GEOMETRICAL CONSTRUCTIONS

CHAPTER 6

6.1 INTRODUCTION

Any engineering drawing, in general consists of a number of geometrical constructions. A knowledge of these constructions is a must for every draughtsman or an engineer. The methods used in these constructions are actually applications of the principles of plane geometry. However, the mathematical proofs for these constructions are omitted here.

6.1.1 To bisect a given arc

Construction (Fig. 6.1)

[1] Draw the given arc AB.

[2] With centre A and radius equal to more than half the chord length AB, draw arcs on either side of AB.

[3] With centre B and the same radius, draw arcs intersecting the above arcs at C and D.

[4] Draw a line through C and D to intersect the given arc at E. The point E bisects the given arc.

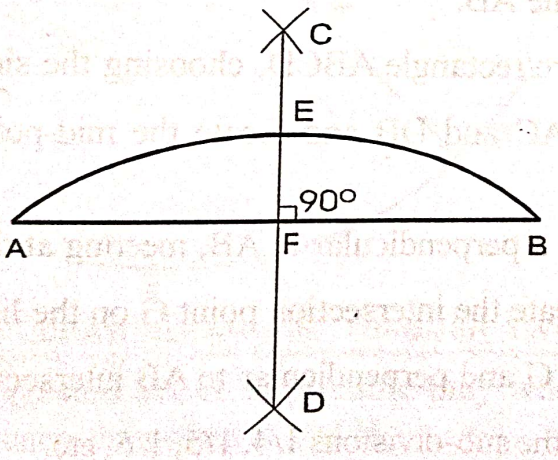


Fig. 6.1 BISECTION OF AN ARC

Note : [1] The above procedure may be followed to bisect the given line AB.

[2] The point F bisects the line AB and the line CD is called the perpendicular bisector of the line AB.

6.1.2 To Divide a straight line into a given number of equal parts, say six

Construction (Fig. 6.2) **(Oct./Nov. 2012, April/May 2015, March/April 2018)**

[1] Draw the given line A B.

[2] Through A, draw a line AC, making any convenient angle with AB.

[3] Set the compass to any convenient length and mark six equal divisions, 1^1, 2^1, 3^1, ... 6^1, along AC.

[4] Join 6^1, B.

[5] Draw lines through 5^1, 4^1, 3^1, etc., and parallel to 6^1 B, to meet the line AB at 5, 4, etc.

The points 1, 2, 3, etc., divide the line AB into six equal parts.

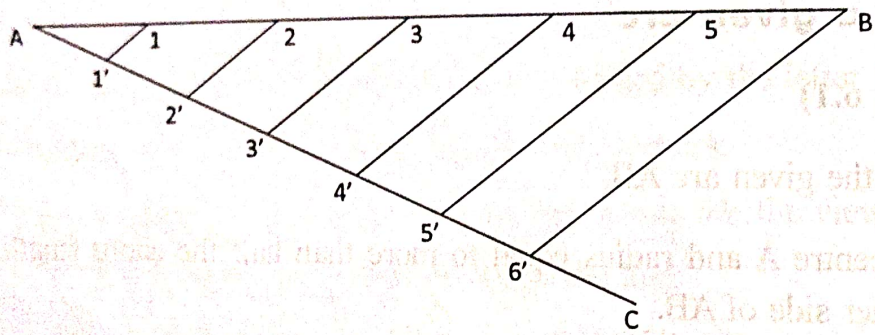


Fig. 6.2 DIVISION OF A LINE INTO A NUMBER OF EQUAL PARTS

6.1.3 To divide a given line into unequal parts by proportioning

Construction (Fig. 6.3)

[1] Draw the given line AB.

[2] Construct a square/rectangle ABCD, choosing the side AD as desired.

[3] Draw diagonals AC and DB and locate the mid-point E at the intersection of diagonals.

[4] Through E, draw a perpendicular to AB, meeting at the mid-point F of the line AI

[5] Join D, F and locate the intersection point G on the line AC.

[6] The line through G and perpendicular to AB intersects it at H (AH = 1/3 AB).

[7] Similarly, locate the sub-divisions 1/4, 1/5, 1/6, etc.

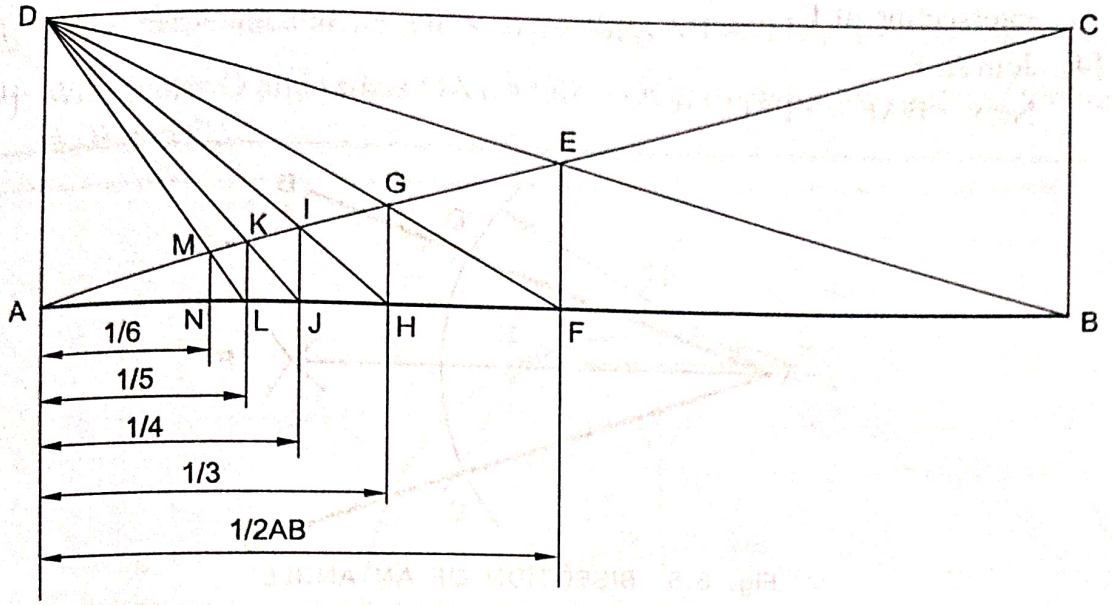


Fig. 6.3 PROPORTIONATE DIVISION OF A LINE

6.1.4 To draw an arc of given radius R through two given points

Construction (Fig. 6.4)

[1] Locate the given points A and B.

[2] With centers A and B and radius equal to R, draw arcs intersecting at O.

[3] With center O and radius R, draw the arc which passes through the given points A and B.

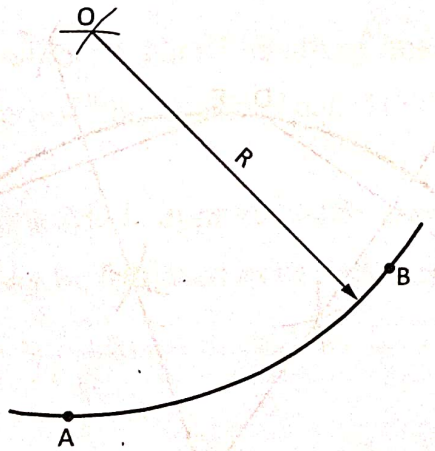


Fig. 6.4 DRAWING AN ARC THROUGH TWO POINTS

Note : The construction suggested above is possible only when the radius R is greater than half the distance between the points A and B.

6.1.5 To bisect a given angle

Construction (Fig. 6.5)

[1] Draw the lines AB and AC, making the given angle.

[2] With centre A and any convenient radius R_1, draw an arc intersecting the sides of the angle at D and E.

[3] With centres D and E and radius larger than half the chord length DE, draw arc intersecting at F.

[4] Join A, F.

Now, <BAF = <FAC.

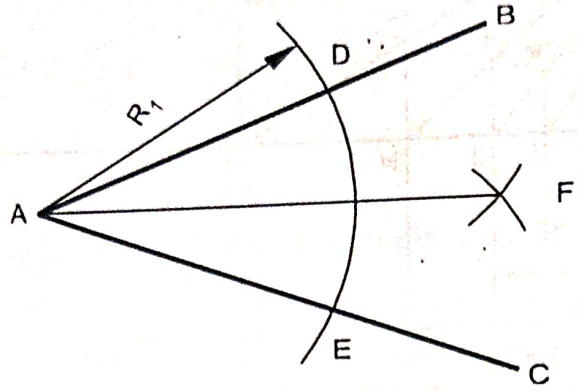


Fig. 6.5 BISECTION OF AN ANGLE

Note : This procedure may be repeated to divide the given angle into 4, 8, 16., equal parts.

6.1.6 To locate the centre of a given arc

Construction (Fig. 6.6)

[1] Draw any two chords CD and EF to the given arc AB.

[2] Draw perpendicular bisectors to CD and EF, intersecting each other at O.

Point O is the required centre.

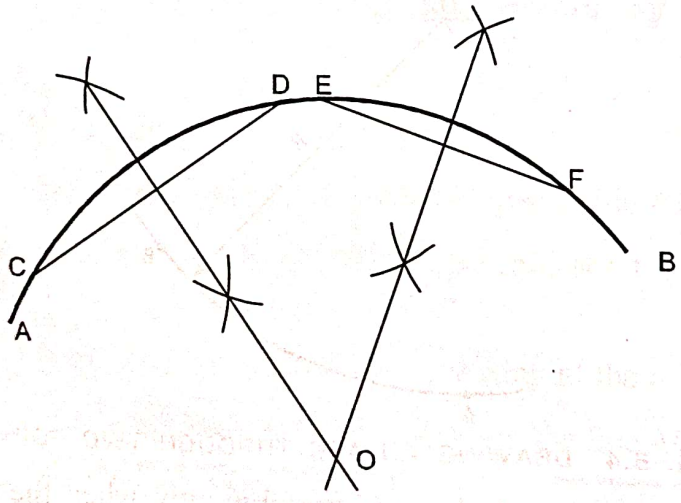


Fig. 6.6 LOCATION OF THE CENTRE OF AN ARC

Note : The above procedure may be followed to locate the centre of a circle.

6.1.7 To Draw an arc passing through three given points, not on straight line

Construction (Fig. 6.7)

[1] Locate the given points A, B and C.

[2] Draw lines AB and BC.

[3] Draw perpendicular bisectors to the lines AB and BC, intersecting at O.

[4] With center O and radius OA (= OB = OC), draw the arc, passing through the points A, B and C.

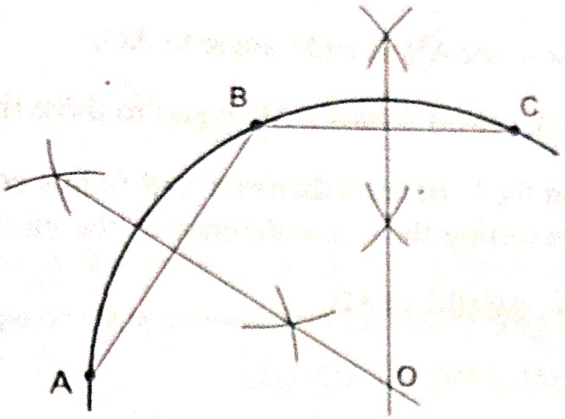


Fig. 6.7 DRAWING AN ARC THROUGH THREE POINTS

6.1.8 To find the length of a given arc

Construction (Fig. 6.8)

[1] With centre O, draw the given arc AB.

[2] Divide the given arc into a number of equal parts (the more the number, the better the accuracy).

[3] Set the divider equal to the chord length of one of the parts.

[4] On any line AC, step-off the divisions equal to the divisions on the curve, reaching the point D on it.

The length of the line AD is approximately equal to the arc length of the curve AB.

Note : The above procedure may be followed to find the approximate length of the circumference of a circle.

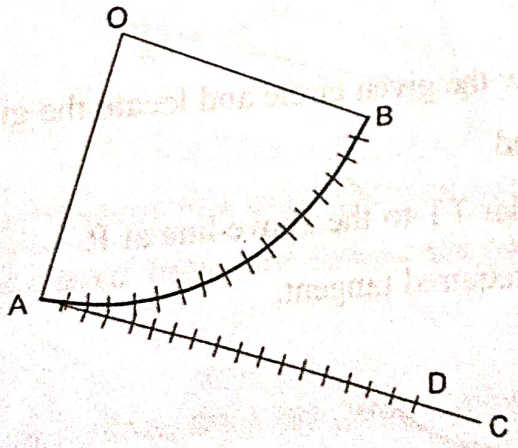


Fig. 6.8 DETERMINATION OF THE LENGTH OF AN ARC

6.1.9 To find the approximate length of the circumference of a given circle

Construction (Fig. 6.9)

[1] With centre O, draw the given circle.

