

UNIT-II Electrostatics - I

Static electricity: The study of behaviour of charge at rest is called static electricity.

In a body, if the electrons are excess or deficient in number, then that body is called charged body.

Electric field: The region (or) a space around a charge in which its influence is felt is called Electric field. According to Coulomb's law, when a charged body is placed near a charge, it will experience a force. That region of the space in which a charged body can experience a force, is called Electric field.

Let a charge 'q' be placed in an uniform electric field of intensity 'E', then the force experienced by the charge 'q' is F , $F = Eq$

$$F = Eq \quad \text{or} \quad E = \frac{F}{q}$$

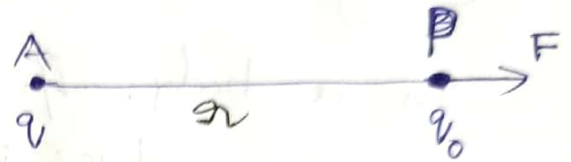
Electric field intensity is defined as the force experienced by unit +ve charge placed at a point in the electric field. $\therefore E = F$ when $q = 1$ coulomb.

It is measured by the force being experienced by a charged particle placed at that point.

Electric field intensity (E) at a point due to an isolated point charge (q) :-

Consider a point charge 'q' at a point 'A'.

Consider a point 'P' in the electric field at a distance of 'r' from 'q'.



Let a test charge 'q_0' (+ve) be placed at 'P'.

Therefore, there exists a force of repulsion between the two charges.

According to Coulomb's law, the Coulombic force of repulsion, $F = \frac{1}{4\pi\epsilon_0} \frac{q \cdot q_0}{r^2}$ — (1)

But, from the definition of electric field intensity $E = \frac{F}{q_0}$ — (2) ($\because F = E q_0$)

From (1) & (2), we have $E = \frac{1}{4\pi\epsilon_0} \frac{q \cdot q_0}{r^2} \times \frac{1}{q_0}$

$$\therefore \boxed{E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}}$$

This is the expression for electric field intensity 'E' at a point due to point charge 'q'.

Gauss law: It states that the total electric flux over a closed surface is equal to $\frac{1}{\epsilon_0}$ times the total charge enclosed by the same surface.

$$\therefore \phi = \int \vec{E} \cdot d\vec{s} = \frac{1}{\epsilon_0} \times q$$

proof:- Consider a closed surface of irregular

Shape with a charge $+q$ at a point A .

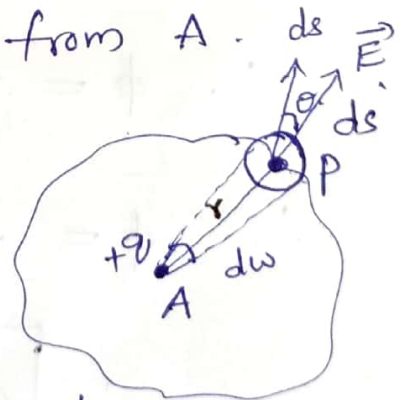
This surface is called Gaussian Surface.

Since the charge is +ve, the electric field is radially outwards. Take a small element of area ds around the point P , which is at a distance of r from A .

A normal to the surface ds is represented by a vector

$\vec{ds}$ which makes an angle θ

with the field direction (E) along AP .



Since the electric field E is uniform, E will have same magnitude & direction at every point on the small element.

$\therefore$ Both $\vec{E}$ and $\vec{ds}$ are parallel to each

- other.

Now, the electric flux through the small element

'ds' is $d\phi = \vec{E} \cdot \vec{ds} = E ds \cos \theta = E ds$ — (1)
($\because \theta = 0$)

By Coulomb's law, the electric field intensity 'E' at a point 'P', because of charge '+q' is

$$E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \quad \text{--- (2)}$$

This is the field intensity at 'P'.

From (1) & (2), $d\phi = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} ds$.

The total electric flux over the entire surface

is $\phi = \int d\phi = \int \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} ds$

$$\therefore \phi = \frac{q}{4\pi\epsilon_0 r^2} \int ds \quad \left\{ \begin{array}{l} \because \int ds = 4\pi r^2 \\ \text{surface area} \\ \text{of sphere.} \end{array} \right.$$

$$\therefore \phi = \frac{q}{4\pi\epsilon_0 r^2} \times 4\pi r^2$$

$$\therefore \boxed{\phi = \frac{1}{\epsilon_0} \times q}$$

where $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2$

$$= 8.85 \times 10^{-12} \text{ F/m (S.I unit)}$$

Consider a small surface area (ds) on the given Gaussian surface. Both $\vec{E}$ and $\hat{n}$ are parallel to each other. Hence $\theta = 0^\circ$.



$\therefore$ The electric flux through the small surface (ds) is $d\phi = E ds \Rightarrow \phi = \int d\phi = \int E ds = E \int ds$

$$\therefore \phi = \frac{1}{4\pi\epsilon_0} \frac{q}{R^2} \int ds$$

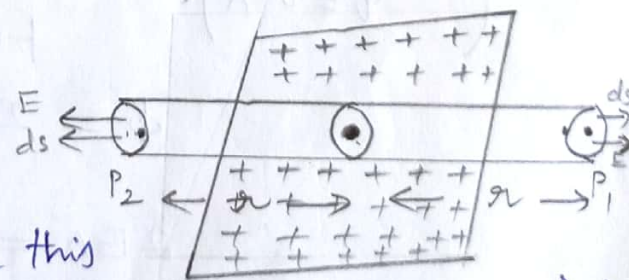
$$\therefore \phi = \frac{1}{4\pi\epsilon_0} \frac{q}{R^2} \cdot 4\pi R^2 \quad (\because \int ds = 4\pi R^2, \text{ surface area of sphere})$$

$$\therefore \boxed{\phi = \frac{1}{\epsilon_0} \cdot q}, \quad \epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N-m}^2$$

Gauss law applications:

(i) Electric field intensity due to uniformly charged infinite plane sheet:-

The following figure shows a portion of thin, non conducting infinite sheet of charge. Let σ be the surface charge density. Let P_1 be a point at a distance r_1 from the sheet where Electric field intensity is to be calculated. For this purpose, we consider a point P_2 symmetrical with P_1 on the other side of the sheet.



on the other side of the sheet.

Construct a cylindrical Gaussian surface with one end on one side and the other end on other side of the sheet. Let A be the area of cross section of each plane end. By symmetry E is everywhere $\perp$ to the plane. For the curved surface of Gaussian surface (cylinder), E and ds are $\perp$ to each other. Thus, the curved surface does not contribute to electric flux. For the end caps, as E and ds are parallel to each other, hence they contribute to electric flux.

$\therefore$ The electric flux through these end caps is given

$$\text{by } \phi = \int E ds + \int E ds \quad (\because \theta = 0)$$

$$\phi = EA + EA \Rightarrow \phi = 2EA$$

If σ be the surface charge density, then the total charge enclosed by the Gaussian cylinder is σA .
($\because q = \sigma A$).

From Gauss law, we have

$$\phi = 2EA = \frac{1}{\epsilon_0} \times \text{charge}$$

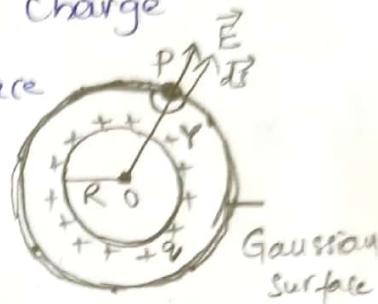
$$\therefore 2EA = \frac{1}{\epsilon_0} \sigma A \Rightarrow 2E = \frac{\sigma}{\epsilon_0}$$

$$\boxed{E = \frac{\sigma}{2\epsilon_0}}$$

Note:- Conducting means charge always only reside on surface.

