

UNIT - 4: Magnetostatics.

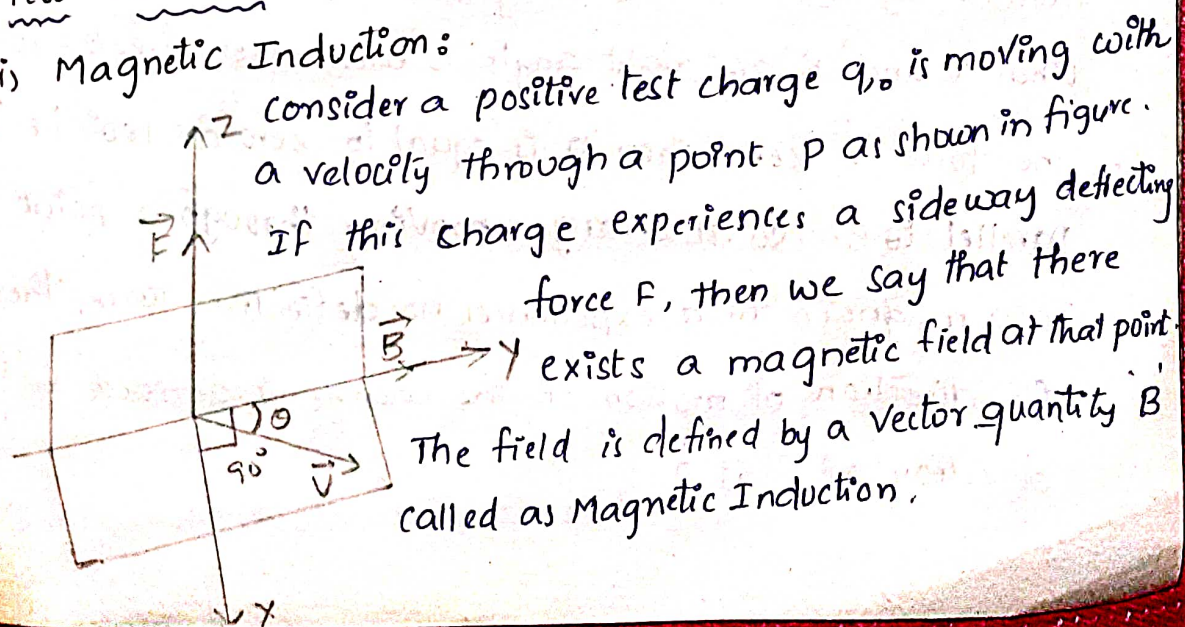
- Magnetic forces
- Biot-Savart's law
- Steady currents
- Ampere's Law
- Magnetic vector potentials
- Magnetization
- permeability
- para, dia, Ferro-Magnetic material properties
- Magnetic Energy
- Boundary conditions
- Scalar & vector fields.

Introduction:

We are familiar with the properties of a magnet. e.g: the existence of poles, opposite poles attract each other while similar poles repel each other, a freely suspended magnet points in north-south direction, etc. There was a time when magnetism was regarded as a separate branch of science having no connection with electricity. In 1820, Oersted discovered that a compass needle suffers a deflection when brought near a current carrying wire. This shows that an electric charge in motion produces a magnetic field in the space around it. It should be remembered that a charge (whether it is stationary or in motion) produces an electric field around it but when it is in motion, then in addition to electric field it also produces a magnetic field. So Magnetism and electricity are two aspects of a single phenomenon. Electricity and Magnetism are related by electro-magnetism.

Few definitions:-

i) Magnetic Induction:



The magnetic induction can be defined in terms of $q_0 v$ and sideway deflecting force F .

Def: If a positive test charge q_0 moving with velocity v through a point 'P' in a magnetic field experiences a deflecting force F , then the magnetic induction B at 'P' is defined by the relation

$$F = q_0 v \times B \longrightarrow (1)$$

$$F = q_0 v B \sin \theta$$

$$\boxed{B = \frac{F}{q_0 v}} \longrightarrow (2)$$

$$B = \frac{F}{q_0 v \sin \theta}$$

If θ is max. q_0

The magnitude of B can be obtained as follows: $B = \frac{F}{q_0 v}$

$$F = q_0 v B \sin \theta$$

$$\boxed{B = \frac{F}{q_0 v \sin \theta}} \longrightarrow (3)$$

Where θ is the angle between v and B .

So, the magnitude of the magnetic induction at a point is the ratio of the force on a moving charge at that point and the product of the charge and component of its velocity along a direction normal to the induction.

The direction of the force is perpendicular to the plane formed by v and B . The magnitude of the force is maximum when v and B are right angle to each other. i.e. $\theta = 90^\circ$.

The force is zero when θ is equal to zero or 180° i.e. is parallel to B . So if charge moving through a point 'P' in a magnetic field experiences no deflecting force, then the direction of motion of the charge is defined as the direction of B at 'P'.

As $B = (F/q_v \sin \theta)$ and hence the unit of B is given by

$$\frac{\text{newton}}{\text{coulomb} \times (\text{metre}/\text{sec})} = \frac{\text{newton}}{(\text{coulomb}/\text{sec}) \times \text{metre}} = \frac{\text{newton}}{\text{ampere} \times \text{metre}}$$

In M.K.S system, the unit of B is given by

$$\text{weber}/\text{metre}^2 \text{ (or) tesla}$$

$$1 \text{ weber}/\text{metre}^2 = 1 \text{ newton}/\text{amp.metre}$$

In c.G.s system, the unit of B is gauss.

$$1 \text{ weber}/\text{metre}^2 \text{ (or) tesla} = 10^4 \text{ gauss.}$$

2. Lorentz force:

The force on a charged particle moving in electromagnetic field (i.e., electric field and magnetic field both) is known as Lorentz force. The force in electric field is $q_0 E$ and in magnetic field is $(q_0 \mathbf{v} \times \mathbf{B})$.

Hence Lorentz force,

$$\mathbf{F} = q_0 \mathbf{E} + q_0 \mathbf{v} \times \mathbf{B} = q_0 [\mathbf{E} + \mathbf{v} \times \mathbf{B}] \longrightarrow (2)$$

Eq'n (1) & (2) are the Magnetic forces.

3. Lines of Magnetic Induction:

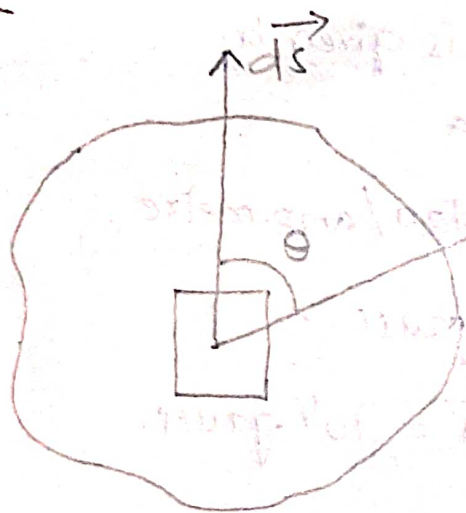
A magnetic field, like an electric field can be represented by lines. The lines are known as 'lines of induction'. There are two important points regarding lines of induction:

i. The tangent to the line of induction gives the direction of B at the point.

(ii) The number of lines per unit area normal to the direction is proportional to the magnitude of B .

Magnetic flux: (Φ_B)

Def: The total number of lines of induction cutting through a surface is called magnetic flux through that surface.



consider an element of surface area represented by vector area ds as shown in figure. Suppose it makes an angle θ with magnetic induction ' B ' on that area. Then the scalar product $B \cdot ds$ will represent the number of lines of induction over that area.

i.e., magnetic flux $d\Phi_B$ over ds . Thus

$$d\Phi_B = B \cdot ds \longrightarrow (1)$$

Now the magnetic flux over the entire surface will be

$$\int d\Phi_B = \int B \cdot ds.$$

$$\Phi_B = \int B \cdot ds \longrightarrow (2)$$

If B is uniform over the entire area, then

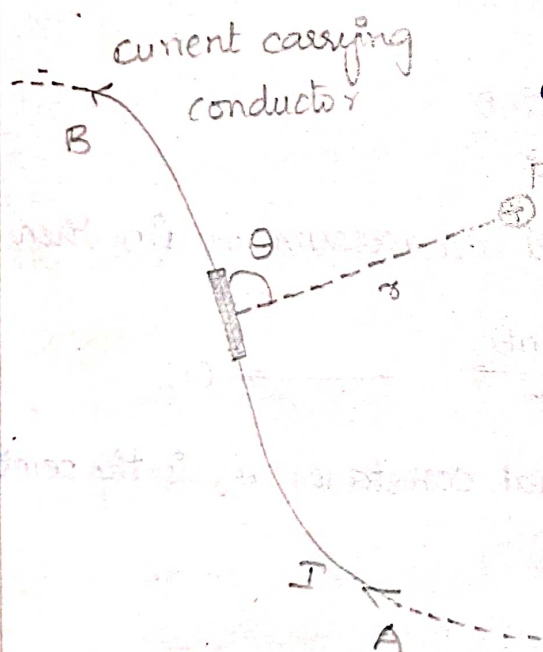
$$\Phi_B = B \oint ds = B \cdot S \longrightarrow (3)$$

When B is parallel to S then $\Phi_B = BS \cos \theta = BS \longrightarrow (4)$

Units for magnetic flux - $(\text{weber/metre}^2) \times \text{metre}^2 = \text{weber}$.

