UNIT - I

Circuit Concepts

Network :- Network is a system with interconnected electrical elements. Network and circuit are the same. The only difference is circuit contain at least one closed path.

Circuit :- A circuit is a closed conducting path through which an electric current flows.

Electric Network :- A combination of various electric elements connected in any manner is called an electric network.

Electric circuits consists of two basic types of elements. They are active elements and passive elements.

Active elements :- It is capable of generating or supplying an electrical energy.

Ex :- voltage sources such as battery or generator and current sources.

passive elements :- A passive element is one which does not generate electricity but either consumes it or store it.

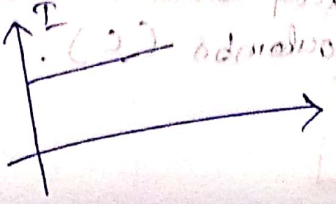
Ex : Resistors, Inductors and Capacitors.

DC circuit :- The closed path in which the direct current flows is called DC circuit. The current flows in only one direction. It is used in low voltage applications.

AC circuit :- The closed path in which the alternating current flows is called AC circuit. It is used in high voltage applications.

Direct current : A current does not change with time.

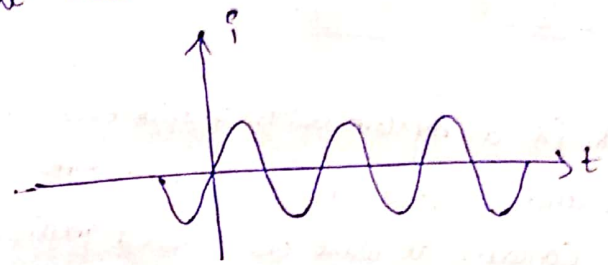
(or) A current that remains constant with time.



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Alternating Current :-

A current that varies sinusoidally with time.



Electrical Devices :- (AC current)

The electrical devices convert the electrical energy into the other form of energy like heat, light, sound etc.
Ex:- generators, motors

Electronic devices :- (DC current)

Electronic devices control the flow of electrons.
Ex Transistors, microprocessors, flipflop, amplifier etc.

Electromotive force :- It is the force which causes to flow the electrons in any closed ckt. The units of emf is volts.

potential Difference :- The potential difference b/w the two points in a ckt is electrical pressure or voltage required to drive the current between them. The units of potential difference is volts.

Voltage :- potential difference is the energy required to move a unit charge through an element measured in volts.

$$V = \frac{dw}{dq}$$

Current :-

⇒ flow of electrons in a circuit is any conductor is called current. It is represented by I. The units of current is Ampere.

⇒ Time rate of change of charge is called current.

$$I = \frac{dq}{dt}$$

Charge :- Charge is an electrical property of the atomic particles of which matter consists is called charge. It is measured in coulombs (C).

$$Q = i \times t$$

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Power :- power is the time rate of expending or absorbing energy. It is measured in watts.

$$p = \frac{dw}{dt}$$

$$p = \frac{dw}{dq} \cdot \frac{dq}{dt} = vi$$

$$\boxed{p = vi}$$

Resistance :- Resistance may be defined as the property of a substance which opposes the flow of electricity. It is represented by R. It is measured in Ω.

The resistance depends upon the following factors.

⟹ The resistance of the conductor varies directly as its length i.e conductor length increases, resistance increases.

$$R \propto L$$

⟹ The resistance of the conductor varies inversely as its cross sectional area i.e cross section area increases, resistance decreases.

$$R \propto \frac{1}{A}$$

⟹ The resistance of a conductor depends on the nature of the material

⟹ The resistance of a conductor depends on its temperature.

formula for calculating the resistance from one point to another point is

$$R \propto L \quad \& \quad R \propto \frac{1}{A}$$

$$R \propto \frac{L}{A}$$

$$R = \rho \frac{L}{A}$$

ρ is constant for the material called its specific resistance

ρ is the resistivity.

Energy :- Energy is the capacity to do work. It is measured in Joules.

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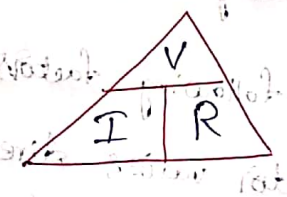
ohm's Law :-

ohm's law states that the voltage across a resistor is directly proportional to the current flowing through the resistor.

$$V \propto I$$

$$V = IR$$

R is the proportionality constant that is resistance.



What ever you want to find out place your thumb on it - the other two will tell the result.

Limitations of ohm's Law

→ ohm's law not applicable for some materials & devices.

Ex: diode

→ Non-linear elements do not support ohm's law

Ex: Thyristor.

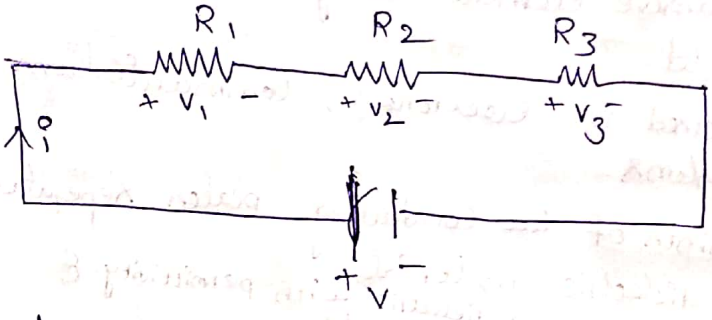
→ Conductors which does not obey ohm's law. It is called non-ohmic conductors.

→ Semiconductors like Ge, Si do not obey ohm's law.

→ It is not applicable for unilateral networks.

→ It is not applied for LED.

Resistance in series ckt :-



→ current will be same in each resistance.

→ voltage drops according to the branch resistance and the voltage will be equal to the applied voltage.

$$V = V_1 + V_2 + V_3$$

$$V = IR_1 + IR_2 + IR_3$$

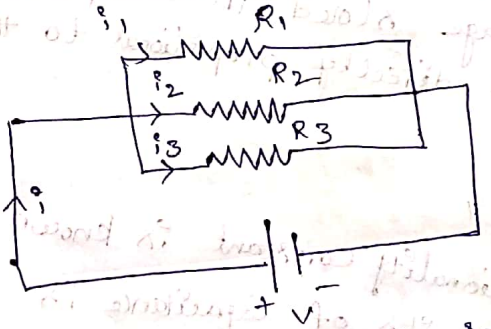
$$V = I(R_1 + R_2 + R_3)$$

$$\frac{V}{I} = R_1 + R_2 + R_3$$

$$\boxed{R_{eq} = R_1 + R_2 + R_3}$$

The total resistance in series ckt will be equal to the sum of all the resistance connected in series.

Resistance in parallel ckt :-



In parallel ckt the voltage is same in each branch.
Total current I divides in seperate branches.

$$I = I_1 + I_2 + I_3$$

$$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$\frac{I}{V} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$\left(R = \frac{V}{I}\right)$

The total resistance in parallel ckt is equal to the sum of the reciprocal of individual resistances.

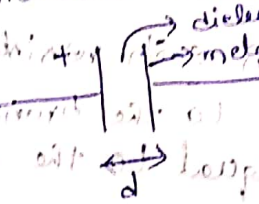
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Capacitor :-

Capacitor is a passive element designed to store energy in its electric field.

Capacitors are used in electronics, communications, computers and power systems.

A capacitor consists of two conducting plates separated by a insulating or dielectric material.



The metal plates may be aluminium foil while the dielectric may be air, paper or mica.

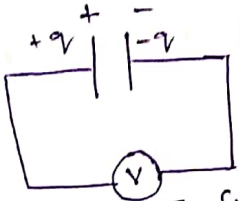


fig: Capacitor with applied voltage.

When a voltage source V is connected to the capacitor, the source deposits a positive charge q on one plate and a negative charge −q on the other plate.

The amount of charge stored in a capacitor, represented by q is directly proportional to the applied voltage

$$q \propto V$$

$$q = CV$$

where C is the proportionality constant is known as the capacitance of the capacitor. The units of capacitance is farad (F).

The capacitance of a capacitor is the ratio of the charge q per plate to the applied voltage V. It does not depends on q or V. It depends on the physical dimensions of the capacitor.

for a parallel plate capacitor the capacitance is

$$C = \frac{\varepsilon A}{d}$$

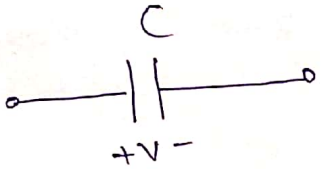
A → surface area

d → distance b/w two plates.

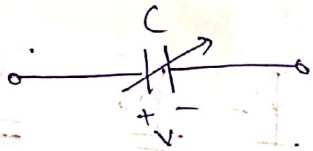
ε → permittivity of the dielectric material b/w the plates.

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Symbolic representation of capacitors



fixed Capacitor



variable capacitor.

Current - voltage relationship of the capacitor :

The current - voltage relationship of the capacitor is

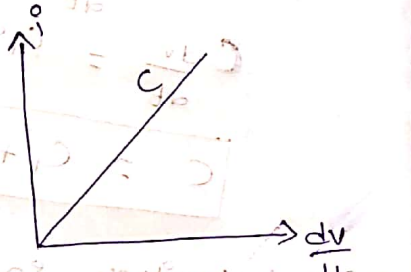
$$q = CV$$

Differentiating on both sides with respect to t

$$\frac{dq}{dt} = C\frac{dv}{dt}$$

$$i = \frac{dq}{dt}$$

$$i = C\frac{dv}{dt}$$



Current - voltage relationship of the capacitor.

The current - voltage relationship of the capacitor can be obtained by integrating the both sides of the equation $i = C\frac{dv}{dt}$

$$\int_{to}^{t} i(t)\,dt = \int_{to}^{t} C\frac{dv}{dt}\,dt$$

$$\frac{1}{C}\int_{to}^{t} i(t)\,dt = \int_{to}^{t} dV$$

$$\boxed{v(t) = \frac{1}{C}\int_{to}^{t} i(t)\,dt}$$

The power delivered to the capacitor is

$$\boxed{P = vi = v \cdot C\frac{dv}{dt}}$$

The Energy stored in the capacitor is

$$P = \frac{dw}{dt}$$

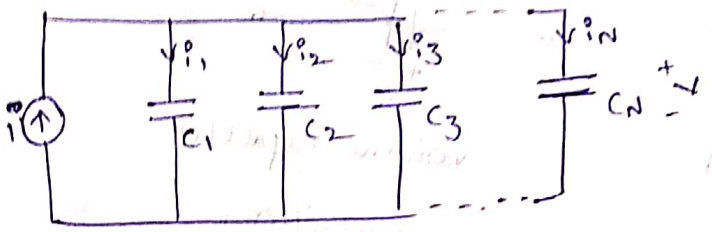
$$\int_{to}^{t} P(t)\,dt = w$$

$$w = \int_{to}^{t} v \cdot C\frac{dv}{dt}\,dt = C\int_{to}^{t} v\,dv$$

$$\boxed{w = \frac{1}{2}Cv^2}$$

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Capacitance in parallel Ckt :-



The capacitors have the same voltage

$$i = i_1 + i_2 + i_3 + \cdots + i_N$$

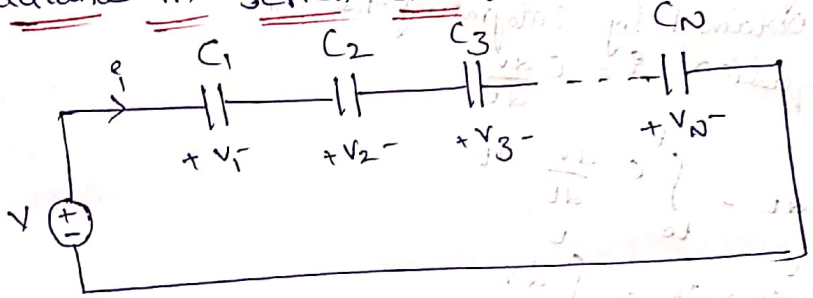
$$i = C\frac{dv}{dt}$$

$$C\frac{dv}{dt} = C_1\frac{dv}{dt} + C_2\frac{dv}{dt} + C_3\frac{dv}{dt} + \cdots + C_N\frac{dv}{dt}$$

$$\boxed{C = C_1 + C_2 + C_3 + \cdots + C_N}$$

Total capacitance in parallel Ckt is the sum of the individual capacitances.

Capacitance in series Ckt :-



$$v = v_1 + v_2 + v_3 + \cdots + v_N$$

$$v = \frac{1}{C}\int_{t_0}^{t} i(t)\,dt$$

$$\frac{1}{C}\int_{t_0}^{t} i(t)\,dt = \frac{1}{C_1}\int_{t_0}^{t} i(t)\,dt + \frac{1}{C_2}\int_{t_0}^{t} i(t)\,dt + \frac{1}{C_3}\int_{t_0}^{t} i(t)\,dt + \cdots + \frac{1}{C_N}\int_{t_0}^{t} i(t)\,dt$$

$$\boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

Total capacitance in the series connected capacitor is the reciprocal of the sum of the reciprocals of the individual capacitance.

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Inductor :- Inductor is a passive element designed to store the energy in its magnetic field.

Inductors are used in electronics, power systems, power suppliers, transformers, radios, TV's, electric motors etc.

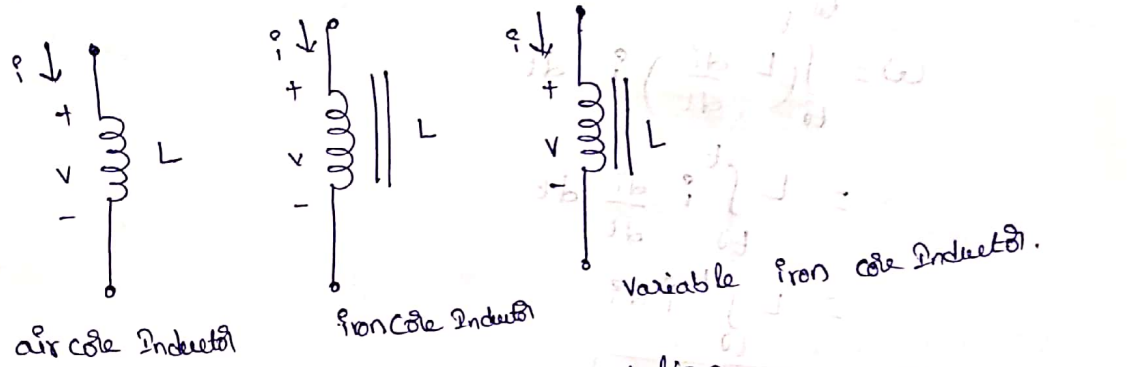


fig: Symbolic Representation.

If current is allowed to pass through an inductor is find that the voltage across the inductor is directly propotional to the time rate of change of the current :-

$$V \propto \frac{di}{dt}$$

$$\boxed{V = L \frac{di}{dt}}$$

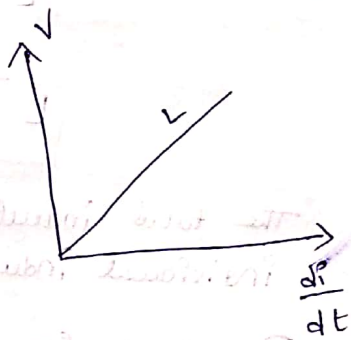
where L is the propotionality constant, it is called Inductance.

The units of Inductance is henrys (H).

Current - voltage relationship of an inductor :-

$$V = L \frac{di}{dt}$$

$$\frac{1}{L} V = \frac{di}{dt}$$



Integrating both sides w.r.t t

$$\int_{t0}^{t} \frac{1}{L} V \, dt = \int_{t0}^{t} \frac{di}{dt} \cdot dt$$

$$\boxed{i(t) = \frac{1}{L} \int_{t0}^{t} V(t) \, dt}$$

power delivered to the inductor is

$$\boxed{P = Vi = \left(L \frac{di}{dt}\right) i}$$

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The energy stored in a inductor is

$$P = \frac{dw}{dt}$$

$$w = \int_{t_0}^{t} P(t)\,dt$$

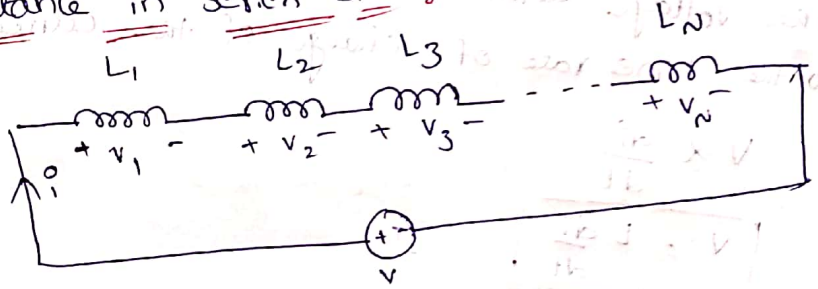
$$w = \int_{t_0}^{t} \left(L \frac{di}{dt}\right) i \, dt$$

$$= L \int_{t_0}^{t} i \frac{di}{dt} dt$$

$$= L \int_{t_0}^{t} i \, di$$

$$\boxed{w = \frac{1}{2} L i^2}$$

Inductance in series ckt :-



$$V = V_1 + V_2 + V_3 + \cdots + V_N$$

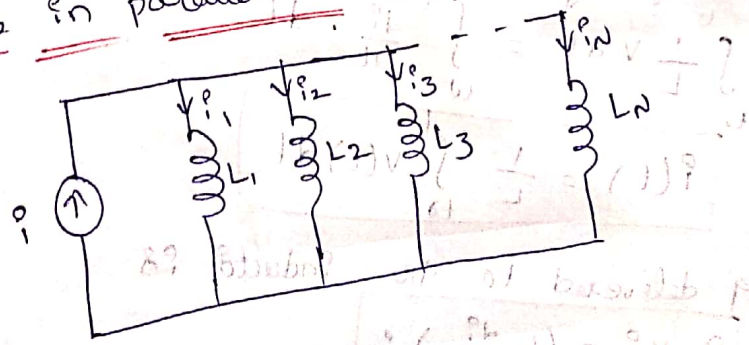
$$V = L \frac{di}{dt}$$

$$L \frac{di}{dt} = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + L_3 \frac{di}{dt} + \cdots + L_N \frac{di}{dt}$$

$$\boxed{L = L_1 + L_2 + L_3 + \cdots + L_N}$$

The total inductance in series ckt is the sum of the individual inductances.

Inductance in parallel ckt :-



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$$i = i_1 + i_2 + i_3 + \cdots + i_N$$

$$i = \frac{1}{L}\int_{t_0}^{t} v\,dt$$

$$\frac{1}{L}\int_{t_0}^{t} v\,dt = \frac{1}{L_1}\int_{t_0}^{t} v\,dt + \frac{1}{L_2}\int_{t_0}^{t} v\,dt + \frac{1}{L_3}\int_{t_0}^{t} v\,dt + \cdots + \frac{1}{L_N}\int_{t_0}^{t} v\,dt$$

$$\boxed{\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_N}}$$

The equivalent inductance of parallel connected inductors is the reciprocal of the sum of the reciprocals of the individual inductance.

Problems :-

① The total charge entering a terminal is given by $q = 5t \sin 4\pi t$ mC. Calculate the current at $t = 0.5$ sec.

Sol

$$\boxed{i = \frac{dq}{dt}} = \frac{d}{dt}(5t \sin 4\pi t)$$

$$= 5\sin 4\pi t + 5t \cos 4\pi t \times 4\pi$$

$$= 5\sin 4\pi t + 20\pi t \cos 4\pi t$$

At $t = 0.5$ sec

$$i = 5\sin 4\pi \times 0.5 + 20\pi (0.5)\cos 4\pi \times 0.5$$

$$= 5\sin 2\pi + 10\pi \cos 2\pi$$

$$= 5 \times 0 + 10\pi \times 1$$

$$i = 10\pi$$

$$\boxed{i = 10 \times 3.14 = 31.4 \text{ mA}}$$

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(2) Determine the total charge entering a terminal between $t = 1\,sec$ and $t = 2\,sec$. If the current passing the terminal is $i = (3t^2 - t)\,A$

Sol

$$Q = \int_{t_0}^{t} i\,dt$$

$$Q = \int_{1}^{2} (3t^2 - t)\,dt$$

$$= \left[\frac{3t^3}{3} - \frac{t^2}{2} \right]_1^2$$

$$= \left[t^3 - \frac{t^2}{2} \right]_1^2$$

$$= \left[2^3 - 1^3 \right] - \frac{1}{2}\left[2^2 - 1^2 \right]$$

$$= [8 - 1] - \frac{1}{2}[4 - 1]$$

$$= 7 - \frac{1}{2}[3]$$

$$= 7 - \frac{3}{2}$$

$$= 7 - 1.5$$

$$Q = 5.5\,C$$

(3) An electric blanket that is connected to 120V outlet consumes 0.140kW. what is the resistance of blanket?

Sol

$V = 120V$

$P = 0.140kW = 140W$

$P = VI$

$P = V \cdot \frac{V}{R}$

$P = \frac{V^2}{R}$

$140 = \frac{120 \times 120}{R}$

$R = \frac{120 \times 120}{140} = 102.8\,\Omega$

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(5) A 12V automobile light is rated at 30W. The total charge that flows through the filament in one minute.

Sol

$V = 12V$ $\quad t = 1 min$

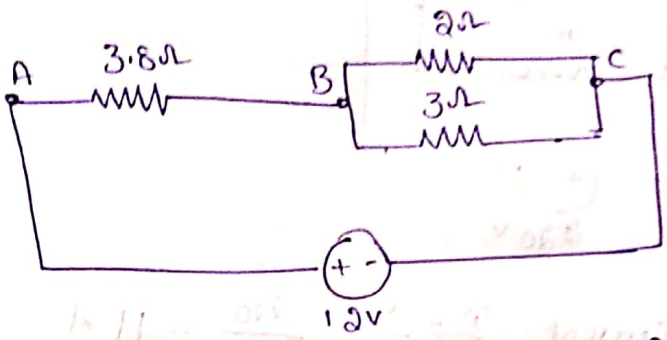
$P = 30 W$ $\quad t = 60 sec$

$P = VI$

$I = \frac{P}{V} = \frac{30}{12} = 2.5 A$

Charge $Q = I \times t = 2.5 \times 60 = 150 C$

(6)



a) Calculate resistance between A & C

b) Calculate potential difference b/w A & B

c) Calculate current in 3Ω branch.

Sol

a) find Resistance b/w A & C

Resistance b/w B & C

$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{2} + \frac{1}{3} = \frac{3+2}{6} = 5/6$

$R = 6/5 = 1.2 \Omega$

Resistance b/w A & C

$R = \frac{6}{5}$ (+) $R = 3.8 + 1.2 = 5 \Omega$

b) Calculate potential difference b/w A & B.

The total current in ckt $I = \frac{V}{R} = \frac{12}{5} = 2.4 A$

potential difference b/w A & B $V = IR$

$V = 2.4 \times 3.8$

$V = 9.2V$

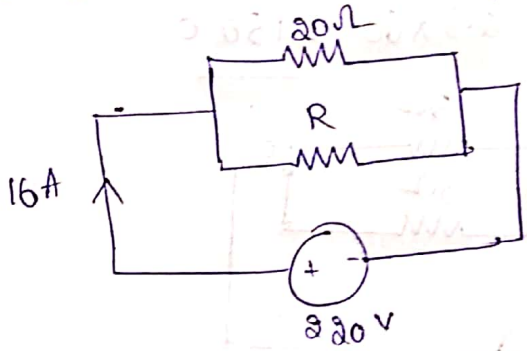
voltage across B & C $V = IR = 2.4 \times 1.2 = 2.88 V$

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c) Current in 3Ω branch resistance

$$I = \frac{V}{R} = \frac{2.88}{3} = 0.96\ A$$

(7) A current of 16 A is distributed in a ckt having two branches connected on 220V supply. If the resistance of one branch is 80Ω. find out the resistance of the other branch and power taken by both branches.



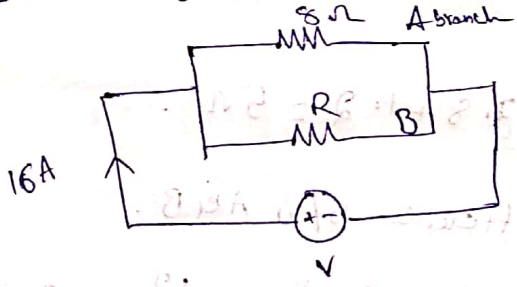
Sol) 20Ω branch current $I = \frac{V}{R} = \frac{220}{20} = 11\ A$

Current in other branch $I = 16 - 11 = 5\ A$.

resistance of other branch $R = \frac{V}{I} = \frac{220}{5} = 44\Omega$

power of the ckt $P = VI = 220 \times 16 = 3520\ W$.

(8) A current of 16 A divides in two branches of circuits in parallel, one branch having a resistance of 8Ω. Determine the current in each branch and power in each branch if the total power wasted in 1280 Watts.



Sol) power $P = 1280\ W$

$$P = V I$$

$$1280 = V \times 16$$

$$V = \frac{1280}{16} = 80 \text{ Volts}.$$

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Current in A branch $I = \frac{V}{R} = \frac{80}{8} = 10A$

Current in B branch $I = 16 - 10 = 6A$

power in A branch $P = VI$

$P = 80 \times 10 = 800W$

power in B branch $P = 80 \times 6 = 480W$

9) find the power delivered to an element at $t = 3msec$ if the current entering its positive terminal is $i = 5\cos 60\pi t$ A and the voltage is a) $v = 3i$ b) $v = 3\frac{di}{dt}$

Sol

a) The voltage is $v = 3i$

$v = 3(5\cos 60\pi t)$

$v = 15\cos 60\pi t$

$p = vi$

$p = 15\cos 60\pi t \times 5\cos 60\pi t$

$p = 75\cos^2 60\pi t$

At $t = 3msec$

$p = 75\cos^2(60\pi \times 3 \times 10^{-3})$

$p = 75\cos^2(0.18\pi)$

$\boxed{p = 53.48W}$

b) $v = 3\frac{di}{dt}$.

$v = 3\frac{d}{dt}(5\cos 60\pi t) = 3(5\sin 60\pi t) \times 60\pi$

$v = 900\pi \sin 60\pi t$

$p = vi$

$p = 900\pi \sin 60\pi t \times 5\cos 60\pi t$

$p = 4500\pi \sin 60\pi t \cos 60\pi t$

At $t = 3msec$

$p = 4500\pi \sin 60\pi \times 3 \times 10^{-3} \cos 60\pi \times 3 \times 10^{-3}$

$\boxed{p = 6.396kW}$

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Circuit elements :-

An element is the basic building block of a ckt

There are two types of elements.

→ Active elements

→ passive elements.

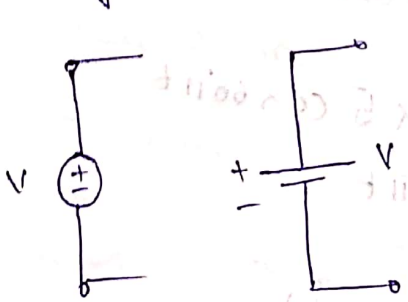
The most important active elements are voltage source and current source.

There are two kinds of sources.

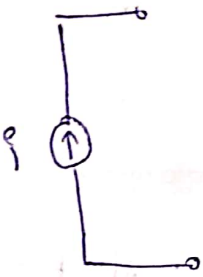
→ Independent sources

→ Dependent sources.

Symbols for Independent sources

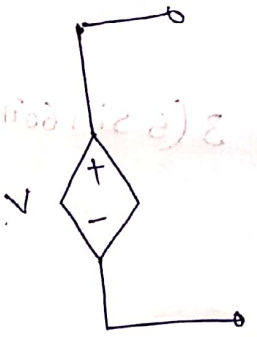


Independent voltage source.

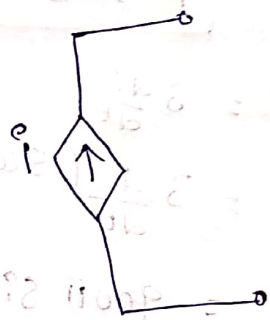


Independent current source.

Symbols for dependent sources.



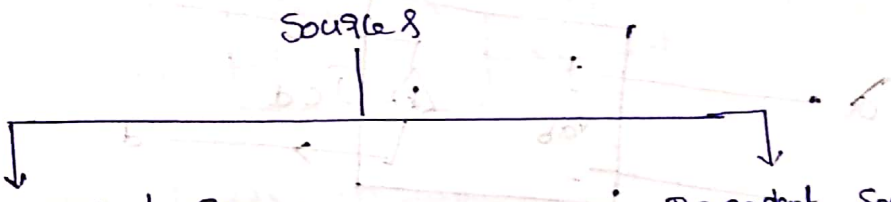
Dependent voltage source.



Dependent current source.

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Sources :-



(The value of independent sources does not depend upon any other voltage & current)

(The value of dependent sources depends upon some voltage or current)

There are two types of Independent Sources

Voltage source:- It will provide a constant voltage irrespective of any amount of current.

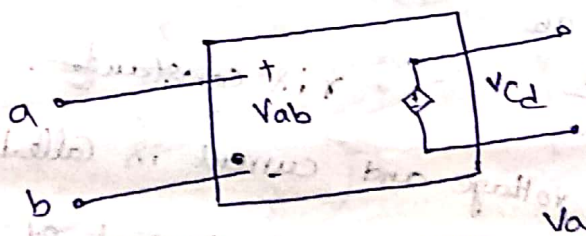
It will maintain a constant voltage across two terminals.

Current Source:- It will always maintain a constant current in the ckt.

There are four types of Dependent Sources.

It depends which voltage and current is being controlled by which voltage & current.

Voltage controlled voltage source (VCVS):-

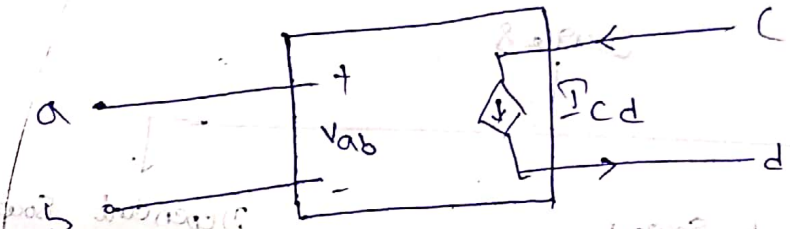


the V_{cd} is depends upon V_{ab} that dependent factor is μ

$V_{cd} = \mu V_{ab} \Rightarrow \mu = \frac{V_{cd}}{V_{ab}}$ (i.e voltage gain).

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Voltage controlled current source (VCCS):-



I_{cd} is depends on V_{ab}.

$$I_{cd} = g_m \cdot V_{ab}$$

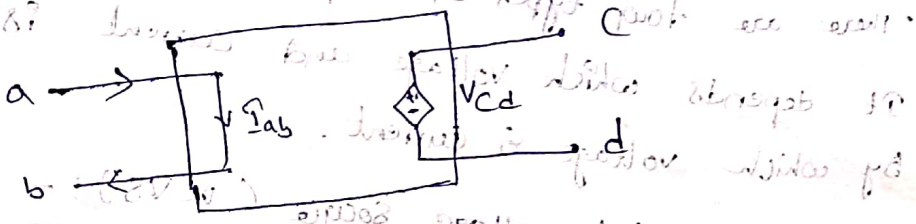
g_m is constant.

$$g_m = \frac{I_{cd}}{V_{ab}} \rightarrow \text{o/p current}, \quad \rightarrow \text{i/p voltage}$$

Ratio of current and voltage is a conductance.

The ratio of o/p current to the i/p voltage is called transconductance.

Current controlled voltage source (CCVS):-



V_{cd} depends of I_{ab}

$$V_{cd} = r \cdot I_{ab}$$

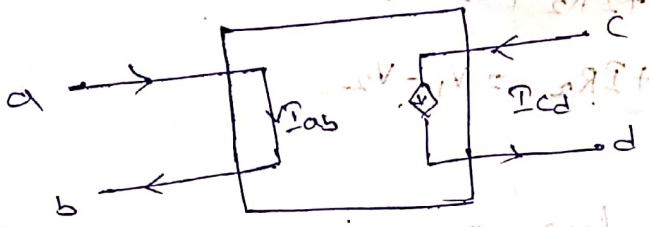
$$r = \frac{V_{cd}}{I_{ab}}$$

r is constant.

The ratio of voltage and current is called resistance.

But the ratio of voltage and current of a different current ports is called transresistance.

Current controlled current source (CCCS) :→



I_{cd} is depends on I_{ab}.

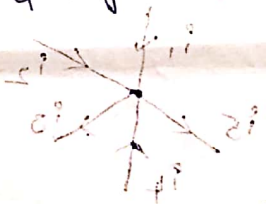
$I_{cd} = \beta I_{ab}$ → β is controlled factor.

$\beta = \frac{I_{cd}}{I_{ab}}$ (i.e current gain)

Significance of dependent sources :-

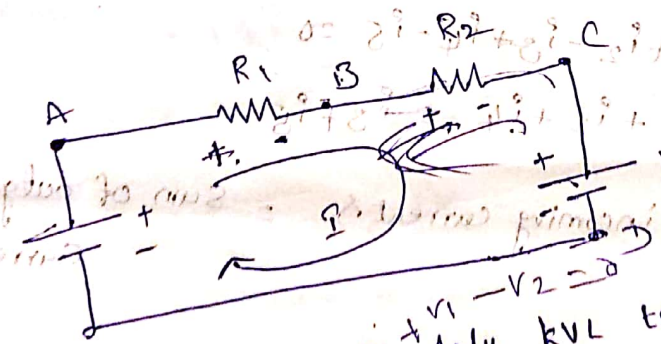
* In practice dependent sources do not exist.
* Dependent sources/Gain are used in the analyse the Ckt.

Ex ; Small signal & large signal amplifier analysis of BJT



KVL

The algebric sum of voltages across all the elements in a closed loop is zero.



$-V_1 - V_2 + V_2 = 0$

$-V_1 + V_2 = V_1 + V_2$

$-V + V_1 + V_2 + V_2 = 0$

$-V_1 + V_1 - V_2 = 0$

Apply KVL to loop ABCDEA

$-V_2 - IR_1 + V_1 - V_2 = 0$

Anti clock $+IR_1 + IR_2 + V_2 - V_1 = 0$

$\therefore -V_1 + V_2 = IR_1 + IR_2$

$V_1 - V_2 = IR_1 + IR_2$

Σ algebric sum of voltage sources = Σ voltage drops across resistors.

Anti clockwise

Sign convension

+ to − ⇒ −ve

− to + ⇒ +ve

clockwise

+ to − ⇒ +ve

− to + ⇒ −ve

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Clock wise

$$-V_1 + IR_1 + IR_2 + V_2 = 0$$

$$IR_1 + IR_2 = V_1 - V_2$$

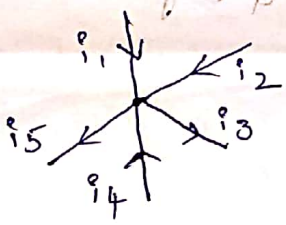
Kirchoff's current law :-

Kirchoff's current law states that the algebric sum of the currents entering a node is zero.

The total current leaving a junction is equal to the current entering that junction.

$$\sum_{m=1}^{m} i_m = 0$$

m — no. of current in loop



The incoming currents to be positive and the outgoing currents to be negative.

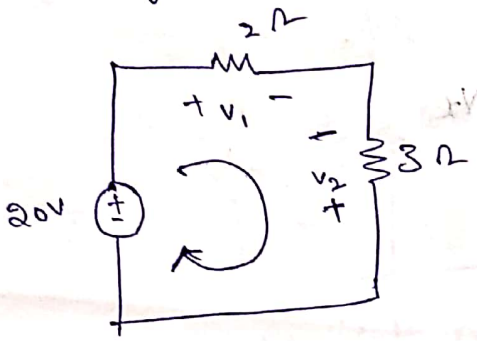
$$i_1 + i_2 - i_3 + i_4 - i_5 = 0$$

$$i_1 + i_2 + i_4 = i_3 + i_5$$

Sum of incoming currents = Sum of outgoing currents.

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(1) for the ckt as shown in below fig. find voltages v_1 & v_2



Sol

$v_1 = 2i$

$-v_2 = 3i$

$v_2 = -3i$

$-20 + v_1 - v_2 = 0$

$-20 + 2i + 3i = 0$

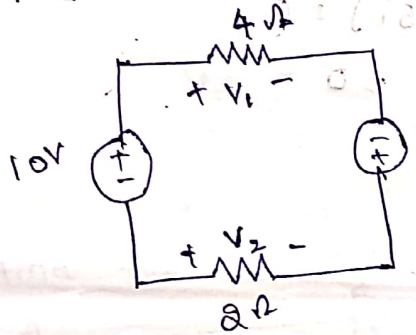
$5i = 20$

$i = 4A$

$v_1 = 2 \times 4 = 8V$

$v_2 = -3 \times 4 = -12V$

(2) find v_1 & v_2 in the ckt



Sol $v_1 = 4i$ $v_2 = -2i$

Apply KVL

$-10 + v_1 - 8 - v_2 = 0$

$-10 + 4i - 8 - (-2i) = 0$

$4i + 2i = 18$

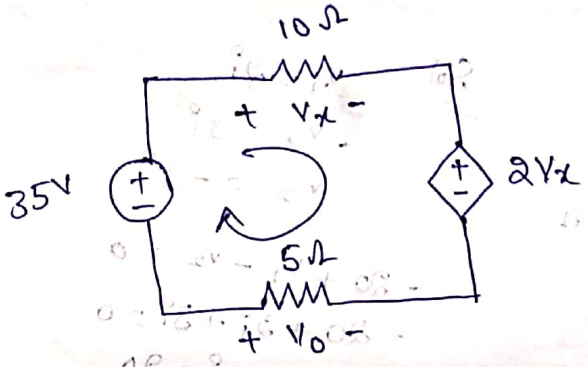
$6i = 18 \Rightarrow i = 3A$

$v_1 = 4 \times 3 = 12V$

$v_2 = -2 \times 3 = -6V$

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3) Find V_x and V_o in the Ckt



Sol

$V_x = 10i$, $V_o = -5i$

Apply KVL

$$-35 + V_x + 2V_x - V_o = 0$$

$$-35 + 10i + 2 \times 10i - (-5i) = 0$$

$$-35 + 10i + 20i + 5i = 0$$

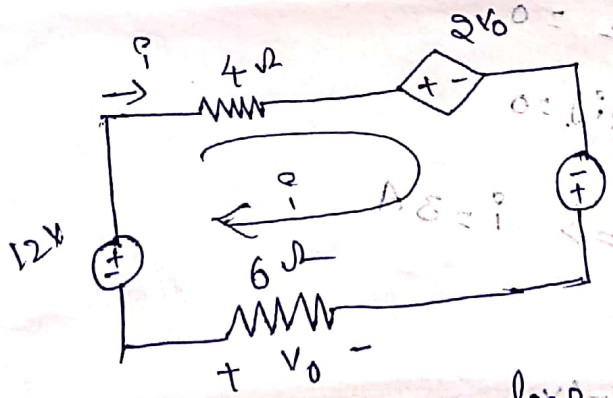
$$35i = 35$$

$$i = 1\ A$$

$$V_x = 10 \times 1 = 10V$$

$$V_o = -5 \times 1 = -5V$$

4) Determine V_o & i in the Ckt



Sol Apply KVL to the loop

$$-12 + 4i + 2V_o - 4 - V_o = 0$$

$$-12 + 4i + 2(-6i) - 4 - (-6i) = 0$$

$$-12 + 4i - 12i - 4 + 6i = 0$$

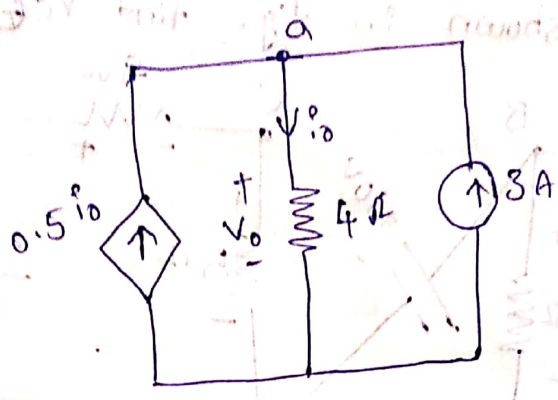
$$-2i = +16$$

$$i = -8\ A$$

$$V_o = -6 \times -8 = 48V$$

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5) find current i_o & voltage v_o in the ckt

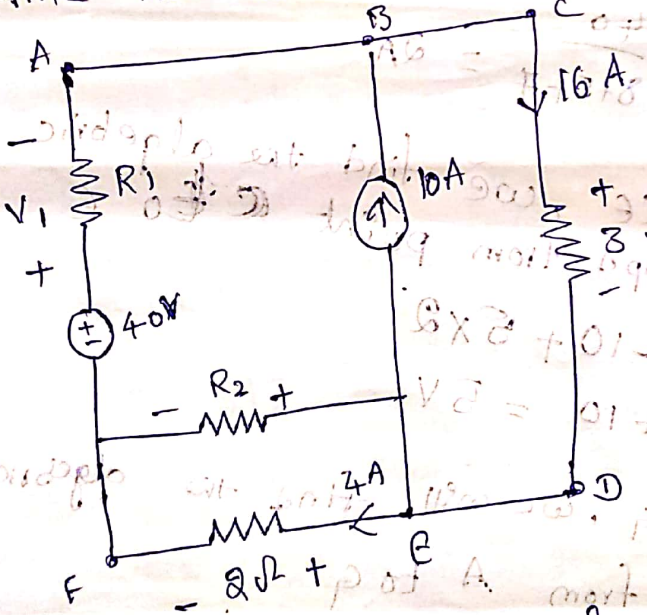


Sol Apply KCL at node a

$$0.5 i_o + 3 = i_o$$
$$3 = i_o - 0.5 i_o$$
$$3 = 0.5 i_o$$
$$i_o = 3/0.5 = 6\,A$$

$$v_o = 4 i_o$$
$$v_o = 4 \times 6 = 24\,V$$

6) find the unknown voltage V_1 in the ckt

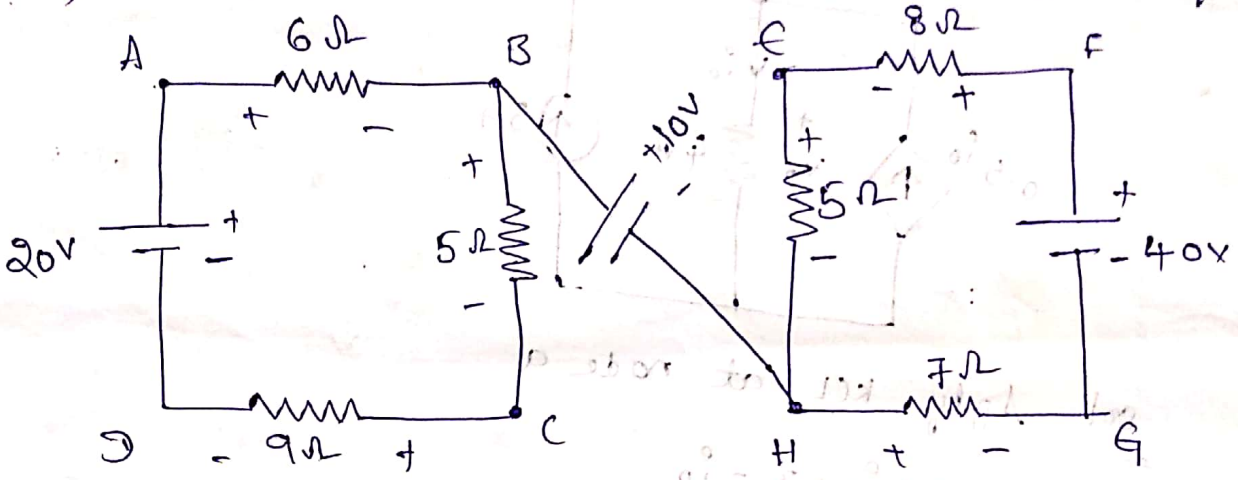


Sol Taking the outer closed loop ABCDEFA and applying KVL to it.

$$16 \times 3 + 4 \times 2 - 40 + V_1 = 0$$
$$48 + 8 - 40 + V_1 = 0$$
$$16 + V_1 = 0$$
$$V_1 = -16\,V$$

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7) for the ckt shown in fig. find V_{CE} & V_{AG}



Sol

Consider the two battery ckts seperately.

Current in the 20V battery ckt ABCD is

$$I = \frac{V}{R} = \frac{20}{6+5+9} = \frac{20}{20} = 1\,A$$

current in the 40V battery ckt EFGH

$$I = \frac{V}{R} = \frac{40}{8+5+7} = 2A$$

for finding V_{CE}, we find the algebric sum of the voltage drops from point C to E,

$$V_{CE} = +5\times1 - 10 + 5\times2$$

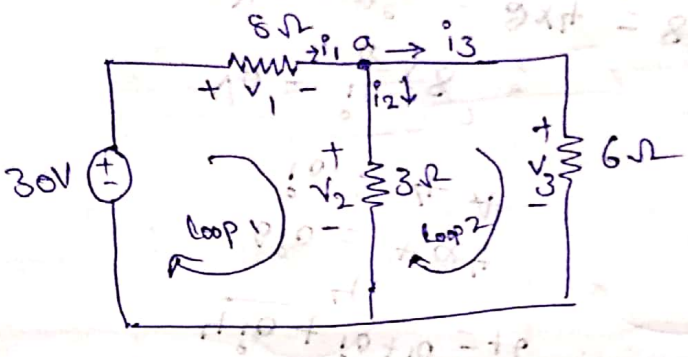
$$V_{CE} = 5 - 10 + 10 = 5\,V$$

for finding V_{AG}, we will find the algebric sum of voltage drops from A to G

$$V_{AG} = 6\times1 + 10 + 7\times2 = 6 + 10 + 14 = 30\,V$$

Scanned with CamScanner

* find currents and voltages in the ckt



Sol

we apply ohm's law

$V_1 = 8i_1 \quad V_2 = 3i_2 \quad V_3 = 6i_3$

Apply KCL to the ckt

$$i_1 - i_2 - i_3 = 0 \longrightarrow (1)$$

Applying KVL to loop 1

$$30 + 8i_1 + 3i_2 = 0$$

$$8i_1 = -3i_2 + 30$$

$$i_1 = \frac{30 - 3i_2}{8} \longrightarrow (2)$$

Apply KVL to loop 2

$$-V_2 + V_3 = 0 \longrightarrow (3)$$

$$V_3 = V_2$$

$$6i_3 = 3i_2 \Rightarrow i_3 = \frac{3i_2}{6} = \frac{i_2}{2}$$

substituting the values in eq (1)

$$\frac{30 - 3i_2}{8} - i_2 - \frac{i_2}{2} = 0$$

$$30 - 3i_2 - 8i_2 - 4i_2 = 0$$

$$30 - 15i_2 = 0$$

$$-15i_2 = -30$$

$$i_2 = 30/15 = 2A$$

$$i_1 = \frac{30 - 3\times 2}{8} = \frac{30-6}{8} = \frac{24}{8} = 3A$$

$$i_3 = i_2/2 = \frac{2}{2} = 1A$$

$V_1 = 8i_1$

$= 8 \times 3 = 24V$

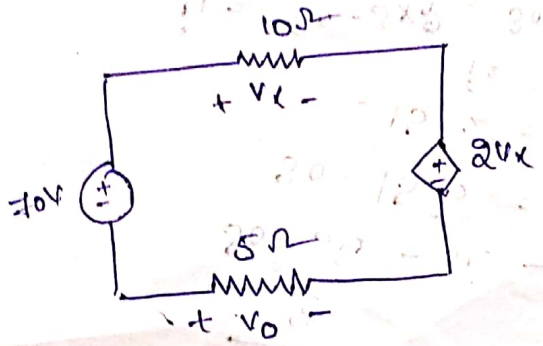
$V_2 = 3i_2$

$= 3 \times 2 = 6V$

$V_3 = 6 \times 1 = 6V$

Scanned with CamScanner

* find V_x & V_o in the ckt



Sol $V_o = -5i$ $\quad V_x = 10i$ $\quad i = \frac{V_x}{10}$

Apply KVL

$V_x + 2V_x - V_o - 70 = 0$

$3V_x + 5i = 70$

$3V_x + 5\left(\frac{V_x}{10}\right) = 70$

$3V_x + \frac{V_x}{2} = 70$

$\frac{7V_x}{2} = 70$

$7V_x = 140$

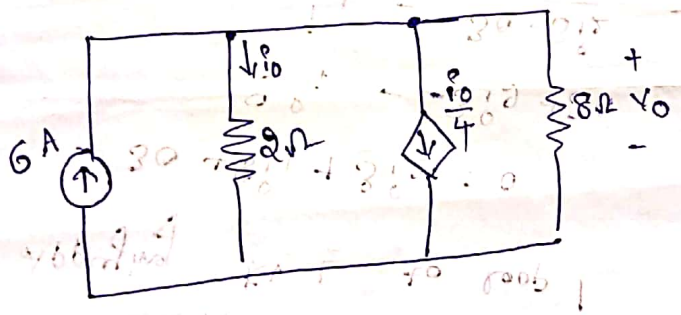
$V_x = \frac{140}{7} = 20V$

$V_x = 10i$

$20 = 10 \times i \Rightarrow i = 2A$

$V_o = -5 \times 2 = -10V$

* find current i_o and voltage V_o in the ckt



Sol Apply KCL

$6 - i_o - \frac{i_o}{4} - \frac{i_o}{4} = 0$

$-i_o - \frac{i_o}{4} - \frac{i_o}{4} = -6$

$-\left(i_o + \frac{i_o}{4} + \frac{i_o}{4}\right) = -6$

$\frac{4i_o + i_o + i_o}{4} = +6$

$6i_o = +24$

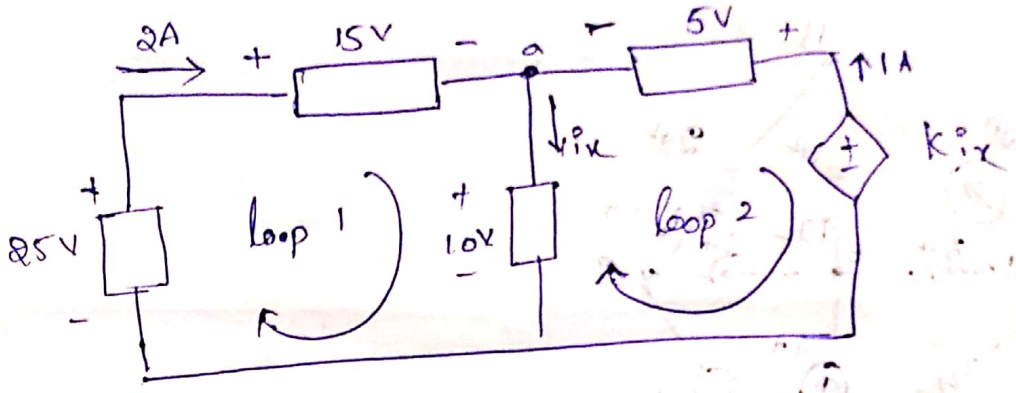
$i_o = 4A$

$V_o = \frac{i_o}{4} \times 8^2$

$V_o = 2i_o = 2 \times 4 = 8V$

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* find the value of k in the ckt



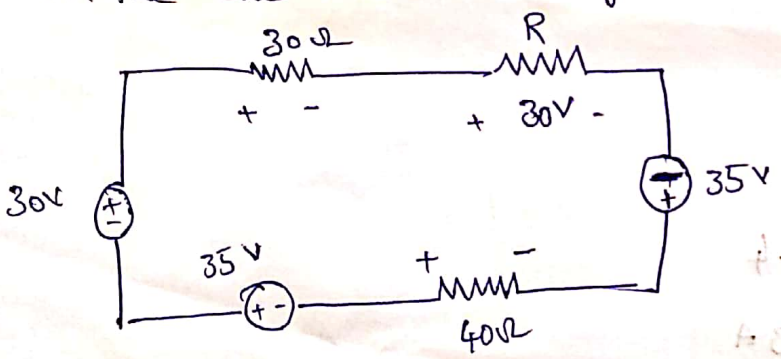
Sol Apply KVL to loop 2

$$-5 + kix - 10 = 0$$
$$kix = 15$$

Applying KCL at node a

$$2 + 1 = ix$$
$$ix = 3$$
$$k(3) = 15$$
$$k = 15/3 = 5$$

For the ckt shown in fig find the value of R



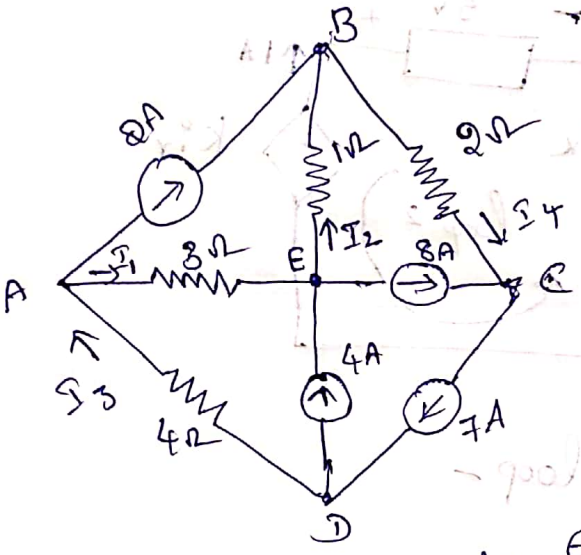
Sol Apply KVL

$$-30 + 30i + 30 - 35 - 40i - 35 = 0$$
$$-10i - 70 = 0$$
$$10i = -70$$
$$i = -7$$

$$V = IR$$
$$30 = -7(R)$$
$$R = -30/7 = -4.2\ \Omega$$

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(4) In the given ckt find I_1 & I_2



Sol Apply KCL at node E

$$I_1 - I_2 - 8 + 4 = 0$$

$$I_1 + 4 = I_2 + 8$$

$$I_2 = I_1 + 4 - 8$$

$$I_2 = I_1 - 4 \longrightarrow (1)$$

Apply KCL at node D

$$4 + I_3 = 7$$

$$I_3 = 3A \longrightarrow (2)$$

At node A

$$I_1 + 2 = I_3$$

$$I_1 = 1\ A$$

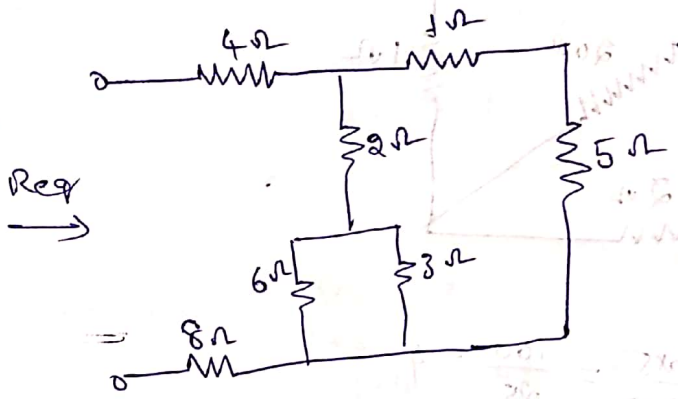
eq (1) $\Rightarrow$ $I_2 = 1 - 4$

$$I_2 = -3\ A$$

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Circuit Reduction techniques

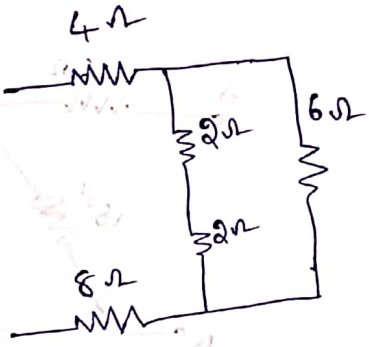
① find equivalent resistance of ckt.



Sol:

6Ω & 3Ω are in parallel

$$R = \frac{R_1 R_2}{R_1 + R_2} = \frac{6 \times 3}{6+3} = \frac{18}{9} = 2\Omega$$

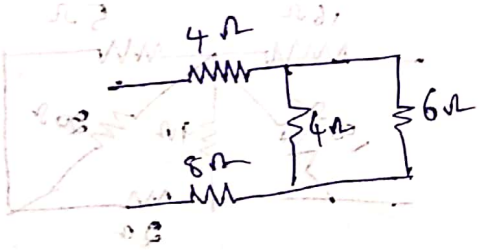


1Ω & 5Ω are in series

$R = 1+5 = 6\Omega$

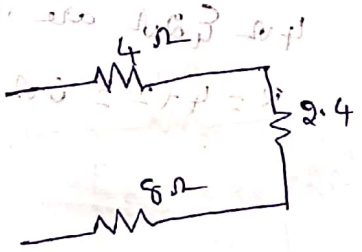
2Ω & 2Ω are in series

$R = 2+2 = 4\Omega$



$4\Omega \,//\, 6\Omega$

$$R = \frac{4 \times 6}{4+6} = \frac{24}{10} = 2.4$$

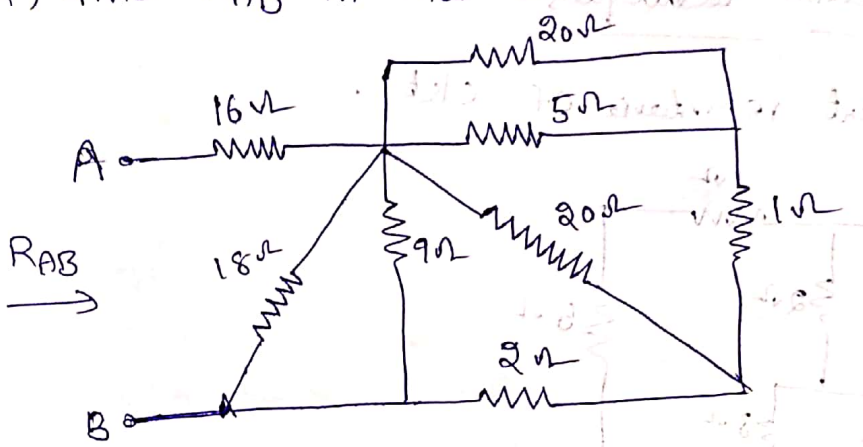


4Ω & 5Ω & 2.4Ω are in series

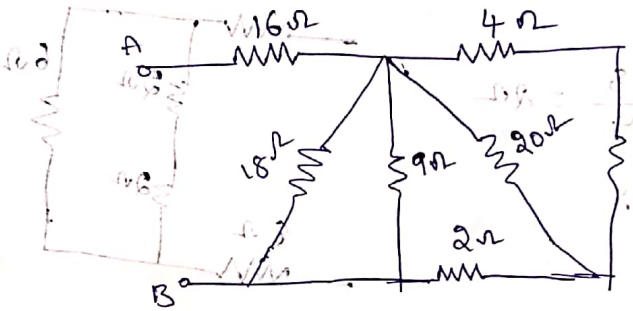
$R = 4 + 2.4 + 8 = 14.4\Omega$

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2) find R_{AB} in the Ckt.

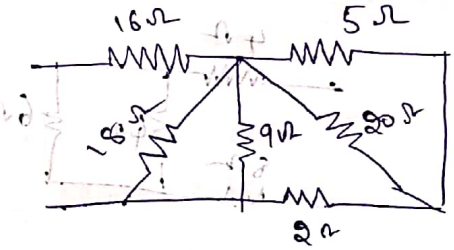


Sol $20\Omega // 5\Omega = \frac{20\times5}{20+5} = \frac{100}{25} = 4$



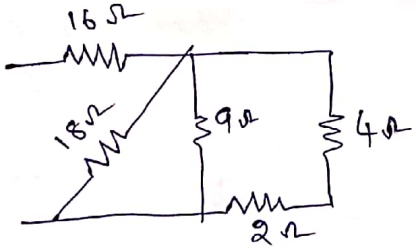
$\therefore 4\Omega$ & 1Ω are in series

$R = 4+1 = 5\Omega$



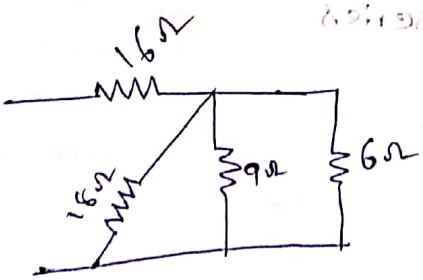
$5\Omega // 20\Omega$

$R = \frac{5\times20}{5+20} = \frac{100}{25} = 4$



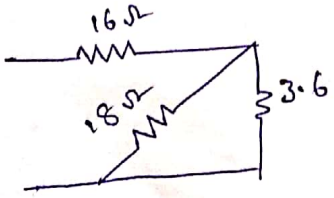
4Ω & 2Ω are in series

$R = 4+2 = 6\Omega$



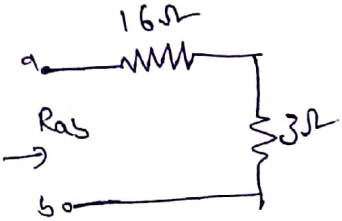
$9\Omega // 6\Omega$

$R = \frac{9\times6}{9+6} = \frac{54}{15} = 3.6$



$18\Omega // 3.6\Omega$

$R = \frac{18\times3.6}{18+3.6} = 3\Omega$

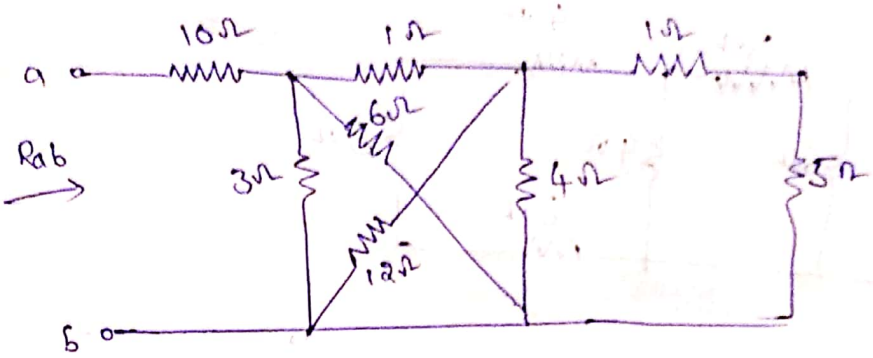


16Ω & 3Ω are in series

$R_{ab} = 16+3 = 19\Omega$

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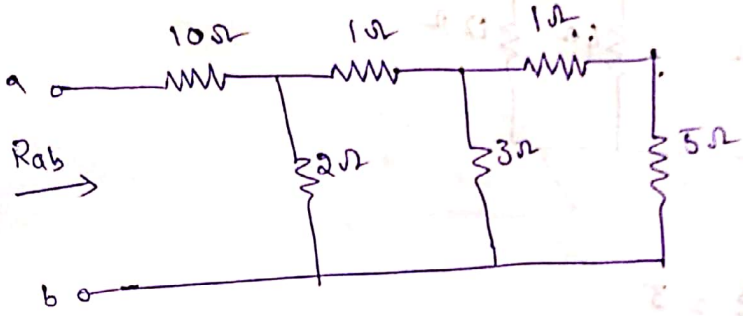
3) Calculate the resistance Rab in the ckt



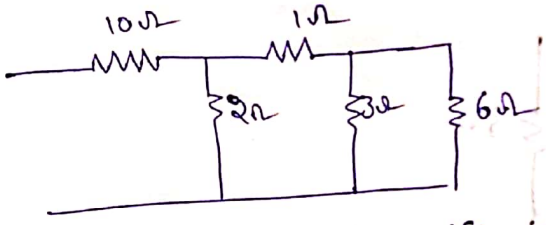
Sol

$3\Omega \,//\, 6\Omega = \frac{3\times 6}{3+6} = \frac{18}{9} = 2$

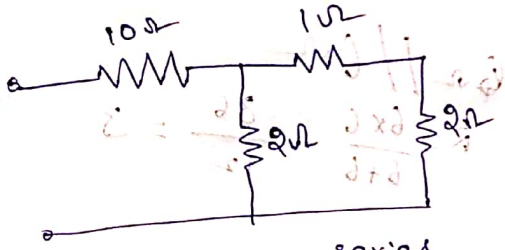
$12\Omega \,//\, 4\Omega = \frac{12\times 4}{12+4} = \frac{48}{16} = 3$



1Ω & 5Ω are in series $R = 1+5 = 6\Omega$

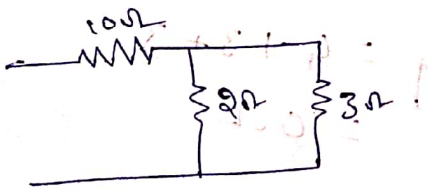


$6\Omega \,//\, 3\Omega = \frac{6\times 3}{6+3} = \frac{18}{9} = 2\Omega$

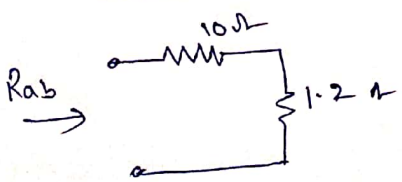


1Ω & 2Ω are in series

$R = 1+2 = 3\Omega$



$2\Omega \,//\, 3\Omega = \frac{2\times 3}{2+3} = \frac{6}{5} = 1.2$



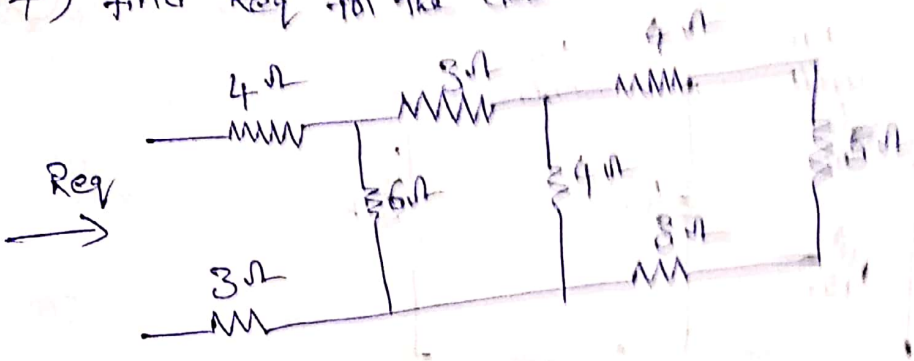
10Ω & 1.2Ω are in series

$R_{ab} = 10 + 1.2$

$R_{ab} = 11.2\Omega$

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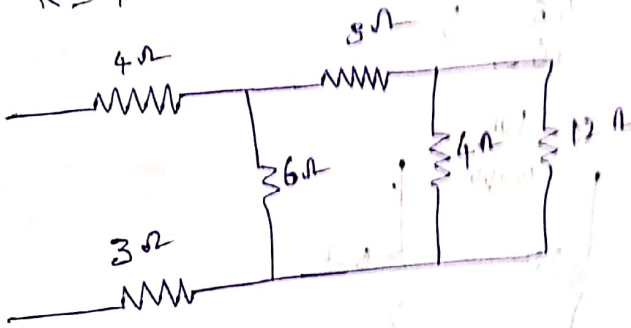
4) find Req for the ckt



Sol

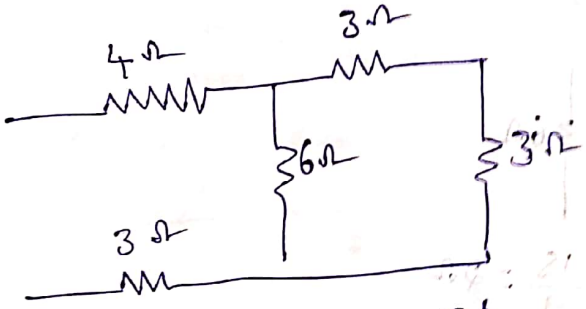
4Ω & 5Ω & 3Ω are in series

$R = 4+5+3 = 12\,\Omega$



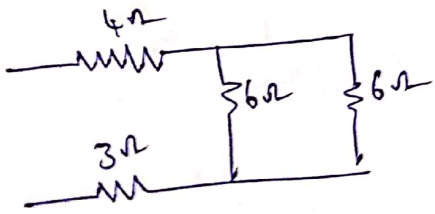
$4\Omega \,//\, 12\Omega$

$R = \frac{4\times12}{4+12} = \frac{48}{16} = 3$



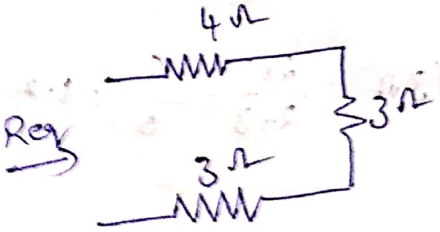
3Ω & 3Ω are in series

$R = 3+3 = 6$



$6\Omega \,//\, 6\Omega$

$R = \frac{6\times6}{6+6} = \frac{36}{12} = 3$



4Ω, 3Ω, 3Ω are in series

$Req = 4+3+3$

$= 10\,\Omega$

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Wye - Delta transformations :-

Star - delta transformations and delta - Star transformations allow us to convert one type of circuit connection into another type of circuit connection inorder to easily analyze the Ckt.

These transformation techniques can be used to good effect for either Star or delta Ckts containing resistances or impedances.

In Ckt analysis when the resistors are neither in parallel nor in series.

Ex: Bridge Ckt

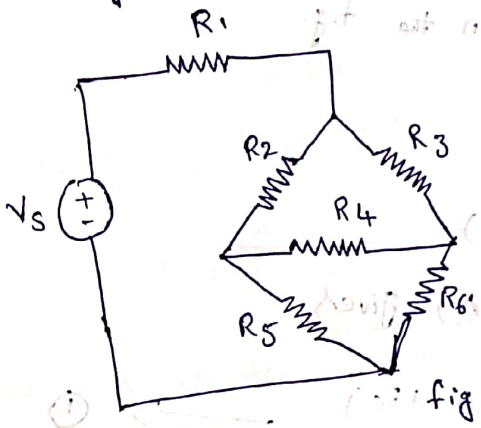


fig: Bridge circuit

Many Ckts are bridge type networks, it can be simplified by using three terminal equivalent networks. These are the Y or T network and Δ or Π network.

These networks are used in the three phase networks, electrical filters and matching networks.

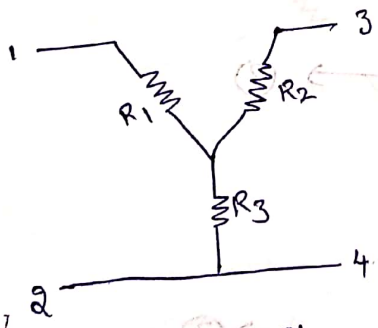


fig: Y - network

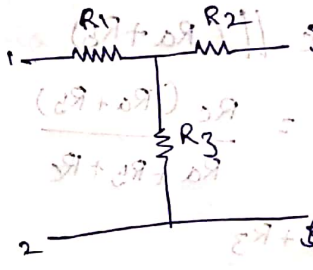


fig: T - network.

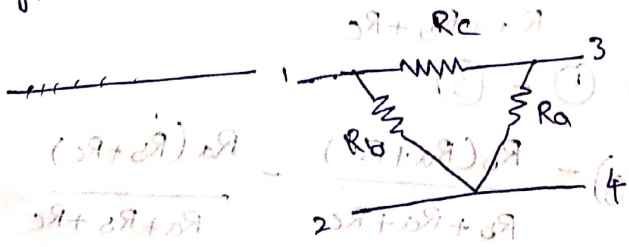


fig: Δ - network

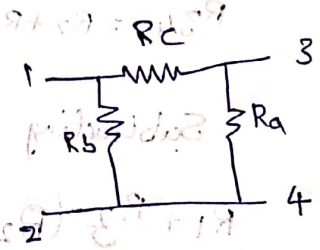


fig: Π - network.

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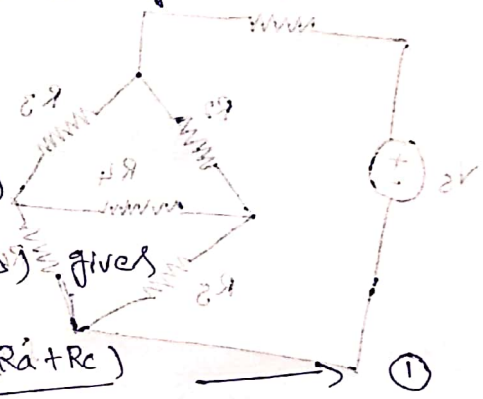
Delta to wye Conversion :-

It is more convinient to work with a wye n/w, in a place where the ckt contains a delta configuration.

we have to superimpose a wye n/w on the existing delta n/w and find the equivalent resistances in the wye n/w.

The resistance b/w each pair of nodes in the Δ or π n/w is the same as the resistance b/w the same pair of nodes in the Y or T n/w.

for the terminals 1 & 2 in the fig.



$$R_{12}(Y) = R_1 + R_3$$

$$R_{12}(\Delta) = R_b \,||\, (R_a + R_c)$$

Equating $R_{12}(Y) = R_{12}(\Delta)$ gives

$$R_{12} = R_1 + R_3 = \frac{R_b(R_a + R_c)}{R_b + (R_a + R_c)} \longrightarrow (1)$$

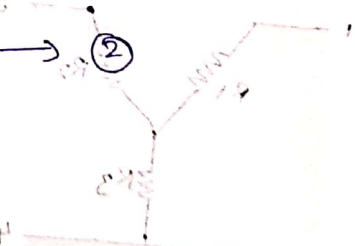
Similarly

$$R_{13}(T) = R_{13}(\pi)$$

$$R_{13}(T) = R_1 + R_2$$

$$R_{13}(\pi) = R_c \,||\, (R_a + R_b)$$

$$R_{13} = R_1 + R_2 = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c} \longrightarrow (2)$$



$$R_{34}(T) = R_2 + R_3$$

$$R_{34}(\pi) = R_a \,||\, (R_b + R_c)$$

$$R_{34} = R_2 + R_3 = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c} \longrightarrow (3)$$

Subtracting eq (1) - (3)

$$R_1 + R_3 - (R_2 + R_3) = \frac{R_b(R_a + R_c)}{R_b + R_a + R_c} - \frac{R_a(R_b + R_c)}{R_a + R_b + R_c}$$

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$$R_1 + R_3 - R_2 - R_3 = \frac{R_a R_b + R_b R_c - R_a R_b - R_a R_c}{R_a + R_b + R_c}$$

$$R_1 - R_2 = \frac{R_c (R_b - R_a)}{R_a + R_b + R_c} \longrightarrow (4)$$

Adding eq (2) + (4)

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} \longrightarrow (5)$$

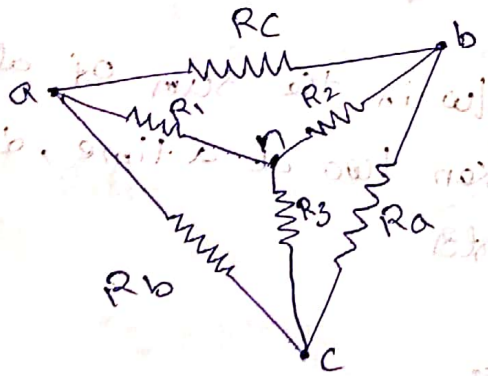
Subtracting eq (4) - (2)

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} \longrightarrow (6)$$

Subtracting equations (5) - (1)

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} \longrightarrow (7)$$

Each resistor in the Y network is the product of the resistor in the two adjacent Δ branches devided by the sum of the three Δ resistors.





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Y to Δ Conversion :-

To obtain the conversion formulas for transforming a Y-n/w to an equivalent delta n/w.

from equations ⑤, ⑥, ⑦

⑤×⑥ + ⑥×⑦ + ⑦×⑤

$$R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_a R_b R_c (R_a + R_b + R_c)}{(R_a + R_b + R_c)^2} \longrightarrow ⑧$$

Dividing eq ⑧ by eq ⑤

$$\therefore R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_aR_bR_c}{R_a + R_b + R_c} \longrightarrow ⑧$$

Dividing eq ⑧ by eq ⑤

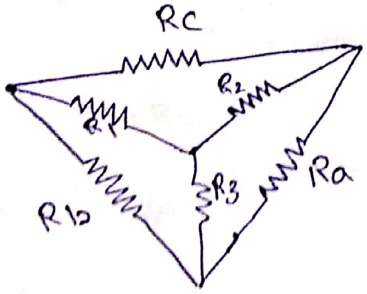
$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} \longrightarrow ⑨$$

eq ⑧/⑥ ⇒ $R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} \longrightarrow ⑩$

eq ⑧/⑦ ⇒ $R_C = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} \longrightarrow ⑪$

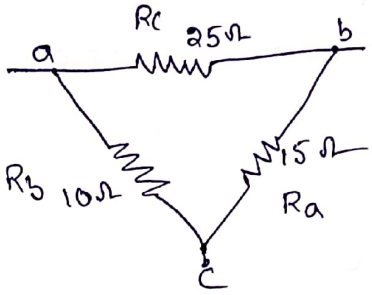
from eq ⑨, ⑩, ⑪ are the conversion rule for Y to Δ network.

Each resistor in the Δ n/w is the sum of all possible products of Y resistors taken two at a time, divided by the opposite Y-resistor.



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(1). Convert the Δ n/w to an equivalent Y n/w

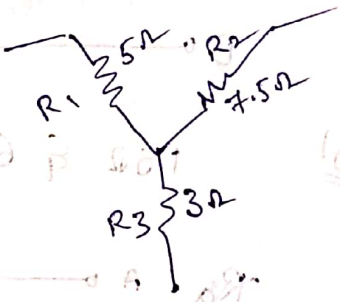


Sol

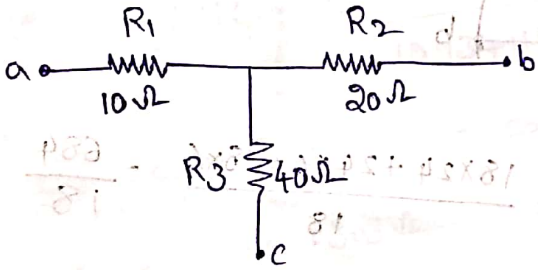
$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{10 \times 25}{25+10+15} = 5\Omega$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} = \frac{25 \times 15}{25+10+15} = 7.5\Omega$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{15 \times 10}{50} = 3\Omega$$



(2) Transform the Y n/w to delta n/w

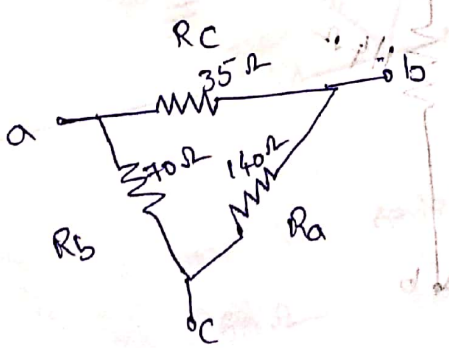


Sol

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} = \frac{10\times20 + 20\times40 + 10\times40}{10} = \frac{1400}{10} = 140$$

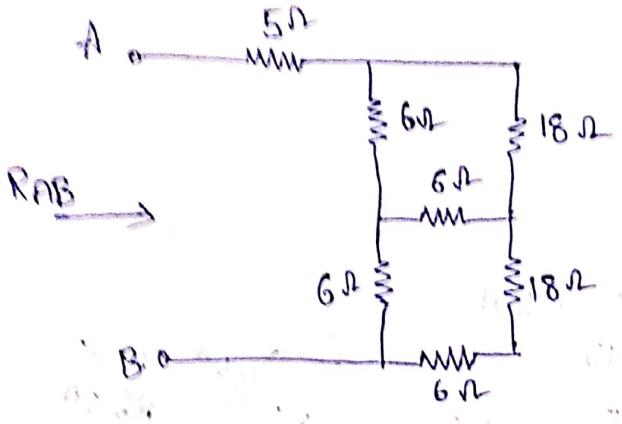
$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} = \frac{1400}{20} = 70\Omega$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} = \frac{1400}{40} = 35\Omega$$

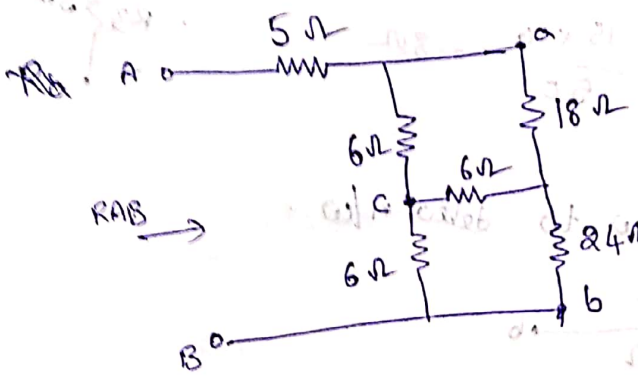


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(9) find equivalent R_{AB} using Y − Δ transformation.



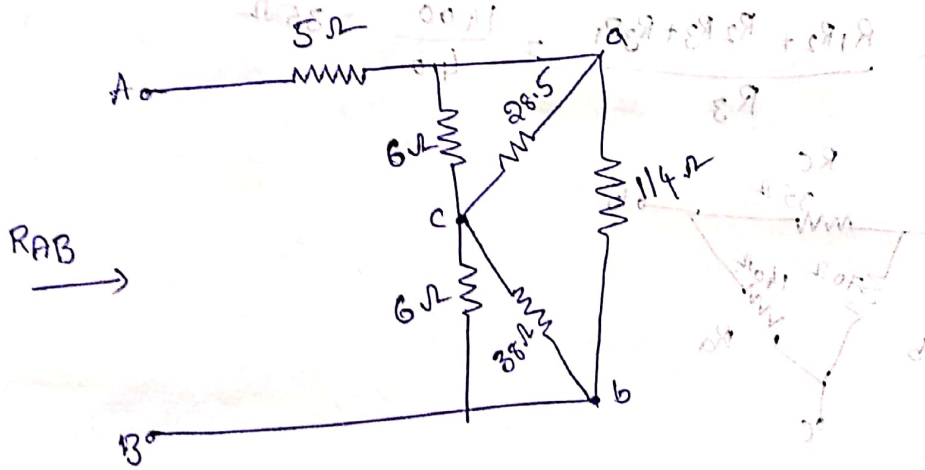
Sol 18Ω & 6Ω are in series.



$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} = \frac{18\times24 + 24\times6 + 18\times6}{18} = \frac{684}{18} = 38\Omega$$

$$R_b = \frac{684}{24} = 28.5\Omega$$

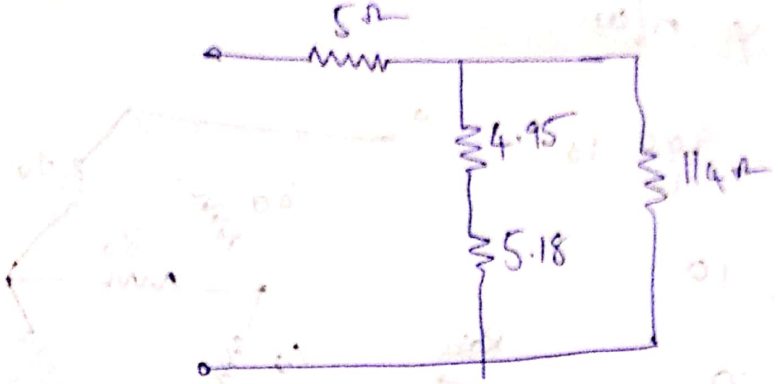
$$R_c = \frac{684}{6} = 114$$



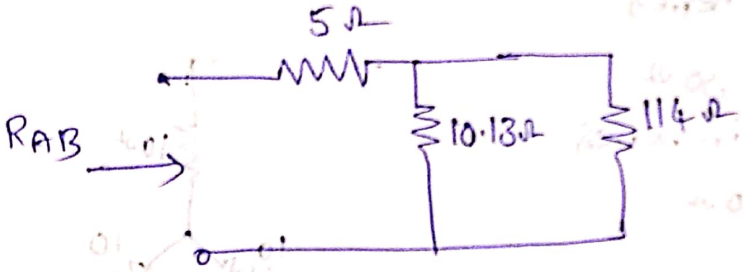
$$6\Omega \,//\, 28.5 \Rightarrow \frac{6\times28.5}{6+28.5} = \frac{171}{34.5} = 4.956$$

$$6\Omega \,//\, 38\Omega \Rightarrow \frac{6\times38}{6+38} = \frac{228}{44} = 5.18$$

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4.95Ω & 5.18 are in series

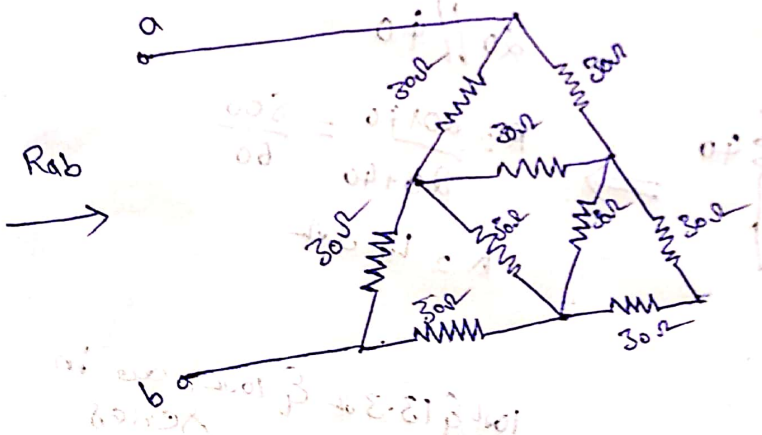


$$10.13\Omega \,||\, 11.4\Omega = \frac{10.13 \times 11.4}{10.13 + 11.4} = \frac{1154.8}{124.13} = 9.303$$

5Ω & 9.303 are in series

$R_{AB} = 14.303\Omega$

(2) find equivalent R_{ab} using wye-delta transformation.



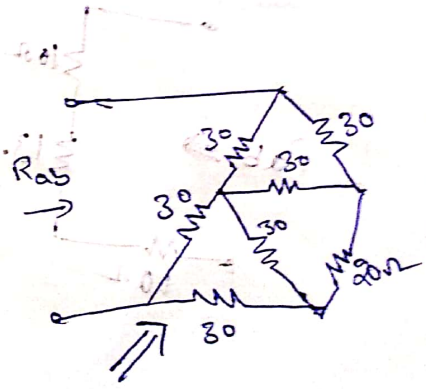
Sol

30Ω & 30Ω are in series

$R = 30 + 30 = 60\Omega$

$60\Omega \,||\, 30\Omega$

$$R = \frac{60 \times 30}{60 + 30} = \frac{1800}{90} = 20\Omega$$



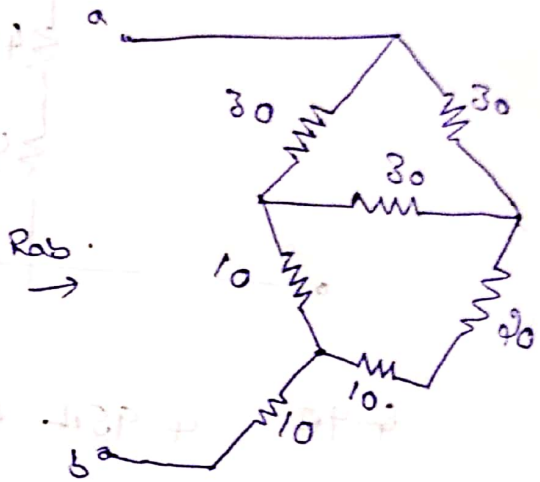
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Convert Δ to Y n/w

$$R_1 = \frac{30 \times 30}{30+30+30} = \frac{900}{90} = 10$$

$$R_2 = \frac{30 \times 30}{30+30+30} = 10$$

$$R_3 = \frac{30 \times 30}{30+30+30} = 10$$



10Ω & 20Ω are in series

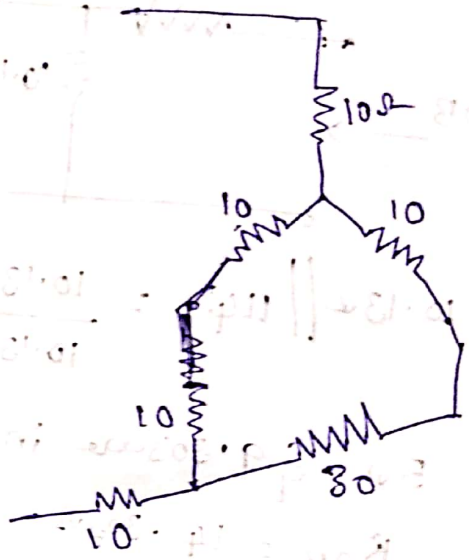
$R = 10 + 20 = 30\Omega$

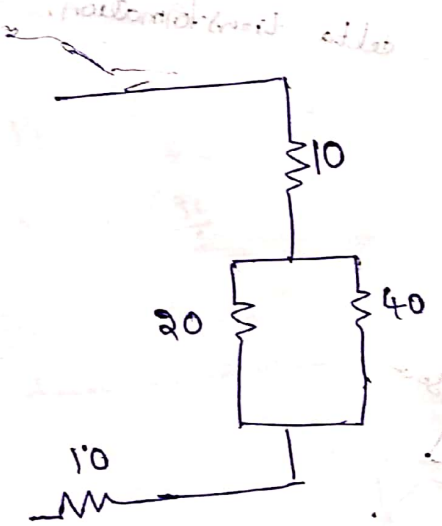
10Ω & 10Ω are in series

$R = 10 + 10 = 20\Omega$

10 & 30 are in series

$R = 10 + 30 = 40$

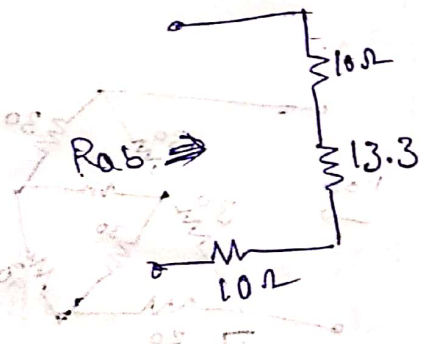




$20 // 40$

$$R = \frac{20 \times 40}{20+40} = \frac{800}{60}$$

$R = 13.3\Omega$



10Ω & 13.3Ω & 10Ω are in series

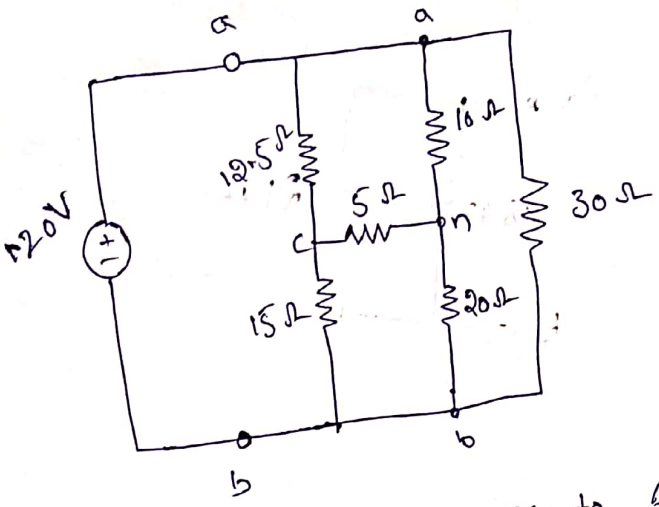
$R = 10 + 13.3 + 10$

$R = 33.3\Omega$

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(*) Obtain the equivalent resistance R_{ab} for the ckt and use it to find current i.

