

UNIT-4 Magneto statics

Fleming's right hand rule: If a current carrying conductor is gripped with the right hand so that the thumb points the direction of current flow and the curled fingers shows the magnetic field.

Magnetic Flux (ϕ)

The no. of magnetic lines of force passing through an area normally ($\perp$ erly) is called magnetic flux, denoted by ϕ . units = 'wb'

Magnetic flux density (or) Magnetic Induction (B)

The magnetic flux per unit area is called magnetic flux density, denoted by 'B'.

$$\therefore B = \frac{\text{magnetic flux}}{\text{Area}} = \phi/A$$

$$\therefore \text{magnetic flux } (\phi) = B \cdot A = \vec{B} \cdot \vec{A} = BA \cos \theta$$

where $\theta \rightarrow$ angle between them.

units are wb/m^2 (α) tesla

$$1 \text{ Tesla} = 10^4 \text{ gauss.}$$

$$\text{Since } \phi = BA \Rightarrow \phi = B \cdot S \text{ (or) } d\phi = \vec{B} \cdot d\vec{S}$$

$$\therefore \boxed{\phi = \int d\phi = \int \vec{B} \cdot d\vec{S}}$$

This is expression for magnetic flux.

Magnetic force:

We know that any stationary charge 'q' experiences no force in a magnetic field. But when the charge moves in a magnetic field, it experiences a force. That force is denoted by $\vec{F}_m$, called magnetic force.

$$\therefore \vec{F}_m = q (\vec{v} \times \vec{B}) = Bqv \sin \theta.$$

Lorentz force:

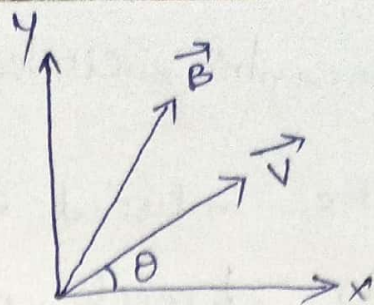
The force on a test charge 'q', when it moves with a velocity $\vec{v}$, through a region in which both electric and magnetic fields are acting is called Lorentz force. It can be denoted

$$\text{by } F_L. \quad \therefore \boxed{F_L = \vec{E}q + q(\vec{v} \times \vec{B})}$$

If the electric field is absent ($\vec{E} = 0$), then

$$F_L = F_m = q(\vec{v} \times \vec{B})$$

ie $\vec{F}_m = \tau (\vec{v} \times \vec{B})$
 $\vec{F}_m = \tau v B \sin \theta$



If $\theta = 90^\circ$, then $\sin 90^\circ = 1$

$$F_m = F_{\max} = B \tau v$$

If $\theta = 0$, then $\sin 0^\circ = 0 \Rightarrow$

$$F = F_{\min} = 0$$

Ampere's law

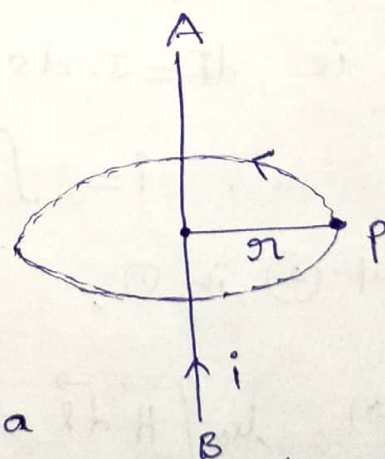
The line integral of the tangential component of magnetic field over any closed path is equal to μ_0 times the conduction current enclosed by the loop.

$$\therefore \oint \vec{B} \cdot d\vec{l} = \mu_0 i$$

Explanation:

According to Ampere,

Consider a straight conductor



AB carrying a current of i amperes, then he observed a magnetic field will be developed around the conductor $\perp$ to the direction of current. The magnetic field produced will be in the form of

Concentric circles with the conductor as the centre.

The induced magnetic field (B) at a point is

- (i) directly proportional to strength of current (i)
- (ii) inversely proportional to 'r' distance of the point under consideration. Let the point be 'P' at a distance of 'r' from conductor.

$\therefore$ According to Ampere, $B \propto i$
 $\propto \frac{1}{r}$

$$\Rightarrow B \propto \frac{i}{r} \Rightarrow B = \frac{\mu_0}{2\pi} \frac{i}{r}$$

$$\therefore B \times 2\pi r = \mu_0 i$$

$$\boxed{\int B \cdot dl = \mu_0 i} \quad \text{--- (1) } \left(\because \int dl = 2\pi r \right)$$

But current density $J = \frac{I}{A} \Rightarrow I = J \cdot A = J \cdot S$

ie $dI = J \cdot ds$

$$\therefore i = \int dI = \int \vec{J} \cdot d\vec{s} \quad \text{--- (2)}$$

put (2) in (1), we get $\boxed{\int \vec{B} \cdot d\vec{l} = \mu_0 \vec{J} \int d\vec{s}} \quad \text{--- (3)}$

$$\text{(a)} \quad \mu_0 \int \vec{H} \cdot d\vec{l} = \mu_0 \int \vec{J} \cdot d\vec{s} \quad \left(\because B = \mu_0 H \right)$$

$$\Rightarrow \boxed{\int \vec{H} \cdot d\vec{l} = \int \vec{J} \cdot d\vec{s}}$$

Equation (3), $\int B \cdot dl = \mu_0 \int \vec{J} \cdot d\vec{s}$ is called
Integral form of Ampere's law.