[2] Draw the vertical diameter through O, meeting the circle at A.

[3] Through A, draw a line AB at right angle to AO.

[4] Along the line AB, set-off distance AC equal to three times the diameter of the circle.

[5] With centre D on the horizontal diameter and radius equal to the radius of the circle, draw an arc intersecting the circumference of the circle at E.

[6] Draw the line EF, parallel to AB.

[7] Join F, C.

The length FC is approximately equal to the circumference of the circle.

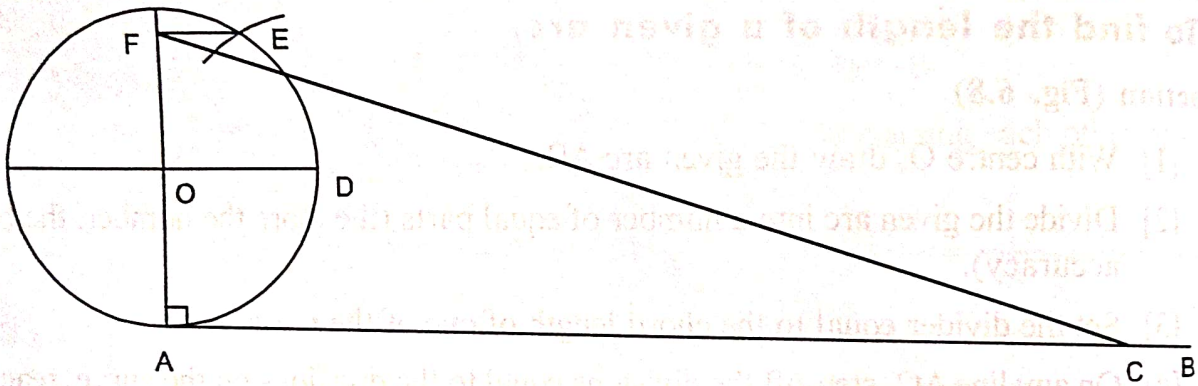


Fig. 6.9 DETERMINATION OF THE CIRCUMFERENCE OF A CIRCLE

6.1.10 Tangent lines

A line, meeting a curve at a point on it is called the tangent to the curve.

6.1.10.1 To draw a tangent to a given circle, at any point on it

Construction (Fig. 6.10)

[1] With centre O, draw the given circle and locate the given point P on it.

[2] Join O, P and extend.

[3] Draw a perpendicular TT to the above line at P.

The line TT is the required tangent.

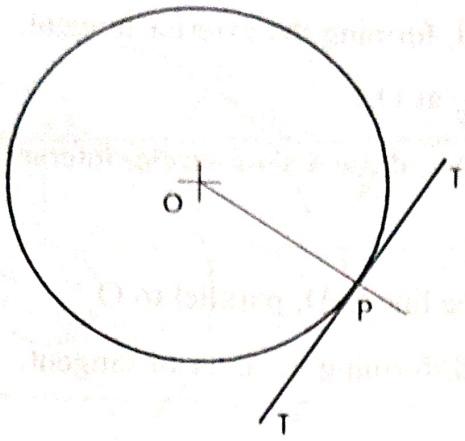


Fig. 6.10 TANGENT TO A CIRCLE AT A POINT ON IT

6.1.10.2 *To draw a tangent to a given circle, from any point outside it*

Construction (Fig. 6.11) **(April/May 2012, March/April 2014, 2016, 2018)**

[1] With centre O, draw the given circle.

[2] Locate the given point P in its correct position.

[3] Join P, O and locate its mid-point A.

[4] With centre A and radius AO, draw an arc intersecting the given circle at B and C.

[5] Join P, B; P, C and extend.

The lines PB and PC extended are the two possible tangents.

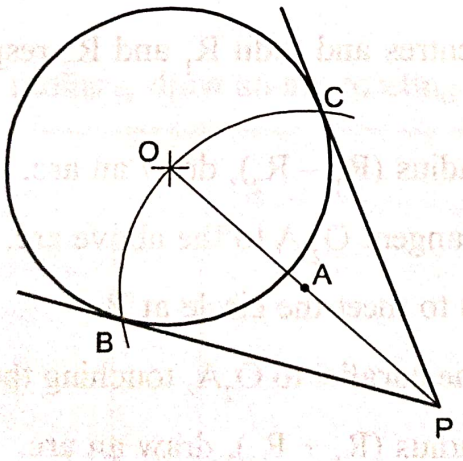


Fig. 6.11 TANGENT TO A CIRCLE FROM AN OUTSIDE POINT

6.1.10.3 *To draw exterior and interior tangents to two given circles of equal radii*

Construction (Fig. 6.12)

[1] With O_1 and O_2 as centres, draw the given circles.

[2] Join O_1, O_2.

6.1.12 To inscribe a square, in a given circle of diameter D

Construction (Fig. 6.15)

[1] With centre O, draw a circle of diameter D.

[2] Through the centre O, draw two diameters, say AC and BD, at right angle to each other.

[3] Join A, B ; B, C ; C , D and D , A.

ABCD is the required square.

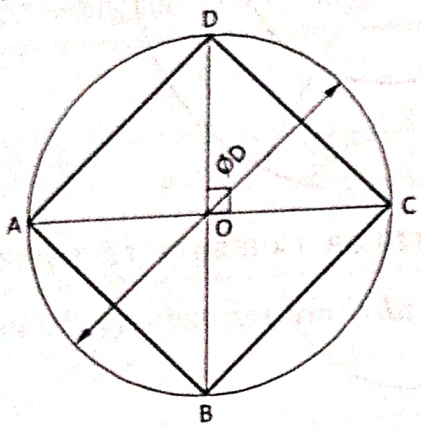


Fig. 6.15

6.1.13 Polygons

A polygon is a plane figure, bounded by straight edges. When all the edges are of equal length the polygon is called a regular polygon.

6.1.13.1 To inscribe a regular polygon of any number of sides, N in a given circle

Construction (Fig. 6.16)

[1] Draw the given circle with AD as diameter.

[2] Divide the diameter AD into N equal parts, say 6.

[3] With AD as radius and A and D as centres, draw arcs intersecting each other at G

[4] Join G, 2 and extend to intersect the circle at B.

[5] Join A, B which is the length of side of the required polygon.

[6] Set the compass to the length AB, and starting from B, mark-off on the circumference of the circle, obtaining the points C, D, etc.

The figure obtained by joining the points A, B, C, etc., is the required polygon.

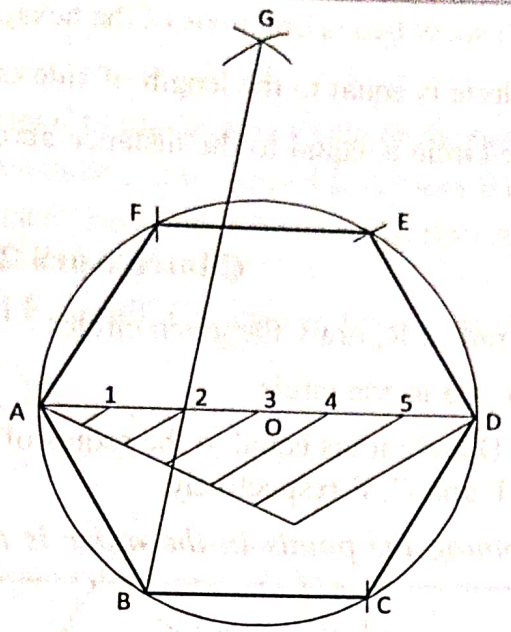


Fig. 6.16 TO INSCRIBE A REGULAR POLYGON

6.1.13.2 To inscribe a hexagon in a given circle of radius R

Case I Using a set-square

Construction (Fig. 6.17)

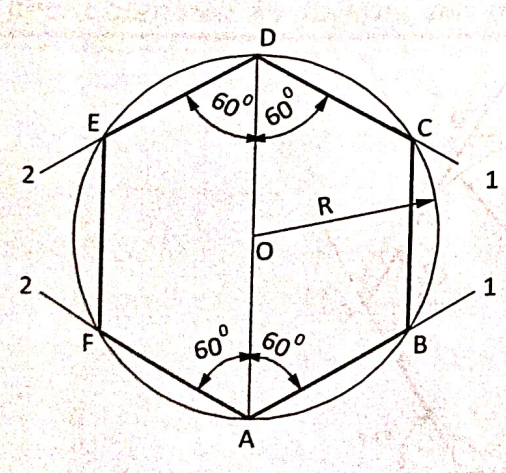
[1] With centre O and radius R, draw the given circle.

[2] Draw any diameter AD to the circle.

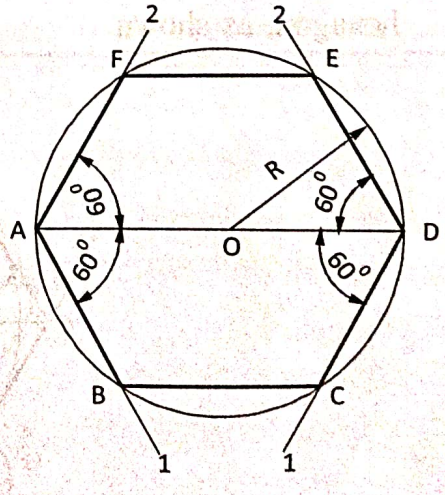
[3] Using 30^0 - 60^0 set-square and through the point A, draw lines A1, A2 at an angle 60^o with AD, intersecting the circle at B and F respectively.

[4] Using 30^0-60^0 set-square and through the point D draw lines D1, D2 at an angle 60^0 with DA, intersecting the circle at C and E respectively.

The figure obtained by joining the points A, B, C, D, E, F and A is the required hexagon.



(a)



(b)

Fig. 6.17 INSCRIBING A HEXAGON - USING SET - SQUARES

6.1.15 To construct a regular polygon of any number of sides N, given the length of its side.

(April/May 2015, Oct./Nov. 2015, March/April 2017)

GENERAL METHOD

Construction (Fig. 6.24)

[1] Draw a line AB equal to the length of the side.

[2] With centre A and radius AB, draw a semi-circle.

[3] Divide the semi-circle into the same number of equal parts, as the number of sides N, say 5.

[4] Draw radial lines through 2, 3, etc. (second division point 2 will always be a vertex of the polygon).

[5] With centre B and radius equal to the side, draw an arc intersecting the radial line through 4 at C.

[6] Repeat the procedure and locate the point D on the radial line through 3.

The figure obtained by joining the points A, B, C, etc., is the required polygon.

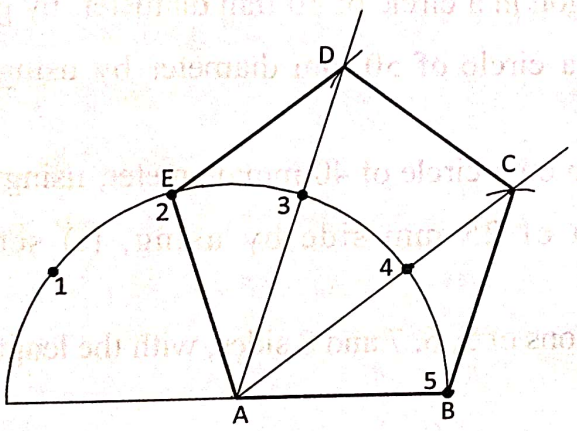


Fig. 6.24 CONSTRUCTION OF A REGULAR POLYGON-GENERAL METHOD

6.1.16 Draw a line, 100 mm long and divide it into 6 equal parts.

Refer Fig. 6.2 for solution.

6.1.17 Draw an exterior common tangent to the given circles of radii 20 mm and 30 mm. The distance between the centres of the circles is 100 mm.

Refer Fig. 6.13 for solution.

6.1.18 Draw internal (interior) tangents to two circles of diameters 50 mm and 75 mm whose centres are at a distance of 80 mm.

Refer Fig. 6.13 for solution.

6.1.19 Construct a regular hexagon, in a circle of diameter 80 mm.

Refer Figs. 6.17 and 6.18 for solution.

CURVES USED IN ENGINEERING PRACTICE

CHAPTER 7

7.1 INTRODUCTION

A cone is formed when a right-angled triangle of an apex angle θ is rotated about its altitude - as the axis. The length or height of the cone is equal to the altitude of the triangle and the radius of the base of the cone is equal to the base of the triangle. The apex angle of the cone is equal to 2 θ.