(4.1)



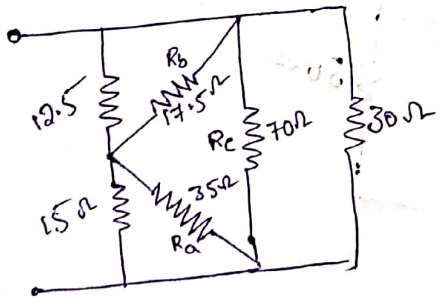
Sol Convert the Y network to Δ n/w.

$R_1 = 10\Omega \quad R_2 = 20\Omega \quad R_3 = 5\Omega$

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} = \frac{10\times20 + 20\times5 + 10\times5}{10} = 35\Omega$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} = \frac{350}{20} = 17.5\Omega$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} = \frac{350}{5} = 70\Omega$$



$70\Omega \,//\, 30\Omega$

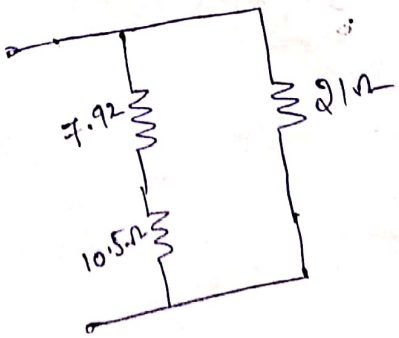
$$R = \frac{70\times30}{70+30} = 21\Omega$$

$12.5\Omega \,//\, 17.5\Omega$

$$R = \frac{12.5\times17.5}{12.5+17.5} = 7.292\Omega$$

$15\Omega \,//\, 35\Omega$

$$R = \frac{15\times35}{15+35} = 10.5\Omega$$

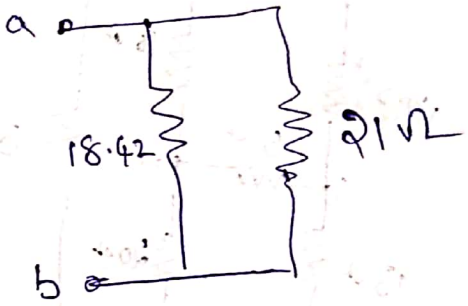


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7.92 Ω & 10.5 Ω is in series

$R = 7.92 + 10.5$

$R = 18.42\Omega$



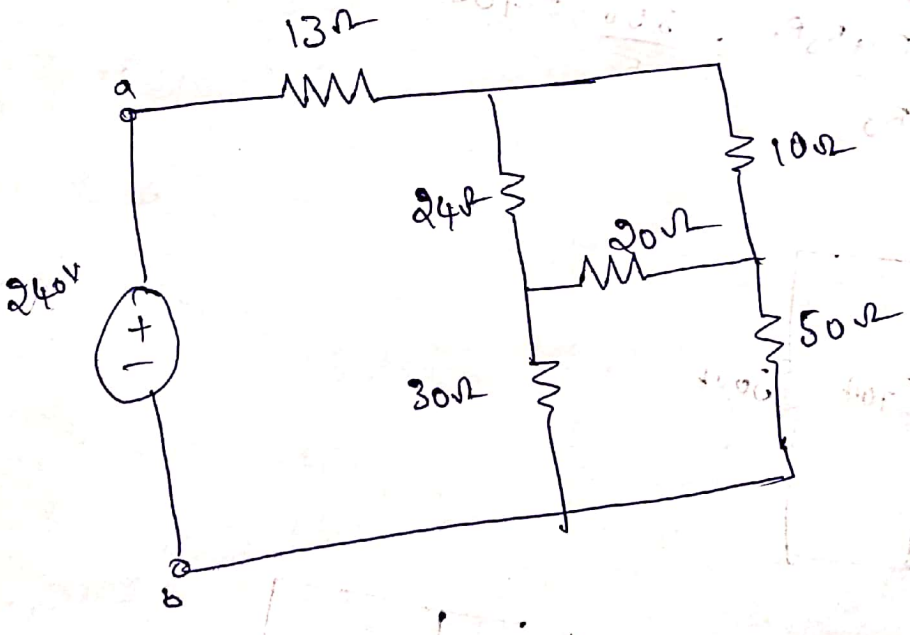
$18.42\Omega \,//\, 21\Omega$

$$R_{ab} = \frac{18.42 \times 21}{18.42 + 21}$$

$$= 9.632$$

$$i = \frac{V_s}{R_{ab}} = \frac{120}{9.632} = 12.458\,A$$

find R_{ab} & i in the ckt



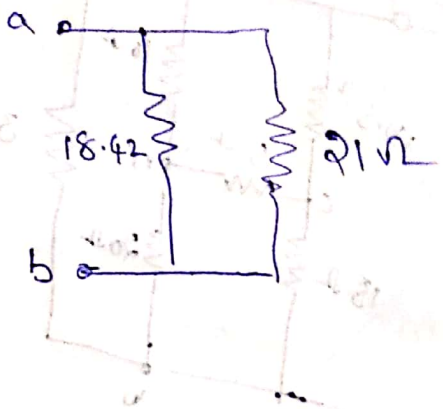
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7.92 Ω & 10.5 Ω is in series

$R = 7.92 + 10.5$

$R = 18.42\Omega$

$18.42\Omega // 21\Omega$

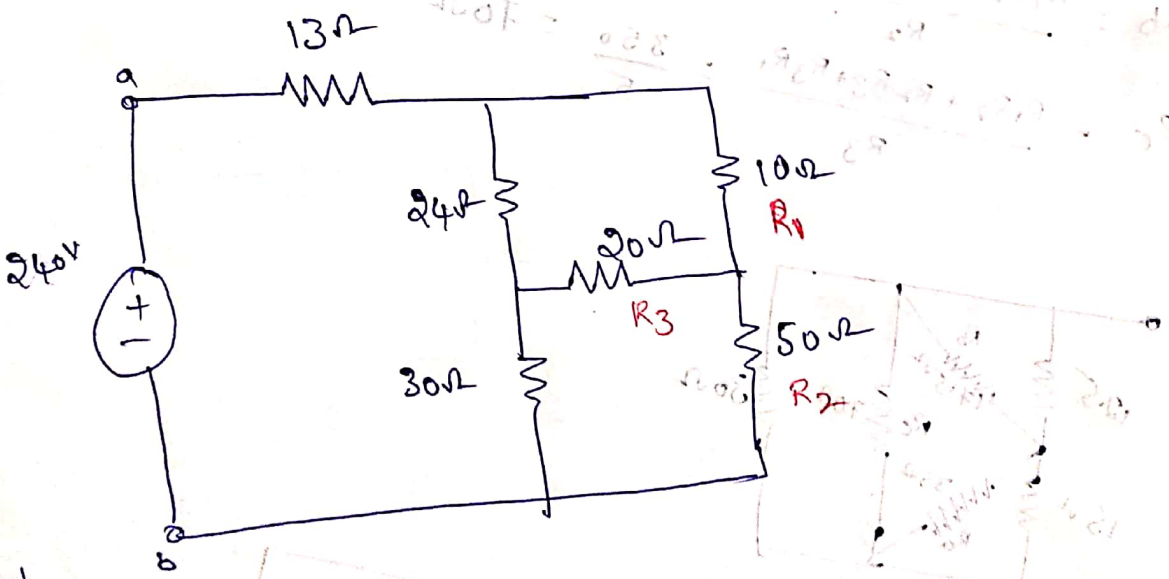


$$R_{ab} = \frac{18.42 \times 21}{18.42 + 21}$$

$$= 9.632$$

$$i = \frac{V_s}{R_{ab}} = \frac{120}{9.632} = 12.458\,A$$

* find R_{ab} & i in the ckt



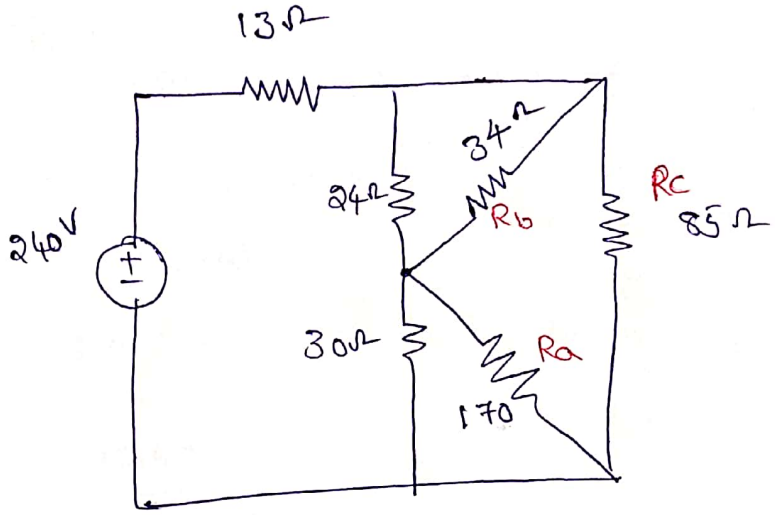
Sol

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} = \frac{10\times50 + 50\times20 + 10\times20}{10} = \frac{1700}{10} = 170$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} = \frac{1700}{50} = 34$$

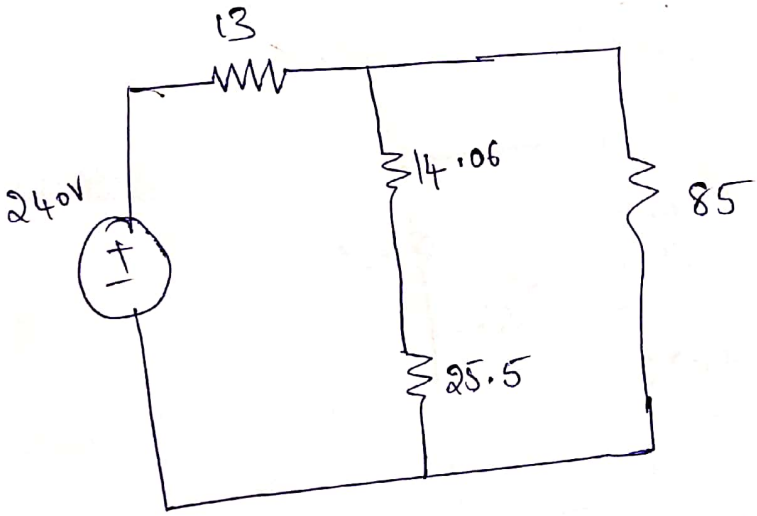
$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} = \frac{1700}{20} = 85$$

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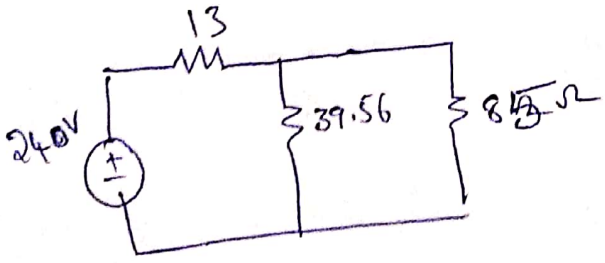
$24\Omega \,//\, 34\Omega = \frac{24 \times 34}{24+34} = \frac{816}{58} = 14.06$

$30\Omega \,//\, 170 = \frac{30 \times 170}{30+170} = \frac{5100}{200} = 25.5$



14.06 & 25.5 is in Series

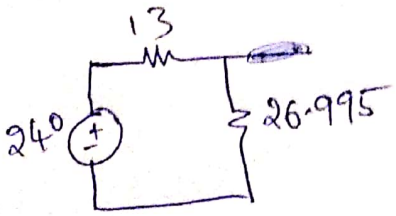
$R = 39.56$



$39.56 \,//\, 85 = \frac{39.56 \times 85}{}$

$= \frac{3362.6}{124.56}$

$= 26.995$



$R_{ab} = 13 + 26.995 = 40\ \Omega$

$I = \frac{240}{40} = 6A$

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Nodal Analysis :- (Branch current method).

A method of determining the voltage between nodes in an electric circuit interms of the branch currents.

Node :- A node is the point of connection between two or more branches. A node is usually indicated by a dot in a circuit.

Nodal analysis provides a general procedure for analysing circuits using node voltages as the ckt variable. Choosing node voltages instead of element voltages as a ckt variable.

The nodal analysis of the ckt involves taking the following three steps.

→ select a node as the reference node. Assign voltages $V_1, V_2 \cdots V_{n-1}$ to the remaining $n-1$ nodes. The voltages are referenced with respect to the reference node.

→ Apply KCL to each of the $n-1$ non reference nodes. Use ohm's law to express the branch currents interms of node voltages.

→ Solve the resulting simultaneous equations to obtain the unknown node voltages by using standard analysis methods such as elimination method or Cramer's rule or matrix inversion method.

The first step in nodal analysis is selecting a node as the reference node. The reference node is commonly called the ground. It is assumed to have zero potential.

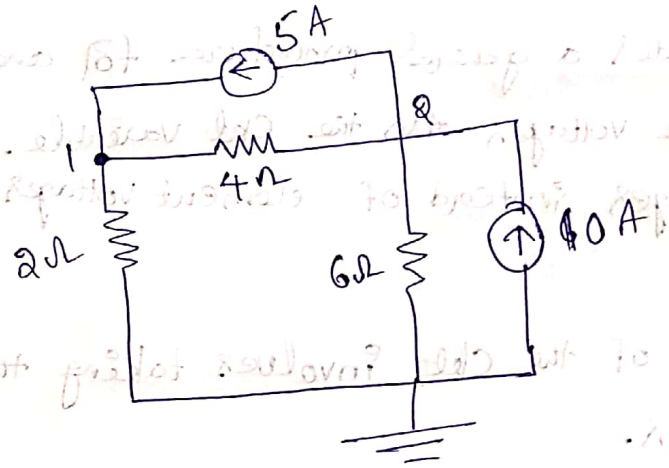
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A reference node is indicated by any of the three symbols.



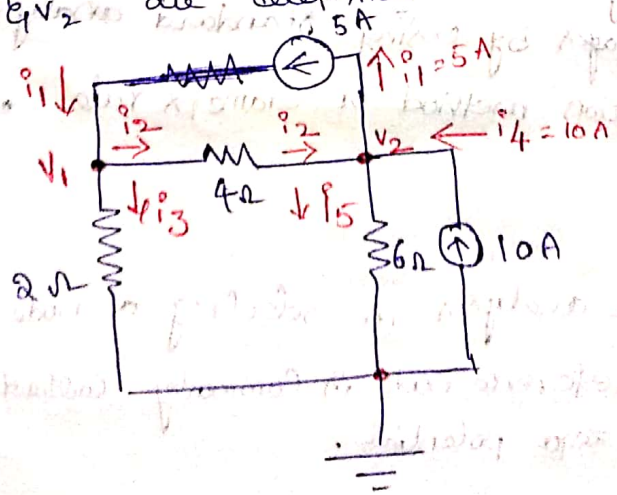
Common symbols for indicating a reference node.

① ~~calculate~~ Calculate the node voltages in the ckt.



Sol The currents are selected for the application of KCL. Except for the branches with current sources, the labeling of the currents is arbitary but consistent. (By consistent, we mean that for example we assume i_2 enters the 4Ω resistor from the left hand side, i_2 must leave the resistor from the right hand side.

The reference node is selected and the node voltages V_1 & V_2 are determined.



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At node 1 Applying KCL & ohm's law

KCL $\Rightarrow i_1 - i_2 - i_3 = 0$

$i_1 = i_2 + i_3$

$i_1 = 5\,A, \; i_2 = \frac{V_1 - V_2}{4}, \; i_3 = \frac{V_1}{2}, \; i_4 = 10\,A, \; i_5 = \frac{V_2}{6}$

$5 = \frac{V_1 - V_2}{4} + \frac{V_1 - 0}{2} \Rightarrow 5 = \frac{V_1 - V_2 + 2V_1}{4}$

$20 = V_1 - V_2 + 2V_1$

$3V_1 - V_2 = 20 \longrightarrow (1)$

At node 2

$i_2 + i_4 = i_1 + i_5$

$\frac{V_1 - V_2}{4} + 10 = 5 + \frac{V_2 - 0}{6}$

$\frac{V_1 - V_2 + 40}{4} = \frac{30 + V_2}{6}$

$\frac{V_1 - V_2 + 40}{4} - \frac{30 - V_2}{6} = 0$

$\frac{3V_1 - 3V_2 + 120 - 60 - 2V_2}{12} = 0$

$3V_1 - 3V_2 + 60 - 2V_2 = 0$

$3V_1 - 5V_2 = -60$

$(-3V_1 + 5V_2) = 60$

$-3V_1 + 5V_2 = 60 \longrightarrow (2)$

we can solve the equations by using Cramer's rule and obtain the values of V_1 & V_2.

(1) $\Rightarrow 3V_1 - V_2 = 20$

(2) $\Rightarrow -3V_1 + 5V_2 = 60$

$$\begin{bmatrix} 3 & -1 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 60 \end{bmatrix}$$

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The determinant of matrix is

$$\Delta = \begin{vmatrix} 3 & -1 \\ -3 & 5 \end{vmatrix} = 15 - 3 = 12$$

To obtain V_1 & V_2 as

$$V_1 = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} 20 & -1 \\ 60 & 5 \end{vmatrix}}{\Delta} = \frac{100+60}{12} = \frac{160}{12} = 13.333\,V$$

$$V_2 = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} 3 & 20 \\ -3 & 60 \end{vmatrix}}{\Delta} = \frac{180+60}{12} = \frac{240}{12} = 20\,V$$

$i_1 = 5\,A$

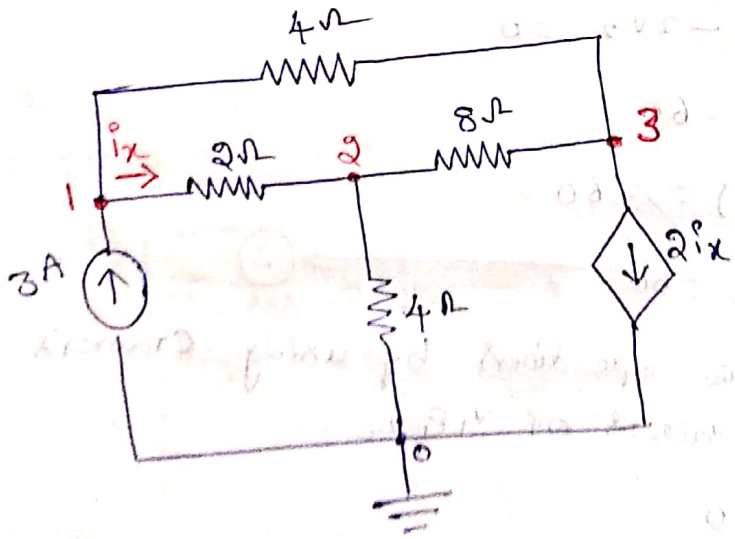
$$i_2 = \frac{V_1 - V_2}{4} = \frac{13.33 - 20}{4} = -1.6668\,A$$

$$i_3 = \frac{V_1}{2} = \frac{13.333}{2} = 6.666\,A$$

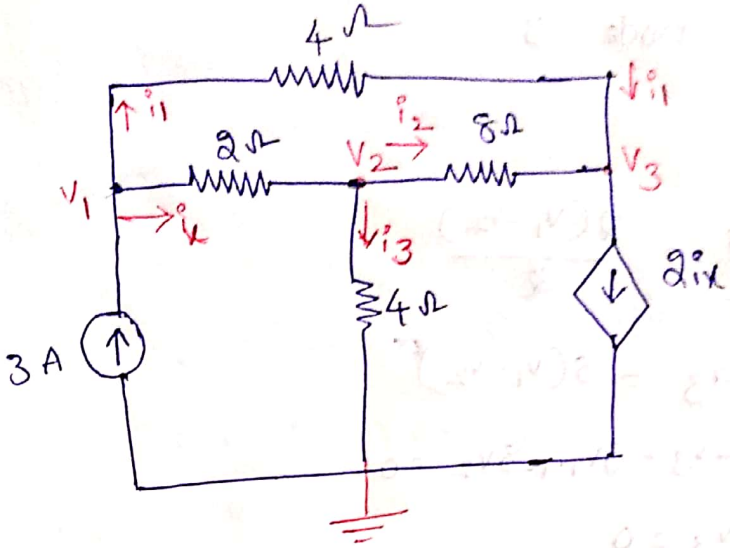
$i_4 = 10\,A$

$$i_5 = \frac{V_2}{6} = \frac{20}{6} = 3.333\,A$$

(2) Determine the voltages at the nodes shown in fig:



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Apply KCL at node 1

$$3 = i_1 + i_x$$

$$i_1 = \frac{v_1 - v_3}{4} \qquad i_x = \frac{v_1 - v_2}{2}$$

$$3 = \frac{v_1 - v_3}{4} + \frac{v_1 - v_2}{2}$$

$$3 = \frac{v_1 - v_3 + 2v_1 - 2v_2}{4}$$

$$12 = 3v_1 - 2v_2 - v_3 \longrightarrow (1)$$

Apply KCL at node 2

$$i_x = i_2 + i_3$$

$$\frac{v_1 - v_2}{2} = \frac{v_2 - v_3}{8} + \frac{v_2 - 0}{4}$$

$$\frac{v_1 - v_2}{2} = \frac{v_2 - v_3 + 2v_2}{8}$$

$$\frac{v_1 - v_2}{2} = \frac{3v_2 - v_3}{8}$$

$$\frac{v_1 - v_2}{2} - \frac{3v_2 + v_3}{8} = 0$$

$$4v_1 - 4v_2 - 3v_2 + v_3 = 0$$

$$4v_1 - 7v_2 + v_3 = 0$$

$$-(-4v_1 + 7v_2 - v_3) = 0 \longrightarrow (2)$$

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Apply KCL at node 3

$$i_1 + i_2 = 2i_x$$

$$\frac{v_1 - v_3}{4} + \frac{v_2 - v_3}{8} = \frac{2(v_1 - v_2)}{2}$$

$$2v_1 - 2v_3 + v_2 - v_3 = 8(v_1 - v_2)$$

$$2v_1 - 2v_3 + v_2 - v_3 - 8v_1 + 8v_2 = 0$$

$$-6v_1 + 9v_2 - 3v_3 = 0$$

$$-3(2v_1 - 3v_2 + v_3) = 0$$

$$2v_1 - 3v_2 + v_3 = 0 \longrightarrow (3)$$

(1) $\Rightarrow$ $3v_1 - 2v_2 - v_3 = 12$

(2) $\Rightarrow$ $-4v_1 + 7v_2 - v_3 = 0$

(3) $\Rightarrow$ $2v_1 - 3v_2 + v_3 = 0$

$$\begin{bmatrix} 3 & -2 & -1 \\ -4 & 7 & -1 \\ 2 & -3 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

$$\Delta = 3(7-3) - (-2)(-4+2) - 1(12-14)$$

$$= 3(4) + 2(-2) - (-2)$$

$$= 12 - 4 + 2 = 12 - 2$$

$$\Delta = 10$$

$$v_1 = \frac{\Delta_1}{\Delta}$$

$$\Delta_1 = \begin{vmatrix} 12 & -2 & -1 \\ 0 & 7 & -1 \\ 0 & -3 & 1 \end{vmatrix}$$

$$= 12(7-3) - (-2)(0-0) - 1(0-0)$$

$$= 12 \times 4 = 48$$

$$v_1 = \frac{48}{10} = 4.8$$

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$$V_2 = \frac{\Delta 2}{\Delta}$$

$$\Delta_2 = \begin{vmatrix} 3 & 12 & -1 \\ -4 & 0 & -1 \\ 2 & 0 & 1 \end{vmatrix}$$

$$= 3(0-0) - 12(-4+2) + (-1)(0-0)$$

$$= 0 - 12(-2) \times 0$$

$$\Delta_2 = 24$$

$$V_2 = \frac{24}{10} = 2.4$$

$$V_3 = \frac{\Delta 3}{\Delta}$$

$$\Delta_3 = \begin{vmatrix} 3 & -2 & 12 \\ -4 & 7 & 0 \\ 2 & -3 & 0 \end{vmatrix}$$

$$= 3(0-0) - (-2)(0-0) + 12(12-14)$$

$$= 0 - 0 + 12(-2)$$

$$= -24$$

$$V_3 = \frac{-24}{10} = -2.4$$

$$i_1 = \frac{V_1 - V_3}{4} =$$

$$i_x = i_2 + i_3$$

$$i_1 = \frac{V_1 - V_3}{4} = \frac{4.8 - (-2.4)}{4} = \frac{4.8 + 2.4}{4} = 1.8$$

$$i_x = \frac{V_1 - V_2}{2} = \frac{4.8 - 2.4}{2} = \frac{2.4}{2} = 1.2$$

$$i_1 + i_2 = 2 i_x$$

$$1.8 + i_2 = 2 \times 1.2$$

$$i_2 = 2.4 - 1.8$$

$$i_2 = 0.6 \text{ A}$$

$$i_x = i_2 + i_3$$

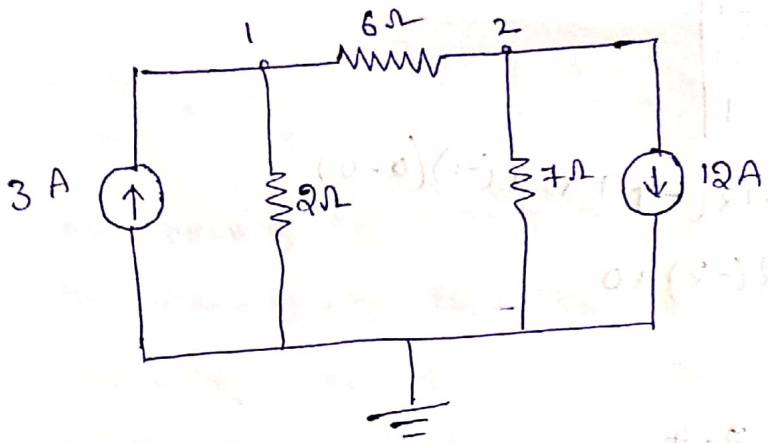
$$1.2 = 0.6 + i_3$$

$$i_3 = 1.2 - 0.6$$

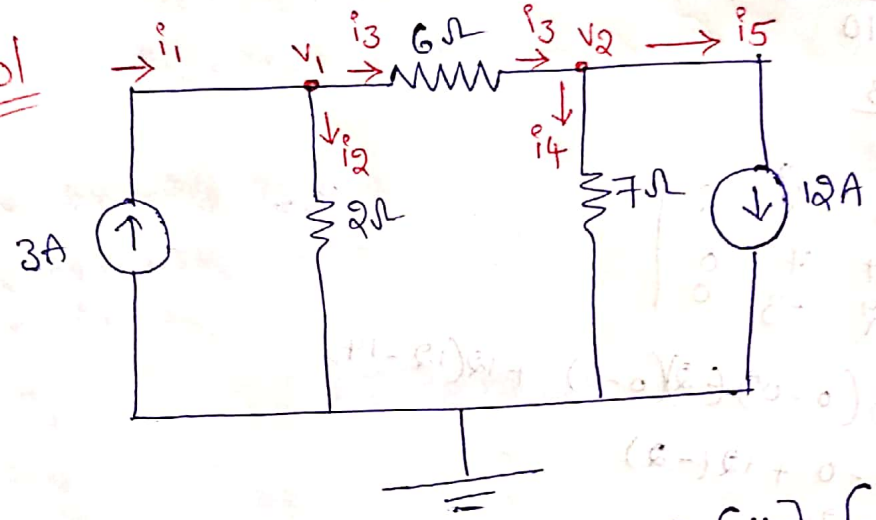
$$i_3 = 0.6 \text{ A}$$

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(*) Obtain the node voltages in the ckt :



Sol



Apply KCL at node ①

$$i_1 = i_2 + i_3$$

$$3 = \frac{v_1 - 0}{2} + \frac{v_1 - v_2}{6}$$

$$3 = \frac{3v_1 + v_1 - v_2}{6}$$

$$18 = 4v_1 - v_2 \longrightarrow ①$$

Apply KCL at node ②

$$i_3 = i_4 + i_5$$

$$\frac{v_1 - v_2}{6} = \frac{v_2}{7} + 12$$

$$\frac{v_1 - v_2}{6} - \frac{v_2}{7} = 12$$

$$\frac{7v_1 - 7v_2 - 6v_2}{42} = 12$$

$$7v_1 - 13v_2 = 504 \longrightarrow ②$$

$$\begin{pmatrix} 4 & -1 \\ 7 & -13 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 18 \\ 504 \end{pmatrix}$$

$$\Delta = ad - bc$$

$$= 4 \times -13 - (-7)$$

$$= -52 + 7 = -45$$

$$v_1 = \frac{\Delta_1}{\Delta}$$

$$\Delta_1 = \begin{vmatrix} 18 & -1 \\ 504 & -13 \end{vmatrix} = -234 + 504$$

$$= 270$$

$$v_1 = \frac{270}{-45} = -6\,V$$

$$v_2 = \frac{\Delta_2}{\Delta}$$

$$\Delta_2 = \begin{vmatrix} 4 & 18 \\ 7 & 504 \end{vmatrix} = 2016 - 126$$

$$= 1890$$

$$v_2 = \frac{1890}{-45} = -42\,V$$

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mesh analysis :-

A mesh is a loop that does not contain any other loop within it.
mesh analysis provides another general procedure for analysing circuits using mesh currents as the circuit variables.

Nodal analysis applies KCL to find unknown voltages in a given ckt, while mesh analysis applies KVL to find unknown currents.

The current through a mesh is known as mesh current.

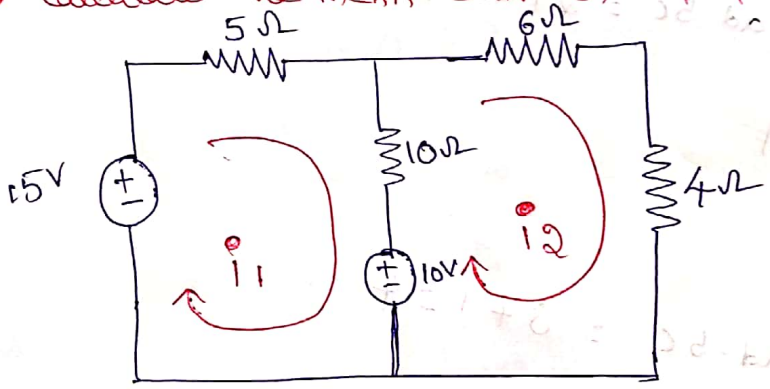
Steps to determine the mesh currents.

→ Assign mesh currents $i_1, i_2 \cdots i_n$ to the n meshes.

→ Apply KVL to each of the n meshes. Use ohm's law to express the voltages interms of the mesh currents.

→ Solve the resulting n simultaneous equations to get the mesh currents.

① Calculate the mesh currents i_1 & i_2



Sol Apply KVL to the mesh 1

$$-15 + 5i_1 + 10(i_1 - i_2) + 10 = 0$$

$$-15 + 5i_1 + 10i_1 - 10i_2 + 10 = 0$$

$$15i_1 - 10i_2 - 5 = 0$$

$$5(3i_1 - 2i_2 - 1) = 0$$

$$3i_1 - 2i_2 = 1 \longrightarrow (1)$$

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Apply KVL to mesh (2)

$$6i_2 + 4i_2 - 10 + 10(i_2 - i_1) = 0$$

$$6i_2 + 4i_2 - 10 + 10i_2 - 10i_1 = 0$$

$$20i_2 - 10i_1 - 10 = 0$$

$$10(-i_1 + 2i_2 - 1) = 0$$

$$-i_1 + 2i_2 = 1 \longrightarrow (2)$$

Use Cramer's rule method

$$\begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 3 & -2 \\ -1 & 2 \end{vmatrix} = ad - bc = 6 - 2 = 4$$

$$i_1 = \frac{\Delta_1}{\Delta}$$

$$\Delta_1 = \begin{vmatrix} 1 & -2 \\ 1 & 2 \end{vmatrix} = ad - bc = 2 + 2 = 4$$

$$i_1 = \frac{4}{4} = 1\,A$$

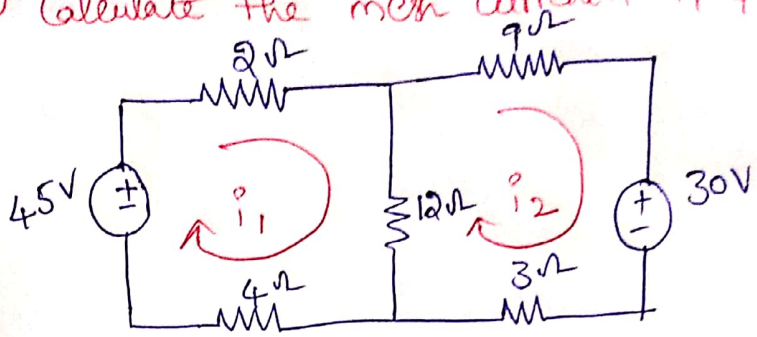
$$i_2 = \frac{\Delta_2}{\Delta}$$

$$\Delta_2 = \begin{vmatrix} 3 & 1 \\ -1 & 1 \end{vmatrix} = ad - bc = 3 + 1 = 4$$

$$i_2 = \frac{4}{4} = 1\,A$$

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② Calculate the mesh currents i_1 & i_2



Sol Apply KVL to mesh ①

$$-45+2i_1+12(i_1-i_2)+4i_1=0$$
$$-45+2i_1+12i_1-12i_2+4i_1=0$$
$$18i_1-12i_2-45=0$$
$$3(6i_1-4i_2-15)=0$$
$$6i_1-4i_2=15 \longrightarrow ①$$

Apply KVL to mesh ②

$$9i_2+30+3i_2+12(i_2-i_1)=0$$
$$9i_2+30+3i_2+12i_2-12i_1=0$$
$$24i_2-12i_1+30=0$$
$$3(8i_2-4i_1+10)=0$$
$$8i_2-4i_1=-10 \longrightarrow ②$$

using cramer's rule

$$\begin{bmatrix} 6 & -4 \\ -4 & 8 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix}=\begin{bmatrix} 15 \\ -10 \end{bmatrix}$$

$$\Delta = ad-bc = 48-16 = 32$$

$$i_1=\frac{\Delta_1}{\Delta} \Rightarrow \Delta_1=\begin{vmatrix} 15 & -4 \\ -10 & 8 \end{vmatrix} = ad-bc = 120-40 = 80$$

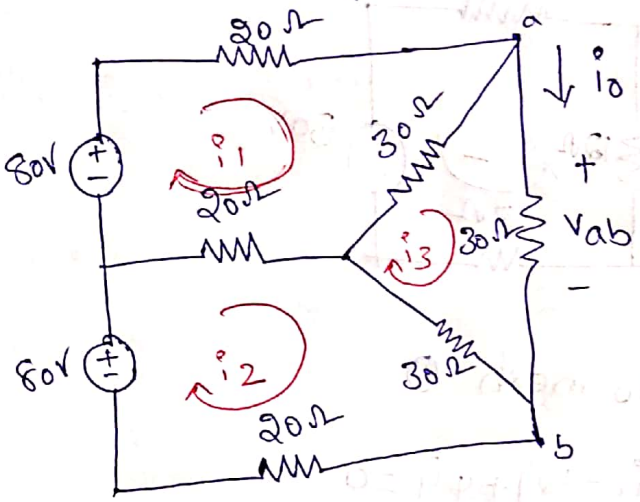
$$i_1=\frac{80}{32}=2.5\,A$$

$$i_2=\frac{\Delta_2}{\Delta} \Rightarrow \Delta_2=\begin{vmatrix} 6 & 15 \\ -4 & -10 \end{vmatrix} = ad-bc = -60+60 = 0$$

$$i_2=\frac{0}{32}=0$$

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3) use mesh analysis to find V_{ab} and i_o in the ckt



Sol

Apply KVL to loop ①

$$-80 + 20i_1 + 30(i_1 - i_3) + 20(i_1 - i_2) = 0$$

$$-80 + 20i_1 + 30i_1 - 30i_3 + 20i_1 - 20i_2 = 0$$

$$-80 + 70i_1 - 20i_2 - 30i_3 = 0$$

$$70i_1 - 20i_2 - 30i_3 = 8 \longrightarrow ①$$

Apply KVL to loop ②

$$-80 + 20(i_2 - i_1) + 30(i_2 - i_3) + 20i_2 = 0$$

$$-80 + 20i_2 - 20i_1 + 30i_2 - 30i_3 + 20i_2 = 0$$

$$-80 - 20i_1 + 70i_2 - 30i_3 = 0$$

$$-20i_1 + 70i_2 - 30i_3 = 80 \longrightarrow ②$$

Apply KVL to loop ③

$$30(i_3 - i_1) + 30i_3 + 30(i_3 - i_2) = 0$$

$$30i_3 - 30i_1 + 30i_3 + 30i_3 - 30i_2 = 0$$

$$-30i_1 - 30i_2 + 90i_3 = 0 \longrightarrow ③$$

Apply Cramer's rule

$$\begin{bmatrix} 70 & -20 & -30 \\ -20 & 70 & -30 \\ -30 & -30 & 90 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 80 \\ 80 \\ 0 \end{bmatrix}$$

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$$\Delta = 70(6300 - 900) - (-20)(-1800 - 900) - 30(600 + 9100)$$

$$= 5400 \times 70 + 20 \times (-2700) - 30(9700)$$

$$\Delta = 243000$$

$$\Delta_3 = \begin{vmatrix} 70 & -20 & 80 \\ -20 & 70 & 80 \\ -30 & -30 & 0 \end{vmatrix}$$

$$= 70(70 \times 0 - (-30 \times 80)) - (-20)(-20 \times 0 - (-30 \times 80)) + 80(-20 \times -30 - (-30 \times 70))$$

$$= 70(30 \times 80) + 20(30 \times 80) + 80(30 \times 70)$$

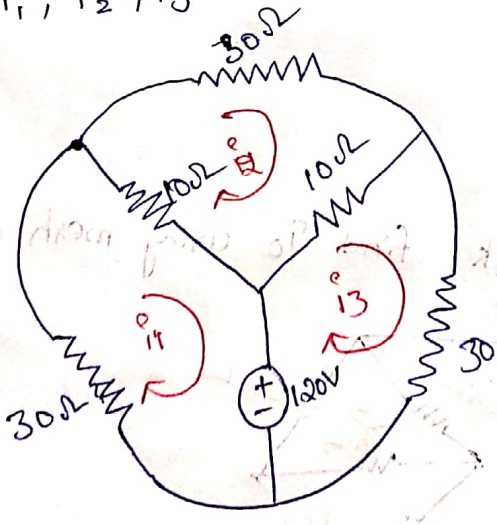
$$= 168000 + 48000 + 216000$$

$$\Delta_3 = 432000$$

$$i_0 = i_3 = \frac{\Delta_3}{\Delta} = \frac{432000}{243000} = 1.777\ A$$

$$V_{ab} = 30 i_3 = 1.777 \times 30 = 53.33\ V$$

* find i_1, i_2, i_3 in the Ckt



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Apply KVL to loop (1)

$$120 + 40i_1 - 10i_2 = 0$$

$40i_1 - i_2$

$$10(12 + 4i_1 - i_2) = 0$$

$$4i_1 - i_2 = -12 \longrightarrow (1)$$

Apply KVL to loop (2).

$$50i_2 - 10i_1 - 10i_3 = 0 \longrightarrow (2)$$

$$-i_1 + 5i_2 - i_3 = 0$$

Apply KVL to loop (3)

$$-120 - 10i_2 + 40i_3 = 0$$

$$-i_2 + 4i_3 = 12 \longrightarrow (3)$$

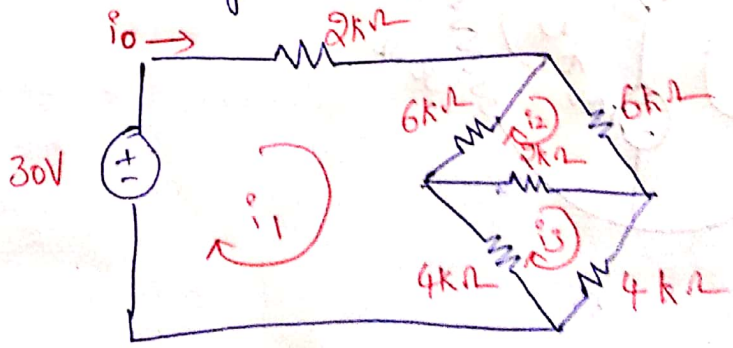
$$\begin{bmatrix} 4 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} -12 \\ 0 \\ 12 \end{bmatrix}$$

$$i_1 = \frac{\Delta_1}{\Delta} = -3A$$

$$i_2 = \frac{\Delta_2}{\Delta} = 0$$

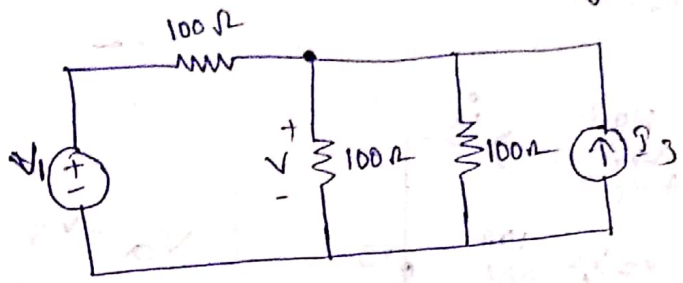
$$i_3 = \frac{\Delta_3}{\Delta} = 3A$$

for the bridge network find i_0 using mesh analysis



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b) If $V = AV_1 + BI_3$ in the following ckt, the value of A+B is



Sol $\frac{V - V_1}{100} + \frac{V}{100} + \frac{V}{100} = I_3$

$3V - V_1 = 100 I_3$

$3V = V_1 + 100 I_3$

$V = \frac{1}{3}V + \frac{100}{3} I_3$

$A = \frac{1}{3}$, $B = \frac{100}{3}$

$A+B = \frac{1}{3} + \frac{100}{3} = \frac{101}{3}$

C) A 12V automobile light is rated at 30W. The total charge that flows through the filament in one minute.

$t = 1 min = 60 sec$

Sol $V = 12V$

$P = 30$

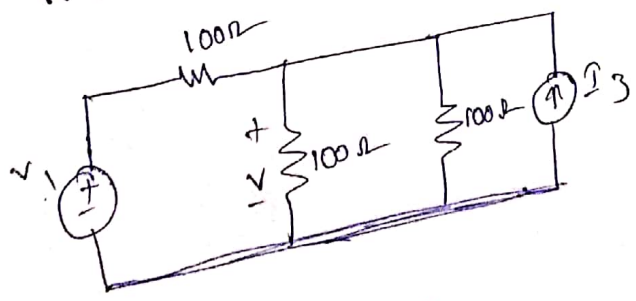
$P = VI$

$I = \frac{P}{V} = \frac{30}{12} = 2.5A$

Charge $Q = I \times t = 2.5 \times 60 = 150 C$

d) If $V_1 = AV + BI_3$ in the following ckt the value of A+B is

Sol



Sol $\frac{V - V_1}{100} + \frac{V}{100} + \frac{V}{100} = I_3$

$3V - V_1 = 100 I_3$

$-V_1 = 100 I_3 - 3V$

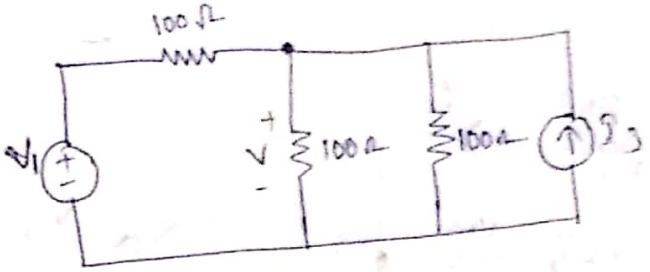
$V_1 = +3V + 100 I_3$

$A = 3 \quad B = -100$

$A+B = 3 - 100 = -97$

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b) If $V = AV_1 + BI_3$ in the following ckt, the value of A+B is



Sol

$$\frac{V - V_1}{100} + \frac{V}{100} + \frac{V}{100} = I_3$$

$$3V - V_1 = 100 I_3$$

$$3V = V_1 + 100 I_3$$

$$V = \frac{1}{3}V + \frac{100}{3} I_3$$

$$A = \frac{1}{3}, \quad B = \frac{100}{3}$$

$$A+B = \frac{1}{3} + \frac{100}{3} = \frac{101}{3}$$

C) A 12V automobile light is rated at 30W. The total charge that flows through the filament in one minute.

$t = 1 \text{ min} = 60 \text{ sec}$

Sol

$V = 12V$

$P = 30$

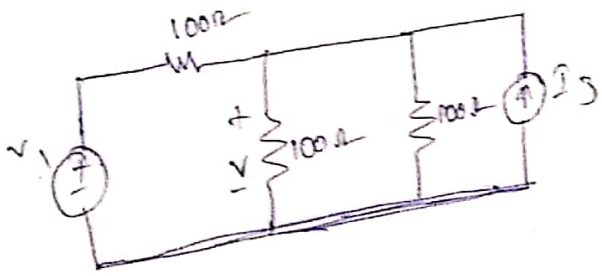
$P = VI$

$I = \frac{P}{V} = \frac{30}{12} = 2.5A$

Charge $Q = It = 2.5 \times 60 = 150\,C$

d) If $V_1 = AV + BI_3$ in the following ckt the value of A+B is

Sol



Sol

$$\frac{V - V_1}{100} + \frac{V}{100} + \frac{V}{100} = I_3$$

$$3V - V_1 = 100 I_3$$

$$-V_1 = 100 I_3 - 3V$$

$$V_1 = +3V - 100 I_3$$

$A = 3 \quad B = -100$

$A + B = 3 - 100 = -97$

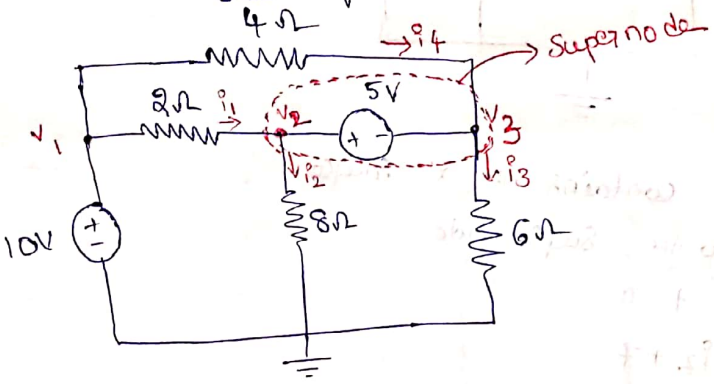
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Nodal Analysis with voltage Sources :-

How the voltage sources affect the nodal analysis. Consider the following two possibilities.

Case I :-

If a voltage source is connected b/w the reference node and a non reference node, we simply set the voltage at the non-reference node equal to the voltage of the voltage source. In the fig $V_1 = 10V$ i.e



Case II :-

If the voltage source (dependent & Independent) is connected b/w two non-reference nodes, the two non reference nodes form a generalised node & supernode, we apply both KCL and KVL to determine the node voltages.

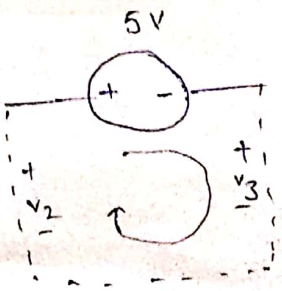
> A Super node is formed by enclosing a (dependent or Independent) voltage source connected b/w two non-reference nodes and any elements connected in parallel with it.

In the above figure node 2 & 3 form a super node. Hence at the super node apply KCL

$$i_1 + i_4 = i_2 + i_3$$

$$\frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{4} = \frac{V_2}{8} + \frac{V_3}{6}$$

Apply KVL to the super node

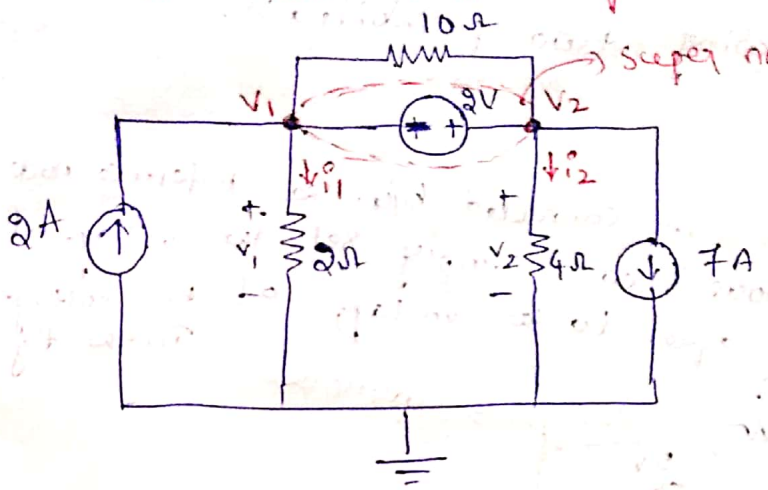


$$-V_2 + 5 + V_3 = 0$$

$$V_2 - V_3 = 5V$$

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For the ckt shown in below fig. find the node voltages.



Sol The super node contains the 2V source.

Apply KCL to the super node

$$2 - i_1 - i_2 - 7 = 0$$

$$2 = i_1 + i_2 + 7$$

$$2 = \frac{V_1}{2} + \frac{V_2}{4} + 7$$

$$2 = \frac{2V_1 + V_2 + 28}{4}$$

$$8 = 2V_1 + V_2 + 28$$

$$V_2 = -20 - 2V_1 \longrightarrow (1)$$

Apply KVL to the ckt

$$-V_1 - 2 + V_2 = 0$$

$$V_2 = V_1 + 2 \longrightarrow (2)$$

from eq (1) & (2)

$$V_1 + 2 = -20 - 2V_1$$

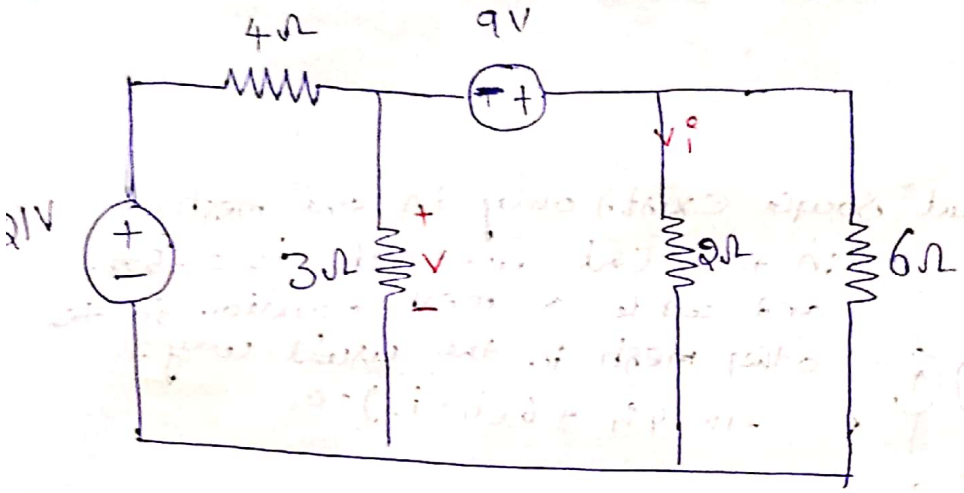
$$3V_1 = -22$$

$$V_1 = -22/3 = -7.333V$$

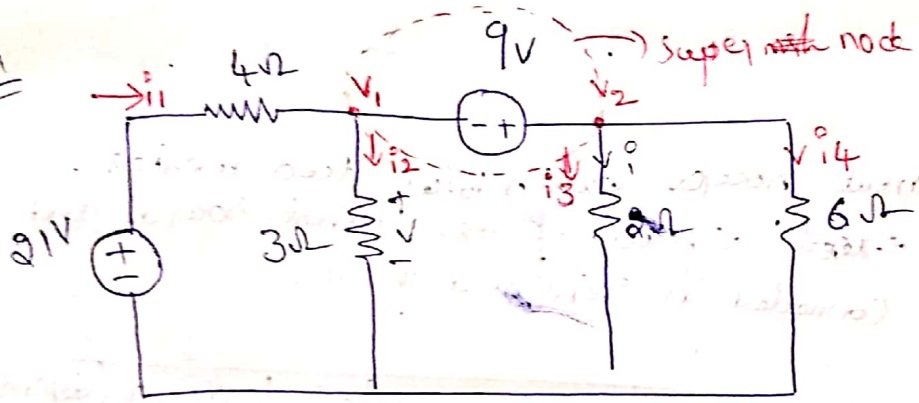
$$V_2 = V_1 + 2 \Rightarrow V_2 = -7.33 + 2 = -5.33$$

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* find v and i in the ckt



Sol



Apply KCL to super node

$$i_1 = i_2 + i_3 + i_4$$

$$\frac{21}{4} = \frac{V_1}{3} + \frac{V_2}{2} + \frac{V_2}{6}$$

$$\frac{21}{4} = \frac{2V_1 + 3V_2 + V_2}{6}$$

$$31.5 = 2V_1 + 4V_2$$

$$31.5 = 2(V_1 + 2V_2)$$

$$\frac{31.5}{2} = V_1 + 2V_2$$

$$15.75 = V_1 + 2V_2 \rightarrow (1)$$

$$V_1 = 15.75 - 2V_2$$

Apply KVL to super node

$$-V_1 - 9 + V_2 = 0$$

$$-V_1 = -V_2 + 9$$

$$V_1 = V_2 - 9 \rightarrow (2)$$

equating (1) & (2)

$$15.75 - 2V_2 = V_2 - 9$$

$$3V_2 = 15.75 + 9$$

$$3V_2 = 24.75$$

$$V_2 = \frac{24.75}{3} = 8.25$$

$$V_1 = V_2 - 9$$

$$V_1 = 8.25 - 9$$

$$V_1 = -0.75$$

$$\boxed{V_1 = V = -0.75}$$

$$i = \frac{V_2}{R} = \frac{8.25}{2}$$

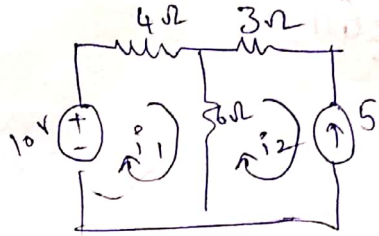
$$\boxed{i = 4.12\ A}$$

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mesh analysis with current sources :-

Case I :-

when a current source exists only in one mesh.



in this ckt we set $i_2 = -5A$.
and write a mesh equation for the other mesh in the usual way.

$$-10 + 4i_1 + 6(i_1 - i_2) = 0$$

Case II :-

when a current source exists b/w two meshes.
A supermesh ~~exists~~ excluding the current source and any elements connected in series with it.

> A supermesh results when two meshes have a (dependent or independent) current source in common.

Ex :-

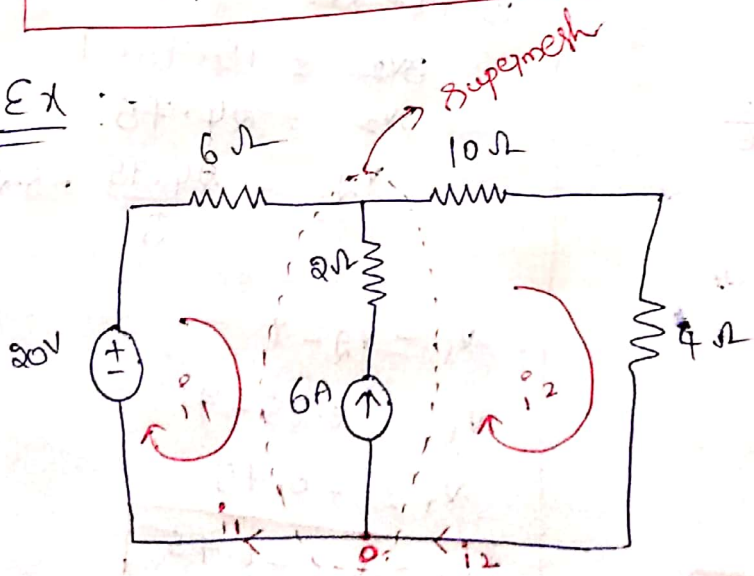


fig: Two meshes having a current source in common.

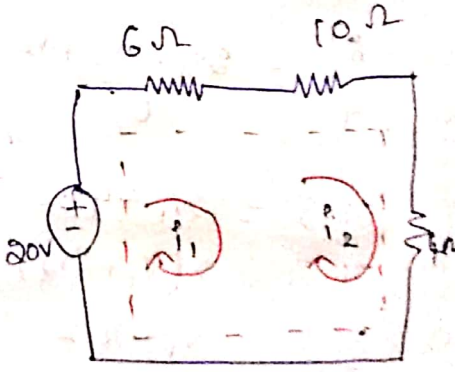


fig: A supermesh created by excluding the current.

Sol

Apply KVL to super mesh

$$-20 + 6i_1 + 10i_2 + 4i_2 = 0$$

$$6i_1 + 14i_2 = 20 \longrightarrow (1)$$

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Apply KCL to a node in the branch where the two meshes intersect.

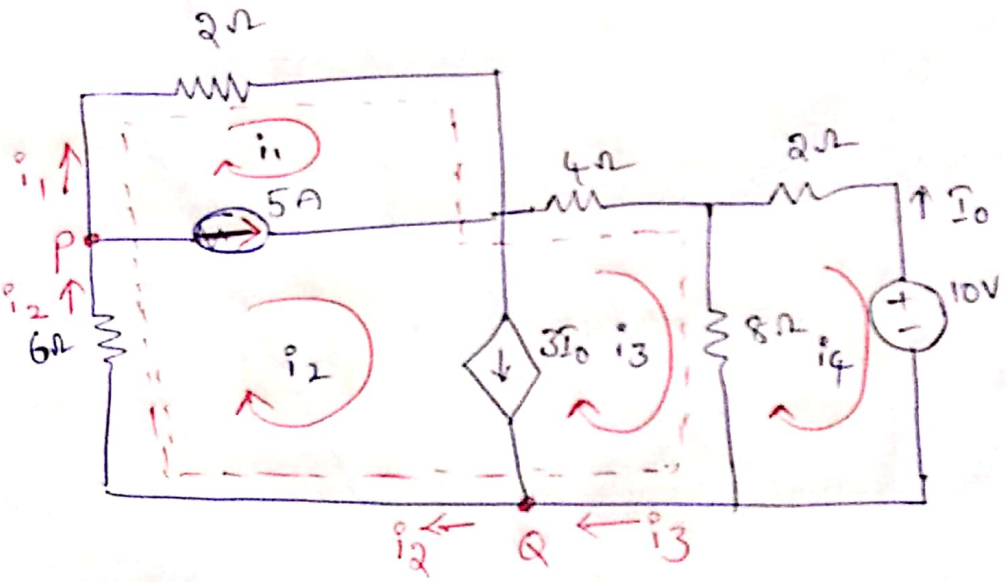
Apply KCL to node O

$$i_2 = i_1 + 6 \longrightarrow (2)$$

Solving the equations (1) & (2)

$$i_1 = -3.2\ A \qquad i_2 = 2.8\ A$$

* For the ckt find i_1 & i_4 using mesh analysis.



Sol

Apply KVL to super mesh

$$2i_1 + 4i_3 + 8(i_3 - i_4) + 6i_2 = 0$$

$$i_1 + 3i_2 + 6i_3 - 4i_4 = 0 \longrightarrow (1)$$

For the independent current source, we apply KCL to node P,

$$i_2 = i_1 + 5 \Rightarrow -i_1 + i_2 = 5 \longrightarrow (2)$$

For the dependent current source, we apply KCL to node Q.

$$i_2 = i_3 + 3I_0$$

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But $I_0 = -I_4$

$i_2 = i_3 - 3i_4 \Rightarrow -i_2 + i_3 - 3i_4 = 0 \longrightarrow (3)$

Apply KVL in mesh (4)

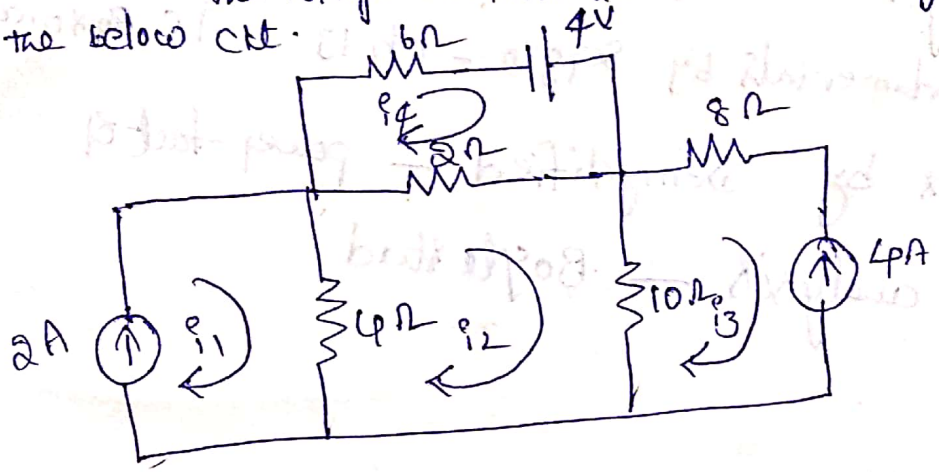
$2i_4 + 8(i_4 - i_3) + 10 = 0$

$5i_4 - 4i_3 = -5 \longrightarrow (4)$

$i_1 = -7.5A \quad i_2 = -2.5A \quad i_3 = 3.93A \quad i_4 = 2.143A$

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(2) b) find the voltage across 8Ω resistor by using MESH analysis for the below ckt.



Sol

from the ckt $i_1 = 2\,A$ $\quad i_3 = -4\,A$

KVL at loop (2) $\Rightarrow$ $2(i_2 - i_4) + 10(i_2 - i_3) + 4(i_2 - i_1) = 0$

$2i_2 - 2i_4 + 10(i_2 + 4) + 4(i_2 - 2) = 0$

$2i_2 - 2i_4 + 10i_2 + 40 + 4i_2 - 8 = 0$

$16i_2 - 2i_4 = -32 \longrightarrow (1)$

KVL at loop (4) $\Rightarrow$ $6i_4 - 4 + 2(i_4 - i_2) = 0$

$6i_4 - 4 + 2i_4 - 2i_2 = 0$

$8i_4 - 2i_2 = 4 \longrightarrow (2)$

from eq (1) & (2)

(1) $\Rightarrow$ $16i_2 - 2i_4 = -32$

(2) $\times 8$ $\Rightarrow$ $-16i_2 + 64i_4 = 32$

$-66i_4 = 0$

$i_4 = 0$

Substitute $i_4 = 0$ in (2)

$-2i_2 = 4$

$i_2 = -2\,A$

The voltage across 8Ω resistor $V = 2(i_2 - i_4)$ or $V = 2(i_4 - i_2)$

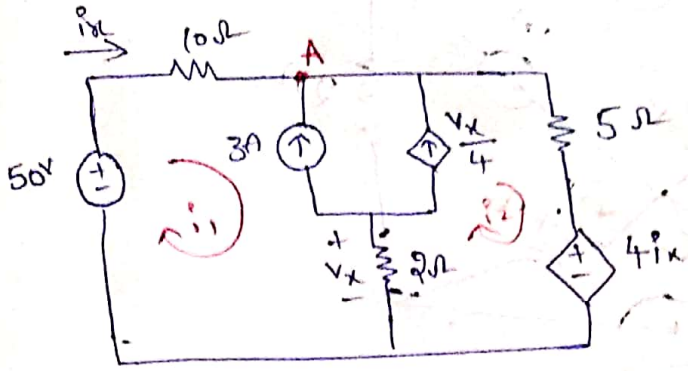
$V = 2(-2)$ (or) $V = 2(0 + 2)$

$V = -4\,A$ $\qquad V = 4\,A$

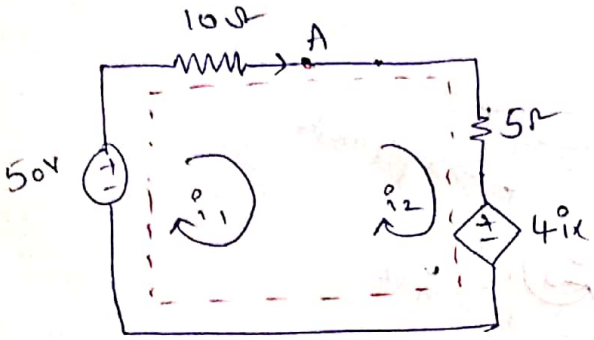
(3) a)

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1) Find v_x and i_x in the ckt.



Sol



For the supermesh Apply KVL

$$-50 + 10i_1 + 5i_2 + 4i_x = 0$$

But $i_x = i_1$

Hence $$+50 = 14i_1 + 5i_2 \longrightarrow (1)$$

At node 'A' Apply KCL

$$i_1 + 3 + \frac{v_x}{4} = i_2$$

But $v_x = 2(i_1 - i_2)$

$$i_1 + 3 + \frac{2(i_1 - i_2)}{4} = i_2$$

$$i_1 + 3 + \frac{i_1 - i_2}{2} = i_2$$

$$2i_1 + 6 + i_1 - i_2 = 2i_2$$

$$3i_1 - 3i_2 + 6 = 0$$

$$3(i_1 - i_2 + 2) = 0$$

$$i_1 + 2 = i_2 \longrightarrow (2)$$

Solving (1) & (2)

$$14i_1 + 5(i_1 + 2) = 50$$

$$14i_1 + 5i_1 + 10 = 50$$

$$19i_1 = 40$$

$$i_1 = \frac{40}{19} = 2.105\ A$$

$$i_2 = 4.105\ A$$

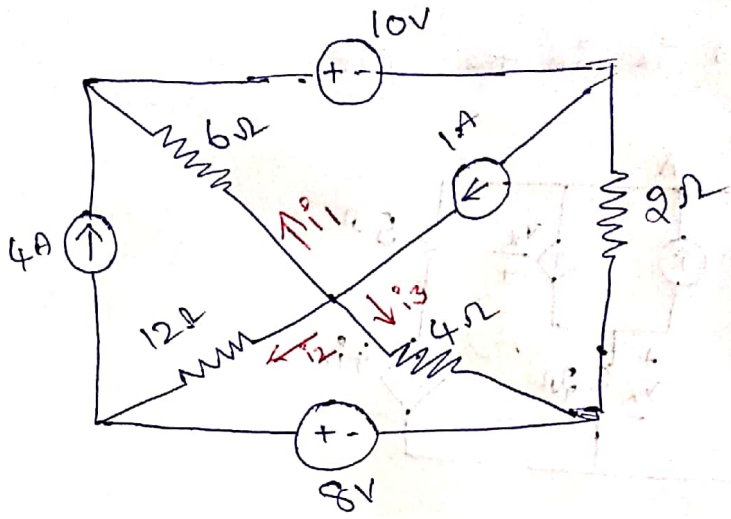
$$v_x = 2(i_1 - i_2)$$

$$v_x = -4\ \text{Volts}$$

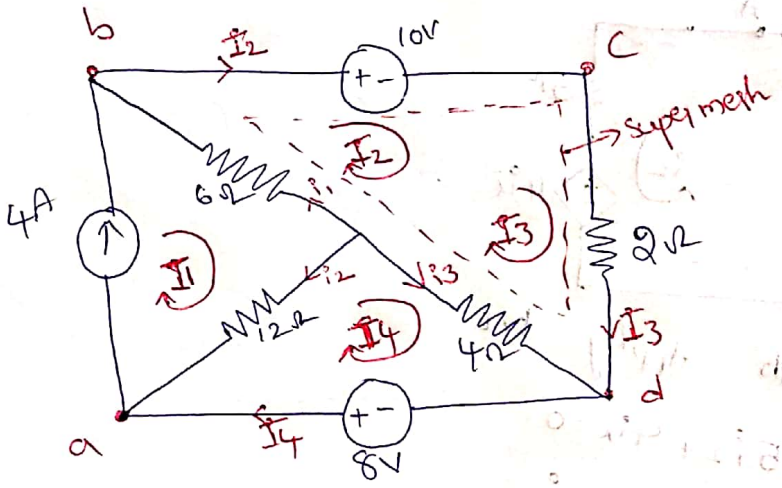
$$i_x = i_1 = 2.105\ A$$

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* In the ckt of fig: Solve i_1, i_2, i_3



Sol



from the fig $I_1 = 4A$ ⟶ (1)

Apply KVL to mesh (4)

$12(I_4 - I_1) + 4(I_4 - I_3) - 8 = 0$

$12I_4 - 12I_1 + 4I_4 - 4I_3 - 8 = 0$

$-12I_1 + 4I_3 + 16I_4 - 8 = 0$

$-3I_1 - I_3 + 4I_4 = 2$ ⟶ (2)

for the supermesh

$6(I_2 - I_1) + 10 + 2I_3 + 4(I_3 - I_4) = 0$

$-3I_1 + 3I_2 + 3I_3 - 2I_4 = -5$ ⟶ (3)

At node C

$I_2 = I_3 + 1$ ⟶ (4)

At node b $\Rightarrow$ $i_1 = I_2 - I_1$

At node a $\Rightarrow i_2 = 4 - I_4$

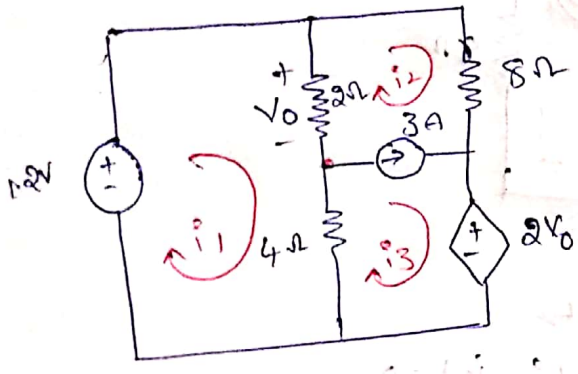
At node d $\Rightarrow i_3 = I_4 - I_3$

Solving equations (1), (2), (3), (4)

$I_1 = 4A$, $I_2 = 3A$, $I_3 = 2A$, $I_4 = 4A$

$$i_1 = I_2 - I_1 = -1\,A$$
$$i_2 = 4 - I_4 = 0\,A$$
$$i_3 = I_4 - I_3 = 2\,A$$

* use mesh analysis to find i_1, i_2, i_3 in the ckt.



Sol Apply KVL to mesh (1)

$$2(i_1 - i_2) + 4(i_1 - i_3) - 12 = 0 \longrightarrow (1)$$
$$3i_1 - i_2 - 2i_3 = 6$$

Apply KVL to super mesh

$$2(i_2 - i_1) + 8i_2 + 2V_o + 4(i_3 - i_1) = 0$$

But $V_o = 2(i_1 - i_2)$

$$2(i_2 - i_1) + 8i_2 + 2\times 2(i_1 - i_2) + 4(i_3 - i_1) = 0$$
$$-i_1 + 3i_2 + 2i_3 = 0 \longrightarrow (2)$$

for the independent current source

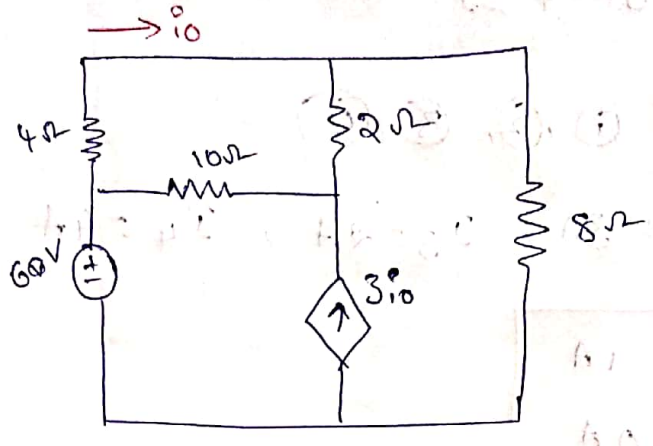
$$i_3 = 3 + i_2 \longrightarrow (3)$$

Solving (1) & (2), (3)

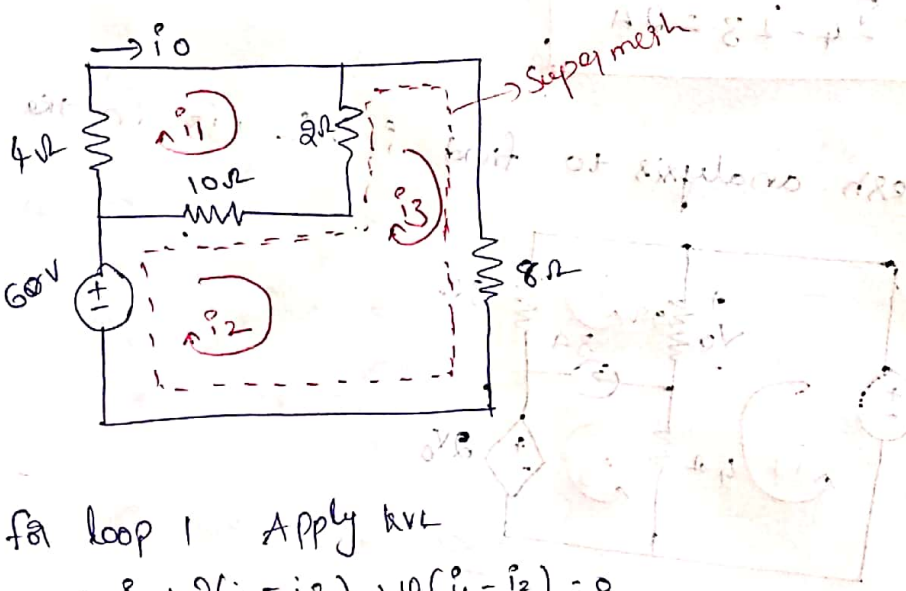
$i_1 = 3.5\,A$ $i_2 = -0.5\,A$, $i_3 = 2.5\,A$

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* use mesh analysis to find the current i_o in the ckt



Sol



Sol for loop 1 Apply KVL

$$4i_1 + 2(i_1 - i_3) + 10(i_1 - i_2) = 0$$
$$4i_1 + 2i_1 - 2i_3 + 10i_1 - 10i_2 = 0$$
$$16i_1 - 10i_2 - 2i_3 = 0$$
$$2(8i_1 - 5i_2 - i_3) = 0$$
$$8i_1 - 5i_2 - i_3 = 0 \longrightarrow (1)$$

for supermesh apply KVL

$$-60 + 10(i_2 - i_1) + 2(i_3 - i_1) + 8i_3 = 0$$
$$-60 + 10i_2 - 10i_1 + 2i_3 - 2i_1 + 8i_3 = 0$$
$$-12i_1 + 10i_2 + 10i_3 - 60 = 0$$
$$2(-6i_1 + 5i_2 + 5i_3 - 30) = 0$$
$$-6i_1 + 5i_2 + 5i_3 = 30 \longrightarrow (2)$$

$$3i_o = i_3 - i_2 \quad \& \quad i_o = i_1$$
$$3i_1 = i_3 - i_2$$

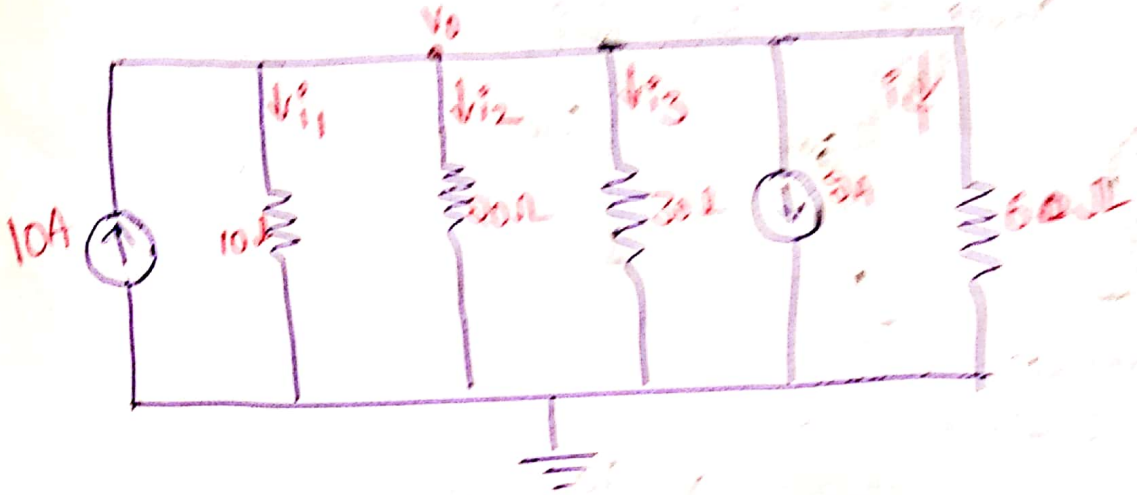
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Solving ①, ②, ③

$i_1 = 1.731$ & $i_0 = i_1 = 1.731\,A$

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* Find the currents i_1, through i_4 and the voltage v_0 in the ckt



Sol

Applying KCL to the upper node.

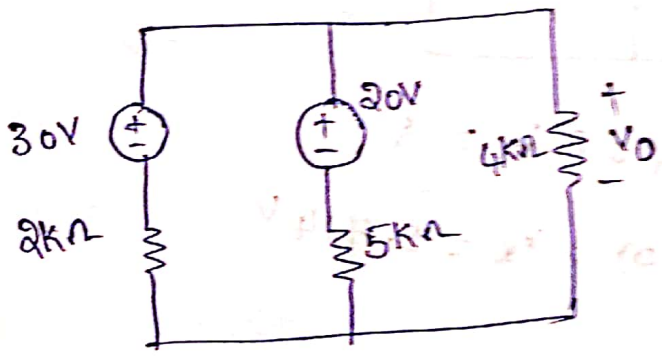
$$10 = \frac{v_0}{10} + \frac{v_0}{20} + \frac{v_0}{30} + 2 + \frac{v_0}{60}$$

$$v_0 = 40V$$

$$i_1 = \frac{v_0}{10} = 4A, \quad i_2 = \frac{v_0}{20}, \quad i_3 = \frac{v_0}{30} = 1.333A$$

$$i_4 = \frac{v_0}{60} = 666.7mA$$

* obtain v_0 in the ckt.



Sol

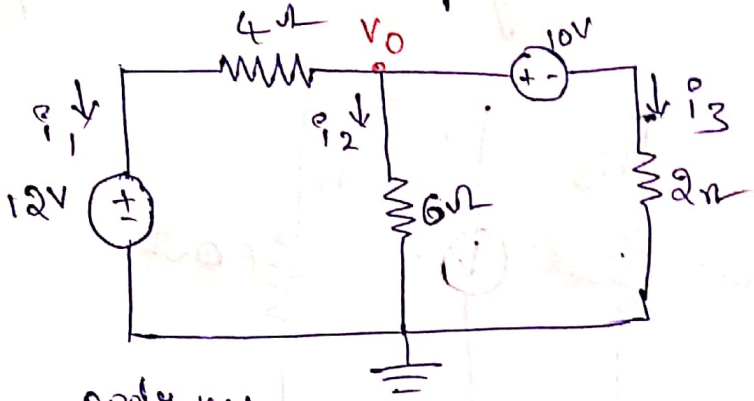
Apply KCL to the top node

$$\frac{30 - v_0}{2k} + \frac{20 - v_0}{5k} = \frac{v_0}{4k}$$

$$v_0 = 20V$$

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* Use nodal analysis to obtain V_o in the ckt.



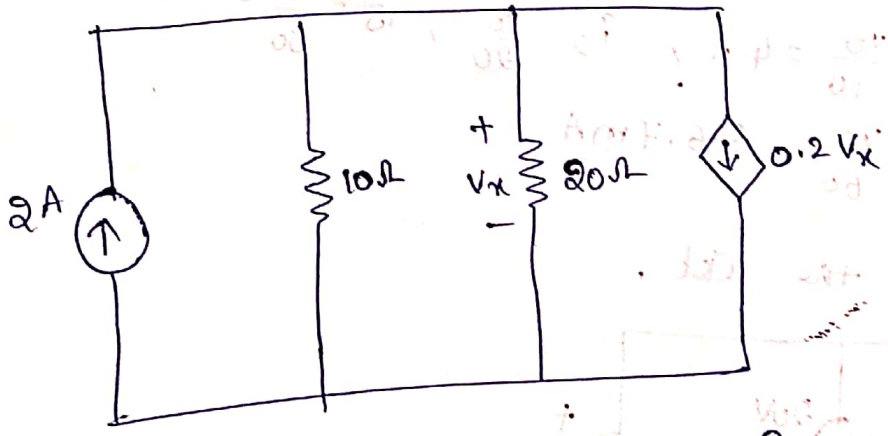
Sol Apply KCL

$$i_1 + i_2 + i_3 = 0$$

$$\frac{V_o - 12}{4} + \frac{V_o}{6} + \frac{V_o - 10}{2} = 0$$

$$V_o = 8.727V$$

* Apply nodal analysis to solve for V_x in the ckt.



Sol

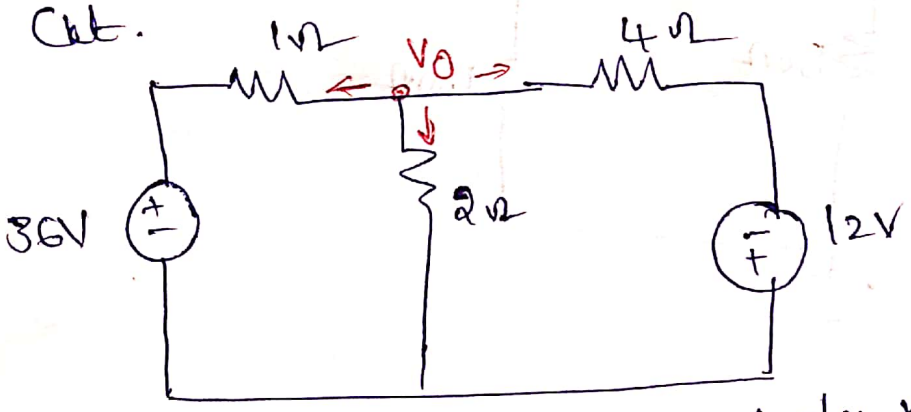
$$\frac{V_x - 0}{10} + \frac{V_x - 0}{20} + 0.2V_x = 2$$

$$0.35V_x = 2 \quad \text{or} \quad V_x = 5.714V$$

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find v_o and power dissipated in all resistors in the Ckt.



Sol: At the top node Apply KCL at node V_o

$$\frac{V_o - 36}{1} + \frac{V_o}{2} + \frac{V_o - (-12)}{4} = 0$$

$$1.75 V_o = 33$$

$$V_o = 18.857 V$$

$$P_{1\Omega} = \frac{(V_o - 36)^2}{1} = \frac{(18.857 - 36)^2}{1} = (-17.143)^2 = 293.882 W$$

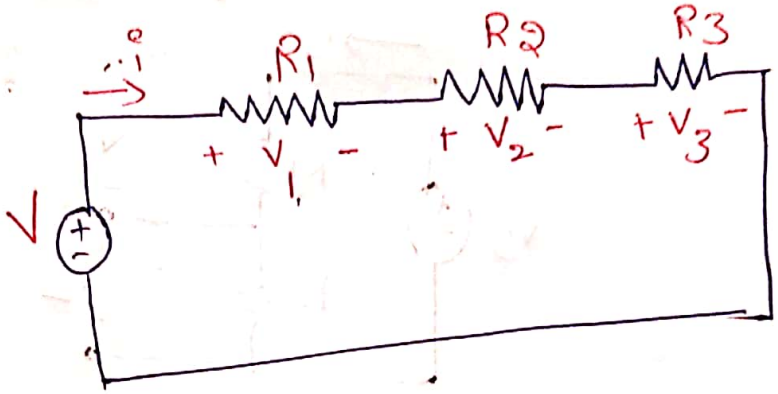
$$P_{2\Omega} = \frac{(V_o)^2}{2} = 177.79 W$$

$$P_{4\Omega} = \frac{(V_o + 12)^2}{4} = 238 W$$

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Voltage and current Division Rule

Voltage Division :



We know

$$i = \frac{V}{R}$$

$$i = \frac{V_{Total}}{R_1+R_2+R_3}$$

So the voltage over the first resistor is

$$V_1 = iR_1$$

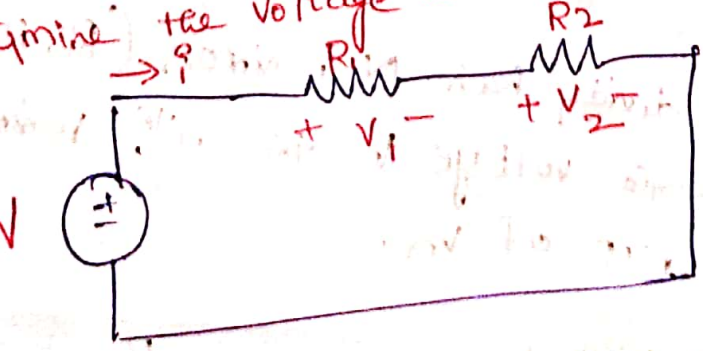
$$V_1 = R_1 \frac{V_{Total}}{R_1+R_2+R_3} \qquad V_{Total} = V$$

$$\boxed{V_1 = V \frac{R_1}{R_1+R_2+R_3}}$$

To find the voltage over an individual resistance in series, take the total series voltage and multiplying by the individual resistance over the total resistance.

Ex

Determine the voltage across each resistor.



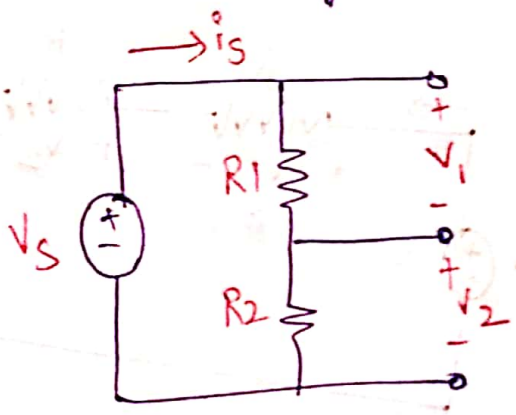
$$V_1 = V \frac{R_1}{R_1+R_2} \qquad V_2 = V \frac{R_2}{R_1+R_2}$$

$$i = \frac{V}{R_1+R_2}$$

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Example :-

Determine the voltage across each resistor.



$$i_S = \frac{V_S}{R_1+R_2}$$

$$V_1 = \frac{R_1}{R_1+R_2} V_S \implies i_S = \frac{V_S}{R_1+R_2} \implies V_1 = i_S R_1$$

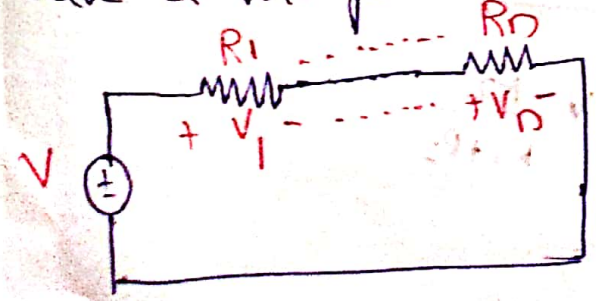
$$V_2 = \frac{R_2}{R_1+R_2} V_S \implies i_S = \frac{V_S}{R_1+R_2} \implies V_2 = i_S R_2$$

→ The source voltage v is divided among the resistors in direct propation to their resistances.

→ The larger the resistance, the larger the voltage drop.

This is called the principle of voltage division.

In a voltage divider has N resistors $(R_1, R_2 \cdots R_N)$ in series with the source voltage V, the nth resistor (R_n) will have a voltage drop of V_n.

