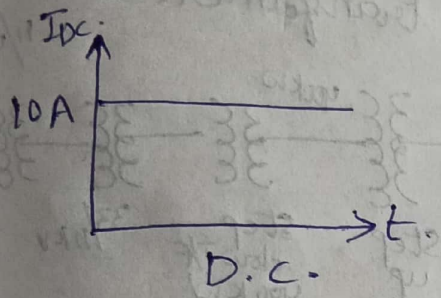
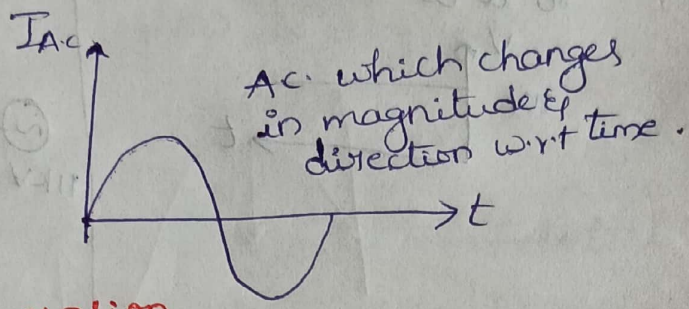
1-ϕ A.C. Circuits.

Introduction :-

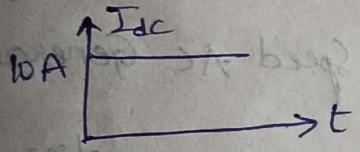
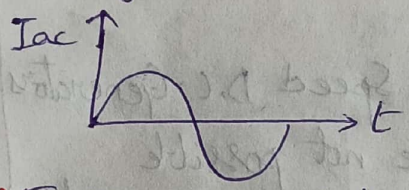
D.C : Direct Current which is constant w.r.t to time.

A.C. : Alternating current which changes polarity or direction w.r.t time.



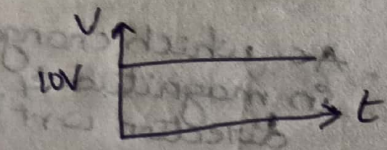
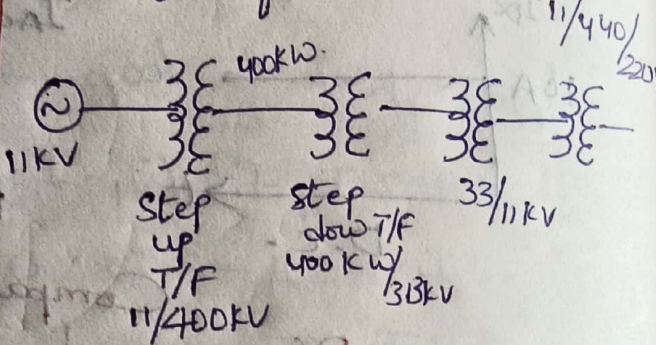


Comparison

DC	AC
1) D.C. voltage can not travel very far	1) A.C. voltage can travel very far distances with safely.
2) D.C. Current is a constant w.r.t to time. 	2) A.C. Current is not constant at any time w.r.t to time. 
3) Frequency is zero for D.C.	3) Frequency is 50Hz for A.C.
4) D.C. contains only Resistances (R)	4) A.C. contains R, L, C ie, Impedance (z).
5) D.C. supply obtained from cell or Battery.	5) A.C is obtained from A.C. Generators.
6) D.C. flows only one direction in circuit.	6) A.C. flows in two directions
7) Flow of electrons in one direction (forward direction).	7) It flows in forward and backward direction.
8) P.f for Dc is 1.	8) P.f for A.C. is always lies b/w 0 & 1.
9) D.C types: Pure Pulsating.	9) It is sinusodial, trapezoidal triangular, square and etc.
10) In D.C. Induction is not possible.	10) In A.C. Induction is possible.
11) Cannot be stepped up or down.	11) Step up or step down is possible with help of T/F.

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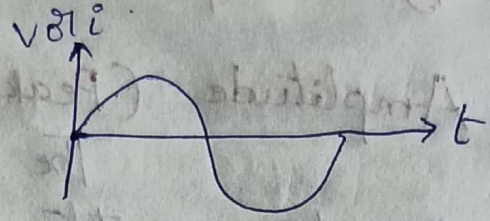
Advantages of A.C. over D.C. :-

D.C.	A.C
1) In DC, voltage can not be stepper up or down. So it is constant 	1) A.C. voltage can be stepped up or stepped down with help of transformer. 
2) In D.C. we can obtain constant losses.	2) In AC, if we go for higher voltages, currents are less & hence losses (I^2R) are less & improves transmission efficiency.
3) High Speed D.C. Generators are not possible	3) High Speed AC Generator
4) D.C. motors are not simple in construction & it occupies more maintaince.	4) A.C. electric motors are simple in construction, cheaper and requires less maintance.
5) D.C. is not easy to generate	5) A.C. is easy to generate.
6) D.C. is not cheaper because it requires rectifiers.	6) A.C is cheaper
7) D.C. Generators has low efficiency.	7) A.C. Generators has higher efficiency

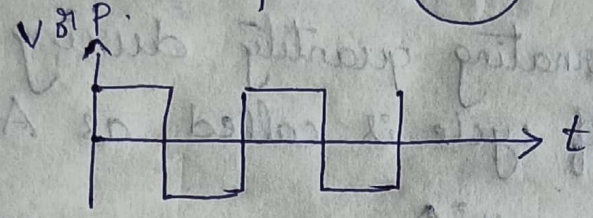
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Types of A.C. Waveforms :-

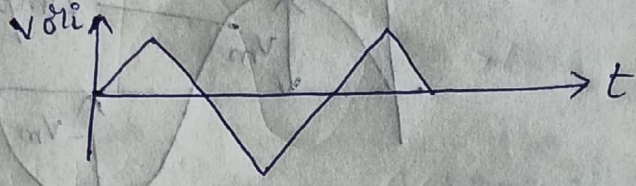
① Sinusodial waveform



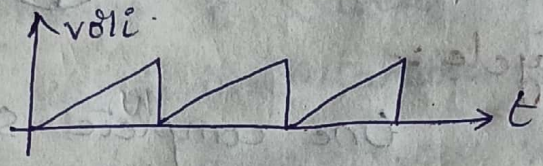
② Square wave



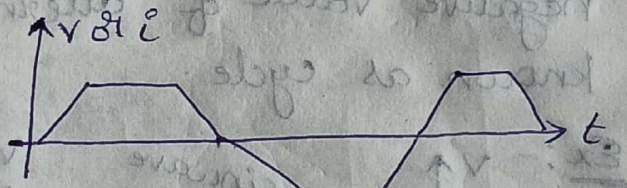
③ Triangular wave



④ Sawtooth wave



⑤ Trapezoidal wave



Among five Sinewave is the better due to following Reasons :-

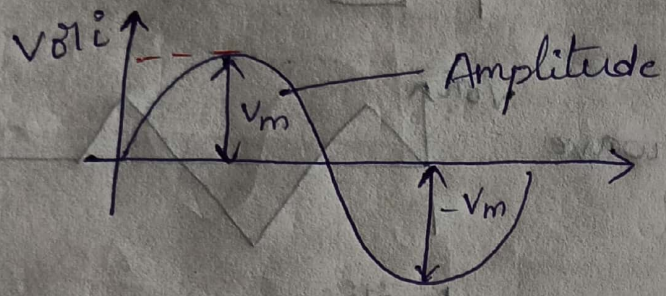
① Any periodic wave can be written in sinusodial function.
② Sinwave integration & derivative is also a sinwave.
③ It is easy to generate.
④ It is easy to analyse.
⑤ It is mostly used in power industry.

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Basic definitions :-

1) Amplitude (Peak Value] :-

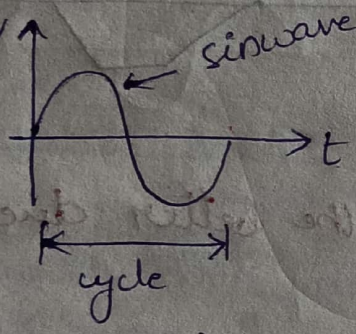
The maximum value of alternating quantity during positive or negative half cycle is called as Amplitude

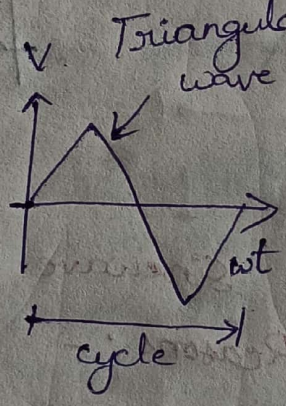


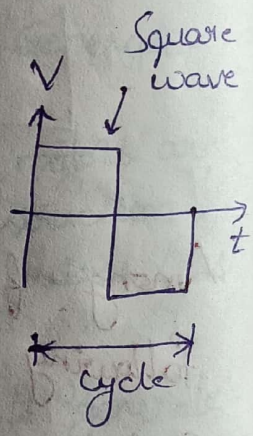
② Cycle :-

One complete set of positive & negative value of alternating quantity is known as cycle.

Ex :-





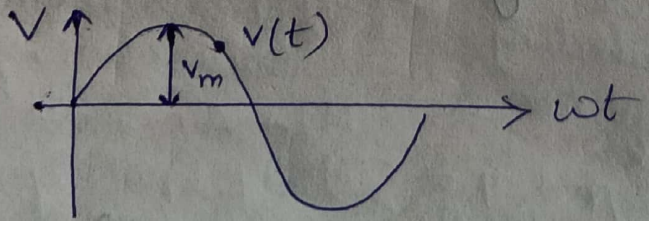


③ Voltage Equation :-

Voltage Equation for sinewave is

$$v(t) = V_m \sin \omega t$$

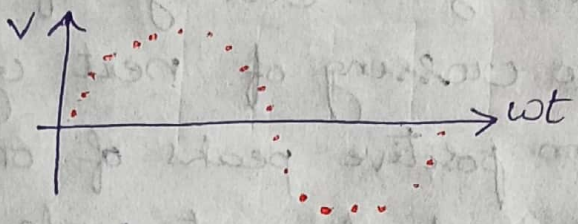
$v(t)$ → any point in the sinewave represented instantaneous value



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④ **Waveform :-**

The waveform is obtained by plotting the instantaneous values of voltage against time is called Waveforms.



⑤ **Instantaneous value :-**

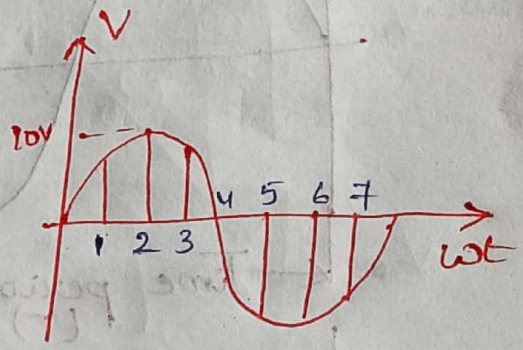
The magnitude of waveform at any instant of time is called instantaneous value.

→ During +ve half cycle, instantaneous values are positive.

→ During -ve half cycle, " " are negative.

Ex:-

At 't'	Instantaneous value v(t)
t = 1	7 V
t = 2	10 V
t = 3	6 V
t = 4	0 V
t = 5	-7 V
t = 6	-10 V



⑥ Time period :-

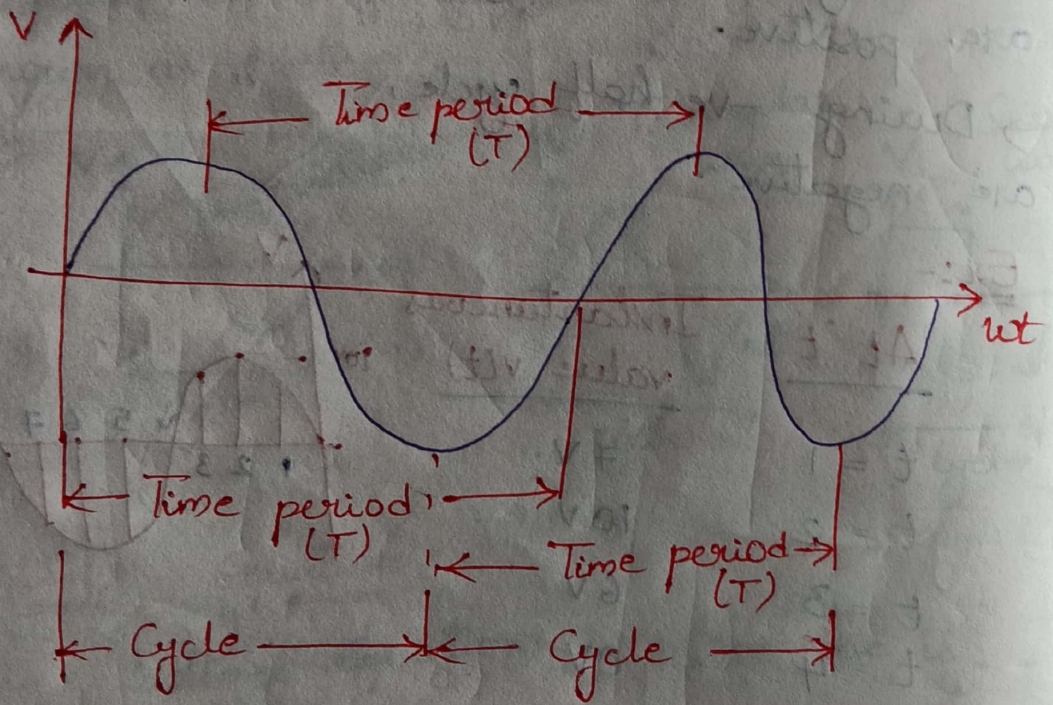
Time taken by the alternating quantity to complete one cycle is called Time period.

→ From zero crossings of one cycle to zero crossing of next cycle.

→ From positive peaks of one cycle to positive peak of next cycle.

→ From -ve peak of one cycle to -ve peak of next cycle.

→ It is denoted by symbol "T".



⑦ Frequency :-

The no. of cycles per second is called frequency

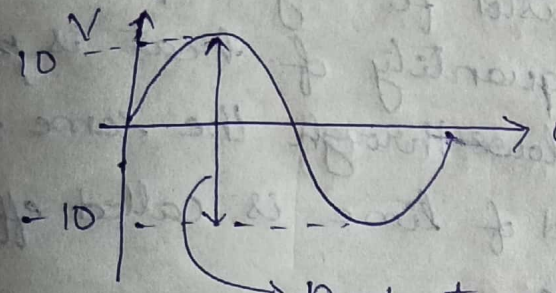
$$f = \frac{1}{T} \text{ Hz}$$

⑧ Angular frequency (ω) :- $\omega = 2\pi f$ rad/sec

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(9) **Peak-to-Peak value :-**

The peak-to-peak value of a sinewave is the peak from +ve to -ve peak.



→ Peak to peak value $10-(-10) = 20V$.

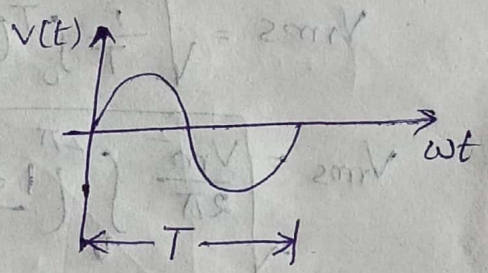
(10) **Average value :-**

It is defined as the "Total area of the waveform divided by the distance of waveform.

$$\text{Average} = \frac{\text{Area of waveform}}{\text{Distance of waveform}}$$

From the fig :

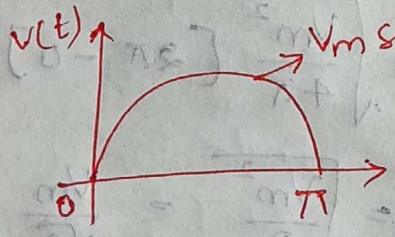
$$V_{avg} = \int_0^T \frac{V(t)\cdot dt}{T}$$



Note :- 1) Average value of symmetrical waveforms is always zero.

2) Average value of one full sinewave is zero.

Ex :-



$$V_{avg} = \int_0^{\pi} \frac{V_m \sin\omega t}{\pi}$$

$$= \frac{V_m}{\pi}\left(-\cos\omega t\right)_0^{\pi}$$

$$= \frac{V_m}{\pi}\left(-(-1)\right)$$

$$= \frac{V_m}{\pi}(1+1) = \frac{2V_m}{\pi}$$

$$V_{avg} = 0.637\,V_m$$

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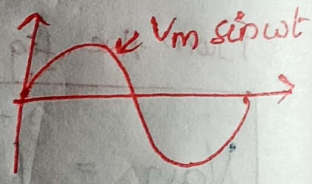
(iv) RMS value or Effective value :-

The steady current (D.C) which when flows through a resistor for a given period of time as a result same quantity of heat is produced by the AC when flows through the same resistor for the same period of time is called effective RMS value of a.c.

→ RMS value of any function V(t) with period of "T" is given by

$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T (V(t))^2 \cdot dt}$$

Ex:- For sinusodial function, find V_{rms}.



$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T (V_m \sin\omega t)^2 \cdot d\omega t}$$

$$V_{rms} = \sqrt{\frac{V_m^2}{2\pi}\int_0^{2\pi} \left(\frac{1-\cos 2\omega t}{2}\right) d\omega t}$$

$\cos 2\omega t = 1 - 2\sin^2\omega t$

$2\sin^2\omega t = 1 - \cos 2\omega t$

$\sin^2\omega t = \frac{1-\cos 2\omega t}{2}$

$$V_{rms} = \sqrt{\frac{V_m^2}{4\pi}\left[\omega t - \frac{\sin 2\omega t}{2}\right]_0^{2\pi}}$$

$\int \cos 2\omega t = \frac{\sin 2\omega t}{2}$

$$V_{rms} = \sqrt{\frac{V_m^2}{4\pi}[2\pi - 0]}$$

$$V_{rms} = \sqrt{\frac{V_m^2}{2}} = \frac{V_m}{\sqrt{2}}$$

$$V_{rms} = 0.707\, V_m$$

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(12) **Form Factor :-**

It is defined as the "ratio of RMS value to the average value of the wave".

$$\text{Form factor } (K_f) = \frac{\text{RMS value}}{\text{Average value}}$$

Ex:- For sinusodial function:

$$\text{Form factor} = \frac{V_m/\sqrt{2}}{\frac{2V_m}{\pi}} = 1.11.$$

(13) **Peak factor or Amplitude factor or Crest factor :-**

It is defined as the ratio of maximum value to rms value.

$$\text{Crest factor} = \frac{\text{Maximum value}}{\text{RMS value}}$$

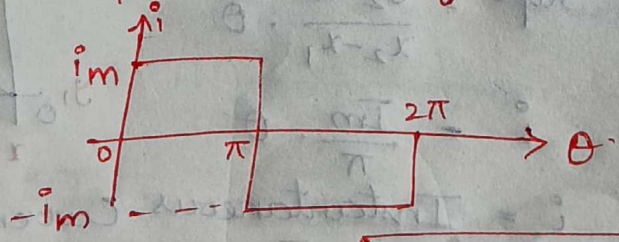
Ex:- For sinusodial function,

$$\text{Crest factor } (K_p) = \frac{V_m}{V_m/\sqrt{2}}$$

$$K_p = 1.414$$

Problems :-

(1) Find the form factor & Peak factor of the Square wave?



Sol:- $I_{avg} = \frac{1}{\pi}\int_0^{\pi} i \cdot d\theta$

$= \frac{1}{\pi}\int_0^{\pi} i_m \cdot d\theta \quad [\because T = \pi]$

$= \frac{i_m}{\pi}[\theta]_0^{\pi} = \frac{i_m}{\pi}[\pi] = i_m.$

$$I_{avg} = i_m$$

$$\text{Avg} = \int_0^T \frac{v(t)}{T} \cdot dt$$

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$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2 \cdot dt}$$

$$= \sqrt{\frac{1}{\pi}\int_0^{\pi} i_m^2 \cdot d\theta}$$

$$= \sqrt{\frac{1}{\pi} i_m^2 [\pi - 0]}$$

$$= \sqrt{\frac{i_m^2}{\pi} \cdot \pi}$$

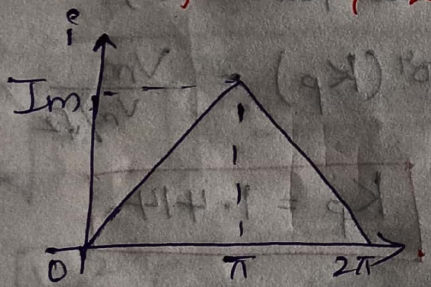
$$\boxed{I_{rms} = i_m}$$

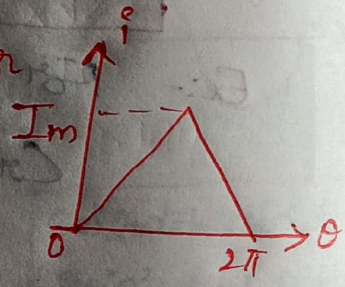
$$\text{Form factor} = \frac{I_{rms}}{I_{avg}} = \frac{i_m}{i_m} = 1.$$

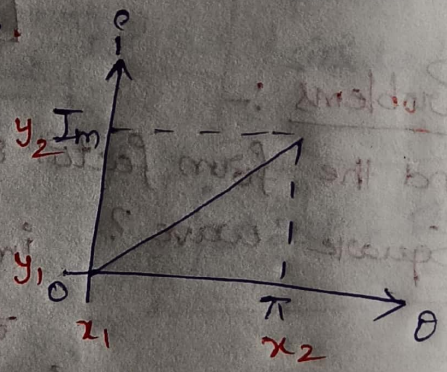
$$\text{Peak factor} = \frac{I_m}{I_{r.m.s}} = \frac{i_m}{i_m} = 1.$$

② Find form factor & Peak factor

Sol:







$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} \cdot \theta$$

$$i = \frac{I_m}{\pi} \cdot \theta$$

i = Instantaneous Current.

$$I_{avg} = \frac{1}{T}\int_0^T i \cdot d\theta = \frac{1}{\pi}\int_0^{\pi} \frac{I_m}{\pi}\theta \cdot d\theta$$

$$= \frac{I_m}{\pi^2}\left(\frac{\theta^2}{2}\right)_0^{\pi}$$

$$= \frac{I_m}{\pi^2}\left(\frac{\pi^2}{2}\right)$$

$$\boxed{I_{avg} = \frac{I_m}{2}}$$

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$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T \frac{I_m^2\theta^2}{\pi^2}\,d\theta}$$

$$= \sqrt{\frac{I_m^2}{\pi^3}\int_0^T \theta^2\,d\theta}$$

$$= \sqrt{\frac{I_m^2}{\pi^3}\left(\frac{\theta^3}{3}\right)_0^{\pi}}$$

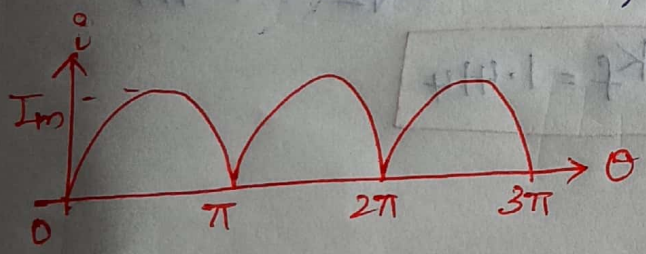
$$= \frac{I_m^2}{\pi^3}\left(\frac{\pi^3}{3}\right)$$

$$\boxed{I_{rms} = \frac{I_m}{\sqrt{3}}}$$

$$K_f = \frac{I_{rms}}{I_{avg}} = \frac{I_m/\sqrt{3}}{I_m/2} = \frac{2}{\sqrt{3}} = 1.155$$

$$K_p = \frac{I_m}{I_{rms}} = \frac{I_m}{I_m/\sqrt{3}} = \sqrt{3} = 1.732$$

(3)

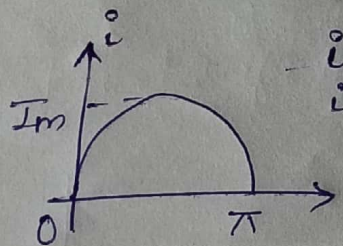


Find K_p & K_f.

Sol:- $I_{avg} = \frac{1}{\pi}\int_0^{\pi} I_m \sin\theta\,d\theta$

$= \frac{I_m}{\pi}(-\cos\theta)_0^{\pi}$

$I_{avg} = \frac{2I_m}{\pi} = 0.637\,I_m$



$i = I_m \sin\omega t$

$i = I_m \sin\theta$

$(\cos\pi = -1)$

$$I_{rms} = \sqrt{\frac{1}{\pi}\int_0^T i\,d\theta}$$

$$= \sqrt{\frac{1}{\pi}\int_0^{\pi} I_m^2 \sin^2\theta\,d\theta} = \sqrt{\frac{I_m^2}{\pi}\int_0^{2\pi}\left(\frac{1-\cos 2\theta}{2}\right)d\theta}$$

$$= \sqrt{\frac{I_m^2}{2\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}} = \sqrt{\frac{I_m^2}{2\pi}\left[\pi - \frac{\sin 2\pi}{2}\right]}$$

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$$I_{rms} = \frac{\sqrt{I_m^2}}{\sqrt{2}}$$

$$I_{rms} = \frac{I_m}{\sqrt{2}}$$

$$I_{rms} = 0.707\ I_m$$

$$= \frac{I_m}{\sqrt{2}} \times \frac{\pi}{2I_m}$$

$$= 1.11$$

$$\text{Form factor} = \frac{I_{rms}}{I_{avg}} = \frac{I_m/\sqrt{2}}{2I_m/\pi}$$

$$K_f = \frac{1}{\sqrt{2}} = 1.11$$

$$\text{Peak factor} = \frac{I_m}{I_{rms}} = \frac{I_m}{I_m/\sqrt{2}}$$

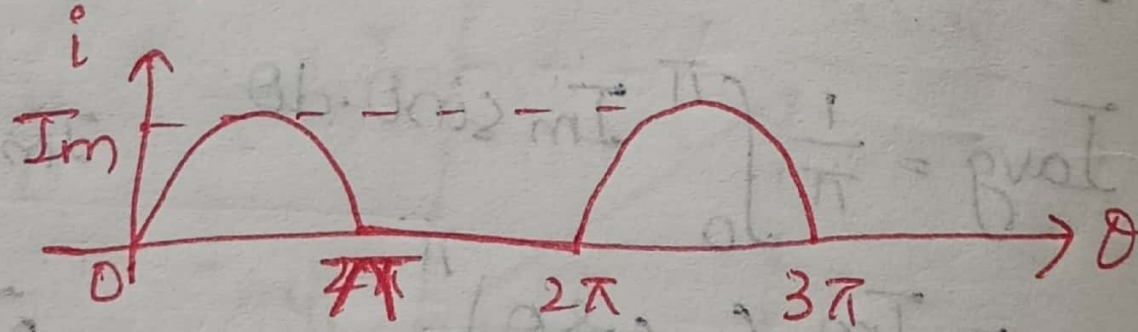
$$= I_m \times \frac{\sqrt{2}}{I_m}$$

$$= \sqrt{2} = 1.414$$

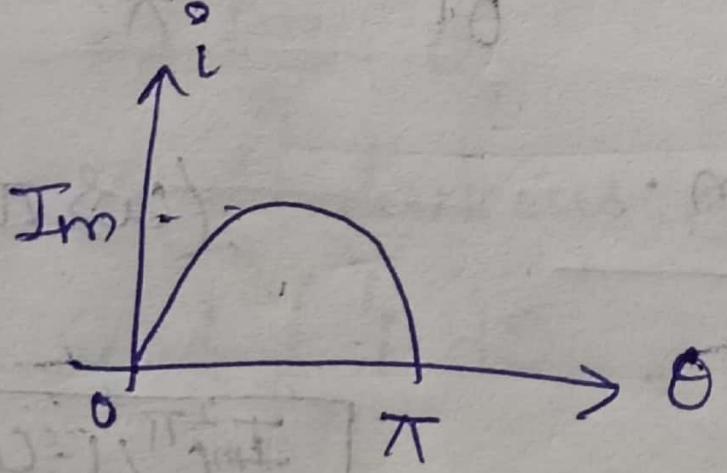
$$K_f = 1.414$$

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④ find K_f and K_p.



Sol:-



This half wave is an Unsymmetrical waveform.

$$I_{avg} = \int_0^T i \cdot d\theta \qquad T$$

$$I_{avg} = \frac{1}{T}\int_0^{\pi} I_m \sin\theta \cdot d\theta$$

$$= \frac{I_m}{T}\left[-\cos\theta\right]_0^{\pi}$$

$\cos\pi = -1$

$$= \frac{2I_m}{T}$$

Unsymmetry $I_{avg} = \frac{2I_m}{2\pi} = \frac{I_m}{\pi} = 0.318\, I_m$

$$I_{rms} = \sqrt{\frac{1}{2\pi}\left[\int_0^{T} I_m^2 \sin^2\theta \cdot d\theta\right]}$$

$$= \sqrt{\frac{I_m^2}{2\pi}\int_0^{\pi}\left(\frac{1-\cos 2\theta}{2}\right)}$$

$$= \sqrt{\frac{I_m^2}{2\pi}\left(\frac{\theta - \sin 2\theta}{2}\right)_0^{\pi}}$$

$$= \sqrt{\frac{I_m^2}{4\pi}(\pi - \sin 2\pi)}$$

$\cos 2\theta = 1 - 2\sin^2\theta$

$2\sin^2\theta = 1 - \cos 2\theta$

$\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$\sin n\pi = 0$

$$I_{rms} = \sqrt{\frac{I_m^2}{4\pi} \times \pi}$$

$$I_{rms} = \frac{I_m}{2} = 0.5\, I_m$$

$$K_f \text{ (Form factor)} = \frac{I_{rms}}{I_{avg}} = \frac{0.5\, I_m}{0.318\, I_m}$$

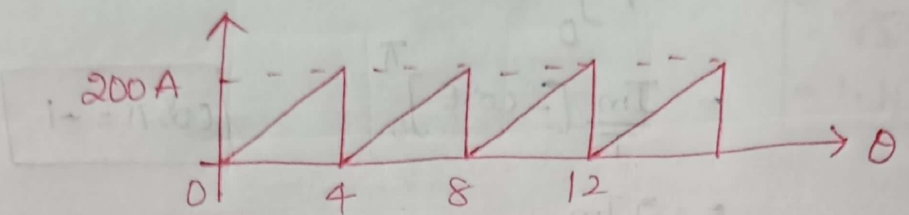
$$K_f = 1.572.$$

$$K_p \text{ (Peak factor)} = \frac{I_m}{I_{rms}} = \frac{I_m}{0.5\, I_m}$$

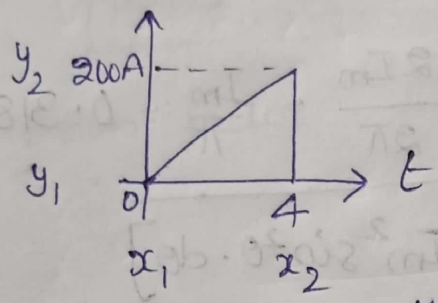
$$K_p = 2$$

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(5) Find K_f & K_p.



Sol:-



$$i = \text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} \cdot t = \frac{200 - 0}{4 - 0} = \frac{200}{4} t.$$

$$\boxed{i = 50t}$$

(i) $$I_{avg} = \frac{1}{T}\int_0^T i(t)\,dt$$

$$= \frac{1}{4}\int_0^4 50t\,dt$$

$$= \frac{50}{4}\left(\frac{t^2}{2}\right)_0^4$$

$$I_{avg} = \frac{50}{4}\left(\frac{16}{2}\right) = 100\,A.$$

(ii) $$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2\,dt}$$

$$= \sqrt{\frac{1}{4}\int_0^4 (50t)^2\,dt} = \sqrt{\frac{2500}{4}\left(\frac{t^3}{3}\right)_0^4}$$

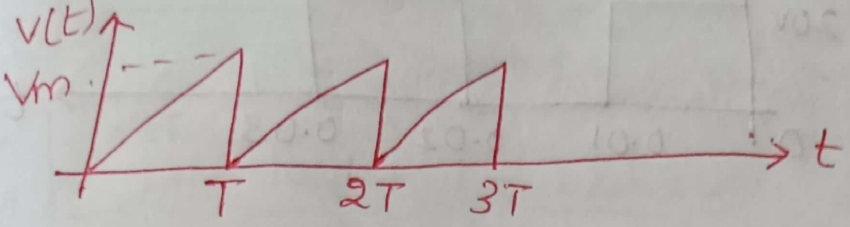
$$= \sqrt{13333.33} = 115.47\,A.$$

$$K_f = \frac{I_{rms}}{I_{avg}} = \frac{115.47}{100} = 1.155$$

$$K_p = \frac{I_m}{I_{rms}} = \frac{200}{115.47} = 1.732$$

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⑥ Find the peak factor and form factor for the waveform given below:



Sol: $V(t) = \frac{V_m}{T} \cdot t$ = slope = $\frac{y_2 - y_1}{x_2 - x_1} t = \frac{V_m - 0}{T - 0} \cdot t$

$= V_m/T \cdot t$

$0 \le t \le T$.

(i) $V_{avg} = \frac{\int_0^T V(t) \cdot dt}{T} = \frac{\int_0^T \frac{V_m}{T} \cdot t \cdot dt}{T}$

$= \frac{V_m}{T^2} \left(\frac{t^2}{2}\right)_0^T = \frac{V_m}{2T^2} (T^2 - 0)$

$V_{avg} = \frac{V_m}{2}$

(ii) $V_{rms} = \sqrt{\frac{1}{T} \int_0^T \left(\frac{V_m}{T} t\right)^2 \cdot dt}$

$= \sqrt{\frac{V_m^2}{T^3} \int_0^T t^2 \cdot dt} = \sqrt{\frac{V_m^2}{T^3} \left(\frac{t^3}{3}\right)_0^T}$

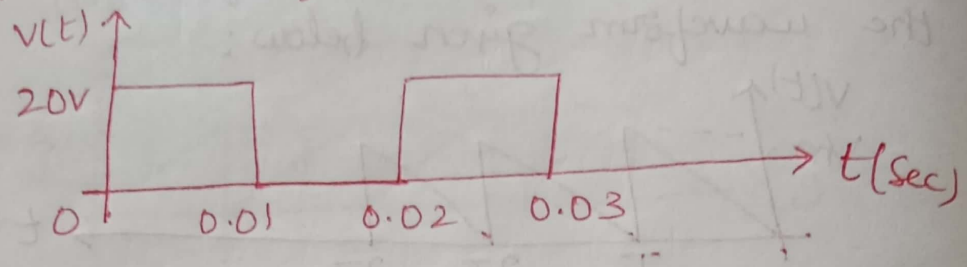
$= \sqrt{\frac{V_m^2}{T^3} \times \frac{T^3}{3}} = \frac{V_m}{\sqrt{3}}$.

Peak factor = $\frac{\text{Peak value}}{\text{RMS value}} = \frac{V_m}{(V_m/\sqrt{3})} = \sqrt{3}$.

Form factor = $\frac{\text{RMS value}}{\text{Average value}} = \frac{V_m/\sqrt{3}}{V_m/2} = \frac{2}{\sqrt{3}}$.

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⑦ Find Peak factor & Form factor



Sol:- $V(t) = 20V \; ; \quad 0 \le t \le 0.01.$

$= 0V \; ; \quad 0.01 \le t \le 0.02.$

(i) $V_{avg} = \dfrac{\int_0^{0.01} V(t).dt}{T} = \dfrac{\int_0^{0.01} 20.dt}{0.02}$

$= \dfrac{20(0.01-0)}{0.02} = 10.$

(ii) $V_{R.M.S} = \sqrt{\dfrac{\int_0^{0.01} (V(t))^2.dt}{T}} = \sqrt{\dfrac{\int_0^{0.01} (20)^2.dt}{0.02}}$

$= \sqrt{\dfrac{(20)^2}{0.02}(0.01)} = \sqrt{\dfrac{400}{2}} = \sqrt{200}$

$= \sqrt{2 \times 100} = \sqrt{2} \times 10$

$= \sqrt{2} \times 10 \times \dfrac{\sqrt{2}}{\sqrt{2}}$

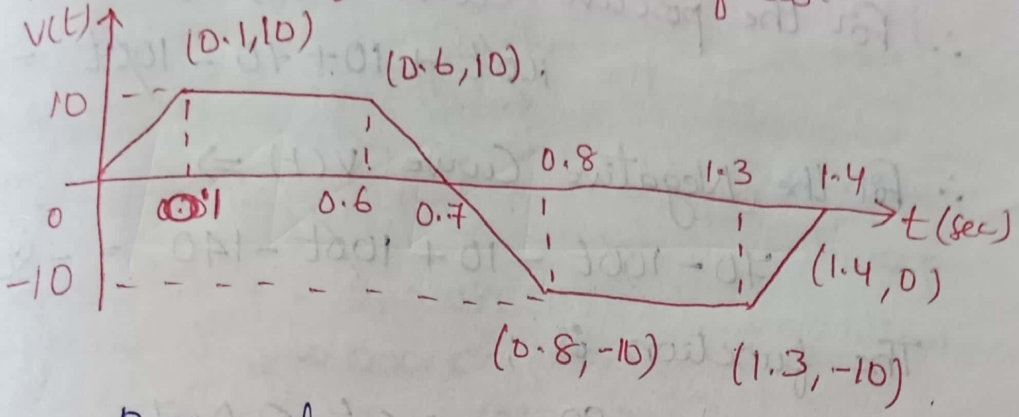
$= \dfrac{20}{\sqrt{2}}$

$\therefore$ Peak factor $= \dfrac{\text{Peak value}}{\text{RMS value}} = \dfrac{20}{20/\sqrt{2}} = \sqrt{2}.$

$\therefore$ Form factor $= \dfrac{\text{RMS value}}{\text{Average value}} = \dfrac{20/\sqrt{2}}{10} = \sqrt{2}.$

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⑧ Find Peak factor & Form factor.



Sol:- Peak value ⇒ $V_m = 10V$.

From points (0, 0) & (0.1, 10), The function is [Slope ⟹ $y = mx$].

(x_1, y_1 ; x_2, y_2)

$$v(t) = \frac{y_2 - y_1}{x_2 - x_1} \cdot t = \left(\frac{10-0}{0.1-0}\right) t = \frac{10}{0.1} t$$

$$= 100t.$$

→ From points (0.1, 10) & (0.6, 10) [Rectangle = 10].

$v(t) = 10$.

→ From points (0.6, 10) & (0.7, 0) [towards x-axis].

(x_1, y_1 ; x_2, y_2)

$$v(t) = \frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}.$$

Here $y = v(t)$; $x = t$.

$$\frac{v(t) - 10}{t - 0.6} = \frac{0 - 10}{0.7 - 0.6} \Rightarrow v(t) - 10 = -100(t - 0.6)$$

$$v(t) - 10 = -100t + 60$$

$$v(t) = 70 - 100t.$$

→ From the points (0.8, -10) to (1.3, -10) [Rectangle].

$v(t) = -10$.

→ From the points (1.3, -10) & (1.4, 0) is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$v(t) + 10 = \frac{0 - (-10)}{1.4 - 1.3}(t - 1.3) \Rightarrow v(t) + 10 = 100(t - 1.3)$$

$$v(t) = 100t - 140$$

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∴ For the positive Curve $v(t) \Rightarrow$

$$100t + 10 + 70 - 100t = 80$$

∴ For the Negative Curve $v(t) \Rightarrow$

$$70 - 100t - 10 + 100t - 140 = -80$$

The function is ;

$$v(t) = 80 \quad , \quad 0 \le t \le 0.7$$
$$= -80 \quad , \quad 0.7 \le t \le 1.4.$$

(i) Average value is

$$V_{avg} = \frac{\int_0^{1.4} v(t)\,dt}{1.4}$$

$$= \frac{1}{1.4}\left[\int_0^{0.7} 80\,dt + \int_{0.7}^{1.4} -80\,dt\right]$$

$$= \frac{1}{1.4}\left[80(0.7) + (-80)(1.4 - 0.7)\right]$$

$$= 0.$$

(ii) ∴ For half wave :-

$$V_{avg} = \frac{1}{0.7}\int_0^{0.7} 80\,dt = \frac{80(0.7)}{0.7} = 80.$$

(iii)

$$V_{RMS} = \sqrt{\frac{1}{1.4}\int_0^{1.4} (v(t))^2\,dt}$$

$$= \sqrt{\frac{1}{1.4}\left[\int_0^{0.7} (80)^2\,dt + \int_{0.7}^{1.4} (-80)^2\,dt\right]}$$

$$= \frac{80}{\sqrt{1.4}}\sqrt{(0.7) + (1.4 - 0.7)}$$

$$= \frac{80}{\sqrt{1.4}} \times \sqrt{0.7 + 0.7} = 80.$$

∴ Peak factor $= \frac{\text{Peak value}}{\text{RMS value}} = 10/80 = 0.125$

Form factor = RMS value / Avg value $= 80/80 = 1.$

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① The equation of alternating quantity $i = 200 \sin 318t$. Determine (a) Max. value. (b) Frequency (c) RMS value (d) Average value. (e) Time period (f) Peak-factor (g) Form factor.

Sol:- Given :- $i = 200 \sin 318t$

$i = 200 \sin \omega t$.

(a) Maximum value $(I_m) = 200\ A$

(b) $\omega = 2\pi f$.

from the problem $\Rightarrow \omega = 318t$

$\therefore 318 = 2\pi f \Rightarrow f = \frac{318}{2\pi} = 50.6\ Hz$.

(c) RMS value $(I_{rms}) = \frac{I_m}{\sqrt{2}} = \frac{200}{\sqrt{2}}$

$= 141.42\ A$

(d) Average value $(I_{avg}) = \frac{2I_m}{\pi} = \frac{2 \times 200}{\pi}$

$= 127.32\ A$.

(e) Time period $(T) = \frac{1}{f} = \frac{1}{50.61} = 0.019$.

(f) Peak factor $(K_p) = \frac{I_m}{I_{rms}} = \frac{200}{141.42} = 1.414$

(g) Form factor $(K_p) = \frac{I_{rms}}{I_{avg}} = \frac{141.42}{127.32}$.

$= 1.11$

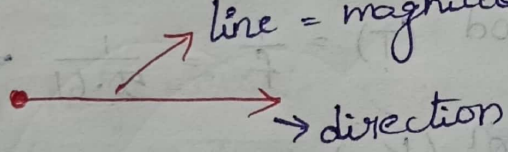


* Phasor :-

In the analysis of a.c. circuits, it is very difficult to solve alternating quantities in terms of waveforms and mathematical equations.