BIOT-SAVART'S LAW :

According to Oersted's experiment, a current carrying conductor produces a magnetic field around it. In 1820, Biot & Savart performed a series of experiments to study the magnetic field produced by various current carrying conductors. They obtained a relation by means of which B can be calculated at any point of space around a conductor in which a current i is passing. The relation is called as Biot and Savart law.



Let AB be a conductor of any arbitrary shape in which a current i is flowing. Let P be a point at which field is to be determined. According to Biot and Savart, the field B at any point can be computed by dividing the conductor into short current elements. The field distribution dB at that point

~~can be~~ due to each element is calculated the total field B is then resultant of the infinitesimal field of all the elements of the conductor. Let dl be the length of one such element and r be the displacement vector of point P from this element. Biot and Savart observed that dB at that point P due to small element dl of a current carrying conductor depends upon the following factors :

i) It is directly proportional to the current i flowing through the conductor $dB \propto i$

ii) It is directly proportional to the length dl of the element considered $dB \propto dl$.

iii) It is directly proportional to the sine of the angle θ between length of element and the line joining the element to the point P $dB \propto \sin \theta$.

(iv) It is inversely proportional to the square of the distance of the point P from the element dl

$$dB \propto \frac{1}{r^2}$$

combining all these factors

$$dB \propto \frac{idl \sin \theta}{r^2}$$

When the conductor is placed in vacuum or air, then

$$dB = \frac{\mu_0}{4\pi} \times \frac{idl \sin \theta}{r^2} \longrightarrow (1).$$

Where $(\mu_0/4\pi)$ is proportional constant, μ_0 is the permeability of free space.

In vector form,

$$dB = \frac{\mu_0}{4\pi} \times \frac{idl \sin \theta}{r^2} \cdot \frac{\vec{r}}{r}$$

$$dB = \frac{\mu_0}{4\pi} \times \frac{idl \times \vec{r}}{r^3} \longrightarrow (2)$$

$$\cancel{dB} = \cancel{\frac{\mu_0}{4\pi}} \times \cancel{idl \times \vec{r}} \quad dB = \frac{\mu_0}{4\pi} \times \frac{idl \times \vec{r}}{r^3}$$

The resultant field at point P can be obtained by integrating eq'n (2)

$$B = \int dB.$$

From eq'n $\frac{\mu_0}{4\pi} = 10^{-7}$ weber / (ampere-meter)

Find the magnetic field $\vec{B}$ due to long straight conductor $\frac{\mu_0 I}{4\pi R}$ at a distance 'R' carrying steady current, I .

(or)
Magnetic field due to long straight conductor carrying steady current

~~Find the magnetic field~~

Consider an infinitely long conductor placed in vacuum and carrying a current i ampere as shown in below figure. The aim is to derive an expression for magnetic induction B due to the conductor at a point P on x -axis using Biot & Savart law.

Let the distance of P from the conductor be R . O is the foot of the perpendicular from P to the conductor.

Consider a small element AB of length dl of conductor at a distance ' l ' from O . Let r be the distance of the element from the point P . Suppose θ be

the angle in clockwise direction which the direction of current makes with the line joining the element to point P . The

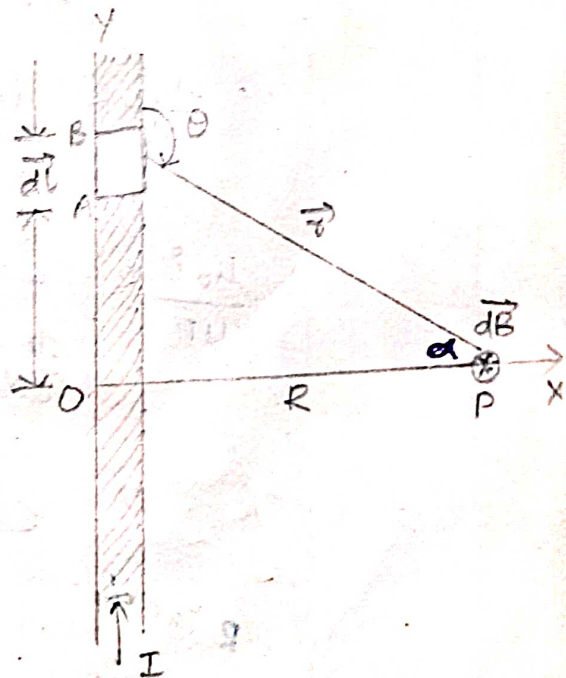
magnitude of the field dB due to element AB at point P

is given by

$$dB = \frac{\mu_0}{4\pi} \times \frac{idl \sin\theta}{r^2} \quad \longrightarrow (1)$$

The field due to the whole conductor is given by

$$B = \int dB = \frac{\mu_0 I}{4\pi} \int_{-\infty}^{+\infty} \frac{\sin\theta dl}{r^2} \quad \longrightarrow (2)$$



From figure,

$$r = \sqrt{L^2 + R^2} \quad \& \quad \sin \theta = \sin(\pi - \theta) = \left(\frac{R}{r}\right) \cdot (\because \frac{\text{opp}}{\text{hyp}} = \sin \theta)$$

$$\therefore \sin \theta = \frac{R}{(L^2 + R^2)^{1/2}}$$

substituting the values of r and $\sin \theta$ in eq (2) we get

$$B = \int dB = \frac{\mu_0 I}{4\pi} \int_{-\infty}^{+\infty} \frac{\frac{R}{(L^2 + R^2)^{1/2}}}{\left(\frac{(L^2 + R^2)^{1/2}}{r}\right)^2} dL$$

$$B = \frac{\mu_0 I}{4\pi} \int_{-\infty}^{+\infty} \frac{R}{\sqrt{L^2 + R^2}} \cdot \frac{1}{(L^2 + R^2)} dL$$

$$B = \frac{\mu_0 I}{4\pi} \int_{-\infty}^{+\infty} \frac{R}{(L^2 + R^2)^{3/2}} dL \rightarrow (3)$$

from figure, $\tan(\alpha) = \frac{L}{R}$

$$\Rightarrow L = R \tan \alpha$$

$$\frac{dL}{d\alpha} = R \sec^2 \alpha \Rightarrow dL = R \sec^2 \alpha d\alpha$$

from Eq'n (3) we get

$$B = \frac{\mu_0 I}{4\pi} \int_{-\infty}^{\infty} \frac{R}{(L^2 + R^2)^{3/2}} dL$$

limits If $L = -\infty$, $-\infty = R \tan \alpha \Rightarrow -\infty = \tan \alpha$
 $\Rightarrow \tan^{-1}(-\infty) = -\pi/2$

If $L = \infty$, $\infty = R \tan \alpha \Rightarrow \infty = \tan \alpha$
 $\Rightarrow \tan^{-1} \infty = \pi/2$

$$B = \frac{\mu_0 I}{4\pi} \int_{-\pi/2}^{\pi/2} \frac{R \cdot (R^2 \sec^2 \alpha) d\alpha}{(R^2 + R^2 \tan^2 \alpha)^{3/2}} = \frac{\mu_0 I}{4\pi} \int_{-\pi/2}^{\pi/2} \frac{R^2 \sec^2 \alpha d\alpha}{(R^2 (1 + \tan^2 \alpha))^{3/2}}$$

$$B = \frac{\mu_0 I}{4\pi} \int_{-\pi/2}^{\pi/2} \frac{R^2 \sec^2 \alpha d\alpha}{R^3 (1 + \tan^2 \alpha)^{3/2}} = \frac{\mu_0 I}{4\pi} \int_{-\pi/2}^{\pi/2} \frac{R^2 \sec^2 \alpha d\alpha}{R^3 \sec^3 \alpha} \quad (\because \sec^2 \alpha - \tan^2 \alpha = 1)$$

$$B = \frac{\mu_0 I}{4\pi} \int_{-\pi/2}^{\pi/2} \frac{1}{R \sec \alpha} d\alpha$$

$$B = \frac{\mu_0 I}{4\pi R} \int_{-\pi/2}^{\pi/2} \cos \alpha d\alpha \quad (\because \frac{1}{\sec \theta} = \cos \theta)$$

$$B = \frac{\mu_0 I}{4\pi R} \left[\sin \alpha \right]_{-\pi/2}^{\pi/2}$$

$$B = \frac{\mu_0 I}{4\pi R} \left[\sin \left[\frac{\pi}{2} \right] - \left[\sin \left[-\frac{\pi}{2} \right] \right] \right]$$

$$B = \frac{\mu_0 I}{4\pi R} \left[\sin \frac{\pi}{2} - (-\sin \frac{\pi}{2}) \right]$$

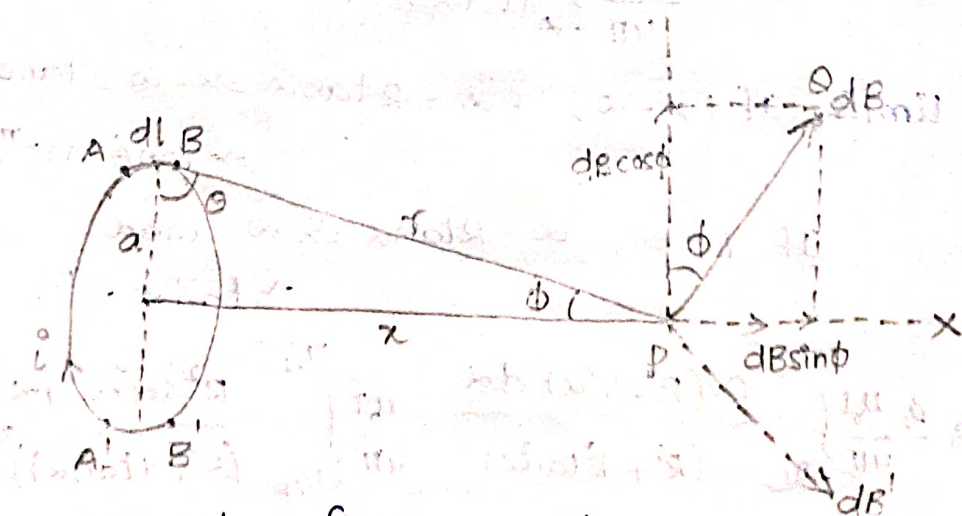
$$B = \frac{\mu_0 I}{4\pi R} [1 - (-1)] = \frac{\mu_0 I}{2\pi R} [2]$$

$$\therefore \boxed{B = \frac{\mu_0 I}{2\pi R}}$$

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Magnetic field on the Axis of A current Loop:

Find the magnetic field at a distance (x) above the centre of a circular loop of radius a , which carries a steady current i .