(ii) Electric field due to a uniformly charged thin Spherical shell (or) conducting Sphere:-

Consider a thin Spherical shell of radius ' R ' with the centre ' O ' as shown. Let ' q ' be the charge on the Spherical shell. Let ' σ ' be the surface charge density of the Spherical shell.



Case-1: When the point ' P ' lies outside the Spherical Shell.

Consider a point ' P ' at a distance of ' r ' from the centre ' O ' where the electric field intensity is to be determined. Draw a Spherical Gaussian Surface passing through ' P ' with radius ' r '. Let $\vec{E}$ be the electric field intensity at ' P ' due to the charge ' q '.

At every point on the surface of Spherical shell, the field has same magnitude and is along normal to the surface.

$\therefore$ The total flux through the Gaussian Surface is $\phi = \int \vec{E} \cdot d\vec{s} = \int E ds \cos 0 = \int E ds = E \times 4\pi r^2$ —①

Since the charge enclosed by the Gaussian surface is ' q '. $\therefore$ According to Gauss law, $\phi = \frac{q}{\epsilon_0}$

$$\therefore \oint = E \times 4\pi r^2 = \frac{q}{\epsilon_0} \quad (i)$$

$$\therefore \boxed{E = \frac{q}{4\pi\epsilon_0 r^2}}$$

but σ be the surface charge density. Hence

$$\sigma = \frac{\text{charge}}{\text{Area}} \Rightarrow \text{charge} = \sigma \times \text{Area of original sphere} = \sigma \times 4\pi R^2$$

$$\therefore E = \frac{1}{4\pi\epsilon_0 r^2} \times \frac{\sigma \times 4\pi R^2}{1}$$

$$\therefore \boxed{E = \frac{\sigma}{\epsilon_0} \frac{R^2}{r^2}} \quad (\text{for } r > R)$$

(ii) when the point 'P' lies on the surface.

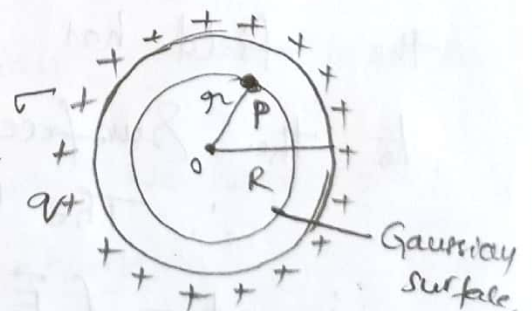
Here $R = r$

$$\therefore E = \frac{\sigma}{\epsilon_0} \frac{R^2}{R^2} \Rightarrow \boxed{E = \frac{\sigma}{\epsilon_0}}$$



(iii) when the point 'P' lies inside the Spherical shell.

when the point 'P' lies inside the shell at a distance of 'r' from 'O'.



As the charge only resides on the surface of shell, $q = 0$ inside the shell. ie no charge is enclosed by Gaussian surface.

∴ The electric flux over the Gaussian Surface is $\phi = \int E \cdot d\vec{s} = \frac{q}{\epsilon_0}$

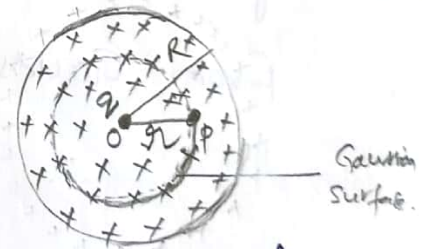
$$\Rightarrow E \times 4\pi r^2 = 0 \Rightarrow \boxed{E=0}$$

(iii) Electric field due to uniformly charged non-conducting solid sphere:

Consider a uniformly charged sphere of radius 'R' with a charge 'q'. Let 'ρ' be the volume charge density. To find the electric field at a point outside the sphere and inside the sphere consider the following cases.

Case-1: When the point 'P' lies inside the solid sphere ($r < R$)

Consider a solid sphere of radius 'R'. Let the charge 'q' be uniformly distributed over a sphere of radius 'R'. Consider a point 'P' inside the sphere at a distance of 'r' from 'O'. Construct a spherical Gaussian surface of radius 'r' passing through the point 'P'. The charge enclosed by the Gaussian surface is equal to charge density (ρ) × volume occupied by the sphere of radius (r).



$$\therefore \text{charge enclosed by Gaussian sphere } (q') = \rho \times \frac{4}{3}\pi r^3$$

$$\therefore q' = \frac{q}{\frac{4}{3}\pi R^3} \times \frac{4}{3}\pi r^3$$

$$\therefore \boxed{q' = q \left(\frac{r^3}{R^3} \right)}$$

$\therefore$ From Gauss law, We have $\int \vec{E} \cdot d\vec{s} = \frac{1}{\epsilon_0} \times q$

$$\therefore E \times 4\pi r^2 = \frac{1}{\epsilon_0} \times q \frac{r^3}{R^3}$$

$$\therefore \boxed{E = \frac{1}{4\pi\epsilon_0} \frac{q r}{R^3}}$$

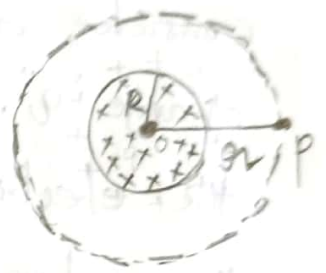
$$\therefore \boxed{E \propto r}$$

Case-2: when 'P' lies outside the sphere. ($r > R$)

Consider a point 'P' at a distance of 'r' from the centre 'O' of the original sphere.

Construct a Spherical Gaussian Surface of radius 'r' passing through 'P'. The Charge

of the gaussian surface is same as that of the original sphere. i.e same amount of charge is distributed.



From Gauss law, we have $\int E \cdot ds = \frac{1}{\epsilon_0} \times \text{charge of gaussian surface}$

$$\therefore E \times 4\pi r^2 = \frac{q}{\epsilon_0}$$

$$\therefore \boxed{E = \frac{q}{4\pi\epsilon_0 r^2}} \Rightarrow \boxed{E \propto \frac{1}{r^2}}$$

Case-3: when the point 'P' lies at the centre of sphere

At the centre of the sphere $r=0$

$$\text{Hence } \boxed{E = 0}$$

on the surface of the sphere $r=R$ and

$$\boxed{E = \frac{1}{4\pi\epsilon_0} \frac{q}{R^2}}$$

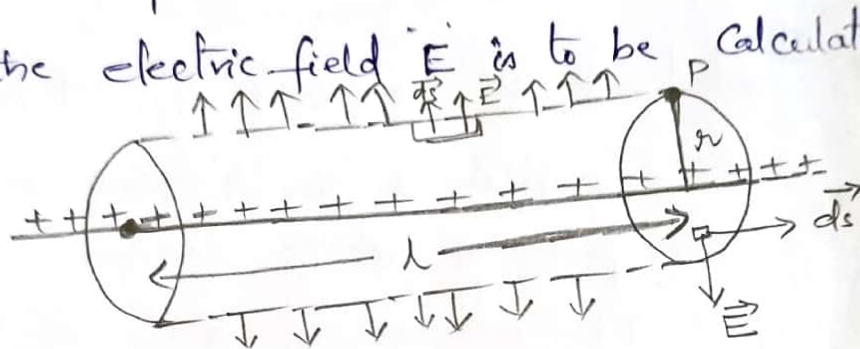


Electric field intensity due to charge distribution (on)

Electric field due to a line of charge :-

Consider a line of charge. Let ' λ ' be the linear charge density (or) charge per unit length of a wire.

