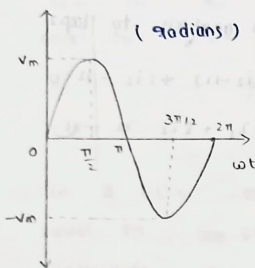


04/09/24

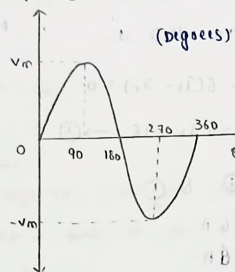
UNIT-3 Single Phase A.C. Circuits.

Representation of A.C wave forms in different formats:

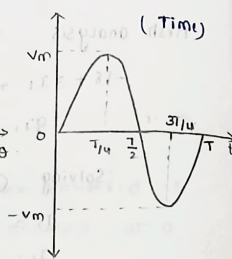
$$V = V_m \sin \omega t$$



$$V = V_m \sin \theta$$



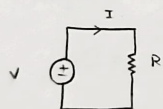
$$V = V_m \sin t$$



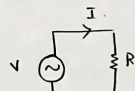
$$T = 360 = 2\pi = 20\pi \text{ sec}$$

RMS VALUE :- (Root Mean Square): (Graphical Method):

Consider a resistive circuit which is connected to d.c supply as well as steady state (ac) A.C supply which shown in the below figures.



Resistor connected to d.c supply



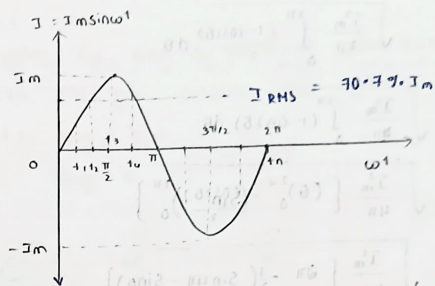
Resistor connected to A.C supply

* The power dissipated by the resistor when it is connected to d.c is $P_{dc} = I^2 R$

* The power dissipated by the resistor when it is connected to A.C supply is

$$P_{ac} =$$

To get this we have to divide the entire sinusoidal wave form into n equal intervals.



$$P_{ac} = \frac{I_1^2 R + I_2^2 R + I_3^2 R + \dots + I_n^2 R}{n}$$

* From the both the circuits it is observed that they are producing same heating effect i.e.,

$$P_{dc} = P_{ac}$$

$$I^2 R = \frac{I_1^2 R + I_2^2 R + \dots + I_n^2 R}{n}$$

$$I_{RMS} = \sqrt{\frac{I_1^2 + I_2^2 + \dots + I_n^2}{n}}$$

(or)

$$I_{RMS} = \sqrt{\frac{1}{T} \int_0^T (i(t))^2 dt} \quad \text{from graph.}$$

ANALYTICAL METHOD:

$$i(t) = i_m \sin \omega t \quad d(\omega t)$$

$$= i_m \sin \theta \cdot d\theta$$

$$I_{RMS} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} (i(t))^2 d\theta}$$

$$= \sqrt{\frac{1}{2\pi} \int_0^{2\pi} (i_m \sin \theta)^2 d\theta}$$

$$= \sqrt{\frac{I_m^2}{2\pi} \int_0^{2\pi} (\sin \theta)^2 d\theta}$$

$$I_{RMS} = \sqrt{\frac{I_m^2}{2\pi} \int_0^{2\pi} \frac{(1 - \cos 2\theta)}{2} d\theta}$$

$$= \sqrt{\frac{I_m^2}{2\pi} \int_0^{2\pi} (1 - \cos 2\theta) d\theta}$$

$$= \sqrt{\frac{I_m^2}{4\pi} \left\{ (\theta)_0^{2\pi} - \left(\frac{\sin 2\theta}{2} \right)_0^{2\pi} \right\}}$$

$$= \sqrt{\frac{I_m^2}{4\pi} \left\{ 2\pi - \frac{1}{2} (\sin 4\pi - \sin 0) \right\}}$$

$$= \sqrt{\frac{I_m^2}{4\pi} (2\pi)}$$

$$= \sqrt{\frac{I_m^2}{2}} = \frac{I_m}{\sqrt{2}}$$

$$I_{RMS} = 0.707 I_m$$

$$I_{RMS} = 70.7\% I_m$$

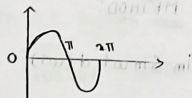
09/09/24

Average Value:

Symmetrical wave form:

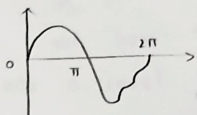
The area under both +ve and -ve half cycles are same then we can call it as a symmetrical wave form.

Ex: Sinusoidal wave form.



Unsymmetrical wave form:

The area under both +ve and -ve half cycles are not same then it is said to be unsymmetrical wave form.

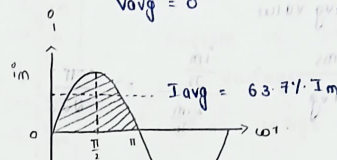


NOTE:

* The average value for any symmetrical wave form should be equal to zero (if you consider full cycle).

$$i.e., i_{avg} = 0$$

$$V_{avg} = 0$$



For Half cycle

$$i_{avg} = \frac{\text{Area under the +ve half cycle}}{\text{Length of that +ve half cycle}}$$

$$i_{avg} = \frac{1}{\pi} \int_0^{\pi} i(t) dt$$

$$= \frac{1}{\pi} \int_0^{\pi} i_m \sin \theta d\theta$$

$$i_{avg} = \frac{i_m}{\pi} [-\cos \theta]_0^{\pi}$$

$$= \frac{i_m}{\pi} [-(\cos \pi - \cos 0)]$$

$$= \frac{i_m}{\pi} (-(-1) - (-1)) = \frac{i_m}{\pi} (2)$$

$$= \frac{2i_m}{\pi} = 0.637 I_m \text{ (or) } 63.7\% \text{ of } I_m$$

$$i_{avg} = 0 \text{ (full cycle)}$$

$$i_{avg} = \frac{2I_m}{\pi} \text{ (Half cycle)}$$

Peak factor (kp): It is defined as the ratio of peak value to the RMS value. i.e.,

$$k_p = \frac{\text{Peak value}}{\text{RMS value}}$$

$$= \frac{I_{peak}}{I_{RMS}} = \frac{i_m}{\frac{i_m}{\sqrt{2}}} = \sqrt{2} = 1.414 = k_p$$

Peak value for the above sinusoidal case is 1.414.

Form factor (k_f): It is defined as the ratio of RMS value to the average value for a given alternating sinusoidal wave. i.e.

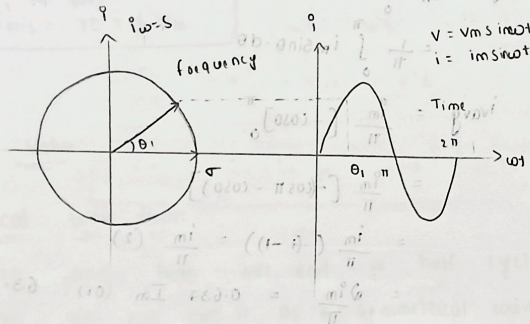
$$k_f = \frac{\text{RMS value}}{\text{avg value}}$$

$$= \frac{i_{rms}}{i_{avg}} = \frac{\frac{i_m}{\sqrt{2}}}{\frac{i_m}{\pi}} \Rightarrow \frac{\pi}{\sqrt{2}} = 1.11 //$$

*** $k_f = 1.11$

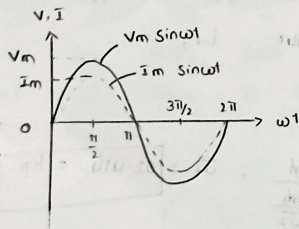
Note: peak and form factor values are same for voltage and current. (V and I)

Phasors



Phase difference

case (i) : when ϕ is 0°



$$V = V_m \sin \omega t$$

$$i = i_m \sin \omega t$$

$V = V_m \sin(\omega t \pm \phi)$
Actual representation.

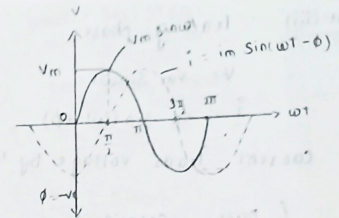
θ	V	I
0°	0	0
90°	+ve	+ve
180°	0	0
270°	-ve	-ve
360°	0	0

case (ii) : when ϕ is $-ve$

$$V = V_m \sin \omega t$$

$$i = i_m \sin(\omega t - \phi)$$

θ	V	I
0°	0	-ve
90°	+ve	0
180°	0	+ve
270°	-ve	0
360°	0	-ve



current is just following the voltage.

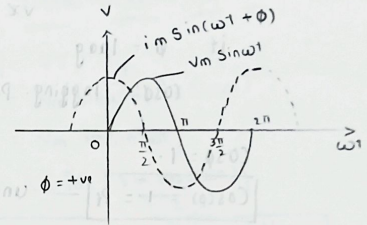
"current lags the voltage by ϕ "

case (iii) : when ϕ is $+ve$

$$V = V_m \sin \omega t$$

$$i = i_m \sin(\omega t + \phi)$$

θ	V	I
0°	0	+ve
90°	+ve	0
180°	0	-ve
270°	-ve	0
360°	0	+ve



voltage following the current.

"current leads the voltage by ϕ "

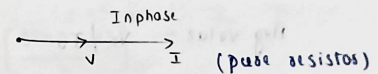
10/02/20

Phasor diagrams

case (i) : $\phi = 0^\circ$

$$V = V_m \sin \omega t$$

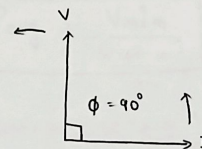
$$i = i_m \sin \omega t$$



Both voltage & current are in phase.

No phase difference.

case (ii) lagging phase.



$$V = V_m \sin \omega t$$

$$i = i_m (\sin(\omega t - \phi))$$

(Pure inductor)

current lags voltage by ϕ

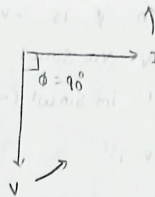
(case iii) Leading phase

$$V = V_m \sin \omega t$$

$$i = i_m \sin(\omega t + \phi)$$

Current leads voltage by ' ϕ '

(pure capacitor).



Note: Power factor (P_f) = $\frac{\text{Active power (P)}}{\text{Apparent power (S)}}$

$$P_f = \frac{V I \cos \phi}{V I} = \cos \phi$$

if $\phi = \text{lag}$.

$\phi = \text{lead}$.

$\cos \phi = \text{lagging } P_f$.

$\cos \phi = \text{leading } P_f$.

$$\cos \phi = 1$$

$$\boxed{\cos \phi = 1 = P_f} \text{ — unity power factor (V.P.F.)}$$

$$V(t) = V_0 + V_{c1} \cos \omega t + V_{c2} \cos 2\omega t + \dots + V_{cn} \cos n\omega t +$$

$$V_{s1} \sin \omega t + V_{s2} \sin 2\omega t + \dots + V_{sn} \sin n\omega t$$

$$\text{Effective / RMS} = \sqrt{V_0^2 + \frac{1}{2}(V_{c1}^2 + V_{c2}^2 + \dots + V_{cn}^2) + \frac{1}{2}(V_{s1}^2 + V_{s2}^2 + \dots + V_{sn}^2)}$$

$$\text{Avg value} = V_0 / I_0$$

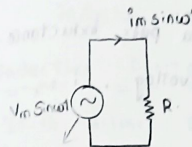
* for current or voltage, both formulas applicable

For ex: $V(t) = 10 + 20 \sin(\omega t + \frac{\pi}{6})$

$$V_{avg} = 10$$

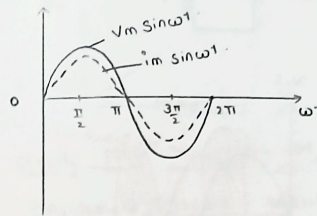
$$V_{RMS} = \sqrt{(10)^2 + \frac{1}{2}(20)^2} =$$

Steady State Analysis of pure resistance



AC supply

V, I



Acc to ohm's law

$$i = \frac{V_m \sin \omega t}{R}$$

$$i = \left(\frac{V_m}{R}\right) \sin \omega t$$

when $\omega t = \frac{\pi}{2}$ to get i_m

$$i_m = \frac{V_m}{R}$$

$$\boxed{i = i_m \sin \omega t}$$

Instantaneous power (P_i)

$$P_i = V I$$

$$= (V_m \sin \omega t) (i_m \sin \omega t)$$

$$= V_m i_m \sin^2 \omega t$$

$$= V_m i_m \left(1 - \frac{\cos 2\omega t}{2}\right)$$

$$\boxed{P = \frac{V_m i_m}{2} - \frac{\cos 2(\omega t) V_m i_m}{2}}$$

Constant factor

fluctuating factor

$$P_{avg} = \frac{1}{T} \int_0^T P_i \cdot dt$$

$$= \frac{1}{2\pi} \int_0^{2\pi} \left[\left[\frac{V_m i_m}{2} - \frac{V_m i_m \cos 2\omega t}{2} \right] d(\omega t) \right]$$