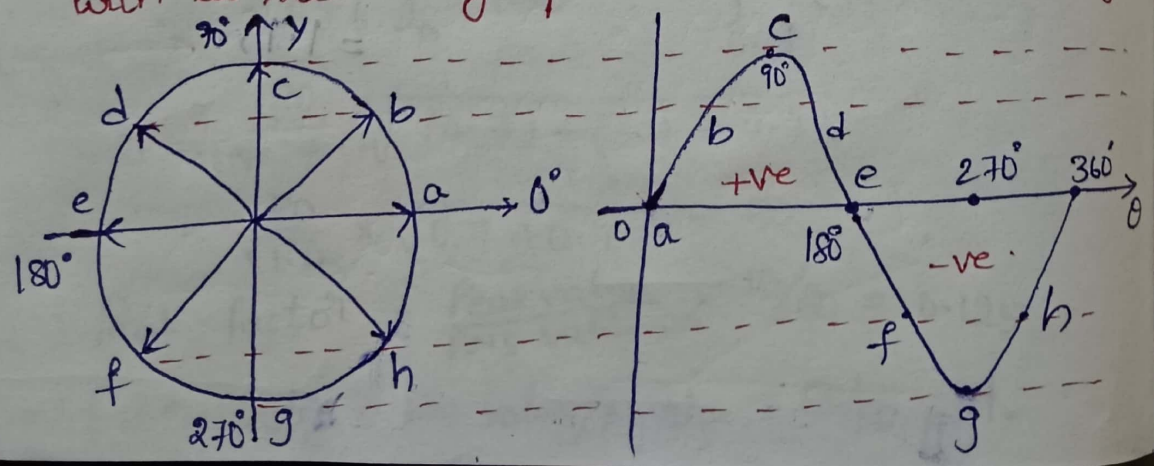
Hence it is necessary to study a method which give an easier way of representing an alternating quantity. Such a representation is called Phasor.

The sinusodial varying alternating quantity can be represented graphically by a straight line with an arrow. The length of the represents the magnitude of the quantity and arrow indicates its direction.



Note :- The phasor are assumed to be rotated in anti-clock wise direction.

* Consider a phasor, rotating in anti-clockwise direction, with uniform angular velocity, with its stationary position "a". as shown in fig :-



From the diagram :-

- O → a is a phasor.

→ At position 'a', $i = 0$

→ At position 'b', $i = I_m \sin\omega t = I_m \sin\theta$

→ " " 'c' = $i = I_m \sin 90^\circ = I_m$.

→ " " 'd' = $i = I_m \sin\theta$

} +ve cycle

→ At position 'f' $\Rightarrow i = -I_m \sin\theta$

→ " " 'g' $i = -I_m$

→ " " h $i = -I_m \sin\theta$

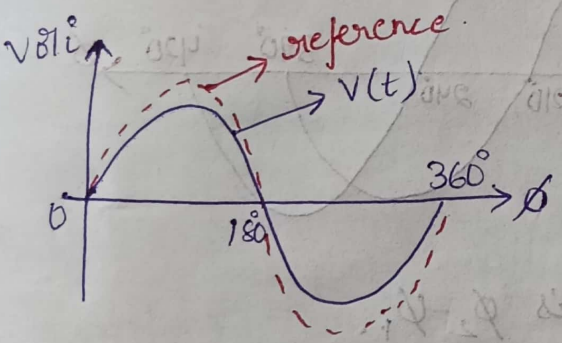
→ " " i $i \Rightarrow 0$.

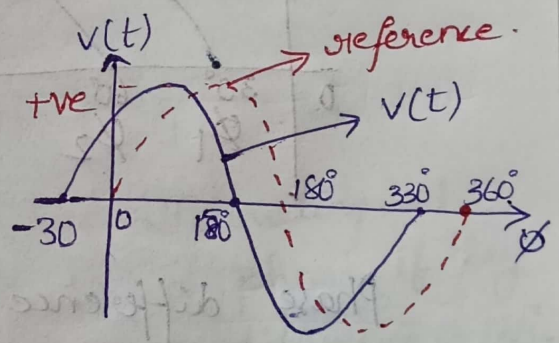
} -ve cycle.

* **Phase :-** It is the fraction of angle through which an alternating quantity is delayed when compared with the reference quantity.

→ In General, the equation of alternating quantity in terms ;

Phase is $i = I_m \sin(\omega t \pm \phi)$ (or) $V = V_m \sin(\omega t \pm \phi)$.



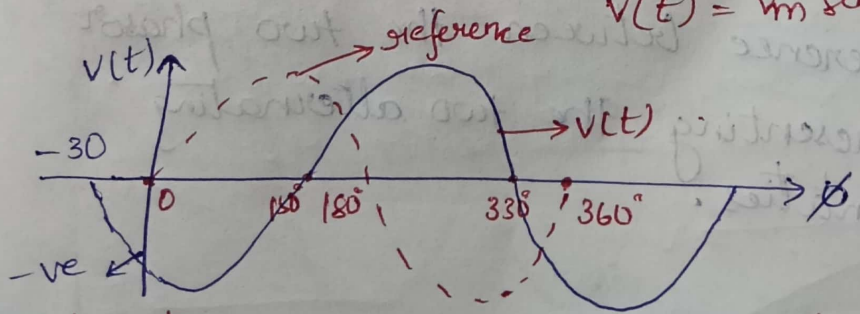


Phase(ϕ) = 0

$\therefore V = V_m \sin\omega t$.

Positive phase.

$V(t) = V_m \sin(\omega t + \phi)$



Negative phase $V(t) = V_m \sin(\omega t - \phi)$.

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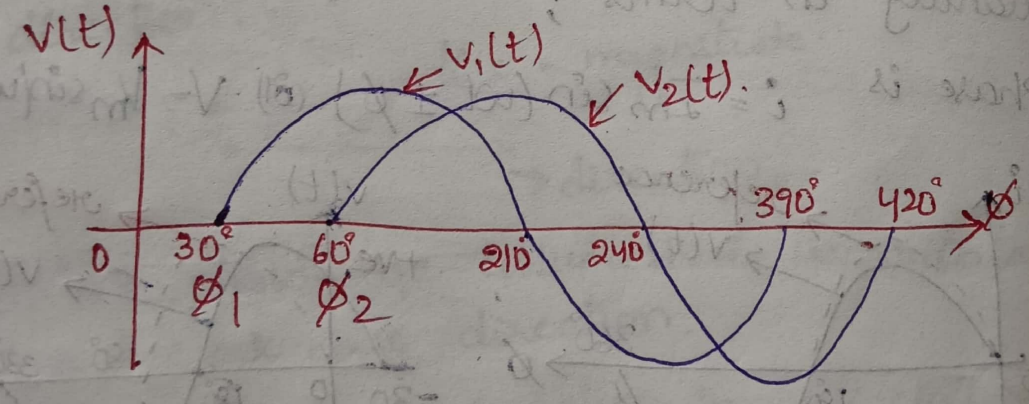
Note :- ① Phase is possible when two alternating quantity have same frequency

② If the frequencies of two alternating quantities are different such quantities phase is not possible.

* Phase difference :-

→ The difference between the phases of the two alternating quantities is called the phase difference.

→ If the two alternating quantities with same frequency have different phase angles, then they have phase difference.



Phase difference is $\phi_2 - \phi_1$.

→ Phase difference is nothing but angle difference between the two phasor representing the two alternating quantities.

① Phase relationships for R, L, C :-

* **In phase :-** When two alternating quantities are said to be in phase then they have same frequency and same phase angles i.e, phase difference is zero.

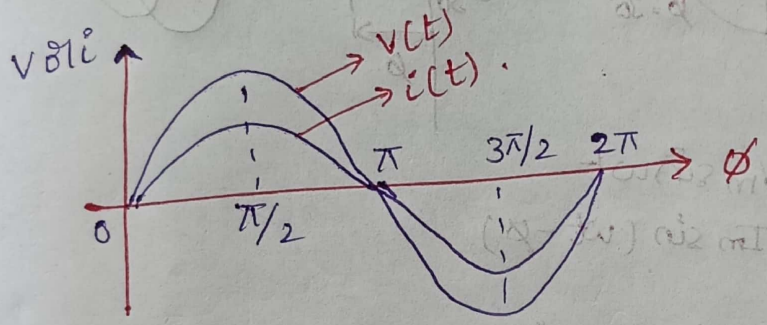
Example :- Resistance (R).

Explanation :- Consider the two alternating quantities having same frequency and different maximum values.

Let $V_0(t) = V_m \sin\omega t$

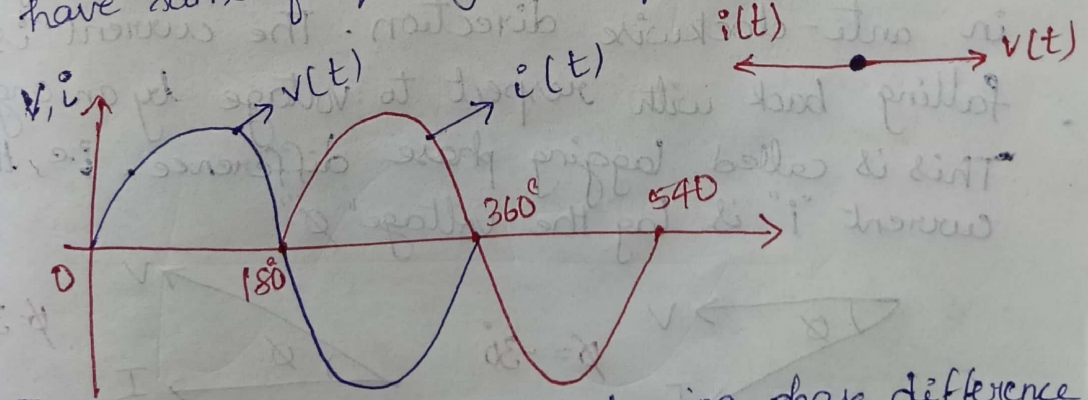
$i(t) = I_m \sin\omega t$

Take $V_m > I_m$.



Phase difference is zero.

* **Out of phase :-** When the two alternating quantities are said to be out of phase if they have same frequency and phase difference is 180°.



From the fig; v(t) & i(t) having phase difference is 180°.

Inductance :-

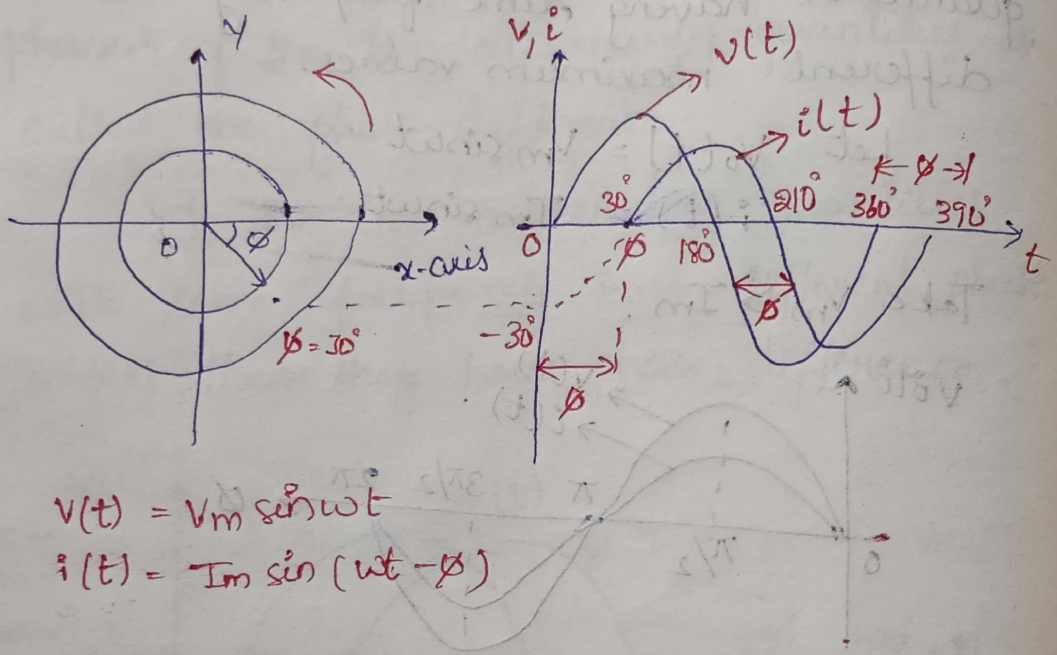
* Lagging Phase :-

If an alternating quantity crosses, its zero position when compared with the zero position of another quantity and if it is later it is called Lagging.

Ex :- Inductance.

Explanation :-

Consider two alternating quantities.

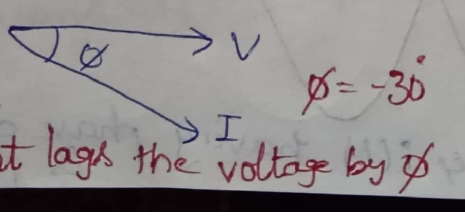


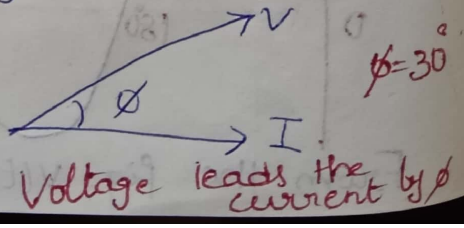
$V(t) = V_m \sin \omega t$

$i(t) = I_m \sin(\omega t - \phi)$

From fig "ϕ" is the phase difference b/w two phasor. When emf "V" at its zero value, current "i" has some negative value.

In the fig, the two phasor are rotating in anti-clockwise direction. The current is falling back with respect to voltage by an angle "ϕ". This is called lagging phase difference. i.e, the current "i" is lag the voltage "ϕ".



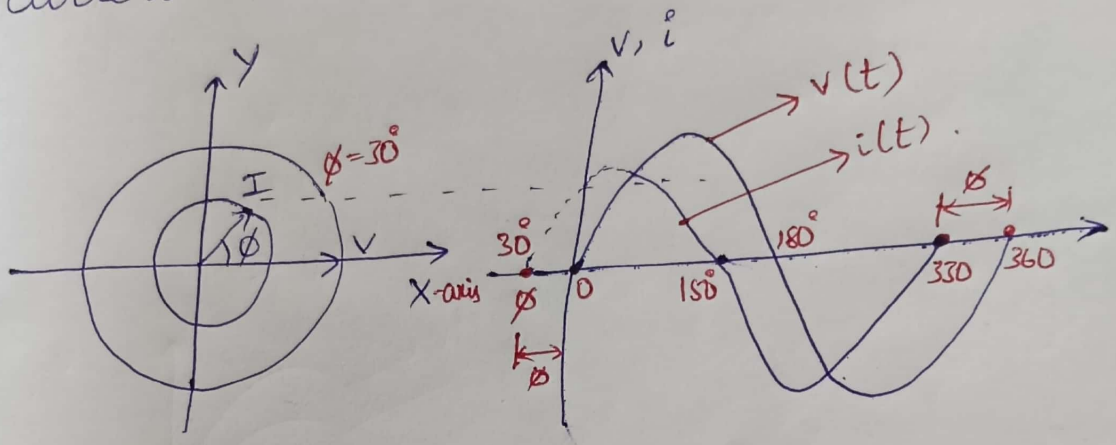


* Leading phase :-

If an alternating quantity crosses its zero position when compared to zero position of another quantity. Then it is before/advanced, it is called leading.

Capacitance.

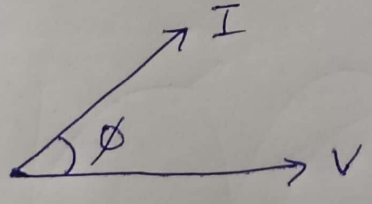
Two alternating quantities $v(t)$ & $i(t)$, when emf of voltage phasor at its zero value, current $i(t)$ has some position value.



here $V(t) = V_m \sin \omega t$

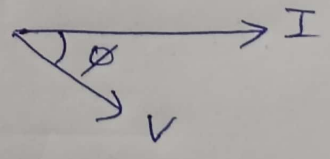
$i(t) = I_m \sin(\omega t + \phi)$

In the above fig, two phasor are rotating anticlockwise direction, current is ahead/advance/before of voltage phasor. Thus current is said to be leading with respect to the voltage and the phasor difference is called leading phase difference.



'I' leads 'V' by ϕ.

$\phi = 30°$



V lags I by ϕ

$\phi = -30°$.

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2.8 Phasor (or) Vector Algebra

A phasor diagram is a graphical representation of the phasors (i.e.voltages and currents) of an a.c circuit and may not yield quick results in case of complex circuits. Hence, the technique used to represent a phasor in an algebraic (i.e., mathematical) form is known as ***phasor algebra* or complex algebra***.

A complex number represents a point in a two dimensional plane located with reference to two distinct axis (i.e., X – axis and Y–axis or horizontal and vertical axis). The horizontal axis is called ***real axis***, while the vertical axis is called the ***imaginary axis*** (see in Fig. 2.39). The symbol j (or some times i) is used to denote an imaginary number

---

* 1. Phasor algebra has provided a relatively simple but power ful tool for obtaining quick solution of a a.c. circuits.

2. In mathematics $\sqrt{-1}$ is denoted by '*i*' but in electrical engineering '*j*' is adopted because letter is used for representing current. This helps to avoid confusion.

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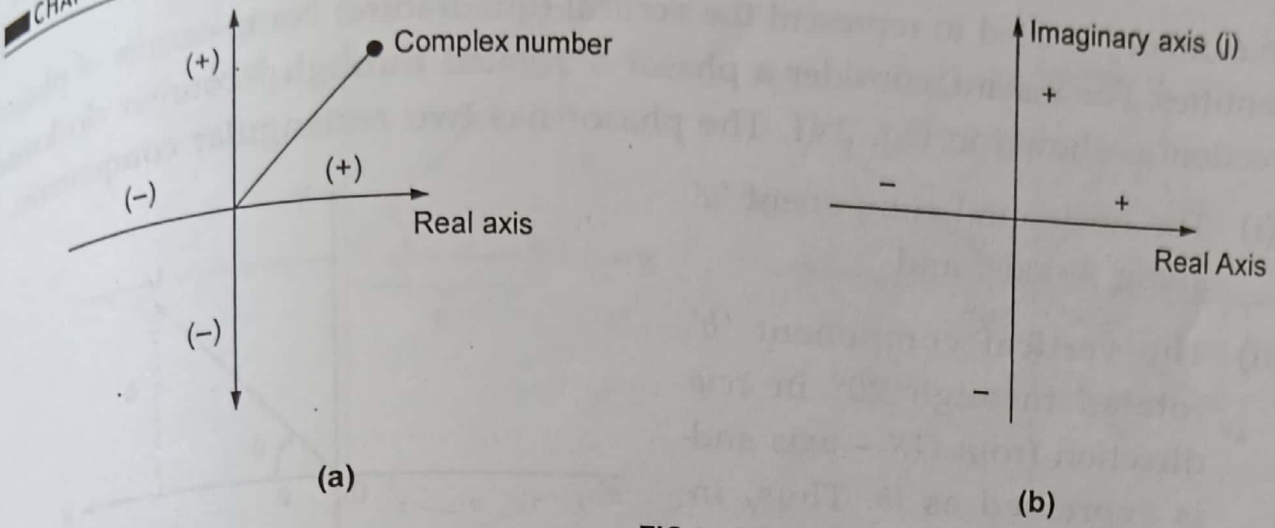


FIG 2.39 :

2.8.1 Operator – J

Consider phasor V lying along X–axis as shown in Fig. 2.40. This phasor 'V' is reversed when it is multiplied by – 1 i.e., the phasor is rotated through 180° in the counter clockwise (ccw) direction. Let us consider j as a factor.

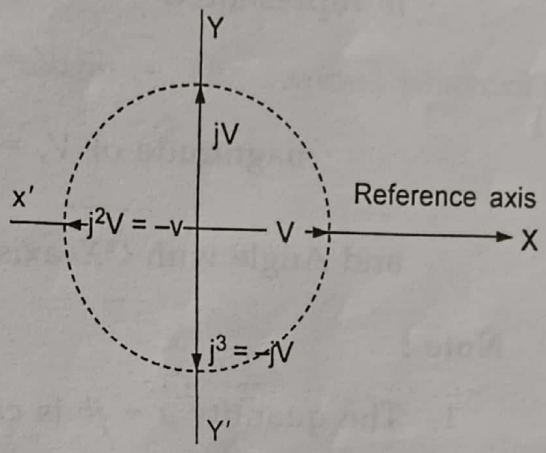


FIG 2.40 :

Which when multiplied by the phasor 'V' the phasor is rotated through 90 in ccw direction. This means that multiplying the phasor by j^2 is the same as multiplying by–1. It follows, therefore, that

$$j^2 = -1 \qquad \text{(or)} \qquad j = \sqrt{-1}$$

Thus, it is concluded that j is just an operator which is when multiplied with a phasor, it shows that the phasor is rotated through 90° in ccw direction. Each successive multiplication of j, rotates the phasor further by 90° as given below.

$j = \sqrt{-1}$ ..... 90° ccw rotation from OX – axis

$j^2 = -1$..... 180° ccw rotation from OX–axis

$j^3 = j^2.j = (-1).\ j = -\ j$..... 270° ccw rotation from OX–axis

$j^4 = j^2.\ j^2 = (-1)\ (-1) = 1$..... 360° ccw rotation from OX–axis

$j^5 = j^3.\ j^2 = (-j)\ (-1) = j$

It should also be noted that,

$$\frac{1}{j} = \frac{j}{j^2} = \frac{j}{-1} = -j$$

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The symbol j is used to represent the vertical (quadrature) comp-onents of phasor quantities. For instant, consider a phasor V rotated through θ^o counter clockwise direction as shown in Fig. 2.41. The phasor has two rectangular components,

(i) The horizontal component 'a' along X–axis, and

(ii) The vertical component 'b' rotated through 90^o in ccw direction from OX – axis and is expressed as jb. Thus, in rectangular form, the phasor V is represented

$$V = a + jb$$

magnitude of V, $= \sqrt{a^2+b^2}$

and Angle with OX–axis, $\theta = tan^{-1}\left(\frac{b}{a}\right)$

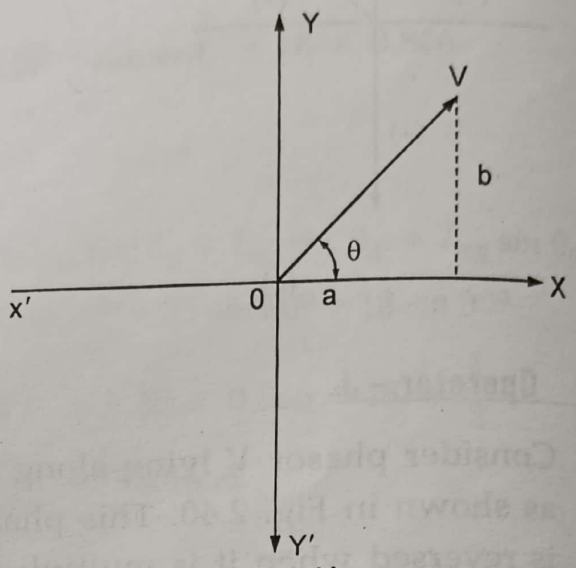


FIG 2.41 :

Note :

1. The quantity $a + jb$ is called a ***complex number*** or ***complex quantity.***
2. The horizontal component i.e. 'a' is called the ***in phase*** or ***active component.***
3. The vertical component i.e., 'b' is called ***quadrature*** or ***reactive component.***

2.8.2 Application of 'J' Notation for Alternating Quantities

There are four principal ways of representing a phasor in the mathematical form viz.

1. Rectangular form or complex form or cartesian form or symbolic notation
2. Trigonometrical form
3. Polar form
4. Exponential form.

1. Rectangular Form : The format for the rectangular form is

$$V = a + jb$$

as shown in Fig. 2.42 (a)

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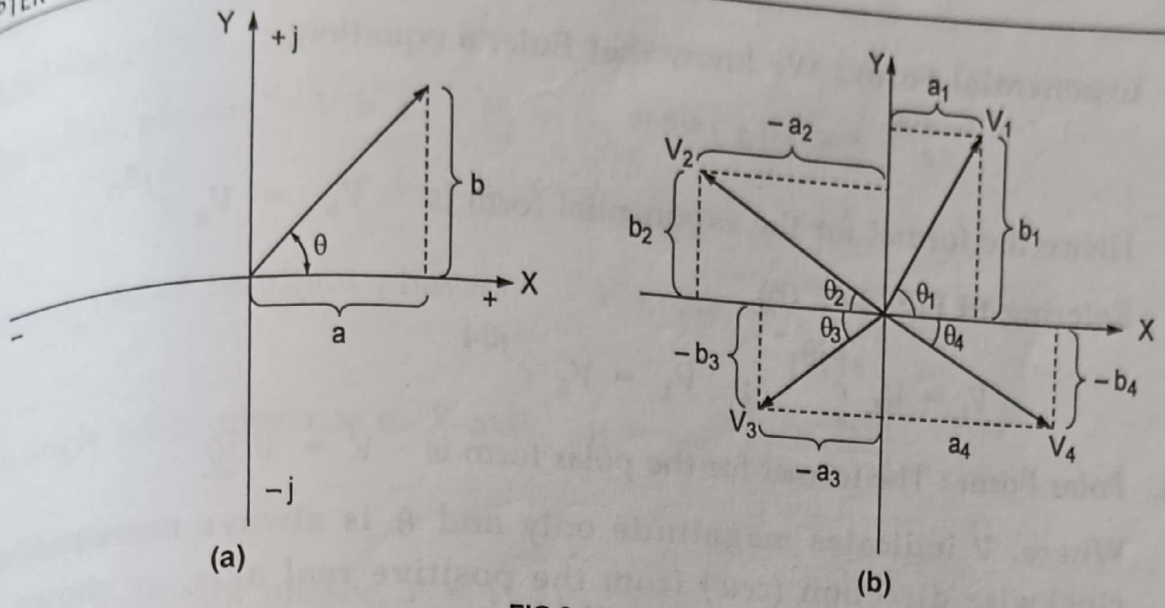


FIG 2.42 :

Fig. 2.54 shows the phasors in different quadrants. The various phasors can be represented in the rectangular form as :

$$V_1 = a_1 + jb_1$$

$$V_2 = -a_2 + jb_2$$

$$V_3 = a_3 - jb_3$$

$$V_4 = a_4 - jb_4$$

Hence, the general format for the rectangular form is also written as

$$V_n = \pm a_n \pm jb_n \text{ where, } n = 1,2,$$

2. **Trigonometrical Form :** The format for the trigonometrical form is

$$V = V(\cos\theta + j\sin\theta) \qquad \ldots\ldots\ldots\ldots (1)$$

as shown in Fig. 2.43.

Equation (1) is equivalent to the rectangular form $V = a + jb$, where $a = V\cos\theta$ and $b = V\sin\theta$. With reference Fig. 2.42 (b)

$$V_1 = V_1(\cos\theta_1 + j\sin\theta_1)$$

$$V_2 = V_2(-\cos\theta_2 + j\sin\theta_2)$$

$$V_3 = V_3(-\cos\theta_3 - j\sin\theta_3)$$

$$V_4 = V_4(\cos\theta_4 - j\sin\theta_4)$$

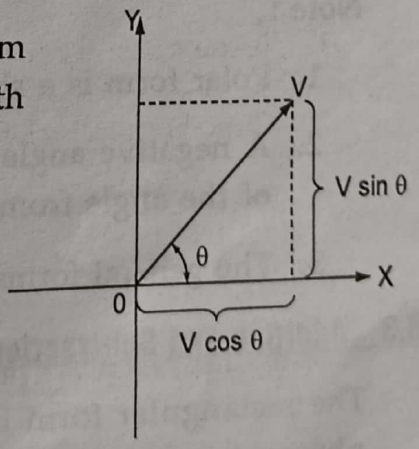


FIG 2.43 :

Hence the general format is writter as

$$V_n = V_n(\pm\cos\theta_n \pm j\sin\theta_n), \text{ where } n = 1, 2, \ldots\ldots\ldots$$

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3. **Exponential Form :** We know that Euler's equation,

$$e^{\pm j\theta} = \cos\theta \pm j \sin\theta$$

Hence the format for the exponential form is $V_n = V_n e^{\pm j\theta_n}$

Refering to Fig. 2.42 (b)

$$V_1 = V_1 e^{+j\theta_1} \quad ; \quad V_4 = V_4 e^{-j\theta 4}$$

4. **Polar Form :** The format for the polar form is $V = V \angle \theta$

Where, V indicates magnitude only and θ is always measured counter clockwise direction (*ccw*) from the positive real axis, as shown in Fig. 2.44 (a). A negative sign has the effect shown in Fig. 2.44 (b).

$$-V = -V\angle\theta = V\angle\theta \pm \pi$$

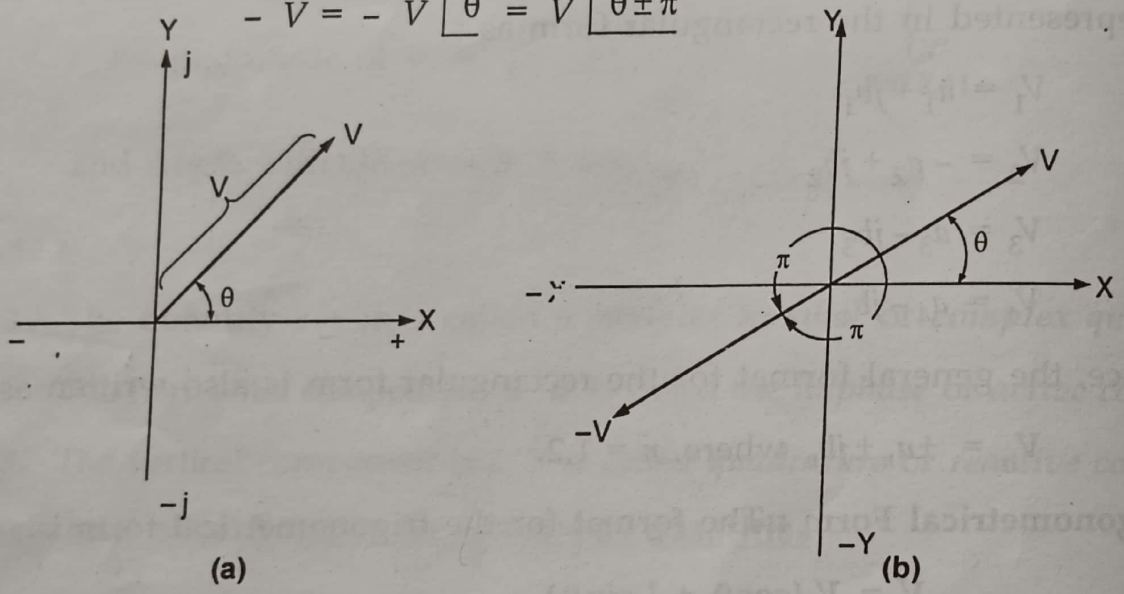


FIG 2.44 :

Note :

1. Polar form is a short hand form of trigonometrical form.
2. A negative angle in the polar form indicates clockwise measurement of the angle from the reference axis.
3. The general format of polar form is, $V = V\angle \pm\theta$

2.8.3 Addition and Subtraction of Alternating Quantities

The rectangular form is the best suited for the addition and subtraction of phasor quantities.

Consider two phasor quantities as :

$$V_1 = a_1 + Jb_1 \text{ and } V_2 = a_2 - jb_2$$

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(i) **Addition :**

Resulting phasor, $V = V_1 + V_2 = (a_1 + jb_1) + (a_2 - jb_2)$

$= (a_1 + a_2) + j\,(b_1 - b_2)$

Magnitude of resultant phasor, $V = \sqrt{(a_1+b_2)^2 + (b_1-b_2)^2}$

Its angle with reference to X-axis, $\theta = tan^{-1}\left[\frac{(b_1-b_2)}{(a_1+a_2)}\right]$

(ii) **Subtraction :**

Resultant phasor, $V = V_1 - V_2$

$= (a_1 + jb_1) - (a_2 - j\,b_2) = (a_1 - a_2) + j\,(b_1 + b_2)$

Magnitude of resultant phasor, $V = \sqrt{(a_1-a_2)^2 + (b_1+b_2)^2}$

Its angle with reference X-axis is, $\theta = tan^{-1}\left[\frac{(b_1+b_2)}{(a_1-a_2)}\right]$

2.8.4 Multiplication and Division of Phasors

The polar form is the best suited for multiplication and division of phasor quantities. Consider two phasor quantities as :

$$V_1 = a_1 + jb_1 \text{ and } V_2 = a_2 - jb_2$$

1. **Multiplication :** To multiple the phasors that are either in polar form or in rectangular form, multiply their magnitues and add the angles (algebraically).

(a) In rectangular form :

Resultant phasor,

$$V = V_1 \times V_2 = (a_1 + jb_1)\,(a_2 - jb_2)$$

$$= a_1\,a_2 + jb_1\,a_2 - ja_1\,b_2 - jb_1{}^2\,b_2$$

$$= (a_1\,a_2 + b_1\,b_2) + j\,(a_1\,b_2 - a_2\,b_1)\ (\because j^2 = -1)$$

Magnitude of resultant phasor, $V = \sqrt{(a_1\,a_2+b_1\,b_2)^2 + (a_1\,b_2-a_2\,b_1)^2}$

Its angle with reference to X-axis, $\theta = tan^{-1}\left[\frac{(a_1\,b_2-a_2\,b_1)}{(a_1\,a_2+b_1\,b_2)}\right]$

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(b) In Exponential Form :

$$V_1 = a_1 + jb_1 = V_1 e^{j\alpha}$$

$$V_2 = a_2 - jb_2 = V_2\, e^{-j\beta}$$

Resultant phasor, $V = V_1 \times V_2$

$$= (V_1 e^{j\alpha}) \times (V_2\, e^{-j\beta}) = V_1\, V_2\, e^{j(\alpha-\beta)}$$

$$V_1 = a_1 + j\, b_1 = V_1 \angle \alpha \text{ and } V_2 = a_2 - jb_2 = V_2 - \angle \beta$$

$\therefore$ Resultant phasor,

$$V = V_1 \times V_2 = (V_1 \angle \alpha)\ (V_2 \angle \beta) = V_1\, V_2 \angle \alpha - \beta$$

2. Division :

(a) In Rectangular form :

Resultant phasor,

$$V = \frac{V_1}{V_2} = \frac{a_1 + jb_1}{a_2 - jb_2} = \frac{(a_1 + jb_1)}{(a_2 - jb_2)} \times \frac{(a_2 + jb_2)}{(a_2 + jb_2)}$$

$$= \frac{(a_1 a_2 - jb_1 b_2) + j(a_1 b_2 + a_2 b_1)}{a_2^2 - (jb_2)^2}$$

$$= \left[\frac{a_1 a_2 - b_1 b_2}{a_2^2 + b_2^2}\right] + j\left[\frac{a_1 b_2 + b_2 b_1}{a_2^2 + b_2^2}\right]$$

$$= a + j\, b \text{ say}$$

$\therefore$ Magnitude of $V = \sqrt{a^2 + b^2}$

Its angle with reference to X – axis is, $\theta = tan^{-1}\left(\frac{b}{a}\right)$

(b) In Exponential Form : To divide the phasors that are in exponential form, divide the magnitude of phasors and subtract the denominator angle from the numerator angle.

$$V = \frac{V_1}{V_2} = \frac{V_1 e^{j\alpha}}{V_2 e^{-jb}} = \frac{V_1}{V_2} . e^{j(\alpha+\beta)}$$

(c) In Polar Form : To divide the phasor that are in polar form, divide the magnitude of phasors and subtract the denominator angle from the numerator angle :

$$V = \frac{V_1}{V_2} = \frac{V_1 \angle \alpha}{V_2 \angle -\beta} = \frac{V_1}{V_2} \angle \alpha + \beta$$

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2.8.5 Conjugate of a Complex Number

The conjugate of a complex number A is defined as one whose real part is equal to the real part of A, and whose imaginary part is equal to the negative of the imaginary part of A. The conjugate of A is denoted by A^*.

For examplex consider a complex number $A = a + j\,b = A\,\theta$.

Then its conjugate will be $A^* = a - j\,b = A\lfloor -\theta$. The two conjugate numbers are shown graphically in Fig. 2.45.

The conjugate numbers have the following properties :

1. **Addition :** $A + A^* = (a + jb)\,(a - jb) = 2a$ the resultant is the sum of two horizontal components only
2. **Subtraction :** $A - A^* = (a + jb) - (a - jb) = j\,2b$. the result is the sum of two vertical components only
3. **Multiplication :** $A \times A^* = (a + jb)\,(a - jb) = a^2 - j^2\,b^2 = a^2 + b^2$ the result contains no quadrature component.

The conjugate of a complex number is used in determining the apparent power of an *a.c* circuit in complex form.

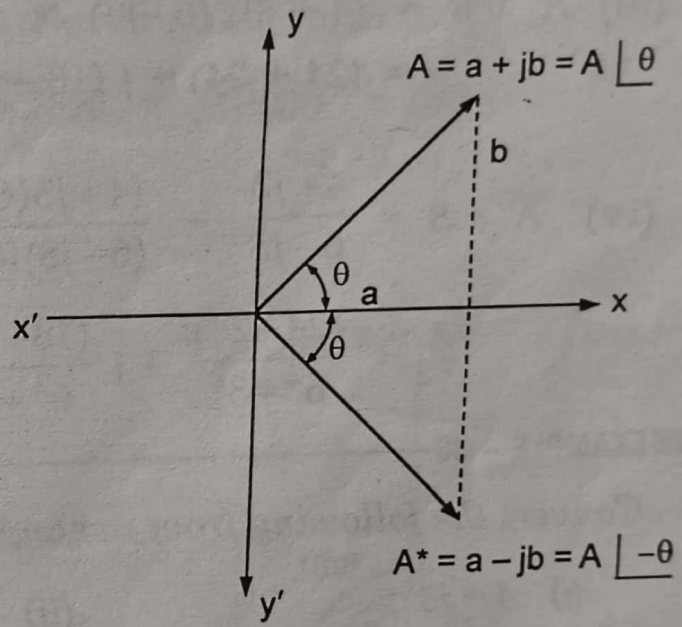


FIG 2. 45 :

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2.5.6 Average Value of Half Wave Rectified Sine Wave

OCT/NOV. 2009

In half wave rectified sine wave, an alternating current flows only during half cycle (i.e.,0 to π) and another half cycle has been suppressed (i.e., nagative half cycle, π to 2π). as shown in Fig. 2.18 (b).

To calculate the average value, the instantaneous value of current have to be carried over the half cycle, i.e. from 0 to π, but averaged out for the whole cycle, i.e.,from 0 to 2π.

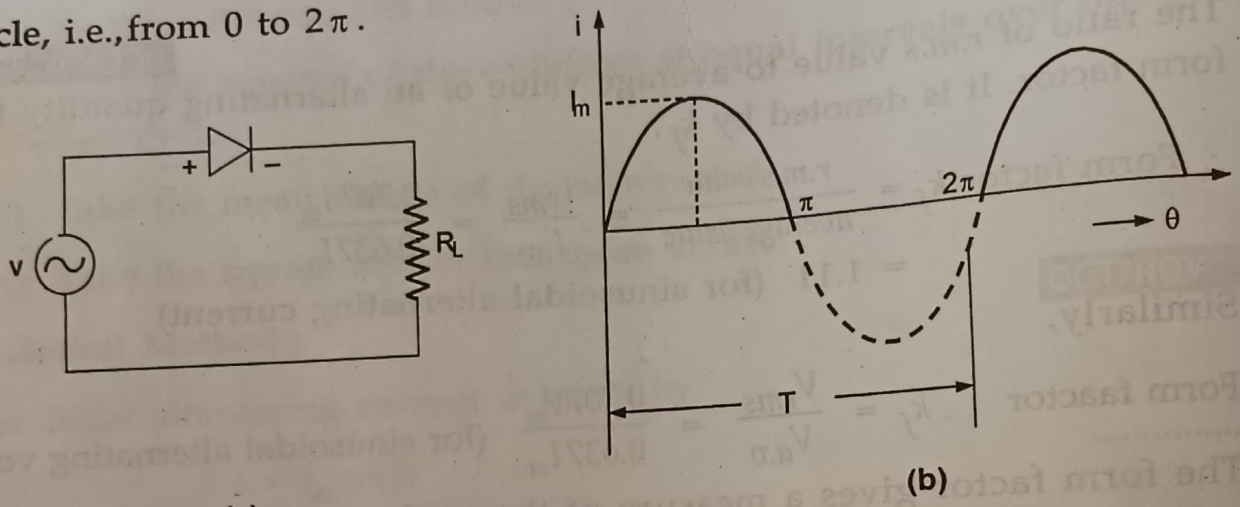


FIG 2.18 :

Sinusoidal alternating current is given by

$$i = I_m \sin\theta \qquad \ldots\ldots\ldots\ldots (1)$$

$$I_{av} = \frac{\text{Area of half cycle}}{\text{Base}} = \frac{\int_0^{\pi} i\, d\theta}{2\pi}$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} I_m \sin\theta\, d\theta\right] = \frac{I_m}{2\pi}\left[-\cos\theta\right]_0^{\pi} = \frac{I_m}{2\pi}\left|(-\cos\pi)-(-\cos 0^\circ)\right|$$

$$= \frac{I_m}{2\pi}\left[-(-1)-(-1)\right] = \frac{I_m}{2\pi}\times 2 = \frac{I_m}{\pi}$$

$$\therefore \quad I_{a.v} = \frac{I_m}{\pi} = 0.318\, I_m$$

Similarly, $V_{a.v} = \frac{V_m}{\pi} = 0.318\, V_m$

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2.5.7 R.M.S Value of Half Wave Rectified Sine Wave

A half wave alternating current wave is shown in Fig.