When a cone is cut by a plane, the curve formed along the section is known as a conic. For this purpose, the cone may be cut by different section planes (Fig. 7.1a) and the conic sections obtained are shown in Fig. 7.1c.

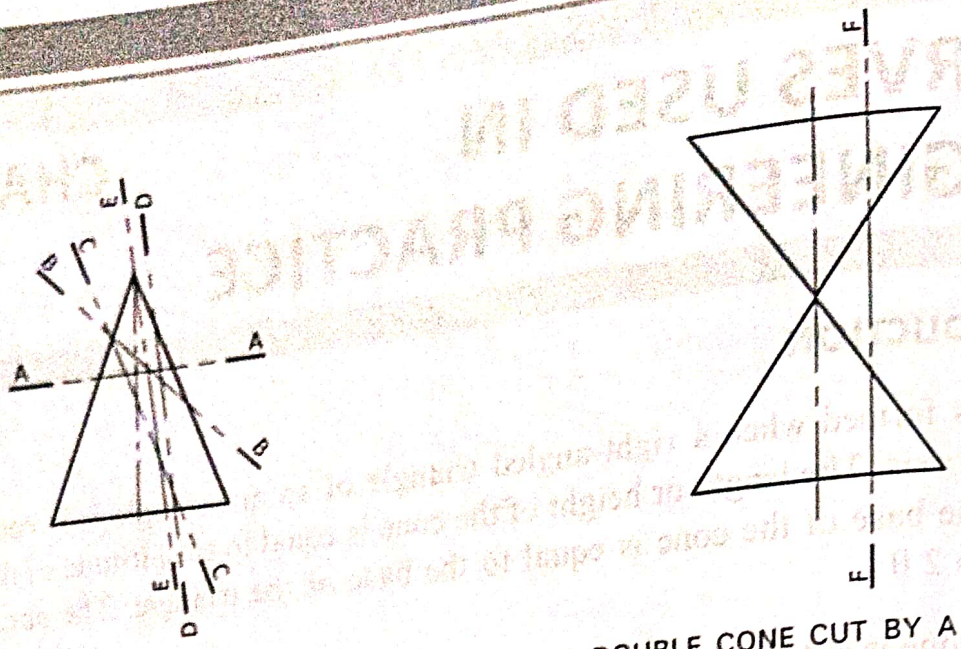
Circle : When a cone is cut by a section plane A-A, making an angle $\alpha = 90^\circ$ with the axis, the section produced is a circle.

Ellipse : When a cone is cut by a section plane B-B, at an angle α, more than half of the apex angle i.e., θ and less than 90°, the curve of intersection is an ellipse. The size of the ellipse depends upon the angle α and the distance of the section plane from the apex of the cone.

Parabola : If the angle α is equal to θ, i.e., when the section plane C-C is parallel to the slant side of the cone, the curve of intersection is a parabola, which is not a closed figure. The size of the parabola depends upon the distance of the section plane from the slant side of the cone.

Hyperbola : If the angle α is less than θ (section plane D-D), the curve of intersection is a hyperbola. The curve of intersection is hyperbola, even if $\alpha = 0$, provided the section plane is not passing through the apex of the cone. However, if the section plane E-E passes through the apex, the section produced is an isosceles triangle.

If a double cone is cut by a section plane F-F on one side of the common axis (Fig. 7.1b), the curves of intersection results in two branches of a hyperbola. In this case, if $\alpha = 0$, the two branches of the curve will be symmetric in form.



(a) A CONE CUT BY DIFFERENT SECTION PLANES (b) A DOUBLE CONE CUT BY A SECTION PLA

Fig. 7.1

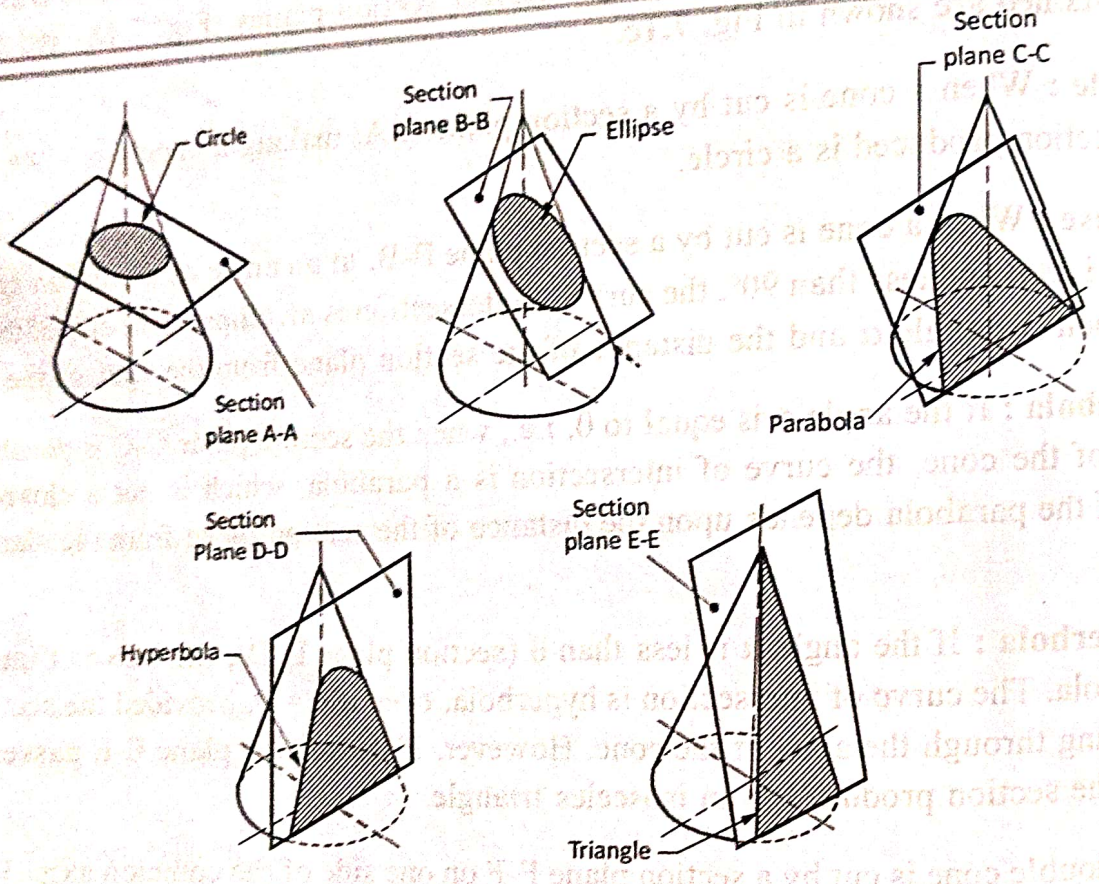


Fig. 7.1 (c) TYPES OF CONIC SECTIONS

7.2 CONIC SECTIONS AS LOCI OF A MOVING POINT

A conic section may be defined as the locus of a point moving in a plane such that the rati its distance from a fixed point (focus) and a fixed straight line (directrix) is always a constant. ratio is called eccentricity. The line passing through the focus and perpendicular to directrix is the axis of the curve. The point at which the conic section intersects the axis is ca the vertex or apex of the curve.

The eccentricity value is less than 1 for ellipse, equal to 1 for parabola and greater than 1 for hyperbola (Fig. 7.2).

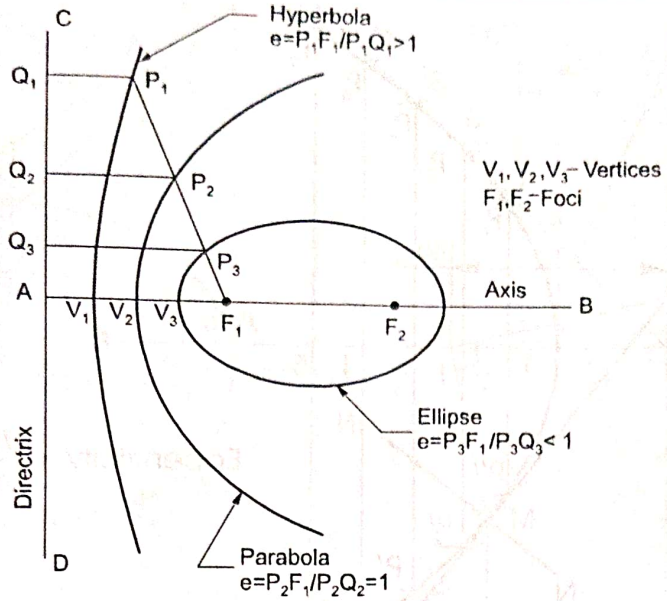


Fig. 7.2 CONIC SECTIONS AS LOCI OF A MOVING POINT

7.2.1 Draw a parabola, with the distance of the focus from the directrix as 50 mm (eccentricity method).

Draw also the curves when the eccentricity values are 2/3 and 3/2 and name the curves. For each curve, draw normal and tangent through a point 40 mm from the directrix.

Construction (Fig. 7.3)

[1] Draw the axis AB and the directrix CD at right angles to it.

[2] Mark the focus F on the axis.

[3] Locate the vertex V on AB such that AV = VF.

[4] Draw a line VE perpendicular to AB such that VE = VF.

[5] Join A,E and extend. By construction,

VE/VA = VF/VA = 1, the eccentricity.

[6] Locate number of points 1, 2, 3, etc., to the right of V on the axis, which need not be equi-distant.

[7] Through the points 1, 2, 3, etc., draw lines perpendicular to the axis and to meet the line AE extended at 1^1, 2^1, 3^1, etc.

[8] With centre F and radius 1- 1^1, draw arcs intersecting the line through 1 at P_1 and P_1^1.

[9] Similarly, locate the points P_2, P_2^1; P_3, P_3^1; etc., on either side of the axis.

Join the points by a smooth curve, forming the required parabola.

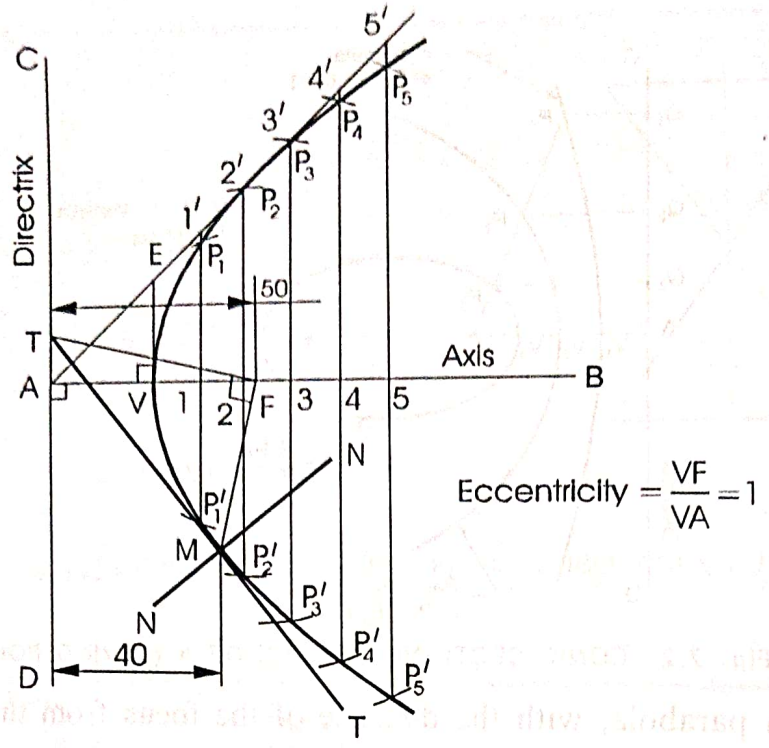


Fig. 7.3 CONSTRUCTION OF A PARABOLA-ECCENTRICITY METHOD

To draw a tangent and normal to the parabola, locate the point M which is at 40 mm from the directrix. Then, join M to F and draw a line through F, perpendicular to M F to meet the directrix at T. The line joining T and M and extended is the tangent and a line NN, through M and perpendicular to TM is the normal to the curve.

Note : [1] The construction, similar to the one given in Fig. 7.3 may be followed to draw an ellipse (Fig. 7.4) and a hyperbola (Fig. 7.5), including tangents and normals.

[2] The ellipse is a closed curve and has two foci and two directrices. A hyperbola is an open curve and has one focus and one directrix.

