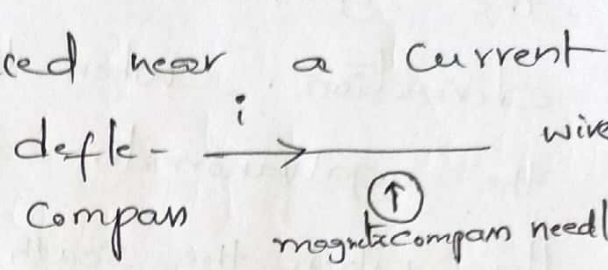
UNIT-5

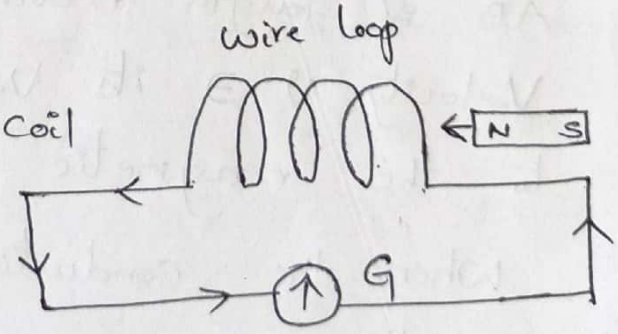
Time Varying fields.

In 1820, Oersted discovered that when a magnetic compass box is placed near a current carrying wire, then a deflection is produced in the compass box. Hence, he concluded that a current carrying wire (conductor) produces magnetic field. The converse of this effect is also true. It was proved by Michael Faraday. According to him, an electric current is induced in a closed circuit by changing magnetic field. This phenomenon is called electromagnetic Induction. i.e the change in the magnetic field in a conductor produces electric field.



Faraday's experiment:

In this experiment a coil is connected to a Galvonameter 'G' as shown.



When north pole of a bar magnet is moved towards the coil, then the pointer in Galvonameter deflects. This indicates the presence of the

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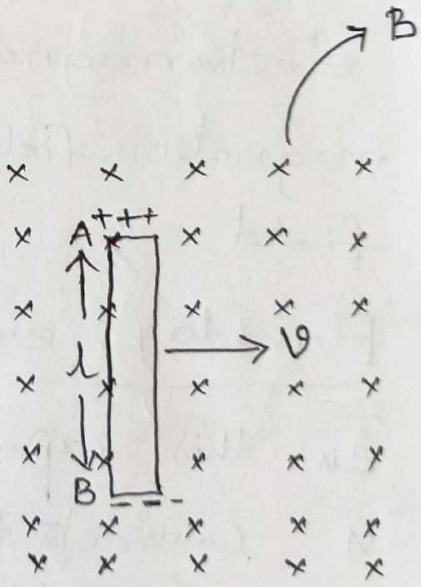
electric current in the coil. But when the bar magnet is pulled away from the coil, the galvanometer shows deflection in opposite direction. when the magnet is held stationary, the galvanometer does not show any deflection. Ily when the South pole of bar magnet is moved towards (or) away from the coil, the deflections will be in opposite direction.

This shows that the relative motion between magnet and the coil is responsible for the generation of electric current. This current is called Induced current in the coil. It is also called as induced emf.

Motional E.M.F:

Consider a thin conducting rod AB of length moving with a velocity v ∋ its velocity is ⊥ to the magnetic field B.



When the conducting rod is moved with a velocity v, the free electrons in the conducting rod gain velocity v and they will experience a force (magnetic force) $F = BQv$ —①
perpendicular to both B and v.