Time period of the wave (i.e. base) is 2π

$$\therefore \quad I_{rms} = \frac{Area\ of\ half\ cycle\ squared\ wave}{Base\ length\ of\ (half\ cycle)}$$

$$= \sqrt{\frac{\int_0^{\pi} i^2\, d\theta}{2\pi}} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} I_m^2 \sin^2\theta\ d\theta} = \sqrt{\frac{I_m^2}{2\pi}\int_0^{\pi} \sin^2\theta\, d\theta}$$

$$= \sqrt{\frac{I_m^2}{2\pi}\int_0^{\pi}\left[\frac{1-\cos 2\theta}{2}\right]} \quad \left[\because \sin^2\theta = \frac{1-\cos 2\theta}{2}\right]$$

$$= \sqrt{\frac{I_m^2}{4\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}} = \sqrt{\frac{I_m^2}{4\pi}\left[\left(\pi - \frac{\sin 2\pi}{2}\right) - \left(0 - \frac{\sin 2\times 0^{\circ}}{2}\right)\right]}$$

$$= \sqrt{\frac{I_m^2}{4}\left[(\pi - 0) - 0\right]} = \sqrt{\frac{I_m^2}{4\pi}\times\pi} = \sqrt{\frac{I_m^2}{4}}$$

$$\therefore \quad I_{rms} = \sqrt{\frac{I_m^2}{4}} = \frac{I_m}{2} = 0.5\, I_m$$

Similarly, $V_{rms} = \frac{V_m}{2} = 0.5\, V_m$

2.5.8 Form Factor and Peak Factor For Half Wave Rectified Sine Wave

$$\text{Form factor } (K_f) = \frac{R.M.S\ value}{Average\ value}$$

$$\therefore \quad K_f = \frac{I_{rms}}{I_{av}} = \frac{I_m/2}{I_m/\pi} = \mathbf{1.57}$$

$$\text{Peak factor } (Ka) = \frac{maximum\ value}{R.M.S\ value}$$

$$\therefore \quad K_a = \frac{I_m}{\left(\frac{I_m}{2}\right)} = \mathbf{2}$$

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The electric main in a house is marked as 230V, 50Hz. Write down the equation for instantaneous voltage in sinusoidal form.

Solution :

Given data :

$$V_{rms} = 230V \; ; \qquad f = 50\ Hz$$

We know that, the formate of the instantaneous voltage in sinusoidal form is

$$v = V_m \sin \omega t$$

$$\therefore \quad \omega = 2\pi f = 2 \times \pi \times 50 = 314$$

$$V_m = \sqrt{2} \times V_{rms} \left[\because V_{rms} = \frac{V_m}{\sqrt{2}} \right] = \sqrt{2} \times 230 = 325.27 \text{ Volt.}$$

$\therefore$ The equation for instantaneous voltage in sinusoidal form is

$$v = 325.27 \sin 314\, t \quad \textbf{Ans}$$

EXAMPLE – 9

An alternating current is represented by $i = 28.2 \sin 100\ \pi t$ amp. Determine the R.M.S value and form factor.

Solution : NOV/DEC. 2003

Given data :

$$i = 28.2 \sin 100\ \pi t \qquad \ldots\ldots\ldots\ldots (1)$$

$$I_{rms} = ?;\ \text{form factor} = ?$$

Compare equation (1) with standard form i.e,

$$i = I_m \sin \omega t = I_m \sin 2\pi ft$$

$$\therefore \quad 2\pi f = 100\pi \Rightarrow f = \frac{100\pi}{2\pi} = 50\ Hz.$$

$$I_m = 28.2\ A$$

$$I_{av} = 0.637\ I_m = 0.637 \times 28.2 = 17.96\ A$$

$$\therefore \quad I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{28.2}{\sqrt{2}} = \mathbf{19.94\ A} \quad \textbf{Ans}$$

Form factor $K_f = \dfrac{I_{rms}}{I_{avg}} = \dfrac{19.96}{17.96} = 1.11$ **Ans**

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EXAMPLE – 10

Calculate the r.m.s value, average value, peak factor and form factor of sinusoidal voltage of maximum value of 100V.

Solution :

Given data :

$V_m = 100V$ Peak factor, K_a = ?

V_{rms} = ? Form factor, K_f = ?

V_{av} = ?

(i) $V_{rms} = \frac{V_m}{\sqrt{2}} = 0.707\ V_m = 0.707 \times 100 = \mathbf{70.7V}$ **Ans**

(ii) $V_{av} = \frac{2V_m}{\pi} = 0.637\ V_m = 0.637 \times 100 = \mathbf{63.7V}$ **Ans**

(iii) Peak factor, $K_a = \frac{V_m}{V_{rms}} = \frac{100}{70.7} = \mathbf{1.414}$ **Ans**

(iv) Form factor, $K_f = \frac{V_{rms}}{V_{av}} = \frac{V_{rms}}{V_{av}} = \frac{70.7}{63.7} = \mathbf{1.11}$ **Ans**

EXAMPLE – 11

An alternating curent is mathematically expressed as i = 150 sin (314t) amp. Determine, (i) maximum value, (ii) r.m.s value, (iii) radian through which its vector has gone, when t = 0.001sec.

Solution :

Given data :

$i = 150\ sin\ (314\ t)$ ..... (1)

$t = 0.001$ sec.

Compare equations (i) with standard equation

$i = I_m\ sin\ \omega t$

$\therefore \quad \omega = 2\pi f = 314\ rad/sec.$

(i) $I_m = \mathbf{150\ A}$ **Ans**

(ii) $I_{rms} = \frac{I_m}{\sqrt{2}} = 0.707\ I_m = 0.707 \times 150 = \mathbf{106.05\ A}$ **Ans**

(iii) $\theta = \omega t = 314 \times 0.001 = \mathbf{0.314\ rad}$ **Ans**

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EXAMPLE – 12

A sine wave has peak value of 50V. Determine

(i) R.M.S value (ii) Average value (iii) Form factor (iv) Peak factor

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Solution :

Given data :

$$V_m = 50V$$

(i) $V_{rms} = 0.707\ V_m = 0.707 \times 50 = 35.36\ V$ **Ans.**

(ii) $V_{av} = 0.637\ V_m = 0.637 \times 50 = 31.86\ V$ **Ans.**

(iii) Form factor $= \dfrac{V_{rms}}{V_{av}} = \dfrac{35.36}{31.86} = 1.11$ **Ans.**

(iv) Peak factor $= \dfrac{V_m}{V_{rms}} = \dfrac{50}{35.36} = 1.414$ **Ans.**

EXAMPLE – 13

An alternating voltage $v = 200 \sin 314t$ is applied to a device which offers an ohmic resistance of 25Ω to the flow of current in one direction while entirely preventing the flow of current in the opposite direction. Calculate the r.m.s value, average value, and form factor.

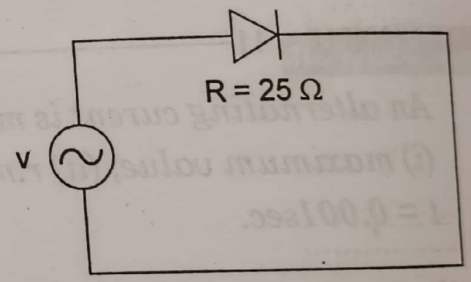
Solution :

Given data :

$$V = 200 \sin 314\ t$$

$$R = 25\ \Omega$$

$$V_{rms} = ?\ V_{av} = ?,\ K_f = ?$$



(a)

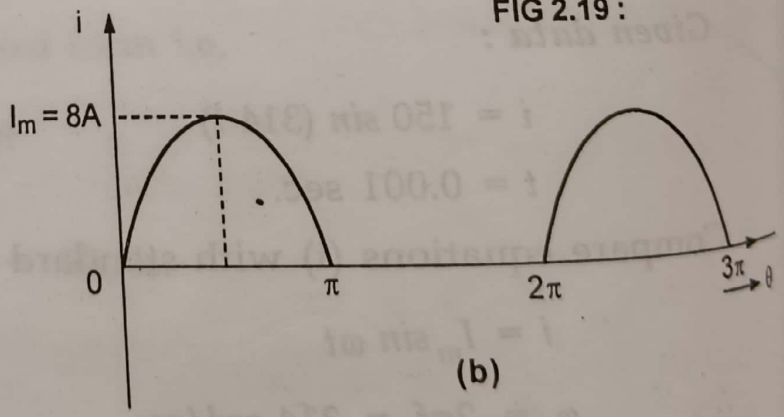
FIG 2.19 :

It is clear that the device is doing half wave rectification.

The maximum value of the rectified current is

$$I_m = \frac{E_m}{R} = \frac{200}{25} = 8\ A$$

Form half wave rectified a.c.



(b)

FIG 2.19 :

(i) $I_{rms} = \dfrac{I_m}{2} = 0.5\ I_m = 0.5 \times 8 = \mathbf{4A}$ **Ans**

(ii) $I_{av} = \dfrac{I_m}{\pi} = 0.318\ I_m = 0.318 \times 8 = \mathbf{2.544\ A}$ **Ans**

(ii) Form factor $= K_f = \dfrac{I_{rms}}{I_{av}} = \dfrac{4}{2.544} = \mathbf{1.572\ A}$ **Ans**

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EXAMPLE – 14

A certain waveform has a form factor of 1.2 and a peak factor of 1.5. If the maximum value is 100, find the r.m.s value and the average value.

Solution :

Given data :

Form factor, $K_f = 1.2$

Peak factor, $K_a = 1.5$ R.M.S value = ?

Maximum value = 100 Average value = ?

we know that,

$$\text{Peak factor, } Ka = \frac{maximum\ value}{R.M.S\ value}$$

$$\therefore \text{ R.M.S value} = \frac{maximum\ value}{K_a} = \frac{100}{1.5} = \mathbf{66.67}\ \mathbf{Ans}$$

$$\text{Average value} = \frac{R.M.S\ value}{form\ factor} = \frac{66.67}{1.2} = \mathbf{55.55}\ \mathbf{Ans}$$

EXAMPLE – 15

Find the form factors of the half - wave rectified sine wave shown in Fig.

OCT/NOV. 2006

Solution :

Given data from Fig.

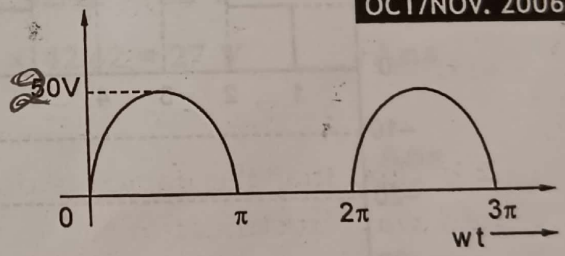


FIG 2.22 :

$$v = V_m \sin wt$$

$$= 20 \sin wt \quad \text{for } 0 < wt < \pi$$

$$= 0 \quad \text{for } \pi < wt < 2\pi$$

$$T = 2\pi$$

$$V_{av} = \frac{1}{T}\int_0^T V_m \sin wt\, d(wt)$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} 20 \sin wt\, d(wt) + \int_0^{\pi} 0\, d(wt)\right]$$

$$= \frac{20}{2\pi}\int_0^{\pi} \sin wt\, d(wt) = \frac{20}{2\pi}\left[-\cos wt\Big|_0^{\pi}\right] = \frac{20}{2\pi} \times 2 = 6.36\ V$$

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$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{\pi}(V_m \sin wt)^2 d(wt)} = \sqrt{\frac{V_m^2}{2\pi}\int_0^{\pi}\sin^2 wt\, d(wt)}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\int_0^{\pi}\left(\frac{1-\cos 2wt}{2}\right) d(wt)} = \sqrt{\frac{V_m^2}{4\pi}\left[wt - \frac{\sin 2wt}{2}\right]_0^{\pi}}$$

$$= \sqrt{\frac{V_m^2}{4\pi}(\pi - 0)} = \sqrt{\frac{V_m^2}{4}} = \frac{V_m}{2} = \frac{20}{2} = 10V$$

$$\text{Form factor} = \frac{V_{rms}}{V_{av}} = \frac{10}{6.36} = 1.572 \quad \textbf{Ans}$$

EXAMPLE – 16

A current has the following steady values in amperes for equal intervals of time changing instantaneously from one value to the next : 0, 10, 20, 30, 20, 10, 0, – 10, 20, – 30, – 20, – 10, 0 etc. *Calculate.*

(i) Everage value (ii) R.m.s value (iii) Form factor (iv) Peak factor

Solution :

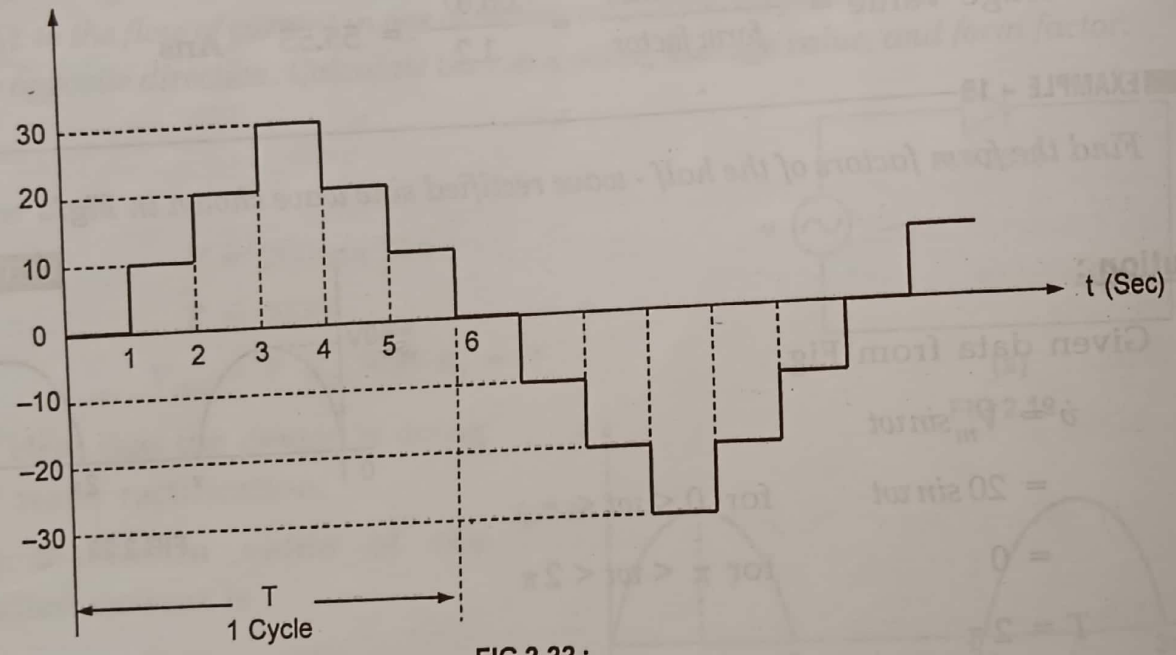


FIG 2.22 :

(i) Since the wave is symmetrical, hence only half cycle is enough to calculate r.m.s value and average value.

we know that,

$$I_{av} = \frac{i_1 + i_2 + \ldots + i_n}{n} = \frac{0+10+20+30+20+10}{6} \quad (\because n = 6)$$

$$= \frac{90}{6} = 15\,A \quad \textbf{Ans.}$$

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(ii) $I_{rms} = \sqrt{\frac{i_1^2 + i_2^2 + \ldots\ldots + i_n^2}{n}} = \sqrt{\frac{0^2 + 10^2 + 20^2 + 30^2 + 20^2 + 10^2}{6}}$

$= \sqrt{\frac{1900}{6}} = 17.79\ A$ **Ans.**

(iii) Form factor $= \frac{I_{rms}}{I_{av}} = \frac{17.79}{15} = 1.186$ **Ans.**

(iv) Peak factor $= \frac{I_m}{I_{rms}} = \frac{30}{17.79} = 1.686$ **Ans.**

EXAMPLE – 17

Determine (a) R.M.S value (b) average value (c) Frequency (d) form factor of an alternating quantity, $v\ (t) = 42.42\ sin\ 628\ t$.

MARCH/APRIL. 2007

Solution :

Given data :

$$v\ (t) = 42.42\ sin\ 628\ t \quad \ldots\ldots\ldots\ldots\ (1)$$

Compare equation (1) with $v(t) = V_m\ sin\ wt$.

$$V_m = 42.42\ volt,\ w = 628.$$

(a) R.M.S value, $V_{rms} = 0.707\ V_m = 0.717 \times 42.42 = 30\ V$ **Ans**

(b) Average value, $V_{av} = 0.637\ V_m = 0.637 \times 42.42 = 27\ V$ **Ans**

(c) $2\pi f = w$ (or) $f = \frac{2}{2\pi} = \frac{628}{2\pi} = 100\ Hz$ **Ans**

(d) Form factor $= \frac{V_{rms}}{V_{av}} = \frac{30}{27} = 1.11$ **Ans**

EXAMPLE – 18

The current in a circuit obeys the load $i = 100\ sin\ wt$. If the frequency is 25 Hz how long will it take for the current to rise to 50A ?

MARCH/APRIL . 2007, SEPT/OCT. 2001

Solution :

Given data :

$$I_m = 100\ A, f = 25\ Hz, i = 50\ A, t = ?$$

given $i = 100\ sin\ wt$

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$$50 = 100 \sin 2\pi ft$$

$$= 100 \sin 2 \times 180^{o} \times 25 \times t = 100 \sin (180 \times 50t)$$

$$\sin (180 \times 50t) = \frac{50}{100} = \frac{1}{2}$$

(or) $$180 \times 50t = \sin^{-1}\left(\frac{1}{2}\right) = 30$$

$$t = \frac{30}{180 \times 50} = \frac{1}{300} S$$ **Ans**

EXAMPLE

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EXAMPLE

Find the phase difference between the respective voltage and current. State also the frequecny of each wave.

(a) $v = 200 \sin (314\,t - \frac{\pi}{3})$, $i = 50 \sin 314\,t$ (b) $v = V_m \sin (\omega t + \frac{\pi}{2})$, $i = I_m \sin \omega t$

(c) $v = V_m \sin 2\omega t$, $i = I_m \sin (2\,\omega t - \frac{\pi}{2})$

(d) $v = V_m \sin (\omega t + \theta)$, $i = \sqrt{\frac{V_m}{R^2+X^2}} \sin (\omega t - tan^{-1} \frac{\times}{R})$

Solution :

(a) Voltage and current equations are given by

$$v = 200 \sin (314\,t - \frac{\pi}{3}) = 200 \sin (314\,t - 60^\circ)$$

$$i = 50 \sin (314\,t)$$

$\therefore$ Phase difference $\phi = (314\,t - 60^\circ) - (314\,t) = \mathbf{-\,60^\circ}$ **Ans**

Thus current wave lags the voltag wave by 60°

Also $\omega = 314 \Rightarrow 2\pi f = 314 \Rightarrow f = \frac{314}{2\pi} = \mathbf{50\ Hz.}$ **Ans**

$\therefore$ frequency, $f = 50\ Hz$

(b) $v = V_m \sin (\omega t + \frac{\pi}{2}) = V_m \sin (\omega t + 90^\circ)$

$$i = I_m \sin \omega t$$

$\therefore$ $\phi = (\omega t + 90^\circ) - \omega t = \mathbf{90^\circ}$ **Ans**

Thus voltage wave leads the current wave by 90°

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$f = \frac{w}{2\pi}$ Hz. **Ans.**

(c) $v = V_m \sin 2\omega t$

$i = I_m \sin (2\omega t - \frac{\pi}{2}) = I_m \sin (2\omega t - 90^o)$

$\therefore \quad \phi = (2\omega t - 90^o) - 2\omega t = -90^o$ **Ans.**

Thus current lags the voltage wave by 90^o

Given, $\omega = 2\omega$

$f = \frac{\omega}{2\pi} = \frac{2\omega}{2\pi} = \frac{\omega}{\pi}$ Hz. **Ans.**

(d) $v = V_m sm (\omega t + \theta)$

$i = \frac{V_m}{\sqrt{R^2 + x^2}} \sin \left[\omega t - \tan^{-1}\left(\frac{x}{R}\right)\right]$

$\therefore \quad \phi = \left(\omega t - \tan^{-1}\frac{x}{R}\right) - (\omega t + \theta) = \tan^{-1}\frac{x}{R} - \theta = -(\tan^{-1}\frac{x}{R} + \theta)$ **Ans**

Thus current wave lags the voltage wave by an angle $(\tan^{-1}\frac{x}{R} + \theta)$.

frequency, $f = \frac{\omega}{2\pi}$ Hz. **Ans**

EXAMPLE - 22

An e.m.f given by, $e = 326 \sin 314\, t$ is applied to a certain circuit and current $i = 2 \sin (314\, t - 1.3736)$. Fin (i) frequency of voltage (ii) phase angle between voltage and current in degrees.

Solution :

Given data :

$e = 326 \sin 314\, t$

$i = 2 \sin (314\, t - 1.3736)$

(ii) $\therefore \phi = (314\, t - 1.3736) - 314\, t = -1.3736$ *rad.*

$= -1.3736 \times \frac{180^o}{\pi} = -78.7^o$ **Ans**

Thus current lags the voltage by 78.70^o.

(i) $\omega = 314 \Rightarrow 2\pi f = \omega$

$f = \frac{\omega}{2\pi} = \frac{314}{2\pi} = 50$ **Hz. Ans.**

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CONVERSION OF POLAR QUANTITIES TO RECTANGULAR QUANTITIES AND VICE-VERSA

EXAMPLE – 28

The two given vectors are $Z_1 = 5 - j2$ *and* $Z_2 = -3 - j8$, *perform (i)* $Z_1 + Z_2$ *(ii)* $Z_1 - Z_2$.

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Solution :

Given data :

$$Z_1 = 5 - j\,2,\ Z_2 = -3 - j\,8$$

$$Z_1 + Z_2 = 5 - j\,2 + (-3) - j\,8 = (5 - 3) + j\,(-2 - 8) = 2 - j\,10 \quad \textbf{Ans}$$

$$Z_1 - Z_2 = 5 - j\,2 - (-3 - j\,8) = 5 - j\,2 + 3 + j\,8$$

$$= (5 + 3) + j\,(-2 + 8) = 8 + j\,6 \quad \textbf{Ans}$$

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$= (5 + 3) + j\,(-2 + 8) = 8 + j\,6$ **Ans**

EXAMPLE – 29

Perform the following operations and express the result in rectangular form. *(i)* $A+B$ *(ii)* $A - B$ *(iii)* $A \times B$ *(iv)* $A \div B$

Where $A = 4 + j3,\ B = 6 - j8.$

tion :

Given data :

$$A = 4 + j3\ ; \qquad B = 6 - j8$$

(i) $A + B = (4 + j3) + (6 - j8) = (4 + 6) + j\,(3 - 8) = \mathbf{10 - j5}$ **Ans**

(ii) $A - B = (4 + j3) - (6 - j8) = (4 - 6) + j\,(3 + 8) = \mathbf{-2 + j\,11}$ **Ans**

(iii) $A \times B = (4 + j3)\,(6 - j8) = 4 \times 6 + j\,3 \times 6 - j\,4 \times 8 - j^2\,3 \times 8$

$= (24 + 24) + j\,(18 - 32) = \mathbf{48 - j14}$ **Ans** $(\because j^2 = -1)$

(iv) $A \div B = \dfrac{4+j3}{6-j8} = \dfrac{(4+j3(6+j8)}{(6-j8)(6+j8)} = \dfrac{24+j18+j32-24}{6^2+8^2}$

$= \dfrac{(24-24)}{6^2+8^2} + j\,\dfrac{(18+32)}{6^2+8^2} = 0 + j\,\dfrac{50}{36+64} = 0 + j\,\dfrac{50}{100} = \mathbf{0 + j0.5}$ **Ans**

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EXAMPLE

Perform the following operations and put the result in a polar form.

(i) $A+B$ *(ii)* $A-B$ *(iii)* $A \times B$ *(iv)* $A \div B$

Where, $A = 6+j8,\ B = 3-j4$

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Solution :

Given data :

$$\left.\begin{aligned} A &= 6 + j8 \ldots\ldots \\ B &= 3 - j4 \end{aligned}\right\} \text{Rectangular form}$$

$$|A| = \sqrt{6^2+8^2} = \sqrt{36+64} = \sqrt{100} = 10$$

$$\theta = tan^{-1}\left(\frac{8}{6}\right) = 53.13^\circ$$

$\therefore$ $A = |A| \angle\theta = 10 \angle 53.13^\circ$ ..... Polar form

Also $B = 3 - j4$ ..... Rectangular form

$$|B| = \sqrt{3^2+(4)^2} = \sqrt{9+16} = \sqrt{25} = 5$$

$$\theta = tan^{-1}\left(-\frac{4}{3}\right) = -53.13^\circ$$

$\therefore$ $B = |B| \angle\theta = 5 \angle -53.13^\circ$

(i) $A+B = (6 + j8) + (3 - j4) = (6 + 3) + j(8 - 4)$

$= 9 + j4$ ..... Rectangular form

$$|A+B| = \sqrt{9^2+4^2} = \sqrt{81+16} = \sqrt{97} = 9.85$$

$$\theta = tan^{-1}\left(\frac{4}{9}\right) = 23.96^\circ$$

$\therefore A+B$ in polar form $= |A+B| \angle\theta = 9.85 \angle 23.96^\circ$ ..... in polar form **Ans.**

(ii) $A-B = (6 + j8) - (3 - j4) = (6 - 3) + j(8 - 4)$

$= 3 + j12$ ..... Rectangualr form

$$|A-B| = \sqrt{3^2+12^2} = \sqrt{9+144} = \sqrt{153} = 12.37$$

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$$\theta = tan^{-1}\left(\frac{12}{3}\right) = 75.96^o$$

$\therefore$ $A - B = |A-B| \angle \theta = 12.37 \angle 75.96^o$ ..... in polar form **Ans.**

(iii) $A \times B = (6 + j8) \times (3 - j4) = 18 + j24 - j24 + 32 = 50 + j0 = 50 \angle 0^o$

Alternating :

$$A \times B = (6 + j8) \times (3 - j4) = 10 \angle 53.13^o \times 5 \angle -53.13^o$$

$$= 10 \times 5 \angle 53.13^o - 53.13^o \quad (\because A \angle \alpha \times B \angle \beta = A \times B \angle \alpha + \beta)$$

$$= 50 \angle 0^o \quad \textbf{Ans}$$

(iv) $$A \div B = \frac{6+j8}{3-j4} = \frac{(6+j8)(3+j4)}{(3-j4)(3+j4)} = \frac{18+j24+j24-32}{3^2+4^2}$$

$$= \frac{(18-32)}{3^2+4^2} + j\frac{(24+24)}{3^2+4^2} = -\frac{14}{25} + j\frac{48}{25}$$

$$= \textbf{- 0.56 + } j \textbf{ 1.92} \ldots\ldots \text{ Rectangular form} \quad \textbf{Ans}$$

$$|A \div B| = \sqrt{(-0.56)^2+(1.92)^2} = 2$$

$$\beta = tan^{-1}\left(\frac{1.92}{0.56}\right) = 73.74^o$$

$\therefore$ $\theta = 180^o - 73.74^o = 106.26^o$

$\therefore$ $A \div B$ in polar form $= |A \div B| \angle \theta = 2 \angle 106.26^o$ **Ans**

Alternatively :

$$A \div B = \frac{6+j8}{3-j4} = \frac{10 \angle 53.15}{5 \angle -53.15} = \frac{10}{5} \angle 53.13^o - (-53.13^o) \quad \left(\because \frac{A \angle \alpha}{B \angle \beta} = \frac{A}{B} \angle \alpha - \beta\right)$$

$$= 2 \angle 106.26^o \quad \textbf{Ans}$$

EXAMPLE – 33

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EXAMPLE

Express in rectangular notation the admittance of the circuit having the following impedonce :

(i) $(4 + j\,6)$ *ohms* *(ii)* $20 \angle 30^\circ$ *ohms*

Solution :

(i) Given, $Z = (4 + j\,6) = \left(\sqrt{4^2 + 6^2}\right) \tan^{-1}\left(\frac{6}{4}\right)$

$= \sqrt{16+36} \angle 56.31^\circ = 7.21 \angle 56.31^\circ\ \Omega$

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$$\therefore \text{Admittance, } Y = \frac{1}{Z} = \frac{1}{7.21 \angle 56.31^\circ} = 0.1387 \angle -56.31^\circ \ \Omega$$

$$= 0.1387 \ (\cos 56.31 - j \sin 56.31)$$

$$= 0.1387 \times 0.5547 - j\, 0.1387 \times 0.832$$

$$= (0.0769 - j0.1154) \text{ ohms} \quad \textbf{Ans.}$$

(ii) Given : $Z = 20 \angle 30^\circ$

$$\therefore \text{Admittance, } Y = \frac{1}{Z} = \frac{1}{20 \angle 30^\circ} = 0.05 \angle -30^\circ$$

$$= 0.05 \ (\cos 30^\circ - j \sin 30^\circ) = 0.05 \times 0.866 - j\, 0.05 \times 0.5$$

$$= (0.0433 - j\, 0.025) \text{ ohms}$$

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Definition of Resistance, Inductance, & Capacitance

1. **Resistance (R) :** "The property of a substance due to which it opposes the flow of electric current through it is called *'resistance'*. It is denoted by a letter 'R' and is measured in ohm (Ω)".

(or)

"The opposition offered by a substance to the flow of electric current is called resistance".

2. **Inductance (L) :** "The property of a coil that opposes any change in current or flux flowing through it is called *'inductance'*. It is denoted by a letter 'L' and is measured in henry (H)".

(or)

"It may also be defined as the flux linkages of the coil per ampere".

$$\therefore \quad L = \frac{\textit{Flux linkages}}{\textit{Ampres}} = \frac{N\phi}{I} = \frac{\psi}{I} \text{ henry.}$$

3. **Capacitance (C) :** "The property of a capacitor to store electric charge is known as its capacitance". Since the charge Q stored by a capacitor is directly proportional to the potential difference applied across it.

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i.e., $Q \propto V$

(or) $\frac{Q}{V} = \text{Constant} = C$

$\therefore$ $C = \frac{Q}{V}$

Hence, capacitance is also defined as "the charge on the capacitor plates per unit potential difference across the plate is called the capacitance of the capacitor".

The unit of capacitance is farad (F).

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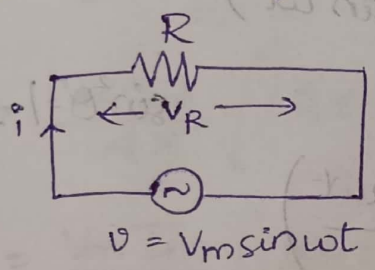
Simple A.C. Circuits :-

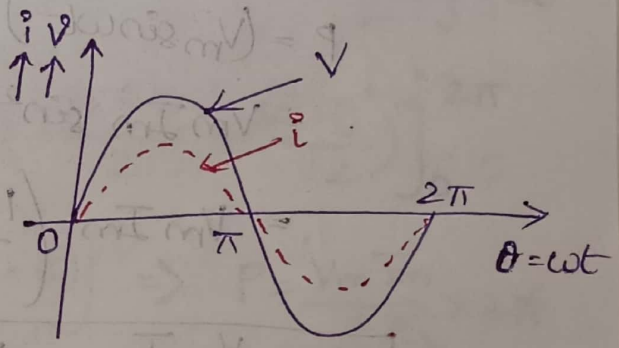
Simple A.C. circuits may be classified according to basic circuit elements (R, L, & C) are:

1) A.C. circuit containing pure resistance only.
2) A.C. circuit containing pure inductance only.
3) A.C. circuit containing pure capacitance only.

(1) A.C. circuit Containing pure resistance only :-

Consider a circuit containing a pure resistance of "R" Ohm.





Let the alternating voltage be given by the equation:

$$v = V_m \sin \omega t \quad \longrightarrow (1)$$

By Ohm's law, the instantaneous current in the circuit will be,

$$i = \frac{v}{R} = \frac{V_m \sin \omega t}{R}$$

$$i = \frac{V_m}{R} \sin \omega t \quad \longrightarrow (2)$$

The value of "i" will be maximum (i.e, I_m) when $\sin \omega t = 1$ or, or $\omega t = 90^\circ$.

$$\therefore I_m = \frac{V_m}{R} \quad \longrightarrow (3)$$

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From equation ③ & ②, we get

$$\boxed{i = I_m \sin \omega t} \longrightarrow ④$$

From equation ① and ④, it is clear that applied voltage and circuit current are in phase with each other because there is no phase difference between voltage and current.

Power :-

Instantaneous power $P = vi$ -- watts

$$P = (V_m \sin \omega t)(I_m \sin \omega t)$$

$$= V_m I_m \sin^2 \omega t \qquad \sin^2\theta = \frac{1 - \cos 2\theta}{2}$$

$$= V_m I_m \left(\frac{1 - \cos 2\omega t}{2}\right)$$

$$\boxed{P = \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \cos 2\omega t}$$

Thus the instantaneous power consists of two parts i.e, constant part $\frac{V_m I_m}{2}$ and a variable part $\frac{V_m I_m}{2} \cos 2\omega t$.

→ For a complete cycle, the average value of $\frac{V_m I_m}{2} \cos 2\omega t$ is zero, because it is a cosine wave of the double frequency.

→ The average power of the cycle is

$$P = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} = V_{rms} \times I_{rms}$$

$$\boxed{P = VI} \text{ -- watts}$$

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where

V = r.m.s value of the applied voltage

I = r.m.s " " " circuit current.

Mathematical Proof for Power :-

Average power consumed over the whole cycle is ;

$$P = \frac{1}{2\pi}\int_0^{2\pi} Vi \cdot d\theta$$

$$= \frac{1}{2\pi}\int_0^{2\pi} (V_m \sin\theta)(I_m \sin\theta)\cdot d\theta$$

$$= \frac{V_m I_m}{2\pi}\int_0^{2\pi} \sin^2\theta \cdot d\theta = \frac{V_m I_m}{2\pi}\int_0^{2\pi}\left[\frac{1-\cos 2\theta}{2}\right]\cdot d\theta$$

$$= \frac{V_m I_m}{4\pi}\left[(2\pi - 0) - \left(\frac{\sin 4\pi}{2} - \frac{\sin 0^\circ}{2}\right)\right]_0^{2\pi}$$

$$P = \frac{V_m I_m}{4\pi}\left[(2\pi - 0) - (0 - 0)\right] \Rightarrow P = \frac{V_m I_m}{4\pi} \times 2\pi$$

$$P = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} = V_{rms} \times I_{rms} = V \cdot I$$

Note :-

1. Current flowing through a pure resistance will be always in phase with the alternating voltage applied.
2. The phase difference (or phase angle) ϕ is zero.
3. Average power, $P = \frac{V_m I_m}{2}$ or $P = VI$.
4. The frequency of the fluctuating power $\left(\frac{V_m I_m}{2}\cos 2\omega t\right)$ is twice that of applied voltage frequency and its average value over one complete cycle is zero.

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The current through a pure resistor of 5 Ω is given by $i = 60 \sin (377t + 60^o)$. Find the sinusoidal expression for the voltage across the resistor.

Solution :

Given data :

$$i = 60 \sin (377\ t + 60^o)$$

$$R = 5\ \Omega$$

$$\therefore \quad v = iR = 5 \times 60 \sin (377\ t + 60^o) = 300 \sin (377\ t + 60^o) \quad \textbf{Ans}$$

EXAMPLE – 2

An a.c. circuit consists of a pure resistance of 20 Ω and is connected across an a.c. supply of 230V, 50Hz.

Find (i) Current

(ii) Power consumed and

(iii) Sinusoial equations for voltage and current.

Solution :

Given data :

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$R = 20\,\Omega$	$I = ?$
$V = 230V$	$P = ?$
$f = 50\ Hz$	$v = ?$
	$i = ?$

(i) $I = \dfrac{V}{R} = \dfrac{230}{20} = 11.5A$ **Ans**

(ii) $P = VI = 230 \times 11.5 = 2645\ W$ **Ans**

(or) $P = I^2R = (11.5)^2 \times 20 = 2645\ W$ **Ans**

(iii) Given $V_{r.m.s} = V = 230V$.

$\therefore \quad V_m = \sqrt{2}\ V = \sqrt{2} \times 230 = 325.27$ volts.

Similarly,

$I_m = \sqrt{2}\ I = \sqrt{2} \times 11.5 = 16.26A$

$\omega = 2\pi f = 2\pi \times 50 = 314$

$\therefore$ Equation for voltage, $V = 325.27\ sin\ 314\ t$ *volts.* **Ans**

Equation for current, $i = 11.5\ sin\ 314\ t$ *volts.* **Ans**

EXAMPLE – 3

In a pure resistive circuit, the instaneous voltage and current are given by

$v = 282.8\ sin\ 377t$

$i = 50\ sin\ 377t$

Determine (i) the peak power and (ii) average power.

Solution :

Given data :

$v = 282.8\ sin\ 377\ t$ ..... (1)

$i = 50\ sin\ 377\ t$ ..... (2)

Compare equations (1) & (2) with standard equations, we get,

$V_m = 282.8V$ and $I_m = 50A$

$\therefore$ (a) Peak power, $P_m = V_m\ I_m = 282.8 \times 50 = 1414$ watt or 14.140 *kW*

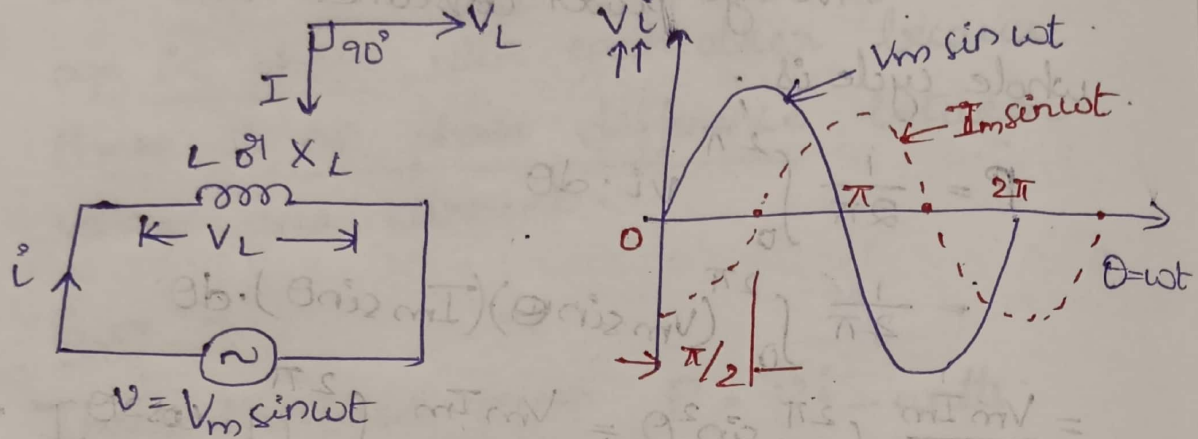
(b) Average power, $P_{av} = \dfrac{V_m I_m}{2} = \dfrac{282.8 \times 50}{2} = \dfrac{14140}{2} = 7070$ **watt.**

Note that it is the average power which is consumed in the circuit.

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② A.C. Circuit Containing Pure inductance only :-

Consider a circuit containing a coil of pure inductance "L" henry;



→ Let an alternating voltage applied across the circuit given by the equation,

$$\boxed{v = V_m \sin\omega t} \longrightarrow (1)$$

→ An alternating current "i" flows through a pure inductive coil, a back emf is induced due to the inductance of the coil.

$$\therefore e = -L\frac{di}{dt}$$

→ L = Self inductance in henry.

This induced emf is equal and opposite to the applied voltage.

$$V = -e$$

$$V = -\left(-L\frac{di}{dt}\right) \Rightarrow V = L\frac{di}{dt}.$$

$$V_m \sin\omega t = L\frac{di}{dt}.$$

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$$di = \frac{V_m}{L} \sin\omega t \cdot dt$$

Integrating on both sides, we get

$$\int di = \int \frac{V_m}{L} \sin\omega t \cdot dt$$

$$i = \frac{V_m}{L} \int \sin\omega t \cdot dt$$

$$= \frac{V_m}{L} \left[-\frac{\cos\omega t}{\omega} \right]$$

$$i = \frac{V_m}{\omega L} (-\cos\omega t)$$

$$\boxed{i = \frac{V_m}{\omega L} \sin(\omega t - \pi/2)} \longrightarrow (2)$$

The value of "i" will be maximum (i.e, I_m) when $\sin(\omega t - \pi/2)$ is unity.

$$\therefore \boxed{I_m = \frac{V_m}{\omega L}} \longrightarrow (3)$$

Substitute eqn (3) in equation (2)

$$\boxed{i = I_m \sin(\omega t - \pi/2)} \longrightarrow (4)$$

Phase angle θ :-

From equation (1) & (4), it is clear that current lags behind the voltage by $\pi/2$ radians or 90°.

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Inductive Reactance :-

→ The quantity "ωL" is called the reactance of a inductor or reactance of the coil.

→ It is represented by X_L and measured in ohms

$$\boxed{X_L = \omega L = 2\pi f L} \quad \text{---(2)}$$

→ From equation (2),

$$i = \frac{V_m}{\omega L} \sin(\omega t - \pi/2)$$

$$i = \frac{V_m}{X_L} \sin(\omega t - \pi/2)$$

$$i = \frac{V_m}{2\pi f L} \sin(\omega t - \pi/2)$$

$$\therefore I_m = \frac{V_m}{\omega L} = \frac{V_m}{X_L} \Rightarrow X_L = \frac{V_m}{I_m} = \frac{V_m/\sqrt{2}}{I_m/\sqrt{2}}$$

$$\boxed{X_L = \frac{V}{I}}.$$

Power :-

Instantaneous power; $p = vi$

$$p = (V_m \sin\omega t)\left(I_m \sin(\omega t - \pi/2)\right)$$

$$= V_m I_m \sin\omega t (-\cos\omega t)$$

$$p = -\frac{V_m I_m}{2} (2 \sin\omega t \cdot \cos\omega t)$$

$$p = -\frac{V_m I_m}{2} \sin 2\omega t.$$

∴ Average power consumed in the circuit over a complete cycle is zero.

$$\boxed{P = \frac{1}{2\pi}\int_0^{2\pi} -\frac{V_m I_m}{2} \sin 2\omega t \, d(\omega t) = 0.}$$

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A pure inductive coil allows a current of 10A to flow from a 230V, 50Hz supply. Find (i) Inductive reactance. (ii) Inductance of the coil (iii) Power absorbed. Write the equation for voltage and current.