As shown in above figure, consider a circular coil of radius a and carrying a current i . P is a point on the axis of the coil distant x from the centre.

Our aim is to calculate the field at point P. Consider a small element AB of length dl .

Let r be the distance of the element from the point P and θ be the angle which the direction of current makes with the line joining the element to the point O. The magnetic field dB at point 'P' due to current element AB of length dl is given by

$$dB = \frac{\mu_0}{4\pi} \times \frac{i dl \times \vec{r}}{r^3} = \frac{\mu_0}{4\pi} \cdot \frac{i dl \sin 90^\circ}{r^2} = \frac{\mu_0}{4\pi} \cdot \frac{i dl}{r^2}.$$

($\because$ angle b/w dl & $r = 90^\circ$)

The vector dB at the point P, due to element dl would be perpendicular to r . This can be resolved into two components:

One of magnitude $dB \cos \phi$ at right angles to axis and other of magnitude $dB \sin \phi$ along the axis of the coil. If we take another element $A'B'$ diametrically opposite to AB of the same length, it will also produce electric field dB at P. The

direction of dB now will be opposite to the previous one. This can also be resolved into components $dB \cos \phi$ and $dB \sin \phi$. The components along the axis will add up while the components perpendicular to the axis will cancel. Similarly, if we divide the whole circular coil into a number of elements, the vertical components will cancel while the components along the axis will add up.

$$\therefore \text{Magnetic field along the axis} = B = \int dB \sin \phi$$

$$\therefore B = \frac{\mu_0 i}{4\pi r^2} \int dl \sin \phi$$

$$B = \frac{\mu_0 i}{4\pi r^2} \int dl \left(\frac{a}{r} \right)$$

$$\because \left(\sin \phi = \frac{a}{r} \right)$$

from figure

$$B = \frac{\mu_0 i}{4\pi r^2} \left(\frac{a}{r} \right) \int dl$$

$$B = \frac{\mu_0 i}{4\pi r^3} a [l]$$

$$B = \frac{\mu_0 i \cdot a \cdot 2\pi a}{4\pi r^3} \quad \left[\because \text{circumference of a circle} \right. \\ \left. l = 2\pi a \right]$$

from figure

$$B = \frac{\mu_0 i a^2}{2 r^3}$$

$$B = \frac{\mu_0 i a^2}{2 (\sqrt{a^2 + x^2})^3} \quad \text{from figure}$$

$$r = \sqrt{a^2 + x^2}$$

$$B = \frac{\mu_0 i a^2}{2 (a^2 + x^2)^{3/2}}$$

If there are N turns in the coil, $B = \frac{\mu_0 N i a^2}{2 (a^2 + x^2)^{3/2}}$ Weber/metre²
(or) Tesla.

Ampere's law :

A magnetic field is one of the forms of an electromagnetic field. Its distinguishing feature is that it acts only on moving particles and bodies having an electric charge, as well as on magnetized bodies regardless of their state of motion. A magnetic field is produced by current carrying conductors, by moving electrically charged particles and bodies, magnetized bodies, or a variable electric field (by displacement current).

The force exerted by the magnetic field on the current carrying conductor is called Ampere's force.

statement :- (Ampere's law)

Oersted, in 1820 discovered the existence of the magnetic field due to the current flowing in a conductor. Ampere derived a relationship between current i and magnetic field. The relationship is known as Ampere's law.

According to Ampere's law, the line integral $\oint B \cdot dl$ for a closed curve is equal to μ_0 times the net current i through the area bounded by the curve. Thus

$$\oint B \cdot dl = \mu_0 i$$

where μ_0 is the permeability constant.

Proof:

Consider a long straight conductor carrying a current i as shown in figure. Let the conductor be perpendicular to the page directed outward. The magnitude of the magnetic induction at a distance R from the conductor is given by

$$B = \frac{\mu_0 i}{2\pi R} \quad (\text{Biot Savart law}) \quad \longrightarrow (1)$$

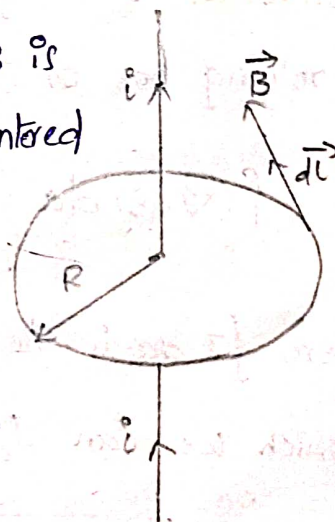
The direction of magnetic induction B is tangent to the circle of radius R centered on the conductor. Regarding B , two points should be remembered:

i) The magnitude of B is constant, at all points on the circle.

(ii) This is parallel to the circuit element dl .

Now we shall calculate the line integral of B and dl , which is given by

$$\begin{aligned} \oint B \cdot dl &= \oint B \, dl \cos 0^\circ = B \oint dl \\ &= B(2\pi R) \quad \longrightarrow (2) \end{aligned}$$



Where $\oint dl = 2\pi R = \text{circumference of the circle}$.

Substituting the values of B from eq'n (1) in eq'n (2) we get

$$\oint B \cdot dl = \frac{\mu_0 i}{2\pi R} \times 2\pi R = \mu_0 i \longrightarrow (3)$$

Thus the line integral $\oint B \cdot dl$ is μ_0 times the current through the area bounded by the circle. This is Ampere's law. This law is true for any assembly of currents and for any closed surface.

The equation for the curl of B ,

$$\boxed{\nabla \times B = \mu_0 J} \longrightarrow (4)$$

is called Ampere's law (in differential form). It can be converted to integral form by the usual device of applying one of the fundamental theorems - i.e., Stokes's theorem:

$$\int (\nabla \times B) \cdot da = \oint B \cdot dl = \mu_0 \int J \cdot da \longrightarrow (5)$$

Here, $\int J \cdot da$ is the total current passing through the surface which we can I_{enclosed} . (the current enclosed by the Amperian loop).

Thus, $\boxed{\oint B \cdot dl = \mu_0 I_{\text{encl.}}} \longrightarrow (6)$

In terms of magnetic field intensity, H :

We know, $B = \mu_0 H$

$$H = \frac{B}{\mu_0}$$

From eq'n (4) $\nabla \times B = \mu_0 J$

$$\nabla \times \frac{B}{\mu_0} = J$$

$$\Rightarrow \boxed{\nabla \times H = J}$$

$$(\because H = \frac{B}{\mu_0})$$

from eq'n (5)

$$\oint B \cdot dl = \mu_0 \oint H \cdot dl = \mu_0 \int J \cdot da = \oint H \cdot dl = \int J \cdot da$$

from eq'n (6)

$$\oint \mu_0 H \cdot dl = \mu_0 I_{enc}$$

$$\mu_0 \oint H \cdot dl = \mu_0 I_{enc} \Rightarrow \oint H \cdot dl = I_{enc}$$

Steady currents :-

Stationary charges produces electric fields that are constant in time hence the term electrostatics.

Steady currents produces magnetic fields that are constant in time, the theory of steady currents is called Magnetostatics.

By steady current I mean a continuous flow that has been going on forever, without change and without charge piling up any where. Formally electrostatics / magnetostatics is

the regime $\frac{\delta \rho}{\delta t} = 0$; $\frac{\delta j}{\delta t} = 0$ at all places and all time.

Of course When a steady current flows in a wire, its magnitude I , must be the same all along the line, otherwise charge would be piling up somewhere and it would not be a steady current.

where & more generally, since $\frac{\delta \rho}{\delta t} = 0$ in magnetostatics, the continuity eq'n is

$$\nabla \cdot \mathbf{J} + \frac{\delta \rho}{\delta t} = 0$$

$$\nabla \cdot \mathbf{J} + 0 = 0$$

$$\boxed{\nabla \cdot \mathbf{J} = 0}$$

Note:-

the Biot-Savart's law in terms of magnetic field intensity (H):

$$\boxed{\mathbf{B} = \mu_0 \mathbf{H}}$$

$$\text{i.e., } d\mathbf{H} = \frac{d\mathbf{B}}{\mu_0} = \frac{\cancel{\mu_0}}{4\pi} \times \frac{idl \sin \theta}{r^2}$$

$$\therefore d\mathbf{H} = \frac{idl \sin \theta}{4\pi r^2}$$

$$\Rightarrow \mathbf{H} = \frac{i}{4\pi} \int \frac{\sin \theta}{r^2} dl$$

$$\mathbf{B} = \frac{\mu_0 I}{2\pi R}$$

$$\mathbf{H} = \frac{I}{2\pi R}$$

$$\mathbf{B} = \frac{\mu_0 I a^2}{2(a^2 + r^2)^{3/2}}$$

In vector form,

$$d\mathbf{B} = \frac{\mu_0}{4\pi} \times \frac{id\mathbf{l} \times \vec{r}}{r^3}$$

$$\mathbf{B} = \frac{ia^2}{2(a^2 + r^2)^{3/2}}$$

$$d\mathbf{H} = \frac{\cancel{\mu_0}}{4\pi} \times \frac{id\mathbf{l} \times \vec{r}}{r^3}$$

$$= \frac{id\mathbf{l} \times \vec{r}}{4\pi r^3}$$

Magnetic field on the Axis of a current loop: (continuation)
the direction of B is along the axis of the coil.