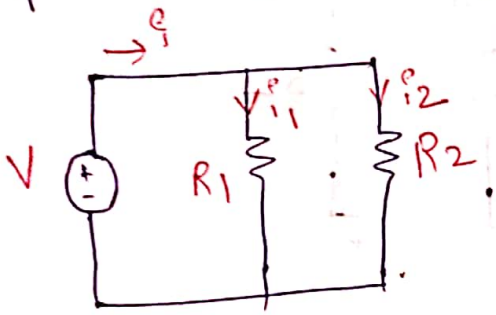


$$V_n = \frac{R_n}{R_1+R_2+\cdots+R_N} \cdot V$$

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Current division :-

If we know the current flowing into two parallel resistors, we can find out how the current will divide up in one step.



$$V = i R_{eq}$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

$$V = i \frac{R_1 R_2}{R_1 + R_2}$$

$$i_1 = \frac{V}{R_1}$$

$$i_1 = \frac{1}{R_1}\left[i \frac{R_1 R_2}{R_1 + R_2} \right] \Rightarrow \boxed{i_1 = i \frac{R_2}{R_1 + R_2}}$$

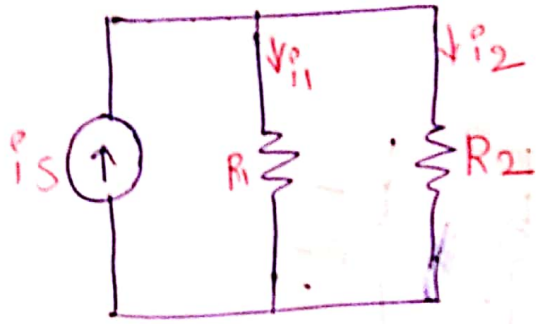
$$i_2 = \frac{V}{R_2}$$

$$i_2 = \frac{1}{R_2}\left[i \frac{R_1 R_2}{R_1 + R_2} \right] \Rightarrow \boxed{i_2 = \frac{R_1 i}{R_1 + R_2}}$$

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Example

find i_1 & i_2 in terms of i_s



$$i_s = i_1 + i_2$$

$$V_1 = V_2$$

$$i_1 R_1 = i_2 R_2$$

$$i_2 = \frac{i_1 R_1}{R_2} \Rightarrow i_s = i_1 + \frac{i_1 R_1}{R_2}$$

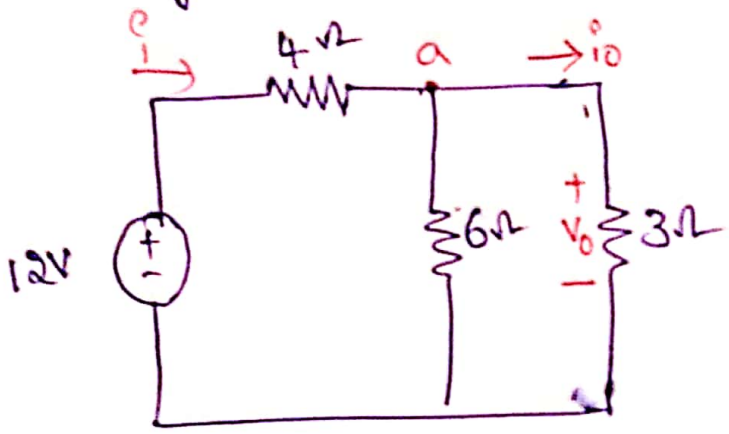
$$i_s = i_1 \left(\frac{R_2 + R_1}{R_2} \right)$$

$$\boxed{i_1 = i_s \left(\frac{R_2}{R_1 + R_2} \right)}$$

$$\boxed{i_2 = i_s \left(\frac{R_1}{R_1 + R_2} \right)}$$

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* use voltage and current division rules to find V_o & i_o

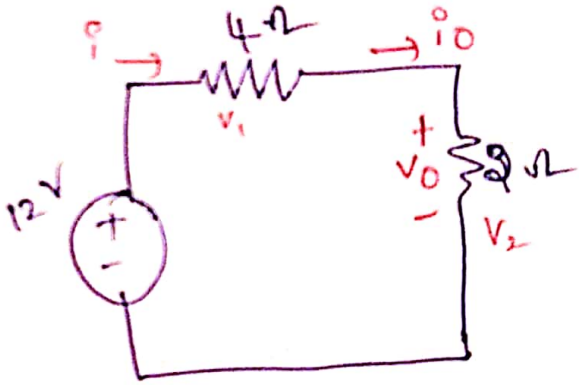


$6 \| 3 = \frac{6 \times 3}{6+3} = \frac{18}{9} = 2\Omega$

Sol

$$i = \frac{V}{R} = \frac{12}{4+2} = 2\ A$$

$$v_2 = V_o = (12)\frac{2}{4+2} = 4V$$



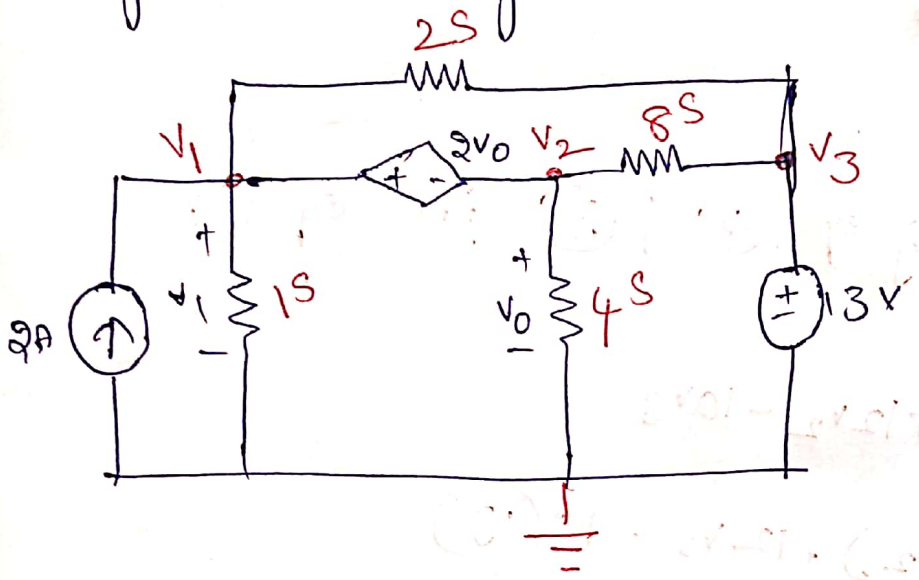
$$V_2 = V\frac{R_2}{R_1+R_2}$$

$$i_2 = i\frac{R_1}{R_1+R_2}$$

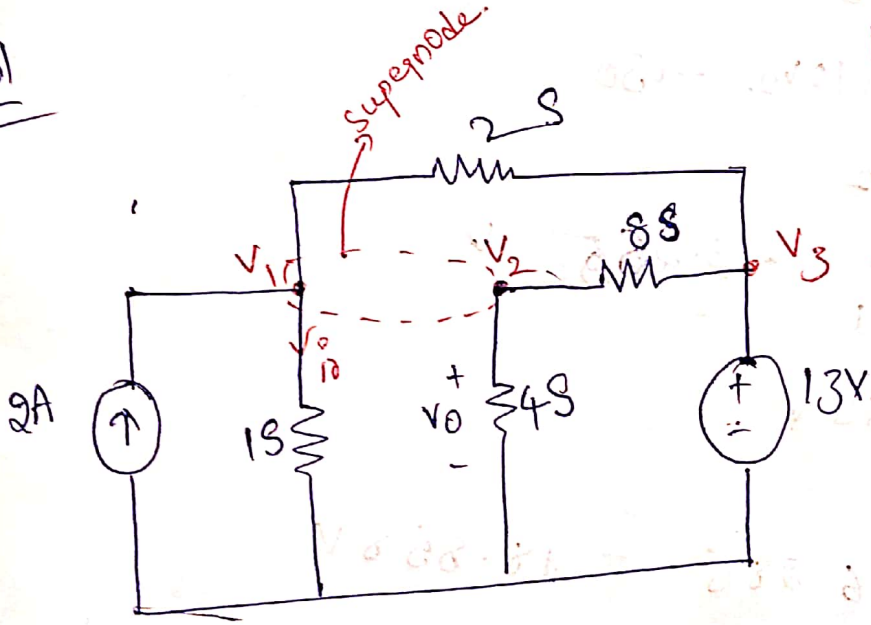
$$i_o = 2 \cdot \frac{6}{6+3} = \frac{4}{3}\ A$$

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Determine the voltages v_1 through v_3 in the ckt using nodal analysis.



Sol



Conductance $G = \frac{1}{R}$

Units of Conductance is Siemen's or $\mho$

Conductance is an expression of the ease with which electric current flows through a substance.

At the supernode Apply KCL

$$2 = v_1 + 2(v_1 - v_3) + 8(v_2 - v_3) + 4v_2$$

$$2 = 3v_1 + 12v_2 - 10v_3 \longrightarrow (1)$$

Apply KVL to the super node loop.

$$-v_1 + 2v_o + v_2 = 0$$

$$v_1 = v_2 + 2v_o$$

$v_o = v_2$

$v_1 = v_2 + 2v_2$

$v_1 = 3v_2 \longrightarrow (2)$

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$V_1 = 3V_2$

$V_3 = 13V$ ⟶ (3)

~~$V_1 = V_2 + 2V_0$~~

~~$3V_2 =$~~

Substituting eq (2) & (3) in eq (1)

(1) $\Rightarrow 2 = 3V_1 + 12V_2 - 10V_3$

$2 = 3(3V_2) + 12V_2 - 10(13)$

$2 = 9V_2 + 12V_2 - 130$

$132 = 21V_2$

$V_2 = \frac{132}{21} = 6.285$ V

$V_2 = 6.285$ V

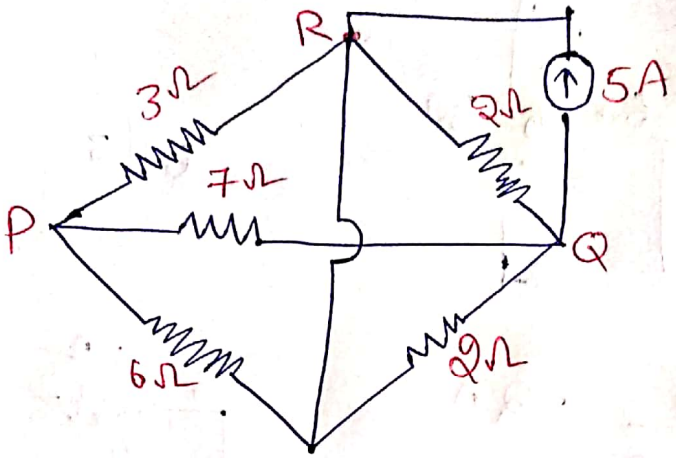
$V_1 = 3V_2$

$V_1 = 3 \times 6.285 = 18.858$ V

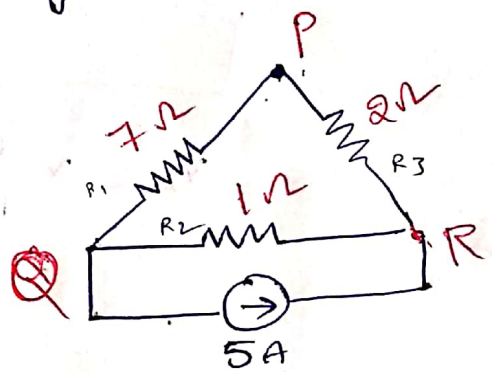
$V_3 = 13$ V

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* Find the current I_{PQ}



Sol Rearrange the Ckt



$3\Omega \,//\, 6\Omega = \frac{3\times 6}{3+6} = \frac{18}{9} = 2\Omega$

$2\Omega \,//\, 2\Omega = \frac{2\times 2}{2+2} = \frac{4}{4} = 1\Omega$

Current division rule

$$I_{PQ} = i \cdot \frac{R_2}{R_1+R_2+R_3} = 5\,\frac{1}{7+2+1} = 0.5A$$

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UNIT - II

Circuit Theorems

Necessity of Theorems :-

In a complicated network with several sources, nodes and meshes, if the response in a single element is desired then network theorems are used.

Tellegens Theorem :-

Algebric sum of powers at any given instant is zero. i.e the power delivered by some elements is equals to the power absorbed by remaining elements present in the network.

mathematically it can be expressed as

$$\sum_{K=1}^{b} V_K I_K = 0$$

$$\sum_{K=1}^{b} P_K = 0$$

where 'b' is the total no. of branches.

$$\sum P_{supply} - \sum P_{absorbed} = 0$$

$$\therefore \sum P_{supply} = \sum P_{absorbed}.$$

$\therefore$ Total power delivered = Total power absorbed.

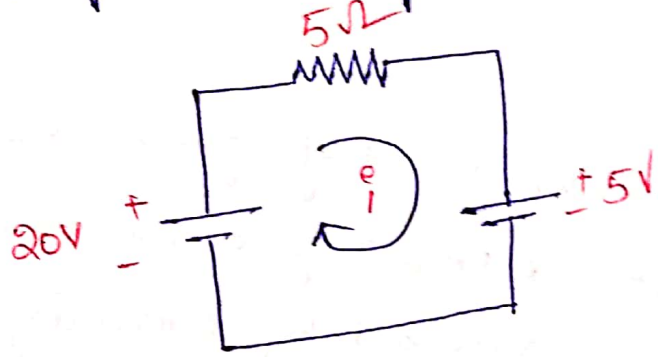
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→ when the current enters at the negative terminal of an element then that element will deliver the power otherwise it will absorb the power.

→ Sources (current & voltage source) can deliver power or absorb power.
whereas the passive elements will always absorb power

Ex :-

* Verify the tellegens theorem



Sol Apply KVL to the ckt

$$-20+5i+5=0$$
$$5i=15$$
$$i=3$$

$$P_{5\Omega} = i^2R = 3^2\times5 = 9\times5 = 45W \Rightarrow \text{absorbed power}$$

$$P_{5V} = vi = 5\times3 = 15W \Rightarrow \text{absorbed power}$$

$$P_{20V} = vi = 20\times3 = 60W \Rightarrow \text{supplied power}$$

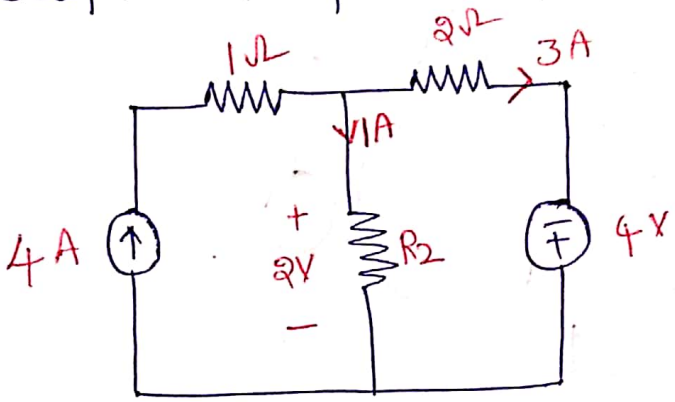
$$\sum P_{supplied} = \sum P_{absorbed}.$$

$$60 = 45+15$$
$$60 = 60$$
$$60-60 = 0$$
$$\sum P = 0$$

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Example :-

* Determine the power supplied by the current source.



Sol

$P_{4A} = ?$

$P_{1\Omega} = i^2R = 4^2 \times 1 = 16W$

$P_{R2\Omega}$ ✱

$P_{R2\Omega} = i^2R$ or $P_{R2} = vi = 9 \times 1 = +9W$

$P_{2\Omega} = i^2R = 3^2 \times 2 = 9 \times 2 = 18W$

$P_{4V} = vi = -4 \times 3 = -12W$

$P_{4A} + P_{1\Omega} + P_{R2} + P_{2\Omega} + P_{4V} = 0 \longrightarrow (1)$

$P_{4A} + 16 + 9 + 18 + (-12) = 0$

$P_{4A} = -24W$

Substitute the value $P_{4A} = -24W$ in eq (1)

$-24 + 16 + 9 + 18 - 12 = 0$

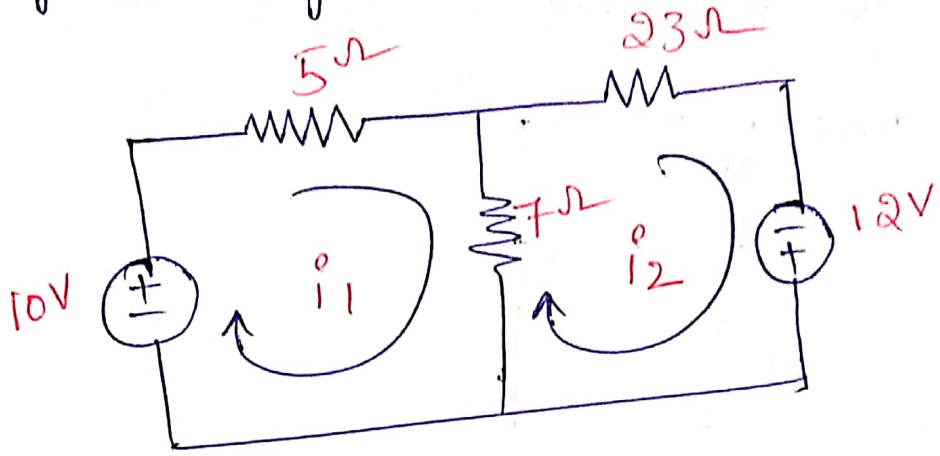
$-36 + 36 = 0$

$-36 = -36$

$36 = 36$

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* Verify the tellegen's theorem for the following Ckt.



Sol Apply KVL to loop ①

$$-10 + 5i_1 + 7(i_1 - i_2) = 0$$

$$-10 + 12i_1 - 7i_2 = 0$$

$$12i_1 - 7i_2 = 10 \longrightarrow ①$$

Apply KVL to loop ②

$$7(i_2 - i_1) + 23i_2 - 12 = 0$$

$$-7i_1 + 30i_2 = 12 \longrightarrow ②$$

Solving the equations ① & ②

$$\begin{bmatrix} 12 & -7 \\ -7 & 30 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 12 \end{bmatrix}$$

$$\Delta = ad - bc = 360 - 49 = 311$$

$$i_1 = \frac{\Delta_1}{\Delta}$$

$$\Delta_1 = \begin{bmatrix} 10 & -7 \\ 12 & 30 \end{bmatrix} = ad - bc = 300 + 84 = 384$$

$$i_1 = \frac{384}{311} = 1.234A$$

$$i_2 = \frac{\Delta_2}{\Delta}$$

$$\Delta_2 = \begin{bmatrix} 12 & 10 \\ -7 & 12 \end{bmatrix} = 144 + 70 = 214$$

$$i_2 = \frac{214}{311} = 0.688A$$

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$I_{7\Omega} = i_1 - i_2 = 1.234 - 0.688 = 0.542\,A$

$P_{7\Omega} = I^2 R = (0.542)^2 \times 7 = 2.05\,W$

$P_{5\Omega} = i^2 R = (1.234)^2 \times 5 = 7.56\,W$

$P_{23\Omega} = i^2 R = (0.688)^2 \times 23 = 10.886\,W$

$P_{12V} = 12 \times i_2 = 12 \times 0.688 = 8.256\,W$

$P_{10V} = 10 \times i_1 = 10 \times 1.25 = 12.5\,W$

$\sum P_{supplied} = \sum P_{absorbed}$

$P_{12V} + P_{10V} = P_{5\Omega} + P_{7\Omega} + P_{23\Omega}$

$8.256 + 12.5 = 7.56 + 2.05 + 10.886$

$20.556 = 20.5$

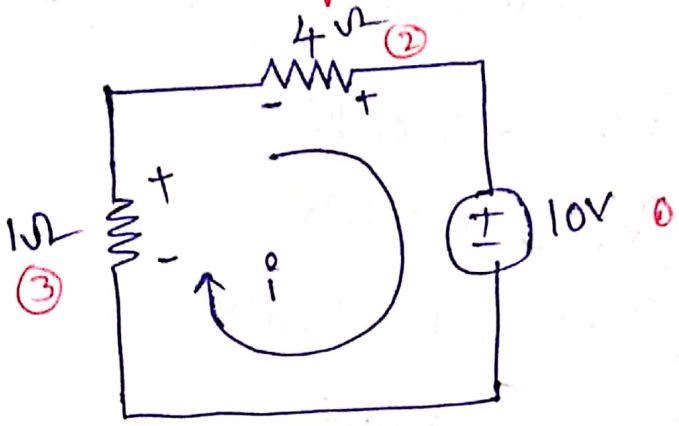
*

Statement of Tellegen's theorem :-

In any linear, non linear, active and passive network the summation of power is zero. i.e sum of powers delivered to the network by an independent source is equals to the sum of power absorbed by all passive elements of the network.

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* verify the tellegen's theorem:



Sol

Apply KVL to the loop

$i_1 = i_2 = i_3 = i$

$-i - 4i + 10 = 0$

$-5i = -10$

$i = 2\ A$

$v_1 = 10V$

$v_2 = -4 \times 2 = -8$

$v_3 = -1 \times 2 = -2$

$$\sum_{K=1}^{b} v_K i_K = 0$$

$$\sum_{K=1}^{3} v_K i_K = 0$$

$$v_1 i_1 + v_2 i_2 + v_3 i_3 = 0$$

$$10 \times 2 + (-8) \times 2 + (-2) \times 2 = 0$$

$$20 - 16 - 4 = 0$$

$$20 - 20 = 0$$

$$20 = 20$$

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Superposition Theorem :-

* The Superposition principle states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or current through) that element due to each independent source acting alone.

Definition:-
In a linear network with several independent sources, the response in a particular branch when all the sources are acting simultaneously is equal to the linear sum of individual responses calculated by taking one independent source at a time.

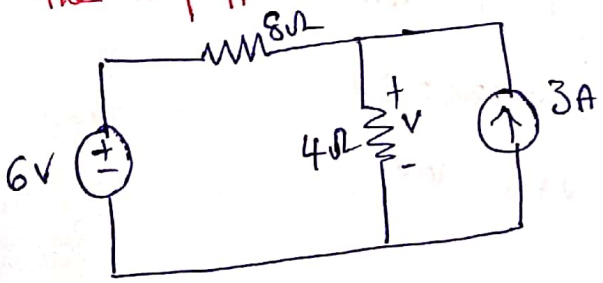
Steps to apply superposition principle:

① → Turn off all independent sources except one source. Find the output (voltage or current) due to that active source.

② → Repeat step ① for each of the other independent sources.

③ → Find the total contribution by adding algebrically all the contributions due to the independent sources.

* Use the superposition theorem to find v in the ckt.



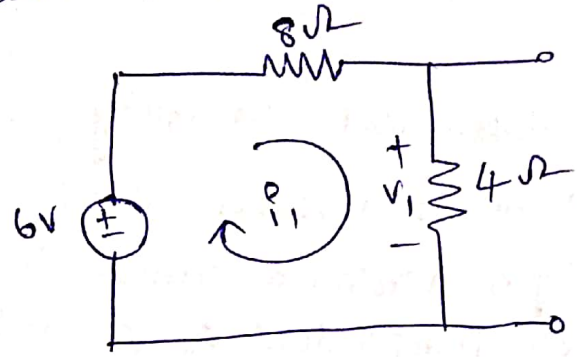
Sol Since there are two sources, let

$$V = V_1 + V_2$$

where V_1 and V_2 are contributions due to the 6V voltage source and 3A current source.

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To obtain v_1, we ~~set~~ set the current source to zero



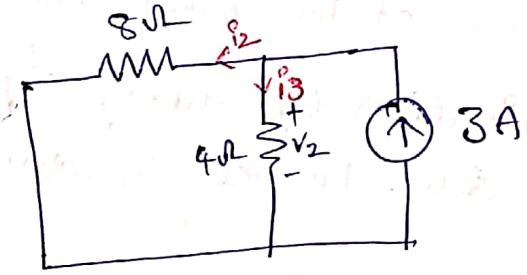
Applying KVL to the loop.

$$8i_1 + 4i_1 - 6 = 0$$
$$12i_1 = 6$$
$$i_1 = 0.5A$$
$$\therefore v_1 = 4i_1 = 4 \times 0.5 = 2V$$

we may use voltage division to get v_1

$$v_1 = \frac{4}{8+4} \cdot 6 = 2V$$

To get v_2, we set the voltage source to zero.



Using current division

$$i_3 = \frac{8}{4+8}(3) = 2A$$
$$v_2 = i_3 \times 4 = 4 \times 2 = 8V$$
$$v = v_1 + v_2 = 2 + 8 = 10V$$

Note :-

Impedance :- (Z)

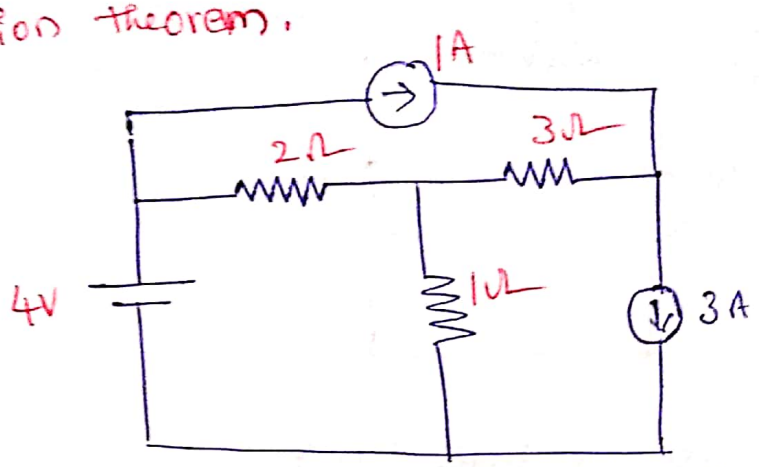
Impedance is a measure of the opposition to electrical flow. It is represented by Z. It is measured in ohms.

→ Ideal voltage source has zero internal impedance.

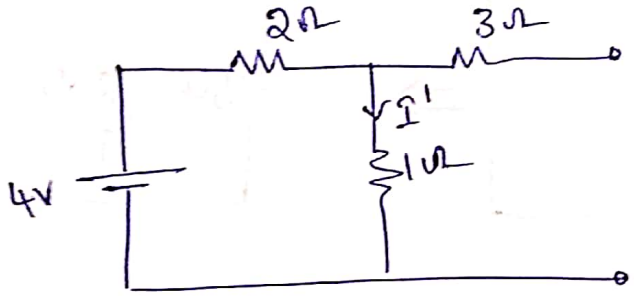
→ Ideal current source has infinite internal impedance.

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* find the current in the 1Ω resistor using superposition theorem.

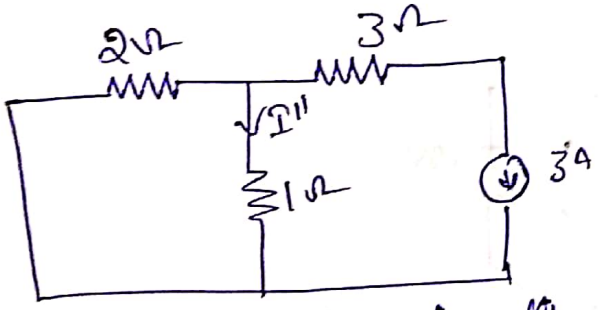


Sol when the 4V source is acting alone.



$$I' = \frac{V}{R} = \frac{4}{2+1} = 1.33\ A$$

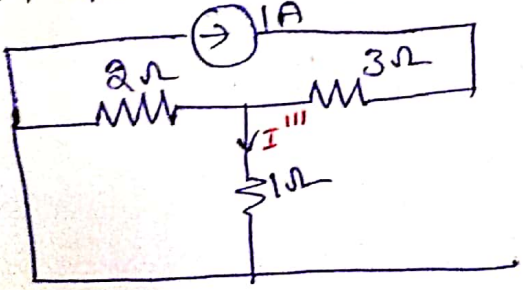
when the 3A source is acting alone



By current division formula

$$I'' = 3 \times \frac{2}{1+2} = 2A$$

when the 1A source is acting alone



By current division rule

$$I''' = 1 \times \frac{2}{2+1} = 0.66\ A$$

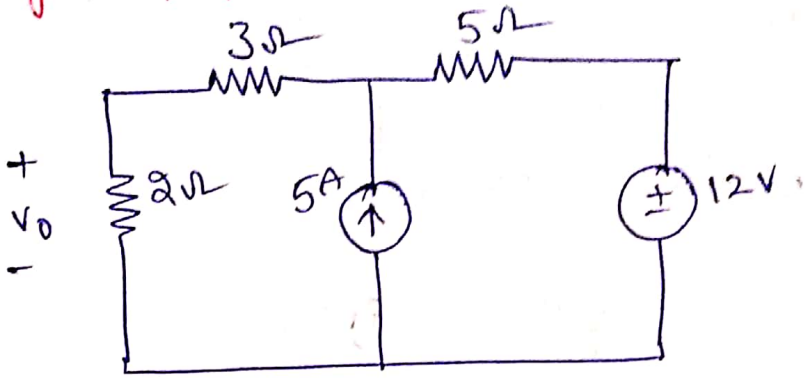
By superposition theorem

$$I = I' + I'' + I'''$$

$$I = 1.33 + 2 + 0.66 = 4\ A$$

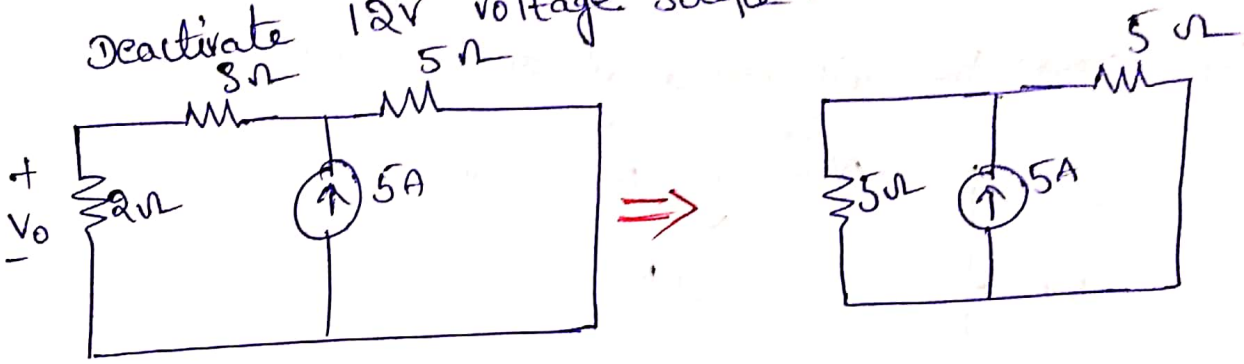
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* Using superposition theorem find V_o in the ckt.



Sol

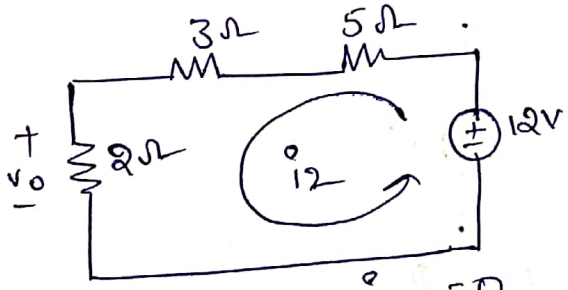
Deactivate 12V voltage source.



$$i_1 = \left(\frac{R_2}{R_1+R_2}\right) i = \left(\frac{5}{5+5}\right) 5 = 2.5$$

$$V_1 = i_1 R_1 = 2.5 \times 2 = 5\ V$$

Deactivate 5A current source.



$$-12 + 10 i_2 = 0$$

$$10 i_2 = 12$$

$$i_2 = 1.2$$

$$V_2 = i_2 \times R = 1.2 \times 2 = 2.4$$

$$V = V_1 + V_2 = 5 + 2.4 = 7.4\ V$$

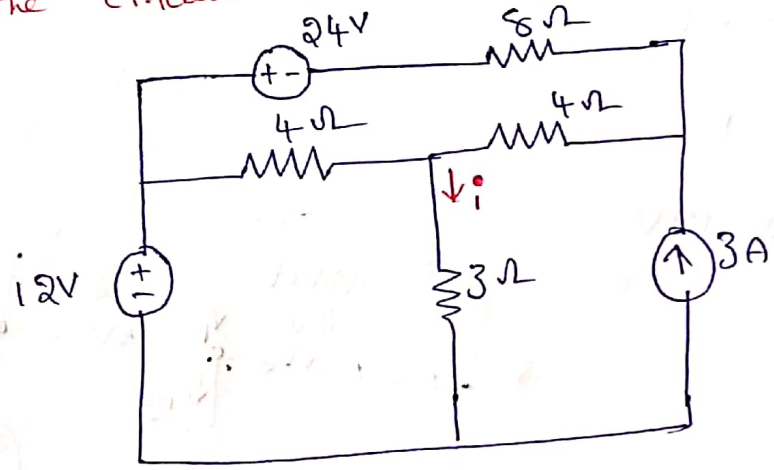
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Linear Circuit :-

A linear circuit is one in which the electronic component's values (such as resistance, Capacitance, Inductance) do not change with the level of voltage or current in the circuit.

Linear circuits are important because they can amplify and process electronic signals without distation.

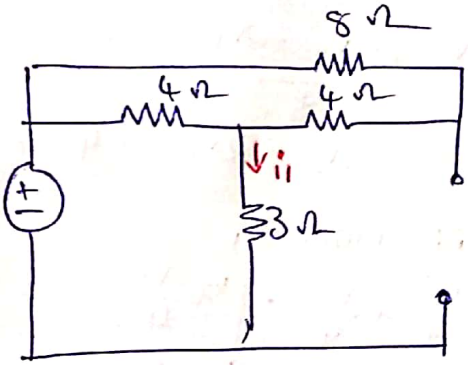
For the circuit use the superposition theorem to find i.

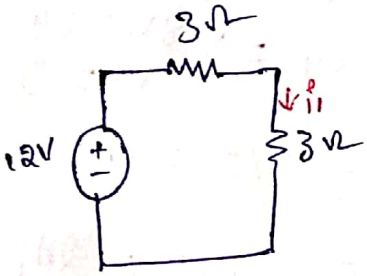


Sol Turn off 24V & 3A Sources.

$$4 // (4+8) = \frac{4 \times 12}{4+12}$$

$$= \frac{48}{16} = 3\Omega$$

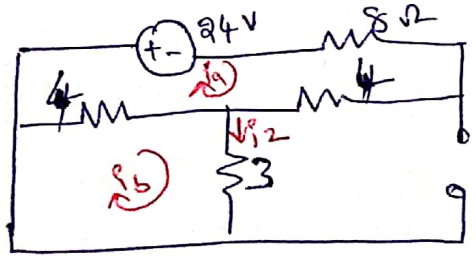




$$R = 3+3 = 6\Omega$$

$$i_1 = \frac{V}{R} = \frac{12}{6} = 2A$$

Turn off 12V & 3A



Apply mesh analysis to loop i_a

$$24 + 8i_a + 4i_a + 4(i_a - i_b) = 0$$

$$24 + 16i_a - 4i_b = 0$$

$$16i_a - 4i_b = -24$$

$$4i_a - i_b = -6 \longrightarrow (1)$$

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Apply mesh analysis to loop 1 is

$4(i_b - i_a) + 3i_b = 0$

$7i_b - 4i_a = 0$

$-4i_a + 7i_b = 0 \longrightarrow (2)$

Adding (1) & (2)

$4i_a - i_b = -6$

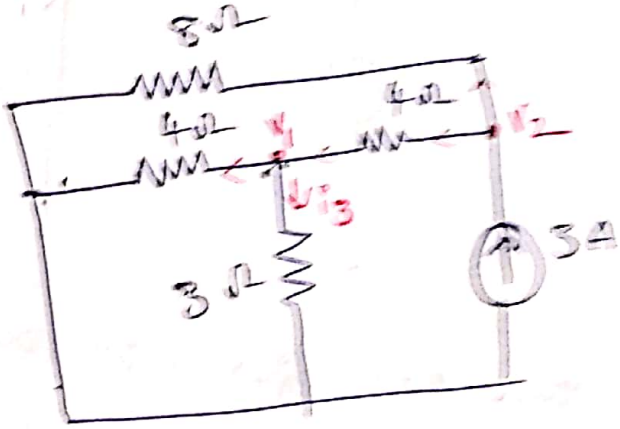
$-4i_a + 7i_b = 0$

$6i_b = -6$

$i_b = -1$

$i_2 = i_b = -1$

Turning off 24V & 12V



At node V_2

$\frac{V_2}{8} + \frac{V_2 - V_1}{4} = 3$

$\frac{V_2 + 2V_2 - 2V_1}{8} = 3$

$3V_2 - 2V_1 = 24 \longrightarrow (1)$

At node V_1

$\frac{V_1}{3} + \frac{V_1}{4} = \frac{V_2 - V_1}{4}$

$\frac{4V_1 + 3V_1}{12} = \frac{V_2 - V_1}{4}$

$4V_1 + 3V_1 = 3V_2 - 3V_1$

$4V_1 + 3V_1 - 3V_2 + 3V_1 = 0$

$10V_1 - 3V_2 = 0$

$-3V_2 = -10V_1$

$V_2 = \frac{+10V_1}{3}$

Substitute V_2 in (1)

$3\left(\frac{10V_1}{3}\right) - 2V_1 = 24$

$10V_1 - 2V_1 = 24 \Rightarrow 8V_1 = 24$

$-V_1 = 3$

$i_3 = \frac{V_1}{3} = 3/3 = 1A$

$i = i_1 + i_2 + i_3 = 2 - 1 + 1 = 2A$

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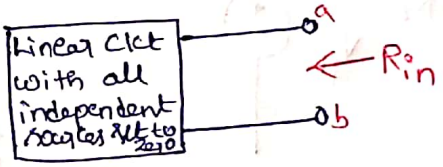
Thevenins Theorem :-

Thevenins theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th}, where V_{Th} is the open circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals when the independent sources are turned off.

Steps to apply Thevenins theorem.

Case (i) :-

If the network has no dependent sources, we turn off all independent sources. R_{Th} is the input resistance of the network looking between terminals a & b.

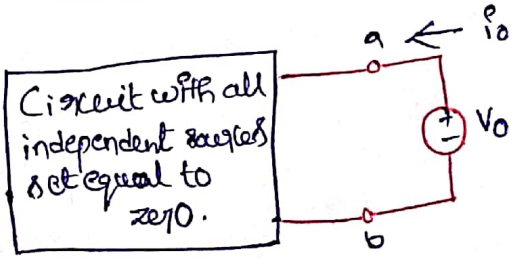


$$R_{Th} = R_{in}$$

Case (ii) :-

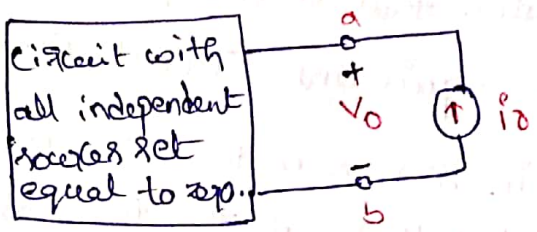
If the network has dependent sources, we turn off all independent sources. As with superposition, dependent sources are not to be turned off because they are controlled by circuit variables. we apply a voltage source V_o at terminals a & b and determine the resulting current i_o. Then

$$R_{Th} = \frac{V_o}{i_o}$$



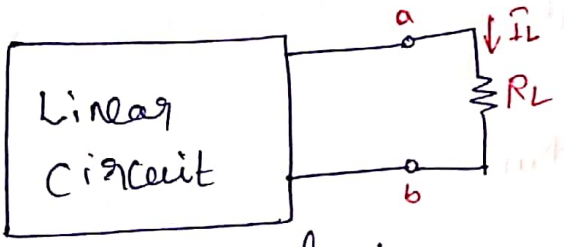
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We may insert a current source i_o at terminals a-b, and find the terminal voltage V_o. Again $R_{Th} = \frac{V_o}{i_o}$.

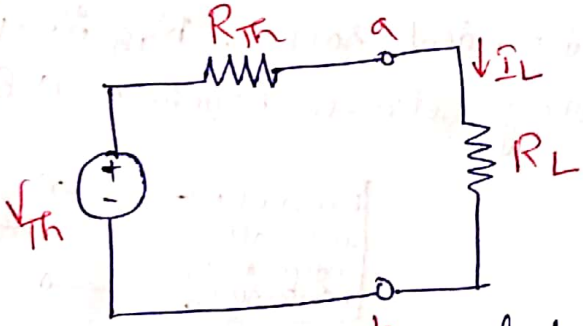


$$R_{Th} = \frac{V_o}{i_o}$$

A linear circuit terminated by a load R_L, the current I_L through the load and the voltage V_L across the load are easily determined once the thevinin equivalent of the circuit at the load's terminals is obtained.



A circuit with a load original Ckt



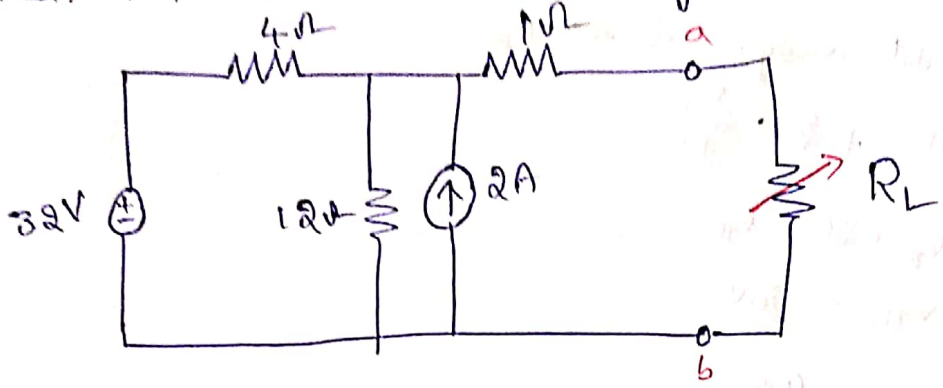
b) Thevenin equivalent Ckt.

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

$$V_L = R_L I_L = \frac{R_L}{R_{Th} + R_L} \cdot V_{Th}$$

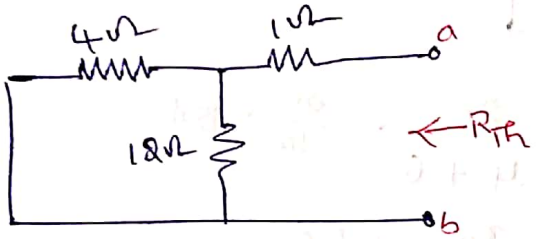
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① find the Thevenin equivalent circuit of the ckt as shown in below fig. to the left of the terminals a-b. Then find the current through R_L = 6, 16, 36 Ω.



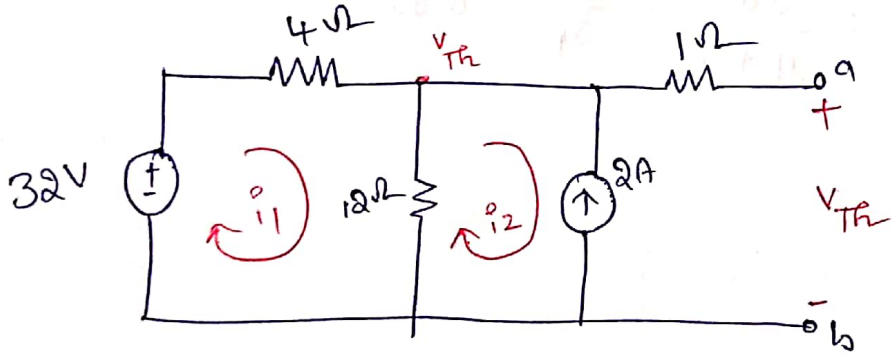
Sol

To obtain R_{Th} turning off the 32V voltage source, 2A current source.



$$R_{Th} = 4 // 12 + 1$$
$$= \frac{4 \times 12}{4+12} + 1$$
$$= \frac{48}{16} + 1 = 4\,\Omega$$

To find V_{Th}, Applying mesh analysis to the two loops.



loop ① ⇒ $-32 + 4i_1 + 12(i_1 - i_2) = 0$

$i_2 = -2A$

~~$V_{Th} = 12(i_1 - i_2)$~~

$-32 + 4i_1 + 12(i_1 - (-2)) = 0$

$-32 + 4i_1 + 12i_1 + 24 = 0$

$16i_1 - 8 = 0$

$i_1 = 8/16 = 0.5\,A$

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$$V_{Th} = 12(i_1 - i_2) = 12(0.5 + 2.0) = 30V.$$

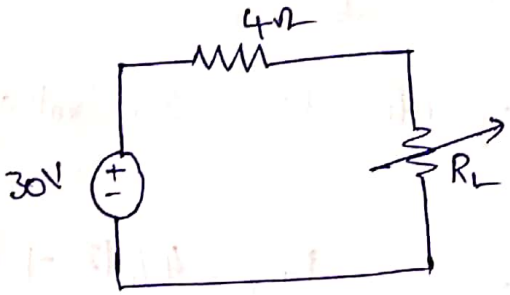
Alternative method:

Apply nodal analysis at node V_{Th}

$$\frac{32 - V_{Th}}{4} + 2 = \frac{V_{Th}}{12}$$

$$36 - 3V_{Th} + 24 = V_{Th}$$

$$V_{Th} = 30V.$$



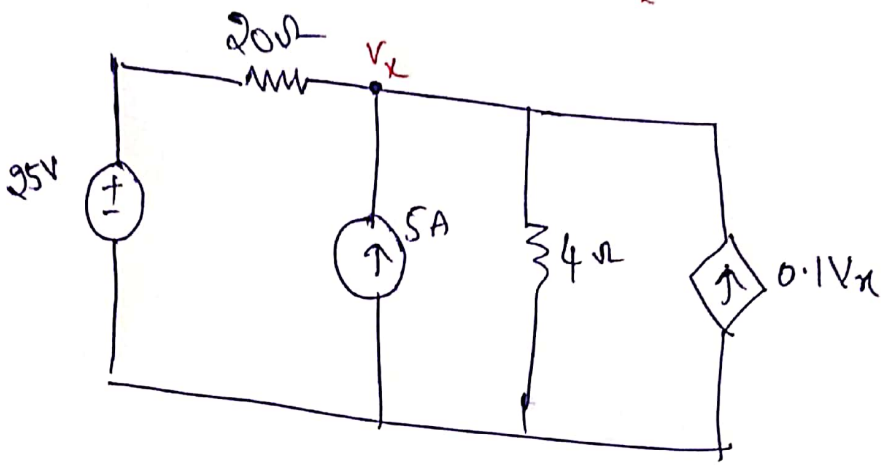
$$R_L = 6 \Rightarrow I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{30}{4 + 6} = \frac{30}{10} = 3A$$

$$R_L = 16\Omega \Rightarrow I_L = \frac{30}{4 + 16} = \frac{30}{20} = 1.5A$$

$$R_L = 30\Omega \Rightarrow I_L = \frac{30}{4 + 30} = \frac{30}{34} = 0.88A.$$

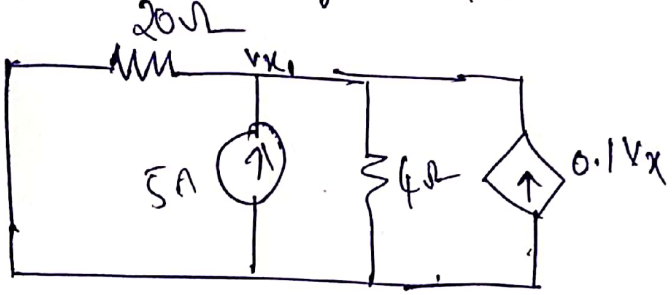
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Use superposition find V_x



Sol

Deactivate voltage source



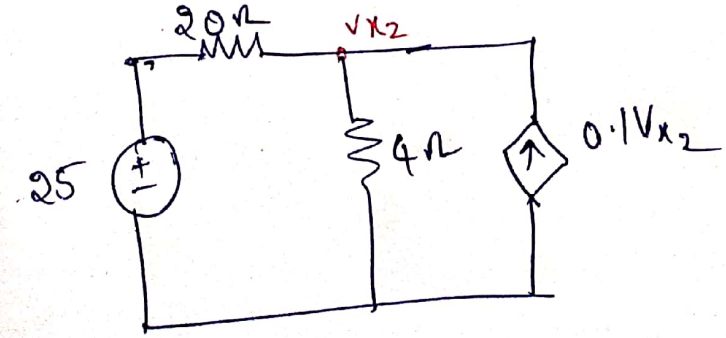
$$\frac{V_{x1}}{20} + \frac{V_{x1}}{4} = 5 + \frac{1}{10}V_{x1}$$

$$\frac{V_{x1} + 5V_{x1}}{20} = \frac{50 + V_{x1}}{10}$$

$$6V_{x1} = 100 + 2V_{x1}$$

$$4V_{x1} = 100 \Rightarrow V_{x1} = \frac{100}{4} = 25$$

Deactivate current source



$$\frac{V_{x2} - 25}{20} + \frac{V_{x2}}{4} = \frac{1}{10}V_{x2}$$

$$\frac{V_{x2} - 25 + 5V_{x2}}{20} = \frac{V_{x2}}{10}$$

$$6V_{x2} - 25 = 2V_{x2}$$

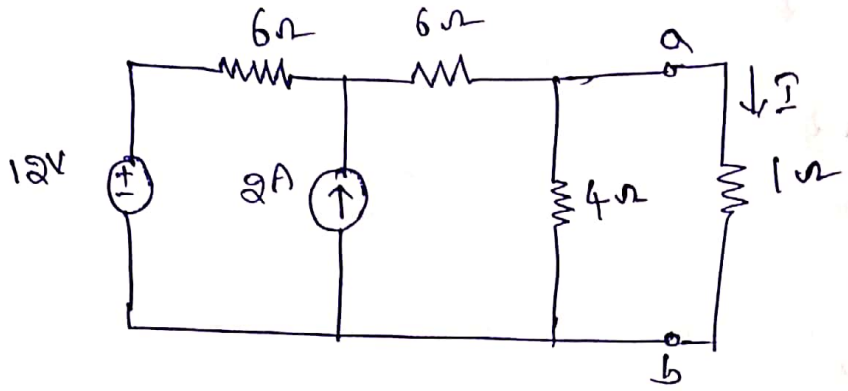
$$V_{x2} = 25/4 = 6.25$$

$$V_x = V_{x1} + V_{x2} = 25 + 6.25$$

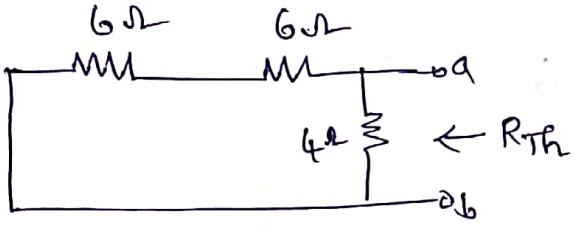
$$\boxed{V_x = 31.25}$$

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Using thevenin's theorem, find the equivalent ckt to the left of the terminals in the ckt. Then find I.

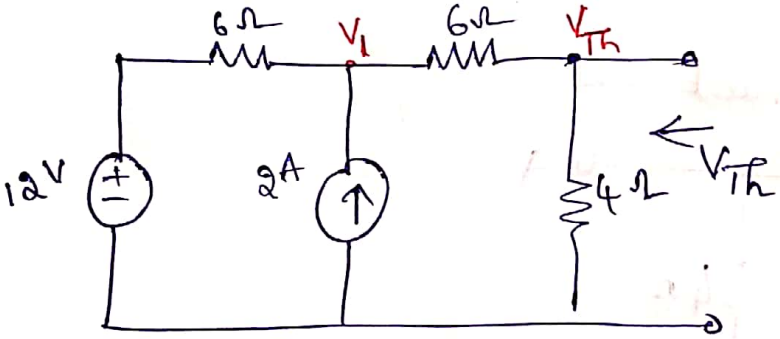


Sol To find R_{Th} turning off the voltage source and 2A current source.



$$R_{Th} = 12\Omega \,||\, 4\Omega = \frac{12\times 4}{12+4} = \frac{48}{16} = 3\Omega$$

To find V_{Th} Consider the ckt



KCL at node V_{Th}

$$\frac{V_{Th}}{4} + \frac{V_{Th} - V_1}{6} = 0$$

$$\frac{3V_{Th} + 2V_{Th} - 2V_1}{12} = 0$$

$$5V_{Th} = 2V_1 \implies V_{Th} = \frac{2V_1}{5}$$

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KCL at node V_1

$$\frac{V_1 - 12}{6} + \frac{V_1 - V_{Th}}{6} = 2$$

$$V_1 - 12 + V_1 - V_{Th} = 12$$

$$2V_1 - V_{Th} = 12 + 12$$

$$2V_1 - \frac{2V_1}{5} = 24$$

$$10V_1 - 2V_1 = 120$$

$$8V_1 = 120$$

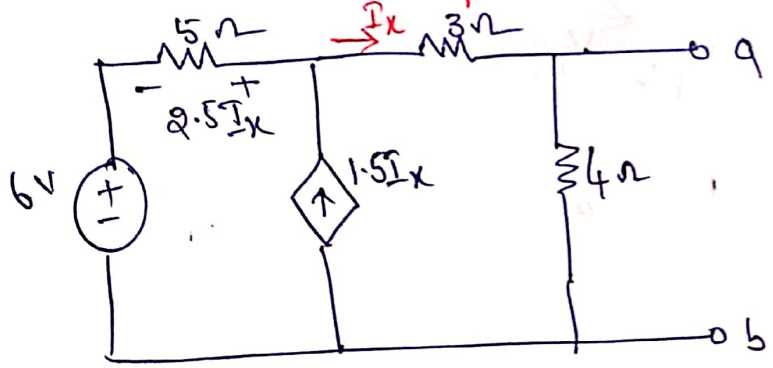
$$V_1 = \frac{120}{8} = 15\ V$$

$$V_{Th} = \frac{2V_1}{5} = \frac{2 \times 15}{5} = 6$$

$$V_{Th} = 6\ V$$

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{6}{3+1} = 6/5 = 1.5\ A$$

find the Thevenin equivalent Ckt.

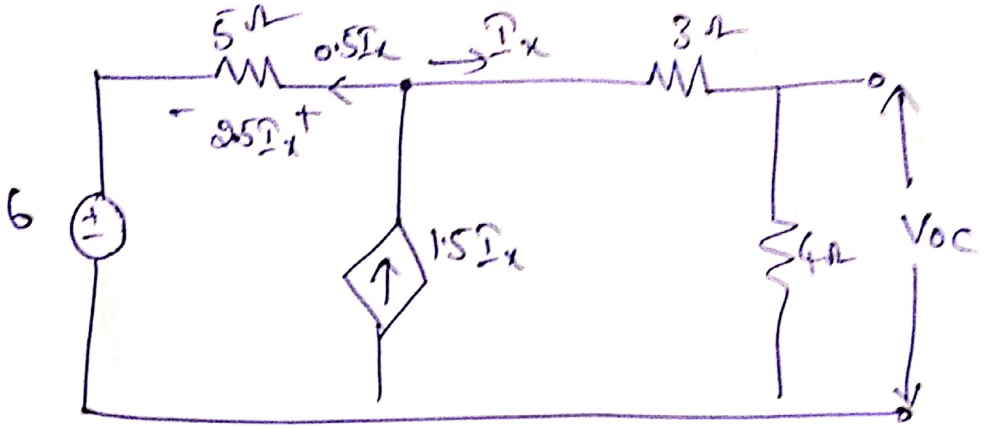


Sol

$$R_{Th} = \frac{V_{OC}}{I_{SC}}$$

$$I_{SC} = I_x$$

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$-6-25$ $\quad -6-2.5I_x+7I_x=0$

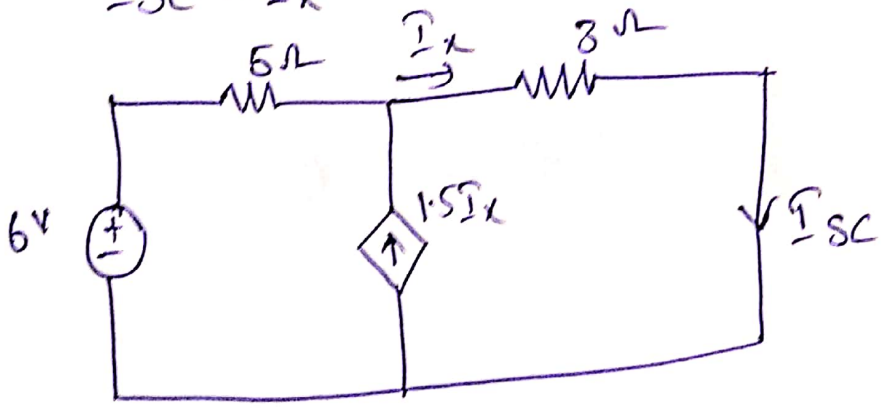
$4.5I_x=6$

~~$I_x = .6\times 10$~~

$$I_x=\frac{6}{4.5}=1.333$$

$$V_{OC}=4\times I_x=4\times 1.333=5.33$$

$I_{SC}=I_x$



$$I_{SC}=-6-2.5I_x+3I_x=0$$

$$0.5I_x=6$$

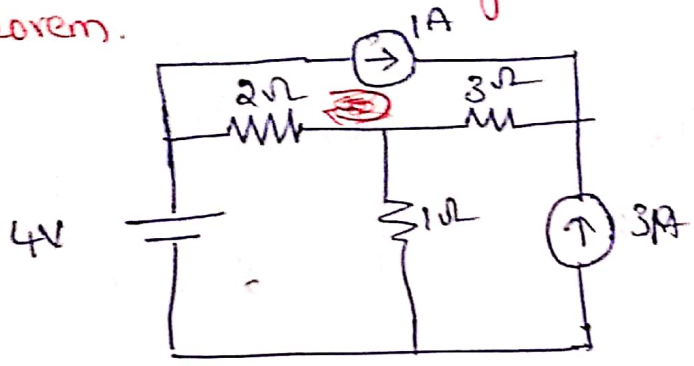
$$I_x=6/0.5=12A$$

$$R_{Th}=\frac{V_{OC}}{I_{SC}}=\frac{5.33}{12}=0.444$$

R_{Th} : 444.4 mΩ

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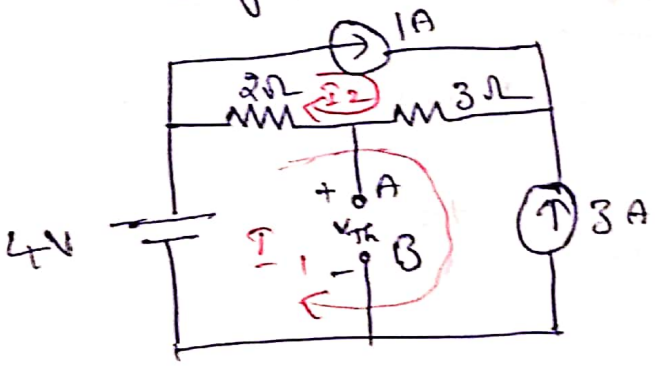
find the current through the 1Ω resistor using Thevenin's Theorem.



Sol

Step I :- Calculation of V_{Th}

Removing the 1Ω resistor from the network



$I_1 = -3A$

$I_2 = 1A$

writing the current equation for mesh 1 & 2

$-4 + 2(I_1 - I_2) + V_{Th} = 0$

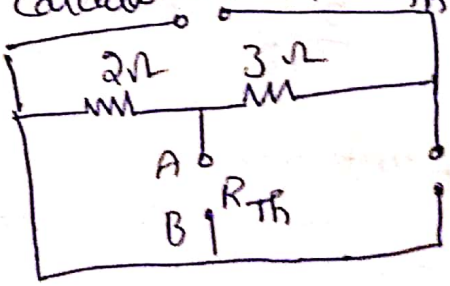
$-4 + 2(-3-1) + V_{Th} = 0$

$-4 + 2(-4) + V_{Th} = 0$

$-4 - 8 + V_{Th} = 0$

$V_{Th} = 12\ V$

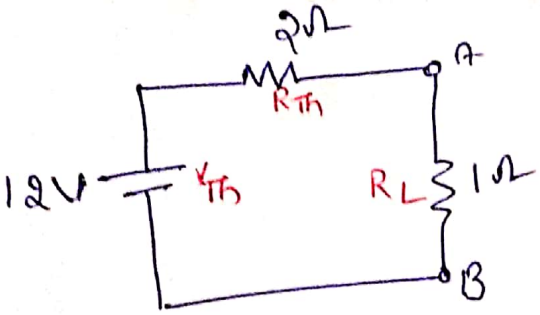
Step II :- Calculation of R_{Th}



$R_{Th} = 2\Omega$

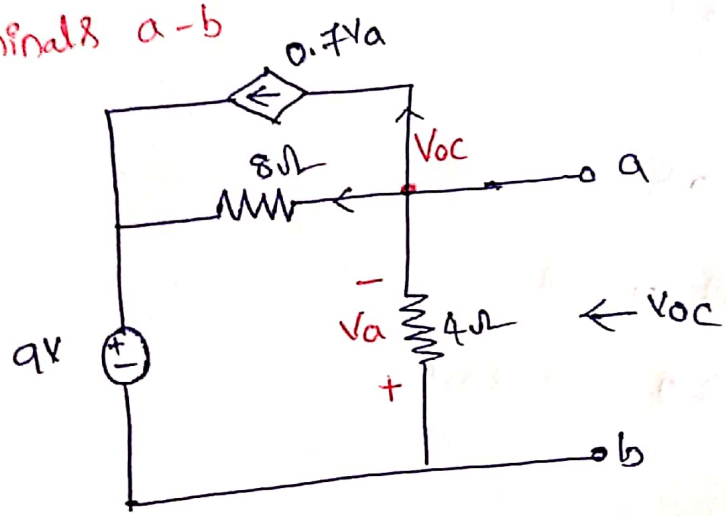
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Step III Calculation of I_L



$$I_L = \frac{V_{Th}}{R_{Th}+R_L} = \frac{12}{2+1} = 4A$$

In the following circuit find the value of R_{Th} at the terminals a-b



Sol

$$V_{oc} = -V_a$$

Apply KCL at node Voc

$$\frac{V_{oc}}{4} + \frac{V_{oc}-9}{8} + 0.75V_a = 0$$

$$\frac{V_{oc}}{4} + \frac{V_{oc}-9}{8} - 0.75V_{oc} = 0$$

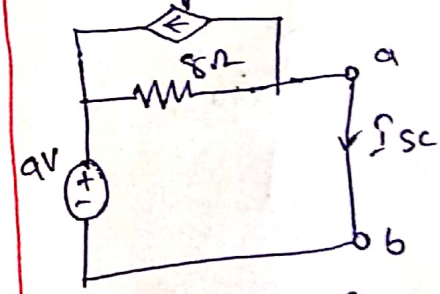
$$\frac{2V_{oc} + V_{oc} - 9 - 6V_{oc}}{8} = 0$$

$$2V_{oc} + V_{oc} - 9 - 6V_{oc} = 0$$

$$-3V_{oc} = 9$$

$$V_{oc} = -3V$$

finding Shat ckt current I_{sc}

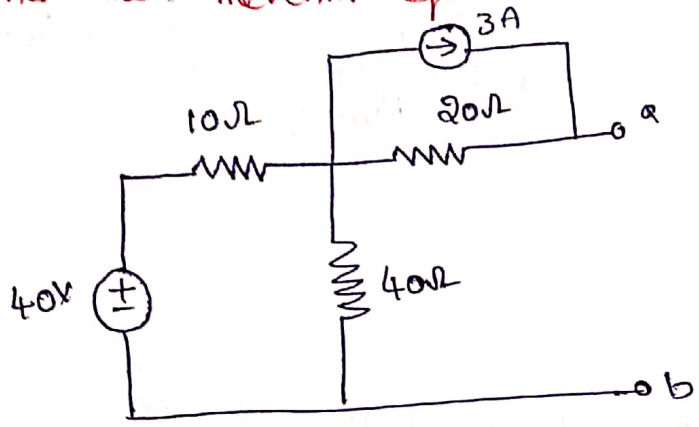


$$I_{sc} = \frac{V}{R} = \frac{9}{8} = 1.125A$$

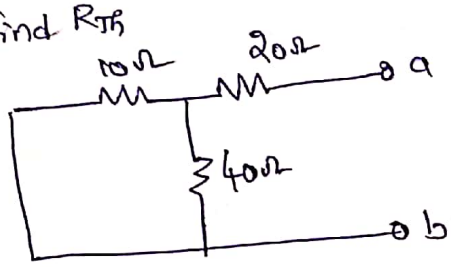
$$R_{Th} = \frac{V_{oc}}{I_{sc}} = \frac{-3}{1.125A} = -2.66\Omega$$

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* find the Thevenin equivalent at terminals a-b of the Ckt.



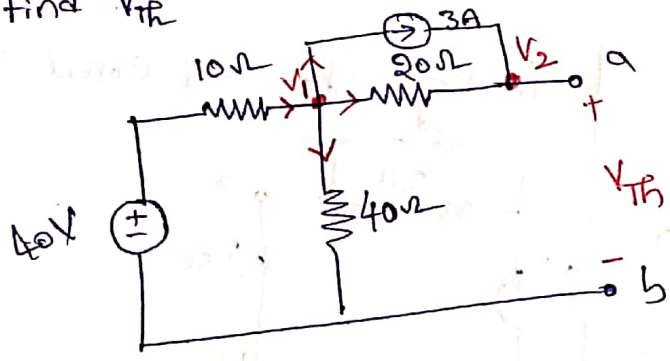
Sol To find R_{Th}



$$R_{Th} = 10\Omega \,||\, 40\Omega + 20\Omega$$

$$= \frac{10 \times 40}{10+40} + 20 = 28\Omega$$

To find V_{Th}



Apply KCL at node ①

$$\frac{40 - V_1}{10} = 3 + \frac{V_1 - V_2}{20} + \frac{V_1}{40}$$

$$40 = 7V_1 - 2V_2 \longrightarrow ①$$

Apply KCL at node ②

$$3 + \frac{V_1 - V_2}{20} = 0$$

$$60 + V_1 - V_2 = 0$$

$$V_1 - V_2 = -60$$

$$V_1 = V_2 - 60 \longrightarrow ②$$

Substitute ② in eq ①

$$40 = 7(V_2 - 60) - 2V_2$$

$$40 = 7V_2 - 420 - 2V_2$$

$$460 = 5V_2$$

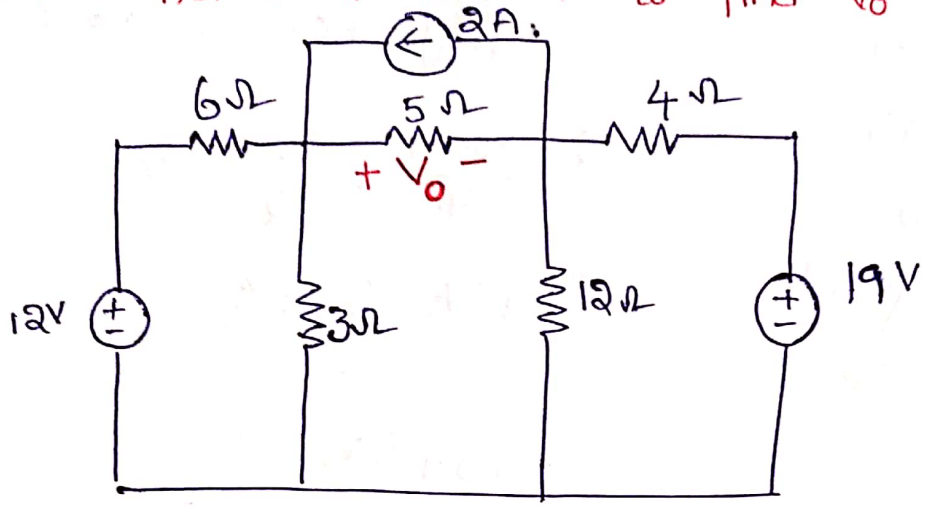
$$V_2 = \frac{460}{5}$$

$$V_2 = 92V$$

$\therefore V_2 = V_{Th} = 92V$

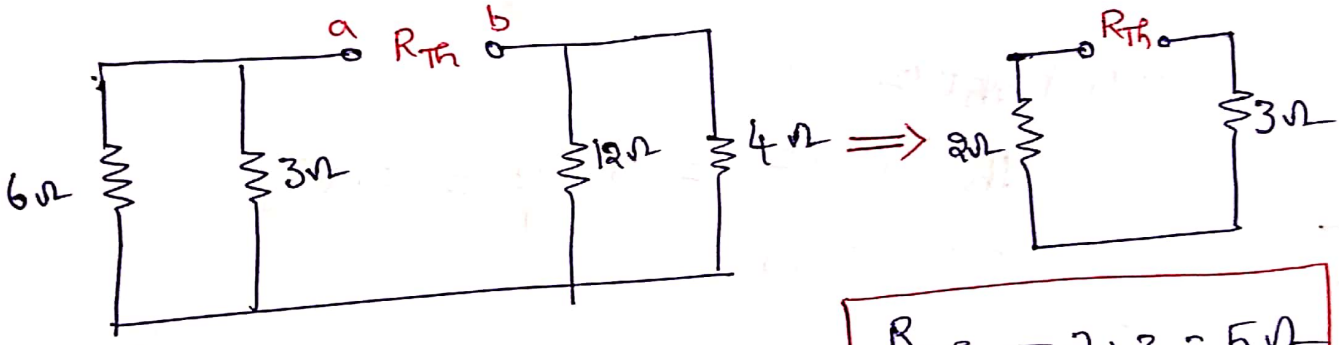
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* Use Thevenin's theorem to find V_o



Sol

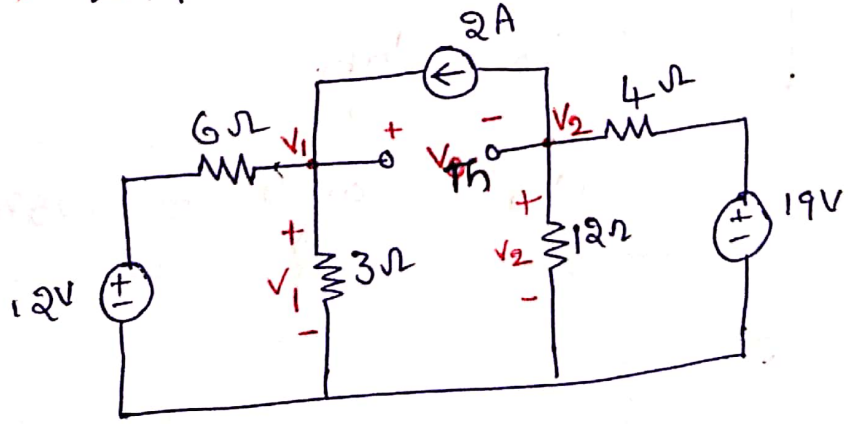
To find R_{Th}



$R_{Th} = 2 + 3 = 5\Omega$

$6//3 \Rightarrow \frac{6 \times 3}{6+3} = \frac{18}{9} = 2\Omega$

$12//4 \Rightarrow \frac{12 \times 4}{12+4} = \frac{48}{16} = 3\Omega$



Apply KCL at node V_1

$$2 = \frac{V_1 - 12}{6} + \frac{V_1}{3}$$

$$2 = \frac{V_1 - 12 + 2V_1}{6}$$

$$12 = 3V_1 - 12$$

$$3V_1 = 24$$

$$V_1 = 8V$$

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$$\frac{24 - V_1}{2} = V_1$$

$$24 - V_1 = 2V_1$$

$$24 - 3V_1 = 0$$

$$-3V_1 = -24$$

$$V_1 = 8V$$

Apply KCL at node V_2

$$2 + \frac{V_2 - 19}{4} + \frac{V_2}{12} = 0$$

$$\frac{24 + 3V_2 - 57 + V_2}{12} = 0$$

$$-33 + 4V_2 = 0$$

$$4V_2 = 33$$

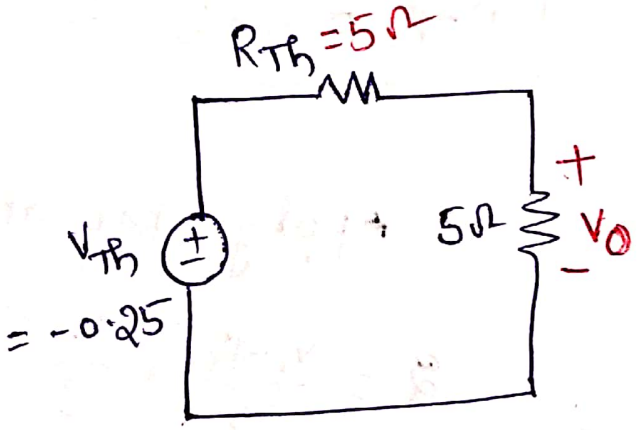
$$V_2 = 33/4$$

Apply KVL to the loop

$$-V_1 + V_{Th} + V_2 = 0$$

$$V_{Th} = V_1 - V_2 = 8 - \frac{33}{4} = -0.25$$

$$V_{Th} = -0.25$$



Voltage division rule

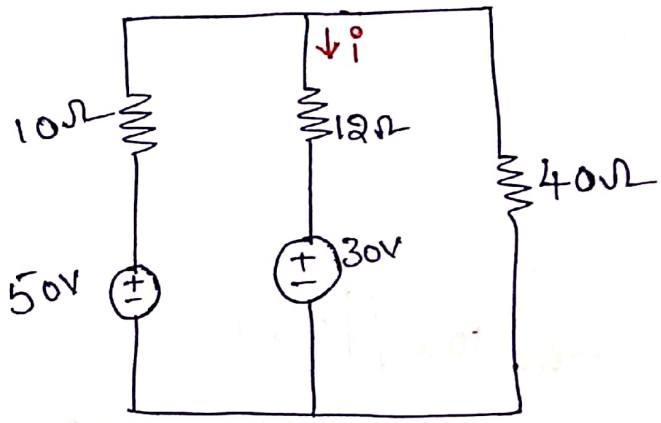
$$V_o = \frac{V_{Th} \times 5}{5+5} = \frac{V_{Th} \cdot 5}{10}$$

$$V_o = \frac{V_{Th}}{2}$$

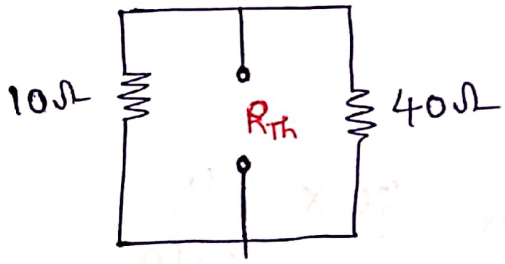
$$V_o = \frac{-0.25}{2} = -125mV$$

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* Use Thevenin's theorem find the current i.

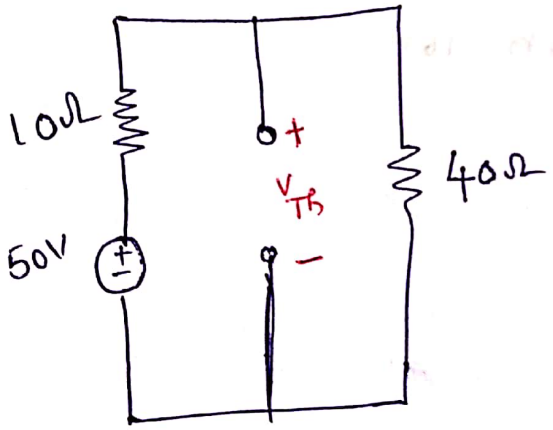


Sol To find R_{Th} remove the 30V voltage source & 50V voltage source



$R_{Th} = 10\Omega // 40\Omega$

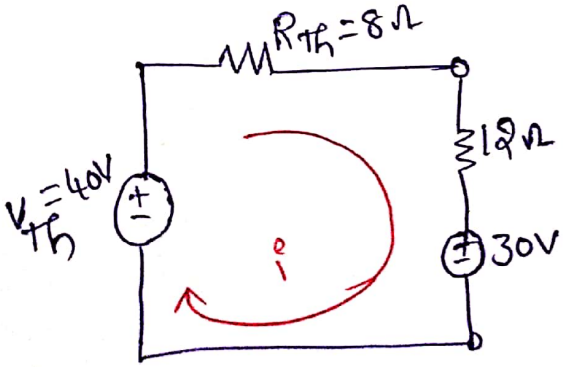
$= \frac{10 \times 40}{10+40} = \frac{400}{50} = 8\Omega$



$V_{Th} = \frac{40}{10+40} \times 50$

$= \frac{40}{50} \times 50$

$V_{Th} = 40V$



Apply KVL

$-40 + 8i + 12i + 30 = 0$

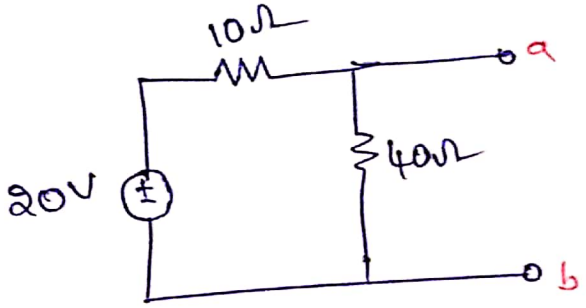
$-40 + 20i + 30 = 0$

$20i = 10$

$i = 10/20 = 0.5\ A$

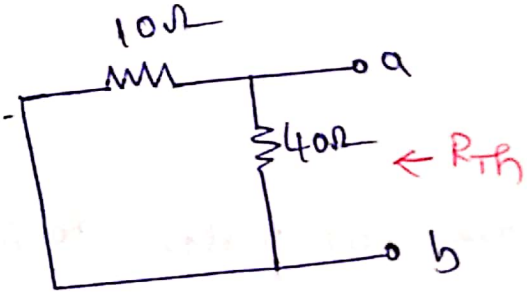
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* Determine R_{Th} & V_{Th} at terminals a-b.



Sol

To find R_{Th}

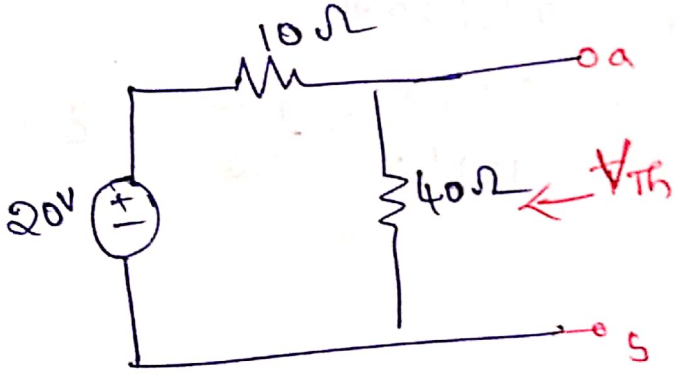


$R_{Th} = 10\Omega \,||\, 40\Omega$

$R_{Th} = \frac{10 \times 40}{10+40} = \frac{400}{50}$

$R_{Th} = 8\Omega$

To find V_{Th}



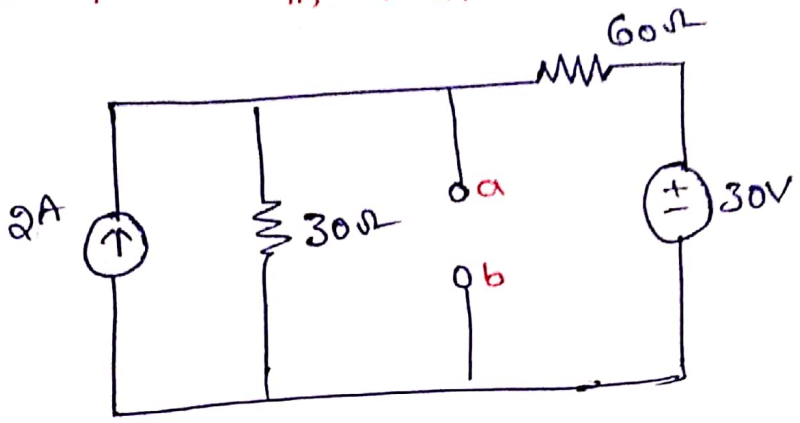
$V_{Th} = 20 \times \frac{40}{40+10}$

$= 20 \times \frac{40}{50}$

$V_{Th} = 16V$

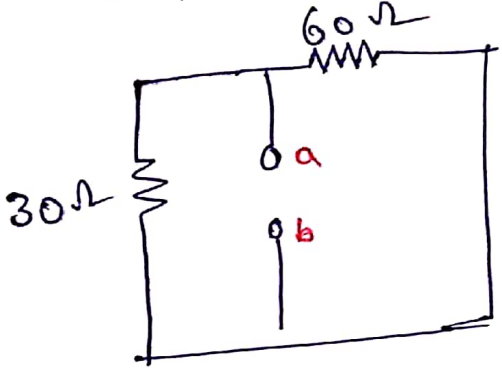
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* Determine R_{Th} & V_{Th} at terminals a-b



Sol

To find R_{Th}

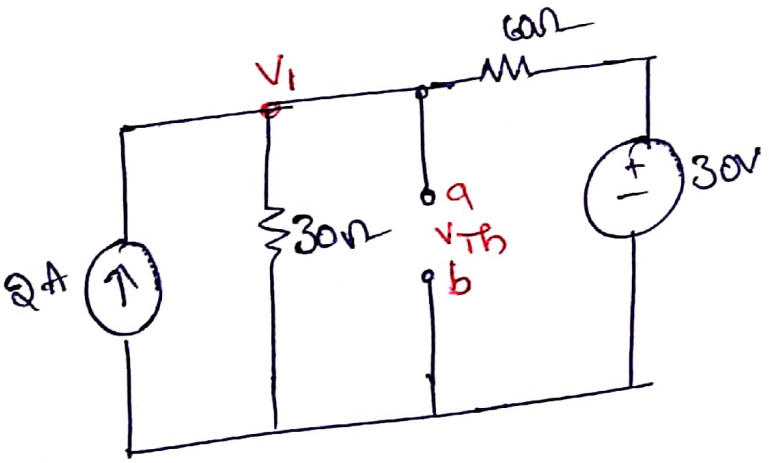


$R_{Th} = 30\Omega \,//\, 60\Omega$

$R_{Th} = \frac{30 \times 60}{30+60} = \frac{1800}{90} = 20\Omega$

$R_{Th} = 20\Omega$

To find V_{Th}



$V_1 = V_{Th}$

Apply KCL at node V_1

$$2 = \frac{V_1 - 30}{60} + \frac{V_1}{30}$$

$$2 = \frac{V_1 - 30 + 2V_1}{60}$$

$$120 = 3V_1 - 30$$

$$150 = 3V_1$$

$$V_1 = 50$$

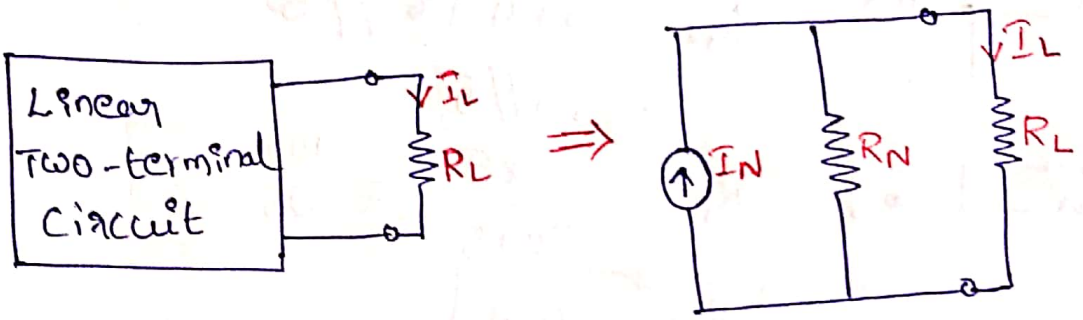
$\therefore V_1 = V_{Th}$

$$\boxed{V_{Th} = 50V}$$

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Norton's Theorem :-

Norton's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N, where I_N is the short circuit current through the load terminals and R_N is the input or equivalent resistance at the load terminals when the independent sources are turned off.



→ Thevenin's and Norton's resistances are equal i.e

$$R_N = R_{Th}$$

→ To find Norton's current I_N we determine the I_{sc} flowing through the terminals.

$$I_N = I_{SC}$$

→ Dependent and independent sources are treated as the same way in thevenin's theorem.

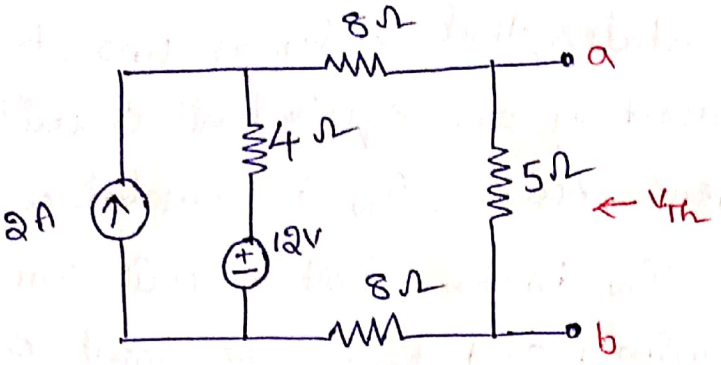
→ The close relationship b/w Thevenin's and Norton's theorem is

$$R_N = R_{Th}$$

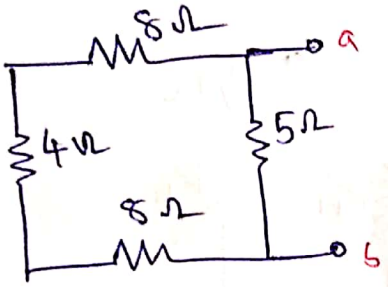
$$I_N = \frac{V_{Th}}{R_{Th}}$$

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* find the Norton's equivalent circuit as shown in figure at terminals a-b.



Sol To find R_N turn off all independent sources

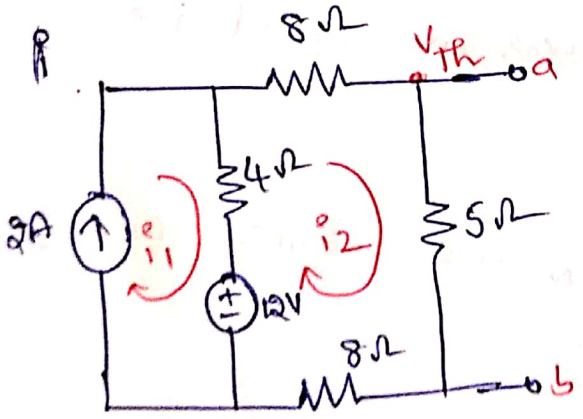


$R_N = 8+4+8 \,//\, 5$

$= 20 \,//\, 5$

$R_N = \frac{20 \times 5}{20+5} = \frac{100}{25}$

$\boxed{R_N = 4\Omega}$



$i_1 = 2A$

Apply KVL to loop i_2

$-12 + 4(i_2 - i_1) + 8i_2 + 5i_2 + 8i_2 = 0$

$-12 + 4i_2 - 4i_1 + 8i_2 + 5i_2 + 8i_2 = 0$

$-12 + 25i_2 - 4i_1 = 0$

$-12 + 25i_2 - 4 \times 2 = 0$

$-12 + 25i_2 - 8 = 0$

$25i_2 = 20$

$i_2 = \frac{20}{25} = \frac{4}{5} = 0.8A$

$i_2 = 0.8A$

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$V_{Th} = 5i_g$

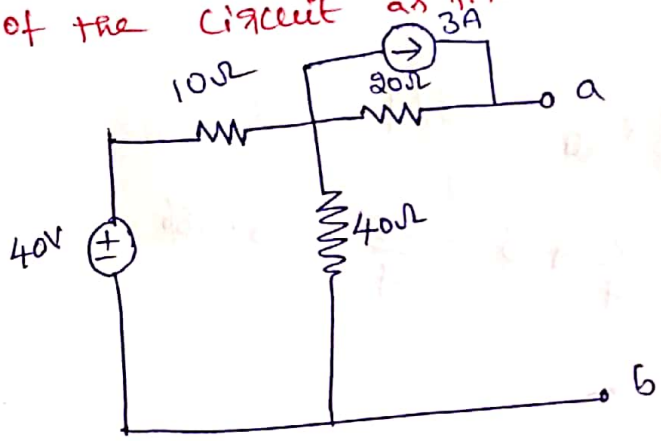
$V_{Th} = 5 \times 0.8$

$\boxed{V_{Th} = 4V}$

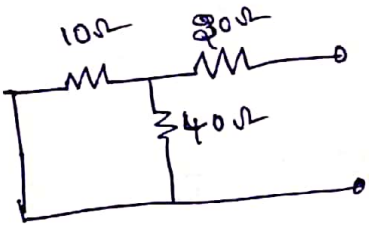
$$I_N = \frac{V_{Th}}{R_{Th}} = \frac{4}{4} = 1A$$

$\boxed{I_N = 1A}$

* find the Norton equivalent looking into terminals a-b of the circuit as shown in fig.

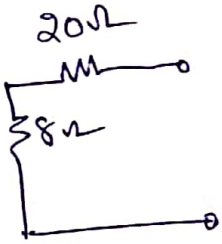


Sol To find R_N

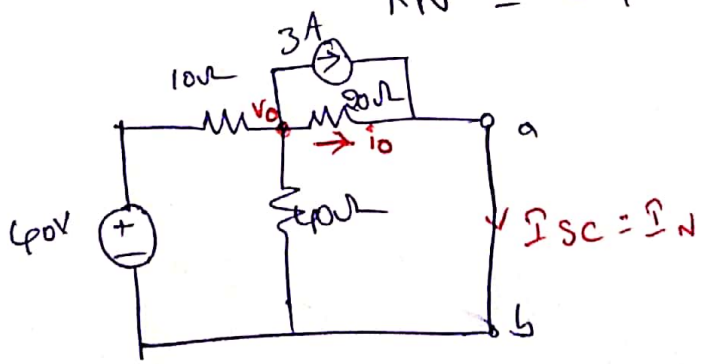


$10\Omega \,||\, 40 = \frac{10 \times 40}{10+40} = \frac{8}{\cancel{400}}$

$= 8\Omega$



$R_N = 8 + 20 = 28\Omega$



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Apply KCL at node V_o

$$\frac{40 - V_o}{10} = 3 + \frac{V_o}{40} + \frac{V_o}{20}$$

$$\frac{40 - V_o}{10} = \frac{120 + V_o + 2V_o}{40}$$

$$40 - V_o = \frac{120 + 3V_o}{4}$$

$$160 - 4V_o = 120 + 3V_o$$

$$-4V_o - 3V_o = 120 - 160$$

$$-7V_o = -40$$

$$V_o = 40/7$$

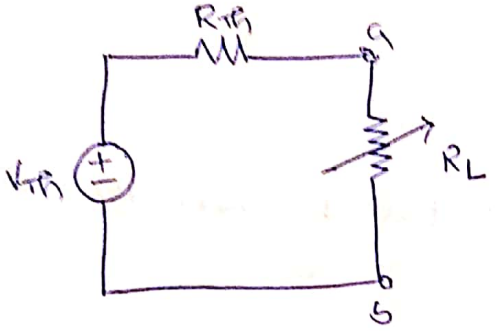
$$i_o = \frac{V_o}{20} = \frac{\frac{40}{7}}{20} = \frac{40}{7} \times \frac{1}{20} = 2/7$$

$$I_N = I_{SC} = i_o + 3 = \frac{2}{7} + 3 = \frac{23}{7} = 3.286$$

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Maximum power transfer theorem :-

Maximum power is transferred to the load when the load resistance is equals the Thevenin's resistance as seen from the load ($R_L = R_{in}$).
This theorem is applicable only when load is variable.
The Thevenin equivalent circuit is useful for find maximum power delivered to the load.



