

Electric charge:

According to electron theory, every substance is made up of atoms. Every atom has a central heavy portion called 'nucleus'. The nucleus contains protons (mass - 1.672×10^{-27} kg and charge 1.60×10^{-19} C) and neutrons (mass - 1.672×10^{-27} kg & charge zero i.e. chargeless). Thus nucleus is positively charged. Around the nucleus, electrons (mass 9.109×10^{-31} kg and charge -1.60×10^{-19} C) revolve in different orbits. The number of electrons in an atom is equal to the number of protons in the nucleus. As the atom has equal quantities of positive and negative charges and hence it is electrically neutral. ^{The number of protons or} the number of electrons in an atom is called the atomic number Z . The mass of the protons and neutrons in the nucleus is called atomic mass number A . Every substance is made up of elements and every element is made up of electrons and central nucleus (containing protons and neutrons). As the number of electrons and protons is the same and hence we conclude that an electron and proton are the natural units of charge i.e., negative and positive charges. If somehow, one or more electrons are extracted from the atom, it becomes positively charged. Conversely, if the atom is given one or more electrons, it becomes negatively charged. Thus a positively charged body means the deficiency of electrons in the atom while a negatively charged body means the excess of electrons in the atom.

Electrostatics:

Electrostatics is a branch of physics that studies electric charges at rest. Since classical physics, it has been known that some materials such as amber attract light weight particles after rubbing. The Greek word for amber is electron, was the source of the word electricity.

Quantisation of charge:

The electric charges is not continuous but it is made up of integral multiples (1, 2, 3, 4 ...) of a certain minimum electric charge. This minimum electric charge is denoted by e (elementary charge $1.6 \times 10^{-19} \text{ C}$). Any physical existing charge in the universe is found to be ne , where n is a positive or negative (1, 2, 3 ...) irrespective of its origin.

"When any physical quantity exists in discrete packets rather than in continuous amount, the quantity is said to be quantised. Hence we can say that charge is quantised because the charge exists in integral multiples rather than in continuous amounts."

Conservation of charge:

When a physical quantity in an isolated system (no matter is allowed to enter or leave the system) does not change as a result of some process taking place in the system, the quantity is said to be conserved. Similarly, the law of conservation of charges states that the total electric charge (algebraic sum of positive & negative charge at any time) in an isolated system never changes.

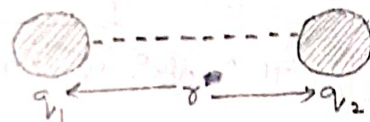
Ex: When a glass rod is rubbed with silk, a positive charge appears on glass rod while a negative charge of equal magnitude appears on silk. so the positive charge acquired by glass rod is equal to the negative charge acquired by the silk. This shows that rubbing merely transfers charge from one object to other and no new charge is created. The net charge is zero.

Coulomb's Law:

Coulomb's law states that the force of attraction ~~between~~ or repulsion between two stationary point charges is directly proportional to the product of the two charges and inversely proportional to the square of the distance between them.

Explanation:

If q_1 and q_2 be the two point charges separated from each other by distance ' r ' (in vacuum or air) then the force F acting between them is given by



(i) The force of attraction or repulsion between two electric charges is directly proportional to the product of two charges.

$$F \propto q_1 q_2 \longrightarrow (1)$$

(ii) The force of attraction or repulsion between two electric charges is inversely proportional to the square of the distance b/w them. This is known as 'inverse square law'

$$F \propto \frac{1}{r^2} \longrightarrow (2)$$

From (1) & (2) $F \propto \frac{q_1 q_2}{r^2} \Rightarrow F = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_2}{r^2}$

Where $\frac{1}{4\pi\epsilon_0}$ is proportionality constant. Its value depends upon the medium between charges & units used for charge, distance and force if q_1 and q_2 are in coulomb's, r in 'metre' and force in newton, then

$$\frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9 \text{ newton-metre}^2 / \text{coulomb}^2$$

ϵ_0 is called the permittivity of free space its value $8.85 \times 10^{-12} \text{ C}^2 / \text{N-m}^2$

Electric Field And Intensity of Electric Field:

Electric field: The region surrounding on electric charge or a group of charges, in which another charge experiences a force is called electric field.

Intensity of electric field ($\vec{E}$): the intensity of electric field at a point in the field is defined as the force experienced by a unit positive charge placed at that point.

Let F be the force experienced by the test charge q_0 placed at a point in the electric field, then the intensity of electric field E at that point is given by

$$\vec{E} = \frac{\vec{F} \text{ newton}}{q_0 \text{ coulomb}}$$

It is obvious that intensity of electric field is a vector quantity. Its direction will be the same as the direction of force. i.e., the direction in which the positive charge placed in the field tends to move. The units of electric field intensity is newton per coulomb.

Let us consider that a unit positive charge is placed at a distance ' r ' from a point charge Q in air. From Coulomb's law, the force on unit positive charge is given by

$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q \cdot 1}{r^2}$$

$$E = \frac{F}{1} = \frac{1}{4\pi\epsilon_0} \times \frac{Q}{r^2}$$

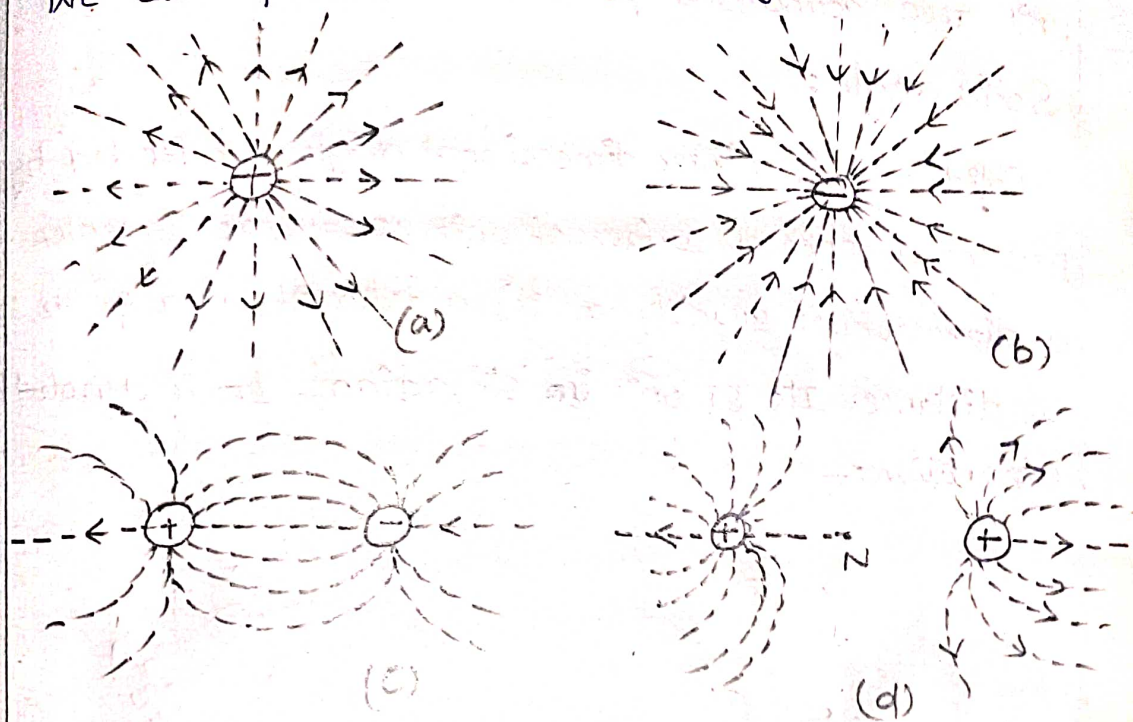
Electric Lines of Force:

The path of free ~~elect~~ positive charge in a electric field is called electric lines of force.

Def: An electric line of force is that, imaginary smooth curve 'drawn' in a electric field along in which a free isolated unit +ve charge will move.

The tangent of any point on it gives the direction of the field at that point.

We can represent an electric field by lines of force.



Explanation:

* In fig (a). The lines of force in an electric field is produced by a positively charged sphere. If a positive charge is left free near the sphere, it will go straight away upto infinity due to repulsive force. Thus the lines of forces due to an isolated +ve charge go straight upto ∞ .

* The lines of forces due to an isolated negative charge comes straight from infinity upto the negative charges.

* In fig (c). The lines of forces due to two equal & opposite charges. The lines start from +ve charge and end at the -ve charge.

* In fig (d) shows the lines of force due to two similar +ve charges. so at the midpoint N the field produced by one charge is equal & opposite to the field produced by the other charge.

* Thus at this point (N) the resultant field is Zero. The point N is called neutral point.

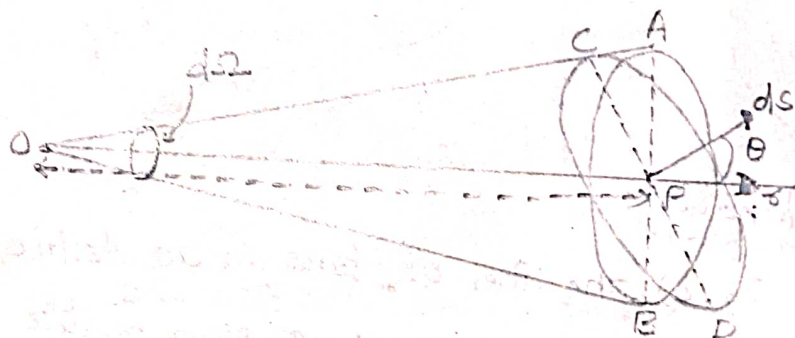
* When any charge is placed at this point, it has no tendency to move in any direction.

Thus, the intensity of electric field at any point can also be defined as the number of lines of force passing through unit area round that point normally.

Solid angle:

Solid angle is a three dimensional angle subtended by any part of a spherical surface of unit radius at its center. It is a dimensionless quantity as it's a ratio of area & the square of distance. Its SI unit is 'steradian'. It is denoted by Ω or ω .

Explanation:-



When every point on the boundary of a surface element
 Ex: CD of area ds is joining to a given point O, then one finds that as a result of it, a cone is formed. Let us imagine a spherical surface with centre at O and of radius r (= distance of surface element from O) then the above cone will intersect this surface in the form of another surface AB of area da . We can easily see that the area da is the projection of the area $ds \perp OP$, i.e., $da = ds \cos \theta$, where θ is the angle between the surface AB and CD (in the fig). obviously, if we consider the surface element as vector whose direction is along the outward normal to the surface element and O is considered as origin, then the angle between ds and r will be θ .

Therefore, $da = \hat{r} \cdot ds = ds \cos \theta$

$:[A \cdot B = AB \cos \theta]$

the angle formed by this surface element 'da' at that point 'O' in three dimensions is termed 'Solid angle'. The solid angle formed by the area 'ds' at a point O is obtained as the ratio of the normal area to the square of the distance, i.e.,

$$d\Omega = \frac{da}{r^2} = \frac{ds \cos \theta}{r^2}$$

If $d\Omega = da$. Thus, the solid angle formed by any surface at point O is equal in magnitude to the area intersected by the cone or pyramid formed by joining every point of that surface to the point O, on a spherical surface of unit radius and having centre at 'O'.

Electric Flux:

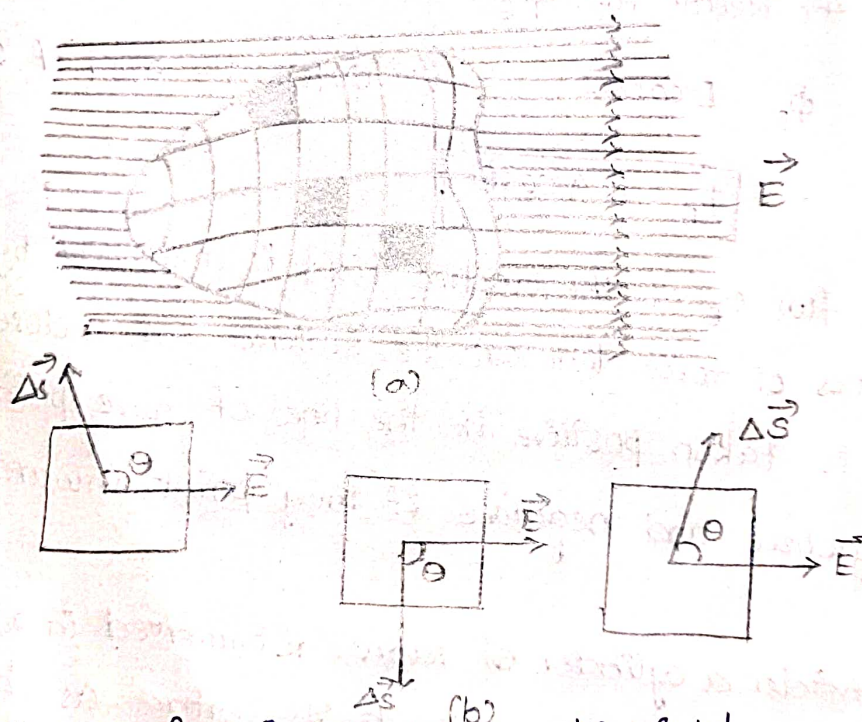


fig (a) surface immersed in electric field.

fig (b) Enlarged view of the three squares on surface.

Let consider an arbitrary closed surface immersed in electric field. Let the surface be divided into a number of elementary squares. [Here we assume that the square is very small so that it may be considered as plane and the electric field over it may be taken as uniform]. Each square on the surface

may be represented by a vector $d\vec{s}$ whose magnitude is equal to its area and the direction taken as the outward normal drawn on this surface.