Ampere's law in differential form:

we have $\int \vec{B} \cdot d\vec{l} = \mu_0 \int \vec{J} \cdot d\vec{s}$

using Stokes's theorem, the LHS of the above equation can be written as $\int \vec{B} \cdot d\vec{l} = \int_S \text{curl } \vec{B} \cdot d\vec{s}$

ie $\int \vec{B} \cdot d\vec{l} = \int_S (\nabla \times \vec{B}) \cdot d\vec{s}$

$\therefore$ The above equation becomes, $\int (\nabla \times \vec{B}) \cdot d\vec{s} = \mu_0 \int \vec{J} \cdot d\vec{s}$

$\therefore \boxed{\nabla \times \vec{B} = \mu_0 \vec{J}}$

This is called Ampere's law in differential form. It can also be written as,

$\nabla \times (\mu_0 \vec{H}) = \mu_0 \vec{J} \quad (\because \vec{B} = \mu_0 \vec{H})$

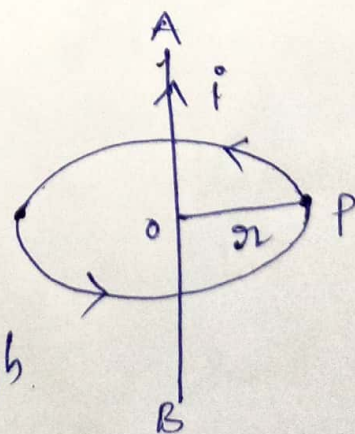
$\mu_0 (\nabla \times \vec{H}) = \mu_0 \vec{J} \Rightarrow \boxed{\nabla \times \vec{H} = \vec{J}}$

Applications of Ampere's law:

(a) Determination of 'B' due to infinitely long

straight thin wire.

Consider an infinitely long straight thin wire. Let 'i' be the current passing through the wire. Let 'B' be the magnetic field induced around the wire which



is in the form of Concentric Circles.
Through out the path of the circle, 'B' will have same magnitude. i.e. $\int B \cdot dl = B \int dl = B \times 2\pi r$.

From Ampere's law, it is equal to $\mu_0 \times i$.

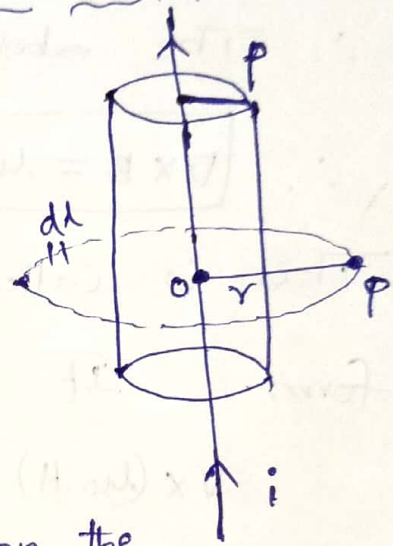
$$\therefore B \times 2\pi r = \mu_0 i \Rightarrow \boxed{B = \frac{\mu_0}{2\pi} \frac{i}{r}}$$

(b) B at a point due to cylindrical conductor carrying a current 'i'

Case-1: When the point 'P' lies out side the cylinder.

Consider a cylindrical conductor with radius 'P'. Let 'i' be the current passing through it.

Consider a small element 'dl' on the closed path.



From Ampere's law, $\int B \cdot dl = \mu_0 i$

$$\therefore B \times 2\pi r = \mu_0 i \Rightarrow \boxed{B = \frac{\mu_0}{2\pi r} i}$$

Case-2: When the point on the cylinder

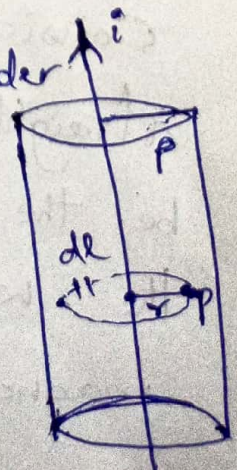
Here $r = P$, then $\boxed{B = \frac{\mu_0}{2\pi P} i}$

Case-3: When the point 'P' inside cylinder

$$\int B \cdot dl = \mu_0 i \left(\frac{\text{desired area}}{\text{Total area}} \right)$$

$$B \times 2\pi r = \mu_0 i \times \frac{\pi r^2}{\pi P^2}$$

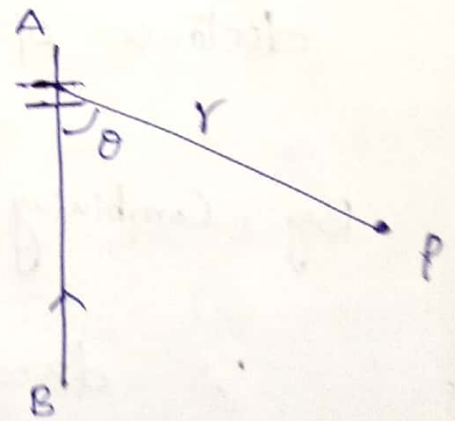
$$\therefore \boxed{B = \frac{\mu_0}{2\pi} \frac{i r}{P^2}}$$



Biot-Savart's law

According to Oersted experiment a current carrying conductor produces a magnetic field around it. In 1820, Biot & Savart performed a series of experiments to study the magnetic field produced by various current carrying conductors. They obtained a relation by means of which $\vec{B}$ can be calculated at any point of space around a conductor in which a current i is passing. This relation is called Biot-Savart's law.

Consider a conductor 'AB' carrying a current of i amp. as shown in figure. Consider an infinite small element of length dl . Consider a point 'P' at a distance r from the element. Let θ be the angle made by the line joining the point under consideration and small element of the conductor.



Let dB be the magnitude of induced magnetic field due to small element at 'P',

According to Biot-Savart's law, ' dB '

(i) is directly proportional to strength of current i.e. $dB \propto i$

(ii) is directly proportional to length of small element i.e. $dB \propto dl$

(iii) is directly proportional to 'sin' of the angle θ between the length of element and the line joining the element to the point 'P'. i.e. $dB \propto \sin \theta$

(iv) is inversely proportional to the square of the distance of the point 'P' from the element.
 $dB \propto \frac{1}{r^2}$

By combining all factors, $dB \propto \frac{idl \sin \theta}{r^2}$

$$\therefore dB = \frac{\mu_0}{4\pi} \frac{idl \sin \theta}{r^2}, \quad \frac{\mu_0}{4\pi} = 10^{-7} \text{ H/m.}$$

$$B = \int dB = \frac{\mu_0}{4\pi} \int \frac{idl \sin \theta}{r^2}$$

The resultant field at 'P' due to entire conductor is

$$B = \int dB = \frac{\mu_0}{4\pi} \int \frac{idl \sin \theta}{r^2}$$

In Vector form, $B = \frac{\mu_0}{4\pi} \int \frac{idl \sin\theta}{r^2} \cdot \frac{\vec{r}}{r}$

Applications: (i) B at a point due to a straight conductor carrying current (I) :-

Consider an infinitely long straight conductor carrying a current 'i' ampere as shown.

