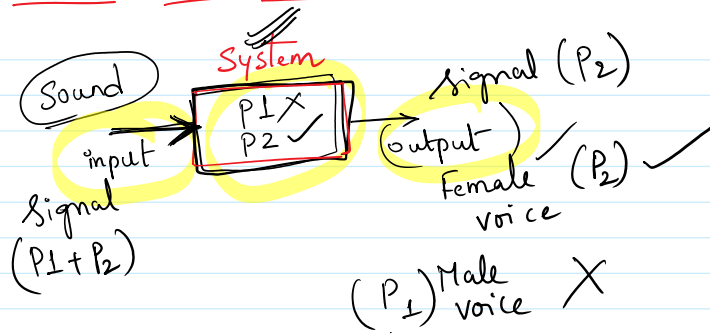


SIGNALS AND SYSTEMS

problem statement



- classification
- properties

- classification
- properties

- properties

Tools

- ① Signals ✓
- ② systems ✓
- ③ Convolution [output of the given system]
- ④ Fourier series [For analysis of signals] → Periodic signals
- ⑤ Fourier Transform [For Analysis of signals] → Both periodic & Aperiodic

Introduction

- ① classification
 - ① Continuous time signals & Discrete time signals
 - ② Continuous valued " & Discrete valued "
 - ③ Multi-channel signals & Multivalued signals

- ② classification
 - ① periodic & Aperiodic signals
 - ② Even & odd signals
 - ③ Energy & power signals
 - ④ Deterministic & Random signals
 - ⑤ Causal, Anti-causal, non-causal signals

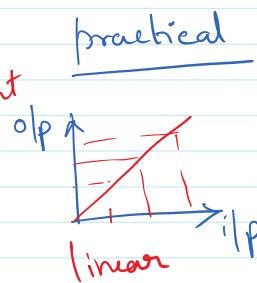
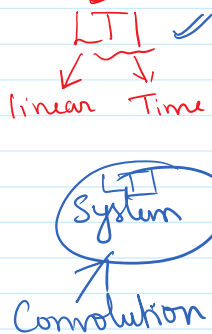
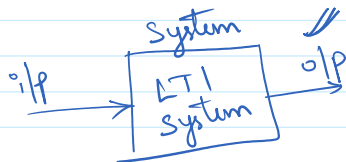
- ② Elementary / Basic signals
 - ① Unit impulse signal
 - ② Unit step signals
 - ③ " ramp signals
 - ④ Real Exponential signals
 - ⑤ Complex " "
 - ⑥ Sigmoid functions

- ③ Transformations of the signals
 - Time shifting
 - Time scaling
 - Time reversal

SYSTEMS

- Introduction
- Classification → Continuous time Systems & Discrete time Systems
- Properties →
 - ① Linear & Non-linear System
 - ② Time Varying & Time-invariant System
 - ③ Causal & Non-causal systems
 - ④ stable & Unstable Systems
 - ⑤ Static & Dynamic Systems
 - ⑥ Invertible & ~~Non~~ Non-invertible systems

CONVOLUTION



practical Linear ✓
Time-invariant

Introduction

Continuous-time LTI System

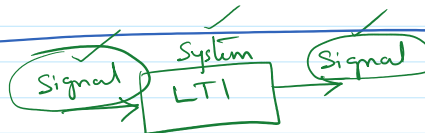
✓ Convolution Integral
" $\int$ "

Discrete-time LTI System

✓ Convolution - Sum.
" $\sum$ "

- properties

Analysis of Signals
Tool



→ Continuous time Fourier Series (CTFS)

periodic signals

→ Continuous time Fourier Transform (CTFT)

periodic & Aperiodic

CTFS

- Introduction
- Trigonometric F.S

✓ polar form representation of F.S

- Introduction

✓ ① Trigonometric F.S

✓ ② Exponential F.S

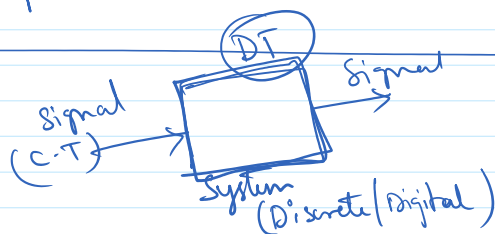
✓ polar form representation of F.S

CTFT

- Introduction

- Aperiodic Signals $\Rightarrow$ CTFT

- properties



Sampling :- Continuous domain $\xrightarrow{\text{Sampling}}$ Discrete domain

SIGNAL: physical quantity that varies with time, space or any independent variable or variables.

Ex:- Speech signal: Acoustic pressure as a function of time.

$f(t)$ $\xrightarrow{\text{independent}}$ $f(\text{time})$ $\xrightarrow{\text{independent variable}}$ time

picture signal: brightness as a function of two independent variables

$f(x, y)$ $\begin{matrix} \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square \end{matrix}$ (x, y)

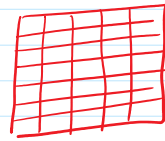
signal

origin

$$f(x, y)$$

brightness

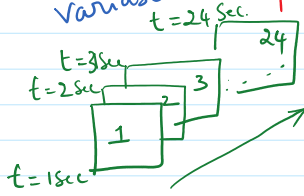
independent variable



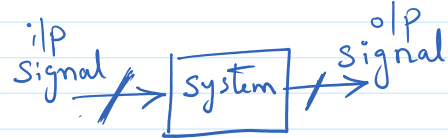
(x, y)

✓ Video signal : $f(x, y, \text{time})$

3-independent variable



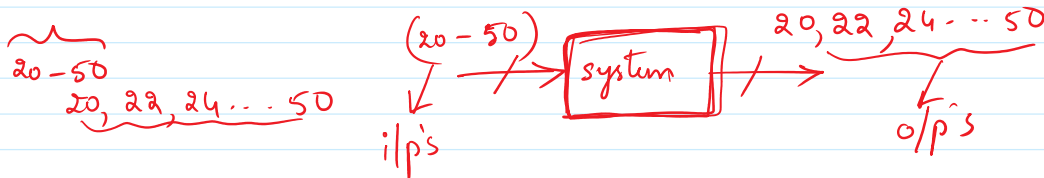
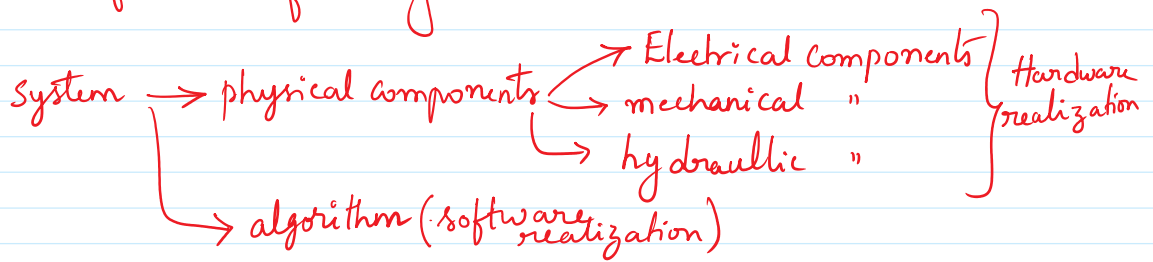
→ single i/p



→ multiple i/p

① System is an entity that processes the set of input signals to yield another set of output signals.