Consider a point 'P' at a distance ' r ' from the axis, where the electric field ' E ' is to be calculated.



Construct a cylindrical Gaussian surface of length ' l ' and radius ' r ' on the line of charge around the point 'P'. The electric flux due to end caps (circular ends) of the cylindrical Gaussian surface comes out as zero, because E and ds are at right angles to each other. i.e. $E ds \cos 90^\circ = 0$. Due to symmetry, the electric field will be the same at all points of the curved surface and is directed radially outward. For the curved surface, E and ds are along the same direction. i.e. the angle between them is zero.

The total flux through the curved surface of the Gaussian cylindrical surface $\phi = \int E ds \cos 0 = \int E ds$

$$\therefore \phi = E \cdot \int ds \Rightarrow \phi = E \times 2\pi r l \quad \text{--- (1)}$$

But from Gauss law, $\phi = \frac{\text{charge}}{\epsilon_0} = \frac{\lambda \times l}{\epsilon_0}$ — (2)

from (1) & (2), $E \times 2\pi r l = \frac{\lambda l}{\epsilon_0}$

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

Coulomb's law from Gauss law

Consider an isolated point charge

'q' as shown. Construct a

spherical Gaussian surface

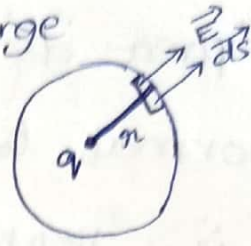
of radius 'r'. At any point on the spherical

surface the electric field intensity E will

have the same magnitude and direction

normal to the surface. Both E and ds are

directed outward.



Thus the total electric flux over a spherical surface is given by $\int E \cdot ds$

According to Gauss law, this must be equal to $\frac{q}{\epsilon_0}$

$$\int E \cdot ds = \frac{q}{\epsilon_0} \Rightarrow \epsilon_0 \int E \cdot ds = q$$

$$(\because E \cdot ds = E ds \cos \theta = E ds \cos 0^\circ = E ds)$$

As E is constant every where at the spherical

surface, hence $\epsilon_0 E \oint ds = q$

$$\epsilon_0 E \times 4\pi r^2 = q$$

$$(\because \oint ds = 4\pi r^2)$$

$$E = \frac{q}{4\pi\epsilon_0 r^2}$$

If a second charge q_0 is placed at the spherical surface, it experiences a force $F = E q_0$

$$\therefore F = \frac{q q_0}{4\pi\epsilon_0 r^2} = \frac{1}{4\pi\epsilon_0} \times \frac{q q_0}{r^2}$$

Electric Potential: The amount of work done on a unit +ve test charge to bring it from ∞ to a point in the electric field is called electric Potential.

$$\therefore V = \frac{W}{q_0} = \frac{U_P - U_\infty}{q_0}, \quad \begin{matrix} U_P \rightarrow \text{P.E at P point.} \\ U_\infty \rightarrow \text{P.E at } \infty = 0. \end{matrix}$$

Let a test charge at ∞ be q_0 , then the work done on it to bring it to a point P in the electric field of charge q .

The electric Potential is against the electric force.



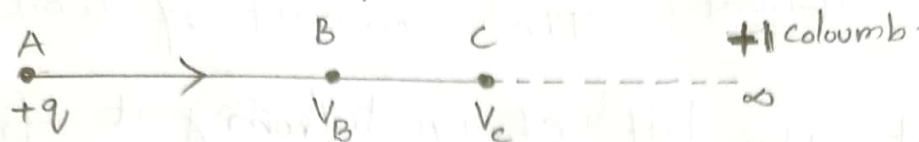
It is a scalar Quantity. S.I unit is $\frac{\text{Joule}}{\text{Coulomb}}$ (or) 1 Volt

Dimensional formula is $[ML^2T^{-3}I^{-1}]$ ($\because V = \frac{W}{q} = \frac{F \times S}{i \cdot t}$)

Volt definition: If 1 Joule of work is done in moving 1 Coulomb of charge from ∞ to a point P in the electric field, then that work is said to be 1 Volt.

Potential Difference:- The amount of work done in bringing a unit +ve test charge from one point to another point in the electric field is called Potential Difference between the two points.

Consider a positive charge '+q' at 'A'. Let V_B & V_C be the electric potentials at B and 'c' respectively.



The work done on unit '+ve' charge in moving from ∞ to 'c' is V_C and from ∞ to 'B' is V_B .

$$\text{Then } V_{CB} = V_B - V_C, \quad V_{BC} = V_C - V_B$$

$$\therefore V_{CB} = -V_{BC} \text{---(1)}$$

→ Work done is zero in the direction of Electric field.

$$\text{Theoretically, } V_{CB} = V_B - V_C, \quad V_{BC} = V_C - V_B$$

$$\therefore V_{CB} + V_{BC} = V_B - V_C + V_C - V_B = 0$$

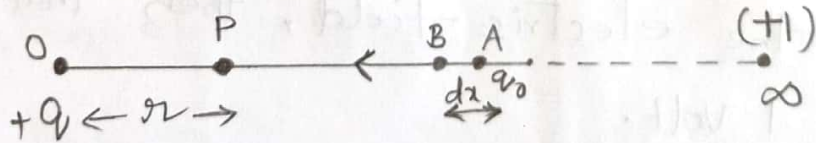
$\therefore$ Theoretically it is zero.

→ Potential difference is always measured opposite to electric field intensity.

Electric Potential due to Point charge:

Consider a +ve charge $\vec{E}$ $\vec{F}$

'+q' at 'O'. Consider a unit charge at ∞ . Consider a point



'P' at a distance of 'r' from 'O' where electric potential is to be measured. Consider a small element 'AB' of length 'dx'. let q_0 be the test charge at 'A'

∴ The force exerted on q_0 at A due to the point charge $+q$ is $F = \frac{1}{4\pi\epsilon_0} \frac{qq_0}{x^2}$ (from Coulomb's law)

if the charge q_0 is displaced through a displacement of dx , then the work done for this small displacement dx is dw . ∴ $dw = F \cdot dx$

$$\therefore dw = \frac{1}{4\pi\epsilon_0} \frac{qq_0}{x^2} dx$$

The total work done

$$w = \int_{\infty}^x dw = \frac{1}{4\pi\epsilon_0} \int_{\infty}^x \frac{qq_0}{x^2} dx = \frac{qq_0}{4\pi\epsilon_0} \int_{\infty}^x \frac{1}{x^2} dx$$

$$\therefore w = \frac{qq_0}{4\pi\epsilon_0} \left[\frac{-2+1}{x} \right]_{\infty}^x \Rightarrow w = -\frac{qq_0}{4\pi\epsilon_0} \left[\frac{1}{x} \right]_{\infty}^x$$

$$\therefore w = -\frac{qq_0}{4\pi\epsilon_0} \left[\frac{1}{x} - \frac{1}{\infty} \right]$$

$$w = -\frac{1}{4\pi\epsilon_0} \times \frac{qq_0}{x}$$

∴ From definition electric potential $V = \frac{w}{q_0}$

$$\therefore V = \frac{-\frac{1}{4\pi\epsilon_0} \frac{qq_0}{x}}{q_0} = -\frac{1}{4\pi\epsilon_0} \frac{q}{x}$$

$$\therefore \boxed{V = -\frac{1}{4\pi\epsilon_0} \frac{q}{x}}$$

The electric potential due to point charge is $\frac{1}{4\pi\epsilon_0} \frac{q}{x}$

