

UNIT - II

Circuit Theorems

Necessity of Theorems :-

In a complicated network with several sources, nodes and meshes, if the response in a single element is desired then network theorems are used.

Tellegen's Theorem :-

Algebraic sum of powers at any given instant is zero. i.e. the power delivered by some elements is equals to the power absorbed by remaining elements present in the network.
mathematically it can be expressed as

$$\sum_{k=1}^b V_k I_k = 0$$

$$\sum_{k=1}^b P_k = 0$$

where 'b' is the total no. of branches.

$$\sum P_{\text{supply}} - \sum P_{\text{absorbed}} = 0$$

$$\therefore \sum P_{\text{supply}} = \sum P_{\text{absorbed}}$$

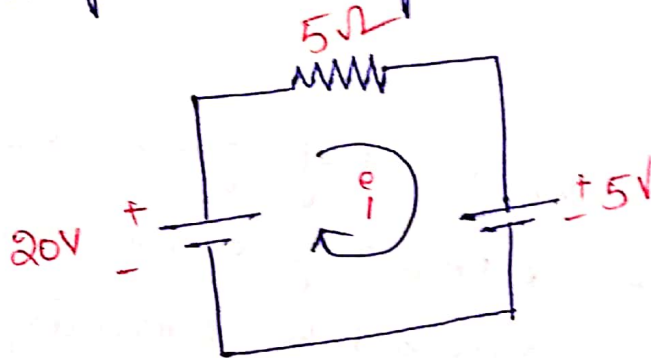
$$\therefore \text{Total power delivered} = \text{Total power absorbed.}$$

→ when the current enters at the negative terminal of an element then that element will deliver the power otherwise it will absorb the power.

→ sources (current & voltage source) can deliver power or absorb power.
whereas the passive elements will always absorb power

Ex :-

* Verify the Tellegen's theorem



Sol

Apply KVL to the ckt

$$\begin{aligned} -20 + 5i + 5 &= 0 \\ 5i &= 15 \\ i &= 3 \end{aligned}$$

$$P_{5\Omega} = i^2 R = 3^2 \times 5 = 9 \times 5 = 45 \text{ W} \Rightarrow \text{absorbed power}$$

$$P_{5V} = v_i = 5 \times 3 = 15 \text{ W} \Rightarrow \text{absorbed power}$$

$$P_{20V} = v_i = 20 \times 3 = 60 \text{ W} \Rightarrow \text{supplied power}$$

$$\sum P_{\text{supplied}} = \sum P_{\text{absorbed}}$$

$$60 = 45 + 15$$

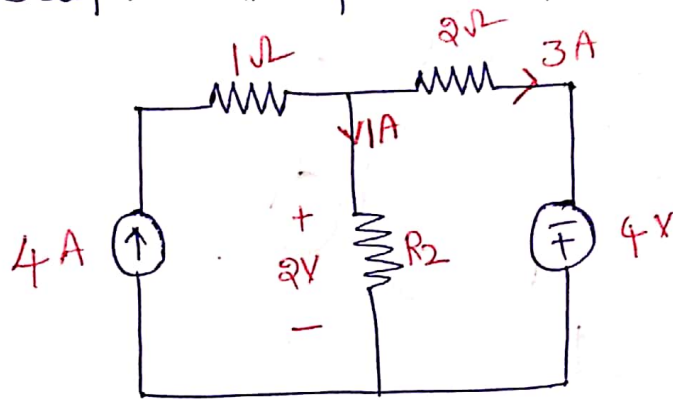
$$60 = 60$$

$$60 - 60 = 0$$

$$\sum P = 0$$

Example :-

* Determine the power supplied by the current source.



Sol

$$P_{4A} = ?$$

$$P_{1\Omega} = i^2 R = 4^2 \times 1 = 16W$$

~~$P_{2\Omega}$~~

$$P_{R2} = i^2 R \text{ or } P_{R2} = v i = 2 \times 1 = +2W$$

$$P_{2\Omega} = i^2 R = 3^2 \times 2 = 9 \times 2 = 18W$$

$$P_{4V} = v i = -4 \times 3 = -12W$$

$$P_{4A} + P_{1\Omega} + P_{R2} + P_{2\Omega} + P_{4V} = 0 \rightarrow \textcircled{1}$$

$$P_{4A} + 16 + 2 + 18 + (-12) = 0$$

$$P_{4A} = -24W$$

Substitute the value $P_{4A} = -24W$ in eq $\textcircled{1}$

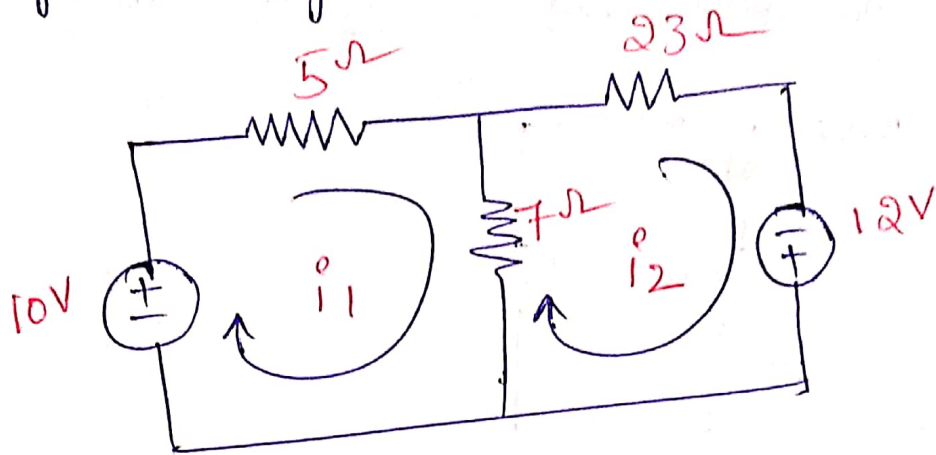
$$-24 + 16 + 2 + 18 - 12 = 0$$

$$-36 + 36 = 0$$

$$-36 = -36$$

$$36 = 36$$

* Verify the Tellegen's theorem for the following circuit.



Sol Apply KVL to loop ①

$$-10 + 5i_1 + 7(i_1 - i_2) = 0$$

$$-10 + 12i_1 - 7i_2 = 0$$

$$12i_1 - 7i_2 = 10 \rightarrow \textcircled{1}$$

Apply KVL to loop ②

$$7(i_2 - i_1) + 23i_2 - 12 = 0$$

$$-7i_1 + 30i_2 = 12 \rightarrow \textcircled{2}$$

Solving the equations $\textcircled{1}$ & $\textcircled{2}$

$$\begin{bmatrix} 12 & -7 \\ -7 & 30 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 12 \end{bmatrix}$$

$$\Delta = ad - bc = 360 - 49 = 311$$

$$i_1 = \frac{\Delta_1}{\Delta}$$

$$\Delta_1 = \begin{vmatrix} 10 & -7 \\ 12 & 30 \end{vmatrix} = ad - bc = 300 + 84 = 384$$

$$i_1 = \frac{384}{311} = 1.236 \text{ A}$$

$$i_2 = \frac{\Delta_2}{\Delta}$$

$$\Delta_2 = \begin{vmatrix} 12 & 10 \\ -7 & 12 \end{vmatrix} = 144 + 70 = 214$$

$$i_2 = \frac{214}{311} = 0.688 \text{ A}$$

$$I_{7\Omega} = I_1 - I_2 = 1.234 - 0.688 = 0.542 \text{ A}$$

$$P_{7\Omega} = I^2 R = (0.542)^2 \times 7 = 2.05 \text{ W}$$

$$P_{5\Omega} = I^2 R = (1.234)^2 \times 5 = 7.56 \text{ W}$$

$$P_{23\Omega} = I^2 R = (0.688)^2 \times 23 = 10.886 \text{ W}$$

$$P_{12V} = 12 \times I_2 = 12 \times 0.688 = 8.256 \text{ W}$$

$$P_{10V} = 10 \times I_1 = 10 \times 1.234 = 12.34 \text{ W}$$

$$\Sigma P_{\text{supplied}} = \Sigma P_{\text{absorbed}}$$

$$P_{12V} + P_{10V} = P_{5\Omega} + P_{7\Omega} + P_{23\Omega}$$

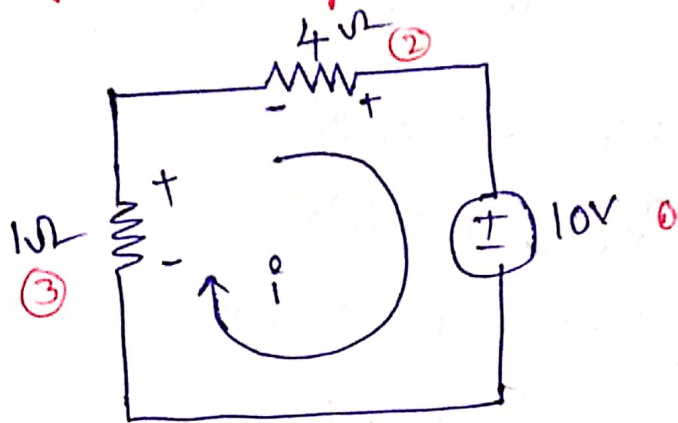
$$8.256 + 12.34 = 7.56 + 2.05 + 10.886$$

$$20.596 = 20.5$$

Statement of Tellegen's theorem :-

In any linear, non linear, active and passive network the summation of power is zero. i.e. sum of powers delivered to the network by an independent source is equals to the sum of power absorbed by all passive elements of the network.

* verify the Tellegen's theorem:



so) Apply KVL to the loop $i_1 = i_2 = i_3 = i$

$$\begin{aligned} -i - 4i + 10 &= 0 \\ -5i &= -10 \\ i &= 2 \text{ A} \end{aligned}$$

$$\begin{aligned} v_1 &= 10 \text{ V} \\ v_2 &= -4 \times 2 = -8 \\ v_3 &= -1 \times 2 = -2 \end{aligned}$$

$$\sum_{k=1}^b v_k i_k = 0$$

$$\sum_{k=1}^3 v_k i_k = 0$$

$$v_1 i_1 + v_2 i_2 + v_3 i_3 = 0$$

$$10 \times 2 + (-8) \times 2 + (-2) \times 2 = 0$$

$$20 - 16 - 4 = 0$$

$$20 - 20 = 0$$

$$20 = 20$$

Superposition Theorem :-

* The Superposition principle states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or current through) that element due to each independent source acting alone.

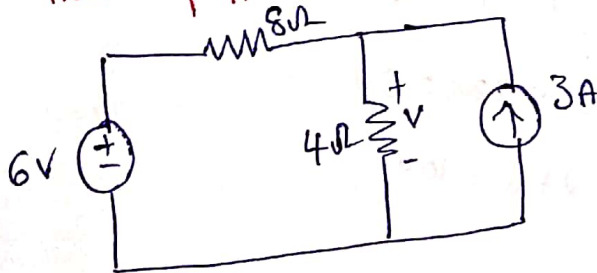
Definition :-

In a linear network with several independent sources, the response in a particular branch when all the sources are acting simultaneously is equal to the linear sum of individual responses calculated by taking one independent source at a time.

Steps to apply superposition principle:

- ① → Turn off all independent sources except one source. Find the output (voltage or current) due to that active source.
- ② → Repeat step ① for each of the other independent sources.
- ③ → Find the total contribution by adding algebraically all the contributions due to the independent sources.

* Use the superposition theorem to find V in the ckt.



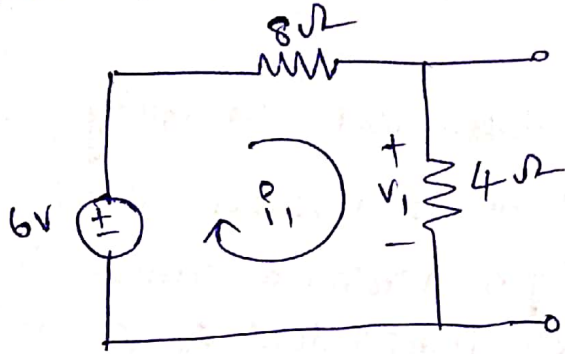
Sol

Since there are two sources, let

$$V = V_1 + V_2$$

where V_1 and V_2 are contributions due to the 6V voltage source and 3A current source.

To obtain v_1 , we ~~set~~ set the current source to zero



Applying KVL to the loop.

$$8i_1 + 4i_1 - 6 = 0$$

$$12i_1 = 6$$

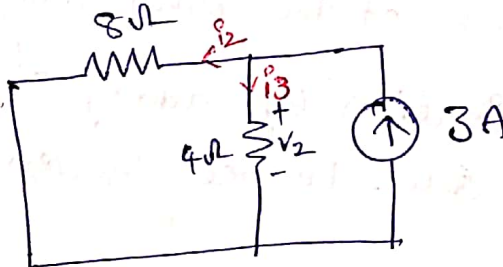
$$i_1 = 0.5 \text{ A}$$

$$\therefore v_1 = 4i_1 = 4 \times 0.5 = 2 \text{ V}$$

we may use voltage division to get v_1

$$v_1 = \frac{4}{8+4} \cdot 6 = 2 \text{ V}$$

To get v_2 , we set the voltage source to zero.



Using current division

$$i_3 = \frac{8}{4+8} (3) = 2 \text{ A}$$

$$v_2 = i_3 \times 4 = 4 \times 2 = 8 \text{ V}$$

$$v = v_1 + v_2 = 2 + 8 = 10 \text{ V}$$

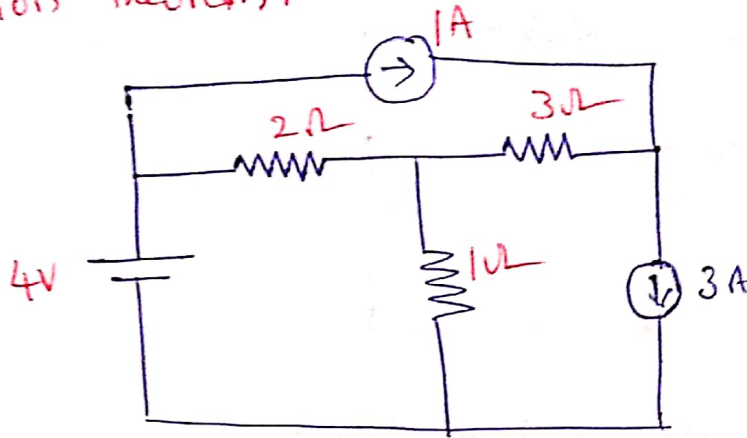
Note :-

Impedance :- (Z)

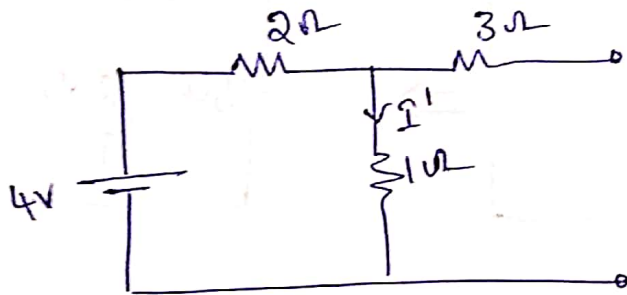
Impedance is a measure of the opposition to electrical flow. It is represented by Z . It is measured in ohms.

- Ideal voltage source has zero internal impedance.
- Ideal current source has infinite internal impedance.

* find the current in the 1Ω resistor using superposition theorem.

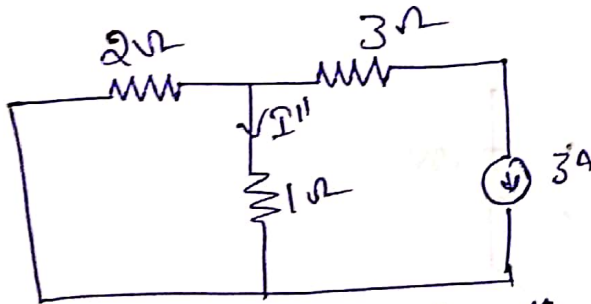


Sol when the 4V source is acting alone,



$$I' = \frac{V}{R} = \frac{4}{2+1} = 1.33 \text{ A}$$

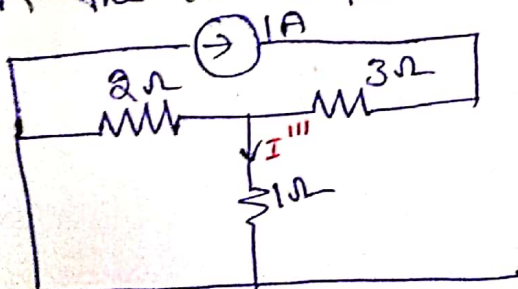
when the 3A source is acting alone



By current division formula

$$I'' = 3 \times \frac{2}{1+2} = 2 \text{ A}$$

when the 1A source is acting alone



By current division rule

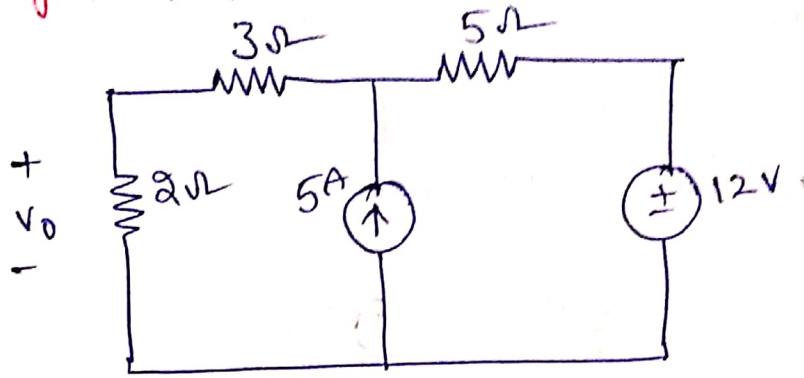
$$I''' = 1 \times \frac{2}{2+1} = 0.66 \text{ A}$$

By superposition theorem

$$I = I' + I'' + I'''$$

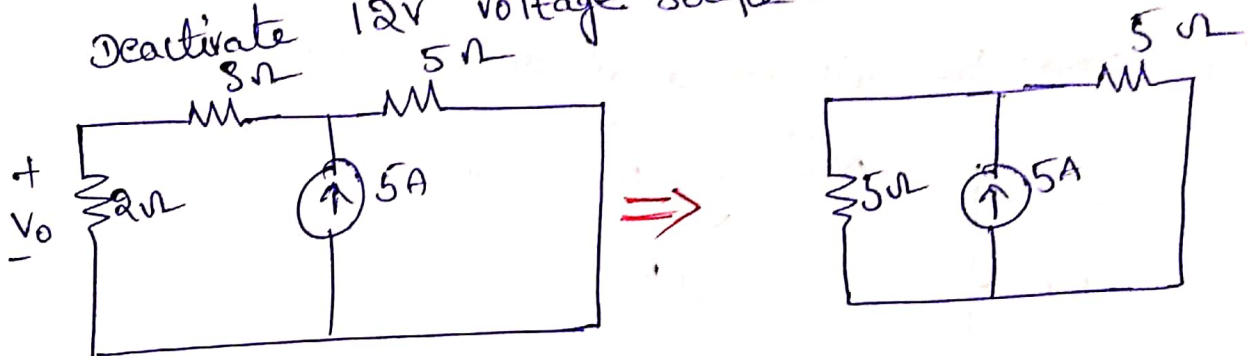
$$I = 1.33 + 2 + 0.66 = \underline{\underline{4 \text{ A}}}$$

* Using superposition theorem find V_0 in the Ckt.