② system processes the signals in order to modify / extract some additional information from signals.



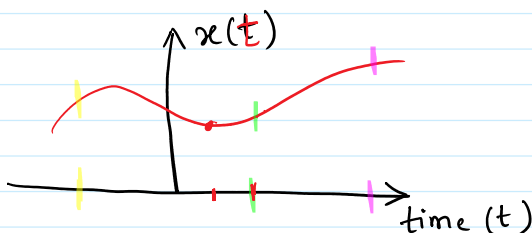
CLASSIFICATION OF SIGNALS

Depending on the independent variable

① Continuous time signal (CTS) and Discrete time signal (DTS)

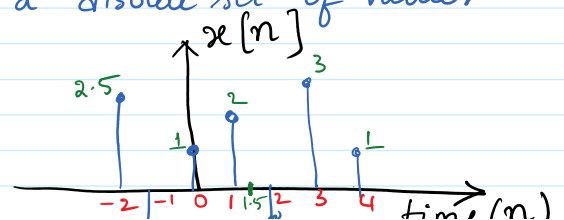
or Analog

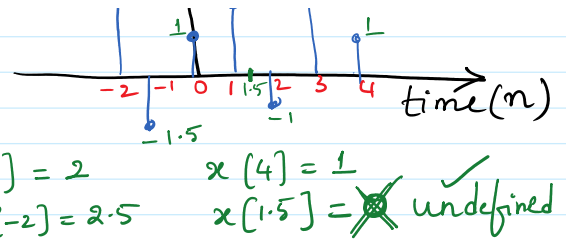
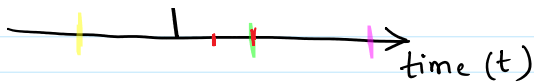
Continuum of values of the independent variable time t



defined only at discrete time n

independent variable takes on only at a discrete set of values.





Note:- The Amplitude/value of the discrete-time signal between two time instants is "UNDEFINED"

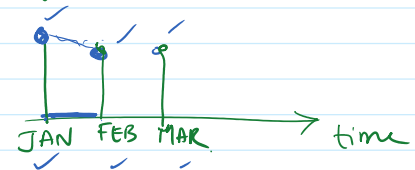
Ex: [CTS]
 $x(t)$

Speech signal is a function time

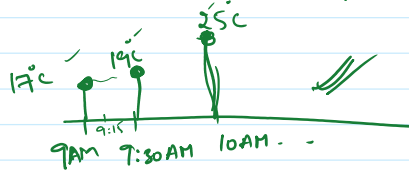
Atmospheric pressure is a function of altitude

[DTS]:
 $x[n]$

GDP [regular interval]
temperature captured at regular interval of time



time
 t (CT)
 n (DT)



② Continuous valued & Discrete value Signal.

Signal takes all possible values on a finite/an infinite range

Signal takes on values from a finite set of possible values

③ Multichannel & Multidimensional signals.

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$

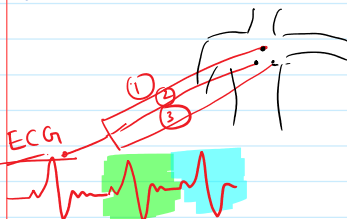
Signal $\Rightarrow$ function of single-independent variable. $\Rightarrow$ one-dimensional

M-independent variable $\Rightarrow$ M-dimensional signal.

Ex: ECG signals, EEG.

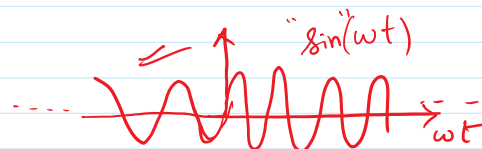
3 lead ECG signals $\Rightarrow$ 3-channel
12 lead ECG signals $\Rightarrow$ 12-channel.

Ex picture $\Rightarrow$ 2-D signal
video $\Rightarrow$ 3-D signal.



Single-channel, one-dimensional Signal

④ Deterministic & Random Signal



④ Deterministic & Random Signal

possible to represent
in mathematical/graphical form

cannot be represented
in mathematical form
(Ex (noise))



SINGULARITY FUNCTIONS/SIGNALS :

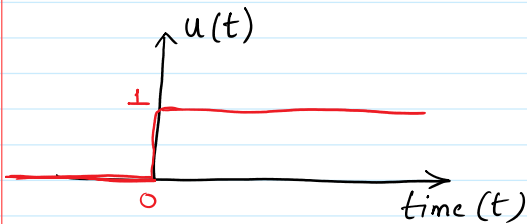
① UNIT STEP FUNCTION :

Continuous time

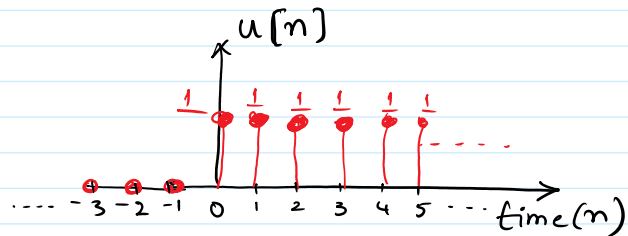
Discrete-time

$$* u(t) = \begin{cases} 1 & ; t > 0 \\ \frac{1}{2} & ; t = 0 \\ 0 & ; t < 0 \end{cases}$$

$$u(t) = \begin{cases} 1 & ; t > 0 \\ 0 & ; t < 0 \end{cases}$$



$$u[n] = \begin{cases} 1 & ; n \geq 0 \\ 0 & ; n < 0 \end{cases}$$

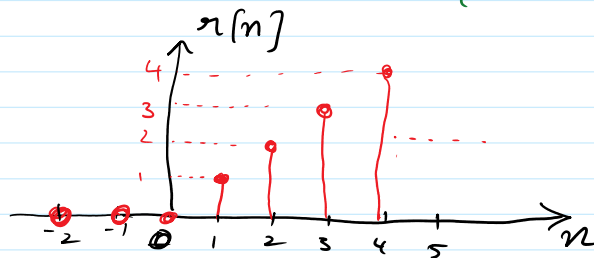
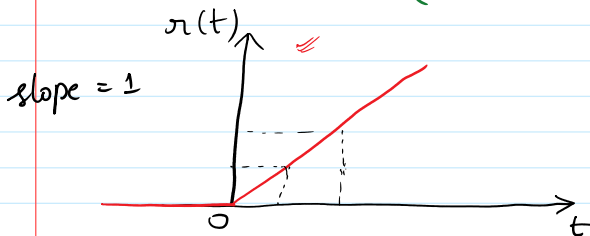


- "SINGULARITY": It is point at which a function does not possess a derivative
- Each singularity function has a singular point at the origin and is zero elsewhere

② UNIT RAMP FUNCTION/SIGNAL

$$r(t) = t \cdot u(t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases}$$

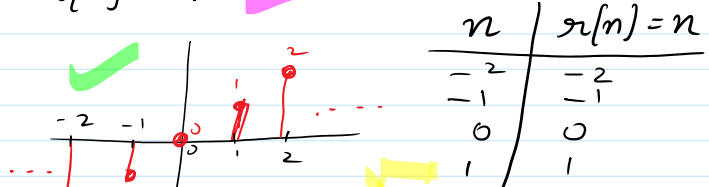
$$r[n] = n \cdot u[n] = \begin{cases} n & n \geq 0 \\ 0 & n < 0 \end{cases}$$