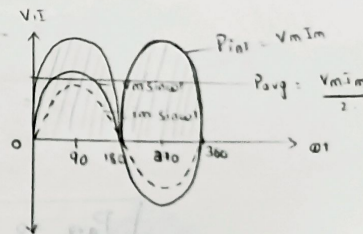
$$= \frac{1}{2\pi} \left[\frac{V_m i_m}{2} (2\pi - 0) \right] - \frac{V_m i_m}{2} \frac{1}{2\pi} \int_0^{2\pi} \cos 2(\omega t) \cdot d(\omega t)$$

$$= \frac{1}{2\pi} \left[\frac{V_m i_m}{2} (2\pi) \right] - \frac{V_m i_m}{4\pi} \left[\frac{\sin 2(\omega t)}{2} \right]_0^{2\pi}$$

$$= \frac{V_m i_m}{2} - \frac{V_m i_m}{8\pi} [\sin 4\pi - \sin 0]$$

$$= \frac{V_m i_m}{2} - 0 \quad (\because \text{also avg value over a complete cycle (or) full cycle is zero})$$

$$\boxed{P_{avg} = \frac{V_m i_m}{2}}$$



Steady State Analysis of pure Inductor:

Consider a circuit which consists of a pure inductance which is connected across a supply voltage, i.e.,

$$V = V_m \sin \omega t \text{ as shown}$$

We know that

$$V = L \frac{di}{dt}$$

$$i = \frac{1}{L} \int v dt$$

$$= \frac{1}{L} \int V_m \sin \omega t$$

$$= \frac{V_m}{L} \left[-\frac{\cos \omega t}{\omega} \right]$$

$$i = \frac{V_m}{\omega L} \left(\sin \left(\omega t - \frac{\pi}{2} \right) \right)$$

To get $i_m \rightarrow \omega t = \pi$

$$i_m = \frac{V_m}{\omega L}$$

$$\frac{V_m}{i_m} = \omega L$$

$$\frac{V_m}{i_m} = X_L \text{ (Inductive Reactance)}$$

$$i = i_m \sin \left(\omega t - \frac{\pi}{2} \right)$$

Average power: $(P_{avg}) = P_{avg} = \frac{1}{T} \int_0^T P_i dt$

$$= \frac{1}{2\pi} \int_0^{2\pi} V_m i_m \sin \omega t (\sin \omega t - \frac{\pi}{2}) \cdot d\omega t$$

$$= \frac{V_m i_m}{2(2\pi)} \left[\int_0^{2\pi} 2 \sin \omega t (-\cos \omega t) \cdot d\omega t \right]$$

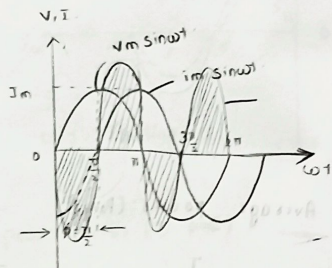
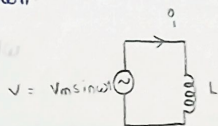
$$= -\frac{V_m i_m}{4\pi} \left[\int_0^{2\pi} (\sin 2\omega t) d(\omega t) \right]$$

$$= -\frac{V_m i_m}{4\pi} \left[-\frac{\cos 2\omega t}{2} \right]_0^{2\pi}$$

$$= \frac{V_m i_m}{8\pi} \left[\cos 2(2\pi) - \cos 2(0) \right]$$

$$= \frac{V_m i_m}{8\pi} [1 - 1] = 0$$

$$P_{avg} = 0$$



NOTE: The average power consumed by the pure inductor is zero.

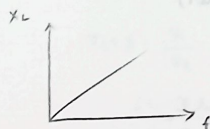
Inductive Reactance (X_L):

It is defined as the opposition offered by the inductance to the current flow in a circuit is called inductive Reactance (X_L).

$$X_L = \omega L$$

$$X_L = 2\pi f L \quad (\omega = 2\pi f)$$

$$X_L \propto f$$



Steady state Analysis of pure capacitor:

Consider a circuit which consists of a pure capacitance which is connected across a supply voltage, i.e.,

$$V = V_m \sin \omega t \text{ as shown}$$

We know that

$$i = C \cdot \frac{dv}{dt}$$

$$i = C \frac{d}{dt} [V_m \sin \omega t]$$

$$= V_m C (\cos \omega t) \omega$$

$$i = \omega C V_m \cos \omega t$$

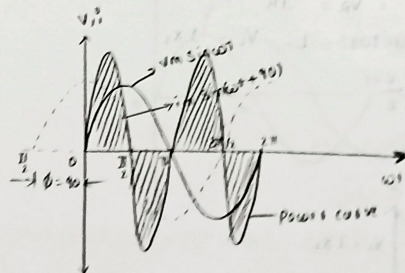
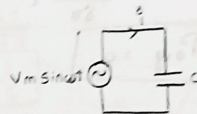
To get $i_m \rightarrow \omega t = 0$

$$i_m = \omega C V_m \Rightarrow \frac{V_m}{i_m} = \frac{1}{\omega C}$$

$$i = i_m \cos \omega t$$

$$i = i_m \sin \left(\frac{\pi}{2} + \omega t \right)$$

$$\frac{V_m}{i_m} = X_C \text{ (Capacitive Reactance)}$$



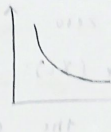
Capacitive Reactance (X_C)

$$X_C = \frac{1}{\omega C}$$

$$X_C = \frac{1}{2\pi f C}$$

$$X_C \propto \frac{1}{f}$$

X_C



$$P_{avg} = \frac{1}{2\pi} \int_0^{2\pi} V_m \sin \omega t \cdot i_m \sin(\omega t) \cdot d(\omega t)$$

$$= \frac{V_m i_m}{(2\pi)2} \int_0^{2\pi} 2 \sin \omega t \cos \omega t \cdot d(\omega t)$$

$$= \frac{V_m i_m}{4\pi} \int_0^{2\pi} \sin 2\omega t \cdot d(\omega t)$$

$$= \frac{V_m i_m}{4\pi} \left[-\frac{\cos 2(\omega t)}{2} \right]_0^{2\pi} = -\frac{V_m i_m}{8\pi} [\cos 2(2\pi) - \cos 2(0)]$$

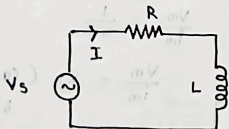
$$= -\frac{V_m i_m}{8\pi} [1 - 1] = 0$$

$$P_{avg} = 0$$

17/09/20

Series RL Circuit:

Consider a circuit which consists of a series combination of resistance and inductance which is connected to a supply source $V_s = V_m \sin \omega t$ as shown in figure.

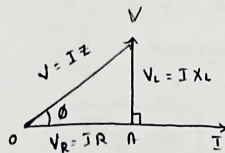


Voltage across 'R', $V_R = IR$.

Voltage across inductor L, $V_L = IX_L$

$$I = \frac{V}{Z}$$

Phasor Diagram:



$$V^2 = V_R^2 + V_L^2$$

$$V = \sqrt{I^2 R^2 + I^2 X_L^2}$$

$$V = I \sqrt{R^2 + X_L^2}$$

$$\frac{V}{I} = Z$$

$$V = IZ$$

The component $\sqrt{R^2 + X_L^2}$ offers opposition to the current flow in circuit is called impedance. Z is impedance $Z = \sqrt{R^2 + X_L^2}$

$$\tan \phi = \frac{V_L}{V_R}$$

$$= \frac{IX_L}{IR}$$

$$\phi = \tan^{-1} \left(\frac{X_L}{R} \right)$$

From the phasor diagram,

$$\text{power factor, } \cos \phi = \frac{V_R}{V}$$

$$\cos \phi = \frac{IR}{IZ}$$

$$\cos \phi = \frac{R}{Z}$$

$$Z = R + jX_L$$

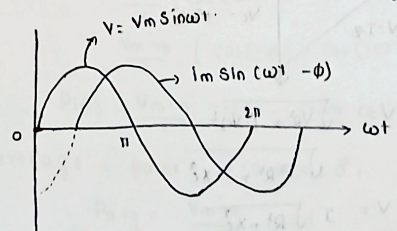
$$|Z| = \sqrt{R^2 + X_L^2}$$

From the phasor diagram it is clear that current lags the voltage by an angle of ϕ .

$$\text{To get phase angle } \phi, \tan \phi = \frac{V_L}{V_R}$$

From Phasor diagram, $V = V_m \sin \omega t$

$$i = i_m \sin(\omega t - \phi)$$



The instantaneous power absorbed by the series RL circuit

$$P_{ins} = VI$$

$$= V_m \sin \omega t + i_m \sin(\omega t - \phi)$$

$$= \frac{V_m i_m}{2} 2 \sin \omega t \cdot \sin(\omega t - \phi)$$

$$= \frac{V_m i_m}{2} [\cos(\omega t - \omega t + \phi) - \cos(\omega t + \omega t - \phi)]$$

$$= \frac{V_m i_m}{2} [\cos \phi - \cos(2\omega t - \phi)]$$

$$P_{ins} = \frac{V_m i_m}{2} \cos \phi - \frac{V_m i_m}{2} \cos(2\omega t - \phi)$$

Constant

Fluctuating D.C.

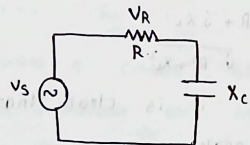
$$\text{Average power, } P_{avg} = \frac{V_m i_m}{2} \cos \phi$$

$$P_{avg} = \frac{V_m}{\sqrt{2}} \cdot \frac{i_m}{\sqrt{2}} \cos \phi$$

$$P_{avg} = VI \cos \phi$$

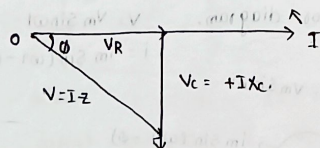
Series R_c Circuit:-

Consider a circuit which consists of a series combination of resistance and capacitance which is connected to a supply source. $V_s = V_m \sin \omega t$.



Voltage across R, $V_R = IR$.

Voltage across 'C', $V_c = IX_c$.



$$V = \sqrt{V_R^2 + (V_c)^2}$$

$$= \sqrt{I^2 R^2 + I^2 X_c^2}$$

$$V = I \sqrt{R^2 + X_c^2}$$

The component $\sqrt{R^2 + X_c^2}$ offers opposition to Impedence the current flow in circuit is called Impedence.

$$Z = \sqrt{R^2 + X_c^2}$$

$$\frac{V}{I} = Z$$

$$V = ZI$$

From phasor diagram, It is clear that current leads the voltage by an angle ϕ .

To get phase angle ϕ , $\tan \phi = \frac{-V_c}{V_R}$

$$= -\frac{IX_c}{IR}$$

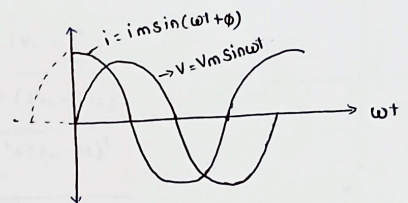
$$\phi = \tan^{-1} \left(-\frac{X_c}{R} \right)$$

Power factor, $\cos \phi = \frac{V_R}{V}$

$$= \frac{IR}{IZ}$$

$$\cos \phi = \frac{R}{Z}$$

$$V = V_m \sin \omega t, I = i_m \sin(\omega t + \phi)$$



Instantaneous power,

$$P_{ins} = VI$$

$$= V_m \sin \omega t \cdot i_m \sin(\omega t + \phi)$$

$$= V_m i_m \sin \omega t \cdot \sin(\omega t + \phi)$$

$$P_{ins} = \frac{V_m i_m}{2} 2 \sin \omega t \cdot \sin(\omega t + \phi)$$

$$= \frac{V_m i_m}{2} [\cos(\omega t - \omega t - \phi) - \cos(\omega t + \omega t + \phi)]$$

$$= \frac{V_m i_m}{2} [\cos(-\phi) - \cos(2\omega t + \phi)]$$

$$P_{ins} = \frac{V_m i_m}{2} \cos \phi - \frac{V_m i_m}{2} \cos(2\omega t + \phi)$$

The average power consumed,

$$P_{avg} = \frac{V_m i_m}{2} \cos \phi$$

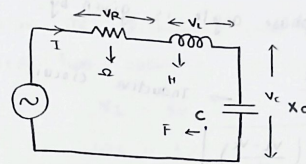
$$= \frac{V_m}{\sqrt{2}} \cdot \frac{i_m}{\sqrt{2}} \cos \phi$$

$$P_{avg} = VI \cos \phi$$

18-09-24

Series R-L-C circuit:

Consider a RLC circuit which is connected to a supply voltage of V_s i.e., $V_s = V_m \sin \omega t$.



Voltage across 'R' $\Rightarrow V_R = IR$

Voltage across 'L' $\Rightarrow V_L = IX_L$

Voltage across 'C' $\Rightarrow V_C = IX_C$

Phasor diagram:

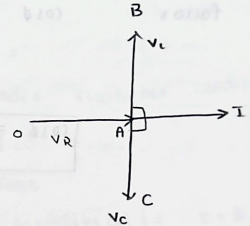
$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$V = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

$$V = I \sqrt{R^2 + (X_L - X_C)^2}$$

$$\frac{V}{I} = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\boxed{\frac{V}{I} = Z}$$



Case (i)

$$V_L = V_C$$

$$IX_L = IX_C$$

$$X_L = X_C$$

$\therefore \boxed{Z = R}$ purely Resistive circuit.

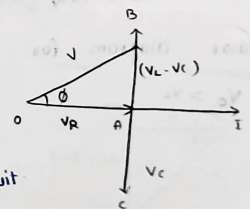
Case (ii)

$$V_L > V_C$$

$$IX_L > IX_C$$

$$X_L > X_C$$

$\therefore X_L > X_C$ is purely Inductive circuit



case (iii) $V_L < V_C$

$$X_L < X_C \quad (\text{capacitive ckt})$$

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

From the phasor phase angle is given by.

$$\tan \phi = \frac{V_L - V_C}{V_R} \rightarrow \text{Inductive circuit.}$$

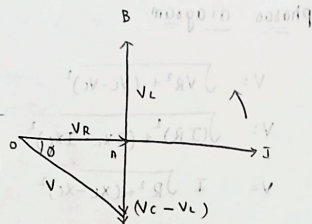
$$\phi = \tan^{-1} \left(\frac{V_L - V_C}{V_R} \right)$$

case (ii) $\rightarrow$ From phasor the voltage V leads the current by ϕ (as I lags the V by ϕ) (for inductive circuit).