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These electrons will flow along the rod, accumulating +ve charge on one side and the -ve charge on the other side of rod. As charges accumulates at the ends of the rod, an electric field 'E' is produced in the rod 'AB' from A to B. This field inturn produces a force on electrons in Conducting rod which is opposite to the force due to magnetic field. ∴ This force $F = Eq$ — ②

When enough charge accumulates, a situation comes, when electric force (Eq) Cancels the magnetic force (BqV). Then the free electrons donot drift any more. Hence no force acting on the free electrons of the rod. ie $Eq = BqV$

$$\therefore E = VB \quad — ③$$

If 'l' is the length of rod, 'E' is electric field produced in the rod, then P.D between the two ends of the rod $V = E \cdot l$ — ④ $\left(\because E = \frac{V}{l}\right)$

put ③ in ④, we get $\boxed{V = VBl}$

This is the emf induced in the rod due to its motion in magnetic field (B). This is Called Motional emf, given by $\boxed{\mathcal{E} = VBl}$

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Let θ be the angle between the velocity of rod and direction of magnetic field B, then motional emf $\mathcal{E} = vBl\sin\theta$ (or) $\boxed{\mathcal{E} = Bl v\sin\theta}$

$v\sin\theta \rightarrow$ Component of $v \perp$ lr to B.

Suppose, if the rod moves parallel to the field (B) direction, then $\theta = 0$, $\therefore \boxed{\mathcal{E} = 0}$

Magnetic flux (ϕ):

The total number of magnetic lines of force crossing (passing) the surface (or) area normally is called magnetic flux. It is denoted by ϕ. It is given by $\phi = \vec{B}.\vec{A} = BA\cos\theta$

$\therefore \phi = BA\cos\theta$, where $\theta \rightarrow$ angle between B and perpendicular drawn on the surface.

It is a scalar Quantity.

Its S.I unit is Weber.

Faraday's laws of e.m Induction:

Faraday stated his experimental observations in the form of laws called Faraday's laws of e.m. Induction.

1st law:- Whenever magnetic flux linked with a coil (or) electric circuit changes, an emf is induced in the coil (or) circuit.

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This induced emf exists as long as the change in magnetic flux continues.

II nd law: It states that, the induced emf produced in the coil (or) electric circuit is equal to the negative rate of change of magnetic flux linked with it.

$\therefore e = -\frac{d\phi}{dt}$, $\phi \rightarrow$ flux through each turn.

If the coil contains 'N' turns, then $e = -N\frac{d\phi}{dt}$

$$\therefore e = -N\frac{d}{dt}(BA\cos\theta)$$

$$\therefore \boxed{e = -\frac{d}{dt}(NBA\cos\theta)}$$

It is also called Neumann's law.

Lenz's law:

The direction of induced emf is always such that which opposes the change in magnetic flux. The '-ve' sign in the above equation is in accordance with Lenz's law.

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Displacement current

Since time Varying magnetic field ($\frac{dB}{dt}$) produces an electric field, it should be expected that a time Varying electric field $\frac{dE}{dt}$ creates a magnetic field. This idea was put forward by Maxwell in 1865. Maxwell argued that Ampere's law was incomplete and an additional term called displacement current was to be included, which can produce magnetic field. The need for inclusion of an additional term can be understood like this.

According to Ampere's law, $\int \vec{B}.dl = \mu_0 i$

By stoke's theorem $\int \vec{B}.dl = \int (\nabla \times B)\, ds$

Then Ampere's law become $\int (\nabla \times B)\, ds = \mu_0 i$

but $i = J \iint ds$

$$\therefore \int (\nabla \times B)\, ds = \mu_0 J \iint ds \Rightarrow \boxed{\nabla \times B = \mu_0 J}$$

Taking divergence on both sides, we get

$$\text{div}\,(\nabla \times B) = \mu_0\, \text{div}\, J$$

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but $\text{div}(\nabla \times B) = 0$, Then $\text{div}\, J = 0 \Rightarrow \boxed{\nabla \cdot J = 0}$

The above equation does not agree with the equation of continuity which states that $\nabla \cdot J + \frac{d\rho}{dt} = 0$.

In General $\nabla \cdot J$ Can't be equal to zero. This contradiction points out that Ampere's law as stated above is valid only for static fields and is not a general law.