Case i)

At the centre of the coil $x=0$. Thus at the centre of the

coil,
$$B = \frac{\mu_0 N i a^2}{2a^3} = \frac{\mu_0 N i}{2a} \quad (\because B = \frac{\mu_0 N i a^2}{2(a^2+0)^{3/2}})$$

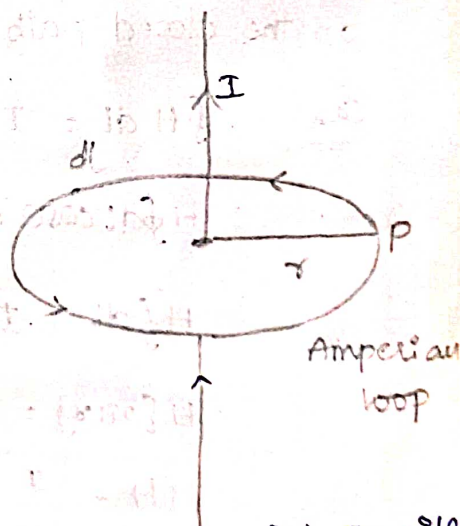
Case ii) At very far off from the loop $x \gg a$ & $(a^2+x^2)^{3/2} \approx x^3$
when x is very much greater than a

$$B = \frac{\mu_0 N i a^2}{2(x^2)^{3/2}} = \frac{\mu_0 N i a^2}{2x^3} \quad (\because a \ll x)$$

$$B = \frac{\mu_0 N i a^2}{2x^3}$$

Find the magnetic field intensity due to long straight conductor by using Ampere's law.

Sol: We know that, magnetic
Consider an infinite conductor carrying current I as shown in figure. Let us consider a point 'P' which is at a distance r from this conductor where we have to find the magnetic field



intensity. Consider a closed circular loop at point P with radius r , this path we call it as amperian path. Consider a small element dl on this closed path.

Apply ampere's law to this loop.

$$\oint H \cdot dl = I_{enc}$$

$$\oint H dl \cos \theta = I$$

$$\oint H \cdot dl \cos 0^\circ = I$$

$$\oint H \cdot dl = I$$

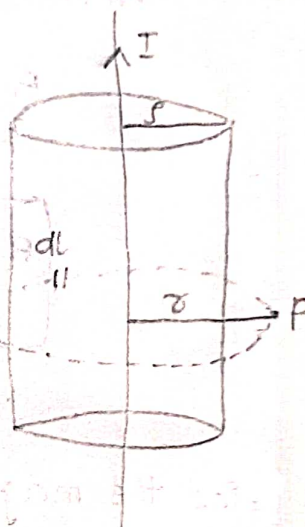
$$H (2\pi r) = I$$

$$H = \frac{I}{2\pi r}$$

($\because$ Total length = circumference = $2\pi r$)

Find the magnetic field intensity due to cylindrical conductor carrying current I using Ampere's law,

Sol: Case:-1 point outside the cylinder.
Let us consider a cylinder conductor with radius r . Consider an infinite conductor carrying current I as shown in figure. Let us consider a point in different positions i.e. outside the cylinder, on the cylinder, inside the cylinder. - considering a small element dl on the closed path i.e. amperian path.



Case:-1 $\oint H \cdot dl = I$.

$$\oint H \cdot dl \cos 0 = I$$

$$\oint H \cdot dl = I$$

$$H (2\pi r) = I$$

$$H = \frac{I}{2\pi r}$$

Case: ii, point on the cylinder.

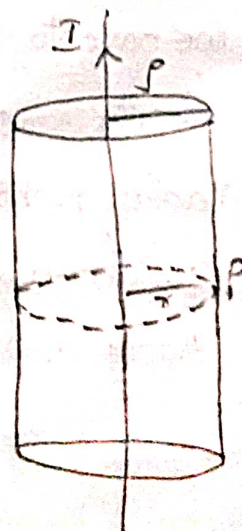
$$\oint H \cdot dl = I$$

$$\oint H \cdot dl \cos 0 = I$$

$$\oint H \cdot dl = I$$

$$H (2\pi r) = I$$

$$H = \frac{I}{2\pi r}$$



case iii) point inside the cylinder.

$$\oint H dl = I.$$

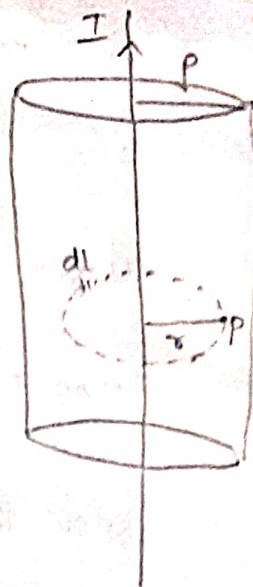
$$H \oint dl \cos 0 = I \left[\frac{\text{desired area}}{\text{total area}} \right]$$

$$H \oint dl = I \left[\frac{\pi r^2}{\pi r^2} \right]$$

$$H [2\pi r] = I \left[\frac{r^2}{r^2} \right]$$

$$H = I \frac{r^2}{r^2} \times \frac{1}{2\pi r}.$$

$$\boxed{H = \frac{I r}{2\pi r^2}} \Rightarrow H = \frac{I r}{2\pi r^2}$$



Magnetic vector potentials:

Scalar magnetic potential:-

In electrostatics the relationship between electric field intensity and electric potential is given by

$$\vec{E} = -\nabla V_e \longrightarrow (1)$$

In the same manner let us relate the magnetic field intensity to a scalar magnetic potential and

$$\vec{H} = -\nabla V_m \longrightarrow (2)$$

From Ampere's law, we know that

$$\nabla \times \vec{H} = \vec{J} \longrightarrow (3)$$

from (2) & (3) we get

$$\nabla \times (-\nabla V_m) = \vec{J} \longrightarrow (4)$$

By using vector Identity $\nabla \times \nabla V = 0 \longrightarrow (5)$

From (3) & (4) we find that $\vec{H} = -\nabla V_m$ is valid only where $\vec{J} = 0$.
Thus the scalar magnetic potential is defined only in the region where $\vec{J} = 0$.

$$\vec{H} = -\nabla V_m$$

$$\vec{H} = -\frac{dV_m}{dl}$$

$$dV_m = -\vec{H} \cdot d\vec{l}$$

$$V_{mAB} = -\int_A^B \vec{H} \cdot d\vec{l} \longrightarrow (6)$$

We know that $\nabla \cdot \vec{B} = 0 \longrightarrow (7)$

($\because$ Non existence of magnetic monopoles)

$$\Rightarrow \nabla \cdot \mu_0 \vec{H} = 0$$

$$\Rightarrow \nabla \cdot \vec{H} = 0$$

$$\Rightarrow \nabla \cdot (-\nabla V_m) = 0$$

$$\boxed{\nabla^2 V_m = 0} \longrightarrow (8) \text{ only for } \vec{J} = 0.$$

$\therefore$ The scalar magnetic potential satisfied the Laplace's equation.
Vector Magnetic potential:

We now introduce the vector magnetic potential which can be used where current density is non zero.

Since $\nabla \cdot \vec{B} = 0$, and we have the vector identity that any vector $\vec{A}$, $\nabla \cdot (\nabla \times \vec{A}) = 0$.

We can write $\vec{B} = \nabla \times \vec{A} \longrightarrow (1)$

Here the vector field $\vec{A}$ is called the magnetic potential. Its S.I units are Wb/m .

"Magnetic vector potential is defined in such a way that its curl gives the Magnetic flux density."

$$\boxed{\vec{B} = \nabla \times \vec{A}}$$

By using vector identity

$$\nabla \times (\nabla \times \vec{A}) = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A} \longrightarrow (2)$$

from eq'n (1) $\vec{B} = \nabla \times \vec{A}$

Applying curl on both sides.

$$\nabla \times \vec{B} = \nabla \times (\nabla \times \vec{A})$$

$$\nabla \times \vec{B} = -\nabla^2 \vec{A} \longrightarrow (3)$$

We know that,

$$\nabla \times \vec{H} = \vec{J}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} \longrightarrow (4) \quad (\because \vec{B} = \mu_0 \vec{H})$$

from eq'n (3) & (4)

$$\boxed{\nabla^2 \vec{A} = -\mu_0 \vec{J}} \longrightarrow (5)$$

Eq'n (5) is called vector Poisson's equation.

$\therefore$ The vector magnetic potential satisfies Poisson's equation.

The solution of the eq'n (5) is given by

$$\vec{A} = \int_L \frac{\mu_0 I d\vec{l}}{4\pi r} \longrightarrow (6)$$

The magnetic flux through a given area S is given by

$$\Phi = \int_S \vec{B} \cdot d\vec{r} \longrightarrow (7)$$

$$\text{since } \vec{B} = \nabla \times \vec{A}$$

$$\Phi = \int_S (\nabla \times \vec{A}) \cdot d\vec{r} = \oint_C \vec{A} \cdot d\vec{l} \longrightarrow (8)$$

Vector Magnetic potential has the physical significance that its integral around any closed path is equal to the magnetic flux through that path.