V_{Th} and R_{Th} are fixed. By varying the load resistance R_L, the power delivered to the load will also be varied.

$$P = V_L I_L$$

$$V_L = I_L R_L$$

$$P = I_L^2 R_L$$

$$P = \left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2 R_L$$

To prove the maximum power transfer theorem we differentiate P with respect to R_L and set the result is equal to zero.

$$\frac{dP}{dR_L} = \frac{d}{dR_L}\left[\frac{V_{Th}}{R_{Th}+R_L}\right]^2 R_L = 0$$

$$= (V_{Th})^2 \frac{d}{dR_L}\left[\frac{R_L}{(R_{Th}+R_L)^2}\right] = 0$$

$$= (V_{Th})^2 \left[\frac{(R_{Th}+R_L)^2 \frac{d}{dR_L} R_L - R_L \frac{d}{dR_L}(R_{Th}+R_L)^2}{(R_{Th}+R_L)^4}\right] = 0$$

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$$\frac{dp}{dR_L} = (V_{Th})^2 \left[\frac{(R_{Th})^2 + R_L^2 + 2R_{Th}R_L - 2R_L(R_{Th}+R_L)}{(R_{Th}+R_L)^4} \right] = 0$$

$$= (R_{Th})^2 + R_L^2 + 2R_{Th}R_L - 2R_L R_{Th} - 2R_L^2 = 0$$

$$= R_{Th}^2 - R_L^2 = 0$$

$$R_{Th}^2 = R_L^2$$

$$\boxed{R_{Th} = R_L}$$

The maximum power can be obtained when load resistance $R_L = R_{Th}$.

$$P = I_L^2 R_L$$

$$P = \left(\frac{V_{Th}}{R_{Th}+R_L} \right)^2 R_L$$

$$P = \frac{V_{Th}^2}{R_{Th}^2 + R_L^2 + 2R_{Th}R_L} \times R_L$$

$$P = \frac{V_{Th}^2}{R_{Th}^2 + R_{Th}^2 + 2R_{Th}^2} \times R_{Th}$$

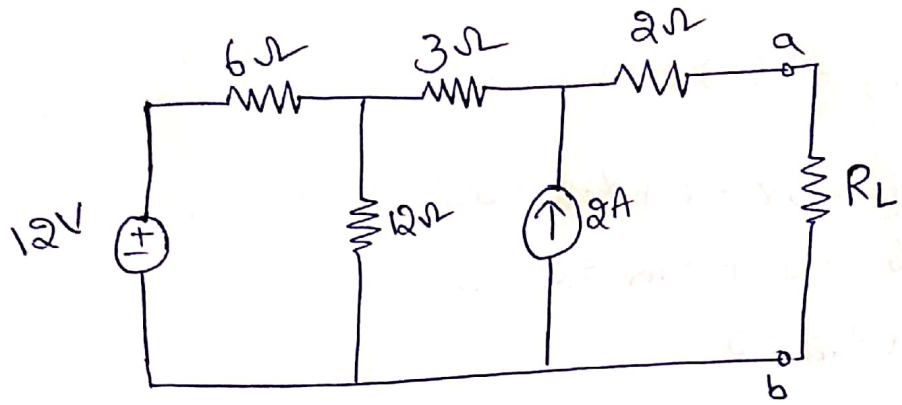
$$P = \frac{V_{Th}^2}{4R_{Th}^2} R_{Th}$$

$$\boxed{P = \frac{V_{Th}^2}{4R_{Th}}}$$

∴ maximum power delivered to the load when $R_{Th} = R_L$.

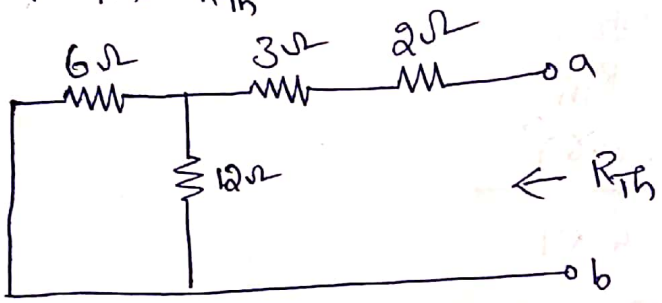
Scanned with CamScanner

* find the valeee of R_L for maximum power transfer in the circuit of fig. find the maximum power.



Sol

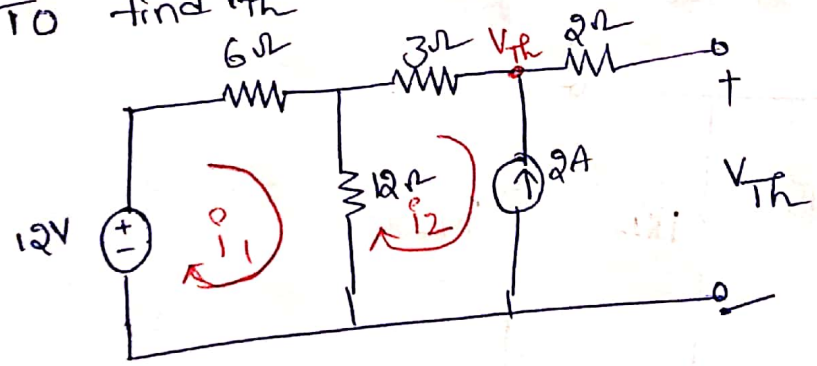
To find R_{Th}



$$R_{Th} = 3+2+\frac{6\times12}{6+12}$$

$$= 5+\frac{72}{18} = 5+4 = 9\Omega$$

To find V_{Th}



Apply mesh analysis to loop ①

$$-12+6i_1+12(i_1-i_2)=0 \qquad i_2=-2$$

$$-12+6i_1+12(i_1-(-2))=0$$

$$-12+6i_1+12i_1+24=0$$

$$18i_1=-12$$

$$i_1=-12/18=-\frac{2}{3}=-0.66$$

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Applying KVL around the outer loop to get V_{Th} across terminals a-b :

$$-12 + 6i_1 + 3i_2 + 2(0) + V_{Th} = 0$$

$$-12 + 6x - 0.66 + 3x - 2 + V_{Th} = 0$$

$$-12 - 3.96 - 6 + V_{Th} = 0$$

$$-21.6 + V_{Th} = 0$$

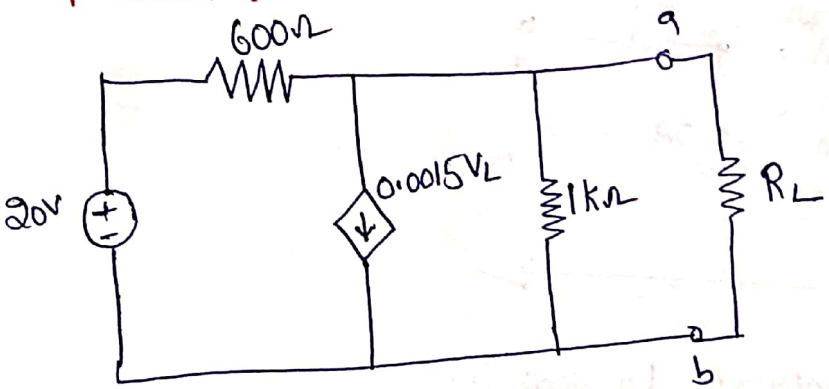
$$V_{Th} = +21.6$$

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}}$$

$$P_{max} = \frac{(21.6)^2}{4 \times 9}$$

$$\boxed{P_{max} = 13\ W}$$

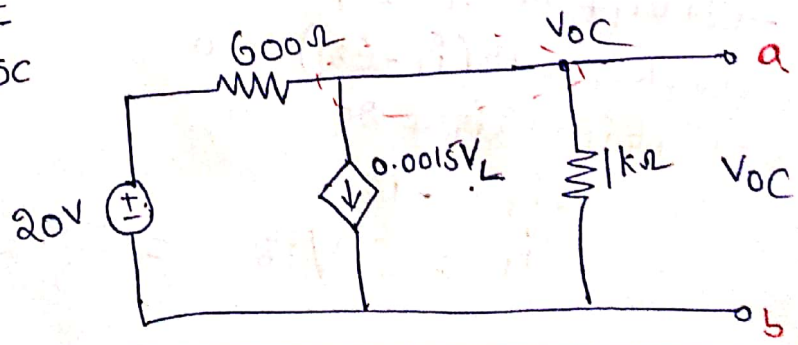
Find the value of load resistance R_L for the maximum power transfer theorem as shown in figure.



Sol'n

$$R_{Th} = \frac{V_{OC}}{I_{SC}}$$

$V_L = V_{OC}$



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Apply KCL to the node V_L

$$\frac{V_L - 20}{600} + 0.0015 V_L + \frac{V_L}{1000} = 0$$

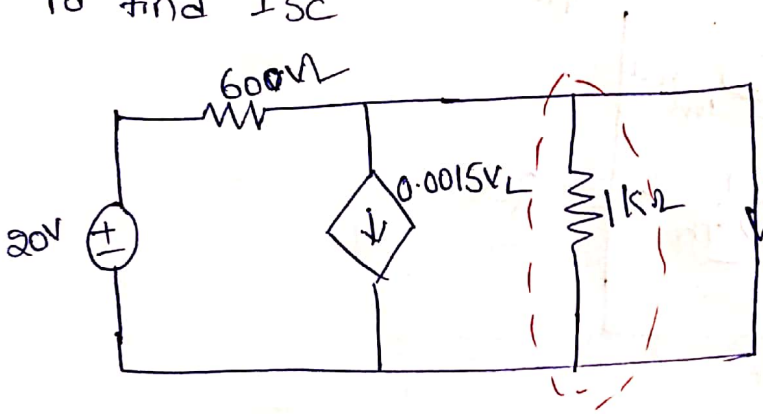
$$\frac{5V_L - 100 + 4.5 V_L + 3V_L}{3000} = 0$$

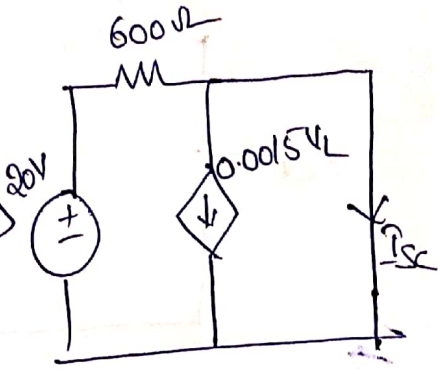
$$12.5 V_L = 100$$

$$V_L = \frac{100}{12.5}$$

$$V_{OC} = V_L = 8V$$

To find I_{SC}





$$I_{SC} = \frac{V}{R} = \frac{20}{600} = 1/30 = 0.033\,A$$

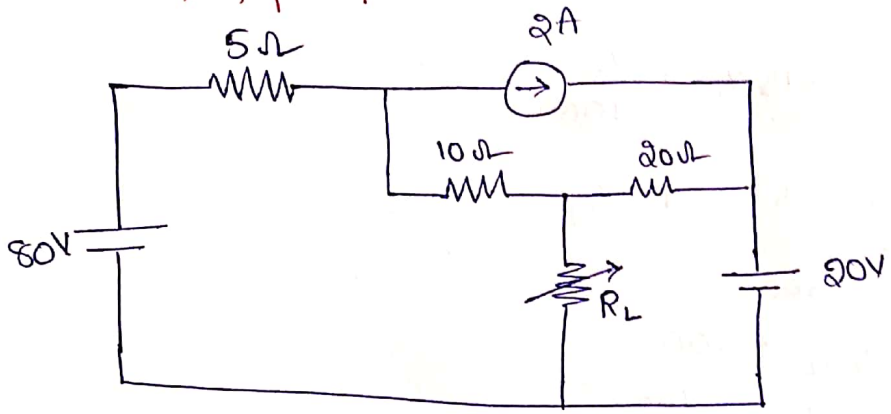
$$R_{TH} = \frac{V_{OC}}{I_{SC}} = \frac{8}{0.033} = 240.2\,\Omega$$

$$\boxed{R_L = R_{TH} = 240.2\,\Omega}$$

$$P_{max} = \frac{V_{TH}^2}{4 R_{TH}} = \frac{8^2}{4 \times 240.2} = \frac{64}{960.8} = 0.0666\,W$$

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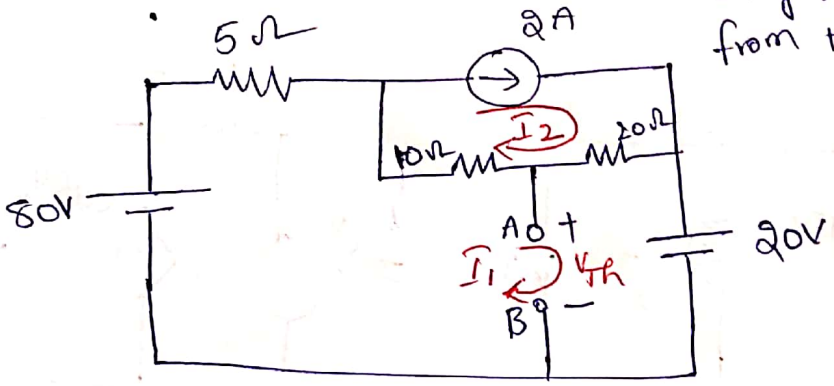
For the Ckt shown, find the value of the resistance R_L for maximum power and Calculate the maximum power.



Sol

Calculation of V_{Th}

Removing the variable resistor R_L from the Ckt.



Applying KVL to mesh 1

$$-80 + 5I_1 + 10(I_1 - I_2) + 20(I_1 - I_2) + 20 = 0$$

$$5I_1 + 10I_1 - 10I_2 + 20I_1 - 20I_2 + 20 = 80$$

$$35I_1 - 30I_2 = 80 - 20$$

$$35I_1 - 30I_2 = 60 \longrightarrow (1)$$

$$I_2 = 2A \longrightarrow (2)$$

Substitute I_2 value in eq (1)

$$35I_1 - 30 \times 2 = 60$$

$$35I_1 = 60 + 60$$

$$I_1 = 120/35 = 3.43A$$

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RL

writing V_{TH} equation

$-V_{TH} + 20(I_1 - I_2) + 20 = 0$

~~$-V_{TH} + 20($~~

$-V_{TH} + 20(3.43 - 2) + 20 = 0$

$-V_{TH} + 20(1.43) + 20 = 0$

$-V_{TH} = -28.6 - 20$

$-V_{TH} = -48.6$

$V_{TH} = 48.6$

(OR)

writing V_{TH} equation.

$-80 + 5I_1 + 10(I_1 - I_2) + V_{TH} = 0$

$-80 + 5 \times 3.43 + 10(3.43 - 2) + V_{TH} = 0$

$-80 + 17.15 + 10 \times 1.43 + V_{TH} = 0$

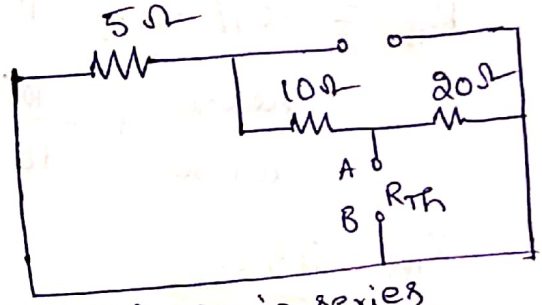
$-80 + 17.15 + 14.3 + V_{TH} = 0$

$-80 + 31.45 + V_{TH} = 0$

$-48.55 + V_{TH} = 0$

$V_{TH} = 48.55$

Calculation of R_{TH}



5Ω, 10Ω are in series

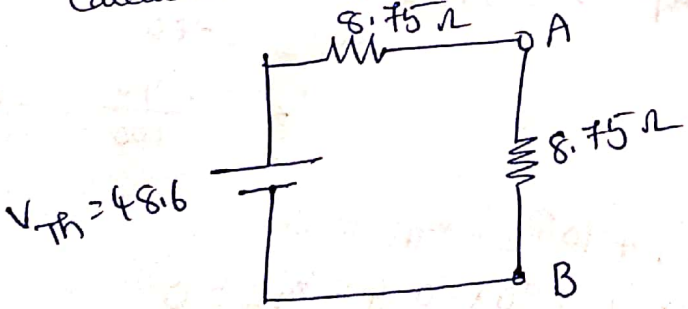
$R_{TH} = 15\Omega \,||\, 20\Omega$

$R_{TH} = \frac{15 \times 20}{15 + 20} = 8.75\Omega$

Calculation of R_L for maximum power transfer

$R_L = R_{TH} = 8.75\Omega$

Calculation of P_{max}



$P_{max} = \frac{V_{TH}^2}{4 \times R_{TH}}$

$= \frac{(48.6)^2}{4 \times 8.75}$

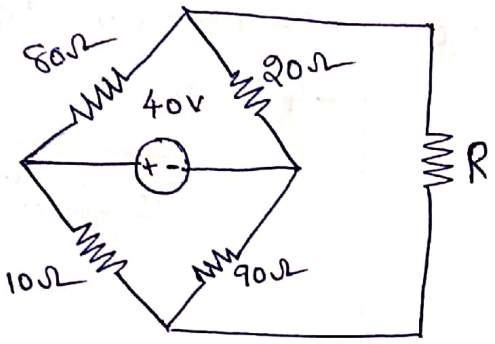
$P_{max} = 68.9\,W$

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The variable resistor R in fig. is adjusted until it absorbs the maximum power from the circuit.

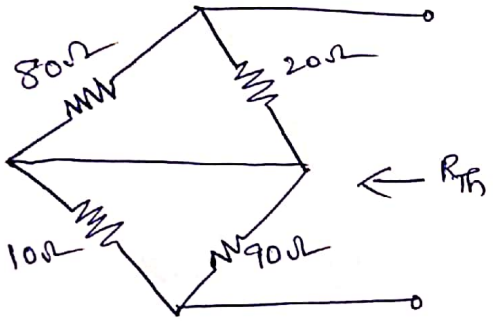
a) Calculate the value of R for maximum power.

b) Determine the maximum power absorbed by R.



Sol

To find R_{Th}

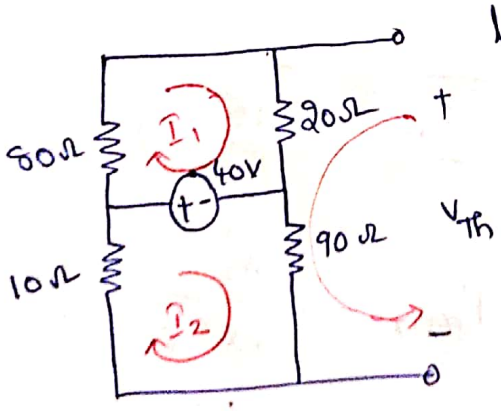


$$R_{Th} = 20 // 80 + 90 // 10$$

$$= \frac{20 \times 80}{20+80} + \frac{90 \times 10}{90+10}$$

$$\boxed{R_{Th} = 25\,\Omega}$$

To find V_{Th}



Loop $I_1 \Rightarrow 80I_1 + 20I_1 - 40 = 0$

$$100I_1 = 40$$

$$I_1 = 40/100 = 0.4$$

Loop $I_2 \Rightarrow 10I_2 + 90I_2 + 40 = 0$

$$100I_2 = -40$$

$$I_2 = \frac{-40}{100} = -0.4$$

$$20I_1 + 90I_2 - V_{Th} = 0$$

$$20 \times 0.4 + 90 \times -0.4 - V_{Th} = 0$$

$$8 - 36 - V_{Th} = 0$$

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$$-28 - V_{Th} = 0$$

$$-V_{Th} = 28$$

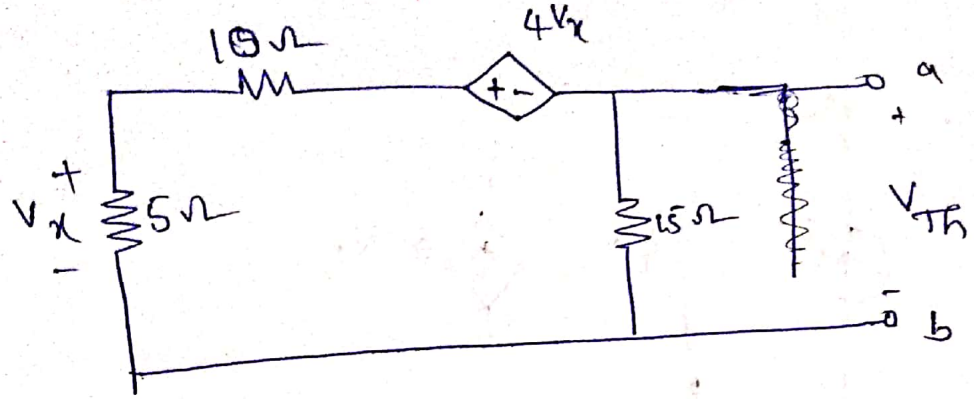
$$\boxed{V_{Th} = -28}$$

a) $R = R_{Th} = 25\,\Omega$

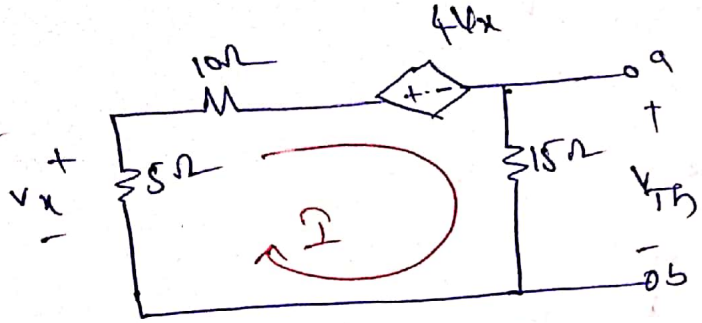
b) $P_{max} = \dfrac{V_{Th}^2}{4R_{Th}} = \dfrac{(28)^2}{4 \times 25} = \dfrac{(28)^2}{100} = 7.84\,W$

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Obtain thevenin equivalent of the Ckt.



Sol'



$V_x = V_{Th}$ $\quad V_x = 5I$

Apply KVL to the loop

$$-V_x + 10I + 4V_x + 15I = 0$$

$$-5I + 10I + 4 \times 5I + 15I = 0$$

$$-5I + 10I + 20I + 15I = 0$$

$$40I = 0$$

$$I = 0$$

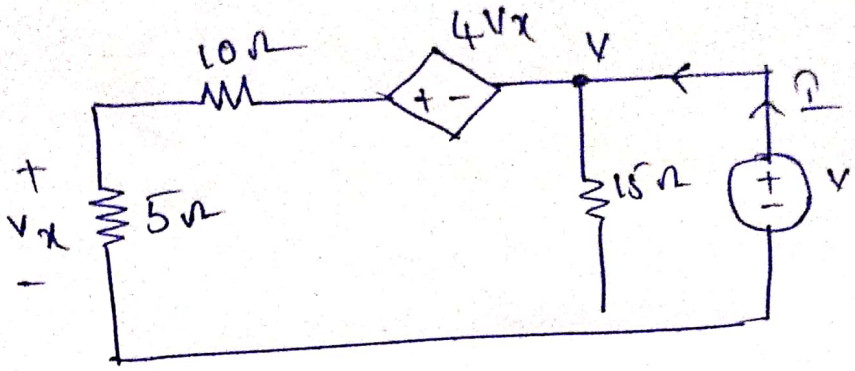
$$V_x = 5 \times 0 = 0$$

$\therefore V_{Th} = 0$

In the Ckt All are dependent sources $V_{Th} = 0$, $I_{SC} = 0$

But resistance is obtained.

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$$\frac{V - 4V_x}{15} + \frac{V}{15} = I$$

$$\frac{V - 4 \times 5I}{15} + \frac{V}{15} - I = 0$$

$$\frac{V}{15} - \frac{4 \cancel{20} I}{\cancel{15}\ 3} + \frac{V}{15} - I = 0$$

$$\frac{V}{15} - \frac{4I}{3} + \frac{V}{15} - I = 0$$

$$\frac{2V}{15} = \frac{7I}{3}$$

$$\frac{2}{5} = \frac{7I}{3}$$

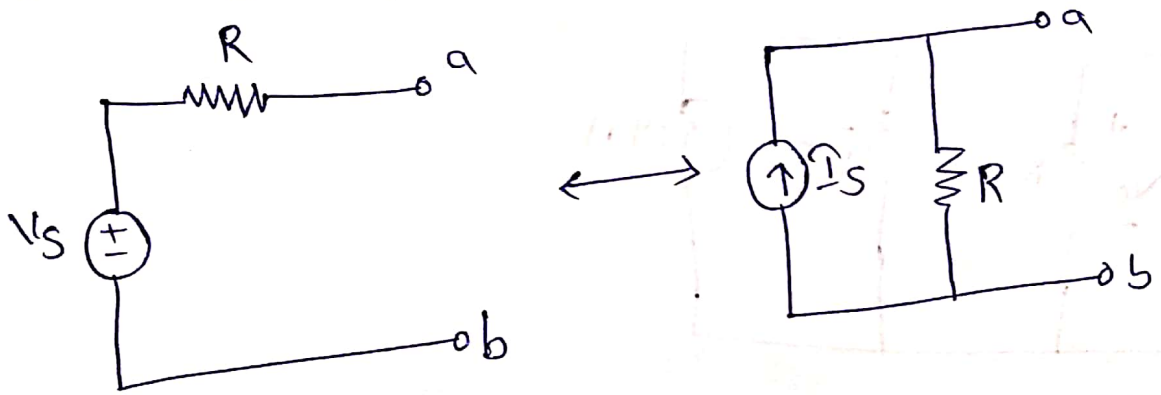
$$R_{Th} = \frac{V}{I} = \frac{35}{2} = 17.5\,\Omega$$

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Source Transformations :-

Source transformation is a tool for simplifying circuits. In circuit analysis is to be able to substitute a voltage source in series with a resistor for a current source in parallel with a resistor.

A source transformation is a process of replacing voltage source V_s in series with a resistor R by a current source I_s in parallel with a resistor R.



Transformation of independent source.

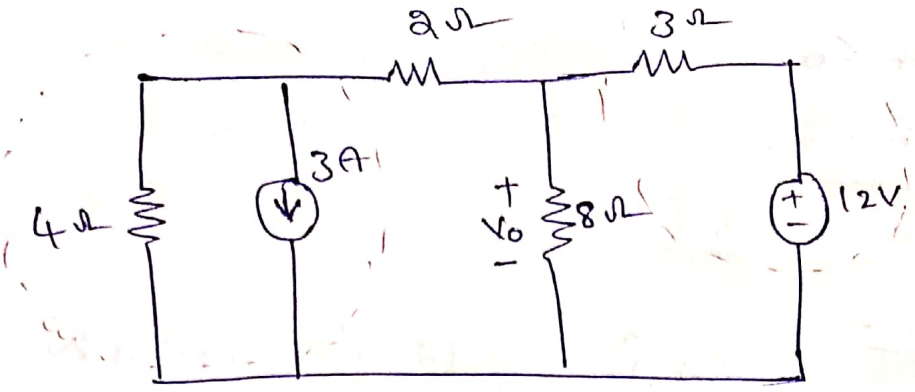
when the terminals a & b are shot circuited, the current flowing from a to b is

$$i_{sc} = \frac{V_s}{R}$$

$$V_{sc} = i_{sc} R$$

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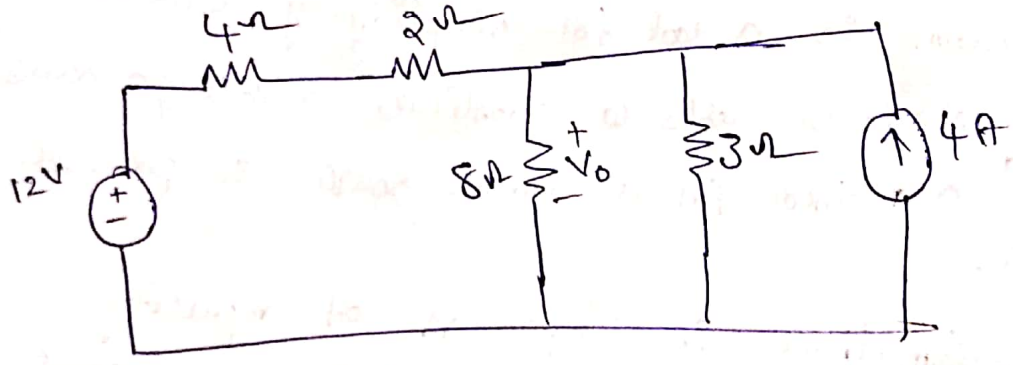
Use Source transformation to find V_o.



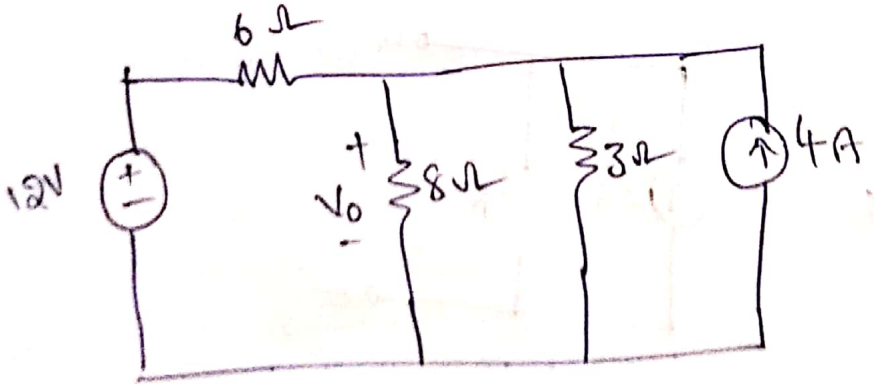
Sol

$$i_S = \frac{V_S}{R} = \frac{12}{3} = 4A$$

$$V_S = i_S R = 4 \times 3A = 12V$$

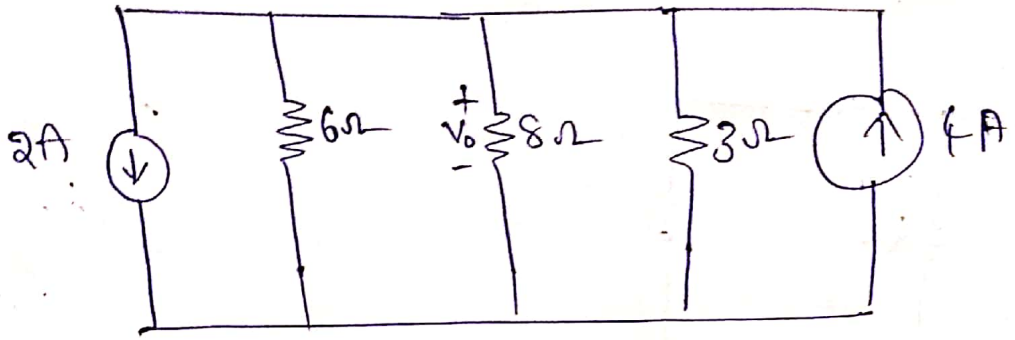


$4\Omega \& 2\Omega \Rightarrow R_{eq} = 4 + 2 = 6\Omega$

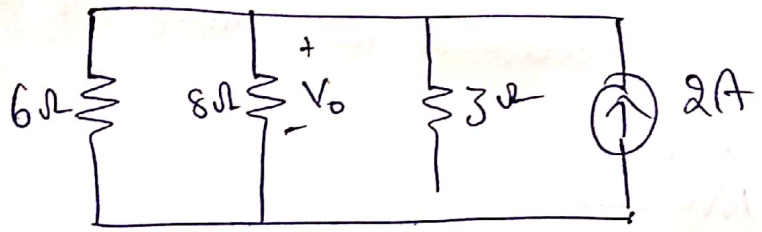


$$i_S = \frac{V}{R} = \frac{12}{6} = 2A$$

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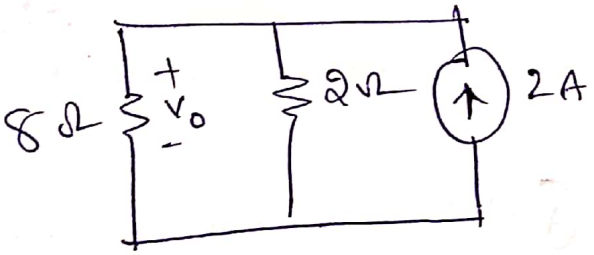


$i = -2 + 4 = 2A$



$6\Omega \,||\, 3\Omega$

$R_{eq} = \frac{6 \times 3}{6+3} = \frac{18}{9} = 2\Omega$



$i_1 = \left(\frac{R_2}{R_1+R_2}\right) i \quad \Rightarrow i_1 = 2 \times \frac{2}{8+2}$

$i_1 = \frac{4}{10}$

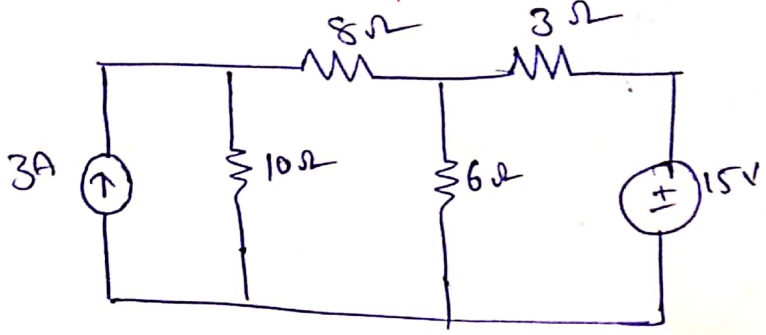
$i_1 = 0.4A$

$V_o = 8 \times 0.4$

$= 3.2\,V$

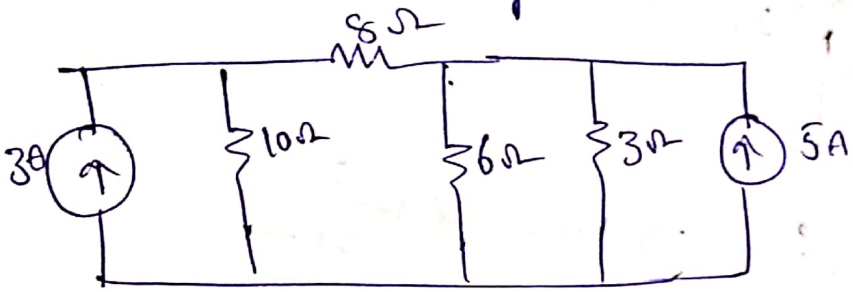
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* Use Source transformation to determine the current and power in the 8Ω resistor.



Sol

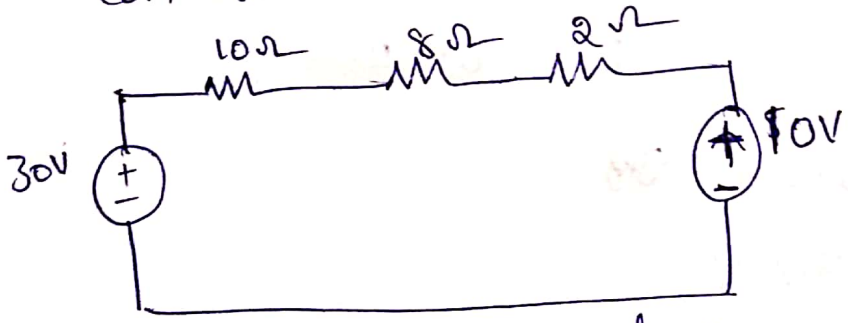
Transform the voltage source



$6\Omega // 3\Omega$

$\frac{6 \times 3}{6+3} = \frac{18}{9} = 2\Omega$

Convert the current source to voltage source



Applying KVL to the loop

$-30 + 10i + 8i + 2i + 10 = 0$

$20i - 20 = 0$

$i = 1\ A$

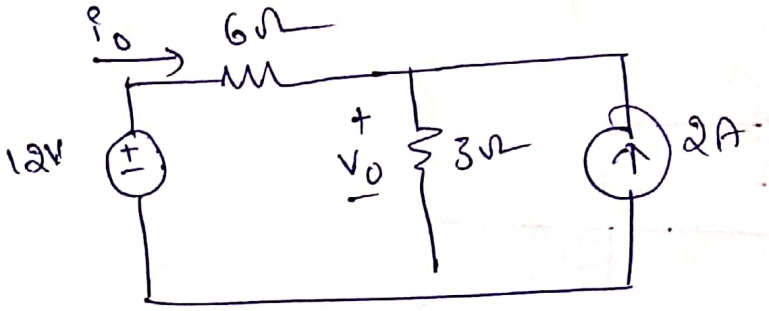
$P = VI$

$P = I^2 R$

$= 1^2 \times 8 = 8W$

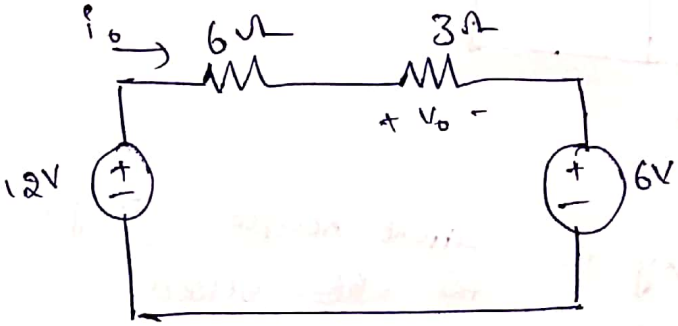
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Apply source transformation to determine V_o & i_o in the ckt



Sol

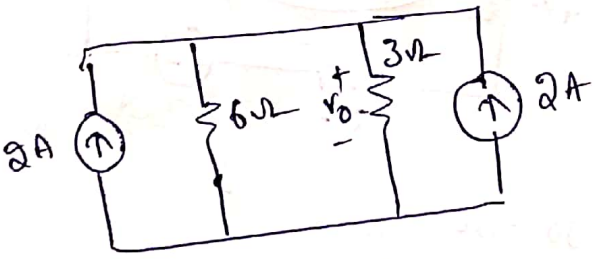
To get i_o transform current source.



$-12 + 9i_o + 6 = 0 \Rightarrow 9i_o = 6, \; i_o = 6/9 = 2/3$

$i_o = 666.7 \text{ mA}$

To get v_o transform the voltage sources.



$$i = \frac{6}{6+3} \times (2+2)$$

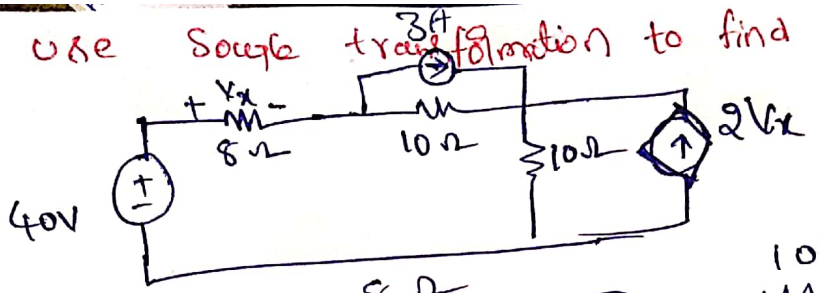
$$= \frac{6}{9} \times 4 = \frac{24}{9} = 8/3$$

$$V_o = 3i = 3 \times \frac{8}{3} = 8V$$

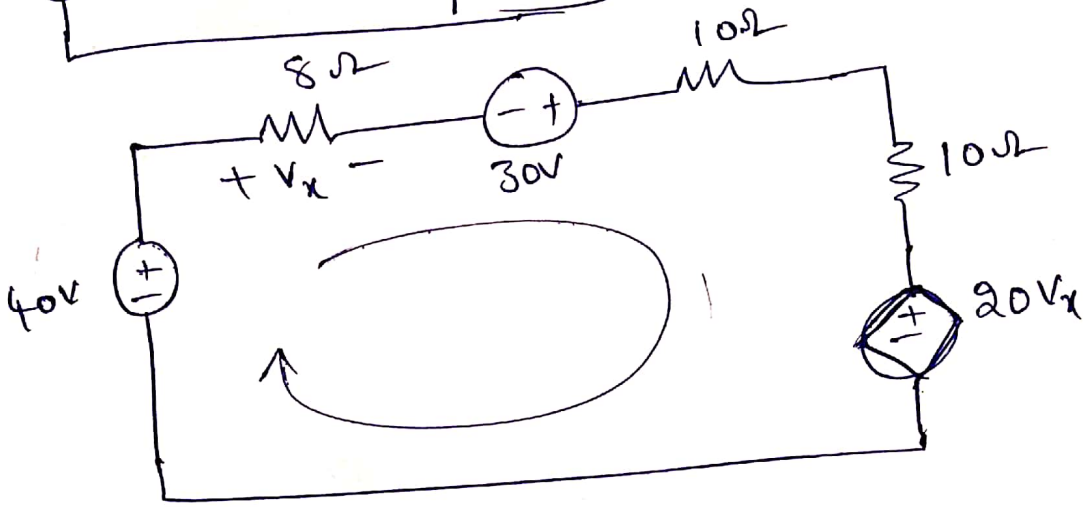
$$V_o = 8V$$

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* use Source transformation to find V_x



Sol



$V_x = 8i$

Sol

$$-40 + 8i - 30 + 10i + 10i + 20V_x = 0$$

$$-70 + 28i + 20 \times 8i = 0$$

$$28i + 160i = 70$$

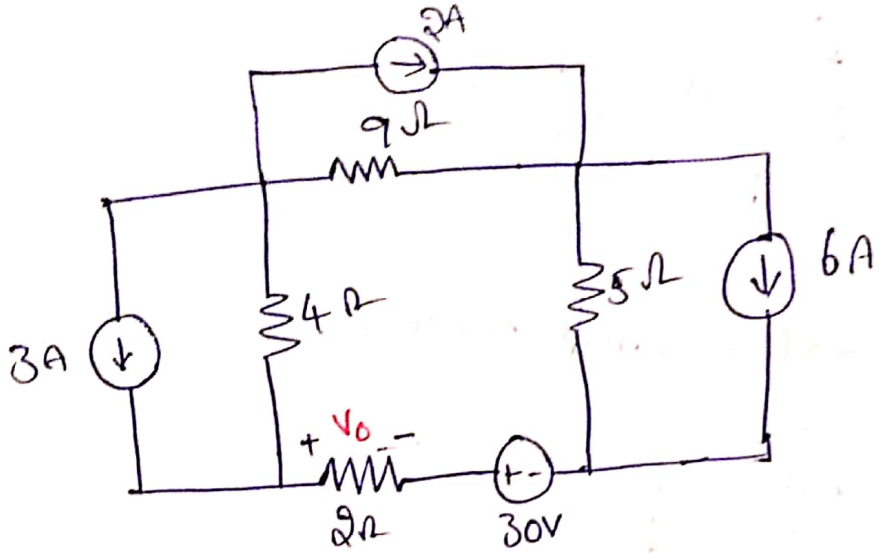
$$i = 0.3723$$

$$V_x = 8 \times 0.3723$$

$$= 2.978\,V$$

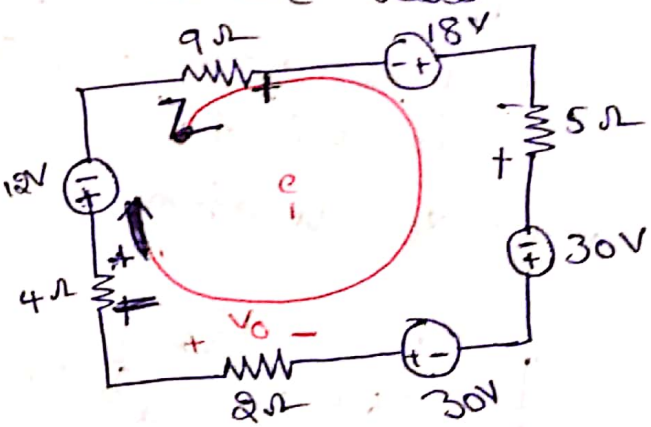
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* obtain V_o in the ckt. using source transformation



Sol

Transforming only the current source it gives the ckt below



① $\Rightarrow V = iR = 3 \times 4 = 12$

② $\Rightarrow V = iR = 2 \times 9 = 18$

③ $\Rightarrow V = iR = 5 \times 6 = 30$

Applying KVL to the loop

$$(4 + 9 + 5 + 2)i + 12 + 18 + 30 + 30 = 0$$

$$+20i + 66 = 0$$

$$20i = -66$$

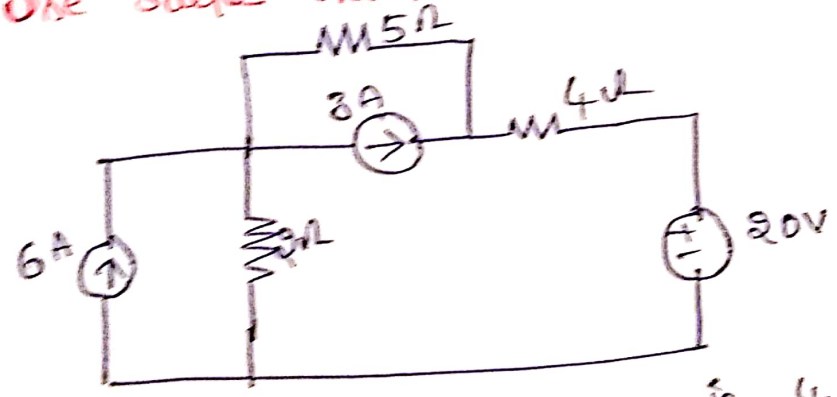
$$i = -66/20 = -3.3$$

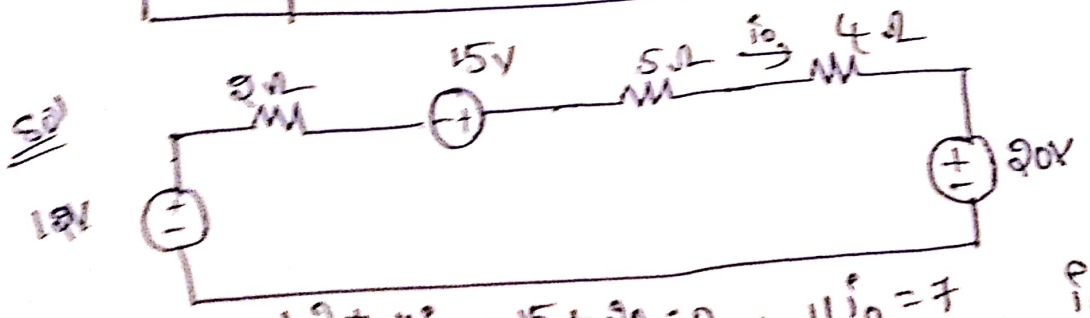
$$V_o = 2i$$

$$V_o = 2 \times -3.3 = -6.6V$$

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One source transformation to find i_o

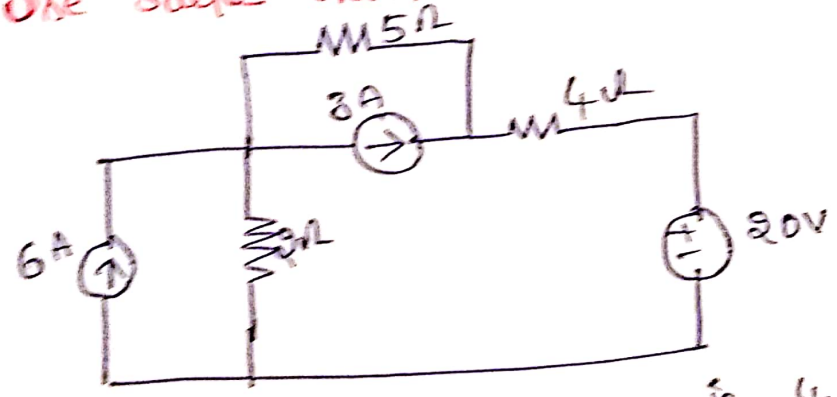


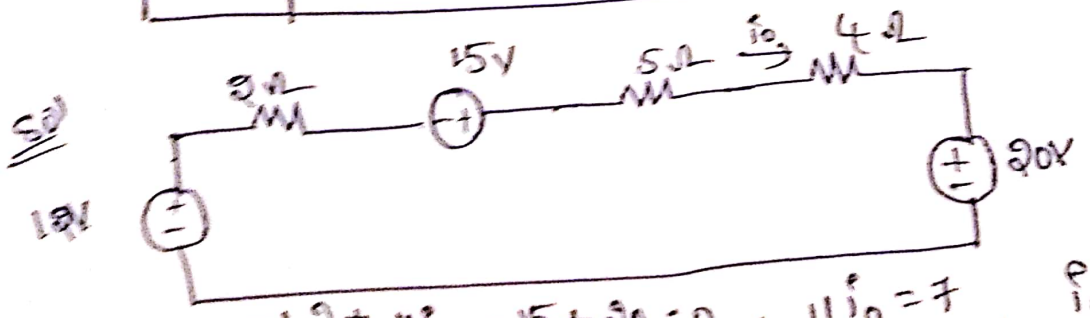


$-12 + 11 i_o - 15 + 20 = 0$, $11 i_o = 7$, $i_o = 636.4\,mA$

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One source transformation to find i_o

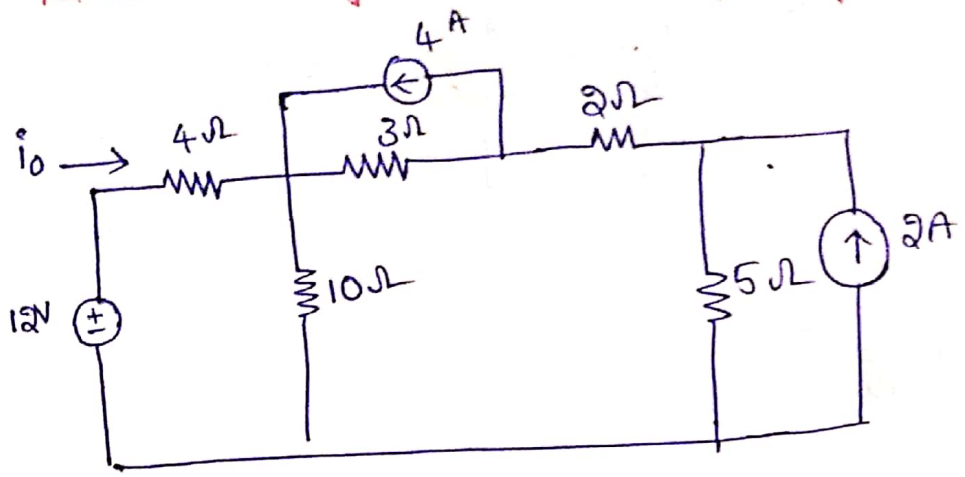




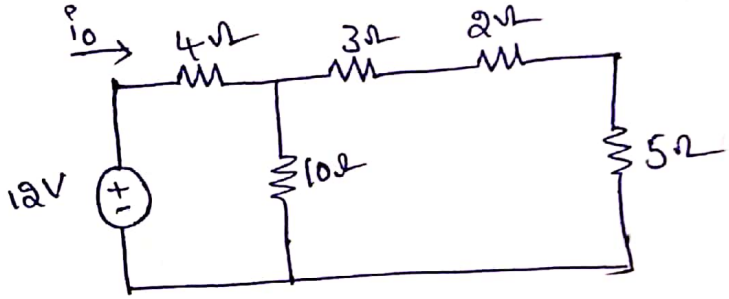
$-12 + 11 i_o - 15 + 20 = 0$, $11 i_o = 7$, $i_o = 636.4\,mA$

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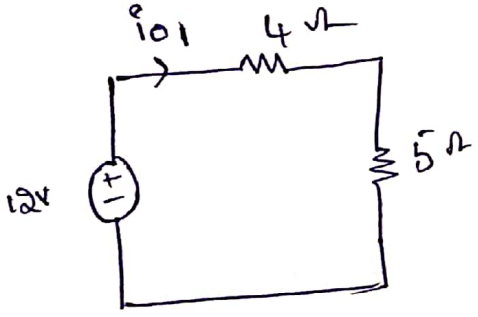
* For the following ckt find i_o by using superposition principle.



Sol 12V voltage source is active alone

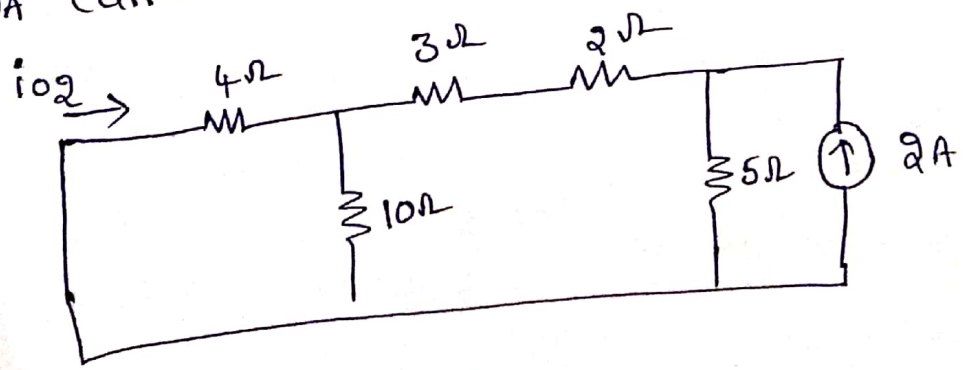


$$10\Omega \,//\, (3+2+5) = \frac{10\times 10}{10+10} = \frac{100}{20} = 5\Omega$$



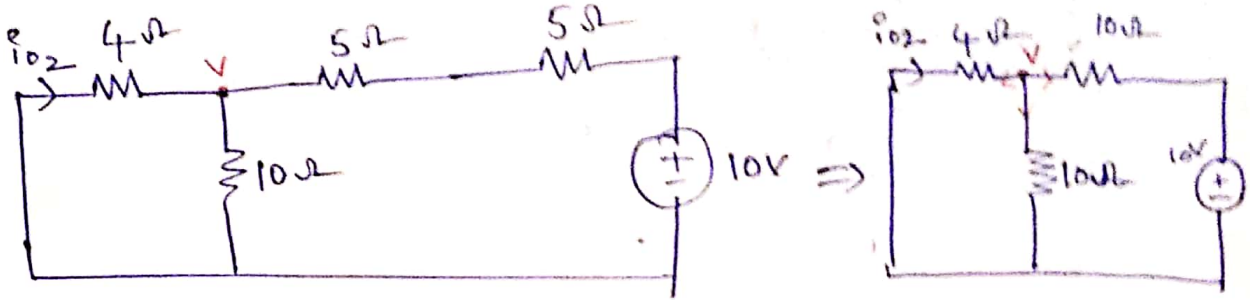
$$i_{o1} = \frac{V}{R} = \frac{12}{9}$$

2A Current Source is active alone



Scanned with CamScanner

Apply Source transformation



Apply KCL at node V

$$\frac{V}{4} + \frac{V}{10} + \frac{V-10}{10} = 0$$

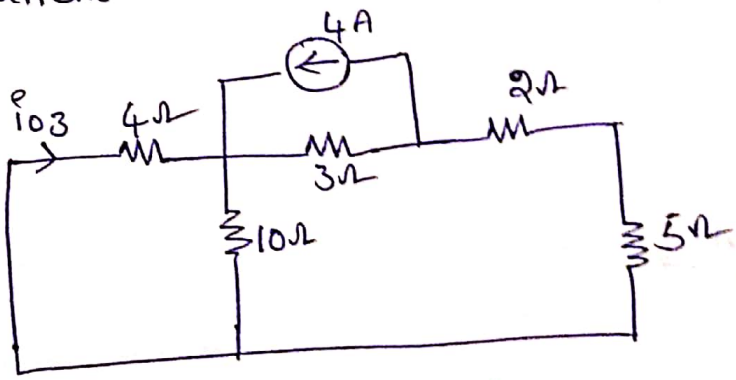
$$\frac{5V + 2V + 2V - 20}{20} = 0$$

$$9V = 20$$

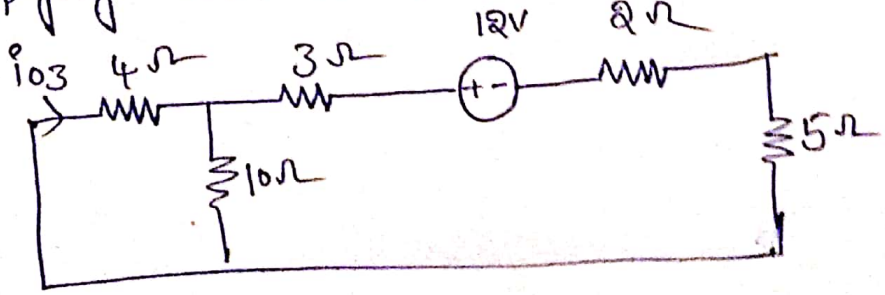
$$V = \frac{20}{9}$$

$$i_{o2} = \frac{V}{R} = \frac{-\frac{20}{9}}{4} = -\frac{5}{9}$$

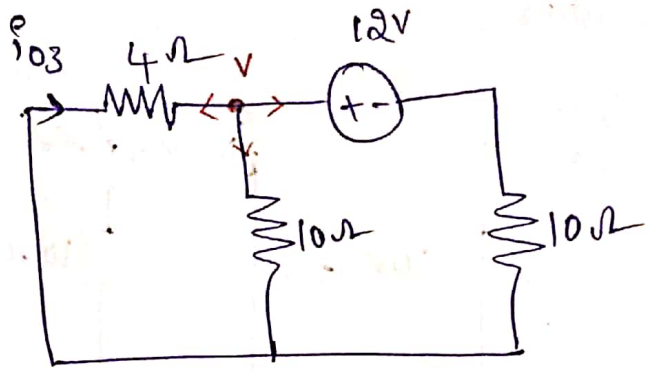
4A current source is active alone.



Applying Source transformation



Scanned with CamScanner



Apply KCL at node V

$$\frac{V}{4} + \frac{V}{10} + \frac{V-12}{10} = 0$$

$$5V + 2V + 2V - 24 = 0$$

$$9V = 24$$

$$V = 24/9$$

$$i_{03} = \frac{-24/9}{4} = -6/9$$

$$i_o = i_{01} + i_{02} + i_{03}$$

$$i_o = \frac{12}{9} - \frac{5}{9} - \frac{6}{9}$$

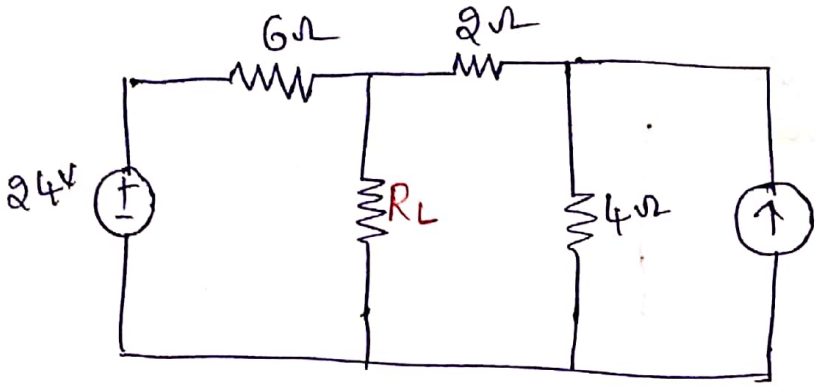
$$i_o = \frac{1}{9}$$

$$i_o = 0.1111$$

$$i_o = 111.1\ mA$$

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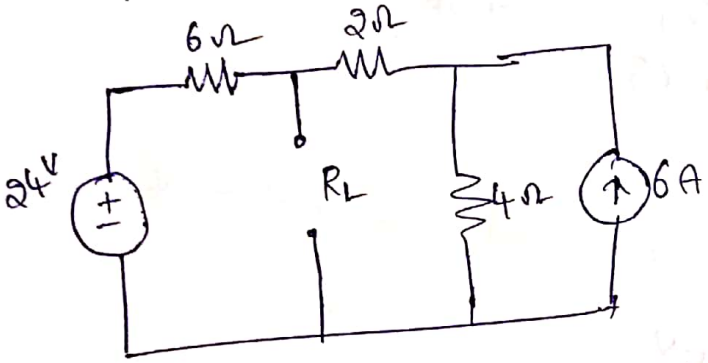
* find the maximum power transfered to R_L in the ckt?



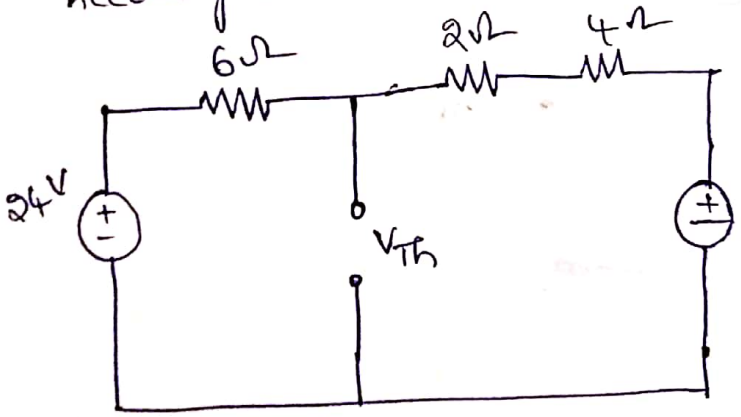
Sol

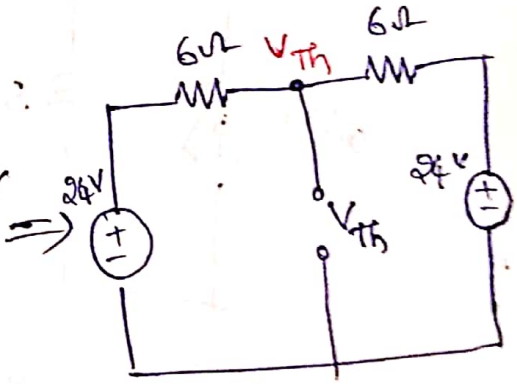
$$P_{max} = \frac{V_{Th}^2}{4 R_{Th}}$$

first remove R_L



According to source transformation (for reducing the ckt).





Apply KCL at node V_{Th}

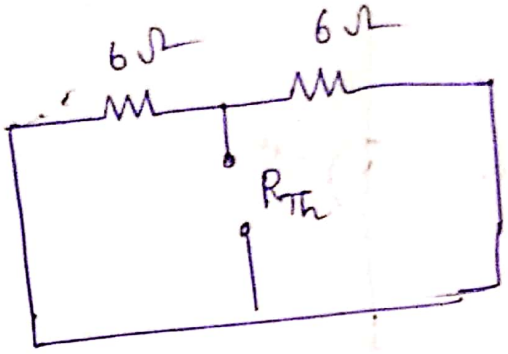
$$\frac{V_{Th} - 24}{6} + \frac{V_{Th} - 24}{6} = 0$$

$$2V_{Th} = 48$$

$$V_{Th} = 24\,V$$

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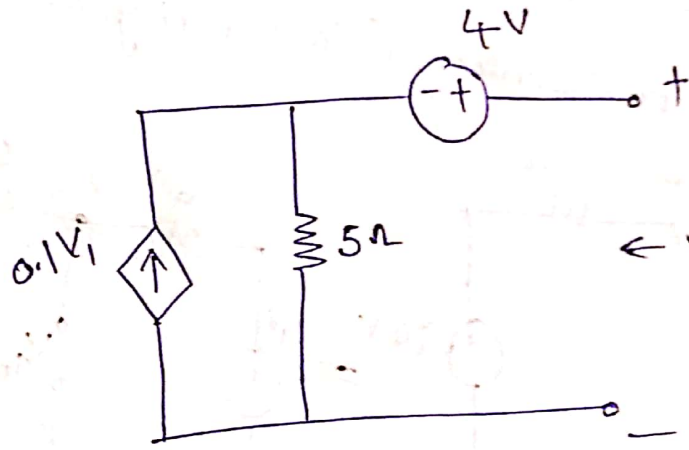
To find thevinien resistance short circuit the voltage sources



$$R_{Th} = \frac{R_1 R_2}{R_1 + R_2} = \frac{6 \times 6}{6+6} = 3\Omega$$

$$P_{max} = \frac{V_{Th}^2}{4 R_{Th}} = \frac{24^2}{4 \times 3} = 48W$$

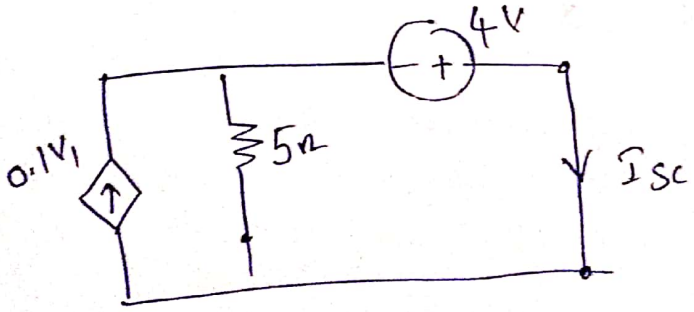
* In the following ckt find the value of V_{Th} & R_{Th}



Sol

$$V_1 = V_{Th} = V_{OC}$$

$$V_1 = 4 + 5 \times 0.1 V_1 = 8V$$



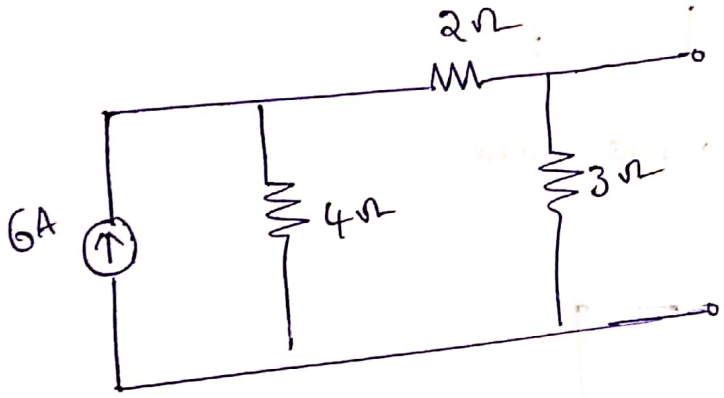
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$$i_{SC} = \frac{V}{R} = \frac{4}{5} = 0.8\ A$$

$$R_{TH} = \frac{V_{OC}}{i_{SC}} = \frac{8}{0.8} = 10\ \Omega$$

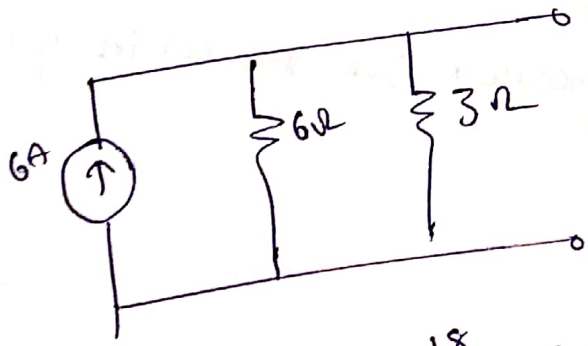
* In the following circuit find the value of i_N & R_N

Sol



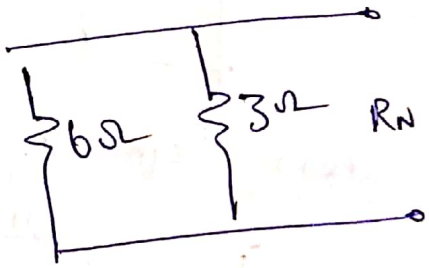
Sol

4Ω & 2Ω are in series

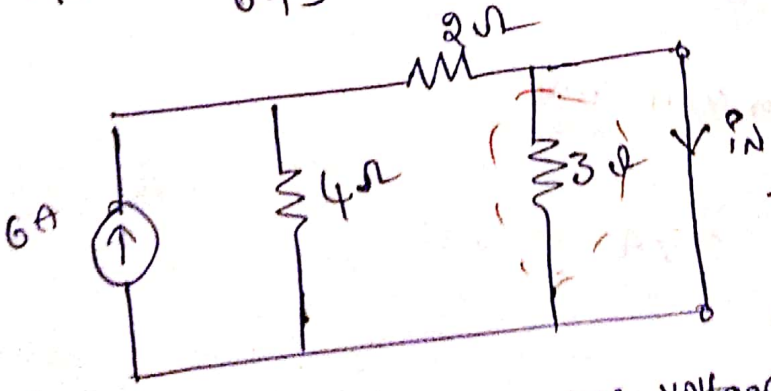


$\Rightarrow$

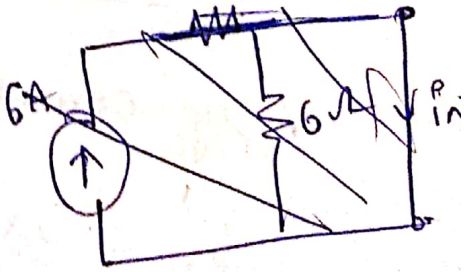
6A source is turn off



$$R_N = \frac{6 \times 3}{6+3} = \frac{18}{9} = 2\Omega$$



$\Rightarrow$



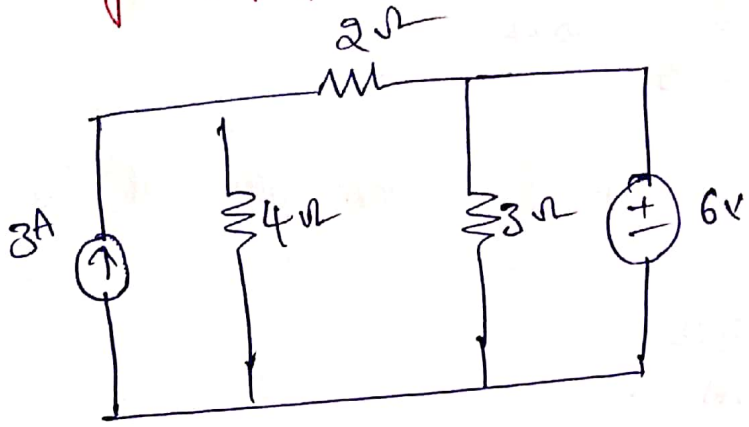
i_N & R_N

4Ω across voltage $= 6 \times 4 = 24\ V$

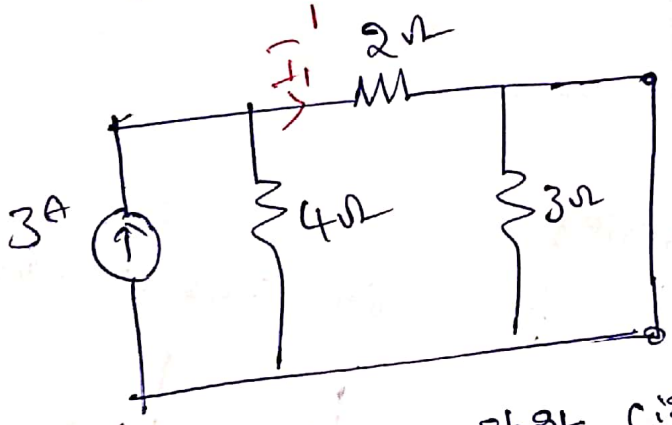
$$i_N = \frac{V}{R} = \frac{24}{4+2} = \frac{24}{6} = 4A$$

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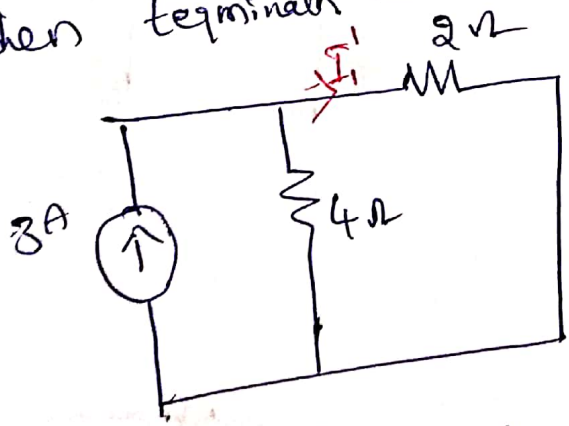
* Calculate the current flowing through 2Ω resistor using superposition principle.



Sol: 3A source is active alone



when terminals are short circuited 3Ω resistor is ignored.

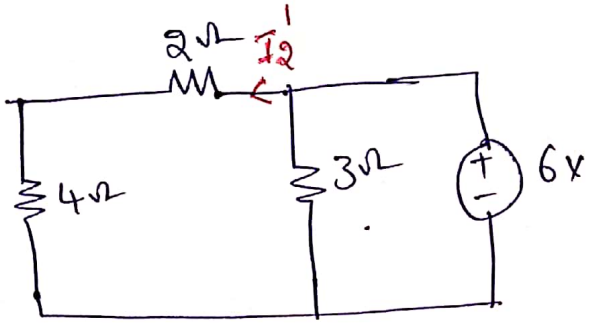


Apply current division rule

$$I_1' = 3 \times \frac{4}{2+4} = 2A$$

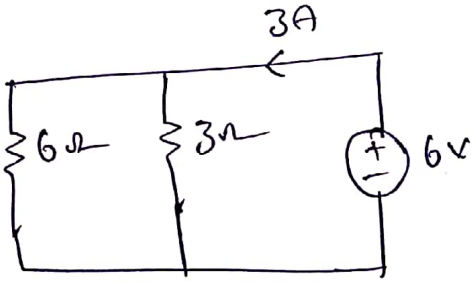
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6V voltage source is active alone



$$Req = \frac{6 \times 3}{6+3} = 2\Omega$$

$$I = \frac{V}{R} = \frac{6}{2} = 3A$$



Apply current division rule

$$I_2' = 3 \times \frac{3}{3+6} = 1A$$

when 3A source is active the current flowing in the 2Ω resistor is I_1'

when 6V source is active the current flowing in the 2Ω resistor is I_2'

$\therefore$ I_1' & I_2' are both are opposite direction

so the current flowing through the 2Ω resistor is

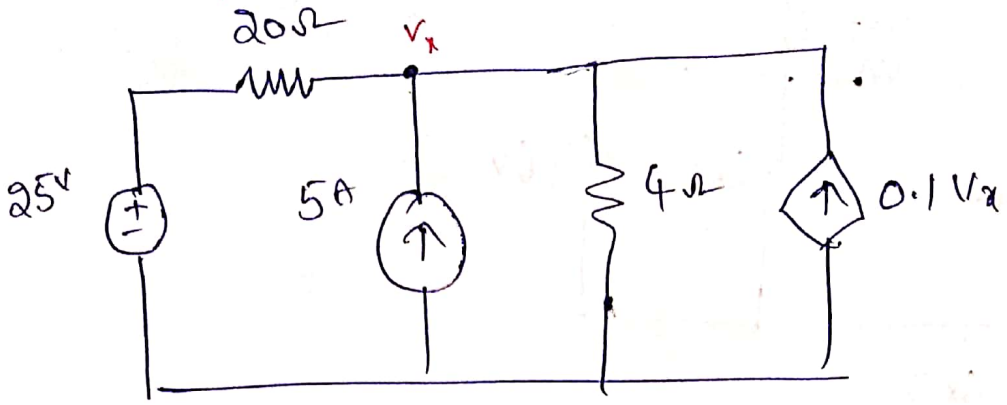
$$I = I_1' - I_2'$$

$$I = 2 - 1$$

$$\boxed{I = 1A}$$

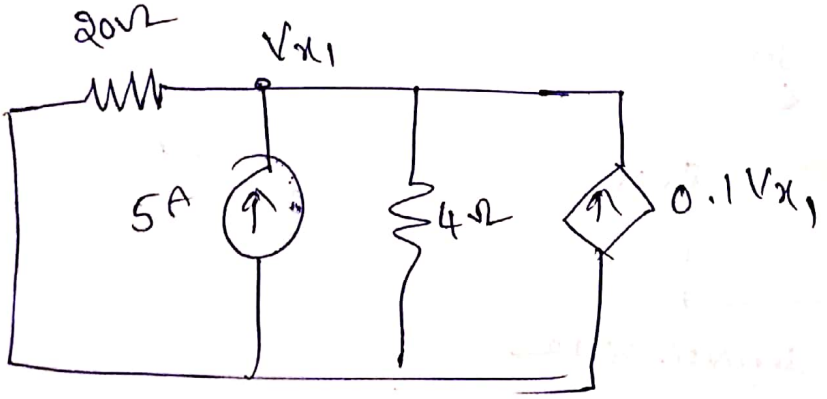
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Use Superposition theorem find V_x



Sol

Deactivate Voltage Source



$$\frac{V_{x1}}{20} + \frac{V_{x1}}{4} = 5 + 0.1V_{x1} \Rightarrow \frac{V_{x1}}{20} + \frac{V_{x1}}{4} = 5 + \frac{1}{10}V_{x1}$$

$$\frac{V_{x1} + 5V_{x1}}{20} = \frac{50 + V_{x1}}{10}$$

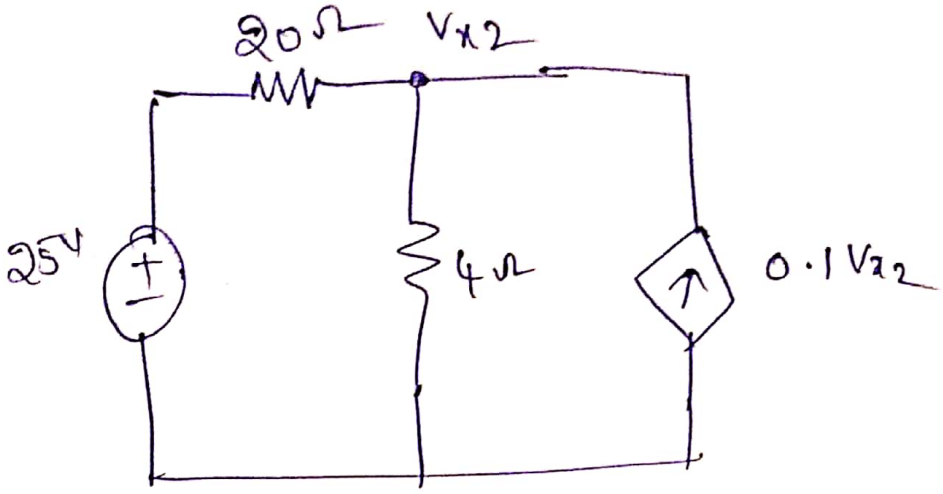
$$6V_{x1} = 100 + 2V_{x1}$$

$$4V_{x1} = 100$$

$$V_{x1} = 25$$

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Deactive 5A current source



$$\frac{V_{x2}-25}{20}+\frac{V_{x2}}{4}=\frac{1}{10}V_{x2}$$

$$\frac{V_{x2}-25+5V_{x2}}{20}=\frac{V_{x2}}{10}$$

$$6V_{x2}-25=2V_{x2}$$

$$4V_{x2}=25$$

$$V_{x2}=\frac{25}{4}=6.25$$

$$V_x=V_{x1}+V_{x2}=25+6.25=31.25V$$

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UNIT - III

AC Circuit Analysis

AC Circuit :-

The path for the flow of alternating current is called an AC circuit.

The alternating current is used for domestic and industrial purposes.

In an AC circuit the value of magnitude and direction of current & voltage is not constant. It changes at a regular interval of time.

The alternating voltage or alternating current travels as a sinusoidal wave completing one cycle.

The sinusoidal alternating quantities are voltage and current which varies according to the sin of angle i.e. $\theta = \omega t$.

For the generation of electric power in all over the world the sinusoidal voltage and current are selected because of the following reasons.

⇒ The sinusoidal voltage and current produce low iron and copper losses in the transformer and rotating electrical machines which is to improve the efficiency of A.C machines.

⇒ Alternating current produce less disturbance in the electrical circuits.

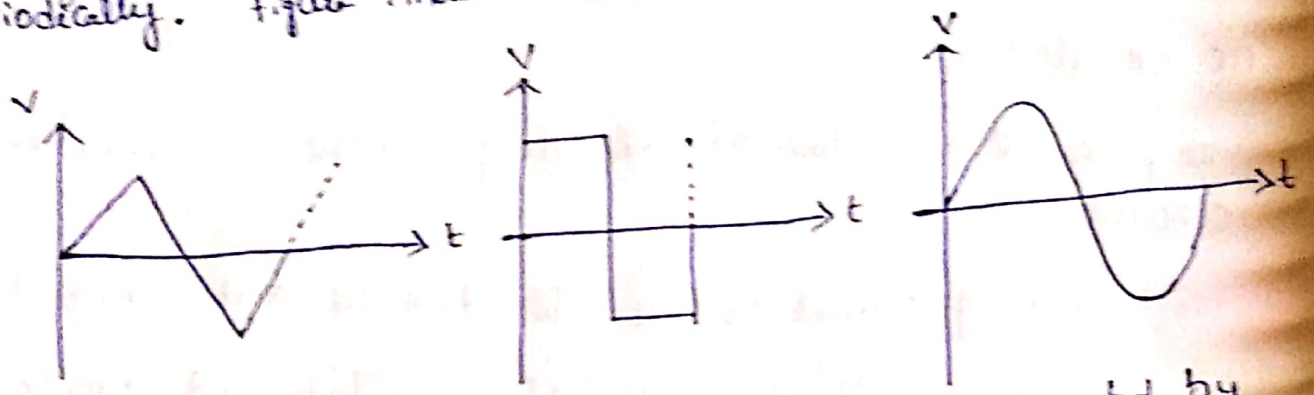
Alternating Voltage :- The voltage that changes its polarity and magnitude at regular intervals of time is called alternating voltage.

Alternating current :- The direction of current and magnitude of current is change with respect to time is called alternating current.

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Introduction

An alternating waveform changes its magnitude and direction periodically. figure shows various AC waveforms.



many times alternating voltages and currents are represented by a Sinusoidal waveforms.

A Sinusoidal voltage can be represented as

$$V = V_m \sin\theta$$
$$v = V_m \sin\omega t$$
$$v = V_m \sin 2\pi f t$$
$$v = V_m \sin \frac{2\pi}{T} t$$

waveform :- A waveform is a graph in which the instantaneous values of any quantity is plotted against time.

Cycle :- one complete set of positive and negative values of an alternating quantity is termed as cycle.

frequency :- The number of cycles per second of an alternating quantity is known as frequency. It is denoted by f and is expressed in hertz (Hz) or cycles per second (c/s).

The phase of an alternating quantity is defined as the fractional part of a cycle through which the quantity moves forward from a selected origin. when the two quantities have the same frequency and their max, min point achieve at the same point, then the quantities are said to have in the same phase.

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Time period :- The time taken by an alternating quantity to complete one cycle is called time period. It is denoted by T and is expressed in seconds.

$$T = \frac{1}{f}$$

Amplitude :- The maximum positive or negative value of an alternating quantity is called the amplitude.

phase :- The phase of an alternating quantity is the time that has elapsed since the quantity has last passed through zero point of reference. phase is an angle by which current leads by or lags behind the voltage.

phase difference :-

This term is used to compare the phases of two alternating quantities. Two alternating quantities are said to be in phase when they reach their maximum and zero values at the same time. Their maximum value may be different in magnitude.

A leading alternating quantity is one which reaches its maximum or zero value earlier as compared to the other quantity.

A lagging alternating is one which attains its maximum or zero value later than the other quantity.

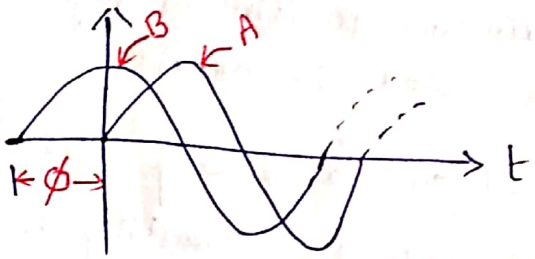
A plus (+) sign when used in connection with the phase difference denotes 'lead' whear a minus (−) sign denotes 'lag'.

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$$V_A = V_m \sin \omega t$$

$$V_B = V_m \sin(\omega t + \phi)$$

Here, quantity B leads A by a phase angle ϕ.



Root mean square value (RMS) or Effective value :-

The root mean square value of sine wave is the measure of the heating effect of wave when resistor is connected across the voltage source.

When a resistor is connected across the DC voltage source a certain amount of heat is produced in resistor in a given time. Similarly a resistor is connected across a AC voltage source, the same amount of heat is produced in that resistor.

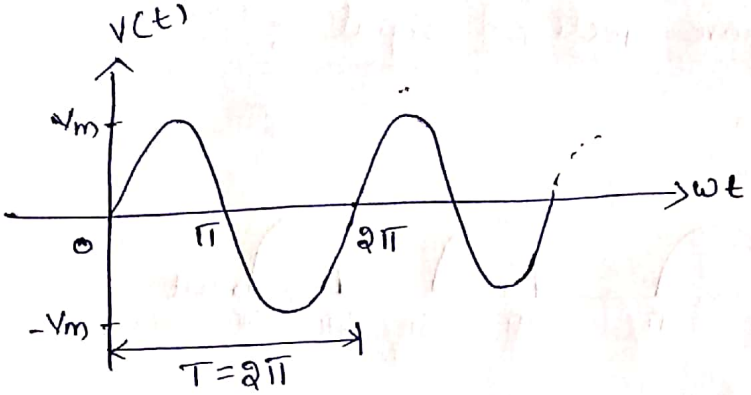
The RMS value of sinewave is equals to the DC voltage source.

The RMS value of any function with period T is given by

$$V_{rms} = \sqrt{\frac{1}{T} \int_0^T v^2(t)\, dt}$$

$$\text{RMS value} = \sqrt{\frac{\text{Area under the squared wave}}{\text{period}}}$$

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$$v(t) = V_m \sin\omega t$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} (V_m^2 \sin^2\omega t)\,d\omega t}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V_m^2 \left(\frac{1-\cos 2\omega t}{2}\right) d\omega t}$$

$$= \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{2\pi}\int_0^{2\pi} (1-\cos 2\omega t)\,d\omega t}$$

$$= \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{2\pi}\left[\omega t - \frac{\sin 2\omega t}{2}\right]_0^{2\pi}}$$

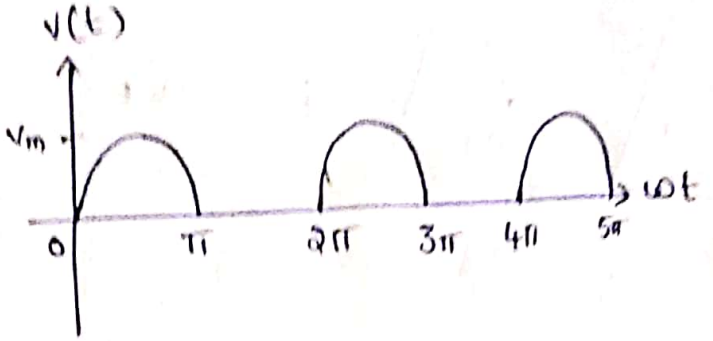
$$= \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{2\pi}\left[2\pi - \frac{\sin 4\pi}{2}\right]}$$

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

$$\boxed{V_{rms} = 0.707 V_m}$$

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For Halfwave rectified Signal :



$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} (V_m \sin\omega t)^2\,d\omega t}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} (V_m^2 \sin^2\omega t)\,d\omega t}$$

$$= \sqrt{\frac{1}{2\pi}\left[\int_0^{\pi} V_m^2 \sin^2\omega t\,d\omega t + \int_{\pi}^{2\pi} V_m^2 \sin^2\omega t\,d\omega t\right]}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{\pi} V_m^2 \sin^2\omega t\,d\omega t}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{\pi} V_m^2 \left(\frac{1-\cos 2\omega t}{2}\right) d\omega t}$$

$$= \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{2\pi}\int_0^{\pi}(1-\cos 2\omega t)\,d\omega t}$$

$$= \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{2\pi}\left[\omega t - \frac{\sin 2\omega t}{2}\right]_0^{\pi}}$$

$$= \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{2\pi}\left[\pi - \sin\pi\right]}$$

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$$= \frac{V_m}{\sqrt{2}} \sqrt{\frac{1}{2\pi}[\pi - 0]}$$

$$= \frac{V_m}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}}$$

$$\boxed{V_{rms} = \frac{V_m}{2}}$$

Average value of Half-rectified Signal

$$V_{avg} = \frac{1}{T}\int_0^T v(t)\,dt$$

$$= \frac{1}{2\pi}\int_0^{\pi} V_m \sin\omega t\, d\omega t$$

$$= \frac{V_m}{2\pi}\left[-\cos\omega t\right]_0^{\pi}$$

$$= \frac{V_m}{2\pi}\left[-\left[\cos\pi - \cos 0\right]\right]$$

$$= \frac{V_m}{2\pi}\left[-\left((-1) - (+1)\right)\right]$$

$$= \frac{V_m}{2\pi} \times 2 = \frac{V_m}{\pi}$$

mean or Avg

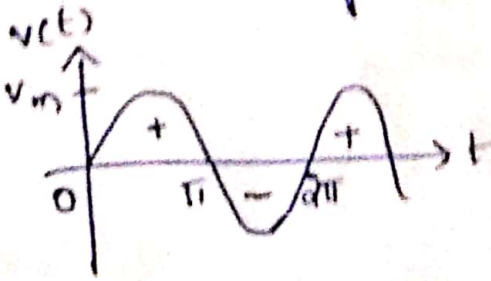
mean or Average value of A.C :-

It is that steady current which when passed through a circuit for half of the time period of the alternating current, sends the same amount of charge as is done by the alternating current in the same time through the same circuit.

It is defined as that value of steady current (DC current) which would generate same amount of charge as it generated by alternating current in the same circuit for half time period.

Average value = DC value.