To find the magnetic field

'B', due to a straight conductor, consider a

point 'P' at a distance of 'R' from 'O' of conductor

and 'r' be the distance between small element and the point 'P'.

Let the small element (dl) be at a distance

'x' from 'O'. Let 'θ' be the angle between small element and the line joining the point 'P'.

The magnetic field due to small element (dl) is

$$dB = \frac{\mu_0}{4\pi} \frac{idl \sin\theta}{r^2} \quad \text{--- (1)}$$

The field due to the whole conductor is given by

$$B = \int_{-\infty}^{+\infty} dB = \frac{\mu_0}{4\pi} \int_{-\infty}^{+\infty} \frac{idl \sin\theta}{r^2} \quad \text{--- (2)}$$

From figure, $r^2 = l^2 + R^2 \Rightarrow r = \sqrt{l^2 + R^2}$ and

$$\sin \theta = \frac{R}{(l^2 + R^2)^{1/2}}$$

put the values of r and $\sin \theta$ in equation ②,

$$B = \frac{\mu_0}{4\pi} \times i \int_{-\infty}^{+\infty} dl \times \frac{R}{(l^2 + R^2)^{1/2}} \frac{1}{[(l^2 + R^2)^{1/2}]^2}$$

$$\therefore B = \frac{\mu_0}{4\pi} i \int_{-\infty}^{+\infty} \frac{R}{(l^2 + R^2)^{3/2}} dl \quad \text{--- ③}$$

From figure, $\tan \alpha = \frac{l}{R} \Rightarrow l = R \tan \alpha$
 $\frac{dl}{d\alpha} = R \sec^2 \alpha$

$$\Rightarrow dl = R \sec^2 \alpha d\alpha \quad \text{--- ④}$$

put ④ in ③, $B = \frac{\mu_0}{4\pi} i \int_{-\infty}^{+\infty} \frac{R}{(l^2 + R^2)^{3/2}} \times R \sec^2 \alpha d\alpha$

when $l \rightarrow -\infty$, $R \tan \alpha \rightarrow -\infty \Rightarrow \tan \alpha = -\infty$
 $\alpha \rightarrow -\pi/2$

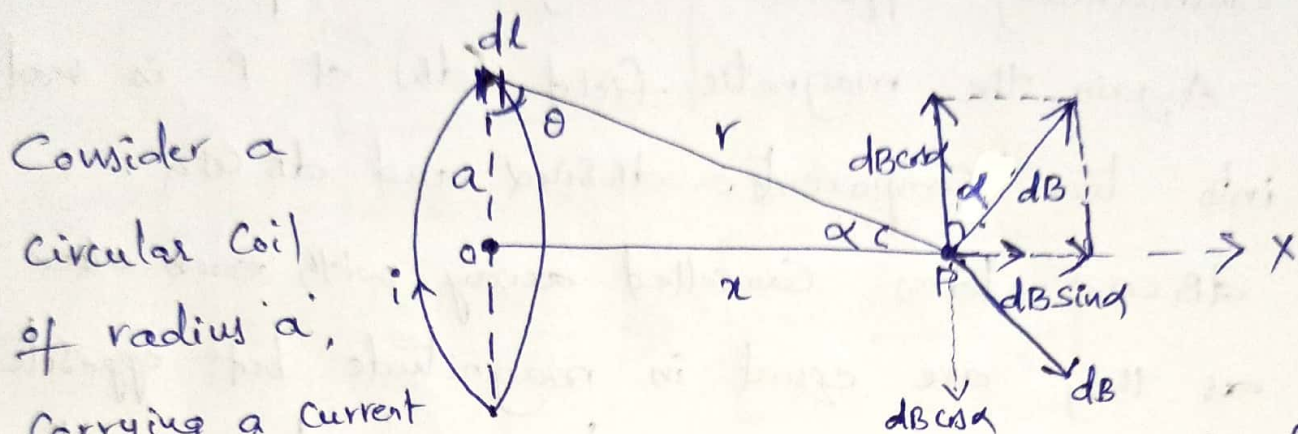
when $l \rightarrow +\infty$, $R \tan \alpha = \infty \Rightarrow \tan \alpha \rightarrow \infty$, $\alpha = \pi/2$.

$$\therefore B = \frac{\mu_0}{4\pi} i \int_{-\pi/2}^{\pi/2} \frac{R^2 \sec^2 \alpha d\alpha}{(R^2 + R^2 \tan^2 \alpha)^{3/2}} = \frac{\mu_0}{4\pi} i \int_{-\pi/2}^{\pi/2} \frac{1}{R \sec \alpha} d\alpha$$

$$\therefore B = \frac{\mu_0}{4\pi} \frac{i}{R} \int_{-\pi/2}^{\pi/2} \cos \alpha d\alpha = \frac{\mu_0}{4\pi} \frac{i}{R} [\sin \alpha]_{-\pi/2}^{\pi/2}$$

$$\therefore B = \frac{\mu_0}{4\pi} \times \frac{i}{R} \times [1 + 1] \Rightarrow \boxed{B = \frac{\mu_0}{2\pi} \frac{i}{R}}$$

(ii) B at a point on the axis of a current
Loop (or) B due to a circular loop of radi-
-us 'a', carrying current 'i':



Consider a circular coil of radius 'a', carrying a current of 'i' amperes. Take a point 'P' on the axis of a coil at a distance of 'x' from the centre of the coil. our aim is to find magnetic field 'B' at a point 'P'. Consider a small element 'dl' on the coil. Let 'r' be the distance of the small element and the point 'P'. Let 'theta' be the angle made by the small element and the line joining the point 'P'. The magnetic field, (dB) at a point 'P' due to small element 'dl' of the coil is given by

$$dB = \frac{\mu_0}{4\pi} \frac{i dl \sin \theta}{r^2} = \frac{\mu_0}{4\pi} \frac{i dl}{r^2} (\because \theta = 90^\circ)$$

The resultant magnetic field 'dB' due to small element 'dl' at 'P' would be $\perp$ to 'r'.