Power factor

$$\cos \phi = \frac{V_R}{V} = \frac{IR}{IZ}$$

$$\cos \phi = \frac{R}{Z}$$



(iii)

I leads V by ϕ

$\cos \phi \Rightarrow$ leading p.f.

$$V = \sqrt{V_R^2 + (V_C - V_L)^2}$$

$$V = I \sqrt{R^2 + (X_C - X_L)^2}$$

$$\frac{V}{I} = \sqrt{R^2 + (X_C - X_L)^2}$$

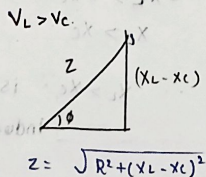
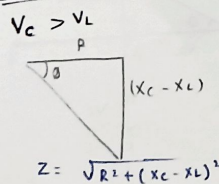
$$i.e. \quad Z = \frac{V}{I}$$

$$\tan \phi = \frac{V_C - V_L}{V_R}$$

$$\phi = \tan^{-1} \left(\frac{V_C - V_L}{V_R} \right)$$

$$p.f. \cos \phi = \frac{V_R}{V} = \frac{IR}{IZ} \Rightarrow \cos \phi = \frac{R}{Z}$$

phasor diagram for impedance:



Resonance

* $Z = R$ (operating on pure resistance)

* $\phi = 0$ in a resistive circuit

* $\cos \phi = 1$ (unity power factor)

* Resonance condition $X_L = X_C$ + Must contains two storage sources

$\Rightarrow$ To obtain the Resonance conditions in electrical circuit we need two opposite storage elements i.e. L & C .

$$X_L = X_C$$

$$\omega L = \frac{1}{\omega C}$$

$$\omega = \frac{1}{LC}$$

$$\omega = \frac{1}{\sqrt{LC}} \quad (\because \omega = 2\pi f)$$

$$2\pi f = \frac{1}{\sqrt{LC}}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

The frequency operating under resonance condition is known as resonant frequency

NOTE: under resonance conditions

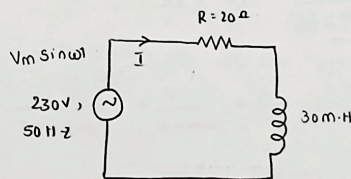
* Circuit behaves like a resistive ckt i.e. $Z = R$.

$$i.e. \quad I = \frac{V}{Z} = \frac{V}{R}$$

$$\cos \phi = 1 \quad (u.p.f.) \quad (\because \phi = 0)$$

problem: A coil of 30 mH is connected in series with resistance of $R = 20\ \Omega$ this connected to a supply of 230 volts , 50 Hz frequency. Find

- Impedance
- current
- phase angle
- power factor
- power consumed by the supply.



i) Impedance $Z = \sqrt{R^2 + X_L^2}$
 $= \sqrt{(20)^2 + (9.42)^2}$
 $Z = 22.1 \Omega$

$X_L = \omega L$
 $= 2\pi f X_L$
 $= 2\pi \times 50 \times 30 \times 10^{-3}$
 $= 9.42 \Omega$

ii) Current $I = \frac{V}{Z} = \frac{230}{22.1} = 10.40 A$

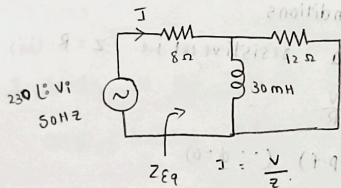
iii) $\phi = \tan^{-1} \left(\frac{X_L}{R} \right)$
 $= \tan^{-1} \left(\frac{9.42}{20} \right)$
 $\phi = 25.22^\circ$

iv) $\cos \phi = \frac{R}{Z}$
 $= \frac{20}{22.1} = 0.904$ lagging
 (Because it is a RL circuit).

v) $P = VI \cos \phi$
 $= 230 \times 10.40 \times 0.904$

$P = 2162.36 \text{ Watts}$

p) Find the value of V for the circuit.

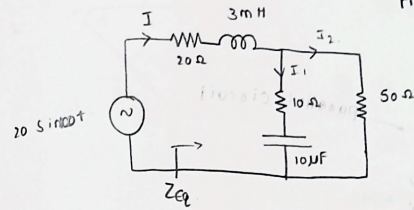


$Z = (12 \parallel j9.42) + 8$

$= \frac{(12 \times j9.42)}{12 + j9.42} + 8$
 $= 12.575 + j5.828$

$Z = 13.859 \angle 24.865^\circ$

$I = \frac{V \angle \phi}{Z(\phi)} = \frac{230 \angle 0^\circ}{13.859 \angle 24.865^\circ}$
 $= 16.595 \angle -24.865^\circ$



Find effective impedance & current through I

$I = \frac{V}{Z_{eq}}$

$30 \text{ mH} \rightarrow jX_L \Rightarrow j\omega L \Rightarrow j100 \times 30 \times 10^{-3}$
 $\Rightarrow 0.3j \Omega$

$10 \mu F \rightarrow -jX_C \Rightarrow -j \frac{1}{\omega C} = \frac{-j}{100 \times 10 \times 10^{-6}}$
 $= -j/10^3 \times 10^{-6}$
 $= -10^3 j (\Omega) = -1000 j \Omega$

Effective impedance

$Z_{eq} = [50 \parallel (10 - 1000j)] + (20 + 0.3j)$

$= \frac{(10 - 1000j)(50)}{50 + 10 - 1000j} + 20 + 0.3j$

$= 20 + 0.3j + 49.85 - 2.49j$

$Z_{eq} = (69.85 - 2.19j) \Omega$

$V = V_m \sin \omega t$

$= 20 \sin 100t$

$V_{rms} = \frac{20}{\sqrt{2}}$ at angle (0°)

$I = \frac{V}{Z_{eq}}$
 $69.85 - j2.19$

$= \frac{20/\sqrt{2} \angle 45^\circ}{69.85 - j2.19}$

$69.85 - j2.19 \rightarrow \text{convert into polar}$

$= 0.202 + 6.3416 \times 10^{-3}j$

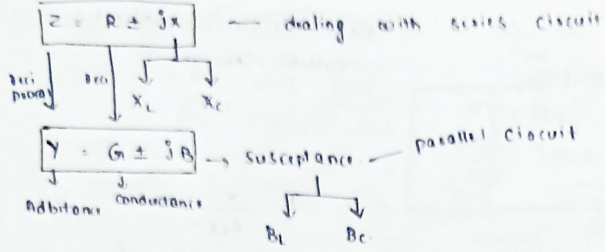
$= \frac{20/\sqrt{2} \angle 45^\circ}{69.88 \angle 1.79^\circ} = \frac{20/\sqrt{2} (45^\circ - 1.79^\circ)}{69.88}$

$I = 0.202 \angle 46.79^\circ$

Current

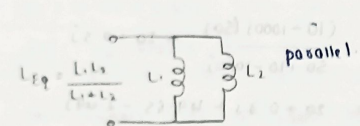
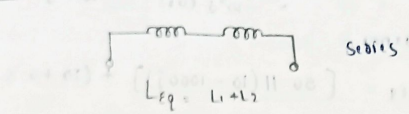
$I_1 = I_T \times \frac{50}{50 + 10 - 1000j}$

$V_1 = \frac{20 \angle 45^\circ}{20 + 0.3j} + \frac{V_1}{10 - 1000j} + \frac{V_1}{50} = 0$

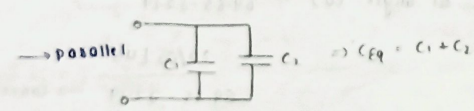
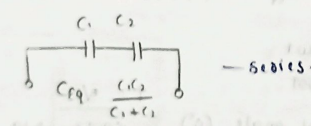


* $V = ZI$
 $I = \frac{V}{Z}$
 $I = YV \Rightarrow Y = \frac{I}{V}$ units: S

Inductor Series:

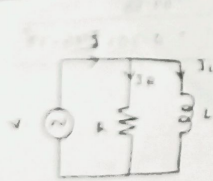


Capacitance:

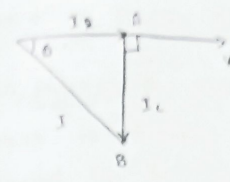


2.3/10/20

Parallel RL Circuit:



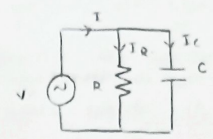
$I_R = \frac{V}{R}$
 $I_L = \frac{V}{jX_L} = \frac{V}{X_L} \angle -90^\circ$
 I lags voltage by an angle ϕ
 $I = \sqrt{I_R^2 + I_L^2}$
 from phasor diagram



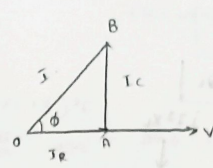
$\tan \phi = \frac{I_L}{I_R} \Rightarrow \phi = \tan^{-1} \left(\frac{I_L}{I_R} \right)$

ϕ is always a lagging angle
 $\cos \phi = \frac{I_R}{I}$ is also a lagging power factor

Parallel R-C circuit:



all are connected in parallel
 So voltage across R & C are same
 I is divided as I_R & I_C
 $I_R = \frac{V}{R}$; $I_C = \frac{V}{-jX_C}$



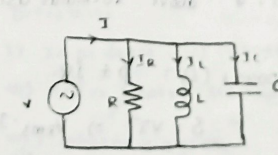
from phasor it is identified that
 I leads voltage by an angle ϕ

$\tan \phi = \frac{I_C}{I_R} \Rightarrow \phi = \tan^{-1} \left(\frac{I_C}{I_R} \right)$

ϕ is always a leading angle in capacitive circuit

Power factor $\cos \phi = \frac{I_R}{I}$ is also a leading P.f

Parallel RLC Circuit:



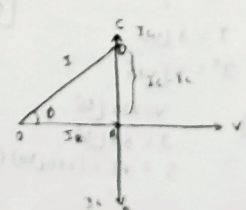
Current I is divided as
 I_R , I_L & I_C

case i: $X_C > X_L$

I leads V by an angle ϕ

$I = \sqrt{I_R^2 + (I_C - I_L)^2}$

$\tan \phi = \frac{I_C - I_L}{I_R} \Rightarrow \phi = \tan^{-1} \left(\frac{I_C - I_L}{I_R} \right)$



$P.f = \cos \phi = \frac{I_R}{I}$ ($\therefore I$ is not same)

$$V_L = -L \frac{di}{dt}$$

for example (for capacitor)

$$V = 20 \sin \omega t$$

$$I_C = \frac{1}{L} \int V dt$$

$$= \frac{1}{L} \int 20 \sin \omega t dt$$

$$I_C = C \frac{dV}{dt}$$

$$= 5 \frac{d}{dt} (20 \sin \omega t)$$

$$I_R = \frac{V}{R}$$

$$= \frac{20 \sin \omega t}{5}$$

In I_R , Resistance doesn't change in

following the $\sin$ function

Solving the parallel RLC circuit

Given values are in terms of Y we can directly calculate $I = \frac{V}{Y}$

In case values are in terms of Z we can Reciprocate

the $R, jX_L, -jX_C$ values into mho (Y)

$$Z = R \pm jX_L$$

$$S = P + jQ$$

$$j^2 (R + jX_L)$$

Multiply on B.S (I^2)

$$VI = I^2 R + j I^2 X_L$$

$$3.636$$

$$ZI^2 = I^2 R + j I^2 X_L$$

$$VI = I^2 R + j I^2 X_L$$

(Apparent) power (Active) power (Reactive / Inactive) power useless

$$Z = R - jX_C$$

$$S = P - jQ$$

$$VI = I^2 R - j I^2 X_C$$

$$ZI^2 = I^2 R - j I^2 X_C$$

$$VI = I^2 R - j I^2 X_C$$

whenever we want calculate

the S, P, Q values by

using V, Z, I, ϕ above formulas used.



$$P = VI \cos \phi$$

$$Q = VI \sin \phi$$

$$S = VI$$

$$\text{Complex power } (S) = P \pm jQ$$

$$S = VI^* \Rightarrow V_{RMS} \cdot I_{RMS}$$

$$[0.9 - 0.1]$$

for Ex:

$$I = 3 \angle 40^\circ$$

$$I^* = 3 \angle -40^\circ$$

$$V = 230 \angle 50^\circ$$

$$I = 60 \angle -38^\circ$$

$$S = VI = (230 \angle 50^\circ) (60 \angle -38^\circ)$$

$$=$$

(conjugate) current is taken

$$\text{apparent } P = |S| = |V_{RMS} \cdot I_{RMS}|$$

TYPES OF POWERS IN AC CIRCUITS:

Active (or) Real power: It is defined as the power which is actually consumed by the circuit and it is denoted by the letter 'P' i.e.

$$P = V(I \cos \phi)$$

Inphase component

units: watts (W) (kW | MW | GW)

Reactive (or) Inactive power (Q): It is defined as the power which is neither consumed nor utilised in a circuit denoted by Q. i.e.

$$Q = V \times I \sin \phi$$

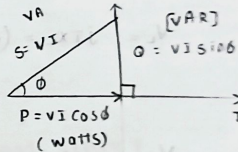
Qua

units: Volt-Amp-reactive [VAR] (KVAR | MVAR)

Apparent / Total power: It is defined as the total power consumed by the circuit and it is denoted by the letter 'S' i.e.