Let E be the electric field vector acting on the surface. The scalar product i.e., $E \cdot d\vec{s}$ is defined as the electric flux for the surface. The total flux Φ_E through the entire surface is given by

$$\boxed{\Phi_E = \int E \cdot d\vec{s} = E \cdot S} \longrightarrow (1).$$

If θ is the angle between E and $d\vec{s}$ then the scalar product is given by $E \cdot d\vec{s} = E ds \cos\theta$.

Now the electric flux $\Phi_E = \oint E ds \cos\theta \Rightarrow \Phi_E = E \cos\theta \oint ds$

or $\Phi_E = E \cos\theta A$ ($\because \oint ds = A = \text{Area of surface}$)

$$\boxed{\Phi_E = E A \cos\theta}$$

The flux Φ_E of the electric field is measured by the number of lines of force that cut the surface. For a closed surface, " Φ_E is taken positive if the lines of force point outward everywhere and negative if they point inward."

Ex:

Consider a cylinder of radius R immersed in a uniform electric field E parallel to its surface as shown in below

fig.

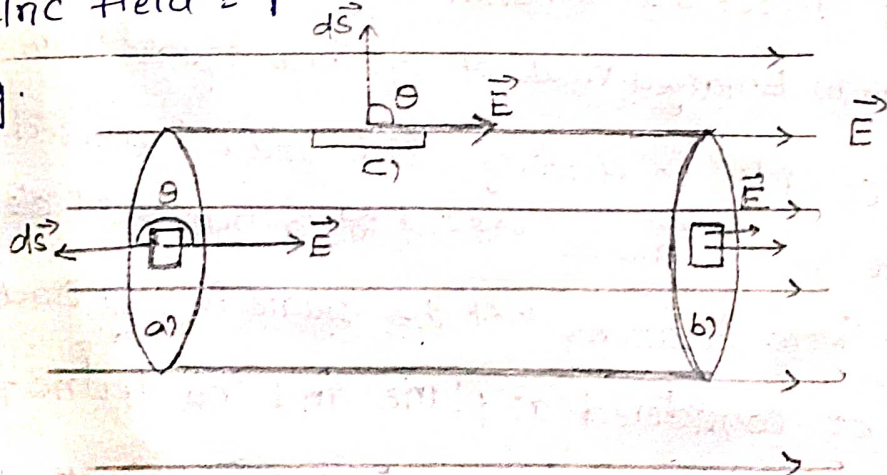


fig:

The flux Φ_E for the entire cylinder is the sum of the fluxes through (a) left face i.e. left cylinder cap, (b) right i.e., right cylinder cap and (c) the cylindrical surface. Thus.

$$\Phi_E = \oint E \cdot ds.$$

$$= \int_{(a)} E \cdot ds + \int_{(b)} E \cdot ds + \int_{(c)} E \cdot ds.$$

For the left cap of the cylinder, the angle between E & ds is 180°

$$\therefore \int_{(a)} E \cdot ds = \int E ds \cos \theta = \int E ds \cos 180^\circ = -E \int ds = -ES.$$

Similarly for right cap of the cylinder, then angle between E and ds is zero.

$$\therefore \int_{(b)} E \cdot ds = \int E ds \cos \theta = \int E ds \cos 0^\circ = E \int ds = +ES.$$

For the curved surface of the cylinder, the angle between E & ds is 90° .

$$\therefore \int_{(c)} E \cdot ds = \int E ds \cos 90^\circ = 0. \quad (\because \cos 90^\circ = 0)$$

$$\text{Hence, } \Phi_E = -E/S + E/S + 0 = 0.$$

Thus the flux through the entire cylinder is zero.
Therefore, the flux through the entire cylinder when its axis is perpendicular to the field is also zero.

GAUSS'S LAW :

Gauss's law is the converse of Coulomb's law. With the help of Coulomb's law we can calculate E for a given charge while Gauss's law enables to determine the charge provided E is known.

Def: Gauss's law states that total normal electric flux Φ_E over a closed surface is $(1/\epsilon_0)$ times the total charge Q enclosed within the surface.

Mathematically it can be expressed as

$$\Phi_E = \oint E \cdot ds = \oint E \cdot ds \cos \theta = (1/\epsilon_0) Q$$

where ϵ_0 is the permittivity of the free space.

proof: (i) when the charge is within the surface

Let a charge $+Q$ is placed at O within a closed surface of irregular shape. Consider a point P on the surface at a distance r from O .

Now take a small area ds around P .

The normal to the surface ds is represented by a vector ds which makes an angle θ with the direction

of electric field E along OP . The

electric flux $d\Phi_E$ outwards ~~from~~ through the area ds given

by

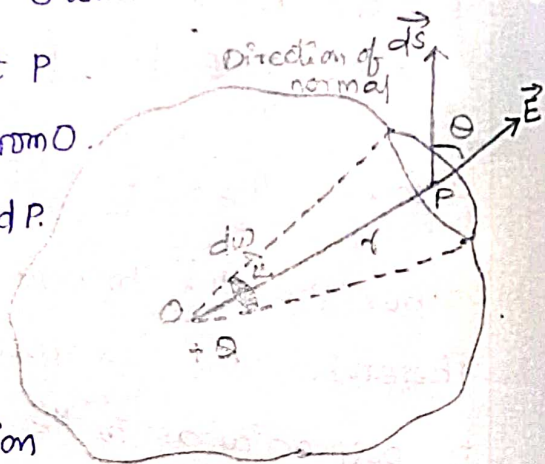
$$E = \frac{1}{4\pi\epsilon_0} \left[\frac{Q}{r^2} \right] \longrightarrow E$$

$$d\Phi_E = E \cdot ds = E ds \cos \theta \longrightarrow (1).$$

where θ is the angle between E and ds .

From Coulomb's law, the electric intensity E at a point P distance r from a point charge Q is given by

$$E = \frac{1}{4\pi\epsilon_0} \left[\frac{Q}{r^2} \right] \longrightarrow (2)$$



From (1) & (2), we get

$$d\phi_E = \frac{Q}{4\pi\epsilon_0 r^2} \cdot ds \cos\theta = \frac{Q}{4\pi\epsilon_0} \left[\frac{ds \cos\theta}{r^2} \right]$$

But $\left[\frac{ds \cos\theta}{r^2} \right]$ is the solid angle $d\omega$ subtended by ds at O . Hence

$$d\phi_E = \frac{Q}{4\pi\epsilon_0} d\omega. \longrightarrow (3)$$

The total flux ϕ_E over the entire whole surface is given by

$$\int d\phi_E = \int \frac{Q}{4\pi\epsilon_0} d\omega$$

$$\Rightarrow \boxed{\phi_E = \frac{Q}{4\pi\epsilon_0} \oint d\omega}$$

where $\oint d\omega$ is the solid angle subtended by the whole surface at O . This is equal to 4π . Hence

$$\phi_E = \frac{Q}{4\pi\epsilon_0} \times 4\pi$$

$$\boxed{\phi = \frac{Q}{\epsilon_0}} \longrightarrow (4)$$

Let the closed surface enclosed several charges say $+Q_1, +Q_2, +Q_3, \dots, -Q_1, -Q_2, -Q_3, \dots$. Now each will contribute to the total electric flux. Here it should be remembered that for positive charges, the electric flux will be outward and hence positive while for the negative charges, the electric flux will be inward and hence negative. Thus the total flux is given by

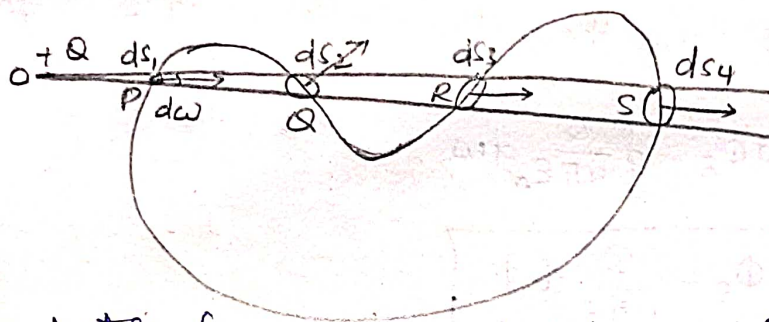
$$\phi_E = \frac{1}{\epsilon_0} [Q_1 + Q_2 + Q_3 + \dots - Q_1, -Q_2, -Q_3, \dots]$$

$$= \frac{1}{\epsilon_0} \sum Q, \text{ where } Q \text{ is algebraic sum of all charges.}$$

So the total normal electric flux over the closed surface is equal to $(1/\epsilon_0)$ times the total charges enclosed within the surface which establishes the Gauss's law.

ii, When the charge is outside the surface

Let a point charge $+Q$ be situated at point O outside the closed surface as shown in fig. Now a cone of solid angle $d\omega$ from O cuts the surface areas ds_1, ds_2, ds_3, ds_4 at point P, Q, R and S respectively.



The electric flux for an outward normal is positive while for inward drawn normal is negative. Therefore, the flux through areas ds_2 and ds_4 are positive while for area ds_1 and ds_3 are negative. Therefore,

$$\Phi_E \text{ through area } ds_1 = (-Q/4\pi\epsilon_0)d\omega$$

$$\Phi_E \text{ through area } ds_2 = (+Q/4\pi\epsilon_0)d\omega$$

$$\Phi_E \text{ through area } ds_3 = (-Q/4\pi\epsilon_0)d\omega$$

$$\Phi_E \text{ through area } ds_4 = (+Q/4\pi\epsilon_0)d\omega$$

$\therefore$ Total electric flux

$$= -\frac{Q}{4\pi\epsilon_0}d\omega + \frac{Q}{4\pi\epsilon_0}d\omega - \frac{Q}{4\pi\epsilon_0}d\omega + \frac{Q}{4\pi\epsilon_0}d\omega = 0$$

So, the total electric flux over the whole surface due to an external charge is zero. This verifies Gauss's law.

Differential Law of Gauss law;

Statement: This states that the divergence of electric field is proportional to the charge at that point or flux of E depends on the magnitude of charge or charge density.

proof:

According to Gauss's law,

$$\oint E \cdot ds = Q/\epsilon_0 \Rightarrow \epsilon_0 \oint E \cdot ds = Q \longrightarrow (1).$$

Let a charge Q be distributed uniformly over a volume V and ρ be the density of charge. Then,

$$Q = \iiint \rho \, dv \longrightarrow (2) \quad \therefore \rho = \frac{Q}{V}.$$

From (1) & (2), we get

$$\epsilon_0 \oint E \cdot ds = \iiint \rho \, dv \longrightarrow (3).$$

Now, we use divergence theorem to convert the surface integral $\oint$ into volume $\iiint$ integral i.e.,

$$\oint_s E \cdot ds = \iiint \text{div } E \, dv \longrightarrow (4).$$

Substituting eq (4) in eq (3) we get

$$\epsilon_0 \iiint \text{div } E \, dv = \iiint \rho \, dv \longrightarrow (5)$$

Eq (5) is true for any arbitrary volume. Hence integrands must be equal i.e.,

$$\epsilon_0 \text{div } E = \rho \quad (\text{or}) \quad \text{div } E = \frac{\rho}{\epsilon_0} \quad (\text{or}) \quad \nabla \cdot E = \rho/\epsilon_0 \longrightarrow (6)$$

In vacuum, $D = \epsilon_0 E$ (or) $E = \frac{D}{\epsilon_0}.$

$$\text{div } D = \rho \quad (\text{or}) \quad \nabla \cdot D = \rho \longrightarrow (7).$$

Eq (6) is the differential form of Gauss's law.

Electric flux density (D): Electric flux passing per unit area is called electric flux density.

$$D = \frac{\Phi}{A} = \frac{Q}{A} \quad \phi = Q \text{ coulombs.}$$

S.I unit of $D = \text{C/m}^2$

Q is the point charge located at the centre of sphere.

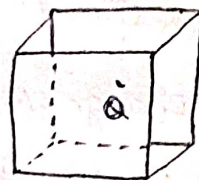
then the flux density, $D = \frac{Q}{A} = \frac{Q}{4\pi r^2} \cdot \frac{\epsilon_0}{\epsilon_0} = \epsilon_0 \cdot E$

$$\therefore \vec{D} = \epsilon_0 \vec{E}$$

problems:

1. When a point charge is placed at the centre of a cube, what is the flux linked

- a) With the cube
b) With each face of the cube.



Soln From Gauss's Law,

$$\phi_E = \oint E \cdot ds = \frac{Q}{\epsilon_0}$$

- a) With the cube

$$\phi_E = \frac{Q}{\epsilon_0}$$

- b) With each face of cube

$$\phi_E = \frac{Q}{6\epsilon_0} \quad [\because \text{cube has 6 faces}]$$

2. S_1 and S_2 are two parallel spherical surfaces enclosing charges q_1 and $2q_1$ respectively as shown in figure. What is the ratio of electric flux through S_1 and S_2 .

Soln From Gauss's Law,

$$\phi = \frac{Q}{\epsilon_0}$$

Flux enclosed by S_1 :

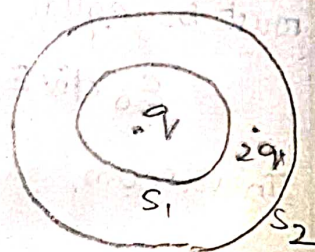
$$\phi_1 = \frac{q_1}{\epsilon_0}$$

Flux enclosed by S_2 :

$$\phi_2 = \frac{q_1 + 2q_1}{\epsilon_0} = \frac{3q_1}{\epsilon_0}$$

Ratio of ϕ_1 and ϕ_2 $\frac{\phi_1}{\phi_2} = \frac{q_1}{\cancel{\epsilon_0}} \cdot \frac{\cancel{\epsilon_0}}{3q_1} = \frac{1}{3}$

$$\boxed{\therefore \frac{\phi_1}{\phi_2} = \frac{1}{3}}$$



3. A conducting sphere of radius 10cm has an unknown charge. If the electric field 20cm from the centre of sphere is $1.5 \times 10^3 \text{ N/C}$ and points inward, what is the charge on the sphere.