$\therefore$ This field can be resolved into two

Components. i.e. Vertical and horizontal components.

they are $dB \cos \alpha$ and $dB \sin \alpha$.

Take another element 'dl' of same length which is diametrically opposite exactly to the previous one.

Again the magnetic field (dB) at 'P' is resolved into two components. $dB \sin \alpha$ and $dB \cos \alpha$.

' $dB \cos \alpha$ ' terms cancelled away with each other as they are equal in magnitude but opposite in direction, whereas ' $dB \sin \alpha$ ' terms will add up with each other.

$$\therefore B = \int dB = \int dB \sin \alpha = \int \frac{\mu_0}{4\pi} \frac{i dl}{r^2} \times \frac{a}{r} \quad (\because \sin \alpha = \frac{a}{r})$$

$$\therefore B = \frac{\mu_0}{4\pi} \frac{i a}{r^3} \int dl \Rightarrow B = \frac{\mu_0}{4\pi} \frac{i a}{r^3} \times 2\pi a$$

$$(\because \int dl = \text{circumference of circle} = 2\pi a)$$

$$\therefore B = \frac{\mu_0}{2} \frac{i a^2}{r^3}$$

$$B = \frac{\mu_0}{2} \frac{i a^2}{(a^2 + x^2)^{3/2}}$$

$$(\because \text{from fig. } r = \sqrt{a^2 + x^2})$$

$\therefore$ If the circular coil contains 'n' turns,

then $B = \frac{\mu_0 n i a^2}{2 (a^2 + x^2)^{3/2}}$ W/m² (or) Tesla.

Cases:- (a):- when 'p' lies at the centre of coil,

then $B = \frac{\mu_0 n i a^2}{2 (a^2)^{3/2}} \quad (\because x=0)$

$$\therefore \boxed{B = \frac{\mu_0 n i}{2a}}$$

Case: (b): when $x \gg a$, then

$$B = \frac{\mu_0 n i a^2}{2 (x^2)^{3/2}} \Rightarrow \boxed{B = \frac{\mu_0 n i a^2}{2x^3}}$$

Magnetic Potentials: There are two magnetic potentials. They are (i) Scalar magnetic potential (ii) Vector magnetic Potential

Scalar magnetic Potential: It is the quantity whose negative gradient gives the magnetic field intensity. It is defined only in a region where $J=0$ (or) no current source is present.

i.e $-\nabla V_m = H$, where $J=0$.

(or) $V_m = -\oint_c H \cdot dl$

Proof:-

In electrostatics, the relationship between electric field intensity (E) and electric potential (V) is given by $E = -\nabla V_e$ — (1)

In the same manner, let us relate the magnetic field intensity (H) to a scalar magnetic Potential

$$\vec{H} = -\nabla V_m \text{ — (2)}$$

From Ampere's law, we know that $\nabla \times \vec{H} = J$ — (3)

put (2) in (3), $\therefore \nabla \times (-\nabla V_m) = J$ — (4)

But, using Vector identity, $\nabla \times \nabla V = 0$ and hence

$\therefore$ Equation (4) becomes $\vec{J}=0$ (or) $J=0$.

From the equations (3) and (4), we find that

$\vec{H} = -\nabla V_m$ is valid only where $J=0$.

$$H = -\nabla V_m \Rightarrow \int H \cdot dl = -\int \nabla V_m$$

$$\therefore \boxed{V_m = -\int \vec{H} \cdot d\vec{l}}$$

From Gauss law in magnetism, we have $\int \vec{B} \cdot d\vec{s} = 0$

$$(a) (\nabla \cdot \vec{B}) dV = 0 \Rightarrow \nabla \cdot (\mu_0 H) = 0 \quad (\because B = \mu_0 H)$$

$$\therefore \boxed{\nabla \cdot H = 0} \text{ --- (5) as } \mu_0 \neq 0.$$

put $H = -\nabla V_m$ in equation (5), we get

$$\nabla \cdot (-\nabla V_m) = 0 \Rightarrow -\nabla^2 V_m = 0 \Rightarrow \boxed{\nabla^2 V_m = 0}$$

$$\Rightarrow \boxed{\nabla^2 V_m = 0}$$

$\therefore$ The scalar magnetic potential satisfied the Laplace's equation.

Vector magnetic Potential ($\vec{A}$):

It is a quantity whose curl gives the magnetic flux density. i.e. $\nabla \times \vec{A} = \vec{B}$, where $J \neq 0$ where $\vec{A}$ is the Vector magnetic potential.

$$\vec{A} = \frac{\mu_0}{4\pi} \int_V \frac{\vec{J}}{r} \cdot dV.$$

Proof:- we have from Gauss law in magnetism

$$\int \vec{B} \cdot d\vec{s} = 0 \quad \text{--- (1)} \quad \left(\because \text{non existence of magnetic mono poles} \right)$$

From Gauss-Divergence theorem, $\int \vec{B} \cdot d\vec{s} = \int \nabla \cdot \vec{B} dV$.