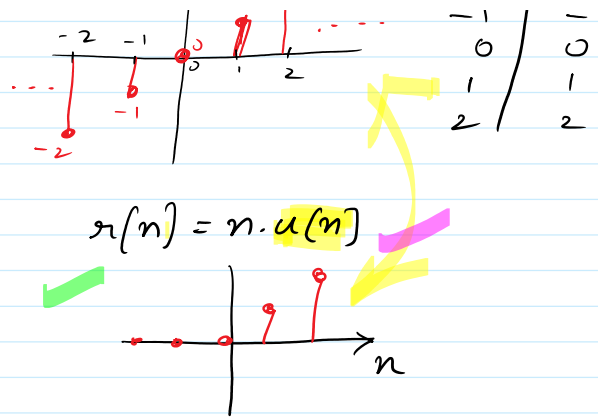
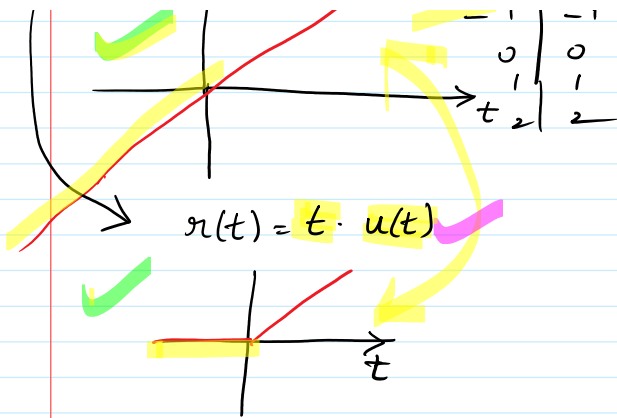


$$r(t) = t$$



$$r[n] = n$$





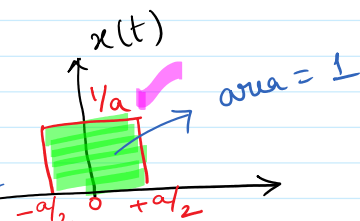
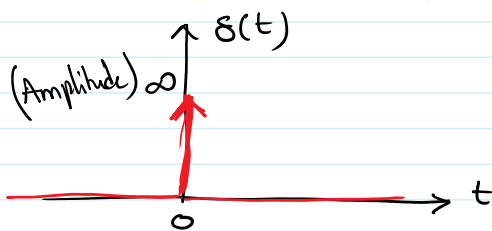
[3] UNIT IMPULSE FUNCTION / SIGNAL

C.T. $\swarrow$ D.T.

(Dirac-Delta-function)

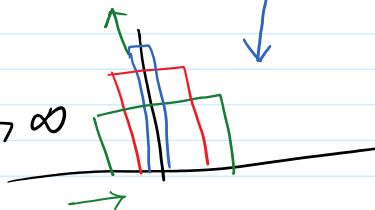
$$\delta(t) = 0 \quad t \neq 0$$

$$\int_{-\infty}^{\infty} \delta(t) dt = 1 \quad \text{area under unit impulse function is "one"}$$



height $\times$ width
 $\frac{1}{a} \times [a/2 - (-a/2)]$
 $\frac{1}{a} \times a = 1$

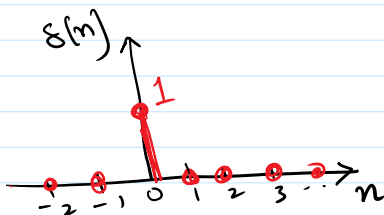
As width $\downarrow$ height $\uparrow$
 As width $\Rightarrow 0$ means height $\Rightarrow \infty$



Unit impulse function

Area = 1 Amplitude = ∞ (infinity)

DT $\delta[n] = \begin{cases} 1 & n=0 \\ 0 & n \neq 0 \end{cases}$



CT $\delta(t) = 0 \quad t \neq 0$
 $\int_{-\infty}^{\infty} \delta(t) dt = 1$

Some more Elementary signals.

Some more Elementary signals.

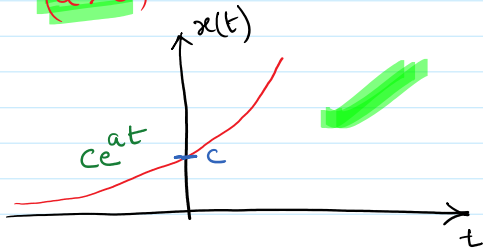
- ④ Real Exponential signals.
 Continuous time Exp signal.
 Discrete time Exp signal.

C.T.

$$x(t) = C \cdot e^{at}$$

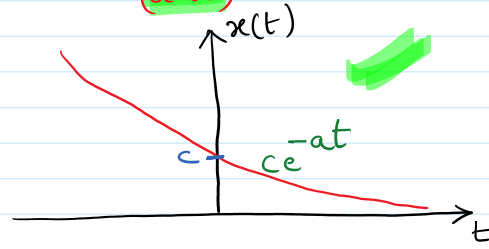
'C' and 'a' $\Rightarrow$ real parameters

a $\Rightarrow$ positive
(a > 0)



Growing Exponential

a $\Rightarrow$ negative
(a < 0)



Decaying Exponential.

Discrete time Exponential signal

real
Exp signal.

$$x[n] = C \cdot r^n$$

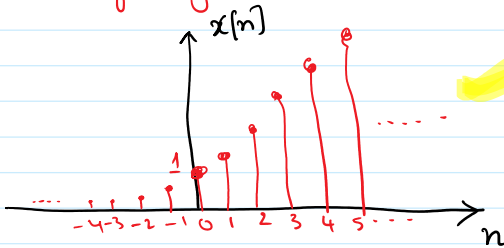
C & r $\Rightarrow$ real

Complex
Exp signal.

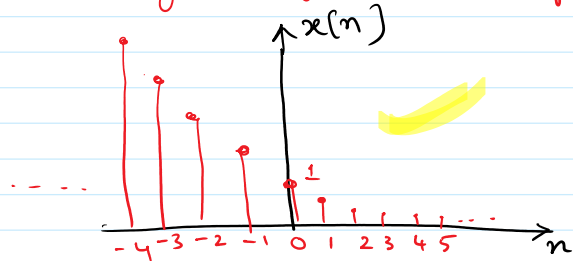
$$x[n] = C \cdot [e^{\alpha}]^n$$

where $r = e^{\alpha}$

if $|r| > 1$
signal grows Exponentially



if $|r| < 1$
signal decays Exponentially



⑤ Complex Exponential signal

$$x(t) = e^{j\omega t}$$

C.T.

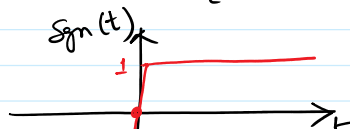
$$x[n] = e^{j\omega n}$$

D.T.

⑥ Signum Function/signal

C.T.