Sol: Given that,

Radius of Gaussian surface,

$$R = 20\text{cm} = \frac{20}{100} = 0.2\text{m}$$

According to Gauss's Law,

$$\Phi = \oint E \cdot ds$$

$$\Phi = E \oint ds$$

$$\Phi = -ES \quad [\because \text{Electric field points inward to surface}]$$

$$\Phi = -E 4\pi r^2 \quad [-E \times \text{area of Gaussian surface}]$$

$$\Phi = -1.5 \times 10^3 \times 4 \times 3.14 \times (0.2)^2$$

$$\Phi = -1.5 \times 10^3 \times 0.5024$$

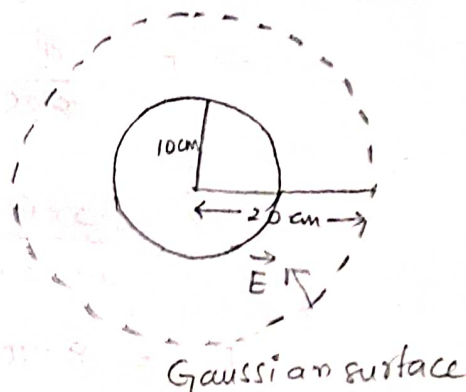
$$\Phi = -753.98$$

We know that,

$$\Phi = \frac{q}{\epsilon_0} \Rightarrow q = \Phi \epsilon_0$$

$$\Rightarrow q = -753.98 \times 8.85 \times 10^{-12}$$

$$\Rightarrow \boxed{q = -6.672 \times 10^{-9} \text{ Coulombs}}$$



4. A point charge $q = 2.0 \times 10^{-7} \text{ coulombs}$ is placed at the centre of spherical cavity of radius 3.0 cm in metal piece. Find electric field intensities at a' and b' as shown in figure.

Sol:

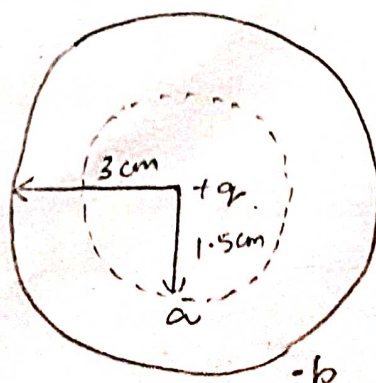
Given data,

$$q = 2.0 \times 10^{-7} \text{ C}$$

$$OA = 1.5\text{cm} = 0.015\text{m}$$

$$= 1.5 \times 10^{-2} \text{ m}$$

$$Ob = 3\text{cm} = 3 \times 10^{-2} \text{ m}$$



a) from Gauss's Law,

$$\Phi = \oint E ds = \frac{q}{\epsilon_0}$$

$$Es = \frac{q}{\epsilon_0}$$

$$\Rightarrow E = \frac{q}{\epsilon_0 s} = \frac{2 \times 10^{-7}}{8.85 \times 10^{-12} \times 4\pi (1.5 \times 10^{-2})^2}$$

$$= \frac{2 \times 10^{-7}}{2.502 \times 10^{-14}} = 7.99 \times 10^6 \text{ N/C} \approx 8 \times 10^6 \text{ N/C}$$

$$E = 8 \times 10^6 \text{ N/C}$$

b) We know that,

charge always resides on the surface of metal conductor.

$\therefore$ Electric field intensity at point 'b' is zero.

Linear charge density (λ):

The charge per unit length, $\lambda = \frac{dq}{dl}$.

Surface charge density (σ):

The charge per unit area, $\sigma = \frac{dq}{ds}$

Volume charge density (ρ):

The charge per unit volume, $\rho = \frac{dq}{dv}$.

Applications of Gauss Law:

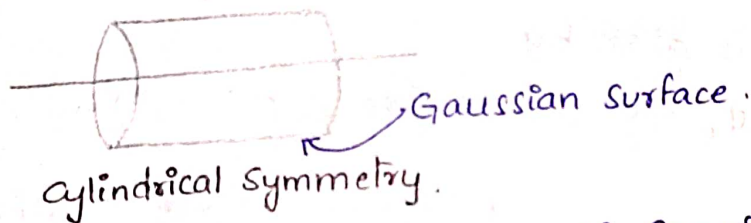
Gauss law provides a powerful method for evaluating the electric field intensity E and potential variation due to simple charge distribution whose symmetry is such that the field is constant and normal over the entire Gaussian surface.

The symmetry is crucial to the application of Gauss's Law.

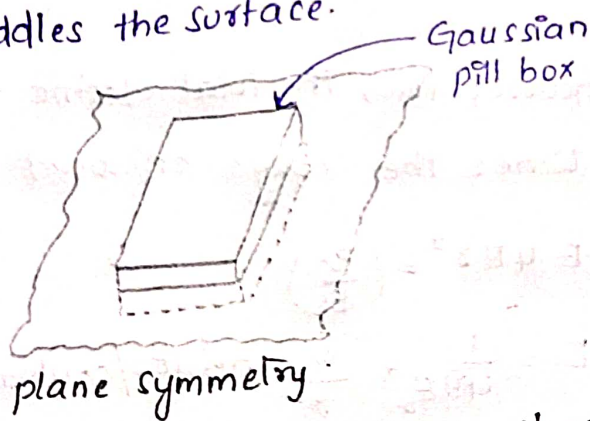
There are 3 kinds of symmetry which are sufficient:

1. spherical symmetry: one can make Gaussian surface a concentric sphere.

2. cylindrical symmetry: one can make Gaussian surface a coaxial cylinder.



3. plane symmetry: one can make use of Gaussian pill box, which straddles the surface.



Electric Field Due to a uniformly charged sphere:

Case i) At a point outside the charged surface.

Consider a sphere A of radius R with centre 'O' as shown in below figure. Let a charge Q be uniformly distributed over it. Suppose 'P' be an external point at a distance 'r' from the centre 'O' of the sphere. We shall find electric field E at this point. So that we construct a Gaussian surface of radius OP .

which is concentric with sphere A. From the symmetry of charge distribution, the electric field at all points of the Gaussian

Surface is the same and will be perpendicular to the surface.

Thus E is directed along the outward normal. For a small

Gaussian surface, ds is also directed radially outward.

So for a small Gaussian surface

$$E \cdot ds = E ds \cos 0^\circ = E ds \quad [\because \text{angle between } E \text{ \& } ds \text{ is zero}]$$

Now the electric ~~field~~ flux through the entire Gaussian surface is given by

$$\Phi_E = \oint E \cdot ds = E \oint ds = E(4\pi r^2) \longrightarrow (1)$$

($\because$ area of Gaussian surface of radius ' r ' $= 4\pi r^2$)
In eq'n (1), E is constant at all points and hence taken out of integral.

According to Gauss's law, the total electric flux over a closed surface is $(1/\epsilon_0)$ times the charge enclosed within the surface.

$$\text{Thus, } E \cdot 4\pi r^2 = \left(\frac{q}{\epsilon_0}\right)$$

$$E = \frac{1}{4\pi\epsilon_0} \times \frac{q}{r^2} \text{ newton/coulomb} \longrightarrow (2)$$

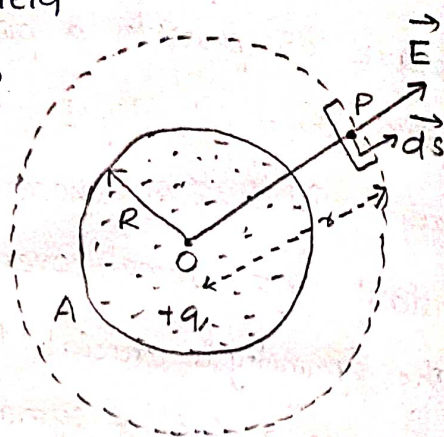
$$\text{In vector form, } E = \frac{1}{4\pi\epsilon_0} \times \frac{q}{r^3} \hat{r} \text{ newton/coulomb} \longrightarrow (3)$$

where $\hat{r}$ is the unit vector along radius.

Case ii, At a point on the surface:

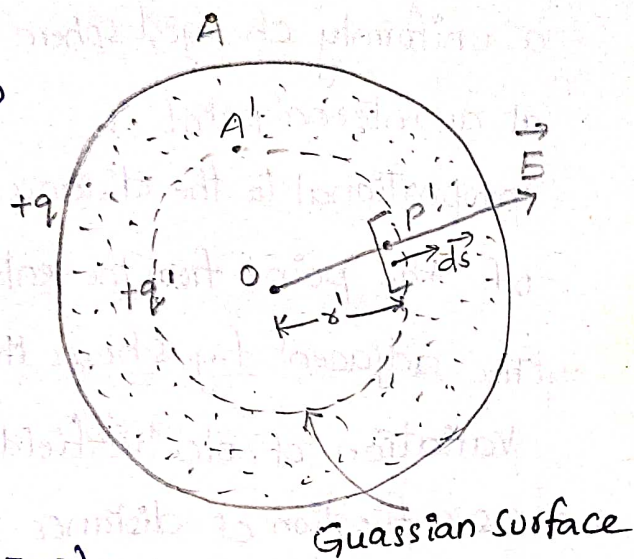
When the point P lies on the surface of the sphere, then $r = R$. In this case the field intensity is given by

$$E = \frac{1}{4\pi\epsilon_0} \times \frac{q}{R^2} \text{ newton/coulomb.}$$



case iii. At a point inside the charged sphere.

the electric field E at a point P which is inside the charged sphere at a distance r from the centre. The Gaussian surface at P (as shown in adjacent figure). The outward flux for a small portion of Gaussian surface will be



$$E \cdot ds = E ds \cos \theta = E \cdot ds$$

($\because$ Angle between E and ds is zero)

The electric flux through the entire Gaussian surface is given by

$$\begin{aligned} \Phi_E &= \oint E ds = E \oint ds \\ &= E(4\pi r^2) \longrightarrow (1) \end{aligned}$$

The total charge enclosed by the Gaussian surface

= Volume enclosed by it $\times$ charge per unit volume

$$= \frac{4}{3}\pi r^3 \times \rho$$

Here a charge q is uniformly distributed over a sphere of radius R . Hence

$$\rho = \frac{\text{total charge}}{\text{volume}} = \frac{q}{(\frac{4}{3})\pi R^3} = \frac{3q}{4\pi R^3}$$

$\therefore$ charge enclosed in Gaussian surface

$$= \frac{4}{3}\pi r^3 \times \frac{3q}{4\pi R^3} = q \left(\frac{r}{R}\right)^3 \longrightarrow (2)$$

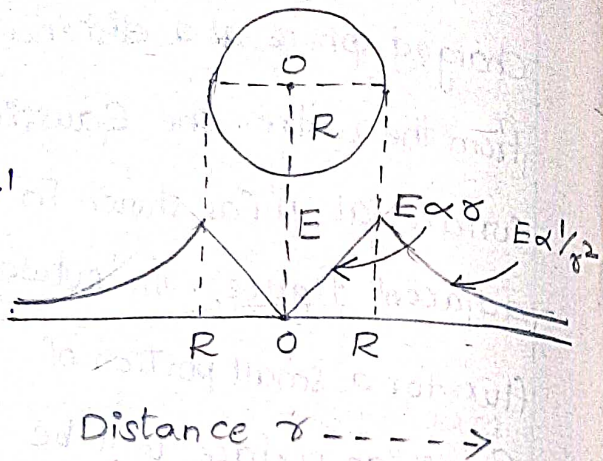
From Gauss's law,

$$E \times (4\pi r^2) = \frac{1}{\epsilon_0} \cdot q \cdot \left(\frac{r}{R}\right)^3$$

$$\therefore E = \frac{1}{4\pi\epsilon_0} \times \frac{q r}{R^3} \longrightarrow (3)$$

This expression shows that the electric field E due to a uniformly charged sphere at an internal point is proportional to the distance r of the point from the centre.

The adjacent fig. shows the variation of electric field as a function of distance r .



It is obvious that electric field outside the sphere is inversely proportional to the square of the distance. It's maximum on the surface of the sphere.

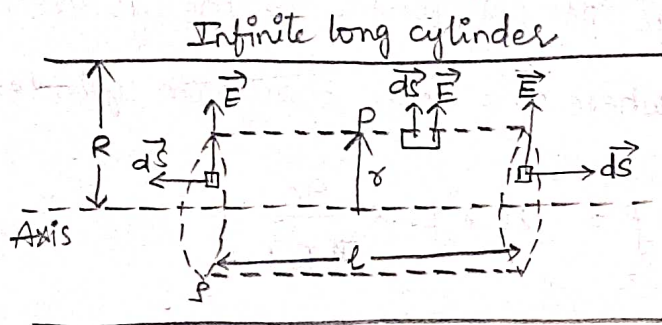
If we consider a point within the conducting sphere, then electric field will be zero because the entire charge will reside over the outer surface.