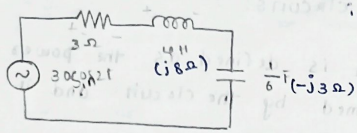
$$S = VI$$

units: Volt-Amp (VA) (KVA | MVA)



P) Determine

- Impedance
- current
- power
- phase angle
- V_R, V_L, V_C
- Active power, reactive, Total power
- power factor
- phasor diagram



$$V_m \sin \omega t = 30 \sin 2t$$

$$V_m = 30, \omega = 2$$

$$V = \frac{V_m}{\sqrt{2}} = 21.21 \text{ V}$$

$$Z = \sqrt{(3)^2 + (8-3)^2}$$

$$= \sqrt{9+25} = \sqrt{34} = 5.83 \Omega$$

$$Z = 5.83 \Omega$$

$$I = \frac{V}{Z} = \frac{21.21}{5.83} = 3.63 \angle -59.03^\circ \text{ A}$$

$$I = 3.63 \angle -59.03^\circ \text{ A}$$

$$I = 3.63 \angle -59.03^\circ \text{ A}$$

III) power phase angle.

$$I = I_R = I_L = I_C$$

$$V_R = I R = 3.63 \times 3 = 10.914 \text{ V}$$

$$V_L = j I X_L = (3.63 \angle -59.03^\circ) (j 8) = 29.104 \angle 30.97^\circ \text{ V}$$

$$V_C = -j I X_C = (3.63 \angle -59.03^\circ) (-j 3) = 10.914 \angle -149.03^\circ \text{ V}$$

Phase angle

$$\phi = \tan^{-1} \left(\frac{X_L - X_C}{R} \right)$$

$$\phi = \tan^{-1} \left(\frac{j8-j3}{3} \right) = \tan^{-1} \left(\frac{5}{3} \right) = 59.03^\circ$$

Impedance

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$X_L = \omega L = 2 \times 4 = 8 \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2 \times \frac{1}{6}} = 3 \Omega$$

$$Z = 3 + j5 = 12.16 \angle 59.03^\circ \Omega$$

$\phi = 59.03^\circ \rightarrow$ lagging angle because $X_L > X_C$
in case the circuit acts like an inductive circuit.

vi) Active power (P)

$$P = VI \cos \phi = 21.21 \times 3.638 \times \cos(59.03^\circ) = 39.66 \text{ Watts}$$

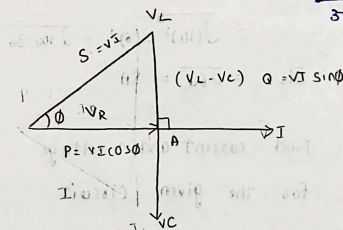
Reactive power (Q) = $I^2 (X_L - X_C)$

$$Q = VI \sin \phi = 21.21 \times 3.638 \times \sin(59.03^\circ) = 66.134 \text{ VAR}$$

$$P = VI \cos \phi = 21.21 \times 3.638 \times \cos(59.03^\circ) = 77.1 \text{ VA}$$

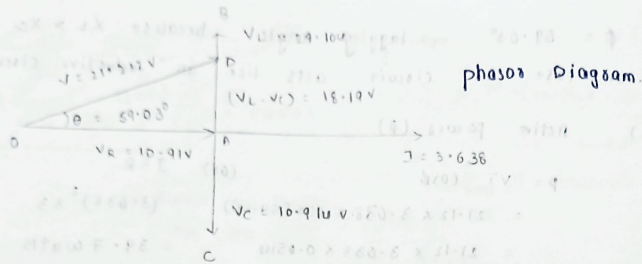
vii power factor = $\cos \phi = 0.514$ lagging power factor.

$$\cos \phi = \frac{R}{Z} = \frac{3}{5.83} = 0.514$$

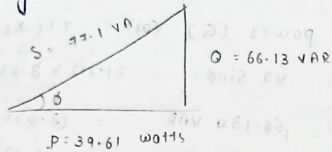


for the same circuit apply kvl and verify?

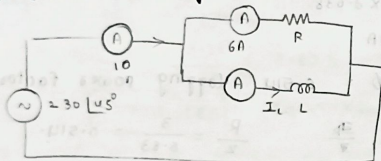
$$30 \sin 2t = 15.435 \sin(2t + 0^\circ) + 41.159 (\sin(2t + 30.97^\circ) + 15.435 \sin(2t - 149.03^\circ))$$



Power triangle.



Find current through Inductor.

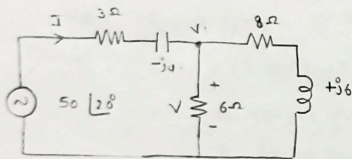


$$I = \sqrt{I_R^2 + I_L^2}$$

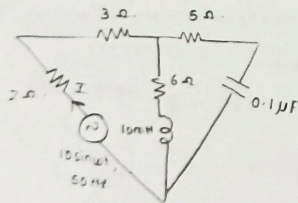
$$I_L = \sqrt{I^2 - I_R^2}$$

$$= \sqrt{(10)^2 - (6)^2} = \sqrt{100 - 36}$$

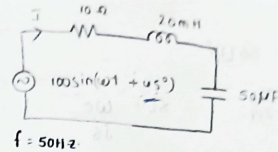
$$I_L = \sqrt{64} = 8 \text{ A}$$



Find current and voltage for the given circuit.



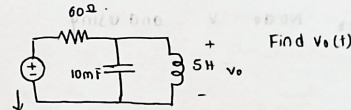
Find I for the given circuit.



3) Find $Z, I, \phi, V_R, V_L, V_C, P, Q, S, \cos \phi$, phasors, verify KVL.

where V_R was considered which angle will be taken when V is the reference angle for $V = 20 \cos(\omega t + 45^\circ)$ and also voltage is given without phase angle $V = 10 \sin \omega t$ Given phase angle θ and from the formula $\theta = \tan^{-1}(\frac{X}{R})$

Alexander's Book - 11 pg. No 389 390.



Find $V_0(t)$

$$V_S = 20 \cos(\omega t - 15^\circ)$$

$$\Rightarrow V_S = 20 \angle -15^\circ$$

$$V_L = ?$$

$$20 \cos(\omega t - 15^\circ) \quad 10 \text{ mF} = \frac{1}{j\omega C}$$

$$I = \frac{V}{Z} = \frac{20 \angle -15^\circ}{116.6 \angle 59.05^\circ} = \frac{1}{j4 \times 10 \times 10^{-3}}$$

$$I = 0.171 \angle -44.03^\circ = \frac{1}{j4 \times 10^{-2}} = \frac{100}{j4} = -j25 \text{ A}$$

$$I_1 = 0.171 \times \frac{5}{-j25 + j20} = j\omega L = j4 \times 5 = j20 \text{ A}$$

$$Z_{eq} = ? \quad (j20 \parallel -j25) + 60 \Omega$$

$$I_1 = 0.855 \angle -44.03^\circ \quad Z = \frac{-j25 \times j20}{-j25 + j20} + 60 = 60 + j100 \Omega$$

$$V = \frac{I}{Z} = \text{we know total voltage } V = 20 \angle -15^\circ$$

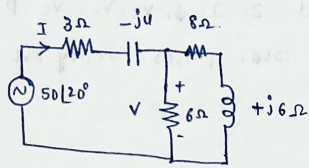
apply voltage division rule

$$V_0 = \frac{Z_2}{Z_1 + Z_2} V_S = \frac{100j}{60 + j100} (20 \angle -15^\circ)$$

$$= \frac{100j}{60 + j100} \times 20 \angle -15^\circ$$

$$= (0.55 + j0.96) \times (20 \angle -15^\circ)$$

$$V_0 = 17.15 \angle 15.96^\circ$$



$$V = 50 \angle 20^\circ$$

$$I = \frac{V}{Z_{eff}}$$

$$Z_{eff} = ?$$

$$X_L = \omega C$$

$$= j6$$

$$X_C = -j4$$

$$(8 + j6) \parallel 6 + 3 - j4$$

$$\frac{(8 + j6)6}{8 + 6 + j6} + 3 - j4 =$$

$$Z_{eff} = 6.827 - 3.068j$$

$$I = \frac{50 \angle 20^\circ}{6.827 - 3.068j}$$

$$= 50 \angle 20^\circ$$

$$= 7.080 \angle -24.19^\circ$$

$$= 6.680 (20 + 24.19^\circ)$$

$$I = 6.680 \angle 44.19^\circ \text{ A}$$

to find V at 6Ω consider Node V and using Nodal analysis:

$$\frac{V - 50 \angle 20^\circ}{3 - j4} + \frac{V}{6} + \frac{V}{8 + j6} = 0$$

$$LCM = 288 - 84j$$

$$\frac{(288 - 84j)}{3 - j4} (V - 50 \angle 20^\circ) + \frac{V(288 - 84j)}{6} + \frac{(288 - 84j)V}{8 + j6} = 0$$

$$(48 + 36j)(V - 50 \angle 20^\circ) + (48 - 14j)V + (18 - 24j)V = 0$$

$$(48 + 36j)V - (46.98 + 17.1j)V + (48 - 14j)V + (18 - 24j)V = 0$$

$$(114 - 2j)V = 1639.4 + 2512.08j$$

$$V = \frac{1639.4 + 2512.08j}{114 - 2j}$$

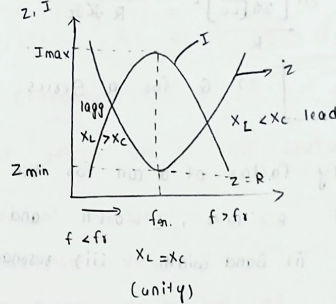
$$V = 13.98 + 22.28j$$

$$V = 26.30 \angle 57.89^\circ \text{ V}$$

25/09/24

UNIT - 4

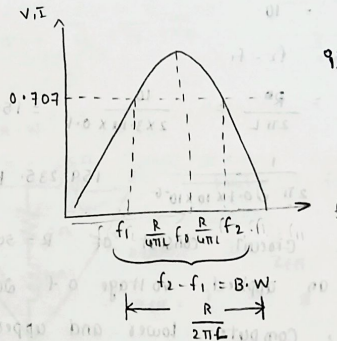
At Resonance Condition:



$$I = \frac{V}{Z}$$

$$\uparrow I = \frac{V}{R} \downarrow \quad (\text{or}) \quad I \downarrow = \frac{V}{R} \uparrow$$

Band width: It is defined as the difference b/w the range of frequencies is known as band width.



$$B.W = f_2 - f_1$$

$$f_2 - f_1 = \frac{R}{2\pi L}$$

B.W units: Hz

$$f_1 = f_0 - \frac{R}{4\pi L} \quad (\text{Lower cut off frequency})$$

$$f_2 = f_0 + \frac{R}{4\pi L} \quad (\text{Higher cut off frequency})$$

Quality factor [Figure of Merit] [Q]:

To check the quality of a coil in the circuit.

Def: It is defined as the ratio of reactance of the coil to the resistance.

$$Q = \frac{X_L}{R} = \frac{2\pi f_1 L}{R}$$

$$Q = \frac{2\pi L}{R} f_0 \Rightarrow Q = \frac{f_0}{B.W} \quad (\because B.W = \frac{R}{2\pi L})$$

$$4) f_0 = \frac{1}{2\pi \sqrt{LC}} \quad \text{resonant frequency}$$

$$Q = \frac{2\pi f_0 L}{R} \Rightarrow 2\pi \left[\frac{1}{2\pi \sqrt{LC}} \right] L = \frac{1}{R} \frac{L}{\sqrt{LC}}$$

$$5) Q = \frac{1}{R} \sqrt{\frac{L}{C}} \Rightarrow Q \text{ for a Series RLC Circuit.}$$

P) Determine the quality factor of a coil for the series circuit consisting of $R = 10 \Omega$, $L = 0.1 \text{ H}$ and $C = 10 \mu\text{F}$

$$Q = \frac{1}{R} \sqrt{\frac{L}{C}}$$

$$Q = \frac{1}{10} \sqrt{\frac{0.1}{10 \times 10^{-6}}}$$

$$Q = \frac{1}{10} \sqrt{\frac{0.1 \times 10^6}{10}} = 10$$

ii) Band width $B.W = f_2 - f_1$

$$= \frac{R}{2\pi L} = \frac{10}{2 \times 3.14 \times 0.1} = 15.92 \text{ Hz}$$

$$\text{iii) } f_0 = \frac{1}{2\pi \sqrt{LC}} = \frac{1}{2\pi \sqrt{0.1 \times 10 \times 10^{-6}}} = 159.235 \text{ Hz}$$

Q) A Series RLC circuit consist of $R = 50 \Omega$, $L = 0.2 \text{ H}$, $C = 10 \mu\text{F}$ with an applied voltage of 20 V . Determine f_0 , Quality factor, Compute lower and upper cutoff frequencies, limits & Band width.