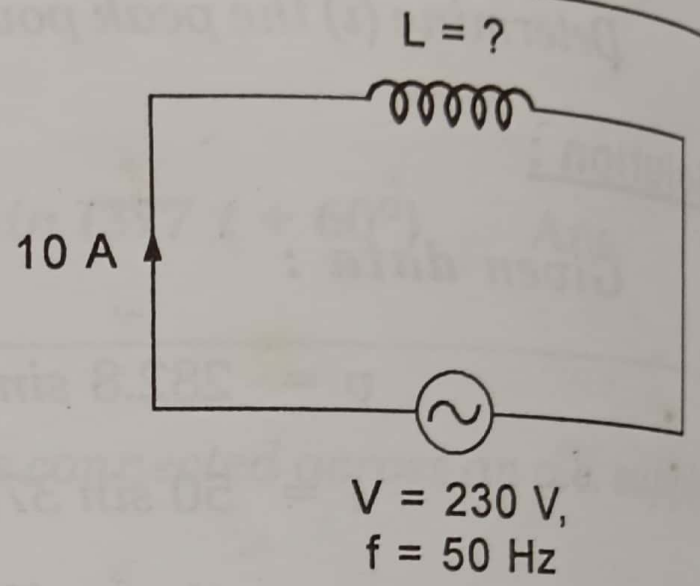


FIG 3.8 :

olution :

Given data :

$I = 10\ A$	$X_L = ?$
$V = 230V$	$L = ?$
$f = 50\ Hz$	$P = ?$

(i) $$X_L = \frac{V}{I} = \frac{230}{10} = 23\,\Omega$$ **Ans**

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(ii) $X_L = 2\pi fL$

$$\therefore \quad L = \frac{X_L}{2\pi f} = \frac{23}{2\pi \times 50} = 0.0732\ H \quad \textbf{Ans}$$

(iii) $P = 0\ (\because R = 0)$ **Ans**

EXAMPLE – 6

The current flowing through a pure inductor is 28.28A. Find the inductance and power consumption when the voltage applied across the inductor is v = 100 sin 314t volts.

Solution :

Given data :

$I = 28.28\ A \quad | \quad L = ?$

$v = 100\ sin\ 314\ t \quad | \quad P = ?$ ..... (1)

From equation (1), $V_m = 100V$, $\omega = 314$

$$\therefore \quad V = V_{rms} = \frac{V_m}{\sqrt{2}} = 0.707\ V_m = 0.707 \times 100 = 70.7\ V.$$

$$X_L = \frac{V}{I} = \frac{70.7}{28.28} = 2.5\,\Omega$$

Also $f = \frac{\omega}{2\pi} = \frac{314}{2\pi} = 50\ Hz.$

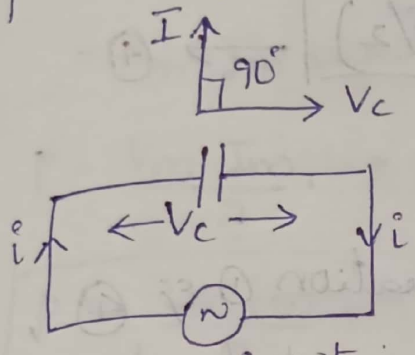
(i) $X_L = \omega_L \Rightarrow L = \frac{X_L}{\omega} = \frac{2.5}{314} = 7.96 \times 10^{-3}\ H$ **Ans**

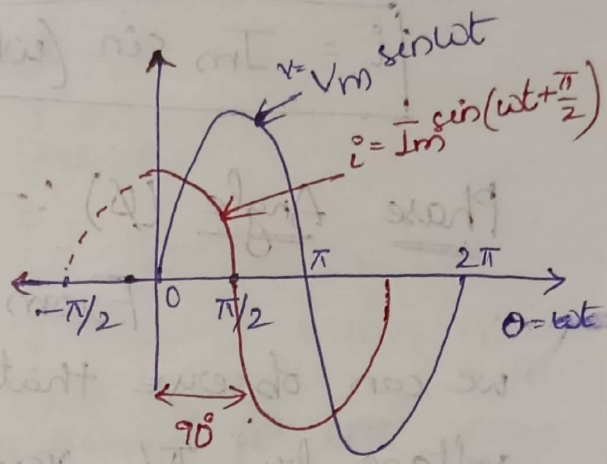
(ii) Power consumed, $P = 0\ (\because R = 0)$ **Ans**

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③ A.C. circuit Containing Pure Capacitance only :-

Consider a circuit containing a pure capacitance "C" farads.





→ Let the alternating voltage applied across the circuit be given by the equation;

$$v = V_m \sin \omega t \quad \longrightarrow (1)$$

→ Charge on a capacitor at any instant,

$$q = CV = CV_m \sin \omega t$$

q = charge on capacitor.

C = capacitance of capacitor.

→ Current flowing through the circuit,

$$i = \frac{dq}{dt} = \frac{d}{dt}(CV_m \sin \omega t)$$

$$i = CV_m \frac{d}{dt}(\sin \omega t)$$

$$= \omega C V_m \cos \omega t.$$

$$i = \omega C V_m \sin\left(\omega t + \pi/2\right) \quad \longrightarrow (2)$$

The value of "i" will be maximum (I_m) when $\sin(\omega t + \pi/2)$ is unity.

$$I_m = \omega C V_m = \frac{V_m}{\left(\frac{1}{\omega C}\right)} \longrightarrow (3)$$

Substitute eqn (3) in eqn (2), we get

$$i = I_m \sin\left(\omega t + \pi/2\right) \longrightarrow (4)$$

Phase Angle (ϕ) :-

From equation (1) & (4), we can observe that current leads the voltage by $\pi/2$ radians or 90°.

Capacitive Reactance :-

The quantity $\frac{1}{\omega C}$ in equation (3) is called reactance of a capacitor.

→ It is represented by X_C

→ " " measured in "ohms"

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} \quad \text{---} \Omega \longrightarrow (5)$$

From the equation (3),

$$I_m = \omega C V_m = \frac{V_m}{\left(\frac{1}{\omega C}\right)}$$

$$\frac{1}{\omega C} = \frac{V_m}{I_m} \text{ or } X_C = \frac{V_m}{I_m}$$

$$I_m = \frac{V_m}{X_C}$$

$$I = \frac{V}{X_C}$$

V & I = R.M.S. value of voltage and current.

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Power :-

Instantaneous power, $p = vi$.

$$p = [V_m \sin\omega t]\left[I_m \sin\left(\omega t + \pi/2\right)\right]$$

$$= V_m I_m \sin\omega t \cdot \cos\omega t$$

$$p = \frac{V_m I_m}{2} \cdot 2 \sin\omega t \cdot \cos\omega t$$

$$\boxed{p = \frac{V_m I_m}{2} \sin 2\omega t}$$

∴ Average power consumed for one completely is

$$\boxed{P = \frac{1}{2\pi}\int_0^{2\pi} \frac{V_m I_m}{2} \sin 2\omega t \cdot d(\omega t) = 0.}$$

Power consumed for one complete cycle is Zero.

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EXAMPLE

The voltage across a μF capacitor is given by $v = 170 \sin 400\, t$. What is the sinusoidal expression for the current? Draw the waveform for v and i.

Solution :

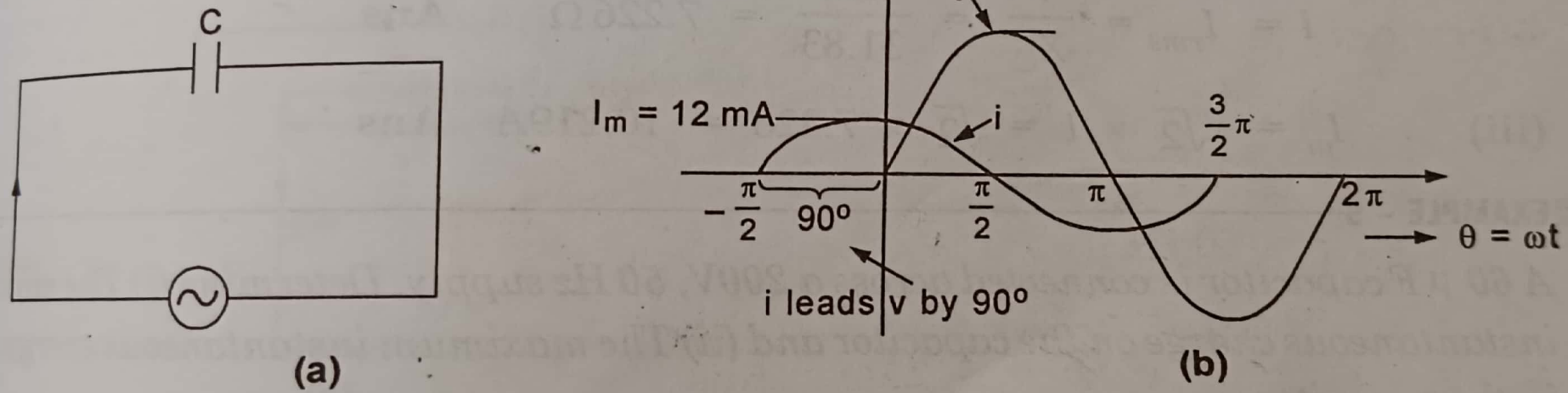


FIG 3.9 :

Given data :

$$v = 170 \sin 400\, t \quad \ldots\ldots$$

$$C = 1\mu F = 1 \times 10^{-6}\, F, \quad i = ?$$

Compare Eqn. (1) with $v = V_m \sin wt$, we get.

$$V_m = 170V$$

$$\omega = 2\pi f = 400$$

$$X_C = \frac{1}{\omega C} = \frac{1}{400 \times 1 \times 10^{-6}} = \frac{10^6}{400} = 2500\,\Omega.$$

$$I_m = \frac{V_m}{X_C} = \frac{170}{2500} = 0.068\, A = 6.8\, mA$$

We know that for a capacitor, i leads v by 90^o.

$\therefore$ $i = I_m \sin (wt + 90^o) = 6.8 \times 10^{-3} \sin (400\, t + 90^o)$ **Ans.**

EXAMPLE – 8

A capacitor of 100 μ F is connected across a 230V, 50Hz a.c. supply. Calculate.

(i) The reactance of the capacitor.

(ii) r.m.s value of current and

(iii) The maximum current.

olution :

Given data :

$C = 100\ \mu F = 100 \times 10^{-6}\, F$ | $X_c = ?$

$V = 230\ V$ | $I = ?$

$f = 50\ Hz$ | $I_m = ?$

(i) $$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}} = 31.83\,\Omega$$ **Ans**

(ii) Given, $V = V_{rms} = 230V$.

$\therefore$ $$I = I_{rms} = \frac{V}{X_C} = \frac{230}{31.83} = 7.226\,\Omega$$ **Ans**

(iii) $$I_m = \sqrt{2} \times I = \sqrt{2} \times 7.226 = 10.219A$$ **Ans**

EXAMPLE – 9

A 60 μ F capacitor is connected across a 200V, 50 Hz supply. Determine (i) The maximum instantaneous charge on the capacitor and (ii) The maximum instantaneous energy stored in the capacitor.

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Solution :

Given data :

$V_{rms} = V = 200\ Volt.$ | $Q_m = ?$

$C = 60 \times 10^{-6}\ F$ | $E_m = ?$

$f = 50\ Hz.$

$$\therefore \quad V_m = \sqrt{2}\ V_{rms} = \sqrt{2} \times 200 = 282.84V$$

1. Maximum instantaneous charge on capacitor.

$$Q_m = C\,V_m = (60 \times 10^{-6}) \times 282.84$$

$$= 16.97 \times 10^{-3}\ C \text{ or } 16.97\ mC \quad \textbf{Ans}$$

2. Maximum instantaneous energy stored.

$$E_m = \frac{1}{2} CV_m^2 = \frac{1}{2} \times (60 \times 10^{-6}) \times (282.84)^2 = 2.4\ J \quad \textbf{Ans}$$

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* Series A.C. circuits :-

In practical, circuits normally consists of the R, L, C elements in combination.

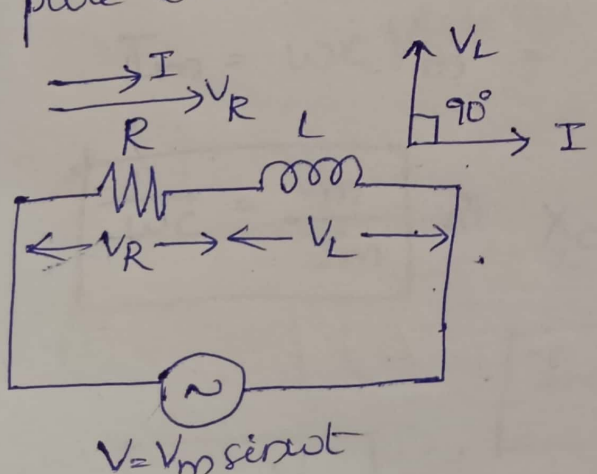
A series circuit in a circuit in which the same alternating current flows through all the circuit elements (R, L, C) is called Series a.c. circuits.

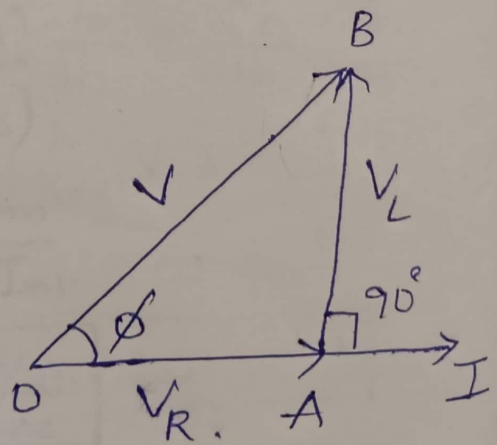
The combination of Series a.c. circuits may be :-

1) R-L Series circuit.
2) R-C Series circuit.
3) L-C Series circuit.
4) R-L-C " "

(1) R-L Series A.C. Circuit :-

Consider a pure resistance "R" ohms connected in series with a coil of pure inductance of "L" henry.





V = R.M.S. value of applied voltage (volt)

I = R.M.S. value of circuit current (ampere)

→ $V_R = IR$ = Voltage drop in resistance.

where V_R is in phase with I.

→ $V_L = IX_L$ = Voltage drop in inductive reactance.

where V_L leads I by 90°.

Phasor diagram :-

→ To draw the phasor diagram for voltage drops of current I (r.m.s. value) is taken as the reference phasor.

→ Phasor OA represents V_R.

→ " AB " V_L.

→ The applied voltage "V" is the phasor sum of V_R and V_L.

→ From the right angle triangle OAB,

$$OB^2 = OA^2 + AB^2$$

$$V^2 = V_R^2 + V_L^2$$

$$V = \sqrt{V_R^2 + V_L^2} = \sqrt{(IR)^2 + (IX_L)^2}$$

$$= \sqrt{I^2(R^2 + X_L^2)}$$

$$V = I\sqrt{(R^2 + X_L^2)}$$

$$\boxed{I = \frac{V}{\sqrt{R^2 + X_L^2}}}$$

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The quantity $\sqrt{R^2+X_L^2}$ offers opposition to the current flow and is called impedance of the circuit.

→ It is represented by "Z" and is measured in ohms (Ω)

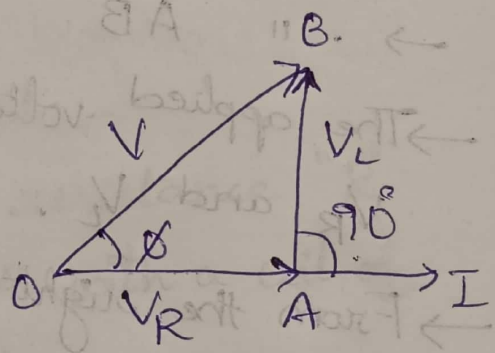
$$I = \frac{V}{Z}$$

$$Z = \sqrt{R^2+X^2}$$

Phase angle :-

It is clear from the phasor diagram that the circuit current "I" lags behind the applied voltage "V" by an angle ϕ is called Phase angle.

From the phasor diagram.



$$\tan\phi = \frac{AB}{OA}$$

$$= \frac{V_L}{V_R}$$

$$= \frac{IX_L}{IR}$$

$$\tan\phi = \frac{X_L}{R}$$

$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$$

Power :- If the alternating voltage applied across the circuit is given by the equation.

$$v = V_m \sin \omega t$$

$$i = I_m \sin(\omega t - \phi)$$

Instantaneous power $P = vi$

$$P = (V_m \sin \omega t)(I_m \sin(\omega t - \phi))$$

$$= \frac{V_m I_m}{2}(2 \sin \omega t \sin(\omega t - \phi))$$

$$P = \frac{V_m I_m}{2}[\cos\phi - \cos(2\omega t - \phi)]$$

$\because$ $\boxed{2 \sin A \sin B = \cos(A-B) - \cos(A+B)}$

$$2 \sin \omega t \cdot \sin(\omega t - \phi) = \cos(\omega t - \omega t + \phi) - \cos(\omega t + \omega t - \phi)$$

$$= \cos\phi - \cos(2\omega t - \phi)$$

$$\boxed{P = \frac{V_m I_m}{2}\cos\phi - \frac{V_m I_m}{2}\cos(2\omega t - \phi)}$$

* Thus Instantaneous Power Consists of Two parts :-

1) Constant part $\frac{V_m I_m}{2}\cos\phi$.

2) A pulsating or variable part $\frac{1}{2} V_m I_m \cos(2\omega t - \phi)$ is double frequency whose average value over one complete cycle is zero.

→ So, the power consists of only constant part.

→ Average power $= \frac{V_m I_m}{2}\cos\phi = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}\cos\phi$

$$P = V_{rms} \times I_{r.m.s} \cos\phi.$$

$\therefore$ $\boxed{P = VI\cos\phi}$.

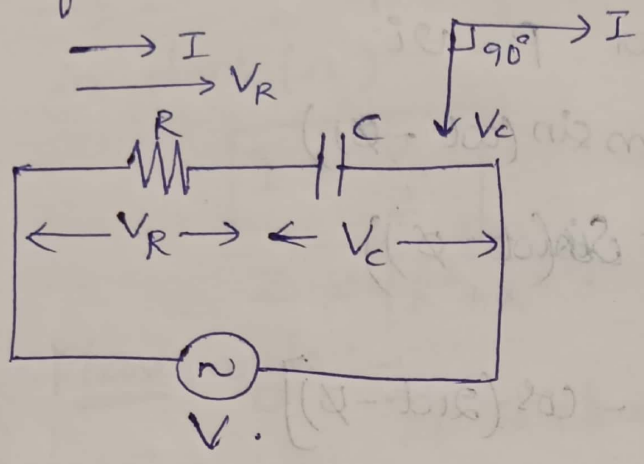
$$\boxed{P = (I Z)\left(\frac{R}{Z}\right) = I^2 R.}$$

$\because \cos\phi = R/Z$

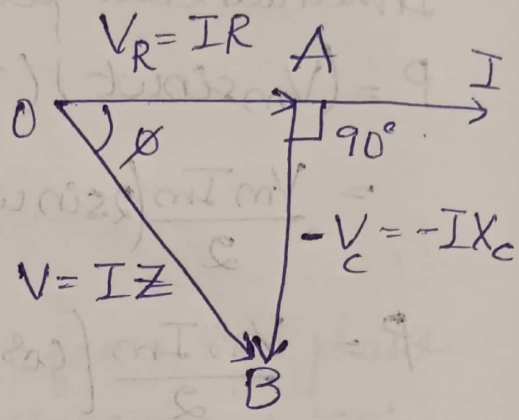
$V = IZ$.

② R-C Series A.C. circuit :-

Consider an a.c. circuit containing resistance "R" ohms and capacitance "C" farads in series shown in fig:



(a)



(b)

V = r.m.s. value of applied voltage.

I = r.m.s. " " circuit current.

→ $V_R = OA = IR$ = Voltage drop across R, where (V_R is in phase with I)

→ $V_c = AB = IX_c$ = Voltage drop across capacitive reactance (where V_c lags I by 90°).

→ Taking "I" as the reference phasor, the phasor diagrams of circuit can be drawn as shown in fig. (b).

$$OB^2 = OA^2 + AB^2$$

$$V^2 = V_R^2 + V_c^2 = (IR)^2 + (-IX_c)^2$$

$$V^2 = I^2(R^2 + X_c^2)$$

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$$V = I\sqrt{R^2 + X_c^2}$$

The current drawn by the circuit

$$I = \frac{V}{\sqrt{R^2 + X_c^2}}$$

The quantity $\sqrt{R^2 + X_c^2}$ offers opposition to the current flow and is called impedance of the circuit, represented by "Z".

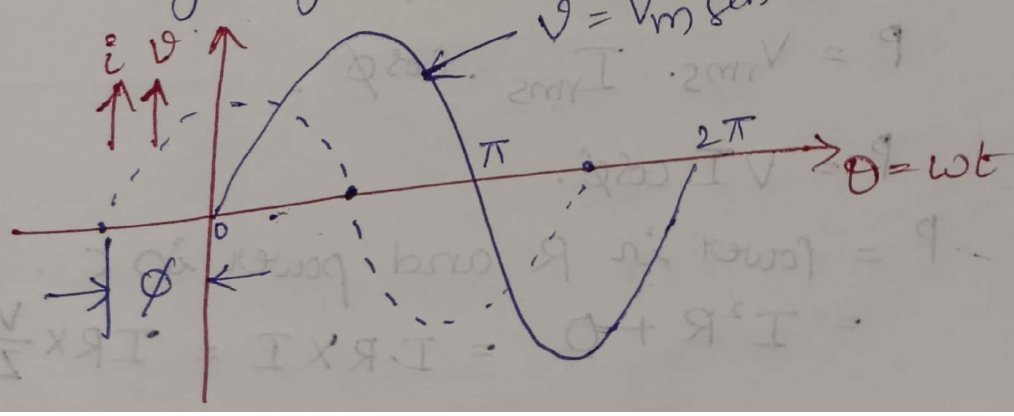
$$\boxed{I = \frac{V}{Z}} \quad \text{where} \quad Z = \sqrt{R^2 + X_c^2}$$

Phase angle (ϕ) :- It is clear from the phasor diagram that the circuit current "I" leads the applied voltage "V" by angle "ϕ" called phase angle.

$$\tan\phi = \frac{AB}{OA} = \frac{-IX_c}{IR} = \frac{-X_c}{R}$$

$$\boxed{\phi = \tan^{-1}\left(\frac{-X_c}{R}\right)}$$

∴ Negative phase angle represents the voltage lags behind the current.



Power :-

If the alternating voltage applied across the circuit is given by

$v = V_m \sin \omega t$

$i = I_m \sin(\omega t + \phi)$ (∵ current leads the voltage).

Instantaneous power $P = vi$

$$P = [V_m \sin \omega t][I_m \sin(\omega t + \phi)]$$

$$P = \frac{V_m I_m}{2}[2 \sin(\omega t + \phi) \sin \omega t]$$

$$P = \frac{V_m I_m}{2}[\cos\phi - \cos(2\omega t + \phi)]$$

$$P = \frac{V_m I_m}{2}\cos\phi - \frac{V_m I_m}{2}\cos(2\omega t + \phi)$$

(∵ power consists only in the constant term)

∴ Average power over a complete cycle,

$$P = \frac{V_m I_m}{2}\cos\phi$$

$$= \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\phi$$

$$P = V_{rms} \cdot I_{rms} \cdot \cos\phi$$

$$P = VI\cos\phi$$

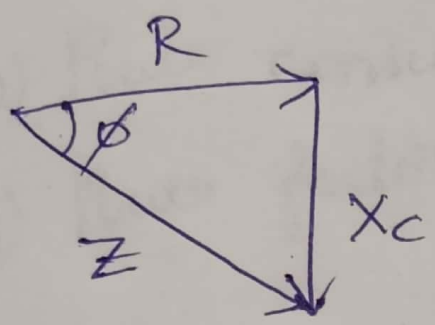
P = Power in R and power in C

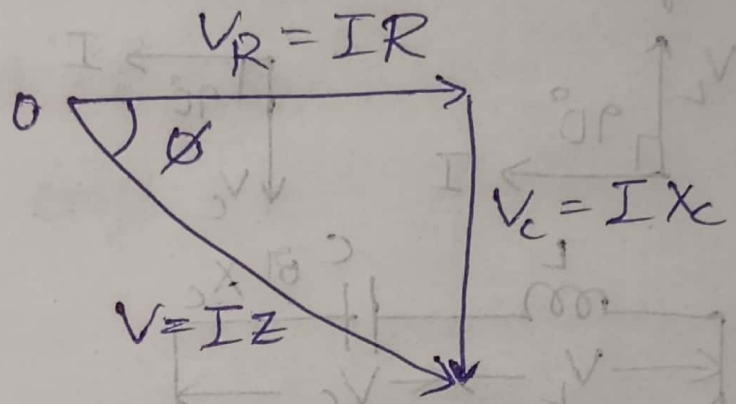
$= I^2R + 0 = I \cdot R \times I = IR \times \frac{V}{Z} = VI\left(\frac{R}{Z}\right)$

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∴ This shows that power is actually consumed in resistance only, capacitor does not consume any power.

→ **Impedance Triangle :-**





1) Impedance $\boxed{Z = \sqrt{R^2 + X_c^2}}$

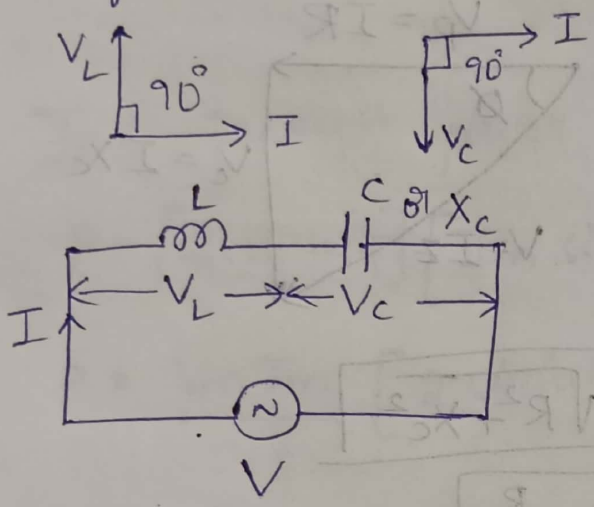
2) Power factor $\boxed{\cos\phi = \frac{R}{Z}}$

3) Phase angle $\boxed{\phi = \tan^{-1}\left(\frac{X_c}{R}\right)}$

$$\phi = \cos^{-1}\left(R/Z\right)$$

③ L-C Series A.C. circuits :-

Consider an a.c. circuit containing "L" henry and Capacitance "C" farad in series as shown in fig:



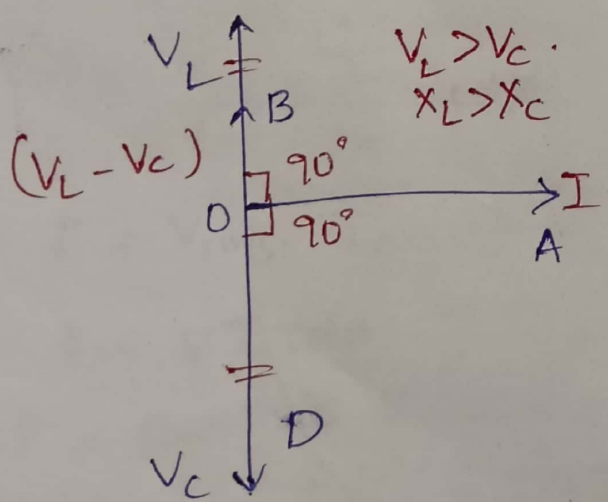
Let $V_L = IX_L$ = Voltage across "L" ... leads I by 90°.

$V_C = IX_C$ = voltage across C ... lags I by 90°.

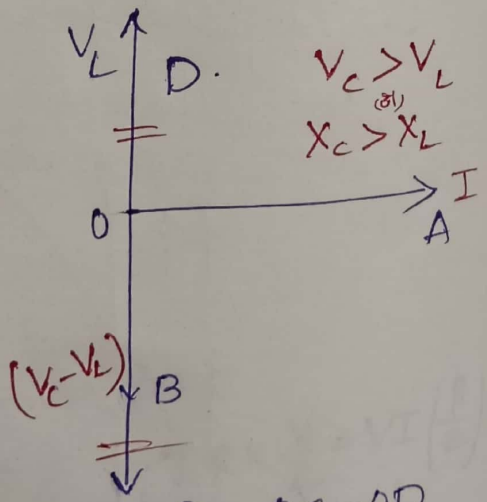
$X = X_L - X_C$ --- net reactance in ohms.

$X = X_L - X_C$ when $V_L > V_C$ (or) $X_L > X_C$

$X = X_C - X_L$ when $V_C > V_L$ (or) $X_C > X_L$



$OB = OC - OD$



$OB = OC - OD$

The net reactance X is the oppositions offered to the flow of current.

$$Z = X = X_L \sim X_C.$$

1) $I = \frac{V}{Z} = \frac{V}{X}$.

2) Power consumed $P = 0$.

3) Power factor $= \cos\phi = 0$.

Note :-

1) If $V_L > V_C$ in effect, the circuit behaves as an inductive circuit.

2) If $V_C > V_L$ in effect, the circuit behaves as an capacitive circuit.

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Impedance Triangle : Consider the phasor diagram of a R–L series a.c. circuit is shown in Fig. 3.12 (a). Divide the each side of the phasor diagram by a common factor I, we get another right angle triangle, as shown in Fig. 3.12 (b), whose sides represented R, X_L and Z. Such a triangle is known as impedance triangle.

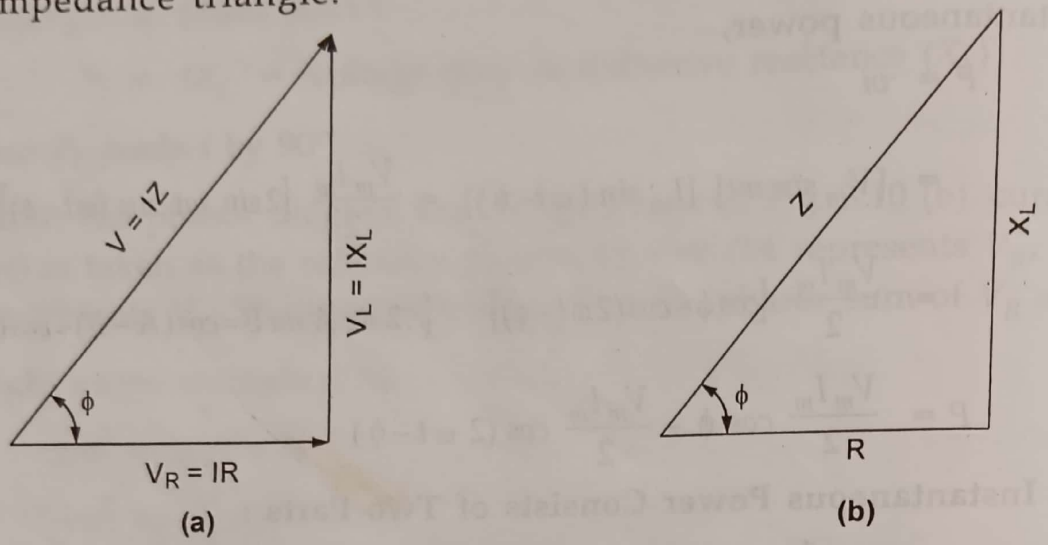


FIG 3.12 :

Note : Impedance triangle is usefull in *a.c.* circuits to calculate the following.

1. Impedance, $Z = \sqrt{R^2 + X_L^2}$
2. Power factor, $\cos \phi = \frac{R}{Z}$
3. Phase angle, $\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$

(or) $\phi = \cos^{-1}\left(\frac{R}{Z}\right)$

Power Factor ($\cos\phi$) : The power factor of a circuit can be defined in one of the following ways.

1. Power factor is defined as the consine of the angle between voltage (V) and current (I) in an a.c circuit.

   i.e., Power factor = $\cos \phi$ = cosine of angle between V and I

2. Power factor may also defined as the ratio of resistance to impedance of an a.c circuit.

   i.e., Power factor, $\cos \phi = \frac{R}{Z}$

3. Power factor is also defined as the ratio of true power (or active power) to the apparent power.

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i.e., Power factor $= \frac{true\,power}{apparent\,power} = \frac{VI\cos\phi}{VI}$

(or) $\cos\phi = \frac{VI\cos\phi}{VI}$

Power Triangle (Active and Reactive Components of Current and Power) :

Let us consider phasor diagram of Fig. 3.10 (a) in which current I lags behind the applied voltage by angle $\phi°$. The current I can be resolved into two components. They are :

1. $I\cos\phi$ which is in phase with V, called ***active component or wattful component*** or in-phase component and is contribut to active or true power of the circuit $I\cos\phi$.
2. $I\sin\phi$, which is 90° out of phase with voltage V, called ***the reactive component*** and contributes to reactive power of the circuit.

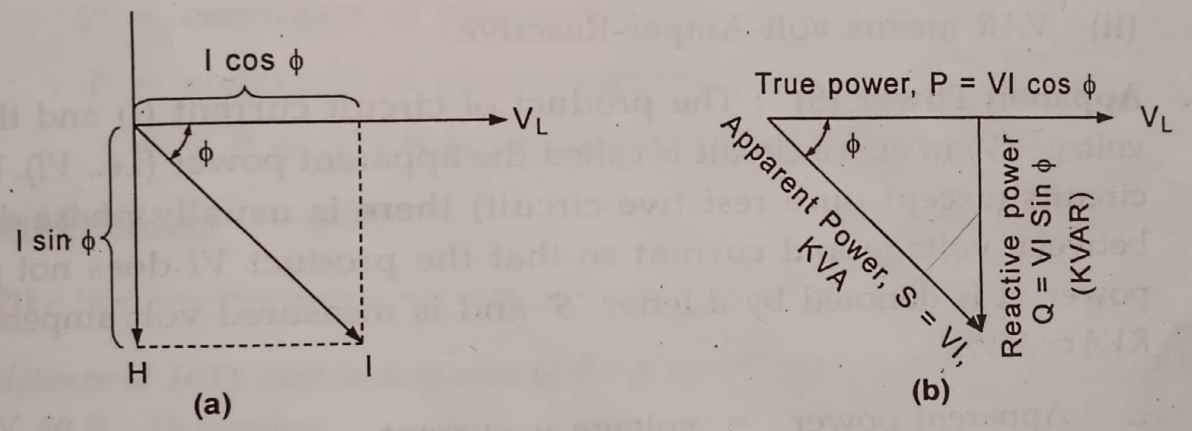


FIG 3.13 :

Now, multiply each of the current phasors in Fig. 3.13 (a) by V, we get the power triangle shown in Fig. 3.13 (b). It is a right angled triangle and sides of this triangle indicates the relation among apparent power true power, and reactive power. From the power triangle of Fig. 3.13 (b), the following points are worth noting.

1. **True Power (or) Active Power (P) :** The power which is actually consumed or utilised in an a.c circuit is called ***true power or active power***. This power relation is obtained by multiplying active component of current with circuit voltage. It is represented by the letter 'P' and is measured in watt (W) or kitowatt (KW) or mega watt (MW).

   ∴ True Power = {voltage} × {active component of current}

   $P = V \times I\cos\phi = VI\cos\phi$ *watts* (or KW)

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(or) $P = (IZ)\, I\left(\frac{R}{Z}\right) = I^2 R \text{ watt} \quad \left(\because V = IZ \text{ and } \cos\phi = \frac{R}{Z}\right)$

2. **Reactive Power (Q) :** The power which flows back and forth (i.e., in both directions in the circuit) or reacts upon itself. This circulating power is called reactive power. This power relation is obtained by multiplying reactive component of current with circuit voltage. It is represented by the letter 'Q' and is measured in *VAR* or *KVAR* or *MVAR*.

$\therefore$ Reactive Power, = [*Voltage*] × [*reactive component of current i.e., current 90° out of phase with voltage*]

$$Q = V \times I \sin\phi = VI \sin\phi \text{ VAR or } KVAR$$

(or) $$Q = (IZ)\, I\left(\frac{X_L}{Z}\right) = I^2 X_L \quad \textbf{(VAR)}$$

Note :

(i) Wattmeter does not measure the reactive power

(ii) *VAR* means volt-Amper-Reactive.

3. **Apparent Power (S) :** The product of circuit current (I) and the circuit voltage (V) in an *a.c* circuit is called the apparent power (i.e., VI). But in a.c circuits (except pure resistive circuit) there is usually phase difference between voltage and current so that the product VI does not give real power. It is denoted by a letter 'S' and is measured volt ampers (VA) or *KVA* or *MVA*.

$\therefore$ Apparent power = voltage × current

$S = VI$ *VA* or *KVA* or *MVA*.

Since, $V = IZ$ or $I = \frac{V}{Z}$

Then, $S = VI = (IZ)\, I = I^2 Z$ (VA)

(or) $S = VI = V\left(\frac{V}{Z}\right) = \frac{V^2}{Z}$ (VA)

The above three power shown in the power triangle of Fig. 3.12 (b) relatad in the vector domain by. $S = P \pm jQ$

For an inductive load, $S = P + jQ_L$ *VA*

For a capacitive load, $S = P - jQ_L$ *VA*

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Since the reactive power and active power are always angled 90^o to each other, the three power are related by.

$$S^2 = P^2 + Q^2 \quad \text{(or)} \quad S = \sqrt{P^2+Q^2}$$

$$\text{(or)} \quad KVA = \sqrt{(KW)^2+(KVAR)^2}$$

Therefore, third power always be found if the other two are known.

Complex Power : Using complex algebra, the apparent power can also express in complex form as

$S = VI^*$ = phasor voltage × conjugate of phasor current

(or) $S = V^* I$ = conjugate of phasor voltage × phasor current.

(or) $P + JQ = VI^*$

Where V = phasor voltage

I = phasor current

V^* = conjugate of phasor voltage

I^* = conjugate of phasor current

$S = V I^*$ or V^*I is apparent power in complex form.

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EXAMPLE

A resistance of 10 Ω and inductance of 0.1 H are connected in series across a supply of 220 V, 50 H_Z. Determine.

(a) The impedance *(b) Current flowing*

(c) Power factor *(d) The phase angle between V and I.*

MARCH/APRIL. 2006 [C-00]

Solution :

Given data :

$R = 10\,\Omega$	$Z = ?$
$L = 0.1\ H$	$I = ?$
$V = 220\ V$	$\cos\phi = ?$
$f = 50\ H_Z$	$\phi = ?$
$Z = ?$	

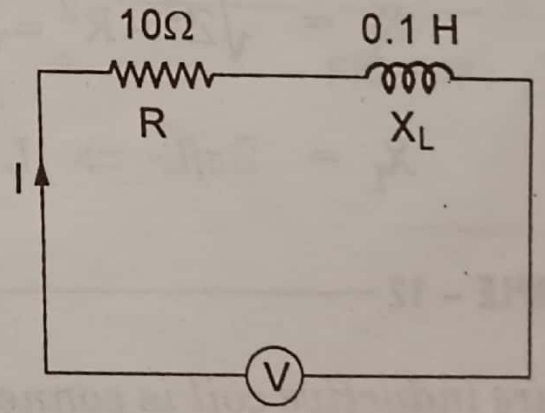


FIG 3.14 :

$$X_L = 2\pi fL = 2\pi \times 50 \times 0.1 = 31.416\,\Omega$$

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(a) $Z = \sqrt{R^2+X_L^2} = \sqrt{10^2+(31.416)^2} = 32.97\,\Omega$

(b) $I = \frac{V}{Z} = \frac{220}{32.97} = 6.673A$

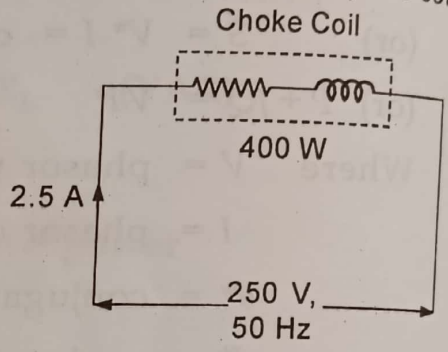
(c) Power factor $= \cos\phi = \frac{R}{Z} = \frac{10}{32.97} = 0.303\ (lag)$

(d) Phase angle $\phi = \cos^{-1}(0.303) = 72.36^\circ$

EXAMPLE – 11

A choke coil takes a current of 2.5 A when connected across 250 V, 50 H_Z mains and consumes 400 watts. Find.

(i) The power factor (ii) Resistance of the coil and (iii) Inductance of the coil.



Solution :

Given data :

$V = 250V$	$\cos\phi = ?$
$I = 2.5\ A$	$R = ?$
$f = 50\ H_Z$	$L = ?$
$P = 400\ W$	

(i) $P = VI\cos\phi$

$\therefore \quad \cos\phi = \frac{P}{VI} = \frac{400}{250\times2.5} = 0.64\ (lag)$ **Ans**

(ii) $V = IZ \Rightarrow Z = \frac{V}{I} = \frac{250}{2.5} = 100\,\Omega$

Also, $\cos\phi = \frac{R}{Z} \Rightarrow R = Z\cos\phi = 100 \times 0.64 = 64\,\Omega$ **Ans**

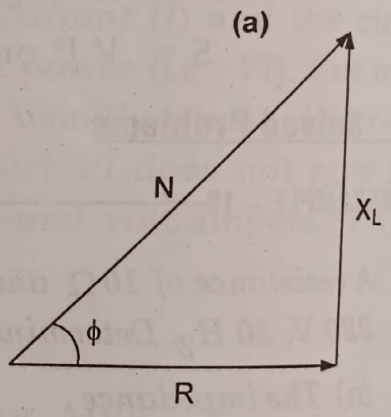


FIG 3.15 :

(iii) $Z = \sqrt{R^2+X_L^2}$

$\therefore \quad X_L = \sqrt{Z^2-R^2} = \sqrt{100^2-64^2} = 76.84\ \Omega$

$\therefore \quad X_L = 2\pi fL \Rightarrow L = \frac{X_L}{2\pi f} = \frac{76.84}{2\pi\times50} = 0.245\ H$ **Ans**

EXAMPLE – 12

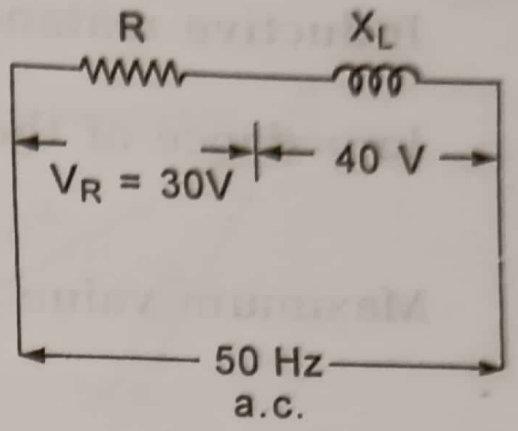
A pure inductive coil is connected in series with a 10 Ω resistor to 50 H_Z a.c source. The voltage across the resistor, and inductor were found to be 30V and 40V respectively. Find the value of the inductive reactance of the coil, the supply voltage and the phase angle.