Electric field due to a charged cylinder:

Consider an infinite circular cylinder of radius R . Let the charge is distributed uniformly along the cylinder and ρ is the charge per unit volume. The electric field at a point P due to this cylinder in the following cases:

Case (i) When P lies inside the cylinder:

Let P be at a distance r from the axis of the cylinder. Now, we consider a cylindrical Gaussian surface of length l and radius r with the same axis as that of the charged cylinder.



the contribution to electric flux due to circular ends of the cylinder comes out to be zero because E and $d\vec{s}$ are at right angles to each other ($\because E \cdot d\vec{s} = E ds \cos 90^\circ = 0$). The only contribution to the electric flux is due to the curved surface of the cylinder. Due to symmetry, the magnitude of electric field will be same at all points of the curved surface of the cylinder and is directed radially outward. For the curved surface of the cylinder, E and $d\vec{s}$ are along the same direction i.e., angle between them is zero. so the electric flux due to entire Gaussian surface is given by

$$\begin{aligned}\Phi_E &= \oint E \cdot d\vec{s} = \oint E ds \cos 0^\circ = \oint E ds \\ &= E \oint ds = E(2\pi r l) \longrightarrow (1)\end{aligned}$$

The total charge enclosed by the Gaussian surface is given by
charge = volume $\times$ volume density

$$\rightarrow \text{charge} = \pi r^2 L \times \rho \longrightarrow (2)$$

According to Gauss's law

$$E(2\pi r l) = \frac{1}{\epsilon_0} \times (\pi r^2 L \rho)$$

$$E = \frac{\rho r}{2\epsilon_0} \longrightarrow (3)$$

Expression in terms of charge per unit length of cylinder, λ

Let λ be the charge per unit length of the cylinder, then

$$\lambda = q/L, \text{ where } q = \text{total charge on cylinder.}$$

$$\begin{aligned} \text{charge} &= \pi r^2 L \times \rho = \pi r^2 L \times \frac{q}{\pi R^2 L} \\ &= \frac{L r^2 \lambda}{R^2} \quad [\because \lambda = q/L] \end{aligned}$$

$$\text{So, } E(2\pi r L) = \frac{1}{\epsilon_0} \times \left(\frac{L r^2 \lambda}{R^2} \right)$$

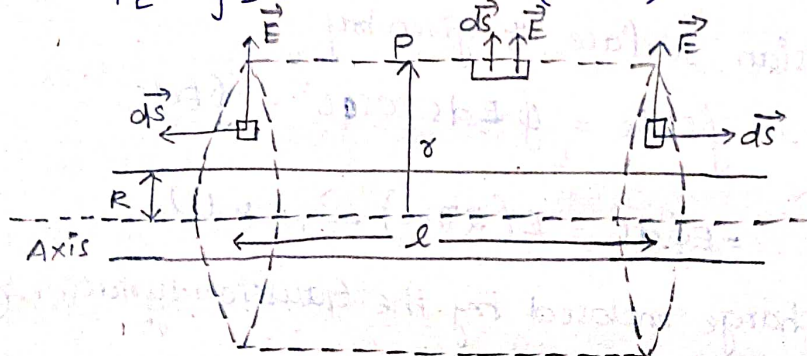
$$\boxed{E = \frac{\lambda r}{2\pi \epsilon_0 R^2}}$$

The above expression shows that electric field inside the cylinder is directly proportional to the distance of the point from the axis of the cylinder.

Case (ii) When P lies outside the cylinder:

The total flux through the Gaussian surface is given by

$$\Phi_E = \oint \vec{E} \cdot d\vec{s} = E ds = E(2\pi r L) \rightarrow (4)$$



The total charge enclosed by the Gaussian surface is given by

charge = charge on the cylinder of actual length L

$\times$ volume density

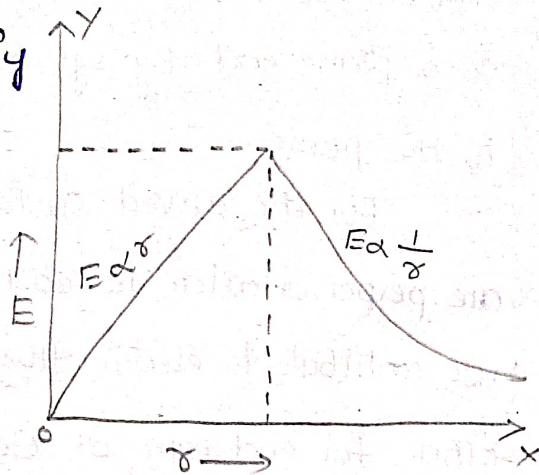
$$= \pi r^2 L \times \rho \rightarrow (5)$$

According to Gauss's law

$$E(2\pi r L) = \frac{1}{\epsilon_0} (\pi R^2 L \rho)$$

$$\boxed{E = \frac{R^2 \rho}{2\epsilon_0 r}} \rightarrow (6)$$

This expression shows that the electric field at a point outside the cylinder is directly proportional to the square of radius of the cylinder and inversely proportional to the distance of the point from the axis of the cylinder.



The variation of electric field strength E with distance r from the axis of the uniform charged cylinder.

$$Q = \frac{q}{\pi R^2 L}$$

$$Q = \frac{1}{\pi R^2} \times \frac{q}{L}$$

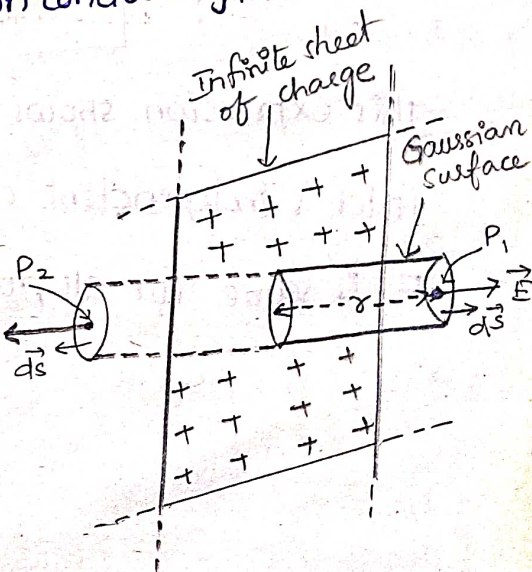
$$Q = \frac{\lambda}{\pi R^2} \quad (\because \lambda = \frac{q}{L})$$

$$\therefore E = \frac{R^2}{2\epsilon_0 r} \times \frac{\lambda}{\pi R^2}$$

$$E = \frac{\lambda}{2\pi \epsilon_0 r}$$

Electric field Due to Infinite sheet of charge:
(Non-conducting)

Consider a portion of the thin, non conducting, infinite sheet of charge. Let σ be the surface charge density (charge per unit area). Let P_1 be a point at a distance r from the sheet where electric field is to be calculated. For this we consider a point P_2 symmetrical with P_1 on the other side of sheet and construct a cylindrical Gaussian surface with one end on one side and other



end on the other side. Let A be the area of cross-section of each plane end. By symmetry E is everywhere perpendicular to the plane.

For the curved surface of Gaussian cylinder, E and ds are perpendicular to each other. Thus the curved surface does not contribute to electric flux. As E and ds are parallel to each other for end caps of Gaussian surface and hence they contribute to electric flux. The electric flux through these caps is given by

$$\begin{aligned}\Phi_E &= \oint E \cdot ds + \oint E \cdot ds \\ &= \oint E ds + \oint E ds \quad (\because \theta = 0^\circ) \\ &= E \oint ds + E \oint E ds \\ &= EA + EA\end{aligned}$$

$$\Phi_E = 2EA.$$

The total charge enclosed by the cylinder $= \sigma A$.

Now, According to Gauss's law

$$2EA = \sigma A / \epsilon_0$$

$$\boxed{E = \frac{\sigma}{2\epsilon_0}}$$

This expression shows that the magnitude of the electric field is independent of the distance from the sheet. Thus E is same for all points on each side of the plane.

Electric potential Difference and Electric potential

The electric charge flows from a point of higher potential to a point at a lower potential.

Consider an electric field due to a positive charge $+q$, as shown in figure. Let there be a positive test charge $+q_0$ in this field.

This test charge experiences

an electric repulsive force F

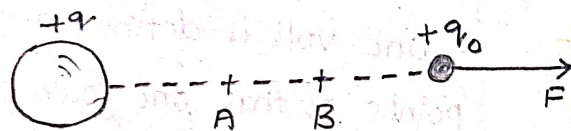


fig:-

due to $+q$. Suppose the test charge is moved, by an external agent, from a point B to another point A. Now some work has to be done against the force of repulsion.

"The ratio of work done in taking a test-charge ~~is defined~~ from one point to other point in an electric field to the magnitude of the test charge is defined as the electric potential difference between these points"

If W be the work done in moving the test charge q_0 from point B to point A, then the potential difference ($V_A - V_B$) between A and B is expressed as

$$V_A - V_B = \frac{W}{q_0} \longrightarrow (1)$$

Suppose the point B (taken as reference point) is chosen at infinity, then the electric potential V_B at this infinite is arbitrarily taken as zero.

$$\text{Now, } V_A = \frac{W}{q_0} \longrightarrow (2)$$

Where, W is the work done by an external agent to carry the test charge q_0 from infinity to point A.

Thus,

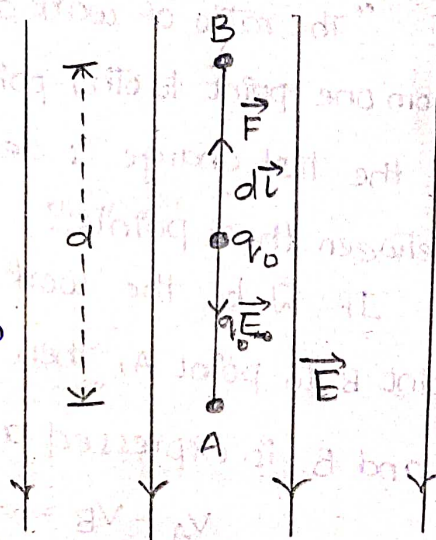
Electrical potential at a point in the electric field is defined as the work done by an external agent in carrying a unit ~~charge~~ positive test charge from infinity to that point against the electric force of the field.

The MKS unit of potential difference is volt. One volt is defined as the difference in potential between the points so that one joule of work is done in carrying one coulomb of positive charge from one point to another point.

$$(1 \text{ volt} = 1 \text{ Joule} / 1 \text{ coulomb}).$$

Potential and Field's strength

Let a uniform electric field E is set up by certain stationary charges as shown in figure. Suppose a positive test charge q_0 is moved by an external agent without acceleration between two points A and B in the electric field. The electric force on the charge q_0 is $q_0 E$. The direction



of this force is in the direction of electric field strength E which points downwards. Now to move the charge in the upward direction by external agent, an equal and opposite force $-q_0 E$ must be applied. Let the charge q_0 is moved through a small distance dl by the external agent. The work done dW by the external agent is given by

$$dW = F \cdot dl = -q_0 E \cdot dl \longrightarrow (1)$$

$\therefore$ The total work done W_{AB} in carrying q_0 from A to B will be

$$W_{AB} = \int_A^B -q_0 E \cdot dl = -q_0 \int_A^B E \cdot dl \longrightarrow (2)$$

Further, $V_B - V_A = \frac{W_{AB}}{q_0} \longrightarrow (3)$

$\therefore$ potential difference between two points A and B will be

$$V_B - V_A = - \int_A^B \vec{E} \cdot d\vec{l} \longrightarrow (4)$$

If the reference point A is taken at infinity, then $V_A = 0$

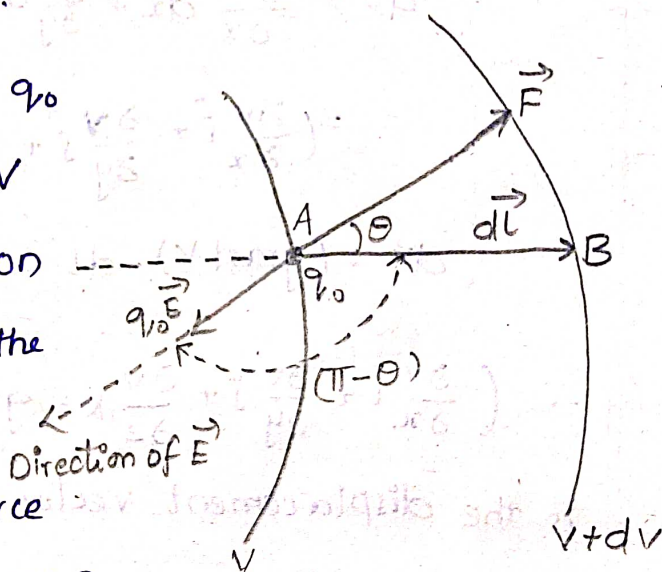
Hence, $V_B = - \int_{\infty}^B \vec{E} \cdot d\vec{l} \longrightarrow (5)$

the integral in equation (5) is called as line integral. so the electric potential at a point in the electric field can be expressed as a line integral of the electric field.

Field as the Gradient of potential (or) Relationship b/w $\vec{E}$ & V :

Consider two surfaces of potentials V and $V + dV$ in an electric field as shown in figure.

Let a positive test charge q_0 at point A on the surface V is moved to the point B on the surface $V + dV$ along the vector displacement $d\vec{l}$ by any external agent. the force experienced by the test charge q_0



figure

at A due to electric field will

be $q_0 E$. This force is in the direction of E which is at right angle to the surface V . In order to move the test charge q_0 without acceleration, the external agent must apply an equal and opposite force F as shown in figure. Thus

$$F = - q_0 E \longrightarrow (1)$$

Further, the workdone by the external agent to move the test charge from A to B along dl is given by

$$dW = F \cdot dl$$

$$dW = -q_0 E \cdot dl$$

$$\therefore \frac{dW}{q_0} = -E \cdot dl \longrightarrow (2)$$

We know that (dW/q_0) is the potential difference dV between points A and B. Thus

$$dV = -E \cdot dl \longrightarrow (3)$$

Let the coordinates of A and B be (x, y, z) and $(x+dx, y+dy, z+dz)$ respectively. The potential difference dV between A and B can be ~~represented~~ expressed as

$$dV = \frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} dy + \frac{\partial V}{\partial z} dz$$

$$= \left(\frac{\partial V}{\partial x} \hat{i} + \frac{\partial V}{\partial y} \hat{j} + \frac{\partial V}{\partial z} \hat{k} \right) \cdot (dx \hat{i} + dy \hat{j} + dz \hat{k})$$

$$dV = (\text{grad } V) \cdot dl = \vec{\nabla} V \cdot dl \longrightarrow (4)$$

$$\therefore \left(\frac{\partial V}{\partial x} \hat{i} + \frac{\partial V}{\partial y} \hat{j} + \frac{\partial V}{\partial z} \hat{k} = \text{grad } V, \text{ and } dx \hat{i} + dy \hat{j} + dz \hat{k} \right)$$

is the displacement vector dl between A and B)

from eq'n (3) & (4) we get,

$$\text{grad } V \cdot dl = -E \cdot dl$$

$$E = -\text{grad } V$$

$$E = -\vec{\nabla} V \longrightarrow (5)$$

If E_x , E_y and E_z be the components of E , then eq'n (5) can be written as

$$E_x \hat{i} + E_y \hat{j} + E_z \hat{k} = - \left(\frac{\partial V}{\partial x} \hat{i} + \frac{\partial V}{\partial y} \hat{j} + \frac{\partial V}{\partial z} \hat{k} \right)$$

Thus, we have,

$$E_x = -\frac{\partial V}{\partial x}, E_y = -\frac{\partial V}{\partial y} \text{ and } E_z = -\frac{\partial V}{\partial z} \longrightarrow (6)$$

Thus,

"The electric intensity at a point in the electric field is equal to the negative potential gradient in that direction."