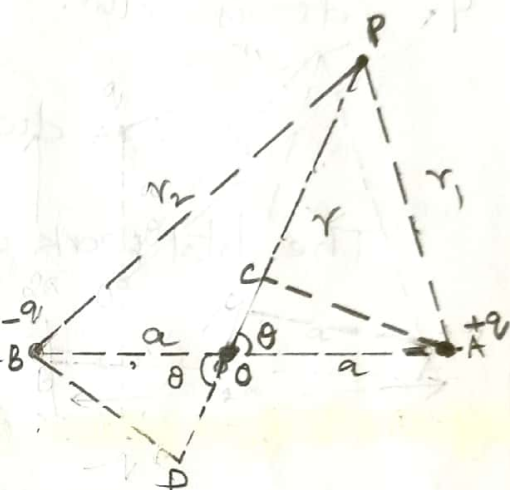
The -ve sign is due to 'V' is always opposite to force.

Electric Potential due to an electric Dipole:-

An electric dipole consists of two equal and opposite charges separated by a very small distance.

Let q be the magnitude of the charge and $2a$ be the length of the dipole, then the electric dipole-moment, $p = q \times 2a$

Consider a dipole of charges $+q$ & $-q$ respectively. Take a point 'P' at a distance r from the centre of the dipole where the electric potential is to be determined.



The potential at the point 'P' due to dipole is the sum of the potentials due to the charges q and $-q$

$$\therefore V = \frac{1}{4\pi\epsilon_0} \left[\frac{q}{r_1} - \frac{q}{r_2} \right] \quad \text{--- (1)}$$

$r_1 \rightarrow$ distance of P from $-q$
 $r_2 \rightarrow$ distance of P from $+q$

From geometry, $CP \approx AP$ and from ΔAOC

$$\therefore \boxed{OC = a \cos \theta} \text{ and } CP \approx AP.$$

$$\cos \theta = \frac{OC}{OA} = \frac{OC}{a}$$

Now extend OP upto D and draw a $\perp$ from B to D

$\therefore PB \approx PD$ and from ΔBOD , we have

$$\cos \theta = \frac{OD}{OB} \Rightarrow \boxed{OD = a \cos \theta}$$

$$r_1 = OP - OC \Rightarrow \boxed{r_1 = r - a \cos \theta} \quad \text{--- (2)}$$

$$r_2 = OP + OD \Rightarrow \boxed{r_2 = r + a \cos \theta} \quad \text{--- (3)}$$

$\therefore$ Equation ① becomes, $V = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{r_1} - \frac{1}{r_2} \right]$

put ② and ③ in equation ①, we get

$$\therefore V = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{r - a \cos \theta} - \frac{1}{r + a \cos \theta} \right] = \frac{q}{4\pi\epsilon_0} \left[\frac{r + a \cos \theta - r + a \cos \theta}{(r - a \cos \theta)(r + a \cos \theta)} \right]$$

$$\therefore V = \frac{q}{4\pi\epsilon_0} \left[\frac{2a \cos \theta}{r^2 - a^2 \cos^2 \theta} \right] = \frac{p \cos \theta}{4\pi\epsilon_0 (r^2 - a^2 \cos^2 \theta)}$$

$\therefore$ The electric potential at 'p' due to dipole is

$$V = \frac{p \cos \theta}{4\pi\epsilon_0 (r^2 - a^2 \cos^2 \theta)}$$

For a short dipole, $r \gg a$, then $a^2 \cos^2 \theta$ can be neglected compared to r^2 . ($\because \cos \theta = 1$, max)

$$\therefore V = \frac{p \cos \theta}{4\pi\epsilon_0 r^2}$$

Special cases:- ① potential on the axial line of dipole when the point 'p' is taken on the axis of dipole, then

$$\theta = 0, \therefore V = \frac{p}{4\pi\epsilon_0 r^2} \text{ volts}$$

② when the point 'p' is on the equatorial line, then

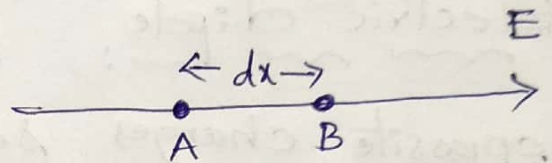
$$\theta = 90^\circ, \therefore V = \frac{p \cos 90^\circ}{4\pi\epsilon_0 r^2} = 0 \Rightarrow V = 0$$

Gradient relationship between 'E' and 'V'

Consider a uniform electric field of intensity 'E'. Take two points A and B with a distance of 'dx' in the given electric field 'E' as shown.

Let a charge 'q' be placed at 'B'. Because of the charge 'q' at 'B', it experiences a force

of $F = Eq$ ($\because F = Eq$)



$\therefore$ The work done to move the charge from B to A is 'dv', is given by $dv = \frac{W_{BA}}{q}$

$$\Rightarrow W_{BA} = dv \cdot q$$

$$\therefore W_{BA} = (V_A - V_B) q$$

$$\text{(or)} \quad V_A - V_B = \frac{W_{BA}}{q} = \frac{\text{Force} \times dx}{q}$$

$$\therefore dv = V_A - V_B = \frac{Eq \times dx}{q}$$

$$\Rightarrow dv = E \cdot dx$$

$$\Rightarrow \boxed{E = -\frac{dv}{dx}}$$

Let E_x, E_y, E_z be the fields in x, y & z axes,

then we have $E_x = -\frac{\partial v}{\partial x}, E_y = -\frac{\partial v}{\partial y}, E_z = -\frac{\partial v}{\partial z}$

Then $\vec{E} = E_x \hat{i} + E_y \hat{j} + E_z \hat{k}$

$$\therefore \vec{E} = -\frac{\partial V}{\partial x} \hat{i} - \frac{\partial V}{\partial y} \hat{j} - \frac{\partial V}{\partial z} \hat{k}$$

$$\vec{E} = -\left(\frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k}\right) V$$

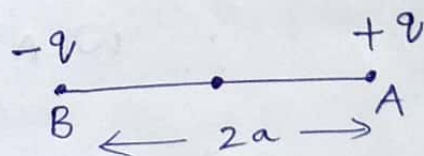
$$\vec{E} = -\nabla \cdot V \Rightarrow \boxed{E = -\nabla V}$$

$$\therefore \boxed{E = -\nabla \cdot V}$$

Electric dipole: A system of two equal and opposite charges separated by a distance is called a dipole. It is purely an imaginary concept but in magnets, there is a reality.

Electric dipole moment: ($\vec{P}$)

The product of any one of the two charges and the distance between them is called dipole-moment. It is denoted by $\vec{P}$.

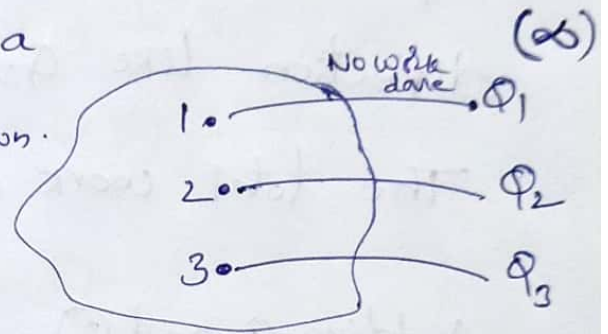


$$\vec{P} = q \times 2a$$

Energy density in Electrostatic fields:

It is defined as the total amount of energy stored in a system per unit volume.

