

Electric current (i): The rate of flow of charge through a cross sectional area is called electric current. (or) the flow of charge per unit time through a cross sectional area. It is denoted by 'i'.

If a net charge 'Q' passes through any cross section of the conductor in time 't', then the average current is given by, $i_{avg} = \frac{Q}{t}$

Instantaneous current is given by $i = \lim_{\Delta t \rightarrow 0} \frac{\Delta Q}{\Delta t}$

$$(or) i = \lim_{\Delta t \rightarrow 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$

Direction of current:

The direction of current is the direction of flow of positive charge (or) opposite to the direction of flow of negative charge.



A current is a fundamental physical quantity.

S.I unit is Ampere (A).

Current has a direction but it is not a Vector. It is a scalar quantity as it does not obey parallelogram law of vectors.

→ i varies with time t . The charge flowing in a time interval from t_1 to t_2 is given by the following way. W.K.T $i = \frac{dq}{dt}$

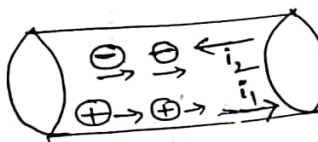
$$\therefore dq = i dt \Rightarrow \int dq = \int_{t_1}^{t_2} i dt$$

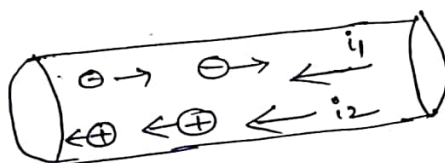
$$\therefore \boxed{q = i \int_{t_1}^{t_2} dt}$$

→ If n charged particles pass through a cross-sectional area in a time t , then $\boxed{i_{avg} = \frac{nq}{t}}$

→ In the above case if the charged particles are electrons, then $\boxed{i = \frac{ne}{t}}$

→

(a)  $i_{net} = i_1 + i_2$

(b)  $i_{net} = i_1 + i_2$

→ If a point charge q is moving in a circle of radius r with uniform speed (v), frequency (f), angular frequency (ω), time period (t), then the ~~curr~~ ~~ent~~ (i) is given by $i = \frac{q}{t} = qf = q \frac{\omega}{2\pi}$ ($\because \omega = 2\pi f$)

$$\therefore \boxed{i = \frac{qv}{2\pi r}} \quad \left(\because \omega = \frac{v}{r} \right)$$

(iv) The direction of conventional current is opposite to the direction of flow of electrons.

(v) The net charge in a current carrying conductor is zero.

Electrical Resistance

Resistance is the opposition offered by a material for the flow of current.

It is defined as the ratio of potential difference 'V' across the conductor to the current (i) flowing through the conductor.

$$\therefore R = \frac{V}{i} \text{ Volts/Ampere. (or) Ohm}$$

Its dimensional formula is $ML^2T^{-3}A^{-2}$

The resistance of a specimen is said to be one ohm if one volt P.D across it causes it a current of one Ampere to flow through it.

$$\frac{1 \text{ Volt}}{1 \text{ Ampere}} = 1 \text{ ohm.}$$

Resistance is a scalar Quantity. It depends on the nature of the material of the specimen, dimension of the specimen and physical conditions like temp., pressure and impurities.

Symbol:-



(a) Fixed Resistor



(b) Variable Resistor.

Drift Velocity (V_d):

The average Velocity with which the free electrons in a conductor get drifted under the influence of external electric field applied across a conductor is called Drift Velocity. It is denoted by $\vec{V}_d$. It is of the order of 10^4 m/s .

Relaxation time (τ):

The short time, for which a free electron accelerates before it undergoes a collision with the $^{+ve}$ ion in the conductor is known as Relaxation time τ .

The relation between drift Velocity and relaxation time (τ) is given by
$$V_d = -\frac{Ee}{m} \tau$$

$$\left\{ \begin{array}{l} \because V = u + at \\ u = 0 \\ a = \frac{F}{m} = \frac{-Ee}{m} \\ t = \tau \end{array} \right.$$

Current density (J):

The current passing through unit area of the conductor is called current density. It is denoted by J .

$$\therefore \boxed{J = \frac{i}{A}}$$

It is a Vector Quantity. S.I unit is Ampere/m^2 .

Mobility (μ): The average drift Velocity acquired by the free electrons per unit electric field is called mobility of the charged carriers. It is denoted by μ .