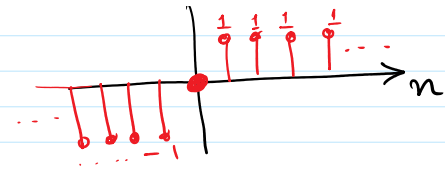
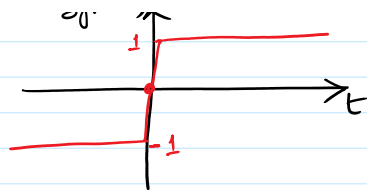
$$\text{Sgn}(t) = \begin{cases} 1 & t > 0 \\ 0 & t = 0 \\ -1 & t < 0 \end{cases}$$



D.T.

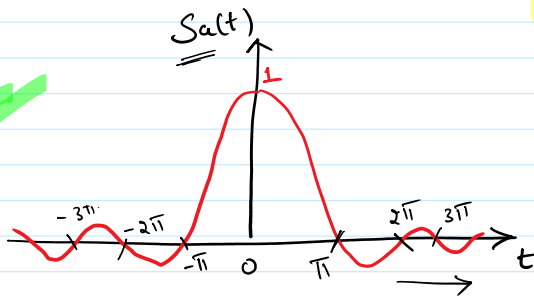
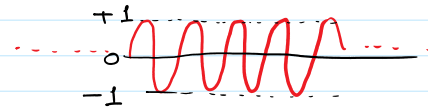
$$\text{Sgn}[n] = \begin{cases} 1 & n > 0 \\ 0 & n = 0 \\ -1 & n < 0 \end{cases}$$





Sampling Function/signal

$$S_a(t) = \frac{\sin t}{t}$$



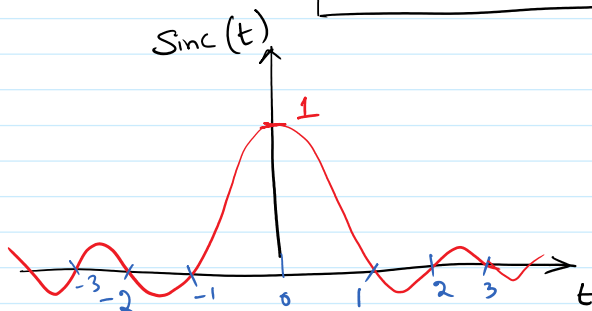
Denominator 't' ↑
numerator (sin t) ⇒ bounded
 $\frac{bt}{-1 \leq t \leq 1}$

Constant = ↓

$$\frac{5}{1} = 5 ; \quad \frac{5}{2} = 2.5, \quad \frac{5}{3} = 1.3 \dots$$

$$\begin{aligned} S_a(t) &= \frac{\sin t}{t} \\ S_{inc}(t) &= \frac{\sin \pi t}{\pi t} = S_a(\pi t) \end{aligned}$$

$$S_{inc}(t) = S_a(\pi t)$$



TRANSFORMATION OF INDEPENDENT VARIABLE (TIME)

- ① Time shifting
- ② Time Scaling
- ③ Time Reversal.

① Time shifting

$$x(t)$$

$$t \rightarrow t - t_0$$

↳ shifting factor

if $t_0 > 0 \Rightarrow$ signal is shifted to right

↳ [or] delaying the signal by t_0 units in time

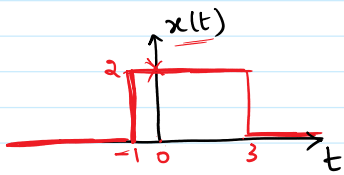
if $t_0 < 0 \Rightarrow$ signal is shifted to left

↳ [or] Advancing the signal by t_0 units in time

$$x(t) \rightarrow x(t - t_0) \begin{cases} t_0 > 0 \Rightarrow \text{right shift} \\ t_0 < 0 \Rightarrow x(t - (-t_0)) = x(t + t_0) \Rightarrow \text{left shift} \end{cases}$$

$$x(t) \rightarrow x(t-t_0) \Rightarrow \text{right shift}$$

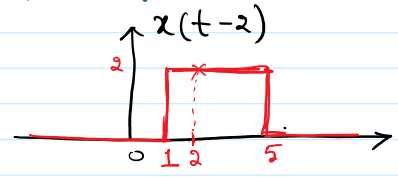
$$t_0 < 0 \rightarrow x(t-(-t_0)) = x(t+t_0) \Rightarrow \text{left shift}$$



$$x(t-t_0) \Rightarrow x(t-2)$$

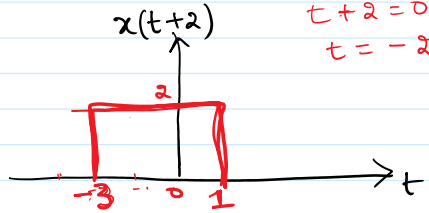
let $t_0 = 2$

$$x(t+t_0) \Rightarrow x(t+2)$$



$$(t-2)=0$$

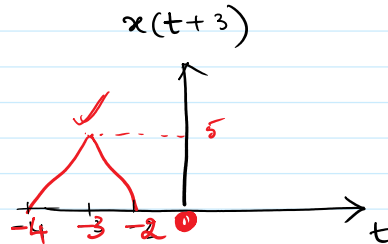
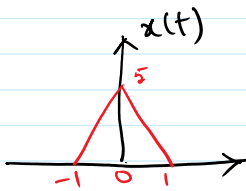
$$t=2$$



$$t+2=0$$

$$t=-2$$

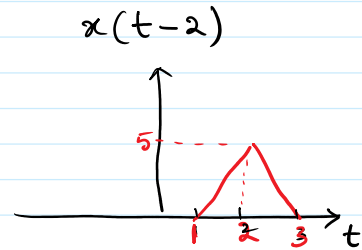
Ex



$$t+3=0$$

$$t=-3$$

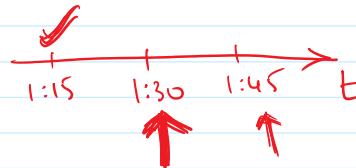
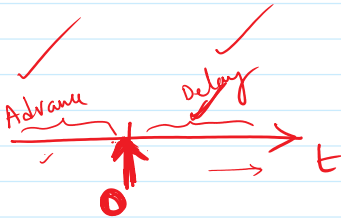
(Advanced)



$$t-2=0$$

$$t=2$$

(Delayed)



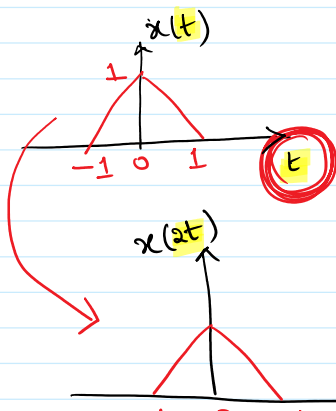
2 Time Scaling

Replacing 't' with 'at'

scaling factor

① if $a \geq 1 \Rightarrow$ Compression [speeded up]

