

SIGNALS AND SYSTEMS

Course Learning Objectives

1. To understand the fundamental characteristics of signal and systems.
2. To understand signal and systems in terms of both the time and transform domains, taking advantage of the complementary insights and tools that these different perspective provide.
3. To develop mathematical skills to solve problems involving convolution, filtering, modulation and sampling.

Learning Resources

Text Books

1. Alan V Oppenheim, Alan V Willsky, S. Hamid Nawab, '*Signals and Systems*', 2nd edition, Pearson/PHI, 2015 ✓
2. B P Lathi, '*Principles of Signal Processing and Linear Systems*', 1st edition, Oxford University press, 2009 ✓

Reference Books

1. Simon Haykin, Van Veen, '*Signals & Systems*', 2nd Edition, Wiley Publications, 2007.
2. Mahamood Nahvi, '*Signals and Systems*', McGraw Hill Publishers, 1st edition, 2015.

Web Resources

1. Prof. Alan V. Oppenheim, Massachusetts Institute of Technology (MIT), '*Signals and System*'. URL: <https://ocw.mit.edu/resources/res-6-007-signals-and-systems-spring-2011/video-lectures/>
2. Prof. K S venkatesh, NPTEL-IIT Kanpur, '*Signals and Systems*'. URL: <http://nptel.ac.in/courses/117104074/>
3. Prof. V.G.K. Murti, NPTEL-IIT Madras, '*Networks and Systems*'. URL: <http://nptel.ac.in/courses/108106075/>

TOPICS COVERD UNDER THIS UNIT :

- INTRODUCTION OF SIGNALS AND SYSTEMS
- ELEMENTARY CONTINUOUS SIGNALS
- BASIC OPERATIONS ON SIGNALS
- CLASSIFICATIONS OF SIGNALS
- MODELING OF SYSTEMS
- CLASSIFICATION AND PROPERTIES OF SYSTEMS

SIGNAL DEFINATION:

A signal is a physical quantity that varies with time or space or any other independent variable or variables (or)

Anything which carries information is a signal

(or)

Signals is a time varying physical phenomenon which is intended to convey information

Ex:- Human voice ✓
 Temperatur ✓
 signals on telephone wires ✓

chirping of birds ✓
 smoke signals ✓
 fragrance of the flowers ✓

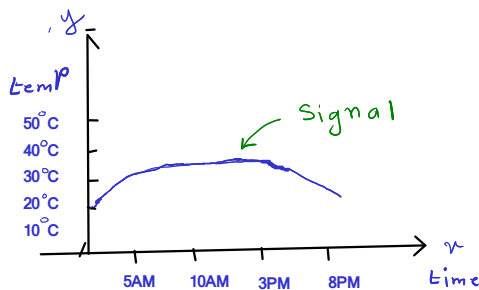


fig:Temp recorded over an interval of time

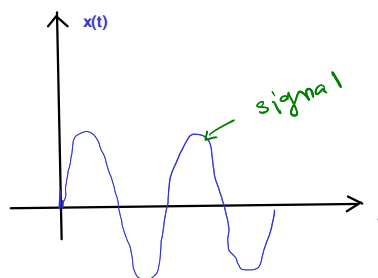


fig: wave form of AC power supply

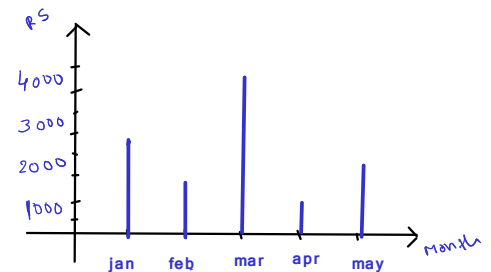


fig : money deposited in bank account every month

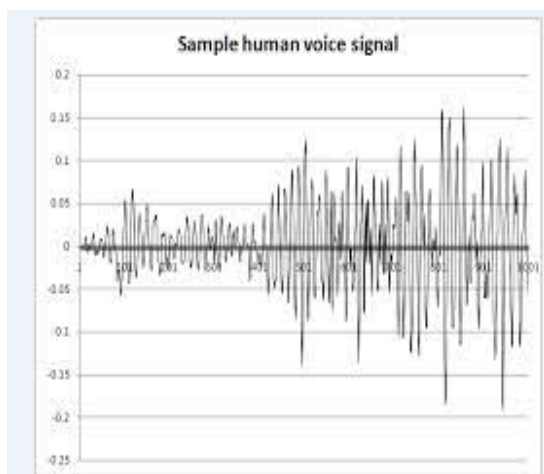


fig:Human voice signal

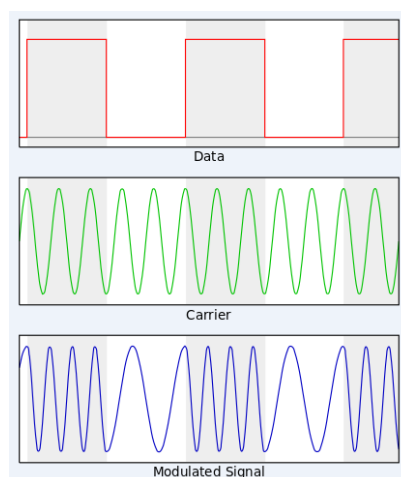


fig:signals on teleohone wires

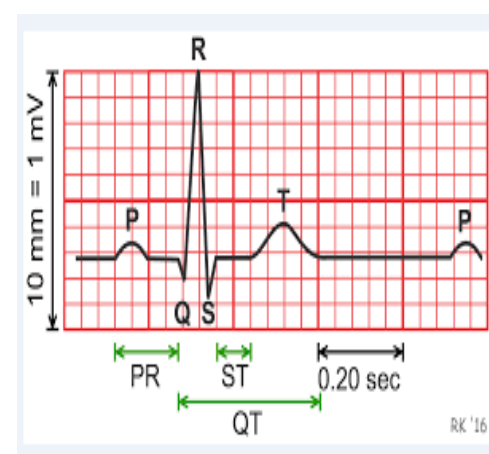
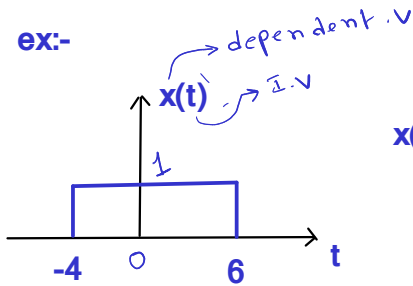


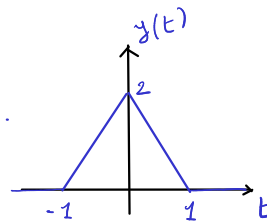
fig:ECG signal

signals are expressed mathematically as a function of independent variable/variables, which is usually time

ex:-



$$x(t) = \begin{cases} 1 & -4 \leq t \leq 6 \\ 0 & \text{otherwise} \end{cases}$$



$$y(t) = \begin{cases} 2 & -1 \leq t \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

if signal depends on only one independent variable then the signal is called as one dimensional signal

✓ $x(t)$, $y(t)$, $temp(t)$

if signal depends on two independent variables then— two dimensional signals Σ

image

system:-

system is an interconnection of different physical components which act together to perform certain task

(or)

A system is defined as a physical device that generates a response or output signal for a given input signal



** the relation b/w the i/p $x(t)$ & o/p $y(t)$ is
 $y(t) = \text{operation on } x(t)$



Examples:-

Motor (converts electrical energy into mechanical energy)

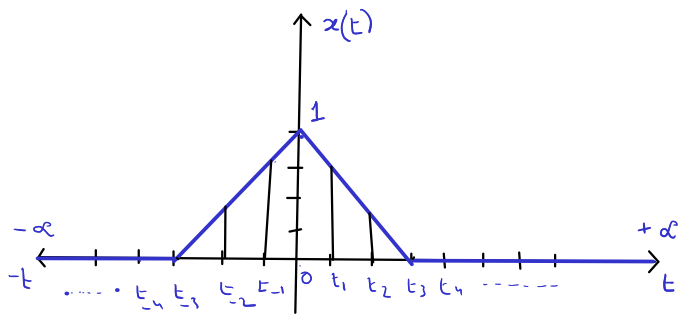
Amplifier (increases the strength of i/p signal)

Filter (rejects the unwanted frequencies present in i/p signal)

Basic classification of signal is:

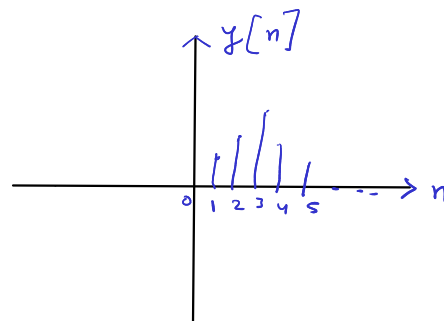
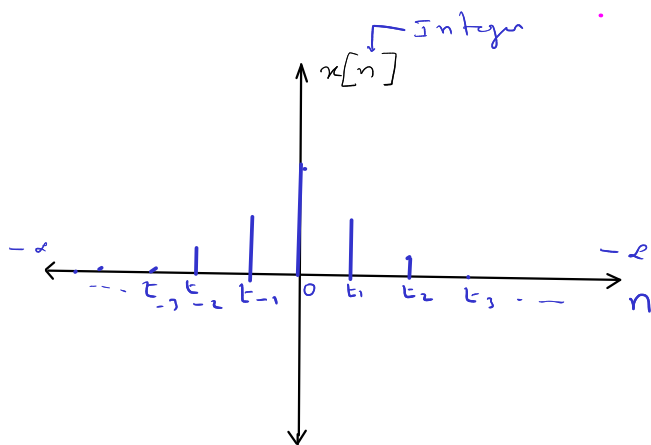
Continuous time signals: cts

Continuous time signals are defined for every value of time 't' and is represented by $x(t)$



Discrete time signals:

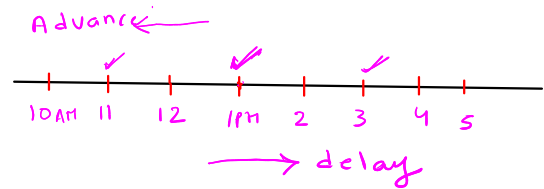
The discrete time signals are defined at discrete instant of time and is represented by $x[n]$



Basic operations on signals

1. shifting ✓
2. scaling ✓
3. reversal ✓

Time shifting:

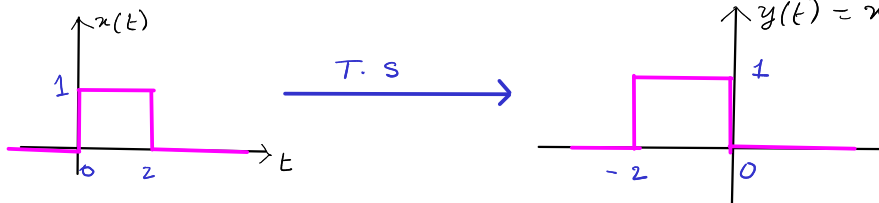


$$x(t) \xrightarrow{T.S} y(t) = x(t + K)$$

$+$ → sig. Adv
 $-$ → sig. delay

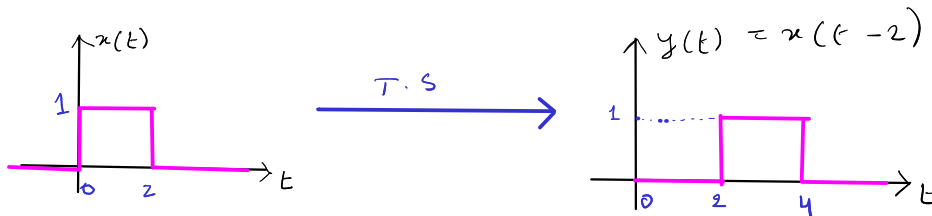
1. Shifting $\begin{cases} \text{Time shifting} \\ \text{Amplitude shifting} \end{cases}$

case (i): when $K > 0$, let $K = 2$



∴ left shifting

case (ii): when $K < 0$, let $K = -2$



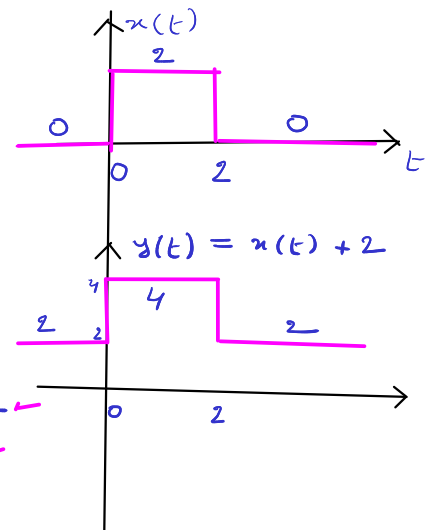
∴ Right shifting

Amplitude shifting:

$$x(t) \xrightarrow{A.S} y(t) = x(t) + K$$

case (i): $K > 0$, let $K = 2$ ∴ upward shifting

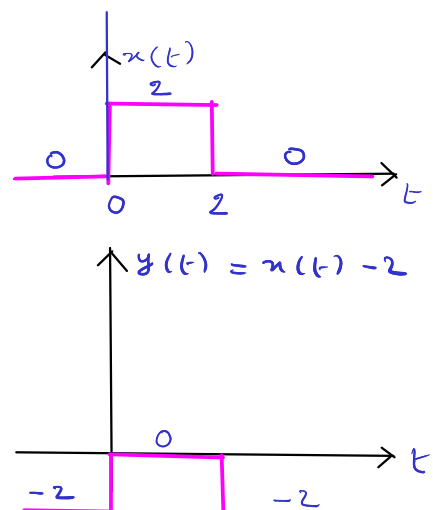
$$x(t) = \begin{cases} 0 & , t < 0 \\ 2 & , 0 \leq t \leq 2 \\ 0 & , t > 2 \end{cases} \quad y(t) = \begin{cases} 0+2 & , t < 0 \\ 2+2 & , 0 \leq t \leq 2 \\ 0+2 & , t > 2 \end{cases}$$



case (ii): $K < 0$, let $K = -2$

$$x(t) \xrightarrow{A.S} y(t) = x(t) - 2$$

∴ downward shifting



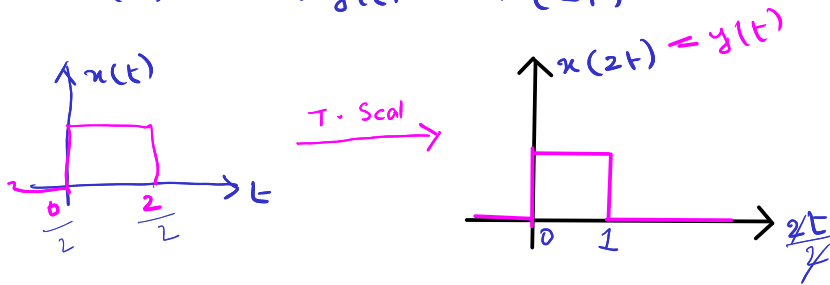
Scaling:

- Time scaling
- Amplitude scaling

$$x(t) \xrightarrow{T \cdot \text{scal}} y(t) = x(at), a \neq 0$$

case (i): when $a > 1$, let $a = 2$

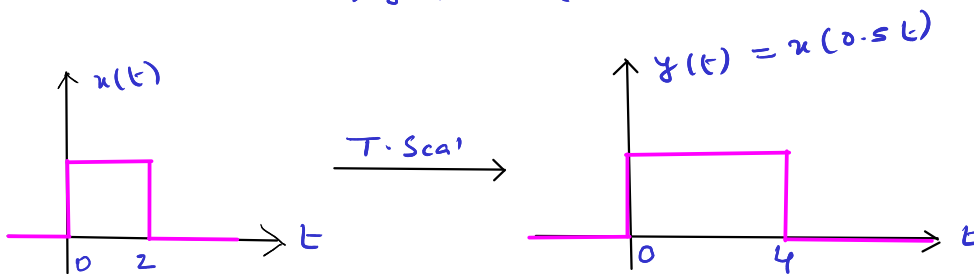
$$x(t) \longrightarrow y(t) = x(2t)$$



$a > 1$
Sig. compressed

case (ii): when $0 < a < 1$, let $a = 0.5$

$$x(t) \longrightarrow y(t) = x(0.5t)$$

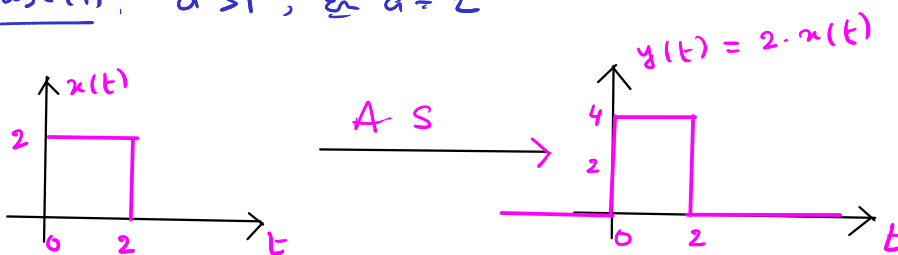


$a < 1$
Sig. Expansion

Amplitude scaling:

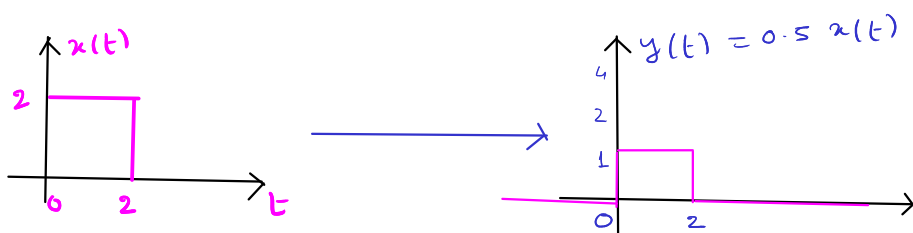
$$x(t) \xrightarrow{A \cdot s} y(t) = a \cdot x(t)$$

case (i): $a > 1$, let $a = 2$

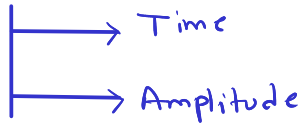


$\therefore a > 1$

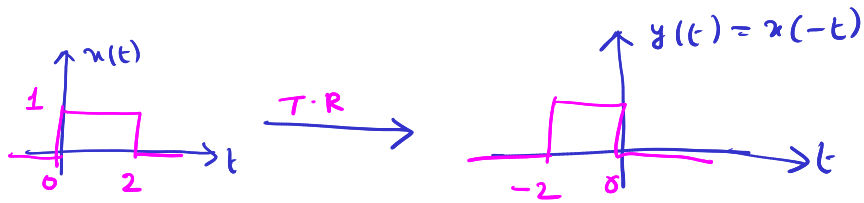
case (ii): $0 < a < 1$, let $a = 0.5$



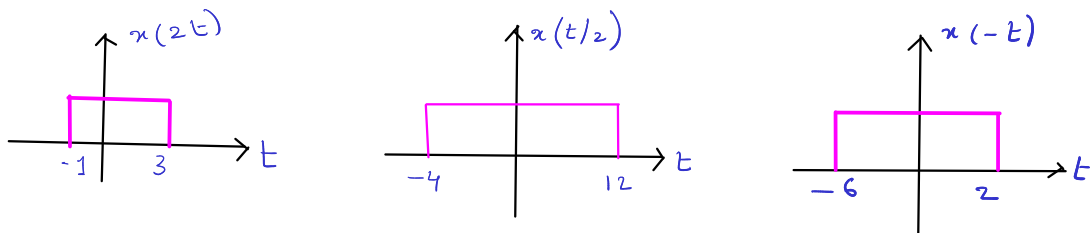
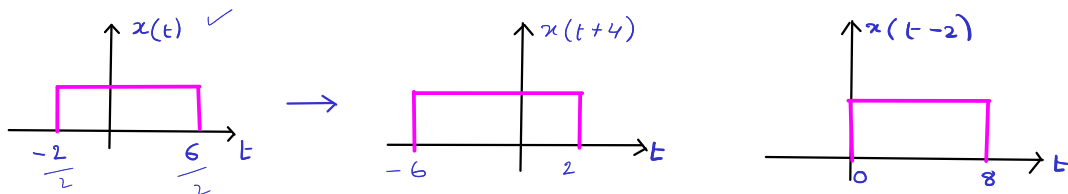
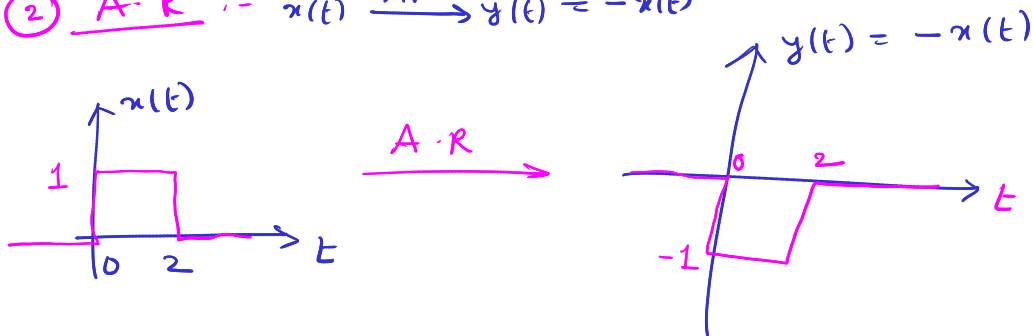
Reversal



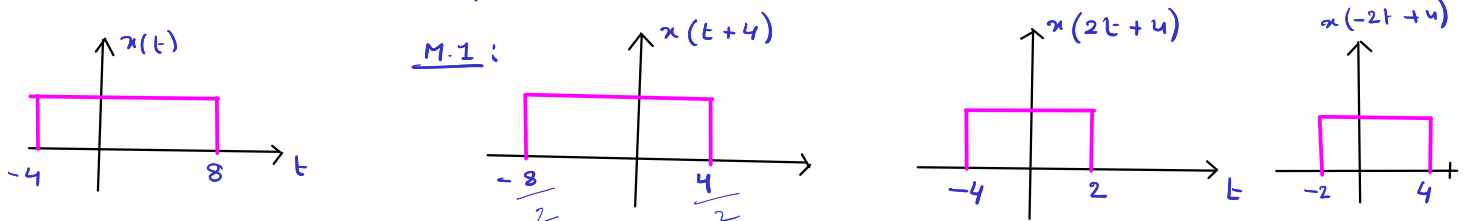
① T.R : $x(t) \xrightarrow{T.R} y(t) = x(-t)$



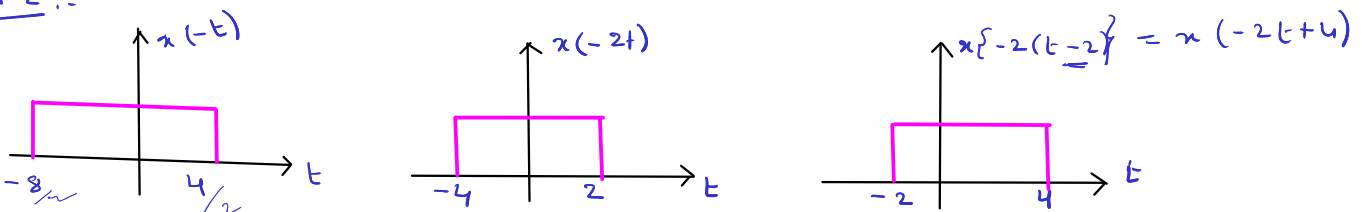
② A.R : $x(t) \xrightarrow{A.R} y(t) = -x(t)$



Q] For given $x(t)$ find $x(-2t+4)$



M.2 :-

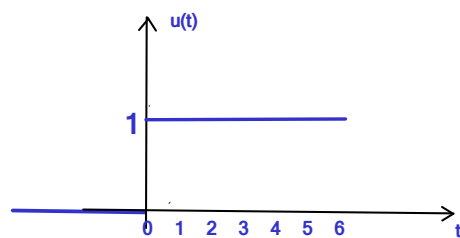


1. unit step function
2. unit ramp function
3. unit impulse function
4. unit parabolic function
5. rectangular pulse function
6. triangular pulse function
7. signum function
8. sinc function
9. gaussian function
10. sinusoidal signal
11. exponential function

1. unit step function

unit step function is denoted by $\underline{u(t)}$ and defined as

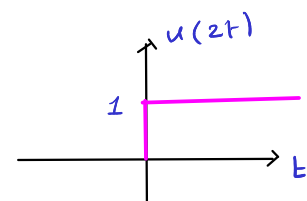
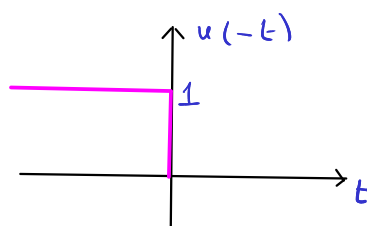
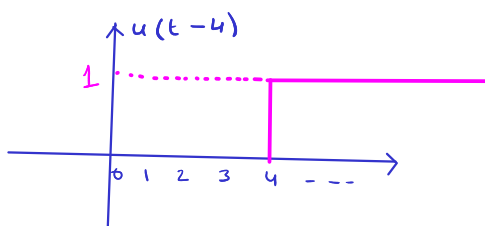
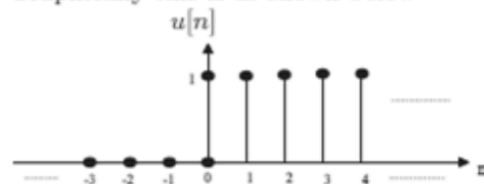
$$u(t) = \begin{cases} 1 & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$



(b) Unit step sequence $u[n]$: Unit step sequence is defined by

$$\underline{u[n]} = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

Graphically this is as shown below



2. unit ramp function

The unit ramp function is denoted by $r(t)$ and defined as

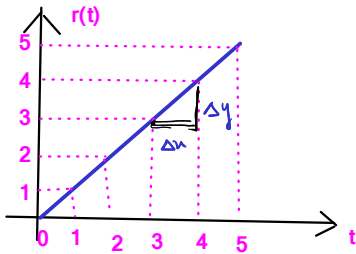
$$r(t) = \begin{cases} t & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

$$r(t) = t \cdot u(t)$$

$$r(t) = \int u(t) dt = \int dt = t \quad \therefore t \geq 0$$

$$u(t) = \frac{d}{dt} r(t)$$

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{4-3}{4-3} = 1$$

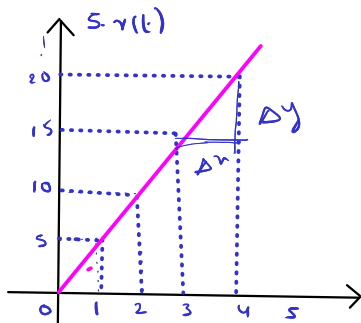


$$r(t) = t$$

$$A \cdot r(t) = A \cdot t$$

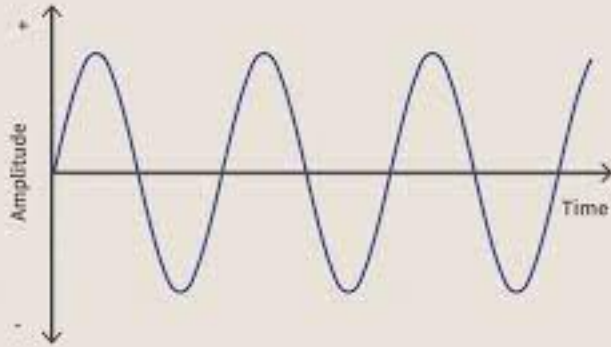
↑ slope $(A=1)$

$$5 \cdot r(t) =$$

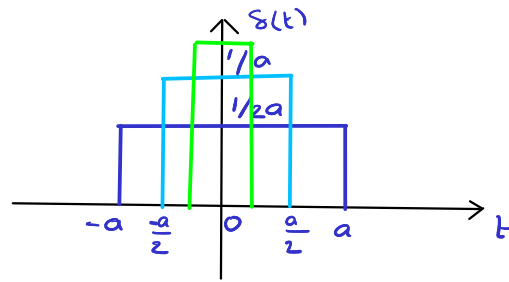


$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{20-15}{4-3} = \frac{5}{1} = 5$$