Then Maxwell generalised Ampere's law, by adding a term (to include time varying electric fields). Then Ampere's law Can be assumed as $\nabla \times B = \mu_0 J +$ Something (a term)

To find that term, Consider Gauss law

$$\int \vec{E} \cdot ds = \frac{q}{\epsilon_0} \Rightarrow \int \vec{D} \cdot ds = \iiint \rho\, dv \quad (\because D = \epsilon_0 E)$$

From Gauss - Divergence theorem, $\iint_S \vec{D} \cdot ds = \iiint \nabla \cdot D\, dV$

$\therefore$ The equation becomes, $\iiint \nabla \cdot D\, dv = \iiint \rho\, dv$

$$\Rightarrow \boxed{\nabla \cdot D = \rho} \Rightarrow \nabla \cdot \frac{\partial D}{\partial t} = \frac{\partial \rho}{dt}$$

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Adding 'div J' on both sides, we get

$$\text{div } J + \nabla \cdot \frac{\partial D}{\partial t} = \text{div } J + \frac{\partial \rho}{\partial t}$$

$$\text{div } J + \nabla \cdot \frac{\partial D}{\partial t} = 0 \qquad \left(\because \text{div } J + \frac{\partial \rho}{\partial t} = 0\right)$$

$$\Rightarrow \nabla \cdot J + \nabla \cdot \frac{\partial D}{\partial t} = 0$$

$$\Rightarrow \nabla \cdot \left[J + \frac{\partial D}{\partial t}\right] = 0.$$ This is the generalised condition.

$\therefore$ $\nabla \cdot J = 0$ is for static fields (steady current)

$$\nabla \cdot \left[J + \frac{\partial D}{\partial t}\right] = \nabla \cdot \left[J + \epsilon_0 \frac{\partial E}{\partial t}\right] = 0,$$ for all currents ie for steady currents and time varying electric fields.

Now Ampere's law may also be changed to

$\nabla \times B = \mu_0 \left[J + \frac{\partial D}{\partial t}\right]$, where $\frac{\partial D}{\partial t}$ is called displacement current density.

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Maxwell's electromagnetic equations:

Maxwell's e.m. equations are based upon the well-known fundamental equations such as (i) Gaun law of electricity (ii) Gaun law of magnetism (iii) Faraday's law of e.m induction and (iv) Ampere's law.

When electric and magnetic fields are changing very rapidly with time, then varying magnetic fields give electric field and vice-versa. We therefore, consider e.m fields by a set of equations known as Maxwell's e.m equations.

(a) Maxwell's equations in integral form:

① Maxwell first equation is nothing but Gaun law in electricity. i.e $\int \vec{E}.\vec{ds} = q/\epsilon_0$

$$\therefore \int \vec{E}.\vec{ds} = \frac{1}{\epsilon_0} \rho \iiint dV \quad \left(\because q = \rho \iiint dV\right) \text{ (or) } \rho = \frac{q}{\iiint dV}$$

$$\Rightarrow \int \epsilon_0 E \, ds = \rho \iiint dV$$

$$\Rightarrow \boxed{\iint_S \vec{D}.ds = \rho \iiint dV} \quad (\because D = \epsilon_0 E)$$

This is 1st equation in integral form.

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② Maxwell's second equation:

From Gaum law in magnetism, we have

$$\int \vec{B}.ds = 0 \Rightarrow \boxed{\iint_S B.ds = 0}$$

This is Second Maxwell's equation.

③ Maxwell's third equation:

From Fasaday's law of e.m induction we have

$$\int E.dl = -\frac{d\phi}{dt}$$

ie $\boxed{\int E.dl = -\frac{d}{dt}\iint_S B.ds}$ $\left(\because \phi = \iint \vec{B}.\vec{ds}\right)$

This is third Maxwell's equation.

④ Maxwell's fourth equation:

From modified form of Ampere's law, we have

$$\int B.dl = \mu_0\left[J + \frac{\partial D}{\partial t}\right]$$

ie, it can be written as $\boxed{\int_l B.dl = \mu_0 \iint_S \left(J + \epsilon_0 \frac{\partial E}{\partial t}\right) ds}$

This is Maxwell's Fourth equation in integral form

(b) In Differential form:

① we have $\iint_S D.ds = \rho \iiint_V dV$

From Gauss - Divergence theorem, $\iint_S D.ds = \iiint_V (\nabla.D)\, dV$.

then the above equation becomes,

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$$\iiint_V (\nabla \cdot D)\, dv = \rho \iiint_V dv$$

$$\therefore \boxed{\nabla \cdot D = \rho} \quad (\text{or}) \quad \boxed{\text{div } D = \rho}$$

This is I st equation in differential form.