Sol

Given $R = 50 \Omega$, $L = 0.2 \text{ H}$, $C = 10 \mu\text{F}$

$V = 20 \text{ V}$ (Rms)

$$f_0 = \frac{1}{2\pi \sqrt{LC}} = \frac{1}{2\pi \sqrt{0.2 \times 10 \times 10^{-6}}} = 112.54 \text{ Hz}$$

$$Q = \frac{R}{2\pi L} = \frac{f_0}{B.W} = \frac{1}{R} \sqrt{\frac{L}{C}} = \frac{1}{50} \sqrt{\frac{0.2}{10 \times 10^{-6}}} = 2.828$$

$$B.W = \frac{R}{2\pi L} = \frac{50}{2 \times 3.14 \times 0.2} = 39.8 \text{ Hz}$$

$$f_1 = f_0 - \frac{R}{4\pi L} = 112.54 - \frac{50}{4 \times 3.14 \times 0.2} = 92.68$$

$$f_2 = 112.54 + \frac{50}{4 \times 3.14 \times 0.2} = 132.49 \text{ Hz}$$

$$f_2 - f_1 = B.W$$

$$132.49 - 92.68 = 39.81 \text{ Hz}$$

3-phase CIRCUITS:

All three phases are at 120° mutually to each other (R-Y-B)

$$V_R = V_m \cos \omega t = V_m \sin \omega t$$

$$V_Y = V_m \cos(\omega t - 120^\circ) = V_m \sin(\omega t - 120^\circ)$$

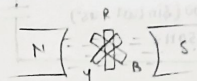
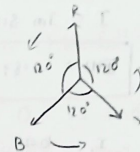
$$V_B = V_m \cos(\omega t - 240^\circ) = V_m \sin(\omega t - 240^\circ)$$

(00)

$$V_B = V_m \cos(\omega t - 240^\circ)$$

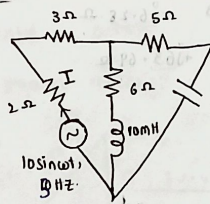
$$V_{\text{Result}} = \sqrt{V_R^2 + V_Y^2 + V_B^2}$$

$$= 0$$



3-UNIT: H.W.

①



Find I for the given circuit

$$I = \frac{V}{Z_{\text{eff}}}$$

$$Z_{\text{eff}} = ?$$

$$-jX_C = \frac{-1}{\omega C} = \frac{-1}{2\pi \times 50 \times 10^{-6}} = -31.847 \times 10^3 j \Omega$$

$$jX_L = \omega L = 2\pi f L = 2 \times 3.14 \times 50 \times 10^{-3} \times 10$$

$$= j3.14 \Omega$$

$$R = 5 \Omega, 6 \Omega, 3 \Omega, 2 \Omega$$

$$(5 + 31.847 \times 10^3 j) \parallel (6 + j3.14) + 5$$

$$\frac{(5 - 31.847 \times 10^3 j)(6 + j3.14)}{5 + 6 - 31.847 \times 10^3 j + j3.14} + 5 = 11.00 + 3.139 j$$

$$V = V_m \sin \omega t = V = \frac{V_m}{\sqrt{2}} = \frac{10}{\sqrt{2}} \text{ at an angle } 0^\circ$$

$$V = 7.07 \angle 0^\circ$$

$$Z_{\text{eff}} = 11.00 + 3.139 j$$

$$I = \frac{V}{Z} = \frac{7.07 \angle 0^\circ}{11.00 + 3.139 j} \rightarrow \text{polar}$$

$$= \frac{7.07 \angle 0^\circ}{11.43 \angle 15.9^\circ}$$

$$I = \frac{7.07 \angle 0^\circ}{11.4 \angle 15.9^\circ} \quad \text{RMS current}$$

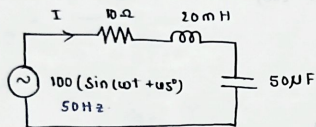
$$I = 0.620 \angle -15.9^\circ \quad \text{In terms of max current and time domain } i(t)$$

$$I_{RMS} \sqrt{2} = I_{max}$$

$$0.620 \times \sqrt{2} = 0.876$$

$$I = I_m \sin(\omega t + \phi)$$

$$i(t) = 0.876 \sin(\omega t - 15.9^\circ)$$



Find Z , I , ϕ , V_R , V_L , V_C ,
 P , Q , S , $\cos \phi$, phasor diagram
 Verify kVL .

$$\text{Given } R = 10 \Omega, L = 20 \text{ mH}, C = 50 \mu\text{F}$$

$$jX_L = j\omega L = 2\pi fL = 2\pi \times 50 \times 20 \times 10^{-3} = j6.28 \Omega$$

$$jX_C = \frac{-j}{\omega C} = \frac{-j}{2\pi \times 50 \times 50 \times 10^{-6}} = -j63.69 \Omega$$

$$Z = R + j(X_C - X_L) = 10 + j(63.69 - 6.28)$$

$$Z = 10 + j(57.41) \Rightarrow Z = 58.27 \angle 80.11^\circ$$

$$i) I = \frac{V}{Z} = \frac{V_m \sin(\omega t + \phi)}{Z} = \frac{100 \angle 45^\circ}{58.27 \angle 80.11^\circ} = 1.715 \angle -35.11^\circ$$

$$I(t) = I_m \sin(\omega t + \phi) = 1.715 \sin(\omega t - 35.11^\circ)$$

$$I_m = I_{RMS} \sqrt{2}$$

$$1.715 \times \sqrt{2} = 2.42$$

$$i(t) = 2.42 \sin(\omega t - 35.11^\circ)$$

$$\phi = \tan^{-1} \left(\frac{X_C - X_L}{R} \right) = \tan^{-1} \left(\frac{63.69 - 6.28}{10} \right) = 80.11^\circ$$

$$i) V_R = IR = 1.715 \times 10 = 17.15$$

Because in resistive circuit angle is b/w V and I .

$$v) V_L = I X_L = 1.715 \angle -35.11^\circ \times 6.28 \angle 90^\circ = 10.77 \angle 54.89^\circ$$

$$v) V_C = I X_C = 1.715 \angle -35.11^\circ \times 63.69 \angle -90^\circ = 109.25 \angle -125.11^\circ$$

$$vii) P = VI \cos \phi = 70.71 \times 1.715 \times \cos(80.11^\circ) = 14.66 \angle 89^\circ \text{ Watts}$$

$$viii) Q = VI \sin \phi = 70.71 \times 1.715 \times \sin(80.11^\circ) = 84.49 \angle 9.39^\circ \text{ VAR}$$

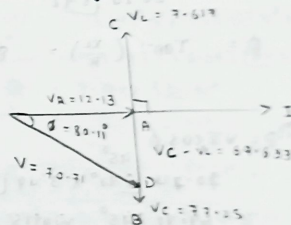
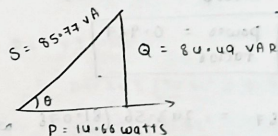
$$ix) S = VI = 70.71 \times 1.715 \angle -35.11^\circ = 85.77 \angle 9.89^\circ \text{ VA}$$

$$S = \sqrt{P^2 + Q^2} = 85.75 \text{ Verified}$$

$$x) \cos \phi = 0.171$$

phasor diagram

power triangle



Verifying kVL

$$-V_S + V_R + V_L + V_C$$

$$-70.71 + 17.15 + 10.77 + 109.25$$

$$-V_S + R i(t) + L \frac{d}{dt} i(t) + \frac{1}{C} \int i(t) dt = 0$$

- 1) A Series RL Circuit $R = 20\Omega$, $L = 10\text{mH}$, when $v(t) = 100\sin(314t + 45^\circ)\text{V}$ is applied to it. Find ϕ , and P (real power) consumed by the load.

Given $R = 20\Omega$, $L = 10\text{mH}$

$V(t) = 100\sin(314t + 45^\circ)$

$\omega = 314$

$X_L = j\omega L = j314 \times 10 \times 10^{-3}$

$X_L = j3.14\Omega$

$I = \frac{V}{Z_{\text{eff}}} = \frac{V_{\text{RMS}}}{Z_{\text{eff}}}$

$Z_{\text{eff}} = j3.14 + 20 = 20.24 \angle 8.92^\circ$

$V = V_m \sin(\omega t + \phi) = \phi = 45^\circ$, $\omega = 314$

$V = \frac{V_m}{\sqrt{2}} \angle \phi = \frac{100}{\sqrt{2}} \angle 45^\circ$

i) $I = \frac{70.71 \angle 45^\circ}{20.24 \angle 8.92^\circ} = 3.49 \angle 36.08^\circ \text{A}$

$\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = 8.92^\circ \Rightarrow \cos\phi = \cos(8.92^\circ)$

power factor = 0.987

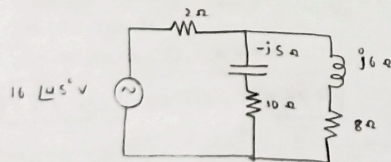
ii) $P = VI \cos\phi$

$= 70.71 \angle 45^\circ \times 3.49 \angle 36.08^\circ \times 0.987 = 243.56 \angle 81.07^\circ \text{ watts}$

$= 69.71 \angle 45^\circ \text{ watts}$

- 2) For the entire circuit in Fig. calculate:

- a) The power factor, b) the average power c) The reactive power



power factor $\cos\phi =$

$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) \times$

$V_{\text{RMS}} = 16$

$X_C = -j5\Omega$

$X_L = j6\Omega$

$Z_{\text{eff}} =$

$(8 + j6) \parallel (10 - j5) + 2$

$= \frac{(8 + j6)(10 - j5)}{8 + 10 + j6 - j5} + 2$

$Z_{\text{eff}} = 8.153 + 0.769j$

$Z_{\text{eff}} = 8.189 \angle 5.38^\circ$, $\phi = 5.38^\circ$

$\cos\phi = \cos(5.38^\circ)$

$= 0.995$

$I = \frac{V}{Z_{\text{eff}}}$

$= \frac{16 \angle 45^\circ}{8.189 \angle 5.38^\circ}$

$I_{\text{RMS}} = 1.953 \angle 39.62^\circ$

$P_{\text{avg}} = VI \cos\phi$

$= V_{\text{RMS}} I_{\text{RMS}}$

$= 16 \angle 45^\circ \times 1.953 \angle 39.62^\circ$

$P_{\text{avg}} = 31.248 \angle 84.62^\circ \times 0.995 = 31.091 \angle 84.62^\circ$

$P = VI \cos\phi$

$= 31.248 \angle 84.62^\circ \times 0.995$

$= 31.091 \angle 84.62^\circ$

The reactive power

$Q = VI \sin\phi$

$= 16 \angle 45^\circ \times 1.953 \angle 39.62^\circ \times \sin\phi$

$= 16 \times 1.953 \times 0.093 \angle 84.62^\circ$

$Q = 2.989 \angle 84.62^\circ$

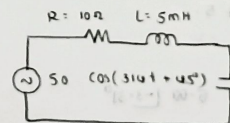
$S = \sqrt{P^2 + Q^2}$

$= \sqrt{(2.989)^2 + (31.091)^2}$

$S = 31.22 \approx P_{\text{avg}}$

- 3) A Series RLC Circuit with $R = 10\Omega$, $L = 5\text{mH}$ and

$C = 20\text{mF}$. when $V_s(t) = 50 \cos(314t + 45^\circ)\text{V}$ is applied to it find the current, power factor and Real power consumed by the load.



$$R = 10 \Omega, \quad L = 5 \text{ mH}, \quad C = 20 \text{ nF}$$

$$V(s) = 50 \cos(314t + 45^\circ)$$

$$i) I = \frac{V}{Z_{\text{eff}}} \quad Z_{\text{eff}} = ?$$

$$X_L = 5 \times 10^{-3} \times 314 = j1.57 \Omega$$

$$X_C = \frac{1}{20 \times 10^{-3} \times 314} = -j0.159 \Omega$$

$$Z = R + j(X_L - X_C)$$

$$= 10 + j(1.57 - 0.159)$$

$$Z = 10 + j(1.411) = 10.09 \angle 8.03^\circ$$

$$I = \frac{35.35 \angle 45^\circ}{10.09 \angle 8.03^\circ} = \boxed{3.503 \angle 36.97^\circ} = I$$

$$\phi = \tan^{-1} \left(\frac{X_L - X_C}{R} \right) = \tan^{-1} \left(\frac{1.411}{10} \right) = 8.031$$

$$\phi = 8.03^\circ$$

$$ii) \cos \phi \text{ (power factor)} = \cos(8.03^\circ)$$

$$\boxed{\cos \phi = 0.990}$$

$$iii) \text{ Real power (P)} = P = VI \cos \phi$$

$$= 35.35 \angle 45^\circ \times 3.503 \angle 36.97^\circ \times 0.990$$

$$\boxed{P = 122.59 \angle 81.97^\circ}$$

$$iv) \text{ Find } v(t) \text{ in the circuit. Supply voltage}$$

$$V_s(t) = 100 \cos(314t) \text{ V}$$

$$\text{Given } V_s(t) = 100 \cos(314t) \text{ V}$$

$$X_L = j\omega L = 314 \times 5 = j1570 \Omega$$

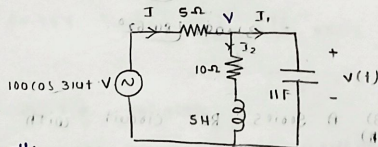
$$X_C = j\frac{1}{\omega C} = \frac{1}{314 \times 11} = -j2.89 \times 10^{-4}$$

$$(j2.89 \times 10^{-4} \parallel 10 + j1570) + 5$$

$$(-j2.89 \times 10^{-4})(10 + j1570) + 5$$

$$-j2.89 \times 10^{-4} \times 10 + 10 + j1570$$

$$Z_{\text{eff}} = 5 - j2.89 \times 10^{-4} j = 5.00 \angle -3.31^\circ$$



$$I = \frac{V}{Z} = \frac{100 \angle 0^\circ}{5.00 \angle -3.31^\circ} = 20 \angle 3.31^\circ$$

division rule

$$I_1 = 14.142 \angle 3.31^\circ \times \frac{Z_2}{Z_1 + Z_2}$$

$$= 14.142 \angle 3.31^\circ \times \frac{j2.89 \times 10^{-4}}{10 + j1570 + j2.89 \times 10^{-4}}$$

$$= 14.142 \angle 3.31^\circ \times 0.999 \angle -1.72 \times 10^{-9} j$$

$$= 14.142 \angle 3.31^\circ \times 0.999 \angle -6.71^\circ$$

$$I_1 = 14.142 \angle -3.4^\circ$$

$$V = I R$$

$$V = I Z = 14.142 \angle 10^\circ$$

Apply nodal analysis: to find $v(t)$.

