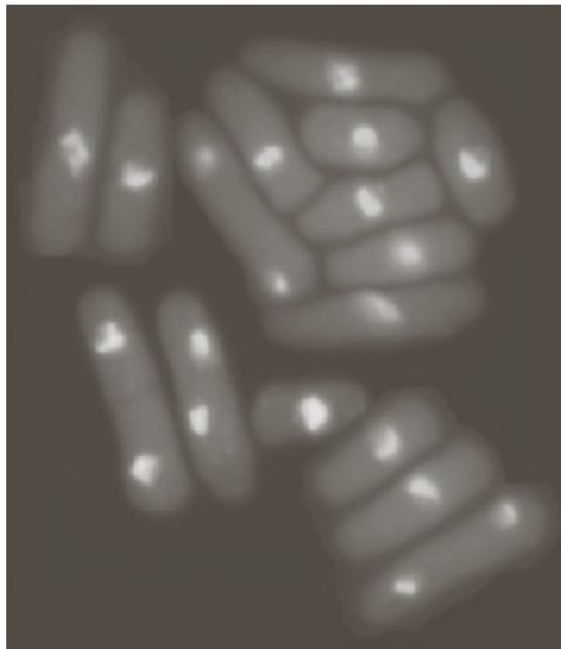
Let σ_{xy} and m_{xy} denote the standard deviation and mean value of the set of pixels contained in a neighborhood S_{xy} , centered at coordinates (x, y) in an image. The local thresholds,

$$T_{xy} = a\sigma_{xy} + bm_{xy}$$

If the background is nearly constant,

$$T_{xy} = a\sigma_{xy} + bm$$

$$g(x, y) = \begin{cases} 1 & \text{if } f(x, y) > T_{xy} \\ 0 & \text{if } f(x, y) \leq T_{xy} \end{cases}$$



a	b
c	d

FIGURE 10.48

(a) Image from Fig. 10.43.

(b) Image segmented using the dual thresholding approach discussed in Section 10.3.6.

(c) Image of local standard deviations.

(d) Result obtained using local thresholding.

$$\begin{aligned} a &= 30 \\ b &= 1.5 \\ m_{xy} &= m_G \end{aligned}$$

Region-Based Segmentation

► Region Growing

1. Region growing is a procedure that groups pixels or subregions into larger regions.
2. The simplest of these approaches is *pixel aggregation*, which starts with a set of “**seed**” points and from these grows regions by appending to each seed points those **neighboring pixels** that have **similar properties** (such as gray level, texture, color, shape).
3. Region growing based techniques are better than the edge-based techniques in noisy images where edges are difficult to detect.

Region-Based Segmentation

Example: Region Growing based on 8-connectivity

$f(x, y)$: input image array

$S(x, y)$: seed array containing 1s (seeds) and 0s

$Q(x, y)$: predicate

$$Q = \begin{cases} \text{TRUE} & \text{if the absolute difference of the intensities} \\ & \text{between the seed and the pixel at } (x, y) \text{ is } \leq T \\ \text{FALSE} & \text{otherwise} \end{cases}$$

Region Growing based on 8-connectivity

1. Find all connected components in $S(x, y)$ and erode each connected components to one pixel; label all such pixels found as 1. All other pixels in S are labeled 0.
2. Form an image f_Q such that, at a pair of coordinates (x, y) , let $f_Q(x, y) = 1$ if the Q is satisfied otherwise $f_Q(x, y) = 0$.
3. Let g be an image formed by appending to each seed point in S all the 1-value points in f_Q that are 8-connected to that seed point.
4. Label each connected component in g with a different region label. This is the segmented image obtained by region growing.

Example

Suppose that we have the image given below.

(a) Use the region growing idea to segment the object. The seed for the object is the center of the image. Region is grown in horizontal and vertical directions, and when the difference between two pixel values is less than or equal to 5.

Table 1: Show the result of Part (a) on this figure.