(2) We know that $\iint_S B \cdot ds = 0$

From Gauss - Divergence theorem, $\iint_S B \cdot ds = \iiint_V (\nabla \cdot B)\, dv$.

$\therefore$ The above equation becomes, i.e

$$\iint_S B \cdot ds = \iiint_V (\nabla \cdot B)\, dv = 0$$

i.e $(\nabla \cdot B) = 0 \Rightarrow \boxed{\nabla \cdot B = 0}$ (or) $\text{div } B = 0$.

This is Second equation in differential form.

(3) We have $\int_L E \cdot dl = -\frac{d}{dt} \iint_S B \cdot ds$

From stoke's theorem LHS can be written as

$$\int_L E \cdot dl = \iint_S (\nabla \times E)\, ds,$$

then the above equation becomes, $\iint_S (\nabla \times E)\, ds = -\frac{d}{dt} \iint_S B \cdot ds$

$$\therefore \nabla \times E = -\frac{dB}{dt} \quad (\text{or}) \quad \nabla \times E = -\frac{d}{dt}(\mu_0 H)$$

$$\Rightarrow \boxed{\nabla \times E = -\mu_0 \frac{\partial H}{\partial t}}$$

This is 3rd equation in differential form.

(4) We have $\int B \cdot dl = \mu_0 \iint_S \left(J + \epsilon_0 \frac{\partial E}{\partial t}\right) ds$.

the LHS can be written as using Stoke's theorem

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ie $\int B \cdot dl = \iint_S (\nabla \times B)\, ds$.

∴ The above equation becomes, $\iint_S (\nabla \times B)\, ds = \mu_0 \iint_S \left(J + \frac{\partial D}{\partial t}\right) ds$

ie $\nabla \times B = \mu_0 \left[J + \frac{\partial D}{\partial t}\right]$ (or) $\boxed{\nabla \times B = \mu_0 J + \epsilon_0 \frac{\partial E}{\partial t}}$

This is Maxwell's fourth equation in differential form.

(I) Maxwell's equations in Free Space:

For free space $\rho = 0$ and $J = 0$

Now the integral and differential forms of Maxwell's equations are as follow.

Integral form: (i) $\iint_S D \cdot ds = 0$ (ii) $\iint_S B \cdot ds = 0$

(iii) $\int_l E \cdot dl = -\frac{d}{dt} \iint_S B \cdot ds$ (iv) $\int B \cdot dl = \mu_0 \iint_S \frac{\partial D}{\partial t}\, ds$.

Differential form:

(i) $\nabla \cdot D = 0$ (ii) $\nabla \cdot B = 0$ (iii) $\nabla \times E = -\frac{dB}{dt}$ (iv) $\nabla \times B = \mu_0 \frac{\partial D}{\partial t}$

(II) Maxwell's equations in Dielectric medium

In di-electric medium, $\rho = 0$, $J = 0$, $B = \mu H$ and $D = \epsilon E$

Integral form:

(i) $\iint_S D \cdot ds = 0$ (ii) $\iint_S B \cdot ds = 0$. (iii) $\int E \cdot dl = -\frac{d}{dt} \iint_S B \cdot ds$

(iv) $\int B \cdot dl = \mu \iint \frac{\partial D}{\partial t}\, ds$ (or) $\int B \cdot dl = \mu\epsilon \iint \frac{\partial E}{\partial t}\, ds$.

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Differential form: $\rho = 0$, $J = 0$

(i) $\nabla \cdot D = 0$ (ii) $\nabla \cdot B = 0$ (iii) $\nabla \times E = -\frac{dB}{dt}$ (iv) $\nabla \times B = \mu\epsilon\frac{\partial E}{\partial t}$

(III) Maxwell's equations in constant field with time.