$\therefore$ Equation (1) can be written as $\int \nabla \cdot \vec{B} dV = 0$

$$\text{Hence } \boxed{\nabla \cdot \vec{B} = 0} \quad \text{--- (2)}$$

$$\Rightarrow \nabla \cdot (\nabla \times \vec{A}) = 0, \quad \text{Here } \vec{B} = \nabla \times \vec{A} \quad \text{--- (3)}$$

$$\text{ie } \vec{B} = \nabla \times \vec{A}$$

Taking 'curl' on both sides,

$$\nabla \times \vec{B} = \nabla \times (\nabla \times \vec{A}) \quad \text{--- (4)}$$

but, using Vector identity $\nabla \times (\nabla \times \vec{A})$ can be written,

$$\nabla \times (\nabla \times \vec{A}) = \nabla \cdot (\nabla \cdot \vec{A}) - (\nabla^2) \vec{A}$$

$$\Rightarrow \boxed{\nabla \times (\nabla \times \vec{A}) = -\nabla^2 \vec{A}} \quad \text{--- (5)}$$

$$\text{put (5) in (4), we get } \boxed{\nabla \times \vec{B} = -\nabla^2 \vec{A}} \quad \text{--- (6)}$$

From differential form of Ampere's law, we have

$$\nabla \times \vec{B} = \mu_0 \vec{J} \quad \text{--- (7)}$$

$$\text{From (6) \& (7) } -\nabla^2 \vec{A} = \mu_0 \vec{J} \Rightarrow \boxed{\nabla^2 \vec{A} = -\mu_0 \vec{J}} \quad \text{--- (8)}$$

∴ This equation $\boxed{\nabla^2 \vec{A} = -\mu_0 \vec{J}}$ is called Vector Poisson's equation.

∴ The Vector magnetic potential satisfies the Poisson's equation.

The solution for the above equation is given by

$$\boxed{\vec{A} = \frac{\mu_0}{4\pi} \int_V \left(\frac{\vec{J}}{r} \right) dv} \quad A$$

It has the physical significance that its integral around any closed path is equal to magnetic flux through that path. It can be proved by the following way.

The magnetic flux through the given area 's' is

$$\phi = \int_s \vec{B} \cdot d\vec{s}$$

∵ $\vec{B} = \nabla \times \vec{A}$, the above equation can be written as

$$\phi = \int_s (\nabla \times \vec{A}) \cdot d\vec{s}$$

$$\boxed{\phi = \int \vec{A} \cdot d\vec{l}} \quad (\because \text{By Stokes's theorem})$$

(or)

$$\int \vec{A} \cdot d\vec{l} = \phi$$

= magnetic flux

Intensity of Magnetization (I):

This is defined as the ratio of the magnetic moment (M) to the volume (V) of the magnetized material, i.e.

$$I = \frac{M}{V}$$

If the given magnetic material is rectangular or cylindrical bar of length 'l' and area of cross-section 'a', then $V = al$ and magnetic moment $M = ml$, where 'm' is designated as pole strength, we have

$$I = \frac{ml}{al} = \frac{m}{a}$$

Thus, the intensity of magnetization may also be defined as the pole strength per unit area at right angles to the direction of magnetization.

Intensity of Magnetization (I) & susceptibility (χ):

For any magnetic material, the intensity of magnetization is found to be proportional to the measure of the capability of magnetizing field H, i.e.

$$I \propto M$$

$$\Rightarrow I = \chi_m H$$

where χ_m is called the magnetic susceptibility of the medium and it is the measure of capability of the material to take up magnetization.

Magnetic Dipoles:

* Magnetism is dipolar, i.e. magnetism is characterized by having two opposite poles north (N) and south (S).

⇒ The strength of a magnetic dipole is measured by the product of the pole strength and the distance between the poles. This is called magnetic moment.

⇒ One source of magnetism in an atom ~~is~~ is the orbital motion of electrons. Each electron revolving around the nucleus in an atom constitutes a circulating electric charge or current and thus produces a small magnetic field. Moreover, each spinning electron on its axis can be conceived as a circulating charge and also produce small magnetic fields.

Magnetic permeability:

permeability describes the nature of the material. It is the ease with which the material allows magnetic lines of force to pass through it or the degree to which magnetic field can penetrate a given medium is called its permeability.

$$\boxed{\mu = \frac{B}{H}} = \frac{\text{Magnetic induction}}{\text{Applied magnetic field}}$$

Units are, $\mu = \frac{\text{Weber}}{\text{m}^2} / \text{Ampere-metre} = \frac{\text{Wb}}{\text{m}^2} \times \frac{\text{m}}{\text{A}} = \frac{\text{Wb}}{\text{mA}}$

$$(\text{or}) = \frac{\text{H}}{\text{m}} \quad (\text{H} \rightarrow \text{Henry})$$

Permeability of free space, $\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$

classification of magnetic materials:

The materials which are affected by the magnets are called magnetic materials. They can be classified into Dia, para and ferro magnetic substances.

Dia magnetic materials:

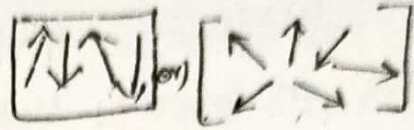
1. The substances which are repelled by a magnet are called Dia magnetic substances. The phenomenon of exhibiting this property is called diamagnetism.
2. They magnetise to a small extent in a direction opposite to the direction of the applied field.
3. The magnetic susceptibility is small and negative.
4. It (χ) represents the ease of magnetisation of a material and is defined as the ratio of intensity of magnetisation to the applied magnetic field intensity (H).
5. The permanent magnetic dipoles are absent in these materials.
6. For these materials the susceptibility is independent of temperature.
7. The relative permeability is always less than 1.
 $\mu_r < 1$
8. Examples: Gold, silver, copper, Bismuth, lead, Zinc, etc.

Para magnetic materials:-

1. The substances which are weakly attracted by the magnets are called para magnetic substances. The phenomenon of exhibiting this property is called Paramagnetism.
2. They magnetise to a small extent along the direction

of the applied field.

3. These materials possess permanent magnetic dipoles but in the absence of the external magnetic field the dipoles are oriented at random. Hence the net magnetisation is zero in a particular direction.



4. The magnetic susceptibility is small and positive.

5. The magnetic susceptibility (χ) is dependent on temperature i.e., χ varies with temperature as follows

$$\chi = \frac{C}{T} \text{ (or) } \chi = \frac{C}{T - \theta}, \text{ For } T > \theta$$

i.e., it obeys Curie's law (or) Curie-Weiss law.