Sol

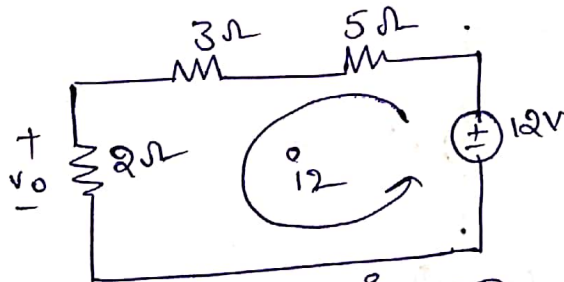
Deactivate 12V voltage source.



$$i_1 = \left(\frac{R_2}{R_1 + R_2} \right) i = \left(\frac{5}{5+5} \right) 5 = 2.5$$

$$V_1 = i_1 R_1 = 2.5 \times 2 = 5 \text{ V}$$

Deactivate 5A current source.



$$-12 + 10 i_2 = 0$$

$$10 i_2 = 12$$

$$i_2 = 1.2$$

$$V_2 = i_2 \times R = 1.2 \times 2 = 2.4$$

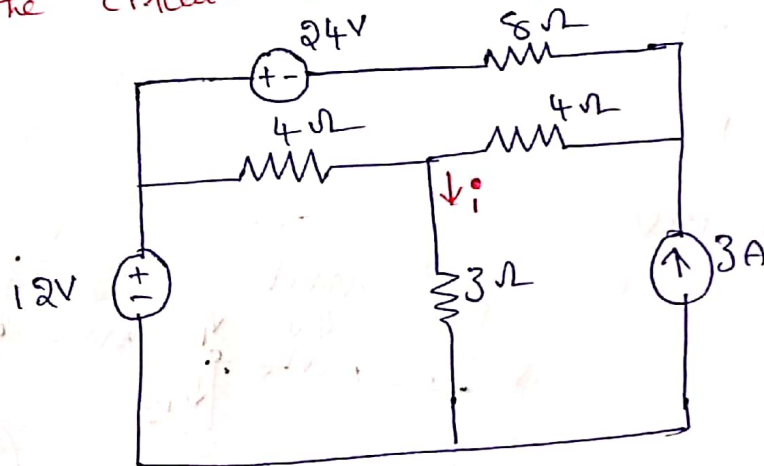
$$V = V_1 + V_2 = 5 + 2.4 = 7.4 \text{ V}$$

Linear Circuit :-

A linear circuit is one in which the electronic components' values (such as resistance, capacitance, inductance) do not change with the level of voltage or current in the circuit.

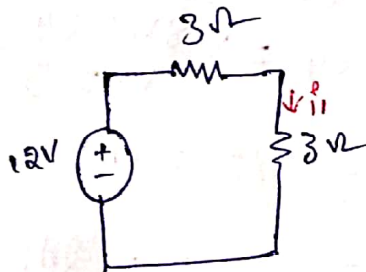
Linear circuits are important because they can amplify and process electronic signals without distortion.

For the circuit use the superposition theorem to find i .



Sol Turn off 24V & 3A sources.

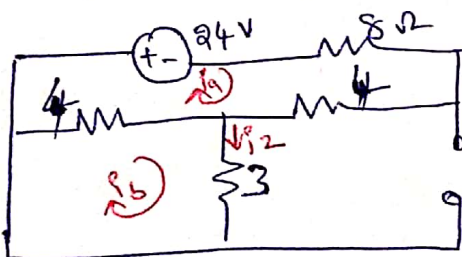
$$4 \parallel (4+8) = \frac{4 \times 12}{4+12} = \frac{48}{16} = 3\Omega$$



$$R = 3 + 3 = 6\Omega$$

$$i_1 = \frac{V}{R} = \frac{12}{6} = 2A$$

Turn off 12V & 3A



Apply mesh analysis to loop i_a

$$24 + 8i_a + 4i_a + 4(i_a - i_b) = 0$$

$$24 + 16i_a - 4i_b = 0$$

$$16i_a - 4i_b = -24$$

$$4i_a - i_b = -6 \rightarrow \textcircled{1}$$

Apply mesh analysis to loop i_b

$$4(i_b - i_a) + 3i_b = 0$$

$$7i_b - 4i_a = 0$$

$$-4i_a + 7i_b = 0 \rightarrow \textcircled{2}$$

Adding $\textcircled{1}$ & $\textcircled{2}$

$$\begin{array}{rcl} 4i_a - i_b & = & -6 \\ -4i_a + 7i_b & = & 0 \\ \hline \end{array}$$

$$6i_b = -6$$

$$i_b = -1$$

$$i_2 = i_b = -1$$

Turning off 24V & 12V

At node v_2

$$\frac{v_2}{8} + \frac{v_2 - v_1}{4} = 3$$

$$\frac{v_2 + 2v_2 - 2v_1}{8} = 3$$

$$3v_2 - 2v_1 = 24 \rightarrow \textcircled{1}$$

At node v_1

$$\frac{v_1}{3} + \frac{v_1}{4} = \frac{v_2 - v_1}{4}$$

$$\frac{4v_1 + 3v_1}{12} = \frac{v_2 - v_1}{4}$$

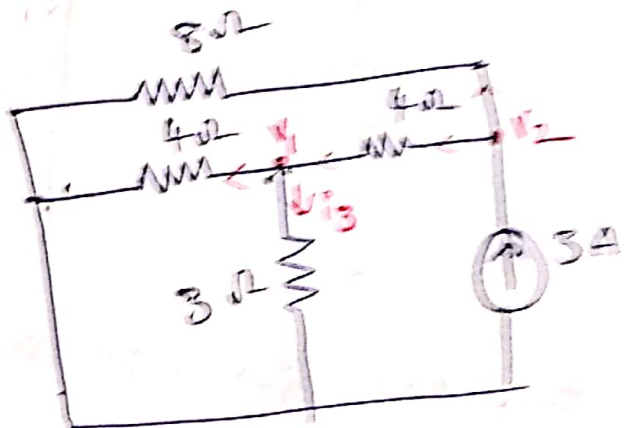
$$4v_1 + 3v_1 = 3v_2 - 3v_1$$

$$4v_1 + 3v_1 - 3v_2 + 3v_1 = 0$$

$$10v_1 - 3v_2 = 0$$

$$-3v_2 = -10v_1$$

$$v_2 = \frac{+10v_1}{3}$$



Substitute v_2 in $\textcircled{1}$

$$3\left(\frac{10v_1}{3}\right) - 2v_1 = 24$$

$$10v_1 - 2v_1 = 24 \Rightarrow 8v_1 = 24$$

$$i_3 = \frac{v_1}{3} = \frac{3}{3} = 1A$$

$$i = i_1 + i_2 + i_3 = 2 - 1 + 1 = 2A$$

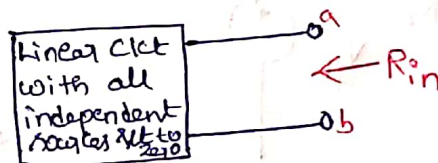
Theremin's Theorem :-

Theremin's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th} , where V_{Th} is the open circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals when the independent sources are turned off.

Steps to apply Theremin's theorem.

Case (i) :-

If the network has no dependent sources, we turn off all independent sources. R_{Th} is the input resistance of the network looking between terminals a & b.

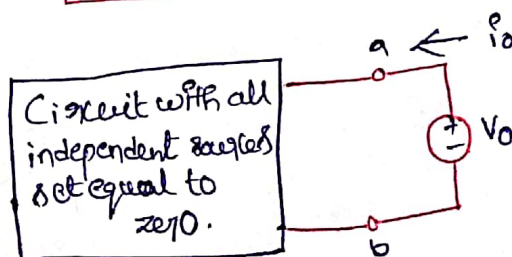


$$R_{Th} = R_{in}$$

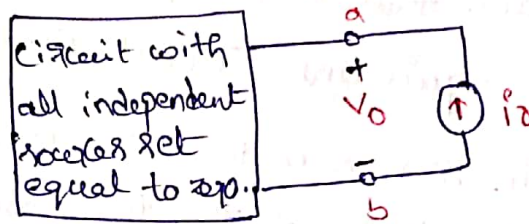
Case (ii) :-

If the network has dependent sources, we turn off all independent sources. As with superposition, dependent sources are not to be turned off because they are controlled by circuit variables. we apply a voltage source V_0 at terminals a & b and determine the resulting current i_0 . Then

$$R_{Th} = \frac{V_0}{i_0}$$

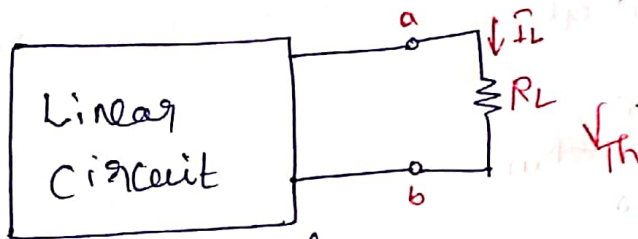


We may insert a current source i_0 at terminals a-b, and find the terminal voltage V_0 . Again $R_{Th} = \frac{V_0}{i_0}$.

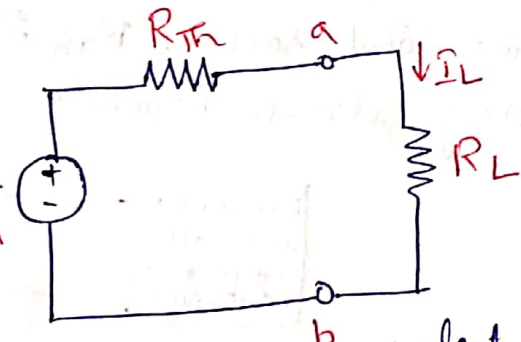


$$R_{Th} = \frac{V_0}{i_0}$$

A linear circuit terminated by a load R_L , the current I_L through the load and the voltage V_L across the load are easily determined once the thevenin equivalent of the circuit at the load's terminals is obtained.



A circuit with a load original ckt

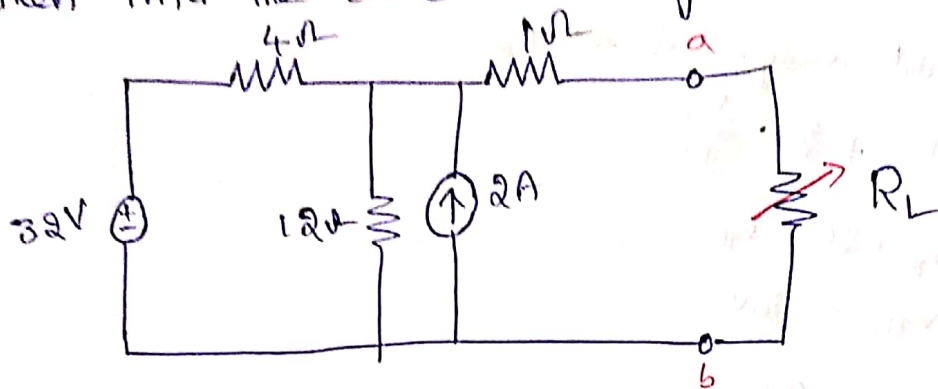


b) thevenin equivalent ckt.

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

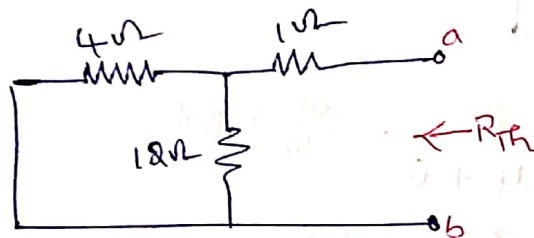
$$V_L = R_L I_L = \frac{R_L}{R_{Th} + R_L} \cdot V_{Th}$$

① find the Thevenin equivalent circuit of the ckt as shown in below fig. to the left of the terminals a-b. Then find the current through $R_L = 6, 16, 36 \Omega$.



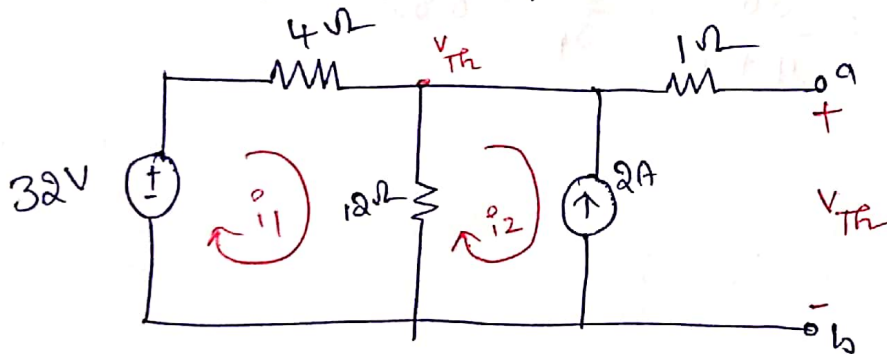
Sol

To obtain R_{Th} turning off the 32V voltage source, 2A current source.



$$\begin{aligned} R_{Th} &= 4 \parallel 12 + 1 \\ &= \frac{4 \times 12}{4 + 12} + 1 \\ &= \frac{48}{16} + 1 = 4 \Omega \end{aligned}$$

To find V_{Th} , Applying mesh analysis to the two loops.



$$\begin{aligned} \text{Loop ①} \Rightarrow -32 + 4i_1 + 12(i_1 - i_2) &= 0 \\ i_2 &= -2A \end{aligned}$$

$$V_{Th} = 12(i_1 - i_2)$$

$$-32 + 4i_1 + 12(i_1 - (-2)) = 0$$

$$-32 + 4i_1 + 12i_1 + 24 = 0$$

$$16i_1 - 8 = 0$$

$$i_1 = 8/16 = 0.5A$$

$$V_{Th} = 12(I_1 - I_2) = 12(0.5 + 2.0) = 30V$$

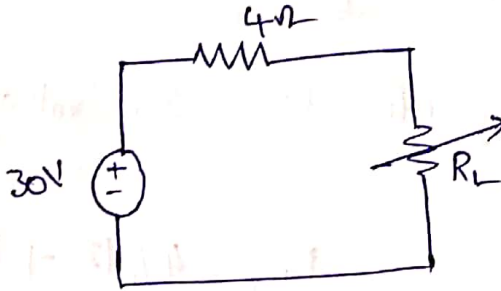
Alternative method:

Apply nodal analysis at node V_{Th}

$$\frac{32 - V_{Th}}{4} + 2 = \frac{V_{Th}}{12}$$

$$36 - 3V_{Th} + 24 = V_{Th}$$

$$V_{Th} = 30V$$

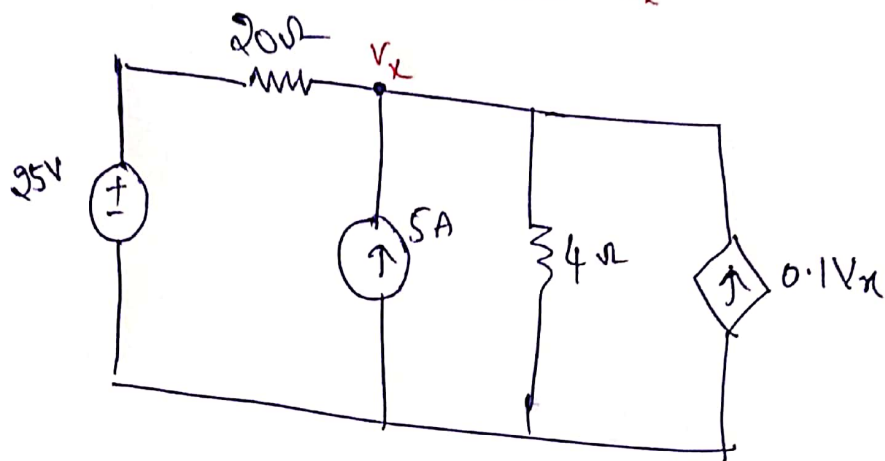


$$R_L = 6 \Rightarrow I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{30}{4 + 6} = \frac{30}{10} = 3A$$

$$R_L = 16\Omega \Rightarrow I_L = \frac{30}{4 + 16} = \frac{30}{20} = 1.5A$$

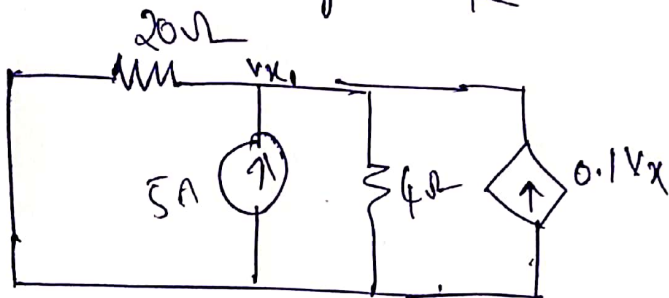
$$R_L = 30\Omega \Rightarrow I_L = \frac{30}{4 + 30} = \frac{30}{34} = 0.88A$$

Use superposition find V_x



Sol

Deactivate voltage source



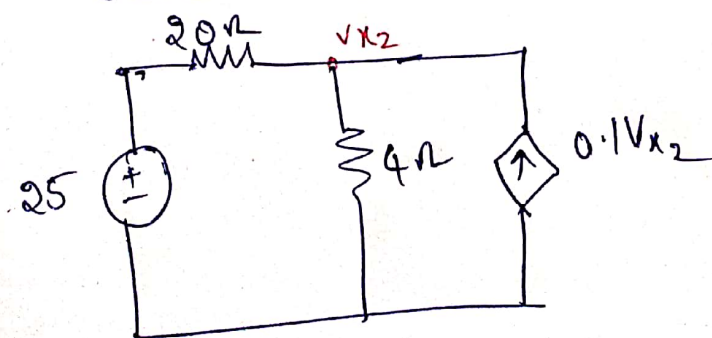
$$\frac{V_{x1}}{20} + \frac{V_{x1}}{4} = 5 + \frac{1}{10} V_{x1}$$

$$\frac{V_{x1} + 5V_{x1}}{20} = \frac{50 + V_{x1}}{10}$$

$$6V_{x1} = 100 + 2V_{x1}$$

$$4V_{x1} = 100 \Rightarrow V_{x1} = \frac{100}{4} = 25$$

Deactivate current source



$$\frac{V_{x2} - 25}{20} + \frac{V_{x2}}{4} = \frac{1}{10} V_{x2}$$

$$\frac{V_{x2} - 25 + 5V_{x2}}{20} = \frac{V_{x2}}{10}$$

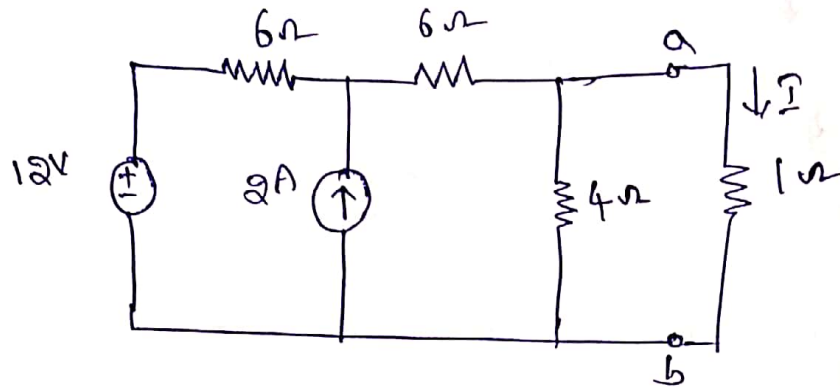
$$6V_{x2} - 25 = 2V_{x2}$$

$$V_{x2} = \frac{25}{4} = 6.25$$

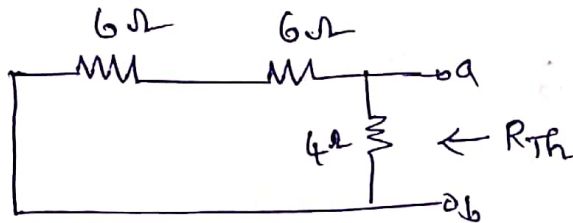
$$V_x = V_{x1} + V_{x2} = 25 + 6.25$$

$$V_x = 31.25$$

Using thevenin's theorem, find the equivalent ckt to the left of the terminals in the ckt. Then find I .