Since the field is constant, then $\frac{\partial B}{\partial t} = 0$, $\frac{\partial E}{\partial t} = 0$

Integral form: (i) $\int D \cdot ds = \int \rho \, dV$ $\left(\because D = \epsilon E\right)$

(ii) $\int B \cdot ds = 0$ (iii) $\int E \cdot dl = 0$ (iv) $\int B \cdot dl = \mu \int J \, ds$

Differential form: (i) $\nabla \cdot D = \rho$ (ii) $\nabla \cdot B = 0$ (iii) $\nabla \times E = 0$

(iv) $\nabla \times B = \mu J$

Maxwell's equations in conducting medium:

For good conducting medium, $J \neq 0$ and $\rho = 0$.

Since the amount of +ve charge and -ve charges are equal. ($\rho = 0$, since charge resides only on surface)

Integral form: (i) $\int \vec{D} \cdot ds = 0$, (ii) $\int B \cdot ds = 0$

(iii) $\int E \cdot dl = -\frac{dB}{dt}\int ds$ (iv) $\int B \cdot dl = \mu_0 \int \vec{J} \cdot ds$

Differential form:

(i) $\nabla \cdot D = 0$ (ii) $\nabla \cdot B = 0$ (iii) $\nabla \times E = -\frac{\partial B}{\partial t}$

(iv) $\nabla \times B = \mu_0 J$.

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Physical Significance of Maxwell's Equations:

① **Maxwell's first equation**: This represents the Gauss law in electrostatics, which states that the total electric flux over a closed surface is $\frac{1}{\epsilon_0}$ times the total charge enclosed with in the surface.

② **Maxwell's Second equation**: This represents the Gauss law in magnetostatics, which states that the net magnetic flux through any closed surface is zero. It is known that a magnetic monopole does not exists, therefore any closed volume will always contain equal and opposite magnetic poles. Thus the magnetic flux entering the region is equal to magnetic flux leaving it.

③ **Maxwell's third equation**: This is the Faraday's law of e.m induction, which signifies that an electric field is produced by a changing magnetic flux $(d\phi/dt)$.

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(4) Maxwell's fourth equation: It is the modified form of Ampere's law. It is Valid for both Steady and time Varying electric fields. This equation states that the amount of work done to move a unit pole around a closed path is equal to Sum of Conduction current and displacement current. This Signifies that a conduction current as well as displacement current (time Varying field) produces magnetic field.

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Energy in an Electromagnetic field:

An electromagnetic wave contains both oscillating electric and magnetic fields. Just like mechanical waves, E.M waves also carry energy over vast distances, when electromagnetic waves propagates. Since the E.M wave contains both electric and magnetic fields, the e.m waves also contains energy as that of the fields.

The energy in an e.m wave is partly carried by electric field and partly by magnetic field.

Let E_T be the energy density (or) energy stored per unit volume in an e.m wave.

$\therefore E_T$ = energy stored per unit Volume by electric field
+
energy stored per unit volume by magnetic field

But we know that,

Energy stored per unit Volume by E.F = $\frac{1}{2}\epsilon_0 E^2$

Energy stored per unit Volume by mag. field = $\frac{B^2}{2\mu_0}$

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$$\therefore \boxed{E_T = \frac{1}{2}\epsilon_0 E^2 + \frac{B^2}{2\mu_0}} \quad -(1)$$

$-(2)$

The Speed of e.m wave is given by $\boxed{c = \frac{E}{B}}$ ie the ratio of electric field to magnetic field.

In general, the Speed of e.m wave is equal to the Velocity of light 'c'. ie $c = 3\times10^8$ m/sec

from maxwell's equations, the Speed of light is given by $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$ $-(3)$

Energy density of an e.m wave can be written interms of electric field and magnetic fields as follows.

(a) Energy density in an e.m wave in terms of Electric field.