② if $0 < a < 1 \Rightarrow$ Expansion [slowed down]



$$t \rightarrow at$$

let $a = 2$

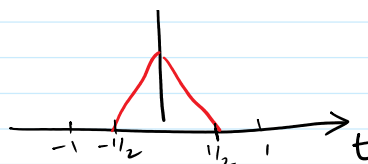


$$2t = +1$$

$$t = +1/2$$

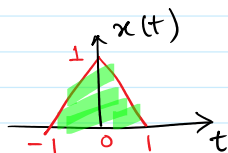


$\Rightarrow$

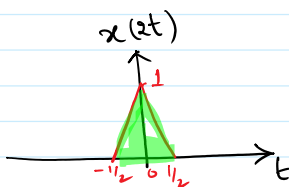


$$2t = +1 \\ t = +1/2$$

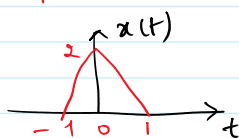
$$2t = -1 \\ t = -1/2$$



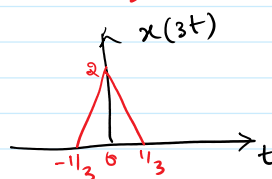
$\Rightarrow$



$a > 1$
Compression

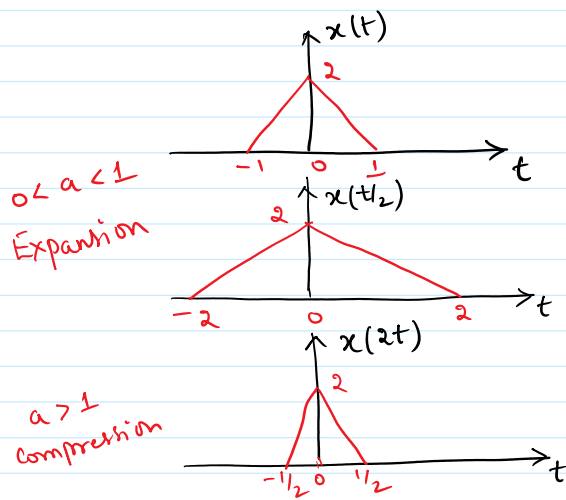


$\Rightarrow$



② if $0 < a < 1$
let $a = 0.5$

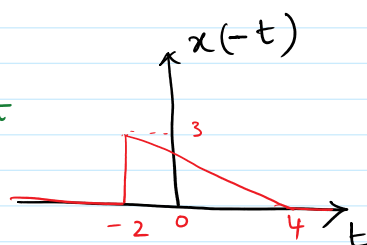
$$t \rightarrow at \\ t \rightarrow 0.5 \cdot t \\ t \rightarrow t/2$$



③ Time Reversal $t \rightarrow -t$ [about the vertical axis / $t=0$ line]
 $\hookrightarrow$ Reflection/Folding



$$t \rightarrow -t$$



C.T

T. shift $x(t) \rightarrow x(t - t_0)$
 $x(t) \rightarrow x(t + t_0)$

T.S $x(t) \rightarrow x(at)$
 $a > 1$
 $0 < a < 1$

D.T

$x[n] \rightarrow x[n - n_0]$ ✓ T. shift
 $x[n] \rightarrow x[n + n_0]$

$x[n] \rightarrow x[an]$ ✓ T. scaling
 $a > 1$
 $0 < a < 1$

T.S / $x(t) \rightarrow x(at)$ $0 < a < 1$

T.R / $x(t) \rightarrow x(-t)$

~~$x[n] \rightarrow x[an]$~~ $a > 1$
 $0 < n < 1$

$x[n] \rightarrow x[-n]$ T.R ✓

Signal

$x(at+b)$

$b \Rightarrow$ Shifting factor

$a \Rightarrow$ Scaling factor

① if $a=1, b < 0$

let $a=1$
 $b=-3$

$x(1 \cdot t - 3) = x(t-3)$

We get time-delayed version of $x(t)$

② if $a=1, b > 0$

let $a=1, b=2$

$x(1 \cdot t + 2) = x(t+2)$

time-advanced version of $x(t)$

③ if $a > 1, b=0$

$x(2t-0) = x(2t)$

let $a=2$

$\hookrightarrow$ Compressed version of $x(t)$

④ if $0 < a < 1, b=0$

$\hookrightarrow$ Expanded version of $x(t)$

⑤ if $a=-1, b=0 \Rightarrow$ time reversed version of $x(t)$

⑥ if $a > 1, b < 0 \Rightarrow$

⑦ if $0 < a < 1, b < 0 \Rightarrow$

⑧ if $-1 < a < 0, b=0 \Rightarrow$

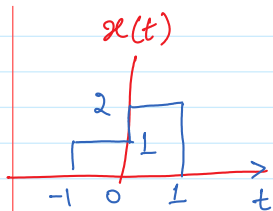
⑨ if $a < -1, b=0 \Rightarrow$

$x(t)$ $\begin{cases} \rightarrow \text{Time shifting} \\ \rightarrow \text{Time Reversal} \\ \rightarrow \text{Time scaling} \end{cases}$

$x(t) \rightarrow x(at+b)$ $\begin{cases} \text{shifting } b \geq 1 \text{ [left shifting]} \\ \text{or } b=0 \text{ [no shifting]} \\ b < 1 \text{ [Right shifting]} \end{cases}$
 $\downarrow$
 scaling / Reversal
 $\begin{cases} a > 1 \\ 0 < a < 1 \end{cases}$ (a = -1)

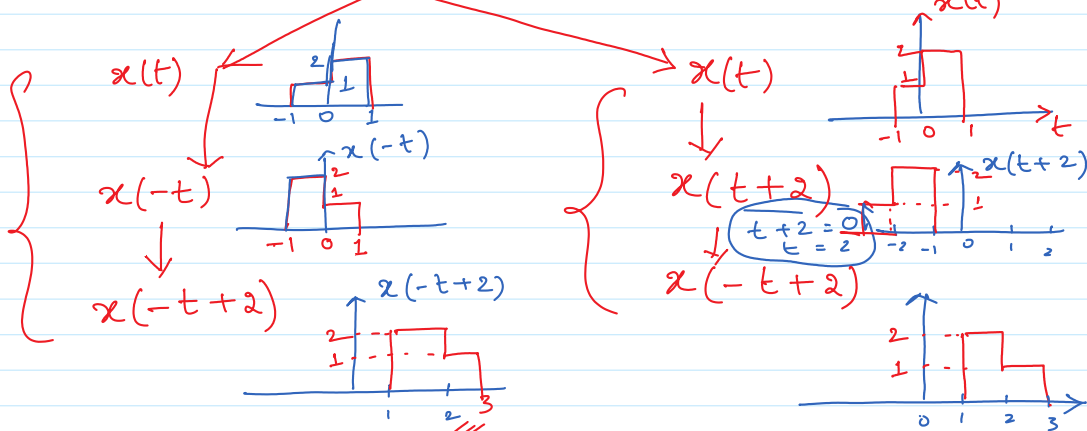
$x(t)$
 |

$x(t) \rightarrow x(-t+2)$
 $\downarrow \quad \downarrow$
 Rev Shifting



$$x(t) \rightarrow x(-t+2)$$

Rev Shifting



$$(-t+2)=0 \Rightarrow t=2$$

Note

- ① Time shifting
- ② Time scaling
- ③ Time Reversal

$$x(\pm at + b)$$

$$x(-at+b)$$

- ① Scaling
- ② Reversal

procedure

Ex ② $x(-3t+6)$

$$x(t) \xrightarrow{\text{T-shf}} x(t+6) \xrightarrow{\text{T-sca}} x(3t+6) \xrightarrow{\text{T-Reversal}} x(-3t+6)$$