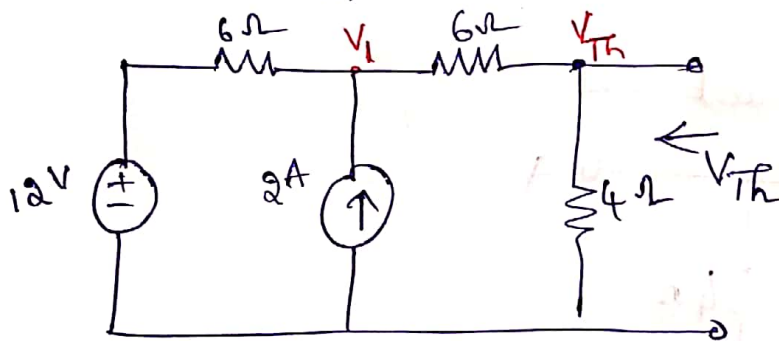


Sol To find R_{Th} turning off the voltage source and 2A current source.



$$R_{Th} = 12\Omega \parallel 4\Omega = \frac{12 \times 4}{12 + 4} = \frac{48}{16} = 3\Omega$$

To find V_{Th} consider the ckt



KCL at node V_{Th}

$$\frac{V_{Th}}{4} + \frac{V_{Th} - V_1}{6} = 0$$

$$\frac{3V_{Th} + 2V_{Th} - 2V_1}{12} = 0$$

$$5V_{Th} = 2V_1 \Rightarrow V_{Th} = \frac{2V_1}{5}$$

KCL at node V_1

$$\frac{V_1 - 12}{6} + \frac{V_1 - V_{Th}}{6} = 2$$

$$V_1 - 12 + V_1 - V_{Th} = 12$$

$$2V_1 - V_{Th} = 12 + 12$$

$$2V_1 - \frac{2V_1}{5} = 24$$

$$10V_1 - 2V_1 = 120$$

$$8V_1 = 120$$

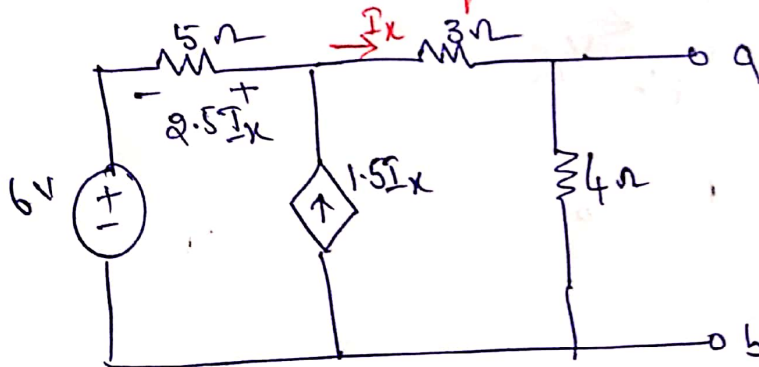
$$V_1 = \frac{120}{8} = 15V$$

$$V_{Th} = \frac{2V_1}{5} = 2 \times \frac{15}{5} = 6$$

$$V_{Th} = 6V$$

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{6}{3 + 1} = \frac{6}{4} = 1.5A$$

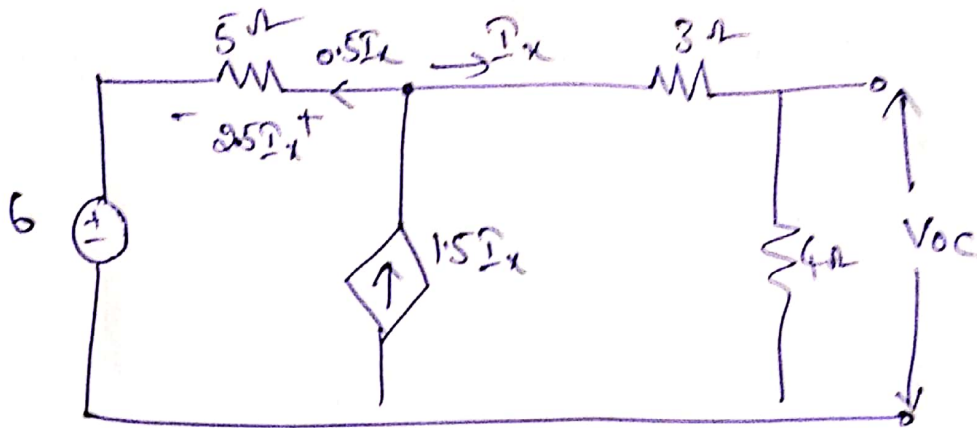
find the Thevenin equivalent Ckt.



Sol

$$R_{Th} = \frac{V_{oc}}{I_{sc}}$$

$$I_{sc} = I_x$$



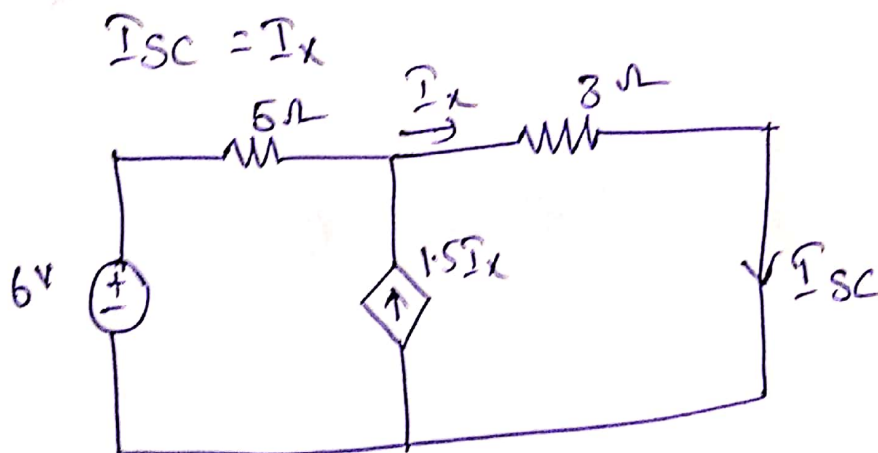
$$-6 - 2.5I_x + 7I_x = 0$$

$$4.5I_x = 6$$

$$I_x = \frac{6 \times 101}{4.5}$$

$$I_x = \frac{6}{4.5} = 1.333$$

$$V_{OC} = 4 \times I_x = 4 \times 1.333 = 5.33$$



$$I_{SC} = -6 - 2.5I_x + 3I_x = 0$$

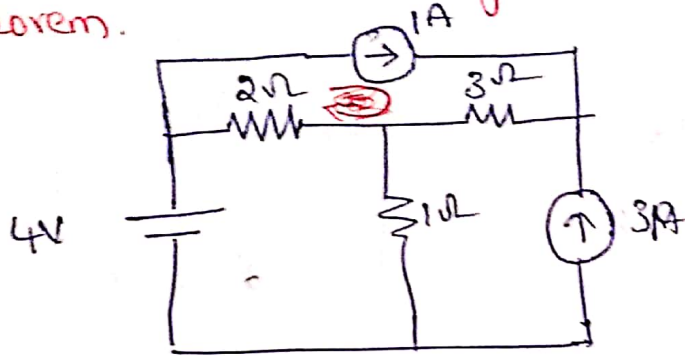
$$0.5I_x = 6$$

$$I_x = 6 / 0.5 = 12A$$

$$R_{TH} = \frac{V_{OC}}{I_{SC}} = \frac{5.33}{12} = 0.444$$

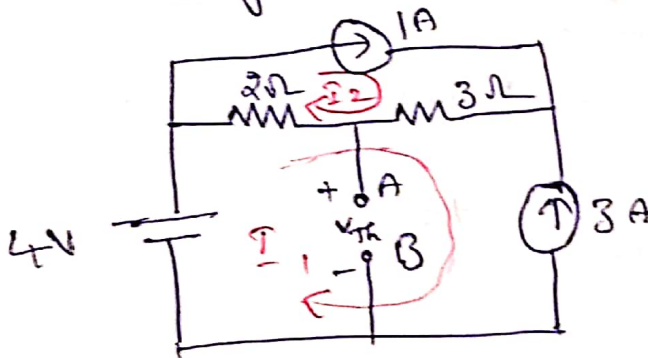
$$R_{TH} = 444.4 \text{ m}\Omega$$

find the current through the 1Ω resistor using Thevenin's theorem.



Sol Step I :- Calculation of V_{Th}

Removing the 1Ω resistor from the network



$$I_1 = -3A$$

$$I_2 = 1A$$

Writing the current equation for mesh 1 & 2

$$-4 + 2(I_1 - I_2) + V_{Th} = 0$$

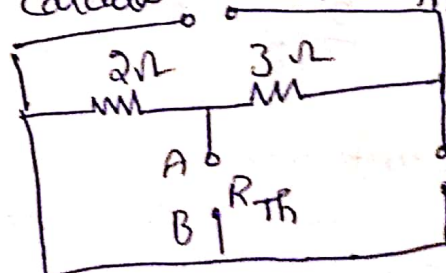
$$-4 + 2(-3 - 1) + V_{Th} = 0$$

$$-4 + 2(-4) + V_{Th} = 0$$

$$-4 - 8 + V_{Th} = 0$$

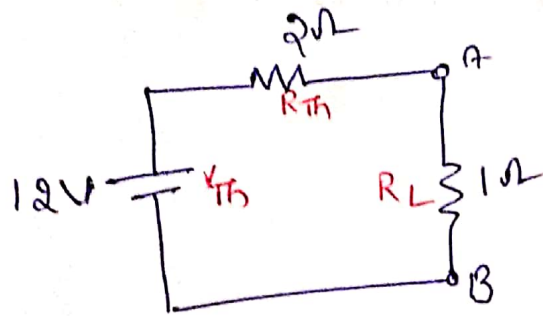
$$V_{Th} = 12V$$

Step II :- Calculation of R_{Th}



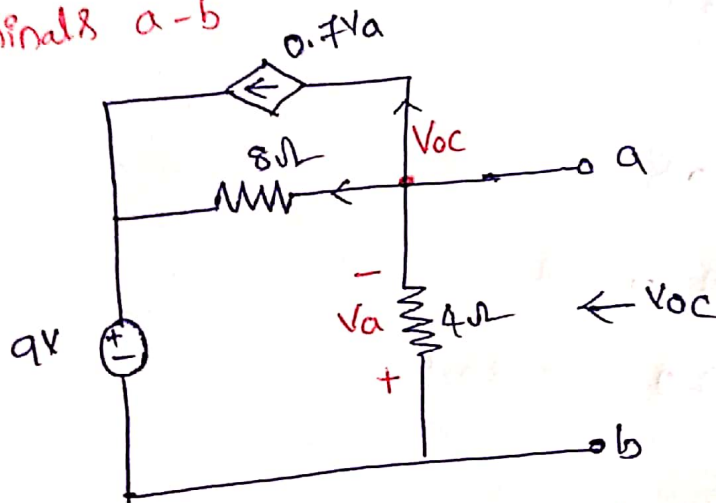
$$R_{Th} = 2\Omega$$

Step III Calculation of I_L



$$I_L = \frac{V_{TH}}{R_{TH} + R_L} = \frac{12}{2+1} = 4A$$

In the following circuit find the value of R_{TH} at the terminals a-b



$$V_{OC} = -V_a$$

Apply KCL at node V_{OC}

$$\frac{V_{OC}}{4} + \frac{V_{OC} - 9}{8} + 0.75V_a = 0$$

$$\frac{V_{OC}}{4} + \frac{V_{OC} - 9}{8} - 0.75V_{OC} = 0$$

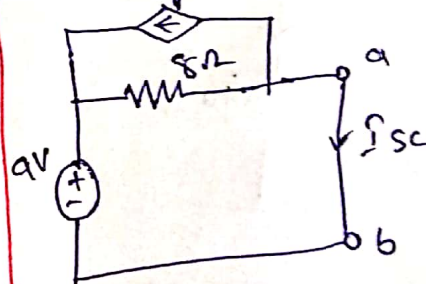
$$\frac{2V_{OC} + V_{OC} - 9 - 6V_{OC}}{8} = 0$$

$$2V_{OC} + V_{OC} - 9 - 6V_{OC} = 0$$

$$-3V_{OC} = 9$$

$$V_{OC} = -3V$$

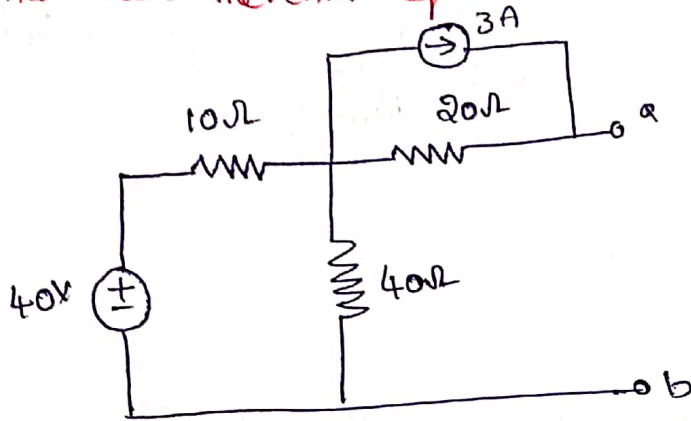
finding short circuit current I_{SC}



$$I_{SC} = \frac{V}{R} = \frac{9}{8} = 1.125A$$

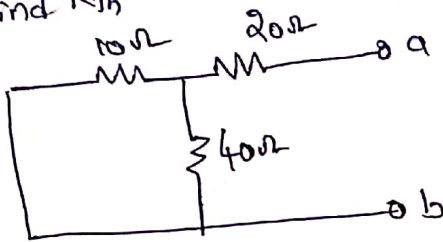
$$R_{TH} = \frac{V_{OC}}{I_{SC}} = \frac{-3}{1.125A} = -2.66\Omega$$

* find the Thevenin equivalent at terminals a-b of the ckt.



Sol

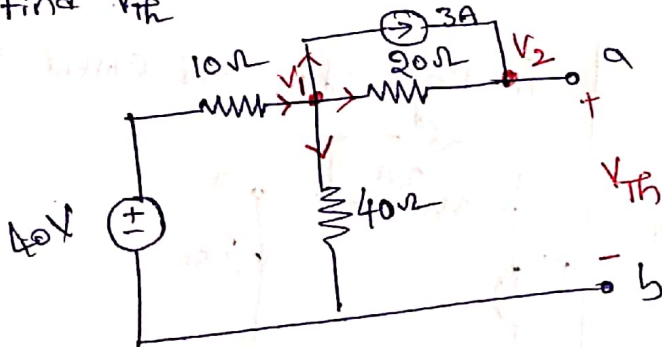
To find R_{Th}



$$R_{Th} = 10\Omega // 40\Omega + 20\Omega$$

$$= \frac{10 \times 40}{10 + 40} + 20 = 28\Omega$$

To find V_{Th}



Apply KCL at node ①

$$\frac{40 - V_1}{10} = 3 + \frac{V_1 - V_2}{20} + \frac{V_1}{40}$$

$$40 = 7V_1 - 2V_2 \rightarrow \textcircled{1}$$

Apply KCL at node ②

$$3 + \frac{V_1 - V_2}{20} = 0$$

$$60 + V_1 - V_2 = 0$$

$$V_1 - V_2 = -60$$

$$V_1 = V_2 - 60 \rightarrow \textcircled{2}$$

Substitute ② in eq ①

$$40 = 7(V_2 - 60) - 2V_2$$

$$40 = 7V_2 - 420 - 2V_2$$

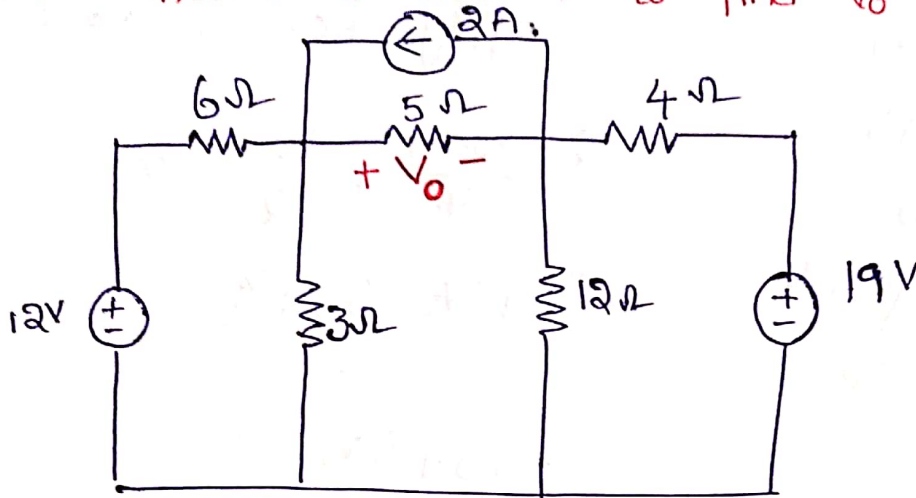
$$460 = 5V_2$$

$$V_2 = \frac{460}{5}$$

$$V_2 = 92V$$

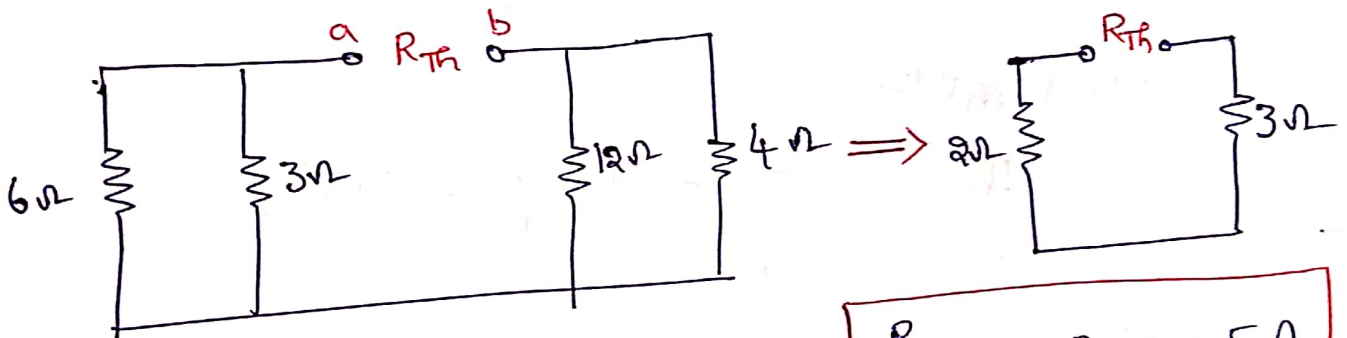
$$\therefore V_2 = V_{Th} = 92V$$

* Use Thevenin's theorem to find V_o



Sol

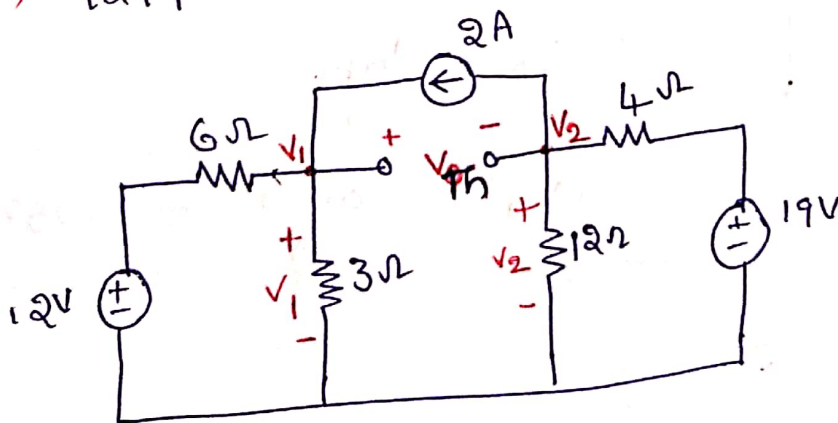
To find R_{Th}



$$R_{Th} = 2 + 3 = 5\Omega$$

$$6//3 \Rightarrow \frac{6 \times 3}{6 + 3} = \frac{18}{9} = 2\Omega$$

$$12//4 \Rightarrow \frac{12 \times 4}{12 + 4} = \frac{48}{16} = 3\Omega$$



Apply KCL at node V_1

$$2 = \frac{V_1 - 12}{6} + \frac{V_1}{3}$$

$$2 = \frac{V_1 - 12 + 2V_1}{6}$$

$$12 = 3V_1 - 12$$

$$3V_1 = 24$$

$$V_1 = 8V$$

$$\begin{aligned}
 24 - V_1 &= V_1 \\
 24 - V_1 &= 2V_1 \\
 24 - 3V_1 &= 0 \\
 -3V_1 &= -24 \\
 V_1 &= 8V
 \end{aligned}$$