Sine Wave



$$\text{let } y(t) = \begin{cases} \frac{1}{2a} & , -a < t < a \\ 0 & , \text{otherwise} \end{cases}$$



$$A = \omega \times h \\ = 2a \times \frac{1}{2a} \\ = 1$$

$$A = \omega \times \frac{1}{\omega} \\ A = 1$$

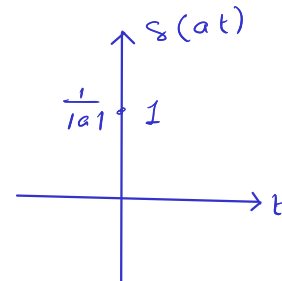
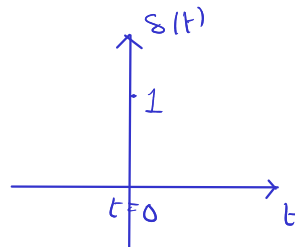
$$A = \int_{-\infty}^{\infty} s(t) dt = 1$$

properties:

time scaling :-

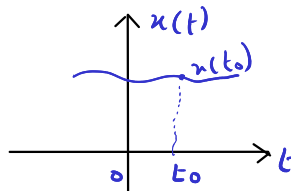
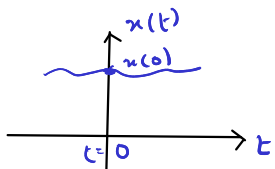
$$s(at) = \frac{1}{|a|} s(t)$$

$$s\{a(t \pm k)\} = \frac{1}{|a|} s(t \pm k)$$



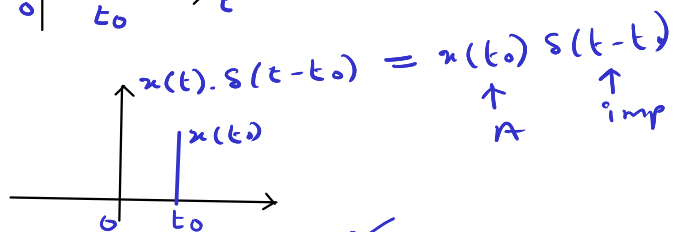
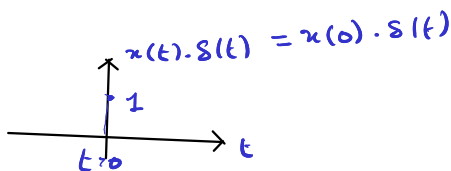
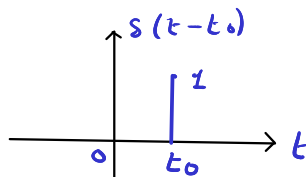
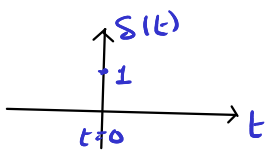
$$\text{Ex ① } s(-2t) = \frac{1}{|-2|} s(t) \Rightarrow \frac{1}{2} s(t)$$

multiplication:



$$\text{Ex ② } s(-2t+3)$$

$$s(-2(t-1.5)) = \frac{1}{|-2|} s(t-1.5) \\ = \frac{1}{2} s(t-1.5)$$



$$x(t) \cdot s(t) = x(0) \cdot s(t)$$

$$x(t) \cdot s(t-t_0) = x(t_0) \cdot s(t-t_0)$$

$$\text{Ex ① } y(t) = \underbrace{2t^2}_{x(t)} \cdot s(t-3) \\ = 2(3)^2 \cdot s(t-3) \\ = 18 \cdot s(t-3) \\ \uparrow A/\omega$$

$$\textcircled{3} \quad \int_{-\infty}^{\infty} x(t) \cdot \delta(t) dt = \int_{-\infty}^{\infty} x(0) \delta(t) dt$$

$$= x(0) \int_{-\infty}^{\infty} \delta(t) dt$$

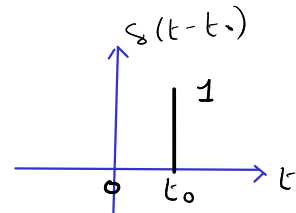
$$\therefore \int_{-\infty}^{\infty} \delta(t) dt = 1$$

$$\boxed{\int_{-\infty}^{\infty} x(t) \cdot \delta(t) dt = x(0)}$$

$$\int_{-\infty}^{\infty} x(t) \delta(t-t_0) dt = \int_{-\infty}^{\infty} x(t_0) \delta(t-t_0) dt$$

$$= x(t_0) \int_{-\infty}^{\infty} \delta(t-t_0) dt$$

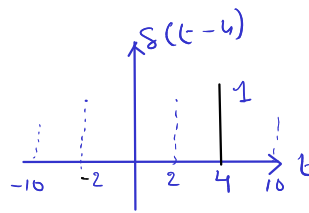
$$\boxed{\int_{-\infty}^{\infty} x(t) \cdot \delta(t-t_0) dt = x(t_0)}$$



Ex ①: $I = \int_{-2}^2 2t^2 \cdot \delta(t-4) dt = ?$

$$= 2(4)^2 \int_{-2}^2 \delta(t-4) dt$$

$$= 0$$



$\textcircled{2'}$

$$\int_{-10}^{10} 2t^2 \cdot \delta(t-4) dt$$

$$= \underline{2(4)^2} \int_{-10}^{10} \delta(t-4) dt$$

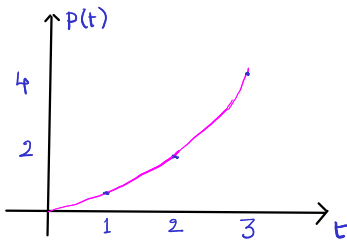
$$= 32 \cdot (1)$$

$$= 32$$

4. unit parabolic function

The unit parabolic function is given by

$$p(t) = \frac{t^2}{2} \text{ for } t \geq 0 \\ = 0 \text{ for } t < 0$$



$$p(t) = \frac{t^2}{2} u(t)$$

The unit parabolic function can be obtained by integrating the ramp function

$$P(t) = \int r(t) dt \Rightarrow \int t dt \Rightarrow \frac{t^2}{2} \quad (t > 0)$$

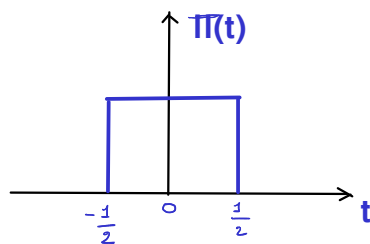
the ramp function is derivative of the parabolic fun.

$$r(t) = \frac{d}{dt} p(t)$$

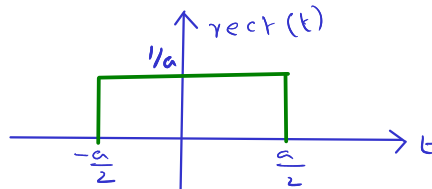
5. rectangular pulse function / Gate fun.

The rectangular pulse function is denoted by $\Pi(t)$ and defined as

$$\Pi(t) = \begin{cases} 1 & \text{for } |t| < 1/2 \\ 0 & \text{otherwise} \end{cases}$$



$$\text{rect}(t) = \begin{cases} 0 & t < -\frac{a}{2} \\ 1/a & -\frac{a}{2} \leq t \leq \frac{a}{2} \\ 0 & t > a/2 \end{cases}$$

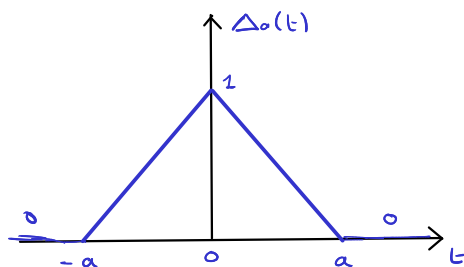


$$A = \cancel{a} \times \frac{1}{\cancel{a}} = 1$$

6. Triangular pulse function

The triangular pulse function is defined as

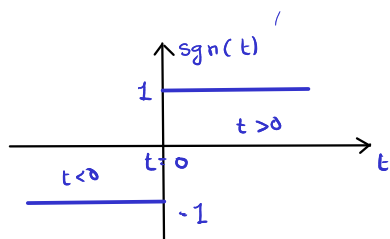
$$\Delta_a(t) = \begin{cases} 1 - \frac{|t|}{a} & \text{for } |t| \leq a \\ 0 & \text{for } |t| > a \end{cases} \quad \text{Trian}(t)$$



7. Signum function

The unit signum function is defined as

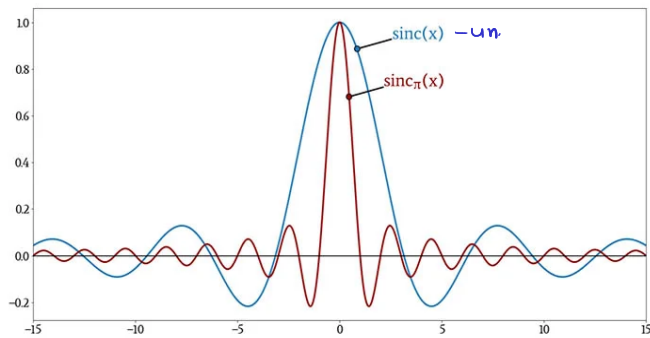
$$\text{sgn}(t) = \begin{cases} 1 & t > 0 \\ 0 & t = 0 \\ -1 & t < 0 \end{cases}$$



8. Sinc function

The sinc function is defined by the expression

- 1. unnormalized sinc ✓
- 2. normalized sinc ✓



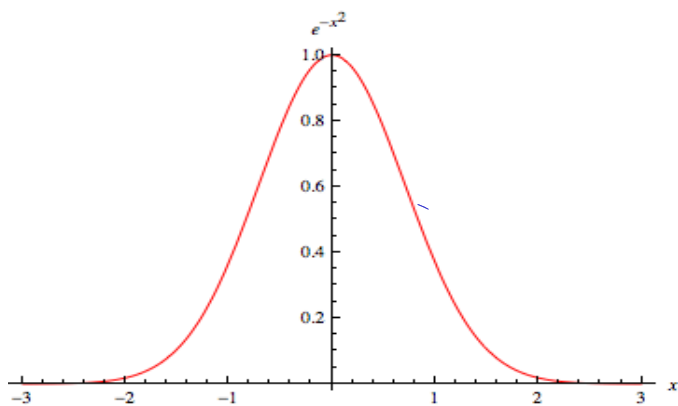
$$\textcircled{1} \quad \text{sinc}(t) = \begin{cases} 1, & t = 0 \\ \frac{\sin t}{t}, & t \neq 0 \end{cases}$$

$$\textcircled{2} \quad \text{sinc}_\pi(t) = \begin{cases} 1, & t = 0 \\ \frac{\sin(\pi t)}{\pi t}, & t \neq 0 \end{cases}$$

9. Gaussian function

The Gaussian function is defined by the expression

$$g_a(t) = e^{-at^2}, \quad -\infty < t < \infty$$



10. Sinusoidal signal

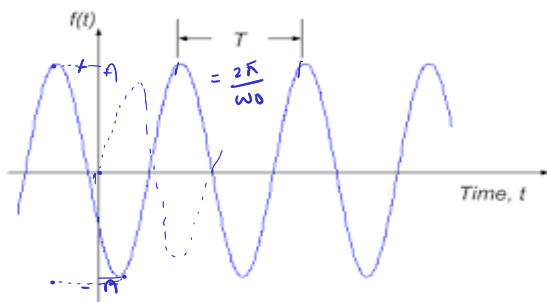
A continuous time sinusoidal signal is given by

$$x(t) = A \sin(\omega_0 t + \phi)$$

amplitude
phase angle in radians

freq in r/s

$$x(t) = A \cos(\omega_0 t + \phi)$$

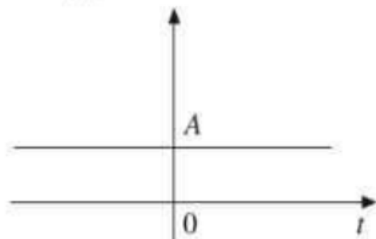


11. Real Exponential signal

A real exponential signal is defined as

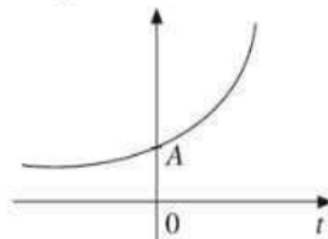
$$\underline{x(t) = Ae^{\alpha t}} \quad \text{where } A, \alpha \rightarrow \text{real}$$

$$x(t) = Ae^{\alpha t} \text{ for } \alpha = 0 \quad \checkmark$$



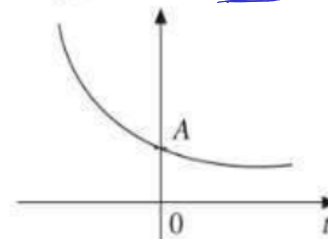
(a)
Dc signal

$$x(t) = Ae^{\alpha t} \text{ for } \alpha > 0$$



(b)
Exponentially growing

$$x(t) = Ae^{\alpha t} \text{ for } \underline{\alpha < 0}$$



(c)
Exponentially decaying

Complex exponential signal

The general form of complex exponential is given by

$$x(t) = e^{st} \rightarrow \textcircled{1}$$

Euler

where s is a complex variable defined as

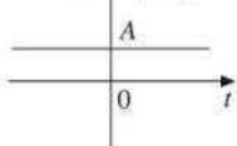
$$s = \sigma + j\omega_0$$

$$e^{jn} = \cos n + j \sin n$$

$$\begin{aligned} x(t) &= e^{(\sigma + j\omega_0)t} \\ &= e^{\sigma t} \cdot e^{j\omega_0 t} \end{aligned}$$

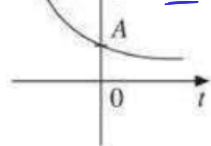
$$x(t) = e^{\sigma t} \cdot [\cos \omega_0 t + j \sin \omega_0 t]$$

$$x(t) \quad s = 0 \quad \checkmark$$



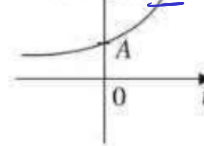
(a)

$$x(t) \quad s = -\sigma$$



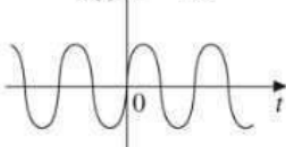
(b)

$$x(t) \quad s = \sigma$$



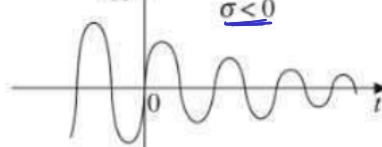
(c)

$$x(t) \quad s = +j\omega$$



(d)

$$x(t) \quad s = \sigma \pm j\omega, \quad \underline{\sigma < 0}$$



(e)

$$x(t) \quad s = \sigma \pm j\omega, \quad \underline{\sigma > 0}$$



(f)