$$\frac{V - 100 \angle 0^\circ}{5} + \frac{V}{10 + j1570} + \frac{V}{-j2.89 \times 10^{-4}} = 0$$

$$\text{LCM} = 2.268 - 0.014 j$$

$$\frac{(2.268 - 0.014j)(V - 100 \angle 0^\circ)}{5} + \frac{(2.268 - 0.014j)V}{10 + j1570} + \frac{(2.268 - 0.014j)V}{-j2.89 \times 10^{-4}} = 0$$

$$(0.453 - 2.8 \times 10^{-3} j)(V - 100 \angle 0^\circ) + (2.83 \times 10^{-7} - 1.04 \times 10^{-3} j)V$$

$$+ (48.44 + 7847.75 j)V = 0$$

$$(0.453 - 2.8 \times 10^{-3} j)V + (100 - 0j)(2.8 \times 10^{-3} j - 0.453) + (1.83 \times 10^{-7} - 1.04 \times 10^{-3} j)V$$

$$+ (48.44 + 7847.75 j)V = 0$$

$$(45.3 - 0.28 j) + (48.89 + 7847.74 j)V = 0$$

$$(48.89 + 7847.74 j)V = -(45.3 - 0.28 j)$$

$$V = \frac{0.28 j + 45.3}{48.89 + 7847.74 j}$$

$$V = -2.818 \times 10^{-7} + 5.77 \times 10^{-3} j$$

Voltage division rule:

$$V = 100 \angle 0^\circ \times \frac{10 + j1570}{10 + j1570 + 5}$$

$$= 0.999 \angle 3.18 \times 10^{-3} \times 100 \angle 0^\circ$$

$$= 0.999 \angle 0.05^\circ \times 100 \angle 0^\circ$$

$$V = 99.9 \angle 0^\circ$$

27/09/20

NOTE: The phasor sum of all the 3 voltages at any instant is equal to zero

$$V_{res} = V_R + V_Y + V_B$$

$$= V_P \angle 0^\circ + V_P \angle -120^\circ + V_P \angle -240^\circ \quad (\text{rotation of phasors } -90^\circ \text{ to } -270^\circ)$$

$$= V_P [1 \angle 0^\circ + 1 \angle -120^\circ + 1 \angle -240^\circ]$$

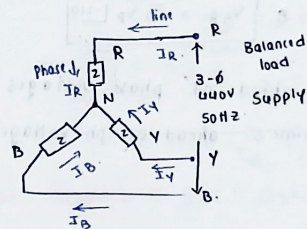
$$= V_P [(1 + j0) - 0.5 - j0.866 - 0.5 + j0.866]$$

$$= V_P [0]$$

$$V_{res} = 0$$

// Hence proved

Star- Connection in 3- ϕ System:



line current = I_L

$$I_L = I_R = I_Y = I_B$$

phase current

$$I_{ph} = I_R = I_Y = I_B$$

$$I_L = I_{ph} \quad (\text{imp obj})$$

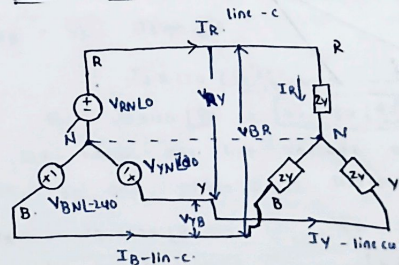
$$\text{line voltage } V_L = V_{RY} = V_{YB} = V_{BR}$$

$$\text{phase voltage } V_{ph} = V_{RN} = V_{YN} = V_{BN}$$

$$V_L \neq V_{ph} \quad **$$

Types of 3- ϕ connections:

1) Y-Y (or) (λ - λ) Star-Star connection



$$V_{ph} = V_{RN} \angle 0^\circ = V_{YN} \angle -120^\circ = V_{BN} \angle -240^\circ$$

$$V_L = V_{RY} = V_{YB} = V_{BR}$$

$$I_L = I_R = I_Y = I_B$$

$$I_{ph} = I_R = I_Y = I_B$$

$$I_N + I_R + I_Y + I_B = 0$$

$$I_N = -(I_R + I_Y + I_B)$$

$$V_{RY} = V_{RN} + V_{NY}$$

$$= V_{RN} - V_{YN}$$

$$= V_{pL0^\circ} - V_{pL-120^\circ}$$

$$= V_p [1 \angle 0^\circ - 1 \angle -120^\circ]$$

$$= V_p [1 + j0 + 0.5 + j0.866]$$

$$= V_p [1.5 + j0.866]$$

$$V_L = \sqrt{3} V_p \angle 30^\circ$$

Hence the line voltage is $\sqrt{3}$ times equal to phase voltage at an angle 30° .

In case V_{YB} and V_{BR} . Same the magnitude but different phase angle because $V_{YB} \angle -120^\circ + 30^\circ = V_{YB} \angle -90^\circ$

So similarly,

$$V_{YB} = \sqrt{3} V_p \angle -90^\circ$$

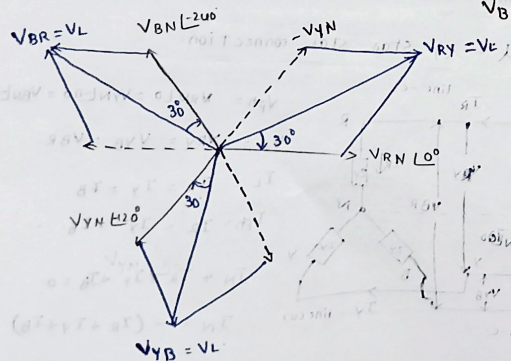
$$\& V_{BR} = \sqrt{3} V_p \angle -210^\circ$$

* line voltages are 30° ahead of phase voltages.

* line voltages are $\sqrt{3}$ times ahead of ph voltages in magnitude.

$$\text{So } V_L \neq V_{ph}$$

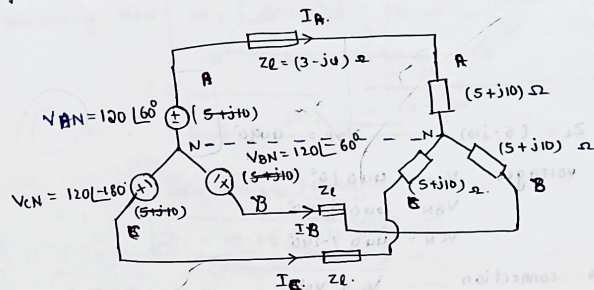
Phasor Diagram (for) line & phase voltages:



Note: ***

1) In 3- ϕ star connection line voltages are 30° ahead of phase voltages

2) A 3- ϕ balanced star-connected load having an impedance of $11 \angle 30^\circ \Omega$. per phase is connected to a 3- ϕ alternator (Generator - AC) with a +ve sequence of $V_{AN} = 120 \angle 60^\circ$ with a line impedance of $Z_L = (3 - j4) \Omega$ per phase. Determine line currents and line voltages



Phase sequence is ABC instead of RYB

Initial phase voltage $V_{AN} = 120 \angle 60^\circ$

Similarly $V_{BN} = 120 \angle -120^\circ + 60^\circ = 120 \angle -60^\circ$

$V_{CN} = 120 \angle -240^\circ + 60^\circ = 120 \angle -180^\circ$

$I_L = I_A = I_B = I_C$ - line currents.

$V_L = V_{AB} = V_{BC} = V_{CA}$ - line voltages.

$$V_{AB} = V_L = \sqrt{3} V_{ph} \angle 30^\circ$$

$$= \sqrt{3} \times 120 \angle 60^\circ + 30^\circ$$

$$V_L = \sqrt{3} \times 120 \angle 90^\circ = 207.84 \angle 90^\circ$$

If we know one line voltage we can derive remaining two by and phase angle difference 120°

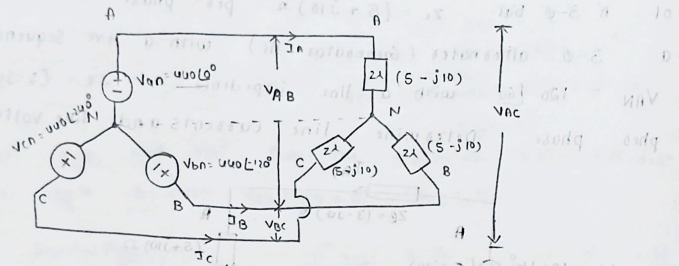
$$V_{BC} = \sqrt{3} \times 120 \angle 90^\circ - 120^\circ$$

$$= 207.84 \angle -30^\circ$$

$$V_{CA} = 207.84 \angle 90^\circ - 240^\circ$$

$$= 207.84 \angle -150^\circ$$

H.W: A 3- ϕ Balanced Star connected load having an impedance of $Z_L = (5 - j10) \Omega$ per phase is connected to a 3- ϕ Alternator with +ve phase sequence having a voltage of $V_{an} = 440 \angle 0^\circ$. Determine line voltages & line currents.



Given $Z_L = (5 - j10) \Omega$, $V_{an} = 440 \angle 0^\circ$
 phase voltages $V_{AN} = 440 \angle 0^\circ$
 $V_{BN} = 440 \angle -120^\circ$
 $V_{CN} = 440 \angle -240^\circ$

In Star connection $V_L \neq V_{ph}$

$$I_L = I_{ph}, \quad I_L = I_A = I_B = I_C$$

$$V_L = \text{line voltage} = \sqrt{3} \times V_{ph} \angle 30^\circ$$

$$V_{AB} = \sqrt{3} \times V_{ph} \angle 30^\circ$$

$$= \sqrt{3} \times 440 \angle 30^\circ$$

$$= \sqrt{3} \times 440 \angle 30^\circ + 30^\circ$$

$$V_{AB} = 762.10 \angle 30^\circ \quad \text{--- line voltage 1st}$$

remaining are differed by 120° phase angle.

$$V_{BC} = 762.10 \angle 30 - 120^\circ$$

$$V_{BC} = 762.10 \angle -90^\circ$$

$$V_{CA} = 762.10 \angle 30 + 240^\circ$$

$$V_{CA} = 762.10 \angle -210^\circ$$

$$I_A = \frac{V_{AN} \angle 0^\circ}{Z_L} = \frac{440 \angle 0^\circ}{5 - j10}$$

$$= \frac{440 \angle 0^\circ}{11.18 \angle -63.43^\circ}$$

$$= \frac{440 \angle 0^\circ - (-63.43^\circ)}{11.18}$$

$$I_A = 39.35 \angle 63.43^\circ$$

Remaining two are differed by 120° from each

$$I_B = 39.35 \angle 63.43 - 120^\circ$$

$$I_B = 39.35 \angle -56.57^\circ$$

$$I_C = 39.35 \angle 63.43 - 240^\circ$$

$$I_C = 39.35 \angle -176.57^\circ$$

For loop AN KVL

$$-440 \angle 0^\circ + I_A(Z_L) = 0$$

$$-440 \angle 0^\circ + 39.35 \angle 63.43^\circ \times (5 - j10) = 0$$

$$-440 \angle 0^\circ + 39.35 \angle 63.43^\circ \times (5 - j10) = 0$$

$$-440 + 0j + (176.75 + 393.5j)(5 - j10) = 0$$

$$-440 + 0j + 439 - 0.5j = 0$$

$$-1 - 0.5j = 0$$

$$-440 \angle -120^\circ + 39.35 \angle -56.57^\circ (5 - j10) = 0$$

$$-440 \angle -120^\circ + 219.65 - 380.7j = 0$$

$$-440 \angle -120^\circ - 219.65 - 380.7j = 0$$

$$+ 220 + 381.5j - 219.65 - 380.7j = 0$$

$$0.35 - 0.8j = 0$$

line current $I_L = I_A = I_B = I_C = I_{ph}$

$$I_A = \frac{V_{AB}}{Z(Y)} \quad Z(Y) = Z_e + Z_L$$

total impedance $= 3 - j4 + 5 + j10$

$$Z(Y) = 8 + j6$$

$$I_A = \frac{207.8u \angle 90^\circ}{8 + j6} = \frac{207.8u \angle 90^\circ}{10 \angle 36.86^\circ}$$

$$= 20.78u \angle 53.14^\circ \text{ A}$$

line current I_A is same at V_{AN} , Z_e and Z_L Because these three are connected in series.

$$I_A = \frac{V_{AN}}{Z(Y)} = \frac{120 \angle 60^\circ}{Z_e + Z_L} = \frac{120 \angle 60^\circ}{8 + j6}$$

$$= \frac{120 \angle 60^\circ}{10 \angle 36.86^\circ} = 12 \angle 23.14^\circ \text{ A}$$

$$I_A = 12 \angle 23.14^\circ \text{ A}$$

$$I_B = 12 \angle 23.14 - 120^\circ$$

$$I_B = 12 \angle 96.86^\circ$$

$$I_C = 12 \angle 23.14 - 240^\circ$$

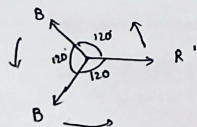
$$I_C = 12 \angle -216.86^\circ \text{ A}$$

$$I_L = I_A = I_B = I_C \quad \& \quad \text{also } I_{ph} = I_B = I_A = I_C$$

$$I_L = I_{ph}$$

H.W: A-3-~~0~~ Balance Phase Sequence:

It is defined as the order of each phase voltage will attain its max. value is called phase sequence.