(4)

$$\mu = \frac{|V_d|}{E}$$

The mobility is 'tve' for both 'tve' and '-ve' charge carriers. S.I unit is $\text{m}^2 \text{s}^{-1} \text{V}^{-1}$ (or) $\frac{\text{C-m}}{\text{N-s}}$

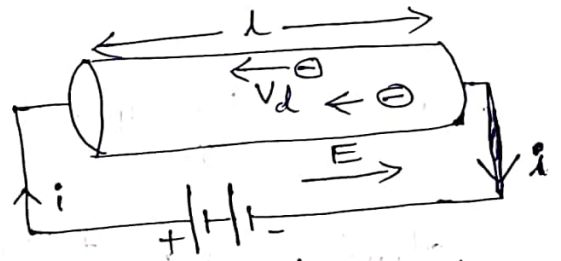
Dimension formula is $\frac{\text{L}^2}{\frac{\text{ML}^2\text{T}^{-2}}{\text{IT}} \times \text{T}} = \text{M}^{-1} \text{T}^2 \text{I}$

Mobility depends on pressure and temperature.

Relation between current and Drift Velocity:

Consider a conductor of length 'l' and of uniform area of cross section 'A'.

$\therefore$ Volume of conductor = Al



Let 'n' be the no. of free electrons per unit volume, then

The total number of free electrons in the conductor of volume 'Al' are 'N'. i.e. $N = n \cdot Al$

If 'e' be the charge of each electron, then the total charge of the conductor due to all electrons is 'q'.

i.e. $q = Ale n$

Suppose when a P.D (V) is applied across the ends of the conductor with the help of a battery, then the electric field 'E' is setup across the conductor.

$\therefore E = \frac{V}{l}$ (magnitude)

Due to this electric field the free e's will begin to move with a drift velocity 'V_d' towards L.H.S.

∴ The time taken by the free e^- s to cross the conductor is $t = \frac{l}{V_d}$.

Hence current $i = \frac{q}{t} = \frac{A l n e}{l/V_d} = \frac{V_d A n e}{t}$

∴ $i = V_d A n e \Rightarrow V_d = \frac{i}{A n e} = \frac{J}{n e}$

∴ $\boxed{V_d = \frac{J}{n e}} \Rightarrow \boxed{J = n e V_d}$

but $V_d = \frac{E e}{m} \tau$, hence the above equation can be written

as $\frac{E e}{m} \tau = \frac{i}{A n e} \Rightarrow \boxed{i = \frac{E n e^2 A \tau}{m}}$

Ohm's law and V-I characteristics:

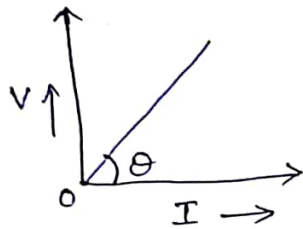
For a given conductor, at a given temperature the strength of the electric current through it is directly proportional to the potential difference applied across it. Let 'V' be the P-D applied across the conductor, 'i' be the current flowing through it.

According to Ohm's law, $i \propto V$ or $V \propto i$

∴ $V = R i$, where 'R' → electrical Resistance of conductor

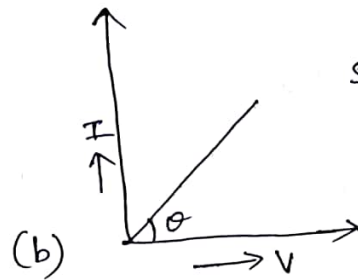
The conductors which obey Ohm's law are called Ohmic conductors. Ex:- Metals and Alloys.

For Ohmic conductors i vs V graphs are straight lines passing through origin. The slope of graphs are shown.



(a) Fig(a)

$$\text{slope} = \tan \theta = \frac{V}{I} = R$$



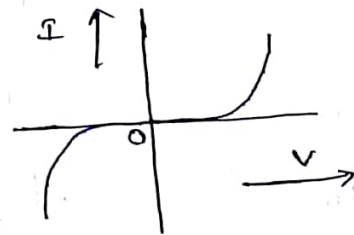
Fig(b)

$$\begin{aligned} \text{slope} &= \tan \theta \\ &= \frac{I}{V} \\ &= \frac{1}{R} \\ &= G \end{aligned}$$

It is observed that the slope of the Ohmic conductors gives Resistance and conductance.

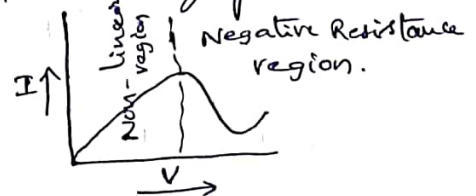
→ In Non Ohmic conductors like Vacuum diode, Semiconductor diode etc the V-I characteristics are as shown.