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Soltuion :

Given data :

$R = 10\,\Omega$	$X_L = ?$
$f = 50\,H_z$	$V = ?$
$V_R = 30\ V$	$\phi = ?$
$V_L = 40\ V$	

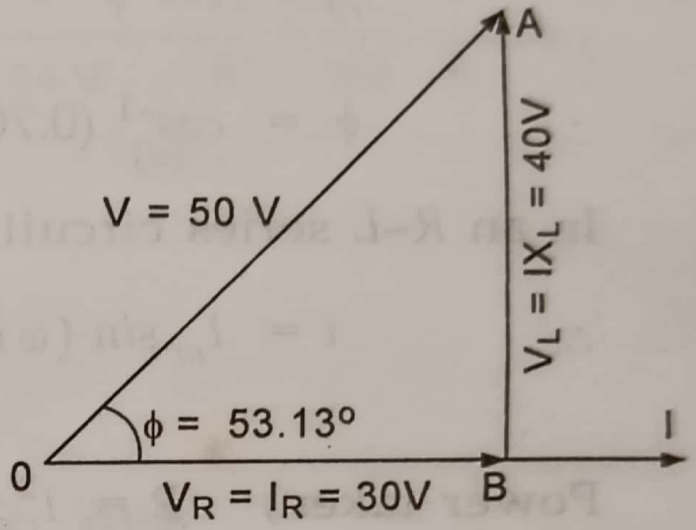


(a)

$$\therefore \quad V = IR \quad \Rightarrow \quad I = \frac{V_R}{R} = \frac{30}{10} = 3A.$$

$$\therefore \quad X_L = \frac{V_L}{I} = \frac{40}{3} = 13.33\,\Omega \quad \textbf{Ans}$$

From Fig. (b),



(b)

FIG 3.16 :

$$V = \sqrt{V_R^2 + L_L^2} = \sqrt{(30)^2 + (40)^2} = 50V \quad \textbf{Ans}$$

$$\phi = tan^{-1}\left(\frac{V_L}{V_R}\right) = tan^{-1}\left(\frac{40}{30}\right) = 53.13\ (lag) \quad \textbf{Ans}$$

$$\text{(or)} \quad \phi = tan^{-1}\left(\frac{X_L}{R}\right) = tan^{-1}\left(\frac{13.33}{10}\right) = 53.13^\circ\ (lag) \quad \textbf{Ans}$$

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EXAMPLE

A voltage $e = 200 \sin 100\pi t$ is applied to a coil having $R = 200\,\Omega$ and $L = 638mH$. Find the expression for the current and also determine the power taken by the coil.

Solution :

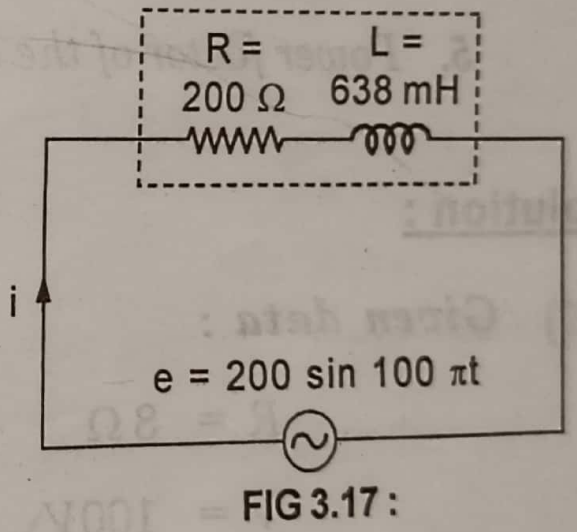


FIG 3.17 :

Given data :

$e = 200 \sin 100\pi$

$R = 200\,\Omega$

$L = 638\ mH = 638 \times 10^{-3}\ H$

$i = ?$

$P = ?$

Compare equation (1) with standard equation ..... (1)

$e = E_m \sin 2\pi ft$; we get.

$E_m = 200V$

$\omega = 2\pi f = 100\pi$

$\therefore \quad f = \dfrac{100\pi}{2\pi} = 50\ H_Z$

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Inductive reatance, $X_L = 2\pi fL = 2\pi \times 50 \times 638 \times 10^{-3} = 200.43\,\Omega$

Impedance of the coil, $Z = \sqrt{R^2 + X_L^2} = \sqrt{(200)^2 + (200.43)^2} = 283.15\,\Omega$

Maximum value of current, $I_m = \frac{E_m}{Z} = \frac{200}{283.15} = 0.7064A$

$$pf = \cos\phi = \frac{R}{Z} = \frac{200}{283.15} = 0.7064 \text{ (lag)}$$

$$\therefore \quad \phi = \cos^{-1}(0.7046) = 45.06^\circ$$

In an R–L series circuit current i lags behind e by an angle ϕ

$$\therefore \quad i = I_m \sin(\omega t - \phi) = 0.7064 \sin(100\pi t - 45.06^\circ) \quad \textbf{Ans.}$$

Power taken, $P = I_{r.m.s}^2 R = \left(\frac{I_m}{\sqrt{2}}\right)R = \left(\frac{0.7064}{\sqrt{2}}\right)^2 \times 200 = 50\,W$ **Ans.**

(or) $P = VI\cos\phi = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}\cos\phi = \left(\frac{200}{\sqrt{2}}\right) \times \left(\frac{0.7064}{\sqrt{2}}\right) \times 0.706 = 50\,W$ Ans.

EXAMPLE –14

A non-inductive resistor of 8 ohms is connected in series with an inductive load, and the combination is connected acorss a 100 V, 50 H_Z A.C. supply. The voltage across the non-inductive resistor is 64 V and that across the inductive load is 48 V. Calculate.

1. *Impedance of the load*
2. *Impedance of the combined circuit*
3. *Power absorbed by the load*
4. *Total power taken from supply*
5. *Power factor of the load and*
6. *Power factor of the whole circuit.*

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olution :

Given data :

$R = 8\,\Omega$

$V = 100V$

$f = 50\,H_Z$

$V_1 = 64\,V$

$V_2 = 48\,V$

1. $Z_L = ?$;
2. $Z = ?$
3. $P_L = ?$
4. $P = ?$
5. $\text{Cos}\,\phi_L = ?$;
6. $\text{Cos}\,\phi = ?$

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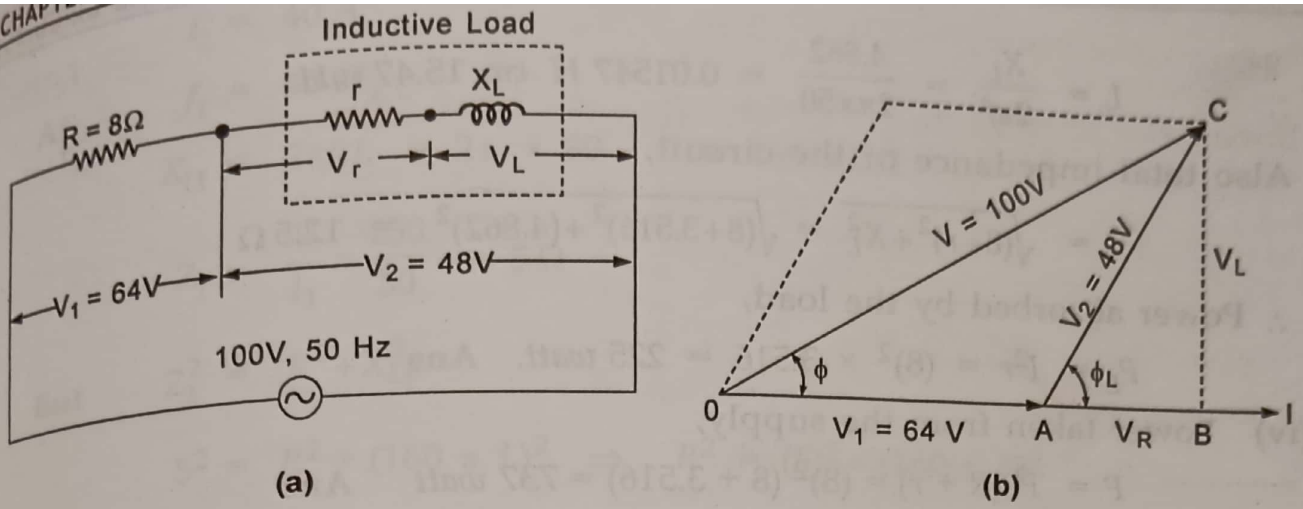


FIG 3.18 :

Circuit Current, $I = \frac{V_1}{R} = \frac{64}{8} = 8A$

(i) Impedance of the load, $Z_L = \frac{V_2}{I} = \frac{48}{8} = 6\,\Omega$ **Ans.**

(ii) Impedance of the combined circuit, $Z = \frac{V}{I} = \frac{100}{8} = 12.5\,\Omega$ **Ans.**

(iii) Impedance of the combined circuit is

$$Z = \sqrt{(R+r)^2 + X_L^2}$$

$$(12.5) = \sqrt{(8+r)^2 + X_L^2}$$

$$(12.5)^2 = \sqrt{(8+r)^2 + X_L^2} \quad \ldots\ldots\ldots\ldots (1)$$

Impedance of the load,

$$Z_L = \sqrt{r^2 + X_L^2}$$

$$6^2 = r^2 + X_L^2$$

$$\therefore \quad X_L^2 = 6^2 - r^2 \quad \ldots\ldots\ldots\ldots (2)$$

Substituting equation (2) in (1), we get

$$(12.5)^2 = (8 + r)^2 + 6^2 - r^2 = 8^2 + 16r + r^2 + 6^2 - r^2$$

$$\therefore \quad 16\,r = (12.5)^2 - 8^2 - 6^2$$

$$r = \frac{(12.5)^2 - 8^2 - 6^2}{16} \Rightarrow r = \frac{56.25}{16} = 3.516\,\Omega.$$

From equation (2), we get

$$X_L = \sqrt{6^2 - (3.516)^2} = \sqrt{23.637}\ \Omega = 4.862\,\Omega$$

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$$\therefore \quad L = \frac{X_L}{2\pi f} = \frac{4.862}{2\pi \times 50} = 0.01547\ H \text{ or } 15.47\ mH.$$

Also total impedance of the circuit,

$$Z = \sqrt{(8+r)^2 + X_L^2} = \sqrt{(8+3.516)^2 + (4.862)^2} = 12.5\,\Omega$$

$\therefore$ Power absorbed by the load,

$$P_L = I^2 r = (8)^2 \times 3.516 = 225\ watt. \quad \textbf{Ans}$$

(iv) Power taken from the supply,

$$P = I^2 (R + r) = (8)^2 (8 + 3.516) = 737\ watt \quad \textbf{Ans}$$

(v) *p.f* of the load,

$$\cos\phi_L = \frac{r}{Z_L} = \frac{3.516}{6} = 0.586\ (lag) \quad \textbf{Ans}$$

(vi) *p.f* of the whole circuit,

$$\cos\phi = \frac{(R+r)}{Z} = \frac{(8+3.516)}{12.5} = 0.92\ (lag) \quad \textbf{Ans}$$

EXAMPLE – 15

A coil is connected across a variable frequency a.c supply, whose voltage is maintained constant at 250 V. When the frequency of the supply is H_Z, the current is 50 A. On increasing the frequency to 120 H_Z, the current falls to 40 A. Calculate (i) the resistance of the coil, and (i) inductance of the coil.

Solution :

Given data :

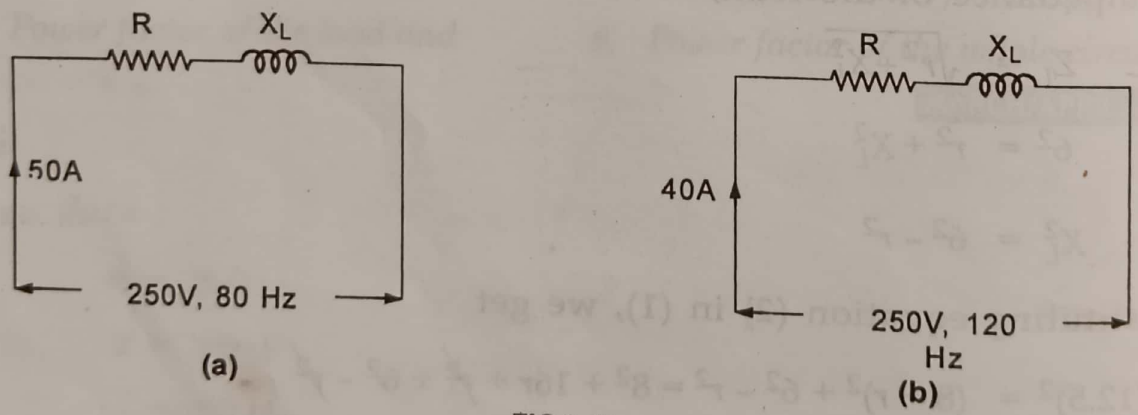


FIG 3.19 :

$V = 250\ V$
$f_1 = 80\ H_Z$
$I_1 = 50\ A$
$f_2 = 120\ H_Z$

$R_L = ?$
$L = ?$

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$I_2 = 40\ A$

$f_1 = 80\ H_Z$

At $X_{L1} = 2\pi f_1 L = 2\pi \times 80 \times L = 160\ \pi L\ \Omega$

$$Z_1 = \frac{V}{I_1} = \frac{250}{50} = 5\,\Omega$$

But $Z_1^2 = R^2 + X_{L1}^2$

$5^2 = R^2 + (160\ \pi\ L)^2 \Rightarrow R^2 = (5)^2 - (160\ \pi\ L)^2$ ..... (1)

$f_2 = 120\ H_Z$

At $X_{L2} = 2\pi f_2 L = 2\pi \times 120L = 240\ \pi\ L\ \Omega$

$$Z_2 = \frac{V}{I_2} = \frac{250}{40} = 6.25\,\Omega$$

But $Z_2^2 = R^2 + X_{L2}^2$

$(6.25)^2 = R^2 + (240\,\pi\,L)^2 \Rightarrow R^2 = (6.25)^2 - (240\,\pi\,L)^2$ ..... (2)

Equating (1) and (2), we get

$$5^2 - (160\,\pi\,L)^2 = (6.25)^2 - (240\,\pi\,L)^2$$

$$(240\,\pi\,L)^2 - (160\,\pi\,L)^2 = (6.25)^2 - 5^2$$

$$(568489.21 - 252661.87)\ L^2 = 39.0625 - 25$$

$$315827.34\ L^2 = 14.0625$$

$$\therefore \quad L = \sqrt{\frac{14.0625}{315827.34}} = 6.673 \times 10^{-3} H \text{ or } 6.673 mH$$

(i) Inductance of the coil, $L = 6.667\ mH$ **Ans**

(ii) Substitute $L = 6.673\ mH$ in Eqn. (1), we get

$$R^2 = 5^2 - (160\,\pi \times 6.673 \times 10^{-3}) = 13.749$$

$$\therefore \quad R = \sqrt{13.749} = 3.708\,\Omega \quad \textbf{Ans}$$

EXAMPLE - 16

A coil A takes 3 A current at power factor of 0.8 lagging with an applied potential difference of 14 V. A second coil B takes 3 A current at power factor of 0.707 lagging with an applied potential difference of 10 V. What p.d will be required to produce a total current of 3 A with coils A and B in series. Find the power factor in this connection.

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Solution :

Since the coils A and B drawing lagging current from supply, hence the given coils are inductive.

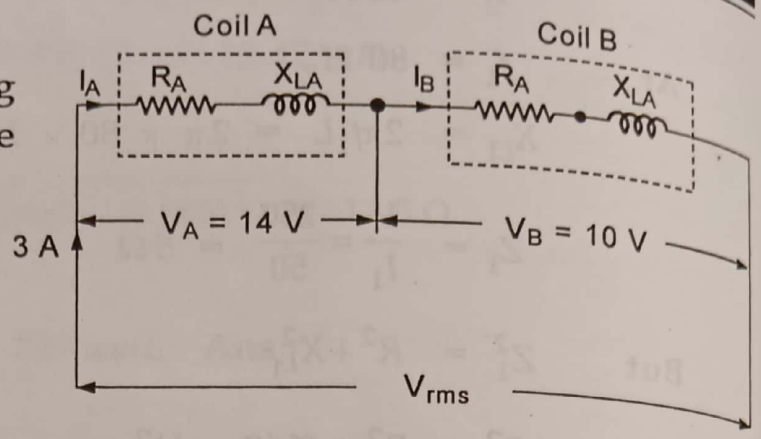


FIG 3.20 :

For Coil – A :

Given data :

$$Z_A = \frac{V_A}{I_A} = \frac{14}{3} = 4.67\,\Omega$$

$$R_A = Z_A \cos\phi_A = 4.67 \times 0.8 = 3.736\,\Omega$$

$$X_A = \sqrt{Z_A^2 - R_A^2} = \sqrt{(4.67)^2 - (3.736)^2} = 2.802\,\Omega$$

For Coil – B :

Given data :

$$Z_B = \frac{V_B}{I_B} = \frac{10}{3} = 3.33\,\Omega$$

$$\therefore \quad Z_B = \frac{R_B}{Z_B} \qquad \therefore R_B = Z_B \cos\phi_B = 3.33 \times 0.707 = 2.354\ \Omega$$

$$X_B = \sqrt{Z_B^2 - R_B^2} = \sqrt{(3.33)^2 - (2.354)^2} = 2.355\,\Omega$$

When Coil–A and Coil–B are Connection in Series :

Total resistance, $R = R_A + R_B = 3.736 + 2.354 = 6.09\,\Omega$

Total reactance, $X = X_A + X_B = 2.802 + 2.355 = 5.157\,\Omega$

Total impedance, $Z = Z_A + Z_B = 4.67 + 3.33 = 8\,\Omega$

(or) $$Z = \sqrt{R^2 + X^2} = \sqrt{(6.09)^2 + (5.157)^2} = 7.98\,\Omega$$

$\therefore$ Voltage applied to the circuit, $V = I\,Z = 3 \times 8 = 24V$ **Ans**

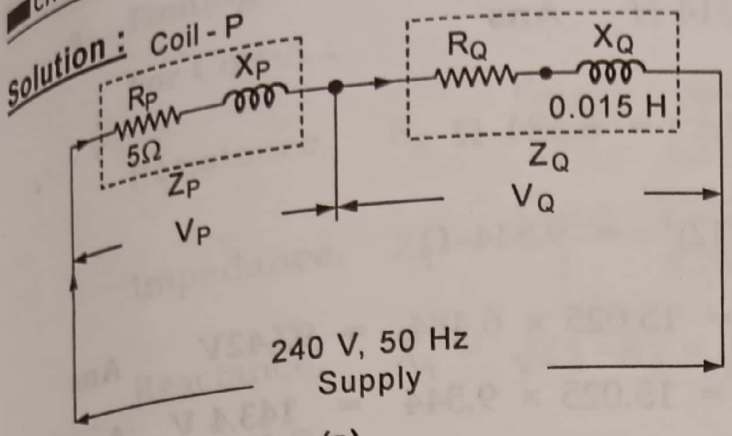
Power factor of the circuit, $\cos\phi = \frac{R}{Z} = \frac{6.09}{8} = 0.761\ (lag)$ **Ans**

EXAMPLE – 17

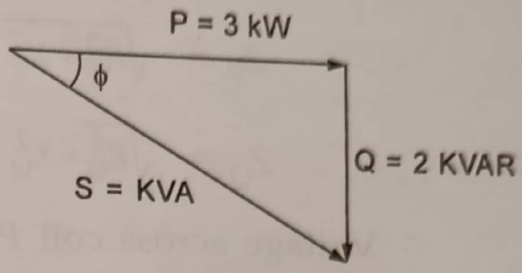
Two coils P and Q are connected in series across a 240 V, 50H_Z supply. The resistance of coil P is 5Ω and the inductance of coil Q is 0.015H. If the input from the supply is 3 KW and 2 KVAR, find the inductance of coil P and the resistance of coil Q. Calculate also the voltage across each coil.

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Solution :



(a)



(b)

FIG 3.21 :

Given data :

$V = 240\ V$ | $L_P = ?$;
$f = 50\ H_Z$ | $R_Q = ?$;
$R_P = 5\ \Omega$ | $V_P = ?$
$L_Q = 0.15\ H$ | $V_Q = ?$
$P = 3\ KW = 3000\ W$
$Q = 2\ KVAR$

From phasor diagram of Fig. 3.24 ; we get

$$S = \sqrt{P^2+Q^2} = \sqrt{3^2+2^2} = \sqrt{9+4} = 3.606\ KVA = 3.606\ VA$$

But $S = VI \Rightarrow I = \frac{S}{V} = \frac{3606}{240} = 15.025\ A$

Also $P = I^2 (R_P + R_Q)$

$\therefore \quad 3000 = (15.025)^2\ (5 + R_Q)$

$\therefore \quad (5 + R_Q) = \frac{3000}{(15.025)^2} = 13.3\ \Omega$

$\therefore \quad R_Q = 13.39 - 5 = 8.3\ \Omega$ **Ans**

$$Z = \frac{V}{I} = \frac{240}{15.025} = 15.97\ \Omega$$

Also, $Z^2 = \sqrt{(R_P+R_Q)^2+(X_P+X_Q)^2}$

$$X_Q = 2\pi f L_Q = 2\pi \times 50 \times 0.015 = 4.712\ \Omega$$

$$\therefore (X_P + X_Q) = \sqrt{Z^2-(R_P+R_Q)} = \sqrt{(15.97)^2-(5+8.3)^2} = \sqrt{(15.97)^2-(13.3)^2} = 8.84\ \Omega$$

$\therefore$

$$X_P = 8.84 - X_Q = 8.84 - 4.712 = 4.128\ \Omega$$

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$$\therefore \quad L_P = \frac{X_P}{2\pi f} = \frac{4.128}{2\pi \times 50} = 0.01314\ H \quad \textbf{Ans}$$

$$Z_P = \sqrt{R_P^2 + X_P^2} = \sqrt{5^2 + (4.128)^2} = 6.484\ \Omega$$

$$Z_Q = \sqrt{R_Q^2 + X_Q^2} = \sqrt{(8.3)^2 + (4.712)^2} = 9.544\ \Omega$$

$\therefore$ Voltage across coil P, $V_P = IZ_P = 15.025 \times 6.484 = \mathbf{97.42V}$ **Ans**

Voltage across coil Q, $V_Q = IZ_Q = 15.025 \times 9.544 = \mathbf{143.4\ V}$ **Ans**

EXAMPLE – 18

Three air cored coils A, B, and C are connected in series when a current of 8.25 A is passed through the circuit, the voltage drops are respectively 30 V, 15, 23V on direct current and 37, 22, 30 V and alternating current. Find for each of the coils.

(i) Parameters *(ii) Power factor*

(iii) Power dissipated when A.C flows through the circuit

(iv) Also determine P.f of the whole circuit and the voltage applied across it.

Solution :

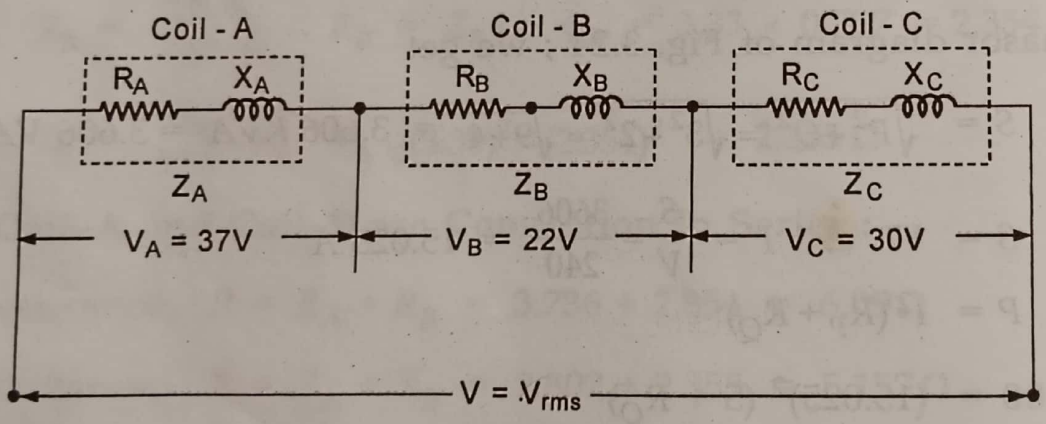


FIG 3.22 :

Given data :

$V_A = 30V$ .....d.c
$V_A = 37V$ .....a.c

$V_B = 15V$ .....d.c
$V_B = 22V$ .....a.c

$V_C = 23V$ .....d.c
$V_C = 30V$ .....a.c

$I = 8.25A$

(i) Parameters : $R_A, X_A = ?$; $R_B, X_B = ?$; $R_C, X_C = ?$

(ii) Power Factors : $\cos\phi_A, \cos\phi_B, \cos\phi_C = ?$

(iii) Power when a.c flows : $P_A, P_B, P_C \ldots = ?$

(iv) *P.f* of the circuit and voltage applied :

$\cos\phi = ?$; $V = ?$

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1. Finding Parameters for Each Coil :

For Coil-A :

Resistance, $R_A = \frac{V_{A(d.c)}}{I} = \frac{30}{8.25} = 3.636\,\Omega$ **Ans**

Impedance, $Z_A = \frac{V_{A(a.c)}}{I} = \frac{37}{8.25} = 4.485\,\Omega$ **Ans.**

Reactance, $X_A = \sqrt{Z_A^2 - R_A^2} = \sqrt{(4.485)^2 - (3.636)^2} = 2.626\,\Omega$ **Ans**

For Coil-B :

$$R_B = \frac{V_{B(d.c)}}{I} = \frac{15}{8.25} = 1.82\,\Omega \quad \textbf{Ans}$$

$$Z_B = \frac{V_{B(a.c)}}{I} = \frac{22}{8.25} = 2.67\,\Omega \quad \textbf{Ans}$$

$$X_B = \sqrt{Z_B^2 - R_B^2} = \sqrt{(2.67)^2 - (1.82)^2} = 1.954\ \Omega \quad \textbf{Ans}$$

For Coil-C :

$$R_C = \frac{V_{C(d.c)}}{I} = \frac{23}{8.25} = 2.79\,\Omega \quad \textbf{Ans}$$

$$Z_C = \frac{V_{C(a.c)}}{I} = \frac{30}{8.25} = 3.636\,\Omega \quad \textbf{Ans}$$

$$X_C = \sqrt{Z_C^2 - R_C^2} = \sqrt{(3.636)^2 - (2.79)^2} = 2.33\,\Omega \quad \textbf{Ans}$$

2. Finding Power Factor for Each Coil :

For Coil-A :

$$p.f = \cos\phi = \frac{R_A}{Z_A} = \frac{3.636}{4.485} = 0.8107\ (lag) \quad \textbf{Ans}$$

$$\therefore \quad \phi_A = \cos^{-1}(0.8107) = 35.84^\circ$$

For Coil-B :

$$\cos\phi_B = \frac{R_B}{Z_B} = \frac{1.82}{2.67} = 0.6816\ (lag) \quad \textbf{Ans}$$

$$\phi_B = (0.6816) = 47.03^\circ$$

For Coil-C :

$$\cos\phi_C = \frac{R_C}{Z_C} = \frac{2.79}{3.636} = 0.7673\ (lag) \quad \textbf{Ans}$$

$$\phi_C = \cos^{-1}(0.7673) = 40^\circ$$

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3. Finding Power Dissipation in each Coil :

For Coil- A :

$$P_A = I^2R_A = (8.25)^2 \times 3.636 = 247.5 \text{ watt} \quad \textbf{Ans}$$

For Coil B : $P_B = I^2R_B = (8.25)^2 \times 1.82 = 124$ watt **Ans**

For Coil C : $P_C = I^2R_C = (8.25)^2 \times 2.79 = 190$ watt. **Ans**

4. Finding p.f of the Whole Circuit and Applied Voltage :

Total resistance, $R = R_A + R_B + R_C = 3.636 + 1.82 + 2.79 = 8.246\,\Omega$

Total reactance, $X = X_A + X_B + X_C = 2.626 + 1.954 + 2.33 = 6.91$

Total impedance, $Z = \sqrt{R^2+X^2} = \sqrt{(8.246)^2(6.91)^2} = \mathbf{10.76\,\Omega}$

$\therefore$ p.f of the circuit, $\cos\phi = \dfrac{R}{Z} = \dfrac{8.246}{10.76} = 0.7664\ (lag)$ **Ans**

$\therefore \phi = \cos^{-1}(0.7664) = 39.96 \approx 40^\circ$

Voltage applied for whole circuit,

$$V = IZ = 8.25 \times 10.76 = \mathbf{88.77\ V} \quad \textbf{Ans}$$

EXAMPLE - 19

A coil having a fixed resistance of 4 Ω and an inductive reactance of 20 Ω is connected in series with a variable resistor. The whole circuit is now connected across a 230 V, 50 H_Z supply mains. For what value of variable resistor is the power consumed in it a maximum. Calculate the current drawn and p.f of the circuit.

Solution :

Given data :

$R_L = 4\,\Omega$	$r = ?$
$X_L = 20\,\Omega$	$I = ?$
$V = 230V$	$\cos\phi = ?$
$f = 50\,H_Z$	

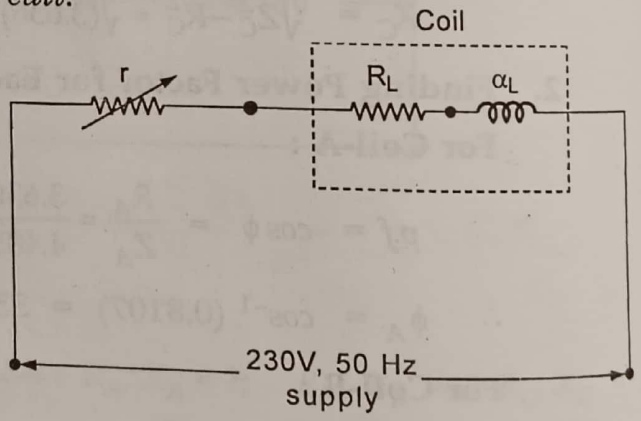


FIG 3.23 :

Total resistance of the circuit,

$$R = r + R_L = (4 + r)\,\Omega$$

$$\therefore \quad Z = \sqrt{R^2+X_L^2} = \sqrt{(4+r)^2+(20)^2}$$

Current drawn by the circuit,

$$I = \frac{V}{Z} = \frac{230}{\sqrt{(4+4)^2+(20)^2}}$$

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Power consumed in variable resistor r,

$$P = I^2 r. \text{ watt}$$

$$\therefore \quad P = \left[\frac{230}{\sqrt{(4+r)^2+(20)^2}}\right]^2 \times r. = \frac{(230)^2 r}{(4+r)^2+(20)^2}$$

To find the value of r corresonding to maximum power in it, the conditions is $\frac{dp}{dr} = 0$

$$\frac{dp}{dr} = \frac{d}{dr}\left[\frac{(230)^2 r}{(4+r)^2+(20)^2}\right] = 0$$

$$(230)^2\left[\frac{\{(4+r)^2+(20)^2\}X1 - r(2r+8)}{(4+r)^2+(20)^2}\right] = 0$$

$$(4+r)^2 + (20)^2 - r(2r+8) = 0$$

$$16 + 8r + r^2 + 400 - 2r^2 - 8r = 0$$

$$r^2 = 416 \Rightarrow r = \sqrt{416} = 20.396\,\Omega \quad \textbf{Ans.}$$

$\therefore$ Current drawn by the circuit,

$$I = \frac{(230)}{\sqrt{(4+r)^2+(20)^2}} = 7.29A \quad \textbf{Ans.}$$

$$\therefore \quad P.f = \cos\phi = \left(\frac{R+r}{Z}\right) = \frac{(4+20.396)}{\sqrt{(4+20.396)^2+(20)^2}} = 0.773\ (lag) \quad \textbf{Ans.}$$

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EXAMPLE – 20

A non inductive resistance of 15Ω is connected in series with a capacitor of 100μF capacitance across a 230V, 50 H_Z single phase a.c supply. Calculate (i) impedance of the circuit, (ii) current (iii) p.f and phase angle, (iv) power consumed in the circuit, (v) equation for the instantaneous value of current.

Solution :

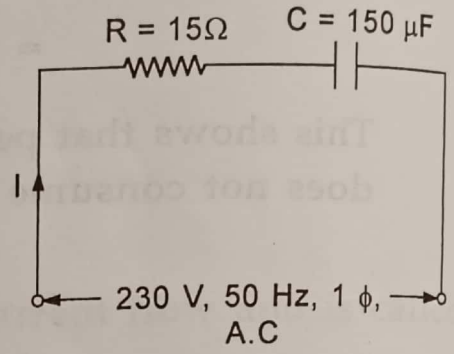


FIG 3.28 :

Given data :

$R = 15\,\Omega$	$Z = ?$
$C = 100\,\mu F = 100 \times 10^{-6} F$	$I = ?$
$V = 230V$	$\cos\phi, = ?$
$f = 50\,H$	$P = ?$

(i) $$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}} = 31.831\,\Omega$$

$$\therefore \quad Z = \sqrt{R^2 + X_C^2} = \sqrt{(15)^2 + (31.831)^2} = 35.12\,\Omega$$

(ii) Current, $$I = \frac{V}{Z} = \frac{230}{35.12} = 6.55\ A$$ **Ans**

(iii) Power factor,

$$\cos\phi = \frac{R}{Z} = \frac{15}{35.12} = 0.4271 \text{ (lead)}$$ **Ans**

$\therefore$ Phase angle, $\phi = \cos^{-1}(0.4271) = 64.72^o$ **Ans**

(iv) Power consumed,

$$P = I^2R = (6.55)^2 \times 15 = 644\ watt$$ **Ans**

(or) $$P = VI\cos\phi = 230 \times 6.55 \times 0.4271 = 644\ watt$$ **Ans**

(v) $$i = I_m \sin(\omega t + \phi) = \sqrt{2} \times I \sin(\omega t + \phi^o) = \sqrt{2} \times 6.55 \sin(\omega t + 64.72^o)$$

$$= 9.263 \sin(\omega t + 64.72^o)$$ **Ans.**

EXAMPLE – 21

A non inductive resistor is connected in series with a condensor across a 100V, 50 H_Z, 1ϕ. A.C supply. The current in the circuit is 3.2 A. The power loss in the resistor is 96.8W and that in the condensor is negligible, calculate the resistance and the capacitance.

Solution :

Given data :

$V = 100\ V$	$R = ?$
$f = 50\ H_Z$	$C = ?$
$I = 3.2\ A$	
$P_R = 96.8\ watt$	

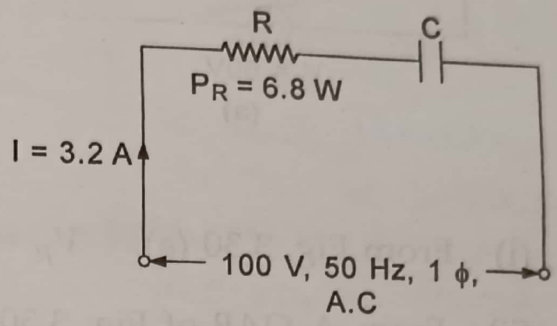


FIG 3.29 :

Power loss in the resistor, $P_R = I^2R$

$$\therefore \quad R = \frac{P_R}{I^2} = \frac{96.8}{(3.2)^2} = 9.453\,\Omega \quad \textbf{Ans}$$

$$Z = \frac{V}{I} = \frac{100}{3.2} = 31.25\,\Omega$$

Also $Z = \sqrt{R^2 + X_C^2} \quad \therefore \quad X_C = \sqrt{Z^2 - R^2} = \sqrt{(31.25)^2 - (9.453)^2} = 29.786\,\Omega$

But $$X_C = \frac{1}{2\pi fC} \Rightarrow C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \times 50 \times 29.786}$$

$= 106.86 \times 10^{-6}\ F$ or $106.86\ \mu F$ **Ans**

EXAMPLE – 22

A 15 Ω resistor and 200 μ F capacitor are connected in series to a 60V sinusoidal supply. The circuit is 3 A. Calculate.

(i) Voltage across the resistor and capacitor.

(ii) The supply frequency

(iii) Phase angle between current and voltage.

Solution :

Given data :

$R = 15\,\Omega$	$V_R = ?$
$C = 200\ \mu F = 200 \times 10^{-6}\ F$	$V_C = ?$
$I = 3\ A$	$f = ?$
	$\phi = ?$

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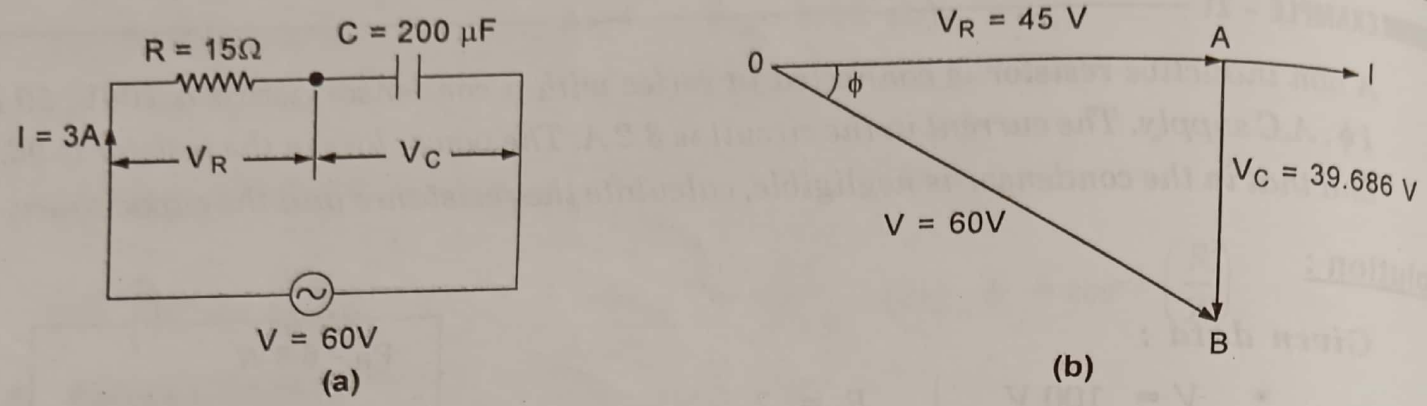


FIG 3.30 :

(i) From Fig. 3.30 (a), $V_R = IR = 3 \times 15 = 45V$ **Ans.**

(ii) From Δ OAB of Fig. 3.30 (b),

$$OB^2 = OA^2 + AB^2$$

$$V^2 = V_R^2 + V_C^2$$

$$\therefore \quad V_C = \sqrt{V^2 - V_R^2} = \sqrt{(60)^2 - (45)^2} = \mathbf{39.686V} \quad \textbf{Ans}$$

(iii) $$V_C = IX_C \Rightarrow X_C = \frac{V_C}{I} = \frac{39.686}{3} = 13.23\,\Omega$$

$$\therefore \text{ Frequency, } f = \frac{1}{2\pi C X_C} = \frac{1}{2\pi \times 200 \times 10^{-6} 13.23} = 60\ H_Z. \quad \textbf{Ans}$$

(iv) From Fig. 330 (b)

$$\cos\phi = \frac{V_R}{V} = \frac{45}{60} = 0.75 \Rightarrow \phi = \cos^{-1}(0.25) = 41.41^\circ \quad \textbf{Ans}$$

EXAMPLE – 23

Determine the capacitance of a condenser to be connected in series with a 100V, 80W lamp in order that the lamp draw its normal current when the combination is connected to a 200V, 50H_Z supply. Also determine p.f of the circuit.

Solution :

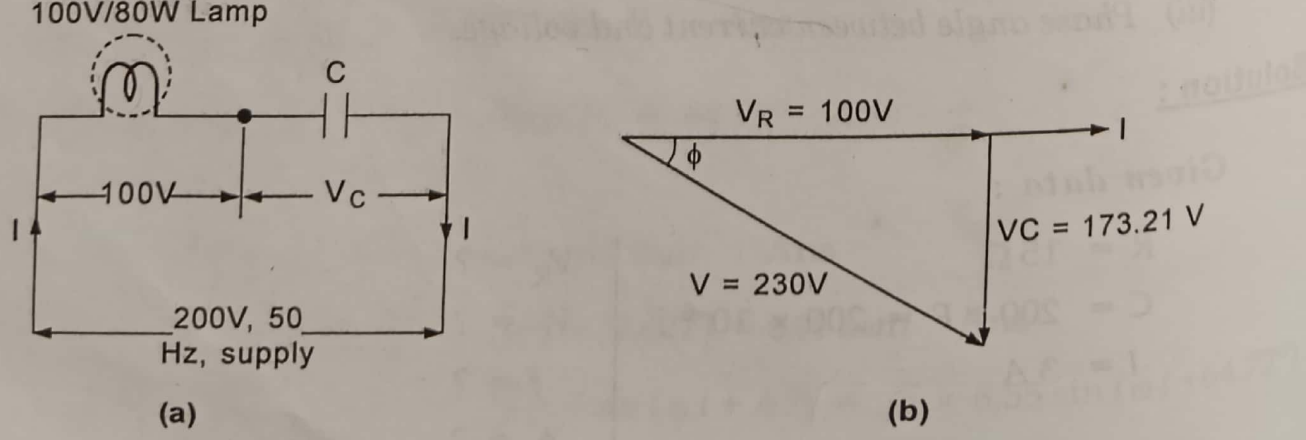


FIG 3.31 :

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Voltage across the bulb, $V_R = 100V$

Power rating of the bulb, $P_R = 80\ \omega$ | $C = ?$

Supply voltage, $V = 200\ V$ | $\cos\phi = ?$

Frequency, $f = 50\ H_Z$

$\therefore$ Rated current of the bulb, $I = \frac{P}{V} = \frac{80}{100} = 0.8A$

From Fig. 3.31 (b),

$$V_C = \sqrt{V^2 - V_R^2} = \sqrt{(200)^2 - (100)^2} = \sqrt{30000} = 173.21\ V$$

$$X_C = \frac{V_C}{I} = \frac{173.21}{0.8} = 216.5\,\Omega$$

$$\therefore \quad C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \times 50 \times 216.5} = 14.703 \times 10^{-6} F \text{ or } 14.703\ \mu F \quad \textbf{Ans}$$

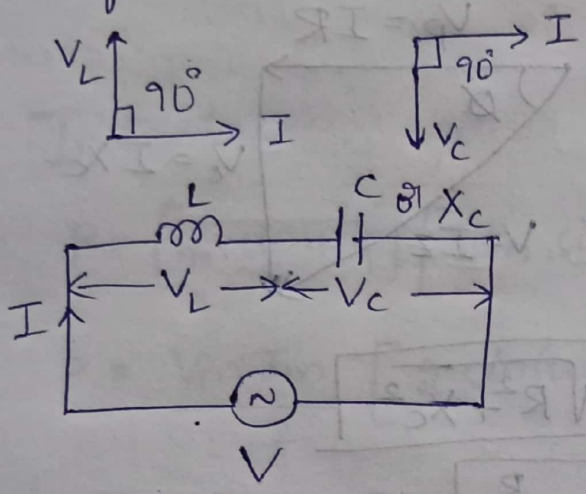
p.f of the circuit, $\cos\phi = \frac{V_R}{V} = \frac{100}{200} = 0.5$ (lead) **Ans**

EXAMPLE 24

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③ L-C Series A.C. circuits :-

Consider an a.c. circuit containing "L" henry and Capacitance "C" farad in series as shown in fig:



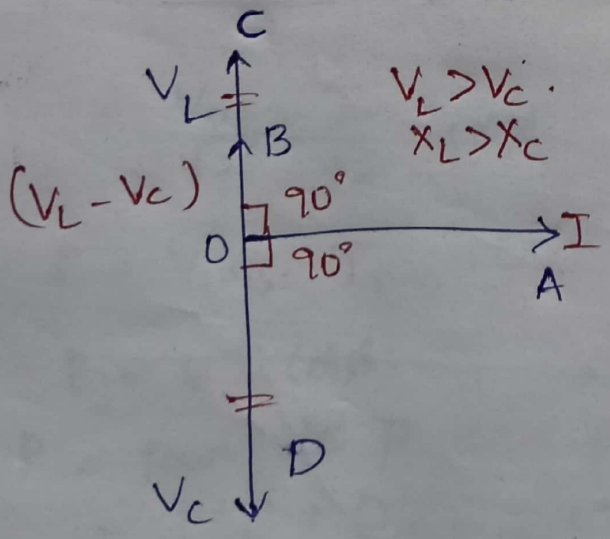
Let $V_L = IX_L$ = Voltage across "L" .. leads I by 90°.