6. The paramagnetism is due to the orbital and spin motion of the electrons.

7. The relative permeability is slightly greater than one ($\mu_r > 1$)

8. Examples: Al, Pt, $MnCl_2$, salts of Iron and Nickel, etc.

Ferro Magnetic Materials:

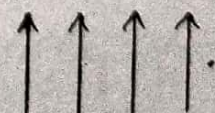
1. The substances which are strongly attracted by the magnets are called ferro magnetic materials.

The phenomenon of exhibiting this property is called ferro magnetism.

2. They magnetise, to a large extent along the direction of the applied magnetic field.

3. These materials possess permanent magnetic dipoles.

4. The permanent magnetic dipoles are as shown



5. The atoms (or) molecules of a ferromagnetic material have magnetic dipole moment (or) magnetic moment due to spin of the electrons.

6. The magnetic susceptibility (χ) is positive and large.

7. The magnetic susceptibility (χ) is dependent on temperature i.e., $\chi = \frac{C}{T - \theta}$

The ferromagnetic material has a characteristic temperature called the ferromagnetic Curie temperature (θ_f).

→ If $T > \theta_f$, ferromagnetic materials exhibit the Paramagnetic behaviour.

→ If $T < \theta_f$, it exhibits ferromagnetic behaviour.

8. The relative permeability is greater than one
 $\mu_r \gg 1$.

9. Examples: Iron, Cobalt, Nickel.

10. These materials possess spontaneous magnetisation hence these materials also exhibit hysteresis.

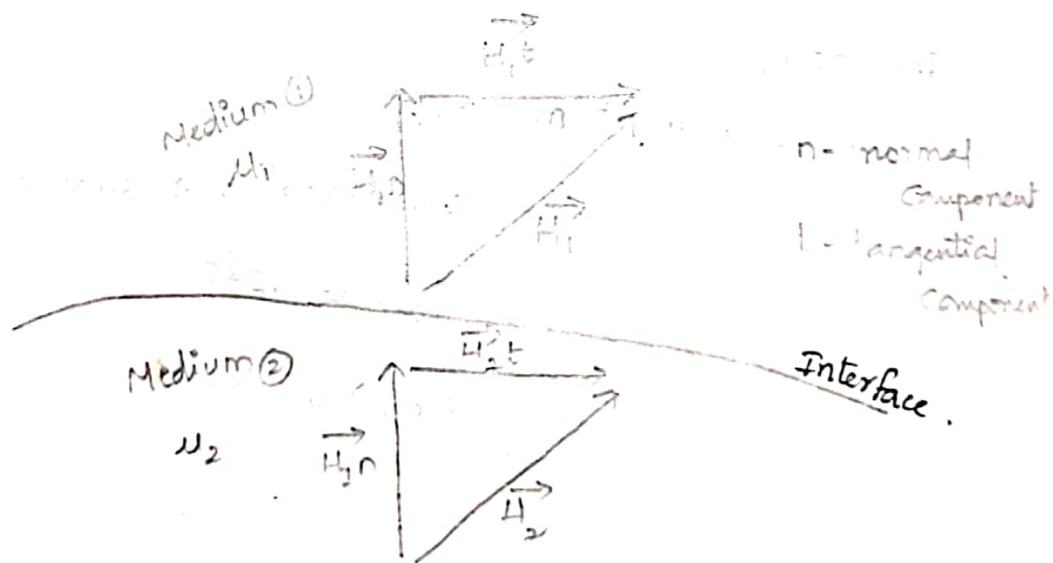
Boundary conditions for B and H at the surface of a Magnetic material:

The aim is consider that how the ^{field} vectors B and H change in passing an interface between two media. The two media may be two materials with different magnetic properties, or a material medium and vacuum.

Consider the interface between two different materials with dissimilar permeabilities.

Let us consider both magnetic field & magnetic ~~line~~ flux density are present in both regions.

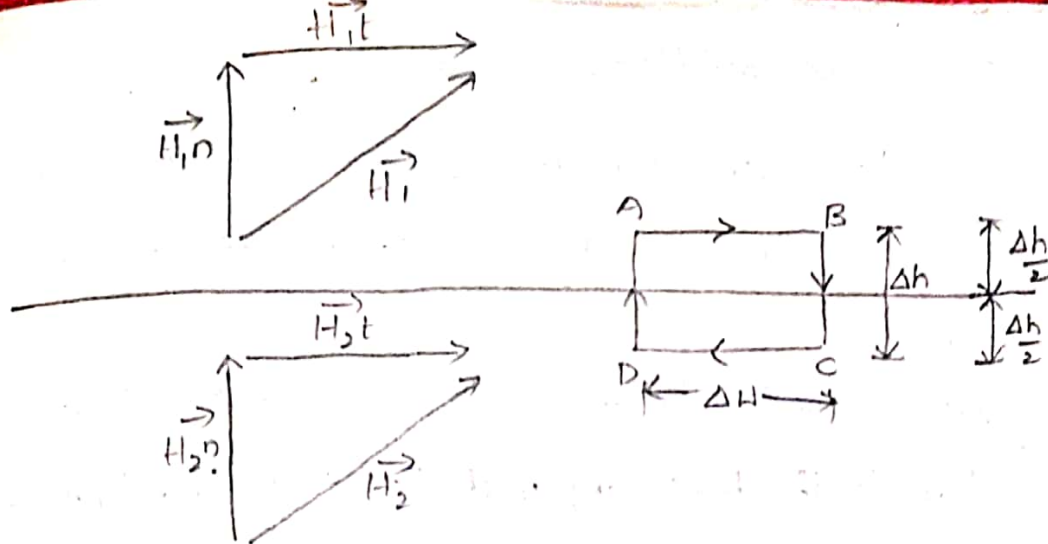
The magnetic fields at the interface in terms of their normal and tangential components as shown in figure.



Boundary condition 1:

It states that the tangential component of the magnetic field intensity is continuous across a boundary.

Let us consider a closed loop ABCD at the boundary between two media of different permeabilities μ_1 & μ_2 as shown in adjacent figure. The height and width of closed loop are Δh and Δw .



Let us suppose that some current is flowing at the boundary.