Apply KCL at node V_2

$$2 + \frac{V_2 - 19}{4} + \frac{V_2}{12} = 0$$

$$\frac{24 + 3V_2 - 57 + V_2}{12} = 0$$

$$-33 + 4V_2 = 0$$

$$4V_2 = 33$$

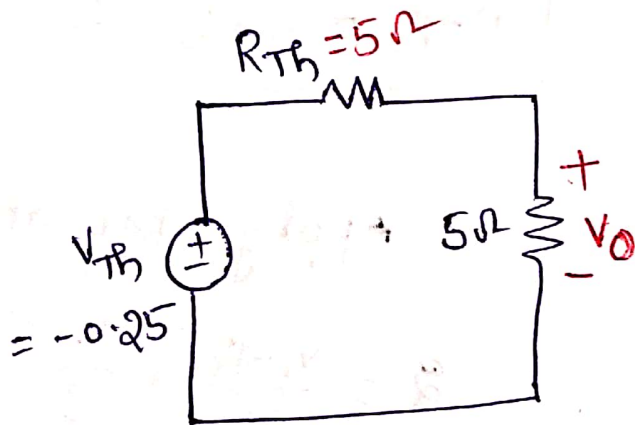
$$V_2 = 33/4$$

Apply KVL to the loop

$$-V_1 + V_{TH} + V_2 = 0$$

$$V_{TH} = V_1 - V_2 = 8 - \frac{33}{4} = -0.25$$

$$V_{TH} = -0.25$$



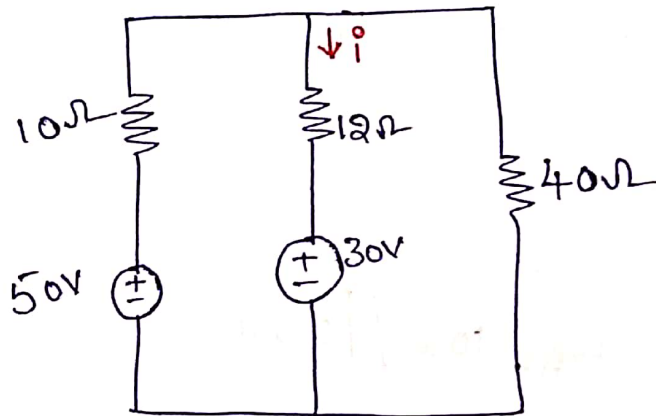
voltage division rule

$$V_0 = \frac{V_{TH} \times 5}{5 + 5} = \frac{V_{TH} \cdot 5}{10}$$

$$V_0 = \frac{V_{TH}}{2}$$

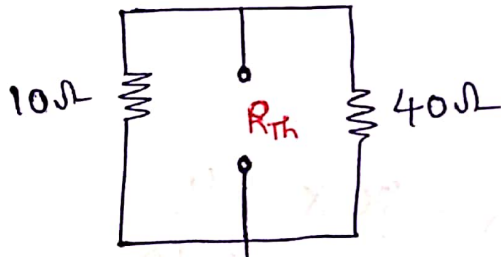
$$V_0 = \frac{-0.25}{2} = -125\text{mV}$$

* Use Thevenin's theorem find the current i .



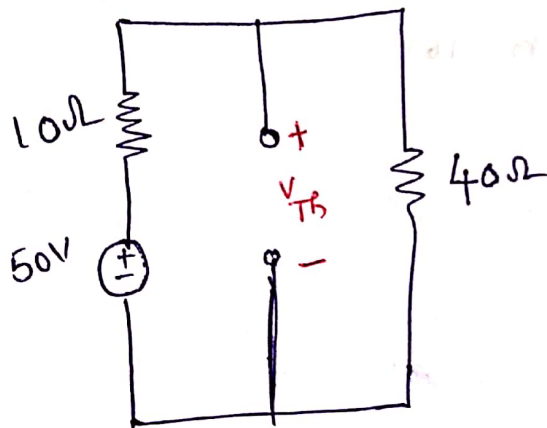
Sol

To find R_{Th} remove the 30V voltage source & 50V voltage source



$$R_{Th} = 10\Omega // 40\Omega$$

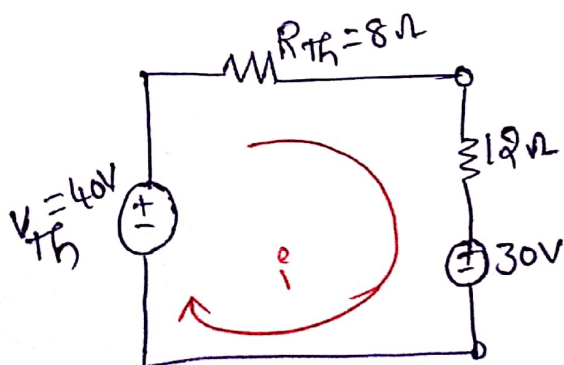
$$= \frac{10 \times 40}{10 + 40} = \frac{400}{50} = 8\Omega$$



$$V_{Th} = \frac{40}{10 + 40} \times 50$$

$$= \frac{40}{50} \times 50$$

$$V_{Th} = 40V$$



Apply KVL

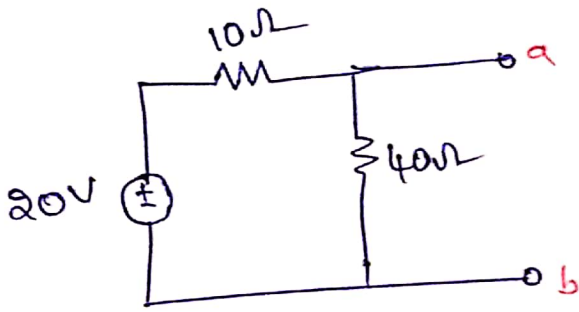
$$-40 + 8i + 12i + 30 = 0$$

$$-40 + 20i + 30 = 0$$

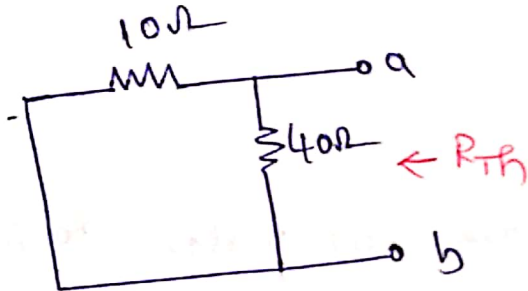
$$20i = 10$$

$$i = 10/20 = 0.5A$$

* Determine R_{TH} & V_{TH} at terminal a-b.



Sol
To find R_{TH}

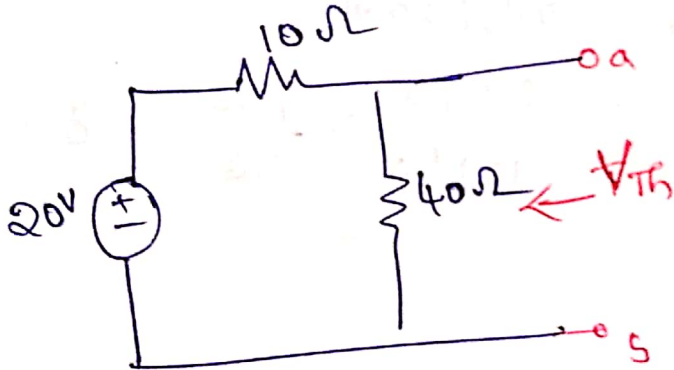


$$R_{TH} = 10\Omega \parallel 40\Omega$$

$$R_{TH} = \frac{10 \times 40}{10 + 40} = \frac{400}{50}$$

$$R_{TH} = 8\Omega$$

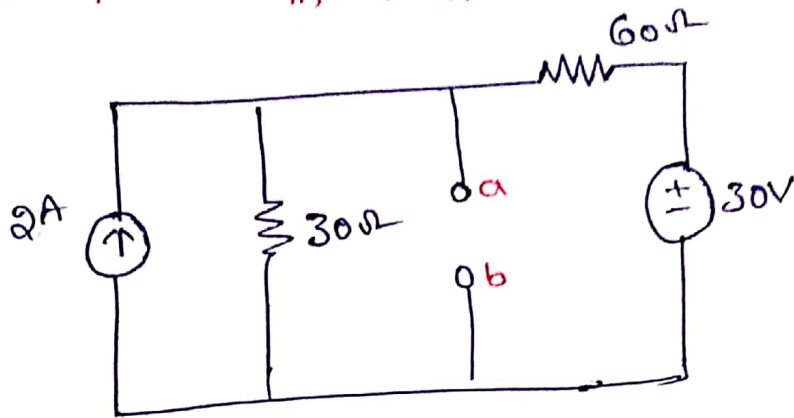
To find V_{TH}



$$V_{TH} = 20 \times \frac{40}{40 + 10}$$
$$= 20 \times \frac{40}{50}$$

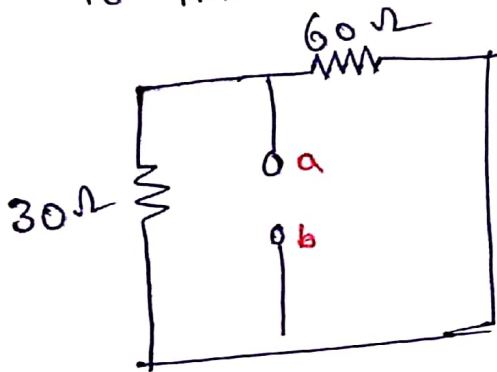
$$V_{TH} = 16V$$

* Determine R_{TH} & V_{TH} at terminals a-b



Sol

To find R_{TH}

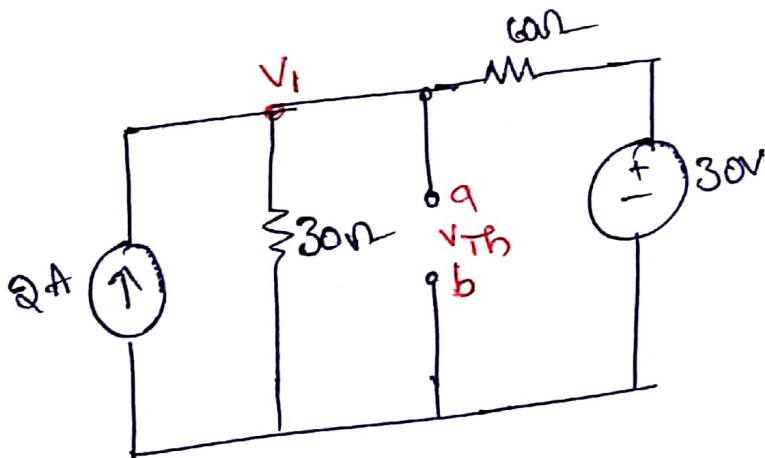


$$R_{TH} = 30\Omega // 60\Omega$$

$$R_{TH} = \frac{30 \times 60}{30 + 60} = \frac{1800}{90} = 20\Omega$$

$$R_{TH} = 20\Omega$$

To find V_{TH}



$$V_1 = V_{TH}$$

Apply KCL at node V_1

$$2 = \frac{V_1 - 30}{60} + \frac{V_1}{30}$$

$$2 = \frac{V_1 - 30 + 2V_1}{60}$$

$$120 = 3V_1 - 30$$

$$150 = 3V_1$$

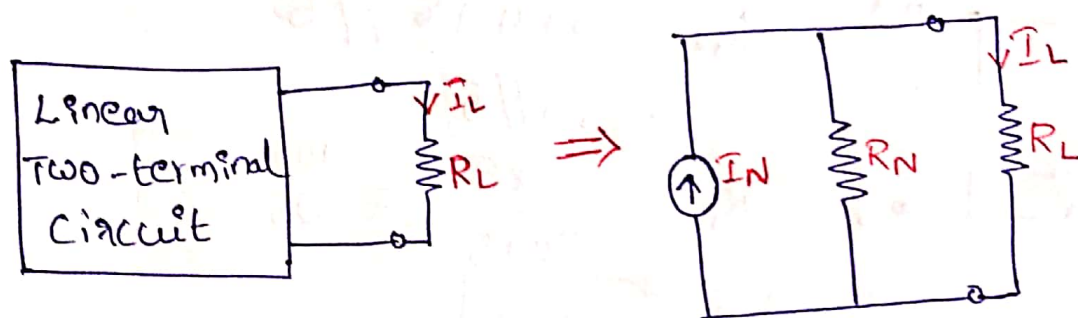
$$V_1 = 50$$

$$\therefore V_1 = V_{TH}$$

$$V_{TH} = 50V$$

Norton's Theorem :-

Norton's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N , where I_N is the short circuit current through the load terminals and R_N is the input or equivalent resistance at the load terminals when the independent sources are turned off.



→ Thevenin's and Norton's resistances are equal i.e.

$$R_N = R_{Th}$$

→ To find Norton's current I_N we determine the I_{sc} flowing through the terminals.

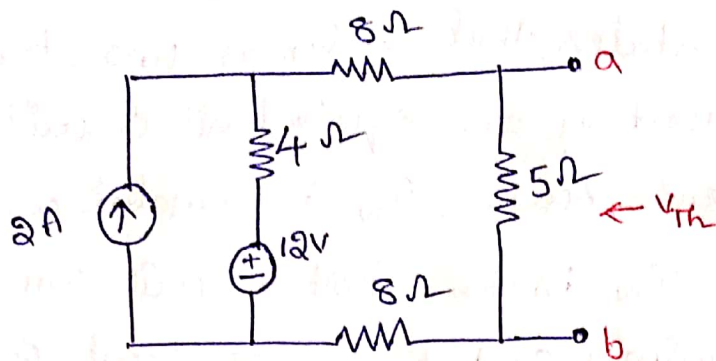
$$I_N = I_{sc}$$

→ Dependent and independent sources are treated as the same way in Thevenin's theorem.

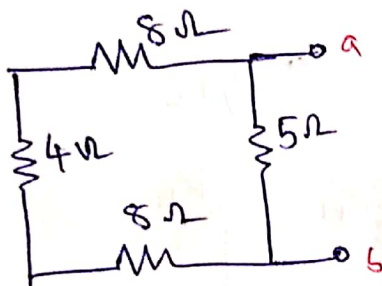
→ The close relationship b/w Thevenin's and Norton's theorem is

$$R_N = R_{Th}$$
$$I_N = \frac{V_{Th}}{R_{Th}}$$

* find the Norton's equivalent circuit as shown in figure at terminals a-b.



Sol To find R_N turn off all independent sources

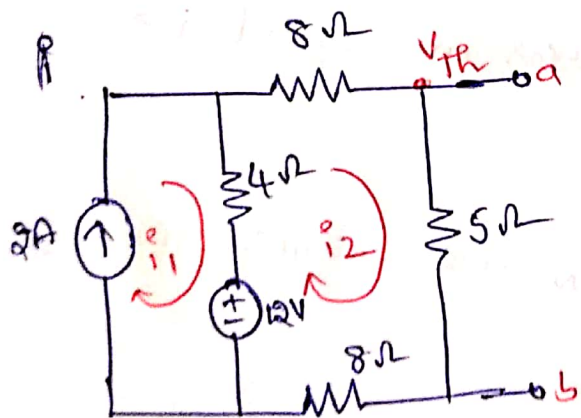


$$R_N = 8 + 4 + 8 \parallel 5$$

$$= 20 \parallel 5$$

$$R_N = \frac{20 \times 5}{20 + 5} = \frac{100}{25}$$

$$R_N = 4 \Omega$$



$$i_1 = 2A$$

Apply KVL to loop i_2

$$-12 + 4(i_2 - i_1) + 8i_2 + 5i_2 + 8i_2 = 0$$

$$-12 + 4i_2 - 4i_1 + 8i_2 + 5i_2 + 8i_2 = 0$$

$$-12 + 25i_2 - 4i_1 = 0$$

$$-12 + 25i_2 - 4 \times 2 = 0$$

$$-12 + 25i_2 - 8 = 0$$

$$25i_2 = 20$$

$$i_2 = \frac{20}{25} = \frac{4}{5} = 0.8A$$

$$i_2 = 0.8A$$

$$V_{Th} = 5 \text{ V}$$

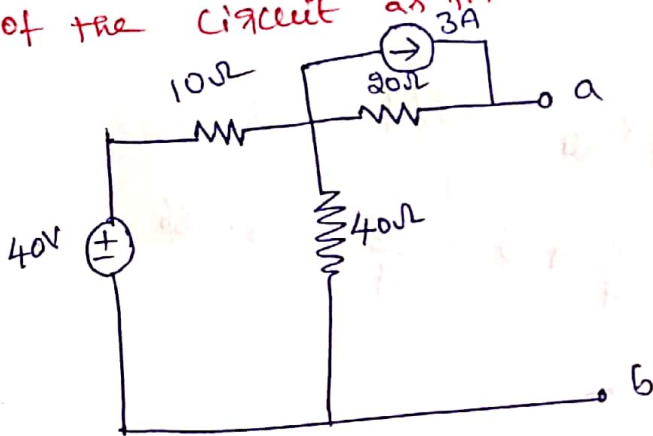
$$V_{Th} = 5 \times 0.8$$

$$V_{Th} = 4 \text{ V}$$

$$I_N = \frac{V_{Th}}{R_{Th}} = \frac{4}{4} = 1 \text{ A}$$

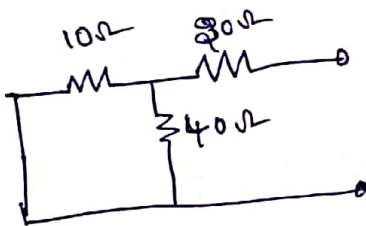
$$I_N = 1 \text{ A}$$

* find the Norton equivalent looking into terminals a-b of the circuit as shown in fig.

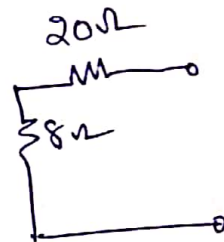


Sol

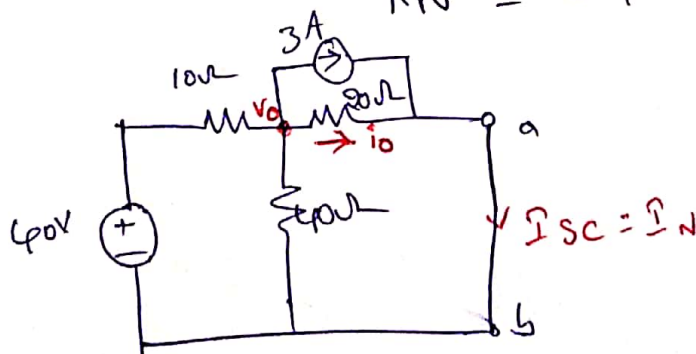
To find R_N



$$10\Omega // 40\Omega = \frac{10 \times 40}{10 + 40} = \frac{400}{50} = 8\Omega$$



$$R_N = 8 + 20 = 28\Omega$$



Apply KCL at node V_0

$$\frac{40 - V_0}{10} = 3 + \frac{V_0}{40} + \frac{V_0}{20}$$

$$\frac{40 - V_0}{10} = \frac{120 + V_0 + 2V_0}{40}$$

$$40 - V_0 = \frac{120 + 3V_0}{4}$$

$$160 - 4V_0 = 120 + 3V_0$$

$$-4V_0 - 3V_0 = 120 - 160$$

$$-7V_0 = -40$$

$$V_0 = 40/7$$

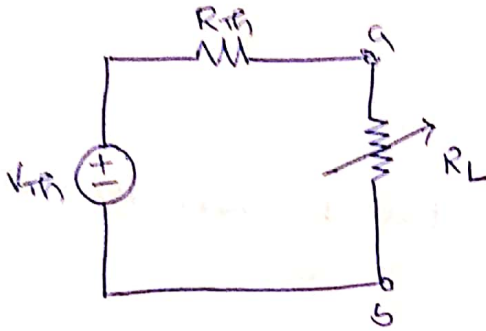
$$i_0 = \frac{V_0}{20} = \frac{\frac{40}{7}}{20} = \frac{40}{7} \times \frac{1}{20} = 2/7$$

$$I_N = I_{SC} = i_0 + 3 = \frac{2}{7} + 3 = \frac{23}{7} = 3.286$$

Maximum power transfer theorem :-

Maximum power is transferred to the load when the load resistance is equal to the Thevenin's resistance as seen from the load ($R_L = R_{Th}$).