Note:

In case of polar coordinates, the components are radial along radius vector and azimuthal perpendicular to the radius vector. These can be calculated as

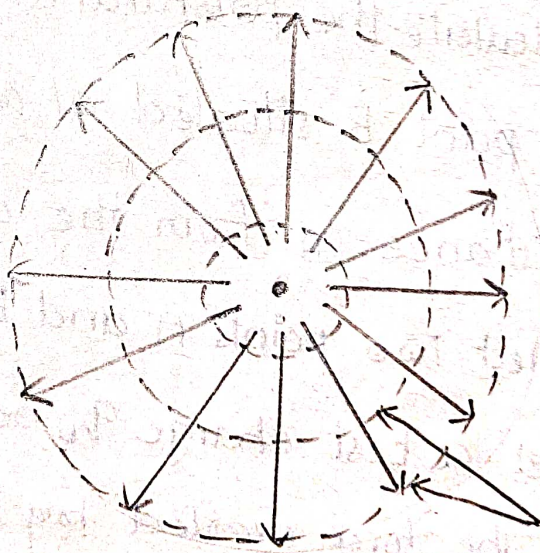
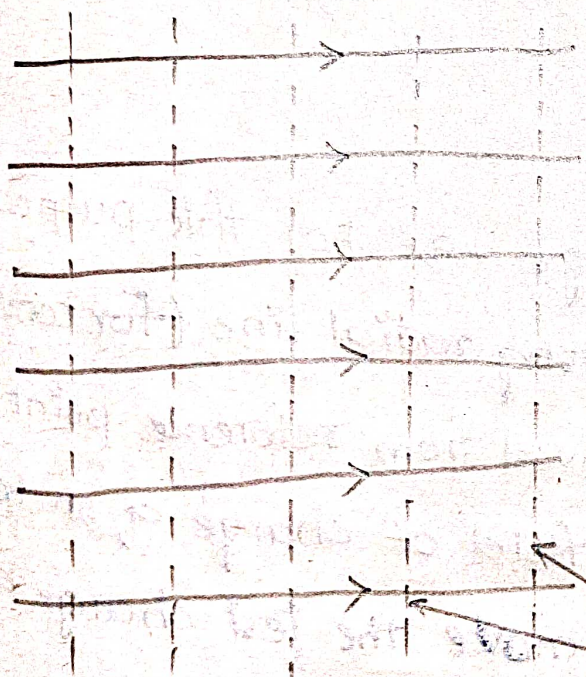
$$E_r = -\left(\partial V / \partial r\right)_{\theta}$$

$$\& E_{\theta} = -\frac{1}{r} \left(\partial V / \partial \theta\right)_r \longrightarrow (7)$$

Equipotential Surfaces:

Def: The locus of all points which have the same electric potential is called equipotential surface.

Equipotential surface in an electric ~~surfa~~ field is a surface on which the potential is same at every point. Hence the lines of force at every point of the equipotential surface are perpendicular to the surface.



Equipotential surfaces.

(a)

(b)

Thus, we have the following points in case of equipotential surfaces:

- (i) The electric field along the equipotential surface is zero.
- (ii) The electric field is always normal to the surface.
- (iii) The work done in moving a charge on the equipotential surface is zero.

$$\therefore \text{Work done } W = \text{potential difference} \times \text{charge}$$

$$= Vq$$

$$= 0 \times \text{charge} = 0$$

(iv) When the charge is infinite, the equipotential surface is plane.

(v) The equipotential surfaces act as wave-fronts in optics.

Potential due to a point charge.

Consider a point charge $+q$, as shown in figure.

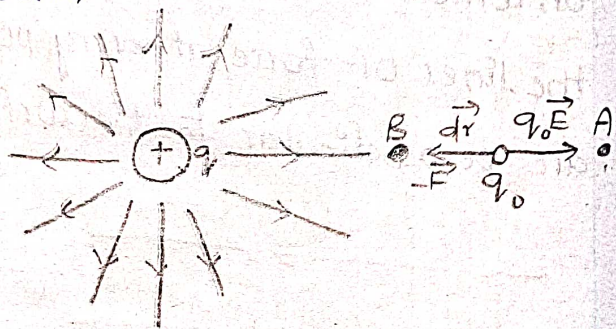
Its electric field E is outward

along radial line. The aim

of this article is to

calculate the potential at

a point B situated at a



distance r_B from the charge $+q$. For this purpose we

select two points A and B along radial line (for convenience).

Let a test charge q_0 be moved from reference point A to B.

The force exerted by the field of charge q on test charge q_0 is $q_0 E$. Now to move the test charge q_0 towards B, a force $-q_0 E$ must be applied.

The work done by external agent to move the charge q_0 through a small distance dr is given by

$$dW = |-q_0 E \cdot dr|$$

$$dW = q_0 E dr \cos 180^\circ$$

$$dW = -q_0 E dr$$

We know, $E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}$

$$\therefore dW = -q_0 \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} dr$$

$$dW = -\frac{1}{4\pi\epsilon_0} \frac{q q_0}{r^2} dr$$

Now the total work done is moving the test charge from A to B.

$$\int dW_{AB} = \int -\frac{1}{4\pi\epsilon_0} \frac{q q_0}{r^2} dr$$

$$W_{AB} = \int_{r_A}^{r_B} -\frac{1}{4\pi\epsilon_0} \frac{q q_0}{r^2} dr$$

where r_A is the distance of point A from q ,

$$W_{AB} = -\frac{1}{4\pi\epsilon_0} q q_0 \int_{r_A}^{r_B} \frac{1}{r^2} dr$$

$$W_{AB} = -\frac{q q_0}{4\pi\epsilon_0} \left[\frac{r^{-2+1}}{-2+1} \right]_{r_A}^{r_B}$$

$$[\because \int x^n dx = \frac{x^{n+1}}{n+1} + C]$$

$$= -\frac{q q_0}{4\pi\epsilon_0} \left[-\frac{1}{r} \right]_{r_A}^{r_B}$$

$$W_{AB} = \frac{q q_0}{4\pi\epsilon_0} \left[\frac{1}{r_B} - \frac{1}{r_A} \right]$$

So the potential difference between two points will be

$$V_B - V_A = \frac{W_{AB}}{q_0} = \frac{q q_0}{4\pi\epsilon_0} \left[\frac{1}{r_B} - \frac{1}{r_A} \right] \cdot \frac{1}{q_0}$$

$$V_B - V_A = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{r_B} - \frac{1}{r_A} \right]$$

$$V_B - V_A = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{r_B} - \frac{1}{r_A} \right]$$

To find the potential at point B, the reference point A is taken at infinity so that $V_A = 0$. Hence

$$V_B = \frac{1}{4\pi\epsilon_0} \frac{q}{r_B}$$

On dropping the suffix, the required expression becomes

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r}$$

This expression shows that at a distance r on all sides of charge q , the potential is the same. So, for an isolated charge, equipotential surfaces are spheres concentric with the point charge.

Potential due to a group of point charges.

The electric potential at any point due to a group of point charges may be calculated

i) We calculate the potential due to each charge as if the other charges are not present, and

ii) then add up all the potentials due to different charges.

It should be remembered that the sum is the algebraic sum.

So the potential due to N point charges is given by

$$V = \sum_{n=1}^N V_n = \frac{1}{4\pi\epsilon_0} \sum_{n=1}^N \frac{q_n}{r_n} \quad \text{where } n=1, 2, 3 \dots N$$

Potential due to a continuous charge distribution.

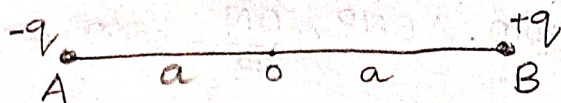
If the charge distribution is continuous, then the summation is replaced by integration. i.e.,

$$V = \frac{1}{4\pi\epsilon_0} \int \left(\frac{dq}{r} \right)$$

Electric dipole

When two charges equal in magnitude but opposite in sign are separated by a small

distance the system is called an electric dipole.

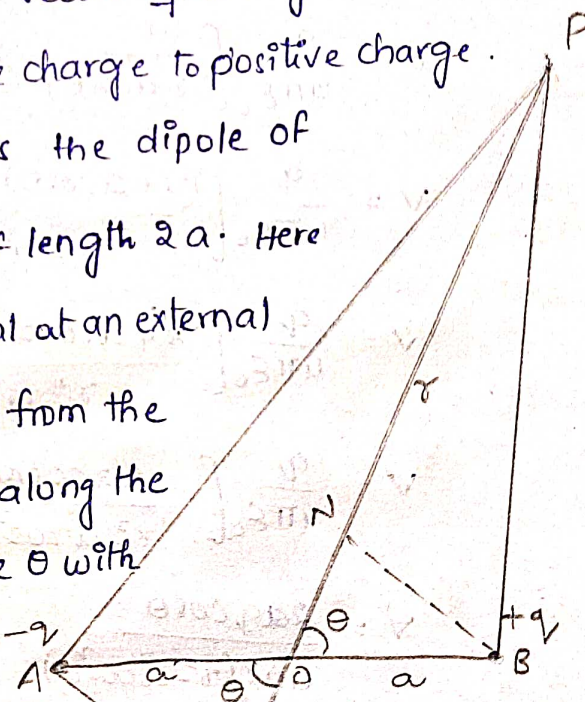


The ^{dipole} moment P of this dipole is $2aq$, where $2a$ is the distance between two charges of magnitude q . The moment P is a vector quantity pointing from negative charge to positive charge.

The S.I unit for dipole moment is coulomb-meter ($C \cdot m$)

Potential due to dipole.

The moment P is a vector quantity pointing from Negative charge to positive charge. The adjacent figure shows the dipole of charges $+q$ and $-q$, of length $2a$. Here we determine the potential at an external point P at a distance r from the centre O of the dipole along the line OP making an angle θ with AB . Let BN and AM be the perpendicular to OP . Now



potential at P due to charge $+q$ at B

$$= \frac{1}{4\pi\epsilon_0} \left[\frac{q}{BP} \right] = \frac{1}{4\pi\epsilon_0} \left[\frac{q}{NP} \right]$$

potential at P due to charge $-q$ at A

$$= \frac{1}{4\pi\epsilon_0} \left[\frac{-q}{AP} \right] = \frac{1}{4\pi\epsilon_0} \left(\frac{-q}{MP} \right)$$

∴ Resultant potential at P is given by

$$V = \frac{1}{4\pi\epsilon_0} \left[\frac{q}{NP} - \frac{q}{MP} \right] \longrightarrow (1)$$

from, $\triangle ONB$, $\frac{ON}{OB} = \cos\theta \Rightarrow ON = OB \cos\theta = a \cos\theta$.

$$\therefore NP = OP - ON = (r - a \cos\theta)$$

Similarly, from $\triangle AMO$,

$$\frac{OM}{AO} = \cos\theta \Rightarrow OM = AO \cos\theta = a \cos\theta$$

$$\therefore MP = MO + OP = (a \cos\theta + r) = (r + a \cos\theta)$$

Substituting these values in eq. (1) we get

$$V = \frac{1}{4\pi\epsilon_0} \left[\frac{q}{(r - a \cos\theta)} - \frac{q}{(r + a \cos\theta)} \right]$$

$$V = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{(r - a \cos\theta)} - \frac{1}{(r + a \cos\theta)} \right]$$

$$V = \frac{q}{4\pi\epsilon_0} \left[\frac{(r + a \cos\theta) - (r - a \cos\theta)}{r^2 - a^2 \cos^2\theta} \right]$$

$$V = \frac{q}{4\pi\epsilon_0} \left[\frac{2a \cos\theta}{r^2 - a^2 \cos^2\theta} \right]$$

$$V = \frac{q}{4\pi\epsilon_0} \left[\frac{2a \cos\theta}{r^2 - a^2 \cos^2\theta} \right]$$

$$V = \frac{2aq \cos\theta}{4\pi\epsilon_0 (r^2 - a^2 \cos^2\theta)}$$

$$V = \frac{p \cos\theta}{4\pi\epsilon_0 (r^2 - a^2 \cos^2\theta)}$$

(∵ $2aq = p = \text{dipole moment}$)

As $r \gg a$, $a^2 \cos^2\theta$ term may be neglected in comparison with r^2 .

$$\text{Hence } V = \frac{p \cos\theta}{4\pi\epsilon_0 r^2} \longrightarrow (3)$$

If $\theta = 90^\circ$ (perpendicular bisector of the line joining the charges) i.e., equatorial plane, $V=0$. Thus the potential vanishes everywhere in the equatorial plane. This means that no work done by an external agent in carrying a test charge from infinity along the perpendicular bisector of the dipole.

i) Equatorial plane is equipotential line

ii) the potential of a dipole depends upon the dipole moment p and not on q or $2a$ separately.