$\vec{K}$ is the current density at the boundary.

Apply Ampere's law, to this closed loop ABCD

$$\oint \vec{H} \cdot d\vec{l} = I_{enc} \longrightarrow (1).$$

$$\Rightarrow H_{1t} \Delta W - H_{1n} \frac{\Delta h}{2} - H_{2n} \frac{\Delta h}{2} - H_{2t} \Delta W + H_{2n} \frac{\Delta h}{2} + H_{1n} \frac{\Delta h}{2} = \vec{K} \Delta W$$

$$\therefore H_{1t} \Delta W - H_{2t} \Delta W = \vec{K} \Delta W.$$

$$\Delta W [H_{1t} - H_{2t}] = \vec{K} \Delta W.$$

$$\Rightarrow \boxed{H_{1t} - H_{2t} = \vec{K}} \longrightarrow (2).$$

If the boundary is free of current then eq'n(2) becomes

$$H_{1t} - H_{2t} = 0$$

$$\boxed{H_{1t} = H_{2t}} \longrightarrow (3)$$

$\therefore$ Tangential component of $\vec{H}$ is continuous at the boundary.

From eq'n(3) $\boxed{\frac{B_{1t}}{\mu_1} = \frac{B_{2t}}{\mu_2}} \longrightarrow (4)$

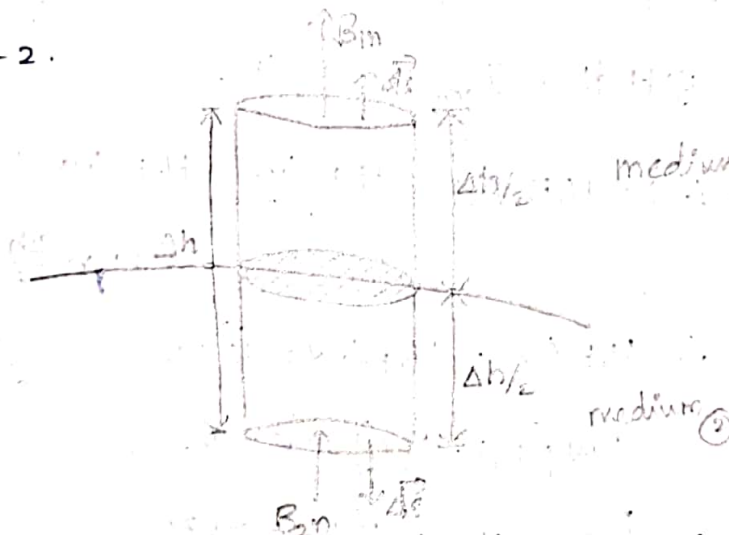
$$\therefore \boxed{B = \mu H}$$

$\therefore$ The tangential component of $\vec{B}$ is discontinuous of Boundary.

Boundary condition 2:

It states that the normal vector component of magnetic flux density is continuous across the magnetic boundary.

Let us consider a small pill box shaped surface which intersects the boundary with its end faces parallel to it. Δh is the height of the pill box is assumed to be very small ($\Delta h \rightarrow 0$). Half of the pill box is present in the medium (1) and half of the pill box is present in medium - 2.



We know that $\oint_S \vec{B} \cdot d\vec{r} = 0 \rightarrow (5)$

The flux through the pill box is given by

$$\int_{\text{top}} \vec{B} \cdot d\vec{r} + \int_{\text{Bottom}} \vec{B} \cdot d\vec{r} + \int_{\text{curve}} \vec{B} \cdot d\vec{r} = 0 \rightarrow (6)$$

Since the height of the pill box negligibly small ($\Delta h \rightarrow 0$)

The only contribution of flux will come through top & bottom face.

$$\therefore B_{1n} \Delta S - B_{2n} \Delta S = 0$$

$$\boxed{B_{1n} = B_{2n}} \rightarrow (7)$$

$\therefore$ The normal component of magnetic flux density is continuous at the boundary.

We know that, $\vec{B} = \mu \vec{H}$

$$\therefore \text{from eq'n (7)} \quad \mu_1 H_{1n} = \mu_2 H_{2n}$$

$$\therefore \boxed{\frac{H_{1n}}{H_{2n}} = \frac{\mu_2}{\mu_1}} \longrightarrow (8)$$

The normal component of magnetic field intensity is discontinuous at the boundary separating two mediums.

Force on a charged particle in a uniform magnetic field:-

Consider a charged particle having a charge ' q ' is moving with a speed ' v ' in uniform magnetic field of induction ' B ' making an angle ' θ ' with the field, then the force acting on the particle

$$F = q (\vec{v} \times \vec{B}) = q v B \sin \theta$$

→ If $v=0$, $\theta=0$ (or) 180 , then $F=0$

i.e. if the particle is at rest, moves parallel or anti parallel to the field then the magnetic field can't apply a force on it.

→ If $\theta=90^\circ$, (i.e. if the particle is moving $\perp$ to the magnetic field), then $F_{\max} = qvB$

$\therefore$ Force will be maximum.

In this situation force is always $\perp$ to the direction of motion of the particle. Hence the particle takes the circular path. (Fleming left hand rule)

The necessary Centripetal force is provided by the magnetic field.

$$\therefore F_{\text{Centripetal}} = F_{\text{mag}}$$

$$\frac{mv^2}{r} = Bqv$$

$$\frac{mv}{r} = Bq = \frac{m(\omega r)}{r}$$

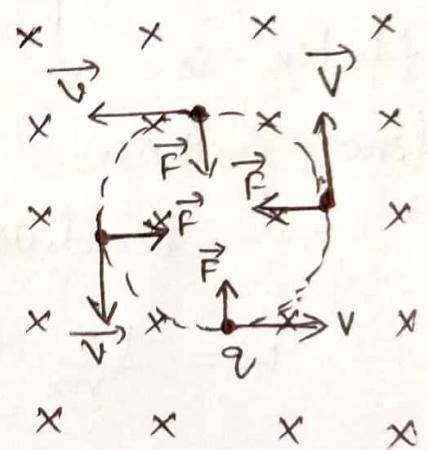
$$\therefore \boxed{r = \frac{mv}{Bq}}$$