$V_C = IX_C$ = voltage across C ... lags I by 90°.

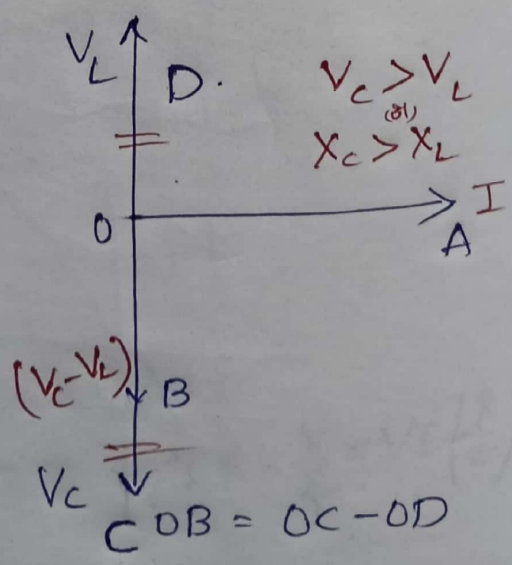
$X = X_L - X_C$ --- net reactance in ohms.

$X = X_L - X_C$ when $V_L > V_C$ (or) $X_L > X_C$

$X = X_C - X_L$ when $V_C > V_L$ (or) $X_C > X_L$



$OB = OC - OD$



$OB = OC - OD$

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The net reactance X is the oppositions offered to the flow of current.

$$Z = X = X_L \sim X_C.$$

1) $I = \frac{V}{Z} = \frac{V}{X}$.

2) Power consumed $P = 0$.

3) Power factor = $\cos\phi = 0$.

Note :-

1) If $V_L > V_C$ in effect, the circuit behaves as an inductive circuit.

2) If $V_C > V_L$ in effect, the circuit behaves as an capacitive circuit.

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A pure inductance coil having an inductance of H is connected in series with a capacitor of 50 μF capacitance across 230 V 50 HZ, 1ϕ, A.C supply. Find (a) the net reactance (b) circuit current, and (c) P.F of the circuit.

Solution :

Given data :

$L = 8H$	$X = ?$
$C = 50\,\mu F$	$I = ?$
$V = 230\ V$	$p.f = ?$
$f = 50\ HZ$	

(a) $X_L = \omega L = 2\pi fL = 2\pi \times 50 \times 8 = 2513.27\,\Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 10^{-6}}$$

$$= 3183.1\,\Omega$$

$\therefore$ $X = X_C - X_L$ $[\because X_C > X_L, X \text{ is a capacitive reactance}]$

$$= 3183.1 - 2513.27$$

$$= 669.83\,\Omega$$

(b) $$I = \frac{V}{X} = \frac{230}{669.83} = 0.343A$$

(c) $$p.f = \cos\phi = \frac{R}{Z} = \frac{R}{X} = \frac{0}{669.83} = 0\ [\because R = 0]$$

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R–L–C SERIES A.C CIRCUIT

Fig. 3.37 shows the R, L and C connected in series across a supply voltage. This is a general seris *a.c* circuit.

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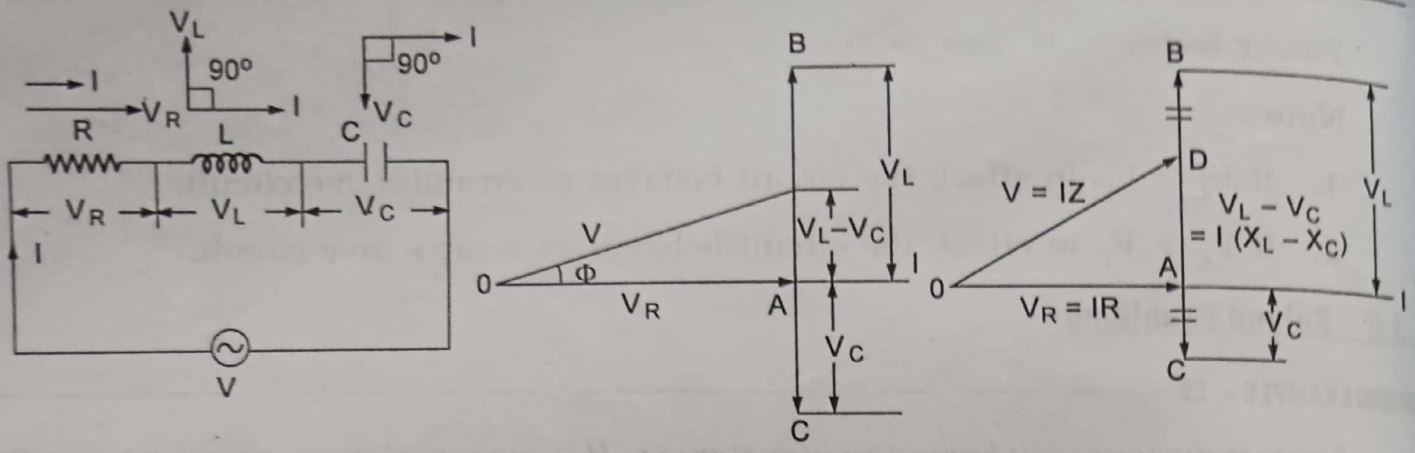


FIG 3.37 :

Let

$OA = V_R$ = voltage drop in resistor **R** (V_R is inphase with I)

$AB = V_L$ = voltage drop in inductance **L** (V_L leads I by 90°)

$AC = V_C$ = voltage drop in capacitance **C** (V_C lags I by 90°)

$OD = V$ = applied voltage.

It should be noted that V_L is in phase opposition to V_C. This means that circuit can be either effectively inductive or capacitive depending upon which voltage drop (V_L or V_C) is predominant. For the case considered. $V_L > V_C$ so that net voltage drop across L–C combination is represented by AD.

$$\therefore \quad AD = AB - AC = V_L - V_C$$

From the voltage triangle of Fig. 3.13 (a),

$$OD^2 = OA^2 + AD^2$$

i.e.,
$$V^2 = V_R^2 + (V_L - V_C)^2$$
$$= (IR)^2 + (IX_L - IX_C)^2$$
$$= I^2 \left[(R^2 + (X_L - X_C)^2\right]$$
$$V = I\sqrt{R^2 + (X_L - X_C)^2}$$

(or)
$$I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$$

The quantity $\sqrt{R^2 + (X_L - X_C)^2}$ offers opposition to current flow and is called impedance of the circuit.

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$$\therefore \quad I = \frac{V}{Z};$$

where $Z = \sqrt{R^2+(X_L-X_C)^2}$

Phase Angle (ϕ) :

From Fig. 3.13,

$$\tan\phi = \frac{AD}{OA} = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R}$$

$$\therefore \quad \phi = \tan^{-1}\left[\frac{(X_L - X_C)}{R}\right]$$

Power : It has already been seen that power in a.c circuit is consumed only in resistance, whereas, in case of pure inductor and capacitor do not consume any power.

∴ Average power,

$$P = VI\cos\phi = (IZ)\, I\,\frac{R}{Z} = I^2R$$

Power factor, $\cos\phi = \frac{V_R}{V} = \frac{R}{Z}$

If the alternating voltage applied across the circuit is given by the equation.

$$v = V_m \sin wt$$

Then $i = Im \sin(wt \pm \phi)$.

The circuit current depends upon the parameters present in the circuit explained below :

Three Cases of *R–L–C* Series A.C Circuit :

Impedance of a *R–L–C* series circuit is given by.

$$Z = \sqrt{R^2+(X_L-X_C)^2}$$

1. When $X_L > X_C$ [i.e., $(X_L - X_C)$ is positive], the phase angle ϕ is positive and the circuit behavies as an *R–L* series circuit. In this case, the circuit *I* lags behind the applied voltage *V* by ϕ and *p.f* is lagging.

   ∴ The current equation is given by

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$$i = I_m \sin(\omega t - \phi)$$

2. When $X_L < X_C$ [i.e., $(X_L - X_C)$ is negative], the phase angle ϕ is negative, and the circuit behaves as an R–C series circuit. In this case, the circuit current I leads the applied voltage V by ϕ and $p.f$ is leading.

   $\therefore$ The current equation is given by,

   $$i = I_m \sin(\omega t + \phi)$$

3. When $X_L = X_C$ [i.e., $(X_L - X_C) = 0$], the phase angle ϕ is zero, and the circuit behaves like a pure resistive circuit. In this case, the circuit current I is in phase with applied voltage 'V' $p.f$ is unity.

   $\therefore$ The current equation is given by

   $$i = I_m \sin wt$$

Hence from the above three points, this is a general series a.c circuit.

3.4 Solved Problems on R–L–C Series Circuit

EXAMPLE – 30

A coil of resistance 50 Ω and inductance 0.1H is connected in series with a capacitor of 150 μF across a 200V, 50 HZ supply. Caculate.

(i) Current
(ii) Power factor
(iii) Power and voltage across the coil and capacitor. Draw the phasor diagram

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Solution :

Given data :

$R = 50\,\Omega$		$I = ?$	
$L = 0.1\,H$		$\phi = ?$;	
$C = 150\,\mu F$	$= 150 \times 10^{-6}\,F$	$P = ?$	
$V = 200\,V$		$V_L = ?$	
$f = 50\,H.$		$V_C = ?$	

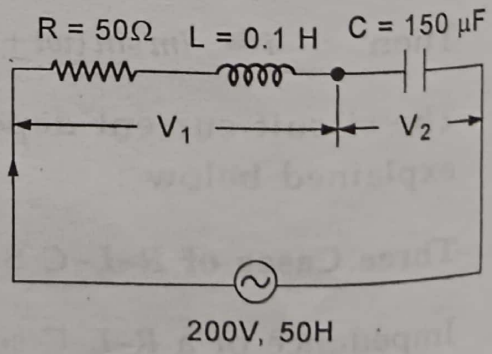


FIG 3.38 :

(i) Inductive reactance, $X_L = 2\pi fL = 2\pi \times 50 \times 0.1 = 31.41\,\Omega$

Capacitvie reactance, $X_C = \dfrac{1}{2\pi fC} = \dfrac{1}{2\pi \times 50 \times 150 \times 10^{-6}} = 21.22\,\Omega$

Here $X_L > X_C$, hence the circuit is inductive. Current lags the voltage by an angle ϕ.

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Impedance of the circuit, $Z = \sqrt{R^2+(X_L-X_C)^2}$

$= \sqrt{50^2+(31.41-21.22)^2} = \sqrt{50^2+(10.19)^2} = 51.03\,\Omega$

$\therefore$ Current, $I = \frac{V}{Z} = \frac{200}{51.03} = 3.92A$ **Ans**

(ii) Power factor, $\cos\phi = \frac{R}{Z} = \frac{50}{51.03} = 0.9798\ (lag)$ **Ans**

$\phi = \cos^{-1}(0.9798) = 11.54^\circ\ (lag)$

Alternatively :

$\tan\phi = \left(\frac{X_L-X_C}{R}\right) = \frac{(31.41-21.22)}{50} = 0.2038$

$\phi = \tan^{-1}(0.2038) = 11.52^\circ$

$\therefore$ $\cos\phi = \cos(11.52^\circ) = 0.9798\ (lag)$ **Ans**

(iii) Power, $P = I^2R = (3.92)^2 \times 50 = 768.32\ watt$ **Ans**

(or) $P = VI\cos\phi = 200 \times 3.92 \times 0.9798$

$= 768.2\ watt$ **Ans.**

Voltage across the coil, $V_1 = IX_L = 3.92 \times 31.41 = 123V\ (lag)$ **Ans.**

Voltage across the capacitor, $V_2 = IX_C = 3.92 \times 21.22 = 83.18V$ **Ans.**

EXAMPLE – 31

A resistance of 12 Ω, an inductance of 0.15 H and a capacitance of 100 μH are connected in series across a 100V, 50 Hz supply. Calculate
(i) Impedance (ii) Current (iii) Power factor (iv) Power consumed.

OCT/NOV. 2009

Solution :

Given data :

$R = 12\,\Omega$
$L = 0.15\ H$
$C = 100 \times 10^{-6}\ F$
$V = 100\ V$
$f = 50\ Hz$

$Z = ?$
$I = ?$
$p.f. = \cos\phi = ?$
$P = ?$

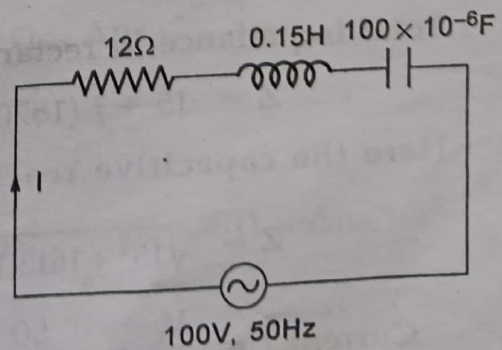


FIG 3.39 :

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(i) $X_L = 2\pi fL = 2\pi \times 50 \times 0.15 = 47.12\,\Omega$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}} = 31.83\Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{12^2 + (47.12 - 30.83)^2} = 19.43\ \Omega$$

(ii) $I = \frac{V}{Z} = \frac{100}{19.43} = 5.145\ A$ **Ans**

(iii) p.f $\cos\phi = \frac{R}{Z} = \frac{12}{19.43} = 0.6176$ (lag) **Ans**

(iv) $P = VI\cos\phi = 100 \times 5.45 \times 0.6176 = 317.75 = 318\ W$ **Ans**

EXAMPLE – 32

In an R – L – C series circuit where $R = 15\,\Omega$, $L = 5H$, and $C = 1\mu F$, Find.

(i) Total impedance

(ii) Line current

(iii) Voltage across

(iv) Power factor of the circuit when the supply voltage is 50V at 50 Hz and draw the phasor diagram.

Solution :

Given data :

$V = 50\ V$	(i) $Z = ?$
$R = 15\,\Omega$	(ii) $I = ?$
$L = 5\ H$	(iii) $V_R, V_L, V_C = ?$
$C = 1\ \mu F$	(iv) $\cos\phi = ?$
$f = 50\ Hz$	

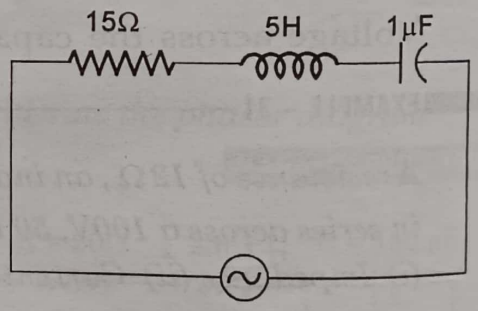


FIG 3.40 :

$X_L = 2\pi fL = 2 \times 50 \times 5 = 1570.79\,\Omega$

Total impedance in rectangular form $Z = R + j\,(X_L - X_C)$

$Z = 15 + j\,(1570.79 - 3183.09) = 15 - j\,1613.1$

Here the capacitive reactance dominated the inductive reactance.

$$Z = \sqrt{15^2 + 1613.1^2} = \sqrt{225 + 2602091.6} = 1{,}613.17\,\Omega$$

Current $I = \frac{V_s}{Z} = \frac{50}{1613.17} = 0.031\ A = 31\ mA$

Phase angle $\theta = tan^{-1}\,[(X_L \sim X_C)/R]$

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$$= \tan^{-1}\left(\frac{-1613.1}{15}\right) = -89.46$$

Voltage across the resistor

$$V_R = I_R = 0.031 \times 15 = 0.465\ V$$

Voltage across the capacitive reactance

$$V_C = IX_C = 0.031 \times 3183.09$$

Voltage across the inductive

$$V_L = IX_L = 0.031 \times 1570.79 = 48.69\ V$$

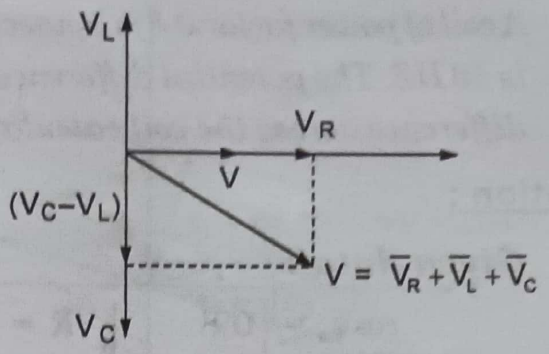


FIG 3.41 :

EXAMPLE – 33

A coil having a resistance of 5Ω and an inductance of 0.7mH is connected in series with a capacitor having a capacitance of 50mF. The circuit is then connected across 240V, 50 HZ a.c supply. Calculate.

(i) Inductive reactance *(ii) Capacitive reactance*
(ii) Total impedance *(iv) Current*
(v) Power consumed *(vi) Voltage drop across the coil.*

Solution :

Given data :

$R = 5\,\Omega$	$X_L = ?$
$L = 0.7\,mH = 7 \times 10^{-3} H$	$Z = ?$
$C = 50\,\mu F = 50 \times 10^{-6}\,F$	$I = ?$
$V = 240\ V$	$P = ?$
$f = 50\ H_Z$	$V_L = ?$

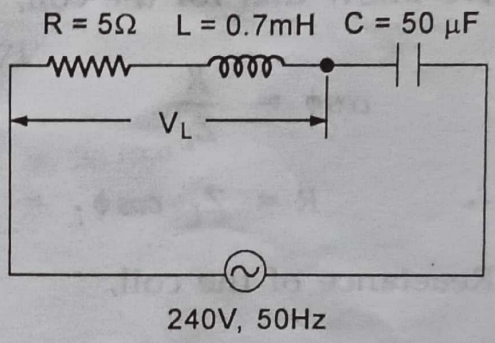


FIG 3.42 :

(i) $X_L = 2\pi fL = 2\pi \times 50 \times 0.7 \times 10^{-3} = 0.2199\,\Omega$ **Ans**

(ii) $X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 50 \times 10^{-6}} = 63.662\,\Omega$ **Ans**

(iii) $Z = \sqrt{R^2 + (X_C - X_L)^2}$ $(\therefore X_C > X_L)$

$= \sqrt{5^2 + (63.662 - 0.2199)^2} = \sqrt{5^2 + (63.44)^2} = 63.636\,\Omega$ **Ans**

(iv) $I = \frac{V}{Z} = \frac{240}{63.636} = 3.77A$ **Ans**

(v) $P = I^2R = (3.77)^2 \times 5 = 71.06\ W$ **Ans**

(vi) $V_L = IX_L = 3.77 \times 0.2199 = 0.829\ V$ **Ans**

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EXAMPLE – 38

Three impedance Z_1, Z_2 and Z_3 are connected in series across a 200 V, 50 HZ supply. If $Z_1 = 20\Omega$, $Z_2 = (8 + j10)\Omega$, $Z_3 = (15 - j15)\Omega$. Determine.

(a) *Total impedance* (b) *Total resistance* (c) *Total reactance*

(d) *The current through the circuit* (e) *The power factor of the circuit,*

(f) *The voltage across each impedance.*

lution :

Given data :

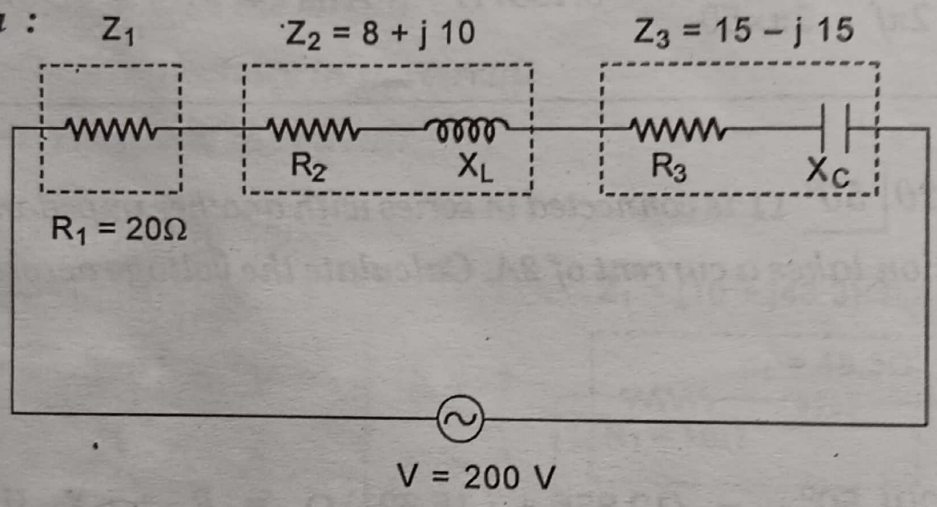


FIG 3.46 :

$V = 200\ V$,

$f = 50\ HZ$

$Z_1 = 20\Omega$ (resistive)

$Z_2 = 8 + J\,10$ (inductive)

$Z_3 = 15 - j\,15$ (capacitive)

$Z = ?$; $R = ?$

$X = ?$; $I = ?$

$\cos\phi = ?$; $V_R = ?$;

$V_2 = ?$; $V_3 = ?$

(a) Total impedance, $Z = Z_1 + Z_2 + Z_3$

$= (R_1 + j\,0) + (R_2 + j\,X_L) + (R_3 - j\,X_C)$

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$$= (20 + j\,0) + (8 + j\,10) + (15 - j\,15)$$

$$= (20 + 8 + 15) + j\,(0 + 10 - 15)$$

$$= (43 - j\,5)\,\Omega = 43.29 \angle -6.63^\circ\ \Omega$$

$\therefore$ $Z = 43.29\,\Omega$ **Ans**

(b) Total resistance, $R = R_1 + R_2 + R_3 = 20 + 8 + 15 = 43\,\Omega$ **Ans**

(c) Total reactance, $X = X_1 + X_2 + X_3 = 0 + 10 - 15 = -5\,\Omega$

$(\because X_C > X_L)$ **Ans**

(d) Taking V as reference, phasor,

i.e., $V = V \angle 0^\circ = 200 \angle 0^\circ\ V$

$$\therefore \quad I = \frac{V}{Z} = \frac{200 \angle 0^\circ}{43.29 \angle -(6.63^\circ)} = 4.62 \angle 6.63^\circ A \quad \textbf{Ans}$$

$\therefore$ $I = 4.62\ A$

(e) Phase angle between V and I, $\phi = 6.63^\circ$

$\therefore$ *p.f*, $\cos\phi = 6.63^\circ = 0.9933^\circ$ (leading) $(\because X_C > X_L)$ **Ans**

(f) Voltage across Z_1 $Z_1, V_1 = I Z_1 = 4.62 \angle 6.63 \times 20 \angle 0^\circ = 92.4 \angle 6.63\,V$ **Ans.**

(g) Voltage across, Z_2, $V_2 = IZ_2 = 4.62 \angle 6.63 \times (8 + j\,10)$

$= 4.62 \angle 6.63^\circ \times 12.81 \angle 51.34 = 59.18 \angle 57.97^\circ\,V$

(h) Voltage across Z_3, $V_3 = I Z_3 = 4.62 \angle 6.63 \times (15 - j\,15)$

$= 4.62 \angle 6.63 \times 21.2 \angle -45^\circ = 97.99 - \angle -38.37^\circ\,V$ **Ans**

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EXAMPLE – 40

An RLC series circuit is connected to 230 V, 50 HZ, supply, and it is found that the current in the circuit is unaltered when the capacitance is short circuited by a switch. If $R = 30\,\Omega$ and the current is 4 A. Determine L and C.

lution :

Given data :

$V = 230V$	$L = ?$
$f = 50\,H_Z$	$C = ?$
$R = 30\,\Omega$	
$I = 4\,A$	

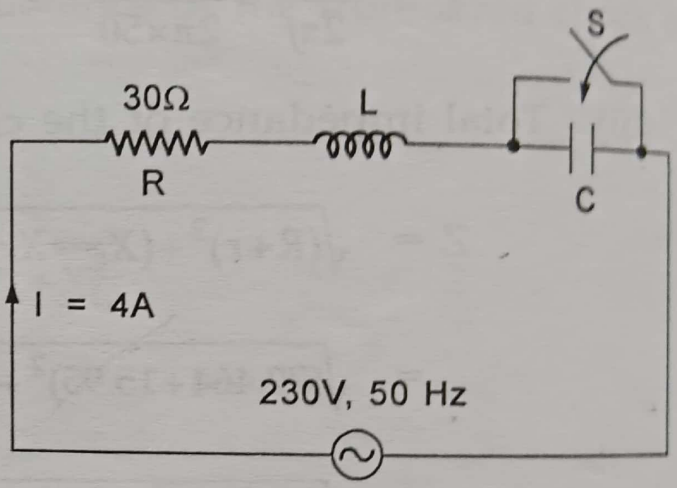


FIG 3.49 :

Impedance, $Z = \frac{V}{I} = \frac{230}{4} = 57.5\,\Omega$

When capacitor is short circuited :

$$X_L = \sqrt{Z^2 - R^2} = \sqrt{(57.5)^2 - (30)^2} = 49.0535\,\Omega$$

$$\therefore \quad L = \frac{X_L}{2\pi f} = \frac{49.0535}{2\pi \times 50} = 0.156\,H \quad \textbf{Ans.}$$

From *RLC* Circuit :

$$Z^2 = R^2 + (X_L - X_C)^2$$

$$(57.5)^2 = (30)^2 + (49.0535 - X_C)^2$$

$$(49.0535 - X_C)^2 = (57.5)^2 - (30)^2 = 2406.25$$

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$$2406.25 + X_C^2 - 98.107\, X_C = 2406.25$$

$$X_C^2 - 98.107\, X_C = 0 \Rightarrow X_C^2 = 98.107\, X_C$$

$$\therefore \quad X_C = 98.107\,\Omega$$

$$\therefore \quad C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \times 50 \times 98.107} = 32.445 \times 10^{-6}\, F \text{ or } 32.445\ \mu F$$

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Parallel A.C Circuits

The discussion for parallel *a.c* circuits will be very similar to that for *d.c* parallel circuits. A parallel circuit consists of two or more branches provides connected in parallel. As in parallel *d.c* circuits, the voltage across each branch is same but current flowing through them is different in magnitude and phase angle depending on the branch impedances.

.5.1 Methods of Solving Parallel A.C Circuits

There are three mains methods, may be applied for solving a.c parallel circuits.

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1. Phasor (or vector) diagram method
2. Admittance method
3. Phasor algebra or complex algebra or symbolic or *j*–method.

The use of a particular method will depend upon the conditions of the problem. However, in general, the method which yields quick results is applied.

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3.5.2 Phasor (or) Vector Diagram Method

Consider a parallel *a.c* circuit consisting of two branches (1) and (2) which is conected to an alternating voltage of V volts (*r.m.s*) as shown in Fig. 3.53.

The following steps to be remembered while solving parallel circuit of Fig. 3.53.

For Branch 1 :

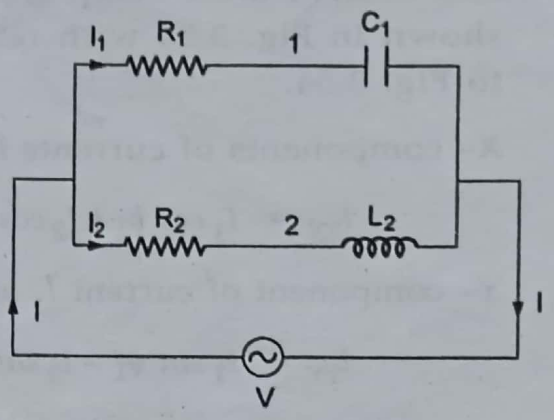


FIG 3.53 :

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

$$Z_1 = \sqrt{R_1^2 + X_C^2}$$

$$\therefore \quad I_1 = \frac{V}{Z_1}$$

and Angle, $\phi_1 = tan^{-1}\left(\frac{X_C}{R_1}\right)$ (leading) for capacitive circuit

(or) $cos\,\phi_1 = \frac{R_1}{Z_1} \Rightarrow \phi_1 = \cos^{-1}\left(\frac{R_1}{Z_1}\right)$ (leading)

The current I_1 leads the applied voltage V by $\phi_1{}^o$ see Fig. 3.56.

For Branch 2 :

$$X_L = \omega L = 2\pi f L$$

$$Z_2 = \sqrt{R_2^2 + X_L^2}$$

$$\therefore \quad I_2 = \frac{V}{Z_2}$$

Angle, $\phi_2 = tan^{-1}\left(\frac{X_L}{R_2}\right)$ (lagging) for inductive circuit

(or) $cos\,\phi_2 = \frac{R_2}{Z_2}$ (lagging)

The current I_2 lags behind V by $\phi_2{}^o$ (see Fig. 3.54).

Step 2 : Draw the phasor diagram by taking voltage as reference vector represent the branch currents on it as shown in Fig. 3.54.

Step 3 : Find the phasor sum of branch currents either by using (i) parallelogram laws of phasor (or) (ii) methods of components method.

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To find the resultant current, resolving the branch currents I_1 and I_2 into their X and Y-components (or active and reactive components respectively) and combin these comp-onents, as shown in Fig. 3.54 with reference to Fig. 3.54.

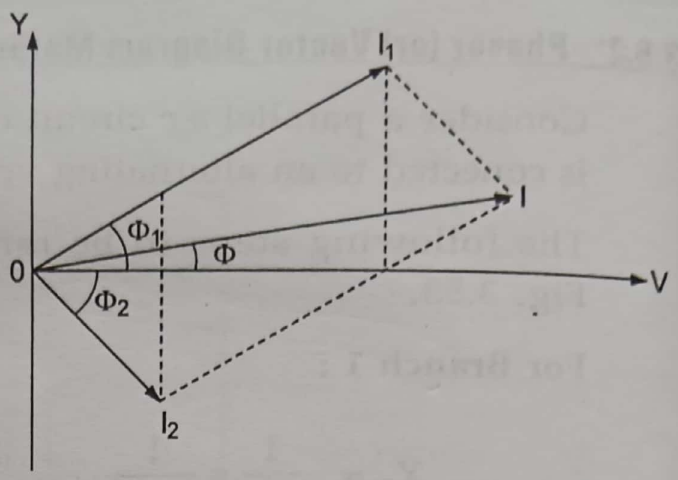


FIG 3.54 :

X- components of currents I_1 and I_2,

$$I_{XX} = I_1 \cos \phi_1 + I_2 \cos \phi_2$$

Y- component of current I_1 and I_2,

$$I_{YY} = I_1 \sin \phi_1 - I_2 \sin \phi_2$$

$$\therefore \quad I = \sqrt{I_{XX}^2 + I_{YY}^2} = \sqrt{(I_1 \cos\phi_1 + I_2 \cos\phi_2)^2 + (I_1 \sin\phi_1 - I_2 \sin\phi_2)^2}$$

Step 4 : Find the phase angle ϕ between the total current I and circuit voltage V.

$$\tan\phi = \frac{Y - component of currents}{X - component of currents}$$

$$\tan\phi = \frac{I_{YY}}{I_{XX}}$$

$$\therefore \quad \phi = \tan^{-1}\left(\frac{I_{YY}}{I_{XX}}\right)\left(\frac{Y - component of currents}{X - component of current}\right)$$

If ϕ is positive, line current I leads the voltage and if ϕ is negative, line current I lags behind the voltage.

$$\therefore \text{ Power factor of the circuit } = \cos\phi = \frac{I_{XX}}{I} = \frac{X - component}{Line\ current}$$

3.5.3 Solved Problems

EXAMPEL – 42

A coil resistance 15 Ω and inductance 0.05 H is connected in parallel with a non-inductive resistor of 20 Ω. Find (a) the current in each branch of the circuit, (b) the total current supplied, (c) the phase angle of the combination and p.f when a voltage of 200V at 50 H_Z is applied. (d) power consumed in the circuit (e) total impedance.

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Solution :

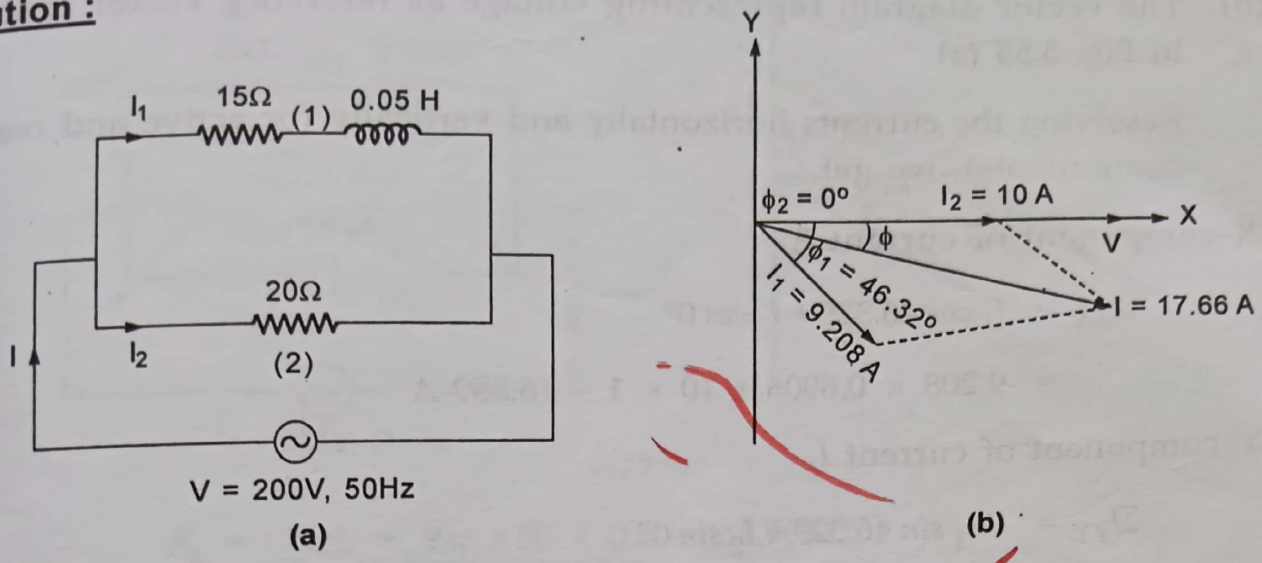


FIG 3.55 :

Branch 1 (R and L Series) :

Resistance, $R_1 = 15$

Inductnace, $L = 0.05\ H$

$\therefore$ Reactance, $X_L = 2\pi fL = 2\pi \times 50 \times 0.05 = 15.708\,\Omega$

Impedance, $Z_1 = \sqrt{R_1^2 + X_L^2} = \sqrt{(15)^2 + (15.708)^2} = 21.72\,\Omega$

$$I_1 = \frac{V}{Z_1} = \frac{200}{21.72} = 9.208\ A$$

Phase angle, $\phi_1 = \cos^{-1}\left(\frac{R_1}{Z_1}\right) = \cos^{-1}\left(\frac{15}{21.72}\right) = 46.32^\circ$ (lagging)

Since it is an RL series circuit, I_1 lags V by angle ϕ_1.

Branch 2 (R only) :

$$R_2 = 20\,\Omega$$

$$\therefore \quad I_2 = \frac{V}{R_2} = \frac{200}{20} = 10\ A$$

$$\phi_2 = \cos^{-1}\left(\frac{R_2}{Z_2}\right) = \cos^{-1}\left(\frac{R_2}{R_2}\right) = \cos^{-1}\left(\frac{20}{20}\right) = 0^\circ$$

Since it is a pure resistive circuit, I_2 is imphase with V

(a) Current in Branch (1), $I_1 = 9.208\ A$ **Ans**

Current in Branch (2), $I_2 = 10\ A$ **Ans**

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(b) The vector diagram representing voltage as reference vector is shown in Fig. 3.55 (a).

Resolving the currents horizontally and vertically (or active and reactive components), we get.

X-component of current I,

$$\Sigma I_{XX} = I_1 \cos 46.32^\circ + I_2 \cos 0^\circ$$

$$= 9.208 \times 0.6906 + 10 \times 1 = 16.359\ A$$

Y-component of current I,

$$\Sigma I_{YY} = -I_1 \sin 46.32^\circ + I_2 \sin 0^\circ$$

$$= -9.208 \times 0.723 + 0 = -6.657\ A$$

$\therefore$ Total current supplied,

$$I = \sqrt{(\Sigma I_{XX})^2 + (\Sigma I_{YY})^2} = \sqrt{(16.359)^2 + (-6.657)^2} = \mathbf{17.66\ A}\ \textbf{Ans.}$$

(c) Phase angle of the combination,

$$\phi = \tan^{-1}\left(\frac{I_{YY}}{I_{XX}}\right) = \tan^{-1}\left(\frac{-6.657}{16.359}\right) = \mathbf{-22.143^\circ}\ \textbf{Ans}$$

Power factor of the circuit,

$$\cos\phi = \cos(-22.143^\circ) = \mathbf{0.9262\ (lagging)}\ \textbf{Ans}$$

(d) Power consumed,

$$P = VI\cos\phi$$

$$= 200 \times 17.66 \times 0.9262$$

$$= 3271.3\ W\ \textbf{Ans}$$

(e) Total impedance,

$$Z = \frac{V}{I} = \frac{200}{17.66} = 11.325\,\Omega$$

EXAMPLE – 43

A coil has a resistance of 20Ω and inductance of 0.05H is connected in parallel with a capacitor of 50μF. If this parallel combination is connected across 200V, 50 H_Z supply, calculate (i) the line current (ii) power factor (iii) the total impedance (iv) power consumed.

Solution :

Branch 1 (R–L Series) :

$$R_1 = 20\,\Omega; \qquad L = 0.05\ H$$

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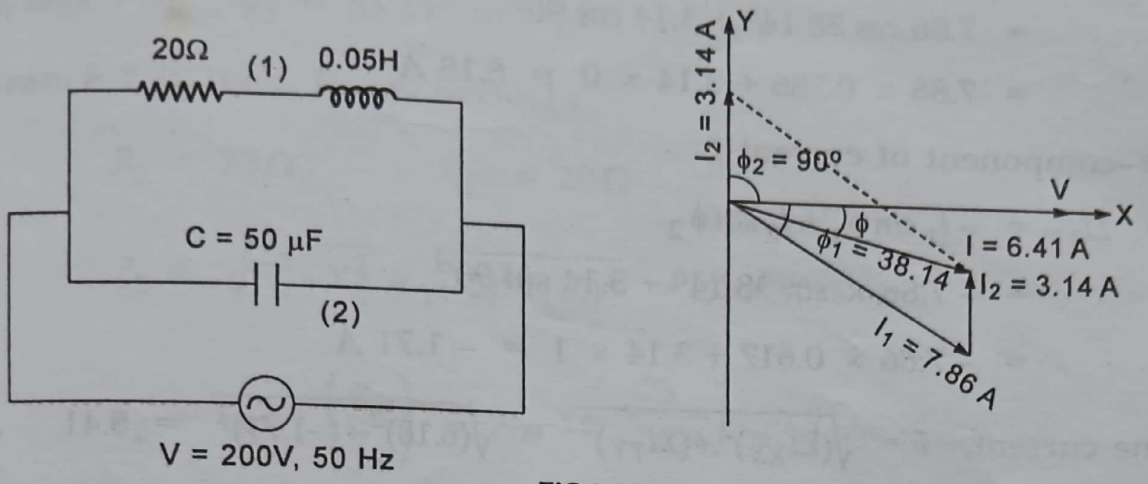


FIG 3.56 :

$$X_L = 2\pi fL = 2\pi \times 50 \times 0.05 = 15.708\,\Omega$$

$$Z_1 = \sqrt{R_1^2 + X_L^2} = \sqrt{(20)^2 + (15.708)^2} = 25.43\,\Omega$$

$$I_1 = \frac{V}{Z_1} = \frac{200}{25.43} = 7.86\ A$$

$$\phi_1 = tan^{-1}\left(\frac{X_L}{R}\right) = tan^{-1}\left(\frac{15.708}{20}\right) = 38.14^\circ \text{ (lag)}$$

(or) $$\phi_1 = cos^{-1}\left(\frac{R_1}{Z_1}\right) = cos^{-1}\left(\frac{20}{25.43}\right) = 38.14^\circ \text{ (lag)}$$

I_1 lags V by $\phi_1 = 38.14^\circ$

Branch 2 (C only) :

$$C = 50\ \mu F = 50 \times 10^{-6}\ F$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 10^{-6}} = 63.67\,\Omega$$

$$I_2 = \frac{V}{X_C} = \frac{200}{63.67} = 3.14\ A$$

$$\phi_2 = cos^{-1}\left(\frac{R_2}{X_C}\right) = cos^{-1}\left(\frac{0}{63.67}\right) = 90^\circ \text{ (leading)}$$

I_2 leads V by $\phi_2 = 90^\circ$

1. The vector diagram representing voltage as reference vector is shown in Fig. 3.56 (b)

   X-component of current I,

   $$\Sigma I_{XX} = I_1 \cos\phi_1 + I_2 \cos\phi_2$$

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$$= 7.86 \cos 38.14^\circ + 3.14 \cos 90^\circ$$

$$= 7.86 \times 0.786 + 3.14 \times 0 = 6.18\ A$$

Y-component of current I,

$$\Sigma I_{YY} = -I_1 \sin\phi_1 + I_2 \sin\phi_2$$

$$= -7.86 \times \sin 38.14^\circ + 3.14 \sin 90^\circ$$

$$= -7.86 \times 0.617 + 3.14 \times 1 = -1.71\ A$$

$\therefore$ Line current, $I = \sqrt{(\Sigma I_{XX})^2 + (\Sigma I_{YY})^2} = \sqrt{(6.18)^2 + (-1.71)^2} = 6.41$ **Ans**

$$\phi = \tan^{-1}\left(\frac{\Sigma I_{YY}}{\Sigma I_{XX}}\right) = \tan^{-1}\left(\frac{-1.71}{6.18}\right) = -15.47^\circ$$

2. Power factor, $\cos\phi = \cos(-15.47^\circ) = 0.964^\circ$ (lag) **Ans**

3. Impedance, $Z = \frac{V}{I} = \frac{200}{6.41} = 31.2\,\Omega$ **Ans**

4. Power consumed, $P = VI \cos\phi = 200 \times 6.41 \times 0.964$

$= 1235.85\ W$ **Ans**

EXAMPLE – 44

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4 Conductance, Impedance and Admitance

Conductance : In d.c circuits, conductance (G) is defined as (equal to) $\frac{1}{R}$. The total conductance of a parallel circuit is then found by adding the conductance of each branch. The total resistance R is simply $\frac{1}{G_T}$

i.e., $$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \text{.....}$$ (Arithmetic sum) **(ohm, Ω)**

(or) $$G = G_1 + G_2 + G_3 + \text{.....}$$ **(mho, ℧)**

Impedance : In *a.c.* series circuits, we seen that the impedance is opposition to the flow of alternating current. It is denoted by Z and is measured ohms.