Electric field at any point due to a dipole (in polar coordinates)

Let (r, θ) be the polar coordinates of P.

The radial component of Electric field E_r along OP is given by

$$E_r = -\frac{\partial V}{\partial r}$$

Substituting the value of V from Eq'n (3), we get

$$E_r = -\frac{\partial}{\partial r} \left[\frac{p \cos \theta}{4\pi \epsilon_0 r^2} \right]$$

$$E_r = -\frac{p \cos \theta}{4\pi \epsilon_0} \left[\frac{\partial}{\partial r} \left[\frac{1}{r^2} \right] \right]$$

$$E_r = -\frac{p \cos \theta}{4\pi \epsilon_0} \left[-2 \frac{1}{r^3} \right]$$

$$E_r = \frac{2p \cos \theta}{4\pi \epsilon_0 r^3} \longrightarrow (4)$$

The transverse component of electric field E_θ is given by

$$E_\theta = -\frac{1}{r} \left[\frac{\partial V}{\partial \theta} \right] = -\frac{1}{r} \frac{\partial}{\partial \theta} \left[\frac{p \cos \theta}{4\pi \epsilon_0 r^2} \right]$$

$$= -\frac{1}{r} \frac{p}{4\pi \epsilon_0 r^2} \left[\frac{\partial}{\partial \theta} \cos \theta \right]$$

$$= -\frac{p}{4\pi \epsilon_0 r^3} [-\sin \theta]$$

$$= \frac{p \sin \theta}{4\pi \epsilon_0 r^3} \longrightarrow (5)$$

p. Calculate the potential due to a dipole of dipole moment $4.5 \times 10^{-10} \text{ C}\cdot\text{m}$ at a distance 1m from it.

(i) on it's axis.

ii, on it's perpendicular bisector.

Sol:- for the potential of a dipole is given by,

$$V = \frac{1}{4\pi\epsilon_0} \frac{P \cos\theta}{r^2}$$

(i)

For a point in the direction of dipole on the axis, $\theta = 0$

$$V = \frac{1}{4\pi\epsilon_0} \frac{P \cos 0}{r^2}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{P}{r^2} = \frac{9 \times 10^9 \times 4.5 \times 10^{-10}}{1} = 9 \times 4.5 \times 10^{-1}$$

$$= \frac{9 \times 4.5}{10} = \frac{40.5}{10} = 4.05 \text{ V}$$

(ii) For a point in the axis perpendicular to dipole $\theta = 90^\circ$

$$V = \frac{1}{4\pi\epsilon_0} \frac{P \cos\theta}{r^2}$$

$$V = \frac{1}{4\pi\epsilon_0} \frac{P(0)}{r^2}$$

$$V = 0$$

p. Electric potential at a distance ' r ' from a point charge ' q ' is given by $V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$. Find the electric field at that point?

Sol:- Given, $V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$

$$\text{We know, } E = -\frac{\partial V}{\partial r} = -\frac{\partial}{\partial r} \left[\frac{1}{4\pi\epsilon_0} \frac{q}{r} \right]$$

$$E = -\frac{q}{4\pi\epsilon_0} \left[\frac{\partial}{\partial r} \left(\frac{1}{r} \right) \right] = -\frac{q}{4\pi\epsilon_0} \left(-\frac{1}{r^2} \right) = \frac{q}{4\pi\epsilon_0 r^2}$$

D. A spherical drop of water carrying a ~~charge~~ $3 \times 10^{-6} \text{ C}$ has a potential of 500V at its surface, what is the radius of water drop?

Sol: Given that,
charge, $q = 3 \times 10^{-6} \text{ C}$

potential, $V = 500 \text{ V}$

Radius, $r = ?$

We know that, $V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$

$$r = \frac{1}{4\pi\epsilon_0} \frac{q}{V}$$

$$r = \frac{9 \times 10^9 \times 3 \times 10^{-6}}{500 \text{ V}}$$

$$r = \frac{27 \times 10^3}{500} = \frac{27 \times 10^2}{5} = 54 \text{ m}$$

$\therefore$ Radius, $r = 54 \text{ m}$

D. A point charge is placed at a point A. The charge is $1.5 \times 10^{-8} \text{ C}$. What is the radius of equipotential surface having a potential of 30V?

Sol: Given data,
charge, $q = 1.5 \times 10^{-8} \text{ C}$

$V = 30 \text{ V}$.

$r = ?$

We know that, $V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$

$$r = \frac{9 \times 10^9 \times 1.5 \times 10^{-8}}{30}$$

$$r = \frac{13.5 \times 10^1}{30}$$

$$r = 4.5 \text{ m}$$

P. Find the intensity of electric field at a point 0.2m on the axis of electric dipole of dipole moment 10^{-9} C.m .

Sol Given that,

$$r = 0.2 \text{ m}$$

$$p = 10^{-9} \text{ C.m}$$

$$\theta = 0 \quad [\because \text{on the axis of dipole}]$$

The relation b/w E and P is,

$$E = \frac{2p \cos \theta}{4\pi \epsilon_0 r^3}$$

$$E = \frac{2 \times 10^{-9} \times \cos 0}{4\pi \epsilon_0 \cdot (0.2)^3}$$

$$E = \frac{9 \times 10^9 \times 10^{-9} \times 2 \times 1}{(0.2)^3}$$

$$E = \frac{18}{(0.2)^3} = \frac{18}{0.008} = 2,250 \text{ N/C}$$

P. What is the electric potential at the surface of nucleus of Gold? The radius of nucleus is $6.6 \times 10^{-15} \text{ m}$. The atomic number of Gold is 79.

Sol Given, radius, $r = 6.6 \times 10^{-15} \text{ m}$

Atomic number, $Z = 79$.

We know that, $V = \frac{1}{4\pi \epsilon_0} \frac{q}{r}$

$$V = \frac{9 \times 10^9}{6.6 \times 10^{-15}} \times q$$

But, we know, $q = ne$

$$q = 79 \times 1.6 \times 10^{-19} \text{ C}$$

$$\therefore V = \frac{9 \times 10^9 \times 79 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-15}}$$

$$V = 172.363 \times 10^{-10} \times 10^{15} \text{ V}$$

$$V = \cancel{58.909} \times 10^5$$

$$V = 172.363 \times 10^5$$

$$V = 17.2 \times 10^6 \text{ V.}$$

P. The charges in a dipole are $5 \times 10^{-6} \text{ C}$ and $-5 \times 10^{-6} \text{ C}$ they are at distance of 2cm apart. calculate the electric intensity at a point which is at a distance of 10cm on its axial line?

Sol:- Given,

$$\text{charges, } q_1 = 5 \times 10^{-6} \text{ C}$$

$$q_2 = -5 \times 10^{-6} \text{ C}$$

$$\text{distance, } 2a = 2 \text{ cm}$$

$$= 2 \times 10^{-2} = 0.02 \text{ m}$$

$$\text{point distance, } r = 10 \text{ cm}$$

$$= 10 \times 10^{-2} = 0.10 \text{ m}$$

$$\theta = 0^\circ \quad [\because \text{the point is on axial line}]$$

$$\text{We know that, } E = -\frac{\partial V}{\partial r}$$

$$E = -\frac{\partial}{\partial r} \left[\frac{kP \cos \theta}{r^2 - a^2 \cos^2 \theta} \right]$$

$$E = -\frac{P \cos \theta}{4\pi\epsilon_0} \left[\frac{\partial}{\partial r} \left[\frac{1}{r^2 - a^2 \cos^2 \theta} \right] \right]$$

$$E = -\frac{P(1)}{4\pi\epsilon_0} \left[\frac{\partial}{\partial r} \left[\frac{1}{r^2 - a^2(1)} \right] \right] \quad [\because \cos 0^\circ = 1]$$

$$E = -\frac{P}{4\pi\epsilon_0} \left[\frac{\partial}{\partial r} \left[(r^2 - a^2)^{-1} \right] \right]$$

$$E = -\frac{P}{4\pi\epsilon_0} \frac{(-1)}{(r^2 - a^2)^2} \cdot \frac{\partial}{\partial r} (r^2 - a^2)$$

$$E = -\frac{P}{4\pi\epsilon_0} \left[\frac{-1}{(r^2 - a^2)^2} (2r) \right]$$

$$E = \frac{2Pr}{4\pi\epsilon_0 (r^2 - a^2)^2}$$

We have, $P = 2aq$.

$$P = (0.02)(5 \times 10^{-6})$$

$$P = (2 \times 10^{-2} \times 5 \times 10^{-6})$$

$$P = 10 \times 10^{-2} \times 10^{-6}$$

$$P = 1 \times 10^{-7} \text{ C.m}$$

$$\text{Therefore, } E = \frac{2(1 \times 10^{-7})(0.1) \times (9 \times 10^9)}{((0.1)^2 - (\frac{0.02}{2})^2)^2}$$

$$E = \frac{18 \times 0.1 \times 10^{-2}}{(0.01 - 0.0001)^2}$$

$$E = \frac{180}{(0.0099)^2}$$

$$E = \frac{180}{9.8 \times 10^{-5}}$$

$$E = 18.36 \times 10^5$$

$$E = 1.83 \times 10^6 \text{ N/C}$$

Energy density in electrostatic fields:

Def:-

Energy density is defined as the total amount of energy in a system per unit volume. It is denoted by U or W_e (or) E_d . The S.I units are Joule/metric³.

Electric and magnetic field can ^{also} store energy. In the case of Electric field (or) capacitor the energy density is given by

$$W_e = \frac{1}{2} \epsilon_0 E^2$$

Where ϵ_0 is permittivity of vacuum &

E is Electric field intensity.

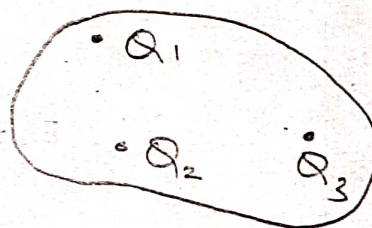
potential energy is the workdone to build the system i.e., to bring the charges together against the coulomb forces. Workdone in moving the charge Q from a to b in electric field is given by

$$W = -Q \int_a^b \mathbf{E} \cdot d\mathbf{L} = Q[V(b) - V(a)]$$

$$\begin{aligned} \because V &= \frac{W}{Q} \text{ \& } E = -\frac{\partial V}{\partial r} \\ W &= VQ \quad -E dr = dV \\ W &= -Q \int E dr \end{aligned}$$

Consider three point charges Q_1 , Q_2 & Q_3 to bring these charges into ~~small~~^{some} region, some work has to be done.

The first charge Q_1 takes no work since there is no field in the region.



$$\therefore W_1 = 0$$

Workdone in bringing Q_2 into the region is given by

$$W_2 = Q_2 V_{21}$$

Workdone in bringing Q_3 into the region is given by

$$W_3 = Q_3 (V_{32} + V_{31})$$

The total workdone to assemble the system of 3 charges is given by

$$W = W_1 + W_2 + W_3$$

$$W = 0 + Q_2 V_{21} + Q_3 (V_{32} + V_{31})$$

$$W = Q_2 V_{21} + Q_3 (V_{32} + V_{31}) \longrightarrow (1)$$

If we assemble the system of charges in reverse direction Q_3 , Q_2 and Q_1 ,

The total workdone is given by

$$W = Q_2 V_{23} + Q_1 (V_{12} + V_{13}) \longrightarrow (2)$$

Adding (1) & (2)

$$W + W = Q_2 V_{21} + Q_3 (V_{32} + V_{31}) + Q_2 V_{23} + Q_1 (V_{12} + V_{13})$$

$$2W = Q_1 (V_{12} + V_{13}) + Q_2 (V_{21} + V_{23}) + Q_3 (V_{31} + V_{32})$$

$$2W = Q_1 V_1 + Q_2 V_2 + Q_3 V_3$$

$$W = \frac{1}{2} (Q_1 V_1 + Q_2 V_2 + Q_3 V_3)$$

For a system of N charges total workdone is given by

$$W = \frac{1}{2} \sum_{n=1}^N Q_i V_i \longrightarrow (3)$$

Where V_i is the electrostatic potential at the location of Q_i due to all other point charges.

The total electrostatic energy W of a system of point charges is the sum of the energies of all possible points of charges.

If the charge is distributed in a volume

$$W = \frac{1}{2} \int \rho V dv$$

$$[\because \rho = \frac{dq}{dv} = dq = \rho dv \Rightarrow q = \int \rho dv]$$

$$W = \frac{1}{2} \int_V (\nabla \cdot \vec{D}) V dv \longrightarrow (4)$$

$$\therefore \nabla \cdot V(\vec{D}) = \nabla V \cdot \vec{D} + V(\nabla \cdot \vec{D})$$

$$V(\nabla \cdot \vec{D}) = \nabla V \cdot \vec{D} - \nabla V \cdot \vec{D}$$

from eqn (4)

$$W = \frac{1}{2} \left[\int_V \nabla \cdot V \vec{D} dv - \int_V \nabla V \cdot \vec{D} dv \right]$$

$$W = \frac{1}{2} \left[\oint_S V \vec{D} \cdot d\vec{s} - \int_V (\nabla V \cdot \vec{D}) dv \right] [\because \text{from Gauss's divergence theorem}]$$

At larger distances from the charge D goes like $\frac{1}{r^2}$ and v like $\frac{1}{r}$, while the surface area grows like r^2 .