Magnetism :-

This comprises physical phenomena involving magnetic fields and their effects upon materials. Magnetic fields may be set up on a macroscopic scale by electric currents or by magnets. On atomic scale, individual atoms cause magnetic fields when their electrons have a net magnetic moment as a result of their angular momentum. A magnetic moment arises whenever a charged particle has an angular momentum. It is the cooperative effect of the atomic magnetic moments which causes the macroscopic magnetic field of a permanent magnet. However the underlying principles mechanisms that explain the magnetic phenomena are complex and subtle, and their understanding has eluded physicists until relatively recent times. Several of our modern technological devices rely on magnetism and magnetic materials; these include electric motors, electric power generators and transformers, components of sound and video reproduction systems, telephones, radios, televisions, computers, etc. Well known examples of magnetic properties are iron, some steels and the naturally occurring mineral lodestone. The important facts about the magnetic materials are:

i) There are some materials which exhibit magnetic properties even without the application of any magnetic field and become more magnetic when a weak magnetic field is applied to them.

(ii) There are many other materials which lose their initially strong magnetism when heated above a certain critical temperature and become comparatively weakly magnetised.

(iii) There are some materials which show a magnetic response in a direction opposite to that of any externally applied field.

(iv) Magnetic materials are all media capable of being magnetised in a magnetic field, i.e. of creating their own magnetic field.

According to their magnetic Properties, such materials are divided into three principle groups diamagnetic, paramagnetic and ferromagnetic materials. From the applications point of view, all the magnetic materials can be placed under two groups.

1. Soft magnetic materials.

2. Hard magnetic materials.

Magnetic Induction (or) Magnetic Flux Density (B):

In the presence of magnetic field in vacuum, the magnetic induction (B) is related to the field strength (H) (in units of $A\bar{m}'$) as follows:

$$B = \mu_0 H$$

Where μ_0 is called the permeability of free space.

B is expressed in units of Tesla (or) Weber per Square meter.

The units of permeability are

$$\mu_0 = \frac{B}{H} = \frac{Wb/m^2}{A/m} = \frac{Wb}{m^2} \times \frac{m}{A} = \frac{Wb}{A \cdot m} = Wb \bar{A}' \bar{m}'$$

$$\mu_0 = H \bar{m}' \left[\begin{array}{l} (\because H = Wb \bar{A}'^{-1} \text{ where } H \rightarrow \text{Henry}) \\ \rightarrow (\because Wb/m^2 = NA^{-1} \bar{m}') \end{array} \right]$$

In SI units the permeability of free space has a value of $4\pi \times 10^{-7} \text{ Hm}^{-1}$

Magnetic Field (H)

It is said to occupy a region when the magnetic effect of an electric current or of a magnet upon a small test magnet which is brought in the vicinity is detectable. Magnetic field strength is denoted by H . When a magnetic material is placed in a magnetic field H , it becomes magnetised, i.e. it itself becomes a magnet. Magnetic field strength (H) is expressed in units of Am^{-1} .

If the magnetic field is applied to a solid medium, the magnetic induction in the solid is given by a relationship

$$\boxed{B = \mu H}$$

Where μ is the permeability of the solid material through which the magnetic lines of force pass.

In general, μ is not equal to μ_0 . The ratio μ/μ_0 is the relative permeability of the medium and designated by μ_r .

Mathematically, $\mu_r = \frac{\mu}{\mu_0}$

Magnetization:

Magnetization may be defined as the process of converting a non-magnetic bar into a magnetic bar. This term is almost analogous to the polarization in dielectric materials.

the flux density,

$$B = \mu H = \mu_0 \mu_r H$$

$$(\because \mu_r = \frac{\mu}{\mu_0})$$

$$= \mu_0 \mu_r H + \mu_0 H - \mu_0 H$$

$$= \mu_0 H + \mu_0 H (\mu_r - 1)$$

$$= \mu_0 H + \mu_0 M$$

$$\boxed{B = \mu_0 (H + M)}$$

This is the relation between B , H & M .

Where $M = H(\mu_r - 1)$ is the magnetization of solid and expressed in ampere/metre.

From the above relation, we find that if a magnetic field is applied to a material, the magnetic flux density is equal to the effect on vacuum and on the material.

The magnetization (M) may be defined as the magnetic dipole moment per unit volume of the bar.

Magnetic susceptibility (χ)

The magnitude of the magnetization, M is proportional to the applied field as follows:

$$\boxed{M = \chi H}$$

Where χ is called the magnetic susceptibility, which is unitless. The magnetic susceptibility and the relative permeability are related as follows:

$$\chi = \mu_r - 1 = \frac{M}{H} \quad (\text{or}) \quad \mu_r = 1 + \chi$$

Note that B , M and H are vectors.

Intensity of Magnetization (I):

This is defined as the ratio of the magnetic moment (M) to the volume (V) of the magnetized material, i.e.

$$I = \frac{M}{V}$$

If the given magnetic material is rectangular or cylindrical bar of length 'l' and area of cross-section 'a', then $V = al$ and magnetic moment $M = ml$, where 'm' is designated as pole strength, we have

$$I = \frac{ml}{al} = \frac{m}{a}$$

Thus, the intensity of magnetization may also be defined as the pole strength per unit area at right angles to the direction of magnetization.

Intensity of Magnetization (I) & susceptibility (χ):

For any magnetic material, the intensity of magnetization is found to be proportional to the measure of the capability of magnetizing field H, i.e.

$$I \propto M$$

$$\Rightarrow I = \chi_m H$$

Where χ_m is called the magnetic susceptibility of the medium and it is the measure of capability of the material to take up magnetization.

Magnetic Dipoles:

→ Magnetism is dipolar, i.e. magnetism is characterized by having two opposite poles north (N) and south (S).

→ The strength of a magnetic dipole is measured by the product of the pole strength and the distance between the poles. This is called magnetic moment.

→ One source of magnetism in an atom ~~is~~ is the orbital motion of electrons. Each electron revolving around the nucleus in an atom constitutes a circulating electric charge or current and thus produces a small magnetic field. Moreover, each spinning electron on its axis can be conceived as a circulating charge and also produce small magnetic fields.

Magnetic permeability:

permeability describes the nature of the material. It is the ease with which the material allows magnetic lines of force to pass through it or the degree to which magnetic field can penetrate a given medium is called its permeability.

$$\boxed{\mu = \frac{B}{H}} = \frac{\text{Magnetic induction}}{\text{Applied magnetic field}}$$

Units are, $\mu = \frac{\text{Weber}}{\text{m}^2} / \text{Ampere-metre} = \frac{\text{Wb}}{\text{m}^2} \times \frac{\text{m}}{\text{A}} = \frac{\text{Wb}}{\text{mA}}$

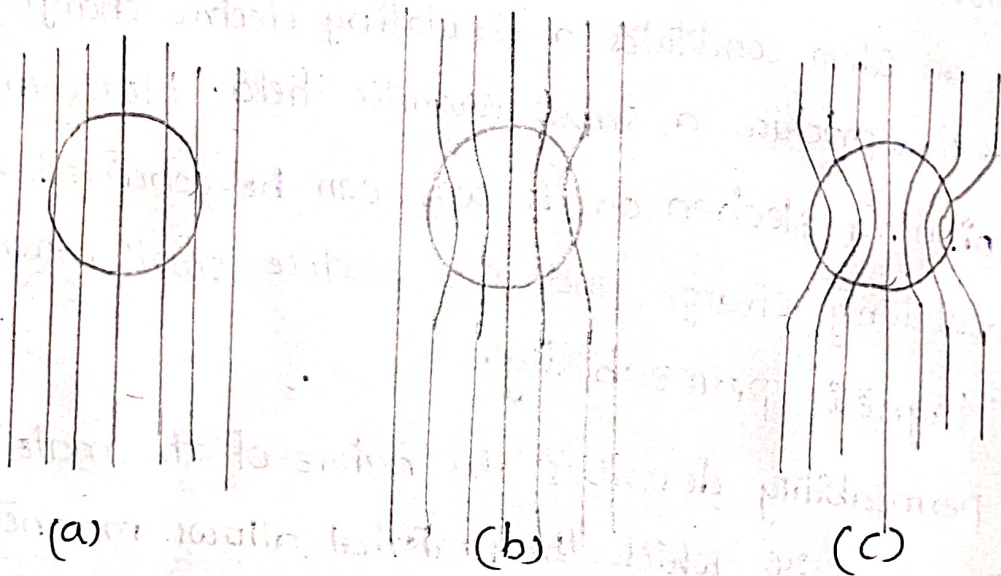
$$(\text{or}) = \frac{\text{H}}{\text{m}} \quad (\text{H} \rightarrow \text{Henry})$$

Permeability of free space, $\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$

Classification of Magnetic Materials:

The three classes into which all the magnetic materials may be grouped according to their magnetic behaviour, although there is some overlap among groups are as follows:

1. Diamagnetic Substances.
2. Paramagnetic Substances.
3. Ferromagnetic substances.



Fig

(a)

(b)

(c)

- a) Diamagnetic solid: Lines of forces are slightly repelled.
 - Relative permeability is less than 1.
- b) Paramagnetic solid: lines of forces are attracted weakly. Relative permeability is slightly greater than 1.
- c) Ferromagnetic solid: lines of forces are attracted very strongly. Relative permeability is much greater than 1.