Ex ① Evaluate the following

$$\int_{-\infty}^{\infty} e^{-at} \delta(t-s) dt = ?$$

$\uparrow$ $x(t)$ $\uparrow$ $s(t-s)$

$$\left[e^{-at} \right]_{t=s} \Rightarrow e^{-as} \checkmark$$

$$\int_{-\infty}^{\infty} x(t) \delta(t-t_0) dt = x(t_0) \checkmark$$

$$\delta(t-s) = \begin{cases} 1 \checkmark, & t=s \\ 0, & \text{other} \end{cases}$$

② $\int_0^{\infty} t^2 \delta(t-6) dt = ?$

$$\delta(t-6) = \begin{cases} 1 & \text{for } t=6 \\ 0 & \text{—} \end{cases}$$

$$t^2 \int_0^{\infty} \delta(t-6) dt \Rightarrow \left[t^2 \right]_{t=6} \Rightarrow 36$$

③ $\int_0^3 \delta(t) \sin 5\pi t dt$

$\uparrow$ $x(t)$

$$\delta(t) = \begin{cases} 1 & t=0 \\ 0 & t \neq 0 \end{cases}$$

$$\left[\sin 5\pi t \right]_{t=0} \Rightarrow \left[\sin 5\pi(0) \right] \Rightarrow 0$$

④ $\int_{-\infty}^{\infty} \delta(t+2) e^{-2t} dt$

$\uparrow$ $x(t)$

$$\Rightarrow \left[e^{-2t} \right]_{t=-2} \Rightarrow e^4$$

⑤ $\int_{-\infty}^{\infty} (t-2)^3 \delta(t-2) dt$

$\uparrow$ $x(t)$ $\uparrow$ $\delta(t-2)$

$$\Rightarrow (t-2)^3 \Big|_{t=2} \Rightarrow 0$$

⑥ $\int_{-\infty}^{\infty} \delta(t) e^{-j\omega t} dt$

$$\Rightarrow e^{-j\omega t} \Big|_{t=0} \Rightarrow e^0 \Rightarrow 1$$

⑦ $\int_{-\infty}^{\infty} [\delta(t) \cos 2t + \delta(t-2) \sin 2t] dt$

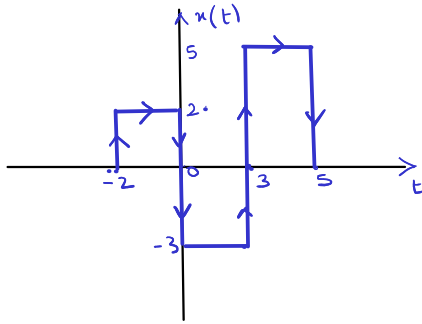
$$= \left[\cos 2t \right]_{t=0} + \left[\sin 2t \right]_{t=2}$$

$$= \left[\cos 2(0) \right] + \left[\sin 2(2) \right]$$

$$= 1 + \sin 4 \checkmark$$

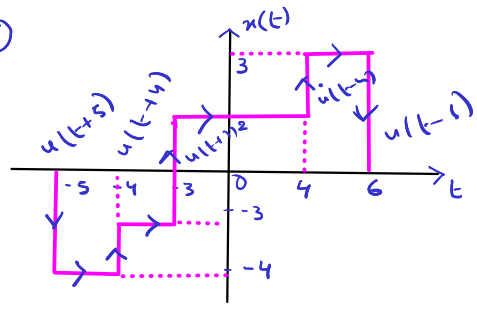
Represent the following signal $x(t)$ using suitable shifted $u(t)$ signal

①



$$x(t) = 2u(t+2) - 5u(t) + 8u(t-3) - 5u(t-5)$$

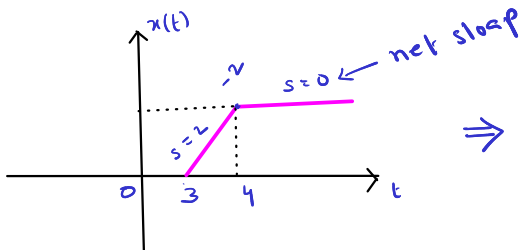
②



$$x(t) = -4u(t+5) + 1u(t+3) + 5u(t+3) + 1u(t-4) - 3u(t-6)$$

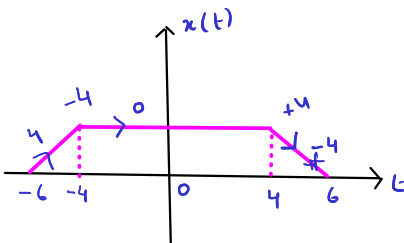
Represent the following signal $x(t)$ using suitable shifted $r(t)$ signal

①



$$\Rightarrow x(t) = 2r(t-3) - 2r(t-4)$$

②



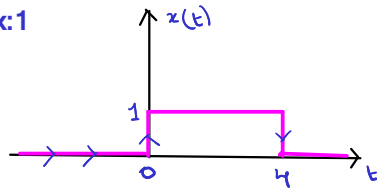
$$x(t) = 4r(t+6) - 4r(t+4) - 4r(t-4) + 4r(t-6)$$

Mathematical representation of a signal waveforms

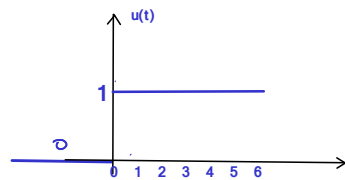
waveform is having no discontinue $\rightarrow$ only ramp

waveform is having discontinue plus constant $\rightarrow$ step

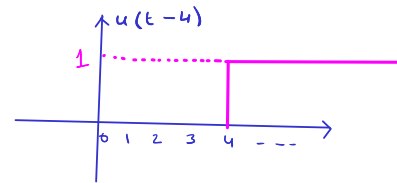
ex:1



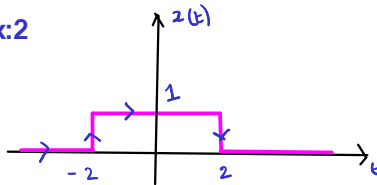
$$x(t) = 1u(t) - 1u(t-4)$$



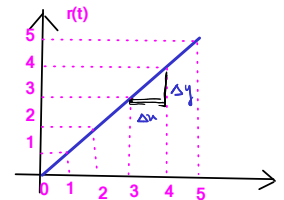
$$u(t) = \begin{cases} 1 & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$



ex:2

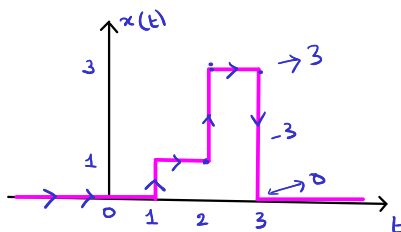


$$x(t) = 1u(t+2) - 1u(t-2)$$



$$r(t) = \begin{cases} t & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

ex:3

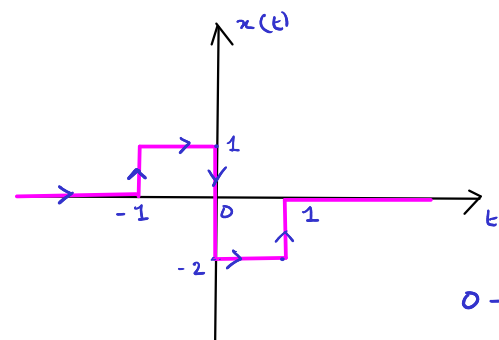


$$x(t) = 1u(t-1) + 2u(t-2) - 3u(t-3) \checkmark$$

$$t = 2.5$$

$$\begin{aligned} x(2.5) &= u(1.5) + 2u(0.5) - 3u(-0.5) \\ &= 1 + 2(1) - \underline{3(0)} \\ &= 3 \end{aligned}$$

ex:4



$$0 - (-2) + 2$$

$$x(t) = 1u(t+1) - 3u(t) + 2u(t-1)$$

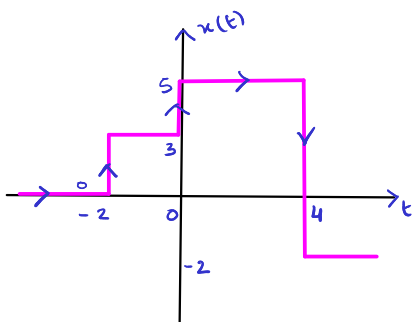
$$1 - p.n$$

$$-2 - p.n$$

$$p.n - p.n$$

$$-2 - 1 = \underline{\underline{-3}}$$

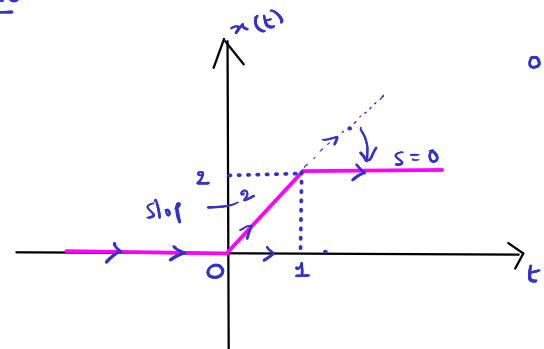
ex:5



$$\begin{aligned} -2 - 5 \\ = -7 \end{aligned}$$

$$x(t) = 3u(t+2) + 2u(t) - 7u(t-4)$$

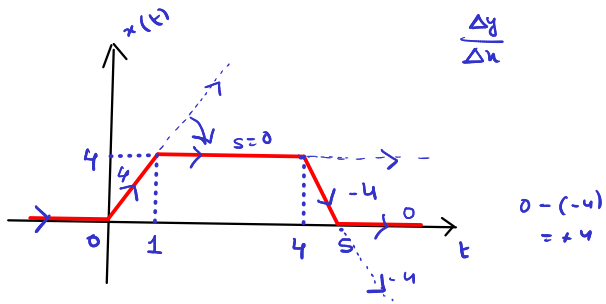
ex:6



$$\begin{aligned} 0 - 2 \\ = -2 \end{aligned}$$

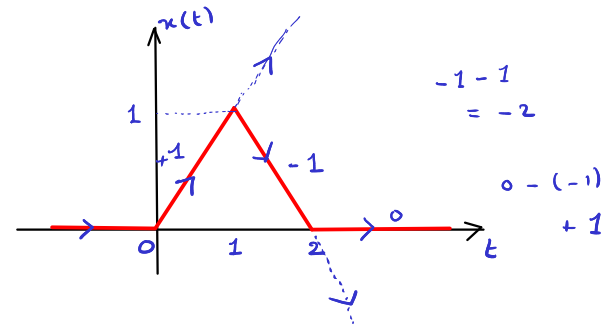
$$x(t) = 2r(t) - 2r(t-1)$$

ex:7



$$x(t) = 4r(t) - 4r(t-1) - 4r(t-4) + 4r(t-5)$$

ex:8

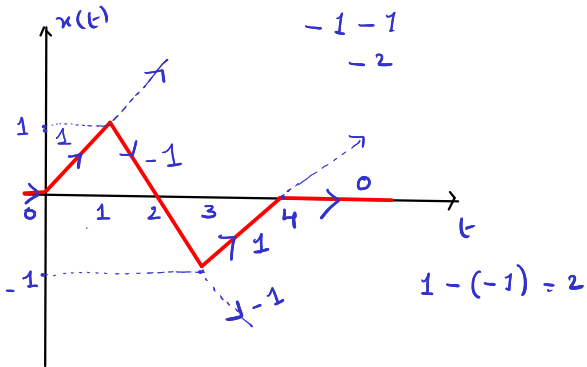


$$x(t) = 1r(t) - 2r(t-1) + 1r(t-2)$$

$$t=1$$

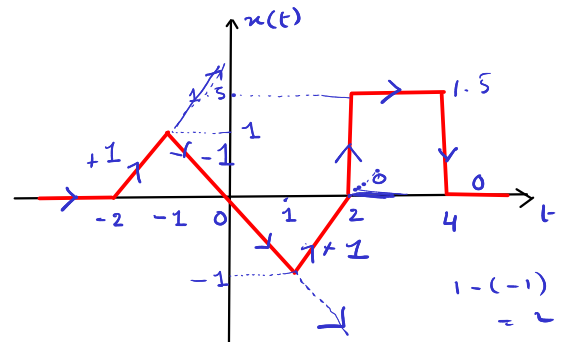
$$x(1) = r(1) - 2r(0) + r(-1) = 1$$

ex:9



$$x(t) = 1r(t) - 2r(t-1) + 2r(t-3) - 1r(t-4)$$

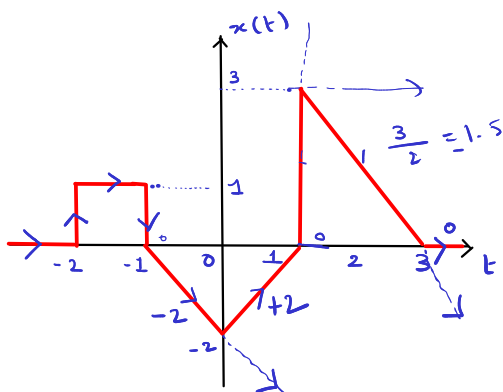
ex:10



$$x(t) = \underline{1r(t+2)} - \underline{2r(t+1)} + \underline{2r(t-1)}$$

$$\underline{-1r(t-2)} + \underline{1.5u(t-2)} - \underline{1.5u(t-4)}$$

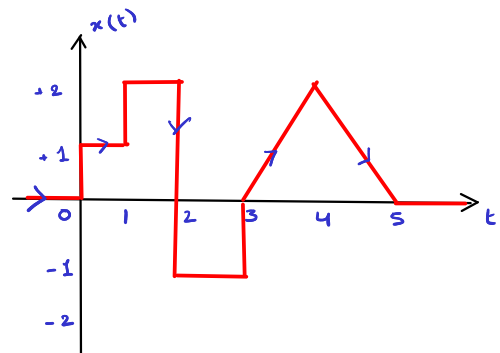
ex:11



$$x(t) = 1u(t+2) - 1u(t+1) - 2r(t+1)$$

$$+ 4r(t) - 2r(t-1) + 3u(t-1) - 1.5r(t-1) + 1.5r(t-3)$$

ex:12



$$x(t) = u(t) + u(t-1) - 3u(t-2)$$

$$+ u(t-3) + 2r(t-3)$$

$$- 4r(t-4) - 2r(t-5)$$

Classification of signals

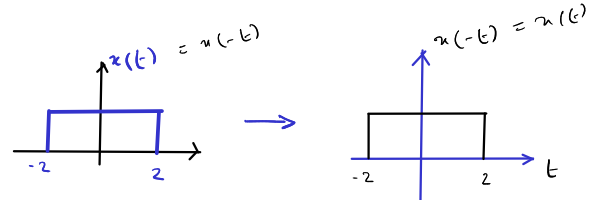
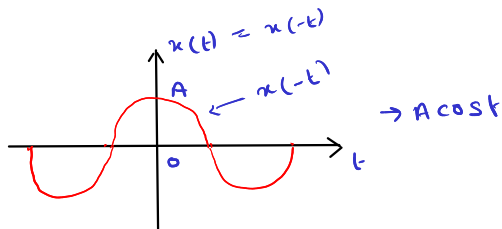
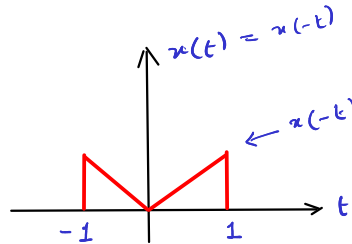
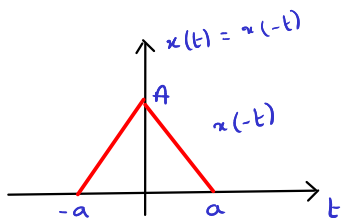
1. Continuous time and discrete time signal
2. Deterministic and random signals
3. Periodic and aperiodic signals
4. Even and odd signals
5. Energy and power signals
6. Causal and non causal signals