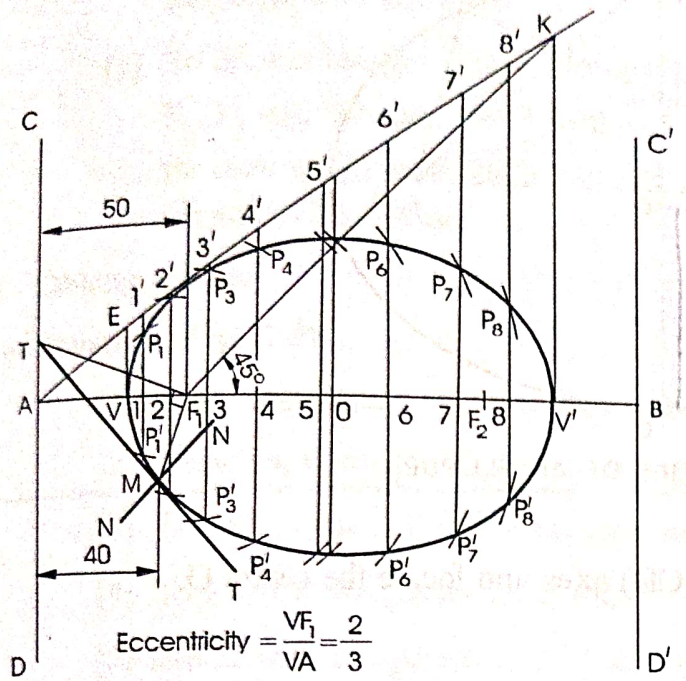


Fig. 7.4 CONSTRUCTION OF AN ELLIPSE-ECCENTRICITY METHOD

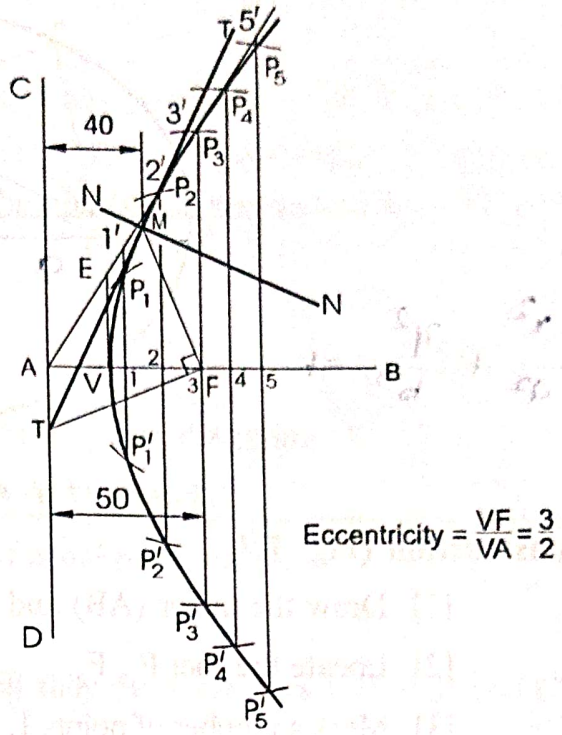


Fig. 7.5 CONSTRUCTION OF A HYPERBOLA-ECCENTRICITY METHOD

7.3 GENERAL METHODS OF CONSTRUCTING ELLIPSE

The major and minor axes of an ellipse are 120 mm and 80 mm. Draw an ellipse by, (i) foci method, (ii) oblong method, (iii) concentric circles method and (iv) trammel method.

I Foci method

The ellipse is a curve traced by a point moving such that the sum of its distances from the two fixed points, known as foci, is constant and equal to the major axis.

Referring Fig. 7.6, F_1 and F_2 are the foci, AB, the major axis and CD, the minor axis. As per the above definition,

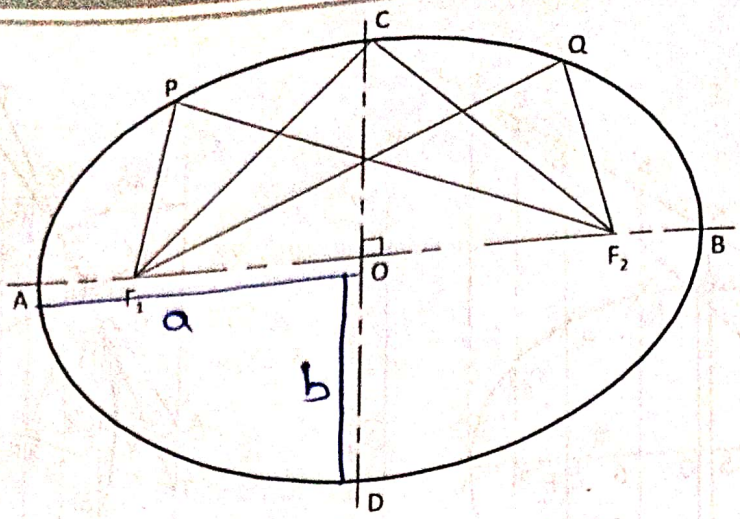
$PF_1 + PF_2 = CF_1 + CF_2 = QF_1 + QF_2 = AB$ (major axis).

It may also be noted that $CF_1 = CF_2 = 1/2\ AB$

e = Distance of the Point from the focus / Distance of the Point from the directrix

e < 1

Ex: Elliptical Gear, Bridges, Arches.



$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Fig. 7.6 PROPERTIES OF AN ELLIPSE

Construction (Fig. 7.7)

[1] Draw the major (AB) and minor (CD) axes and locate the centre O.

[2] Locate the foci F_1, F_2.

[3] Mark a number of points 1, 2, 3, etc., between F_1 and O, which need not be equi-dista

[4] With centres F_1 and F_2 and radii A1 and B1 respectively, draw arcs intersecting at points P_1 and P'_1.

[5] With centres F_1 and F_2 and radii B1 and A1 respectively, draw arcs intersecting at points Q_1 and Q'_1.

[6] Repeat the steps 4 and 5 with the remaining points 2,3,4, etc., and obtain additio points on the curve.

Join the points by a smooth curve, forming the required ellipse.

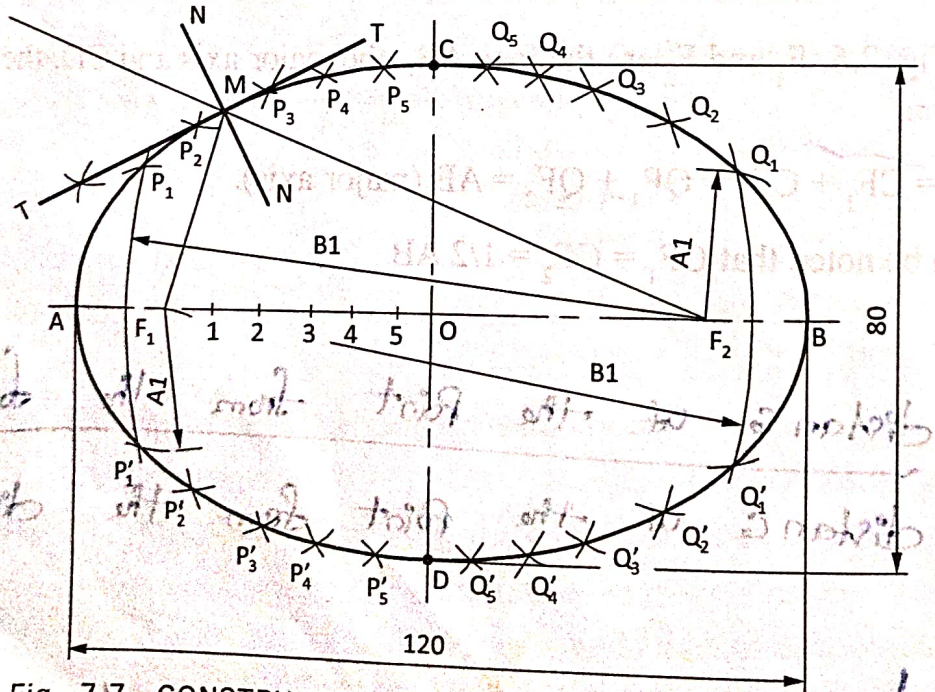


Fig. 7.7 CONSTRUCTION OF AN ELLIPSE - FOCI METHOD

Note : [1] To locate the foci, with centre C (or D) and radius OA (= OB), draw arcs intersecting the major axis at F_1 and F_2.

[2] To draw a tangent and normal to the ellipse at any point, say M on it, join the foci F_1, F_2 with M and extend F_2M to E and bisect $\angle EMF_1$. The bisector TT represents the required tangent and a line NN, drawn through M and perpendicular to TT is the normal to the ellipse.

II Oblong method

Construction (Fig. 7.8)

[1] Draw the major (AB) and minor (CD) axes and locate the centre O.

[2] Draw the rectangle KLMN, passing through A, D, B and C.

[3] Divide A O and AN into the same number of equal parts, say 4.

[4] Join C with the points 1^1, 2^1 and 3^1.

[5] Join D with the points 1, 2, 3 and extend till they meet the lines $C1^1$, $C2^1$ and $C3^1$ respectively at P_1, P_2 and P_3.

[6] Repeat steps 3 to 5 to obtain the points in the remaining quadrants.

Join the points by a smooth curve, forming the required ellipse.

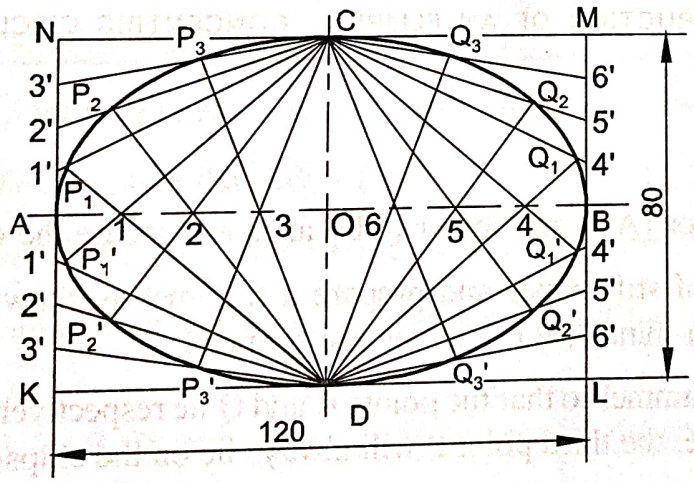


Fig. 7.8 CONSTRUCTION OF AN ELLIPSE - OBLONG METHOD

III Concentric circles method

Construction (Fig. 7.9) **[April/May 2015, Oct./Nov. 2015, March/April 2016]**

[1] Draw the major (AB) and minor (CD) axes and locate the centre O.

[2] With centre O and major and minor axes as diameters, draw two concentric circles.

[3] Divide both the circles into an equal number of parts, say 12 and draw the radial lines.

[4] Considering the radial line O - 1' - 1, draw a horizontal line from 1' to meet the verti line from 1 at P_1.

[5] Repeat the step 4 and obtain other points P_2, P_3, etc.

Join the points by a smooth curve, forming the required ellipse.

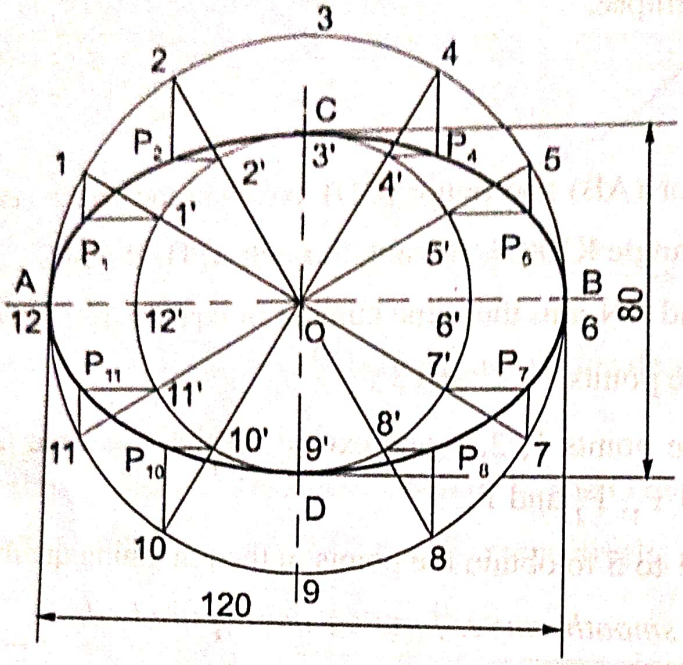


Fig. 7.9 CONSTRUCTION OF AN ELLIPSE - CONCENTRIC CIRCLES METHOD

IV Trammel method

Construction (Fig. 7.10)

[1] Draw the major (AB) and minor (CD) axes and locate the centre O.