Consider an empty region when there is no electric field at all. Consider a set of 3-charges as shown.



To bring the charges from ∞ to some region, some work has to be done.

Initially the charge Q_1 is moved from ∞ to a point 1 in the region. Then it requires no work because E is absent initially (i.e. charge free)

$$\therefore \text{work done to move } Q_1 = 0$$

$$\therefore \boxed{W_1 = 0}$$

work done in moving Q_2 into the region to the point 2 is given by $W_2 = Q_2 V_{21}$, ($\because V = \frac{W}{Q}$)

Here $V_{21} \rightarrow$ Potential at 2 due to the charge Q_1 (at 1).

When a charge Q_3 is brought to the point 3

the work done (W_3) = $Q_3 V_{32} + Q_3 V_{31}$
($\because$ there are electric fields due to Q_1 and Q_2).

∴ The total work done (W) = $W_1 + W_2 + W_3$

$$\therefore W = 0 + Q_2 V_{21} + Q_3 (V_{32} + V_{31})$$

$$\therefore W = Q_2 V_{21} + Q_3 (V_{32} + V_{31}) \text{ --- (1)}$$

If we assemble the system of charges in reverse direction like Q_3 , Q_2 and Q_1

$$\text{The total work done } (W) = Q_2 V_{23} + Q_1 (V_{13} + V_{12}) \text{ --- (2)}$$

Adding (1) and (2)

$$W + W = Q_2 V_{21} + Q_3 (V_{32} + V_{31}) + Q_2 V_{23} + Q_1 (V_{13} + V_{12})$$

$$\therefore 2W = Q_1 (V_{12} + V_{13}) + Q_2 (V_{21} + V_{23}) + Q_3 (V_{31} + V_{32})$$

$$2W = Q_1 V_1 + Q_2 V_2 + Q_3 V_3$$

$$\therefore W = \frac{1}{2} (Q_1 V_1 + Q_2 V_2 + Q_3 V_3)$$

$$\left\{ \begin{array}{l} \therefore \text{let } V_{12} + V_{13} = V_1 \\ V_{21} + V_{23} = V_2 \\ V_{31} + V_{32} = V_3 \end{array} \right\}$$

$$\text{for } n\text{-point charges, } W = \frac{1}{2} \sum_{i=1}^n Q_i V_i \text{ --- (3)}$$

∴ The total electrostatic energy (W) of a system of point charges is the sum of energies of all possible points of charges.

If the charge is distributed in a volume,

$$\text{then } W = \frac{1}{2} \int \rho V \, dv.$$

$$\left(\because p = \frac{dq}{dv} \Rightarrow dq = p dv \text{ (or)} q = \int_V p dv \right)$$

$$\therefore W = \frac{1}{2} \int_V p dv$$

$$W = \frac{1}{2} \int_V (\nabla \cdot \vec{D}) v dv \quad \text{--- (4)} \quad \left\{ \begin{array}{l} \because \nabla \cdot \vec{D} = \rho \\ \text{and } \vec{D} = \epsilon_0 \vec{E} \end{array} \right.$$

$$\text{we have } \nabla \cdot v(\vec{D}) = \nabla v \cdot \vec{D} + v(\nabla \cdot \vec{D})$$

$$\Rightarrow (\nabla \cdot \vec{D}) v = \nabla \cdot v(\vec{D}) - \nabla v \cdot \vec{D} \quad \text{--- (5)}$$

$$\text{put (5) in (4), } W = \frac{1}{2} \int_V (\nabla \cdot v(\vec{D}) - \nabla v \cdot \vec{D}) dv$$

$$\therefore W = \frac{1}{2} \left[\int_V \nabla \cdot v \vec{D} dv - \int_V \nabla v \cdot \vec{D} dv \right]$$

$$\therefore W = \frac{1}{2} \left[\int_S (v \cdot \vec{D}) ds - \int_V \nabla v \cdot \vec{D} dv \right] \quad \left\{ \begin{array}{l} \because \text{By Gauss divergence theorem} \\ \text{ie } \int_V \nabla \cdot (v \vec{D}) dv = \int_S v \vec{D} \cdot \vec{n} ds \end{array} \right.$$

'V' varies as $\frac{1}{r^2}$ and 'D' as $\frac{1}{r^3}$ for dipoles, hence VD in first term on RHS must be at least as $\frac{1}{r^3}$, while ds varies as r^2 .

$$\text{For larger surfaces, } \frac{1}{2} \int_S (v \cdot \vec{D}) ds = 0$$

$$\therefore W = -\frac{1}{2} \int_V (\nabla v \cdot \vec{D}) dv = \frac{1}{2} \int_V (-\nabla v) \cdot \vec{D} dv$$

$$\therefore W = \frac{1}{2} \int_V (\vec{E} \cdot \epsilon_0 \vec{E}) dv \quad (\because \vec{D} = \epsilon_0 \vec{E})$$

$$\left(\because p = \frac{dq}{dv} \Rightarrow dq = p dv \text{ (a)} \quad q = \int_V p dv \right)$$

$$\therefore W = \frac{1}{2} \int p dv$$

$$W = \frac{1}{2} \int_V (\nabla \cdot \vec{D}) v dv - (4) \quad \left\{ \begin{array}{l} \because \nabla \cdot \vec{D} = \rho \\ \text{and } \vec{D} = \epsilon_0 \vec{E} \end{array} \right\}$$

$$\text{we have } \nabla \cdot v(\vec{D}) = \nabla v \cdot \vec{D} + v(\nabla \cdot \vec{D})$$

$$\Rightarrow (\nabla \cdot \vec{D}) v = \nabla \cdot v(\vec{D}) - \nabla v \cdot \vec{D} - (5)$$

$$\text{put (5) in (4), } W = \frac{1}{2} \int_V (\nabla \cdot v(\vec{D}) - \nabla v \cdot \vec{D}) dv$$

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$$\therefore W = \frac{1}{2} \left[\int_S (v \cdot \vec{D}) ds - \int_V \nabla v \cdot \vec{D} dv \right] \quad \left\{ \begin{array}{l} \because \text{By Gauss divergence theorem} \\ \text{ie } \int_V \nabla \cdot (v \vec{D}) dv = \int_S v \vec{D} \cdot \vec{n} ds \end{array} \right\}$$

'V' varies as $\frac{1}{r^2}$ and 'D' as $\frac{1}{r^3}$ for dipoles, hence VD in first term on RHS must be at least as $\frac{1}{r^3}$, while ds varies as r^2 .

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$$\therefore W = \frac{1}{2} \int_V (\vec{E} \cdot \epsilon_0 \vec{E}) dv \quad (\because \vec{D} = \epsilon_0 \vec{E})$$

$$\therefore W = \frac{1}{2} \epsilon_0 \int E^2 dv$$

Differentiating the above equation,

$$dw = \frac{1}{2} \epsilon_0 E^2 dv$$

$$\therefore \frac{dw}{dv} = \frac{1}{2} \epsilon_0 E^2$$

This $\frac{dw}{dv}$ is called energy density.

$$W_e = U = \frac{1}{2} \epsilon_0 E^2 = \frac{1}{2} \frac{D^2}{\epsilon_0}$$

$$\therefore \text{Energy density } \left[U = \frac{1}{2} \epsilon_0 E^2 = \frac{1}{2} \frac{D^2}{\epsilon_0} \right]$$

Fields inside a perfect Conductor:

(i) $E=0$ inside a conductor why?