10	10	10	10	10	10	10
10	10	10	69	70	10	10
59	10	60	64	59	56	60
10	59	10	<u>60</u>	70	10	62
10	60	59	65	67	10	65
10	10	10	10	10	10	10
10	10	10	10	10	10	10

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Table 1: Show the result of Part (a) on this figure

4-connectivity

10	10	10	10	10	10	10
10	10	10	69	70	10	10
59	10	60	64	59	56	60
10	59	10	60	70	10	62
10	60	59	65	67	10	65
10	10	10	10	10	10	10
10	10	10	10	10	10	10

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10	59	10	<u>60</u>	70	10	62
10	60	59	65	67	10	65
10	10	10	10	10	10	10
10	10	10	10	10	10	10

8-connectivity

Region Splitting and Merging

R : entire image R_i : entire image Q : predicate

1. For any region R_i , If $Q(R_i) = \text{FALSE}$,
we divide the image R_i into quadrants.
2. When no further splitting is possible,
merge any adjacent regions R_j and R_k
for which $Q(R_j \cup R_k) = \text{TRUE}$.
3. Stop when no further merging is possible.

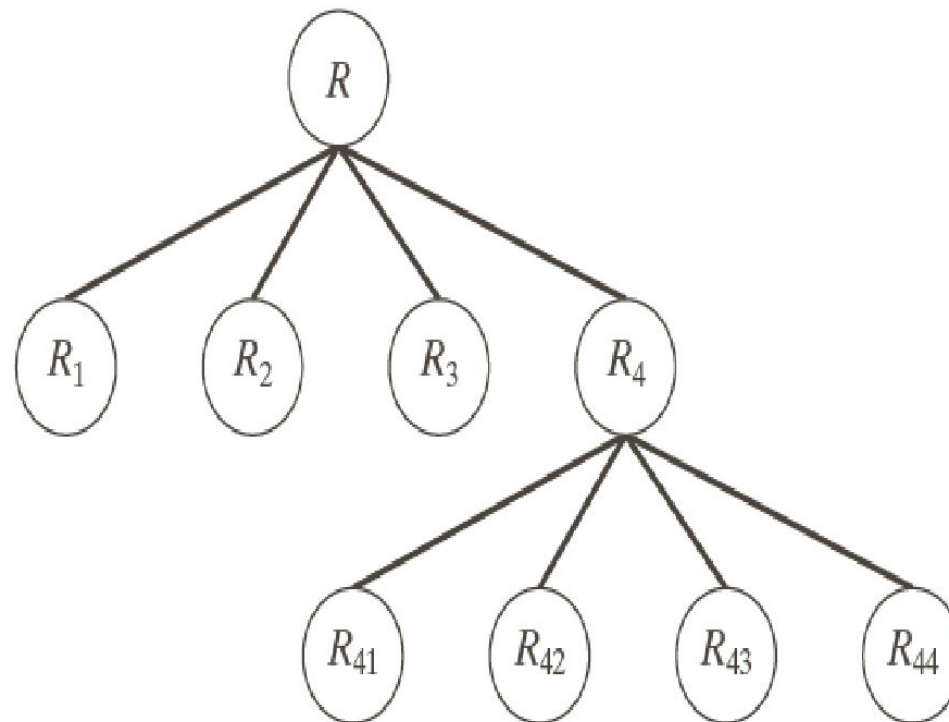
$$Q = \begin{cases} \text{TRUE} & \text{if } \sigma > a \text{ and } 0 < m < b \\ \text{FALSE} & \text{otherwise} \end{cases}$$