In Non-ohmic materials Ohm's law does not hold even for low values of current. i.e. V is propor-



-tional to I and hence these are called Non-Ohmic conductors. Ex:- Diode, Semiconductors, transistors, thermistor, electrolytes etc.

It should be noted that this type of behaviour is not unique in Non-ohmic materials. In 'Ga As' the graph is as shown. ∴ Ohm's law is not a Universal law.



Electron ...

Poisson's and Laplace equations

According to Gauss law in electrostatics

$$\int E \cdot ds = \frac{q}{\epsilon_0} \Rightarrow \boxed{\epsilon_0 \int E \cdot ds = q} \quad \text{--- (1)}$$

Let ρ be the volume charge density. i.e. $\rho = \frac{q}{V}$

$$\therefore q = \rho V \text{ (or)} \boxed{q = \rho \int dv} \quad \text{--- (2)}$$

$$\text{put (2) in (1), we get } \epsilon_0 \int E \cdot ds = \rho \int dv. \quad \text{--- (3)}$$

Using Gauss - Divergence theorem, $\int E \cdot ds$ on LHS can be written as $\int E \cdot ds = \int (\nabla \cdot E) dv$ --- (4)

$$\therefore \text{put (4) in (3), we get } \epsilon_0 \int (\nabla \cdot E) dv = \rho \int dv.$$

$$\Rightarrow \epsilon_0 (\nabla \cdot E) = \rho \quad \text{or} \quad \nabla \cdot E = \rho / \epsilon_0 \quad \text{--- (5)}$$

$$\text{but we know that } E = -\nabla \cdot V \quad \text{--- (6)}$$

$$\text{put (6) in (5), } \nabla \cdot (-\nabla \cdot V) = \rho / \epsilon_0 \Rightarrow \boxed{\nabla^2 V = -\rho / \epsilon_0}$$

The equation $\boxed{\nabla^2 V = -\rho / \epsilon_0}$ is called Poisson's equation.

→ If there is no free charge present in the surface, then $\rho = 0$, hence the above equation becomes

$$\boxed{\nabla^2 V = 0}$$

This is known as Laplace's equation.

Boundary Conditions for Electric field

Boundary conditions for electric field are calculated to understand the continuity and discontinuity of electric field when it passes from one medium to another medium through a surface.

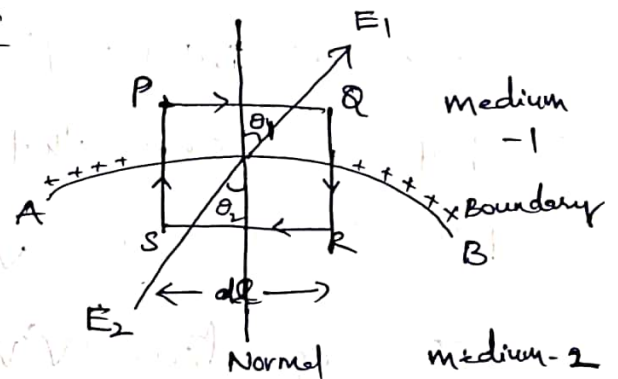
Following are the boundary conditions:

- ① The tangential components of electric field intensities are same (continuous) on both sides of the boundary
- ② The normal components of electric field intensities are discontinuous (not same) at the charged surface (or) continuous across the charged free surface.

Consider a rectangle PQRS of small width and its length dl .

The length of PQRS is parallel to boundary. Let E_1, E_2 be the electric field intensities

in two media. Let E_1, E_2 the field vectors make angles θ_1 and θ_2 with the normal to the boundary.



In electrostatics, we know that the voltage across a closed path is zero.

$$\text{Hence } V = - \int E \cdot dl = 0 \quad \left\{ \begin{array}{l} \because E = - \frac{dv}{dl} \text{ (or)} \\ V = - \int E \cdot dl \end{array} \right.$$

$\therefore \int E \cdot dl$ for the rectangle PQRS should be zero.