[2] Take a strip of stiff paper and prepare a trammel as shown. Mark the semi-ma (PR) and semi-minor (PQ) axes on the trammel.

[3] Position the trammel so that the points R and Q lie respectively on the minor and ma axes. As a rule, the third point P will always lie on the ellipse required.

[4] Keeping R on the minor axis and Q on the major axis, move the trammel to oth positions and locate other points on the curve.

Join the points by a smooth curve, forming the required ellipse.

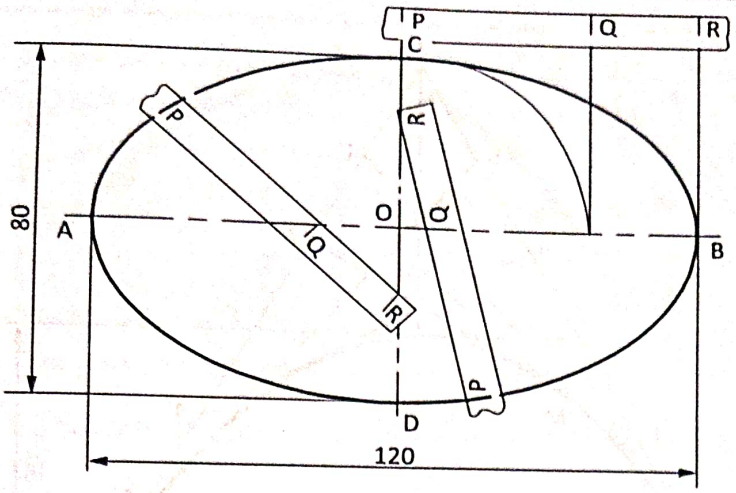


Fig. 7.10 CONSTRUCTION OF AN ELLIPSE-TRAMMEL METHOD

7.4 GENERAL METHODS OF CONSTRUCTING PARABOLA

Draw a parabola with 70 mm as the base and 30 mm as the length of the axis, by using (i) tangent method and (ii) rectangle method.

1. TANGENT METHOD

Construction (Fig. 7.11)

[1] Draw the base AB and locate its mid-point C.

[2] Through C, draw CD perpendicular to AB, forming the axis.

[3] Produce CD to E such that DE = CD.

[4] Join E, A and E, B. These are the tangents to the parabola at A and B.

[5] Divide AE and BE into the same number of equal parts and number the points as shown.

[6] Join 1, 1^1; 2, 2^1; 3, 3^1; etc., forming the tangents to the required parabola.

a) Draw an isosceles triangle of 100mm long base & 110mm long altitude Inscribe a Parabola in it by method of tangent.

A smooth curve passing through A, D and B and tangential to the above lines is the required parabola.

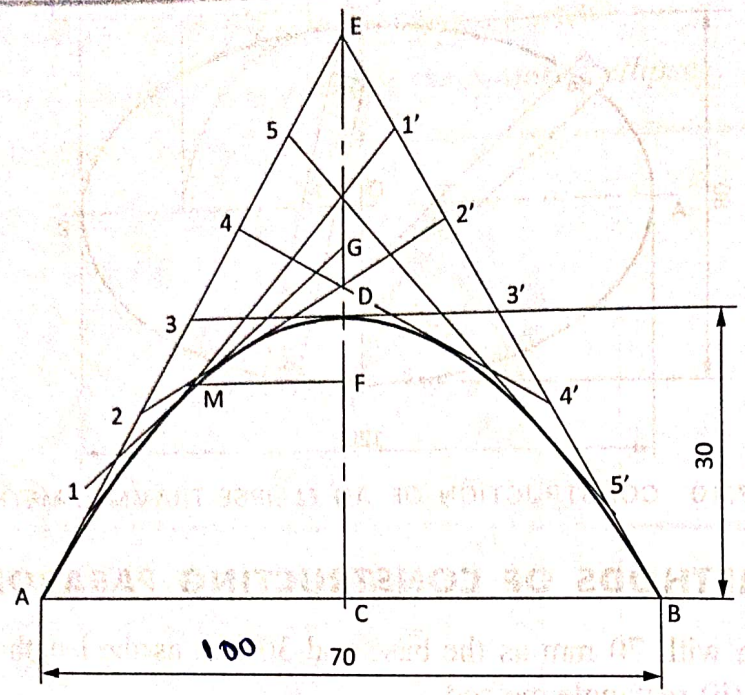


Fig. 7.11 CONSTRUCTION OF A PARABOLA-TANGENT METHOD

Note : To draw a tangent to the curve at a point, say M on it, draw a horizontal through M, meeting the axis at F. Mark G on the extension of the axis such that DG = FD. Join G M and extend, forming the tangent to the curve at M.

2. RECTANGLE METHOD

[Oct./Nov. 2015, March/April 2016, 2018

Construction (Fig. 7.12)

[1] Draw the base AB and axis CD such that CD is a perpendicular bisector to AB.

[2] Construct a rectangle A B E F, passing through C.

[3] Divide AC and AF into the same number of equal parts and number the points as shown.

[4] Join 1, 2 and 3 to D.

[5] Through 1^1, 2^1 and 3^1, draw lines parallel to the axis, intersecting the lines 1D, 2D and 3D at P_1, P_2 and P_3 respectively.

[6] Obtain the points P_1^1, P_2^1 and P_3^1 which are symmetrically placed to P_1, P_2 and P_3 respectively, with respect to the axis CD.

Join the points by a smooth curve, forming the required parabola. Fig. 7.12 also shows the method of drawing a tangent at any point M.

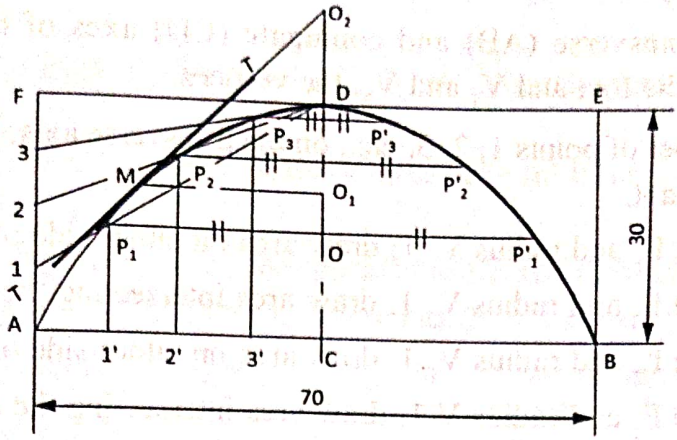


Fig. 7.12 CONSTRUCTION OF A PARABOLA-RECTANGLE METHOD

7.5 METHOD OF CONSTRUCTING A HYPERBOLA, GIVEN THE FOCI AND THE DISTANCE BETWEEN THE VERTICES

A hyperbola is a curve generated by a point moving such that the difference of its distances from two fixed points called the foci is always constant and equal to the distance between the vertices of the two branches of hyperbola. This distance is also known as the major axis of the hyperbola. Referring Fig. 7.13,

$$P_1F_1 \sim P_1F_2 = P_2F_2 \sim P_2F_1 = V_1\ V_2 \text{ (major axis)}$$

Note : The axes AB and CD are respectively known as transverse and conjugate axes of the hyperbola. The curve has two branches which are symmetric about the conjugate axis.

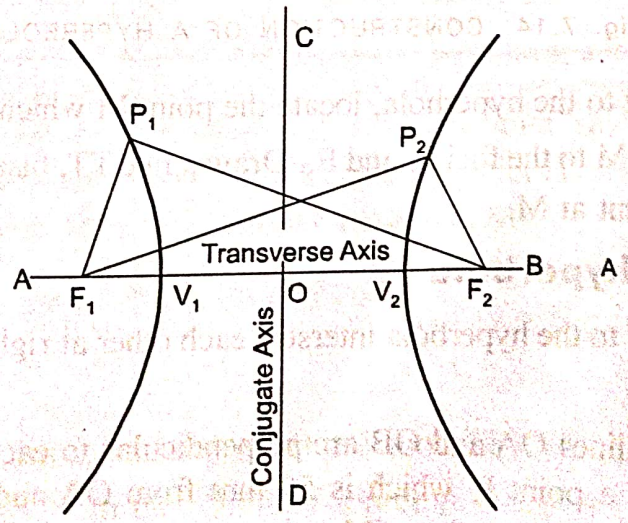


Fig. 7.13 PROPERTIES OF HYPERBOLA

7.5.1 Construct a hyperbola with its foci 70 mm apart and the major axis (distance between the vertices) as 40 mm. Draw a tangent to the curve at a point 20 mm from the focus.

Construction (Fig. 7.14)

[1] Draw the tansverse (AB) and conjugate (CD) axes of the hyperbola and locate F_1 and F_2, the foci and V_1 and V_2, the vertices.

[2] Mark number of points 1, 2, 3, etc., on the transverse axis, beyond F_2 which need not be equi-distant.

[3] With centre F_1 and radius V_1 1, draw arcs on either side of the transverse axis.

[4] With centre F_2 and radius V_2 1, draw arcs intersecting the above arcs at P_1, P_1^1.

[5] With centre F_2 and radius V_1 1, draw arcs on either side of the transverse axis.

[6] With centre F_1 and radius $V_2$1, draw arcs intersecting the above arcs at Q_1, Q_1^1.

[7] Repeat the steps 3 to 6 and obtain other points P_2, P_2^1, etc., and Q_2, Q_2^1, etc.

Join the points P_1, P_2, etc., P_1^1, P_2^1, etc., and Q_1, Q_2, etc., Q_1^1, Q_2^1, etc., forming the two branches of hyperbola.

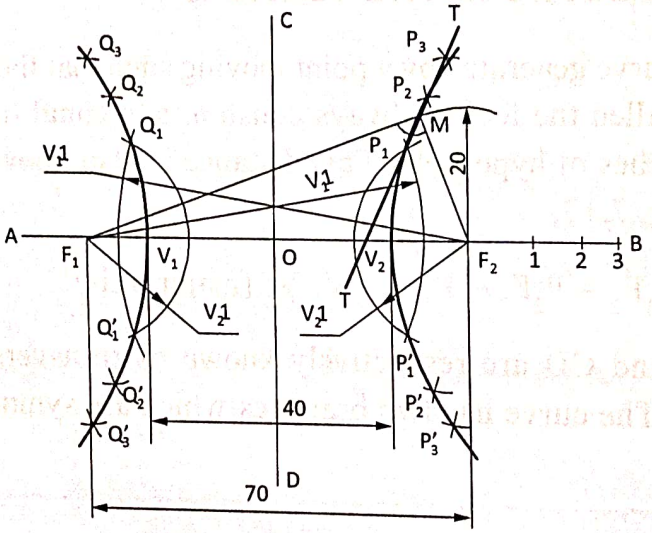


Fig. 7.14 CONSTRUCTION OF A HYPERBOLA

Note : To draw a tangent to the hyperbola, locate the point M which is at 20mm form the focu say F_2. Then, join M to the foci F_1 and F_2. Draw a line TT, bisecting the $\angle F_1 MF_2$, formir the required tangent at M.

7.5.2 Rectangular hyperbola

When the asymptotes to the hyperbola intersect each other at right angles, the curve is know as a rectangular hyperbola.

7.5.2.1 Two straight lines OA and OB are perpendicular to each other. Draw a rectangul hyperbola, passing through a point P, which is 50 mm from OA and 25 mm from OB. Draw tangent to the curve at a point 35 mm from OA.

Construction (Fig. 7.15)

[1] Draw the asymptotes OA and OB at right angles to each other and locate the giv point P.

[2] Draw the lines CD and EF, passing through P and parallel to OA and OB respectively.

[3] Locate a number of points 1, 2, 3, etc., on the line CD, which need not be equi-distant.

[4] Join 1, 2, 3, etc., to O and extend if necessary till these lines meet the line EF at the points 1^1, 2^1, 3^1, etc.

[5] Draw lines through 1, 2, 3, etc., parallel to EF and through 1^1, 2^1, 3^1, etc., parallel to CD, intersecting respectively at the points P_1, P_2, P_3, etc.

A smooth curve passing through P_1, P_2, P_3, etc., is the required rectangular hyperbola.