Even and Odd signals

Even (symmetric) signal: A CTS $x(t)$ is said to be even signal if it satisfies the condition

$$x(-t) = x(t) \text{ for all } t$$

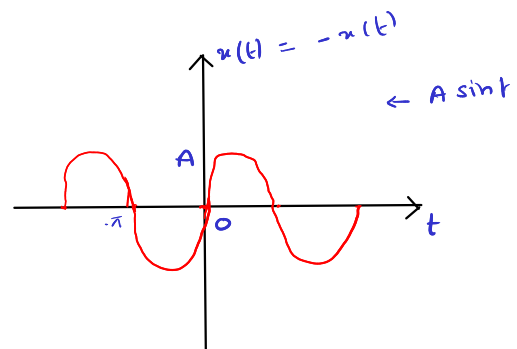
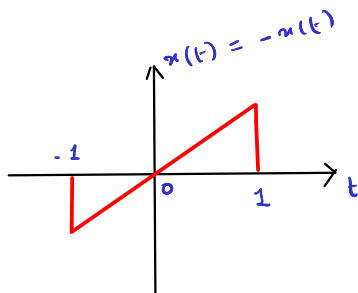
ex:



Odd (Anti symmetric) signal: A CTS said to be Odd signal if it satisfies the condition

$$x(-t) = -x(t) \text{ for all } t$$

ex:



→ Any CTS can be represented as sum of even and odd signals/component

→ A signal is neither even nor odd, but has both even and odd components

Let $x(t) \rightarrow$ CTS

$x_e(t) \rightarrow$ Even component

$x_o(t) \rightarrow$ odd component

$$x(t) = x_e(t) + x_o(t) \rightarrow (1)$$

T.R on eq (1) $t \rightarrow -t$

$$x(-t) = x_e(-t) + x_o(-t)$$

$$x(-t) = x_e(t) - x_o(t) \rightarrow (2)$$

Let (1) + (2) eq

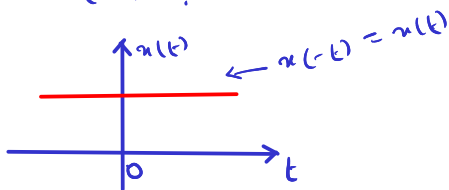
$$x(t) + x(-t) = 2x_e(t)$$

$$x_e(t) = \frac{1}{2} [x(t) + x(-t)]$$

properties:

(1) Let $x(t) = 10$

$$x(-t) = ?$$



DC Value is an even signal

(2) When we add DC + E

$$\underline{\underline{ex}} \quad 10 + t^2 = x(t)$$

$$10 + (-t)^2 = x(-t)$$

$$10 + t^2 = x(-t) = x(t)$$

$$\begin{aligned} \text{DC} + E &= E \\ E + E &= E \end{aligned}$$

(5) $O \times O = ?$

$$x(t) = t^3 \times t^5 = t^8$$

$$O \times O = E$$

(6) $O \times E = ?$

$$x(t) = t^3 \times t^6 = t^9$$

$$O \times E = O$$

(7)

$$\frac{d}{dt}(E) = 0$$

Let eq (2) - eq (1)

$$x(t) - x(-t) = 2x_o(t)$$

$$x_o(t) = \frac{1}{2} [x(t) - x(-t)]$$

$$\rightarrow x(t) = t, t^3, t^5, t^7, \dots \quad \text{odd}$$

$$x(t) = t^2, t^4, t^6, t^8, \dots \quad \text{even}$$

(3) DC value + odd = ?

$$x(t) = 10 + t^3$$

$$\begin{aligned} x(-t) &= 10 + (-t)^3 \\ &= 10 - t^3 \end{aligned}$$

$$\text{DC} + O = \text{neither E nor O}$$

$$E + O = \text{neither E nor O}$$

(4) $E \times E = ?$

$$x(t) = t^2 \times t^4 = t^6$$

$$x(-t) = (-t)^6 = t^6 = x(t)$$

$$\therefore E \times E = E$$

$$(8) \frac{d}{dt}(O) = E$$

$$(9) \int E dt = O$$

$$(10) \int O dt = E$$

$$(11) \frac{1}{O} = O$$

$$(12) \frac{1}{E} = E$$

$$\boxed{Q} \quad x(t) = \underbrace{\cos t}_E + \underbrace{\sin t}_O + \underbrace{\cos t \sin t}_O$$

$$x(-t) = \cos(-t) + \sin(-t) + \cos(-t) \sin(-t)$$

$$x(-t) = \cos t - \sin t - \cos t \sin t$$

$$x_e(t) = \frac{1}{2} [x(t) + x(-t)]$$

$$x_o(t) = \frac{1}{2} [x(t) - x(-t)]$$

$$x_e(t) = \frac{1}{2} [\cancel{\cos t} + \cancel{\sin t} + \cancel{\cos t \sin t} + \cancel{\cos t} - \cancel{\sin t} - \cancel{\cos t \sin t}]$$

$$\cos(-\theta) = \cos \theta \checkmark$$

$$\sin(-\theta) = -\sin \theta \checkmark$$

$$x_e(t) = \frac{1}{2} [2 \cos t] = \cos t$$

$$x_o(t) = \frac{1}{2} [\cancel{\cos t} + \cancel{\sin t} + \cancel{\cos t \sin t} - \cancel{\cos t} + \cancel{\sin t} + \cancel{\cos t \sin t}]$$

$$= \sin t + \cos t \cdot \sin t$$

$$\textcircled{2} \quad x(t) = e^{-t}$$

$$x(-t) = e^t$$

$$x_e(t) = \frac{x(t) + x(-t)}{2} = \frac{e^{-t} + e^t}{2} = \cosh t$$

$$x_o(t) = \frac{x(t) - x(-t)}{2} = \frac{e^{-t} - e^t}{2} = -\sinh t$$

$$\frac{e^x + e^{-x}}{2} = \cosh x$$

$$\frac{e^x - e^{-x}}{2} = \sinh x$$

$$\textcircled{3} \quad x(t) = 2 - 3t + 4t^2 \rightarrow \textcircled{1}$$

$$x(-t) = 2 - 3(-t) + 4(-t)^2$$

$$x(-t) = 2 + 3t + 4t^2 \rightarrow \textcircled{2}$$

$$x(-t) = x(t) \rightarrow \text{even}$$

$$x(-t) = -x(t) \rightarrow \text{odd}$$

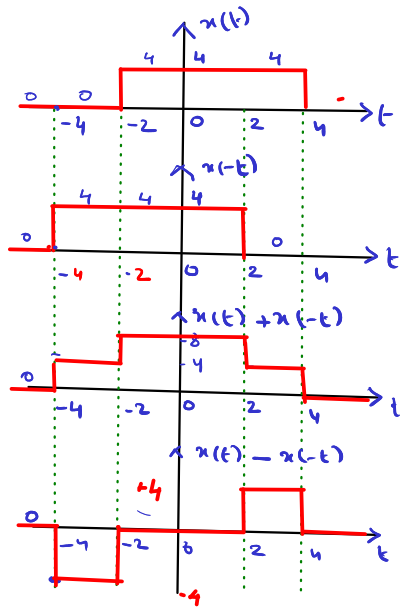
$$\text{put } t=1 \quad \textcircled{1} \text{ eq}$$

$$x(1) = 2 - 3(1) + 4(1)^2 \Rightarrow 3$$

$$\text{put } t=1 \quad \textcircled{2} \text{ eq}$$

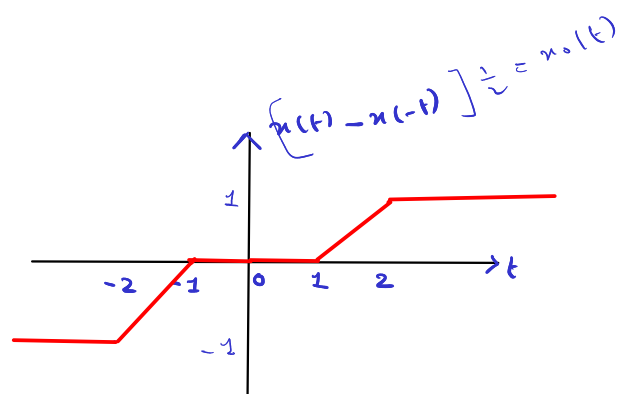
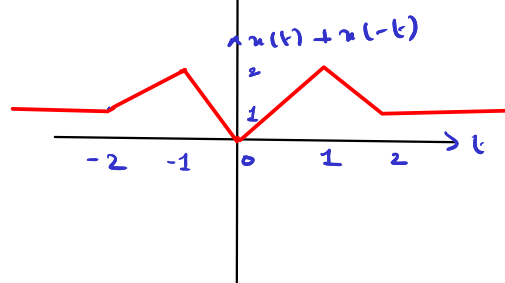
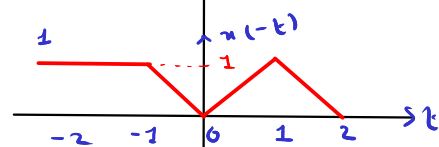
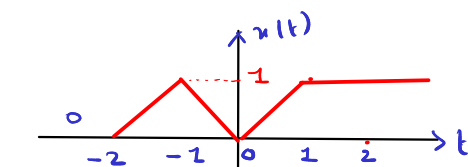
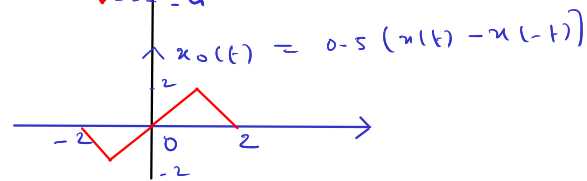
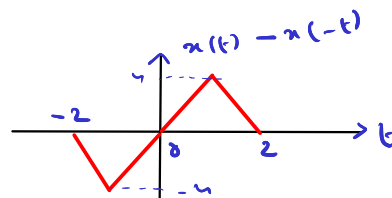
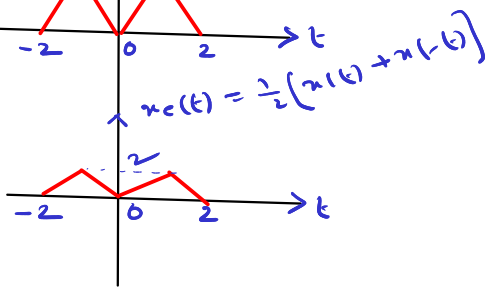
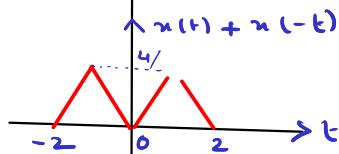
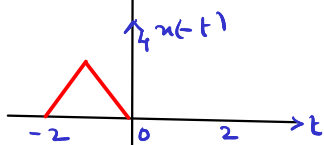
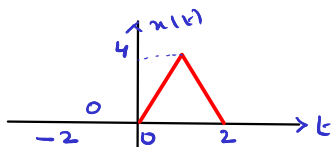
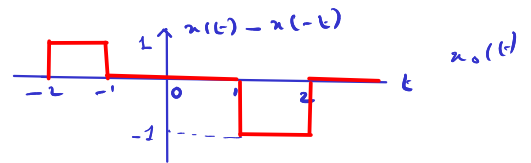
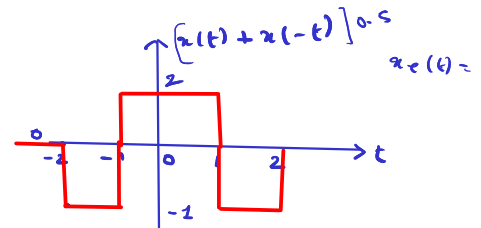
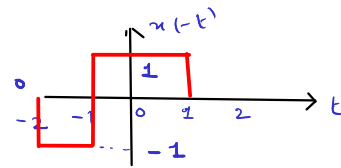
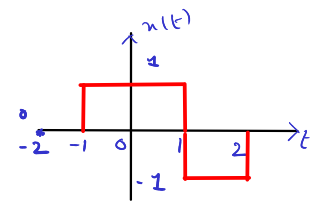
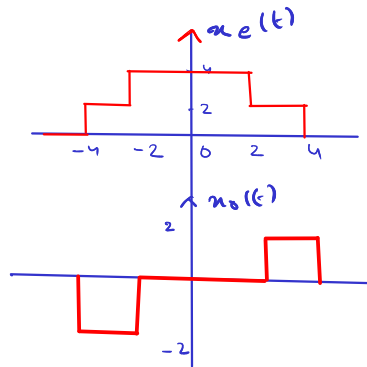
$$x(1) = 2 + 3(1) + 4(1)^2 \Rightarrow 9$$

Find even and odd components of given signals



$$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$$

$$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$



Periodic and aperiodic signals:

A signal is said to be periodic, if it satisfies following two conditions

1. It must exist for $-\infty \leq t \leq \infty$

2. It must repeat itself after some constant amount of time 'T'

$$x(t) = x(t+T) \text{ for all } t \quad \checkmark$$

The smallest value of T that satisfies the above condition is known as fundamental period

$$T = \frac{2\pi}{\omega_0} \rightarrow \text{fundamental freq (rad/sec)}$$

$$T = \frac{1}{f} \rightarrow \text{Hz}$$

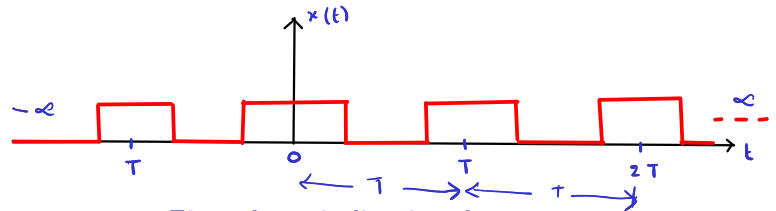


Fig: A periodic signal

General representation of sinusoidal signal is

$$x(t) = A \sin(\omega_0 t + \phi)$$

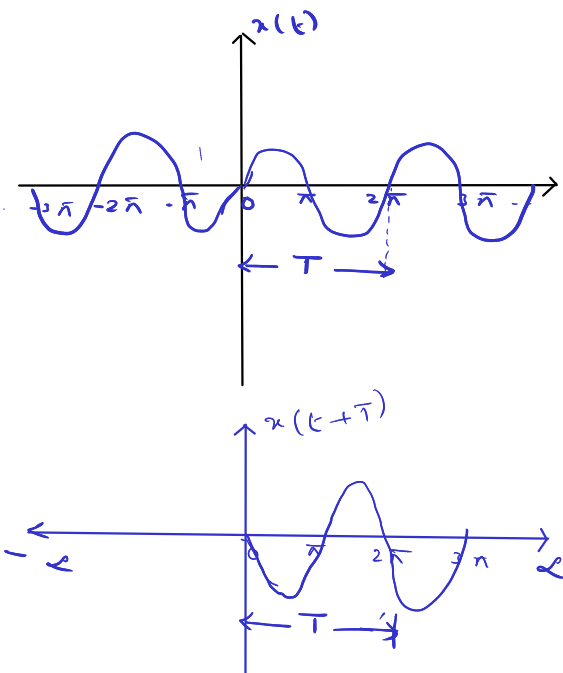
↓ ↓ ↘
Amp phase angle phase shift
angular freq

$$\frac{d}{dt}(\text{phase}) = \frac{d}{dt}(\omega_0 t) \Rightarrow \omega_0 \rightarrow \text{freq}$$

$$x(t) = A \sin \omega_0 t$$

$$x(t) = x(t \pm T)$$

$$\begin{aligned} x(t+T) &= A \sin \omega_0(t+T) \\ &= A \sin(\omega_0 t + \omega_0 T) \\ &= A \sin\left(\omega_0 t + \frac{2\pi}{T} \cdot T\right) \\ &= A \sin(\omega_0 t + 2\pi) \\ &= A \left(\sin \omega_0 t \cos 2\pi + \cos \omega_0 t \sin 2\pi \right) \\ &= A \sin \omega_0 t \quad \left(\begin{array}{l} \cos 2\pi = 1 \\ \sin 2\pi = 0 \end{array} \right) \\ &= x(t) \end{aligned}$$
$$T = \frac{2\pi}{\omega_0}$$
$$\omega_0 = \frac{2\pi}{T}$$



Ex ① $x(t) = 4 \sin 300\pi t$ $| \underbrace{A \sin \omega_0 t}$

$$\omega_0 = 300\pi$$

$$T = \frac{2\pi}{\omega_0} \Rightarrow \frac{2\pi}{300\pi} \Rightarrow \frac{1}{150} \text{ sec}$$

$$\textcircled{2} \quad x(t) = \underset{A}{5} \cos \left(\underset{\omega_0}{20\pi t} + \underset{\phi}{\frac{\pi}{4}} \right)$$

$$T = \frac{2\pi}{\omega_0} = \frac{2\pi}{20\pi} = \frac{1}{2} = 0.5 \text{ sec}$$

$$x(t) \neq x(t+T)$$



$$\textcircled{3} \quad x(t) = j e^{j5t} \Rightarrow e^{j\omega_0 t}$$

$$\omega_0 = 5$$

$$T = \frac{2\pi}{\omega_0} = \frac{2\pi}{5} = 0.4\pi \text{ sec}$$

$$\textcircled{4} \quad x(t) = \sin^2(4\pi t)$$

We know

$$\cos 2\theta = 1 - 2 \sin^2 \theta$$

$$\sin^2 \theta = \frac{1}{2} [1 - \cos 2\theta]$$

$$\theta = 4\pi t$$

$$x(t) = \sin^2 4\pi t = \frac{1}{2} [1 - \cos 8\pi t]$$

$$= \frac{1}{2} - \frac{\cos 8\pi t}{2}$$

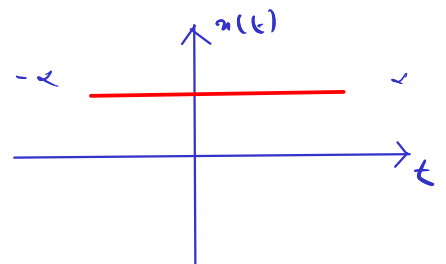
↑
periodic
but u.f.p

$$\omega_0 = 8\pi$$

$$T = \frac{2\pi}{\omega_0}$$

$$= \frac{2\pi}{8\pi}$$

$$= 1/4 \text{ sec} \Rightarrow 0.25 \text{ sec}$$



$$f_0 = \frac{1}{T} = \frac{1}{0.25} = 4 \text{ Hz}$$

The sum of two periodic signals $x_1(t)$ and $x_2(t)$ with periods T_1 and T_2 may or may not be periodic depending on the relation between T_1 and T_2

sum to be periodic, the ratio of the periods T_1/T_2 must be rational number or ratio of two integers

$$x(t) = x_1(t) + x_2(t)$$

$$\begin{matrix} \downarrow & \downarrow \\ T_1 & T_2 \end{matrix}$$

$$\frac{T_1}{T_2} \rightarrow \text{should be rational}$$

Ex ① $x(t) = \sin 6\pi t + \cos 5\pi t$

$$x_1(t) = \sin 6\pi t \quad x_2(t) = \cos 5\pi t$$

$$\omega_1 = 6\pi$$

$$\omega_2 = 5\pi$$

$$T_1 = \frac{2\pi}{6\pi} = \frac{1}{3}$$

$$T_2 = \frac{2\pi}{5\pi} = \frac{2}{5}$$

$$\frac{T_1}{T_2} = \frac{\frac{1}{3}}{\frac{2}{5}} \Rightarrow \frac{5}{6} \text{ (rational)}$$

$$\Rightarrow 0.8333 \dots$$

$$\left| \begin{array}{l} T = \frac{T_1}{T_2} = \frac{5}{6} \\ T = 6T_1 = 5T_2 \\ T = \cancel{6}\left(\frac{1}{\cancel{3}}\right) \\ T = 2 \end{array} \right.$$

$$T = \text{LCM}\{T_1, T_2\} \Rightarrow \text{LCM}\left\{\frac{1}{3}, \frac{2}{5}\right\} \Rightarrow 2 \text{ sec} \checkmark$$

$$f_0 = \frac{1}{T} = \frac{1}{2} = 0.5 \text{ Hz}$$

Ex :- $x(t) = 4 \cos\left(\frac{\pi}{100}\right)t + 2 \cos\left(\frac{2\pi}{180}\right)t$

$$x(t) = x_1(t) + x_2(t)$$

$$x_1(t) = 4 \cos\left(\frac{\pi}{100}\right)t \quad x_2(t) = 2 \cos\left(\frac{2\pi}{180}\right)t$$

$$\omega_1 = \frac{\pi}{100}$$

$$\omega_2 = \frac{2\pi}{180}$$

$$T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{\pi/100} = 200 \text{ sec}$$

$$T_2 = \frac{2\pi}{2\pi/180} \Rightarrow 180 \text{ sec}$$

$$\frac{T_1}{T_2} = \frac{200}{180}$$

$$180T_1 = 200T_2$$

Ex :- $x(t) = \cos^2(2\pi t)$

$$1 + \cos \theta = 2 \cos^2 \theta$$

$$\cos^2 \theta = \frac{1}{2} [1 + \cos 2\theta]$$

$$x(t) = \frac{1}{2} + \frac{1}{2} \cos(4\pi t)$$

$$\omega_0 = 4\pi$$

$$T = \frac{2\pi}{4\pi} = \frac{1}{2} \text{ sec}$$

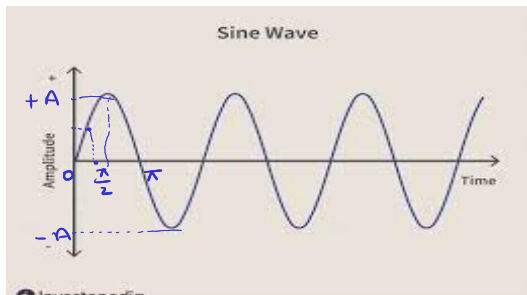
$$f = \frac{1}{T} = \frac{1}{1/2} \Rightarrow 2 \text{ Hz}$$

Deterministic and random signals

Deterministic : A signal is said to be deterministic , if there is no uncertainty with respect to its values at any instants of time

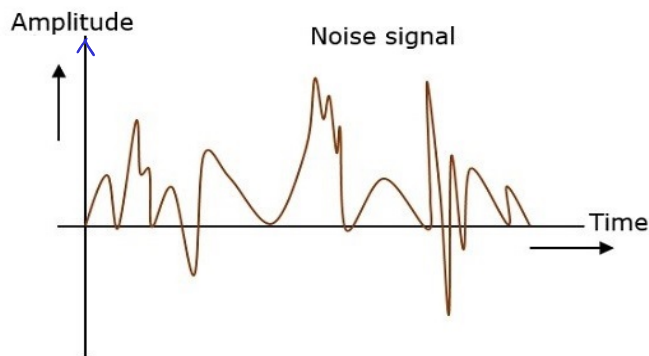
↓
mag
phase

Ex: sine wave



Non deterministic / random : A signal is said to be random, if there is uncertainty at any particular instant of time

Ex: Noise signal

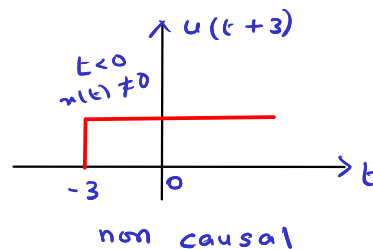
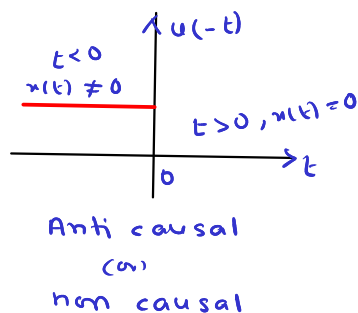
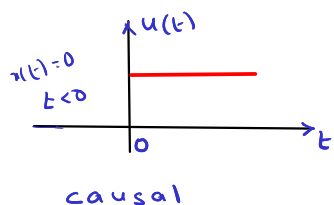


Causal and non causal signals

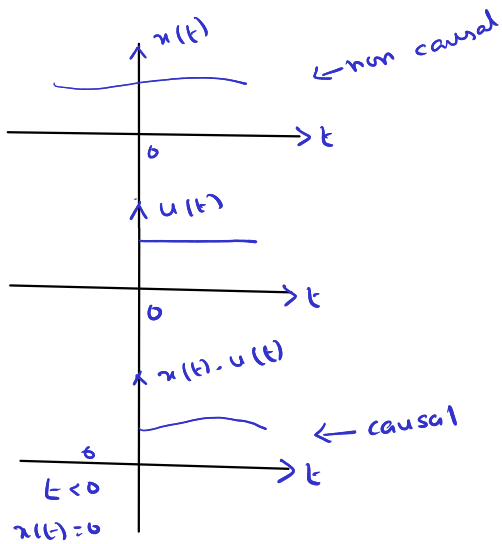
A CTS $x(t)$ is said to be causal if $x(t) = 0$ for $t < 0$, ✓

anti causal if $x(t) = 0$ for $t > 0$, ✓

non causal if $x(t) \neq 0$ for $t < 0$. ✓

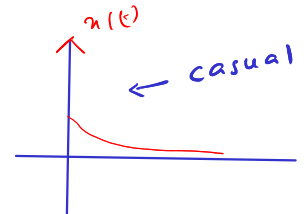
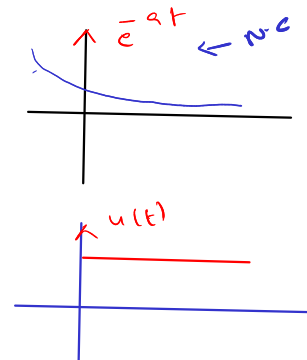


All anti causal signals are non causal but all non causal signals need not to be anti causal

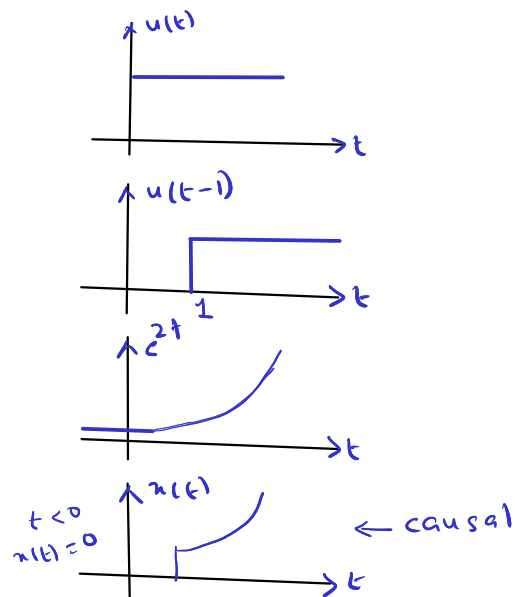


Any non causal signal is multiplied with $u(t)$, it generates causal signal

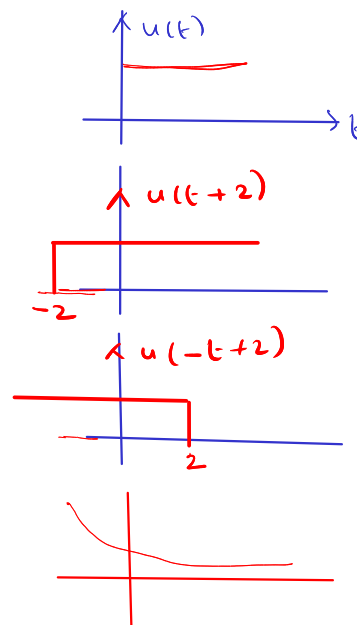
$$\underline{\text{Ex}} \quad x(t) = e^{-at} \cdot u(t)$$



$$\underline{\text{Ex}} \quad (1) \quad x(t) = e^{2t} u(t-1)$$

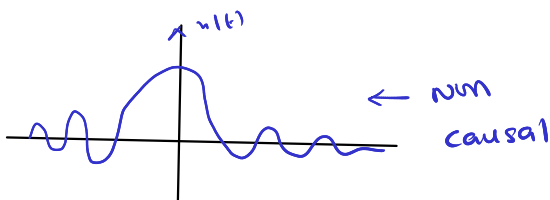


$$\underline{\text{Ex}} \quad (2) \quad x(t) = e^{-3t} u(-t+2) \quad \leftarrow$$