Consider $E_T = \frac{1}{2}\epsilon_0 E^2 + \frac{B^2}{2\mu_0}$

Replace B, with $\frac{E}{c}$ using the formula

$c = \frac{E}{B}$,

Then, $E_T = \frac{1}{2}\epsilon_0 E^2 + \frac{E^2}{2\mu_0 c^2}$ $-(4)$

put (3) in (4), we get

$$E_T = \frac{1}{2}\epsilon_0 E^2 + \frac{E^2}{2\mu_0} \times \frac{\epsilon_0 \mu_0}{1}$$

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$\therefore E_T = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\epsilon_0 E^2$

$\Rightarrow \boxed{E_T = \epsilon_0 E^2}$ — (5)

This is energy density in an e.m wave in terms of Electric field.

(b) Energy density in terms of magnetic field for e.m wave.

Again Consider $E_T = \frac{1}{2}\epsilon_0 E^2 + \frac{B^2}{2\mu_0}$

But from $c = \frac{E}{B}$ we have $\boxed{E = B \cdot c}$ — (6)

put this value in E_T

$\therefore E_T = \frac{1}{2}\epsilon_0 B^2 c^2 + \frac{B^2}{2\mu_0}$

but $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$, hence $E_T = \frac{1}{2}\epsilon_0 B^2 \times \frac{1}{\mu_0 \epsilon_0} + \frac{B^2}{2\mu_0}$.

$\therefore E_T = \frac{B^2}{2\mu_0} + \frac{B^2}{2\mu_0} \Rightarrow \boxed{E_T = \frac{B^2}{\mu_0}}$.

$\therefore$ The expression for energy density in e.m wave interms of magnetic field is given by $\boxed{E_T = \frac{B^2}{\mu_0}}$

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Equation of continuity :

Let S be the closed surface enclosing a volume V and ds be the small element of this surface. If 'J' is the current density at a point on the surface element ds, then the divergence of J is the net current coming out from the surface.

The net outward current = $\oint_S \vec{J} \cdot \vec{ds}$

If the volume has charge density ρ, the rate of change of charge in the volume = $-\frac{\partial Q}{\partial t}$ $\quad Q \rightarrow$ total charge enclosed in the volume V.

$$= -\frac{\partial}{\partial t}\left(\int_V \rho \, dV\right)$$

∴ The net flow of current from the surface = Rate of change of charge in the volume.

$$\oint_S J \cdot ds = -\frac{\partial}{\partial t}\left(\int_V \rho \, dV\right) = -\int_V \left(\frac{\partial \rho}{\partial t}\right) dV.$$

By using divergence theorem to L.H.S,

$$\int_V (\nabla \cdot \vec{J}) \, dV = -\int \frac{\partial \rho}{\partial t} \, dV$$

$$\int_V \left(\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t}\right) dV = 0$$

$$\nabla \cdot J = -\frac{\partial \rho}{\partial t}.$$

This is called continuity equation (or) equation of continuity.

For time varying charges $\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$

For steady charges $\nabla \cdot \vec{J} = 0$.

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Poynting theorem: This theorem is used to find the power of electromagnetic wave using Maxwell's equations known as Poynting theorem. This theorem is analogous to work energy theorem in classical mechanics and mathematically similar to the continuity equation.

Suppose let us have some charge at a time t in the electric field E and magnetic field B. Let the charge be moved a bit in a time dt

The work done on the element of charge dq is time dt is $dw = \vec{F}\cdot d\vec{l} = \left[\vec{E} + (\vec{v}\times\vec{B})\right] dq \times \vec{v}\,dt$

$\left(\because dl = \vec{v}\,dt\right)$

$\therefore dw = \vec{E}\cdot dq\,\vec{v}\,dt + (\vec{v}\times\vec{B})\,\vec{v}\,dq\times dt$

but $(\vec{v}\times\vec{B})\cdot\vec{v} = 0$, then $\boxed{dw = \vec{E}\cdot\vec{v}\,dq\,dt}$ — ①

Let ρ → volume charge density, J → Current density

then $\boxed{dq = \rho\,dv}$ — ② $(\because q = \rho v)$

put ② in ①, we get

$$dw = \vec{E}\cdot\vec{v}\cdot\rho\,dv\,dt$$

The total work done on all charges per second is

$$\frac{dw}{dt} = \int(\vec{E}\cdot\vec{v})\cdot\rho\,dv \quad — ③$$

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put $\vec{\nabla}.\rho = J$, $J \rightarrow$ current density