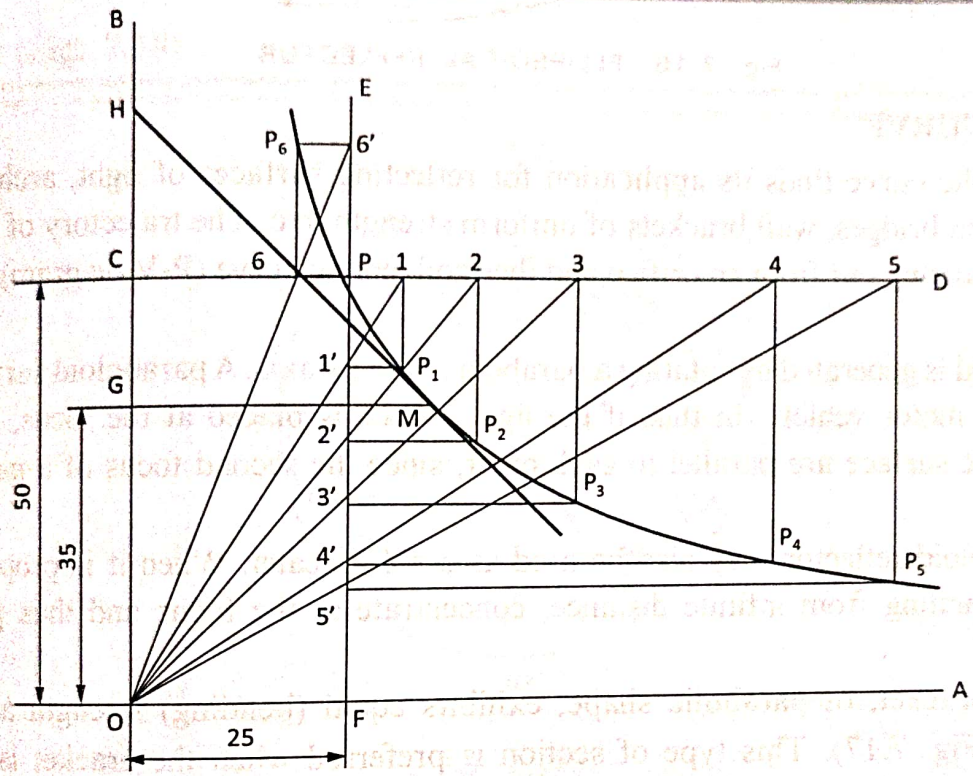


FIG. 7.15 RECTANGULAR HYPERBOLA

To draw a tangent to the rectangular hyperbola at any point, say M on it, draw a line through M and parallel to OA, to meet the line OB at G. Locate the point H on OB such that HG = OG. The line joining H and M and extended is the required tangent at M.

7.6 APPLICATIONS OF CONIC SECTIONS

ELLIPTICAL CURVE

An ellipsoid is generated by rotating an ellipse about its major axis. An ellipsoidal surface is used as a head-lamp or reflector. In this, the light source is placed at the first focus F_1 (Fig. 7.16). This works effectively, if the second focus F_2 is at a sufficient distance from the first focus. With this arrangement, the light rays reflecting from the surface are almost parallel to each other.

CHAPTER 8
SPECIAL CURVES

8.1 INTRODUCTION

In engineering drawing practice, certain non-circular curves may be required to be drawn for representing machine parts, instruments, arches, etc. These curves can not be constructed by using a compass. These are generally drawn after locating various intermediate points and joining them by using french curves. Cycloid, involute and helix are the curves, the constructional procedures of which are discussed in this chapter.

8.2 CYCLOIDAL CURVES

Cycloidal curves are generated by a fixed point on the circumference of a circle when it rolls without slipping along a fixed straight line or circular path. The rolling circle is called the generating circle, the fixed straight line, the directing line and the fixed circle, the directing circle.

8.2.1 Cycloid

A cycloid is a curve generated by a fixed point on the circumference of a circle, when it rolls without slipping along a straight line.

8.2.1.1 To draw a cycloid, given the radius R of the generating circle

(March/April 2018, Oct./Nov. 2018)

Construction (Fig. 8.1)

[1] With centre O and radius R, draw the given generating circle.

[2] Assuming P to be the initial position of the generating point, draw a line PA, tangential and equal to the circumference of the circle. PA is the directing line.

[3] Divide the line PA and the circle into the same number of equal parts and number the points.

[4] Draw the line OB, parallel and equal to PA. OB is the locus of the centre of the generating circle.

[5] Erect perpendiculars at 1^1, 2^1, 3^1, etc., meeting OB at O_1, O_2, O_3, etc.

[6] Through the points 1,2,3, etc., draw lines parallel to PA.

[7] With centre O_1 and radius R, draw an arc intersecting the line through 1 at P_1. P_1 is the position of the generating point, when the centre of the generating circle moves to O_1.

[8] Similarly, locate the points P_2, P_3, etc.

A smooth curve passing through the points P, P_1, P_2, P_3 etc., is the required cycloid.

Note : [1] To draw a tangent and normal to the cycloid at a point, say M on it, with centre M and radius R, draw an arc intersecting the line OB at C. Through C, draw a line perpendicular to PA, meeting it at D. The line DM extended is the normal and a line TT perpendicular to it and passing through M is the tangent.

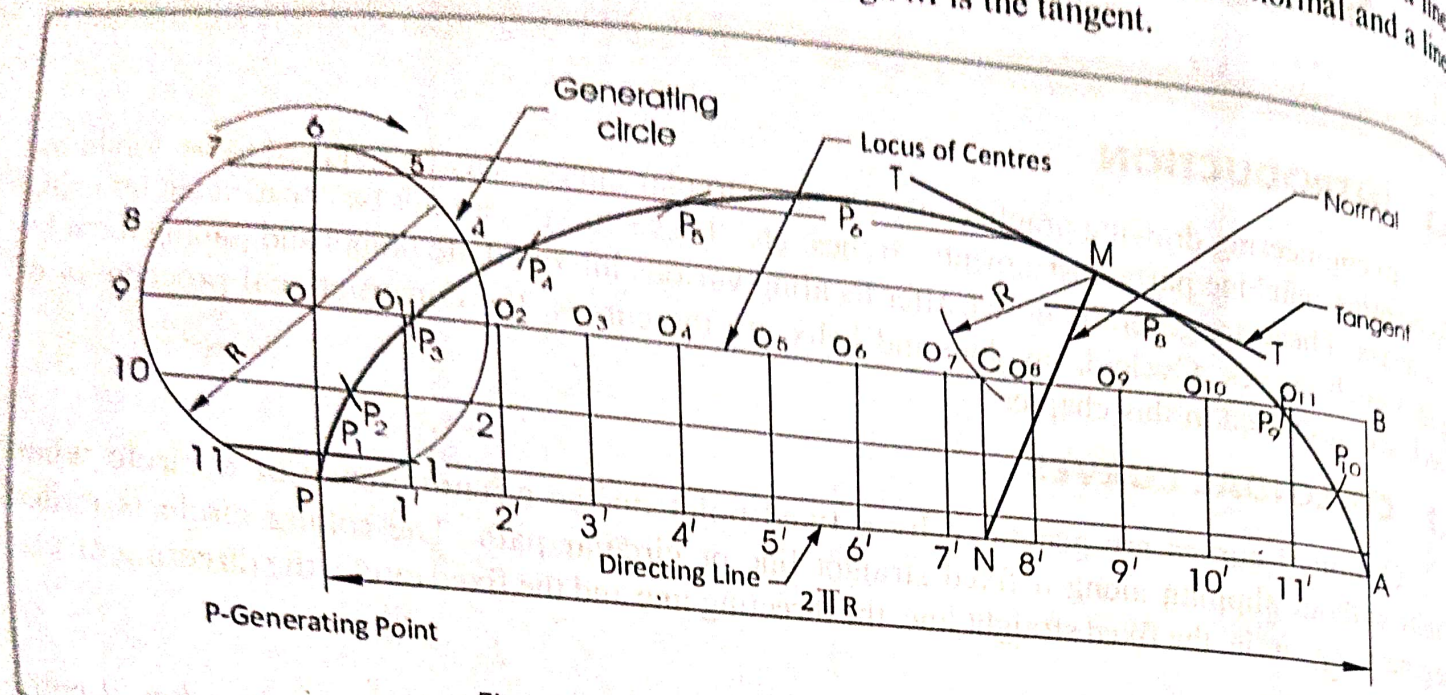


Fig. 8.1 CONSTRUCTION OF A CYCLOID

[2] Figure 8.1 represents the cycloid, generated by clockwise rotation of the generating circl

8.2.2 Epi-cycloid and hypo-cycloid

An epi-cycloid is a curve traced by a point on the circumference of a generating circle, wh it rolls without slipping on another circle (directing circle) outside it. If the generating circle ro inside the directing circle, the curve traced by the point is called hypo-cycloid.

8.2.2.1 To draw an epi-cycloid, given the radius r of the generating circle and t radius R of the directing circle

Construction (Fig. 8.2)

[1] With centre O^1 and radius R, draw a part of the directing circle.

[2] Draw the generating circle, by locating the centre O of it, on any radial line O extended such that O P = r.

[3] Assuming P to be the generating point, locate the point A on the directing circle su that the arc length PA is equal to the circumference of the generating circle. T angle subtended by the arc PA at O^1 is given by,

$$\theta = \angle PO^1 A = 360^\circ \times r/R$$

[4] With centre O^1 and radius O^1 O, draw an arc intersecting the line O^1 A produced at B. The arc OB is the locus of the centre of the generating circle.

[5] Divide the arc PA and the generating circle into the same number of equal parts and number the points.

[6] Join O^1, 1^1 ; O^1, 2^1; etc., and extend to meet the arc OB at O_1, O_2, etc.

[7] Through the points 1,2,3, etc., draw circular arcs with O^1 as centre.

[8] With centre O_1 and radius r, draw an arc intersecting the arc through 1 at P_1.

[9] Similarly, locate the points P_2, P_3, etc.

A smooth curve through the points P, P_1, P_2, etc., is the required epi-cycloid.

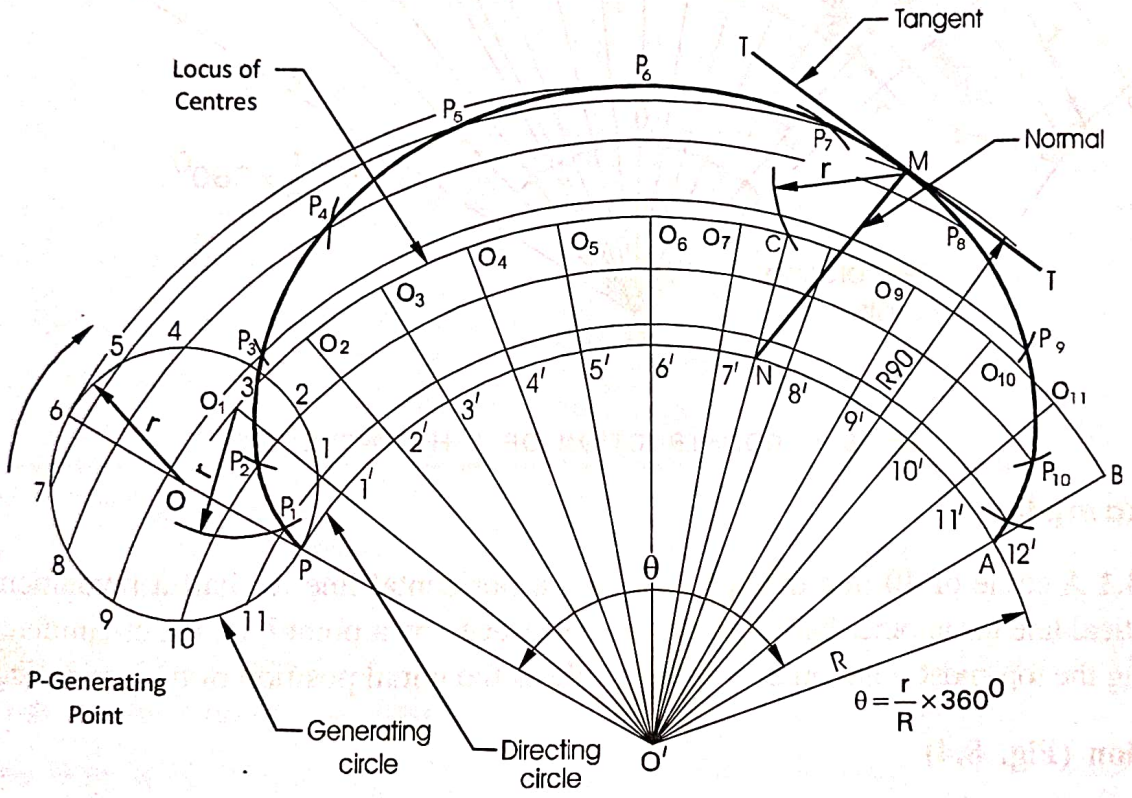


Fig. 8.2 CONSTRUCTION OF AN EPI-CYCLOID

Note : [1] To draw a tangent and normal to the epi-cycloid at a point, say M on it, with centre M and radius r, draw an arc intersecting the arc OB at C. Join C, O^1, intersecting the directing circle at D. The line DM extended, is the normal and a line TT, perpendicular to it and passing through M is the tangent.