But $mv = p$, hence $r = \frac{p}{Bq}$

$$\text{also } p = \sqrt{2m(K.E)} \Rightarrow r = \frac{\sqrt{2m(K.E)}}{Bq}$$

Hence, the radius of circular path is given by

$$r = \frac{mv}{Bq} = \frac{p}{Bq} = \frac{\sqrt{2m \cdot KE}}{Bq}$$



We also know that $V = r\omega$

$$\therefore V = \frac{mV}{Bq} \omega \quad \left(\because r = \frac{mV}{Bq} \right)$$

$$\Rightarrow \boxed{\omega = \frac{Bq}{m}} \quad \text{This is the formula for angular frequency } \omega.$$

$$\text{Time period } T = \frac{2\pi}{\omega}$$

$$\therefore T = \frac{2\pi}{\left(\frac{Bq}{m}\right)} \Rightarrow \boxed{T = \frac{2\pi m}{Bq}}$$

This is the time period.

$$f = \frac{1}{T} = \frac{Bq}{2\pi m} \Rightarrow \boxed{f = \frac{Bq}{2\pi m}}$$

→ If $\theta \neq 90^\circ$ (i.e. other than $0^\circ, 180^\circ, 90^\circ$)

Then the particle takes the Helix path (or) Ellipse.
Helix is a combination of circular and straight line paths.

$$\text{Radius of helix } (r) = \frac{mV \sin \theta}{Bq}$$

$$\omega = \frac{Bq}{m}, \quad T = \frac{2\pi m}{Bq}, \quad f = \frac{Bq}{2\pi m}$$

Faraday's laws of electro magnetic Induction

Faraday's laws:

- ① Emf is induced in a coil because of the change in magnetic flux associated with the coil.
- ② Emf induced in a coil is directly proportional to the rate of decrease of magnetic flux associated with the coil. i.e. $E \propto -\frac{d\phi}{dt}$
 $\therefore E = -\frac{d\phi}{dt}$

Lenz's law:- The direction of induced e.m.f in a coil is such that it always opposes the change in flux which induces it.

Inductance: The process of inducing emf in a coil due to the change in magnetic flux associated with the coil is called Inductance (or) Self inductance.

The magnetic flux associated with a coil ' ϕ ' is directly proportional to the current passing through the coil ' i '. Then $\phi \propto i$

$\phi = Li$, where ' L ' is called

Coefficient of self inductance of the coil.

$$\therefore \phi = Li \Rightarrow \boxed{L = \frac{\phi}{i}}$$

The Coefficient of Self inductance can be defined as the ratio between the magnetic flux associated with the coil and the current passing through the coil.

(or)

$$\text{EMF induced in a coil } E = - \frac{d\phi}{dt}$$

$$\therefore E = - \frac{d}{dt}(Li)$$

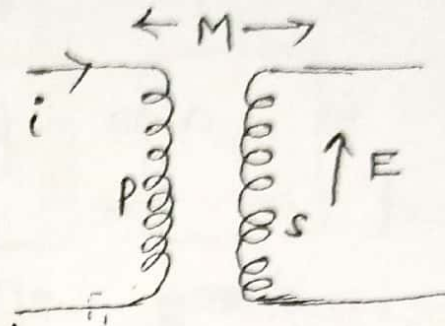
$$\Rightarrow E = -L \frac{di}{dt}$$

$$\Rightarrow \boxed{L = \frac{E}{-\frac{di}{dt}}}$$

It is also the ratio between emf induced in the coil and the rate of decrease of current in the coil.

Mutual Inductance

consider two coils P and S
placed nearer to one another



When a current is passed through
the coil 'P', it produces magnetic flux, some
of which will be linked to the coil 'S'. A change
in flux (or) current in the coil 'P' will cause
a change in the flux linkage with the coil 'S'.
This will produce an induced emf in the
Coil S.

The process of inducing emf in a secondary coil is due to the change in the current in the primary coil, i_p , is called Mutual Inductance.

When a current ' i ' passes through the primary coil, then the emf induced in the secondary coil is ϕ , then $\phi_s \propto i_p$

$$\therefore \phi = M i \Rightarrow \boxed{M = \frac{\phi}{i}}$$

The ratio between the flux associated in the secondary coil to the current passing through the primary coil.

(or)

It is also the ratio between the emf induced in the secondary coil to the rate of decrease of current in the primary coil.

$$\text{i.e. } M = \frac{E}{-\frac{di}{dt}}$$

Energy stored in a magnetic field:

consider a coil of inductance ' L ' at any instant of time. Let ' $\mathcal{E}$ ' be the emf induced in the coil due to the current ' i ' in the coil.

Work done to pass the current ' i ' for a

$$\text{time } dt = \mathcal{E} i dt$$

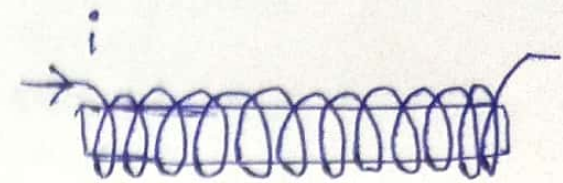
$$\left(\because p = \frac{W}{t} \Rightarrow W = Pt \right. \\ \left. \Rightarrow W = \mathcal{E} i t \right)$$

$$\therefore \text{Work done} = E i dt$$

$$= - \frac{d\phi}{dt} \cdot i \cdot dt$$

$$= -L \frac{di}{dt} \cdot i \cdot dt$$

$$= -L i di$$



If the current in the coil increases from 0 to i_0 in a time 't', then work done in this

$$\text{time} = - \int_0^{i_0} i di$$

$$= -L \frac{i_0^2}{2}$$

$$\therefore \text{work done in a time 't'} = -\frac{1}{2} L i_0^2$$

This work done is stored as the energy of a coil

$$\text{Energy stored in a magnetic field} = -\frac{1}{2} L i_0^2$$