$\left(\because \frac{x}{t} \times \frac{v}{A \times l} = \vec{\nabla} \times \rho = \frac{i}{A} = J\right)$

Then equation ③ becomes, $\boxed{\frac{dW}{dt} = \int (\vec{E}.\vec{J})\, dV}$ — ④

Consider Maxwell's 4th equation,

$$\nabla \times \vec{B} = \mu\left[\vec{J} + \frac{\partial D}{\partial t}\right] = \left(\mu \vec{J} + \mu\epsilon \frac{\partial E}{\partial t}\right)$$

$\epsilon \rightarrow$ permittivity of a medium

$\mu \rightarrow$ permeability.

$$\Rightarrow \nabla \times \vec{B} = \mu \vec{J} + \mu\epsilon \frac{\partial E}{\partial t}$$

$$\vec{J} = \frac{\nabla \times B}{\mu} - \epsilon \frac{\partial E}{\partial t}$$

Now $\vec{E}.\vec{J}$ can be written as,

$$\vec{E}.\vec{J} = \frac{\vec{E}.(\nabla \times \vec{B})}{\mu} - \epsilon \vec{E} \frac{\partial E}{\partial t} \quad — ⑤$$

To find $E.(\nabla \times \vec{B})$ in equation ⑤ Consider

$$\vec{\nabla}.(\vec{E} \times \vec{B}) = \vec{B}(\nabla \times \vec{E}) - \vec{E}(\nabla \times \vec{B})$$

$$\Rightarrow \vec{E}(\nabla \times \vec{B}) = \vec{B}(\nabla \times E) - \nabla(\vec{E} \times \vec{B}) \quad — ⑥$$

put ⑥ in ⑤, we get

$$\vec{E}.\vec{J} = \frac{\vec{B}(\nabla \times \vec{E})}{\mu} - \frac{\nabla(\vec{E} \times \vec{B})}{\mu} - \epsilon \vec{E} \frac{\partial E}{\partial t} \quad — ⑦$$

but $\nabla \times E = -\frac{\partial B}{\partial t}$, from Maxwell's 3rd equation

then ⑦ becomes, $\vec{E}.\vec{J} = \frac{\vec{B}}{\mu}\left(-\frac{\partial B}{\partial t}\right) - \nabla\frac{(\vec{E} \times \vec{B})}{\mu} - \epsilon E \frac{\partial E}{\partial t}$

but $\vec{B}.\frac{\partial \vec{B}}{\partial t} = \frac{1}{2}\frac{\partial}{\partial t}(B^2)$ and $\vec{E}\frac{\partial \vec{E}}{\partial t} = \frac{1}{2}\frac{\partial}{\partial t}(\vec{E}^2)$

then ⑦ becomes, $\vec{E}.\vec{J} = -\frac{1}{2\mu}\frac{\partial}{\partial t}(B^2) - \frac{\epsilon}{2}\frac{\partial}{\partial t}(E^2) - \nabla\frac{(\vec{E} \times \vec{B})}{\mu}$

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$$\therefore \frac{dW}{dt} = \int (\vec{E} \cdot \vec{J})\, dV$$

$$= -\frac{1}{2} \frac{d}{dt} \iiint_V \left(\epsilon E^2 + \frac{B^2}{\mu}\right) dV - \frac{1}{\mu} \int_V \nabla \cdot (\vec{E} \times \vec{B})\, dV.$$

$$\therefore \boxed{\frac{dW}{dt} = P = -\frac{d}{dt} \int_V \frac{1}{2} \left(\epsilon E^2 + \frac{B^2}{\mu}\right) dV - \frac{1}{\mu} \int_S (\vec{E} \times \vec{B})\, ds}$$

(∵ From Gauss-Divergence theorem)

This equation is called Poynting's theorem.

Note :- $$\iint_S (\vec{E} \times \vec{B})\, ds = \iiint_V \nabla \cdot (\vec{E} \times \vec{B})\, dV$$

Gauss – Divergence theorem.

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