$$\therefore E_1 \sin \theta_1 dl - E_2 \sin \theta_2 dl = 0$$

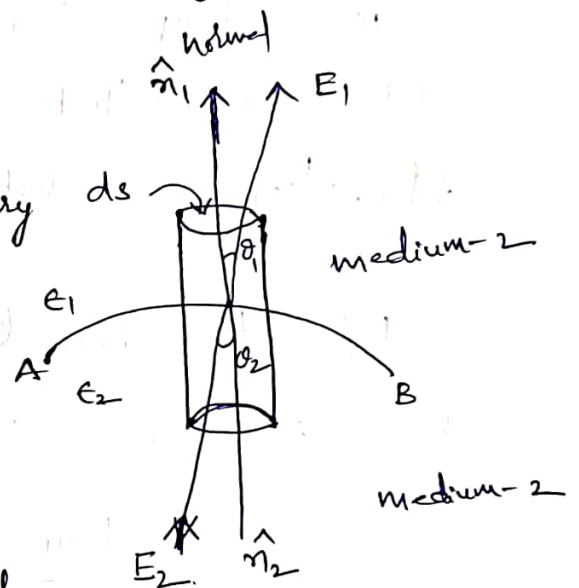
Here we have neglected the contribution of, short sides QR and SP as they are taken very small.

$$\therefore \boxed{E_{t1} = E_{t2}}$$

$\therefore$ The tangential component of electric field in medium 1 is same as the tangential component of $\vec{E}$ in medium-2.

To find the secondary boundary condition, let us construct a small pill box shaped surface (cylindrical).

The height of the pill box is assumed to be negligibly small in comparison with diameter of base diameter.



Apply Gauss theorem to pill box surface,

$$\int \vec{E} \cdot d\vec{s} = q$$

let E_{n1} , E_{n2} be the normal components of electric field intensities in medium 1 and medium-2 respectively.

Here, it should be noted that E_{n2} is inward normal. Hence, from Gauss law we have

$$\text{The total electric flux} = q \Rightarrow \boxed{E_{n1} ds - E_{n2} ds = q} \quad \left(\frac{q}{2} \right)$$

$$E_{n1} - E_{n2} = \frac{q}{ds} = \sigma \quad (\because \text{flux} = E \cdot ds)$$

$$\therefore \boxed{E_{n1} - E_{n2} = \sigma} \quad \sigma \rightarrow \text{Surface charge density}$$

ie $E_{n1} - E_{n2} \neq 0$ and hence $E_{n1} \neq E_{n2}$

It shows that normal components of field intensities are not same for a charged surface.

If the boundary is free from charge, then $\sigma = 0$, $\therefore E_{n1} - E_{n2} = 0$

$$\boxed{E_{n1} = E_{n2}}$$

In this case, the normal components are same across the boundary.

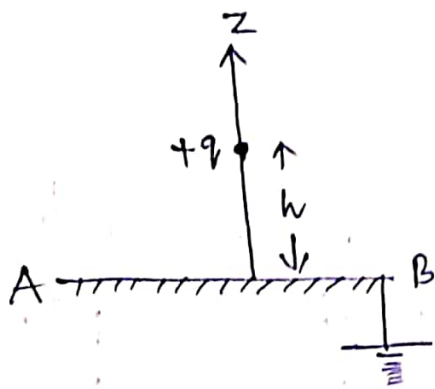
Method of Images

Method of images is the another way to determine the electric field intensity (E), potential (V) and surface charge density (σ) due to the charges in the presence of conductors. using method of images we can determine E , V and ρ_s without solving Poisson's and Laplace's equations.

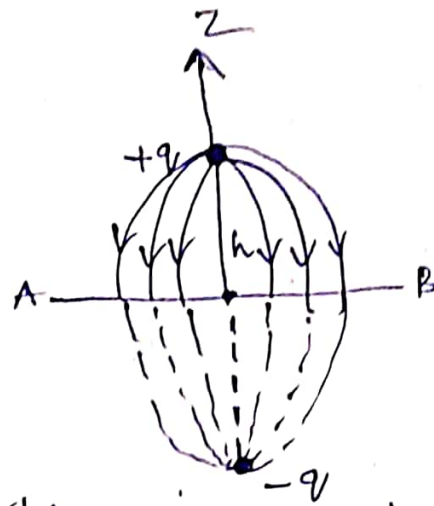
It states that a given charge (+ q) configuration above the infinite grounded conducting plane may be replaced by its image ($-q$) and that conducting plane get replaced by an equipotential surface having potential (V) is zero. (ie $V=0$).

Example: Point charge near a grounded conducting plane.

Consider a conducting plane which is grounded as shown. Since it is grounded, its potential on the plane is zero ($V=0$). Let ' $+q$ ' be the point charge at a distance of ' h ' from the plane along z -axis.