Because when we put a conductor in an external electric field of intensity E_0 . Then Electric field exerts a force on the charges of the conductor. As a

result -ve charges of a conductor move towards the left edge of the conductor and

+ve charges moves towards the other edge of the conductor. Therefore, no charges are there

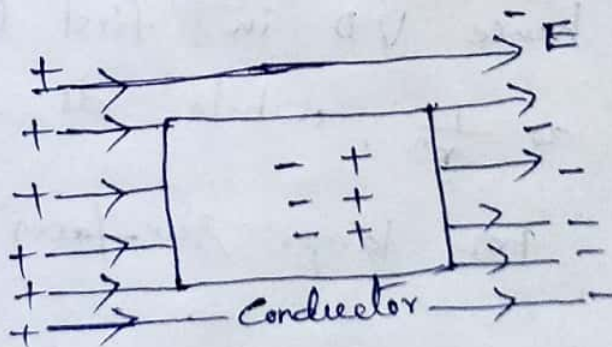
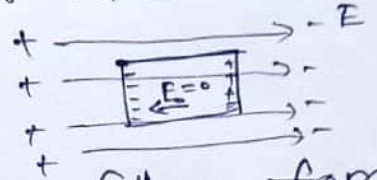


fig (a).

inside a conductor. These induced charges at the two edges in turn produce a new electric field which is opposite to the external electric field. This means that the field of the induced charges tends to cancel the original electric field. Hence the resultant electric field inside the conductor is zero.



(ii) $\rho = 0$ inside a conductor: This follows from Gauss law. i.e. $\nabla \cdot \mathbf{E} = \rho / \epsilon_0$ ($\because \nabla \cdot \mathbf{D} = \rho$)

If $\mathbf{E}$ is zero, then ρ is also zero.

There is still as much as +ve and -ve charge around. So, the net charge density in the interior is zero.

(iii) Any net charge resides on the surface.

(iv) A conductor is an equipotential:

If a, b are any two points within (or) at the surface of a conductor. Then

$$V(b) - V(a) = - \int_a^b \mathbf{E} \cdot d\mathbf{l} = 0, \text{ and hence } V(a) = V(b)$$

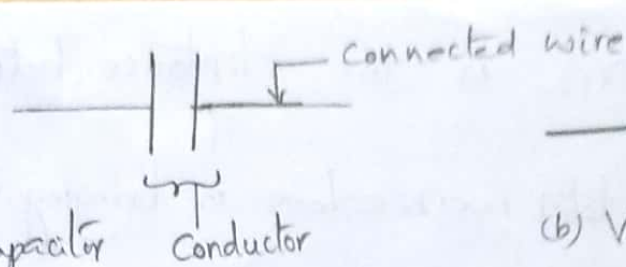
(v) Outside the conductor, the field is not zero, for here $\mathbf{E}$ and $\mathbf{E}_0$ do not tend to cancel.



Capacitor

It is a device to store large amount of charge at low potential. It is a system of two conductors (plates) separated by an insulating medium. A Capacitor is made up of two plates of any shape. For convenience we use only two straight plates of conductor, one should be the '+ve' and other is '-ve'. The space between them may be a medium, vacuum (or) free space.

The symbol to represent the Capacitor is



(b) Variable Capacitor



The charge on the '+ve' plate is called charge of the Capacitor.

The Potential difference between the two plates is called potential of Capacitor. ($V = V_1 - V_2$).

Capacitance:

The Capacity to store the charge inside a Capacitor is called Capacitance.

Here charge stored proportional to Potential difference

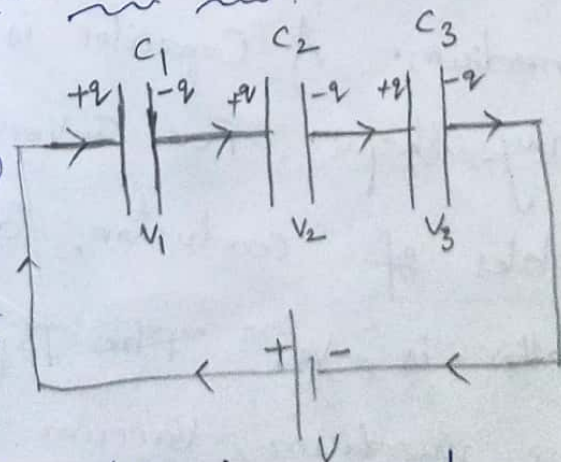
between the two plates. $q \propto V \Rightarrow \boxed{q = CV}$

where 'c' is a constant called proportionality constant, known as Capacitance. $C = \frac{q}{V} = \frac{\text{charge}}{\text{potential}}$

Capacitance depends on (i) Size (ii) shape (iii) medium between the plates. S.I unit of Capacitance is farad (or) Coulomb/volt.

Combination of Capacitors in Series:

If the Second plate of the Capacitor-1 is connected to the 1st plate of the 2nd Capacitor, Second plate of the Second Capacitor is connected to the first plate of the 3rd Capacitor



and 2nd plate of last capacitor is connected to 1st plate of 1st capacitor across a source (battery) of potential difference 'V'.

As the capacitors system is connected to a source, left plate of 1st Condenser acquires +ve charge due to conduction. This in turn will produce -ve charge on the second plate of 1st capacitor due to induction. This process continues for the remaining two condensers. Hence, the charge acquired by all three capacitors will be the same.

As the capacitors are different, the potentials developed across them will be different.

Let V_1, V_2, V_3 be the potentials and C_1, C_2, C_3 be the capacitances of the 3 capacitors respectively.

$$\text{Then } q = C_1 V_1 = C_2 V_2 = C_3 V_3$$

$$\therefore V_1 = \frac{q}{C_1}, V_2 = \frac{q}{C_2}, V_3 = \frac{q}{C_3}$$

$\therefore$ The total potential across the combination is given by 'V', $V = V_1 + V_2 + V_3$

$$\therefore \frac{q}{C} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3} \Rightarrow$$

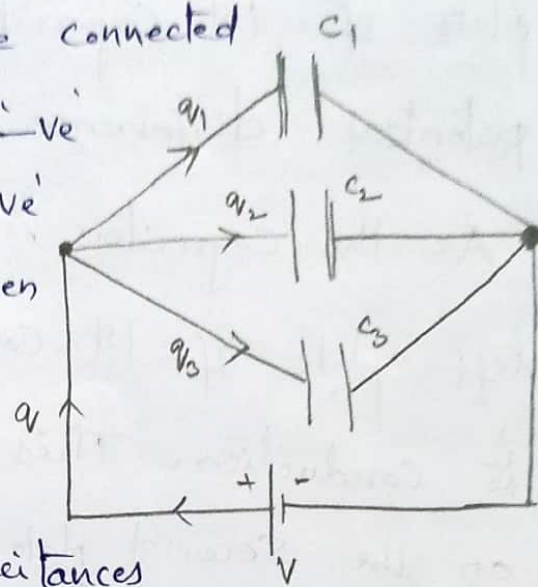
$$\boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

$\therefore$ For 'n' capacitors in series, then

$$\boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots + \frac{1}{C_n}}$$

Combination of Capacitors in Parallel

If all the '+ve' plates are connected to '+ve' terminal and all '-ve' plates are connected to '-ve' terminal of the source, then the connection is said to be Parallel Combination.