For larger surfaces at $r = \infty$, $\oint \vec{v} \cdot \vec{D} \cdot d\vec{s}$ becomes zero.

$$W = -\frac{1}{2} \int_V (\nabla v \cdot \vec{D}) dv$$

$$W = \frac{1}{2} \int_V (-\nabla v \cdot \vec{D}) dv$$

$$W = \frac{1}{2} \int_V (\vec{E} \cdot \epsilon_0 \vec{E}) dv$$

$$W = \frac{1}{2} \epsilon_0 \int_V E^2 dv.$$

Differentiating the above equation

$$dw = \frac{1}{2} \epsilon_0 E^2 dv.$$

$$\Rightarrow \frac{dw}{dv} = \frac{1}{2} \epsilon_0 E^2$$

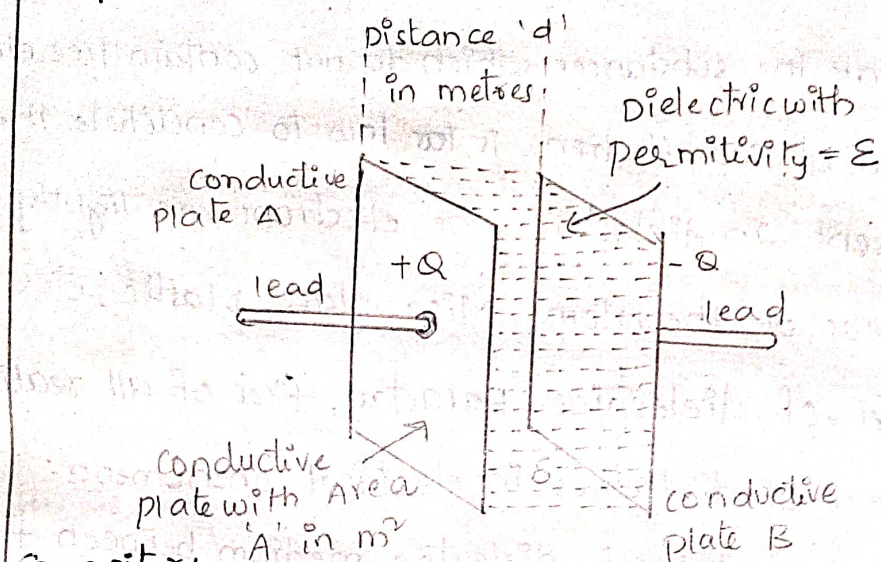
$$\Rightarrow \boxed{W_e = \frac{1}{2} \epsilon_0 E^2} \quad (\because \text{Energy density})$$

$$\Rightarrow \boxed{W_e = \frac{D^2}{2 \epsilon_0}} \quad (\because D = \epsilon_0 E)$$

$$\Rightarrow W_e = \frac{1}{2} \epsilon E^2 = \frac{1}{2} \frac{D^2}{\epsilon_0} \longrightarrow (s)$$

Energy stored per unit volume is known as Energy density.
Eq'n(s) represents the energy density in electrostatic field.

Capacitor and Capacitance:



Capacitor:-

This is an arrangement consisting of two insulating conductors near to each other and carrying equal and opposite charge.

The conductors are known as the plates of the capacitor.

A device or an arrangement which can store considerable amount of charge is called a capacitor or condenser.

This can also be defined as a device for storing energy in the electric field between the plates.

Capacitance:

The capacitance is the ratio of the magnitude of charge q , on either conductor to the potential V between the plates. It is denoted by 'C'.

$$C = q/V. \text{ SI units of Capacitance is } C/V \text{ or Farad (F)}$$

When $V=1$ then $C=q$. Hence the capacity of a conductor may be defined as the charge required to raise its potential by unity.

The farad is too large for practical purposes and hence two other units are generally used. They are micro farad, pico farad.

$$1 \text{ micro farad} = 1 \mu F = 10^{-6} \text{ farad}$$

$$1 \text{ pico farad} = 1 \text{ pF} = 10^{-12} \text{ farad}$$

Dielectrics:

Dielectrics are the substances which do not contain free electrons or the number of such electrons is too low to constitute the electric current. In dielectrics, the electrons are tightly bound to the nucleus of the atom. Mica, glass, plastic, etc. are examples of dielectrics. Faraday, first of all realised the importance of dielectrics in electrical phenomena.

He introduced a slab of dielectric medium between the plates of a parallel plate condenser and found that the charge on the capacitor with dielectric is greater than that without. Thus the capacitance of the capacitor increases because the charge is larger. The dielectric helps in the following three ways:

i) It helps in maintaining two large metal plates at very small separation.

ii) It increases the potential difference which a capacitor can withstand without breakdown.

iii) It increases the capacitance of a capacitor.

Differences between dielectrics and conductors:

Dielectrics	conductors
i) A Dielectric or an insulator is a material which doesn't contain free electrons. The electrons are tightly bound to the nuclei of atoms.	i) Conductors contain a fairly large no. of free electrons. The free electrons wander through the conducting material.

ii) The dielectric doesn't conduct electricity. On the application of an external electric field, the electrons may be able to move to and fro about their equilibrium positions but they do not leave the vicinity of their atoms.

ii) Conductors conduct electricity.

iii) The charge given to a dielectric remains localised.

iii) Conductor resides the charge on the outer surface.

iv) For a particular field strength, the dielectric loses its insulation character.

* The field strength applied to the dielectric is known as breakdown strength.

Uses of dielectrics in the field of electricity.

i) For large dielectric constant and for large dielectric strength glass and paper are used.

ii) In making high capacity condensers, paper and mica are used.

iii) Quartz, mica, glass, paraffin etc. are used for high insulation.

iv) For mechanically supporting and insulating the various electrical devices, solid dielectrics are used.

Non-polar Dielectric in an Electric field:

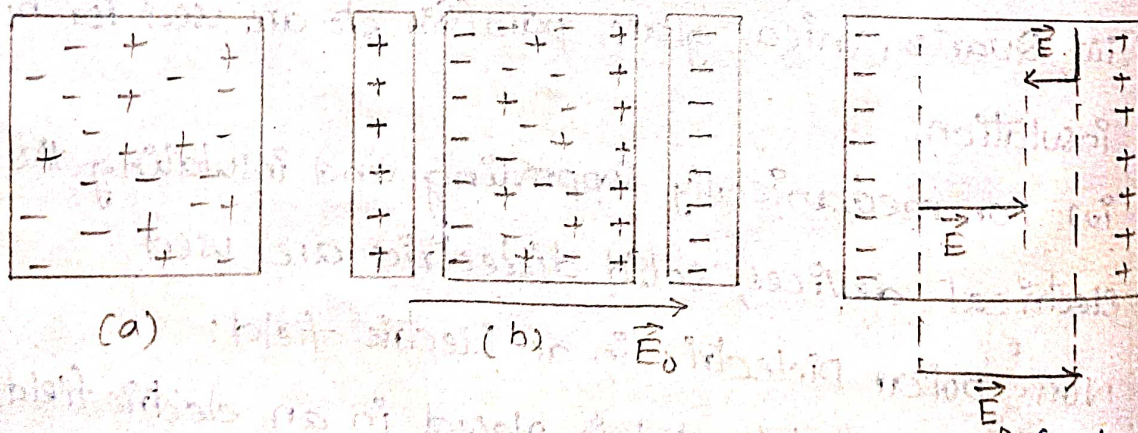
When a dielectric slab is placed in an electric field, say between the plates of a charged condenser, the positive and negative charges are reoriented i.e. the centre of gravity of positive charges is pulled towards the negative plate of the condenser and vice versa. Thus the net

effect of the applied field is to separate the positive charges from the negative charges. This effect is known as polarisation of dielectric.

"The dielectrics which are polarised only when they are placed in an electric field are called non-polar dielectric."

fig (a) shows the random distribution of plus and minus charges in a non-polar dielectric. When this dielectric is placed in an electric field E_0 (say between the plates of a parallel plate condenser), surface charges appear (positive charge on one surface and negative charge on the other surface) as shown in fig (b). As shown in fig (c) the induced surface charges appear in such a way that the electric field set up by them (E') opposes the external electric field E_0 . The resultant field E in the dielectric is the vector sum of E' and E_0 .

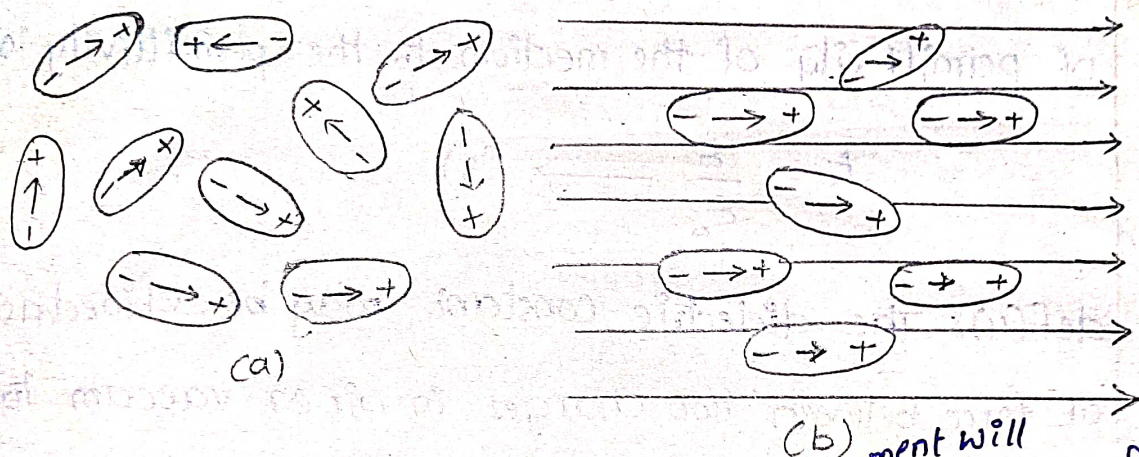
$$E = E_0 - E'$$



thus if the dielectric is placed in an electric field, induced surface charges appear which tend to weaken the original field within the dielectric.

polar dielectric in Electric field:

We know that polar dielectrics have permanent dipole moments with their random orientations as shown in below figure (a). In presence of an electric field, the partial alignment of dipoles takes place as shown in figure (b).



As the molecules are always in thermal agitation, the alignment will never be perfect. The alignment increases with the increase of electric field or with the decrease of temperature. The dipole moment of a polar molecule in an electric field will be $P_p + P_i$, where P_p is the permanent dipole moment and P_i is the induced dipole moment.

Thus the non-polar molecules in an electric field become induced dipoles while polar molecules are re-oriented with dipole moment increased.

Dielectric constant

defn: the ratio of the capacitance of a condenser with dielectric to the capacitance of the same condenser without dielectric is defined as dielectric constant.

$$k = \frac{C \text{ (Capacitance of the condenser with dielectric)}}{C_0 \text{ (Capacitance of the condenser without dielectric)}} \rightarrow (1)$$

def(2): The ratio of potential difference without dielectric to the potential difference with dielectric is defined as dielectric constant.

$$k = \frac{V_0}{V_d} \longrightarrow (2)$$

def(3): The dielectric constant may be defined as the ratio of permittivity of the medium to the permittivity of free space.

$$k = \frac{\epsilon}{\epsilon_0} \longrightarrow (3)$$

def(4): The dielectric constant may be defined as the ratio of force between two charges in air or vacuum to the force between the same charges in dielectric medium.

$$k = \frac{F_0}{F} \longrightarrow (4)$$

Note: Dielectric constant is a ratio. Its value is one for vacuum and ∞ for metals.

Dielectric strength: (E_d)

The maximum electric field that a dielectric medium can withstand without breakdown is called its dielectric strength.

$$\text{For air, } E_d = 3 \text{ kV/mm} = 3 \times 10^3 \text{ V} / 10^{-3} \text{ m}$$

$$= 3 \times 10^3 \times 10^3 \frac{\text{V}}{\text{m}}$$

$$= 3 \times 10^6 \frac{\text{V}}{\text{m}}$$

$$= 3 \text{ MV/m}$$

$$\text{For mica, } E_d = 200 \text{ MV/m}$$

Fields inside perfect conductor:

(i) $E=0$ inside a conductor why?

Because when we put a conductor into an external electric field E_0 . Initially, the field will drive any free positive charges to the right, and negative ones to the left. When they come to the edge of the material, the charges pile up; plus on the right side, minus on the left. Now, these induced charges produce a field of their own, E_i , is in the opposite direction to E_0 .

That's the crucial point, for it means that the field of the induced charges tends to cancel the original field. Charge will continue to flow until this cancellation is complete, and the resultant field inside the conductor is precisely zero.

ii) $\rho=0$ inside a conductor: This follows from Gauss's law: $\nabla \cdot E = \rho / \epsilon_0$. If E is zero, so also is ρ . There is still charge around, but exactly as much plus as minus, so the net charge density in the interior is zero.

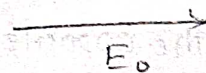
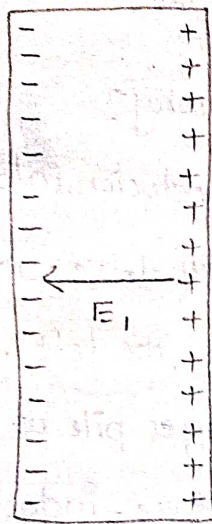
iii) Any net charge resides on the surface: That's the only place left.

iv) A conductor is an equipotential: For if a and b are any two points within (or at the surface of) a given conductor,

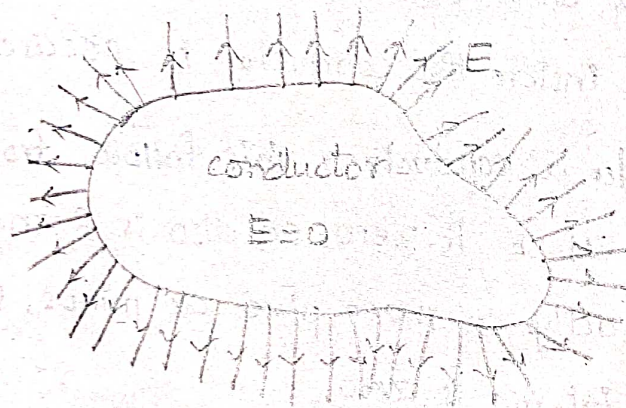
$$V(b) - V(a) = - \int_a^b E \cdot dl = 0, \text{ and hence } V(a) = V(b)$$

v) E is perpendicular to the surface, just outside a conductor.