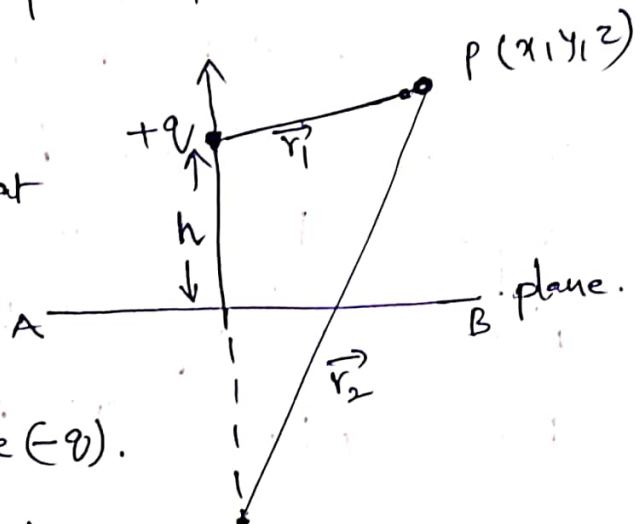
(a) Point charge and grounded plane



(b) Image and field lines.

To find the 'E' at a point, consider a point $P(x, y, z)$ at a distance of r_1 from charge $+q$ and at a distance of r_2 from the image charge $-q$ (expected).

- ① The electric field at the point P is due to the charge $+q$ and due to the charge $(-q)$.



Let E_+ ; E_- be the electric fields at P due to $+q$ and $-q$ respectively.

$$\therefore \vec{E} = E_+ + E_- = \frac{q}{4\pi\epsilon_0 r_1^2} \cdot \vec{a}_{r_1} + \frac{-q}{4\pi\epsilon_0 r_2^2} \cdot \vec{a}_{r_2}$$

$$\therefore \vec{E} = \frac{q}{4\pi\epsilon_0} \frac{\vec{r}_1}{r_1^3} + \frac{-q}{4\pi\epsilon_0} \cdot \frac{\vec{r}_2}{r_2^3}$$

$$\therefore E = \frac{q}{4\pi\epsilon_0} \frac{x\vec{a}_x + y\vec{a}_y + (z-h)\vec{a}_z}{(\sqrt{x^2 + y^2 + (z-h)^2})^3} - \frac{x\vec{a}_x + y\vec{a}_y + (z+h)\vec{a}_z}{(\sqrt{x^2 + y^2 + (z+h)^2})^3}$$

Here $\vec{r}_1 = (x, y, z) - (0, 0, h) = (x, y, z-h)$

$\vec{r}_2 = (x, y, z) - (0, 0, -h) = (x, y, z+h)$

$$\therefore \vec{E} = \frac{q}{4\pi\epsilon_0} \left[\frac{x\vec{a}_x + y\vec{a}_y + (z-h)\vec{a}_z}{(\sqrt{x^2+y^2+(z-h)^2})^3} - \frac{x\vec{a}_x + y\vec{a}_y + (z+h)\vec{a}_z}{(\sqrt{x^2+y^2+(z+h)^2})^3} \right]$$

This is the electric field $\vec{E}$ at 'P'.

Note: When $z=0$, $\vec{E}$ has only z -components as x, y components cancelled.

$$\therefore \vec{E} = \frac{q}{4\pi\epsilon_0} \frac{(-2h)\vec{a}_z}{(\sqrt{x^2+y^2+h^2})^3} \Rightarrow E = \frac{-q \times 2h}{4\pi\epsilon_0 r^3}$$

In this way we can determine the electric field $\vec{E}$ using method of images as $E = -\frac{q \times 2h}{4\pi\epsilon_0 r^3}$.

② To find the potential 'V' at 'P'.

We have $V = - \int \vec{E} \cdot d\vec{l}$

Total potential $V = V_{+q} + V_{-q}$

$$\therefore V = \frac{q}{4\pi\epsilon_0 r_1} + \frac{-q}{4\pi\epsilon_0 r_2}$$

$$\therefore V = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{\sqrt{x^2+y^2+(z-h)^2}} - \frac{1}{\sqrt{x^2+y^2+(z+h)^2}} \right]$$

- ③ To find the Surface charge density (σ).
 let σ be the surface charge density on infinite conducting plane, then electric field intensity E at any point P on the plane is given by

$$E = \frac{\sigma}{\epsilon_0} \quad \text{--- (1)}$$

$$\text{we have } E = \frac{-q/h}{2\pi\epsilon_0 r^3} \quad \text{--- (2)}$$

$$\text{From (1) \& (2), } \frac{-q/h}{2\pi\epsilon_0 r^3} = \frac{\sigma}{\epsilon_0} \Rightarrow \boxed{\sigma = \frac{-q/h}{2\pi r^3}}$$