This theorem is applicable only when load is variable. The Thevenin equivalent circuit is useful to find maximum power delivered to the load.



V_{Th} and R_{Th} are fixed. By varying the load resistance R_L , the power delivered to the load will also be varied.

$$P = V_L I_L$$

$$V_L = I_L R_L$$

$$P = I_L^2 R_L$$

$$P = \left(\frac{V_{Th}}{R_{Th} + R_L} \right)^2 R_L$$

To prove the maximum power transfer theorem we differentiate P with respect to R_L and set the result is equal to zero.

$$\frac{dP}{dR_L} = \frac{d}{dR_L} \left[\frac{V_{Th}}{R_{Th} + R_L} \right]^2 R_L = 0$$

$$= (V_{Th})^2 \frac{d}{dR_L} \left[\frac{R_L}{(R_{Th} + R_L)^2} \right] = 0$$

$$= (V_{Th})^2 \left[\frac{(R_{Th} + R_L)^2 \frac{d}{dR_L} R_L - R_L \frac{d}{dR_L} (R_{Th} + R_L)^2}{(R_{Th} + R_L)^4} \right] = 0$$

$$\frac{dp}{dR_L} = (V_{Th})^2 \left[\frac{(R_{Th})^2 + R_L^2 + 2R_{Th}R_L - 2R_L(R_{Th} + R_L)}{(R_{Th} + R_L)^4} \right] = 0$$

$$= (R_{Th})^2 + R_L^2 + \cancel{2R_{Th}R_L} - \cancel{2R_L R_{Th}} - 2R_L^2 = 0$$

$$= R_{Th}^2 - R_L^2 = 0$$

$$R_{Th}^2 = R_L^2$$

$$\boxed{R_{Th} = R_L}$$

The maximum power can be obtained when load resistance $R_L = R_{Th}$.

$$P = I_L^2 R_L$$

$$P = \left(\frac{V_{Th}}{R_{Th} + R_L} \right)^2 R_L$$

$$P = \frac{V_{Th}^2}{R_{Th}^2 + R_L^2 + 2R_{Th}R_L} \times R_L$$

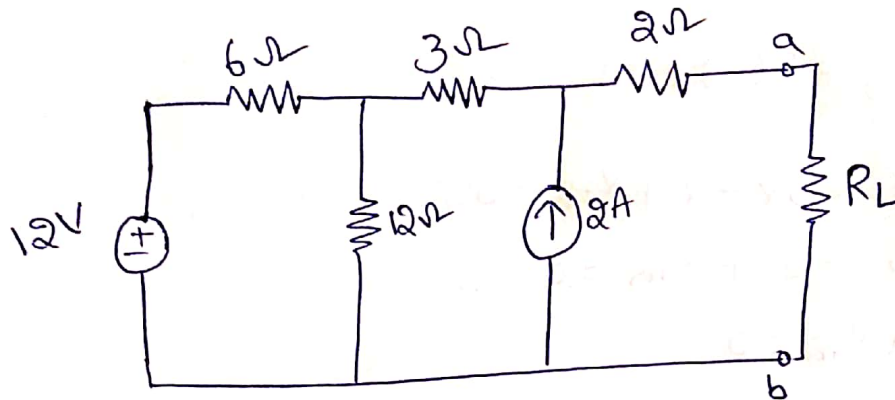
$$P = \frac{V_{Th}^2}{R_{Th}^2 + R_{Th}^2 + 2R_{Th}^2} \times R_{Th}$$

$$P = \frac{V_{Th}^2}{4R_{Th}^2} R_{Th}$$

$$\boxed{P = \frac{V_{Th}^2}{4R_{Th}}}$$

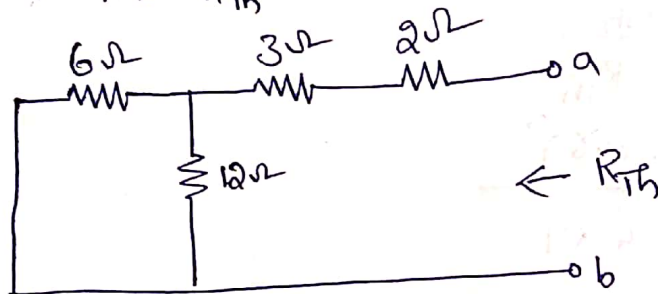
∴ maximum power delivered to the load when $R_{Th} = R_L$.

* find the value of R_L for maximum power transfer in the circuit of fig. find the maximum power.



Sol

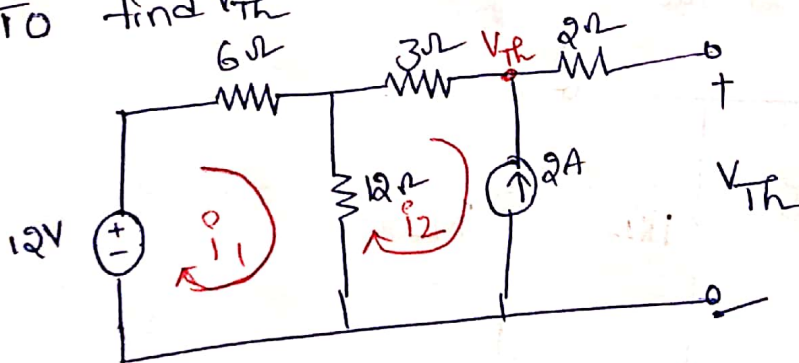
To find R_{Th}



$$R_{Th} = 3 + 2 + \frac{6 \times 12}{6 + 12}$$

$$= 5 + \frac{72}{18} = 5 + 4 = 9\Omega$$

To find V_{Th}



Apply mesh analysis to loop ①

$$-12 + 6i_1 + 12(i_1 - i_2) = 0$$

$$= 12 + 6i_1 + 12(i_1 - (-2)) = 0$$

$$-12 + 6i_1 + 12i_1 + 24 = 0$$

$$18i_1 = -12$$

$$i_1 = -12/18 = -\frac{2}{3} = -0.66$$

$$i_2 = -2$$

Applying KVL around the outer loop to get V_{Th} across terminals a-b.

$$-12 + 6i_1 + 3i_2 + 2(0) + V_{Th} = 0$$

$$-12 + 6 \times -0.66 + 3 \times -2 + V_{Th} = 0$$

$$-12 - 3.96 - 6 + V_{Th} = 0$$

$$-21.6 + V_{Th} = 0$$

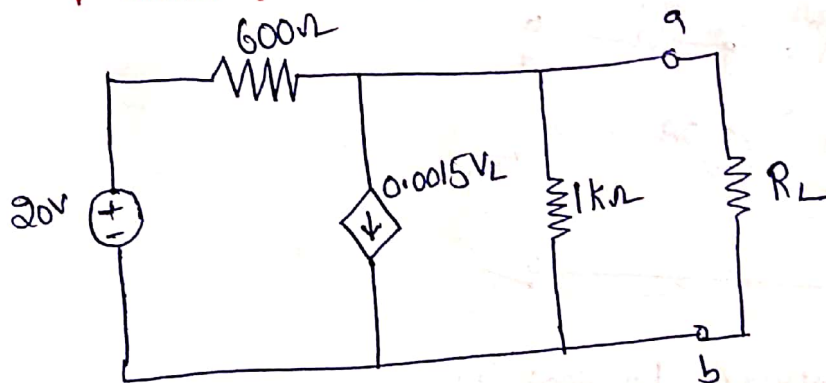
$$V_{Th} = +21.6$$

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}}$$

$$P_{max} = \frac{(21.6)^2}{4 \times 9}$$

$$P_{max} = 13 \text{ W}$$

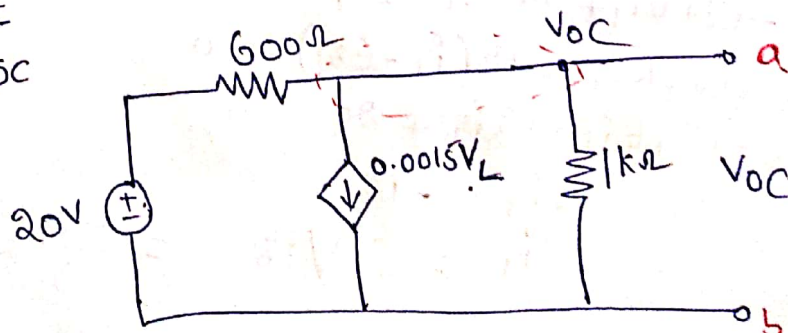
find the value of load resistance R_L for the maximum power transfer theorem as show in figure.



sol

$$R_{Th} = \frac{V_{OC}}{I_{SC}}$$

$$V_L = V_{OC}$$



Apply KCL to the node V_L

$$\frac{V_L - 20}{600} + 0.0015V_L + \frac{V_L}{1000} = 0$$

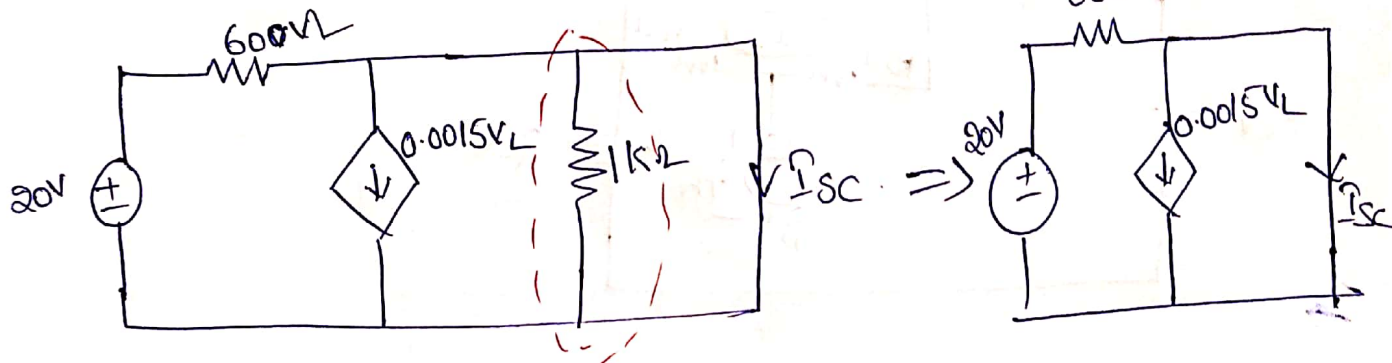
$$\frac{5V_L - 100 + 4.5V_L + 3V_L}{3000} = 0$$

$$12.5V_L = 100$$

$$V_L = \frac{100}{12.5}$$

$$V_{OC} = V_L = 8V$$

To find I_{SC}



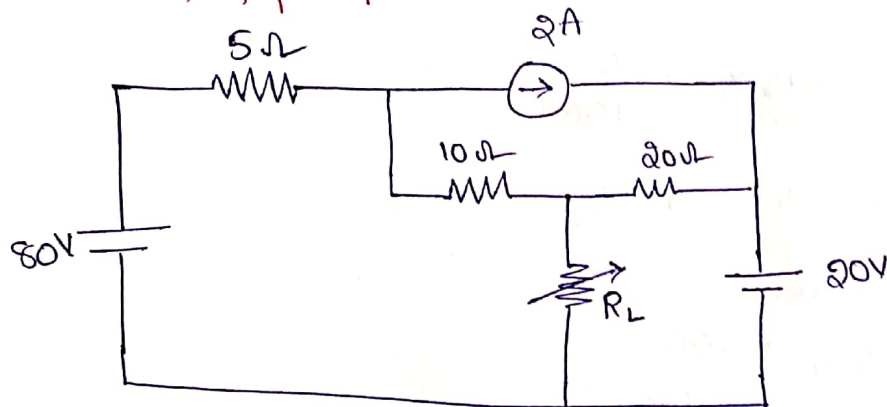
$$I_{SC} = \frac{V}{R} = \frac{20}{600} = \frac{1}{30} = 0.033 A$$

$$R_{TH} = \frac{V_{OC}}{I_{SC}} = \frac{8}{0.033} = 240.2 \Omega$$

$$R_L = R_{TH} = 240.2 \Omega$$

$$P_{max} = \frac{V_{TH}^2}{4 R_{TH}} = \frac{8^2}{4 \times 240.2} = \frac{64}{960.8} = 0.0666 W$$

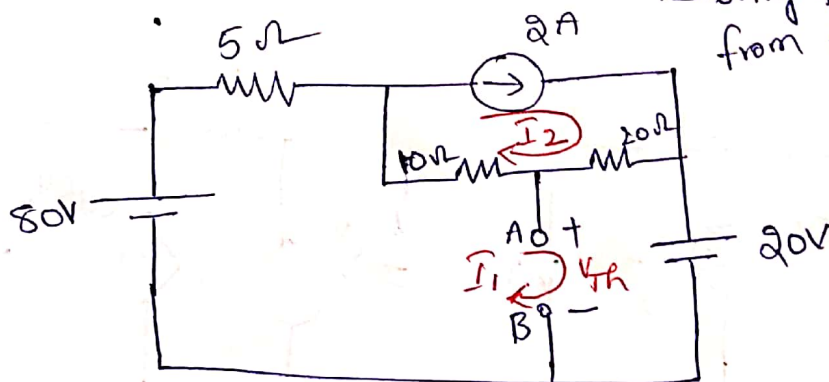
for the Ckt shown, find the value of the resistance R_L for maximum power and calculate the maximum power.



Sol

Calculation of V_{Th}

Removing the variable resistor R_L from the Ckt.



Applying KVL to mesh 1

$$-80 + 5I_1 + 10(I_1 - I_2) + 20(I_1 - I_2) + 20 = 0$$

$$5I_1 + 10I_1 - 10I_2 + 20I_1 - 20I_2 + 20 = 80$$

$$35I_1 - 30I_2 = 80 - 20$$

$$35I_1 - 30I_2 = 60 \rightarrow \textcircled{1}$$

$$I_2 = 2A \rightarrow \textcircled{2}$$

Substitute I_2 value in eq $\textcircled{1}$

$$35I_1 - 30 \times 2 = 60$$

$$35I_1 = 60 + 60$$

$$I_1 = 120/35 = 3.43A$$

writing V_{TH} equation

$$-V_{TH} + 20(I_1 - I_2) + 20 = 0$$

$$-V_{TH} + 20(3.43 - 2) = 0$$

$$-V_{TH} + 20(3.43 - 2) + 20 = 0$$

$$-V_{TH} + 20(1.43) + 20 = 0$$

$$-V_{TH} = -28.6 - 20$$

$$-V_{TH} = -48.6$$

$$V_{TH} = 48.6$$

(OR)

writing V_{TH} equation.

$$-80 + 5I_1 + 10(I_1 - I_2) + V_{TH} = 0$$

$$-80 + 5 \times 3.43 + 10(3.43 - 2) + V_{TH} = 0$$

$$-80 + 17.15 + 10 \times 1.43 + V_{TH} = 0$$

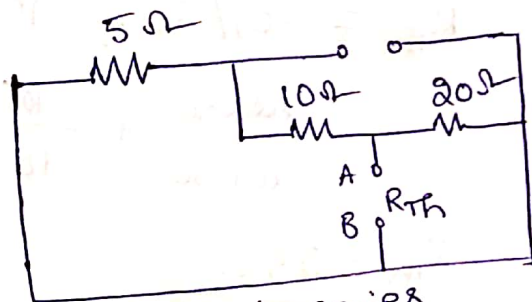
$$-80 + 17.15 + 14.3 + V_{TH} = 0$$

$$-80 + 31.45 + V_{TH} = 0$$

$$-48.55 + V_{TH} = 0$$

$$V_{TH} = 48.55$$

Calculation of R_{TH}



$5\Omega, 10\Omega$ are in series

$$R_{TH} = 15\Omega \parallel 20\Omega$$

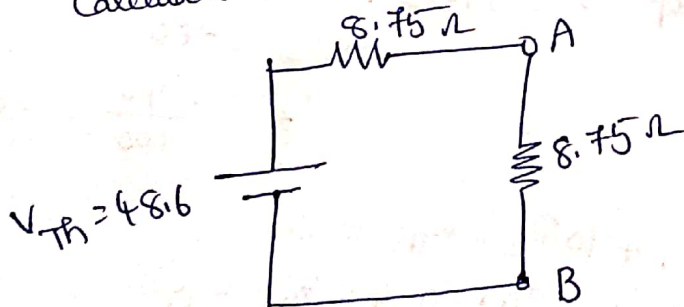
$$R_{TH} = \frac{15 \times 20}{15 + 20} = 8.75\Omega$$

Calculation of R_L

for maximum power transfer

$$R_L = R_{TH} = 8.75\Omega$$

Calculation of P_{max}



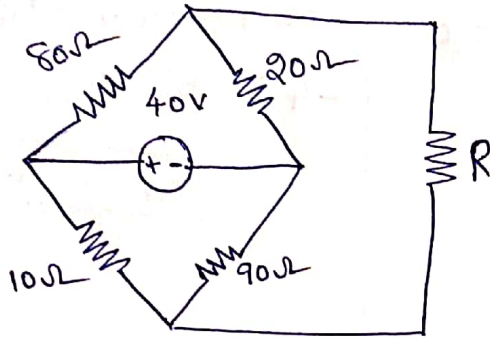
$$P_{max} = \frac{V_{TH}^2}{4 \times R_{TH}}$$
$$= \frac{(48.6)^2}{4 \times 8.75}$$

$$P_{max} = 68.9W$$

The variable resistor R in fig. is adjusted until it absorbs the maximum power from the circuit.

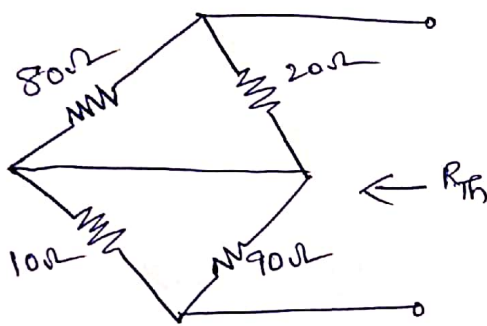
a) Calculate the value of R for maximum power.

b) Determine the maximum power absorbed by R .