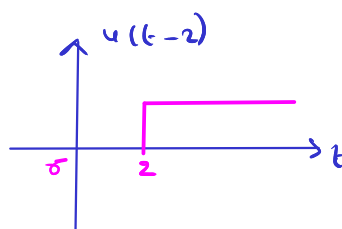
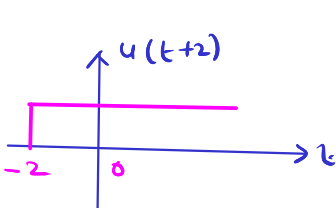


$\rightarrow$ non causal

$$(3) \quad x(t) = 3 \sin 2t$$



$$(4) \quad x(t) = u(t+2) - u(t-2)$$



non causal

Energy and power signals :

Energy signal: A signal is said to be an energy signal, if its total energy E is finite i.e $0 < E < \infty$

For an energy signal, the average power p=0

power signal: A signal is said to be a power signal, if its average power p is finite, i.e $0 < p < \infty$

For a power signal, the total energy E= ∞

for a signal $x(t)$, the total energy is defined as

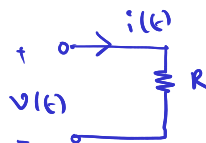
$$E = \lim_{T \rightarrow \infty} \int_{-T}^T |x(t)|^2 dt \quad \text{Joules} \quad \checkmark$$

average power is defined as

$$P = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |x(t)|^2 dt \quad \text{Watts}$$

$$v(t) = i(t) \times R \rightarrow (1)$$

$$P(t) = i^2(t) \times R \rightarrow (2)$$



$$i(t) = \frac{v(t)}{R}$$

$$P(t) = \frac{v^2(t)}{R} \times \cancel{R}$$

$$P(t) = \frac{v^2(t)}{R} \quad \text{but } R=1 \Omega$$

$$P(t) = v^2(t) \rightarrow (3)$$

$$(1) \quad x(t) = e^{j(2\pi t + \frac{3\pi}{4})}$$

$$E = \lim_{T \rightarrow \infty} \int_{-T}^T |x(t)|^2 dt$$

$\omega \cdot t$

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$|e^{j\theta}| = \sqrt{\cos^2\theta + \sin^2\theta}$$

$$|e^{j\theta}|^2 = \cos^2\theta + \sin^2\theta = 1$$

$$\left| e^{j(2\pi t + \frac{3\pi}{4})} \right|^2 = 1$$

$$E = \lim_{T \rightarrow \infty} \int_{-T}^T (1) dt$$

$$= \lim_{T \rightarrow \infty} [t]_{-T}^T$$

$$= \lim_{T \rightarrow \infty} [T - (-T)]$$

$$= \lim_{T \rightarrow \infty} 2T$$

$$\boxed{E = \infty}$$

$$P = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |x(t)|^2 dt$$

$$P = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T (1) dt$$

$$P = \lim_{T \rightarrow \infty} \frac{1}{2T} \times 2T$$

$$P = 1 \text{ watt}$$

$\therefore x(t)$ power signal

② $x(t) = \cos t$

$$E = \lim_{T \rightarrow \infty} \int_{-T}^T |x(t)|^2 dt$$

$$= \lim_{T \rightarrow \infty} \int_{-T}^T \cos^2 t dt$$

w.k.t

$$\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$$

$$E = \lim_{T \rightarrow \infty} \int_{-T}^T \left(\frac{1 + \cos 2t}{2} \right) dt$$

$$= \frac{1}{2} \lim_{T \rightarrow \infty} \int_{-T}^T (1 + \cos 2t) dt$$

$$= \frac{1}{2} \lim_{T \rightarrow \infty} \left\{ \int_{-T}^T 1 dt + \int_{-T}^T \cos 2t dt \right\}$$

$$= \frac{1}{2} \lim_{T \rightarrow \infty} \left\{ \left(t \right)_{-T}^T + \left(\frac{\sin 2t}{2} \right)_{-T}^T \right\}$$

$$= \frac{1}{2} \lim_{T \rightarrow \infty} \left\{ 2T + \frac{1}{2} \underbrace{(\sin 2T - \sin(-2T))}_0 \right\}$$

$$\boxed{E = \infty}$$

$$P = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |x(t)|^2 dt$$

$$P = \frac{1}{2} \text{ watt}$$

$\therefore$ given $x(t)$ is a power signal

③ $x(t) = e^{-3t} \cdot u(t)$

$$E = \lim_{T \rightarrow \infty} \int_{-T}^T |x(t)|^2 dt$$

$$= \lim_{T \rightarrow \infty} \int_{-T}^T |e^{-3t} \cdot u(t)|^2 dt$$

w.k.t $u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$

$$E = \lim_{T \rightarrow \infty} \int_0^T |e^{-3t}|^2 dt$$

$$= \lim_{T \rightarrow \infty} \int_0^T e^{-6t} dt$$

$$= \lim_{T \rightarrow \infty} \left[\frac{e^{-6t}}{-6} \right]_0^T \rightarrow \textcircled{1}$$

$$= -\frac{1}{6} \lim_{T \rightarrow \infty} (e^{-6T} - e^0)$$

$$= -\frac{1}{6} (-1)$$

$$\boxed{E = \frac{1}{6} \text{ J}}$$

$$P = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T |x(t)|^2 dt$$

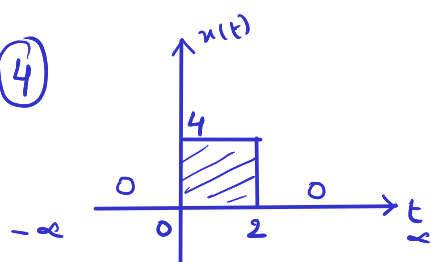
$$P = \lim_{T \rightarrow \infty} \frac{1}{2T} \left[\frac{e^{-6t}}{-6} \right]_0^T$$

$$P = \lim_{T \rightarrow \infty} \frac{1}{2T} \times \left(-\frac{1}{6} \right) (e^{-6T} - e^0)$$

$$\boxed{P = 0}$$

$\therefore x(t)$ is a Energy Signal

④

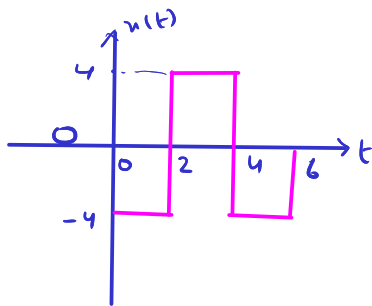


$$E = \int_{-T}^T |x(t)|^2 dt$$

$$= \int_0^2 |4|^2 dt$$

$$= 16[2 - 0] \Rightarrow 32 \text{ J}$$

5



$$\begin{aligned}
 E &= \int_{-T}^T |x(t)|^2 dt \Rightarrow \int_{-2}^0 0 dt + \int_0^2 (4)^2 dt + \int_2^4 (4)^2 dt + \int_4^6 (4)^2 dt + \int_6^8 0 dt \\
 &\Rightarrow 0 + 16[t]_0^2 + 16[t]_2^4 + 16[t]_4^6 + 0 \\
 &\Rightarrow 32 + 32 + 32 \\
 E &\Rightarrow 96 \text{ J}
 \end{aligned}$$

system:-

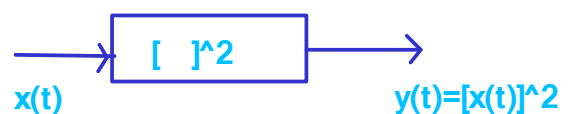
system is an interconnection of different physical components which act together to perform certain task

(or)

A system is defined as a physical device that generates a response or output signal for a given input signal



** the relation b/w the i/p $x(t)$ & o/p $y(t)$ is
 $y(t) = \text{operation on } x(t)$



Examples:-

(converts electrical energy into mechanical energy)

(increases the strength of i/p signal)

(rejects the unwanted frequencies present in i/p signal)

Classification of systems

1. Static and dynamic systems
2. Causal and non causal systems
3. Time invariant and time variant systems
4. Linear and non linear systems
5. Invertible and non invertible systems
6. stable and unstable systems



$$\begin{aligned} y(t) &= x(t) \rightarrow t=0 \quad y(0) = x(0) \xrightarrow{\text{pr}} \textcircled{1} \\ y(t) &= x(t-1) \rightarrow y(0) = x(-1) \xrightarrow{\text{past}} \textcircled{2} \\ y(t) &= x(t+1) \rightarrow y(0) = x(1) \xrightarrow{\text{future}} \textcircled{3} \end{aligned}$$

1. Static and dynamic systems

Static(memory less) system:- Output of system depends only on present value of inputs

Dynamic (with memory) system:- output of system depends on past or future values of i/p at any instant of time.

Ex $\textcircled{1} \quad y(t) = 2 \cdot x(t)$
 $\uparrow$
co-eff

$$t=0, \quad y(0) = 2 \cdot x(0)$$

$$t=1, \quad y(1) = 2 \cdot x(1)$$

$\therefore y(t)$ is static

$\textcircled{2} \quad y(t) = x(t-2)$
 $t=0, \quad y(0) = x(-2)$

$\therefore$ dynamic

$\textcircled{3} \quad y(t) = x(t+2)$
 $t=0, \quad y(0) = x(2)$
 $\uparrow$ f.v

$\therefore$ dynamic

$\textcircled{4} \quad y(t) = x(t) + x(t-1)$
 $\uparrow \quad \quad \uparrow$
pr past

$\therefore$ dynamic

$\textcircled{5} \quad y(t) = x(2t)$

$$t=0, \quad y(0) = x(0) \text{ pr i/p}$$

$$t=-1, \quad y(-1) = x(-2) \text{ past i/p}$$

$$t=1, \quad y(1) = x(2) \text{ future i/p}$$

$\therefore$ Dynamic system

$\textcircled{7} \quad y(t) = x(\sin t)$

$$t=0, \quad y(0) = x(\sin 0)$$

$$y(0) = x(0)$$

$$t=\pi \quad y(\pi) = x(\sin \pi)$$

$$y(3.14) = x(0)$$

 $\uparrow$ past i/p

$\therefore$ Dynamic system

$\textcircled{6} \quad y(t+1) = x(t+1)$

$$t=0, \quad y(1) = x(1) \text{ pr i/p}$$

$$t=1, \quad y(2) = x(2) \text{ pr i/p}$$

$\therefore$ static system

$\textcircled{8} \quad y(t) = x(t^3)$

$$t=0, \quad y(0) = x(0)$$

$$t=1, \quad y(1) = x(1)$$

$$t=2, \quad y(2) = x(8) \text{ f.v} \quad \therefore \text{dynamic}$$

2. Causal and non causal systems

Causal system:- output of system is independent of future values of i/p

Ex: $y(t) = x(t)$

$$y(t) = \underset{\substack{\uparrow \\ \text{p}}}{x(t)} + \underset{\substack{\uparrow \\ \text{past}}}{x(t-1)}$$

Non causal system:- output of system depends on future values of i/p at any instant of time.

Ex $y(t) = x(t+2)$
 $t=0, y(0) = x(2)$
 $\uparrow$ future i/p

Ex $y(t) = x(t) + x(t-1) + x(t+1)$
 $\uparrow \quad \uparrow \quad \uparrow$ future i/p

① $y(t) = x(3t)$

$t=0, y(0) = x(0)$

$t=1, y(1) = x(3)$
 $\uparrow$ future i/p

$\therefore$ N.C

② $y(t) = \begin{cases} x(3t) & , t < 0 \\ x(t-1) & , t \geq 0 \end{cases}$

for $t < 0$ $y(t) = x(3t)$

$t=-1, y(-1) = x(-3)$
 $\uparrow$ past i/p

for $t \geq 0$

$t=0, y(0) = x(-1)$

$t=1, y(1) = x(0)$
 $\uparrow$ past i/p

$\therefore$ causal

③ $y(t) = \underbrace{\sin(t+1)}_{\text{co-ctf}} \cdot x(t-1)$

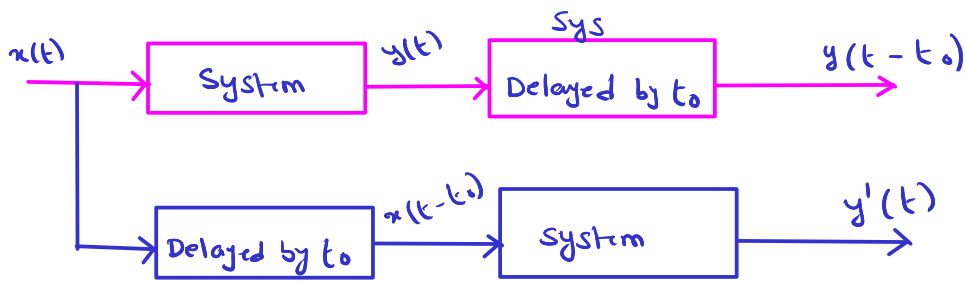
$t=0, y(0) = \sin(1) \cdot x(-1)$
 $\uparrow$ past i/p

$t=-1, y(-1) = \sin(0) \cdot x(-2)$
 $\uparrow$ past i/p

$t=1, y(1) = \sin(2) \cdot x(0)$
 $\uparrow$ past i/p

$\therefore$ causal

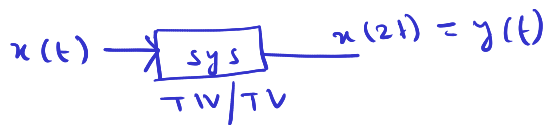
3. Time invariant and time variant systems



$$y'(t) = y(t-t_0) \quad \text{TIV}$$

$$y'(t) \neq y(t-t_0) \quad \text{TV}$$

Ex ① $y(t) = x(2t)$



S.1 $y(t) \xrightarrow{t_0} y(t-t_0) = x[2(t-t_0)] = x(2t-2t_0)$

S.2 $x(t) \xrightarrow{t_0} x(t-t_0) \rightarrow \text{sys} \rightarrow x(2t-2t_0)$