Let C_1, C_2, C_3 be the Capacitances of 3 capacitors connected in parallel across a source.

The charge drawn from source (battery) is shared to 3 branches (or) 3 capacitors. Hence, let q_1, q_2, q_3 be the charges across 3-capacitors.

As all capacitors are connected in parallel, the potential across any of the capacitors is same.

$$\therefore V = V_1 = V_2 = V_3$$

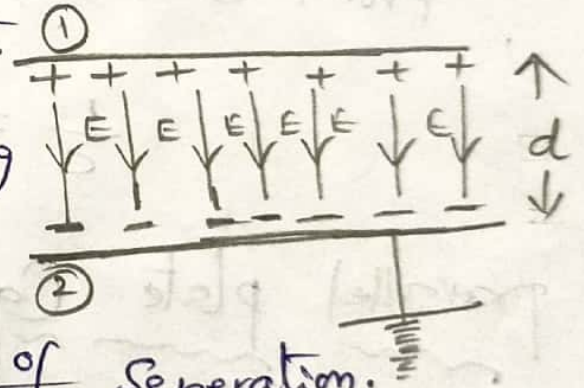
$$\text{But } q = q_1 + q_2 + q_3 \Rightarrow CV = C_1V + C_2V + C_3V$$

$$\therefore \boxed{C = C_1 + C_2 + C_3}$$

For n -Capacitors connected in parallel, then effective capacitance $\boxed{C = C_1 + C_2 + C_3 + \dots + C_n}$.

Parallel plate Capacitor (with out dielectric)

Consider a parallel plate Capacitor. It consists of two thin conducting plates of area of cross section 'A' each. Let 'd' be the distance of separation.



Let $+q$ be the charge on the inner surface of plate ① and the bottom face of the plate ② is connected to earth. Because of $+ve$ charge on the inner surface of plate ① a $-ve$ charge is induced on the inner surface of plate ②. Since the plates are uniform and parallel and hence a uniform electric field is established.

Let σ be the surface charge density. (ie $\sigma = \frac{q}{A}$)

As they are in the form of thin sheets & hence

$\therefore$ The electric field in the region between the two plates

$$\text{is } E = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} \Rightarrow E = \frac{2\sigma}{2\epsilon_0} \Rightarrow \boxed{E = \frac{\sigma}{\epsilon_0}}$$

$$\therefore E = \frac{q}{A\epsilon_0} \quad \text{--- (1)}$$

But if V is the potential difference between the plates and d is the distance between them, then $E = \frac{V}{d}$ --- (2)

$$\therefore \frac{V}{d} = \frac{q}{\epsilon_0 A} \Rightarrow \boxed{V = \frac{q}{\epsilon_0 A} d} \quad \text{--- (3)}$$

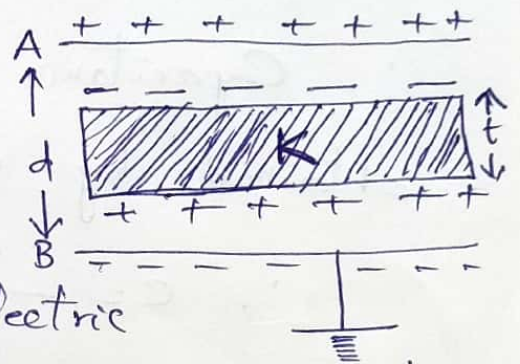
$$\text{but } V = \frac{q}{C} \Rightarrow \text{--- (4)}$$

From (3) & (4), we have $\frac{q}{C} = \frac{q}{\epsilon_0 A} d$

$$\frac{1}{C} = \frac{d}{\epsilon_0 A} \Rightarrow \boxed{C = \frac{\epsilon_0 A}{d}}$$

Parallel plate Capacitor (with dielectric) :

Consider a parallel plate capacitor. Let 'A' be the area of cross-section of each plate and 'Q' be the charge on the plates of the capacitor. Let 'd' be the distance between the two plates. Let ' σ ' be the surface charge density.



Let E_0 be the electric field produced due to the plates in the region. Consider a dielectric slab of thickness 't' and of dielectric constant 'K' is introduced between the plates. Then charges are induced on the surface of dielectric. As a result a new electric field ' E ' is induced due to dielectric. Hence the dielectric material is polarised.

We know that Capacitance of a parallel plate capacitor with out dielectric is $C_0 = \frac{\epsilon_0 A}{d}$

Electric potential between the plates $V = E \times \text{distance}$

$$\therefore V = (E \times t) + E_0 (d-t)$$

$$V = \frac{E_0}{K} t + E_0 (d-t) \quad (\because K = \frac{E_0}{E})$$

$$V = E_0 \left[\frac{t}{K} + d-t \right] \quad \text{--- ①}$$

but we know that $E_0 = \frac{\sigma}{\epsilon_0}$ (from Gauss law)

put ② in ①, we get $V = \frac{\sigma}{\epsilon_0} \left[\frac{t}{K} + (d-t) \right]$

$$\therefore V = \frac{\sigma}{\epsilon_0} \left[\frac{t + K(d-t)}{K} \right] = \frac{q}{\epsilon_0 A} \left[\frac{t + K(d-t)}{K} \right]$$

$$\therefore V = \frac{q}{\epsilon_0 A} \left[d + \frac{t}{K} - t \right]$$

$\therefore$ Capacitance of parallel plate Capacitor can be obtained by using $C = \frac{q}{V}$

$$\therefore C = \frac{q}{\frac{q}{\epsilon_0 A} \left[d - t + \frac{t}{K} \right]} = \frac{\epsilon_0 A}{d - t + \frac{t}{K}}$$

$$\therefore \boxed{C = \frac{\epsilon_0 A}{d - t \left[1 - \frac{1}{K} \right]}}$$

→ When the dielectric is completely filled between the plates of a Capacitor, then $d=t$

$$\therefore C = \frac{\epsilon_0 A}{d - d + \frac{d}{K}} = \frac{K \epsilon_0 A}{d}$$

$$\therefore \boxed{C = \frac{K \epsilon_0 A}{d}}$$

Dielectric: A dielectric is electrically a non-conducting (or) insulating material. When these materials are placed in an electric field, no current flows through it because they have no free electrons. But when dielectrics are subjected to external electric field, dipoles are produced. Due to the dipoles, dipole-moment induced and hence polarisation takes place.

Ex:- Rubber, waxed paper, mica, wood, Ceramic, porcelain

Dielectrics are of two types. (a) Polar dielectrics (b) Non-polar dielectrics.

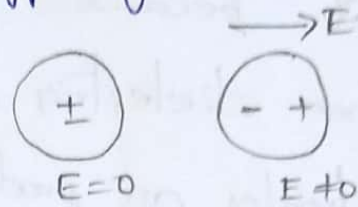
polar dielectric materials: In polar dielectrics the centre of gravity of +ve charge and centre of gravity of -ve charge may not coincide with each other. i.e. there exist an effective separation between them. Hence these molecules (or) materials possess permanent dipoles and dipole moment and hence the permanent polarisation. Ex:- H_2O , HCl etc. They have asymmetric shape.

Non-Polar dielectrics: In non polar dielectrics the centre of +ve charge and centre of -ve charge coincide with each other. i.e. no separation between the charges.

Hence dipole moment is zero. So, these materials do not possess permanent dipole moment.