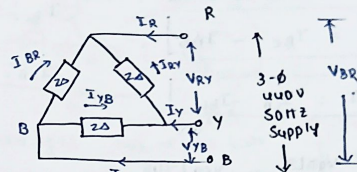


$$R-Y-B \rightarrow (+ve)$$

$$R-B-Y \rightarrow (-ve)$$

$$A-B-C \rightarrow A-C-B (-ve)$$

3- ϕ Delta Connection:



In case of Delta connection

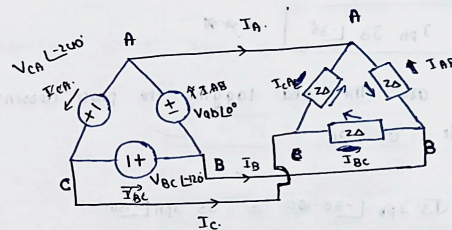
$$\text{Line voltage} = \text{phase voltage} \quad \text{i.e. } V_L = V_{ph} = V_{RY} = V_{YB} = V_{BR}$$

$$I_L = \text{line current} = I_R = I_Y = I_B$$

$$I_{ph} = \text{phase current} = I_{RY} = I_{YB} = I_{BR}$$

Hence $I_L \neq I_{ph}$

Delta-Delta connection ($\Delta-\Delta$):



all 3 phase voltages are displaced by an angle of 120°
 $V_{AB} \angle 0^\circ$; $V_{BC} \angle -120^\circ$; $V_{CA} \angle -240^\circ$

* The direction of Ph voltage is considered for phase currents

$$\text{phase voltages } V_{ph} = V_{AB} \angle 0^\circ = V_{BC} \angle -120^\circ = V_{CA} \angle -240^\circ$$

$$\text{line current } I_L = I_A = I_B = I_C$$

$$\text{phase current } I_{ph} = I_{AB} = I_{BC} = I_{CA}$$

$$\text{KCL at 'A' } I_A + I_{AB} = I_{CA} \quad \text{to verify KCL}$$

kco at 'A'

$$I_A + I_{CA} = I_{AB}$$

$$I_A = I_{AB} - I_{CA}$$

$$I_B = I_{BC} - I_{AB}$$

$$I_C = I_{CA} - I_{BC}$$

phase 3 phase-V

$$I_{AB} = \frac{V_{AB} L_0}{Z(\Delta) \text{ Total Imp}}$$

$$I_{BC} = \frac{V_{BC} L_{120}}{Z(\Delta)}$$

$$I_{CA} = \frac{V_{CA} L_{120}}{Z(\Delta)}$$

line current $I_A = \frac{V_{AB} L_0}{Z(\Delta)} - \frac{V_{CA} L_{120}}{Z(\Delta)}$ magnitude of ph-V is same.

$$= \frac{V_p}{Z(\Delta)} [1 \angle 0^\circ - 1 \angle -240^\circ]$$

$$= \frac{V_p}{Z(\Delta)} [1 + 0.5 + 0.5 \angle -0.866j]$$

$$= \frac{V_p}{Z(\Delta)} [1.5 - j0.866]$$

$$= \frac{V_p}{Z(\Delta)} [\sqrt{3} \angle -30^\circ]$$

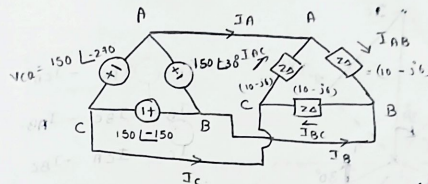
$$I_L = I_{ph} \sqrt{3} \angle -30^\circ \quad \star \star \star$$

* line currents are the ph lagging the phase currents by an angle of 30°

$$I_B = \sqrt{3} I_{ph} \angle -30 - 120 = \sqrt{3} I_{ph} \angle -150$$

$$I_C = \sqrt{3} I_{ph} \angle -30 - 240 = \sqrt{3} I_{ph} \angle -270$$

P) A Delta connected balanced source with +ve ph. Sequence $V_{AB} = 150 \angle -30^\circ$ V is connected to a balanced Δ -connected load having an impedance of $Z_L = (10 - j6) \Omega$ per phase. Determine line currents and phase currents.



phase voltages $V_{AB} = 150 \angle -30^\circ$
 $V_{BC} = 150 \angle -150^\circ$
 $V_{CA} = 150 \angle -270^\circ$ } equal line voltages

$$V_L = V_{ph}$$

Line currents $\neq$ phase currents.

phase currents $I_{AB} = \frac{V_{AB}}{Z(\Delta)} = \frac{150 \angle -30^\circ}{10 - j6} = \frac{150 \angle -30^\circ}{11.66 \angle -30.96^\circ}$

$$I_{AB} = 12.86 \angle 0.96^\circ \text{ A}$$

1st phase current.

directly write I_{BC} & I_{CA} .

$$I_{BC} = 12.86 \angle 0.96 - 120$$

$$I_{BC} = 12.86 \angle -119.04^\circ \text{ A}$$

$$I_{CA} = 12.86 \angle 0.96 - 240$$

$$I_{CA} = 12.86 \angle -239.04^\circ \text{ A}$$

line current = 2 Methods (i) Apply KCL (ii) formula

$$I_L = I_{ph} \sqrt{3} \angle -30^\circ$$

$$I_A = \sqrt{3} \times 12.86 \angle 0.96^\circ \angle -30^\circ$$

$$I_A = 22.27 \angle -29.04^\circ \text{ A} \quad \text{phase current}$$

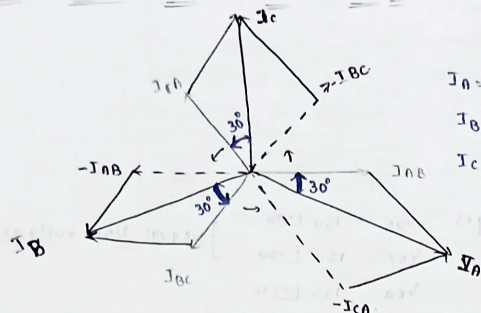
$$I_B = 22.27 \angle -29.04^\circ - 120$$

$$I_B = 22.27 \angle -149.04^\circ \text{ A}$$

$$I_C = 22.27 \angle -29.04^\circ - 240$$

$$I_C = 22.27 \angle -269.04^\circ \text{ A}$$

phasor diagram for line & phase currents:

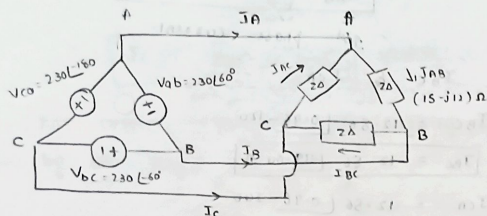


$$I_A = I_{AC} - I_{CB}$$

$$I_B = I_{BC} - I_{AB}$$

$$I_C = I_{CA} - I_{BC}$$

p) A Balanced Delta connected Source with +ve phase Sequence of $V_{ab} = 230 \angle 60^\circ$ is connected to a A Balanced Δ -connected load of $Z_L = (15 - j12) \Omega$. per phase. Determine Line currents & phase currents?



In Δ connection $V_{load} = V_{ab} = V_{bc} = V_{ca} = V_{ph}$

$$V_{ph} = V_{AB} = 230 \angle 60^\circ$$

$$V_{BC} = 230 \angle -60^\circ$$

$$V_{CA} = 230 \angle 180^\circ$$

$$I_L = I_A = I_B = I_C = I_{ph} = I_{AB} = I_{BC} = I_{CA}$$

$$I_{AB} = \frac{V_{AB}}{Z(\Delta)} = \frac{230 \angle 60^\circ}{15 - j12} = \frac{230 \angle 60^\circ}{19.2 \angle -38.65^\circ} = 11.979 \angle 98.65^\circ \text{ A}$$

$$\text{Similarly } I_{BC} = \frac{V_{BC}}{Z(\Delta)} = 11.979 \angle -21.35^\circ \text{ A}$$

$$I_{CA} = \frac{V_{CA}}{Z(\Delta)} = 11.97 \angle -141.35^\circ \text{ A}$$

$$I_A = \sqrt{3} I_{ph} \angle -30^\circ$$

$$= \sqrt{3} I_{AB} \angle -30^\circ$$

$$= \sqrt{3} \times 11.979 \angle 98.65^\circ - 30^\circ$$

$$I_A = 20.74 \angle 68.65^\circ$$

$$I_C = \sqrt{3} I_{CA} \angle -30^\circ (\text{os})$$

$$= 20.74 \angle 68.65^\circ - 240^\circ$$

$$I_C = 20.74 \angle -171.35^\circ$$

$$I_B = \sqrt{3} I_{BC} \angle -30^\circ (\text{os})$$

$$I_B = 20.74 \angle -51.35^\circ$$

$$\text{Kcl at } n' : \frac{20.74}{\sqrt{3}} \angle 68.65^\circ + 11.979 \angle -141.35^\circ$$

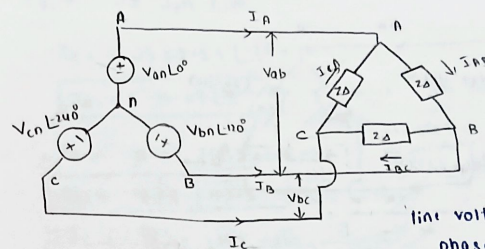
$$7.55 - j19.316j$$

$$+ 9.35 - j7.48j = -1.80 + j11.84j$$

$$-1.80 + j11.84j = 11.80 \angle 98.65^\circ$$

to check whether the obtained answers correct or not apply Kcl at any point of the load connection.

source load.
Star - Delta connection:



line voltages of star - phase voltages of delta.

$$\text{In star connection } V_L = \sqrt{3} V_{ph} \angle 30^\circ$$

$$V_{ab} = \sqrt{3} V_{an} \angle 30^\circ$$

$$V_{bc} = \sqrt{3} V_{bn} \angle -90^\circ (\because 30^\circ - 120^\circ)$$

$$V_{ca} = \sqrt{3} V_{cn} \angle -210^\circ (\because 30^\circ - 240^\circ)$$

$$\text{In delta connection } V_L = V_{ph}$$

line voltages of star becomes phase voltages of delta. Now we can calculate I_{ab} , I_{bc} , I_{ca} phase current.

$$\text{phase currents } I_{AB} = \frac{V_{AB}}{Z(\Delta)} ; I_{BC} = \frac{V_{BC}}{Z(\Delta)} ; I_{CA} = \frac{V_{CA}}{Z(\Delta)}$$

$$\text{Line currents } I_L = \sqrt{3} I_{ph} \angle -30^\circ$$

$$I_A = \sqrt{3} I_{AB} \angle -30^\circ$$

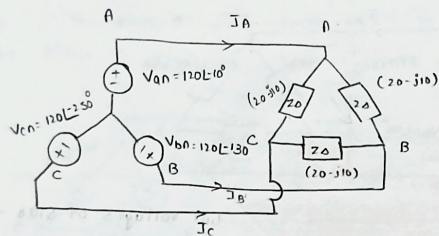
$$I_B = \sqrt{3} I_{ph} \angle -30^\circ - 120^\circ$$

$$I_B = \sqrt{3} I_{ph} \angle -150^\circ$$

$$I_C = \sqrt{3} I_{ph} \angle -30^\circ + 240^\circ$$

$$I_C = \sqrt{3} I_{ph} \angle -270^\circ$$

P) A Balanced Star-connected source alternates having a positive phase sequence ABC, $V_{an} = 120 \angle -10^\circ$ is connected to a balanced Δ -connected load. of $Z_L = (20 - j10) \Omega$ per phase. Determine line voltage, phase current, line current. (oo) only determine line currents directly.



phase Line voltage $V_L = \sqrt{3} V_{ph} \angle 30^\circ$

$$V_{AB} = \sqrt{3} V_{an} \angle 30^\circ$$

$$= \sqrt{3} \times 120 \angle -10^\circ + 30^\circ$$

$$V_{AB} = 207.84 \angle 20^\circ \text{ V}$$

Fast line voltage in star.

remaining line voltages are differ by $\phi 120^\circ$

$$V_{BC} = 207.84 \angle 20^\circ - 120^\circ$$

$$V_{BC} = 207.84 \angle -100^\circ$$

$$V_{CA} = 207.84 \angle 20^\circ + 240^\circ$$

$$V_{CA} = 207.84 \angle -236^\circ$$

$I_N(\Delta)$ $V_L = V_{ph}$ so V_L of Δ becomes V_{ph} in Δ . Now phase currents I_{AB} , I_{BC} , I_{CA} calculated

$$I_{AB} = \frac{V_{AB}}{Z(\Delta)} = \frac{207.84 \angle 20^\circ}{20 - j10}$$

$$= \frac{207.84 \angle 20^\circ}{22.36 \angle -26.5^\circ} = 9.29 \angle 46.5^\circ \text{ A}$$

$$I_{AB} = 9.29 \angle 46.5^\circ \text{ A} \quad 1^{st} \text{ ph. current}$$

$$I_{BC} = 9.29 \angle 46.5^\circ - 120^\circ$$

$$I_{BC} = 9.29 \angle -73.5^\circ \text{ A}$$

$$I_{CA} = 9.29 \angle 46.5^\circ - 240^\circ$$

$$I_{CA} = 9.29 \angle -193.5^\circ \text{ A} \rightarrow I_{ph}$$

& also calculated using formula $\frac{V_{BC}}{Z(\Delta)}$

$$I_A = \sqrt{3} I_{ph} \angle -30^\circ$$

$$I_A = \sqrt{3} \times 9.29 \angle -30^\circ + 46.5^\circ$$

$$I_A = 16.09 \angle 16.5^\circ \text{ A}$$

reference line current

$$I_B = 16.09 \angle 16.5^\circ - 120^\circ$$

$$I_B = 16.09 \angle 103.5^\circ$$

↓ remaining two I_B , I_C differ by 120°

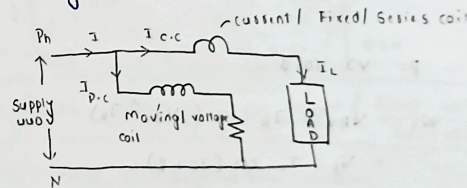
$$I_C = 16.09 \angle 16.5^\circ - 240^\circ$$

$$I_C = 16.09 \angle -223.5^\circ$$

18/10/20:

Measurement of Three phase power using two wattmeter Method.