- ④ Induced charge on the plane:

consider a small element ds in the form of a ring of radius x and of thickness dx on the plane.

$$\text{Then } ds \text{ (area of angular ring)} = 2\pi \cdot x \cdot dx$$

The induced charge on the surface element ds of conducting plane $= \sigma ds$ ($\because \sigma = q/ds$)

$$\therefore \text{ Induced charge} = \int_0^\infty \sigma ds = \int_0^\infty \frac{-q/h}{2\pi r^3} \times 2\pi x dx$$

$$\therefore \text{ Induced charge} = -q/h \int_0^\infty \frac{x dx}{r^3}$$

$$\therefore \int_0^\infty \frac{x}{r^3} dx \text{ becomes } \frac{1}{h} \text{ on solving:}$$

$$\therefore \text{ Induced charge} = -q \times h \times \frac{1}{h} \\ = -q$$

Energy of a charge distribution and its expression in terms of electric field:

For a continuous charge distribution in a given volume, the expression for work done or energy

$$W = \frac{1}{2} \int \rho V d\tau \quad \text{--- (1)} \quad \left\{ \begin{array}{l} \because W = VQ \text{ or } V \times \rho d\tau \\ \text{where } d\tau - \text{volume} \\ \rho \rightarrow \text{vol. charge density} \end{array} \right.$$

We know that $\nabla \cdot E = \rho / \epsilon_0$

$$\Rightarrow \epsilon_0 \nabla \cdot E = \rho \quad \text{--- (2)}$$

put (2) in (1), $W = \frac{1}{2} \int \epsilon_0 (\nabla \cdot E) V d\tau = \frac{\epsilon_0}{2} \int (\nabla \cdot E) V d\tau \quad \text{--- (3)}$

According to product rule, $\nabla \cdot (EV) = (\nabla \cdot E)V + E \cdot (\nabla V)$

$$\Rightarrow (\nabla \cdot E)V = \nabla \cdot (EV) - E \cdot (\nabla V) \quad \text{--- (4)}$$

put (4) in (3), $W = \frac{\epsilon_0}{2} \int [\nabla \cdot (EV) - E \cdot (\nabla V)] d\tau \quad \text{--- (5)}$

but we have $E = -\nabla V \quad \text{--- (6)}$

put (6) in (5), $W = \frac{\epsilon_0}{2} \left[\int \nabla \cdot (EV) d\tau + \int E^2 d\tau \right]$

$$\therefore W = \frac{\epsilon_0}{2} \left[\int \nabla \cdot (EV) d\tau + \int E^2 d\tau \right] \quad \text{--- (7)}$$

using Gauss - Divergence theorem the 1st term of RHS can be written as $\int \nabla \cdot (EV) d\tau = \oint_S EV ds$

$$\therefore W = \frac{\epsilon_0}{2} \oint_S VE ds + \frac{\epsilon_0}{2} \int_V E^2 d\tau$$

$$W = \frac{\epsilon_0}{2} \int_S V E \, ds + \frac{\epsilon_0}{2} \int_V E^2 \, d\tau$$

Here $\int E^2$ can only increase, as E^2 being positive.

So, the other term $\int V E \, ds$ must decrease to leave the sum un-altered.

Hence, for all spaces if we integrate, the surface integral term will reduce to zero. [Since, for $r \rightarrow \infty$

$V \propto \frac{1}{r}$, $E \propto \frac{1}{r^2}$ and $ds = 4\pi r^2$ ($\because$ in case of a sphere)

$$\therefore V E \, ds = \frac{1}{r} \times \frac{1}{r^2} \times r^2 = \frac{1}{r}$$

$$\therefore V E \, ds = \frac{1}{r}, \text{ for } r \rightarrow \infty, \text{ then } \frac{1}{r} \rightarrow 0$$

$$\therefore \text{the term } \boxed{\frac{\epsilon_0}{2} \int_S V E \, ds = 0}$$

$$\therefore W = \frac{\epsilon_0}{2} \int_V E^2 \, d\tau$$

The above equation is the expression for energy in a given volume.

The energy per unit volume (α) Energy density $U = W =$

$$U = \frac{\epsilon_0}{2} E^2 \Rightarrow \boxed{U = \frac{1}{2} \epsilon_0 E^2}$$