a b

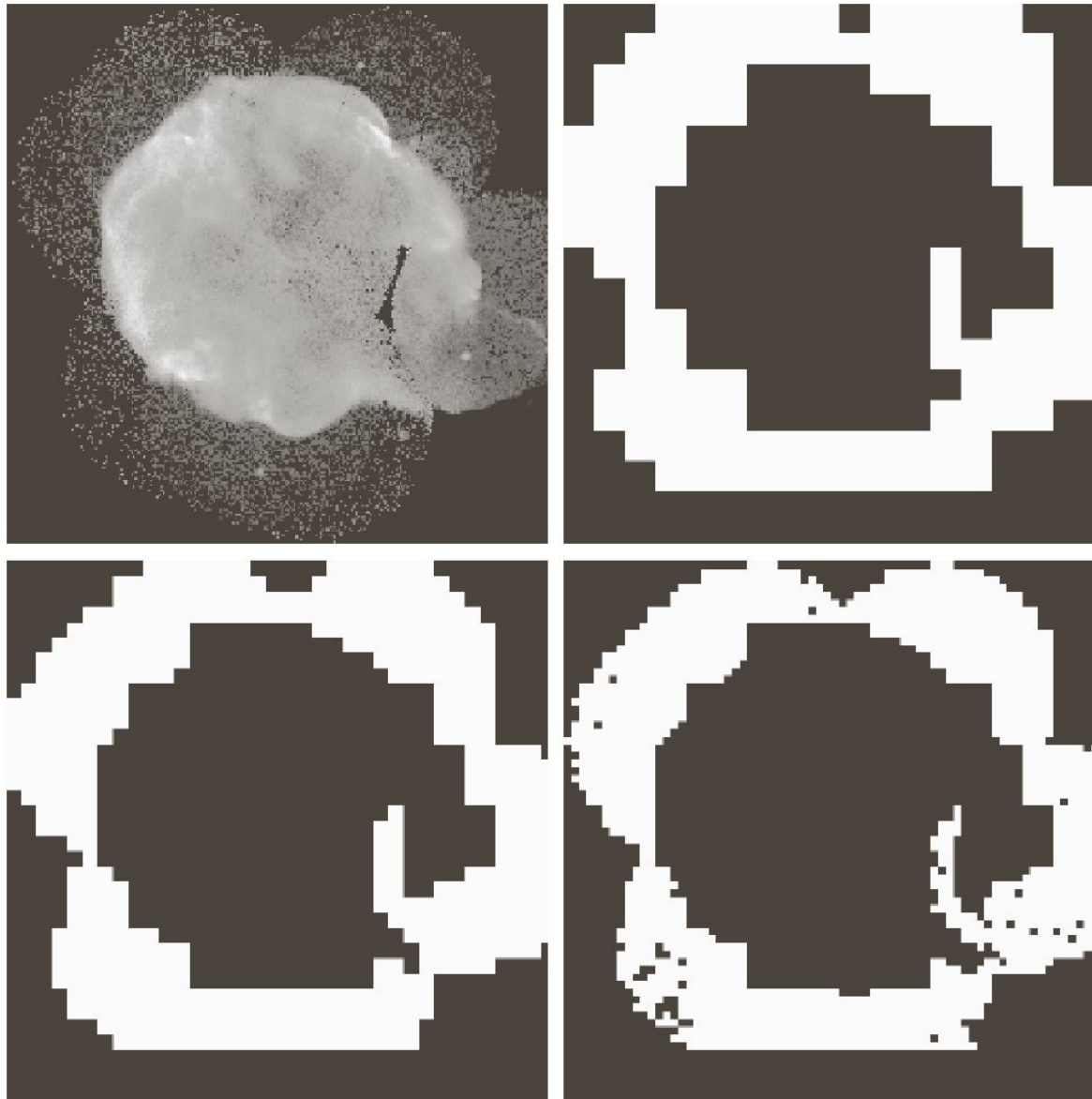
FIGURE 10.52

(a) Partitioned image.

(b) Corresponding quadtree. R represents the entire image region.