Shift
Scaling
Reversal

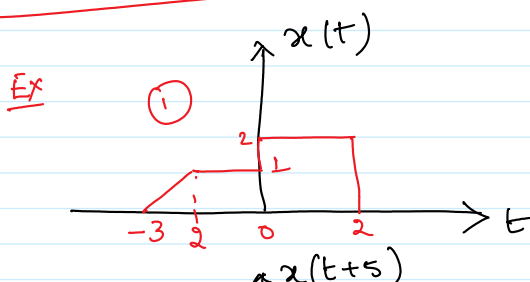
Ex ③ $x(-t+7)$

$$x(t) \xrightarrow{\text{T-R}} x(t+7) \xrightarrow{\text{T-sh}} x(-t+7)$$

Ex 4 $x(-(t-8))$

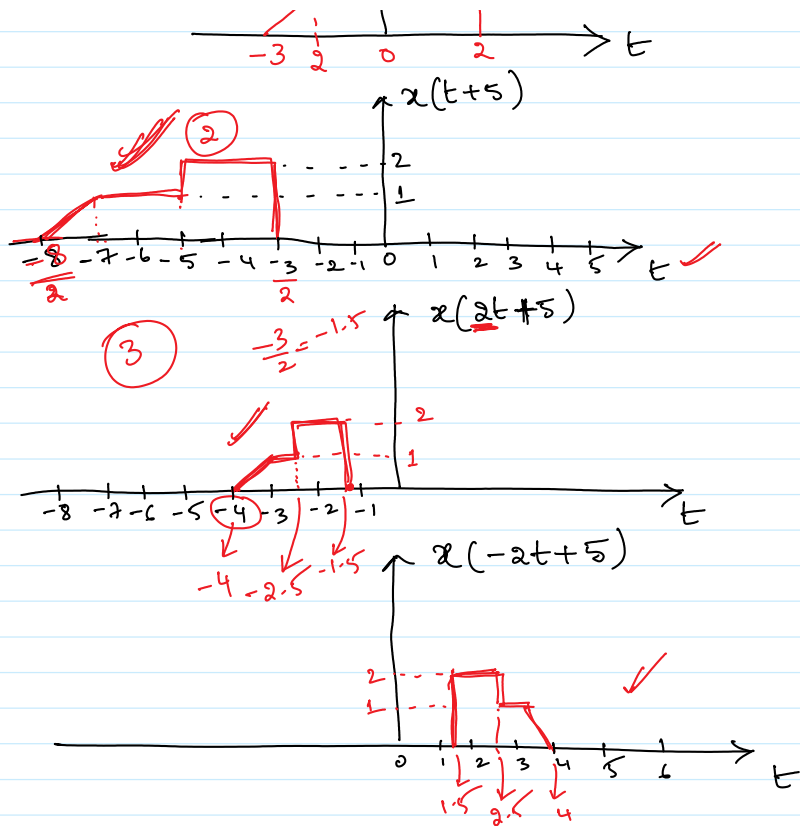
- ① $x(-t+8)$ ✓
- ② $x(-(t-8))$

- ① $x(t) \rightarrow x(t+8) \rightarrow x(-t+8)$ ✓
- ② $x(t) \rightarrow x(t-8) \rightarrow x(-(t-8))$ ✗



$$x(-2t+5)$$

- ① $x(t) \rightarrow x(t+5) \rightarrow x(2t+5)$
- ②
- ③
- ④



Comp

t

$\frac{1}{2}t$

$\frac{t}{2}$

Exp

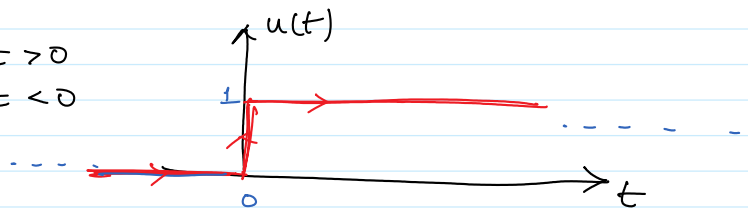
t

$0.5t$

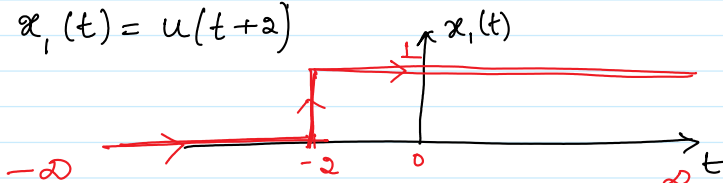
$$\frac{t}{0.5} = \frac{t}{1/2} = 2t$$

$$x(-2t+5)$$

Ex Given $u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$

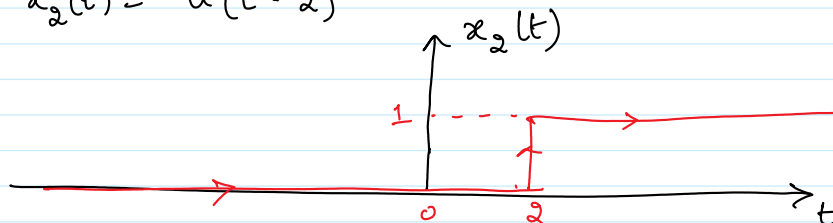


(a) $x_1(t) = u(t+2)$



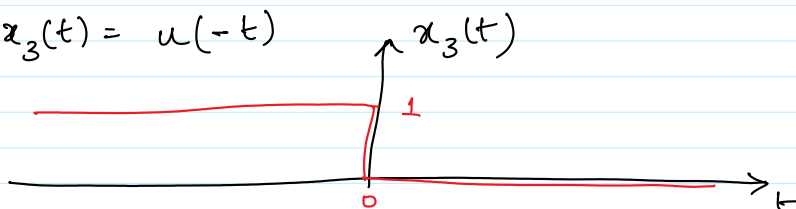
$$t+2=0 \\ t=-2$$

(b) $x_2(t) = u(t-2)$



$$t-2=0 \\ t=2$$

(c) $x_3(t) = u(-t)$



(4) $x_4(t) = u(-2t+1)$

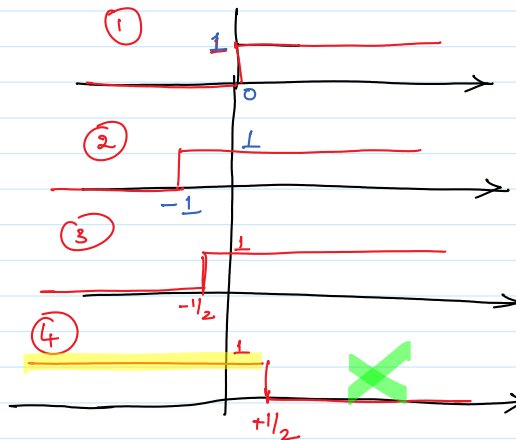
$$u(t) \rightarrow u(t+1) \rightarrow u(2t+1) \rightarrow u(-2t+1)$$