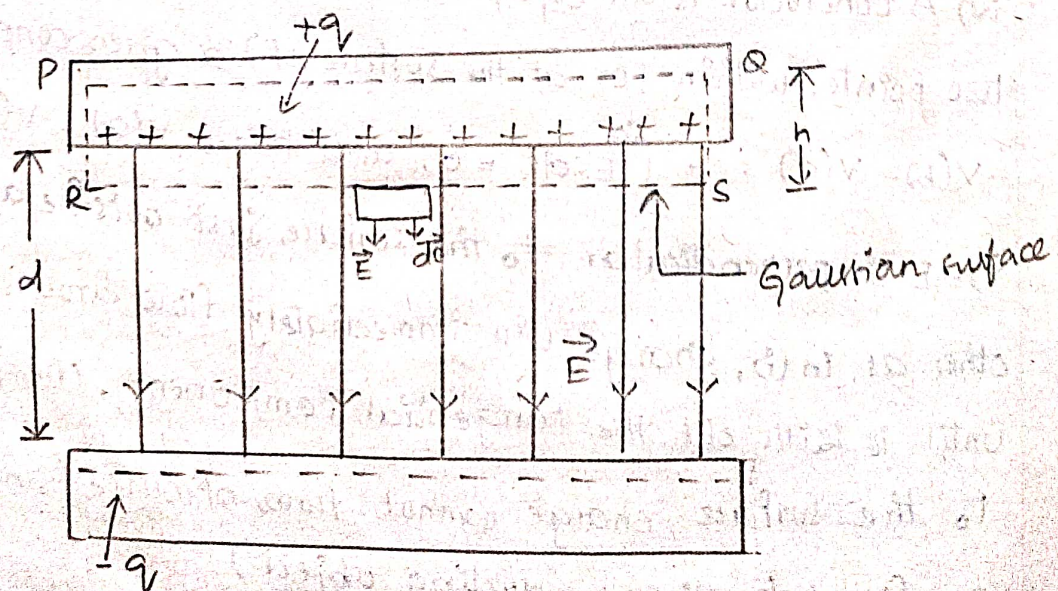
Other as in (i), charge will immediately flow around the surface until it kills off the tangential component. (i.e. perpendicular to the surface; charge cannot flow, of course, since it is confined to the conducting object).



outside the conductor the field is not zero, for here E_0 and E_1 do not tend to cancel.



Capacitance of parallel plate capacitor:



As shown in above fig. a parallel plate capacitor consists of two parallel conducting plates of area A and separated by a distance d . When these two plates are connected to the terminals of a battery, let a charge $+q$ appears on one plate which a charge $-q$ on the other plate. If d is small compared with the dimensions of the plate, then electric field strength E between the plates is uniform i.e., the lines of force are parallel and equally spaced. Here we take the medium as vacuum or air between the plates.

In order to calculate the capacitance, we first calculate the value of electric field E between the plates by using Gauss's law.

For this purpose, we consider a Gaussian surface PQRS in the form of a closed cylinder of height h and area of cross section A . ^{The flux through the ends PQRS of Gaussian surface is zero because E & ds are $\perp$ to each other.} The flux through the wall PQ of the Gaussian surface is also zero because ' E ' inside a conductor carrying a static charge is zero. The only contribution to the flux is due to the surface RS. so the flux through the entire Gaussian surface is given by

$$\Phi_E = \oint E \cdot ds = \oint E ds \cos \theta$$

$$\Phi_E = \oint E ds = E \oint ds = EA \quad [\because \cos \theta = 1]$$

Applying the Gauss's Law,

$$\Phi_E = EA = \frac{1}{\epsilon_0} q$$

$$\Rightarrow EA = \frac{q}{\epsilon_0} \Rightarrow E = \frac{q}{\epsilon_0 A} \rightarrow (1)$$

The potential difference V between the plates can be written as

$$V = Ed = \frac{q}{\epsilon_0 A} d \longrightarrow (2) \text{ from (1)}$$

The capacity of a parallel plate condenser is

$$\therefore C = \frac{q}{V} \Rightarrow C = \frac{q}{(q/\epsilon_0 A)}$$

$$C = \frac{q \cdot \epsilon_0 A}{q d}$$

$$\boxed{C = \frac{\epsilon_0 A}{d}} \longrightarrow (3)$$

Eq (3) gives the capacity of a parallel plate capacitor.

(medium is air or vacuum)

Energy stored in a capacitor:

The energy of a charged capacitor is the amount of work done in charging it. A charged capacitor stores an electric potential energy in it.

The energy is identical to the potential energy stored in a compressed spring. The energy can be recovered when the capacitor is allowed to discharge. When the charging of the capacitor is done by a battery, then the electrical energy stored in it is at the expense of chemical energy of the battery.

Consider a capacitor of capacitance C and carrying a charge q , at any instant. Let the potential difference between the plates is V . Then

$$V = q/C \longrightarrow (1)$$

If an additional charge dq is to be given to this capacitor then some work must be done against the potential difference. So work done in increasing charge by dq is given by

$$dW = V dq = (q/c) dq \longrightarrow (2) \text{ from (1)}$$

$\therefore$ Total work to charge a capacitor to a charge q_0

$$W = \int dW = \int_0^{q_0} \left(\frac{q}{c}\right) dq = \frac{1}{c} \int_0^{q_0} q dq$$

$$W = \frac{1}{c} \left[\frac{q^2}{2} \right]_0^{q_0} = \frac{1}{c} \left[\frac{q_0^2}{2} - 0 \right]$$

$$\boxed{W = \frac{q_0^2}{2c}} \longrightarrow (3)$$

Now the energy stored by a charged capacitor

$$U = W = \frac{1}{2} \frac{q^2}{c} = \frac{1}{2} \left(\frac{q}{c} \right) q = \frac{1}{2} V q$$

$$U = \frac{1}{2} CV^2 \quad (\because q = CV) \longrightarrow (4)$$

For a parallel plate capacitor of area A and plate separation d , then the capacitance C is given by

$$C = \frac{\epsilon_0 A}{d} \quad \text{and} \quad V = Ed.$$

$$\begin{aligned} \therefore \text{Energy stored } U &= \frac{1}{2} \times \frac{\epsilon_0 A}{d} \times (Ed)^2 \\ &= \frac{1}{2} \times \epsilon_0 E^2 A d \text{ Joules.} \longrightarrow (5) \end{aligned}$$

$$\text{Energy stored per unit volume} = \frac{1}{2} \epsilon_0 E^2 \text{ Joules/m}^3.$$

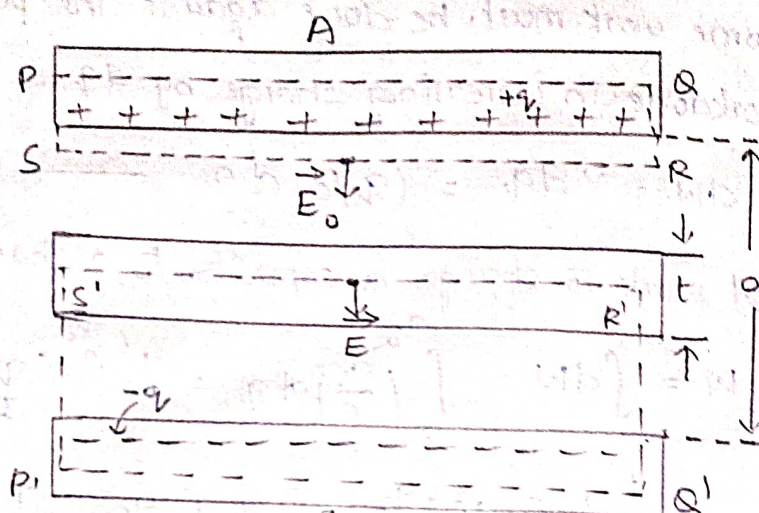
If the capacitor is placed in a medium of dielectric constant k , then the energy stored per unit volume

$$\text{is given by } U = \frac{1}{2} k \epsilon_0 E^2 = \frac{1}{2} \epsilon E^2$$

$$\therefore \boxed{U = \frac{\epsilon E^2}{2}}$$

Where ' ϵ ' is the permittivity of the medium.

Parallel plate capacitor with Dielectric slab:



The above figure shows a capacitor partially filled with a dielectric material.

Let A and B be the plates of the capacitor with area A and separation d. The upper plate A is given a charge +q, while the lower plate B is given a charge -q. Now a dielectric slab with thickness t and dielectric constant k is introduced between the plates. In order to consider the electric field E_0 and E in air and dielectric respectively, we consider the Gaussian surfaces PQRS and P'Q'R'S' as shown in figure.

$$\int E_0 \cdot ds = \frac{q}{\epsilon_0} \text{ or } E_0 A = \frac{q}{\epsilon_0}$$

$$\Rightarrow E_0 = \frac{q}{\epsilon_0 A} \longrightarrow (1)$$

Similarly for the Gaussian surface P'Q'R'S', we have

$$\int k E \cdot ds = \frac{q}{\epsilon_0} \text{ or } k E A = \frac{q}{\epsilon_0}$$

$$E = \frac{q}{k \epsilon_0 A} \longrightarrow (2)$$

The potential difference V between the two plates is the workdone in carrying a unit charge from one plate to another. Thus

$$V = E_0(d-t) + Et \longrightarrow (3)$$

(Here the electric field E_0 is over a distance $(d-t)$ while the field E is over a distance t)

Substituting the value of E_0 and E from eqⁿ(1) & (2) in eqⁿ(3), we have

$$\begin{aligned} V &= \frac{q}{\epsilon_0 A} (d-t) + \frac{q}{K \epsilon_0 A} \cdot t \\ &= \frac{q}{\epsilon_0 A} \left[(d-t) + \frac{t}{K} \right] \\ &= \frac{q}{\epsilon_0 A} \left[d - t \left(1 - \frac{1}{K} \right) \right] \longrightarrow (4) \end{aligned}$$

The capacitance of the capacitor is given by

$$C = \frac{q}{V} = \frac{q}{\frac{q}{\epsilon_0 A} \left[d - t \left(1 - \frac{1}{K} \right) \right]} = \frac{\epsilon_0 A}{\left[d - t \left(1 - \frac{1}{K} \right) \right]}$$

$$C = \frac{\epsilon_0 A}{d - t \left(\frac{K-1}{K} \right)} = \frac{\epsilon_0 A}{\frac{Kd - t(K-1)}{K}} = \frac{\epsilon_0 A}{Kd - t(K-1)} \cdot K$$

$$\therefore \boxed{C = \frac{\epsilon_0 A K}{Kd - t(K-1)}} \longrightarrow (5)$$

When the dielectric is completely filled, then $t=d$. In this case, the capacitance is given by

$$C = \frac{\epsilon_0 A K}{[Kd - t(K-1)]} = \frac{\epsilon_0 A K}{Kd - Kd + d} = \frac{\epsilon_0 A K}{d}$$

$$\boxed{C = \frac{\epsilon_0 A K}{d}} \longrightarrow (6)$$

Problems :-

- p. The capacitance of a parallel plate capacitor is $0.2 \mu\text{F}$ and potential difference between the plates is 2V . Calculate the energy stored by the charged capacitor?

Sol:- Given, capacitance, $C = 0.2 \mu\text{F}$

potential difference, $V = 2\text{V}$

$$\text{Energy, } U = \frac{1}{2} CV^2 = ?$$

$$\therefore U = \frac{1}{2} \times 0.2 \times 10^{-6} \times (2)^2$$

$$= 0.4 \times 10^{-6}$$

$$\therefore U = 4 \times 10^{-7} \text{ J}$$

- p. Calculate the capacitance and energy stored in a parallel plate capacitor which consists of two metal plates of area 60cm^2 separated by a dielectric 1.5mm thickness and $\epsilon_r = 3.5$. If a voltage of 1000V is applied across each plate.

Sol:- Given data,

$$\text{Area, } A = 60\text{cm}^2 = 60 \times 10^{-4} \text{ m}^2$$

$$\text{potential } V = 1000\text{V}$$

$$\text{thickness of dielectric, } t = 1.5\text{mm} = 1.5 \times 10^{-3} \text{ m}$$

$$\epsilon_r = k = 3.5$$

a) capacitance,

$$C = \frac{\epsilon_0 A k}{t} = \frac{8.85 \times 10^{-12} \times 60 \times 10^{-4} \times 3.5}{1.5 \times 10^{-3}}$$

$$C = \frac{8.85 \times 60 \times 3.5 \times 10^{-13}}{1.5} = \frac{1858.5 \times 10^{-13}}{1.5}$$

$$C = 1239 \times 10^{-13}$$

$$C = 1.23 \times 10^{-10} \text{ F}$$

b) Energy, $U = \frac{1}{2} CV^2$

$$= \frac{1}{2} \times 1.23 \times 10^{-10} \times (1000)^2$$

$$= \frac{1}{2} \times 1.23 \times 10^{-10} \times 10^6$$

$$= 0.5 \times 1.23 \times 10^{-4}$$

$$= 0.615 \times 10^{-4}$$

$$U = 6.15 \times 10^{-5} \text{ J}$$

$$\therefore \text{Energy} = 6.15 \times 10^{-5} \text{ J}$$

$$\begin{array}{r} 123 \times 5 \\ \hline 0615 \end{array}$$