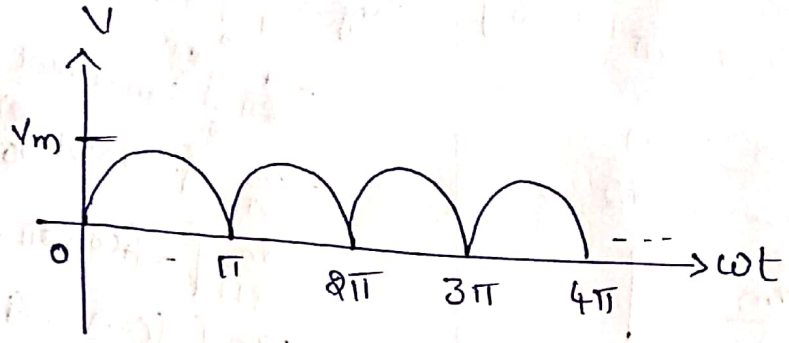
$$X_{avg} = X_{dc} = \frac{1}{T}\int_0^T x(t)\,dt$$



Average value = $\frac{\text{Area under the curve}}{\text{period}}$

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for full rectified signal:



$V(t) = V_m \sin\omega t$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T V^2(t)\,dt}$$

$$= \sqrt{\frac{1}{\pi}\int_0^{\pi} V_m^2 \sin^2\omega t\,d\omega t}$$

$$= V_m\sqrt{\frac{1}{\pi}\int_0^{\pi}\left(\frac{1-\cos 2\omega t}{2}\right)d\omega t}$$

$$= \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{\pi}\int_0^{\pi}(1-\cos 2\omega t)\,d\omega t}$$

$$= \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{\pi}\left(\omega t - \frac{\sin 4\omega t}{2}\right)_0^{\pi}}$$

$$= \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{\pi}\left(\pi - \sin 2\pi\right)}$$

$$= \frac{V_m}{\sqrt{2}}\sqrt{\frac{1}{\pi}(\pi - 0)}$$

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

$$\boxed{V_{rms} = 0.707\,V_m}$$

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Average value of full rectified signal.

$$V(t) = V_m \sin\omega t$$

$$V_{avg} = \frac{1}{T}\int_0^T v(t)\,dt$$

$$= \frac{1}{\pi}\int_0^{\pi} V_m \sin\omega t\, d\omega t$$

$$= \frac{V_m}{\pi}\left[-\cos\omega t\right]_0^{\pi} = \frac{V_m}{\pi}\left[-\cos\pi - \cos 0\right]$$

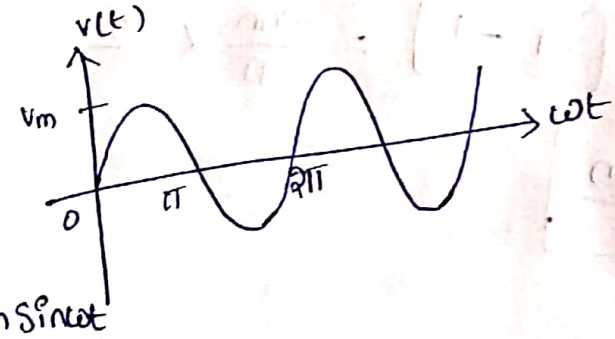
$$= \frac{V_m}{\pi}\left[-\{\cos\pi - \cos 0\}\right] = \frac{V_m}{\pi}\left[-\{(-1) - (+1)\}\right]$$

$$= \frac{V_m}{\pi}\left[-(-1) + (+1)\right]$$

$$= \frac{V_m}{\pi}\left[1 + 1\right]$$

$$\boxed{V_{avg} = \frac{2V_m}{\pi}}$$

Average value of full Sine wave:



$$V(t) = V_m \sin\omega t$$

$$V_{avg} = \frac{1}{T}\int_0^{T} v(t)\,dt$$

$$= \frac{1}{2\pi}\int_0^{2\pi} V_m \sin\omega t\, d\omega t$$

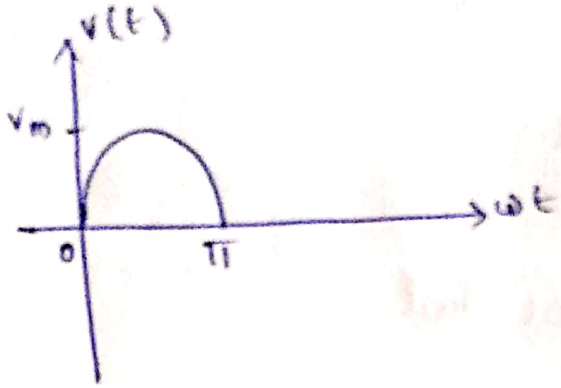
$$= \frac{V_m}{2\pi}\int_0^{2\pi} \sin\omega t\, d\omega t = \frac{V_m}{2\pi}\left[-\cos\omega t\right]_0^{2\pi}$$

$$= \frac{-V_m}{2\pi}\left(\cos 2\pi - \cos 0\right) = -\frac{V_m}{\pi}\left[1 - 1\right]$$

$$\boxed{V_{avg} = 0}$$

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Average value of Half Sine wave:



$$v(t) = V_m \sin\omega t$$

$$V_{avg} = \frac{1}{T}\int_0^T v(t)\,dt$$

$$= \frac{1}{\pi}\int_0^{\pi} V_m \sin\omega t\, d\omega t$$

$$= \frac{V_m}{\pi}\int_0^{\pi} \sin\omega t\, d\omega t$$

$$= \frac{V_m}{\pi}\left[-\cos\omega t\right]_0^{\pi}$$

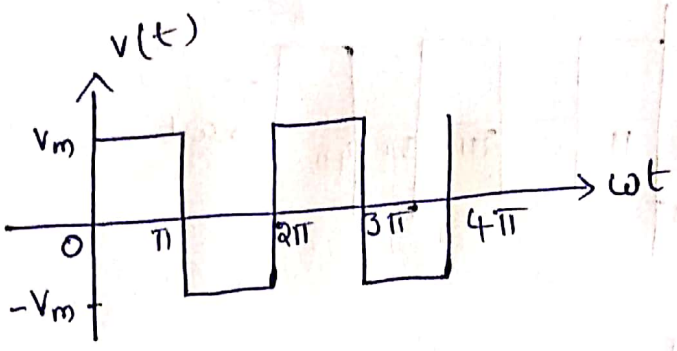
$$= \frac{-V_m}{\pi}\left[\cos\pi - \cos 0\right]$$

$$= \frac{-V_m}{\pi}\left[-1-1\right] = \frac{-V_m}{\pi} \times -2$$

$$\boxed{V_{avg} = \frac{2V_m}{\pi}}$$

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RMS value of Square wave



$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$$

$$= \sqrt{\frac{1}{2\pi}\left[\int_0^{2\pi} v^2(t)\,dt\right]}$$

$$= \sqrt{\frac{1}{2\pi}\left[\int_0^{\pi} V_m^2\,d\omega t + \int_{\pi}^{2\pi}(-V_m)^2\,d\omega t\right]}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\left[(\omega t)_0^{\pi} + (\omega t)_{\pi}^{2\pi}\right]}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\left[(\pi - 0) + (2\pi - \pi)\right]}$$

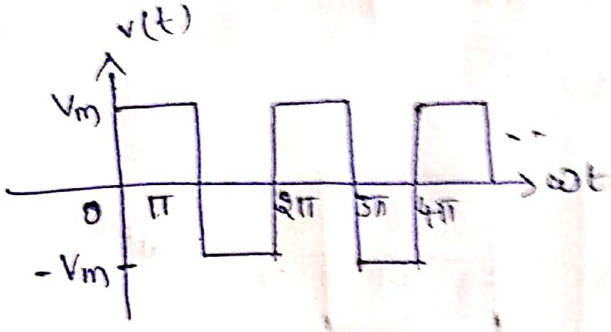
$$V_{rms} = \sqrt{\frac{V_m^2}{2\pi}(\pi + \pi)}$$

$$V_{rms} = \sqrt{\frac{V_m^2}{2\pi} \times 2\pi}$$

$$\boxed{V_{rms} = V_m}$$

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Average value of square wave



$$V_{avg} = \frac{1}{T}\int_0^T v(t)\,dt$$

$$= \frac{1}{2\pi}\int_0^{2\pi} v(t)\,d\omega t$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} V_m\,d\omega t + \int_{\pi}^{2\pi} -V_m\,d\omega t\right]$$

$$= \frac{V_m}{2\pi}\int_0^{\pi} d\omega t - \frac{V_m}{2\pi}\int_{\pi}^{2\pi} d\omega t$$

$$= \frac{V_m}{2\pi}\left[\omega t\right]_0^{\pi} - \frac{V_m}{2\pi}\left[\omega t\right]_{\pi}^{2\pi}$$

$$= \frac{V_m}{2\pi}(\pi - 0) - \frac{V_m}{2\pi}(2\pi - \pi)$$

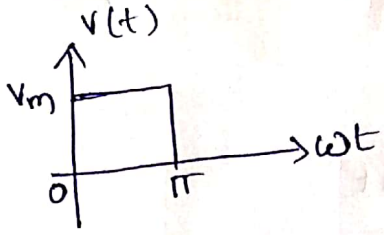
$$= \frac{V_m}{2} - \frac{V_m}{2\pi}(\pi)$$

$$= \frac{V_m}{2} - \frac{V_m}{2}$$

$$\boxed{V_{avg} = 0}$$

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Average value of Half Square wave



$$V_{avg} = \frac{1}{T}\int_0^T v(t)\,dt$$

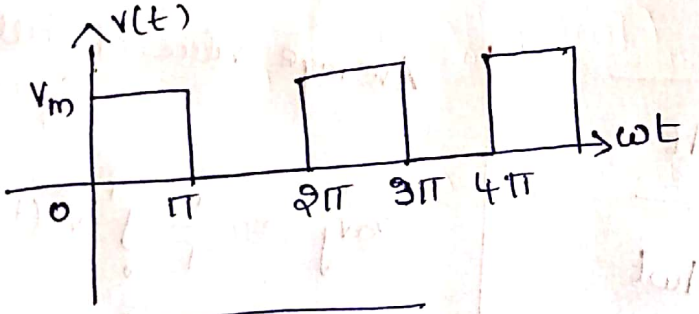
$$= \frac{1}{\pi}\int_0^{\pi} V_m\,d\omega t$$

$$= \frac{V_m}{\pi}\left(\omega t\right)_0^{\pi}$$

$$= \frac{V_m}{\pi}(\pi - 0)$$

$$\boxed{V_{avg} = V_m}$$

RMS value of Half rectified square wave:



π to $2\pi \Rightarrow V_m$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V_m^2\,d\omega t}$$

$$= \sqrt{\frac{1}{2\pi}\left(\int_0^{\pi} V_m^2\,d\omega t + \int_{\pi}^{2\pi} V_m^2\,d\omega t\right)}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\int_0^{\pi} d\omega t} = \sqrt{\frac{V_m^2}{2\pi}(\pi - 0)}$$

$$\boxed{V_{rms} = \frac{V_m}{\sqrt{2}}}$$

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Average value of Half-rectified Square wave

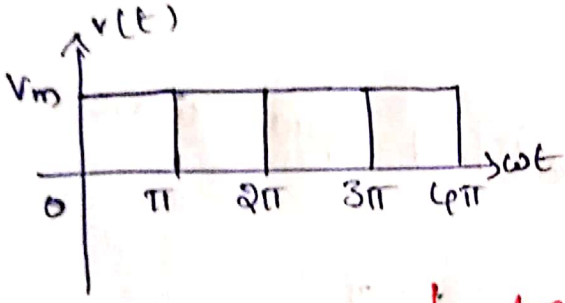
$$V_{avg} = \frac{1}{T}\int_0^T v(t)\,dt$$

$$= \frac{1}{2\pi}\int_0^{\pi} V_m\,d\omega t$$

$$= \frac{V_m}{2\pi}(\pi - 0)$$

$$\boxed{V_{avg} = \frac{V_m}{2}}$$

RMS value of full-rectified square wave



$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$$

$$= \sqrt{\frac{1}{\pi}\int_0^{\pi} V_m^2\,d\omega t}$$

$$= \sqrt{\frac{V_m^2}{\pi}(\pi - 0)}$$

$$\boxed{V_{rms} = V_m}$$

Average value of full-rectified Square wave

$$V_{avg} = \frac{1}{T}\int_0^T v^2(t)\,dt$$

$$= \frac{1}{\pi}\int_0^{\pi} V_m\,d\omega t$$

$$= \frac{V_m}{\pi}(\pi - 0)$$

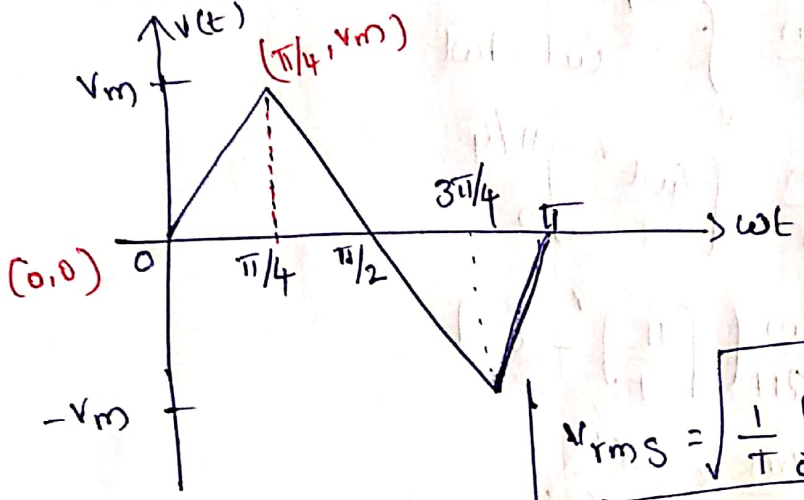
$$\boxed{V_{avg} = V_m}$$

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RMS Value of triangular wave :-

Consider a triangular voltage waveform that is bi-polar, has a 50% duty-cycle (symmetrical about V=0) and a frequency f. let the maximum voltage is V_m and the minimum voltage is $-V_m$.

Because of the symmetry, integrate from $0 - \pi/4$ to calculate RMS value.



(0,0) $(\pi/4, V_m)$

$x_1\ y_1 \quad x_2\ y_2$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{V_m - 0}{\frac{\pi}{4} - 0} = \frac{4V_m}{\pi}$$

$$y - y_1 = m(x - x_1)$$

$$V(t) - 0 = \frac{4V_m}{\pi}(\omega t - 0)$$

$$V(t) = \frac{4V_m}{\pi} \times \omega t$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$$

$$= \sqrt{\frac{1}{\pi/4}\int_0^{\pi/4}\left(\frac{4V_m}{\pi} \times \omega t\right)^2 d\omega t}$$

$$= \sqrt{\frac{1}{\frac{\pi}{4}}\int_0^{\pi/4}\left(\frac{16V_m^2}{\pi^2} \times \omega t^2\right) d\omega t}$$

$$= \sqrt{\frac{4}{\pi} \times \frac{16V_m^2}{\pi^2}\left(\frac{\omega t^3}{3}\right)_0^{\pi/4}}$$

$$= \sqrt{\frac{4}{\pi} \times \frac{16V_m^2}{3\pi^2}(\pi/4)^3}$$

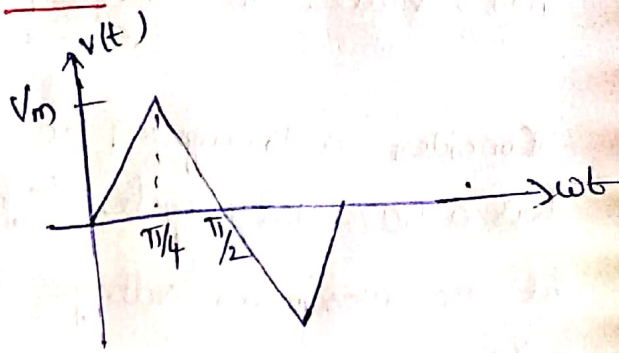
$$= \sqrt{\frac{4}{\pi} \times \frac{16V_m^2}{3\pi^2} \times \frac{\pi^3}{64} \times 4}$$

$$= \sqrt{\frac{4}{\pi} \times \frac{V_m^2}{3} \times \frac{\pi}{4}} = \frac{V_m}{\sqrt{3}}$$

$$V_{rms} = \frac{V_m}{\sqrt{3}}$$

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Average value of triangular wave :-



$$V_{avg} = \frac{1}{T}\int_0^T v(t)\,dt$$

$$v(t) = \frac{4V_m}{\pi}\times\omega t$$

$$V_{avg} = \frac{1}{\pi/4}\int_0^{\pi/4}\frac{4V_m}{\pi}\,\omega t\,d\omega t$$

$$= \frac{4}{\pi}\,\frac{4V_m}{\pi}\int_0^{\pi/4}\omega t\,d\omega t$$

$$= \frac{4}{\pi}\,\frac{4V_m}{\pi}\left[\frac{\omega t^2}{2}\right]_0^{\pi/4}$$

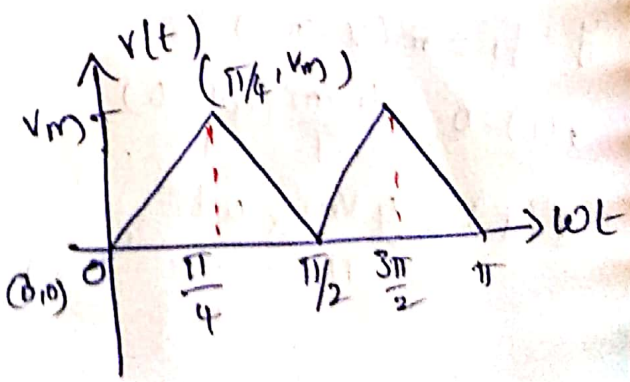
$$= \frac{4}{\pi}\times\frac{4V_m}{2\pi}\left(\left[\frac{\pi}{4}\right]^2 - 0\right)$$

$$= \frac{4}{\pi}\times\frac{4V_m}{2\pi}\times\frac{\pi^2}{16}$$

$$= \frac{4}{\pi}\times\frac{4V_m}{2\pi}\times\frac{\pi^2}{16}$$

$$\boxed{V_{avg} = \frac{V_m}{2}}$$

full rectified triangular signal

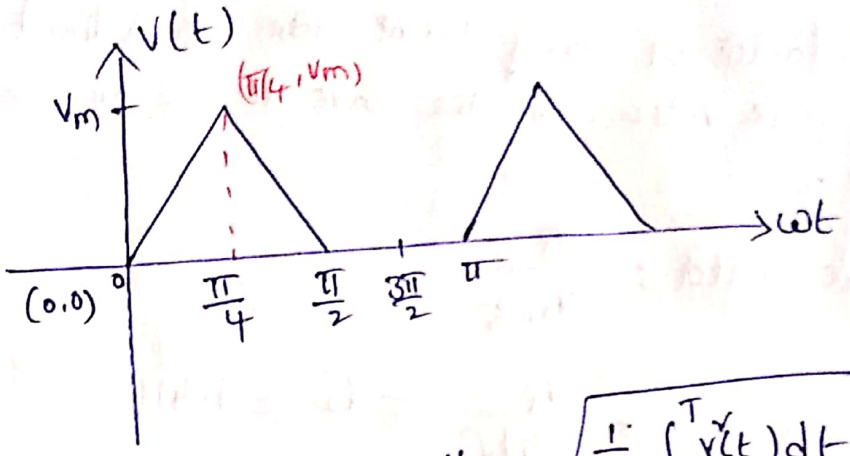


$$\boxed{V_{rms} = \frac{V_m}{\sqrt{3}}}$$

$$\boxed{V_{avg} = \frac{V_m}{2}}$$

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RMS value of Half rectified Triangular wave :-



$(0,0)\ (\frac{\pi}{4}, V_m)$

$x_1\ y_1 \quad x_2\ y_2$

$$m = \frac{V_m - 0}{\frac{\pi}{4} - 0} = \frac{4V_m}{\pi}$$

$$y - y_1 = m(x - x_1)$$

$$v(t) - 0 = \frac{4V_m}{\pi}(\omega t - 0)$$

$$v(t) = \frac{4V_m}{\pi}\omega t$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$$

$$= \sqrt{\frac{1}{\pi/2}\int_0^{\pi/4}\left(\frac{4V_m}{\pi}\times\omega t\right)^2 d\omega t}$$

$$= \sqrt{\frac{2}{\pi}\,\frac{16V_m^2}{\pi^2}\int_0^{\pi/4}\omega t^2\,d\omega t}$$

$$= \sqrt{\frac{2}{\pi}\cdot\frac{16V_m^2}{\pi^2}\left[\frac{\omega t^3}{3}\right]_0^{\pi/4}}$$

$$= \sqrt{\frac{2}{\pi}\times\frac{16V_m^2}{3\pi^2}\left[\frac{\pi}{4}\right]^3}$$

$$= \sqrt{\frac{2}{\pi}\times\frac{16V_m^2}{3\pi^2}\times\frac{\pi^3}{64}}$$

$$V_{rms} = \sqrt{\frac{V_m^2}{6}}$$

$$\boxed{V_{rms} = \frac{V_m}{\sqrt{6}}}$$

Average value of half-rectified triangular wave :-

$$V_{avg} = \frac{1}{T}\int_0^T v(t)\,dt$$

$$= \frac{1}{\pi/2}\int_0^{\pi/4}\frac{4V_m}{\pi}\omega t\,d\omega t$$

$$= \frac{2}{\pi}\,\frac{4V_m}{\pi}\left[\frac{\omega t^2}{2}\right]_0^{\pi/4} = \frac{2}{\pi}\times\frac{4V_m}{2\pi}\left[\frac{\pi}{4}\right]^2$$

$$= \frac{2}{\pi}\times\frac{4V_m}{2\pi}\times\frac{\pi^2}{16} = \frac{V_m}{4}$$

$$\boxed{V_{avg} = \frac{V_m}{4}}$$

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Peak factor :- (V_p)

The peak factor of any wave form is defined as the ratio of peak value of the wave to the rms value of the wave.

$$\text{Peak factor} = \frac{V_p}{V_{rms}}$$

$$= \frac{V_p}{V_p/\sqrt{2}} = \sqrt{2} = 1.414$$

form factor :- form factor is defined as the ratio of RMS value ~~of the~~ to the average value of the wave.

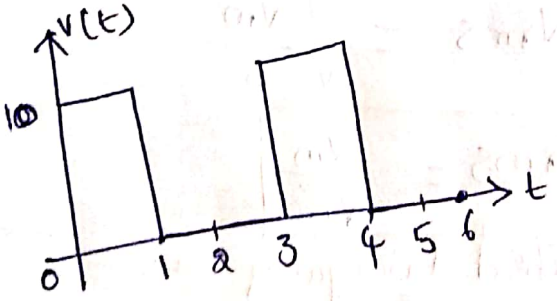
$$\text{form factor} = \frac{V_{rms}}{V_{avg}} = \frac{V_p/\sqrt{2}}{0.637 V_p}$$

$$= \frac{1/\sqrt{2}}{0.637}$$

$$= \frac{1}{\sqrt{2} \times 0.637}$$

$$= 1.11$$

* Determine the RMS value of voltage in the below fig-



Sol

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$$

$$= \sqrt{\frac{1}{3}\int_0^1 (10)^2\,dt}$$

$$= \sqrt{\frac{100}{3}\int_0^1 1\cdot dt}$$

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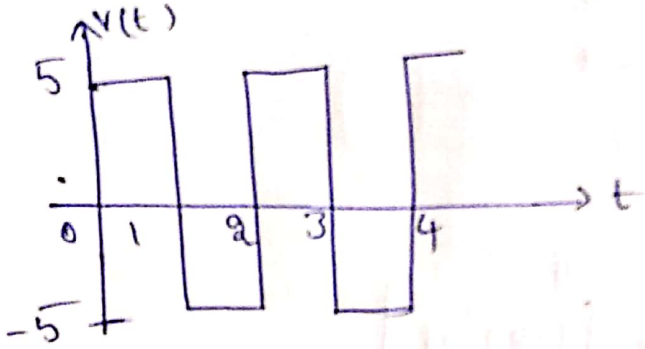
$$V_{rms} = \sqrt{\frac{100}{3}[1-0]}$$

$$= \sqrt{\frac{100}{3}}$$

$$V_{rms} = \sqrt{33.333}$$

$$\boxed{V_{rms} = 5.77}$$

* Determine the RMS value of the wave-form.



So!

$$V(t) = \begin{cases} 5 & 0<t<1 \\ -5 & 1<t<2 \end{cases}$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T V^2(t)\,dt}$$

$$= \sqrt{\frac{1}{2}\left(\int_0^1 (5)^2\,dt + \int_1^2 (-5)^2\,dt\right)}$$

$$= \sqrt{\frac{1}{2}\left(\int_0^1 25\,dt + \int_1^2 25\,dt\right)}$$

$$= \sqrt{\frac{25}{2}[1-0] + \frac{25}{2}[2-1]}$$

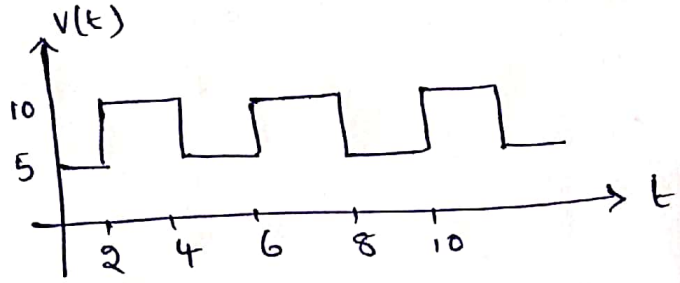
$$= \sqrt{\frac{25}{2} + \frac{25}{2}} = \sqrt{\frac{50}{2}}$$

$$= \sqrt{25}$$

$$V_{rms} = 5V$$

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* find the effective value of the voltage



Sol

$T = 4$

$$v(t) = \begin{cases} 5, & 0 < t < 2 \\ 10, & 2 < t < 4 \end{cases}$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$$

$$= \sqrt{\frac{1}{4}\left(\int_0^2 (5)^2\,dt + \int_2^4 (10)^2\,dt\right)}$$

$$= \sqrt{\frac{1}{4}\left(\int_0^2 25\,dt + \int_2^4 100\,dt\right)}$$

$$= \sqrt{\frac{25}{4}[2-0] + \frac{100}{4}[4-2]}$$

$$= \sqrt{\frac{25}{42} \times 2 + \frac{100}{42} \times 2}$$

$$= \sqrt{\frac{25}{2} + \frac{100}{2}}$$

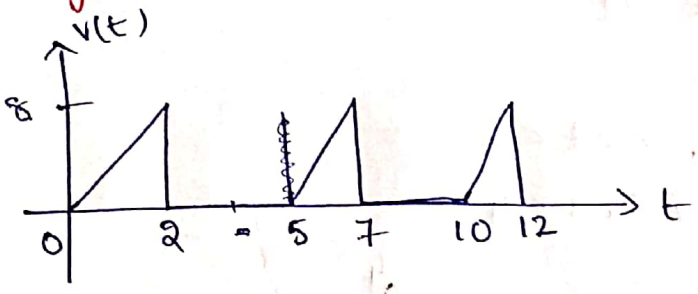
$$= \sqrt{\frac{125}{2}}$$

$$V_{rms} = \sqrt{62.5}$$

$$\boxed{V_{rms} = 7.905\ V}$$

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* Calculate the rms value of the voltage and average power through 2Ω resistor.



Sol

$T = 5$

$$v(t) = \begin{cases} 4t & 0 < t < 2 \\ 0 & 2 < t < 5 \end{cases}$$

$$v_{rms} = \sqrt{\frac{1}{5}\left[\int_0^2 (4t)^2 dt + \int_2^5 0 \, dt\right]}$$

$$= \sqrt{\frac{1}{5}\int_0^2 16t^2 dt}$$

$$= \sqrt{\frac{16}{5}\left[\frac{t^3}{3}\right]_0^2}$$

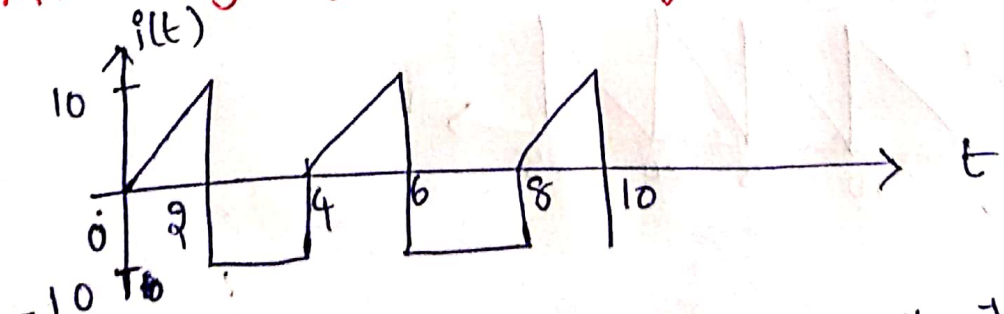
$$= \sqrt{\frac{16}{15}[8]} = \sqrt{\frac{128}{15}} = \sqrt{8.533}$$

$$v_{rms} = 2.92$$

Average power through 2Ω resistor is

$$P = \frac{V_{rms}^2}{R} = \frac{8.533}{2} = 4.267 W$$

* find i_{rms} & power through 2Ω resistor.



Sol

$T = 4$

$(0, 0)$ $(2, 10)$

x_1, y_1 x_2, y_2

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10 - 0}{2 - 0} = 5$$

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$$y - y_1 = m(x - x_1)$$

$$i(t) - 0 = 5(t - 0)$$

$$i(t) = 5t$$

$$i(t) = \begin{cases} 5t, & 0 < t < 2 \\ -10, & 2 < t < 4 \end{cases}$$

$$I_{rms} = \sqrt{\frac{1}{4}\int_0^2 (5t)^2 dt + \int_2^4 (-10)^2 dt}$$

$$= \sqrt{\frac{25}{4}\left[\frac{t^3}{3}\right]_0^2 + \frac{100}{4}\left[t\right]_2^4}$$

$$= \sqrt{\frac{25}{12}\left[8 - 0\right] + \frac{100}{4}\left[4 - 2\right]}$$

$$= \sqrt{\frac{25}{3} \times 8 + \frac{100}{4} \times 2}$$

$$= \sqrt{\frac{50}{3} + \frac{100}{2}} \quad \therefore = \sqrt{\frac{50}{3} + \frac{50}{2}}$$

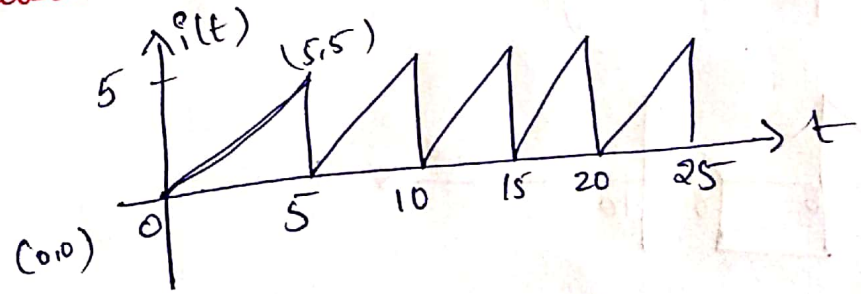
$$= \sqrt{16.66 + 25}$$

$$= \sqrt{41.66} = 6.454$$

Power through 2Ω resistor

$$P = I_{rms}^2 \times R = (6.454)^2 \times 2 = 83.308\ W$$

* Calculate the RMS value of current waveform.



$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 0}{5 - 0} = 5/5 = 1$$

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$y - y_1 = m(x - x_1)$

$i(t) - 0 = 1(t - 0)$

$i(t) = t$

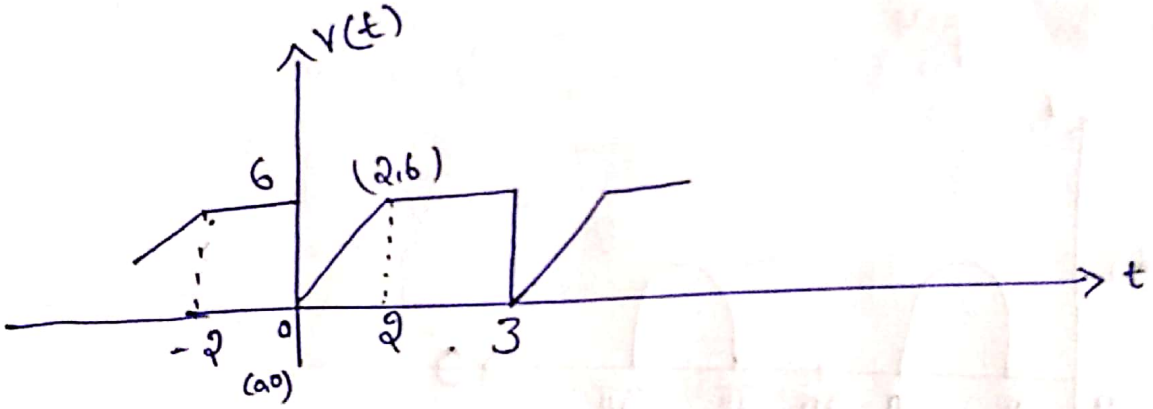
$$i_{rms} = \sqrt{\frac{1}{5}\int_0^5 (t^2)\,dt}$$

$$= \sqrt{\frac{1}{5}\left[\frac{t^3}{3}\right]_0^5} = \sqrt{\frac{1}{5\times 3}\left[5^3 - 0\right]}$$

$$= \sqrt{\frac{1}{15} \times 125} = \sqrt{\frac{125}{15}} = \sqrt{8.33}$$

$$i_{rms} = 2.88\ A$$

* find the effective value of the following signal



Sol

(0,0) (2,6)

$x_1\ y_1\ \ x_2\ y_2$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-0}{2-0} = 3$$

$$y - y_1 = m(x - x_1)$$

$$v(t) - 0 = 3(t - 0)$$

$$v(t) = 3t$$

$$v(t) = \begin{cases} 3t & 0 < t < 2 \\ 6 & 2 < t < 3 \end{cases}$$

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Effective value $= \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$

$= \sqrt{\frac{1}{3}\int_0^2 (3t)^2 dt + \int_2^3 6^2 dt}$

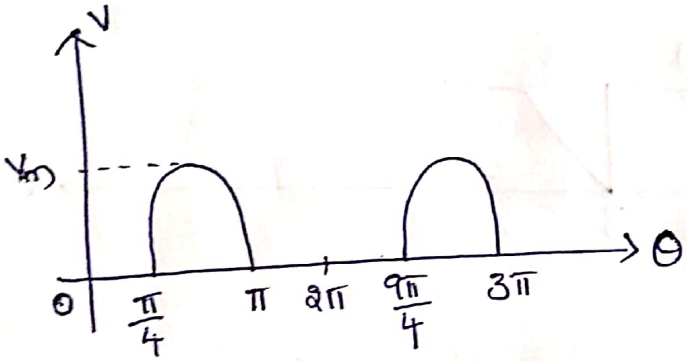
$= \sqrt{\frac{1}{3}\left[9\left[\frac{t^3}{3}\right]_0^3 + 36[t]_2^3\right]}$

$= \sqrt{\frac{1}{3}\left[3[8-0] + 36[3-2]\right]}$

$= \sqrt{\frac{1}{3}[24+36]} = \sqrt{\frac{60}{3}} = \sqrt{20}$

$= 4.472$

* find the average value and rms value of the waveform shown in figure.



Sol

$$V = \begin{cases} 0 & , 0 < \theta < \frac{\pi}{4} \\ V_m \sin\theta & \frac{\pi}{4} < \theta < \pi \\ 0 & \pi < \theta < 2\pi \end{cases}$$

$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v^2(\theta)\,d\theta}$

$= \sqrt{\frac{1}{2\pi}\int_{\pi/4}^{\pi} V_m^2 \sin^2\theta\,d\theta}$

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$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_{\pi/4}^{\pi} V_m^2 \sin^2\theta \, d\theta}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\int_{\pi/4}^{\pi} \sin^2\theta \, d\theta}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\int_{\pi/4}^{\pi} \left(\frac{1-\cos 2\theta}{2}\right) d\theta}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\left[\left(\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right)\right]_{\pi/4}^{\pi}}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\left(\frac{\pi}{2} - \frac{\sin 2\pi}{4} - \frac{\pi}{8} + \frac{\sin \pi/2}{4}\right)}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\times\frac{\pi}{2} - \frac{V_m^2}{2\pi}\times\frac{\sin 2\pi}{4} - \frac{V_m^2}{2\pi}\times\frac{\pi}{8} + \frac{V_m^2}{2\pi}\times\frac{\sin\pi/2}{4}}$$

$$= \sqrt{\frac{V_m^2}{4} - \frac{V_m^2}{16} + \frac{V_m^2}{2\pi}\times\frac{1}{4}}$$

$$= \sqrt{0.227\, V_m^2} = 0.476\, V_m$$

$$V_{avg} = \frac{1}{2\pi}\int_0^{2\pi} v(\theta)\, d\theta$$

$$= \frac{1}{2\pi}\int_{\pi/4}^{\pi} V_m \sin\theta \, d\theta$$

$$= \frac{V_m}{2\pi}\left(-\cos\theta\right)_{\pi/4}^{\pi}$$

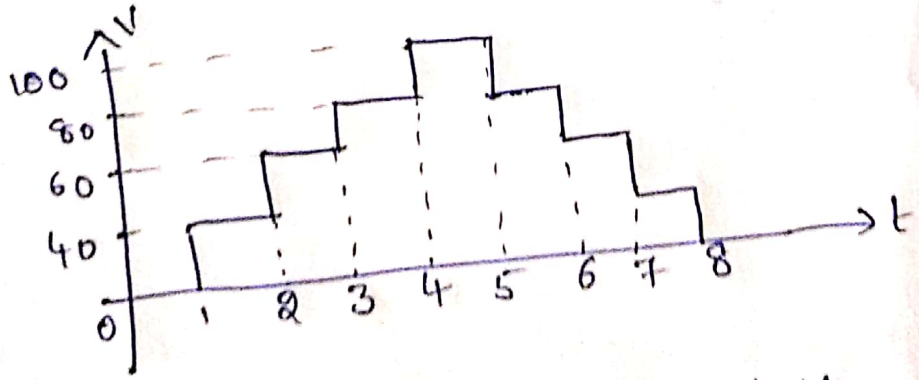
$$= \frac{-V_m}{2\cdot\pi}\left(\cos\pi - \cos\frac{\pi}{4}\right)$$

$$= \frac{-V_m}{2\pi}\left[-1 - \frac{1}{\sqrt{2}}\right]$$

$$= \frac{V_m}{2\pi}\left[1 + 0.707\right]$$

$$V_{avg} = 0.272\, V_m$$

* find the rms and average values of the waveform.



Sol

$$V_{avg} = \frac{0+40+60+80+100+80+60+40}{8} = 57.5\,V$$

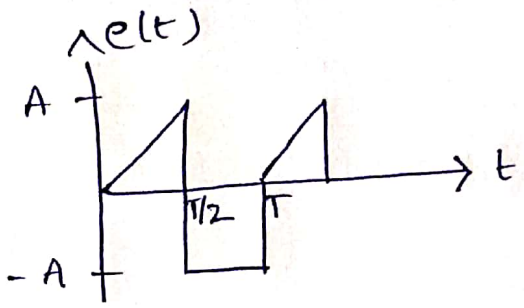
$$V_{rms} = \sqrt{\frac{0^2+40^2+60^2+80^2+100^2+80^2+60^2+40^2}{8}} = 64.42\,V$$

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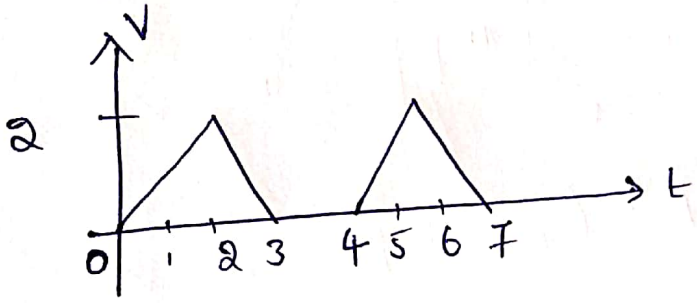
obtain the rms valu

1) The RMS Value of the periodic waveform e(t).

Ans
$A\sqrt{(2/3)}$



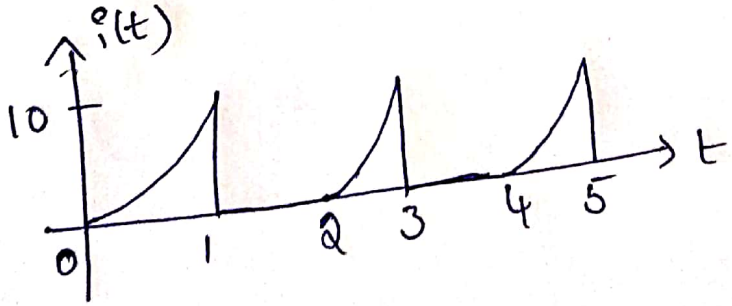
2) find the average power delivered to a 1Ω resistor with a voltage across it as shown in the figure.



3) The current waveform as shown in fig is passed through resistor of 100Ω. what is the power dissipation in the resistor.



4) Obtain the rms value of the waveform



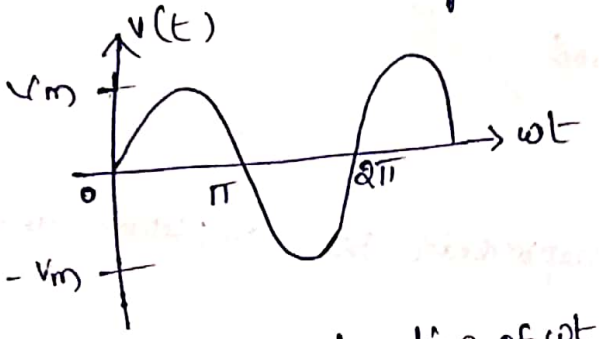
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Steady State analysis of R, L, C elements (in series, Parallel, series-parallel combinations) with Sinusoidal excitation:-

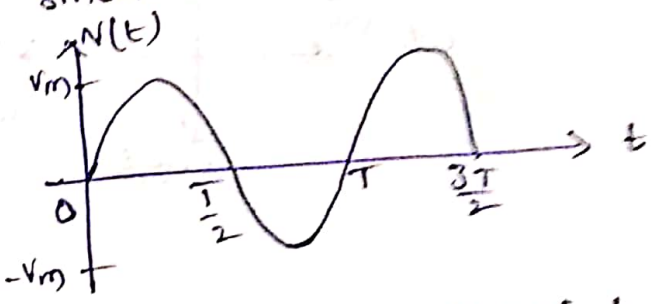
Sinusoid:- The sinusoid repeats itself at a regular interval of time (T sec). This is called a period of sinusoid.

$$v(t) = V_m \sin \omega t$$

V_m is the amplitude of sinusoid.
ω is the angular frequency (radians/sec)
ωt is the argument of sinusoid.



Time as a function of ωt



* Time as a function of t

A Sinusoidal Signal v(t) repeats itself every T seconds

$$v(t) = v(t+T)$$
$$v(t) = V_m \sin \omega t$$
$$v(t+T) = V_m \sin \omega (t+T)$$
$$= V_m \sin \omega \left(t + \frac{2\pi}{\omega}\right)$$
$$= V_m \sin \omega \left(\frac{\omega t + 2\pi}{\omega}\right)$$
$$= V_m \sin(\omega t + 2\pi)$$
$$v(t+T) = V_m \sin \omega t.$$

$\omega = 2\pi f$

$f = \frac{1}{T}$

$\omega = \frac{2\pi}{T}$

$T = \frac{2\pi}{\omega}$

Periodicity property:- A periodic function satisfies the condition $v(t) = v(t+T)$ for all t conditions.

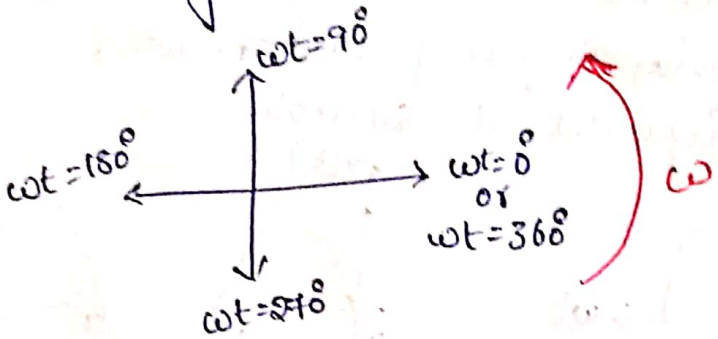
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Phasor Diagrams:-

A graphical or pictorial representation of phasor is known as phasor diagram.

phasor diagram is a pictorial representation of all the phase voltages and their respective currents in a network.

phasor is a rotating vector, which rotates in the anti-clockwise direction with angular frequency ω in the time domain.



→ phasor is a complex number that represents the amplitude and phase of a sinusoid.

→ An alternating voltage or current may represents in the form of waves and equations.

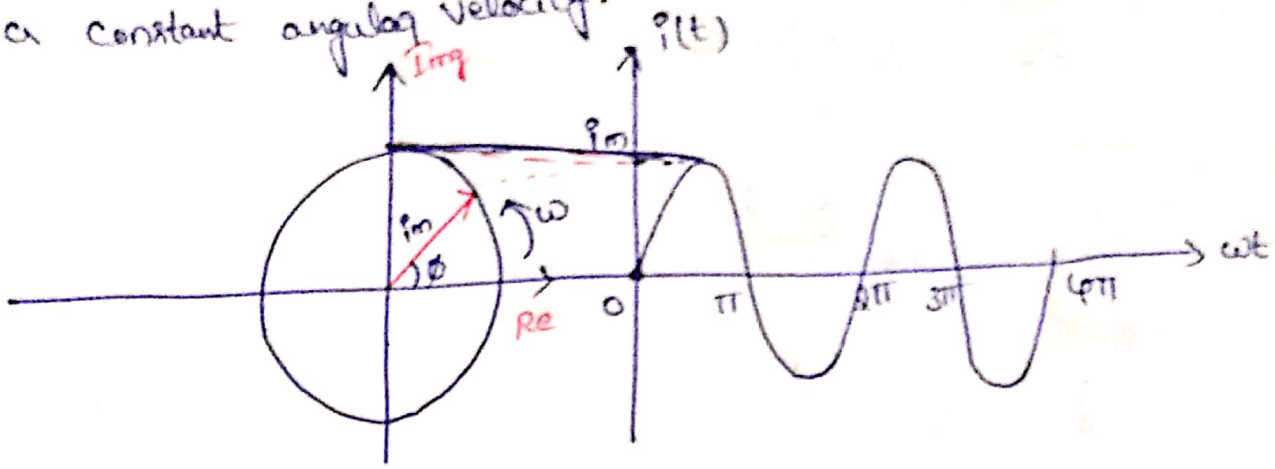
→ The waveform presents to the eye a very definite picture what is happening at every instant of time. but it is difficult to draw the waveform accurately.

→ The current flowing at any instant of time can be determine from the equations is very difficult.

The above two difficulties has been overcome by representing sinusoidal alternating voltage or current by a line of definite length rotating in acticlockwise direction at constant angular velocity ω. Such rotating line is called phasor.

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The length of the phasor is taken, it is equal to the maximum value of alternating quantity with a constant angular velocity.



$$i(t) = i_m \sin(\omega t + \phi)$$
$$i(t) = i_m \angle \phi$$

$z = x + jy$ ⟶ Rectangular form

$r = \sqrt{x^2 + y^2}$

$\phi = \tan^{-1}\left(\frac{y}{x}\right)$

$z = r e^{j\phi}$ ⟶ Exponential form

$z = r \angle \phi$ ⟶ polar form

↳ $z = r(\cos\phi + j\sin\phi)$

Addition and subtraction of complex numbers are better performed in rectangular form.

multiplication and division are better done in polar form.

$$z = x + jy = r \angle \phi$$
$$z_1 = x_1 + jy_1 = r_1 \angle \phi_1$$
$$z_2 = x_2 + jy_2 = r_2 \angle \phi_2$$

Addition:

$$z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$$

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Subtraction:

$$z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$$

multiplication:

$$z_1 z_2 = r_1 r_2 \angle \phi_1 + \phi_2$$

Division:

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} \angle \phi_1 - \phi_2$$

Reciprocal:

$$\frac{1}{z} = \frac{1}{r} \angle -\phi$$

Square root:

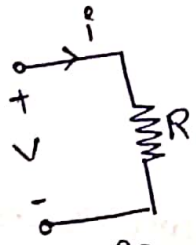
$$\sqrt{z} = \sqrt{r} \angle \phi/2$$

Complex Conjugate:

$$z^* = x - jy = r \angle -\phi = r e^{-j\phi}$$

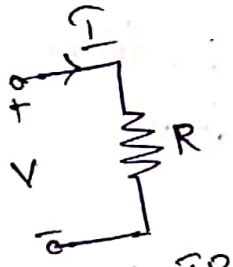
phasor relationship for the circuit elements :-

Resistor :-



$v = iR$

time domain representation



$V = IR$ frequency domain representation.

$$i = i_m \sin(\omega t + \phi)$$

$$V = iR$$

$$V = R\, i_m \sin(\omega t + \phi)$$

$$V = R\, i \angle \phi$$

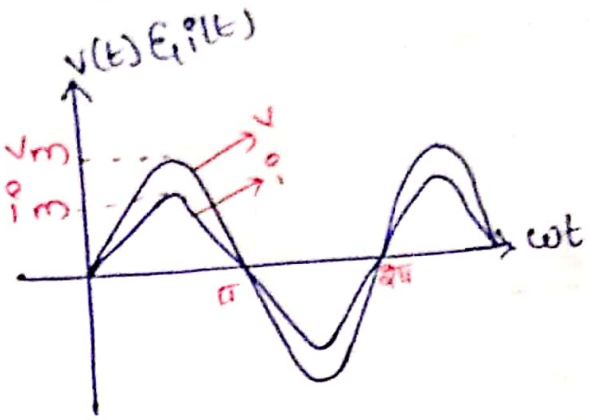
$$\frac{V}{R} = i \angle \phi$$

$$I = i \angle \phi$$

$$V = IR$$

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V & I are in phase rotating with some phase angle.



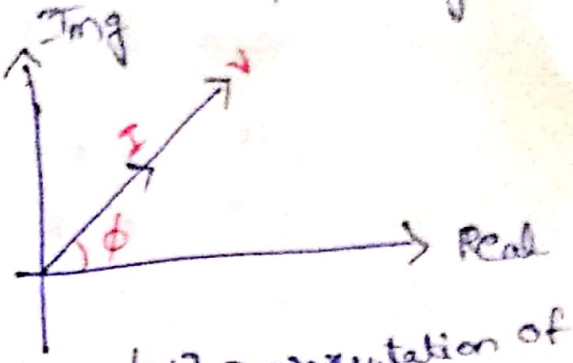
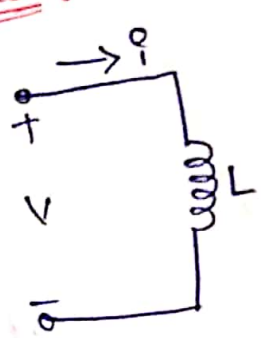
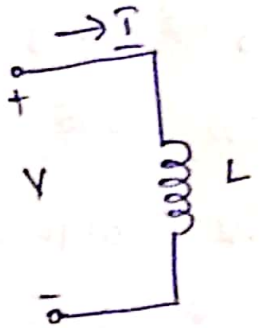


fig: phasor representation of R & RL.

Inductor :-



$V = L \frac{di}{dt}$



$V = j\omega I L$

The current flowing through the inductor is

$i = i_m \cos(\omega t + \phi)$

The voltage & current relationship of inductor is

$$V = L \frac{di}{dt}$$

$$V = L \frac{d}{dt} i_m \cos(\omega t + \phi)$$

$$V = L i_m [-\sin(\omega t + \phi) \cdot \omega]$$

$$V = \omega L i_m [-\sin(\omega t + \phi)]$$

$(\because -\sin\theta = \cos(\theta + 90^\circ))$

$$V = \omega L i_m \cos(\omega t + \phi + 90^\circ)$$

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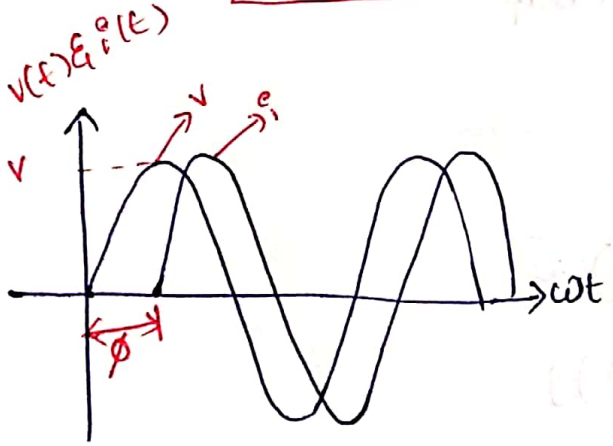
∵ Euler's identity

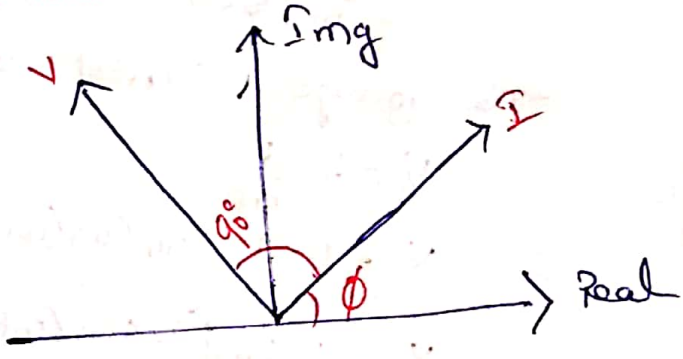
$$e^{jx} = \cos x + j \sin x$$
$$e^{-jx} = \cos x - j \sin x$$
$$e^{\pm j\phi} = \cos\phi \pm j \sin\phi$$
$$\cos\phi = \text{Real}(e^{j\phi})$$
$$\sin\phi = \text{Img}(e^{j\phi})$$

$$V = \omega L I_m e^{j(\phi + 90^\circ)}$$
$$= \omega L I_m e^{j\phi} \cdot e^{j90^\circ}$$
$$= \omega L I_m e^{j\phi} (\cos 90^\circ + j \sin 90^\circ)$$
$$= \omega L I_m e^{j\phi} (0 + j)$$
$$= j\omega L I_m e^{j\phi}$$
$$= j\omega L I_m \angle\phi$$
$$V = j\omega L \bar{I}$$

∵ $I_m e^{j\phi} = I \angle\phi$

$\boxed{\omega L = X_L}$ Inductive reactance.

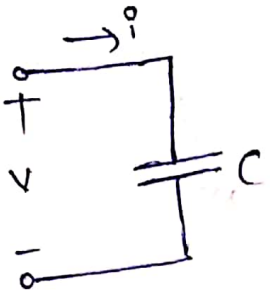




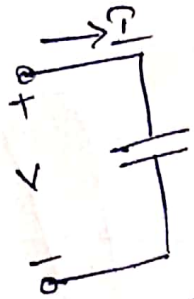
$$V = \omega L I_m \angle\phi + 90^\circ$$

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Capacitor :-



$$i = C\frac{dv}{dt}$$



$$V = \frac{I}{j\omega C}$$

$$I = V j\omega C$$

$$i = C\frac{dv}{dt}$$

The voltage across capacitor is

$$V = V_m \cos(\omega t + \phi)$$

$$i = C\frac{d}{dt} V_m \cos(\omega t + \phi)$$

$$= C V_m [-\sin(\omega t + \phi) \cdot \omega]$$

$$= \omega C V_m [-\sin(\omega t + \phi)]$$

$$= \omega C V_m \cos(\omega t + \phi + 90^\circ)$$

$$= \omega C V_m e^{j(\phi + 90^\circ)}$$

$$= \omega C V_m e^{j\phi} \cdot e^{j90^\circ}$$

$$= \omega C V_m e^{j\phi} (\cos 90 + j \sin 90)$$

$$= \omega C V_m e^{j\phi} \cdot (0 + j)$$

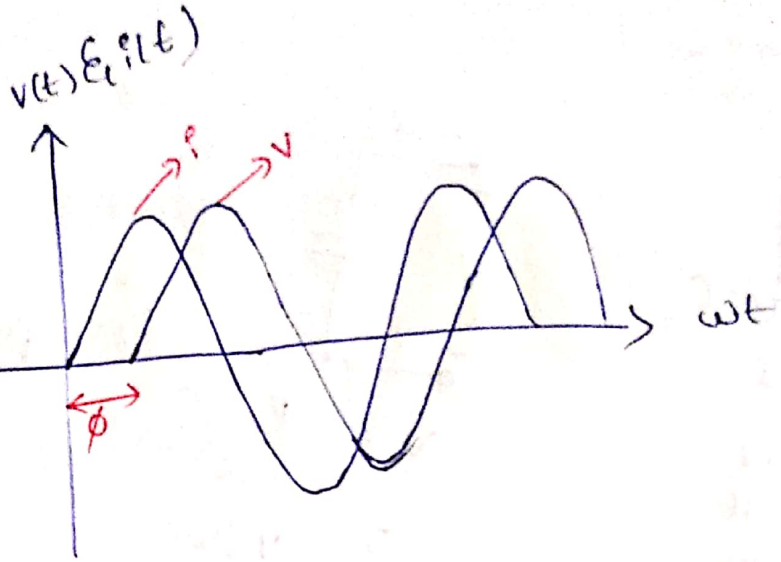
$$= j\omega C V_m e^{j\phi}$$

$$i = j\omega C V_m \angle\phi$$

$$I = j\omega C V$$

$$\frac{I}{j\omega C} = V \qquad \therefore \frac{1}{j\omega C} = X_C = \text{Capacitive reactance}$$

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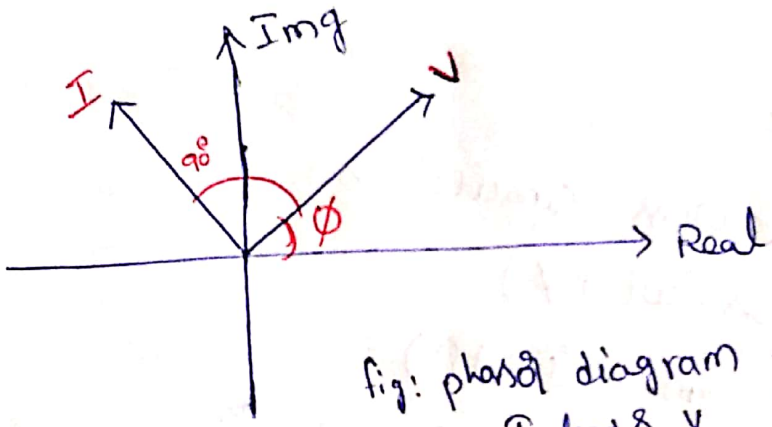


fig: phasor diagram for the capacitor
I leads V

Impedance (Z) :-

Impedance is a measure of the opposition to electrical flow. It is measured in ohm. It is represented by the symbol Z. For DC systems, impedance and resistance are the same.

Impedance of the AC circuit is the ratio of phasor voltage to phasor current. $Z = \frac{V}{I}$

Resistor	Inductor	Capacitor
$V = IR$	$V = I j\omega L$	$V = \frac{I}{j\omega C}$
$\frac{V}{I} = Z_R = R$	$\frac{V}{I} = Z_L = j\omega L$	$\frac{V}{I} = Z_C = \frac{1}{j\omega C}$

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Admittance (Y) :-

The admittance of circuit is the reciprocal of the impedance. It is measured in Simen's or mho ℧.

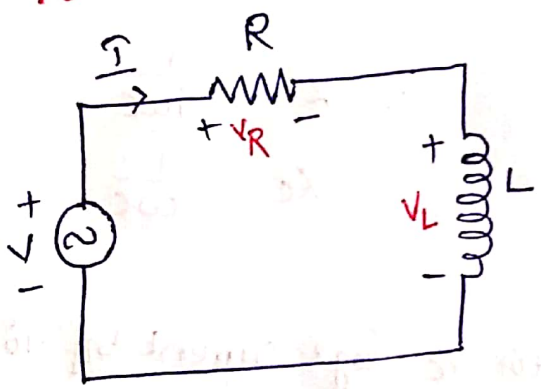
$$Y = \frac{1}{Z}$$

power factor :-

The power factor is defined as the ratio of the resistance to the impedance. It is denoted as $\cos\phi$.

$$\cos\phi = \frac{R}{Z}$$

Series RL circuit :-



$V_R = I \cdot R$

$V_L = I \cdot Z_L$

$V_L = I\, j\omega L$

$\therefore V_L = I\, j\omega L \Rightarrow j\omega L = Z_L$

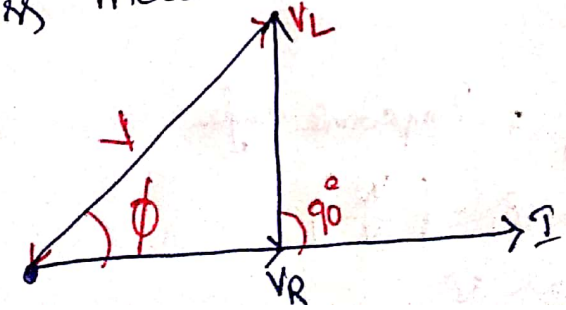
$V_L = I\, Z_L$

$\omega L = X_L$

$V_L = I\, j\omega L$

$V_L = I\, X_L \angle 90^\circ$

Voltage drop across inductor V_L leads current by 90°



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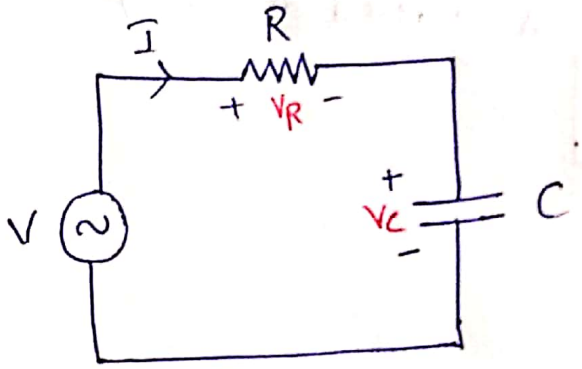
$$V = \sqrt{V_R^2 + V_L^2}$$

$$\phi = \tan^{-1}\left(\frac{V_L}{V_R}\right) = \text{Impedance angle}.$$

$$\cos\phi = \text{pow}^{\text{r}}\text{ factor} = \left(\frac{V_R}{V}\right)$$

i.e I lags V

Series RC circuit :-



$$V_R = I \cdot R$$

$$V_C = I\, Z_C$$

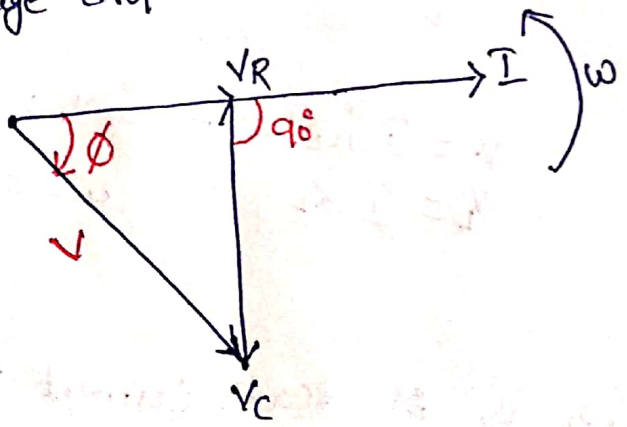
$$V_C = I \cdot \frac{1}{j\omega C}$$

$$V_C = I \cdot X_C \angle -90^\circ$$

$\because Z_C = \frac{1}{j\omega C}$

$X_C = \frac{1}{\omega C}$

Voltage drop across capacitor V_C lags current by 90°



$$V = \sqrt{V_R^2 + V_C^2}$$

$$\phi = \tan^{-1}\left(\frac{V_C}{V_R}\right) = \text{Impedance angle}$$

$$\cos\phi = \text{power factor} = \left(\frac{V_R}{V}\right).$$

i.e I leads V

$$\frac{1}{j} = \frac{-(-1)}{j} = \frac{-(j^2)}{j}$$

$$= -j$$

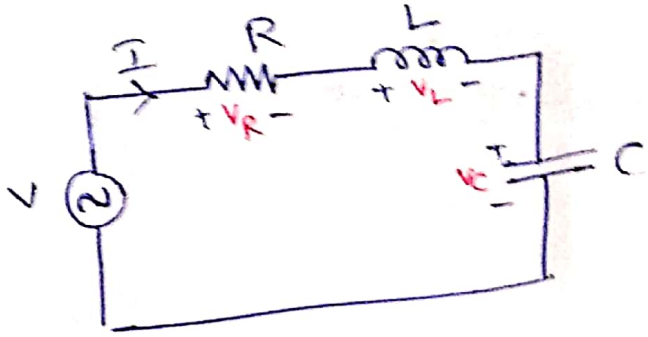
$$\cos\pi/2 - j\sin\pi/2$$

$$= 0 - j$$

$$= -j$$

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Series RLC circuit :-

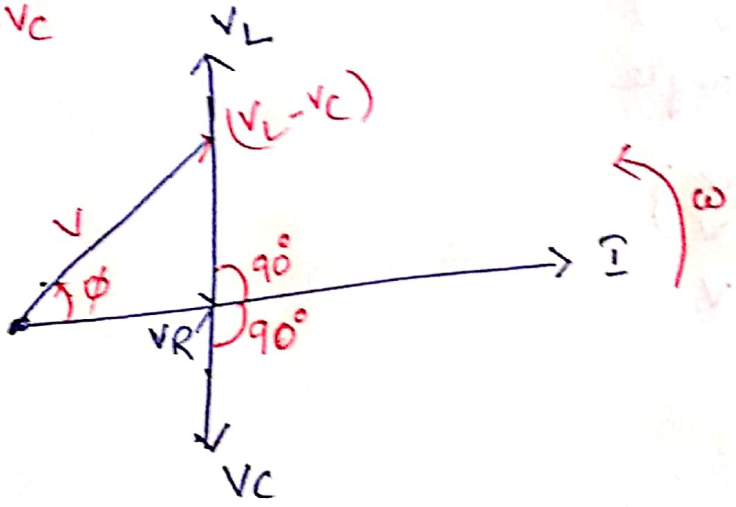


$V_R = I R$

$V_L = I . X_L \angle 90^\circ$

$V_C = I X_C \angle -90^\circ$

(i) $V_L > V_C$



$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$\phi = \tan^{-1}\left(\frac{V_L - V_C}{V_R}\right)$$

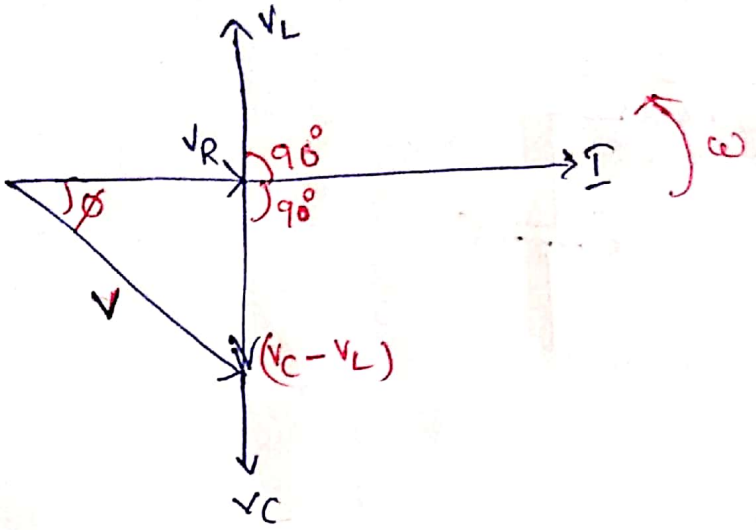
= Impedance angle

$$\cos\phi = \text{P.f} = \left(\frac{V_R}{V}\right)$$

i.e I lags V

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(ii) $V_C > V_L$



$$V = \sqrt{V_R^2 + (V_C - V_L)^2}$$

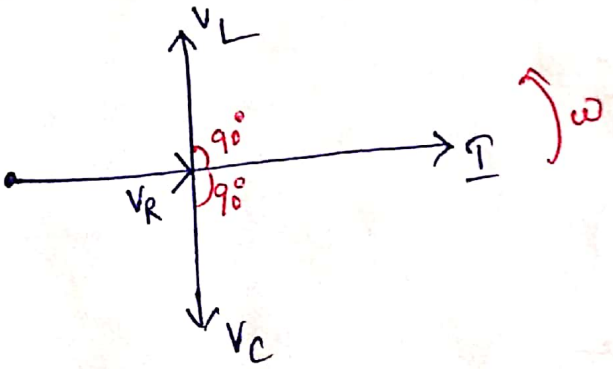
$$\phi = \tan^{-1}\left(\frac{V_C - V_L}{V_R}\right)$$

= Impedance angle

$$\cos\phi = \text{p.f} = \left(\frac{V_R}{V}\right)$$

i.e I leads V

(iii) $V_L = V_C$

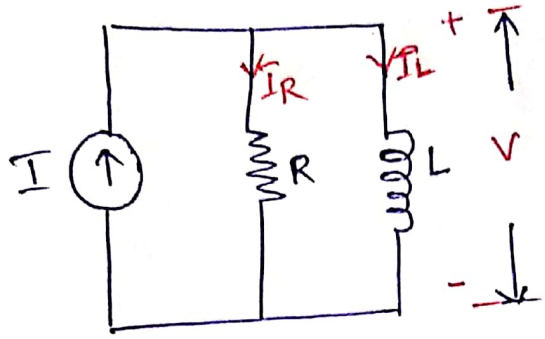


In the above phasor diagram, Inductor and capacitor voltages are equal in magnitude and opposite in phase. So they cancel to each ~~other~~ other and this is called resonance.

$\phi = 0^\circ$ = unity power factor.

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Parallel RL Circuit :-

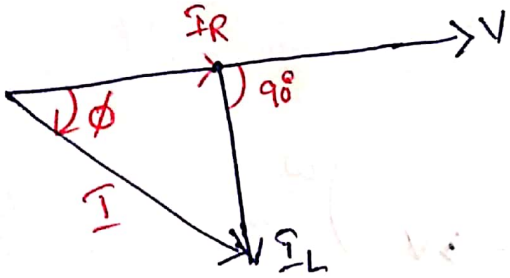


$$I_R = \frac{V}{R}$$

$$I_L = \frac{V}{Z_L}$$

$$I_L = \frac{V}{j\omega L}$$

$$I_L = \frac{V}{jX_L} = \frac{V}{X_L} \angle -90^\circ$$



$$I = \sqrt{I_R^2 + I_L^2}$$

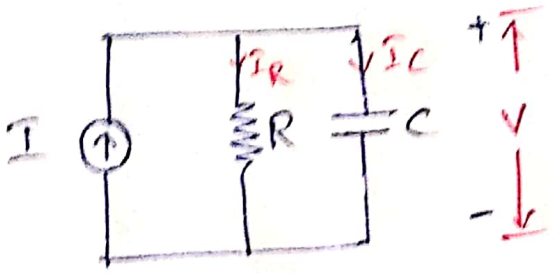
$$\phi = \tan^{-1}\left(\frac{I_L}{I_R}\right) = \text{impedance angle}$$

$$\cos\phi = \text{p.f} = \left(\frac{I_R}{I}\right)$$

i.e I lags V

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Parallel RC circuit :-

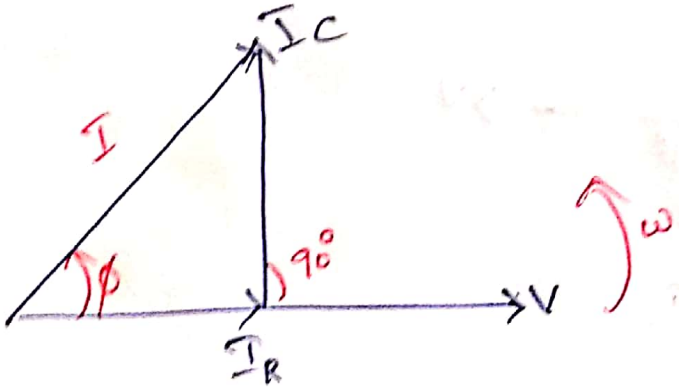


$$I_R = \frac{V}{R}$$

$$I_C = \frac{V}{Z_C}$$

$$I_C = \frac{V}{\frac{1}{j\omega C}} = \frac{j V}{\frac{1}{\omega C}} = \frac{j V}{X_C}$$

$$I_C = \frac{V}{X_C} \angle 90^\circ$$



$$I = \sqrt{I_R^2 + I_C^2}$$

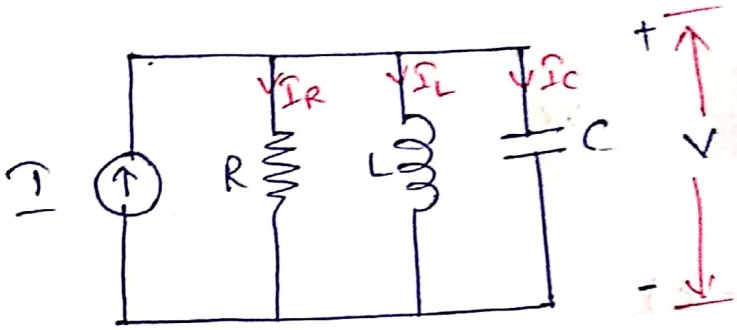
$\phi = \tan^{-1}\left(\frac{I_C}{I_R}\right)$ = Impedance angle

$\cos\phi$ = p.f = $\left(\frac{I_R}{I}\right)$

i.e I lead's V

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Parallel RLC circuit :-

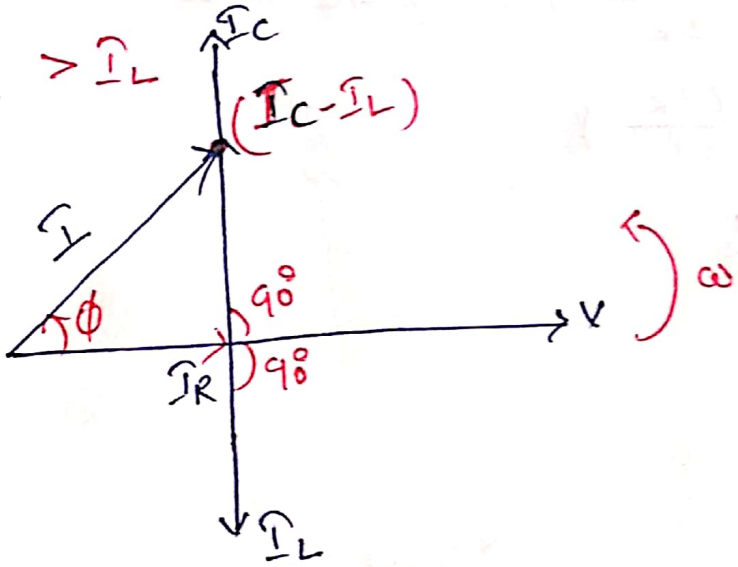


$$\bar{I}_R = \frac{V}{R}$$

$$\bar{I}_L = \frac{V}{X_L} \angle -90^\circ$$

$$\bar{I}_C = \frac{V}{X_C} \angle 90^\circ$$

(i) $\bar{I}_C > \bar{I}_L$



$$\bar{I} = \sqrt{\bar{I}_R^2 + (\bar{I}_C - \bar{I}_L)^2}$$

$$\phi = \tan^{-1}\left(\frac{\bar{I}_C - \bar{I}_L}{\bar{I}_R}\right)$$

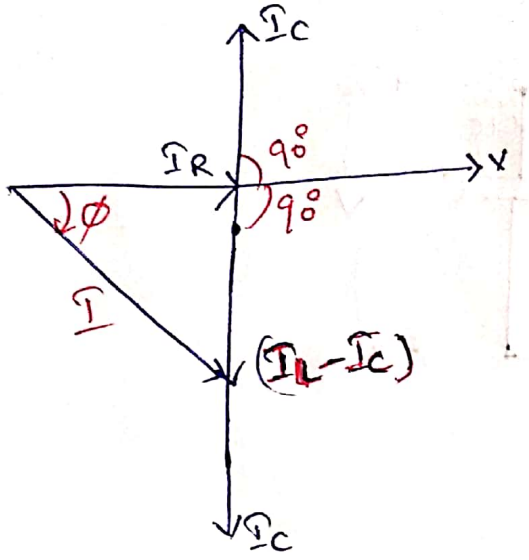
= Impedance angle

$$\cos\phi = \text{p.f} = \left(\frac{\bar{I}_R}{\bar{I}}\right)$$

i.e $\bar{I}$ lead & V

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(ii) $\underline{I}_L > \underline{I}_C$



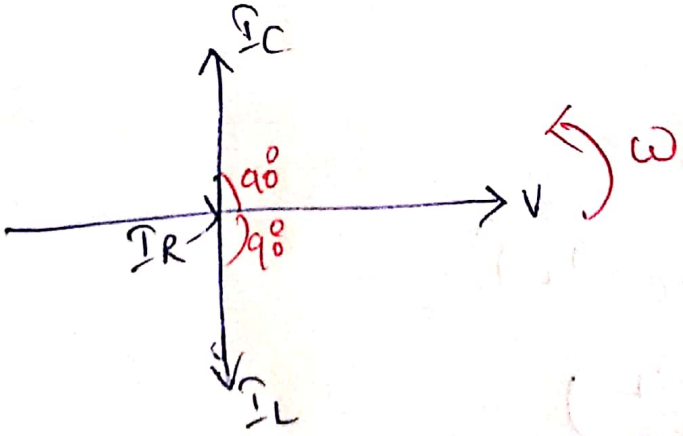
$$\underline{I} = \sqrt{\underline{I}_R^2 + (\underline{I}_L - \underline{I}_C)^2}$$

$$\phi = \tan^{-1}\left(\frac{\underline{I}_L - \underline{I}_C}{\underline{I}_R}\right) = \text{Impedance angle}$$

$$\cos\phi = \text{P.f} = \left(\frac{\underline{I}_R}{\underline{I}}\right)$$

i.e $\underline{I}$ lags V

(iii) $\underline{I}_L = \underline{I}_C$



In the above phasor diagram inductor and capacitor current's are equal in magnitude and opposite in phase. So they cancel to each other and this is called resonance.

$\phi = 0^\circ$ = unity power factor.

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Admittance (Y) :-

Admittance is reciprocal of impedance.

$$Y = \frac{1}{Z} =$$

$$\because Z = \frac{V}{I}$$

$$Y = \frac{1}{V/I} = \frac{I}{V}$$

It is measured in Siemens(S) or mho(℧).

* The advantage of using admittance is if the impedance of any branch is higher, the current flowing through that branch is lower.

If the admittance is higher definitely the current flowing through that branch is higher.

$$I = V \cdot Y \Rightarrow I \propto Y$$

* Impedance is a combination of resistance and reactance. Both the parameters resistance and reactance are use to restrict the opposition of flow of current.

Resistance is offered by a resistor. (Irrespective of time at any instant of time resistance will remains same).

Reactance is decided by the time parameter, if time is changing reactance is changing. (The opposition of current flow changing with respect to time).

$$Z = R + jX$$

R → Resistance

X → Reactance.

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Admittance is a combination of Conductance and Susceptance.

$$Y = G + jB.$$

G → Conductance
B → Susceptance.

Conductance and susceptance are to allow the flow of current.
If the conductance and susceptance increased the flow of current will also be increased.

* Conductance : The rate of ~~flow~~ of increase of flow of current will remain same.
* Susceptance :- Rate of flow of current will depends upon time.

Conductance (G) :-

Conductance is reciprocal of resistance.

$$G = \frac{1}{R}$$

It is measured in siemens or mho (℧)
Conductance is always positive.
Susceptance may be positive & negative.

Susceptance (B) :-

Susceptance is reciprocal of reactance. (X_L or X_C).

$$B = \frac{1}{X}$$

There are two types of susceptances
→ Inductive susceptance (B_L) → always -ve
→ Capacitive susceptance (B_C) → always +ve.

How to identify the inductive susceptance & Capacitive susceptance
If the susceptance is -ve i.e B_L
If the susceptance is +ve i.e B_C

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Units of susceptance is siemens or mho. (℧)

Reactance (X) :-

Reactance is defined as the opposition to the flow of current from a circuit element due to its inductance or capacitance.

$X_C \rightarrow$ Capacitive reactance.
$X_L \rightarrow$ Inductive reactance.

Components of Admittance :-

The impedance can be written in a rectangular form

$$Z = R \pm jX$$

~~+ ⇒ Capacitive reactance.~~

+ ⇒ Inductive reactance
− ⇒ Capacitive reactance.

$$Z = R + jX_L$$
$$Z = R - jX_C$$

$$Y = G \pm jB$$

+ ⇒ Capacitive susceptance
− ⇒ Inductive susceptance.

$$Y = G + jB_C$$
$$Y = G - jB_L$$

The polar form representation of admittance is

$Y = |Y| \angle \phi \rightarrow$ polar form.

polar form consists of magnitude and phase angle.

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$|Y| = \sqrt{G^2 + B^2}$ $\Rightarrow$ $|Y| = \sqrt{G^2 + (-B_L)^2}$ or $|Y| = \sqrt{G^2 + B_C^2}$

$\phi = \tan^{-1}\left(\frac{B}{G}\right)$ $\Rightarrow \phi = \tan^{-1}\left(\frac{-B_L}{G}\right)$ or $\phi = \tan^{-1}\left(\frac{B_C}{G}\right)$

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Power Calculations :-

* Power Calculations in AC circuits :-

for a DC circuit, the power is given by.

$$P_{dc} = VI = I^2R = \frac{V^2}{R}$$

The power at any instant in an AC circuit is called instantaneous power.
It is defined by the product of voltage and current at that instant. i.e

$$P = vi$$

As the voltage and current are not constant in an AC circuit, the power value will also change with time.
Therefore the average power taken over a period T is

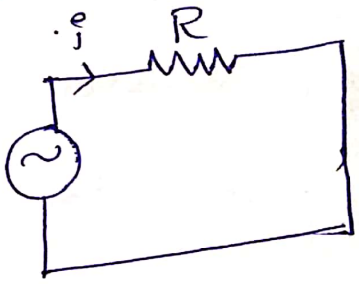
$$P = \frac{1}{T}\int_0^T vi\, dt$$

$$P = \frac{1}{2\pi}\int_0^{2\pi} vi\, dt$$

The average power is also called the active power or true power or real power and is measured in watts.

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* Power in pure resistive circuits :-



The voltage and current in a pure resistive circuit are in phase and may be respectively represented by

$$V = V_m \sin\omega t$$

$$i = I_m \sin\omega t$$

The instantaneous power is given by

$$P = vi$$

$$= V_m \sin\omega t \times I_m \sin\omega t$$

$$= V_m I_m \sin^2\omega t$$

$$P = V_m I_m \left(\frac{1-\cos 2\omega t}{2}\right)$$

$$\boxed{P = \frac{V_m I_m}{2}(1-\cos 2\omega t)}$$

The active power is given by

$$P = \frac{1}{T}\int_0^T vi\, dt$$

$$= \frac{V_m I_m}{2T}\int_0^T (1-\cos 2\omega t)\, dt$$

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$$P = \frac{V_m I_m}{2T} \int_0^T (1 - \cos 2\omega t)\, d\omega t$$

$$= \frac{V_m I_m}{2T} \int_0^T 1 \, d\omega t - \frac{V_m I_m}{2T} \int_0^T \cos 2\omega t \, d\omega t$$

$$= \frac{V_m I_m}{2T} [T - 0] - \frac{V_m I_m}{2T} \left[\frac{\sin 2\omega t}{2\omega} \right]_0^T$$

$$= \frac{V_m I_m}{2} - \frac{V_m I_m}{2T} \left[\frac{\sin 2\omega [T] - \sin 2\omega (0)}{2\omega} \right]$$

$$= \frac{V_m I_m}{2} - \frac{V_m I_m}{2T} \left[\frac{\sin 2\phi \cdot \frac{2\pi}{\omega} - 0}{2\omega} \right] \quad \because \sin n\pi = 0$$

$$= \frac{V_m I_m}{2}$$

$$P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} = V_{rms} I_{rms} \text{ Watts.}$$

The power waveform is positive throughout the cycle. This is because voltage and current are in phase. The power flows only in one direction. It is Consumed in R and appears in the form of heat.

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* Power in a pure inductive circuit :-

The voltage leads the current by 90° in a pure inductive circuit.

let the voltage and current at any instant can be represented by

$$V = V_m \sin(\omega t + 90^\circ)$$
$$i = I_m \sin\omega t$$

The instantaneous power is given by

$$p = vi$$
$$= V_m \sin(\omega t + 90^\circ)\, I_m \sin\omega t \qquad \because \sin(90+\theta) = \cos\theta$$
$$= \frac{V_m I_m}{2} 2\cos\omega t \sin\omega t \qquad \because 2\cos A \sin A = \sin 2A$$
$$\boxed{p = \frac{V_m I_m}{2} \sin 2\omega t}$$

Active power (or) Average power

$$P_L = \frac{1}{T}\int_0^T vi\, dt$$
$$= \frac{1}{T}\int_0^T \frac{V_m I_m}{2} \sin 2\omega t\, dt$$
$$= \frac{1}{2T}\int_0^T V_m I_m \sin 2\omega t\, dt$$
$$= \frac{V_m I_m}{2T}\int_0^T \sin 2\omega t\, dt$$
$$= \frac{V_m I_m}{2T}\left[\frac{-\cos 2\omega t}{2\omega}\right]_0^T$$
$$= \frac{-V_m I_m}{2T \times 2\omega}\left(\cos 2\omega T - \cos 0\right)$$
$$= \frac{-V_m I_m}{4\omega T}\left(\cos 2\varphi\left(\frac{2\pi}{\varphi}\right) - 1\right)$$

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$$P_L = \frac{-V_m I_m}{4\omega T}(\cos 4\pi - 1) \qquad \because \cos 4\pi = 1$$

$$= \frac{-V_m I_m}{4\omega T}(1-1)$$

$$\boxed{P_L = 0}$$

Therefore, in a pure inductive circuit the active power supplied over a complete cycle is zero.

Reactive power:-

The power supplied to a reactance (inductive or capacitive) is termed as reactive power.

The active power is denoted by symbol 'P'

The Reactive power is denoted by symbol 'Q'.

The peak value of instantaneous power is given by

$$Q_L = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} = V_{rms} I_{rms}.$$

for a pure inductive circuit

$$V_L = V_{rms} = X_L I_L$$

$$I_L = I_{rms}$$

$$\therefore Q_L = V_L I_L$$

$$Q_L = I_L^2 X_L$$

Also: $$Q_L = \frac{V_L^2}{X_L}$$

Here Q_L is the rate of change of reactive energy b/w the load and the source.

By convention Q_L is taken as +ve and is called the lagging reactive power.

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* power in a pure Capacitive circuit:-

The voltage lags the current by 90° in a pure capacitive circuit.

let the voltage and current any instant can be represented respectively by

$$V = V_m \sin \omega t$$

$$i = I_m \sin(\omega t + 90^\circ)$$

The instantaneous power can be written as

$$P = Vi = V_m \sin\omega t \; I_m \sin(\omega t + 90^\circ)$$

$$= \frac{V_m I_m 2 \cos\omega t \sin\omega t}{2}$$

$$P = \frac{V_m I_m}{2} \sin 2\omega t$$

Active power or Average power

$$P_C = \frac{1}{T}\int_0^T P\,dt$$

$$= \frac{1}{T}\int_0^T \frac{V_m I_m}{2} \sin 2\omega t \, dt$$

$$= \frac{V_m I_m}{2T}\int_0^T \sin 2\omega t \, dt$$

$$= \frac{V_m I_m}{-2T}\left[\frac{-\cos 2\omega t}{2\omega}\right]_0^T$$

$$= \frac{-V_m I_m}{2T \times 2\omega}\left[\cos 2\omega T - \cos 0\right]$$

$$= \frac{-V_m I_m}{4T\omega}\left[\cos 2\omega\left(\frac{2\pi}{\omega}\right) - 1\right]$$

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$$P_C = \frac{-V_m I_m}{4\omega T} \quad (1-1)$$

$$\boxed{P_C = 0}$$

Reactive power :-

The peak peak value of instantaneous power can be written as

$$P_C = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} = V_{rms} \cdot I_{rms}$$

for a purely capacitive circuit-

$$V_{rms} = V_C = X_C I_C$$

$$Q_C = V_C I_C$$

$$= I_C^2 X_C$$

or $$Q_C = \frac{V_C^2}{X_C}$$

Here Q_C is the rate of change of reactive power b/w the load and the source. By convention, Q_C is taken as negative and is called the leading reactive power.

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Power in a general series circuit :-

for a passive ac circuit, let the voltage and current at any instant to be given by

$$V = V_m \sin\omega t$$
$$i = I_m \sin(\omega t - \phi)$$

where ϕ is the phase difference b/w the voltage and current. If ϕ is -ve current leads the voltage.

The instantaneous power p is given by

$$p = vi$$
$$= V_m I_m \sin\omega t \sin(\omega t - \phi)$$
$$= \frac{1}{2} V_m I_m \left(2 \sin\omega t \sin(\omega t - \phi)\right)$$
$$= \frac{1}{2} V_m I_m \left(\cos\phi - \cos(2\omega t - \phi)\right)$$

$\because 2\sin A \sin B = \cos(A-B) - \cos(A+B)$

The average power P is given by

~~$P = \frac{1}{T} \int_0^T \sin\omega t \, dt$~~

$$P = \frac{1}{T} \int_0^T vi \, dt$$
$$= \frac{1}{T} \int_0^T \frac{1}{2} V_m I_m \left(\cos\phi - \cos(2\omega t - \phi)\right) dt$$
$$= \frac{V_m I_m}{2T} \left[\int_0^T \cos\phi \, dt - \int_0^T \cos(2\omega t - \phi) \, dt \right]$$
$$= \frac{V_m I_m}{2T} \cos\phi \int_0^T 1 \, dt \qquad \because \int_0^T \cos(2\omega t - \phi) dt = 0$$
$$= \frac{V_m I_m}{2T} \times \cos\phi \times T$$
$$= \frac{V_m I_m}{2} \cos\phi = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\phi$$

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$P = V_{rms} I_{rms} \cos\phi$

The product of V_{rms} and I_{rms} is called the apparent power and P is called the true or real power.

$\therefore P$ = Apparent power $\times \cos\phi$

(or)

True (real power) = Apparent power × power factor.

The product of rms values of voltage and current with the cosine of the angle b/w them (i.e the power factor) is called the real power or true power denoted by Symbol P.

Reactive power :-

We can now generalize our previous definition and state that the product of rms values of voltage and current with the sine of the angle b/w them is called the reactive power Q.

$\therefore Q = V_{rms} \cdot I_{rms} \sin\phi$

The reactive power is also called the wattless power or quadrature power.

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Circuit	power factor $\cos\phi = pf$	Real power $P = V_{rms} \cdot I_{rms} \cos\phi$	Reactive power $Q = V_{rms} \cdot I_{rms} \sin\phi$	Apparent power $S = \sqrt{P^2 + Q^2}$
R	1	$V_{rms} \cdot I_{rms}$	0	$V_{rms} \cdot I_{rms}$
L	0	0	$V_{rms} I_{rms}$ (lag)	$V_{rms} \cdot I_{rms}$
C	0	0	$V_{rms} I_{rms}$ (lead)	$V_{rms} \cdot I_{rms}$
RL	$\cos\phi$ (lag)	$V_{rms} I_{rms} \cos\phi$	$V_{rms} I_{rms} \sin\phi$ (lag)	$V_{rms} I_{rms}$
RC	$\cos\phi$ (lead)	$V_{rms} I_{rms} \cos\phi$	$V_{rms} I_{rms} \sin\phi$ (lead)	$V_{rms} \cdot I_{rms}$
RLC	$\cos\phi$ (lag)	$V_{rms} I_{rms} \cos\phi$	$V_{rms} I_{rms} \sin\phi$ (lag)	$V_{rms} I_{rms}$
	$\cos\phi$ (lead)	$V_{rms} I_{rms} \cos\phi$	$V_{rms} I_{rms} \sin\phi$ (lead)	$V_{rms} I_{rms}$
	$\cos\phi = 1$ inphase	$V_{rms} I_{rms}$	0	$V_{rms} I_{rms}$

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power triangle :-

The geometrical representation of the apparent power S, the average (true power / real power) P and the reactive power Q is called the power triangle.

fig (a) :- It shows the power triangle for an inductive load.

fig (b) : It shows the power triangle for a capacitive load.