Diamagnetic substances:—

The substances which when placed in a magnetic field acquire feeble magnetism opposite to the direction of the applied field are known as diamagnetic substances.

Diamagnetism:-

* Diamagnetism is a very weak form of magnetism exhibited by substances with a negative magnetic susceptibility ($\chi = \mu_0 M/B$), i.e., by substances which magnetize in a direction opposite to that of an applied magnetic field. A diamagnetic substance has a magnetic permeability less than 1, and is repelled when placed near a magnet.

* Examples - Organic solids like Naphthalene, benzene etc.
Metals like silver, gold and ~~atoms~~ ^{atoms} copper. ~~metals~~ ^{atoms} like

rare gas configurations like Ar, He, Ne, etc.

* The magnetization of diamagnetic substances is associated with the field.

* According to Lenz's law, the flow of an induced current is in such a direction as to oppose the change of flux of inducing field; this accounts for the negative susceptibility.

* The diamagnetic susceptibility is invariably small, of the order of $10^{-5} \text{ cm}^3/\text{mole}$.

* When placed between the poles of a strong electromagnet, diamagnetic materials are attracted toward regions where the field is weak.

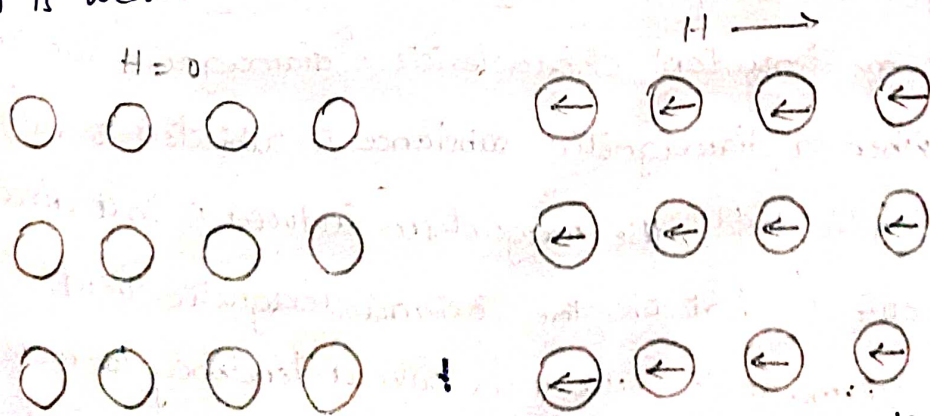


Fig: Schematic illustration of the atomic dipole configuration for a diamagnetic material with and without a magnetic field. No dipole exists in the absence of an external field, whereas in the presence of a field, dipoles are induced that are aligned opposite to the field direction.

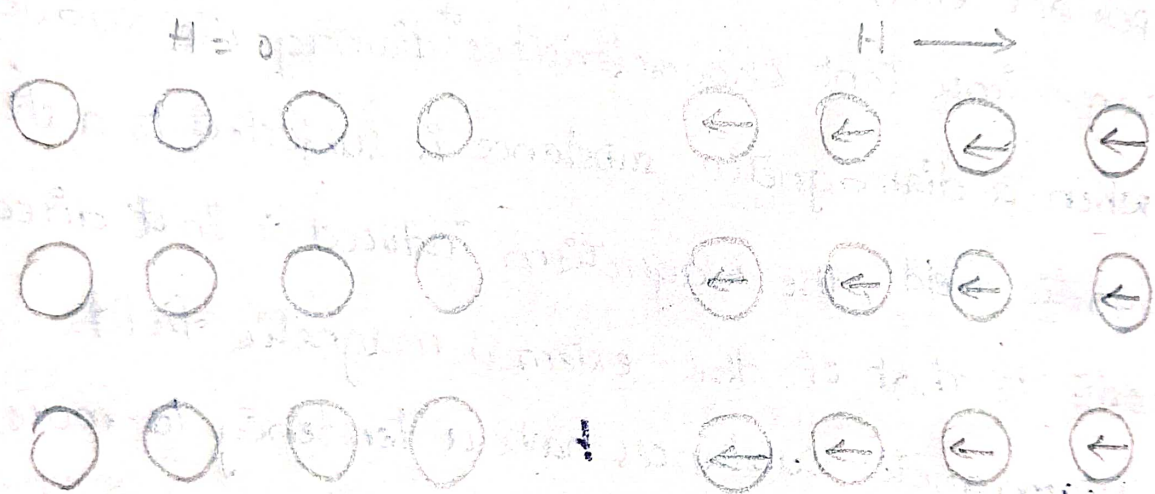


Fig: Schematic illustration of the atomic dipole configuration for a diamagnetic material with and without a magnetic field. No dipole exists in the absence of an external field, whereas in the presence of a field, dipoles are induced that are aligned opposite to the field direction.

Table: Magnetic susceptibilities for diamagnetic materials at room temperature.

Material	Susceptibility χ (volume) (SI units)
Copper	-0.96×10^{-5}
Gold	-3.44×10^{-5}
Mercury	-2.85×10^{-5}
Silicon	-0.41×10^{-5}
Silver	-2.38×10^{-5}
Zinc	-1.56×10^{-5}
Aluminium oxide	-1.81×10^{-5}
Sodium chloride	-1.41×10^{-5}

Diamagnetism is found in all materials, but because it is so weak, it can be observed only when other two types of magnetism, i.e. para or ferromagnetism are totally absent. The condition for pure diamagnetism is that all electronic spins be paired and all orbital moments either be zero or effectively cancel one another. Nearly all molecules with an even number of electrons satisfy this condition; an important exception is O_2 .

Some important characteristics of diamagnetic substances are

- i. When a diamagnetic substance is subjected to a strong magnetic field, the magnetism induced is in a direction opposite to that of the external magnetic field.
- ii. Diamagnetic substances have a tendency to move from stronger to weaker part of the magnetic field, when the field is non-uniform.

iii) These substances are characterised by negative susceptibility.

iv) When a bar of diamagnetic substance is suspended between the pole pieces of a magnet, the bar stays at right angles.

v) A diamagnetic liquid in a 'U' tube shows depression when placed between the pole pieces of a strong magnet.

vi) The diamagnetic susceptibility (χ) is independent of temperature.

Note: Superconductor is also a diamagnetic material.

Paramagnetism :-

A property exhibited by the substances which when placed in a magnetic field, are magnetized parallel to the field to an extent proportional to the field (except at low temperatures or in extremely large magnetic fields). Paramagnetic materials always have permeabilities greater than 1, but the values are in general not nearly so great as those of ferromagnetic materials.

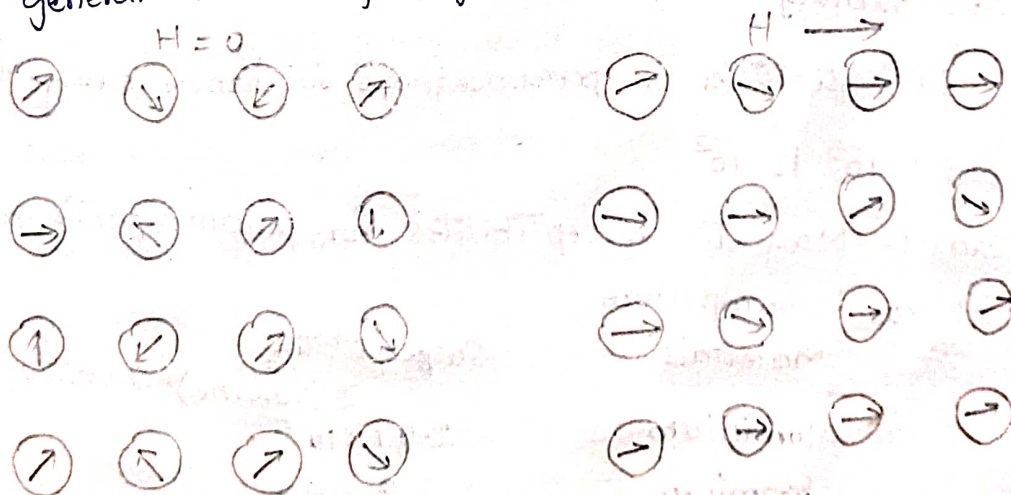


Fig: Atomic dipole configuration with and without an

external magnetic field for a paramagnetic material.

Paramagnetic substances :-

The substances which when placed in a magnetic field acquire feeble magnetism in the direction of applied magnetic field are known as paramagnetic substances.

* For some solid materials, each atom possesses a permanent dipole moment by virtue of incomplete cancellation of electron spin and/or orbital magnetic moments.

* In the absence of an external magnetic field, the orientation of the atomic magnetic moments are random, such that a piece of material possesses no net macroscopic magnetization.

* These atomic dipoles are free to rotate, and paramagnetism results when they preferentially align, by rotation, with an external field.

* These magnetic dipoles are acted on individually with no mutual interaction between adjacent dipoles.

* In as much as the dipoles align with the external field, they enhance it, giving rise to a relative permeability μ , that is greater than 1, and to a relatively small and positive susceptibility.

* Susceptibilities for paramagnetic substances range from about 10^5 to 10^2 .

Table:- Magnetic susceptibilities for paramagnetic materials at room temperature.