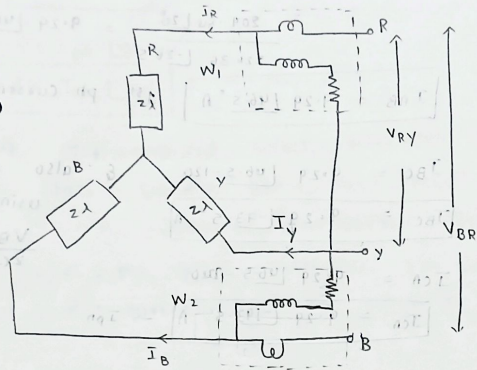
Single line diagram of a wattmeter.



$$3 \phi \text{ power } P = 3 V_{ph} I_{ph} \cos \phi$$

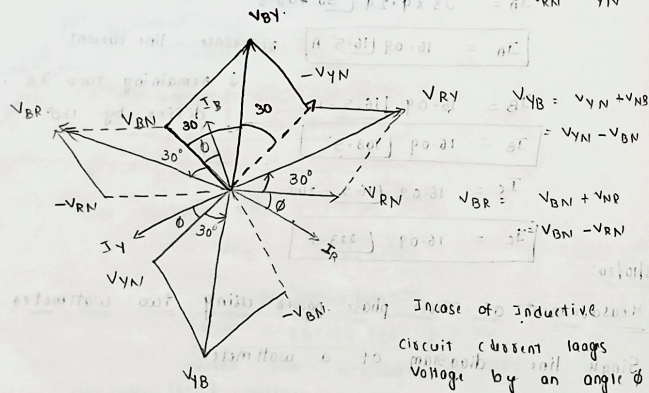
Derivation:

Fig(i)



$$V_L = \sqrt{3} V_{ph} \angle 30^\circ$$

$$V_{RY} = V_{RN} + V_{NY}$$



In case of inductive load, circuit current lags voltage by an angle ϕ

From (i) $P = VI \cos \phi$

$$W_1 = V_{RY} \cdot I_R \cos (V_{RY} \angle I_R)$$

$$= V_L \cdot I_L \cos (30 + \phi)$$

$$W_2 = V_{BY} \cdot I_B \cos (V_{BY} \angle I_B)$$

$$= V_L \cdot I_L \cos (30 - \phi)$$

Total watt meter reading is $W_1 + W_2 = W$

$$\Rightarrow V_L I_L \cos (30 + \phi) + V_L I_L \cos (30 - \phi)$$

$$\Rightarrow V_L I_L [2 \cos 30 \cos \phi] \quad (\because \cos(A+B) + \cos(A-B) = 2 \cos A \cos B)$$

$$\Rightarrow V_L I_L \sqrt{3} \times \cos \phi$$

$$*** \boxed{W_1 + W_2 = \sqrt{3} V_L I_L \cos \phi} \quad \text{3-}\phi \text{ Active power (P)} \quad \text{--- (1)}$$

To get 3- ϕ reactive power take the difference of 2 wattmeters

$$W_1 - W_2 = V_L I_L \cos (30 + \phi) - V_L I_L \cos (30 - \phi)$$

$$= V_L I_L [\cos (30 + \phi) - \cos (30 - \phi)]$$

$$= V_L I_L [-2 \sin 30 \sin \phi] \quad (\because \cos(A+B) - \cos(A-B) = -2 \sin A \sin B)$$

$$= V_L I_L -1 \times \frac{1}{2} \sin \phi$$

$$\boxed{W_1 - W_2 = -V_L I_L \sin \phi} \rightarrow \text{Single } \phi \text{ reactive power}$$

Multiply $\sqrt{3}$ on both sides. we will get the 3- ϕ

reactive power.

$$*** \boxed{\sqrt{3} (W_2 - W_1) = \sqrt{3} V_L I_L \sin \phi} \quad \text{3-}\phi \text{ reactive power} \quad \text{--- (2)}$$

Divide Eq (2) by (1)

$$\frac{\sqrt{3} (W_2 - W_1)}{W_1 + W_2} = \frac{\sqrt{3} V_L I_L \sin \phi}{\sqrt{3} V_L I_L \cos \phi}$$

$$\tan \phi$$

$$\tan \phi = \frac{\sqrt{3} (W_2 - W_1)}{W_1 + W_2}$$

$$\boxed{\phi = \tan^{-1} \left(\frac{\sqrt{3} (W_2 - W_1)}{W_1 + W_2} \right)} \quad ***$$

$$\text{power factor } \cos \phi = \cos \left(\tan^{-1} \left(\frac{\sqrt{3} (W_2 - W_1)}{W_1 + W_2} \right) \right) \quad ***$$

ϕ	$\cos \phi$	Relation b/w W_1 & W_2
0	1	$W_1 = W_2$
30	0.8666	$W_2 = 2W_1$
60	0.5	$W_1 = 0, W_2 = \text{Total power}$
90	0	$W_1 = -W_2$

① In 2 wattmeter method the wattmeter readings are -5 kW and 20 kW which is connected to a 3ϕ induction motor.

i) Find power absorbed by the machine.

ii) Power factor.

iii) Reactive power absorbed by the load taken

Sol Given $W_1 = -5 \text{ kW}$
 $= -5 \times 10^3 \text{ watts}$
 $W_2 = 20 \text{ kW} = 20 \times 10^3 \text{ watts}$

$$\phi = \tan^{-1} \left(\frac{\sqrt{3}(\omega_2 - \omega_1)}{\omega_1 + \omega_2} \right)$$

$$= \tan^{-1} \left(\frac{\sqrt{3}(20)}{20 \times 10^3 - 5 \times 10^3} \right)$$

$$\omega_1 + \omega_2 = W = 20 \times 10^3 - 5 \times 10^3$$

$$= 15 \times 10^3 \text{ watts}$$

$$= 15 \text{ kW}$$

$$\cos \phi = \cos \left(\tan^{-1} \left(\frac{\sqrt{3}(\omega_2 - \omega_1)}{\omega_1 + \omega_2} \right) \right)$$

$$= \cos \left(\tan^{-1} \left(\frac{\sqrt{3}(20 \times 10^3 - 5 \times 10^3)}{20 \times 10^3 - 5 \times 10^3} \right) \right)$$

$$= \cos \left(\tan^{-1} \left(\frac{\sqrt{3}(15)}{15} \right) \right)$$

$$= \cos \left(\tan^{-1} \left(\frac{\sqrt{3} \times 5}{3} \right) \right)$$

$$= 70.89^\circ \text{ lag}$$

$$\omega_2 - \omega_1 = V_L I_L \sin \phi \rightarrow \text{Reactive power}$$

$$\sqrt{3}(\omega_2 - \omega_1) = \sqrt{3} V_L I_L \sin \phi$$

$$= \sqrt{3}(20 + 5) = 43.30 \text{ VAR}$$

A 3ϕ induction motor load has 0.5 power factor lagging. 2 wattmeters are connected to measure power. Show the input has 30 kW find the reading of each wattmeter

$$\cos \phi = 0.5$$

$$\phi = \cos^{-1}(0.5) = 60^\circ$$

$$\omega_1 + \omega_2 = 30 \text{ kW}$$

$$\omega_1 = V_L I_L \cos(30 + \phi) \times$$

$$=$$

$$\omega \tan \phi = \frac{\sqrt{3}(\omega_2 - \omega_1)}{\omega_1 + \omega_2}$$

$$\sqrt{3} = \frac{\sqrt{3}(\omega_2 - \omega_1)}{\omega_1 + \omega_2}$$

$$\omega_1 + \omega_2 = \omega_2 - \omega_1$$

$$30 = \omega_2 - \omega_1 \Rightarrow \omega_2 = 30 + \omega_1$$

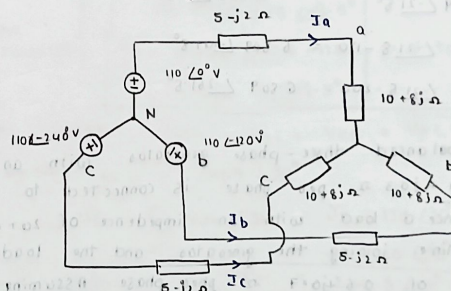
$$\omega_1 + 30 + \omega_1 = 30$$

$$2\omega_1 = 0$$

$$\omega_1 = 0 \text{ kW}$$

$$\omega_2 = 30 \text{ kW}$$

① Calculate the line currents in the three wire Y-Y system of Fig 12-13



It is observed that it is a star to star connection. In star-star line current are equal to phase currents and also line voltages are not equal to phase voltages.

Line voltages = $V_{ab} = V_{bc} = V_{ca}$

phase voltages given $V_{an} = 110 \angle 0^\circ$ V
 $V_{bn} = 110 \angle -120^\circ$ V
 $V_{cn} = 110 \angle -240^\circ$ V } V_{ph} differs by 120°

from this we know the formula for line voltage

$$V_L = \sqrt{3} V_{ph} \angle 30^\circ$$

$$V_{ab} = \sqrt{3} V_{an} \angle 30^\circ$$

$$= \sqrt{3} \times 110 \angle 0^\circ + 30^\circ$$

$$= \sqrt{3} \times 110 \angle 30^\circ$$

$$V_{ab} = 190.52 \angle 30^\circ$$

remaining two are differed by angle 120°

$$V_{bc} = 190.52 \angle 30^\circ - 120^\circ$$

$$V_{bc} = 190.52 \angle -90^\circ$$

$$V_{ca} = 190.52 \angle 30^\circ - 240^\circ$$

$$= 190.52 \angle -210^\circ$$

line currents = $I_a = \frac{\text{line voltage}}{\text{line impedance} + \text{load impedance}}$

$$= \frac{V_{an} \angle 0^\circ}{Z_L + Z_L} = \frac{110 \angle 0^\circ}{5 - j2 + 10 + j8} = \frac{110 \angle 0^\circ}{15 + j6}$$

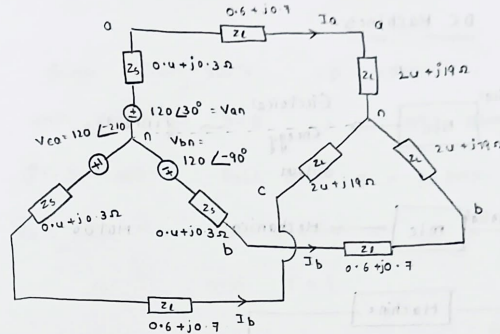
$$= \frac{110 \angle 0^\circ}{16.155 \angle 21.8^\circ} = 6.809 \angle -21.8^\circ$$

$$I_a = 6.809 \angle -21.8^\circ$$

$$I_b = 6.809 \angle -21.8^\circ - 120^\circ = 6.809 \angle -141.8^\circ$$

$$I_c = 6.809 \angle -21.8^\circ - 240^\circ = 6.809 \angle -261.8^\circ$$

A - Star connected balanced three-phase generator with an impedance of $0.4 + j0.3 \Omega$ per phase is connected to a star-connected balanced load with an impedance of $2 + j19 \Omega$ per phase. The line joining the generator and the load has an impedance of $0.6 + j0.7 \Omega$ per phase. Assuming a positive phase sequence for the source voltages and that $V_{an} = 120 \angle 30^\circ$ V, find a) the line voltages, b) the line currents.



we know in star to star $I_L = I_p$ & $V_L \neq V_{ph}$

given phase voltages are $V_{an} = 120 \angle 30^\circ$

$$V_{bn} = 120 \angle -90^\circ$$

$$V_{cn} = 120 \angle -210^\circ$$

$$Z_s = 0.4 + j0.3 \Omega$$

$$Z_L = 2 + j19 \Omega$$

$$Z_L = 2 + j19 \Omega$$

we have to know that line currents = $\frac{V_L \angle \theta}{Z_s + Z_L + Z_L}$

$$I_a = I_b = I_c$$

$$I_a = \frac{V_{an} \angle 0^\circ}{Z_s + Z_L + Z_L} = \frac{120 \angle 30^\circ}{0.4 + j0.3 + 0.6 + j0.7 + 2 + j19}$$

$$= \frac{120 \angle 30^\circ}{2.5 + j20} = \frac{120 \angle 30^\circ}{20.015 \angle 38.5^\circ} = \frac{120 \angle 30^\circ - 38.5^\circ}{20.015}$$

$$\text{line currents} \begin{cases} I_a = 3.748 \angle -8.5^\circ \\ I_b = 3.748 \angle -8.5^\circ - 120^\circ = 3.748 \angle -128.5^\circ \\ I_c = 3.748 \angle -8.5^\circ - 240^\circ = 3.748 \angle -248.5^\circ \end{cases}$$

Line voltages $V_{ab} = V_{bc} = V_{ca} = V_p \angle$

we know $V_L = \sqrt{3} V_{ph} \angle 30^\circ$

$$V_{ab} = \sqrt{3} V_{an} \angle 30^\circ = \sqrt{3} \times 120 \angle 30^\circ + 30^\circ$$

$$= \sqrt{3} \times 120 \angle 60^\circ$$

$$V_{ab} = 207.8 \angle 60^\circ$$

$$V_{bc} = 207.8 \angle 60^\circ - 120^\circ = 207.8 \angle -60^\circ$$

$$V_{ca} = 207.8 \angle 60^\circ - 240^\circ = 207.8 \angle -180^\circ$$

line voltages