Sol

To find R_{Th}

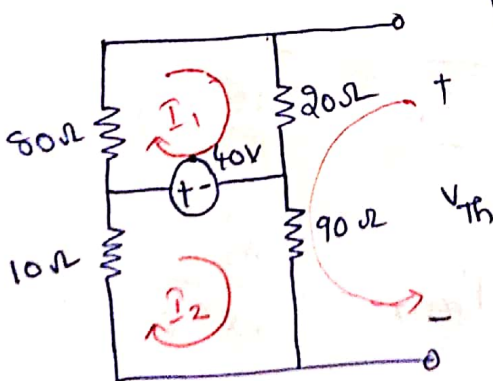


$$R_{Th} = 20 \parallel 80 + 90 \parallel 10$$

$$= \frac{20 \times 80}{20 + 80} + \frac{90 \times 10}{90 + 10}$$

$$R_{Th} = 25 \Omega$$

To find V_{Th}



$$\text{Loop } I_1 \Rightarrow 80I_1 + 20I_1 - 40 = 0$$

$$100I_1 = 40$$

$$I_1 = 40/100 = 0.4$$

$$\text{Loop } I_2 \Rightarrow 10I_2 + 90I_2 + 40 = 0$$

$$100I_2 = -40$$

$$I_2 = -\frac{40}{100} = -0.4$$

$$20I_1 + 90I_2 - V_{Th} = 0$$

$$20 \times 0.4 + 90 \times -0.4 - V_{Th} = 0$$

$$8 - 36 - V_{Th} = 0$$

$$-28 - V_{TH} = 0$$

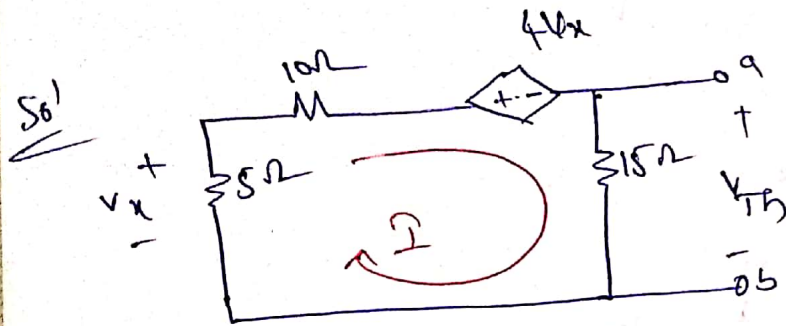
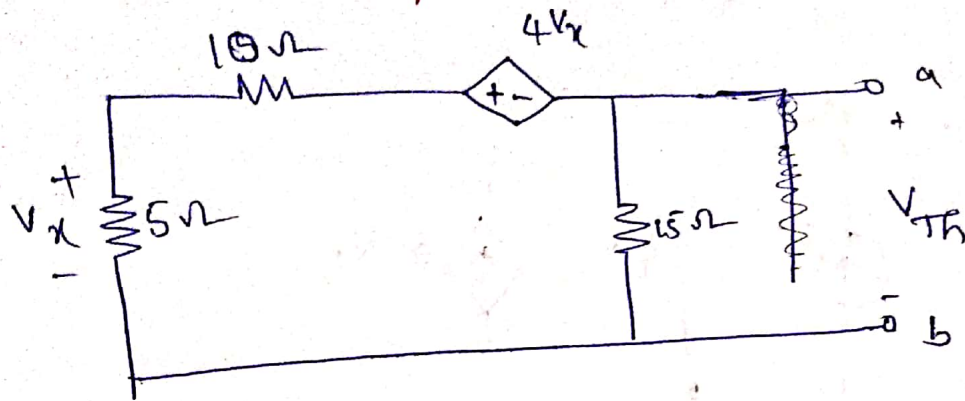
$$-V_{TH} = 28$$

$$V_{TH} = -28$$

$$a) \quad \underline{R = R_{TH} = 25 \Omega}$$

$$b) \quad P_{max} = \frac{V_{TH}^2}{4R_{TH}} = \frac{(28)^2}{4 \times 25} = \frac{(28)^2}{100} = \underline{\underline{7.84 W}}$$

Obtain thevenin equivalent of the Ckt.



$$V_x = V_{th}$$

$$V_x = 5I$$

Apply KVL to the loop

$$-V_x + 10I + 4V_x + 15I = 0$$

$$-5I + 10I + 4 \times 5I + 15I = 0$$

$$-5I + 10I + 20I + 15I = 0$$

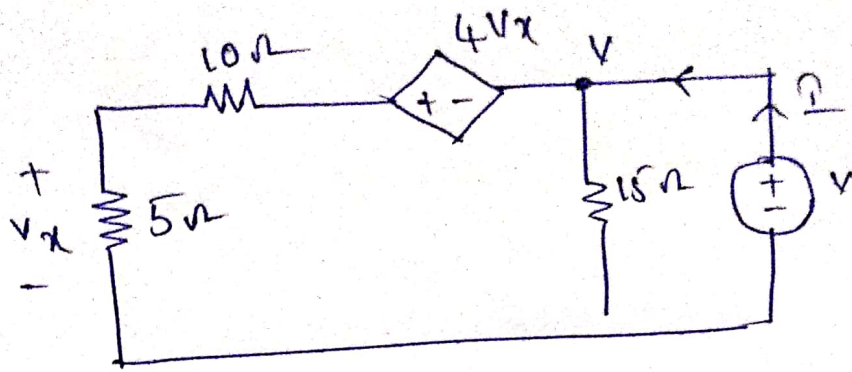
$$40I = 0$$

$$I = 0$$

$$V_x = 5 \times 0 = 0$$

$$\therefore V_{th} = 0$$

In the ckt All are dependent sources $V_{th} = 0$, $I_{sc} = 0$
But resistance is obtained.



$$\frac{V - 4V_x}{15} + \frac{V}{15} = I$$

$$\frac{V - 4 \times 5I}{15} + \frac{V}{15} - I = 0$$

$$\frac{V}{15} - \frac{4 \times 5I}{15} + \frac{V}{15} - I = 0$$

$$\frac{V}{15} - \frac{4I}{3} + \frac{V}{15} - I = 0$$

$$\frac{2V}{15} = \frac{7I}{3}$$

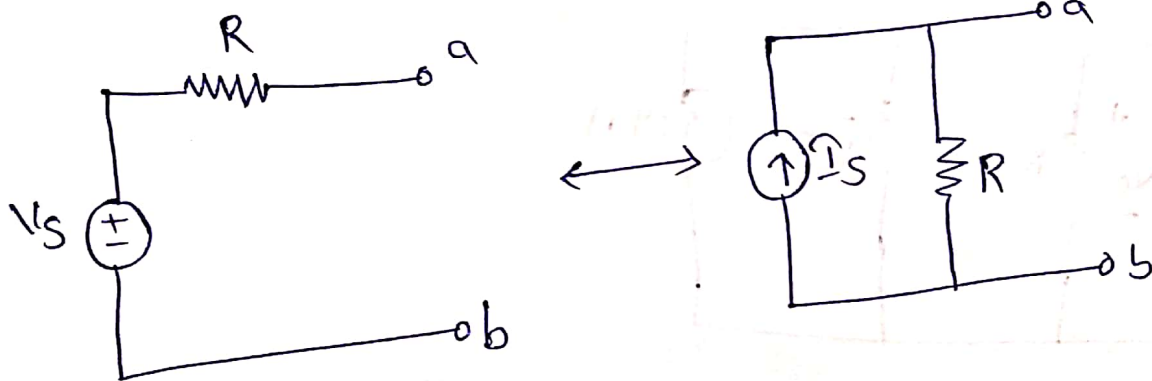
$$R_{Th} = \frac{V}{I} = \frac{35}{2} = 17.5 \Omega$$

Source Transformations :-

Source transformation is a tool for simplifying circuits.

In circuit analysis it is to be able to substitute a voltage source in series with a resistor for a current source in parallel with a resistor.

A source transformation is a process of replacing voltage source V_S in series with a resistor R by a current source I_S in parallel with a resistor R .



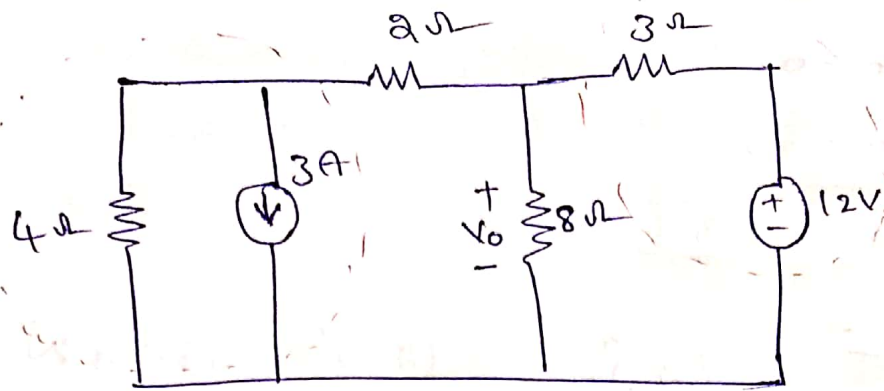
Transformation of independent source.

When the terminals a & b are short circuited, the current flowing from a to b is

$$I_{SC} = \frac{V_S}{R}$$

$$V_{SC} = I_{SC} R$$

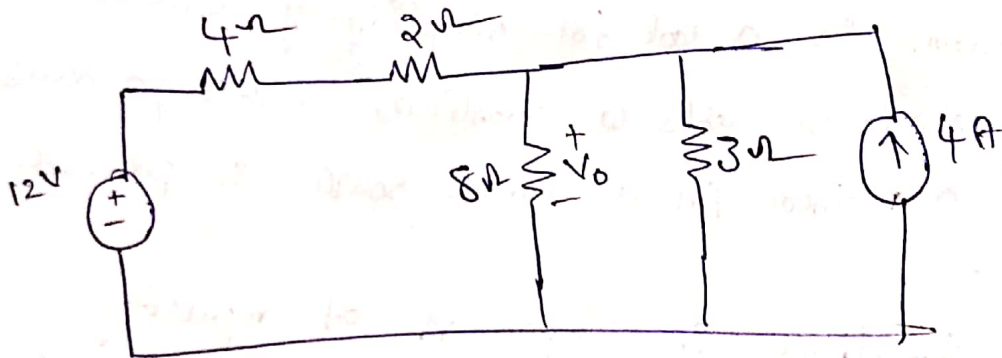
Use source transformation to find V_o .



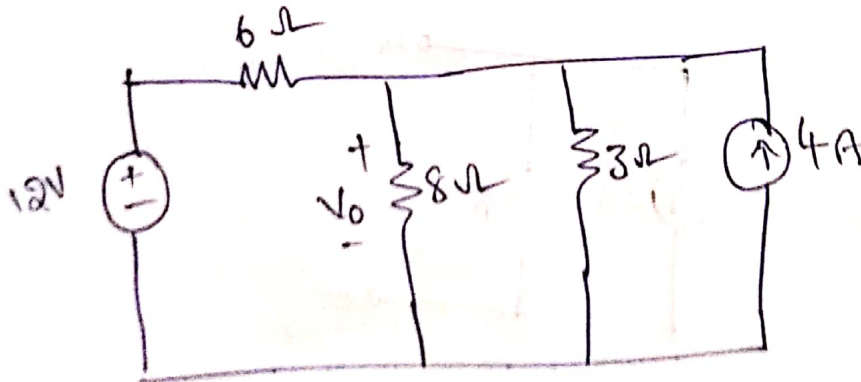
Sol

$$I_S = \frac{V_S}{R} = \frac{12}{3} = 4A$$

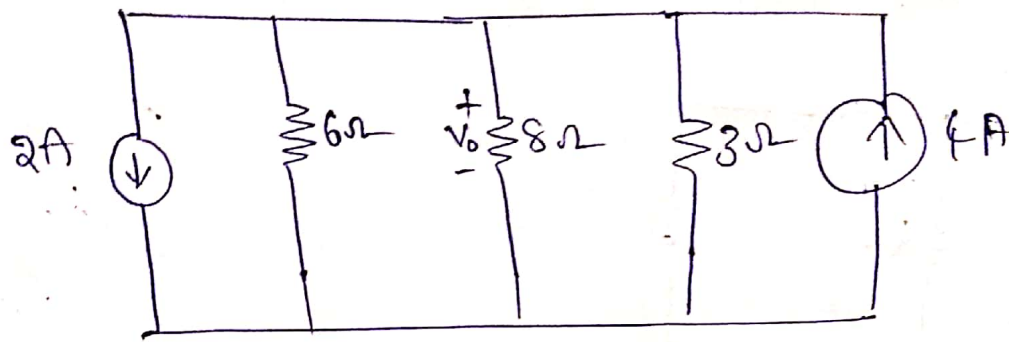
$$V_S = I_S R = 4 \times 3A = 12V$$



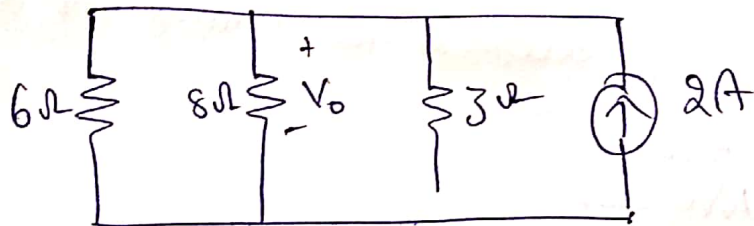
$$4\Omega \text{ \& } 2\Omega \Rightarrow R_{eq} = 4 + 2 = 6\Omega$$



$$I_S = \frac{V}{R} = \frac{12}{6} = 2A$$

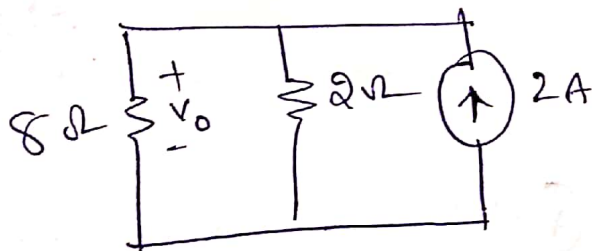


$$i = -2 + 4 = 2A$$



$$6\Omega \parallel 3\Omega$$

$$R_{eq} = \frac{6 \times 3}{6 + 3} = \frac{18}{9} = 2\Omega$$



$$i_1 = \left(\frac{R_2}{R_1 + R_2} \right) i \Rightarrow i_1 = 2 \times \frac{2}{8 + 2}$$

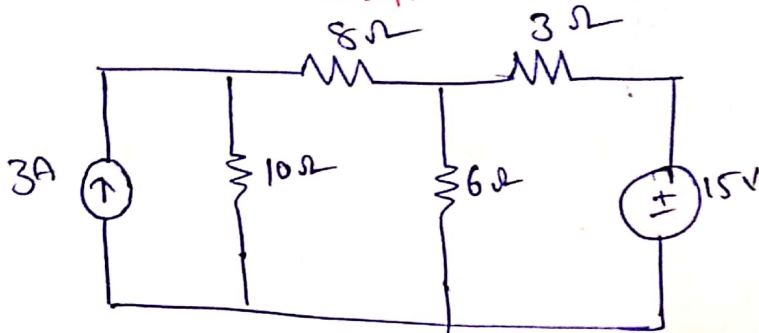
$$i_1 = \frac{4}{10}$$

$$i_1 = 0.4A$$

$$V_0 = 8 \times 0.4 = 3.2V$$

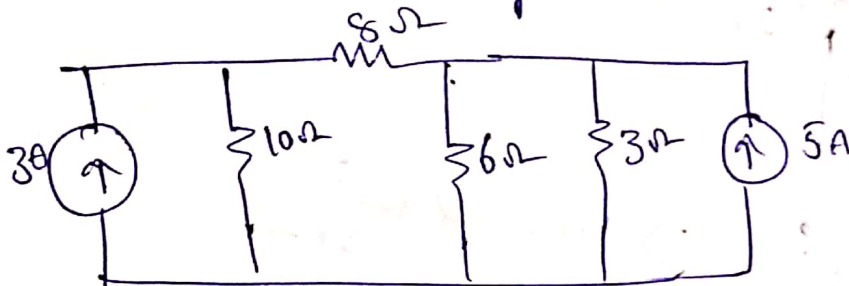
$$= 3.2V$$

* Use source transformation to determine the current and power in the 8Ω resistor.



Sol

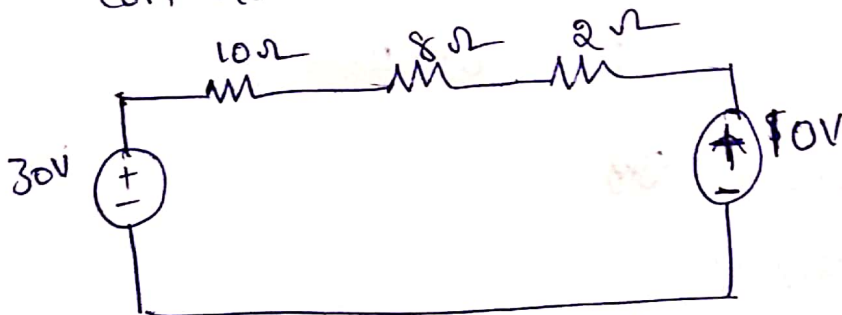
Transform the voltage source



$$6\Omega // 3\Omega$$

$$\frac{6 \times 3}{6 + 3} = \frac{18}{9} = 2\Omega$$

Convert the current source to voltage source



Applying KVL to the loop

$$-30 + 10i + 8i + 2i + 15 = 0$$

$$20i - 15 = 0$$

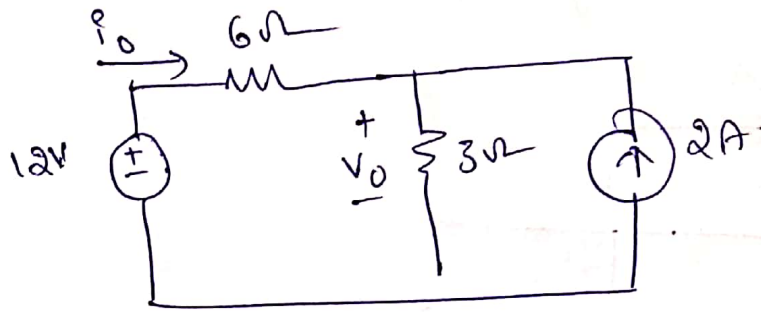
$$i = 1 \text{ A}$$

$$P = VI$$

$$P = I^2 R$$

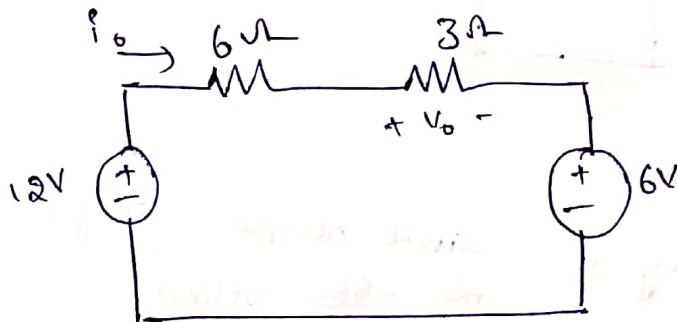
$$= 1^2 \times 8 = 8 \text{ W}$$

Apply source transformation to determine V_0 & i_0 in the circuit.



1181

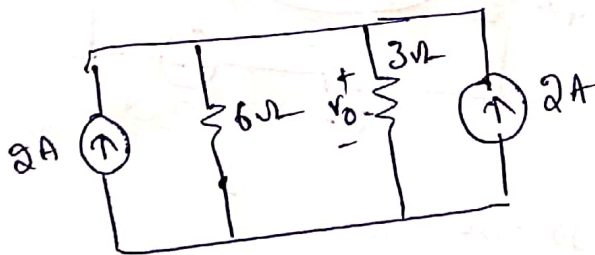
To get i_0 transform current source.



$$-12 + 9i_0 + 6 = 0 \Rightarrow 9i_0 = 6, i_0 = \frac{6}{9} = \frac{2}{3}$$

$$i_0 = 666.7 \text{ mA}$$

To get V_0 transform the voltage source.