Materials	Susceptibility χ (volume) (SI units)
Aluminium	2.07×10^{-5}
Chromium	3.13×10^{-4}
Chromium chloride	1.51×10^{-3}
Manganese	3.70×10^{-3}
Molybdenum	1.19×10^{-4}
Sodium	8.48×10^{-6}
Titanium	1.81×10^{-4}
Zirconium	1.09×10^{-4}

The following types of substances are paramagnetic:

a) All atoms and molecules which have an odd number of electrons. According to quantum mechanics, such a system cannot have a total spin equal to zero; therefore each atom or molecule has a net magnetic moment which arises from the electron spin angular momentum. Examples are organic free radicals and gaseous nitric oxide.

b) All free atoms and ions with unfilled inner electron shells and many of these ions when in solids or in solution. Examples are transition, rare-earth and actinide elements and many of their salts. This includes ferromagnetic and antiferromagnetic materials above their transition temperatures.

c) Several miscellaneous compounds including molecular oxygen and organic biradicals.

d) Metals, in this case, the paramagnetism arises from the magnetic moments associated with the spins of the conduction electrons and is called the Pauli paramagnetism.

Both paramagnetic and diamagnetic substances are considered to be non-magnetic because they exhibit magnetization only in the presence of an external field. Moreover, for both, the flux density B within them is almost the same as it would be in vacuum.

Salient features of paramagnetic substances are as follows:

i) The relative permeability is slightly greater than unity and susceptibility is of the order of 10^{-6} and positive.

ii) When a bar of paramagnetic substance is suspended between the pole pieces of a magnet, it aligns parallel to the magnetic lines of force.

iii) When a paramagnetic liquid in a U-tube is placed between pole pieces of a strong magnet, liquid shows a rise in U-tube.

iv) The paramagnetic substance tends to move from weaker part to stronger part of a non-uniform magnetic field.

v) A paramagnetic substance gas, when allowed to ascend between the pole pieces of a magnet, the gas spreads along the magnetic field.

vi) The susceptibility of paramagnetic substances varies inversely to absolute temperature (T) i.e.

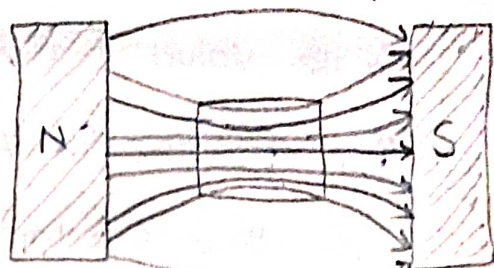
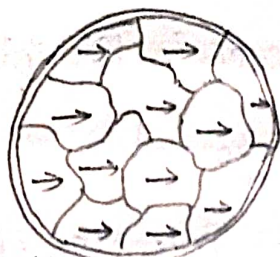
$$\chi \propto \frac{1}{T}$$

This law is known as Curie law.

Ferromagnetism:-

The substances which when placed in a magnetic field become strongly magnetised in the direction of the applied field are known as ferromagnetic substances.

In the materials like iron, cobalt, nickel and some alloys like alnico (Al, Ni and Co), there is a strong resultant magnetic moment in the atoms. In fact, there will be localized regions known as 'magnetic domains' in which all the atomic magnets are already oriented in a particular direction. But these 'domains' may be randomly oriented making the resultant magnetic moment of the material zero.



Ferromagnetic domains.

a) at random

b) in external field

ferromagnetic material in external field.

When placed in an external magnetic field, these domains get arranged in the direction of the field. If the external field is removed, the domains do not get deoriented completely as in the case of paramagnetic materials and the material remains magnetised permanently to some extent. This nature of matter is known as ferromagnetism.

Properties of ferromagnetic materials:

Ferromagnetic materials are strongly attracted by magnets. Their relative permeability values are very high ($\mu_r = 10$ to $10,000$). In other words, they possess all the paramagnetic properties but to a very high degree. When a ferromagnetic material is placed in an external magnetic field, the lines of force through the material will be very large in number, as if they are sucked into it.

As temperature is increased, the material loses its ferromagnetism and at a particular temperature known as Curie temperature the ferromagnetic substance turns paramagnetic in nature.

Magnetic materials are employed in making electro-magnets, transformers, relay systems etc. They are also used in making permanent magnets.

The susceptibility of these substances remain constant for smaller values of H , increases in the intermediate region and then decreases for very large values of H .

For these substances, the intensity of magnetization (I) is proportional to the small value of magnetising field (H) and becomes constant for large values of H .

Below the curie point, a ferromagnetic substance is composed small, spontaneously magnetised regions called domains. The total magnetic moment of the material is the vector sum of the magnetic moments of the individual domains. It is now known that these assumed domains really exist and are usually 0.1 cm and 0.01 cm across.

The magnetic moment of a magnet is 1 Am^2 . What is the intensity of magnetisation? (The dimension of magnet are $10 \text{ cm} \times 2 \text{ cm} \times 1 \text{ cm}$)

Sol. Given that,

$$M = 1 \text{ Am}^2$$

$$I = ?$$

$$\text{Volume, } V = 10 \times 10^{-2} \times 2 \times 10^{-2} \times 1 \times 10^{-2}$$

$$\text{We know, } I = \frac{M}{V}$$

$$I = \frac{1 \text{ Am}^2}{20 \times 10^{-6} \text{ m}^3} = 0.05 \times 10^6 \text{ A/m}$$

$$I = 50,000 \text{ A/m}$$

A specimen of iron is uniformly magnetized by a magnetizing field of 500 A/m . If the magnetic induction in the specimen is 0.2 wb/m^2 . Find the relative permeability and the susceptibility.

Given data, $H = 500 \text{ A/m}$

$$B = 0.2 \text{ wb/m}^2$$

$$\frac{\text{wb}}{\text{m}^2} \cdot \frac{\text{m}}{\text{A}}$$

$$\frac{\text{wb}}{\text{A} \cdot \text{m}}$$

We know, $B = \mu H$

$$\therefore \mu_r = \frac{\mu}{\mu_0}$$

We know, $\mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2 \text{ (or) H/m}$

$$\mu = \frac{0.2}{500} = 4 \times 10^{-4}$$

a) $\therefore \mu_r = \frac{\mu}{\mu_0} = \frac{4 \times 10^{-4}}{4\pi \times 10^{-7}} = \frac{1}{3.14 \times 10^{-3}} = 318.47$

b) $\chi = \mu_r - 1 = 318.47 - 1 = 317.47$

An iron rod of cross sectional area 4 cm^2 is placed with its length parallel to a magnetic field of intensity 1600 A/m . The flux through the rod is $4 \times 10^{-4} \text{ wb}$. What is the permeability of the material of the rod?

Given data,

Area, $a = 4 \text{ cm}^2 = 4 \times (10^{-2})^2 \text{ m}^2$

Magnetic field, $H = 1600 \text{ A/m}$

Flux, $\phi = 4 \times 10^{-4} \text{ wb}$

$$\begin{aligned} \phi &= BA \cos \theta \\ &= BA \cos 0 \\ \phi &= BA \end{aligned}$$

We know, $B = \frac{\phi}{A} = \frac{4 \times 10^{-4}}{4 \times 10^{-4}} = 1 \text{ wb/m}^2$

$\therefore \mu = ? \quad \therefore B = \mu H$

$$\mu = \frac{B}{H} = \frac{1}{1600} = 0.000625 \frac{\text{wb}}{\text{A} \cdot \text{m}}$$

$$\mu = 6.25 \times 10^{-4} \frac{\text{wb}}{\text{A} \cdot \text{m}}$$

The magnetic susceptibility of medium is 948×10^{-11} . Calculate the permeability and relative permeability.

Sol:- Given data,

$$\chi = 948 \times 10^{-11}$$

$$\mu = ?$$

$$\mu_r = ?$$

$$\text{We know, } \mu_0 = 4\pi \times 10^{-7} \frac{\text{N}}{\text{A}^2} \text{ (or) H/m}$$

$$a) \mu_r = \chi + 1$$

$$= 948 \times 10^{-11} + 1$$

$$= 1.00000000948$$

$$b) \boxed{\mu_r = \frac{\mu}{\mu_0}}$$

$$\mu = \mu_0 \mu_r$$

$$= 4\pi \times 10^{-7} \times 1.00000000948$$

$$= 4 \times 3.14 \times 10^{-7} \times 1.00000000948$$

$$= 12.5600 \times 10^{-7}$$

$$= 0.000001256 \text{ H/m (or) } \frac{\text{Wb}}{\text{Am}}$$

In the bohr model of hydrogen atom the electrons circulate around nucleus in a path radius $5.1 \times 10^{-11} \text{ m}$ at a frequency of 6.8×10^{15} revolutions per second. Find the equivalent dipole moment?