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The vector sum of resistances and reactances (i.e., the total opposition) connected in an a.c circuit is called impedance.

$$\therefore \quad Z = \sqrt{R^2+X^2} = \sqrt{R^2+(X_L-X_C)^2}$$

$$= \sqrt{R^2+(2\pi fL-\frac{1}{2\pi fC})^2}$$

In a.c series circuits, when impedances Z_1, Z_2 Z_3, etc., are connected in series, the total impedance.

$$Z = Z_1 + Z_2 + Z_3 + \ldots\ldots\ldots \text{ (phasor sum)}$$

Admittance : Admittance is defined as the reciprocal of impedance. It is denoted by a letter Y and is measured in mho (℧) or siemens (S).

$$\therefore \text{ Admittance, } Y = \frac{1}{Z} = \frac{I}{V} = \frac{R.M.S\,ampere}{r.m.s\,volts} \quad \left[\because Z = \frac{V}{I}\right]$$

Admittance is a measure of how well an *a.c.* circuit will admit, or allow, current to flow in the circuit. The larger the value of admittance, therefore, the heavier the current flow for the same applied potential.

When impedances Z_1, Z_2, Z_3 etc., are connected in parallel, the total impedance of the circuit is given by.

$$\frac{1}{Z} = \frac{1}{Z_1}+\frac{1}{Z_2}+\frac{1}{Z_3} + \ldots\ldots\ldots\ldots\ldots \text{ (Vector sum)}$$

$$Y = Y_1 + Y_2 + Y_3 + \ldots\ldots\ldots\ldots$$

Like impedance, admittance also, has two components. One component is known as *conductance* (G) and the second component is *susceptance* (B). *The reciprocal of reactance* $\left(i.e.\frac{1}{x}\right)$ *is called susceptance and is a measure of how susceptible an element is to passage of current through it.*

Both conductance and susceptance are measured in mhos (℧) or siemens (S).

We know that the complex form of impedance is,

$$Z = R \pm jx$$

Similarly, the complex form of admittance is

$$Y = G \pm jB$$

Where G = Conductance of the circuit

B = Suceptance of the circuit.

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1. For Resistance,

$$Y = \frac{1}{Z} = \frac{1}{R} = \frac{1}{R\angle 0^\circ} = G\angle 0^\circ$$

so that $G = G\angle 0^\circ$

2. For inductor, an increase in frequency or inductance will result in a decrease in suceptance or, correspondingly, in admittance.

For Inductor,

$$Y = \frac{1}{Z} = \frac{1}{X_L} = \frac{1}{X_L\angle 90^\circ} = \frac{1}{\omega_L\angle 90^\circ} = \frac{1}{\omega_L}\angle -90^\circ$$

Defining $B_L = \frac{1}{X_L}$, we have., $B_L = B_L\angle -90 = -j\,B_L$

3. For capacitor, an increase in frequency or capacitance will result in an increase in its susceptibility.

For Capacitor,

$$Y = \frac{1}{Z} = \frac{1}{X_C} = \frac{1}{X_C\angle -90^\circ} = \frac{1}{(1/\omega C)\angle -90^\circ} = \omega C\angle -90^\circ = J\,\omega C$$

Defining, $B_C = \frac{1}{X_C}$,

We have, $B_C = B_C\angle 90^\circ = j\,BC$

4. From the above points (2) and (3), we conclude that
   - Inductive susceptances is taken as negative
   - Capacitive susceptance is taken as positive.

3.5.5 Admitetance Method

Consider a parallel circuit shown in Fig. 3.62 (a)

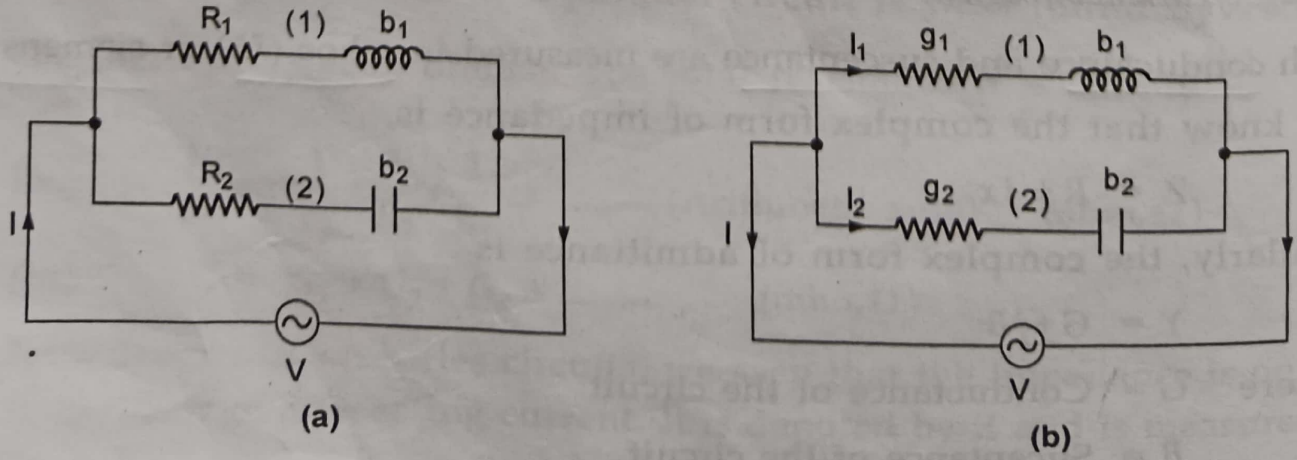


FIG 3.62

For Branch 1 :

It is an inductive branch.

$$Y_1 = \frac{1}{Z_1} = \frac{1}{R_1 + jX_L} = \frac{1}{R_1 + jX_L} \times \frac{R_1 - jX_L}{R_1 - jX_L}$$

$$= \frac{R_1}{R_1^2 + X_L^2} - j\frac{X_L}{R_1 + X_L^2} = g_1 - jb_1 \quad \ldots\ldots\ldots\ldots (1)$$

Thus for inductive branch,

$$g_1 = \frac{R_1}{R_1^2 + X_L^2} = \frac{R_1}{Z_1^2} \left[if \times = 0, g_1 = \frac{1}{R_1} \right]$$

$$b_1 = \frac{X_L}{R_1^2 + X_L^2} = \frac{X_L}{Z_1^2} \left[if\ R = 0, g_1 = \frac{1}{X_L} \right]$$

From equation (1), note that the inductive susceptance is negative.

For Branch (2) : It is a capacitive branch,

$$Y_2 = \frac{1}{Z_2} = \frac{1}{R_2 - jX_C} = \frac{1}{R_2 - jX_C} \times \frac{R_2 + jX_C}{R_2 + jX_C} = \frac{R_2}{R_2^2 + X_C^2} + j\frac{X_C}{R_2 + X_C^2}$$

$$Y_2 = g_2 + Jb_2 \quad (\because j^2 = -1) \ldots\ldots\ldots\ldots (2)$$

The for capacitive branch,

$$g_2 = \frac{R_2}{R_2^2 + X_C^2} = \frac{R_2}{Z_2^2} \left(if\ X_C = 0, g_2 = \frac{1}{R_2} \right)$$

$$b_2 = \frac{X_C}{R_2^2 + X_C^2} = \frac{X_C}{Z_2^2} \left(if\ R_2 = 0,\ b_2 = \frac{1}{X_C} \right)$$

From equation (2), note that the capacitive susceptance is positive.

∴ Total admittance,

$$Y = Y_1 + Y_2 = (g_1 - j\,b_1)(g_2 + j\,b_2)$$
$$= (g_1 + g_2) + j(-b_1 + b_2) = G + jB$$

Where Conductance $G = g_1 + g_2$ and $B = -b_1 + b_2$

∴ $Y = \sqrt{G^2 + B^2}$ in magnitude

Phase angle, $\phi = tan^{-1}\left(\frac{B}{G}\right) = tan\left(\frac{-b_1 + b_2}{g_1 + g_2}\right)$

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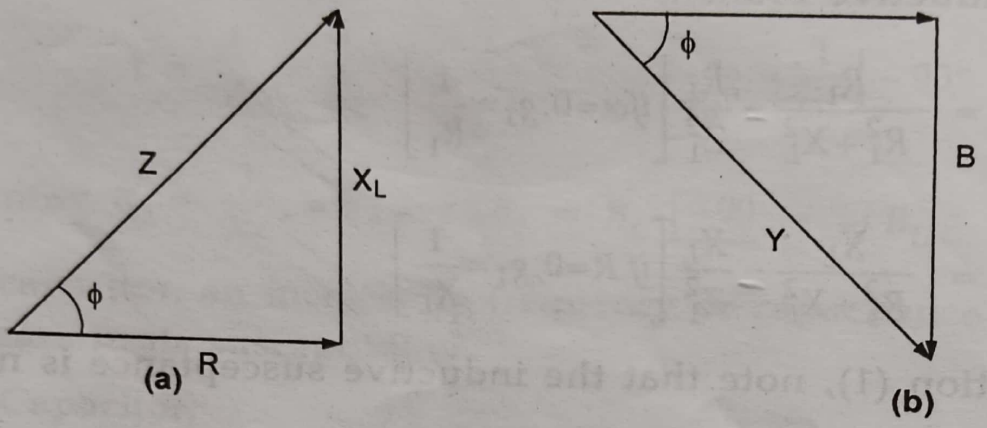
Line current $I = VY$

Power consumed,

$$P = VI \cos \phi$$

Admittance Triangle : Admittance can also be represented by a triangle similar to that of impedance 'Z'.Admittance has two rectangular components i.e., conductance g and susceptance b as shown in Fig. 3.62.

1. **For an Inductive Circuit :**



2. **For a Capacitive Circuit :**

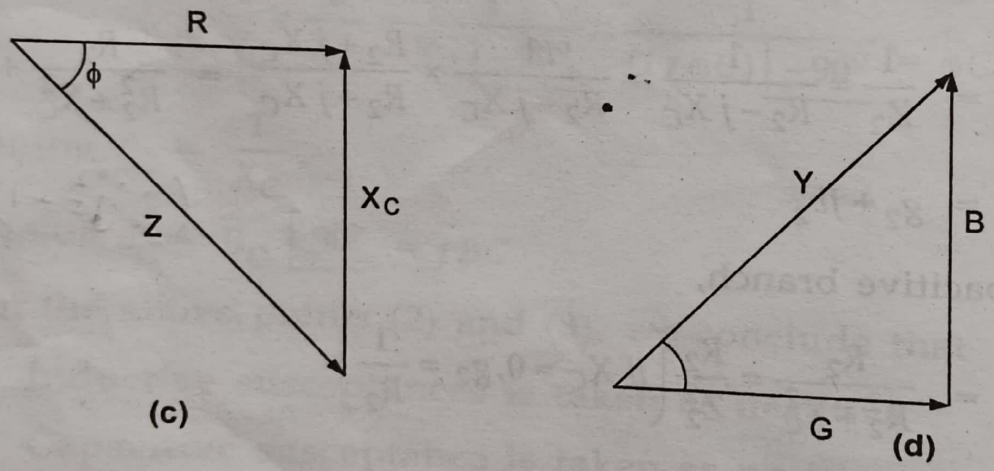


FIG 3.62 :

- Susceptance is positive for capacitive reactance.
- Susceptance is negative for inductive reactnace.

Conductance, $G = Y \cos \phi = \frac{1}{Z}\frac{R}{Z} = \frac{R}{Z^2} = \frac{R}{R^2+X^2}$

- Conductance is always positive irrespective of the circuit, parameters.

Conductance, $G = Y \cos \phi = \frac{1}{Z} = \frac{R}{Z} = \frac{R^2}{Z^2} = \frac{R}{R^2+X^2}$

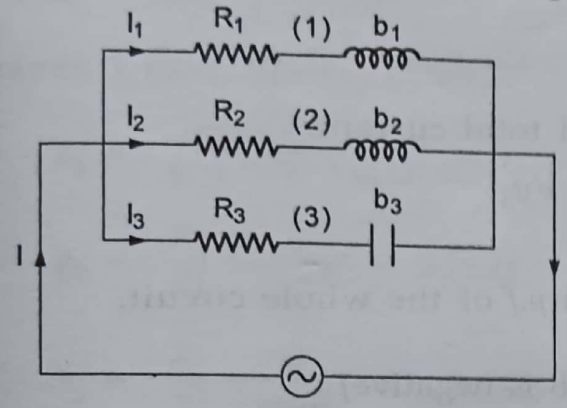
Susceptance, $B = Y \sin \phi = \frac{1}{Z}\frac{X}{Z} = \frac{X}{Z^2} = \frac{X}{R^2+X^2}$

Total admittance, $Y = \sqrt{G^2+B^2}$

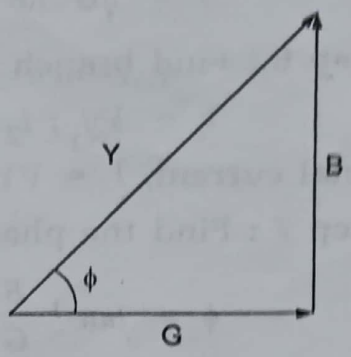
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Steps to Solve the Parallel A.C Circuits by Admittance Method : The following steps to be followed while solving parallel a.c circuits by admittance method.

Step 1 : Draw the circuit diagram as per the given problem. Here, consider the parallel circuit as shown in Fig. 3.63 (a).



(a)



(b)

FIG 3.63 :

Step 2 : Find the impedance and phase angle of each branch.

For Branch (1) : $Z_1 = \sqrt{R_1^2 + X_{L_1}^2}$; $\phi_1 = tan^{-1} \frac{X_{L1}}{R_1}$

For Branch (2) : $Z_2 = \sqrt{R_2^2 + X_{L_2}^2}$; $\phi_2 = tan^{-1} \left(\frac{X_{L2}}{R_2} \right)$

For Branch (3) : $Z_3 = \sqrt{R_3^2 + X_C^2}$; $\phi_3 = tan^{-1} \left(\frac{X_C}{R_3} \right)$

Step 3 : Find conductance, susceptance and admittance of each branch.

For Branch (1) : $g_1 = \frac{R_1}{Z_1^2}$; $b_1 = \frac{X_{L_1}}{Z_1^2}$ (negative)

$\therefore \quad y_1 = \sqrt{g_1^2 + b_1^2}$

For Branch (2) : $g_2 = \frac{R_2}{Z_2^2}$, $b_2 = \frac{X_{L2}}{Z_2^2}$ (negative)

$\therefore \quad y = \sqrt{g_2^2 + b_2^2}$

For Branch (3) : $g_3 = \frac{R_3}{Z_3^2}$, $b_3 = \frac{X_C}{Z_3^2}$ (positive)

$\therefore \quad y = \sqrt{g_3^2 + b_3^2}$

Step 4 : Find the algebraic sum of conductance and susceptance.

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Total conductnace, $G = g_1 + g_2 + g_3$

Total susceptance, $B = -b_1 - b_2 + b_3$

[$\because$ $-ve$ sign for inductive susceptance, and $+ve$ sign for capacitive susceptance.]

Step 5 : Find the total admittance of the circuit.

$$Y = \sqrt{G^2 + B^2}$$

Step 6 : Find branch currents and total current.

$$I_1 = Vy_1 ; I_2 = Vy_2 ; I_3 = Vy_3$$

Total current, $I = VY$

Step 7 : Find the phase angle and $p.f$ of the whole circuit.

$$\phi = tan^{-1} \frac{B}{G} \text{ (lagging of } B \text{ is negative)}$$

$$\therefore \quad p.f = cos\,\phi = \frac{G}{Y}$$

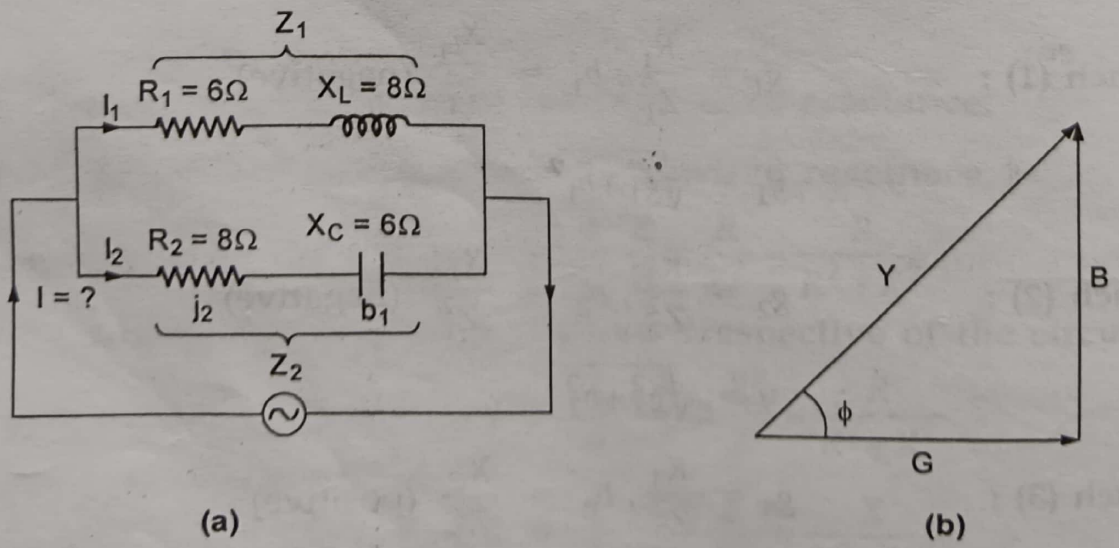
3.5.6 Solved Problems

EXAMPLE – 48

A circuit having a resistance of 6 Ω and inductive reactance of 8 Ω is connected in parallel with another circuit having a resistance of 8 Ω and a capacitive reactance of 6 Ω. The parallel circuit is connected across 200 V, 50 Hz supply. Calculate :

1. *Supply current*
2. *Power factor of whole circuit*
3. *Power consumed.*

Solution :



(a) (b)

FIG 3.64 :

Given data :

$R_1 = 6\,\Omega$ | $V = 200\,V$
$X_L = 8\,\Omega$ | $f = 50\,H_Z$
$R_2 = 8\,\Omega$ | $I = ?\,;\quad \cos\phi = ?\,;\quad P = ?$
$X_C = 6\,\Omega$ |

For Branch 1 (*R–L* Series) :

$$Z_1 = \sqrt{R_1^2 + X_L^2} = \sqrt{(6)^2 + (8)^2} = 10\,\Omega$$

$$g_1 = \frac{R_1}{Z_1^2} = \frac{6}{(10)^2} = 0.06\,\mho$$

$$-b_1 = \frac{X_L}{Z_1^2} = \frac{8}{(10)^2} = -0.08\,\mho \text{ (inductive)}$$

For branch 2 (*R–C* Series) :

$$Z_2 = \sqrt{R_2^2 + X_C^2} = \sqrt{(8)^2 + (6)^2} = 10\,\Omega$$

$$\therefore \quad g_2 = \frac{R_2}{Z_2^2} = \frac{8}{(10)^2} = 0.08\,\mho$$

$$b_2 = \frac{X_C}{Z_2^2} = \frac{6}{(10)^2} = 0.06\,\mho \text{ (capacitive)}$$

1. Total conductance, $G = g_1 + g_2 = 0.06 + 0.08 = 0.14\,\mho$

   Total suceptance, $B = -b_1 + b_2 = -0.08 + 0.06 = -0.02$ ..... inductive

   $\therefore$ Total Admittance, $Y = \sqrt{G^2 + B^2} = \sqrt{(0.14)^2 + (-0.02)^2} = 0.1414\,\mho$

   Supply current, $I = Vy = 200 \times 0.1414 = 28.28\ A$ **Ans**

2. $p.f = \cos\phi = \frac{G}{Y} = \frac{0.14}{0.1414} = 0.99\ (lag)$ **Ans**

3. Power consumed, $P = V\,I\cos\phi = 200 \times 28.28 \times 0.99 = 5599.44\ watt$

   $= 5600\ watt$

   (or) $P = V^2G = (200)2 \times 0.14 = 5600\ watt$ **Ans**

EXAMPLE – 49

Two impedance, $Z_1 = (8 + j6)\,\Omega$ and $Z_2 = (3 - j4)\,\Omega$ are connected in parallel across a single phase, 50 H_Z supply. Find the current and power taken by each branch if the total current drawn is 25A.

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Solution :

Given data :

$$Z_1 = (8 + j6)\,\Omega = (R_1 + j\,X_L)\,\Omega$$

$$Z_2 = (3 - j4)\,\Omega = (R_2 - JX_C)\,\Omega$$

$$I = 25A$$

$$f = 50\,H_Z$$

I_1 and I_2 = ?

P_1 and P_2 = ?

For Branch 1 (*R–L* Series) :

$$Z_1 = \sqrt{R_1^2 + X_L^2} = \sqrt{(8)^2 + (6)^2} = 10\,\Omega$$

$$g_1 = \frac{R_1}{Z_1^2} = \frac{8}{(10)^2} = 0.08\,\Omega$$

$$-b_1 = \frac{X_L}{(Z_1)^2} = \frac{6}{(10)^2} = -\,0.06\,\Omega \text{ ..... inductive}$$

$$y_1 = \sqrt{g_1^2 + (-b_1)^2} = \sqrt{(0.08)^2 + (-\,0.06)^2} = 0.1\,\Omega$$

For Branch 2 (*R – C* Series) :

$$Z_2 = \sqrt{R_2^2 + X_C^2} = \sqrt{(3)^2 + (4)^2} = 5\,\Omega$$

$$b_2 = \frac{X_C}{Z_2^2} = \frac{4}{(5)^2} = 0.16\,\text{℧} \text{ ..... Capacitive}$$

$$y_2 = \sqrt{g_2^2 + b_2^2} = \sqrt{(0.12)^2 + (0.16)^2} = 0.2\,\text{℧}$$

For Combined Parallel Circuit :

∴ Total conductance, $G = g_1 + g_2 = 0.08 + 0.12 = 0.2\,\text{℧}$

Total suceptance, $B = -\,b_1 + b_2 = -\,0.06 + 0.16 = 0.1\,\text{℧}$

∴ Total admittance, $Y = \sqrt{G^2 + B^2} = \sqrt{(0.2)^2 + (0.1)^2} = 0.2236\,\text{℧}$

Applied voltage, $V = \frac{I}{Y} = \frac{25}{0.2236} = 111.81V$

∴ Current in branch 1, $I_1 = \frac{V}{Z_1} = \frac{111.81}{10} = 11.181\,A$ **Ans**

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Current in branch 2, $I_2 = \frac{V}{Z_2} = \frac{111.81}{5} = 22.362\ A$ **Ans**

Power taken in branch 1, $P_1 = I_1^2 R_1 = (11.181)^2 \times 8 = 1000\ W$ **Ans**

Power in branch 2, $P_2 = I_2^2 R_2 = (22.362)^2 \times 3 = 1500\ W$ **Ans**

Alternatively :

$\therefore \quad I_1 = Vy_1 = 111.81 \times 0.1 = 11.181\ A$ **Ans**

$I_2 = Vy_2 = 111.81 \times 0.2 = 22.362\ A$ **Ans**

$P_1 = V^2 g_1 = (111.81)^2 \times 0.08 = 1000\ W$ **Ans**

$P_2 = V^2 g_2 = (111.81)^2 \times 0.12 = 1500\ W$ **Ans**

EXAMPLE – 50

A parallel circuit having three branches is connected across 100V, single phase 50 H_Z a.c supply. The parameters of the branches are given below.

Branch 1 : Inductive ; resistance 8Ω reactance 31.4Ω

Branch 2 : Capacitive ; resistance 20Ω , reactance 63.7Ω

Branch 3 : Purely capacitive ; capacitance, 120 μF.

Calculate (i) Total admittance, conductance and susceptance of the whole circuit (ii) Total current, power factor and power.

Solution :

For Branch 1 : Given, *R–L* in Series :

$R_1 = 8\Omega; \quad X_1 = 31.4\Omega$

$\therefore \quad Z_1 = \sqrt{(8)^2 + (31.4)^2} = 32.4\Omega$

$\therefore \quad g_1 = \frac{R_1}{Z_1^2} = \frac{8}{(32.4)^2} = 0.00762\ \text{℧}$

$-b_1 = \frac{X_1}{Z_1^2} = \frac{31.4}{(32.4)^2} = (inductive) - 0.0299$

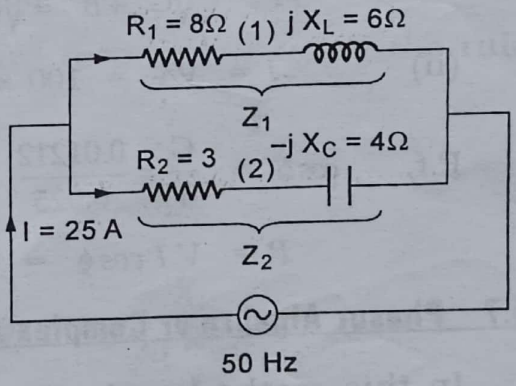


FIG 3.65 :

For Branch 2 : Given *R–C* in Series :

$R_1 = 20\Omega; \quad X_2 = 63.7\Omega$

$Z_2 = \sqrt{(20)^2 + (63.7)^2} = 66.76\Omega$

$\therefore \quad g_2 = \frac{R_2}{Z_2^2} = \frac{20}{(66.76)^2} = 0.0045\ \text{℧}$

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$$b_2 = \frac{X_2}{Z_2^2} = \frac{63.7}{(66.76)^2} = 0.0143\,\text{℧} \quad \text{(capacitive)}$$

For Branch 3 Given, C only :

$$R_3 = 0; \qquad C = 120\,\mu F = 120 \times 10^{-6} F$$

$$X_3 = X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 120 \times 10^{-6}} = 26.53\ \Omega$$

$$\therefore \quad Z_3 = X_3 = 26.53\,\Omega$$

$$\therefore \quad g_1 = \frac{R_3}{Z_3^2} = \frac{0}{26.53} = 0\,\text{℧}$$

$$b_1 = \frac{X_3}{Z_3^2} = \frac{26.53}{(26.53)^2} = 0.0376\,\text{℧} \quad \text{(Capacitive)}$$

Combined Parallel Circuit :

(i) $G = g_1 + g_2 + g_3$

$= 0.00762 + 0.0045 + 0 = 0.01212\,\text{℧}$

$B = -b_1 + b_2 + b_3$

$= 0.0299 + 0.0143 + 0.0376 = 0.022\,\text{℧}$

$$Y = \sqrt{G^2 + B^2} = \sqrt{(0.01212)^2 + (0.022)^2} = \mathbf{0.025\,\text{℧}} \quad \textbf{Ans}$$

(ii) $I = VY = 100 \times 0.025 = \mathbf{2.5A}$ **Ans**

P.f, $\cos\phi = \frac{G}{Y} = \frac{0.01212}{0.025} = \mathbf{0.4848\ (lead)}$ **Ans**

$P = V I \cos\phi = 100 \times 2.5 \times 0.4848 = \mathbf{121.2W}$ **Ans**

3.5.7 Phasor Algebra or Complex Algebra or j Method

In this method, voltages currents and impedances are expressed in the complex form.

1. Rectangular form
2. Polar form.

By using the rules of phasor algebra, the solution of parallel circuits problems can be obtained mathematically. This method eliminates the need of phasor diagram. Consider a parallel circuit consists of two parallel branches as shown in Fig. 3.66.

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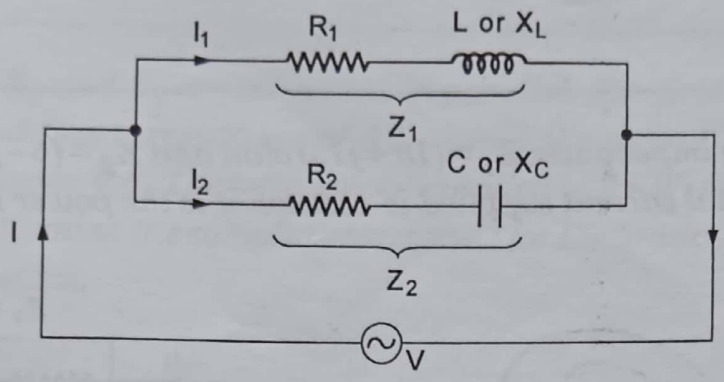


FIG 3.66 :

Referring to the Fig. 3.66 we have,

$V = V + j\,0 = V$ ..... reference phasor

$Z_1 = R_1 + jX_L$

$Z_2 = R_2 - jX_C$

1. Rectangular Form :

Current in first branch, $I_1 = \dfrac{V}{Z_1} = \dfrac{V}{R_1 + jX_L}$

Current in second branch $I_2 = \dfrac{V}{Z_2} = \dfrac{V}{R_2 - JX_C}$

Line current, $I = I_1 + I_2 = \dfrac{V}{R_1 + jX_L} + \dfrac{V}{R_2 - jX_C}$

The solution I can be obtained in the standard form $a \pm jb$ using the rules of phasor algebra.

2. Polar Form :

$V = V \angle 0^o$ ..... Reference phasor

$Z_1 = Z_1 \angle \phi_1^0$, where $Z_1 = \sqrt{R_1^2 + X_L^2}$; $\phi_1 = tan^{-1}\dfrac{X_L}{R}$

$Z_2 = Z_2 \angle -\phi_2^0$, where $Z_2 = \sqrt{R_2^2 + X_C^2}$; $\phi_2 = tan^{-1}\left(\dfrac{X_C}{R}\right)$

$\therefore \quad I_1 = \dfrac{V}{Z_1} = \dfrac{V \angle 0^o}{Z_1 \angle \phi_1^0} = \dfrac{V}{Z_1} \angle -\phi_1^0$

$I_2 = \dfrac{V}{Z_2} = \dfrac{V\,0^o}{Z_2 \angle -\phi_2^0} = \dfrac{V}{Z_2} \angle \phi_2^0$

$\therefore \quad I_1 + I_2 = \dfrac{V}{Z_1} \angle -\phi_1^0 + \dfrac{V}{Z_2} \angle \phi_2^0$

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EXAMPLE – 51

Two circuits having impedances $Z_1 = (10 + j15)$ ohm and $Z_2 = (6 - j8)$ ohm are connected in parallel. If the total current supplied is 15A; what is the power taken by each branch?

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Solution :

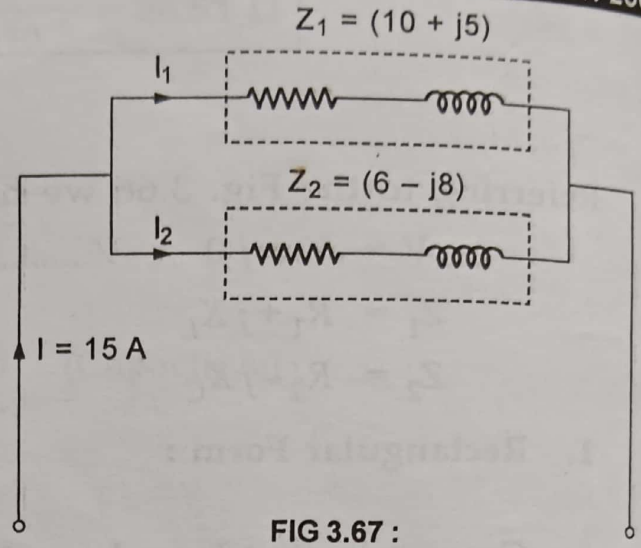


FIG 3.67 :

Given data :

$$Z_1 = (10 + j15)\,\Omega = 18.03 \angle 56.31^\circ\,\Omega$$

$$Z_2 = (6 - j\,8)\,\Omega = 10 \angle -53.13^\circ\,\Omega$$

$$y_1 = \frac{1}{Z_1} = \frac{1}{10 + j15} = \frac{1}{18.03 \angle 56.31^\circ}$$

$$= 0.0555 \angle -56.31^\circ$$

$$= (0.03078 - j\,0.0462)\,\mho$$

$$y_2 = \frac{1}{Z_2} = \frac{1}{6 - j8} \quad \frac{1}{10 \angle -53.13^\circ} = 0.1 \angle 53.13^\circ = (0.06 + j\,0.08)\,\Omega$$

∴ Total admittance,

$$Y = y_1 + y_2 = (0.03078 - j0.0462) + (0.06 + j0.08)$$

$$= (0.09078 + j0.0338)\,\mho = 0.0968 \angle 20.42^\circ$$

$$\therefore \quad V = \frac{I}{Y} = \frac{15 \angle 0^\circ}{0.0968 \angle 20.42^\circ} = 155 \angle -20.42^\circ$$

∴ Supply voltage,

$$V = 155\,V$$

$$I_1 = \frac{V}{Z_1} = \frac{155 \angle 20.42^\circ}{18.03 \angle 56.31^\circ} = 8.597 \angle -76.73^\circ$$

∴ Current in branch 1,

$$I_1 = 8.597\,A \quad \textbf{Ans}$$

$$I_2 = \frac{V}{Z_2} = \frac{155 \angle 20.42^\circ}{10 \angle -53.13^\circ} = 15.5 \angle 32.71^\circ$$

∴ Current in branch 2,

$$I_2 = 15.5\,A \quad \textbf{Ans}$$

$$\therefore \quad P_1 = I_1^2\,R_1 = (8.597)^2 \times 10 = 739\,W \quad \textbf{Ans}$$

$$P_2 = I_2^2\,R_2 = (15.5)^2 \times 6 = 1441\,W \quad \textbf{Ans}$$

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EXAMPLE – 32

Two impedances, Z_1 and Z_2 are connected in parallel. The first branch takes a leading current of 16A and has a resistance of 5Ω, while the second branch takes a lagging current at power factor 0.8. The total power supplied is 5 KW, the applied voltage being (100 + j 200) V. Determine the complex expressions for the branch and total currents, and for the circuit constant.

Solution :

Given that :

$$V = (100 + j\,200)\ V = 223.61 \angle 63.43^\circ\ V$$

For Branch 1 :

Since the first branch takes a leading current, it contains a capacitive reactance X_C in series with $R = 5\,\Omega$

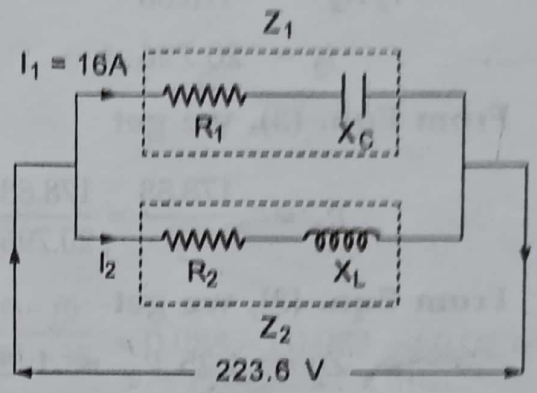


FIG 3.68 :

$$\therefore \quad Z_1 = \frac{V}{I_1} = \frac{223.6}{16} = 13.975\,\Omega$$

$$Z_1^2 = R_1^2 + X_C^2$$

$$\therefore (13.975)^2 = (5)^2 + X_C^2$$

$$\therefore \quad X_C = \sqrt{(13.975)^2 - (5)^2} = 13.05\,\Omega$$

In complex form,

$$Z_1 = R_1 - jX_C = 5 - j\,13.05 = 13.975 \angle -69^\circ\,\Omega \quad \textbf{Ans}$$

$$\therefore \quad I_1 = \frac{V}{Z_1} = \frac{223.6 \angle 63.43^\circ}{13.975 \angle -69^\circ} = 16 \angle -132.43^\circ$$

$$= (-10.795 + j\ 11.809)\ A \quad \textbf{Ans}$$

For Branch 2 :

Given total power consumed $P = 5\ kW = 5000\ W$

Also, total power consumed in both the branches.

$$P = I_1^2\ R_1 + I_2^2\ R_2$$

$$5000 = (16)^2 \times 5 + I_2^2\ R_2$$

$$\therefore \quad I_2^2\ R^2 = 5000 - (16)^2 \times 5 = 3720\ W \quad \ldots\ldots\ldots\ldots (1)$$

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Since the $p.f$ is 0.8 lagging for branch 2, hence the circuit is inductive.

$$\cos\phi = \frac{R_2}{Z_2} \Rightarrow Z_2 = \frac{R_2}{\cos\phi_2} = \frac{R_2}{0.8} = 1.25\,R_2 \quad \ldots\ldots\ldots\ldots (2)$$

$$I_2 = \frac{V}{Z_2} = \frac{223.6}{1.25\,R_2} = \frac{178.88}{R_2}\,A$$

$$I_2 R_2 = 178.88\ V \quad \ldots\ldots\ldots\ldots (3)$$

Dividing Eqn. (1) by Eqn (3), we get,

$$\frac{I_2^2 R_2}{I_2 R_2} = \frac{3720}{178.88}$$

$$\therefore \quad I_2 = 20.796\ A$$

From Eqn. (3), we get

$$R_2 = \frac{178.88}{I_2} = \frac{178.88}{20.796} = 8.60\,\Omega$$

From Eqn. (3), we get

$$Z_2 = 1.25\,R_2 = 1.25 \times 8.6 = 10.75\,\Omega$$

$$X_L = \sqrt{Z_2^2 - R_2^2} = \sqrt{(10.75)^2 - (8.6)^2} = 6.45\,\Omega$$

$\therefore$ In complex form,

$$Z_2 = R_2 + jX_L = (8.6 + j\,6.45)\,\Omega = 10.75\angle 36.87^\circ\ \Omega \quad \textbf{Ans}$$

$$I_2 = \frac{V}{Z_2} = \frac{223.6\angle 63.43^\circ}{10.75\angle 36.87^\circ}$$

$$= 20.8\angle 26.56^\circ\ A = (18.6 + j\,9.3)\ A \quad \textbf{Ans}$$

Total circuit current,

$$I = I_1 + I_2 = (-10.795 + j\,11.809) + (18.6 + j\,9.3)$$

$$= (7.805 + j\,21.109)\ A \quad \textbf{Ans}$$

$$= 22.5\angle 69.7^\circ\ A \quad \textbf{Ans}$$

Total impedance of the circuit,

$$Z = \frac{V}{I}$$

$$\therefore \quad Z = \frac{223.6\angle 63.43^\circ}{22.5\angle 69.7^\circ} = 9.938\angle -6.27^\circ = (9.878 - j1.085)\,\Omega$$

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3.9 Series – Parallel A.C Circuits

The solution of problems in series – parallel a.c circuits is done in the same way that series parallel d.c circuits. While solving the series - parallel circuits, the alternating quantities are represented in complex form. The quantities are changed from rectangular to polar from or vice-versa frequently as per requirements to make the solution quick.

Consider a series - parallel A.C circuit shown in Fig. 3.70

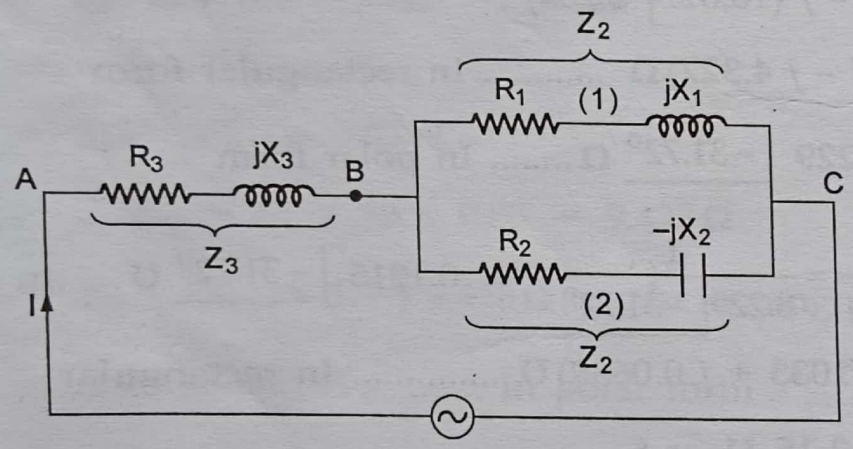


FIG 3.70 :

Step 1 : First reduce all parallel circuits to equivalent series circuits.