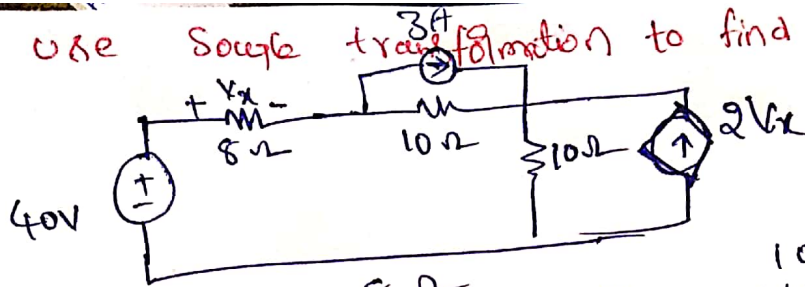
$$i = \frac{6}{6+3} \times (2+2)$$

$$= \frac{6}{9} \times 4 = \frac{24}{9} = \frac{8}{3}$$

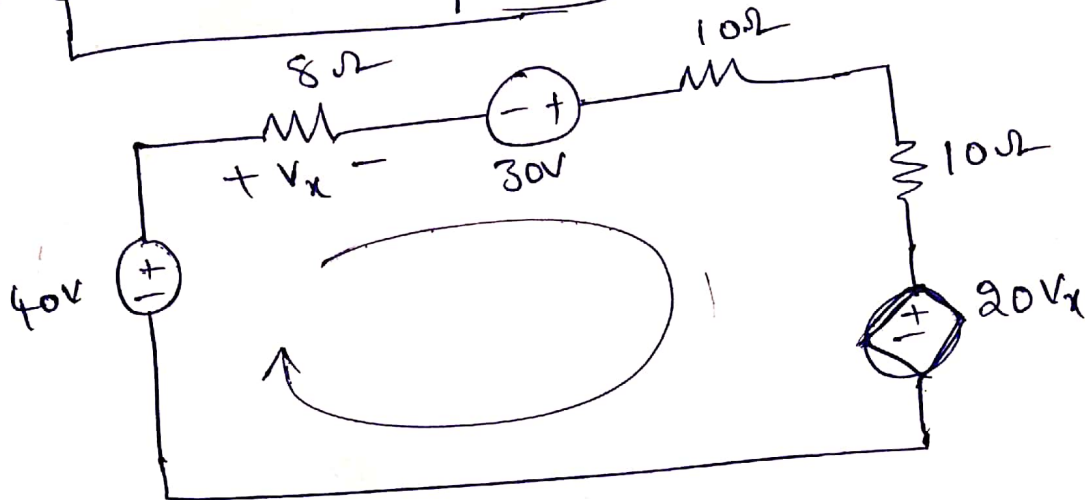
$$V_0 = 3i = 3 \times \frac{8}{3} = 8 \text{ V}$$

$$V_0 = 8 \text{ V}$$

* use source transformation to find V_x



Sol



$$V_x = 8V$$

Sol

$$-40 + 8i - 30 + 10i + 10i + 20V_x = 0$$

$$-70 + 28i + 20 \times 8i = 0$$

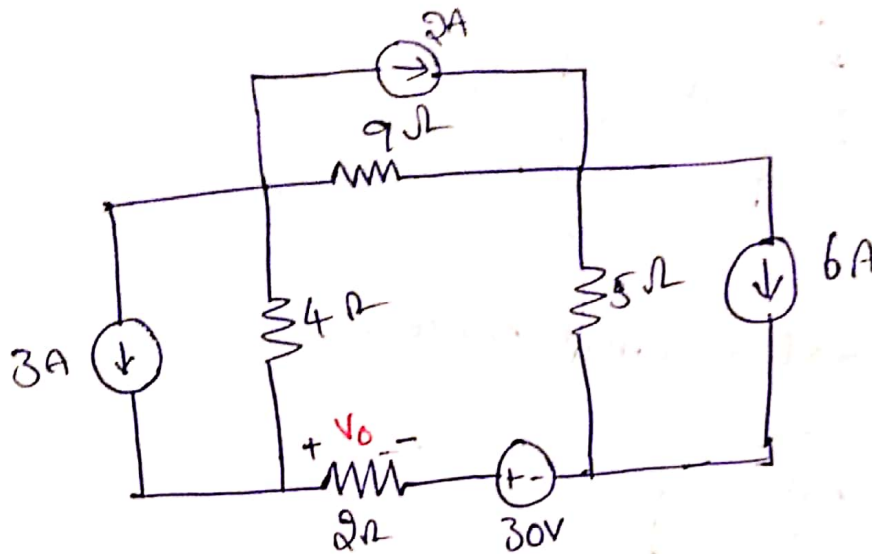
$$28i + 160i = 70$$

$$i = 0.3723$$

$$V_x = 8 \times 0.3723$$

$$= 2.978V$$

* obtain V_0 in the ckt. using source transformation



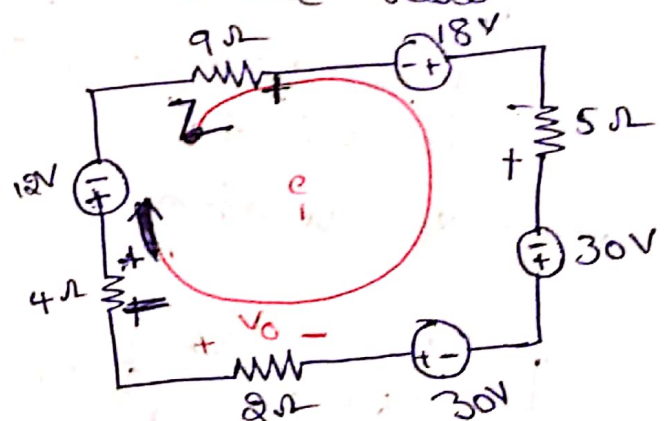
Sol

~~V = IR~~ Transforming only the current source it gives the ckt below

$$\textcircled{1} \Rightarrow V = IR = 3 \times 4 = 12$$

$$\textcircled{2} \Rightarrow V = IR = 2 \times 9 = 18$$

$$\textcircled{3} \Rightarrow V = IR = 5 \times 6 = 30$$



Applying KVL to the loop

$$(4 + 9 + 5 + 2) i + 12 + 18 + 30 + 30 = 0$$

$$+20i + 66 = 0$$

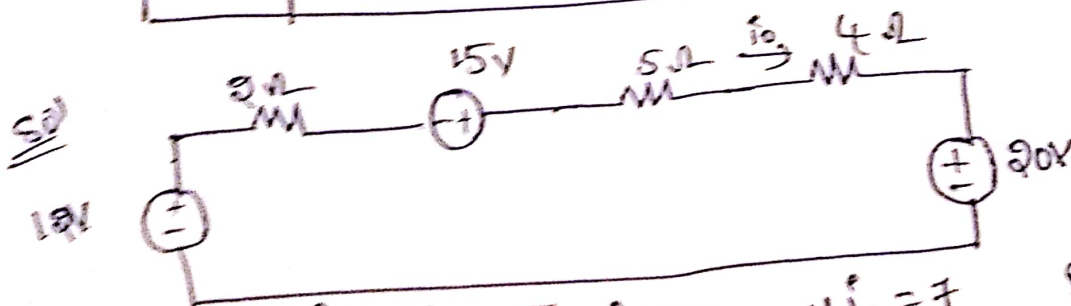
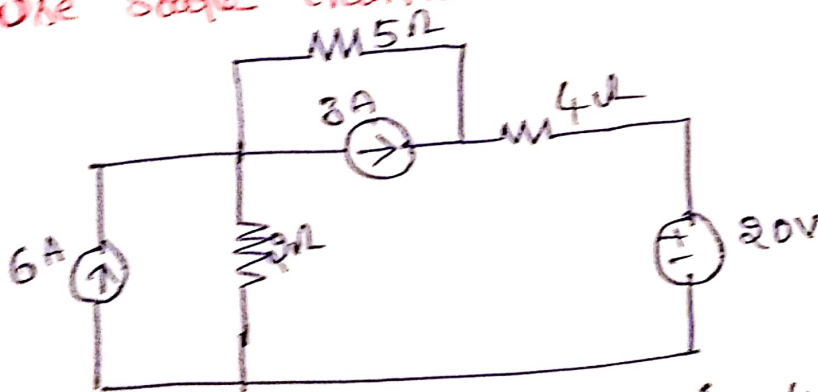
$$20i = -66$$

$$i = -66/20 = -3.3$$

$$V_0 = 2i$$

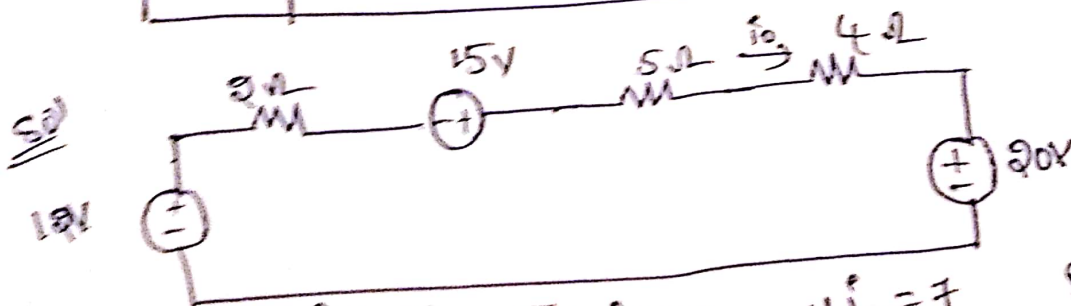
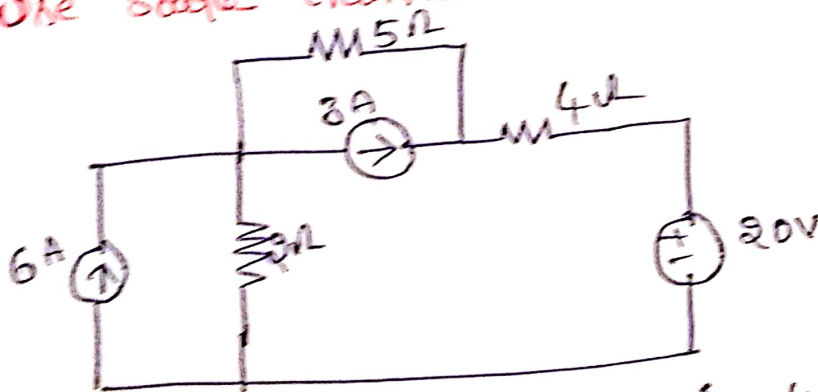
$$V_0 = 2 \times -3.3 = -6.6V$$

Use source transformation to find i_o



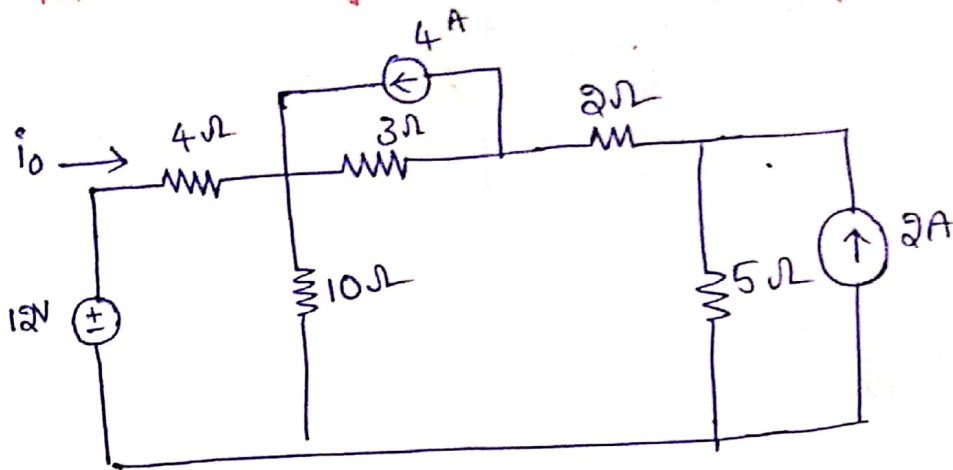
$$-12 + 4i_o - 15 + 20 = 0, \quad 11i_o = 7, \quad i_o = 636.4 \text{ mA}$$

Use source transformation to find i_o

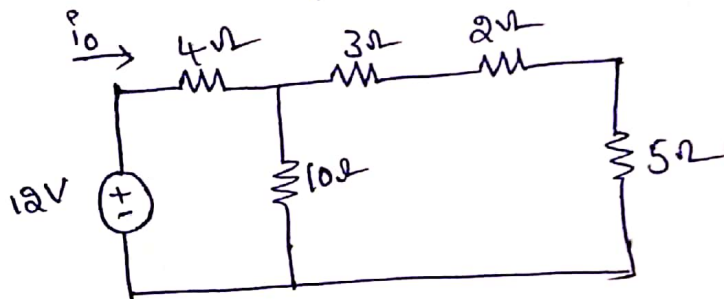


$$-12 + 4i_o - 15 + 20 = 0, \quad 11i_o = 7, \quad i_o = 636.4 \text{ mA}$$

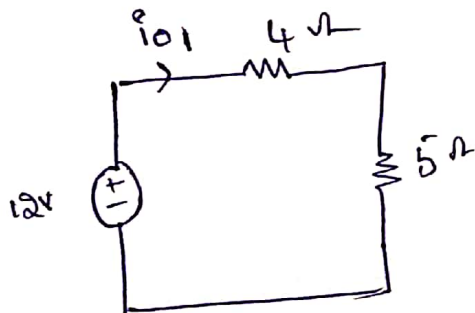
* for the following circuit find i_o by using superposition principle.



Sol 12V voltage source is active alone

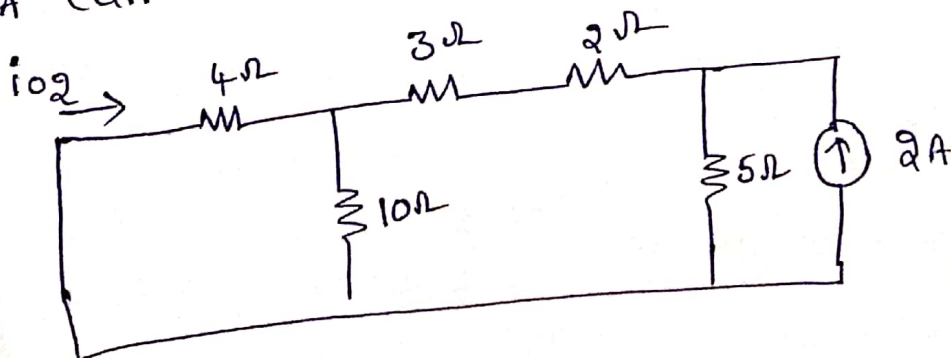


$$10\Omega \parallel (3+2+5) = \frac{10 \times 10}{10+10} = \frac{100}{20} = 5\Omega$$

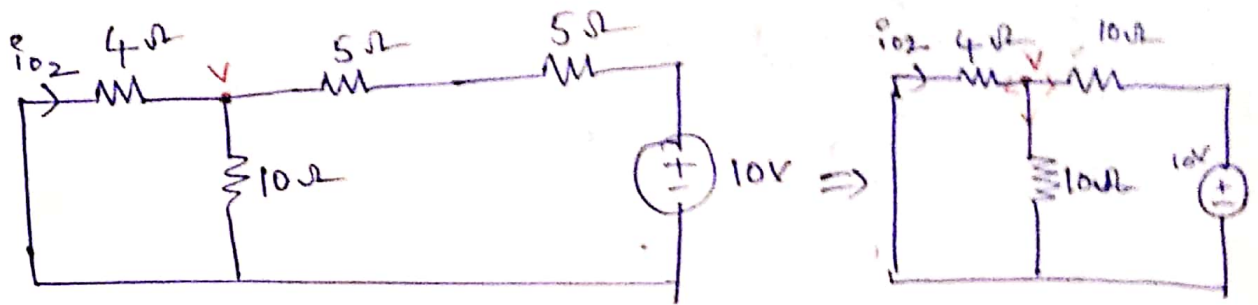


$$i_{o1} = \frac{V}{R} = \frac{12}{9}$$

2A Current source is active alone



Apply source transformation



Apply KCL at node V

$$\frac{V}{4} + \frac{V}{10} + \frac{V-10}{10} = 0$$

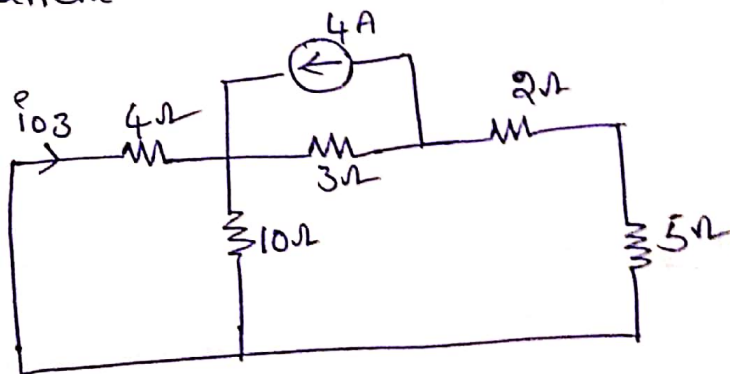
$$\frac{5V + 2V + 2V - 20}{20} = 0$$

$$9V = 20$$

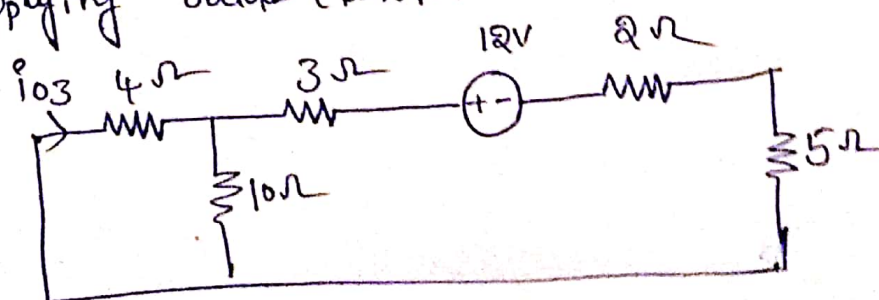
$$V = \frac{20}{9}$$

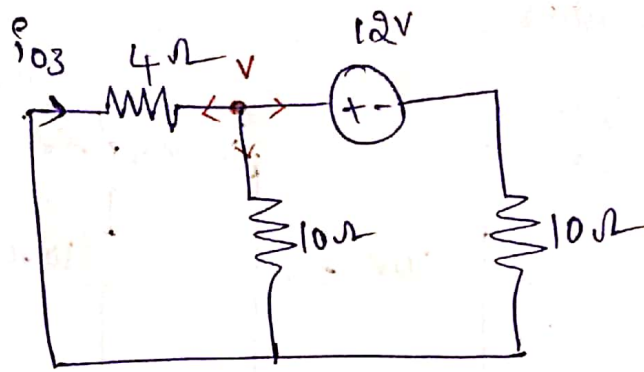
$$i_{02} = \frac{V}{R} = \frac{-\frac{20}{9} \times 5}{4} = -\frac{5}{9}$$

4A current source is active alone.



Applying source transformation





Apply KCL at node v

$$\frac{v}{4} + \frac{v}{10} + \frac{v-12}{10} = 0$$

$$5v + 2v + 2v - 24 = 0$$

$$9v = 24$$

$$v = 24/9$$

$$i_{03} = \frac{-24/9}{4} = -6/9$$

$$i_0 = i_{01} + i_{02} + i_{03}$$

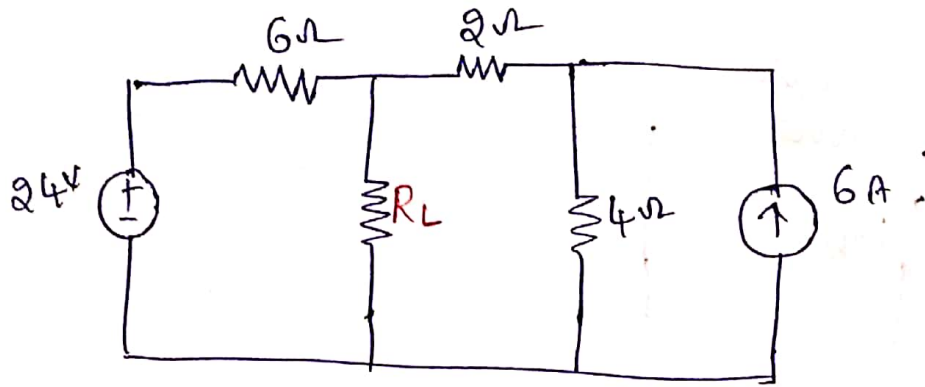
$$i_0 = \frac{12}{9} - \frac{5}{9} - \frac{6}{9}$$

$$i_0 = \frac{1}{9}$$

$$i_0 = 0.1111$$

$$i_0 = 111.1 \text{ mA}$$

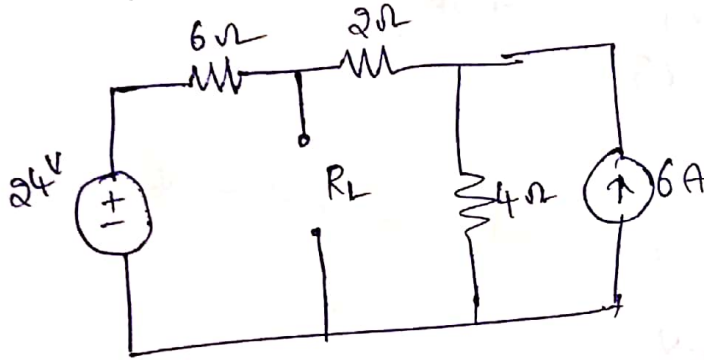
* find the maximum power transferred to R_L in the ckt?