④ $x_4(t) = u(-2t+1)$

$u(t) \xrightarrow{(1)} u(t+1) \xrightarrow{(2)} u(\frac{1}{2}t+1) \xrightarrow{(3)} u(-2t+1)$

$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$

$u(-2t+1) \Rightarrow ?$



$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$

$u(-2t+1) = \begin{cases} 1 & -2t+1 > 0 \\ & -2t > -1 \\ & t < 1/2 \\ 0 & -2t+1 < 0 \\ & -2t < -1 \\ & t > 1/2 \end{cases}$

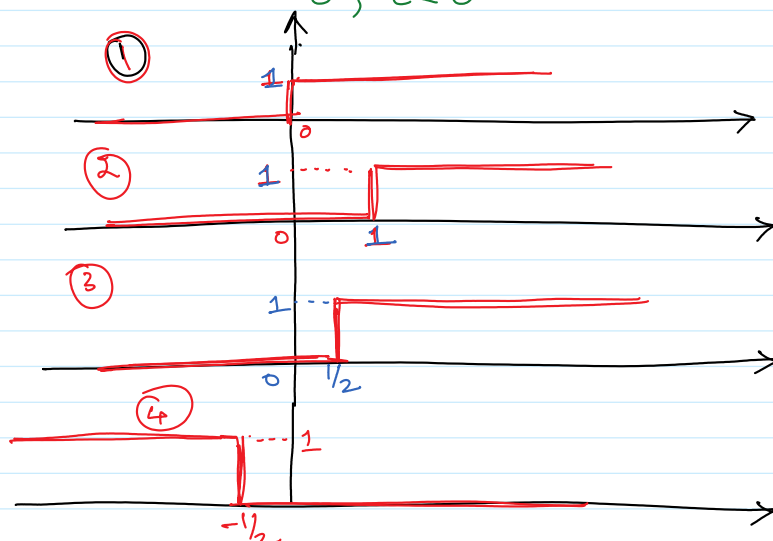
$u(-2t+1) = \begin{cases} 1 & t < 1/2 \\ 0 & t > 1/2 \end{cases}$

⑤ $x_5(t) = u(-2t-1)$

$u(t) \xrightarrow{(1)} u(t-1) \xrightarrow{(2)} u(2t-1) \xrightarrow{(3)} u(-2t-1)$

$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$

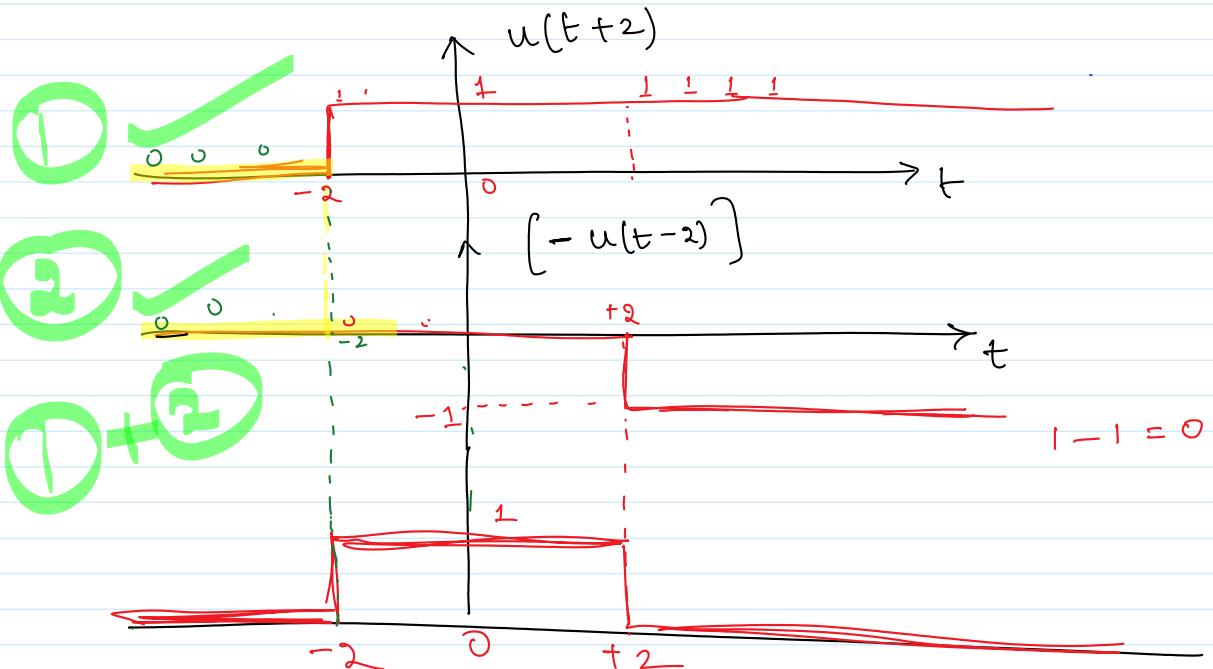
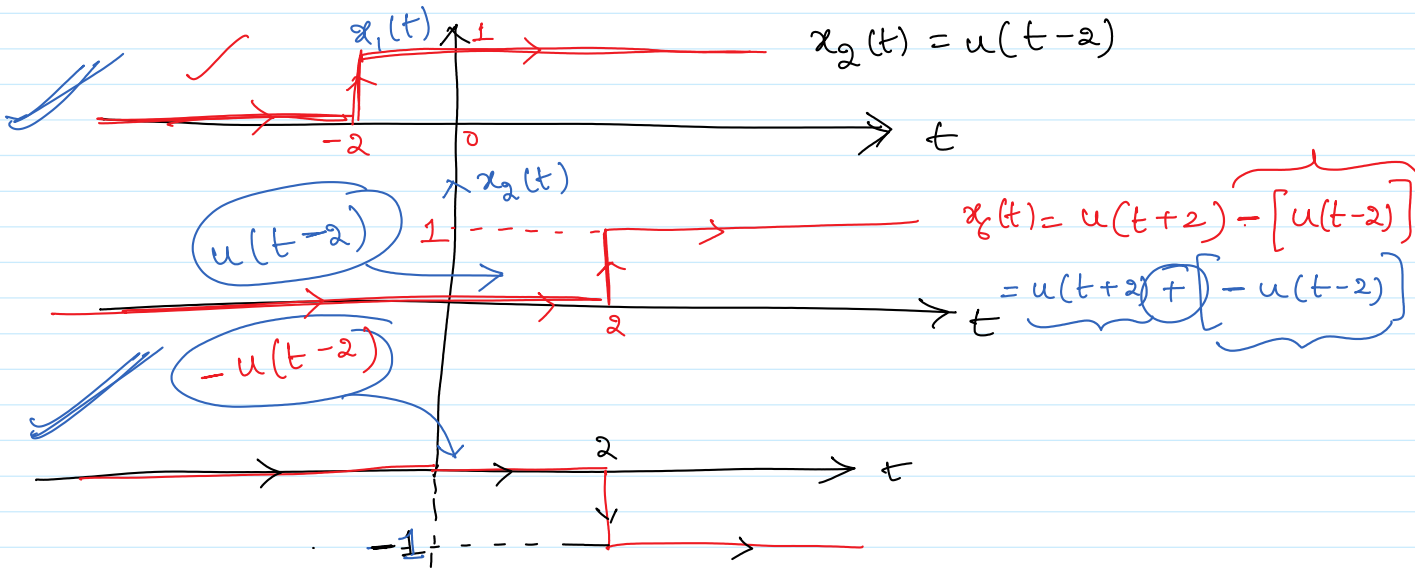
$u(-2t-1) = \begin{cases} 1 & -2t-1 > 0 \\ & -2t > 1 \\ & t < -1/2 \\ 0 & -2t-1 < 0 \\ & -2t < 1 \\ & t > -1/2 \end{cases}$