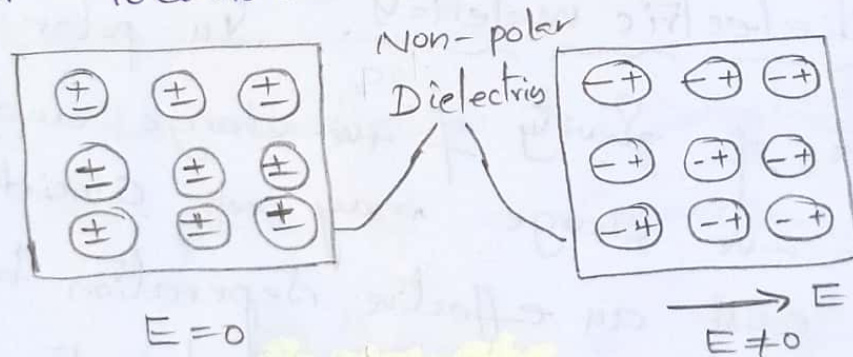
But dipole moment can be induced in non-polar dielectric materials by applying electric field.

Ex:- CO_2 , H_2 etc.



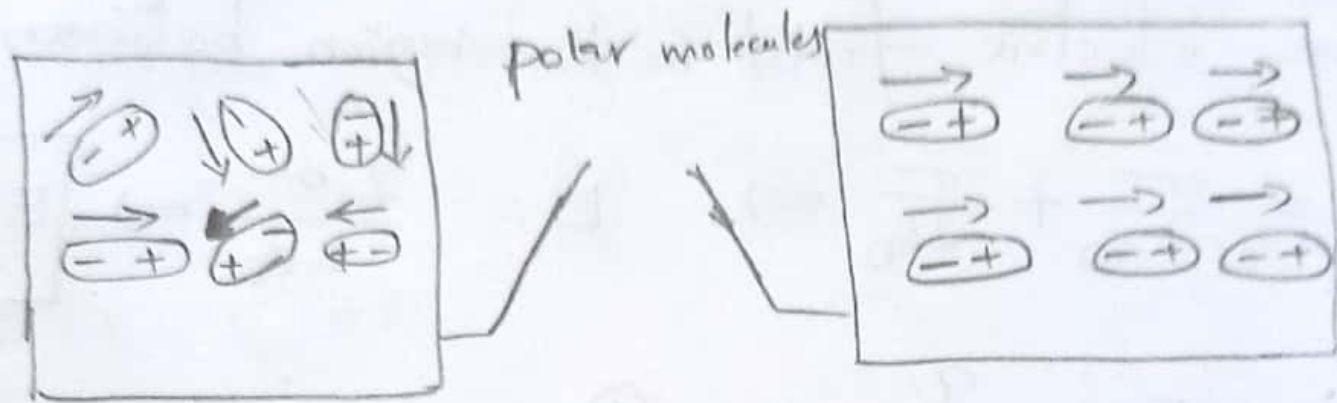
Dielectric Polarisation

When a non-polar dielectric is placed in an electric field $\vec{E}$, the centre of +ve charge gets displaced along the field direction whereas the centre of -ve charge displaced in the opposite direction. As a result Polarisation is induced.



All molecules in Polar dielectrics possess permanent dipole moment even in the absence of electric field. Because of their orientation as Randomly within the material the net dipole moment is zero. When the electric field is applied, each and every molecule try to re-orient along the field direction. Thus the material as a whole will be polarised. This process occurs throughout the material.

Thus the material as a whole will be acquiring ⁽⁷⁾ polarisation. Hence Polarisation is induced.



Electric Susceptibility (χ):

The dielectric Polarisation in a dielectric material is directly proportional to electric field applied (E)

$$\therefore P \propto E \Rightarrow \boxed{P = \chi E} \text{ where } \chi \text{ is}$$

Called electric susceptibility of the material.

Dielectric constant (K): It can be defined as,

(i) The ratio of the Capacitance of a condenser with dielectric to the Capacitance of the same condenser without dielectric is called Dielectric Constant. $\therefore K = \frac{C}{C_0}$ (Capacitance with dielectric) / (Capacitance without dielectric)

(ii) The ratio of potential difference without dielectric to the P.D with dielectric is called Dielectric Constant. $K = \frac{V_0}{V}$

(iii) The ratio of the permittivity of the medium to the permittivity of the free space is also called Dielectric constant. $\therefore K = \frac{\epsilon}{\epsilon_0}$

(iv) The ratio of the force between the two charges in air (or) vacuum to the force between the same charges. $\therefore K = \frac{F_0}{F}$

Dielectric break down: When the strength of the electric field applied to a dielectric exceeds a critical value, very large current flows through it. The dielectric loses its insulating property and becomes a conductor. This phenomenon is known

- as dielectric break down.

Dielectric strength: The electric field strength at which dielectric break down occurs is known as Di-electric strength. It is the break down voltage per unit thickness of the material.

$$\text{Di electric strength} = \frac{\text{Di electric break down Voltage}}{\text{Thickness of dielectric}}$$

for example, for air, $E_{ds} = 3 \text{ MV/m}$

for mica, $E_{ds} = 200 \text{ MV/m}$

Derive the expression for the energy of a condenser:-

' q ' is the charge on conductor, v is its potential. The capacity of the conductor, $C = \frac{q}{v}$

When a small amount of charge dq is added to the conductor, work done $= v dq$.

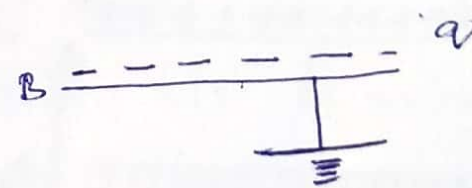
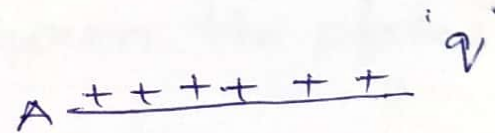
The total work done to have a charge q on the conductor

$$W = \int_0^q v dq$$

But

$$v = \frac{q}{C}$$

$$W = \int_0^q \frac{q}{C} dq = \frac{q^2}{2C}$$



This work done is stored as the energy of the condenser E

$$E = \frac{q^2}{2c}$$

substituting $q = CV$, $E = \frac{1}{2} CV^2$

Applying this equation for a parallel plate condenser, energy of the condenser, $E = \frac{1}{2} CV^2$

In the parallel plate condenser,

intensity of electric field, $E = \frac{V}{d}$

$$V = Ed.$$

Substituting $V = Ed$ and $C = \frac{a\epsilon_0}{d}$

$$\begin{aligned} \text{Energy} &= \frac{1}{2} \frac{a\epsilon_0}{d} \times E^2 d^2 \\ &= \frac{1}{2} a\epsilon_0 E^2 d \end{aligned}$$

$ad = \text{volume.}$

$$\text{Energy per unit volume} = \frac{1}{2} \times \epsilon_0 \times E^2$$

Derive the expression for the energy of a condenser:-

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When a small amount of charge dq is added to the conductor, work done $= v dq$.

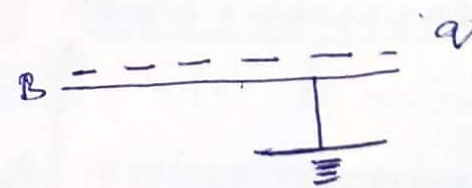
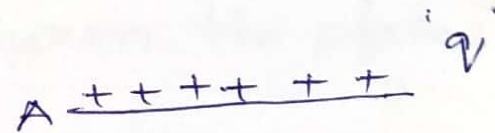
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