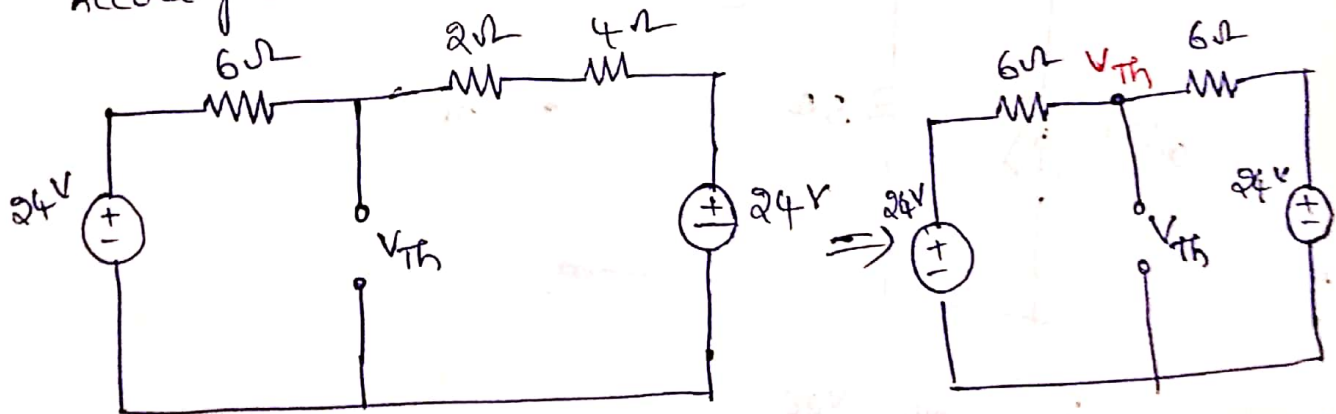
Sol

$$P_{\max} = \frac{V_{Th}^2}{4 R_{Th}}$$

first remove R_L



According to source transformation (for reducing the ckt).



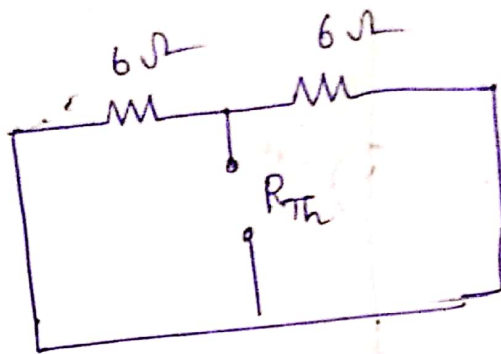
Apply KCL at node V_{Th}

$$\frac{V_{Th} - 24}{6} + \frac{V_{Th} - 24}{6} = 0$$

$$2V_{Th} = 48$$

$$V_{Th} = 24V$$

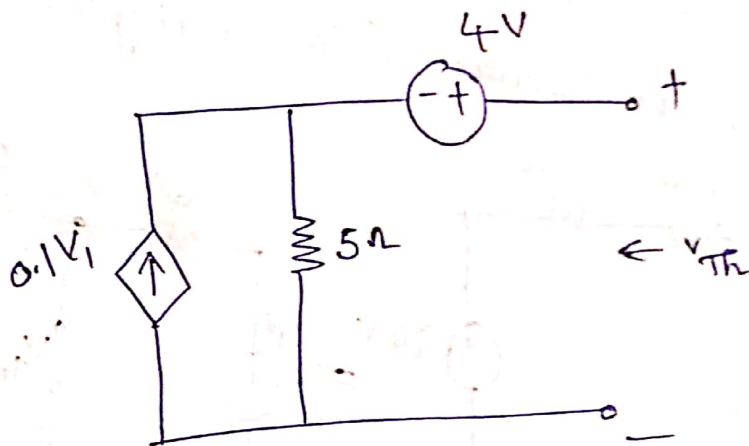
To find thevenin resistance short circuit the Voltage sources.



$$R_{Th} = \frac{R_1 R_2}{R_1 + R_2} = \frac{6 \times 6}{6 + 6} = 3\Omega$$

$$P_{max} = \frac{V_{Th}^2}{4 R_{Th}} = \frac{24^2}{4 \times 3} = 48W$$

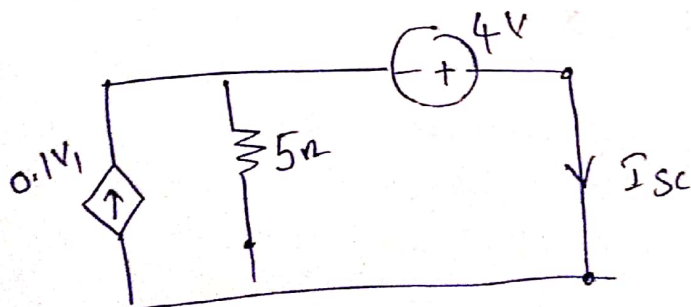
* In the following ckt find the value of V_{Th} & R_{Th}



Sol

$$V_1 = V_{Th} = V_{OC}$$

$$V_1 = 4 + 5 \times 0.1V_1 = 8V$$

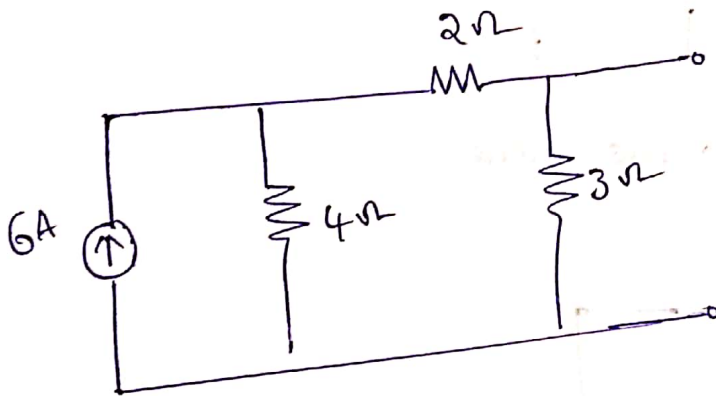


$$i_{SC} = \frac{V}{R} = \frac{4}{5} = 0.8 \text{ A}$$

$$R_{Th} = \frac{V_{OC}}{i_{SC}} = \frac{8}{0.8} = 10 \Omega$$

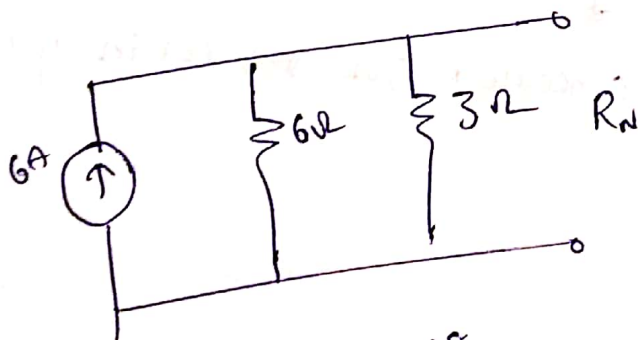
* In the following circuit find the value of i_N & R_N

Sol

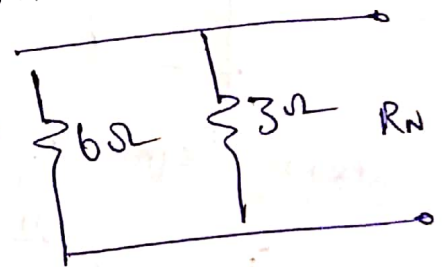


Sol

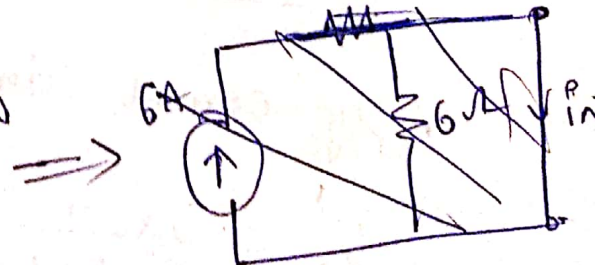
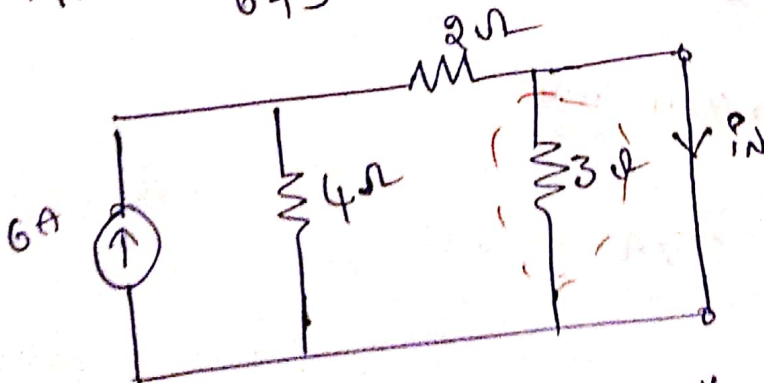
4Ω & 2Ω are in series



6A source is turn off



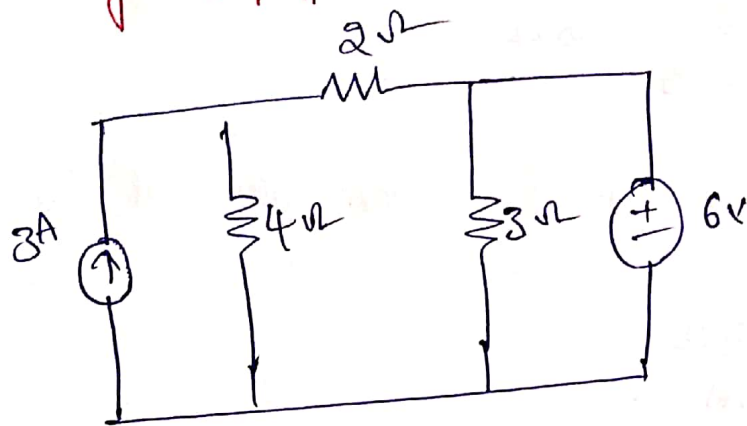
$$R_N = \frac{6 \times 3}{6 + 3} = \frac{18}{9} = 2 \Omega$$



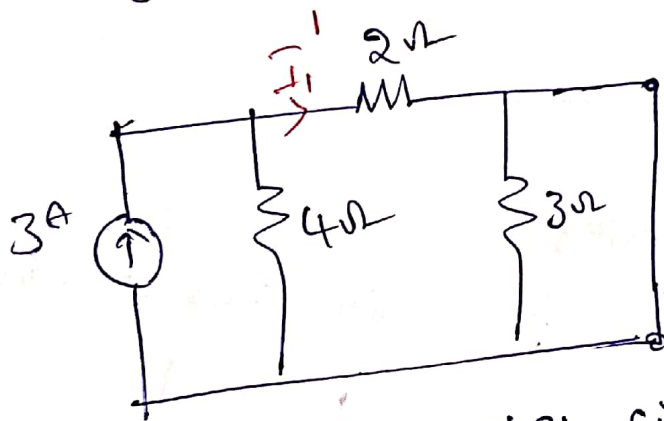
4Ω across voltage = $6 \times 4 = 24 \text{ V}$

$$i_N = \frac{V}{R} = \frac{24}{4 + 2} = \frac{24}{6} = 4 \text{ A}$$

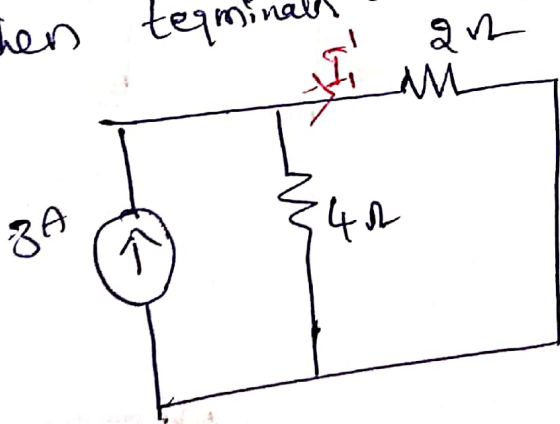
* Calculate the current flowing through 2Ω resistor using superposition principle.



Sol 3 A source is active alone



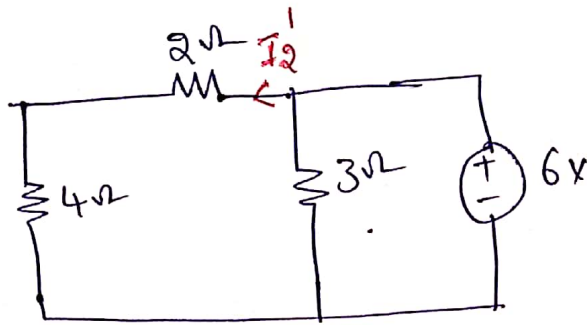
when terminals are short circuited 3Ω resistor is ignored.



Apply current division rule

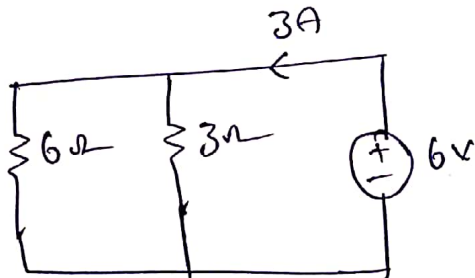
$$I' = 3 \times \frac{4}{2+4} = 2A$$

6V voltage source is active alone



$$R_{eq} = \frac{6 \times 3}{6 + 3} = 2\Omega$$

$$I = \frac{V}{R} = \frac{6}{2} = 3A$$



Apply current division rule

$$I_2' = 3 \times \frac{3}{3+6} = 1A$$

when 3A source is active the current flowing in the 2Ω resistor is I_1'

when 6V source is active the current flowing in the 2Ω resistor is I_2'

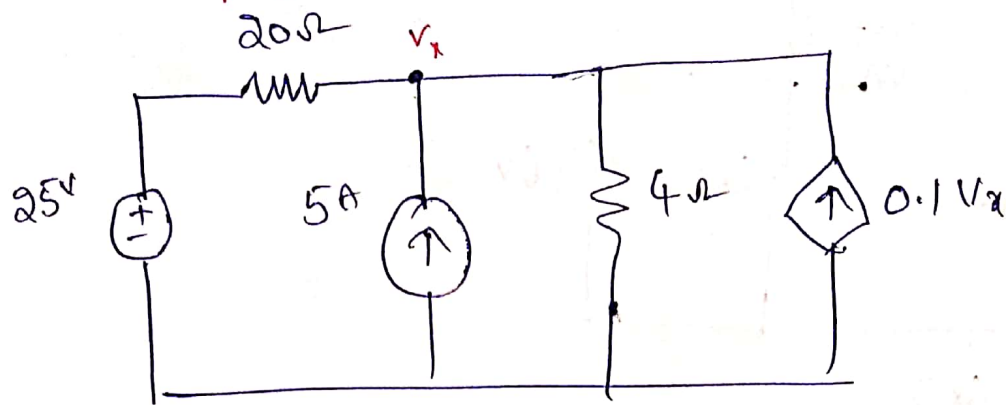
$\therefore I_1'$ & I_2' are both in opposite direction
so the current flowing through the 2Ω resistor is

$$I = I_1' - I_2'$$

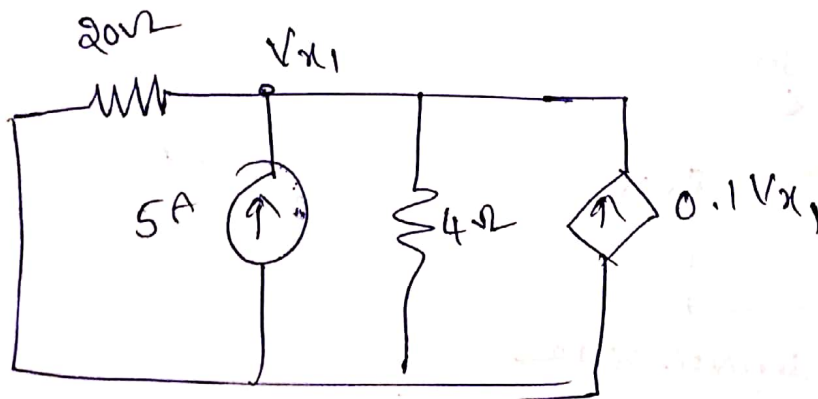
$$I = 2 - 1$$

$$I = 1A$$

Use Superposition theorem find V_x



Sol Deactivate voltage source



$$\frac{V_{x1}}{20} + \frac{V_{x1}}{4} = 5 + 0.1V_{x1} \Rightarrow \frac{V_{x1}}{20} + \frac{V_{x1}}{4} = 5 + \frac{1}{10}V_{x1}$$

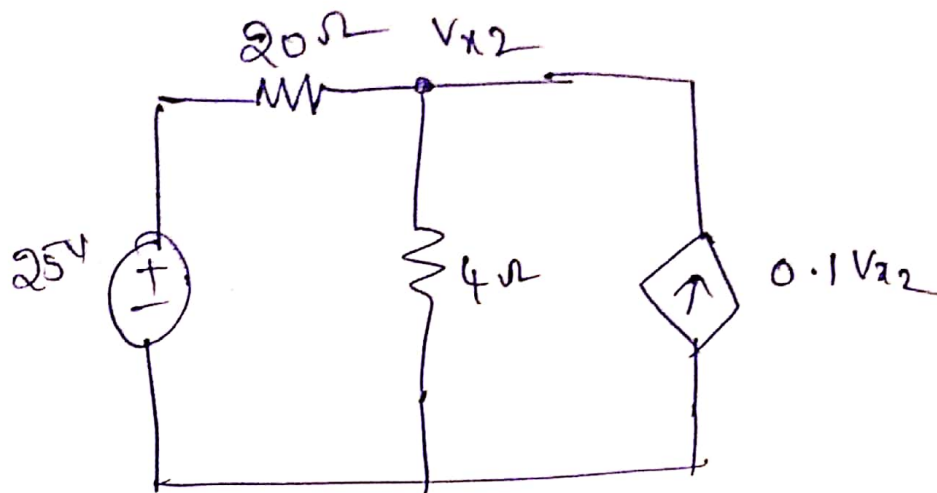
$$\frac{V_{x1} + 5V_{x1}}{20} = \frac{50 + V_{x1}}{10}$$

$$6V_{x1} = 100 + 2V_{x1}$$

$$4V_{x1} = 100$$

$$V_{x1} = 25$$

Deactive 5 A current source



$$\frac{V_{x2} - 25}{20} + \frac{V_{x2}}{4} = \frac{1}{10} V_{x2}$$

$$\frac{V_{x2} - 25 + 5V_{x2}}{20} = \frac{V_{x2}}{10}$$

$$6V_{x2} - 25 = 2V_{x2}$$

$$4V_{x2} = 25$$

$$V_{x2} = \frac{25}{4} = 6.25$$

$$V_x = V_{x1} + V_{x2} = 25 + 6.25 = 31.25 \text{ V}$$