[2] A procedure similar to the above may be followed to construct a hypo-cycloid (Fig. 8.3), knowing the fact that the generating circle rolls inside the directing circle.

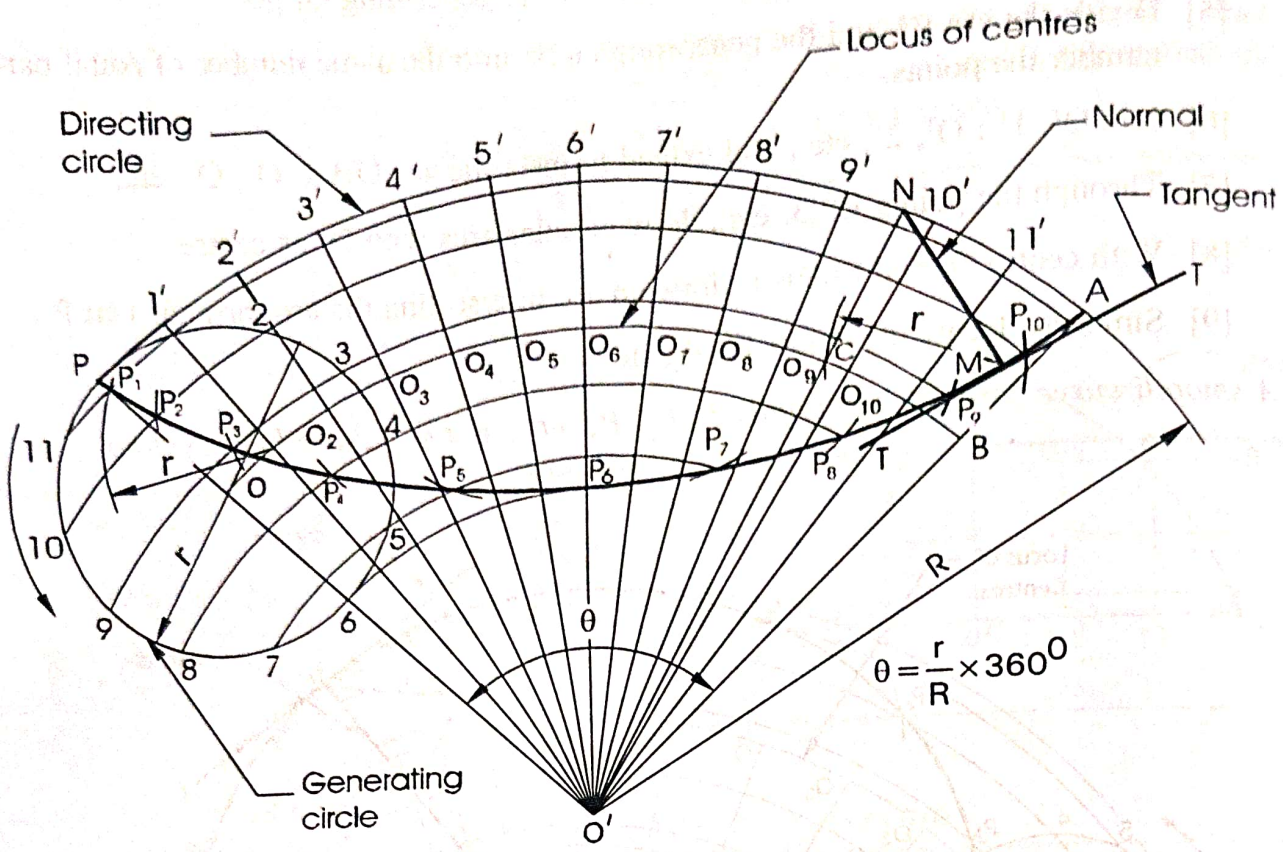


Fig. 8.3 CONSTRUCTION OF A HYPO-CYCLOID

8.2.3 Examples

8.2.3.1 A circle of 40 mm diameter rolls on a horizontal line for half a revolution and then along a vertical line for another half. Draw the curve traced by a point P on the circumference of the circle, taking the top-most point on the rolling circle as the initial position of the generating point.

Construction (Fig. 8.4)

[1] With centre O and diameter 40 mm, draw a circle.

[2] Divide the circle into a number of equal parts, say 12.

[3] Mark the division points on the circle and draw a tangent (directing line) passing through 6.

[4] Mark half the circumference along the tangent and divide it into 6 equal parts.

[5] Locate the initial position of the generating point P.

[6] Draw a smooth curve, representing the path of P, as the circle rolls along the directing line for half a revolution, following the Construction : Fig. 8.1.

[7] With O_6 as centre, draw the circle, representing the position of the rolling circle after half revolution.

[8] Draw a line, tangential to the circle and perpendicular to the above directing line. This becomes the directing line for the next half revolution of the generating circle.

[9] Trace the path of P for the next half revolution of the generating circle, as shown.

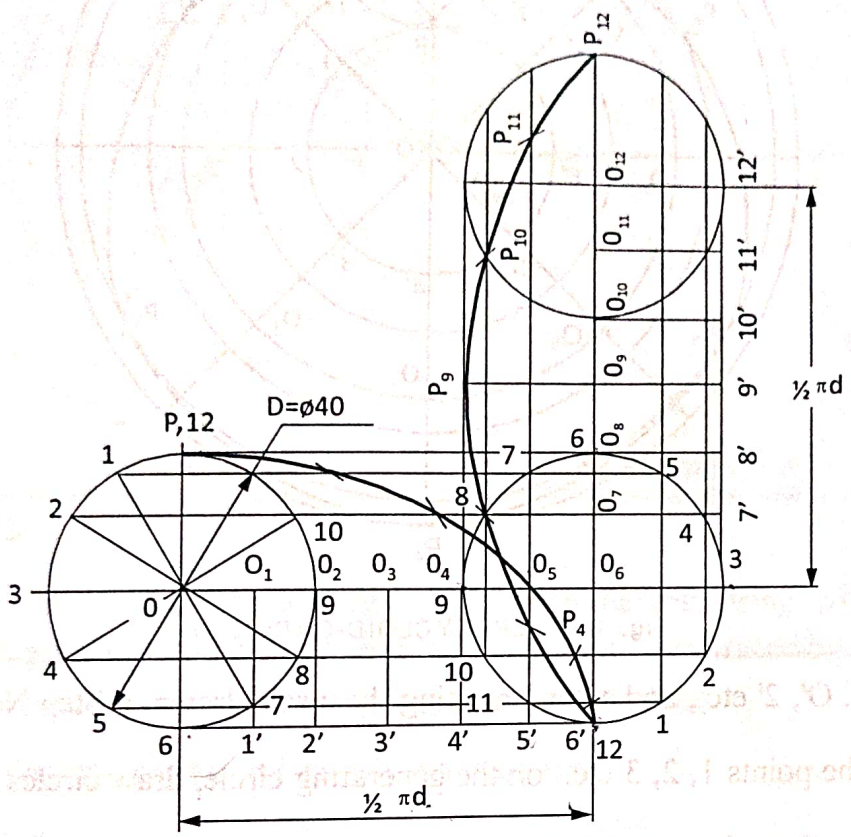


Fig. 8.4

8.2.3.2 Draw an epi-cycloid of a circle of 40 mm diameter, which rolls outside on another circle of 40 mm diameter for one revolution.

Construction (Fig. 8.5)

1. With O' as centre, draw a circle of 40 mm diameter, representing the directing circle.
2. Draw any radial line O' P and extend it.
3. Locate the point O on the above line such that OP = 20 mm, the radius of the generating circle.
4. With O as centre and radius 20 mm, draw the generating circle.
5. With O' as centre and radius O'O draw a circle, representing the locus of the centre of the generating circle.
6. Divide the generating circle and the above circle into the same number of equal parts and number as shown.

epi-cycloid traced is known as a cardiod.

8.2.3.3 A circle of 40 mm diameter rolls on the concave side of another circle of 40 mm radius. Draw the path traced by a point on the generating circle for one complete revolution.

Construction (Fig. 8.6)

1. Construct the hypo-cycloid, following the procedure under construction : Fig. 8.3.
2. It may be noted that when the diameter of the generating circle is equal to the radius of the directing circle, the hypo-cycloid traced is a straight line.

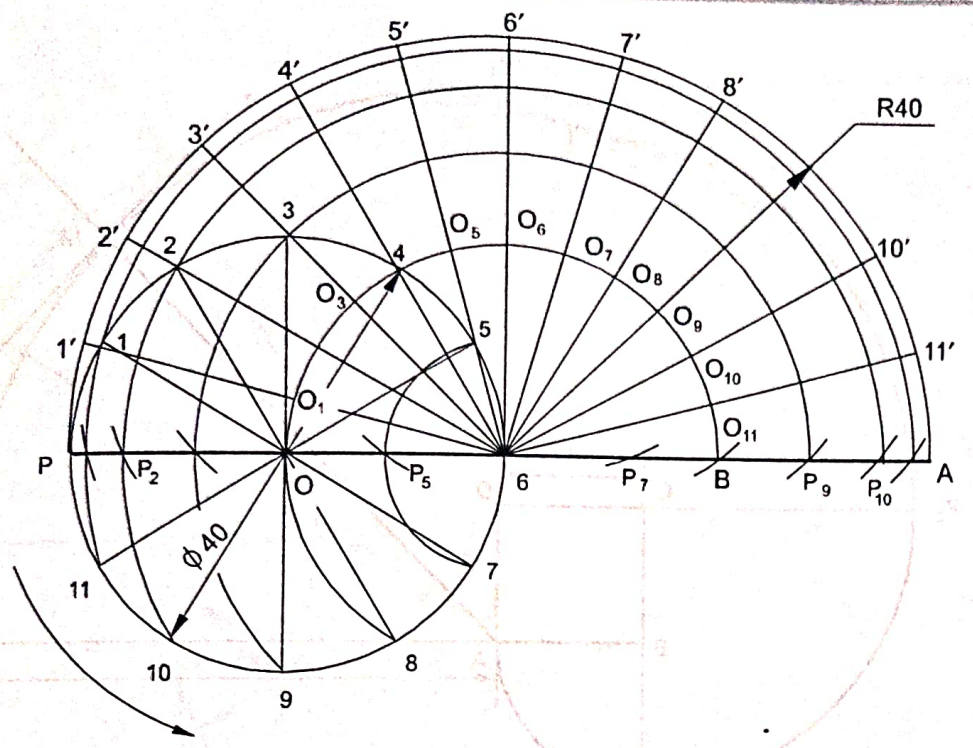


Fig. 8.6 HYPO-CYCLOID-STRAIGHT LINE

8.3 INVOLUTES

An involute is a curve traced by a point on a perfectly flexible string, while unwinding from around a circle or polygon; the string being kept tight. It is also a curve traced by a point on a straight line while the line is rolling around a circle or polygon without slipping.

8.3.1 To draw an involute of a given square

Construction (Fig.8.7)

[1] Draw the given square A B C D of side a.

[2] Assuming A as the starting point, with centre B and radius BA (= a), draw an arc to intersect the line CB produced at P_1.

[3] With centre C and radius CP_1 (= 2a), draw an arc to intersect the line DC produced at P_2.

[4] Similarly, locate the points P_3 and P_4.

The curve through A, P_1, P_2, P_3 and P_4 is the required involute.

Note : [1] The length A P_4 is equal to the perimeter of the square.

[2] To draw a normal and tangent to the curve at any point, say M on it, as M lies on the arc P_3 P_4, with its centre at A, the line AMN is the normal and the line TT drawn through M and perpendicular to M A is the tangent to the curve.