From Fig. $Z_1 = R_1 + jX_1$ and $Z_2 = R_2 - jX_2$

$$\therefore \quad Z_{BC} = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

$$\therefore \quad Y_{BC} = \frac{1}{Z_{BC}}$$

Step 2 : Find the total impedance of the circuit by taking phasor sum of all series impedances

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From Fig. 3.70 Total impedance,

$$Z = Z_3 + Z_{BC}$$

$\therefore$ Total admittance, $Y = \frac{1}{Z}$

Step 3 : Total circuit current is found by $I = \frac{V}{Z}$ or $I = V\,Y$

Step 4 : Each voltage drop is calculated by

$$V_1 = I\,Z_1 \text{ or } V_1 = \frac{I}{y_1} \text{ where } y_1 = \frac{I}{Z_1}$$

$$V_2 = I\,Z_2 \text{ or } V_2 = \frac{I}{y_2} \text{ where } y_2 = \frac{I}{Z_2}$$

$$V_3 = I\,Z_3 \text{ or } V_3 = \frac{I}{y_3} \text{ where } y_3 = \frac{I}{Z_3}$$

Step 5 : Individual branch currents are calculated by

$$I_1 = \frac{V_1}{Z_1} \text{ or } I_1 = V_1\,y_1$$

$$I_2 = \frac{V_2}{Z_2} \text{ or } I_2 = V_2\,y_2$$

$$I_3 = \frac{V_3}{Z_3} \text{ or } I_3 = V_3\,y_3$$

EXAMPLE – 55

In the network shown in Fig. 3.62 determine

(i) The total impedance and admittance

(ii) The total current

(iii) The current in each branch

(iv) The power factor whole network

(v) Apparaent

(vi) Active and reactive power.

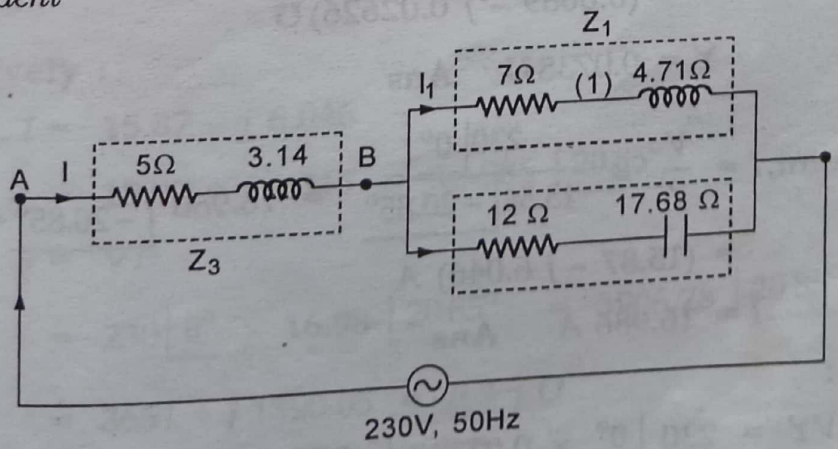


FIG 3.73 :

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Solution :

From Branch (1) and Branch (2),

$$Z_1 = R_1 + jX_{L1} = 7 + j\,4.71 = 8.437 \angle 33.9^\circ\ \Omega$$

$$Z_2 = R_2 - jX_C = 12 - j\,17.68 = 21.37 \angle -55.83^\circ\ \Omega$$

Since Z_1 and Z_2 are in parallel, thus the equivalent impedance is given by

$$Z_{BC} = Z_1 \| Z_2 = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{8.437\angle 33.93^\circ \times 21.37\angle -55.83^\circ}{(7+j4.71)+(12-j17.68)}$$

$$= \frac{180.3\angle -21.9^\circ}{19 - j12.97} = \frac{180.3\angle -21.9^\circ}{23\angle -34.3^\circ}$$

$$= 7.839 \angle 12.4^\circ\ \Omega \quad \ldots\ldots\ldots \text{In polar form}$$

$$= (7.656 + j\ 1.68)\,\Omega \quad \ldots\ldots\ldots \text{In rectangular form}$$

From Branch (3),

$$Z_3 = R_3 + jX_{L3} = 5 + j\,3.14 = 5.9 \angle 32.13^\circ\ \Omega$$

Since Z_3 and Z_{BC} are in parallel, thus the total impedance,

(i) $$Z = Z_3 + Z_{BC} = (5 + j\,3.14) + (7.656 + j\ 1.68)$$

$$= (12.656 + j\ 4.82)\,\Omega \quad \ldots\ldots\ldots\ldots \text{In rectangular form}$$

$$= 13.54 \angle 20.85^\circ\ \Omega \quad \ldots\ldots\ldots\ldots \text{In polar form}$$

$\therefore$ Total impedance, $Z = 13.54\,\Omega$ **Ans.**

Total admittance, $Y = \frac{1}{Z} = \frac{1}{13.54\angle 20.85^\circ} = 0.0738 \angle -20.85^\circ$ ℧

$$= (0.0689 - j\ 0.02626)\ ℧$$

$\therefore$ $Y = 0.0738$ ℧ **Ans**

(ii) Total current, $I = \frac{V}{Z} = \frac{230\angle 0^\circ}{13.54\angle -20.85^\circ} = 16.986 \angle -20.85^\circ\ A$ ..... (1)

$$= (15.87 - j\ 6.046)\ A$$

$\therefore$ $I = 16.986\ A$ **Ans**

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In the circuit shown in Fig. Determine what 50 H_Z voltage must be applied across AB in order that a current 10 A may flow in the capacitor.

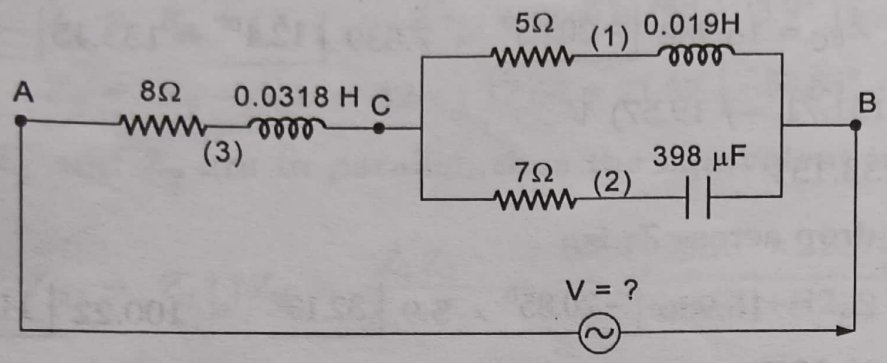


FIG 3.74 :

Solution :

For Branch (1) :

$$R_1 = 5\Omega, \; L_1 = 0.0191\ H$$

$$\therefore \quad X_{L1} = 2\pi f L_1 = 2\pi \times 50 \times 0.0191 = 6\Omega$$

$$Z_1 = R_1 + jX_{L1} = (5 + j\,6) = 7.81 \angle 50.19^\circ\ \Omega$$

$$Y_1 = \frac{1}{Z_1} = \frac{1}{7.81 \angle 50.19^\circ} = 0.12804 \angle 50.19^\circ\ \mho$$

For Branch (2) :

$$R_2 = 7\Omega$$

$$C = 398\,\mu F = 398 \times 10^{-6}\ F$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 398 \times 10^{-6}} = 8\Omega$$

$$Z_2 = R_2 - j\,X_C = (7 - j\,8)\,\Omega = 10.63 \angle -48.81^\circ\ \Omega$$

$$Y_2 = \frac{1}{Z_2} = \frac{1}{10.63 \angle -48.81^\circ} = 0.09407 \angle -48.81^\circ\ \mho$$

For Branch (3) :

$$R_3 = 8\Omega, \; L_3 = 0.0318\ H$$

$$X_{L3} = 2\pi f L_3 = 2\pi \times 50 \times 0.0318 = 10\Omega$$

$$Z_3 = R_3 + jX_{L3} = (8 + j\,10)\,\Omega = 12.8 \angle 51.34^\circ\ \Omega$$

$$Y_3 = \frac{1}{Z_3} = \frac{1}{12.8 \angle 51.34^\circ} = 0.0781 \angle -51.34^\circ\ \Omega$$

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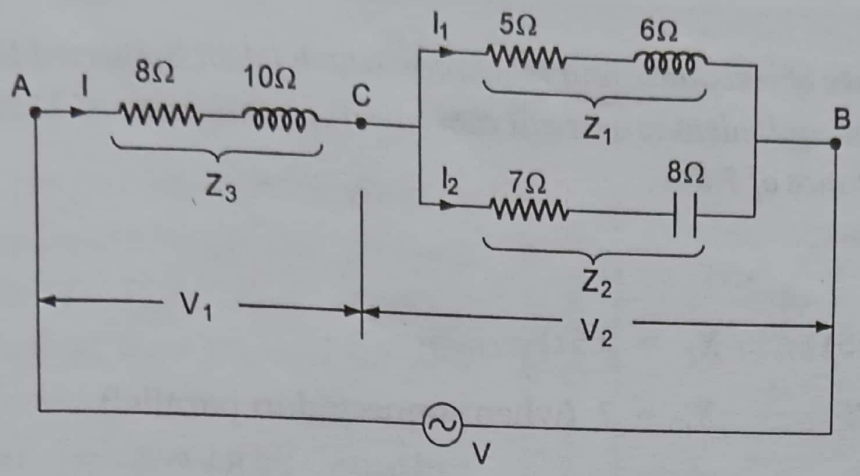


FIG 3.75 :

Given, $I_2 = 10\,A$

Taking I_2 as reference vector,

$$\therefore \quad I_2 = 10\angle 0^\circ = 10 + j\,0$$

$$V_2 = \frac{I_2}{Y_2} = \frac{10\angle 0^\circ}{0.09407\angle 48.81^\circ} = 106.3\angle -48.81^\circ\ V = (70 - j\,80)V$$

$$\therefore \quad I_2 = V_2\,Y_1 = 106.3\angle -48.81^\circ \times 0.12804\angle 50.19^\circ$$

$$= 13.61\angle -99^\circ\ A = (-2.13 - j\,13.44)\ A$$

$\therefore$ Total current flowing through impedance Z_3

$$I = I_1 + I_3$$

$$= (-2.13 - j\,13.44) + 10 + j\,0$$

$$= (7.87 - j\,13.44)\ A = 15.57\angle -59.65^\circ\ A$$

Similarly,

Voltage across branch AC,

$$V_1 = \frac{I}{Y_3} = \frac{15.57\angle -59.65^\circ}{0.0781\angle -51.34^\circ} = 199.3\angle 8.31^\circ\ V = (197.2 - j\,28.8)V$$

$\therefore$ Applied voltage across AB,

$$V_{AB} = V_1 + V_2$$

$$\therefore \quad V_{AB} = 70 - j\,80 + 197.2 - j\,28.8 = (267.2 - j\,108.8)\ V = 288.5\angle 22.15^\circ\ V$$

$$\therefore \quad V_{AB} = \mathbf{288.5V}\ \textbf{Ans.}$$

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EXAMPLE – 57

Calculate the values of resistance and reactance, which when connected in paralell with one another, will be equivalent to a circuit consisting of a resistance of 15 Ω in series with an inductive reactance of 7.5 Ω.

Solution :

Given data :

$R = 15\,\Omega$; $X_L = 7.5\,\Omega$

$R = ?$; $X_L = ?$ (when connected in parallel)

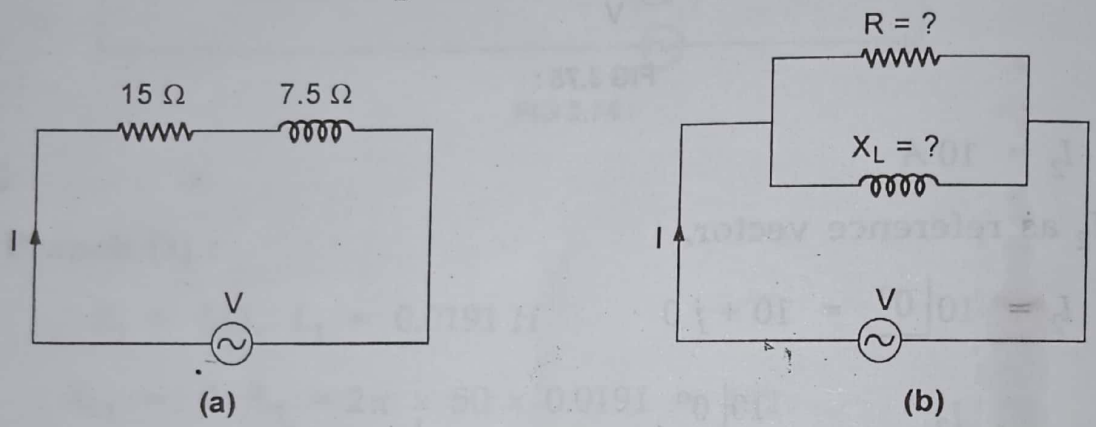


FIG 3.75 :

From *R – L* Series Circuit :

$$Z = R_1 + jX_1 = (15 + j\,7.5) = 16.77 \angle 26.56^\circ$$

$$\therefore \quad Z = 16.77\,\Omega$$

$$\text{Conductance, } G = \frac{R}{Z^2} = \frac{15}{(16.77)^2} = 0.053 \text{ ℧ or } S$$

$$\text{Susceptance, } B = \frac{X}{Z^2} = \frac{7.5}{(16.77)^2} = 0.027 \text{ ℧ or } S$$

For Parallel Circuit : Since the parallel circuit is to have the same conductance and susceptance as the series circuit.

$$\therefore \text{ Resistance, } R = \frac{1}{G} = \frac{1}{0.053} = 18.87\,\Omega \quad \textbf{Ans}$$

$$\text{Reactance, } X_L = \frac{1}{B} = \frac{1}{0.02T} = 37.037\,\Omega \quad \textbf{Ans}$$

3.6 Resonance in A.C Circuits

The resonant electrical circuit must have both inductance and capacitance. In addition, resistance will always be present due to either the lack of ideal elements or to the control offered on the shape of the resonance curve. Thus, "*resonance is defined as the condition in a circuit containing atleast one inductor*

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and one capacitor, when the supply voltage and supply current are in phase". Therefore, the resonance circuit is a combination of R, L, and C elements having a frequency response characteristic as shown in Fig. 3.76.

When response occurs due to the application of the proper frequency (f_r), the energy absorbed at any instant by one reactive element is exactly equal to that released by another reactive element, within the system. Therefore, once the system has reached a state of resonance, it required no further reactive power since it is self sustaining. The total apparent power is then simply equal to the average power dissipated by the resistive elements. The average power absorbed by the system will also be a maximum at resonance.

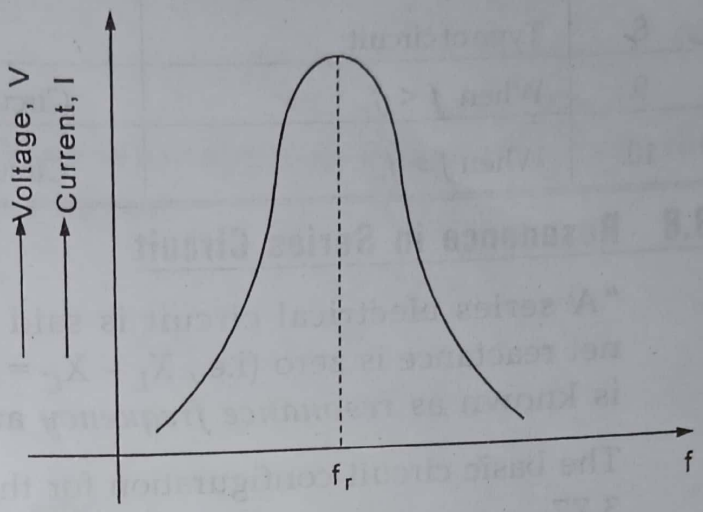


FIG 3.76 :

The study of resonance finds application in tuning circuits like in radio, television, and telephones and also in machine control systems.

There are two types of resonant circuit, namely.

1. Series Resonant Circuits
2. Parallel Resonant Circuits.

3.7 Difference Between Series and Parallel Resonant Circuits

OCT/NOV. 2009, 2006, MARCH/APRIL. 2007

S.No.	Particular	Series Circuit	Parallel Circuit
1.	Impedance at resonance	Minimum ($Z_r = R$)	Maximum $\left(Z_r = \frac{L}{CR}\right)$
2.	Current at resonance	Maximum $\left(I_r = \frac{V}{R}\right)$	Minimum $\left(I_r = \frac{V}{\left(\frac{L}{CR}\right)}\right)$ $\left(\frac{L}{CR}\right)$
3.	Effective impedance	R	Unity
4.	P.f at resonance	Unity	$\frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$
5.	Resonant frequency	$\frac{1}{2\pi\sqrt{LC}}$	

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6.	It magnifies	Voltage	Current
7.	Magnification or Q-factor	$\frac{X_L}{R}$ or $\frac{\omega L}{R}$	$\frac{X_L}{R}$ or $\frac{\omega L}{R}$
8.	Type of circuit	Acceptor	Rejector
9.	When $f < f_r$	Circuit is capacitive	Circuit is inductive
10.	When $f > f_r$	Circuit is inductive	Circuit is capacitive

3.8 Resonance in Series Circuit

"A series electrical circuit is said to be in electrical resonance when its net reactance is zero (i.e., $X_L - X_C = 0$). The frequency at which this happens is known as ***resonance frequency*** and is denoted by the letter f_r.

The basic circuit configuration for the series resonant circuit is shown in Fig. 3.77.

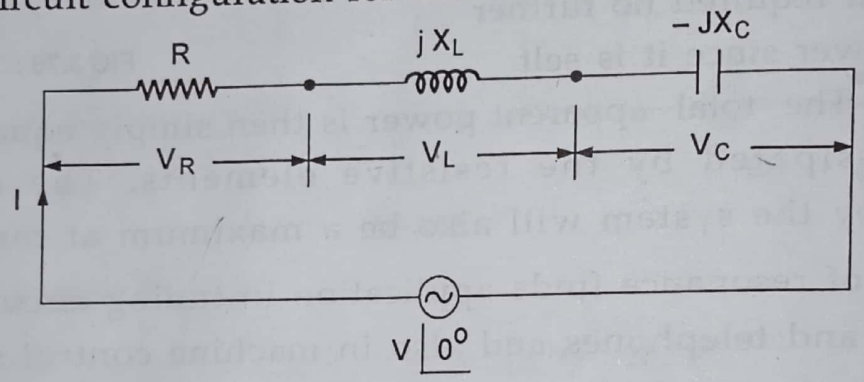


FIG 3.77 :

Let R = the total resistance in the circuit

$X_L = 2\pi fL$ = reactance of the coil

$X_C = \frac{1}{2\pi fC}$ = reactance of the capacitor.

∴ The total impedance at any frequency,

$$Z = \sqrt{R^2+(X_L-X_C)^2} \quad \text{..... (1)}$$

$$= \sqrt{R^2+(2\pi fL-\frac{1}{2\pi fC})^2}$$

In the earlier topics, it is seen that the situation from equation (1) where $X_L > X_C$ and $X_C > X_L$. The left over condition $X_L = X_C$ leads to the present topics.

i.e., series resonance will occur when.

$$X_L = X_C \quad \text{..... (2)}$$

∴ The total impedance at resonance,

$$Z_r = R \quad \text{..... (3)}$$

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The circuit impedance at resonace is called the dynamic impedance Z_r and it has minimum value. Hence, the circuit current has maximum value and is given by.

$$I_r = \frac{V}{Z_r} = \frac{V}{R} \quad [\because Z_r = R] \qquad \ldots\ldots\ldots\ldots (4)$$

Since the circuit current is in-phase with applied voltage. Hence, the circuit behaves like a pure resistive circuit.

At resonance,

Phase angle, $\phi_r = 0^o$

Power factor, $\cos\phi_r = 1$ (unity)

(or) $$\cos\phi_r = \frac{R}{Z_r} = \frac{R}{R} = 1 \text{ (unity)}$$

Also, the power factor has a maximum value (i.e., $\cos\phi_r = 1$) the power observed by the circuit is also maximum at resonance.

$\therefore$ The power at resonance,

$$P_r = VI_r \cos\phi_r = V \times \frac{V}{R} \times 1 = \frac{V^2}{R} \text{ watt} \qquad \ldots\ldots\ldots\ldots (5)$$

At resonance,

$$X_L = X_C$$

$$I_r X_L = I_r X_C$$

$$\boldsymbol{V_L = V_C} \text{ (or) } V_L = V_C = 0$$

Thus at resonance, the voltage across the inductance is equal in magnitude and opposite in phase to the voltage across the capacitance.

Also, $V = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{V_R^2 + 0}$

$$\therefore \boldsymbol{V = V_R}$$

and $$\boldsymbol{V_R = I_r R}$$

Accepter Circuit : Since a series resonant circuit draw a maximum current and power from the mains due to minimum impedance it is after regarded as ***acceptor circuit.***

(or)

The series resonant circuit is also called as an ***acceptor circuit*** since it accepts frequencies close to the resonant frequency when its impedance is a minimum. Resonance in sereis circuit is also aften referred to as voltage resonance.

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If as series circuit resonates at a frequency f_r, then,

1. Net reactance, $X_N = 0$
2. Impedance of the circuit, $Z = R$
3. The current flowing through the circuit is maximum, and in phase with the applied voltage.

   The magnitude of the current will be equal to $\left(\frac{V}{R}\right)$
4. The voltage drop across the inductance is equal to the voltage drop across capacitance and is sometimes more than the applied voltage
5. The power factor is unity.

3.8.1 Resonant Frequency

OCT/NOV. 2006, MARCH/APRIL. 2006

The frequency at which the resonance occurs is called the ***resonant frequency*** or frequency of resonance. It is denoted by f_r.

At resonance, the net reactance of an R–L–C circuit is.

$$X_L - X_C = 0 \quad \text{(or)} \quad X_L = X_C$$

$$\omega_r L = \frac{1}{\omega_r C}; \qquad 2\pi f_r L = \frac{1}{2\pi f_r C}$$

$$\therefore \quad f_r^2 = \frac{1}{(2\pi)^2 LC} \quad \therefore \quad f_r = \frac{1}{2\pi\sqrt{Lc}} \; H_Z$$

Where f_r = resonant frequency, in H_z

L = inductance in herby

C = capacitance in farads

3.8.2 Bandwidth of a Series Resonant Circuit

" The band width of a series circuit is defined as the range of frequency over which circuit current is equal or greater than 70.7% of maximum current (i.e., I_r, current at resonance) Fig. 3.68 shown a resonance curve of a typical R–L–C circuit where the circuit current is equal to or greater than 70.7% of maximum current (i.e., $I_r = \frac{V}{R}$) between frequency range f_1 to f_2

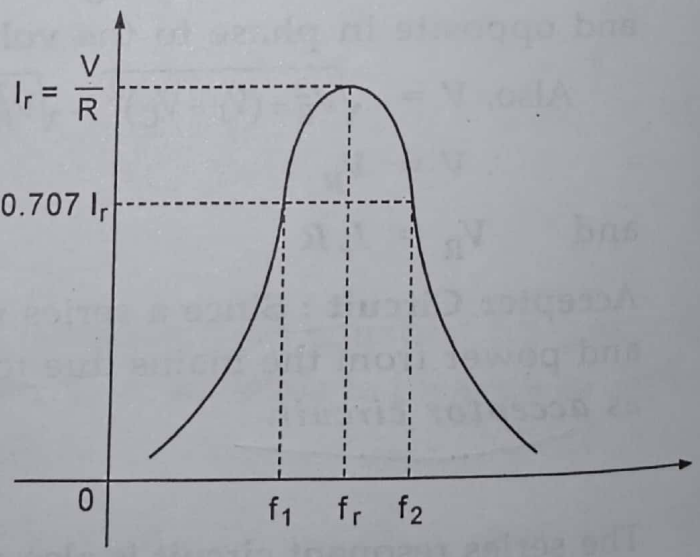


FIG 3.78 :

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$\therefore$ Band width

$$BW = f_1 - f_2$$

Where, f_1 = the lower cut-off frequency, and

f_2 = the upper cut-off frequency.

The bandwidth represents the frequency range at which the circuit offers low impedance to circuit current.

The following points may be noted :

1. If the resonant frequency is not centrally located w.r.t lower and upper cut off frequencies, then,

$$f_r = \sqrt{f_1 f_2}$$

2. If Q of a circuit is ≥ 10 and the resonant frequency is sufficiently near to the centre if the two cut off frequencies. Then

$$f_1 = f_r - \frac{BW}{2}$$

$$f_2 = f_r + \frac{BW}{2}$$

3. The relation between Q–factor and bandwidth of a series resonant circuit.

$$Q = \frac{f_r}{BW}$$

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EXAMPLE – 58

A coil of resistance 40 ohm and inductance 0.75 H forms part of a series circuit for which the resonant frequency is 55 H_Z. If the supply is 250V, 50 H_Z, find.

(i) The line current *(ii) The power factor* *(iii) The voltage across the coil.*

NOV/DEC. 2003 [C-96]

Solution :

Given data :

$R = 40\,\Omega$

$L = 0175\ H$

$f_r = 55\ H_Z$	$I = ?$
$V = 250\ V$	$\cos\phi = ?$
$f = 50\ H$	$V_L = ?$

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At 55 *HZ* resonant frequency.

$$X_L = 2\pi f_r L = 2\pi \times 55 \times 0.75 = 259.18\,\Omega$$

But at resonance,

$$X_L = X_C = 259.18\,\Omega$$

$$\therefore \quad C = \frac{1}{2\pi f_r X_C} = \frac{1}{2\pi \times 55 \times 259.18} = 11.165 \times 10^{-6}\,F = 11.165\,\mu F$$

At 250 *V*, 50*HZ* Supply :

$$X_L = 2\pi fL = 2\pi \times 50 \times 0.75 = 235.62\,\Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 11.165 \times 10^{-6}} = 285.1\ \Omega$$

$$\therefore \quad Z = \sqrt{R2 + (X_C - X_L)^2} \quad (\therefore X_C > X_L)$$

$$= \sqrt{(40)^2 + (285.1 - 235.62)^2} = 63.63\,\Omega$$

1. $$I = \frac{V}{Z} = \frac{250}{63.63} = 3.93\ A$$ **Ans**

2. $$\cos\phi = \frac{R}{Z} = \frac{40}{63.63} = 0.628 \text{ (leading)}$$ **Ans**

3. $$Z_L = \sqrt{R^2 + X_L^2} = \sqrt{(40)^2 + (235.62)^2} = 239\,\Omega$$

$$\therefore \quad V_L = I\,Z_L = 3.93 \times 239 = 939.27\ V$$ **Ans**

EXAMPLE – 59

A coil of resistance 10 Ω and inductance 5 mH is connected in series with a capacitor of C farads. When the circuit is connected across 100V, 50 H_Z a.c supply, the current is found to be maximum calculate (i) the value of C, (ii) the current, and (iii) power consumed.

SEPT/OCT. 1992

Solution :

Given data :

$R = 10\,\Omega$

$L = 5\ mH = 5 \times 10^{-3}\ H$

$V = 100\ V$

$f = 500\ H_Z$

$C = ?$

$I = ?$

$P = ?$

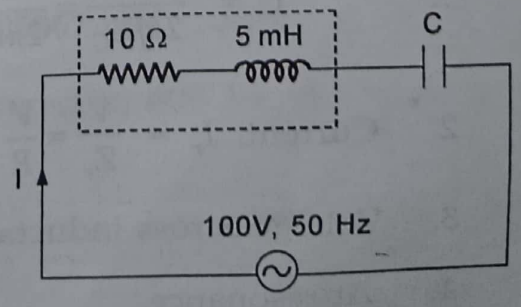


FIG 3.80 :

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Since the current is found to be maximum it means that the circuit attains resonance condition.

1. Inductance $X_L = 2\pi fL = 2\pi \times 50 \times 5 \times 10^{-3} = 1.57\,\Omega$

At resonance, $X_L = X_C = 1.57\,\Omega$

$$\therefore \quad C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \times 50 \times 1.57} = 2027.45 \times 10^{-6}\,F = 2027.45\,\mu F \quad \textbf{Ans}$$

2. Circuit current, $I_r = \frac{V}{R} = \frac{100}{10} = 10\,A$

3. Power consumed, $P_r = \frac{V^2}{R} = \frac{(100)^2}{10} = 1000\ watts$ **Ans**

EXAMPLE – 60

A circuit having a resistance of 5 Ω, an inductance of 0.4 H and a variable capacitance in series, is connected across a 110 V, 50 H_Z supply. Calculate at resonance condition (i) The value of capacitance (ii) Current, (iii) Voltage across the inductance (iv) Voltage across the capacitance, and (v) Q–factor of the circuit.

Solution :

Given data :

$R = 5\,\Omega$	$C = ?$
$L = 0.4\,H$	$I_r = ?$
$V = 110\,V$	$V_L = ?$
$f = 50\,H_Z$	$V_C = ?$
	Q–factor $= ?$

1. $X_L = 2\pi fL = 2\pi \times 50 \times 0.4 = 125.66\,\Omega$ **Ans**

At resonance, $X_L = X_C = 125.66\,\Omega$

$$\therefore \quad C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \times 50 \times 125.66} = 25.33 \times 10^{-6}\,F = 25.33\,\mu F \quad \textbf{Ans}$$

2. Current, $I_r = \frac{V}{Z_r} = \frac{V}{R} = \frac{110}{5} = 22\,A$ **Ans**

3. Voltage across inductance, $V_L = I X_L = 22 \times 125.66 = 2764.52V$ Ans

4. At resonance,

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$$V_L = V_C = 2764.52V \quad \textbf{Ans}$$

5. Q–factor $= \dfrac{\omega_r L}{R} = \dfrac{2\pi f_r L}{R} = \dfrac{2\pi \times 50 \times 0.4}{5} = 25.13$ **Ans**

(or) $$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{5} \times \sqrt{\frac{0.4}{25.33 \times 10^{-6}}} = 25.13$$ **Ans.**

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3.8.5 Q-Factor of a Series Circuit

It is defined as the *voltage magnification* in the series circuit at resonance.

At resonance, $V_L = V_c$

$\therefore$ Voltage across L or C, $V_L = V_c = I_r X_L$

Supply voltage, $V = I_r R$

Q - factor = Voltage magnification

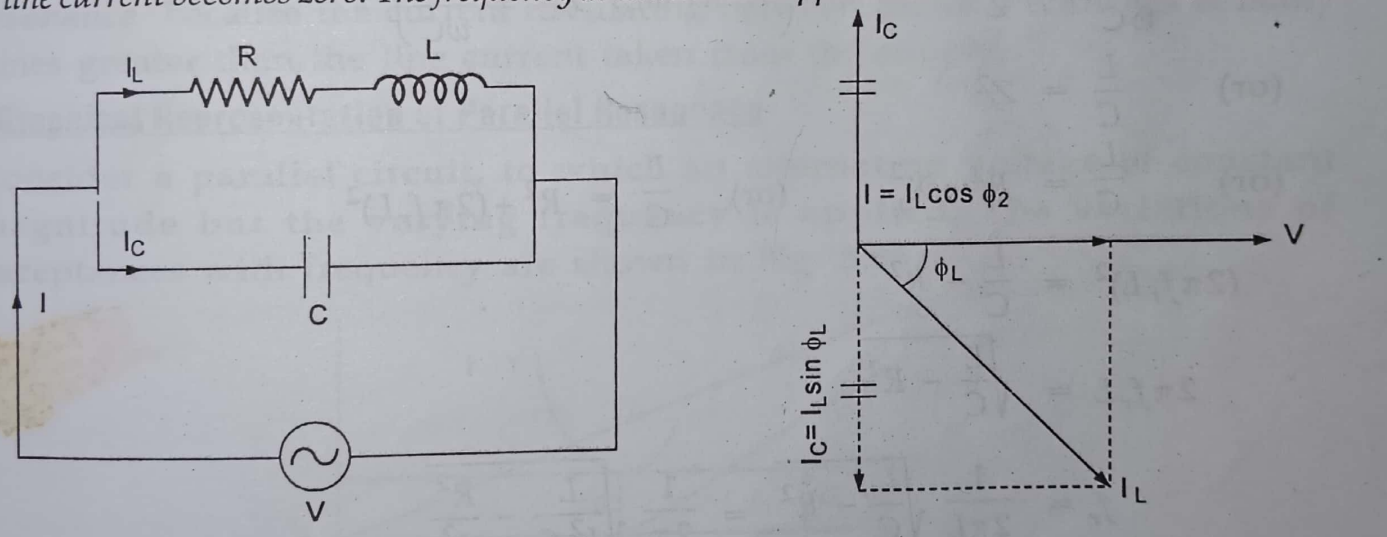
$$= \frac{\text{Voltage across } L \text{ or } C}{\text{Supply voltage}} = \frac{V_L}{V} = \frac{I_r X_L}{I_r R} = \frac{X_L}{R} = \frac{\omega L}{R} = \frac{2\pi f_r L}{R}$$

$$= \left(\frac{2\pi L}{R}\right) \times \frac{1}{2\pi\sqrt{LC}} \quad \left(\because f_r = \frac{1}{2\pi\sqrt{LC}}\right) = \frac{1}{R}\sqrt{\frac{L^2}{LC}} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Q-factor (quality factor) of a series circuit indicates how many times p.d across L or C greater than the applied voltage at resonance.

3.9 Resonance in Parallel Circuits

"A parallel circuit is said to be in electrical resonance when the reactive (or wattless) component of line current becomes zero. The frequency at which this happens is known as resonant frequency."



(a) Circuit Diagram (b) Vector Diagram

FIG 3.84 :

The most commonly used practical resonant parallel circuit is that a coil with internal resistance (i.e., an inductor) in parallel with a capacitor as shown in Fig. 3.84 (a). This circuit is often called the tank circuit due to the storage of energy by the inductor and capacitor. The phasor diagram of the circuit is shown in Fig. 3.84 (b).

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3.9.1 Resonance Frequency

Net reactive component of current,

$$= I_C - I_L \sin\phi_L$$

At parallel resonance,

The net reactive component of line current is zero.

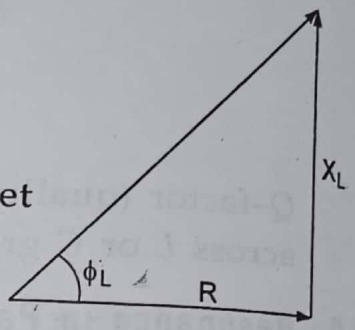
i.e. $I_C - I_L \sin\phi_L = 0$

(or) $I_C = I_L \sin\phi_L$ ..... (1)

But, $I_L = \frac{V}{Z}$; $I_C = \frac{V}{X_C}$

By the impedance triangle of Fig. 3.84 (c)

$$\sin\phi_L = \frac{X_L}{Z}$$



(c) Impedance triangle

FIG 3.84 :

Substituting the above values in equation (1), We get

$$\frac{V}{X_C} = \frac{V}{Z} \cdot \frac{X_L}{Z}$$

$$X_L X_C = Z^2$$

$$\frac{\omega L}{\omega C} = Z^2 \qquad \left(\because X_L = \omega L,\ X_C = \frac{1}{wC}\right)$$

(or) $\frac{L}{C} = Z^2$ ..... (2)

(or) $\frac{L}{C} = R^2 + X_L^2$ (or) $\frac{L}{C} = R^2 + (2\pi f_r L)^2$

$$(2\pi f_r L)^2 = \frac{L}{C} - R^2$$

$$2\pi f_r L = \sqrt{\frac{L}{C} - R^2}$$

$$f_r = \frac{1}{2\pi L}\sqrt{\frac{L}{C} - R^2} = \frac{1}{2\pi}\sqrt{\frac{L}{L^2 C} - \frac{R^2}{L^2}}$$

$$\therefore \quad \boxed{f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}} \quad \text{..... (3)}$$

This is the resonant frequency and is given in *Hz*. if R is *Ohm*, L is in *henry*, and C is in *farad*. If R is negligibly small, then

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} = \frac{1}{2\pi}\frac{1}{\sqrt{LC}} \quad \text{..... (4)}$$

This is same as series resonance frequency.

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3.9.2 Impedance at Resonance

The circuit current,

$$I = I_L \cos \phi_L \qquad (\because \text{ Wattless component is zero})$$

$$= \frac{V}{Z} \cdot \frac{R}{Z} \qquad \left[\because \cos \phi_L = \frac{R}{Z_L} \text{ from fig (b)}\right]$$

$$= \frac{VR}{Z^2} \qquad \ldots\ldots\ldots\ldots (5)$$

Substituting equation (2) in (5), We get,

$$I = \frac{VR}{\left(\frac{L}{C}\right)} = \frac{V}{\left(\frac{L}{CR}\right)} = \frac{V}{Z_r}$$

where $Z_r = \frac{L}{CR}$ is known as the ***dynamic impedance*** of the parallel circuit at resonance, and it will be maximum. Hence the current will minimum.

Rejector Circuit : Since a parallel resonant circuit can draw a very small current and power from mains due to maximum impedance, it is often regarded as ***rejector circuit***. Resonance in parallel circuit is also often referred to as ***current resonance*** because the current circulating between the two branches is many times greater than the line current taken from the supply.

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3.9.4 Q - Factor of a Parallel Resonant Circuit

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It is defined as the ratio of the current circulating between the two branches to the line current in the parallel circuit at resonance.

(or)

It is simply defined as equal to the *current magnification* in the parallel circuit at resonance.

Q - factor = current magnification

$$= \frac{\text{Circulatin g current between L and C}}{\text{Line current } (I_r)} = \frac{I_C}{I_R} \quad \ldots\ldots\ldots\ldots \quad (1)$$

Now $I_C = \dfrac{V}{X_C} = V\,\omega_r\,C$

And $I_r = \dfrac{V}{\left(\dfrac{L}{CR}\right)} = \dfrac{VCR}{L}$

Substituting the above values in equation (1), We get,

$$\therefore Q \text{ - factor} = \frac{V\,\omega_r\,C}{\left(\dfrac{VCR}{L}\right)} = \frac{\omega_r\,L}{R}$$

$$= \frac{2\pi f_r L}{R} = \frac{2\pi L}{R} \times f_r \ldots\ldots \text{same as series circuit.}$$

$$= \frac{2\pi L}{R} \cdot \frac{1}{2\pi\sqrt{LC}} \qquad \left(\because\ f_r = \frac{1}{2\pi\sqrt{LC}}\right)$$

$$= \frac{1}{R}\sqrt{\frac{L^2}{LC}} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

This is same as for series circuit Q - factor.

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3.9.5 Solved Problems

EXAMPLE – 64

A coil of 10 Ω resistance and 0.1 H inductance is connected in parallel with a capacitor of 100 m F capacitance. Calculate the frequency at which the circuit will act as a non-inductive resistance of R Ohm. Find also the value of R.

Solution :

Given data :

Resistance of the coil, $R = 10\,\Omega$

Inductnace of the coil, $L = 0.1\ H$

Capacitance, $C = 100\ \mu F = 100 \times 10^{-6}\ F$

Resonant frequency, $f_r = ?$

Dynamic resistance, $R = ?$

Resonant frequency,

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \left(\frac{R}{L}\right)^2}$$

$$= \frac{1}{2\pi}\sqrt{\frac{1}{0.1\times 100\times 10^{-6}} - \left(\frac{10}{0.1}\right)^2} = \frac{\sqrt{90000}}{2\pi} = \frac{300}{2\pi} = \mathbf{47.75\ Hz.}$$

Dynamic Resistance, $R = \frac{L}{CR} = \frac{0.1}{100\times 10^{-6}} = 100\ \Omega$

EXAMPLE – 65

An inductive circuit of resistance 2 Ω and inductance of 0.01 H is connected to a 250 V, 50 Hz. supply. What capacitance placed in parallel with it will produce resonance? Find the total current taken from supply and the current in the branch circuits.

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Solution :

Given data :

$R = 2\,\Omega$	$C = ?$
$L = 0.01\ H$	$I = ?$
$V = 250\ V$	
$f = 50\ Hz.$	

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$$X_L = 2\pi f L = 2\pi \times 50 \times 0.01 = 3.14$$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{(2)^2 + (3.14)^2} = 3.723\,\Omega$$

$\therefore$ At resonance,

$$C = \frac{L}{Z^2} = \frac{0.01}{(3.723)^2} = 721 \times 10^{-6}\,F \quad \text{(or)} \quad 721\,\mu F$$

Total current taken from supply at resonance,

$$I = \frac{V}{\left(\frac{L}{CR}\right)} = \frac{250}{\left(\frac{(0.01)}{721 \times 10^{-6} \times 2}\right)} = 36.05\ A$$

Current in inductive branch,

$$I_L = \frac{V}{Z} = \frac{250}{3.723} = 67.15A$$

Current in capacitive branch,

$$I_C = \frac{V}{X_C} = 2\pi f C V = 2\pi \times 50 \times 721 \times 10^{-6} \times 250 = 56.63\ A$$

EXAMPLE - 66

A coil has an impedance of 20 Ω at 50 Hz. and its power factor is 0.8. Determine the value of the capacitor to be shunted with the coil to produce resonance at 100 Hz. also calculate the Q - factor of the circuit.

Solution :

Given data :

$Z = 20\ \Omega$	$f_r = 100\ Hz.$
$f = 50\ Hz.$	Q - factor = ?
$\cos\phi = 0.8$	C = ?

Resistance, $R = Z\cos\phi = 20 \times 0.8 = 16\,\Omega$

$\therefore$ Inductive reactance, $X_L = \sqrt{Z^2 - R^2} = \sqrt{(20)^2 - (16)^2} = 12\ \Omega$

$$\therefore \quad f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \left(\frac{R}{L}\right)^2}$$

$$100 = \frac{1}{2\pi}\sqrt{\frac{1}{0.0382 \times C} - \left(\frac{16}{0.0382}\right)^2}$$

$$200\,\pi = \sqrt{\frac{1}{0.0382 \times C} - 175433.79}$$

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Squaring on both sides

$$(200\pi)^2 = \frac{1}{0.0382\,C} - 175433.79$$

$$\frac{1}{0.0382\,C} = (200\pi)^2 + 175433.79$$

$$\frac{1}{0.0382\,C} = 570217.966$$

$$\therefore \quad C = \frac{1}{0.0382 \times 570217.966} = \mathbf{45.9 \times 10^{-6}\ F} \quad \text{(or)} \quad \mathbf{45.9\ \mu F}$$

$$Q\text{ - factor} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{16}\sqrt{\frac{0.0382}{45.9 \times 10^{-6}}} = \mathbf{1.803} \quad \textbf{Ans}$$

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