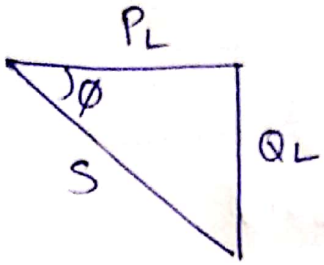


fig (a)

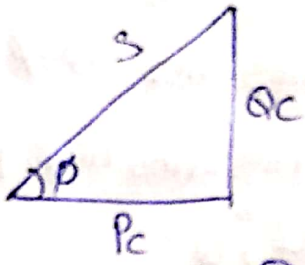


fig (b)

Complex power :-

The powers S, P and Q of a power triangle can be obtained from the product VI^*, where I^* is the conjugate of I. It is called the complex power S.

for a capacitive load

$$V = V_{rms}\, e^{j\theta} \quad \& \quad I = I_{rms}\, e^{+j(\theta+\phi)}$$

$$S = VI^*$$

$$= V_{rms}\, e^{j\theta}\, I_{rms}\, e^{-j(\theta+\phi)}$$

$$= V_{rms}\, I_{rms}\, e^{-j\phi}$$

$$= V_{rms} I_{rms} (\cos\phi - j\sin\phi)$$

$$S = V_{rms} I_{rms} \cos\phi - j\, V_{rms} I_{rms} \sin\phi$$

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$S = P - jQ$

Similarly for an inductive load

$V = V_{rms} e^{j\theta}$ & $\bar{I} = I_{rms} e^{+j(\theta+\phi)}$

$S = V\bar{I}^*$

$= V_{rms} e^{j\theta} \cdot I_{rms} e^{-j(\theta-\phi)}$

$= V_{rms} I_{rms} e^{j\phi}$

$= V_{rms} I_{rms} (\cos\phi + j\sin\phi)$

$= V_{rms} I_{rms} \cos\phi + jV_{rms} I_{rms} \sin\phi$

$S = P + jQ$

By convention $+Q$ indicates lagging reactive power.

$-Q$ indicates leading reactive power.

The inductive load absorbs the lagging reactive power and capacitive load supplies the lagging reactive power.

In other words, the inductive load supplies the leading reactive power whereas the capacitive load absorbs the leading reactive power.

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A 318μf capacitor is connected across 230V, 50Hz system. Determine (i) capacitive reactance (ii) r.m.s values of current (iii) equations for voltage and current.

(i) Capacitive reactance $X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 318 \times 10^{-6}}$

$= \frac{10^6}{2\pi \times 50 \times 318} = 10\Omega$

(ii) rms value of current $I = \frac{V}{X_C} = \frac{230}{10} = 23A$

(iii) $V_{rms} = \frac{V_m}{\sqrt{2}}$

$V_m = V_{rms}\sqrt{2} = 230\sqrt{2} = 325.27$ volts

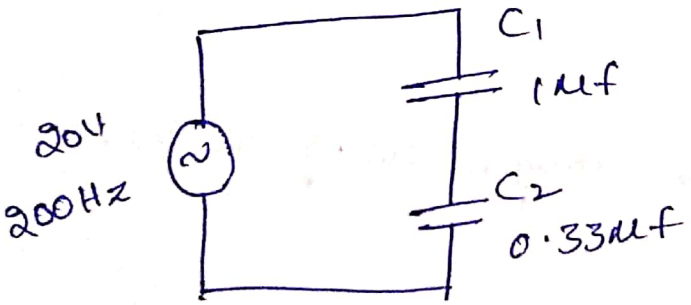
$I_{rms} = \frac{I_m}{\sqrt{2}} \Rightarrow I_m = I_{rms}\sqrt{2} = 23\sqrt{2} = 32.53A$

$\omega = 2\pi f = 2\pi \times 50 = 314$ rad/sec.

equation for voltage $V(t) = 325.27 \sin 314t$.

equation for current $i(t) = 32.53 \sin\left(314t + \frac{\pi}{2}\right)$.

Determine the total capacitive reactance for the ckt as shown in below fig.



Sol The total capacitive reactance X_{C_T} is

$X_{C_T} = X_{C_1} + X_{C_2} + X_{C_3} + \cdots + X_{C_n}$

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$$X_{C_1} = \frac{1}{2\pi f C_1} = \frac{1}{2\pi \times 200 \times 1 \times 10^{-6}}$$

$$X_{C_1} = 796\,\Omega$$

$$X_{C_2} = \frac{1}{2\pi f C_2} = \frac{1}{2\pi \times 200 \times 0.33 \times 10^{-6}}$$

$$X_{C_2} = 2411\,\Omega$$

$$X_{C_T} = X_{C_1} + X_{C_2}$$

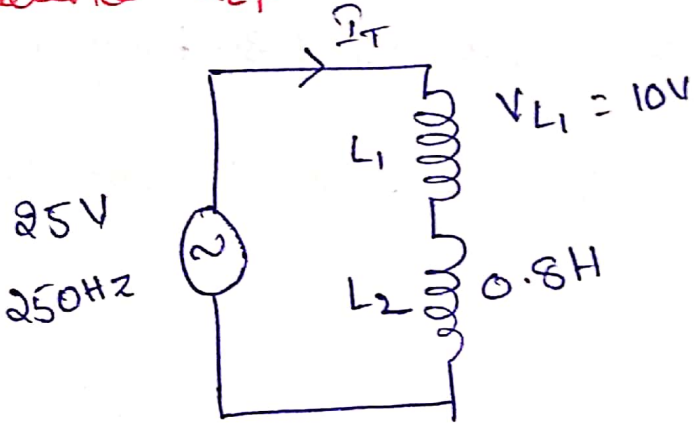
$$\boxed{X_{C_T} = 3207\,\Omega}$$

(Or)

$$C_T = \frac{C_1 C_2}{C_1 + C_2} = \frac{1 \times 0.33}{1 + 0.33} = 0.248\,\mu f = 0.248 \times 10^{-6}\,f$$

$$X_{C_T} = \frac{1}{2\pi f C_T} = \frac{1}{2\pi \times 200 \times 0.248 \times 10^{-6}} = 3207\,\Omega.$$

For the ckt shown in fig. Determine L_1 and total inductive reactance X_{L_T}



So series inductive circuits are analysed using the series resistive circuits.

$$X_{L_T} = X_{L_1} + X_{L_2} + \cdots + X_{L_n}$$

$$X_{L_2} = 2\pi f L_2 = 2\pi \times 250 \times 0.8 = 1256\,\Omega$$

$$V_{L_2} = V_T - V_{L_1} = 25 - 10 = 15V$$

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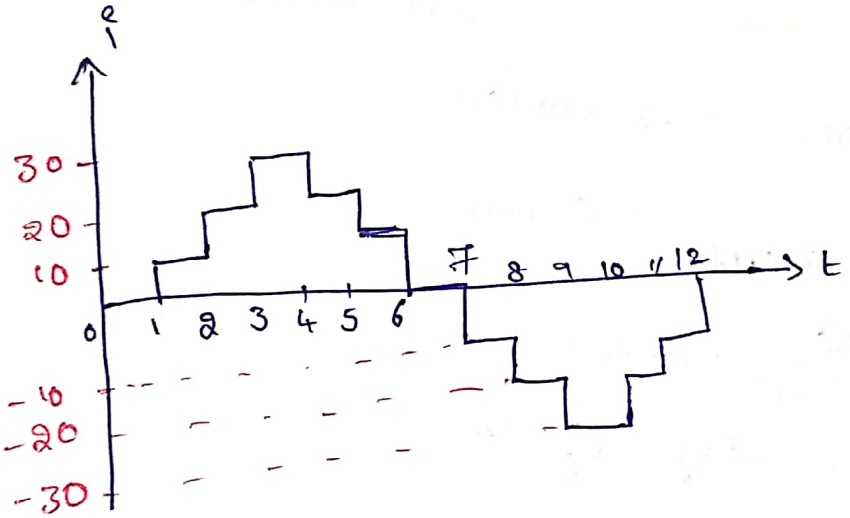
$$I_{L_1} = I_T = I_{L_2} = \frac{V_{L_2}}{X_{L_2}} = \frac{15V}{125} = 11.94mA$$

$$X_{L_1} = \frac{V_{L_1}}{I_{L_1}} = \frac{10V}{11.94mA} = 837.5\ \Omega$$

$$X_{L_T} = X_{L_1} + X_{L_2} = 837.5 + 1256 = 2093.5\ \Omega$$

$$L_1 = \frac{X_{L_1}}{2\pi f} = \frac{837.5}{2\pi \times 250} = 0.53\ H$$

A current has the following steady values in amperes for equal intervals of time changing instantaneously from one value to the next 0, 10, 20, 30, 20, 10, 0, -10, -20, -30, -20, -10, 0 etc. Calculate (i) average value
(ii) RMS value
(iii) form factor
(iv) peak factor.



Sol i) $I_{avg} = \frac{i_1 + i_2 + i_3 + i_4 + i_5 + i_6}{6} = \frac{0+10+20+30+20+10}{6} = \frac{90}{6} = 15A$

ii) $I_{rms}^2 = \frac{i_1^2 + i_2^2 + i_3^2 + i_4^2 + i_5^2 + i_6^2}{6} = \frac{1900}{6} = 316.67$

$I_{rms} = \sqrt{316.67} = 17.8A$

iii) form factor $= \frac{I_{rms}}{I_{avg}} = \frac{17.8}{15} = 1.19$

iv) peak factor $= \frac{I_m}{I_{rms}} = \frac{30}{17.8} = 1.68$

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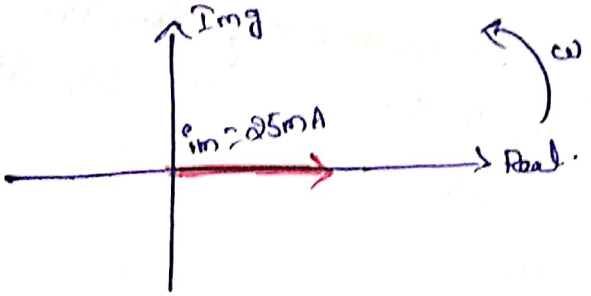
Draw the phasor and waveform for current $i = 25 \sin \omega t$ mA for $f = 1000$ Hz

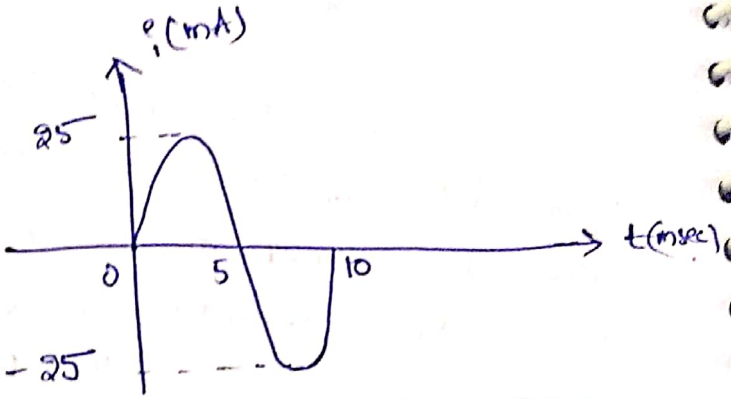
Sol The phasor has a length of 25 mA and is drawn at its t=0 position. which is zero degrees

$I_m = 25$ mA

$f = 1000$ Hz

$T = 1/f = \frac{1}{1000} = 10$ msec.





The voltage across a coil of capacitance is $v_C = 100 \sin(\omega t - 40^\circ)$ V and f = 1000 Hz. Determine i_C and sketch its waveform.

Sol

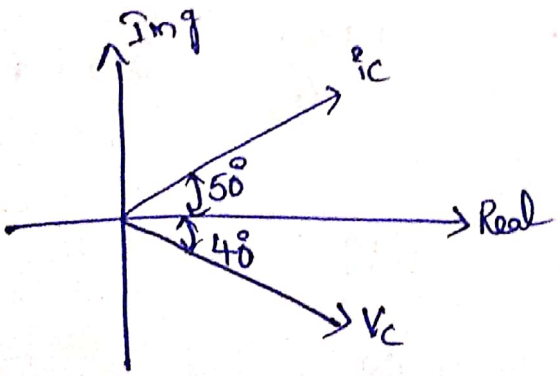
$\omega = 2\pi f = 2\pi \times 1000 = 6283$ rad/sec.

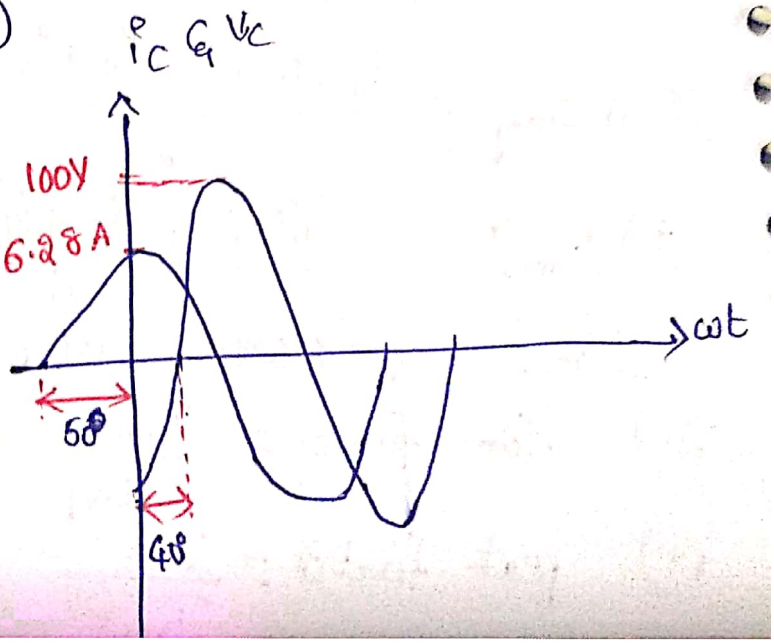
$X_C = \frac{1}{\omega C} = \frac{1}{6283 \times 10\mu f} = 15.92\Omega$

$I_m = \frac{V_m}{X_C} = \frac{100}{15.92} = 6.28\Omega$

The current leads voltage by 90°.

$i_C = 6.28 \sin(6283t + 50^\circ)$





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The current in a circuit is given by $(4.5 + j12)$ A. calculate the applied voltage if $(100 + j150)$ V. Determine the impedance and phase angle.

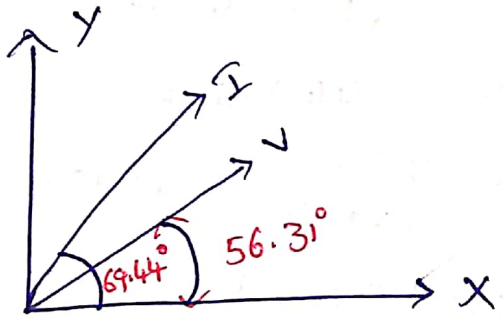
Sol $V = (100 + j150)\ V = 180.28 \angle 56.31^\circ$

$I = (4.5 + j12)\ A = 12.82 \angle 69.44^\circ$

$$Z = \frac{V}{I} = \frac{180.28 \angle 56.31^\circ}{12.82 \angle 69.44^\circ} = 14.06 \angle -13.13^\circ$$

$Z = 14.06\ \Omega$

$\phi = -13.13^\circ$



The current is leading the voltage by 13.13°. Therefore ckt is Capacitive ckt.

The voltage $V = 12\cos(60t + 45^\circ)$ is applied to a 0.1H inductor. find the steady state current through inductor.

Sol $V = 12 \angle 45^\circ$

$\omega = 60$ rad/sec.

$$I = \frac{V}{X_L} = \frac{V}{j\omega L} = \frac{12 \angle 45^\circ}{j60 \times 0.1} = \frac{12 \angle 45^\circ}{60 \times 0.1 \angle 90^\circ} = 2A \angle -45^\circ$$

$i = 2\cos(60t - 45^\circ)$ A

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* A 230V, 50Hz ac supply is applied to a coil of 0.06 H inductance and 2.5 Ω resistance is connected in series with a 6.8 μf capacitor.

Calculate : i) Impedance (ii) current (iii) phase angle b/w current and voltage (iv) power factor.

Sol $X_L = 2\pi fL = 2\pi \times 50 \times 0.06 = 18.84\Omega$

$X_C = \frac{1}{2\pi fc} = \frac{1}{2\pi \times 50 \times 6.8 \times 10^{-6}} = 468\Omega$

i) Impedance $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(2.5)^2 + (18.84 - 468)^2}$

$Z = 449.2\Omega$

ii) $I = \frac{V}{Z} = \frac{230}{449.2} = 0.512A$

series RLC Circuit

$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{18.84 - 468}{2.5}\right)$

$= \tan^{-1}\left(\frac{-449.16}{2.5}\right) = -89.7$

power factor $= \cos\phi = \cos(-89.7) = 0.0056$

* A resistance R_1 and inductance $L = 0.01H$ and capacitance C are connected in series. when an alternating voltage $V = 400\sin(3000t - 20^\circ)$ is applied to the series combination. The current flowing is $10\sqrt{2}\sin(3000t - 65^\circ)$. find the values of R & C.

Sol $\phi = 65^\circ - 20^\circ = 45^\circ$

$X_L = 2\pi fL = \omega L = 3000 \times 0.01 = 30$

$\tan(\phi) = \tan(45^\circ) = 1$

$\tan(\phi) = \frac{X_L - X_C}{R} = 1$

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$$Z = \frac{V_m}{I_m} = \frac{400}{10\sqrt{2}} = 28.28$$

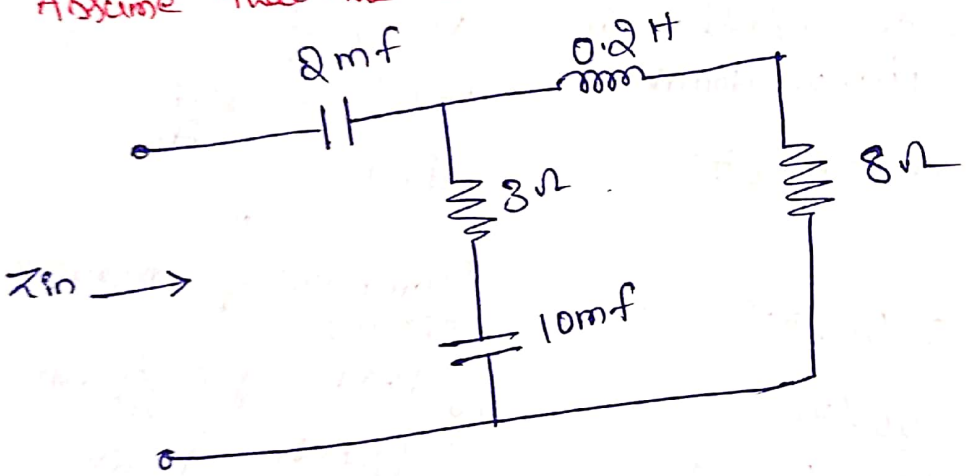
$$Z = R = 28.28$$

$$X_L - X_C = R$$

$$X_C = X_L - R = 30 - 28.28 = 1.72$$

$$X_C = \frac{1}{2\pi fC} \Rightarrow C = \frac{1}{\omega X_C} = \frac{1}{3000 \times 1.72}$$

* find the input impedance of the ckt shown in fig. Assume that the ckt operates at $\omega = 50$ rad/sec.



Sol let

Z_1 = impedance of the 2mf capacitor.

Z_2 = impedance of the 3Ω resistor in series with the 10mf capacitor

Z_3 = impedance of the 0.2H inductor is in series with 8Ω resistor.

$$Z_1 = \frac{1}{j\omega C} = \frac{1}{j50 \times 2 \times 10^{-3}} = -j10\ \Omega$$

$$Z_2 = 3 + \frac{1}{j\omega C} = 3 + \frac{1}{j50 \times 10 \times 10^{-3}} = (3 - j2)\ \Omega$$

$$Z_3 = 8 + j\omega L = 8 + j50 \times 0.2 = (8 + j10)\ \Omega$$

The input impedance is

$$Z_{in} = Z_1 + Z_2 // Z_3 = -j10 + \frac{(3 - j2)(8 + j10)}{11 + j8}$$

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$$= -j10 + \frac{44+j14}{11+j8} \times \frac{11-j8}{11-j8}$$

$$= -j10 + \frac{(44+j14)(11-j8)}{11^2+8^2} = -j10 + 3.22 - j1.07\Omega$$

$$Z_{in} = 3.22 - j11.07\Omega$$

* using the phasor approach, determine the current $i(t)$ in a circuit described by integrodifferential equation.

$$4i + 8\int i\,dt - 3\frac{di}{dt} = 50\cos(2t+75^\circ)$$

Sol

We can transform the equation in time domain to phasor domain.

$$4I + \frac{8I}{j\omega} - 3Ij\omega = 50\angle 75^\circ$$

$\omega = 2$

$$I[4 - j4 - j6] = 50\angle 75^\circ$$

$$I = \frac{50\angle 75^\circ}{4-j10} = \frac{50\angle 75^\circ}{10.77\angle -68.2^\circ}$$

$$I = 4.642\angle 143.2^\circ$$

$$i(t) = 4.642\cos(2t+143.2^\circ)\ A$$

Current - voltage relationship of Inductor.

$$V = L\frac{di}{dt}$$

$$j\omega L I = L\frac{di}{dt}$$

$$j\omega I = \frac{di}{dt}$$

Current-voltage relationship of Capacitor

$$\int i(t)dt = \int C\frac{dv}{dt}\cdot dt$$

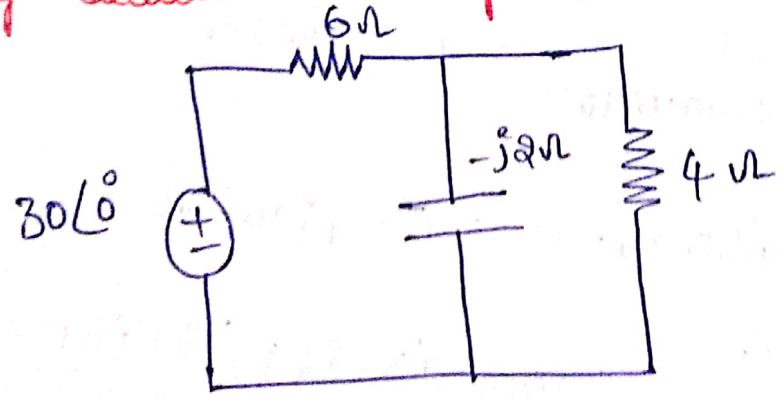
$$\frac{1}{C}\int i(t)dt = \int 1\,dv$$

$$\frac{1}{C}\int i(t)dt = V$$

$$\frac{1}{C}\int i(t)dt = \frac{I}{j\omega C}$$

$$\int i(t)dt = \frac{I}{j\omega}$$

* Determine the power factor of the entire ckt shown in fig. Calculate the average power delivered by the source.



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Sol

The total impedance is

$$Z = 6 + 4 // -j2 = 6 + \frac{4 \times -j2}{4 - j2} = 6.8 - j1.6$$

$$= 7 \angle -13.24^\circ$$

The power factor $\cos\phi = \cos(-13.24) = 0.9734$

The RMS value of the current is

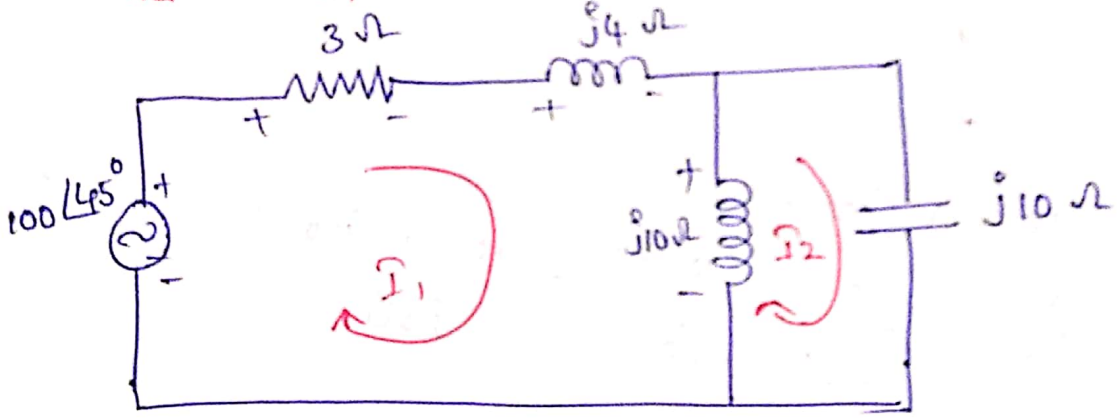
$$I_{rms} = \frac{V_{rms}}{Z} = \frac{30\angle 0^\circ}{7\angle -13.24^\circ} = 4.286 \angle 13.24^\circ$$

The average power supplied by the source is

$$P = V_{rms} I_{rms}$$

$$P = I_{rms}^2 R = (30)(4.286)(0.9734) = 125\,W.$$

* find the mesh currents I_1 & I_2 in the n/w.



Sol Apply KVL to mesh (1)

$$-100\angle 45^\circ + (3 + j4) I_1 + j10 (I_1 - I_2) = 0$$

$$(3 + j14) I_1 - j10 I_2 = 100\angle 45^\circ \longrightarrow (1)$$

Apply KVL to mesh (2)

$$-j10 (I_2 - I_1) + j10 I_2 = 0$$

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$$-j10\bar{I}_2 + j10\bar{I}_1 + j10\bar{I}_2 = 0$$

$$j10\bar{I}_1 = 0$$

$$\boxed{\bar{I}_1 = 0}$$

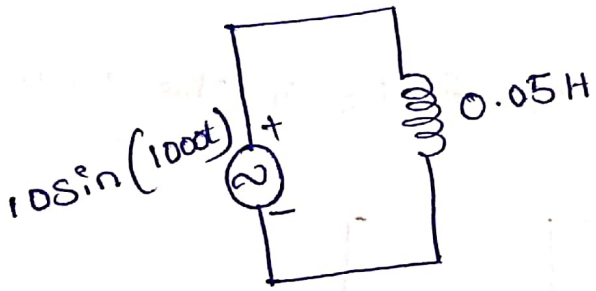
$\bar{I}_1 = 0$ Substitute in eq (1)

$$-j10\bar{I}_2 = 100\angle 45^\circ$$

$$\bar{I}_2 = \frac{100\angle 45^\circ}{-j10} = \frac{10}{10\angle -90}\ 100\angle 45^\circ = 10\angle 45^\circ + 90$$

$$\boxed{\bar{I}_2 = 10\angle 135^\circ\ A}$$

In the given ckt find the current in Amps.



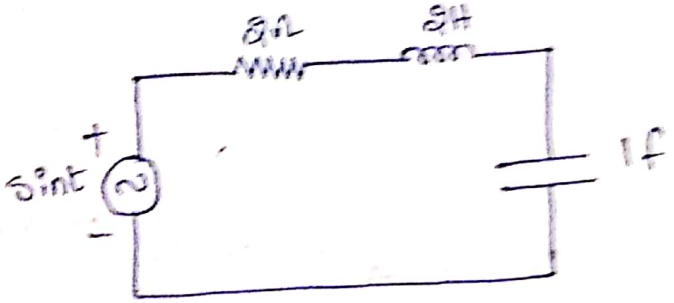
Sol $V = 10\sin(1000t)$

$$\bar{I} = \frac{V}{Z} = \frac{10\sin(1000t)}{j(1000)(0.05)} = \frac{10\sin(1000t)}{j50} = \frac{0.2}{j}\sin(1000t)$$

$$\bar{I} = \frac{0.2\sin(1000t)}{j} = -0.2j\sin(1000t)$$

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find the differential equation for the current $i(t)$ shown in fig.



Sol

$$-\sin t + 2i(t) + 2\frac{di(t)}{dt} + \frac{1}{C}\int i\,dt = 0$$

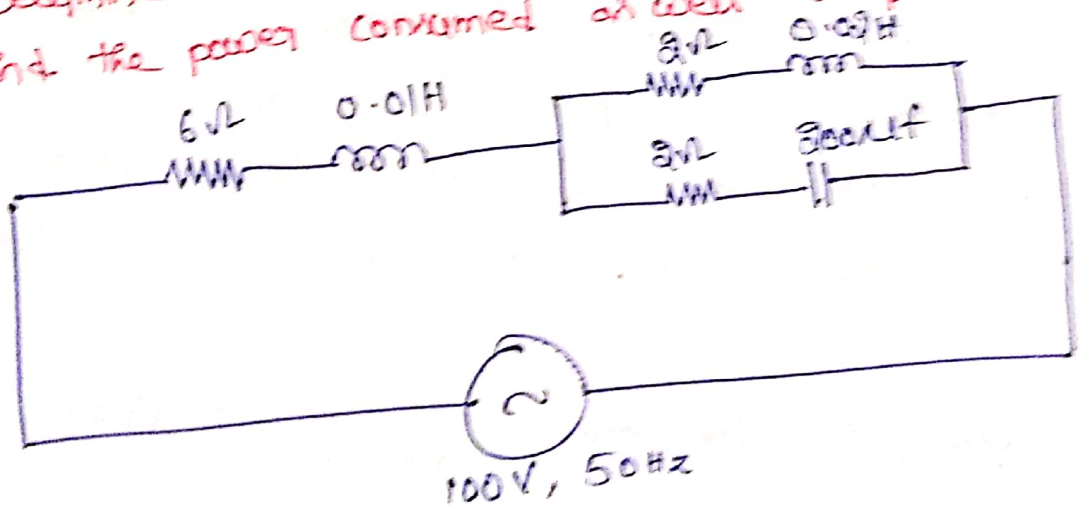
$$2i + 2\frac{di}{dt} + \frac{1}{1}\int i\,dt = \sin t$$

differentiating on both sides

$$2\frac{di}{dt} + 2\frac{d^2i}{dt^2} + i = \cos t$$

$$\boxed{2\frac{d^2i}{dt^2} + 2\frac{di}{dt} + i = \cos t}$$

* Determine the current in the circuit of figure. And also find the power consumed as well as power factor.



Sol

$$X_{L1} = \omega L_1 = 2\pi \times 50 \times 0.01 = 3.14\,\Omega$$

$$X_{L2} = \omega L_2 = 2\pi \times 50 \times 0.02 = 6.28\,\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi \times 50 \times 200 \times 10^{-6}} = 15.92$$

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$$Z_1 = (R_1 + jX_{L1}) = 6 + j3.14$$

$$Z_2 = (R_2 + jX_{L2}) = 4 + j6.28$$

$$Z_3 = 2 - j15.92$$

$$Z = Z_1 + \frac{Z_2 X Z_3}{Z_2 + Z_3} = (6 + j3.14) + \frac{(4+j6.28)(2-j15.92)}{(4+j6.28)+(2-j15.92)}$$

$$Z = 17.27 \angle 30.75^\circ$$

$$I = \frac{V}{Z} = \frac{100\angle 0^\circ}{17.27\angle 30.75^\circ} = 5.79 \angle -30.75^\circ$$

$$P = VI\cos\phi$$

$$= 100 \times 5.79 \times \cos(30.75^\circ)$$

$$P = 497.94 W$$

$$pf = \cos\phi = \cos(30.75^\circ) = 0.86.$$

A series connected load drawn a current $i(t) = 4\cos(100\pi t + 10^\circ)A$ when the applied voltage is $v(t) = 12\cos(100\pi t - 20^\circ)V$. find the apparent power and the power factor of the load. Determine the element values that form the series connected load.

Sol Apparent power $S = V_{rms} I_{rms} = \frac{120}{\sqrt{2}} \frac{4}{\sqrt{2}} = 240 VA$

The power factor is $= \cos\phi$

$$= \cos(\theta_v - \theta_i) = \cos(-20-10)$$

$$= 0.866 \text{ leading}$$

The power factor is leading because the current leads the voltage.

Pf may also be obtained from the load impedance.

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$$Z = \frac{V}{I} = \frac{120\angle -20^\circ}{4\angle 10^\circ} = 30\angle -38 = 25.98 - j15$$

$$Pf = \cos(-30^\circ) = 0.866 \text{ leading.}$$

The load impedance Z can be modeled by a 25.98 Ω resistor in series with a capacitor.

$$X_C = -15 = -\frac{1}{\omega C}$$

(or) $$C = \frac{1}{15\omega} = \frac{1}{15 \times 100\pi} = 212.2\,\mu C$$

$Z = R - jX_C$.

$I = \frac{V}{X_C}$

$X_C I = V$

$X_C = \frac{V}{I}$

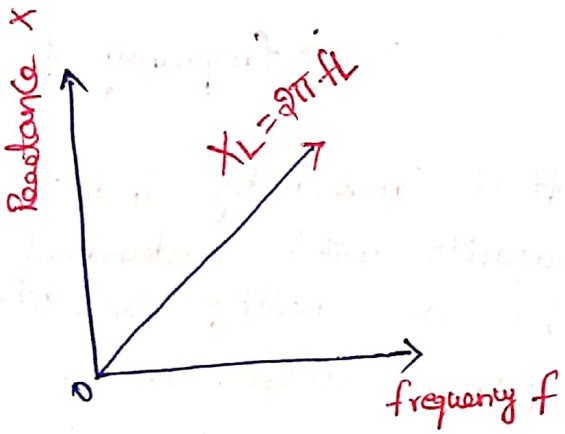
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Resonance in Series RLC circuit :-

Resonance in an AC ckt implies a special frequency determined by the values of the resistance, Capacitance and inductance.

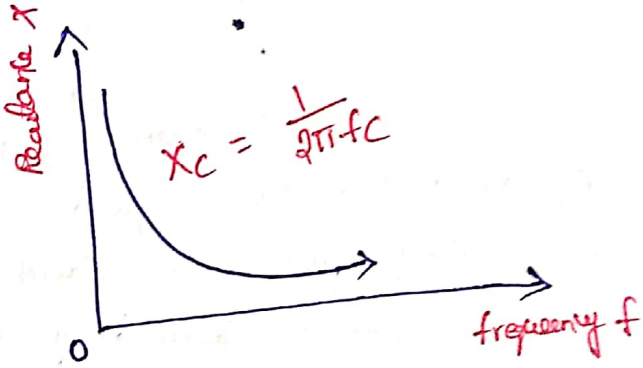
Total voltage applied and current flowing through the circuit is inphase the resultant ckt is a resistive circuit.

For series ~~ckt~~ resonance the condition of resonance is straight forward and it is characterized by minimum impedance and zero phase. when the frequency f is varied both X_L and X_C also varied as both are the functions of f.



$X_L = 2\pi fL$

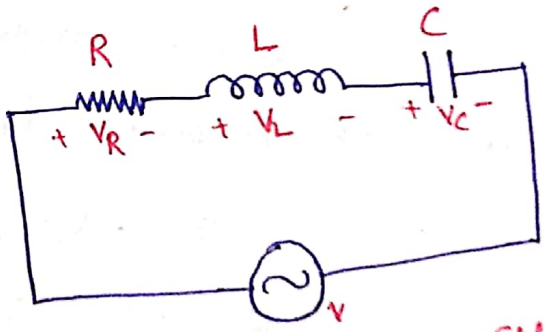
X_L is directly proportional to f



$X_C = \frac{1}{2\pi fC}$

X_C is inversely proportional to f

* At a certain frequency X_L becomes equal to X_C. Such a condition when $X_L = X_C$ for a certain frequency is called <u>series resonance</u>.



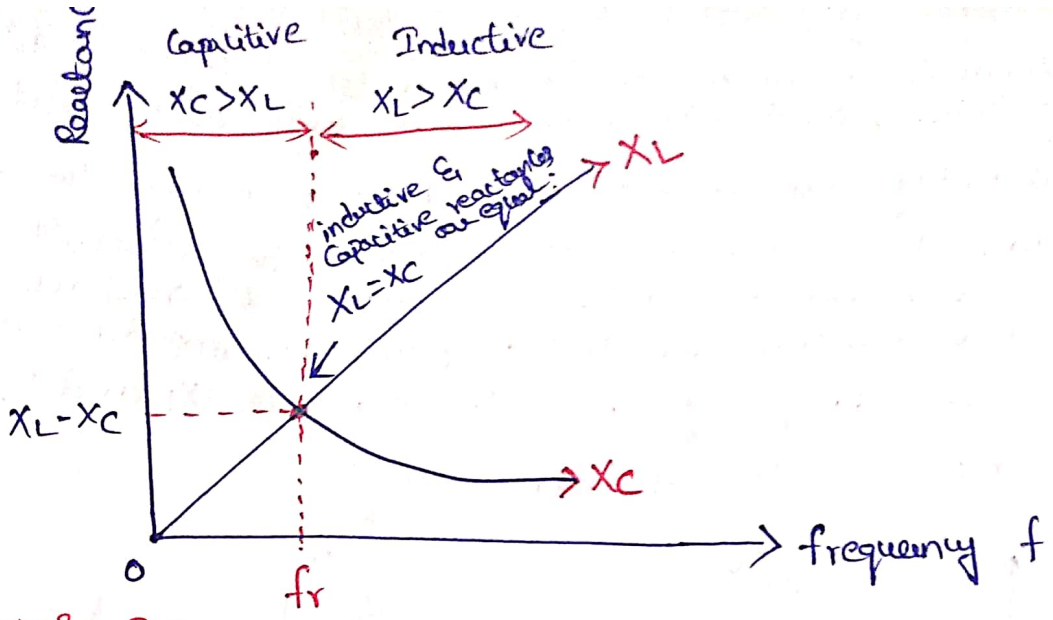
Series RLC Ckt.

(i) For frequencies which are less than resonant frequency $f < f_r$ circuit behaves like capacitive circuit.

(ii) For frequencies which are more than resonant frequency $f > f_r$ circuit behaves like inductive ckt.

(iii) At resonant frequency $f = f_r$ the circuit behaves like resistive ckt.

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Series Resonance.

X_L and X_C are equal at that point the effect caused by inductor will be nullify; the effect caused by Capacitor will be redundant. So the effect caused by L & C are nullify each other, only the passive element i.e resistor is active alone. Capacitor and inductor will act as a redundant leaving behind only resistive circuit.

Expression for Resonant frequency:

Let fr be the resonant frequency

$$X_L = X_C$$

$$\omega L = \frac{1}{\omega C}$$

$$2\pi f_r L = \frac{1}{2\pi f_r C}$$

$$f_r^2 = \frac{1}{4\pi^2 LC}$$

$$\boxed{f_r = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}}$$

$$\omega_r = \frac{1}{\sqrt{LC}} \text{ rad/sec.}$$

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At Resonance:

→ Impedance is minimum.
→ Current is maximum
→ phase angle is 0°
→ power factor is unity.

At resonance impedance become minimum, before resonance and after resonance impedance is given by

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

At resonance $X_L = X_C$

$$Z = R$$

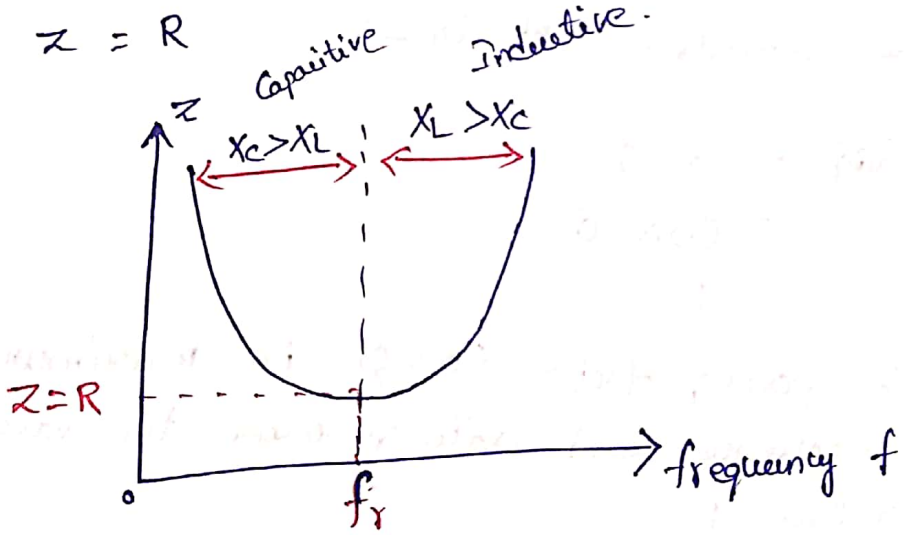
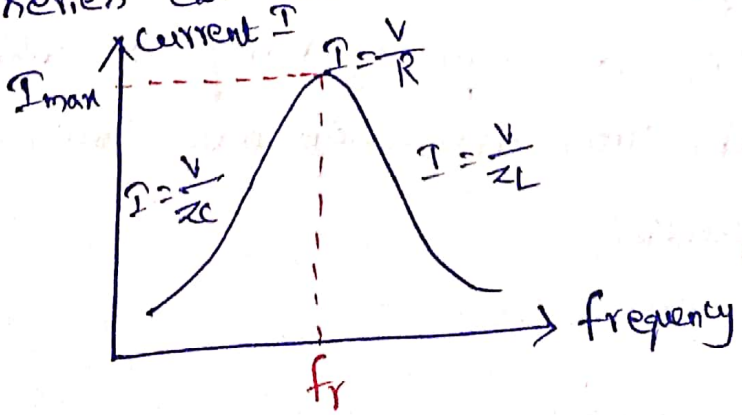


fig: Impedance Vs frequency.

In a series ckt current is maximum the impedance is minimum



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phase angle 0°

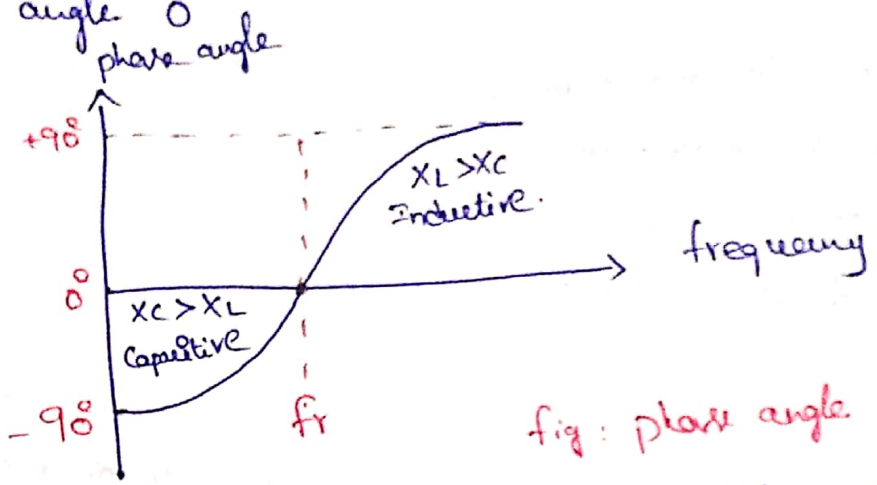


fig: phase angle Vs frequency.

At resonance the Ckt will be a resistive Ckt & the angle will be 0°.

Beyond resonance the Ckt will be inductive Ckt. The angle for the inductive Ckt is +ve.

Before resonance the Ckt will be Capacitive Ckt. The angle for the Capacitive Ckt is -ve.

$$\text{power factor} = \cos\phi$$
$$= \cos 0^\circ$$
$$= 1$$

At resonance power factor $\cos\phi$ is maximum that is 1. Before resonance and after resonance the value of $\cos\phi$ is less than 1

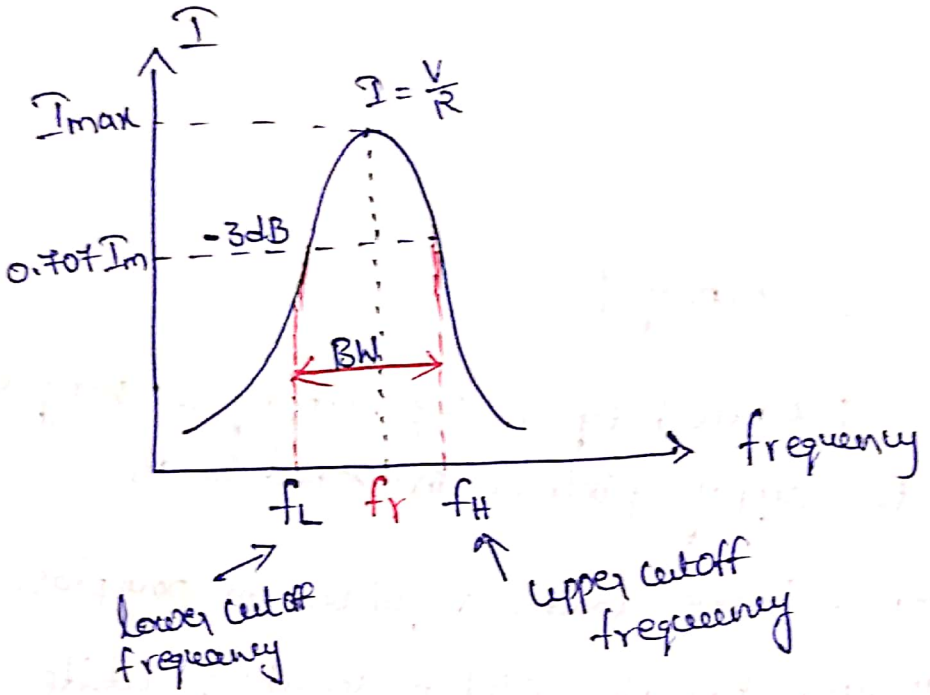
Bandwidth :-

Bandwidth is a selectivity of a Ckt to receive or transmit the signal.

The bandwidth is a frequency range in which the signal is accepted. Those frequencies are lower cutoff and upper cutoff frequencies.

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The difference between the lower cutoff frequency and upper cutoff frequency at which current is 0.707 I_m (corresponds to half of maximum power).



Bandwidth = $f_H - f_L$

Lower cutoff frequency $f_L = f_r - \frac{BW}{2}$

Upper cutoff frequency $f_H = f_r + \frac{B.W}{2}$

Quality factor:-

The quality factor of R-L-C series circuit is the voltage magnification in the ckt at resonance.

$$\text{Voltage magnification} = \frac{\text{Voltage across L or C}}{\text{Supply voltage}} = \frac{V_L}{V}$$

$V_L = I \omega_r L$ and $V = IR$

$$Q = \frac{\omega_r L}{R} = \frac{\omega_r}{B.W} \qquad BW = \frac{R}{L}$$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} \qquad \omega_r = \frac{1}{\sqrt{LC}}$$

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Need of 3-ϕ supply :-

→ Voltage produced by a 1-ϕ supply is very less and not enough to supply practical industrial loads.

→ Industrial heavy loads require a polyphase supply. phase means branch, Ckt or winding while poly means many.

→ In polyphase system there are many independent ac voltages present simultaneously (depends on no. of many phases).

3-ϕ system :-

→ The magnitude & frequencies of a.c voltages are same and phase difference of $\frac{360^\circ}{3} = 120^\circ$ with respect to each other.

Advantages of 3-ϕ system over 1-ϕ system :-

→ The o/p of 3-ϕ machine is always greater than o/p of 1-ϕ machine of same size.

→ Transmission of 3-ϕ power is very much economical as it needs less copper or less conducting material as compared to 1-ϕ power.

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→ Rotating magnetic field (RMf) can be generated in 3-ϕ supply. hence 3-ϕ machines are self starting. with the help of 1-ϕ supply we can not produced RMf. hence 1-ϕ machines are not self started.

→ 3-ϕ supply give rise to 3 1-ϕ supplies but vice versa is not possible. (we cannot obtained 3-ϕ supply from 1-ϕ supply).

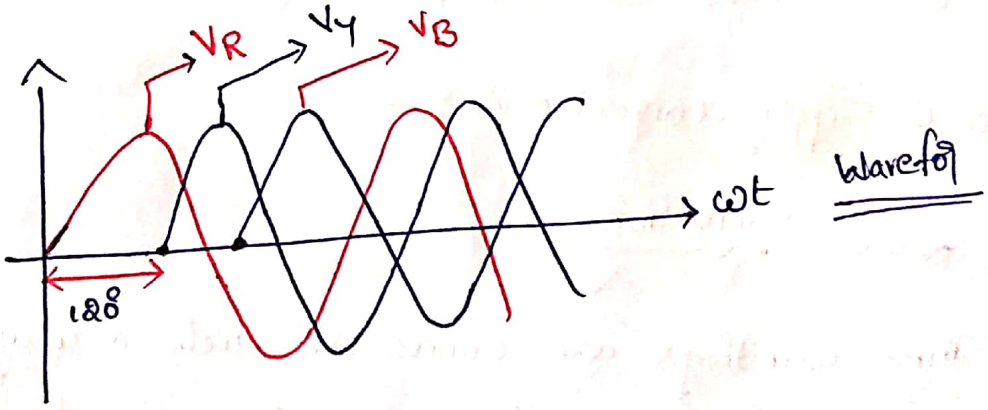
→ Rectifier DC output voltage is smoother (having less ripple) if no. of phases are increased at the supply side.

Generation of 3-ϕ supply :-

$V_R = V_m \sin \omega t$

$V_Y = V_m \sin(\omega t - 120^\circ)$

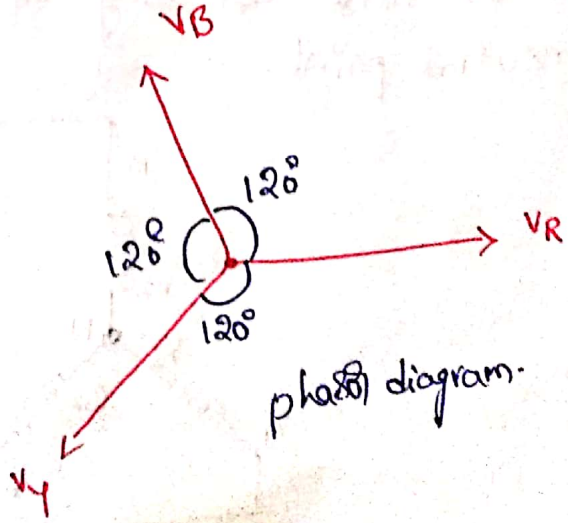
$V_B = V_m \sin(\omega t - 240^\circ)$



waveform

V_Y lags V_R by 120°

V_B lags V_R by 240°.



phasor diagram.

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Symmetrical System.

If in a 3-ϕ system the three voltages are of same magnitude and frequency & displaced from each other by 120° phase angle it is defined as symmetrical system.

phase sequence :-

→ The sequence in which the voltages in 3-ϕ supply reach their maximum positive values is called phase sequence.

→ Generally the phase sequence is R-Y-B.

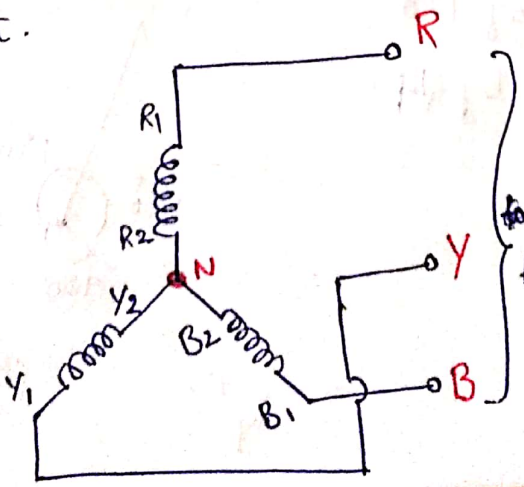
Need of phase sequence:

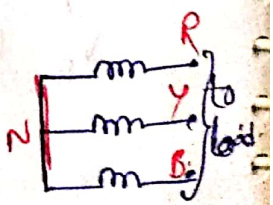
The phase sequence is important in determining the direction of rotation of a.c motors and parallel operation of ~~alternators~~. alternators.

3-ϕ Supply Connections :-

① STAR Connection :-

Three windings are connected such a way that one end of each winding is connected together to form a neutral point.

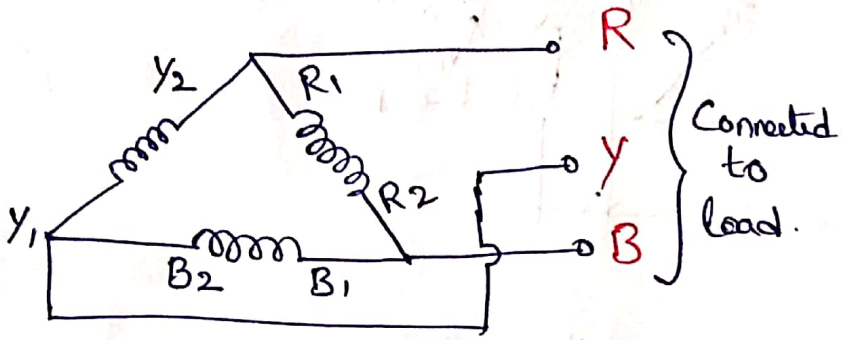


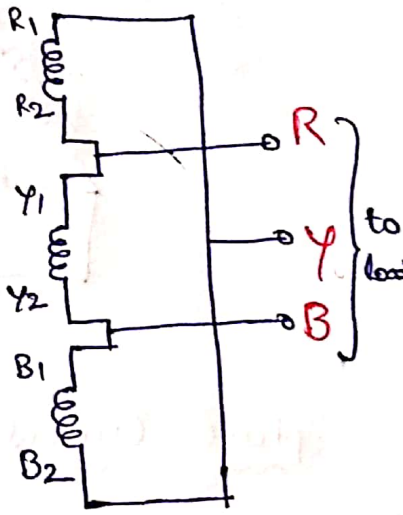


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(2) DELTA Connection :-

In delta connection three windings are connected in such a way that they will form a loop.

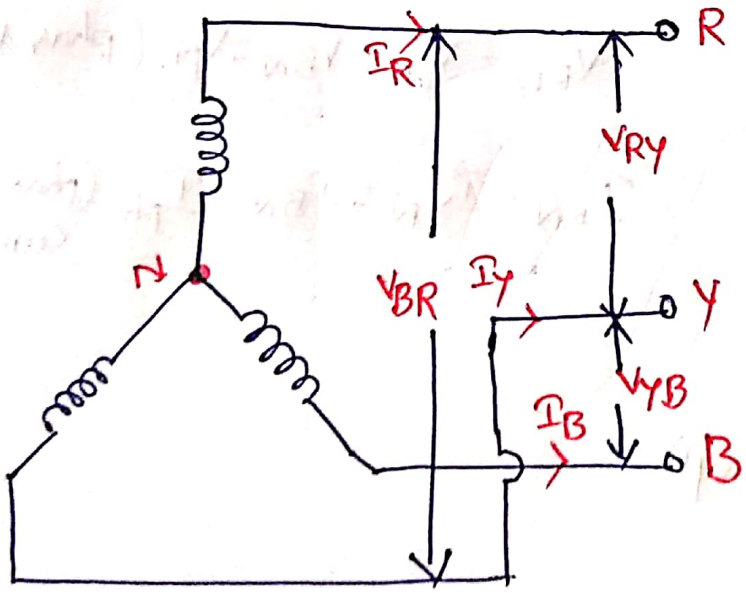




Line current and Line voltage :-

* Current flowing through any line is called line current.

* The voltage b/w the two any lines is called line voltage.

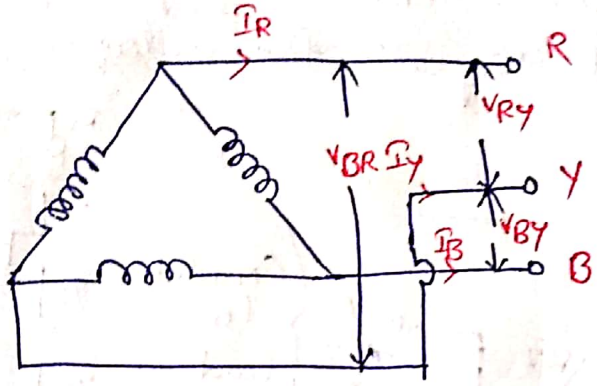


$V_{RY} = V_{YB} = V_{BR} = V_L$ (line voltage)

$I_R = I_Y = I_B = I_L$ (line current)

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Delta Connection :



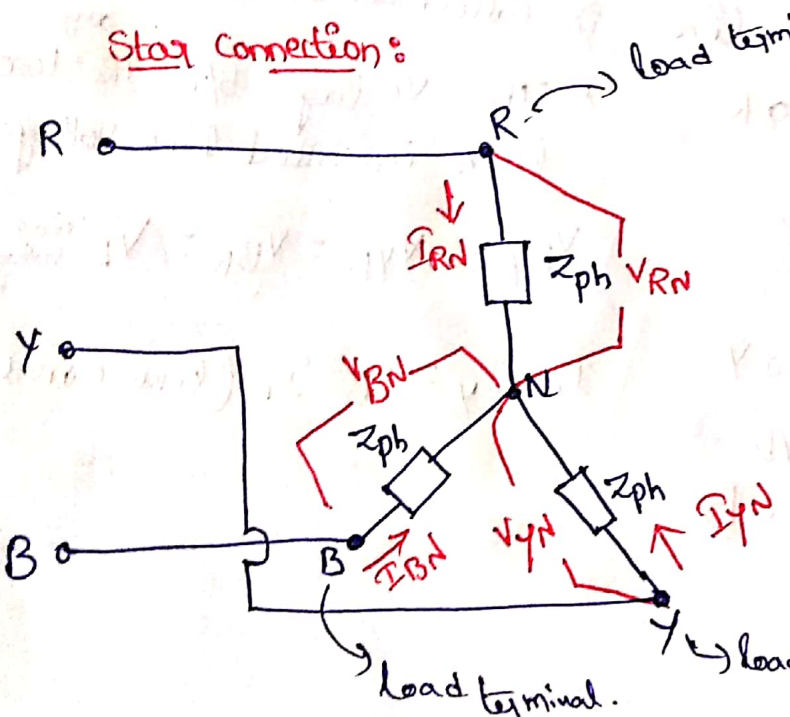
$$I_R = I_Y = I_B = I_L$$

$$V_{RY} = V_{BY} = V_{BR} = V_L$$

phase Current & phase voltage :-

Voltage across phase impedance is called phase voltage
Current carrying by the phase impedance is called phase current.

Star Connection :



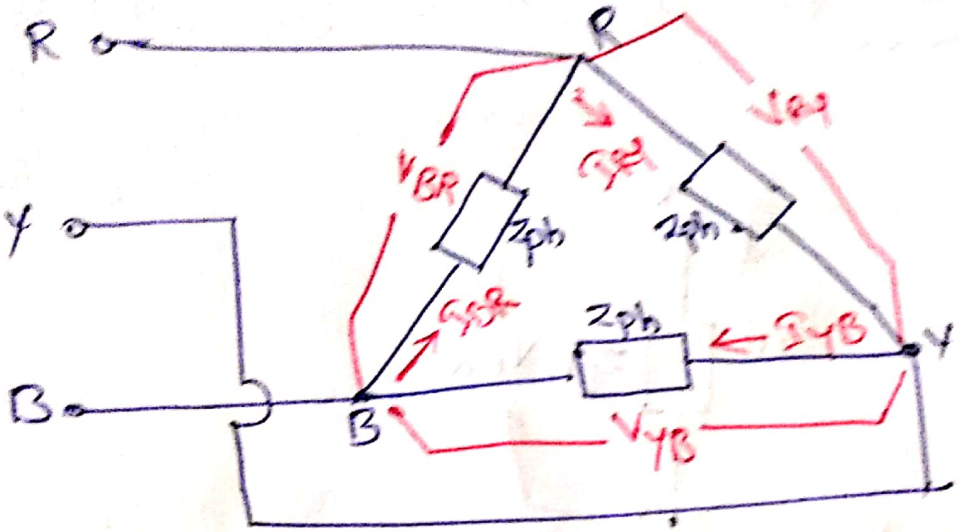
$$V_{RN} = V_{YN} = V_{BN} = V_{ph} \text{ (phase voltage)}$$

$$I_{RN} = I_{YN} = I_{BN} = I_{ph} \text{ (phase current).}$$

Balanced 3-ϕ load i.e the magnitude of phase impedance is same and nature of the impedance is also same.
Nature means the power factor of all the three impedances are same.

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Delta connected load:

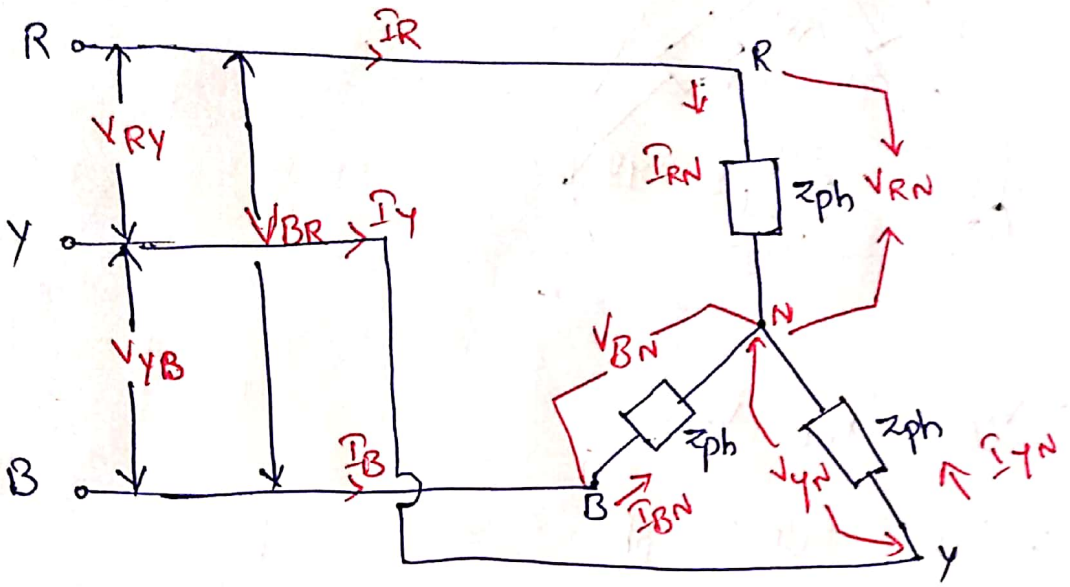


$$V_{RY} = V_{YB} = V_{BR} = V_{ph}$$

$$I_{RY} = I_{YB} = I_{BR} = I_{ph}$$

Scanned with CamScanner

Relationship between the Line & phase quantities for Star (Y) Connected load :-



$V_{RY} = V_{YB} = V_{BR} = V_L$ (line voltage)

$I_R = I_Y = I_B = I_L$ (line current).

$V_{RN} = V_{YN} = V_{BN} = V_{ph}$ (phase voltage)

$I_{RN} = I_{YN} = I_{BN} = I_{ph}$ (phase current).

Z_{ph} is the phase impedance.

from the observations

$I_R = I_{RN}$

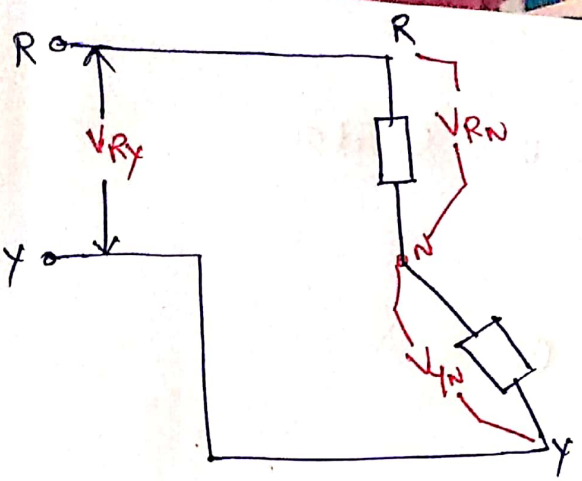
$I_Y = I_{YN}$

$I_B = I_{BN}$

∴ Line currents is equal to the phase currents for the star connection load.

$$I_L = I_{ph}$$

Scanned with CamScanner



Apply KVL to the loop

$$V_{RY} = V_{RN} + V_{NY}$$

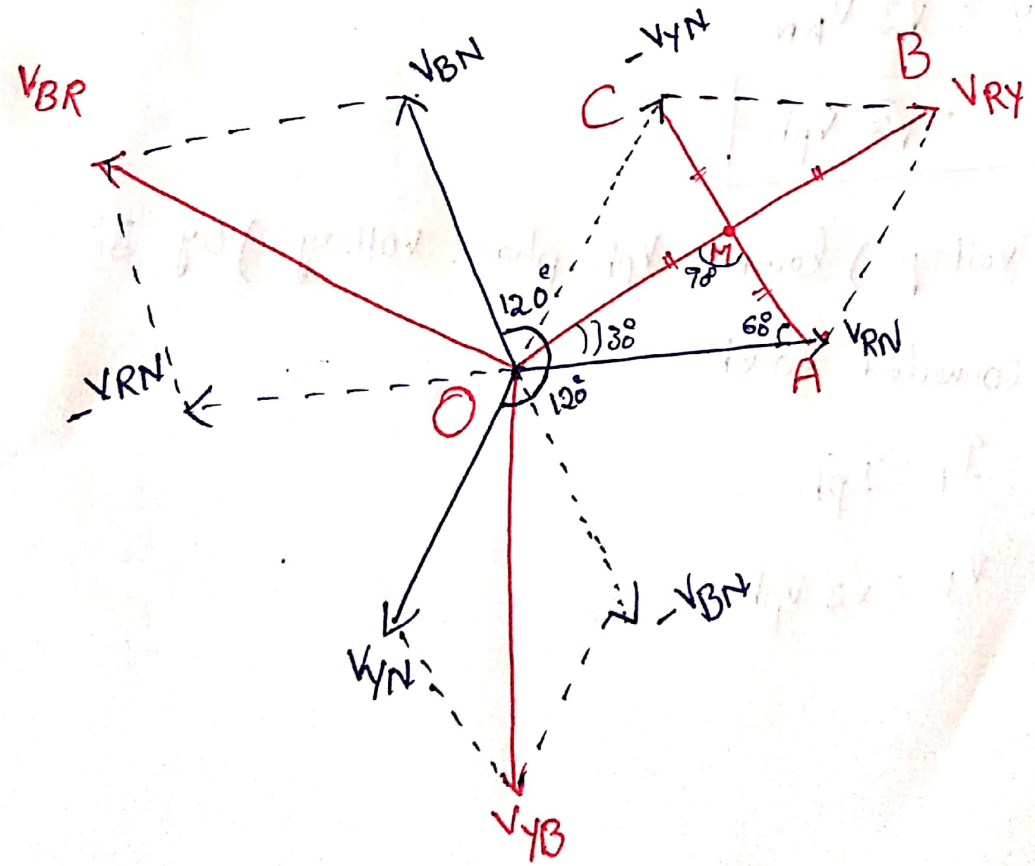
$$V_{NY} = -V_{YN}$$

$$V_{RY} = V_{RN} + (-V_{YN})$$

$$V_{YB} = V_{YN} + (-V_{BN})$$

$$V_{BR} = V_{BN} + (-V_{RN})$$

phasor Diagram :-



Scanned with CamScanner

OABC is a Rhombus

Rhombus is having the perpendicular bisector

$OB \perp AC$

$$OB = 2(OM) \longrightarrow (1)$$

In ΔOMA

measure angle $OAM = 60^\circ$

In the Right angle triangle the side which is opposite to 60° angle. $\sin 60^\circ = \frac{\text{opposite side}}{\text{hypotenuse}} \Rightarrow \frac{\sqrt{3}}{2} = \frac{OM}{OA}$

$$OM = \frac{\sqrt{3}}{2} \times (OA) \longrightarrow (2)$$

$\sin 60^\circ = \frac{\sqrt{3}}{2}$

from (1) & (2)

$$OB = 2 \times \frac{\sqrt{3}}{2} OA$$

$$OB = \sqrt{3}\, OA$$

$$V_{RY} = \sqrt{3}\, V_{RN}$$

$$\boxed{V_L = \sqrt{3}\, V_{ph}}$$

V_L (line voltage) leads V_{ph} (phase voltage) by 30°.

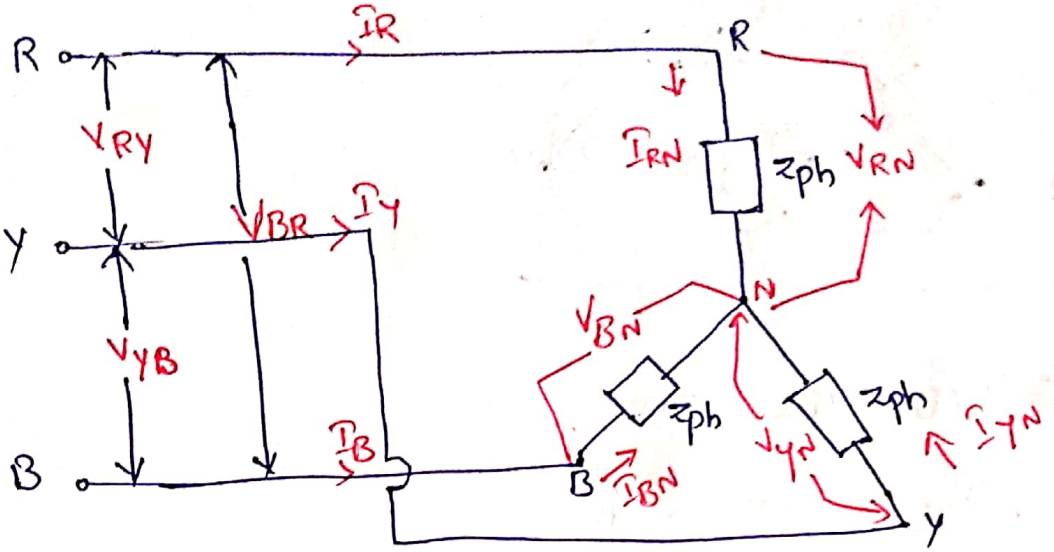
for star connected load.

$$I_L = I_{ph}$$

$$V_L = \sqrt{3}\, V_{ph}$$

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Relationship between the Line & phase quantities for Star (Y) connected load :-



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Z_{ph} is the phase impedance.

from the observations

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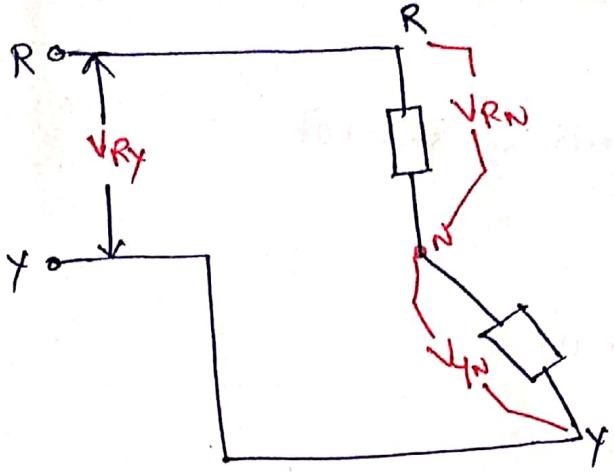
$I_Y = I_{YN}$

$I_B = I_{BN}$

∴ Line currents is equal to the phase currents for the star connection load.

$$I_L = I_{ph}$$

Scanned with CamScanner



Apply KVL to the loop

$$V_{RY} = V_{RN} + V_{NY}$$

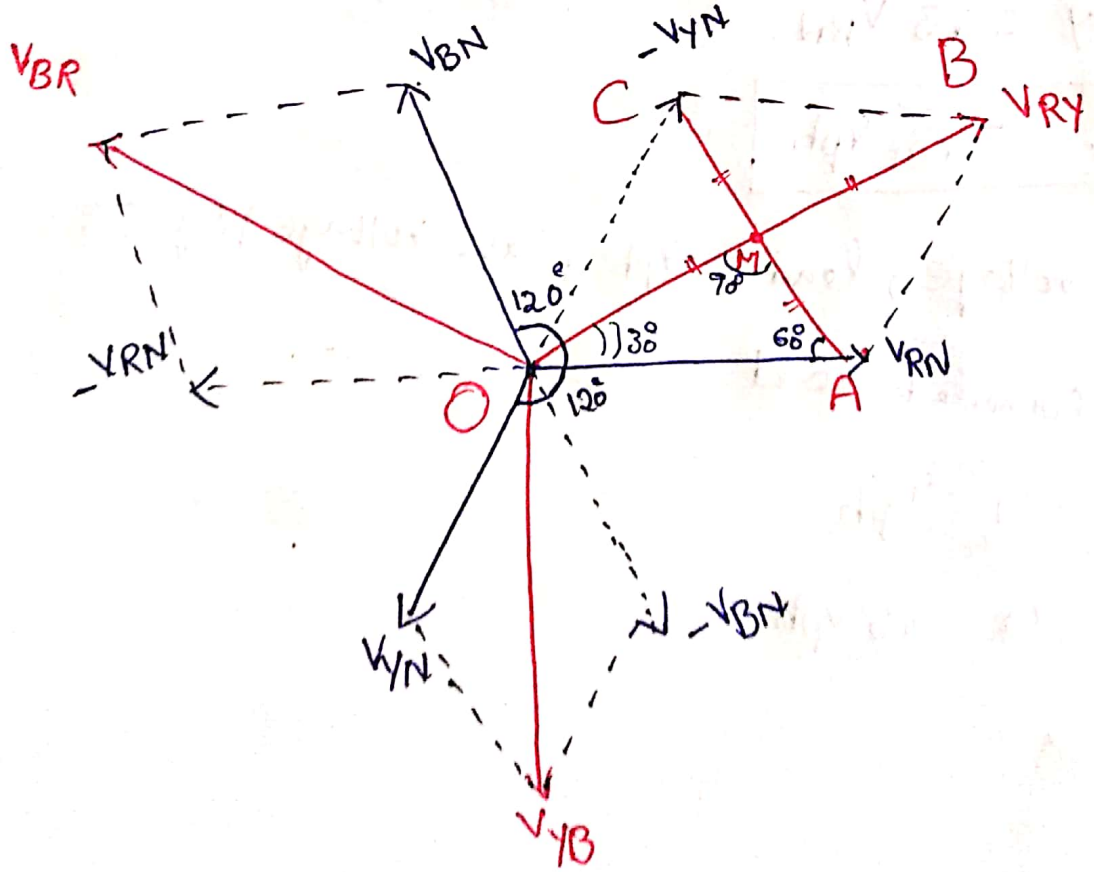
$$V_{NY} = -V_{YN}$$

$$V_{RY} = V_{RN} + (-V_{YN})$$

$$V_{YB} = V_{YN} + (-V_{BN})$$

$$V_{BR} = V_{BN} + (-V_{RN})$$

phasor Diagram :-



Scanned with CamScanner

OABC is a Rhombus

Rhombus is having the perpendicular bisector

$OB \perp AC$

$$OB = 2(OM) \longrightarrow (1)$$

In ΔOMA

measure angle $OAM = 60^\circ$

In the Right angle triangle the side which is opposite to 60° angle. $\sin 60^\circ = \frac{\text{opposite side}}{\text{hypotenuse}} \Rightarrow \frac{\sqrt{3}}{2} = \frac{OM}{OA}$

$$OM = \frac{\sqrt{3}}{2} \times (OA) \longrightarrow (2)$$

$\sin 60^\circ = \frac{\sqrt{3}}{2}$

from (1) & (2)

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$$OB = \sqrt{3}\, OA$$

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$$\boxed{V_L = \sqrt{3}\, V_{ph}}$$

V_L (line voltage) leads V_{ph} (phase voltage) by 30°.

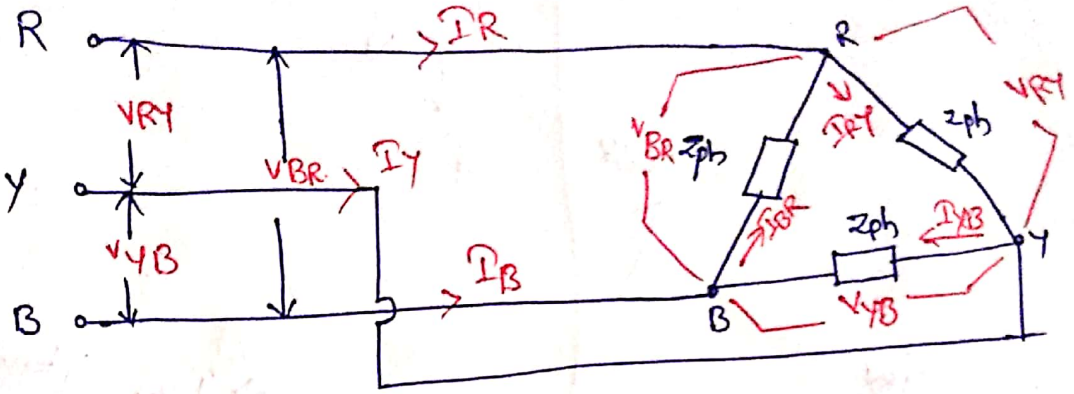
for star connected load.

$$I_L = I_{ph}$$

$$V_L = \sqrt{3}\, V_{ph}$$

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Delta connected load :-



* $V_{RY} = V_{YB} = V_{BR} = V_L$ (line voltage)

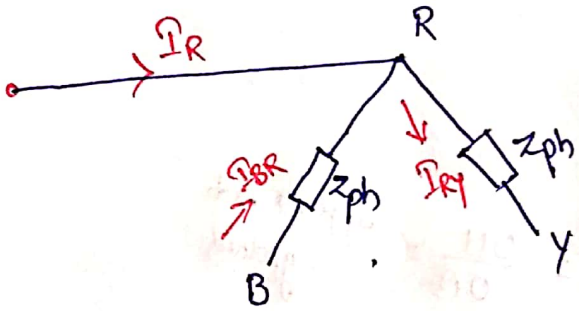
$I_R = I_Y = I_B = I_L$ (line current).

* $V_{RY} = V_{YB} = V_{BR} = V_{ph}$ (phase voltage).

$I_{RY} = I_{YB} = I_{BR} = I_{ph}$ (phase current)

$Z_{ph} \Rightarrow$ phase impedance

$\boxed{V_L = V_{ph}}$ for Delta connected load.



Apply KCL at node R

$I_R + I_{BR} = I_{RY}$

$\therefore I_R = I_{RY} - I_{BR}$

Apply KCL at node Y

$I_Y + I_{RY} = I_{YB}$

$\therefore I_Y = I_{YB} - I_{RY}$

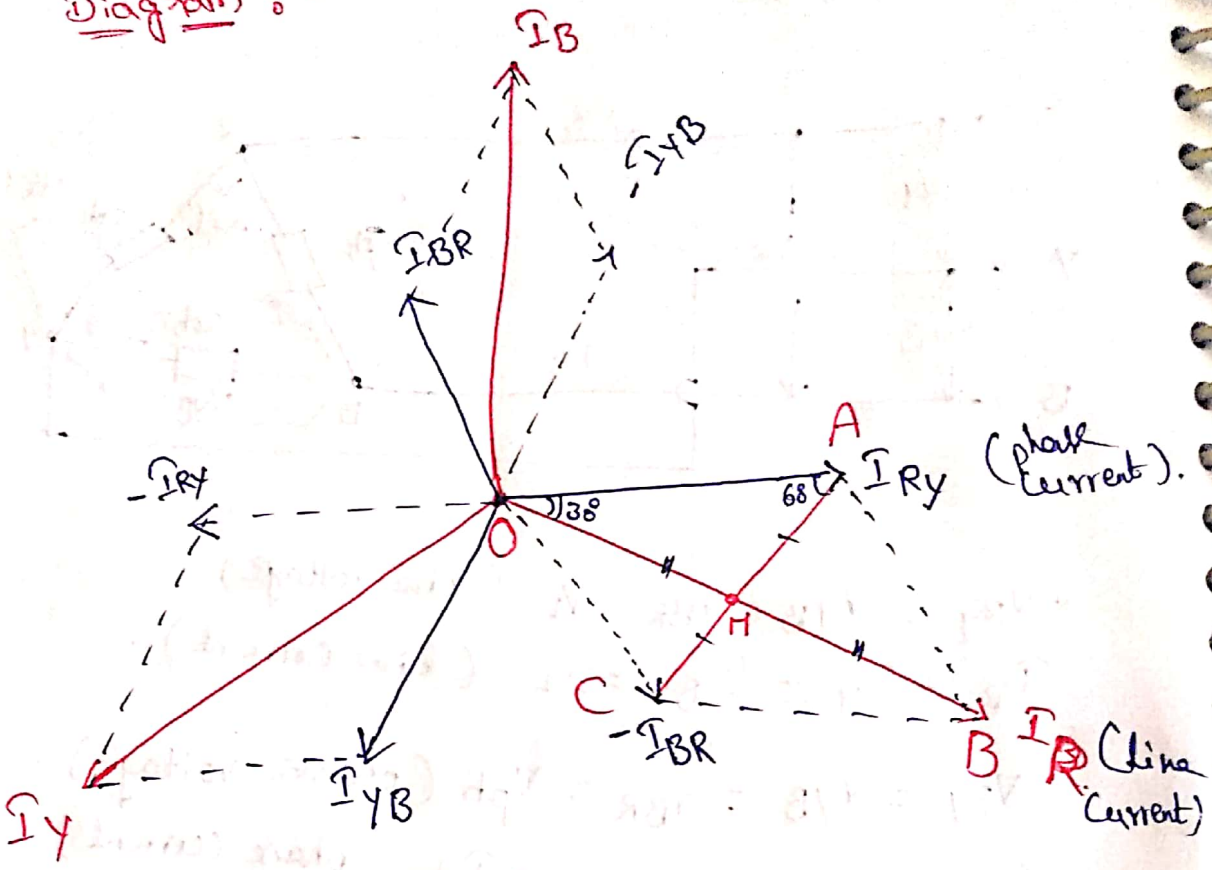
Apply KCL at node B

$I_B + I_{YB} = I_{BR}$

$\therefore I_B = I_{BR} - I_{YB}$

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phasor Diagram :-



□ OABC is a Rhombus

In a Rhombus Diagonals are perpendicular bisector of each other.

$OB \perp AC$

$OB = 2(OM)$

In Δ OMA

measure angle is 60°

$\sin 60° = \frac{OM}{OA} = \frac{\text{opposite side}}{\text{hypotenuse}}$

$OM = \frac{\sqrt{3}}{2} \times OA$

$OB = 2\,OM.$

$OB = \cancel{2} \times \frac{\sqrt{3}}{\cancel{2}} \times OA$

$OB = \sqrt{3}\,OA$

$I_R = \sqrt{3}\, I_{Ry}$

$\boxed{I_L = \sqrt{3}\, I_{ph}}$

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Line current I_L lags I_{ph} by 30°

In delta connected load

$$V_L = V_{ph}$$

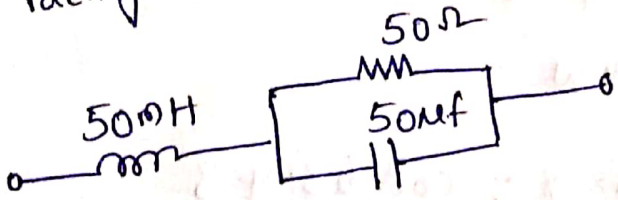
$$I_L = \sqrt{3}\, I_{ph}$$

Problem:

Each phase of a delta connected load consists of a 50mH inductor in series with a parallel combination of 50Ω resistor and 50μf capacitor. The load is connected to 3-phase 550V, 800 rad/sec ac supply. find

a) phase current
b) line current
c) power
d) power factor
e) Reactive power
f) KVA rating of the load.

Sol



$$X_L = \omega L = 800 \times 50 \times 10^{-3} = 40\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{800 \times 50 \times 10^{-6}} = 25\Omega$$

~~To~~ $$z_L = j\omega L = jX_L$$

$$z_C = \frac{1}{j\omega C} = -jX_C$$

~~To~~ phase impedance

$$z_{ph} = jX_L + (R // -jX_C)$$

$$z_{ph} = j40 + \frac{50 \times (-j25)}{50 + (-j25)}$$

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$Z_{ph} = 22.3607 \angle 63.43^\circ \Omega$

$\phi = 63.43^\circ$

$V_L = 550V$

Delta connected load

$V_{ph} = V_L$

$V_{ph} = 550V$

$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{550}{22.3607}$ I

$I_{ph} = 24.5967\ A$

$I_L = \sqrt{3}\, I_{ph} = \sqrt{3} \times 24.5967$

$I_L = 42.6028\ A$

$P = 3\, V_{ph} I_{ph} \cos\phi$

$P = 3 \times 550 \times 24.5967 \times \cos 63.43^\circ$

$P = 18.1531\ kW$

Power factor $= \cos\phi = \cos 63.43$

$pf = 0.4473$ (lagging)

coherence the impedance angle is +ve that ckt is called inductive.

Reactive power $Q = 3 V_{ph} I_{ph} \sin\phi$

$Q = 3 \times 550 \times 24.5967 \times \sin 63.43$

$Q = 36.2984\ KVAR$

KVAR → Kilovolt ampere reating.

KVA rating of the load $S = 3 V_{ph} I_{ph}$

$= 3 \times 550 \times 24.596$

$S = 40.5846\ KVA$

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power Consumed in a 3-ϕ circuit :-

Active / Real / True power :-

$P = 3 V_{ph} I_{ph} \cos\phi_{ph}$

$P = \sqrt{3}\, V_L I_L \cos\phi_{ph}$

> ϕ_{ph} is the angle b/w V_{ph} & I_{ph}

↳ Interms of line quantities.

units for active power is W / KW / MW

Reactive power

$Q = 3 V_{ph} I_{ph} \sin\phi_{ph}$

$Q = \sqrt{3}\, V_L I_L \sin\phi_{ph}$

> ϕ_{ph} is the angle b/w V_{ph} & I_{ph}.

units for reactive power is VAR / KVAR / MVAR

Apparent power :-

$S = 3 V_{ph} I_{ph}$

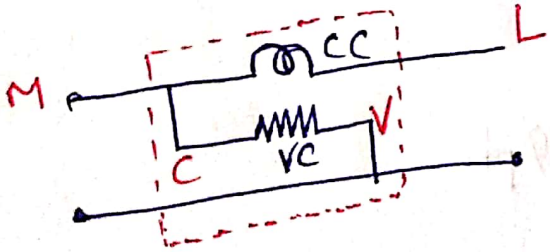
$S = \sqrt{3}\, V_L I_L$

units for apparent power is VA / KVA / MVA

power measurements in 3-ϕ wattmeters :-

power is measured in a.c circuit with the help of wattmeter.

wattmeter connections.



M - main
L - load
C - common
V - voltage.

CC - Current coil → to sense current.
VC - voltage coil → to sense voltage.

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Current coil is acts like ammeter. and ammeter has a very low resistance in series with a galvanometer.
So the current coil should have the very low resistance.
the current coil is madeup of a thick coire and also a less no. of turns. $R = \rho \frac{L}{A}$. (Increasing the area and decreasing the length by having less no. of turns).

voltage coil is like a ~~voltage~~ volt meter. It is having high resistance in parallel with a galvanometer. It is having more no. of turns. It is a thin coire. If the no. of turns is high compared to current coil the resistance will be high.

$$P = V I \cos\phi$$

Relationship b/w P_{star} & P_{delta} connected loads :-

V_L is same for star & delta connected load.

Z_{ph} is same for star & delta connected load.

star load.

$$P_{star} = 3 V_{ph} I_{ph} \cos\phi_{ph}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}}$$

$$P_{star} = 3 V_{ph} \frac{V_{ph}}{Z_{ph}} \cos\phi_{ph}$$

$$P_{star} = \frac{3 V_{ph}^2 \cos\phi_{ph}}{Z_{ph}}$$

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$$V_{ph} = \frac{V_L}{\sqrt{3}}$$

$$P_{Star} = 3\frac{\left(\frac{V_L}{\sqrt{3}}\right)^2}{Z_{ph}} \cos\phi_{ph}$$

$$P_{Star} = 3\frac{V_L^2}{3Z_{ph}} \cos\phi_{ph}$$

$$\therefore P_{Star} = \frac{V_L^2 \cos\phi_{ph}}{Z_{ph}}$$

$$P_{Delta} = 3 V_{ph} I_{ph} \cos\phi_{ph}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}}$$

$$P_{Delta} = 3 V_{ph} \cdot \frac{V_{ph}}{Z_{ph}} \cos\phi_{ph}$$

$$P_{Delta} = 3\frac{V_{ph}^2}{Z_{ph}} \cos\phi_{ph}$$

$$V_{ph} = V_L$$

$$P_{Delta} = \frac{3V_L^2}{Z_{ph}} \cos\phi_{ph}$$

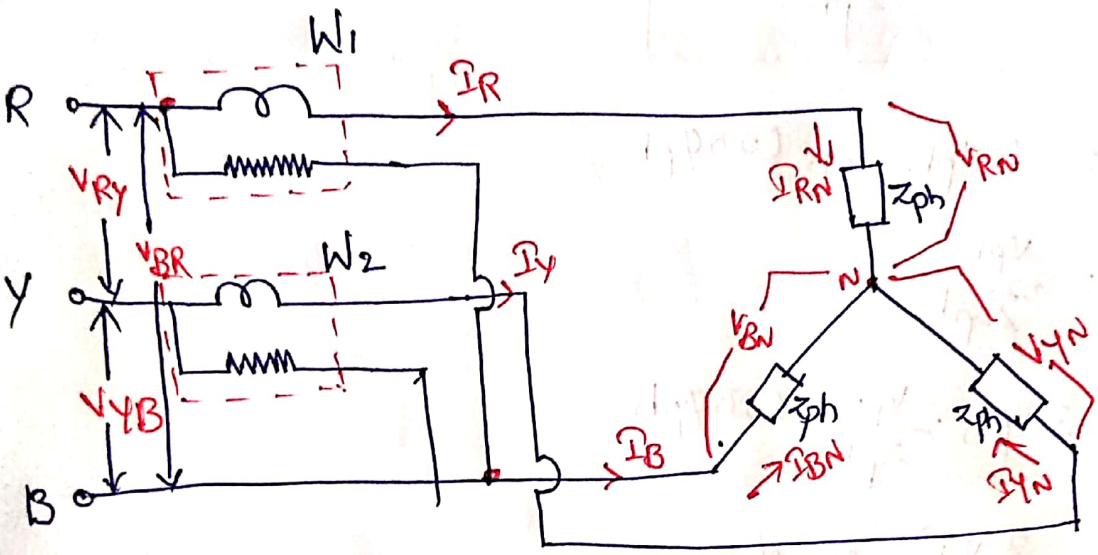
$$P_{Delta} = 3 P_{Star}$$

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Two wattmeter method for 3-ϕ power measurement :-

→ Star connected lagging power factor (Inductive) load.
→ Star connected leading p.f (capacitive) load.
→ Delta connected Lagging p.f (Inductive) load.
→ Delta connected leading p.f (capacitive) load.