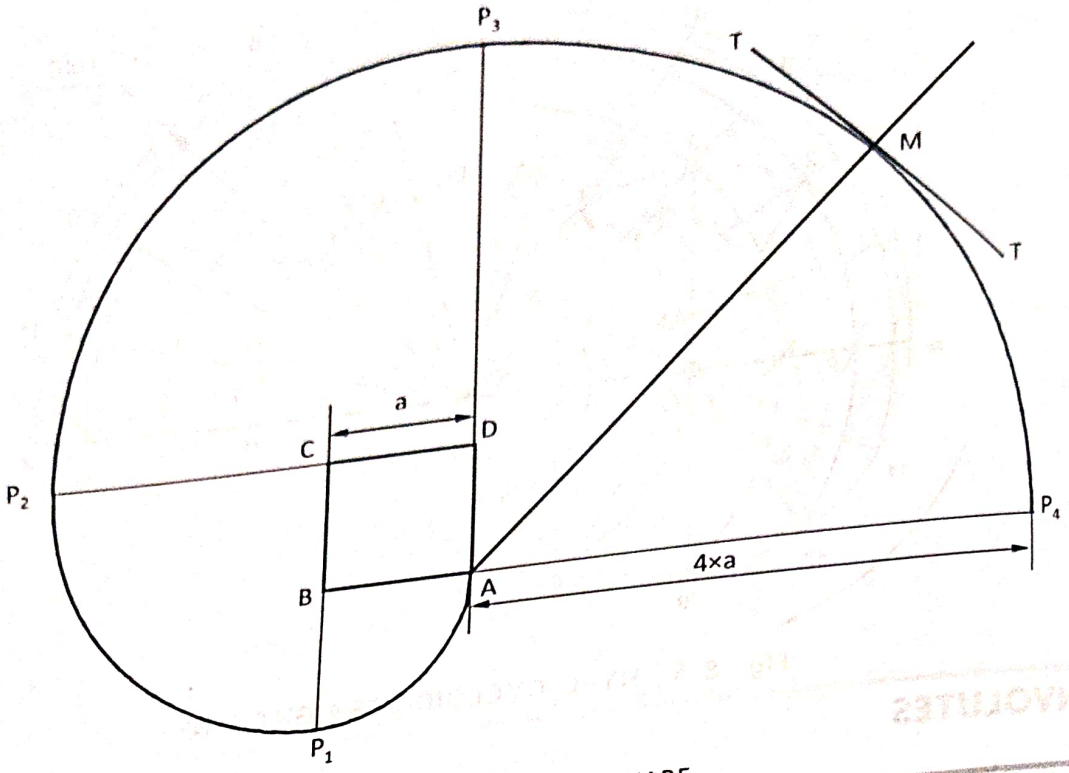


Fig. 8.7 INVOLUTE OF A SQUARE

8.3.2 To draw an involute of a given circle of radius R

[March/April 2017, 2018, Oct./Nov. 2

Construction (Fig. 8.8)

[1] With centre O and radius R, draw the given circle.

[2] Assuming P as the starting point, draw the tangent PA equal in length t circumference of the circle.

[3] Divide the line PA and the circle into the same number of equal parts and numb points.

[4] Draw tangents to the circle at the points 1, 2, 3, etc., and locate the points P_1, P etc., such that $1P_1 = P1^1$; $2P_2 = P2^1$; $3P_3 = P3^1$; etc.

A smooth curve through the points P, P_1, P_2, etc., is the required involute.

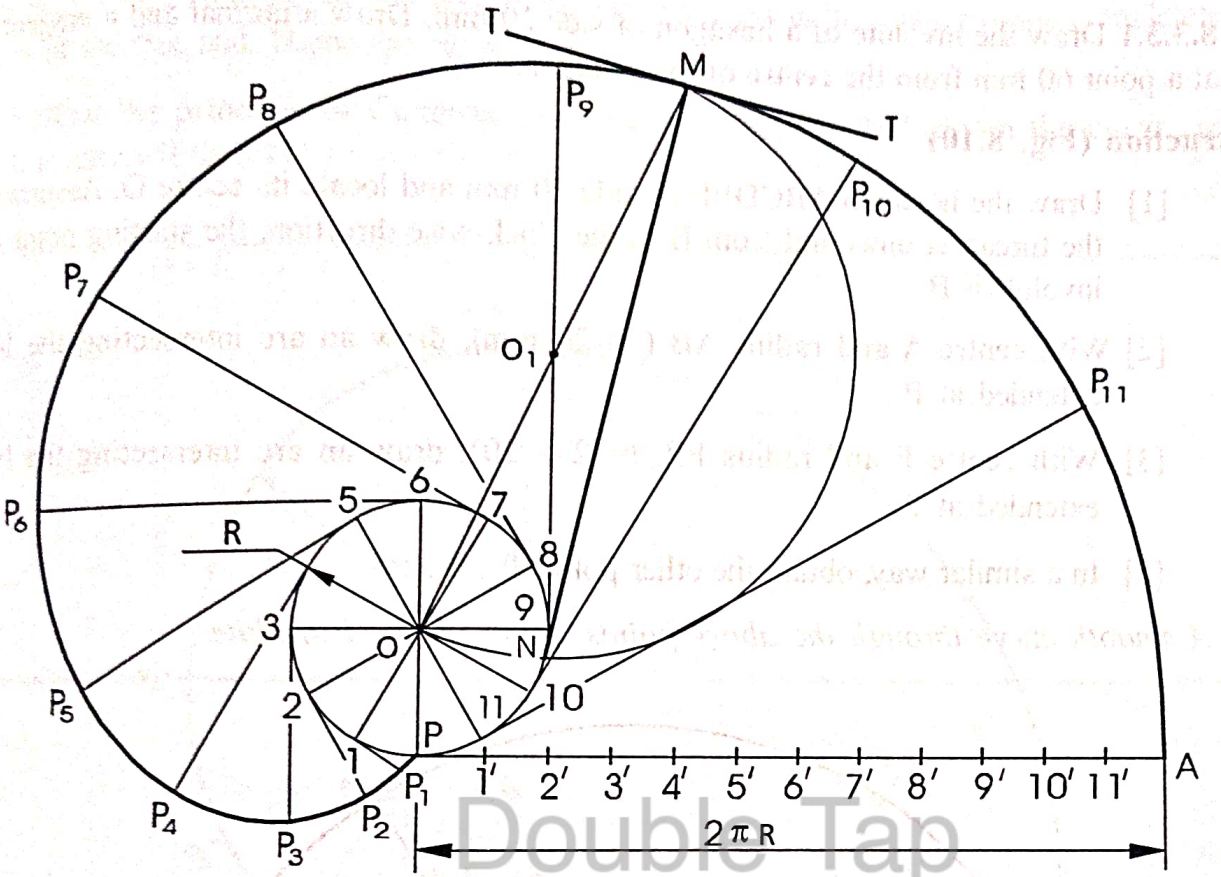


Fig. 8.8 INVOLUTE OF A CIRCLE

Note : [1] It is obvious from the above construction that a tangent to the circle is a normal to the involute. Hence, to draw a normal and tangent at a point, say M on it, first draw the tangent BMN to the circle. This is the normal to the curve and a line TT drawn through M and perpendicular to BM is the tangent to the curve.

[2] In involute system of gears, involute (of a circle) curve is used for gear tooth profile (Fig. 8.9).

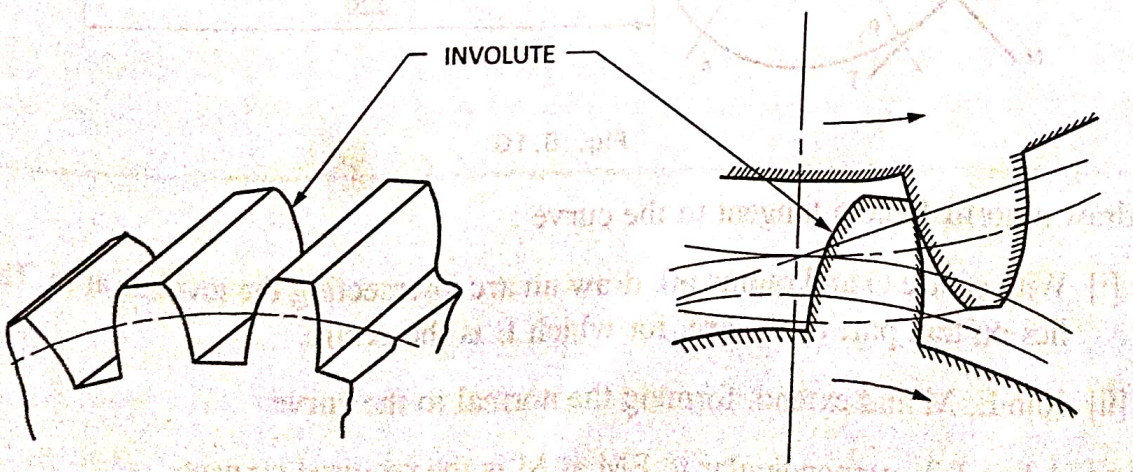


Fig. 8.9 INVOLUTE OF A CIRCLE AS GEAR TOOTH PROFILE

8.3.3 Examples

8.3.3.1 Draw the involute of a hexagon of side 20 mm. Draw a normal and a tangent to curve at a point 60 mm from the centre of the hexagon.

Construction (Fig. 8.10)

[1] Draw the hexagon ABCDEF of side 20 mm and locate its centre O. Assuming the thread is unwound from B in the clock-wise direction, the starting point for involute is B.

[2] With centre A and radius AB (= 20 mm), draw an arc intersecting the line extended at P_1.

[3] With centre F and radius FP_1 (= 2 × 20), draw an arc intersecting the line extended at P_2.

[4] In a similar way, obtain the other points P_3, P_4, etc.

A smooth curve through the above points is the required involute.

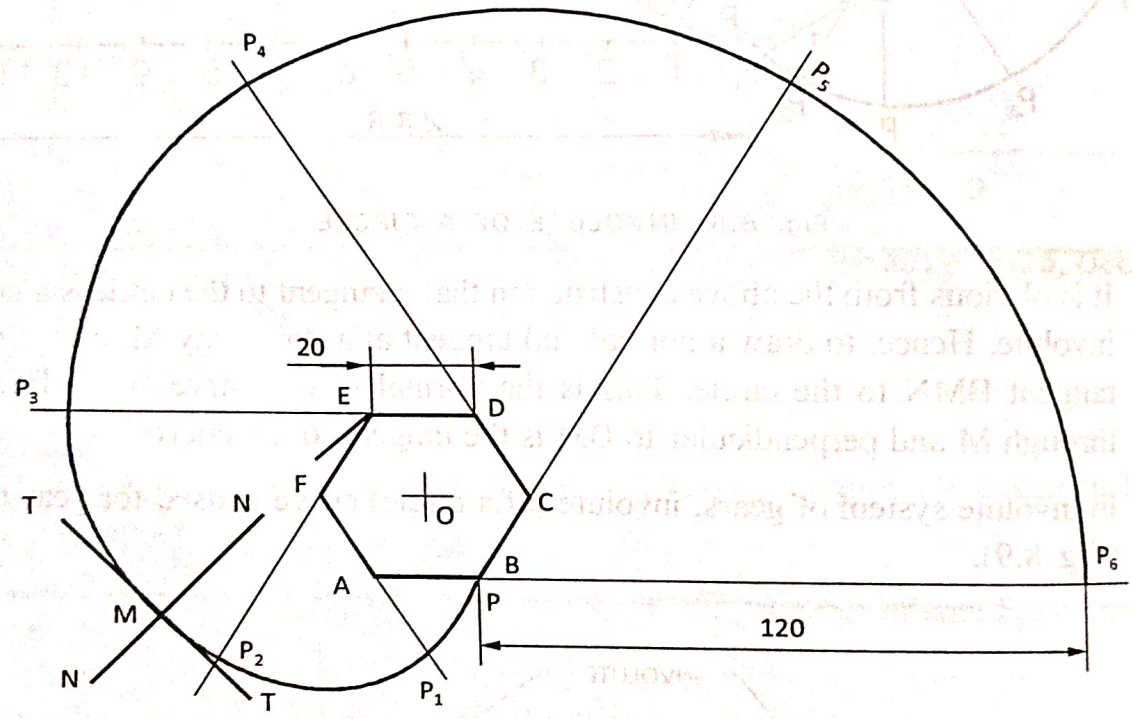


Fig. 8.10

To draw a normal and a tangent to the curve :

[i] With centre O and radius 60, draw an arc intersecting the involute at M. The po lies on that part of the arc, for which E is the centre.

[ii] Join E, M and extend, forming the normal to the curve.

[iii] A line T-T, perpendicular to EM at M is the required tangent.