Sol:- Given data, Hydrogen = $1e^{\ominus}$

$$\text{radius, } r = 5.1 \times 10^{-11} \text{ m}$$

$$\text{frequency, } f = 6.8 \times 10^{15} \text{ rev/sec}$$

$$\text{We know, dipole moment, } \mu = I \cdot A$$

$$P = \frac{1}{T}$$

$$T = \frac{1}{f} = \frac{1}{6.8 \times 10^{15}} = 0.147 \times 10^{-15} \text{ sec.}$$

$$\text{Area, } A = \pi r^2$$

$$= 3.14 \times (5.1 \times 10^{-11})^2$$

$$= 3.14 \times 26.01 \times 10^{-22}$$

$$= 8.167 \times 10^{-21} \text{ m}^2$$

$$\text{We know, } I = \frac{qV}{T}$$

$$I = \frac{1.602 \times 10^{-19}}{0.147 \times 10^{-15}}$$

(we know,
 $q \times e^- = 1.6 \times 10^{-19} \text{ x } 1$
 $\text{for H} = 1.602 \times 10^{-19} \text{ C})$

$$I = 10.897 \times 10^{-4} \text{ A}$$

Therefore,

$$\text{dipole moment, } m = I \cdot A$$

$$= 10.897 \times 10^{-4} \times 8.167 \times 10^{-21}$$

$$= 88.995 \times 10^{-25}$$

$$= 8.89 \times 10^{-24} \text{ A} \cdot \text{m}^2$$

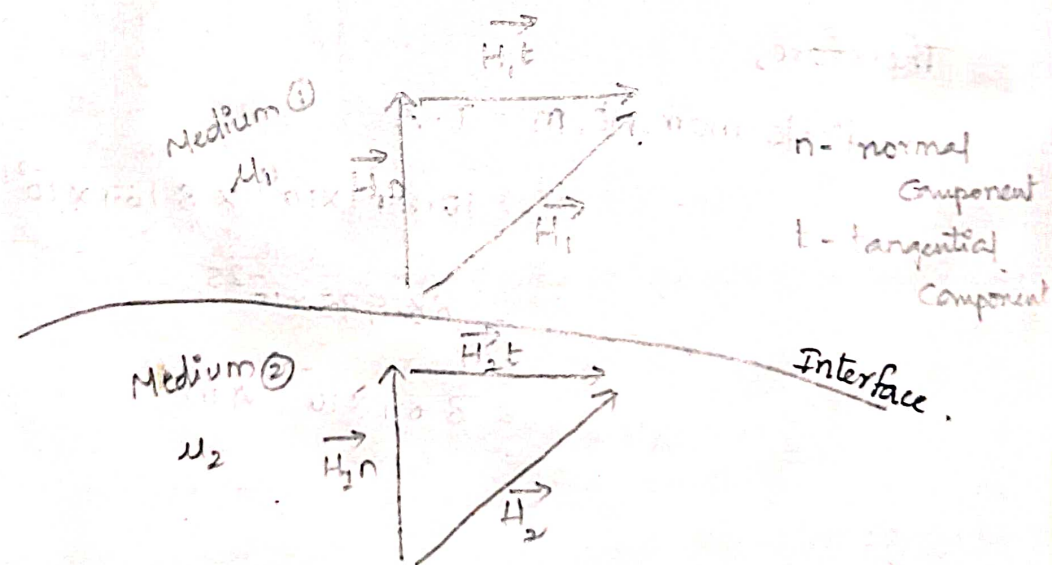
Boundary conditions for B and H at the surface of a Magnetic material:

The aim is consider that how the ^{field} vectors B and H change in passing an interface between two media. The two media may be two materials with different magnetic properties, or a material medium and vacuum.

Consider the interface between two different materials with dissimilar permeabilities.

Let us consider both magnetic field & magnetic ~~line~~ flux density are present in both regions.

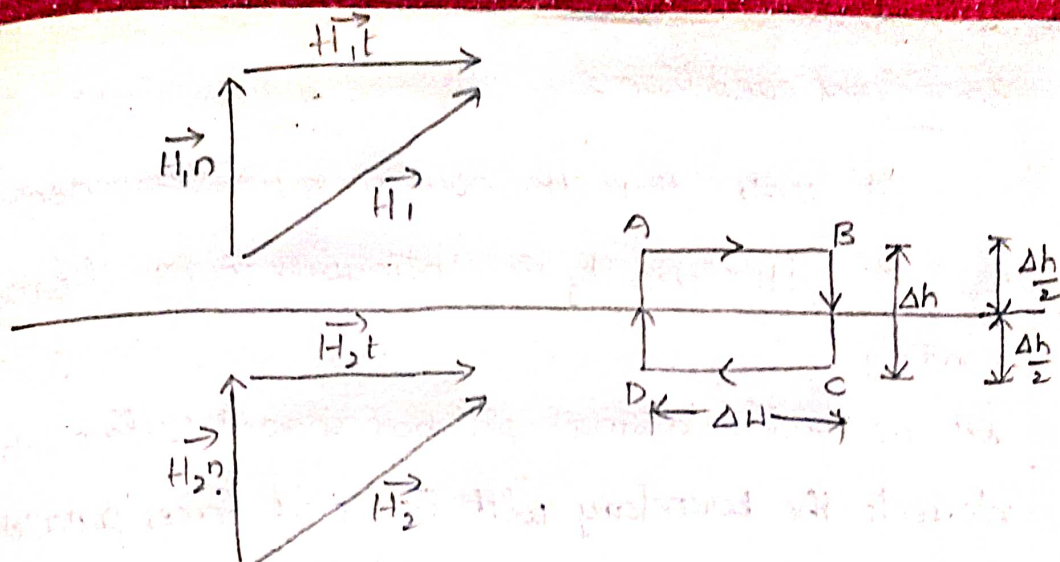
The magnetic fields at the interface in terms of their normal and tangential components as shown in figure.



Boundary condition 1:

It states that the tangential component of the magnetic field intensity is continuous across a boundary.

Let us consider a closed loop ABCD at the boundary between two media of different permeabilities μ_1 & μ_2 as shown in adjacent figure. The height and width of closed loop are Δh and Δw .



Let us suppose that some current is flowing at the boundary.
 $\vec{K}$ is the current density at the boundary.

Apply Ampere's law, to this closed loop ABCD

$$\oint \vec{H} \cdot d\vec{l} = I_{enc} \longrightarrow (1).$$

$$\begin{aligned} \Rightarrow H_{1t} \Delta w - H_{1n} \frac{\Delta h}{2} - H_{2n} \frac{\Delta h}{2} - H_{2t} \Delta w + H_{2n} \frac{\Delta h}{2} \\ + H_{1n} \frac{\Delta h}{2} = \vec{K} \Delta w \end{aligned}$$

$$\therefore H_{1t} \Delta w - H_{2t} \Delta w = \vec{K} \Delta w.$$

$$\Delta w [H_{1t} - H_{2t}] = \vec{K} \Delta w.$$

$$\Rightarrow \boxed{H_{1t} - H_{2t} = \vec{K}} \longrightarrow (2).$$

If the boundary is free of current then eq'n(2) becomes

$$H_{1t} - H_{2t} = 0$$

$$\boxed{H_{1t} = H_{2t}} \longrightarrow (3)$$

$\therefore$ Tangential component of $\vec{H}$ is continuous at the boundary.

From eq'n(3)

$$\boxed{\frac{B_{1t}}{\mu_1} = \frac{B_{2t}}{\mu_2}} \longrightarrow (4)$$

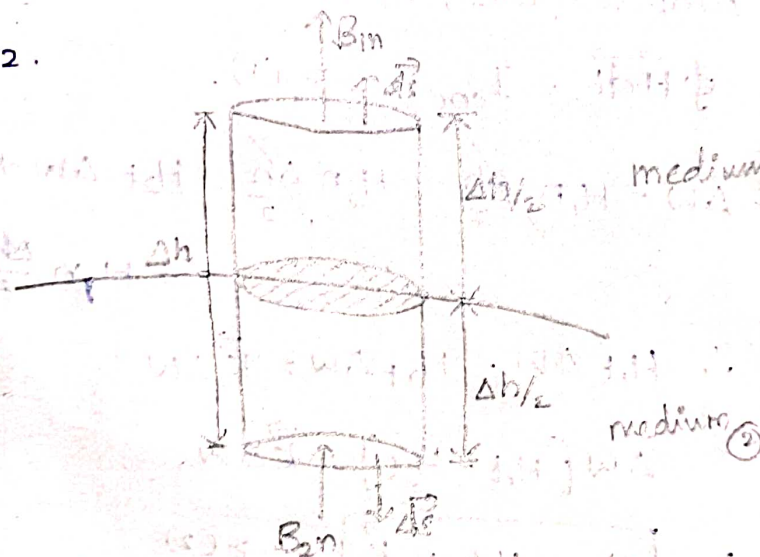
$$\therefore \boxed{B = \mu H}$$

$\therefore$ The tangential component of $\vec{B}$ is discontinuous of Boundary.

Boundary condition 2:

It states that the normal vector component of magnetic flux density is continuous across the magnetic boundary.

Let us consider a small pill box shaped surface which intersects the boundary with its end faces parallel to it. Δh is the height of the pill box is assumed to be very small ($\Delta h \rightarrow 0$). Half of the pill box is present in the medium (1) and half of the pill box is present in medium - 2.



We know that $\oint_S \vec{B} \cdot d\vec{r} = 0 \rightarrow (5)$

The flux through the pill box is given by

$$\int_{\text{top}} \vec{B} \cdot d\vec{r} + \int_{\text{Bottom}} \vec{B} \cdot d\vec{r} + \int_{\text{curve}} \vec{B} \cdot d\vec{r} = 0 \rightarrow (6)$$

Since the height of the pill box negligibly small ($\Delta h \rightarrow 0$)

The only contribution of flux will come through top & bottom face.

$$\therefore B_{1n} \Delta S - B_{2n} \Delta S = 0$$

$$\boxed{B_{1n} = B_{2n}} \rightarrow (7)$$

$\therefore$ The normal component of magnetic flux density is continuous at the boundary.

We know that, $\vec{B} = \mu \vec{H}$

$$\therefore \text{from eq'n (7)} \quad \mu_1 H_{1n} = \mu_2 H_{2n}$$

$$\therefore \boxed{\frac{H_{1n}}{H_{2n}} = \frac{\mu_2}{\mu_1}} \longrightarrow (8)$$

The normal component of magnetic field intensity is discontinuous at the boundary separating two mediums.