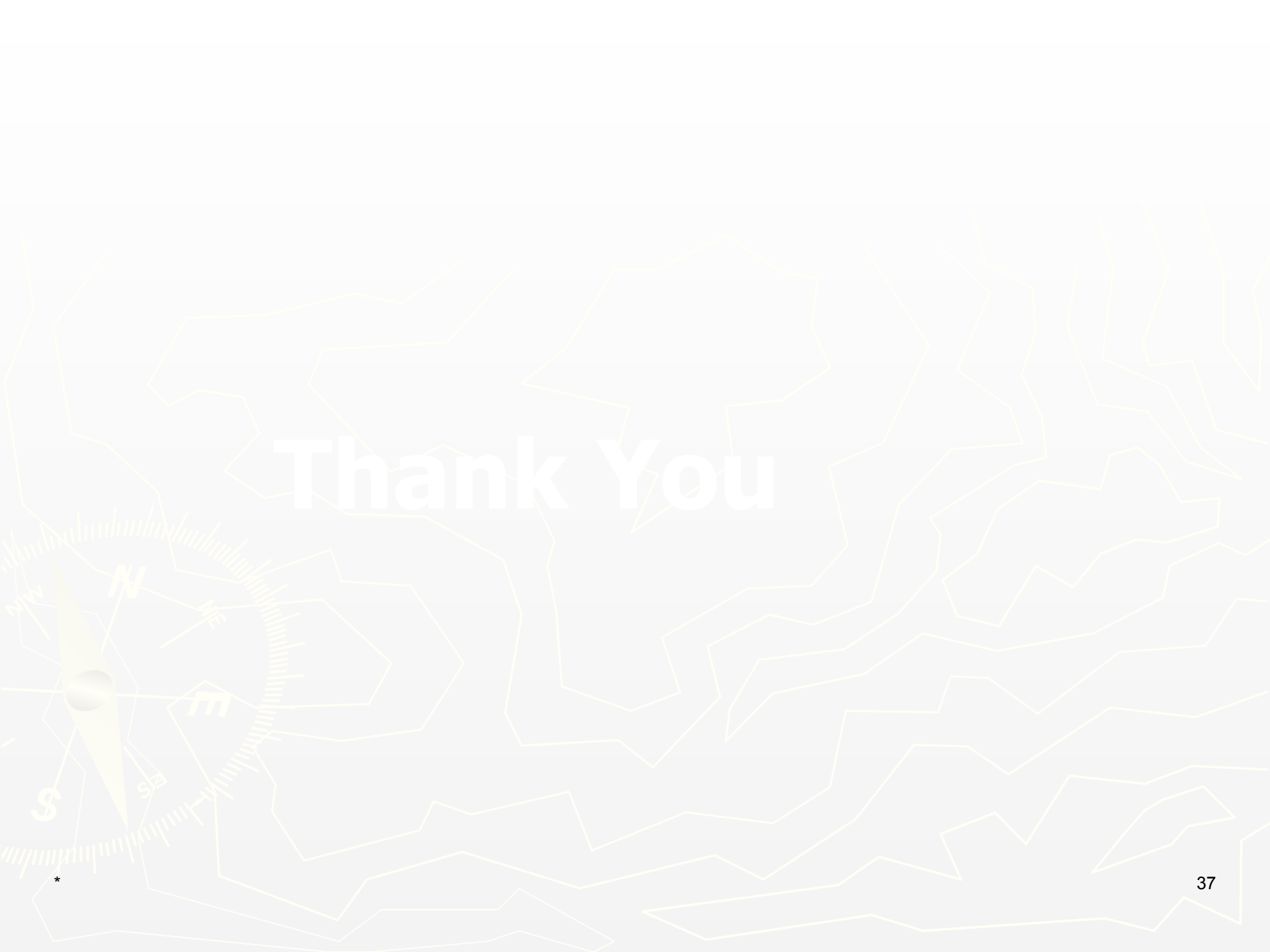


R_1	R_2	
R_3	R_{41}	R_{42}
	R_{43}	R_{44}



a	b
c	d

FIGURE 10.53
 (a) Image of the Cygnus Loop supernova, taken in the X-ray band by NASA's Hubble Telescope. (b)–(d) Results of limiting the smallest allowed quadregion to sizes of 32×32 , 16×16 , and 8×8 pixels, respectively. (Original image courtesy of NASA.)

The background of the slide is a light gray topographic map with yellow contour lines. In the bottom-left corner, there is a yellow compass rose with a black needle pointing towards the bottom-left. The compass rose has labels for North (N), South (S), East (E), and West (W).

Thank You