1) Star connected lagging p.f load :



V_{RY}, V_{YB}, V_{BR} = V_L (line voltage)

I_R, I_Y, I_B = I_L (line current)

V_{RN}, V_{YN}, V_{BN} = V_{ph} (phase voltage)

I_{RN}, I_{YN}, I_{BN} = I_{ph} (phase current)

Z_{ph} = phase Impedance.

wattmeter reading :-

wattmeter 1

current = I_R

voltage = V_{RB}

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$$W_1 = V_{RB} I_R \cos(V_{RB} \wedge I_R)$$

$$W_1 = V_L I_L \cos(V_{RB} \wedge I_R)$$

wattmeter 2

Current = I_Y

Voltage = V_{YB}

$$W_2 = V_{YB} \cdot I_Y \cos(V_{YB} \wedge I_Y)$$

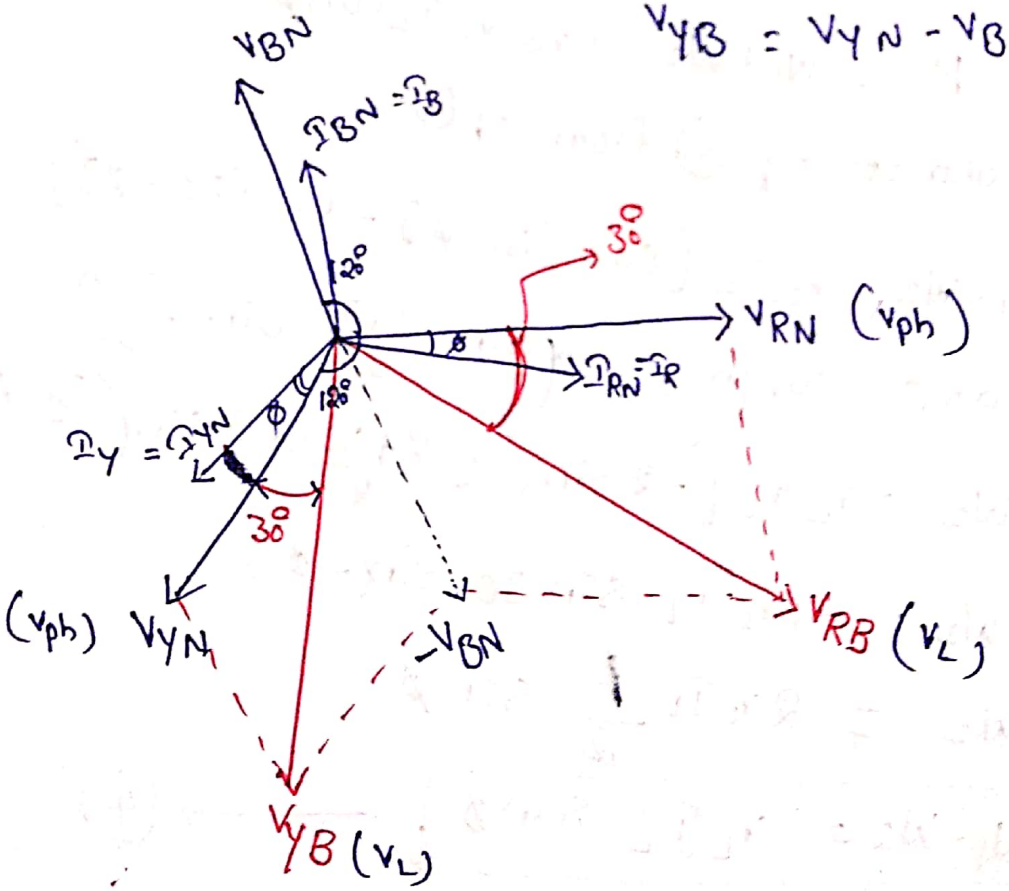
$$W_2 = V_L I_L \cos(V_{YB} \wedge I_Y)$$

Angle determination can be done with the help of phasor diagram

phasor diagram:-

$$V_{RB} = V_{RN} - V_{BN}$$

$$V_{YB} = V_{YN} - V_{BN}$$



Angle b/w $V_{RB} \wedge I_R = 30° - \phi$

$V_{YB} \wedge I_Y = 30° + \phi$

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$$W_1 = V_L I_L \cos(30-\phi) \longrightarrow (1)$$

$$W_2 = V_L I_L \cos(30+\phi) \longrightarrow (2)$$

(1) + (2)

$$W_1 + W_2 = V_L I_L \left[\cos(30-\phi) + \cos(30+\phi)\right]$$

$$\because \cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$$

$$W_1 + W_2 = V_L I_L \, 2\cos\left(\frac{30-\phi+30+\phi}{2}\right)\cos\left(\frac{30-\phi-30-\phi}{2}\right)$$

$$W_1 + W_2 = 2 V_L I_L \cos 30^\circ \cos(-\phi)$$

$$W_1 + W_2 = 2 V_L I_L \frac{\sqrt{3}}{2}\cos\phi$$

$$\boxed{W_1 + W_2 = \sqrt{3} V_L I_L \cos\phi}$$

$$P = W_1 + W_2 = \sqrt{3} V_L I_L \cos\phi \longrightarrow (3)$$

Subtract eq (2) from eq (1)

$$W_1 - W_2 = V_L I_L \left[\cos(30-\phi) - \cos(30+\phi)\right]$$

$$\because \cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$$

$$W_1 - W_2 = V_L I_L \left[-2\sin\left(\frac{30-\phi+30+\phi}{2}\right)\sin\left(\frac{30-\phi-30-\phi}{2}\right)\right]$$

$$W_1 - W_2 = -2 V_L I_L \sin 30 \cdot \sin(-\phi)$$

$$W_1 - W_2 = 2 V_L I_L \frac{1}{2}\sin\phi$$

$$\boxed{W_1 - W_2 = V_L I_L \sin\phi} \longrightarrow (4)$$

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Divide eq (4) by eq (3)

$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{\cancel{V_L} \cancel{I_L} \sin\phi}{\sqrt{3}\cancel{V_L}\cancel{I_L}\cos\phi}$$

$$\sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right) = \tan\phi$$

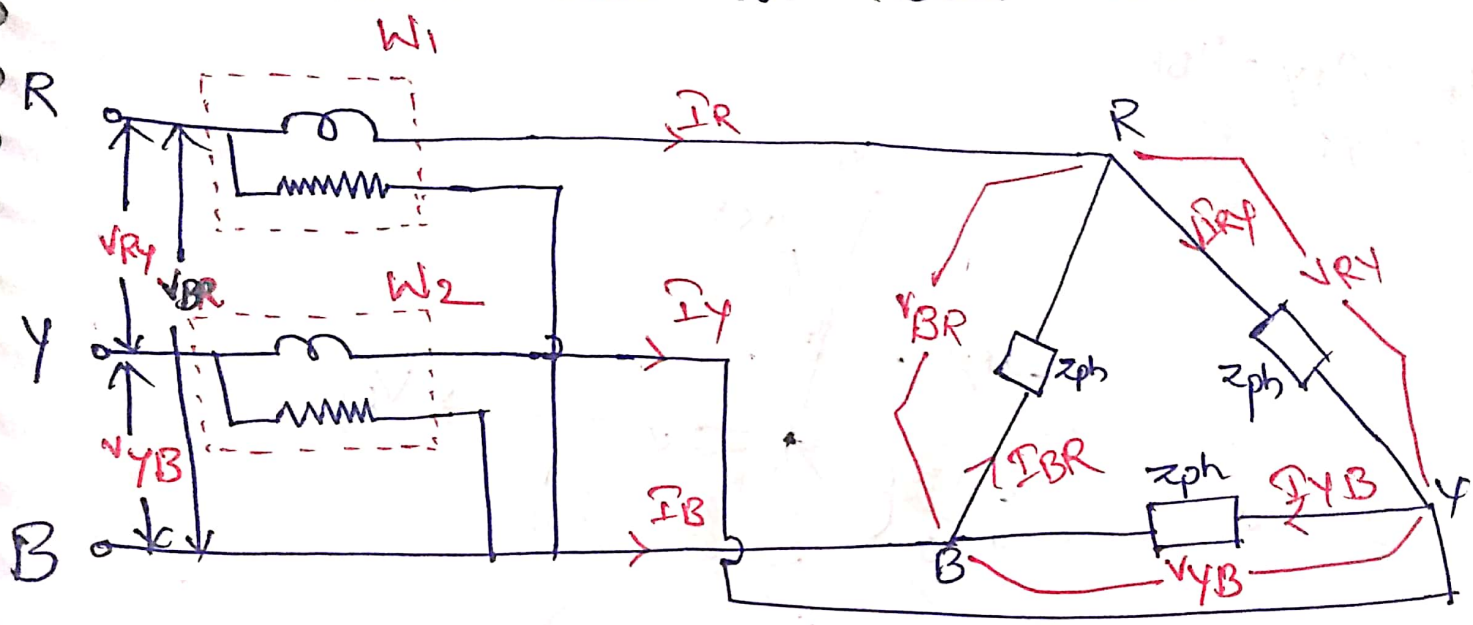
$$\phi = \tan^{-1}\left[\sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right)\right]$$

power factor $= \cos\phi$

$$pf = \cos\left[\tan^{-1}\sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right)\right]$$

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Delta connected lagging pf (Inductive) load :-



$V_{RY}, V_{YB}, V_{BR} = V_L$ (line voltage)

$I_R, I_Y, I_B = I_L$ (line current)

$V_{RY}, V_{YB}, V_{BR} = V_{ph}$ (phase voltage)

$I_{RY}, I_{YB}, I_{BR} = I_{ph}$ (phase current)

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$\bar{Z}_{ph}$ = phase impedance (lagging pf load).

wattmeter readings:

wattmeter 1

current = $\bar{I}_R$

voltage = V_{RB}

$W_1 = V_{RB} I_R \cos(V_{RB} \wedge \bar{I}_R)$

wattmeter 2

current = $\bar{I}_y$

voltage = V_{YB}

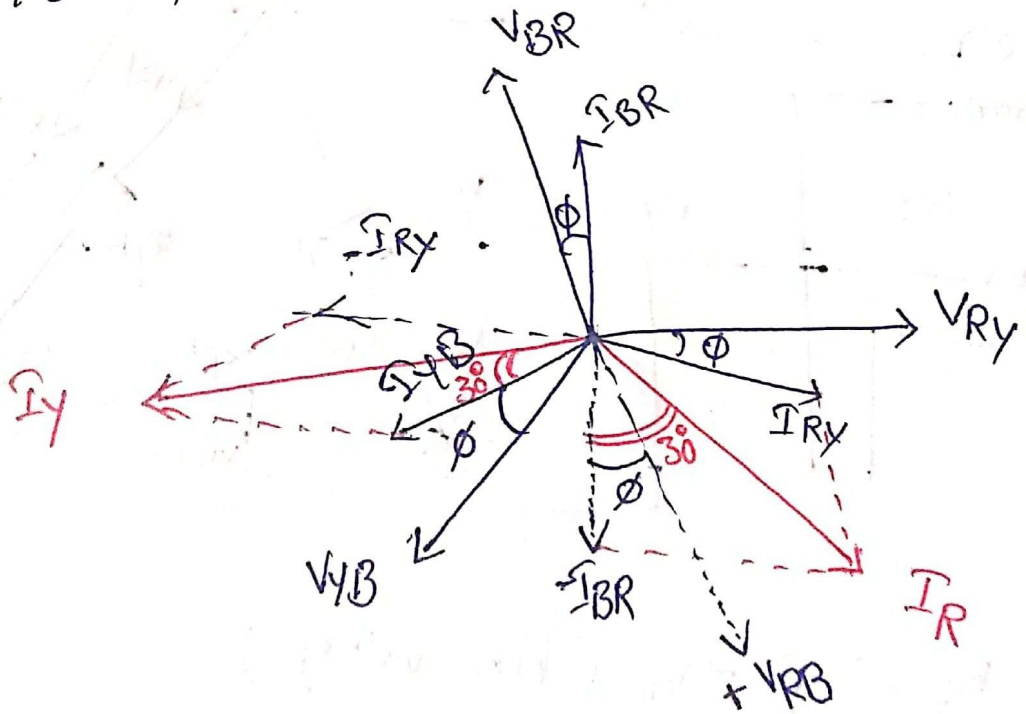
$W_2 = V_{YB} I_y \cos(V_{YB} \wedge \bar{I}_y)$

$W_1 = V_{RB} \bar{I}_R \cos(V_{RB} \wedge \bar{I}_R)$

$W_2 = V_{YB} \bar{I}_y \cos(V_{YB} \wedge \bar{I}_y)$

$\bar{I}_R = \bar{I}_{RY} - \bar{I}_{BR}$

$\bar{I}_Y = \bar{I}_{YB} - \bar{I}_{RY}$



$V_{BR} \wedge \bar{I}_R = 30° - \phi$

$V_{YB} \wedge \bar{I}_y = 30° + \phi$

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$$W_1 = V_{RB} \cdot I_R \cos(V_{RB} \wedge I_R)$$

$$W_1 = V_L I_L \cos(30-\phi) \longrightarrow (1)$$

$$W_2 = V_{YB} I_Y \cos(V_{YB} \wedge I_Y)$$

$$W_2 = V_L I_L \cos(30+\phi) \longrightarrow (2)$$

eq (1) + eq (2)

$$W_1+W_2 = V_L I_L [\cos(30-\phi) + \cos(30+\phi)]$$

$$\because \cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$$

$$W_1+W_2 = V_L I_L \left[2\cos\frac{30-\phi+30+\phi}{2} + \cos\frac{30-\phi-30-\phi}{2}\right]$$

$$= 2V_L I_L [\cos 30 \cos(-\phi)]$$

$$= 2V_L I_L \frac{\sqrt{3}}{2}\cos\phi$$

$$W_1+W_2 = \sqrt{3} V_L I_L \cos\phi \longrightarrow (3)$$

$$\therefore P = W_1+W_2 = \sqrt{3} V_L I_L \cos\phi$$

eq (1) − eq (2)

$$W_1-W_2 = V_L I_L [\cos(30-\phi) - \cos(30+\phi)]$$

$$\cos C - \cos D = 2\sin\frac{C+D}{2}\sin\frac{D-C}{2}$$

$$W_1-W_2 = V_L I_L \left[2\sin\frac{30-\phi+30+\phi}{2}\sin\frac{30+\phi-30+\phi}{2}\right]$$

$$= 2V_L I_L \sin 30^\circ \sin\phi$$

$$= 2V_L I_L \frac{1}{2}\sin\phi$$

$$W_1-W_2 = V_L I_L \sin\phi \longrightarrow (4)$$

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eq ④ ÷ eq ③

$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{V_L I_L \sin\phi}{\sqrt{3} V_L I_L \cos\phi}$$

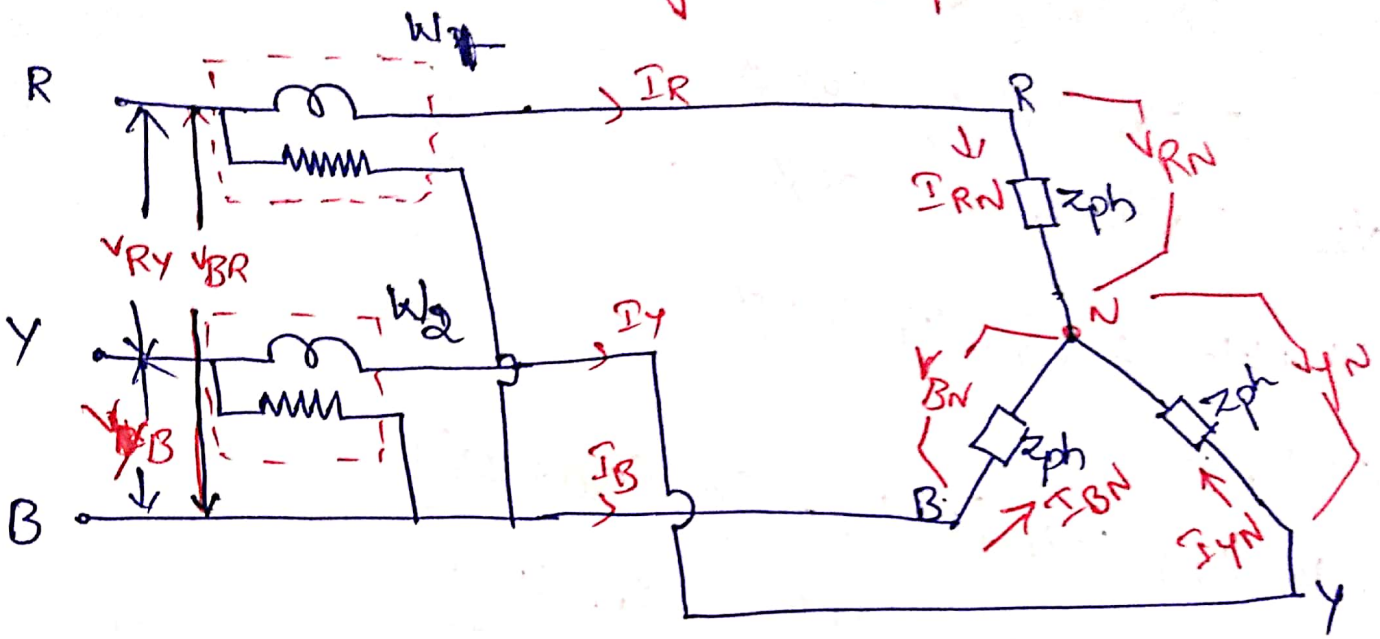
$$= \frac{1}{\sqrt{3}} \tan\phi$$

$$\boxed{\phi = \tan^{-1}\left[\sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right)\right]}$$

$$pf = \cos\phi = \cos\left\{\tan^{-1}\left[\sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right)\right]\right\}$$

Scanned with CamScanner

Star connected leading Pf (capacitive) load :-



$V_{RY}, V_{YB}, V_{BR} = V_L$ (line voltage)

$I_R, I_Y, I_B = I_L$ (line current)

$V_{RN}, V_{YN}, V_{BN} = V_{ph}$ (phase voltage)

$I_{RN}, I_{YN}, I_{BN} = I_{ph}$ (phase current)

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Z_{ph} = phase impedance (leading P.f load).

Wattmeter Readings :

Wattmeter 1

Current = I_R

Voltage = V_{RB}

Wattmeter 2

Current = I_Y

Voltage = V_{YB}

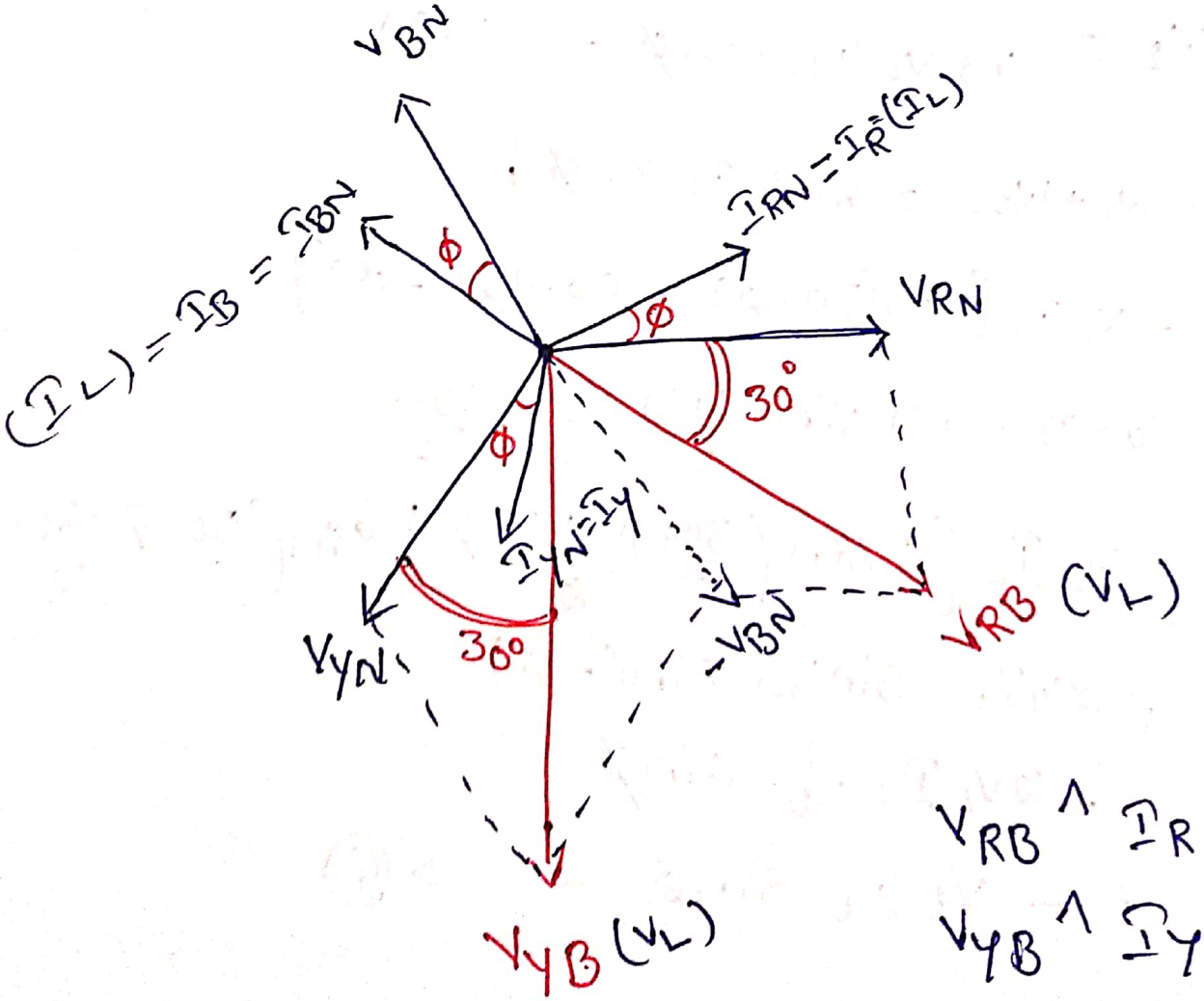
$$W_1 = V_{RB} I_R \cos(V_{RB} \wedge I_R)$$

$$W_2 = V_{YB} I_Y \cos(V_{YB} \wedge I_Y)$$

$V_{RB} \wedge I_R$, $V_{YB} \wedge I_Y$ angle can be measured with the help of phasor diagram.

$$V_{RB} = V_{RN} - V_{BN}$$

$$V_{YB} = V_{YN} - V_{BN}$$



$$V_{RB} \wedge I_R = 30^\circ + \phi$$

$$V_{YB} \wedge I_Y = 30^\circ - \phi$$

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$$W_1 = V_{RB} \cdot I_R \cos(V_{RB} \wedge I_R)$$

$$W_1 = V_L I_L \cos(30^\circ + \phi) \longrightarrow (1)$$

$$W_2 = V_{YB} \cdot I_Y \cos(V_{YB} \wedge I_Y) \longrightarrow (2)$$

$$W_2 = V_L I_L \cos(30^\circ - \phi) \longrightarrow (2)$$

(1) + (2)

$$W_1 + W_2 = V_L I_L [\cos(30+\phi) + \cos(30-\phi)]$$

$$\because \cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$$

$$W_1 + W_2 = V_L I_L \left[2\cos\frac{30+\phi+30-\phi}{2}\cos\frac{30+\phi-30+\phi}{2}\right]$$

$$W_1 + W_2 = 2V_L I_L [\cos 30 \cos\phi]$$

$$= 2V_L I_L \frac{\sqrt{3}}{2}\cos\phi$$

$$W_1 + W_2 = \sqrt{3} V_L I_L \cos\phi \longrightarrow (3)$$

$$\therefore P = W_1 + W_2 = \sqrt{3} V_L I_L \cos\phi$$

$$W_1 - W_2 = V_L I_L [\cos(30+\phi) - \cos(30-\phi)]$$

$$\cos C - \cos D = +2\sin\frac{C+D}{2}\sin\frac{D-C}{2}$$

$$W_1 - W_2 = V_L I_L \left[2\sin\left(\frac{30+\phi+30-\phi}{2}\right)\sin\left(\frac{30-\phi-30}{2}\right)\right.$$

$$W_1 - W_2 = 2V_L I_L \sin 30 \sin(-\phi)$$

$$W_1 - W_2 = -2V_L I_L \frac{1}{2}\sin\phi$$

$$W_1 - W_2 = -V_L I_L \sin\phi \longrightarrow (4)$$

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Equation (4) ÷ equation (3)

$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{-V_L I_L \sin\phi}{\sqrt{3} V_L I_L \cos\phi}$$

$$\therefore \frac{W_1 - W_2}{W_1 + W_2} = \frac{-1}{\sqrt{3}} \tan\phi$$

$$\tan\phi = -\sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right)$$

$$\phi = \tan^{-1}\left\{-\sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right)\right\}$$

$$\text{pf of load} = \cos\phi$$

$$\text{pf} = \cos\left\{\tan^{-1}\left[-\sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right)\right]\right\}$$

problem :-

Two wattmeter's connected to measure the power input to 3-ϕ circuit, indicate 2500 W and 500 W respectively. find the power factor of the circuit.

i) when both the readings are positive.

ii) when later reading is obtained after reversing the connection to the current coil of one instrument?

Sol

i) Both reading are positive

$W_1 = 2500\ W$

$W_2 = 500\ W$

$P = W_1 + W_2 = 2500 + 500 = \underline{3000}\ W$

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$$\phi = \tan^{-1}\left\{\sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right)\right\}$$

$$= \tan^{-1}\left\{\sqrt{3}\left(\frac{2500 - 500}{3000}\right)\right\}$$

$$= \tan^{-1}\left\{\sqrt{3}\left(\frac{2000}{3000}\right)\right\}$$

$$= \text{too}$$

$$\phi = 49.11^\circ$$

$$pf = \cos\phi = \cos(49.11^\circ)$$

when pf is +ve

$$\boxed{pf = 0.6547 \text{ (lag)}}$$ ∴ Inductive load.

ii) Reversing connection of current coil

$$W_1 = 2500\,W$$

$$W_2 = -500\,W$$

$$P = W_1 + W_2 = 2500 + (-500) = 2000\,W$$

$$\phi = \tan^{-1}\left\{\sqrt{3}\left(\frac{W_1 - W_2}{W_1 + W_2}\right)\right\}$$

$$= \tan^{-1}\sqrt{3}\left[\frac{2500 - (-500)}{2000}\right]$$

$$= \tan^{-1}\left\{\sqrt{3}\left(\frac{3000}{2000}\right)\right\}$$

$$\phi = 68.95^\circ$$

$$pf = \cos\phi = \cos(68.95^\circ)$$

$$pf = 0.3592 \text{ (lagging)}$$

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* find the values of circuit elements and reactive Volt-ampere drawn for a balanced 3-ϕ load connected in delta and draws a power of 12kW at 440V. The power factor is 0.7 leading.

Sol

Delta connected load

$P = 12kW = 12000\ W$

$V_L = 440V$

$\cos\phi = 0.7$ (leading)

$P = \sqrt{3}\, V_L I_L \cos\phi$

$12000 = \sqrt{3} \times 440 \times I_L \times (0.7)$

$I_L = 22.4942\ A$

In delta connected load $I_{ph} = \frac{I_L}{\sqrt{3}} = \frac{22.4942}{\sqrt{3}}$

$I_{ph} = 12.987\ A$

In delta Connected load $V_{ph} = V_L = 440V$

$$Z_{ph} = \frac{V_{ph}}{I_{ph}} = \frac{440}{12.987}$$

$$\boxed{Z_{ph} = 33.88\Omega}$$

$\cos\phi = 0.7$

$\phi = \cos^{-1}(0.7)$

$\phi = 45.57$

$\frac{R_{ph}}{Z_{ph}} = \cos\phi$

$R_{ph} = Z_{ph}\cos\phi = 33.88 \times 0.7 = 23.716\ \Omega$

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Reactance $X_{ph} = Z_{ph} \sin\phi$

$X_{ph} = 33.88 \times \sin(45.57)$

$X_{ph} = 24.1939\ \Omega$

Reactive power $Q = 3 V_{ph} I_{ph} \sin\phi$

$Q = 3 \times 440 \times 12.987 \times \sin(45.57)$

$Q = 12.2424$ KVAR

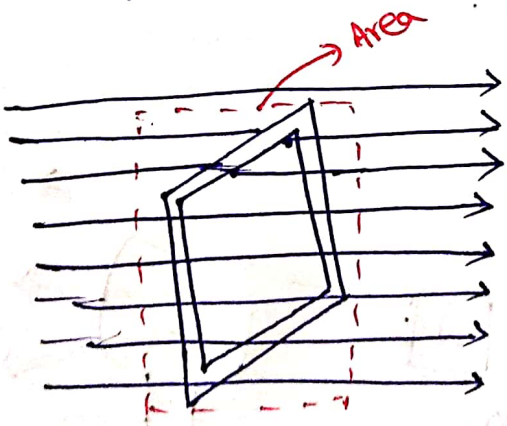
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UNIT - V

Magnetic field :- It is defined as the space around the magnet in which the other charges or objects can experience a magnetic force. It is denoted by 'B'.
units of magnetic field is "Tesla"
magnetic field is a vector quantity.

Magnet :- A magnet is a material & object that produces magnetic field.

Magnetic flux :- Quantity of magnetic field linked to a surface area is is known as "magnetic flux".
It is given by $\phi_B = \vec{B} \cdot \vec{A}$
The units of magnetic flux is "weber".

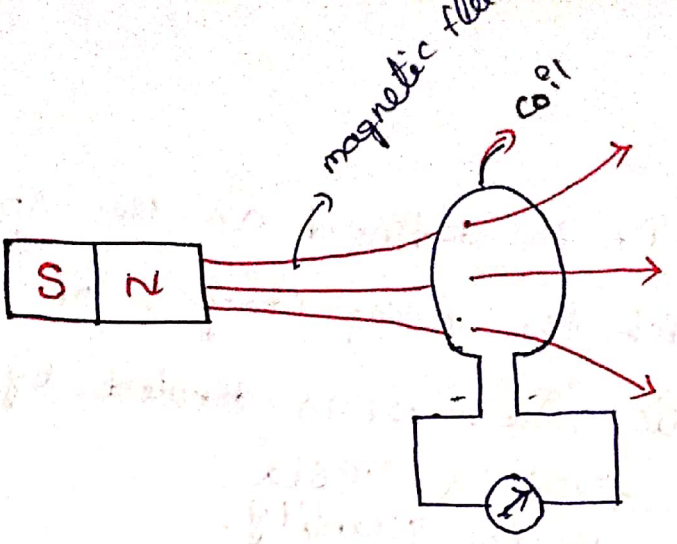


B → magnetic field
A → Area of the magnet.

Faraday's laws of Electromagnetic Induction :-

Law 1 : Change of magnetic flux linked to a coil induce an emf across a coil.

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Law 2 :-

The emf induced across the coil is equal to the rate of change of flux in the coil.

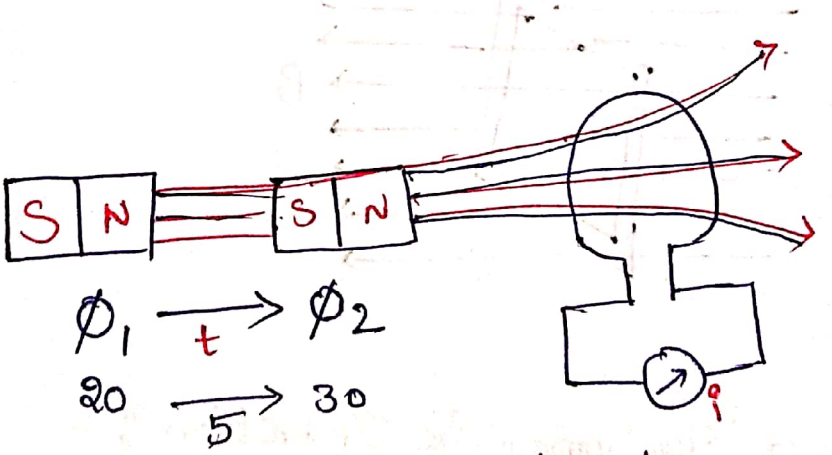
$$e \propto d\phi_B/dt$$

$$e = -N \frac{d\phi_B}{dt}$$

N is no. of turns.

-ve sign shows that emf induced always opposes the change in flux.

Example :-



Rate of change of flux is $\frac{\phi_2 - \phi_1}{t}$

$$\text{Emf } e = -\frac{\phi_2 - \phi_1}{t}$$

$$= -\left(\frac{30-20}{5}\right)$$

$$e = -2V$$

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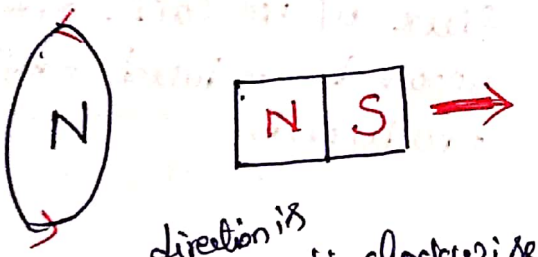
$$i = \frac{e}{R}$$

$$i = -\frac{1}{R}\frac{d\phi_B}{dt}$$

Lenz's law :-

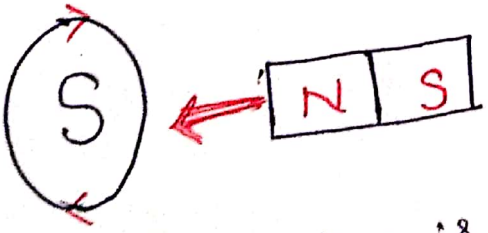
According to lenz's law the polarity of induced emf is such which oppose the change in magnetic flux.

Case 1 :-



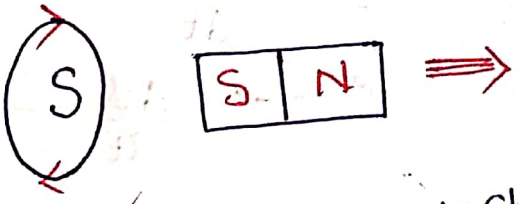
Current is anti clockwise direction

Case 2 :-



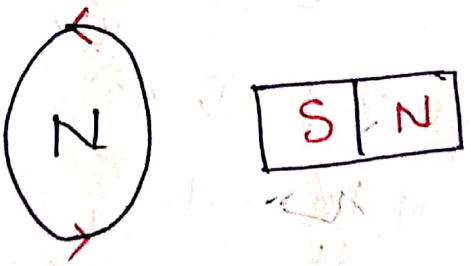
current direction is clockwise.

case 3 :-



current direction is Clock wise direction.

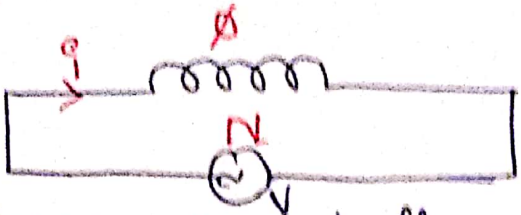
case 4 :-



Current direction is anti clockwise direction.

Scanned with CamScanner

Self Inductance :-



If changing current flowing a coil produces emf in the same coil, it is called self inductance.

When the current flowing through the coil changes flux also changes, emf is induced in the same coil is known as self induced emf. It is given by

$$e \propto -\frac{d\phi}{dt}$$

$$e = -N\frac{d\phi}{dt} \longrightarrow (1)$$

when there is a change in the current or magnetic flux of the coil, an opposed induced emf is produced.

$$\phi \propto i$$

$$e \propto -\frac{di}{dt}$$

$$e = -L\frac{di}{dt} \longrightarrow (2)$$

Equating (1) & (2).

$$N\frac{d\phi}{dt} = -L\frac{di}{dt}$$

$$-L = -N\frac{d\phi}{dt} \times \frac{dt}{di}$$

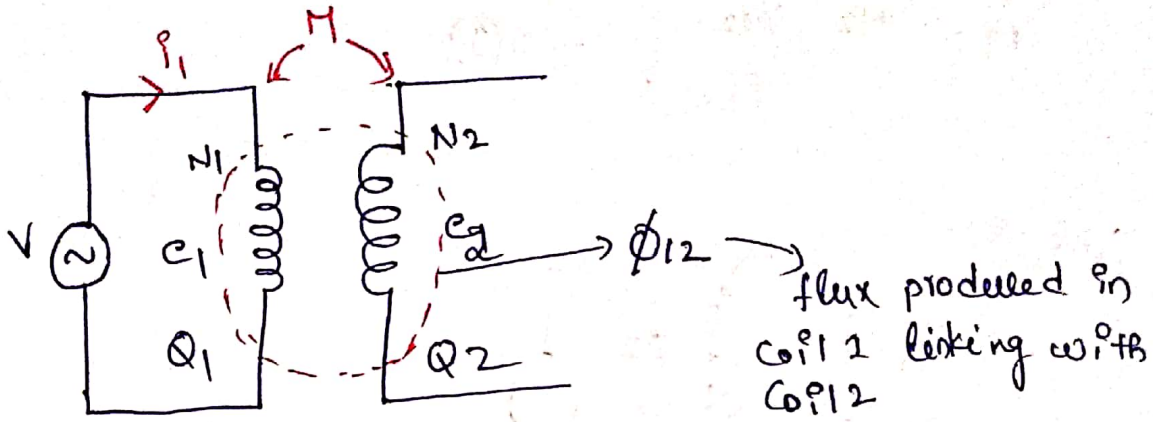
$$L = N\frac{\phi}{i}$$

L → Inductance.

It is the ratio of flux linkage to current.

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Mutual Inductance :-



when there is a change in the current or magnetic flux linked with two coils an opposing emf is produced across each coil, this phenomenon is called mutual inductance.

The changing current through one coil produces emf in another coil is called mutual inductance.

$$e_2 \propto -\frac{d\phi_{12}}{dt}$$

$$e_2 = -N_2 \frac{d\phi_{12}}{dt} \longrightarrow (1)$$

$$e_2 \propto -\frac{di_1}{dt}$$

$$e_2 = -M \frac{di_1}{dt} \longrightarrow (2)$$

equating (1) & (2)

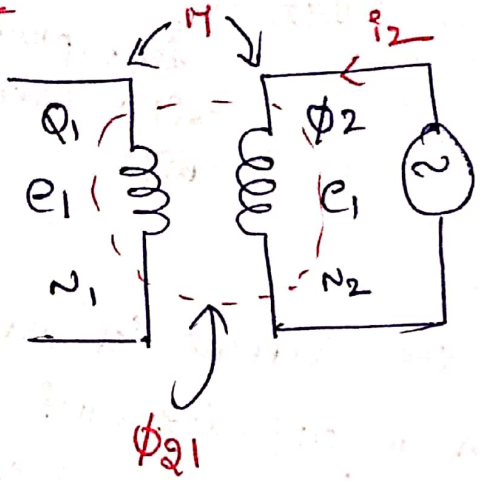
$$-M \frac{di_1}{dt} = -N_2 \frac{d\phi_{12}}{dt}$$

$$M = N_2 \frac{d\phi_{12}}{dt} \times \frac{dt}{di_1}$$

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$$M = \frac{N_2 \phi_{12}}{i_1} \longrightarrow (3)$$

Case 2:-



$$e_1 \propto -\frac{d\phi_{21}}{dt}$$

$$e_1 = -N_1 \frac{d\phi_{21}}{dt} \longrightarrow (4)$$

$$e_1 \propto -\frac{di_2}{dt}$$

$$e_1 = -M \frac{di_2}{dt} \longrightarrow (5)$$

equating (4) & (5)

$$-M \frac{di_2}{dt} = -N_1 \frac{d\phi_{21}}{dt}$$

$$M = N_1 \frac{d\phi_{21}}{dt} \times \frac{dt}{di_2}$$

$$M = N_1 \frac{\phi_{21}}{i_2}$$

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Single phase Transformer :-

Need of a transformer :-

→ The a.c voltages can be raised or lowered as per requirements in different stages of electrical network (Generation, Transmission, Distribution & utilization) with the help of static device called as transformer.

→ Transformer works on the principle of mutual Induction

→ Transfers an electrical energy from one circuit to other when there is no electrical connection b/w two circuits.

Definition of transformer :-

The transformer is a static device by means of which an electrical power is transferred from one a.c circuit to another with the desired change in voltage & current without any change in the frequency.

Working principle of a Transformer :-

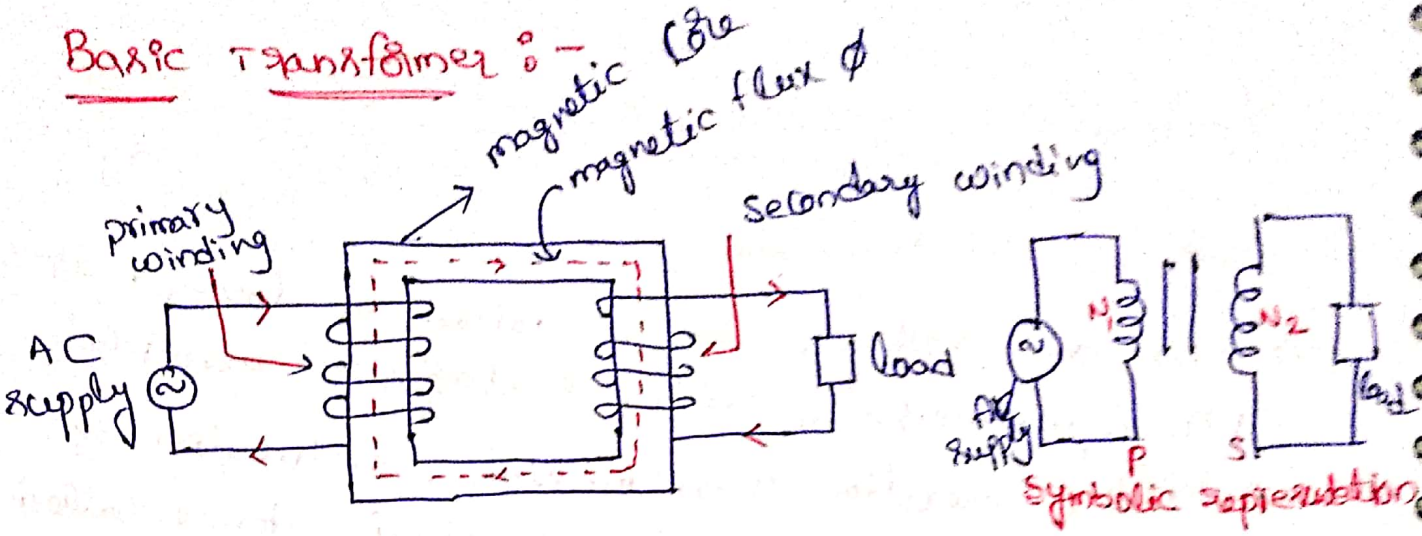
Transformer works on the principle of mutual induction.

mutual Induction :-

The principle of mutual induction states that, when two coils are inductively coupled & if current in one coil changed uniformly then an e.m.f gets induced in the other coil.

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Basic Transformer :-



where ever the supply is connected to the coil is called primary winding. wherever the load is connected to coil is called secondary winding.

Due to the A.C supply the current will pass through the primary windings. This current will gives raise to the magnetic flux through the magnetic core. The magnetic flux changing with respect to time gets linked with a primary winding and it will gives raise to emf by the faraday's law of an electromagnetic induction. That emf is called self induced emf

Getting the voltage becox of the supply which is connected to same coil is called self induced emf.

Same magnetic flux is linked with the secondary windings. So the flux is changing with respect to time gets linked with secondary windings, its gives rise to emf. This emf is induced bcox of the supply at primary side. Therefore it is called mutually induced emf. So the current will pass through the secondary windings.

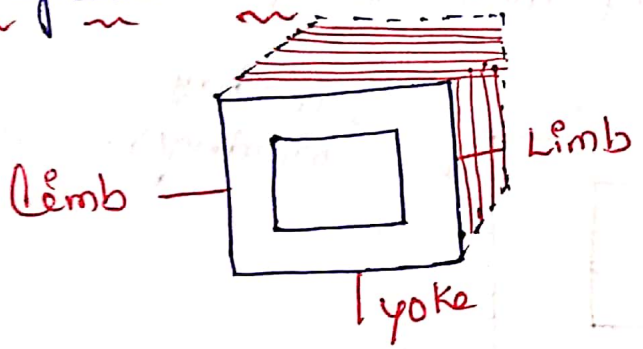
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Construction of a Transformer :-

There are two basic parts of the transformer.

1) magnetic core
2) windings & coils.

magnetic core :-



Magnetic core provides path for magnetic flux (ϕ).

Two types of losses occur in the magnetic core.

1) Hysteresis loss & 2) Eddy current loss.

These two losses together is are called magnetic losses or iron losses.

We need to construct a core in such a way that these two losses as low as possible through design.

Generally core is made up of high grade silicon steel. this is the best magnetic material (magnetic material with high permeability).

By using the high permeability materials in order to minimize the Hysteresis losses.

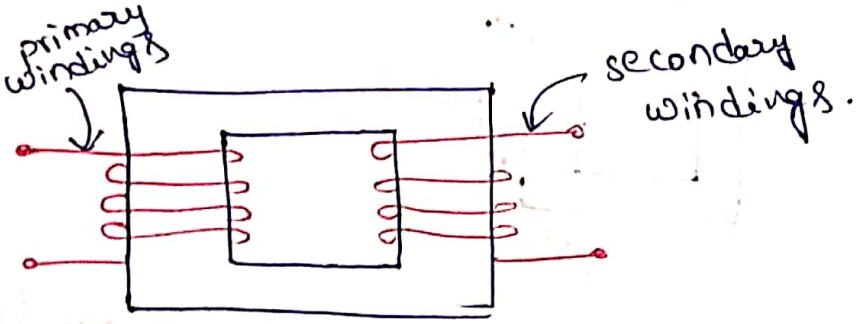
Core is made up of thin laminations to minimize eddy current losses.

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We are providing a thin laminations of 0.5 mm or 0.7 mm. So the width is very less.
These laminations are insulated from each other by insulation. This insulation is called as varnish.

Winding or Coils

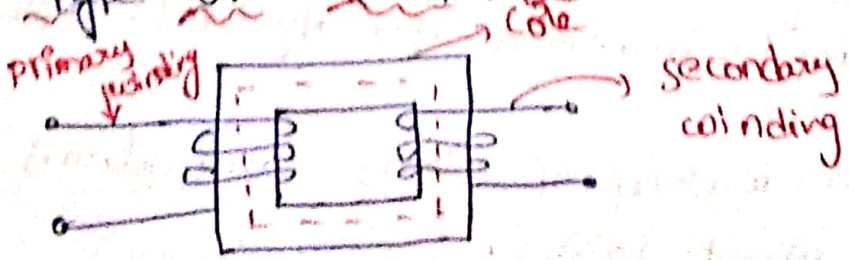
Windings are wound over limbs of core.



Windings carry primary and secondary current.
i.e electric current passes through windings.
We need a material which will have less resistivity.
Less resistivity materials are copper & Iron.
Winding are made up of copper and are wound on the limbs of magnetic core. windings are insulated from each other.
windings are subjected to I^2R losses. (copper losses).

Based on the arrangement of core and windings the transformers are devided into the two types.

→ Core type of Transformer :-



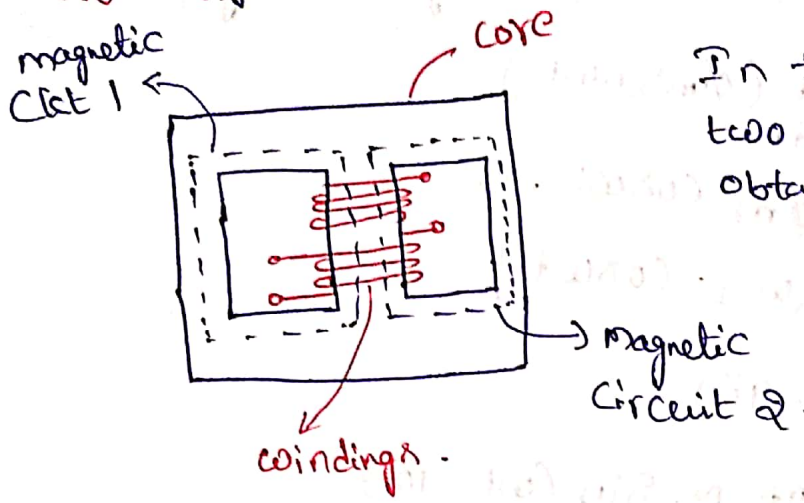
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The winding encircles the core (or)

Core is surrounded by windings.

In this core type transformer only one magnetic ckt is obtained

→ Shell type Transformer :-



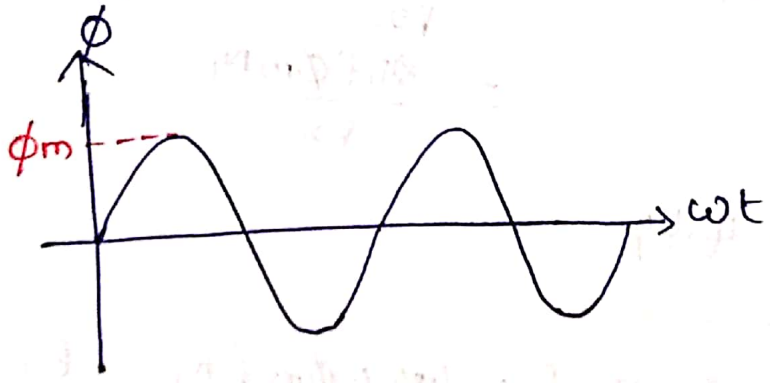
In this type of transformer two magnetic circuits are obtained.

The core encircles the winding (or)

windings are surrounded by core.

A very high voltage transformers are shell type.

EMF equation of a Transformer :-



$$\phi = \phi_m \sin\omega t$$

By faraday's law of an electromagnetic induction the induced emf is

$$e = -N\frac{d\phi}{dt}$$

By Lenz's law

$$e = -N\frac{d\phi}{dt}$$

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E.M.f induced at the primary side

$$e_1 = -N_1 \frac{d\phi}{dt}$$

$N_1 \rightarrow$ no. of turns of primary winding.

$$e_1 = -N_1 \frac{d}{dt}(\phi_m \sin\omega t)$$

$$e_1 = -N_1 \phi_m \cos\omega t \cdot \omega$$

$$e_1 = N_1 \phi_m \omega [-\cos\omega t]$$

$$-\cos\theta = \sin(\theta - 90^\circ)$$

$$e_1 = \phi_m 2\pi f N_1 \sin(\omega t - 90^\circ)$$

Comparing with

$$e_1 = E_{m1} \sin(\omega t + \varphi)$$

$E_{m1} = 2\pi f \cdot \phi_m N_1 \quad \varphi = -90^\circ$

R.M.S value of emf $= \frac{E_{m1}}{\sqrt{2}}$

$$= \frac{2\pi f \phi_m N_1}{\sqrt{2}}$$

$\frac{2\pi}{\sqrt{2}} = 4.44$

RMS value of emf $= 4.44 \phi_m f N_1 = E_1$

Similarly secondary side emf

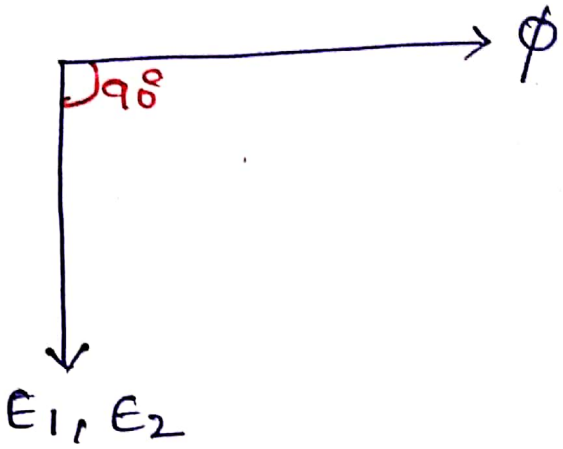
$$E_2 = 4.44 \phi_m f N_2$$

E_1 & E_2 lag flux ϕ by 90° or $\frac{\pi}{2}^\circ$

frequency will be same in the both the windings.

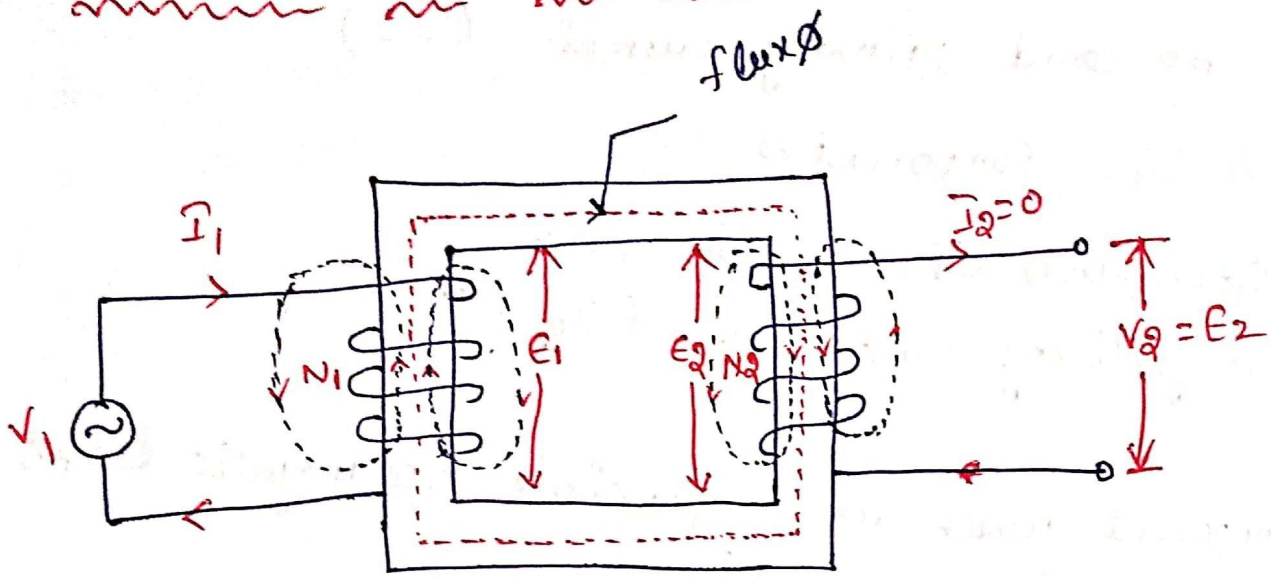
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phasor diagram :-



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Transformer on No load :-



The primary current I_1 flowing through the primary windings it will gives raise to the ~~ft~~ magnetic flux, it will pass through the magnetic core.

The magnetic flux Ø is linked with the primary windings, N_1 turns it will gives the emf E_1.

The maximum flux is ~~generated~~ passed through the ~~inside~~ the core but Some amount of flux is generated in the outside the core. It is called leakage flux.

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Same flux ϕ is linked with the secondary windings and at the secondary side some amount of flux is going to be coasted in the air.
The maximum part of the flux is linked with the win
it will gives raise to ~~the~~ emf E_2.

~~The~~ SS

At the secondary side the ckt will be open ckt the current flowing through the secondary windi
$I_2 = 0$ becoz of the terminal voltage V_2 which is same as the secondary side induced emf E_2.

I_1 = No load primary current (I_o)

I_o has two components

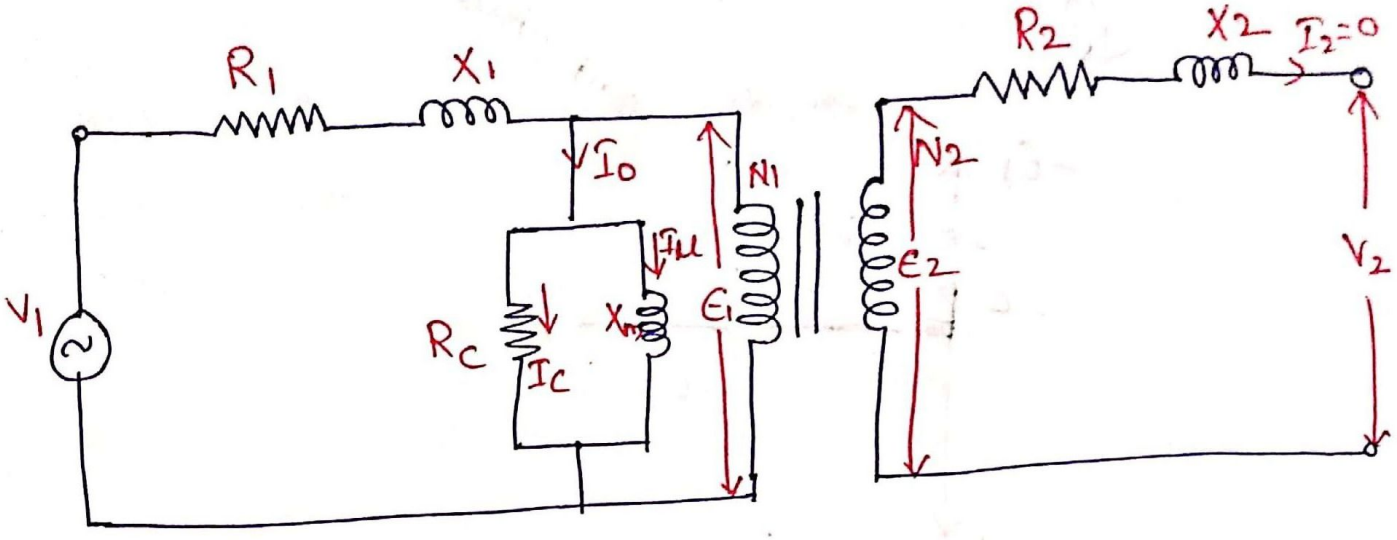
i) Generation of magnetic flux.
magnetising components of I_o (I_M)

(ii) Component which is responsible for magnetic losses
[hysterisis & eddy current]

primary winding I^2R losses in called as core loss
Component of I_o (I_c)

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Equivalent ckt of a Transformer no load :-



V_1 → primary voltage

R_1 → primary winding resistance

X_1 → primary leakage reactance

I_0 → no load primary current.

I_c → core loss component of I_0

I_μ → magnetizing component of I_0

R_c → core loss resistance.

X_m → magnetizing Reactance.

E_1 → primary induced emf

N_1 → no. of primary winding turns

R_2 → secondary winding resistance.

X_2 → secondary leakage reactance.

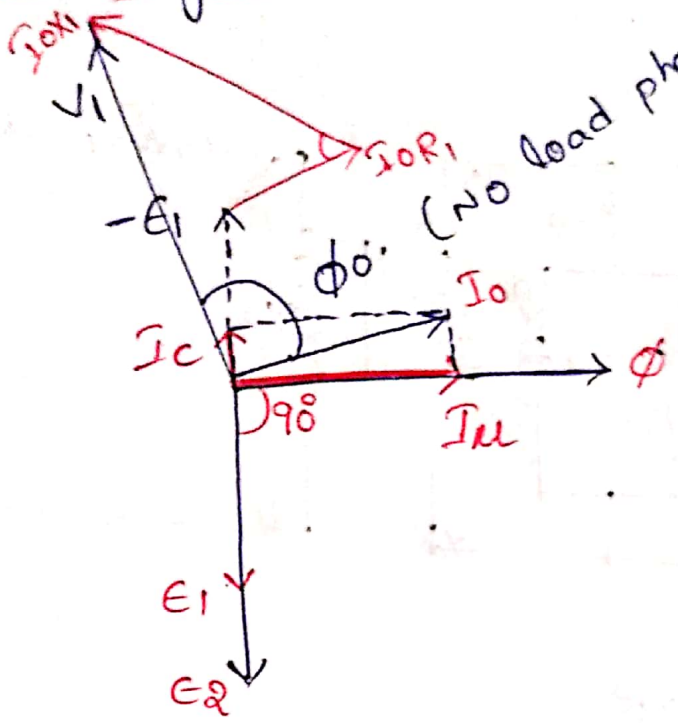
N_2 → no. of secondary winding turns.

E_2 → secondary induced emf

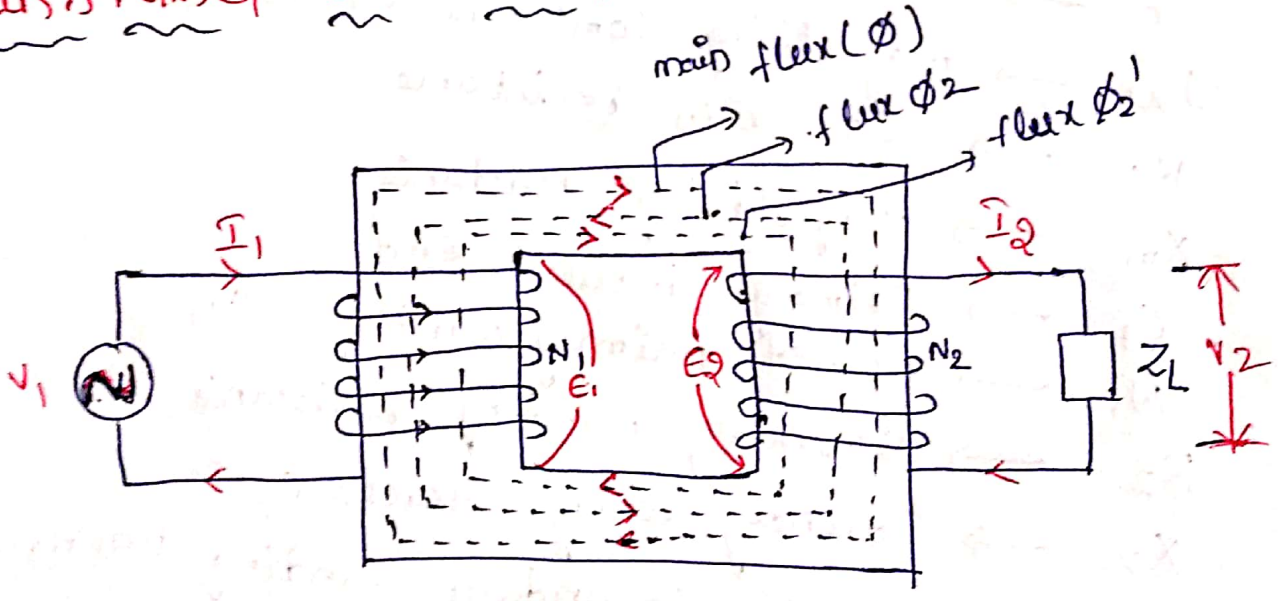
V_2 → Terminal voltage (secondary voltage).

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phasor diagram :-



Transformer on Load :-



windings are wound over the limbs of the core. where ever the supply is giving to the windings that windings are called primary windings.

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Because of the primary voltage V_1, current will be flow pass through the primary windings. Inside the core the flux will be produced. This flux will gives raise to the emf at the primary side i.e E_1 same magnetic flux is linked with the N_2 number of turns, it will gives raise to the emf at the secondary side i.e E_2.

The secondary side winding is closed through Z_L, which will gives raise to the secondary side current I_2, it will flows through the Z_L and coe will get the terminal voltage V_2.

I_2 is flowing through the windings, inside the core. one more flux is produced because of this I_2. The direction of the flux ϕ_2 is opposite to the main flux.

Inside the core there will be a reduction of flux and the flux which is equivalent to the in the core that is $\phi - \phi_2$

~~The jobs of the ϕ_2 is to nullify the effect of ϕ_2 Inside the core there will be a three fluxes, But the resultant is only main flux that is ϕ~~

Inside the core main flux is reduced becoz of that emf is directly proportional to ϕ which we assume as a constant.

In the core there will be only ϕ, there won't be effect of the I_2. It will take of care of the I_1.

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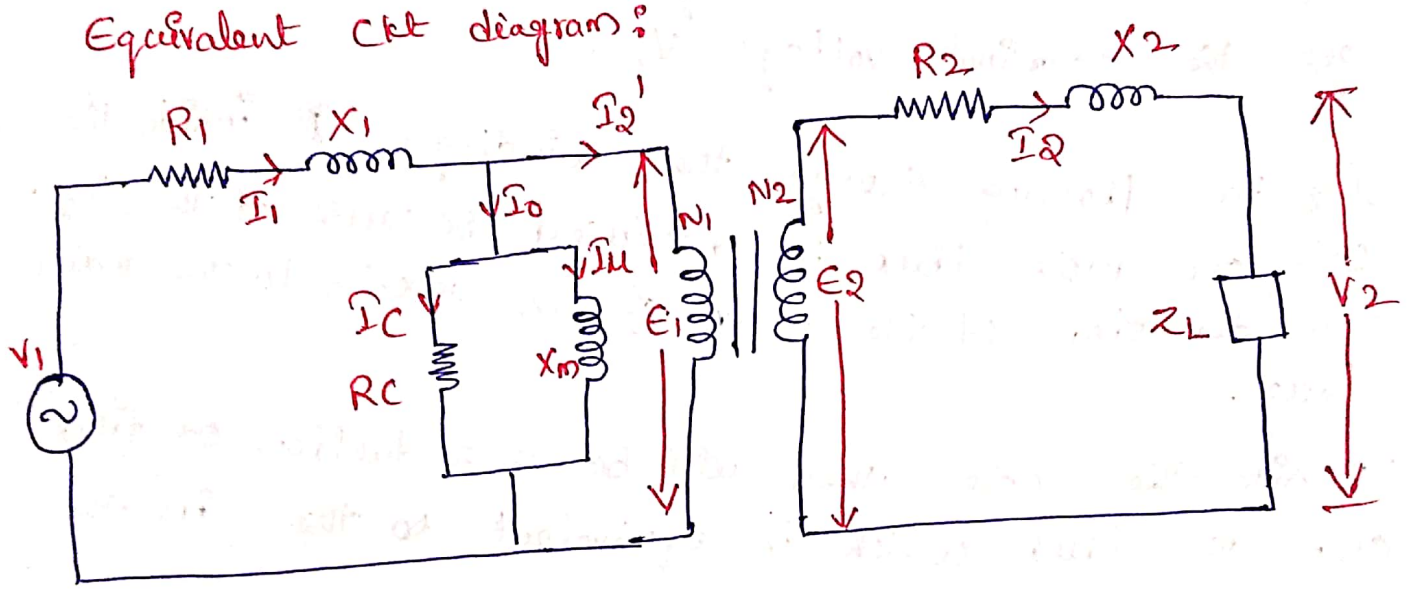
I_1 will give some extra current from supply in order to nullify the effect of ϕ_2.

So one more flux is produced inside the core. i.e ϕ_2'. The direction of ϕ_2' is opposite to ϕ_2.

The effect of the ϕ_2' is to nullify (reduce) the effect of ϕ_2.

Inside core three fluxes are there but the resultant is only one flux i.e main flux ϕ.

Equivalent ckt diagram:



V_1 → primary or supply voltage

I_1 → primary current

R_1 → primary winding resistance.

X_1 → primary leakage reactance

I_0 → No load primary current.

I_C → core loss component of I_0

I_M → magnetising component of I_0

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$\bar{I}_2'$ → Component of $\bar{I}_1$ to nullify effect of ϕ_2

R_c → core loss resistance.

X_m → magnetising reactance.

N_1 → primary turns.

E_1 → primary induced emf

N_2 → Secondary turns

E_2 → secondary induced emf

R_2 → secondary winding resistance

X_2 → secondary leakage reactance.

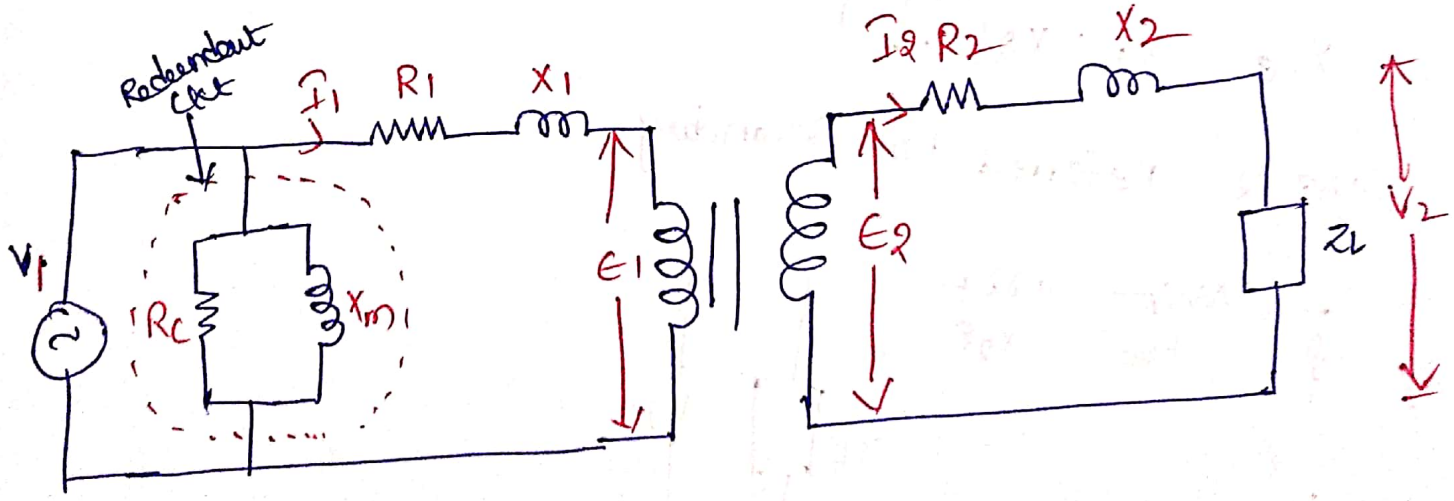
$\bar{I}_2$ → secondary current.

V_2 → secondary voltage or terminal voltage.

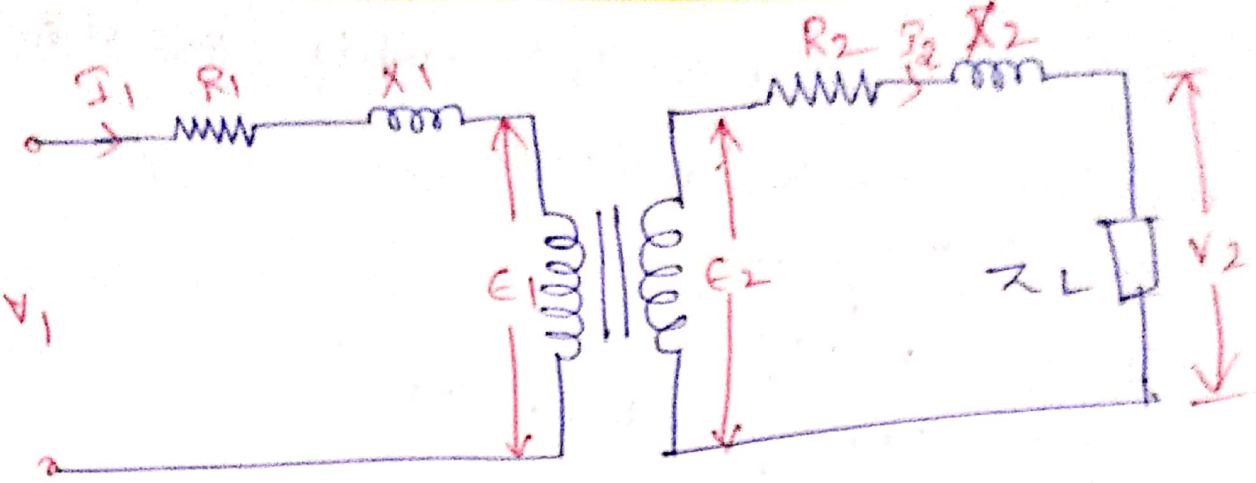
$$\bar{I}_1 = \bar{I}_0 + \bar{I}_2'$$

$\bar{I}_0$ → No load primary current
very low (5-10% of full load current).

$$\bar{I}_1 \simeq \bar{I}_2'$$

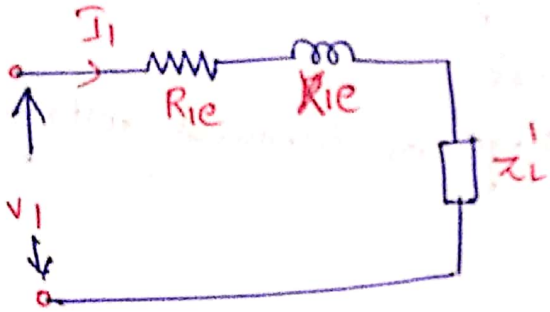


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Equivalent ckt diagram of 1-ϕ transformer.

Case 1 :- Referred to primary (All the element at the primary side).



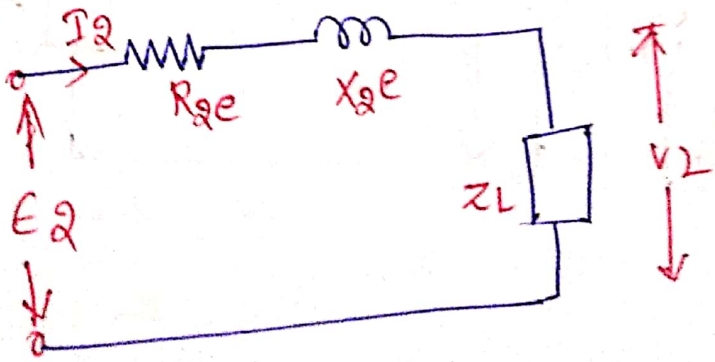
R_{1e} (equivalent resistance referred to primary).

$$R_{1e} = R_1 + R_2 \left(\frac{N_1}{N_2}\right)^2$$

X_{1e} (equivalent reactance referred to primary.

$$X_{1e} = X_1 + X_2 \left(\frac{N_1}{N_2}\right)^2$$

Case 2: Referred to secondary



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R_{2e} (equivalent resistance refered to secondary)

$$R_{2e} = R_2 + R_1 \left(\frac{N_2}{N_1}\right)^2$$

X_{2e} (equivalent reactance refered to secondary)

$$X_{2e} = X_2 + X_1 \left(\frac{N_2}{N_1}\right)^2$$

Losses in a Transformer :-

Losses in a Transformer classified into two types.

1) Core loss (or) Iron loss (or) Constant loss:

Core gets subjected to alternating flux causing core losses.

Types of core losses

⇒ Hysteresis loss :

Due to AC flux setup in the magnetic core of the transformer, it undergoes a cycle of magnetisation and demagnetisation. Due to hysteresis effect there is a loss of energy in this process, which is called as hysteresis loss.

$$P_h = k_h B_m^{1.67} f.V$$

$P_h \rightarrow$ hysteresis loss

$k_h \rightarrow$ hysteresis constant depends on material

$B_m \rightarrow$ maximum flux density

$f \rightarrow$ frequency

$V \rightarrow$ volume of the core.

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⇒ Eddy current losses :

The induced emf in the core tries to setup eddy currents in the core and hence responsible for eddy current losses.

$$P_e = K_e B_m^2 f^2 t^2$$

P_e → eddy current loss
K_e → eddy current constant
t → thickness of the core
B_m → maximum flux density
f → frequency.

flux in the core is almost constant as supply voltage V, at rated frequency f is always constant.

P_h & P_e are constant losses.

How to Reduce these core losses:

* High grade core material like silicon steel with laminations of smallest thickness.

2) Copper losses (or) variable losses :-

The windings carry currents when transformer is loaded Causing copper losses.

The copper losses are due to power wasted in the form of I^2R loss due to the resistances of the primary and secondary windings.

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$$P_{cu} = I_1^2 R_1 + I_2^2 R_2$$

Copper losses depend upon the amount of load current which can be changed depends upon the load connected.

Hence copper losses are variable losses.

Transformer total loss is

$$\text{Total losses} = P_i + P_{cu}$$

$$\text{Total losses} = (P_h + P_e) + (I_1^2 R_1 + I_2^2 R_2)$$

Rating of a transformer is in VA / kVA :-

→ Rating of any device depends upon the losses occuring in that device.

→ In transformer two types of losses present.

Core loss : depends on the supply voltage

Copper loss : depends on the current through windings.

→ In order to know total losses in a transformer by knowing VA of a transformer.

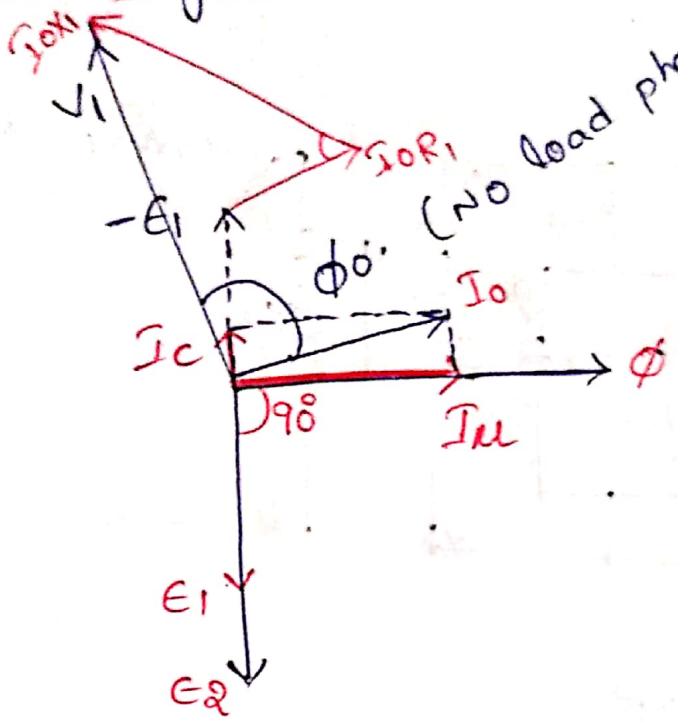
→ At the time of design nature of the load is not known. (i.e power factor $\cos\phi$).

Transformer rating is a apparent power.

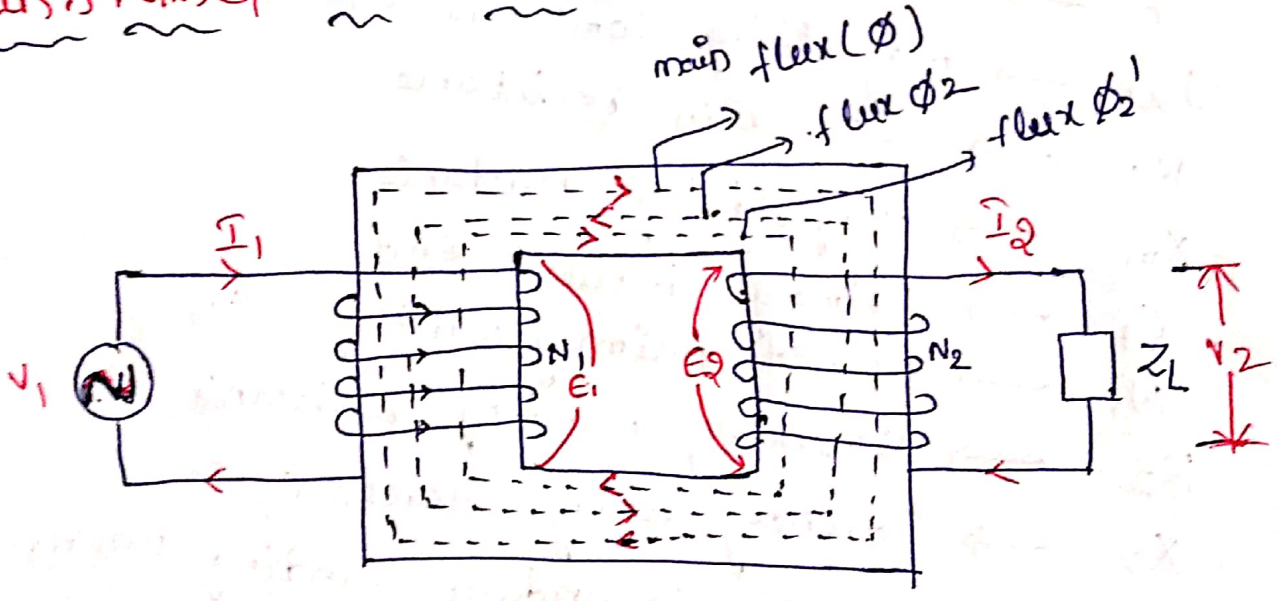
losses in a transformer depends only on voltage and current but not depend on power factor.

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phasor diagram :-



Transformer on Load :-



windings are wound over the limbs of the core. where ever the supply is giving to the windings that windings are called primary windings.

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I_2 is flowing through the windings, inside the core. one more flux is produced because of this I_2. The direction of the flux ϕ_2 is opposite to the main flux.

Inside the core there will be a reduction of flux and the flux which is equivalent to the in the core that is $\phi - \phi_2$

~~The jobs of the ϕ_2 is to nullify the effect of ϕ_2 Inside the core there will be a three fluxes, But the resultant is only main flux that is ϕ~~

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In the core there will be only ϕ, there won't be effect of the I_2. It will take of care of the I_1.

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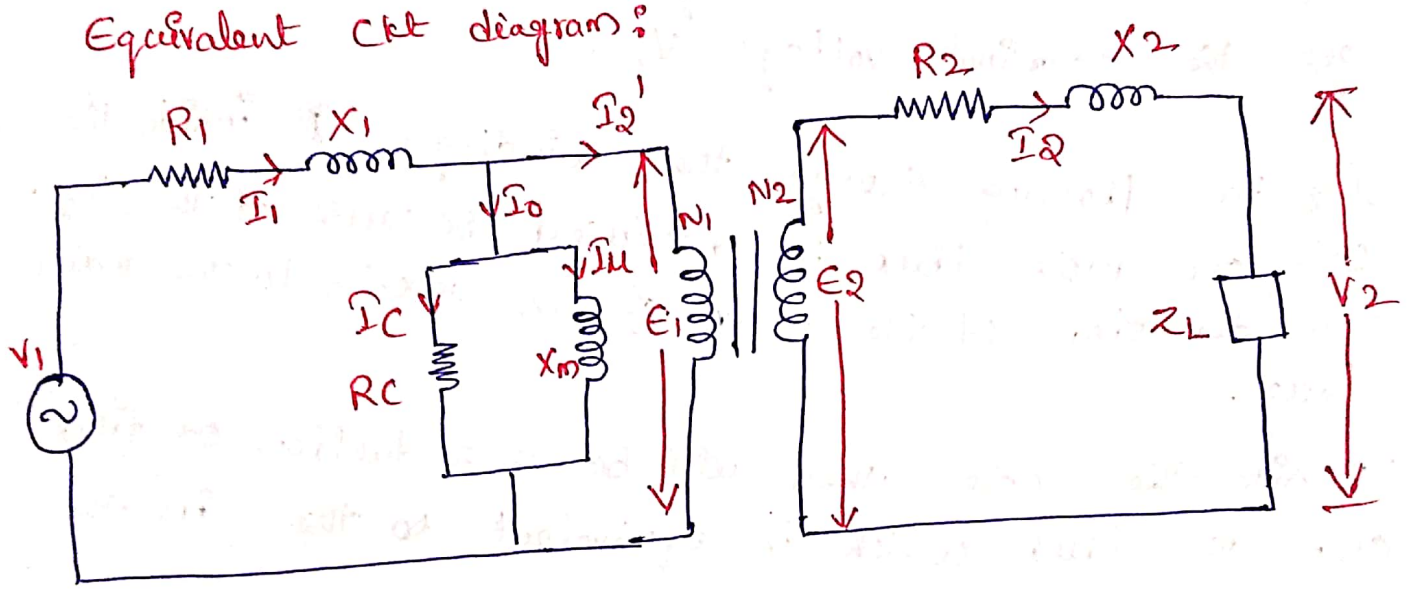
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Inside core three fluxes are there but the resultant is only one flux i.e main flux ϕ.

Equivalent ckt diagram:



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I_1 → primary current

R_1 → primary winding resistance.

X_1 → primary leakage reactance

I_0 → No load primary current.

I_C → core loss component of I_0

I_M → magnetising component of I_0

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I_2' → Component of I_1 to nullify effect of ϕ_2

R_c → core loss resistance

X_m → magnetising reactance.

N_1 → primary turns.

E_1 → primary induced emf

N_2 → Secondary turns

E_2 → p secondary induced emf

R_2 → secondary winding resistance

X_2 → secondary leakage reactance.

I_2 → Secondary current

V_2 → Secondary voltage or terminal voltage.

$$\bar{I}_1 = \bar{I}_0 + \bar{I}_2'$$

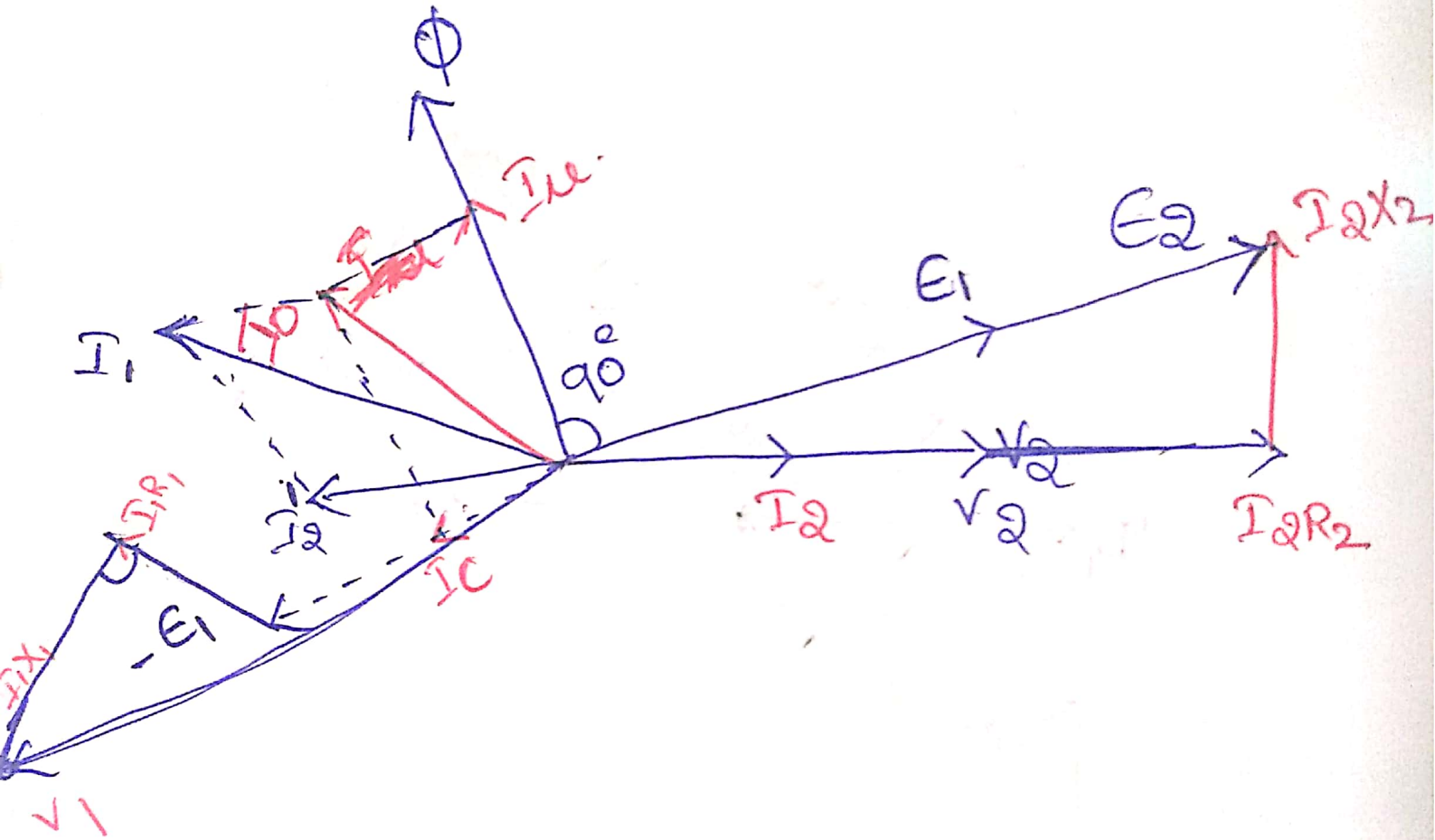
$\bar{I}_0$ → No load primary current
very low (5-10% of full load current).

$$\bar{I}_1 \simeq \bar{I}_2'$$

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Resistive load :-



$$\bar{I}_C + \bar{I}_\mu = \bar{I}_0$$

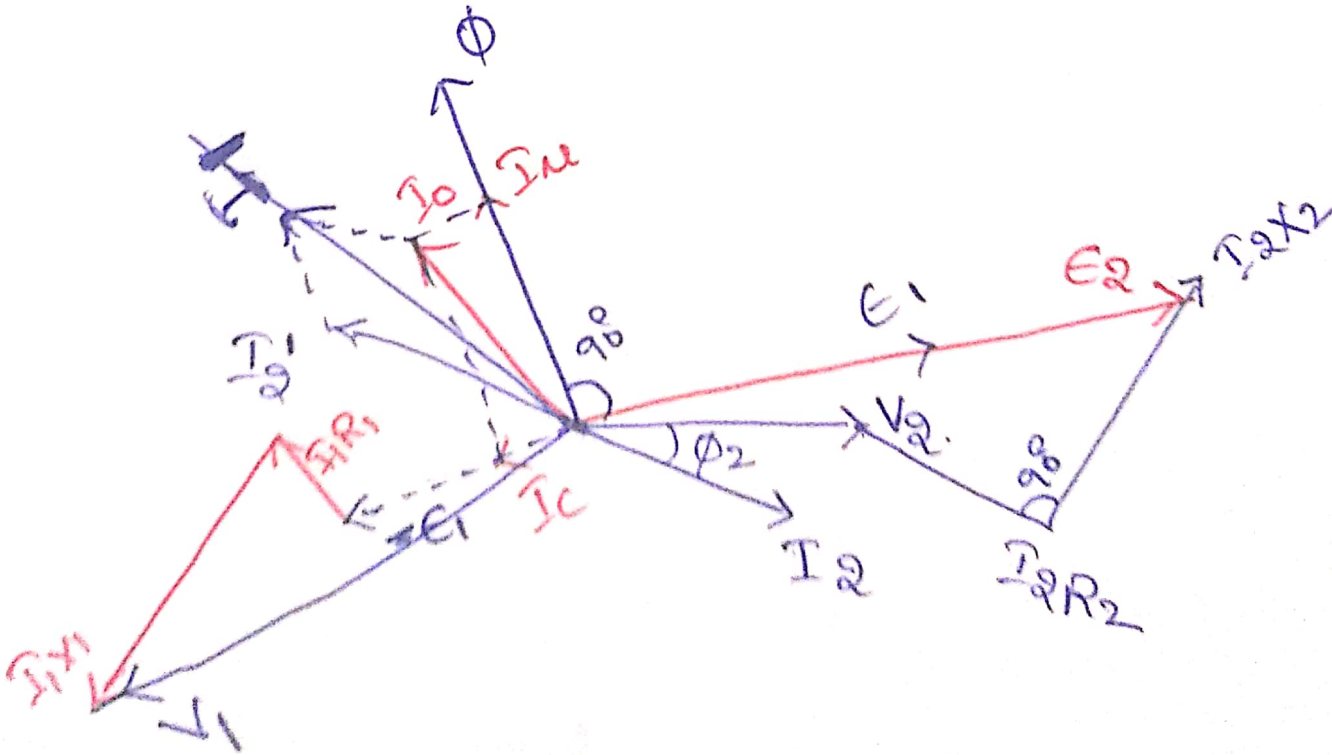
$$\bar{I}_0 + \bar{I}_2' = \bar{I}_1$$

$$-\bar{E}_1 + \bar{I}_1\bar{R}_1 + \bar{I}_1\bar{X}_1 = \bar{V}_1$$

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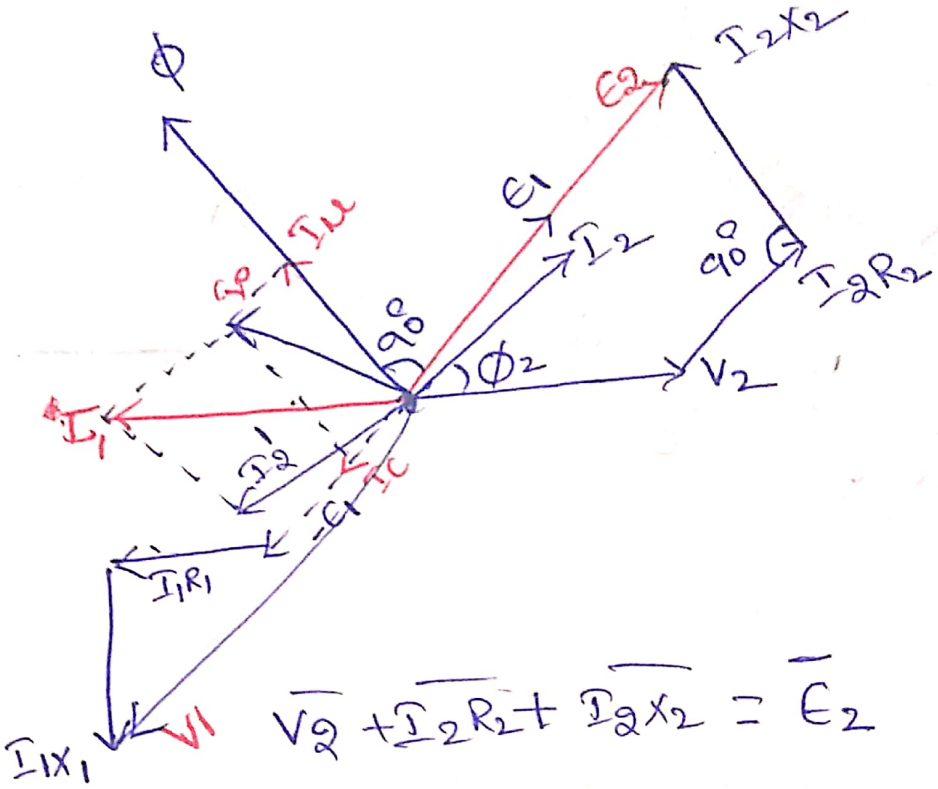
Inductive load

$$\bar{V}_2 + \bar{I}_2\bar{R}_2 + \bar{I}_2\bar{X}_2 = \bar{E}_2$$



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Capacitive load :-



$$\overline{V_2} + \overline{I_2}\overline{R_2} + \overline{I_2}\overline{X_2} = \overline{E_2}$$

$$-\overline{E_1} + \overline{I_1}\overline{R_1} + \overline{I_1}\overline{X_1} = \overline{V_1}$$

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Losses in a Transformer :-

Losses in a Transformer classified into two types.