$\therefore$ T.V system

Ex ② $y(t) = 2 + x(t)$

S.1 $y(t) \xrightarrow{t_0} y(t-t_0) = 2 + x(t-t_0)$

S.2 $x(t) \xrightarrow{t_0} x(t-t_0) \rightarrow \text{sys} \rightarrow 2 + x(t-t_0)$

$\therefore$ TIV

Ex ③ $y(t) = x(\cos t)$

S.1 $y(t) \xrightarrow{t_0} y(t-t_0) = x(\cos(t-t_0))$

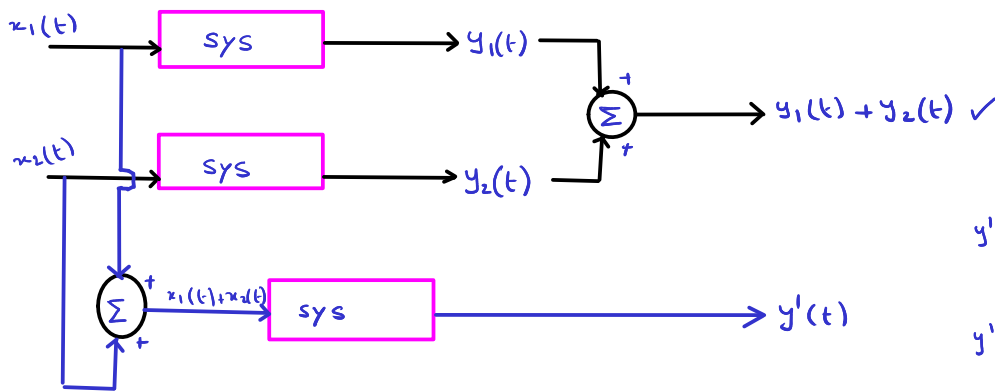
S.2 $x(t) \xrightarrow{t_0} x(t-t_0) \rightarrow \text{sys} \rightarrow x(\cos t - t_0)$

$\therefore$ T.V system

4. Linear and non linear systems

Linear systems:- The system which follows the principle of superposition is known as linear system

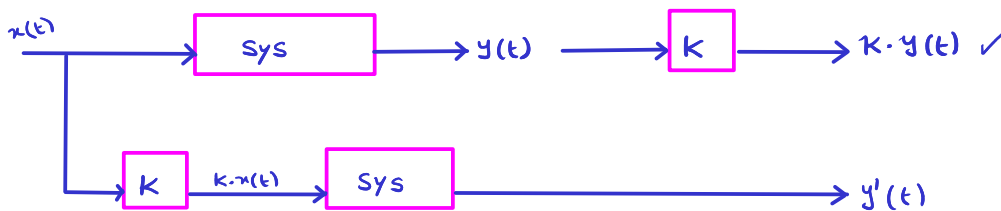
1. Law of additivity



$$y'(t) = y_1(t) + y_2(t) \quad \leftarrow \text{Law of additivity}$$

$$y'(t) \neq y_1(t) + y_2(t) \quad \nearrow \times \text{LoA}$$

2. Law of homogeneity



$$y'(t) = K \cdot y(t) \quad \leftarrow \text{follows LOH}$$

$$y'(t) \neq K \cdot y(t) \quad \nearrow \times \text{LOH}$$

Ex ① $y(t) = x(\sin t)$

a) LoA $y_1(t) = x_1(\sin t)$
 $y_2(t) = x_2(\sin t)$

$$y_1(t) + y_2(t) = \underline{x_1(\sin t) + x_2(\sin t)} \quad \leftarrow \text{same}$$

$$x_1(t) + x_2(t) \rightarrow \text{sys} \rightarrow \underline{x_1(\sin t) + x_2(\sin t)}$$

b) LOH :- $y(t) = x(\sin t)$

$$K \cdot y(t) = K \cdot \underline{x(\sin t)} \rightarrow \textcircled{1'}$$

$$K \cdot x(t) \rightarrow \text{sys} \rightarrow \underline{K \cdot x(\sin t)} \rightarrow \textcircled{2'}$$

∴ Given sys linear

Ex (2) $y(t) = x(t^2)$

a) LOA: $x_1(t) \rightarrow \text{sys} \rightarrow y_1(t) = x_1(t^2)$

$x_2(t) \rightarrow \text{sys} \rightarrow y_2(t) = x_2(t^2)$

$y_1(t) + y_2(t) = x_1(t^2) + x_2(t^2) \checkmark$

$x_1(t) + x_2(t) \rightarrow \text{sys} \rightarrow y'(t) = x_1(t^2) + x_2(t^2) \checkmark$

$y'(t) = y_1(t) + y_2(t)$

b) LOH:

$x(t) \rightarrow \text{sys} \rightarrow y(t) \rightarrow K \rightarrow K \cdot y(t) = K x(t^2) \checkmark$

$x(t) \rightarrow K \rightarrow K \cdot x(t) \rightarrow \text{sys} \rightarrow y'(t) = K \cdot x(t^2) \checkmark$

$\therefore$ Linear System

Ex (3): $y(t) = x^2(t)$

$x(t) \rightarrow \text{sys} \rightarrow y(t) = x^2(t) = (ip)^2$

① LOA: $y_1(t) = x_1^2(t)$, $y_2(t) = x_2^2(t)$

$y_1(t) + y_2(t) = \underline{x_1^2(t)} + x_2^2(t) \checkmark$

$x_1(t) + x_2(t) \rightarrow \text{sys} \rightarrow y'(t) = \underline{[x_1(t) + x_2(t)]^2} \checkmark$

LOA is not satisfying

$\therefore$ sys is a non linear

Ex (4) $y(t) = \sin t \cdot x(t)$
 $\quad \quad \quad \uparrow \quad \quad \uparrow \text{ip}$
 $\quad \quad \quad \text{co-rft}$

$x(t) \rightarrow \text{sys} \rightarrow y(t) = \sin t \cdot x(t)$

a) LOA: $x_1(t) \rightarrow \text{sys} \rightarrow y_1(t) = \sin t \cdot x_1(t)$

$x_2(t) \rightarrow \text{sys} \rightarrow y_2(t) = \sin t \cdot x_2(t)$

$y_1(t) + y_2(t) = \sin t [\underline{x_1(t)} + x_2(t)] \checkmark$

$x_1(t) + x_2(t) \rightarrow \text{sys} \rightarrow \sin t [\underline{x_1(t) + x_2(t)}] \checkmark$

$\therefore$ LOA satisfy

b) LOH

$$x(t) \rightarrow \text{sys} \rightarrow y(t) \rightarrow K \rightarrow K \cdot y(t) \rightarrow K \cdot \sin t \cdot x(t) \checkmark$$

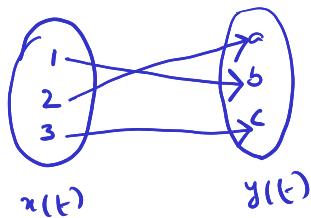
$$K \cdot x(t) \rightarrow \text{sys} \rightarrow \sin t \cdot K \cdot x(t) \\ = K \cdot \sin t \cdot x(t) \checkmark$$

$\therefore$ sys linear

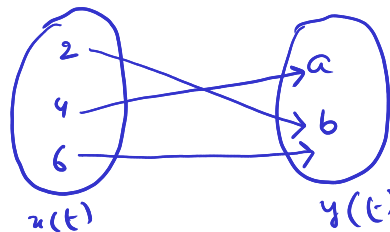
5. Invertible and non invertible systems

For any invertible system, there should be one to one mapping b/w i/p and o/p at each and every instant of time

one to one mapping



many to one mapping



Ex:- $y(t) = x^2(t)$

$x(t)$	$y(t) = x^2(t)$
-2	4
2	4

many to one
 $\therefore$ non invertible

Ex $y(t) = x(t) + 2$

$x(t)$	$y(t)$
0	2
1	3
-1	1

one to one mapp
 $\therefore$ Invertible

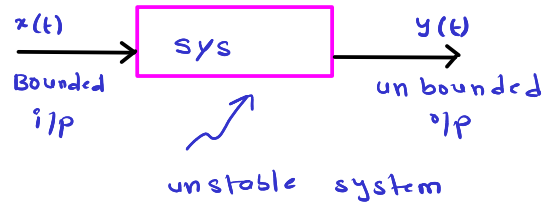
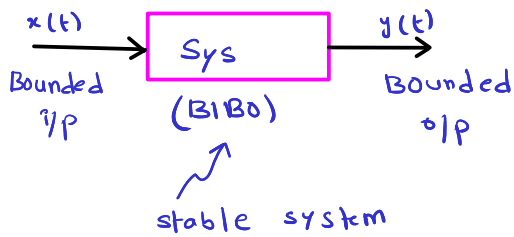
Ex $y(t) = |x(t)|$

$x(t)$	$y(t)$
-2	2
2	2
$2j$	2
$-2j$	2

$\therefore$ many to one
 $\therefore$ NI

Stable and unstable system

For a stable system, o/p should be bounded for bounded i/p (BIBO) at each and every instant of time



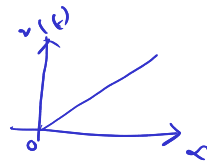
Ex :- DC, $\sin t$, $\cos t$, $u(t)$
 -1 to 1 -1 to 1 0 or 1

Ex ① $y(t) = t \cdot x(t)$

$$x(t) \rightarrow \text{sys} \rightarrow y(t) = t \cdot x(t)$$

$$u(t) \rightarrow \text{sys} \rightarrow y'(t) = t \cdot u(t) = v(t)$$

bounded



$\therefore$ unstable system

Ex ② $y(t) = x(t) + 2$

$$x(t) \rightarrow \text{sys} \rightarrow y(t) = x(t) + 2$$

$$4 \rightarrow \text{sys} \rightarrow y'(t) = 4 + 2 = 6$$

DC
bounded
i/p

DC
bounded o/p

$\therefore$ stable

Ex ③ $y(t) = \sin[x(t)]$

$$x(t) \rightarrow \text{sys} \rightarrow y(t) = \sin[x(t)]$$

$$x(t) \rightarrow \text{bounded}$$

$$y(t) = \sin[\text{finite value}]$$

$$-1 \leq \sin(\quad) \leq 1$$

$$-1 \leq y(t) \leq 1$$

$\therefore$ stable

Ex ④ $y(t) = \frac{x(t)}{t}$

$$x(t) \rightarrow \text{sys} \rightarrow y(t) = \frac{x(t)}{t}$$

$$2 \rightarrow \text{sys} \rightarrow y'(t) = \frac{2}{t}$$

DC

$$\begin{aligned} t &= 0 \\ t &= 0^+ \\ t &= 0^- \end{aligned}$$

$$= \frac{2}{0} = \infty$$

$\therefore$ unstable

Interconnection of system:

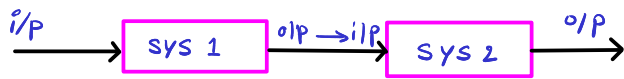


Fig a: series(cascade) interconnection

Ex:-A radio receiver followed by an amplifier

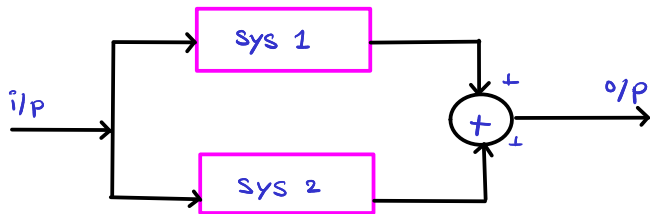


Fig b: parallel interconnection

Ex:simple audio system with several microphones feeding into a single amplifier and speaker system

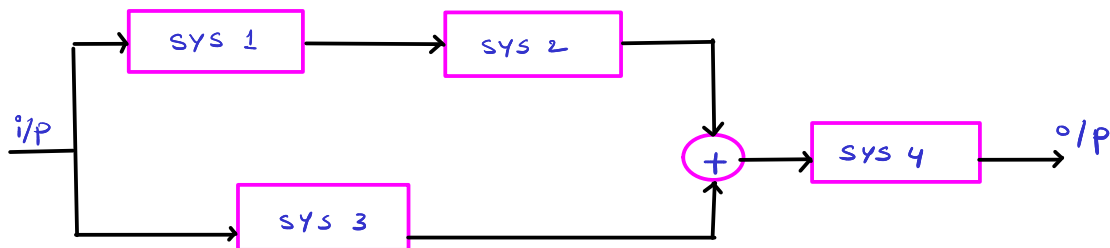


Fig c: series-parallel interconnection

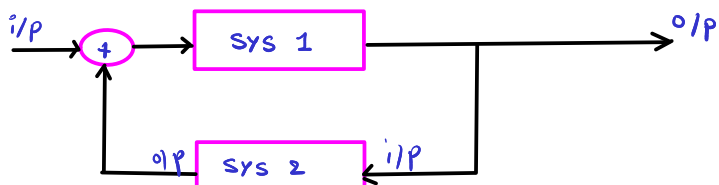


Fig d:feedback interconnection

Ex:-a cruise control system on an automobile senses the vehicle's velocity and adjust the fuel flow in order to keep the speed at desired level

Absolutely integrable:-

For $x(t)$ is to be absolutely integrable, if

$$\int_{-\infty}^{\infty} |x(t)| dt < \infty \quad \checkmark$$

Absolutely summable:-

$$\sum_{n=0}^{\infty} |x(n)| < \infty \quad \checkmark$$

Ex :- $e^{-\alpha t} \cdot u(t) = x(t)$

$$\begin{aligned} \int_{-\infty}^{\infty} e^{-\alpha t} \cdot u(t) dt &\Rightarrow \int_0^{\infty} e^{-\alpha t} dt \\ &\Rightarrow -\frac{1}{\alpha} \left[e^{-\alpha t} \right]_0^{\infty} \Rightarrow \frac{1}{\alpha} \quad \checkmark \end{aligned}$$

$\therefore x(t)$ is absolutely integrable $\checkmark$

Ex :- $\int_{-\infty}^{\infty} u(t) dt \Rightarrow \int_0^{\infty} 1 \cdot dt \Rightarrow [t]_0^{\infty} \Rightarrow \infty \quad \checkmark$

$\therefore u(t)$ is not absolutely integrable

Square integrable:

A signal is said to be square integrable, if

$$\int_{-\infty}^{\infty} |x(t)|^2 dt < \infty$$