1) Core loss (or) Iron loss (or) Constant loss:

Core gets subjected to alternating flux causing core losses.

Types of core losses

⇒ Hysteresis loss :

Due to AC flux setup in the magnetic core of the transformer, it undergoes a cycle of magnetisation and demagnetisation. Due to hysteresis effect there is a loss of energy in this process, which is called as hysteresis loss.

$$P_h = k_h B_m^{1.67} f.V$$

$P_h \rightarrow$ hysteresis loss

$k_h \rightarrow$ hysteresis constant depends on material

$B_m \rightarrow$ maximum flux density

$f \rightarrow$ frequency

$V \rightarrow$ volume of the core.

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⇒ Eddy current losses :

The induced emf in the core tries to setup eddy currents in the core and hence responsible for eddy current losses.

$$P_e = K_e B_m^2 f^2 t^2$$

P_e → eddy current loss
K_e → eddy current constant
t → thickness of the core
B_m → maximum flux density
f → frequency.

flux in the core is almost constant as supply voltage V_1 at rated frequency f is always constant.

P_h & P_e are constant losses.

How to Reduce these core losses.

* High grade core material like silicon steel with laminations of smallest thickness.

2) Copper losses (or) variable losses :-

The windings carry currents when transformer is loaded Causing copper losses.

The copper losses are due to power wasted in the form of I^2R loss due to the resistances of the primary and secondary windings.

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$$P_{cu} = I_1^2 R_1 + I_2^2 R_2$$

Copper losses depend upon the amount of load current which can be changed depends upon the load connected.

Hence copper losses are variable losses.

Transformer total loss is

Total losses = $P_i + P_{cu}$

Total losses = $(P_h + P_e) + (I_1^2 R_1 + I_2^2 R_2)$

Rating of a transformer is in VA | KVA :-

→ Rating of any device depends upon the losses occuring in that device.

→ In transformer two types of losses present.

Core loss : depends on the supply voltage.

Copper loss : depends on the current through windings.

→ In order to know total losses in a transformer by knowing VA of a transformer.

→ At the time of design nature of the load is not known. (i.e power factor $\cos\phi$)

Transformer rating is a apparent power.

losses in a transformer depends only on voltage and current but not depend on power factor.

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Efficiency of a Transformer :-

Efficiency is the ratio of the output power to input power.

$$\eta = \frac{\text{output power}}{\text{Input power}}$$

$$\eta = \frac{\text{output power}}{\text{output power + Losses.}}$$

for full load

$$\eta = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + W_i + W_{cu}}$$

$$\eta = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + W_i + I_2^2 R_{2e}}$$

for fractional load (half load or 60% load).

$$x = \frac{\text{actual load}}{\text{full load}} = \frac{I_2}{I_2 \text{ (full load)}}$$

$$I_2 = x \cdot I_{2(FL)}$$

$$\eta = \frac{x I_{2(F.L)} V_2 \cos\phi_2}{x V_2 I_{2(FL)} \cos\phi_2 + W_i + x^2 I_{2(FL)}^2 R_{2e}}$$

$$\eta = \frac{x (\text{VA Rating}) \cdot pf}{x (\text{VA Rating}) \cdot pf + W_i + x^2 W_{cu(FL)}}$$

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Maximum efficiency :-

$$\eta = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + W_i + I_2^2 R_{2e}}$$

differentiate η w.r.t I_2

$$\frac{d\eta}{dI_2} = \frac{(V_2 I_2 \cos\phi_2 + W_i + I_2^2 R_{2e})\, V_2 \cos\phi_2 - V_2 I_2 \cos\phi_2 (V_2 \cos\phi_2 + 0 + 2 I_2 R_{2e})}{(V_2 I_2 \cos\phi_2 + W_i + I_2^2 R_{2e})^2}$$

To get η_{max}

$$\frac{d\eta}{dI_2} = 0$$

$$(V_2 I_2 \cos\phi_2 + W_i + I_2^2 R_{2e})\, V_2 \cos\phi_2 - V_2 I_2 \cos\phi_2 (V_2 \cos\phi_2 + 2 I_2 R_{2e}) = 0$$

$$\therefore V_2 \cos\phi_2 \left[V_2 I_2 \cos\phi_2 + W_i + I_2^2 R_{2e} - V_2 I_2 \cos\phi_2 - 2 I_2^2 R_{2e} \right] = 0$$

$$W_i - I_2^2 R_{2e} = 0$$

$$W_i = I_2^2 R_{2e}$$

Iron losses = copper losses

This is the condition for maximum efficiency.

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load current at η_{max}

$$I_a^2 R_{ae} = W_i$$

$$I_a = x \cdot I_{a(FL)}$$

$$x^2 \cdot I_{a(FL)}^2 R_{ae} = W_i$$

$$x^2 W_{cu(FL)} = W_i$$

fractional full load

$$x = \sqrt{\frac{W_i}{W_{cu(FL)}}}$$

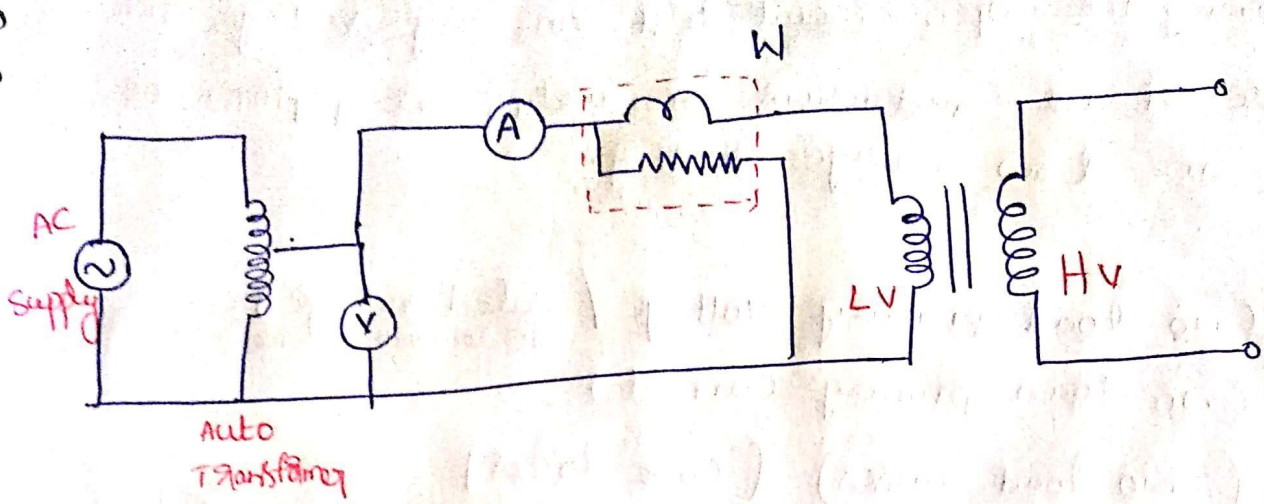
kVA supplied at η_{max} = x * (kVA Rating)

maximum Efficiency

$$\eta_{max} = \frac{x(\text{VA Rating}) \cdot Pf}{x(\text{VA Rating}) Pf + 2 W_i}$$

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Open circuit test (OC Test) :-



OC Test is performed on low voltage side. (low voltage winding side) keeping high voltage winding open circuited.

In the open circuit test the current measured at the ammeter is very less. So if KVA rating of the transformer is same from primary and secondary side.

Very less current is getting if we are applying voltage to the high voltage side.

Example: if KVA is 1KVA

Low voltage side : 100V
High voltage side is 10,000V

If we are performing the test on high voltage side very very low current is getting, which we can't measure at the ammeter.

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Again a very high voltage is applied at the HV side, that high voltage can't be measured at the voltmeter.

Because of the restrictions on meters we are not performing the open circuit test on high voltage side.

Because of the restrictions on meters we perform OC test on low voltage winding.

V_o (No load primary voltage / Rated voltage for low voltage side)

I_o (No load primary current)

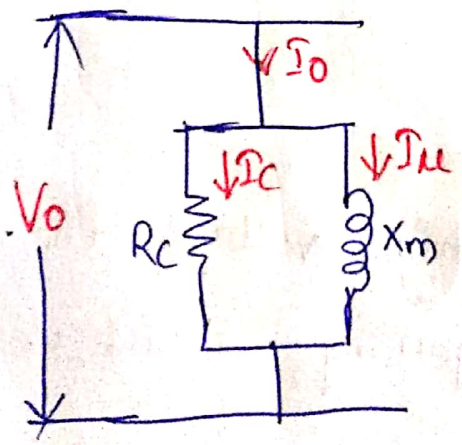
W_o (No load losses) [Core losses]

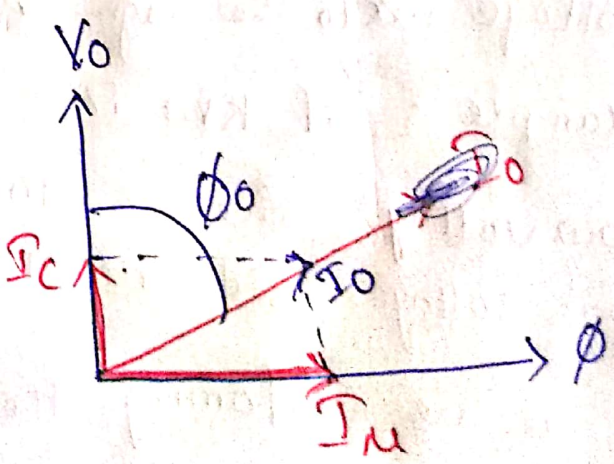
I_o is the no load primary current which is very very low. Copper losses occuring in the primary winding will be very less.

Hence the power measured in the wattmeter will be core losses only. in a Transformer only.

$$W_o = V_o I_o \cos\phi_o$$

$$\therefore \cos\phi_o = \frac{W_o}{V_o I_o} \Rightarrow \phi_o \text{ (No load pf angle)}$$





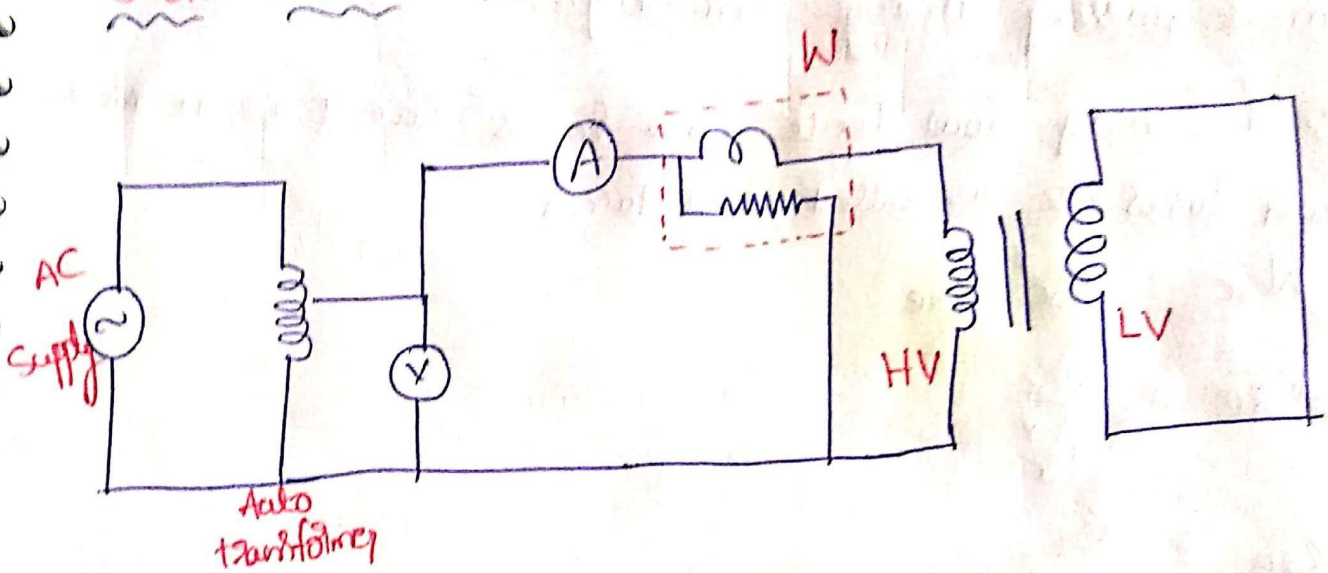
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$I_C = I_0 \cos\phi_0$

$I_M = I_0 \sin\phi_0$

$R_C = R_0 = \frac{V_0}{I_C}$ $\quad$ $X_m = X_0 = \frac{V_0}{I_M}$

Short circuit Test (SC Test) :-



Normally SC Test is performed on H.V winding, short circuiting LV winding. Because of the reason of the ammeter.

If we perform a test on low voltage side a very high current is flowing by the ammeter, it can't be measured by this ammeter.

that is the only reason behind this, otherwise both the tests are performed on the same windings.

V_{sc} (short ckt voltage)

I_{sc} (short ckt current)

W_{sc} (short ckt power).

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V_{sc} is very low compared to rated voltage

If voltage is very low, the losses depends on voltage will also be less.

losses will depends on voltage that losses are called core losses. Hence the wattmeter will not measure the core losses.

I_{sc} is comparable with full load current. Wattmeter will read the copper losses because of high current passing through windings.

V_{sc} is very low hence core losses can't be recorded.

power losses W_{sc} are called I^2R losses

$$W_{sc} = I_{sc}^2 R_{eq}$$

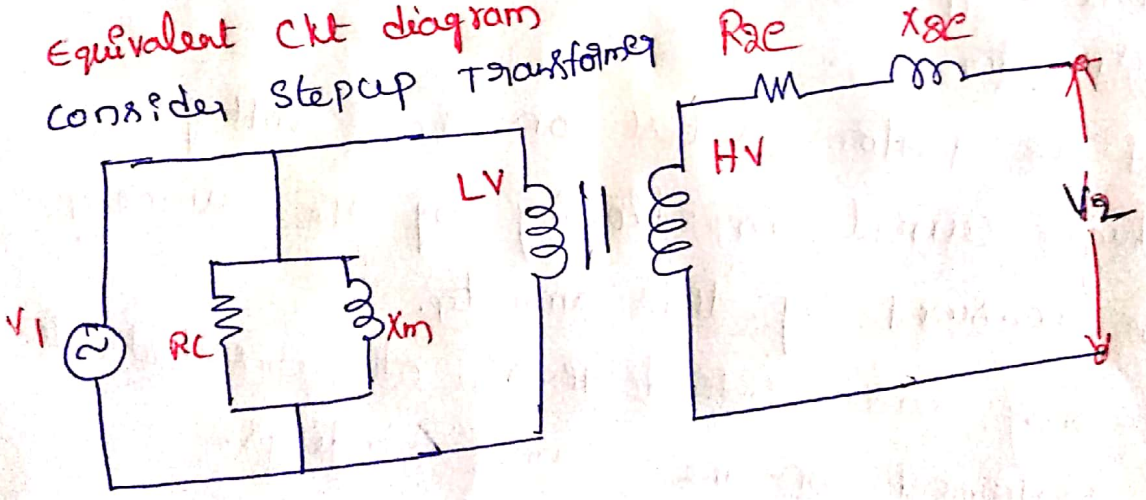
$$R_{eq} = \frac{W_{sc}}{I_{sc}^2}$$

$$Z_{eq} = \frac{V_{sc}}{I_{sc}}$$

$$X_{eq} = \sqrt{Z_{eq}^2 - R_{eq}^2}$$

Equivalent ckt diagram

consider stepup transformer



Core losses (OC test)

Copper losses (SC test)

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The use of OC & SC test we can find the transformer specifications i.e efficiency and voltage regulation of a transformer.

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DC Machines :-

Transfer of electrical energy from one form to another form.

It converts electrical energy to mechanical energy and mechanical to electrical energy. This conversion is called as electromechanical conversion.

Conversion of mechanical energy into electrical energy is nothing but a Generator.

Conversion of Electrical energy into mechanical energy is nothing but a motor.

Electrical machines are two types.

→ AC machines : Electrical machine which works on the alternating supply is called AC machines.

→ DC machines : Electrical machines which works on the direct type supply is called DC machines.

Principle of operation of DC Generator :-

DC Generator working principle is based on dynamically induced emf i.e faraday's law of an electromagnetic induction.

faraday's law of an electromagnetic induction :-

It states that whenever the flux linking with a conductor changes, an electromotive force (emf) is setup in that conductor.

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Changing a flux is having two types.

Relative motion b/w the conductor and flux

Relative motion can be achieved in two ways.

i) flux is stationary, conductor is moving.

ii) Conductor is stationary, flux is moving.

In the DC Generator, flux required to produce a motion or emf induced in a dc generator will be generated from electromagnet as a effect of DC current.

Electromagnet :-

A conductor wound over a magnetic material that conductor is carrying a current. That current is responsible for production flux around it.

Conductor can be rotated with the help of prime mover prime mover is a external device, which will rotates the conductor.

Examples of prime mover.

Diesel Engine, steam turbine, Diesel turbine etc.

These are the external agents which will produce a torque, which will be given to a conductor which will be placed inside the magnetic flux inorder to have a rotation of a conductor and that conductor. when it is cutting with study magnetic flux there will be a relative motion b/w flux and conductor and by faraday's law of an electromagnetic induction emf is induced in that conductor.

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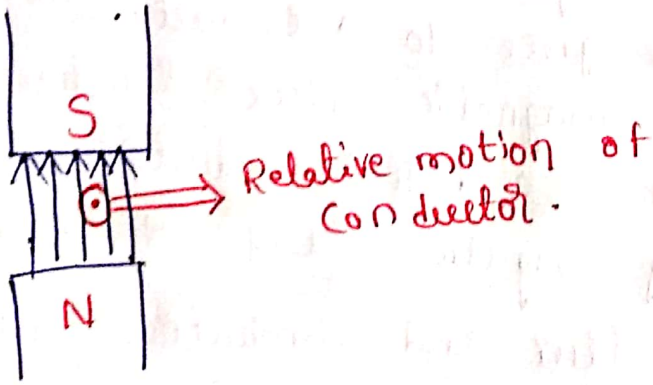
Since for a single turn the emf is very less. So we form a conductor of several wires or several coils are ultimately forming a winding. That winding is called a Armature winding.

In DC machine emf armature is induced in the armature. and the winding which is responsible for the production of this is called armature winding.

Fleming's Right hand Rule :-

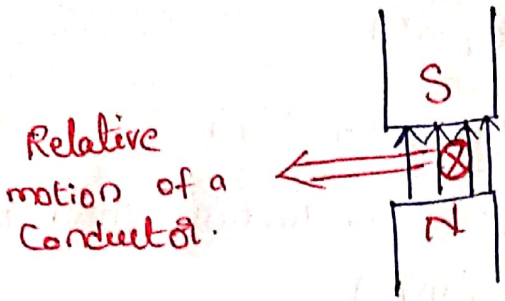
If three fingers of a righ hand, namely thumb, index finger and middle finger are outstrected so that everyone of them is at right angles with the remaining two and if in this position index finger is made to point in the direction of flux, thumb in the direction of the relative motion of the conductor with respect to flux then the outstretched middle finger gives the direction of emf induced in the conductor.

Direction of induced e.m.f :-

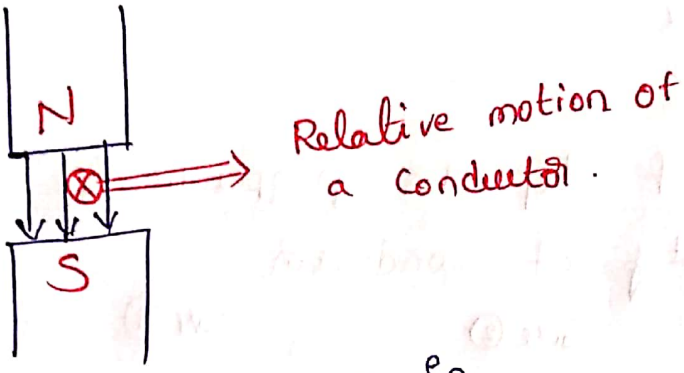


* current is coming out

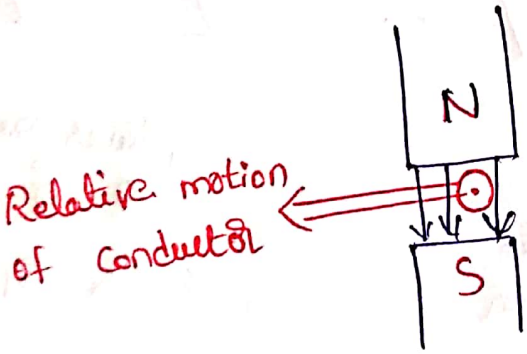
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* current going in



* current going in



* current coming out.

magnitude of induced emf :-

$e = Blv \sin\theta$

$e = E_m \sin\theta$ volts

$\therefore$ Blv constant $= E_m$

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where

B = flux density (wb/m²)

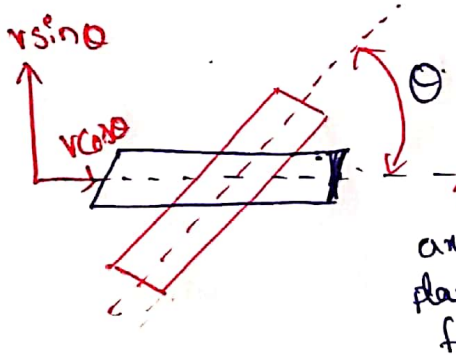
l = active length of conductor (m)

v = relative velocity of conductor with respect to magnetic flux (m/s)

θ = angle b/w plane of rotation and plane of magnetic flux measure from axis of plane of flux.

Magnitude of emf depends perpendicular component of relative velocity of conductor.

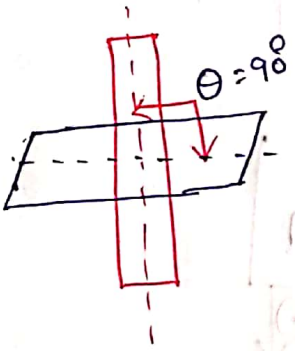
Case ①



$e \propto v \sin\theta$

v sinθ is perpendicular component of v

Case ②

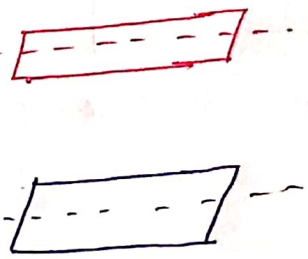


planes are perpendicular

θ = 90°

e is maximum

Case ③

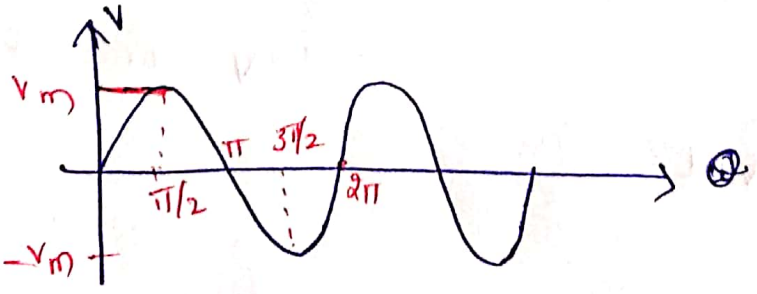


planes are parallel

θ = 0°

e is zero.

Emf induced in a DC machine is alternating nature.



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Inside a generator one rectifier is there, which converts emf induced which is in AC to DC. That rectifier is not electronic. (i.e no semiconductor is used). It is mechanical and the device which is responsible for conversion of AC to DC. That device is called Commutator.

Commutator:-

A device is used in dc generator to convert alternating induced emf to unidirectional DC emf. It also called as mechanical rectifier.

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Yoke :-

Yoke is the outermost protective cover for a DC machine which is responsible for protection of insulating materials that are used inside.

It protects all the components inside from dust, moisture and all other gases and any other particles from the external environment.

Yoke is made up of a magnetic material which is cast iron or cast steel.

The magnetic flux is required for a production of emf is passed through this yoke.

yoke is providing a low reluctance path for the flux.

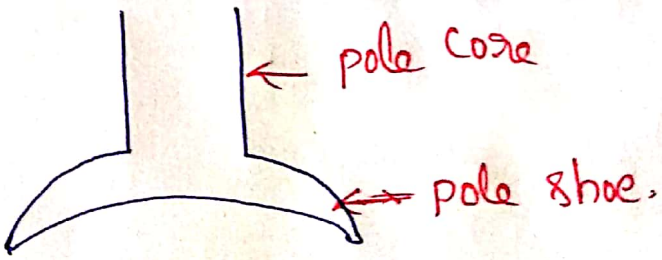
maximum flux will be passing through this yoke, hence low reluctance is required so that will save a magnetic power.

pole :-

pole has two parts.

→ pole core

→ pole shoe



The job of pole core is ~~[illegible]~~ to carry field winding.

field winding is a winding which is responsible for

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production of flux.

pole core is to direct the magnetic flux through air gap, armature and to the next pole.

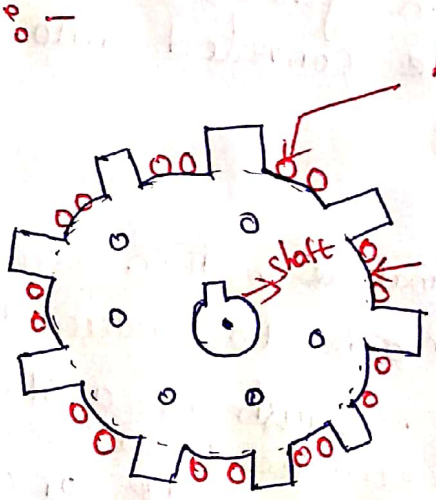
- pole shoe cover maximum armature conductors in its periphery. So the pole shoe area is required, it will cover maximum armature conductors to cut a flux. so that emf induced is more.

pole is madeup of laminations.

field windings :-

field windings are carry current which is responsible for the production of magnetic flux. It should be madeup of good conducting material i.e aleminium or copper. A better ductility is providing by the copper.

Armature :-



Armature slots are carry armature conductors. Emf got induced in the armature conductors.

Air dugs are provide for the cooling purpose.

Armature carrying a magnetic flux.

Armature is madeup of a magnetic material.

Armature conductor is madeup of a copper, it is to off current.

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Commutator :-

Commutator is a mechanical rectifier.
So whatever the currents has been developed or whatever the emf is induced in the armature current that need to be taken out for that purpose it is first to connect to a commutator.

Commutator is made up of a segments and that segments is made up of a copper bcoz it has to carry a current.
Two segments are seperated by a mica ie thin layer of mica.

One segment is associated with the one conductor of a armature winding - ~~card~~

The job of commutator is take out the voltage and current from the armature conductors for the outside the world and secondly serve ~~the~~ as a purpose of mechanical rectifier.
So that alternating DC is generated inside the armature conductor can be converted into unidirectional DC.

Brushes :-

Commutator is fit with armature, this armature is rotating to cut the flux in order to induce the emf commutator is also rotating. Brush is a stationary part.
Brush is a stand steel : it is connected to a every segment.
The job of brush is to take a voltage or current is there inside the commutator to outside the world.

The DC voltage taken out from the commutator to through the brush to the terminal box.

Brush is made up of a carbon.

Shaft is a mechanical piece in a DC machine that provide the rotational of a force known as torque and brings about rotation.

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E.M.F equation of DC generator :-

P = Number of poles of the generator.

ϕ = flux produced by each pole (wb)

N = Speed of armature (rpm)

Z = Total no. of conductors in armature.

A = No. of parallel paths in which conductors are distributed.

e = Rate of cutting the flux

$e = \frac{d\phi}{dt}$ → as per faraday's law of electromagnetic induction.

Total flux = flux produced by each pole × No. of poles

Total $\phi = \phi \times P$

Time required for a conductor to complete one revolution is equal to $\frac{60}{N}$

$e = \frac{\phi P}{\frac{60}{N}} = \frac{\phi P N}{60}$ → emf induced by one conductor

Z conductors are distributed in A parallel paths.

Effectively $\frac{Z}{A}$ Conductors need to be multiplied with emf induced in conductor

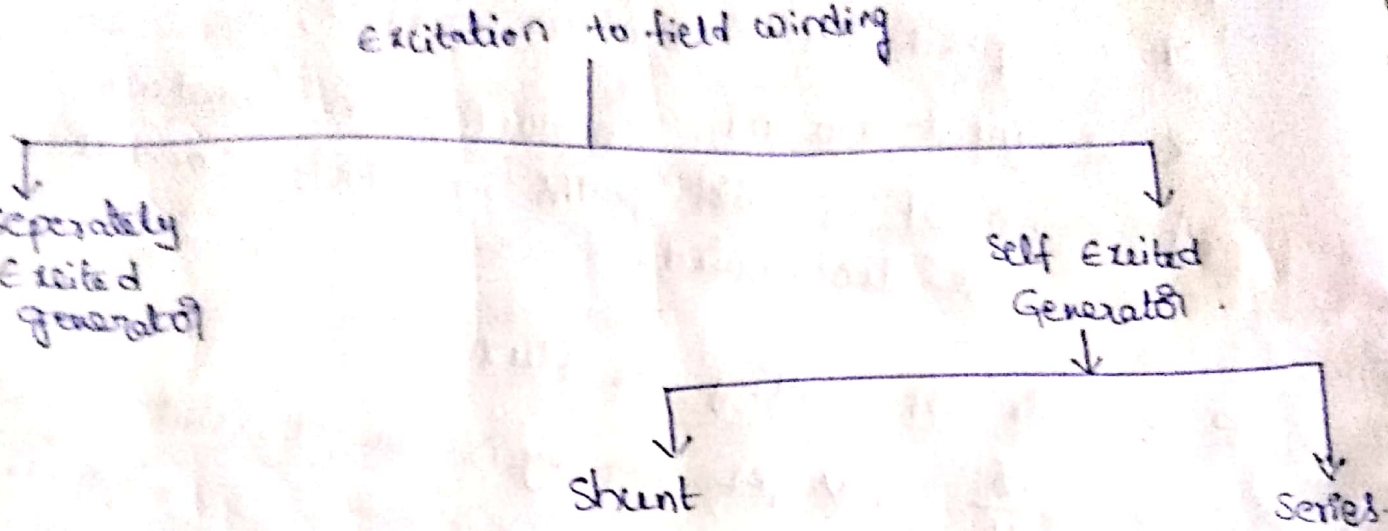
$$e = \frac{\phi P N}{60} \times \frac{Z}{A}$$

→ emf equation for DC Generator.

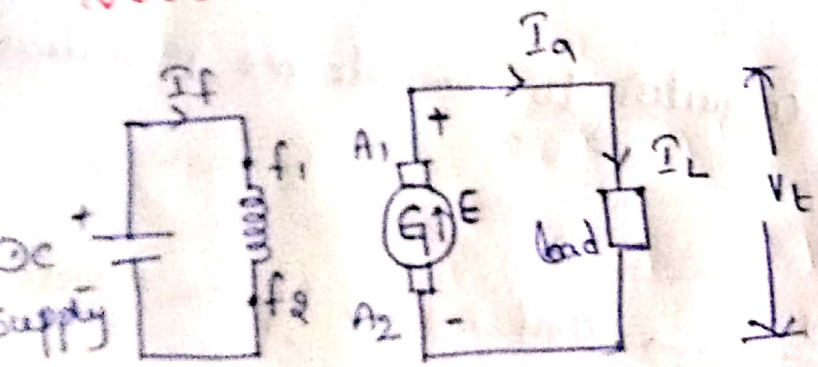
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Types of DC Generator :-

Types of DC Generator depends on the connection of a field windings in a generator and how it is excited.



Seperately Excited Generator :-



Different DC supply given to the field winding and from the armature we are taking the voltage out.

Voltage - Current relations

$I_a = I_L$

V_t =

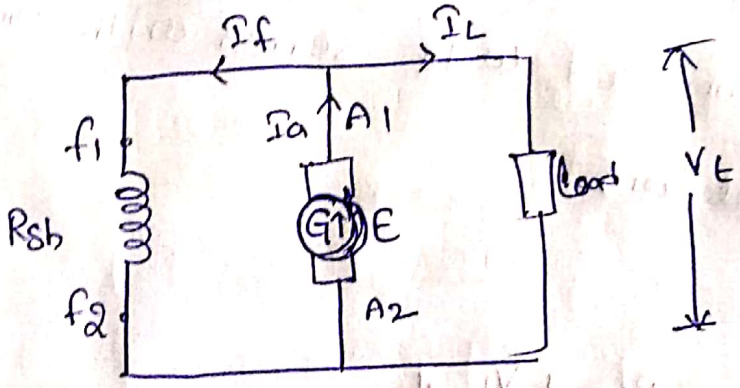
$$E = V_t + I_a R_a + V_{brush}$$

flux ϕ is independent of I_a or e.m.f got induced in generator. It's purely dependent on seperate DC supply.

Self Excited DC Generator :-

A DC generator whose field windings are excited by current supplied by generator is called a self excited DC generator. Here there is a physical connection b/w field winding and armature winding.

Shunt Generator :-



field winding is parallel with armature winding. It is called a shunt generator.

whatever the terminal voltage or whatever the emf induced in armature it will serve a field current I_f.

current & voltage Relations.

$$I_a = I_L + I_f$$

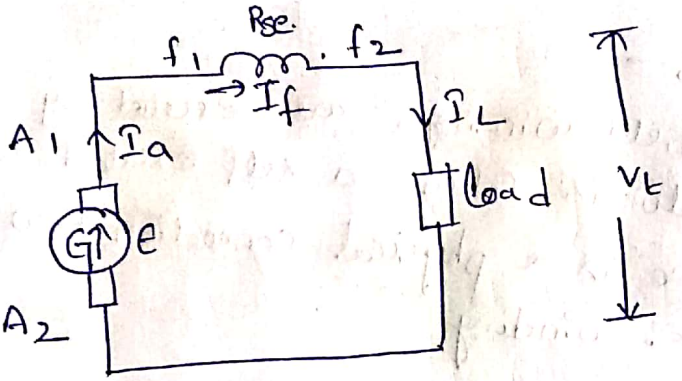
$$I_f = \frac{V_t}{R_{sh}}$$

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e.m.f induced in the armature

$E = V_t + I_a R_a + V_{brush}$

Series Generator:-



~~$I_a = I_f = I_L$~~

field winding is connected in series with armature.

Voltage & current Relations

$I_a = I_f = I_L$

$E = V_t + I_a R_a + I_a R_{se} + V_{brush}$

$E = V_t + I_a (R_a + R_{se}) + V_{brush}$

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DC motor:-

Working principle of a DC motor:-

When a current carrying conductor is placed in a magnetic field, it experiences a mechanical force.

In a DC motor the flux is created by field winding Armature winding plays a role of current carrying conductor.

Armature windings are carry current, those windings are or conductors carry placing in a magnetic field caused by field winding, hence there will be a mechanical force on each conductor of a armature of winding. This armature conductors are placed in a slot in the periphery of armature. Hence every conductor experience a mechanical force, that will produce a twisting action which will generate a torque. Hence armature starts rotating.

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Direction of rotation of motor :-

Magnitude of the force experienced by armature conductor is

$$F = BlI \quad (N)$$

where

B = magnetic flux density produced by field winding.

l : Active length of the conductor.

I = Magnitude of the current carrying by the conductor.

Direction of rotation is determined by fleming's left hand rule.

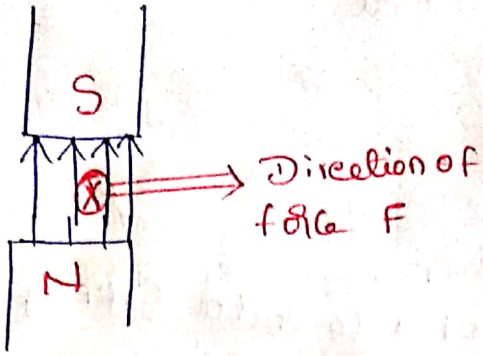
Fleming's left hand Rule:

The rule states that, the three fingers of the left hand namely the thumb finger, index finger and middle finger, they are mutually perpendicular to each other.

The index finger is the direction of magnetic field, middle finger is in the direction of current then the thumb finger gives the direction of the force experienced by the conductor.

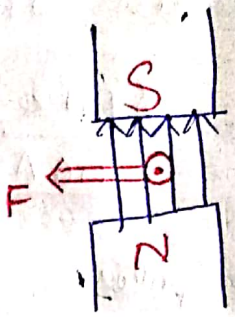
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Case ①



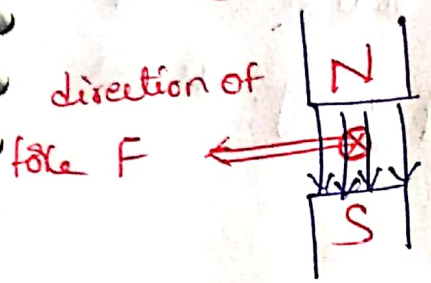
Current coming in

Case ②

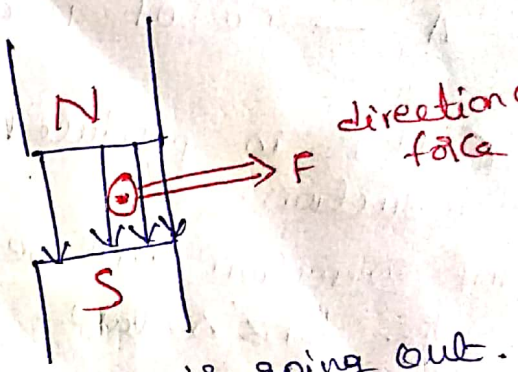


Current going out.

Case ③



Current is going in



Current is going out.

To reverse the direction of rotation either direction of main field produced by the field winding is reverse (Case 1, 3) or direction of the current passing through armature is reverse (case 1, 2)

Back E.M.F (E_b) :-

once motor starts rotating its conductor will cut by the magnetic flux produced by field winding. So by faraday's law of an electromagnetic induction there will be a emf induced. This emf opperes the supply voltage.

Back emf is generated by the generating action (moving conductor cutting the magnetic flux).

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E.m.f equation of a motor

$$E_b = \frac{\Phi P N Z}{60 A}$$

P — no. of poles.

N — speed of rotation of a conductor. (rpm)

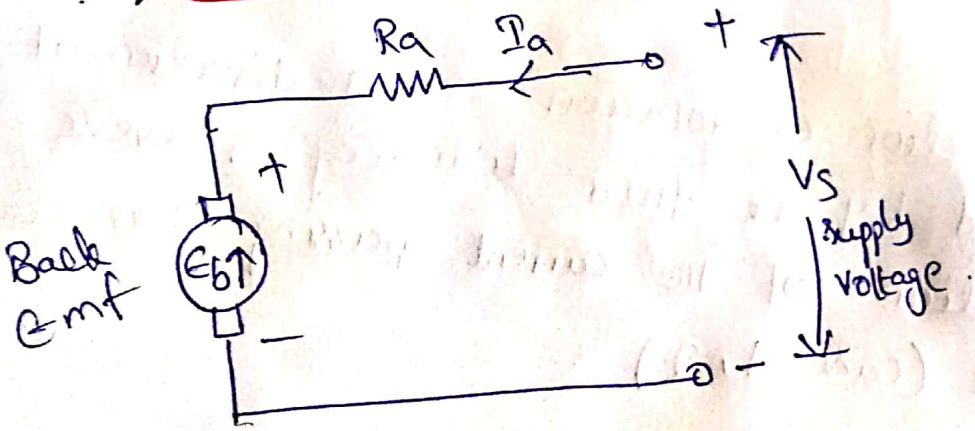
Z — total no. of conductors.

A — no. of parallel paths of a conductor.

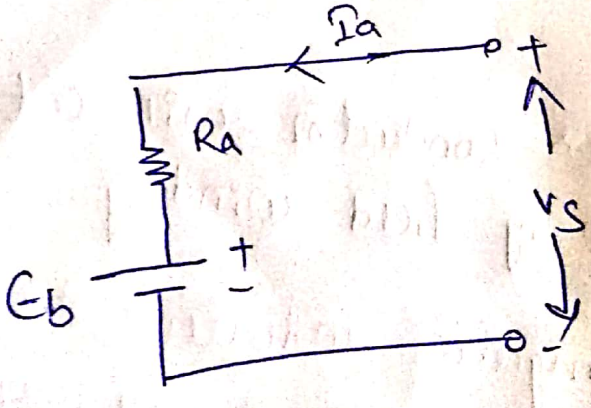
E.M.F is produced because of faraday's law of electromagnetic induction.

Emf direction will be opposite to the supply voltage as per the lenz's law.

Equivalent Ckt of a motor



The back emf is generated such a way that it will oppose the armature current it will produced by the supply voltage Vs.



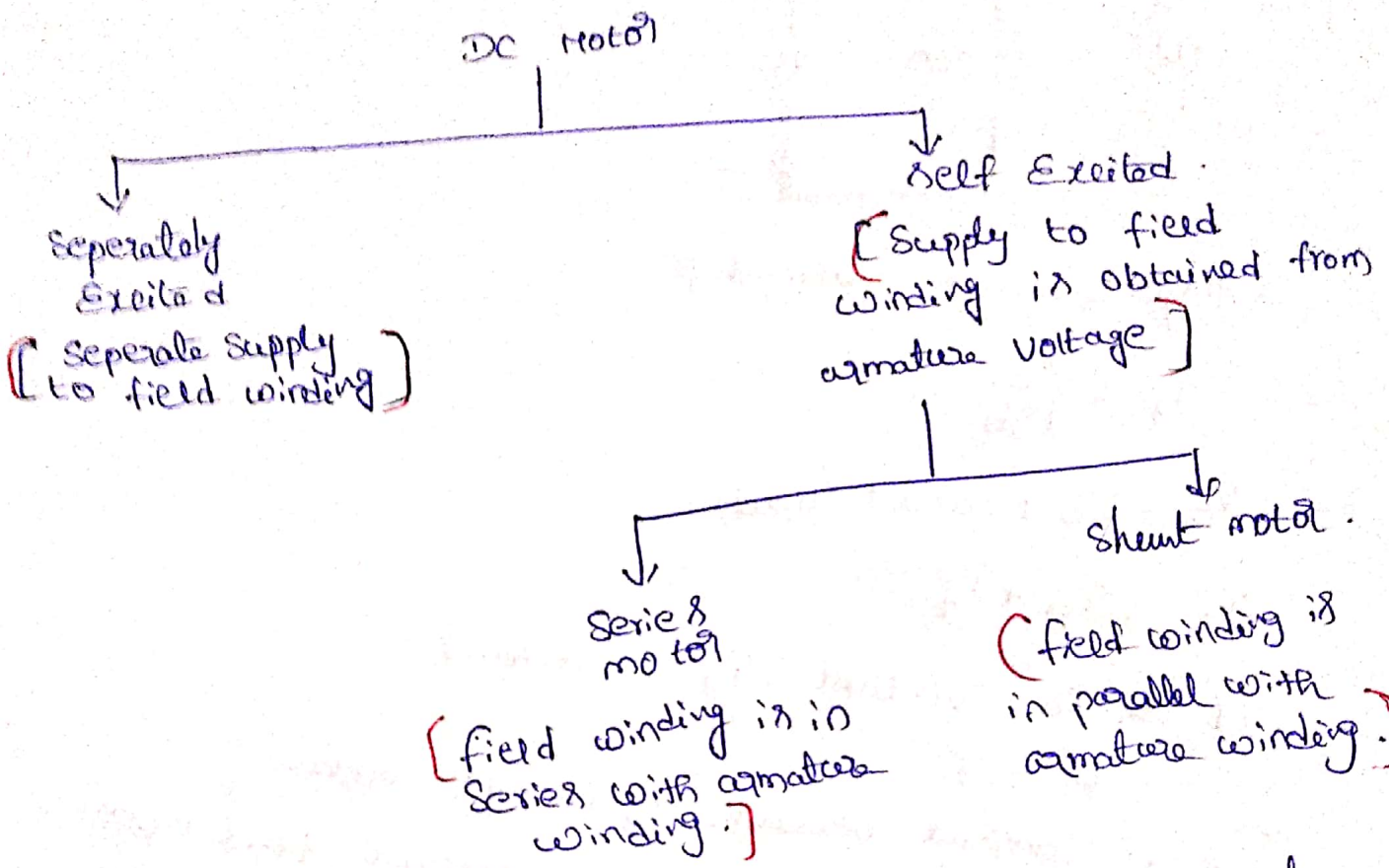
voltage equation

$$V_S = I_a R_a + E_b + V_{brush}$$

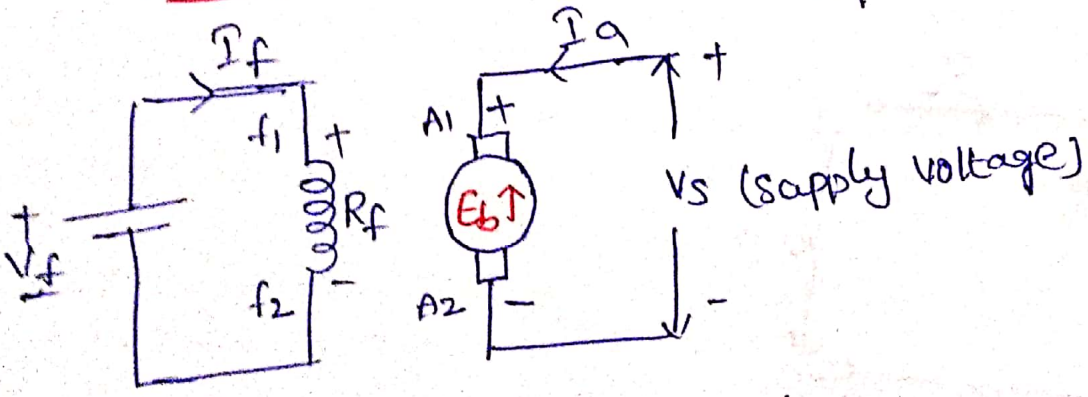
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Types of DC motor:-

As per the excitation given to the field winding types of dc motors are classified into two types.



Seperately Excited motor:- field winding is excited through a seperate supply



Voltage & current relationship

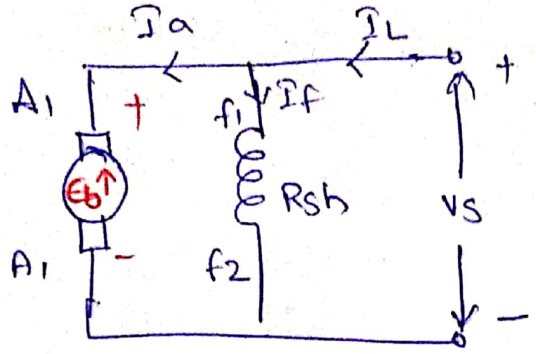
Constant flux ϕ , ϕ only depends of V_f

$V_S = E_b + I_a R_a + V_{brush}$

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Shunt motor :-

field winding is in parallel with armature winding.



Voltage & current relationship.

$$I_f = \frac{V_s}{R_{sh}}$$

$$V_s = E_b + I_a R_a + V_{brush}$$

$$I_L = I_a + I_f$$

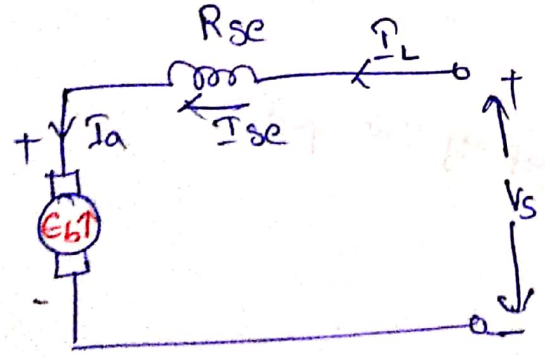
If V_s is constant, I_f is constant.

$$\phi \propto I_f$$

I_f is constant because V_s is kept constant.

This motor is also called as constant flux motor.

Series motor:-



voltage & current relationship

$$I_L = I_{se} = I_a$$

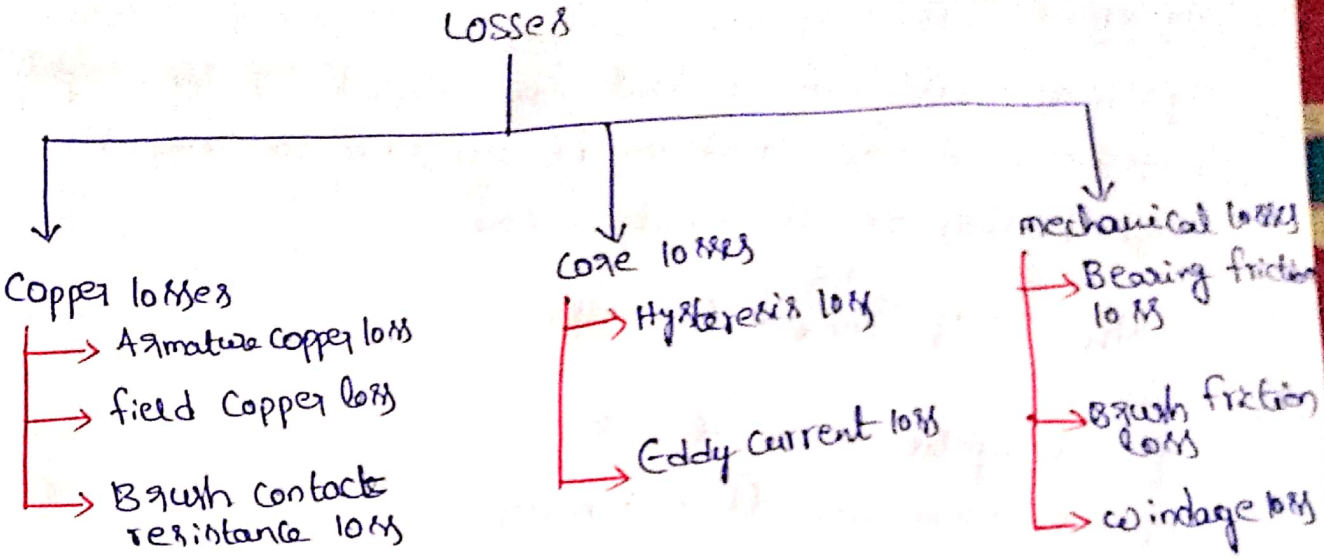
$$V_s = E_b + I_a R_a + I_a R_{se} + V_{brush}$$

$$V_s = E_b + I_a (R_a + R_{se}) + V_{brush}$$

$$\phi \propto I_{se} \propto I_a.$$

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Losses in a DC motor:-



Mechanical losses :-

The losses produced by the mechanical friction of the machine are called mechanical losses.
These losses occur due to friction in the moving parts of the motor like bearings, brushes etc.

Bearing friction loss depends on type of bearings and their lubrication.

Brush friction loss occur in b/w the carbon brush and Commutator.

For cooling purpose air is passing through a ventilating dugs of a motor, it also causes some friction it is known as windage loss or wind friction loss.

Core losses :- Core losses are presenting in the all rotating electrical machines.

Iron Core of the armature

In the motor iron core of the armature is rotating in the magnetic field. So Core losses occured normally motor is operated in constant speed. So these

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losses are almost constant.

Hysterxis loss :-

Hysterxis loss is the heat loss caused by the magnetic properties of the armature. It occurs due to reversal magnetisation of the armature core.

$$W_h = \eta B_{max}^{1.6} f V \text{ watts}$$

$\eta \rightarrow$ hysterxis co-efficient.
$B \rightarrow$ maximum flux density
$f \rightarrow$ frequency.
$V \rightarrow$ volume of core.

Eddy current loss :-

If armature of a motor rotates in magnetic field it will cut the magnetic flux and produce current in the armature coil due to this in the armature core also a small amount of emf is induced and it produces a small amount of current with last magnitude due to eddy current. called eddy current.

The power loss due to flow of this current is called eddy current loss.

The power is dissipated in the form of heat.

$$W_e = k B_{max}^{2} f^{2} t^{2} V \text{ watt}$$

$t \rightarrow$ thickness of lamination
$k \rightarrow$ eddy current constant

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Copper losses :-

Copper losses are the winding losses that take place during the current flowing through the winding. That occur due to the resistance of winding. They generate heat that is proportional to the resistance of the conductor.

motor has two windings.

→ Armature winding
→ field winding.

Armature copper loss :-

Armature Copper loss is equal to $I_a^2 R_a$

where
R_a → armature resistance
I_a → Armature current.

This loss is about 20-30% of full load losses.

field copper loss for shunt motor = $I_{sh}^2 R_{sh}$

field Copper loss for series motor = $I_{se}^2 R_{se}$.

Brush contact loss occurs due to resistance b/w the surface of brush and commutator.

Magnetic losses and mechanical losses are together is known as stray losses. These losses are known as rotational losses.

Total losses =	Variable armature Copper loss $I_a^2 R_a$	+	Constant losses W/C

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Efficiency :-

It is the ratio of the motor output to the input.

$$\text{Efficiency } \eta = \frac{\text{output power}}{\text{Input power}}$$

$$\text{mechanical efficiency } \eta_m = \frac{\text{Useful mechanical power output}}{\text{mechanical power developed}}$$

$$= \frac{\text{Useful mechanical power output}}{E_b I_a}$$

$$\text{electrical efficiency } \eta_e = \frac{\text{mechanical power developed}}{\text{Total electrical power input}}$$

$$= \frac{E_b I_a}{V I}$$

$V \rightarrow$ terminal voltage

$I \rightarrow$ load current.

$$\text{overall efficiency } \eta = \frac{\text{useful mechanical opp power}}{\text{Total electrical power i/p}}.$$

$$\text{overall efficiency } \eta = \eta_m \times \eta_e$$

motor input = output + Total losses.

$$= V I + I_a^2 R_a + W_c$$

$$I_a = I + I_{Sh}$$

$$= V I + (I + I_{Sh})^2 R_a + W_c$$

field current is negligible compare to load current.

$$= V I + I^2 R_a + W_c$$

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$$\eta = \frac{o/p}{i/p}$$

$$= \frac{VI}{VI + I^2 Ra + Wc}$$

$$= \frac{VI}{VI\left(1 + \frac{I Ra}{V} + \frac{Wc}{VI}\right)}$$

$$\eta = \frac{1}{1 + \frac{I Ra}{V} + \frac{Wc}{VI}}$$

efficiency is maximum when denominator is minimum.

$$\frac{I Ra}{V} + \frac{Wc}{VI} = 0 \Rightarrow$$

or

$$I_a^2 Ra = Wc.$$

variable loss = constant loss.

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Speed control of DC shunt motor:-

The emf equation of a motor

$$E_b = \frac{\phi P N}{60} \times \frac{Z}{A}$$

$$N = \frac{60 A E_b}{P \phi Z} \rightarrow (1)$$

$$N \propto \frac{E_b}{\phi}$$

for the shunt motor equation is given by

$$V = E_b + I_a R_a \rightarrow (2)$$

$$E_b = V - I_a R_a \rightarrow (2)$$

from eq (1) & (2) speed is given by

$$N \propto \frac{V - I_a R_a}{\phi} \rightarrow (3)$$

From the above equation hence it is observe that the speed of motor depends on various factors such as flux per pole (ϕ), armature resistance Ra and voltage V

1) field control methods:-

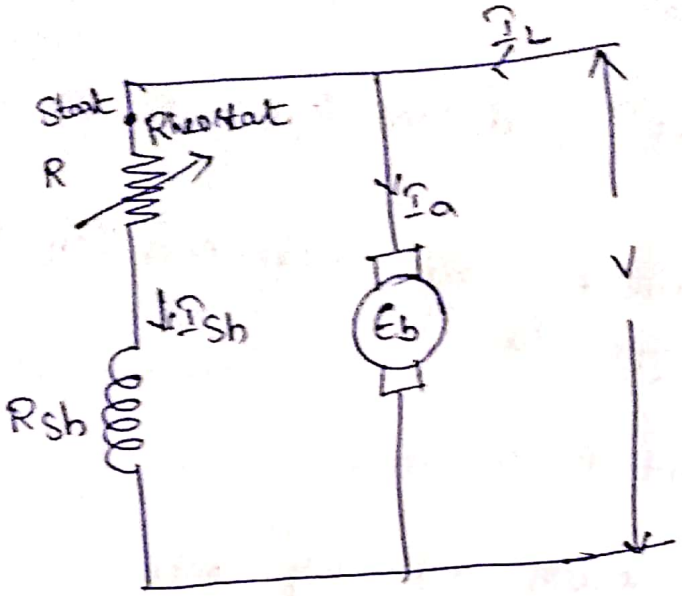
from equation (3) $N \propto \frac{1}{\phi}$

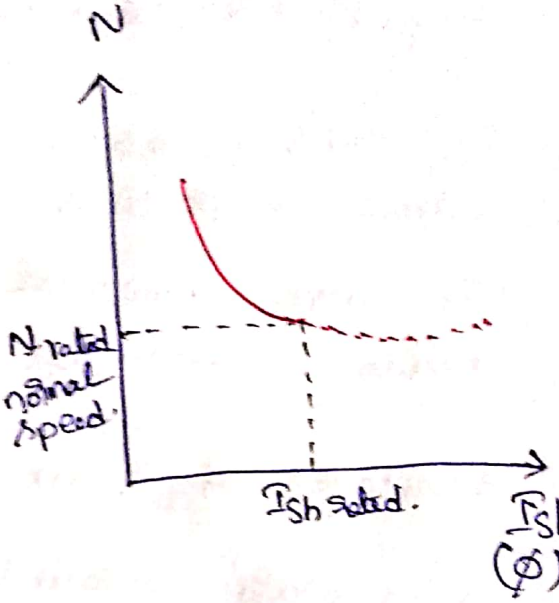
The speed is inversely proportional to the flux.

→ In this method the speed can be increased by decreasing the flux & vice-versa.

→ The speed is controlled by controlling the field flux.

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The flux is dependent on the current through the shunt field winding.
The flux can be controlled by adding a rheostat (variable resistor) in series with the shunt field winding.
At the begining the rheostat R is kept minimum indicated as start.

The supply voltage is at its rated value. So the current through shunt field winding is also rated value.
Hence the speed is also rated speed, also called as normal speed.

The resistance R is increased due to which shunt field current I_{sh} decreases and also decreasing flux will be produced.

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As $N \propto \frac{1}{\phi}$, The speed of the motor increases beyond its rated value.

In this method, the speed control above the rated value is possible.

The curve shows the inverse relation b/w N & ϕ_{sh} as its nature is rectangular hyperbola.

Advantages of flux control method :-

→ It provides relatively smooth and easy control

→ Speed control above rated speed is possible.

→ As the field winding resistance is high, the field current is small. Hence the power loss ($I_{sh}^2 R$) in the external resistance is very small. It is more economical and efficient

→ As the field current is small, the size of the rheostat required is small.

Disadvantages :-

→ The speed control below normal rated speed is not possible as flux can be increased only upto its rated value.

→ As flux reduces, speed increases. But high speed affects the commutation making motor operation is unstable.

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Armature voltage control method or Rheostatic control method.

from eq (5)

$$N \propto E_b$$

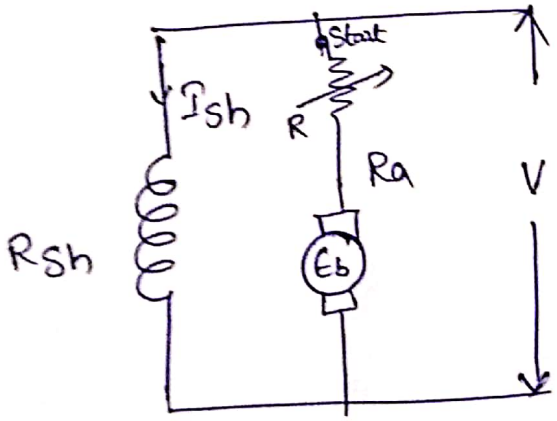
$$N \propto (V - I_a R_a)$$

The speed is directly proportional to the voltage applied across the armature.

The speed can be controlled by controlling the armature resistance.

The speed can be increased by decreasing the armature resistance.

As the supply voltage is normally constant, the voltage across the armature can be controlled by adding a variable resistance in series with the armature.



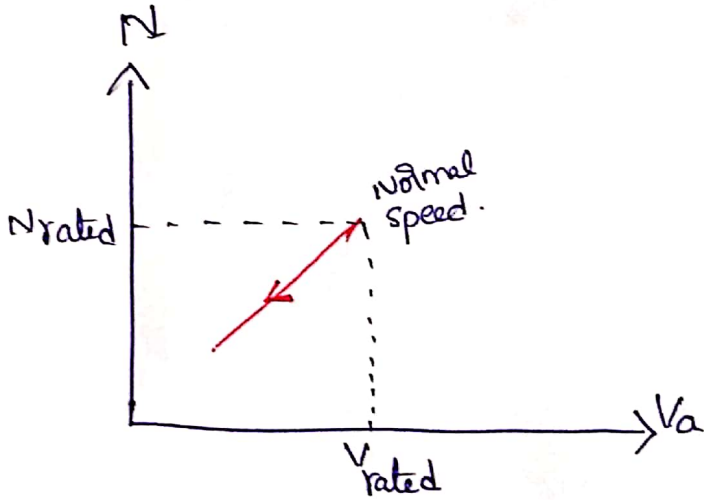


fig: Speed Vs voltage across armature.

The field winding is excited by the normal voltage. Hence I_{sh} is rated and constant in this method.

Initially the rheostat position is minimum and rated voltage gets applied across the armature and speed is also rated.

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When extra resistance is added in the armature circuit, I_a remains same and there is a voltage drop across the resistance added (I_aR)

Hence voltage across the armature decreases, decreasing the speed below a normal value.

By varying extra resistance various speeds below a rated value can be obtained.

So for a constant load torque. the speed is directly proportional to the voltage across armature.

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UNIT - 6

Polyphase Induction motors -

Three phase currents flowing within the stator create a rotating magnetic field which induces a current and consequent magnetic field in the rotor. Stator is a stationary part, rotor is a rotating part.

working principle of 3-phase Induction motor :-

When a three phase supply is connected to the three phase winding of the stator, rotating magnetic field is produced. Stator winding is called as field winding, rotor winding is called as armature winding. This magnetic field rotates in the clock wise direction.

This rotating magnetic field comes in the contact with stationary rotor conductor. Rotor conductor is stationary b/w the stator winding of the stator and rotor air gap is present. through the air gap the rmf of the stator comes in contact with the rotor conductor. ~~As per faraday's law of electromagnetic induction~~ when there are two parameters one is moving another one is stationary.

In this case magnetic field is rotating, where as rotor conductor is stationary.

So As per faraday's law of an electromagnetic induction EMF is induced in the rotor conductor.

Since the rotor conductors are short circuited at both the ends by the end rings, this induced voltage causes the current to flow through the rotor conductor. which is called as induced current.

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By fleming's right hand rule, direction of the induced current in the conductor can be determined.

The current in the rotor ~~conductor~~ conductor produces its own magnetic flux.

Now, when the current carrying conductor is placed in the magnetic field, force is exerted on that conductor. This direction can be determined by fleming left hand rule.

Initially the rotor conductor was not carry any current, as the emf induced in the rotor conductor by the faradays laws of an electromagnetic induction and the rotor circuit being shot circuited at both the ends, the induced current start to flow in the rotor conductor.

Since when the current carrying conductor is placed in the magnetic field force is exerted on that conductor.

Since the rotor conductor is placed in the slot on the circumference of the rotor, this force acts in the tangential direction to the rotor and torque is developed on the rotor.

Similar torque is produced on all the rotor conductors.

Since the rotor is free to rotate, it starts rotating in the same direction as the rotating magnetic field.

Thus a 3-phase induction motor is self starting.

As the operation of this motor depends upon the induced voltage in the rotor conductors, it is called as induction motor.

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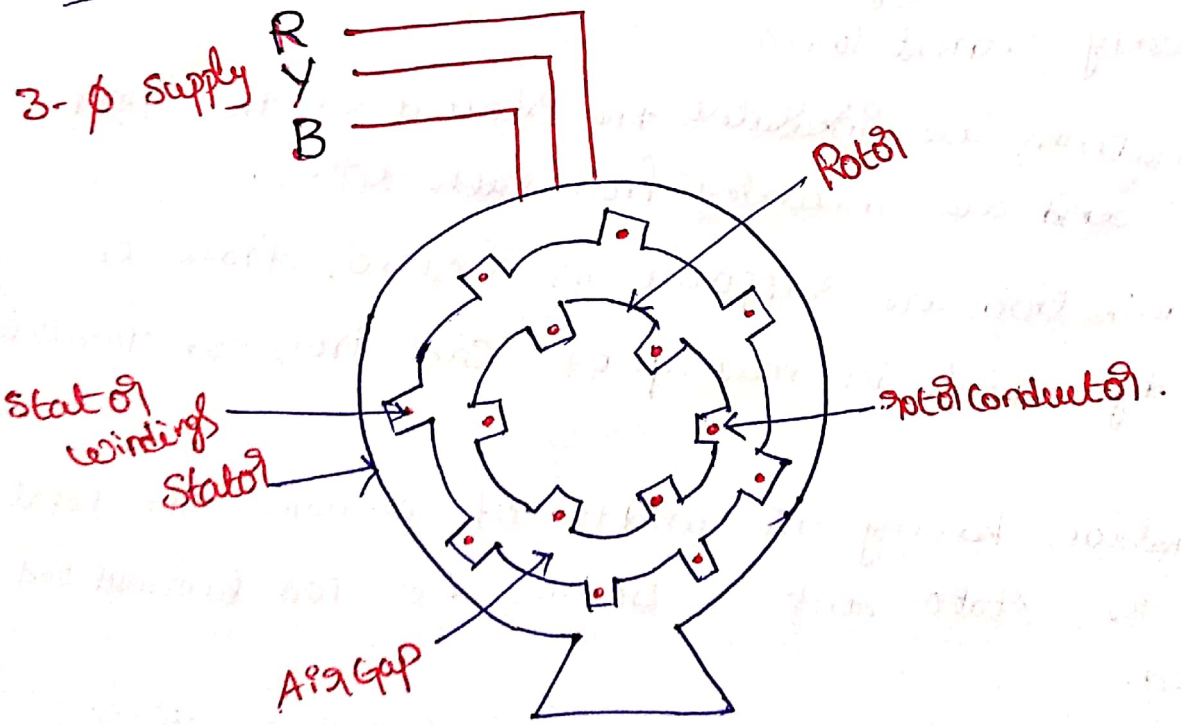
Advantages of 3-phase Induction motor:-

Three phase induction motor is the most popular type of the ac motor.

It is very commonly used in industrial drives due to following reasons.

→ It is cheap
→ It is efficient
→ It is reliable.
→ It has good speed regulation.
→ It has high starting torque.
→ It requires low maintenance.

Construction of a 3-ϕ Induction motor:-



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Three phase induction motor consists of the following parts.

→ Stator
→ Rotor
→ Stator frame or motor Body
→ Fan
→ fan Cover.

Stator :-

The stator is the stationary part of the motor.
It is made of high-grade alloy steel laminations to reduce eddy current losses.
The laminations are insulated and slotted on the inner periphery and are insulated from each other.
These laminations are supported in a stator frame or motor body which is madeup of cast iron or fabricated steel.
One lamination having the width of 0.5mm. The total length of the stator stack is 50mm i.e 100 laminations are required.
The insulated stator conductors are placed in these slots.
They form the three phase stator windings.
Stator windings can be connected in star connection or delta connection.

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Rotor :-

Rotor is the rotating part of the motor.
It is also built up of thin laminations of the same material as the stator.

The laminated cylindrical core is mounted directly on the shaft.

These laminations are slotted on the outer periphery to receive the rotor conductors.

Types of Rotors.

There are two types of the induction motor rotors.

→ Squirrel cage rotor or Cage rotors.

→ Wound rotors or Slip ring rotors.

Squirrel Cage rotor :-

It consists of a cylindrical laminated core with slots nearly parallel to the shaft axis.

It is called as skewing.

Rotor structure is the cylindrical laminated core, it is also consists of no. of slots in which the conductors are placed. This slots are parallel to each other, but they are not perpendicular to end ring there is an angle present. that angle is called skewing.

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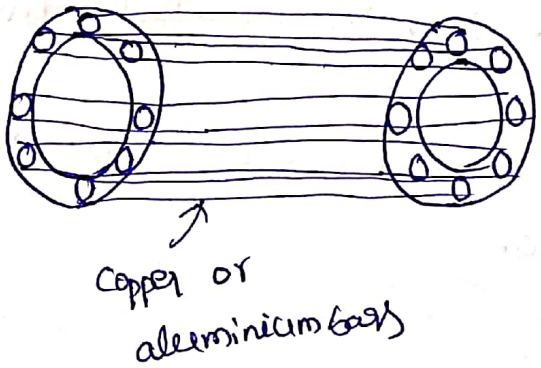
Each slot contains an uninsulated bar conductor of aluminium or copper.

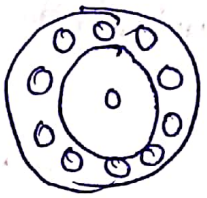
At each end of the rotor, the rotor bar conductors are short circuited by the heavy end rings which are made up of same material.

Advantages of skewing the rotor conductor:

→ Due to skewing smoother uniform torque is produced and the noise is reduced during the operation.

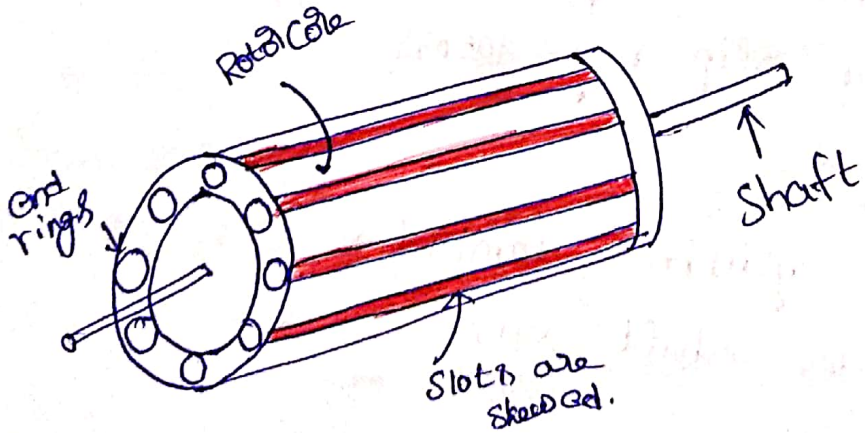
→ The locking tendency of the rotor is reduced.
Rotor locking tendency occurs when rotor teeth remain directly under stator teeth thus they might be magnetically attracted.





Symbolic representation

fig Cage type structure of rotor.



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Wound or Slip ring rotor :-

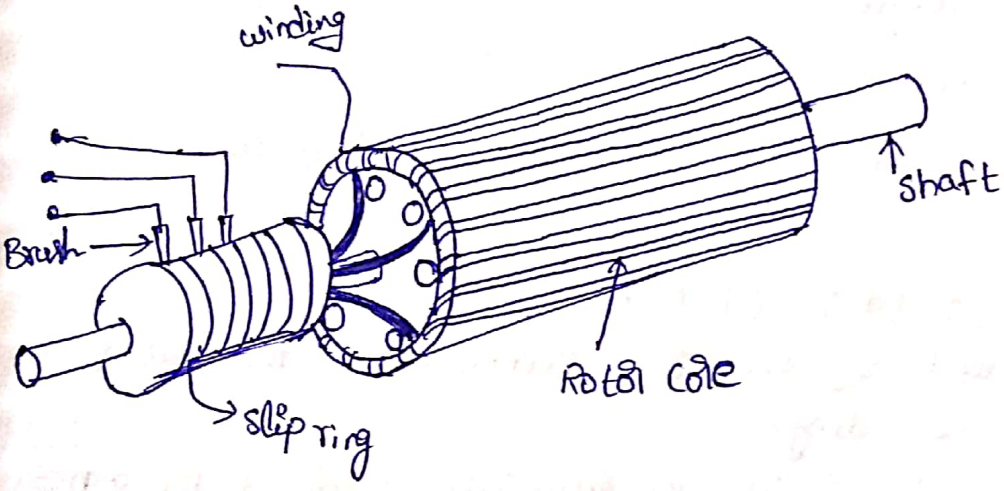
The wound rotor consists of slotted armature. Armature is nothing but rotor.

Insulated conductors are put in the slots and connected to form the three phase double layer distributed winding.

It is similar to the stator winding.

The rotor windings are connected in star.

The open ends of the star circuit are brought outside the rotor and connected to slip rings.



The slip rings are mounted on the shaft with brushes resting on them.

The brushes are connected to three variable resistors connected in star.

The purpose of the slip rings and brushes is to provide a means for connecting external resistors to the rotor ckt.

The resistors enable the variation of each rotor phase resistance to serve following purposes.

To increase the starting torque and decrease the starting current.

Speed control of motor.

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Advantages of squirrel cage rotor :-

→ Robust construction and cheaper.
→ The absence of brushes reduces the risk of sparking.
→ Less maintenance.
→ High efficiency.
→ Higher power factor.

Advantages of slip ring Rotor:

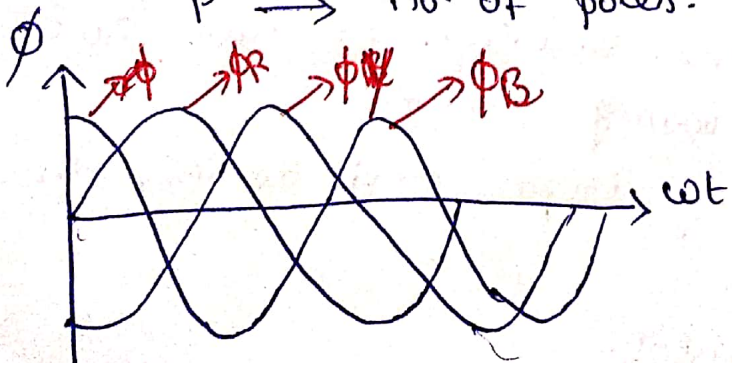
→ High starting torque.
→ Low starting current.
→ Easier speed control due to presence of resistors.

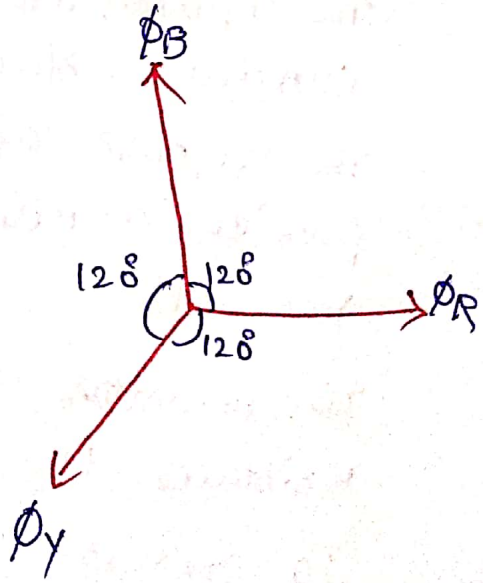
Rotating magnetic field :- (R.M.F)

R.M.F is produced by the 3-ϕ current of the stator in the stator windings.
Balanced 3-ϕ windings i.e mechanically displaced 120 degrees from each other fed by balanced three phase source.
A rotating magnetic field with constant magnitude is produced, it is rotating in a clock wise direction with a speed

$$N_S = \frac{120f}{P} \text{ r.p.m}$$

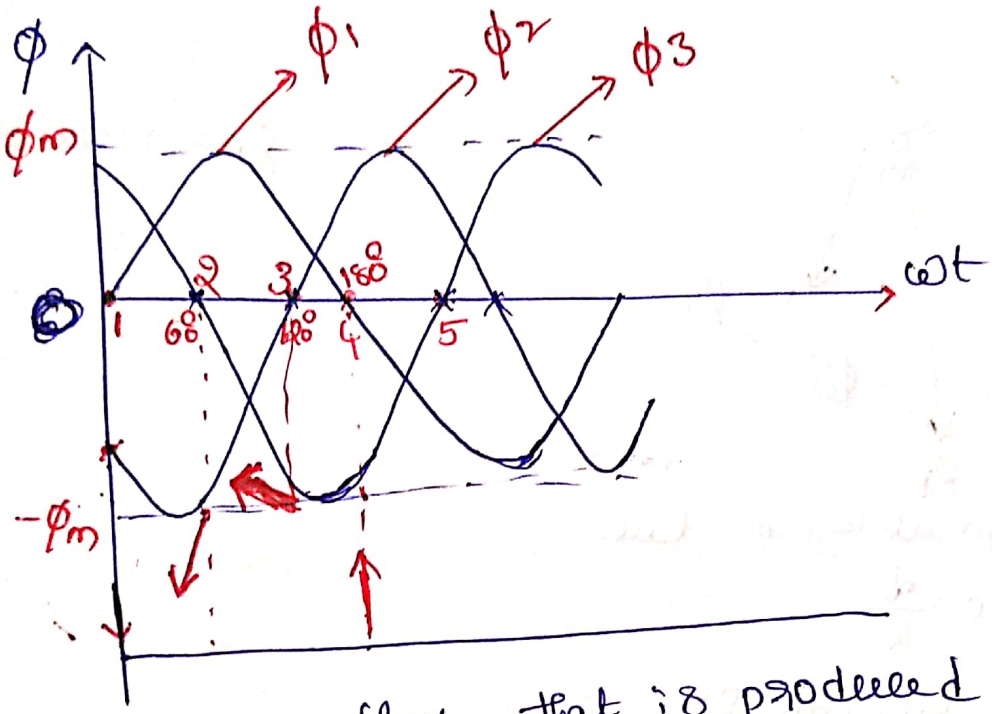
where N_S → Synchronous speed
f → supply frequency
P → no. of poles.





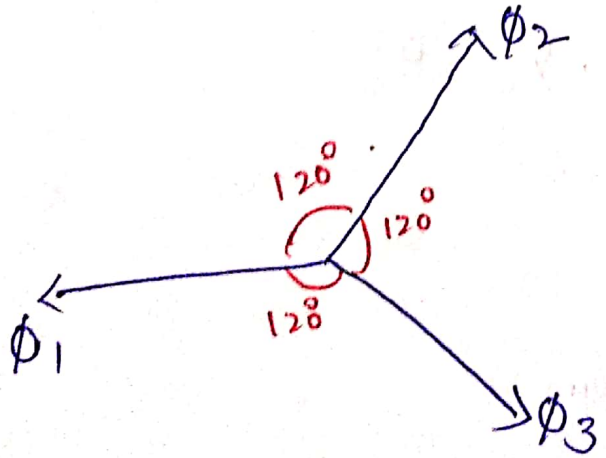
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Production of Rotating magnetic field :-



Three phase flux that is produced when we give 3-ϕ supply to the Stator winding.

$$\phi_1 = \phi_m \sin \omega t$$
$$\phi_2 = \phi_m \sin(\omega t - 120^\circ)$$
$$\phi_3 = \phi_m \sin(\omega t - 240^\circ)$$

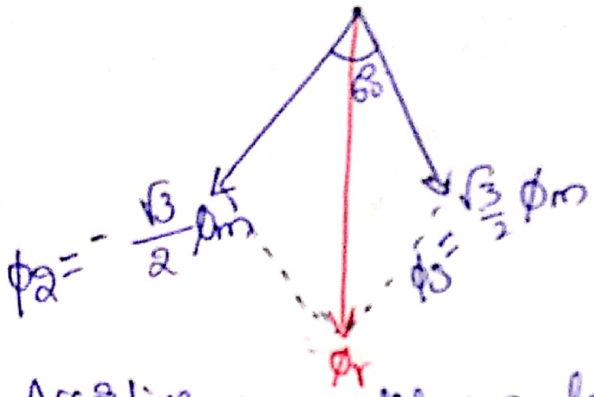


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At position 1, $\omega t = 0^\circ$

$\phi_1 = 0 \qquad \phi_2 = -\frac{\sqrt{3}}{2}\phi_m \qquad \phi_3 = \frac{\sqrt{3}}{2}\phi_3$

ϕ_r = resultant.



According to parallelogram law

$\phi_r = 2a \cos\frac{\alpha}{2}$

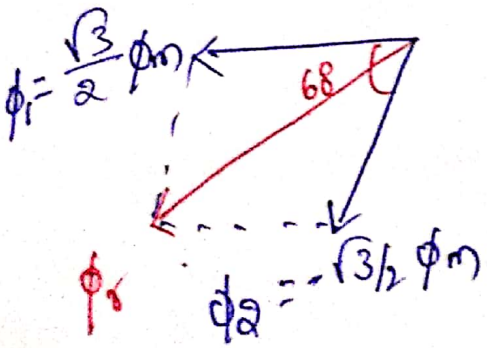
$= 2 \times \frac{\sqrt{3}}{2}\phi_m \cos\frac{60}{2}$

$= 2 \times \frac{\sqrt{3}}{2}\phi_m \frac{\sqrt{3}}{2}$

$\phi_r = \frac{3}{2}\phi_m$

At position 2, $\omega t = 60^\circ$

$\phi_1 = \frac{\sqrt{3}}{2}\phi_m, \quad \phi_2 = -\frac{\sqrt{3}}{2}\phi_m, \quad \phi_3 = 0$

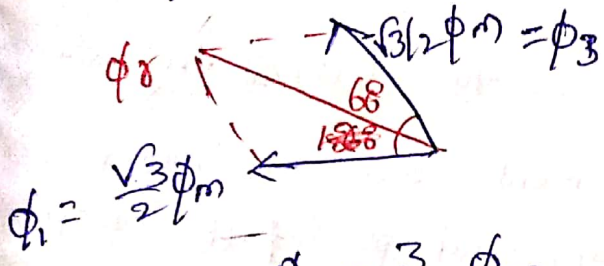


$\phi_r = \frac{3}{2}\phi_m$

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position 3, $\omega t = 120^\circ$

$\phi_1 = \frac{\sqrt{3}}{2}\phi_m$ $\phi_1 = 0$ $\phi_3 = -\frac{\sqrt{3}}{2}\phi_m$



$\phi_1 = \frac{\sqrt{3}}{2}\phi_m$

$\phi_r = \frac{3}{2}\phi_m$

The resultant vector rotates at synchronous speed

$$N_S = \frac{120f}{P}$$

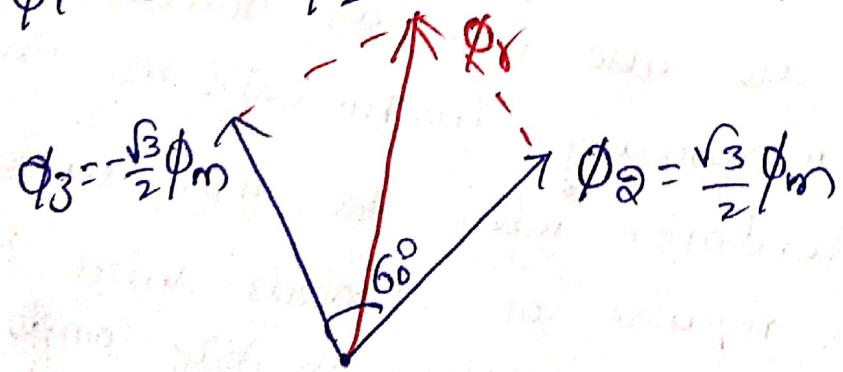
$f \rightarrow$ frequency of the supply

$P \rightarrow$ no. of poles

when we give 3-ϕ supply to the stator winding there will be a production rotating magnetic field. That will rotate at synchronous speed $N_S = \frac{120f}{P}$.

position 4, $\omega t = 180^\circ$

$\phi_1 = 0$ $\phi_2 = +\sqrt{3}/2\,\phi_m$ $\phi_3 = -\sqrt{3}/2\,\phi_m$



$\phi_r = 3/2\,\phi_m$

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Electrical measuring Instruments :-

* measuring instrument is some instrument which is measure some quantity.

measuring instrument is the heart of the electrical engineering.

Various electrical measuring instruments are voltmeter, ammeters, wattmeters, frequency meters and powerfactor meters.

Measuring instruments are basically classified into two categories.

1) primary or Absolute instraments.

2) Secondary Instraments.
   - → Indicating instraments
   - → Integrating instraments
   - → Recording instruments.

primary or Absolute instraments :-

primary instruments are also called as standard instruments. These measuring instruments are used in research laboratories where the measurement of precise nature is require or absolute value is require. The construction of these meters are very complex. It is very costly and handle with care.
In this instruments, they don't have dail, pointer moving. The reading it gives in meter constant.

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Ex :- potentiometer, Tangent Galvanometer, Absolute Electrometer, Raileigh current balance meter.
The values obtained in absolute nature (potentiometer).

Secondary instruments :-

Secondary instruments are also called as direct type reading type instruments. which we are used in the day to day life.
The secondary type instruments are not give the accurate values.

The primary instruments are used to calibarate the secondary instruments.

Calibaration means the proces of finding the error.

Secondary instruments are avaible in very cheap cost, & Construction is also a very simple.

Indicating instruments :-

Indicating instruments are the quantity which is being measure as a ~~continuous~~ Continuous function of time.

It represents the instantaneous value of the quantity under measurement. It measures as a continuous function of time.

Ex Ammeters, voltmeters, wattmeters, frequency meters, power factor meters.

Indicating instruments are too types.

→ Electrical instruments (Actuating quantity is current).
→ Electronic instruments. (Actuating quantity is voltage).

Actuating quantity means the quantity which is responsible for actuating this measuring instruments.

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Integrating Instruments :-

They integrate or they give cummulative valee of the quantity being measured.
They add up the existing valee to the next measured valee.
Ex:- Again the measured valee will be add up the existing valee. Then we get a total valee.

Ex :- Energy meters.

Recording Instruments :-

Recording type of instruments are which indicate as well as they record. It may or may not indicate but it records on a piece of paper on a graph.

Ex ECG, Substations (monitoring the voltages and currents in time to time).

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