⑥ $x_6(t) = x_1(t) - x_2(t)$

$x_1(t) = u(t+2)$

$x_2(t) = u(t-2)$

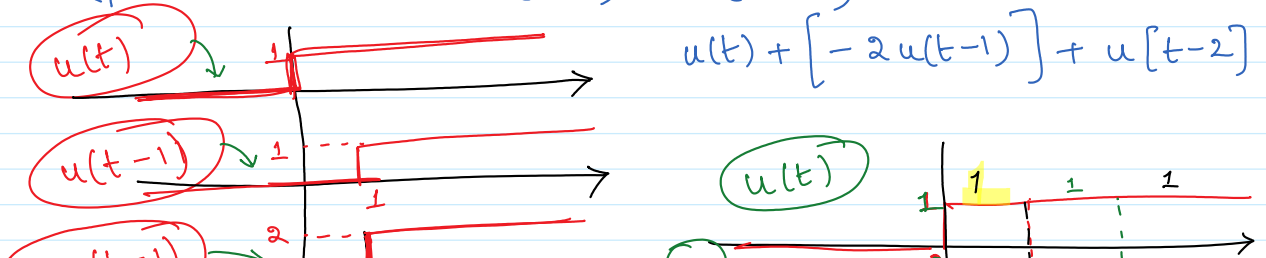


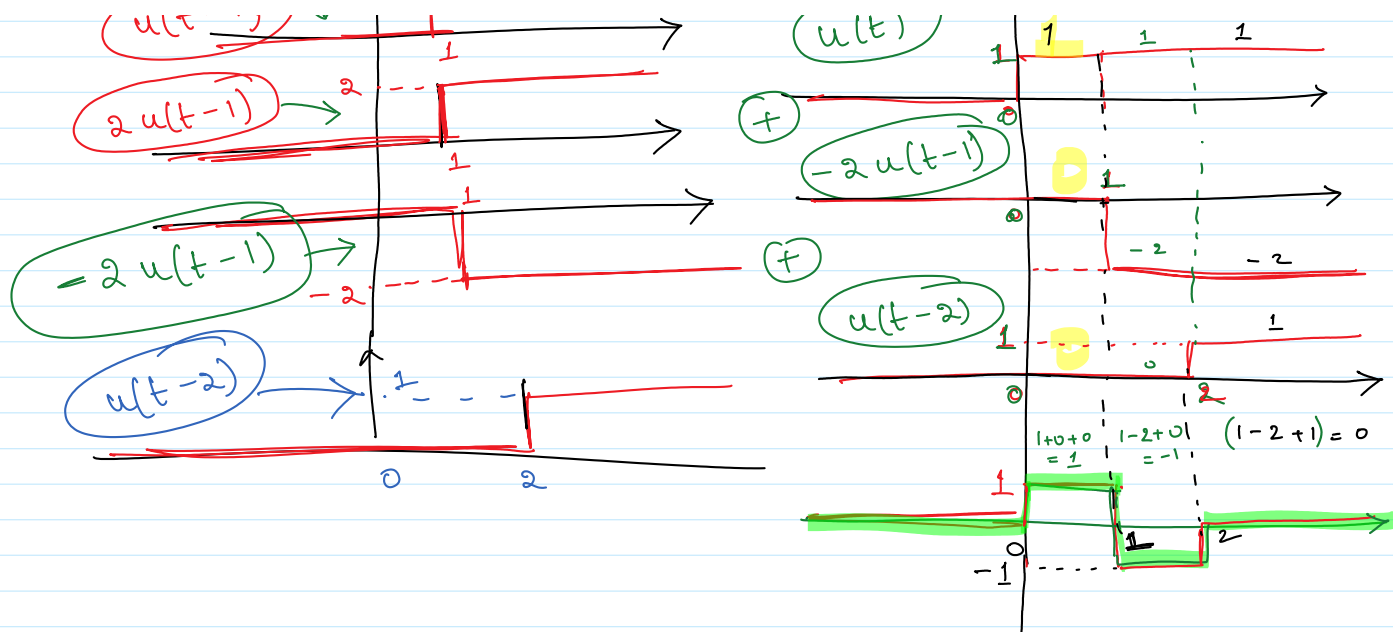
*

$$u(t+2) - u(t-2) = \begin{cases} 1 & ; -2 < t < 2 \\ 0 & \text{otherwise} \end{cases}$$

⑦ $x_7(t) = u(t) - 2u(t-1) + u(t-2)$

$u(t) + [-2u(t-1)] + u[t-2]$





Energy Signalif $0 < E_x < \infty$ ie., E_x finite and $P_x = 0$ then $x(t)/x[n] \Rightarrow$ Energy signalPower Signalif $E_x = \infty$ but $0 < P_x < \infty$ ie $P_x \Rightarrow$ finitethen $x(t)/x[n] \Rightarrow$ power signal $x(t) \Rightarrow$ Energyor
 $x[n]$

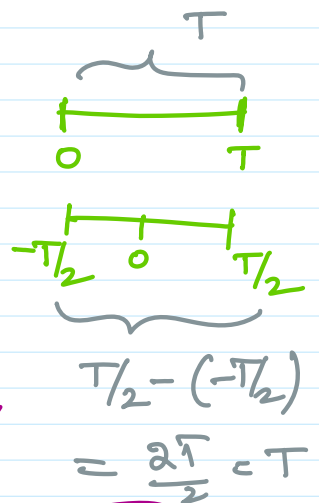
$$\begin{cases} E_{x(t)} = \lim_{T \rightarrow \infty} \int_{-T}^T |x(t)|^2 dt \\ E_{x[n]} = \lim_{N \rightarrow \infty} \sum_{n=-N}^N |x[n]|^2 \end{cases}$$

Avg power $\Rightarrow x(t)/x[n]$

$$P_{avg, x(t)} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

or

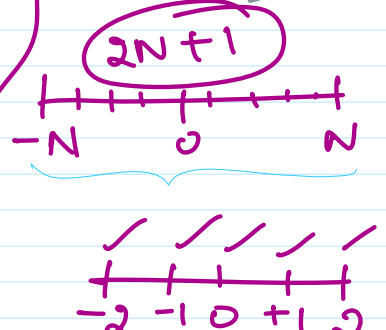
$$P_{avg, x(t)} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T |x(t)|^2 dt$$



$$P_{avg, x[n]} = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^{+N} |x[n]|^2$$

1 $\Rightarrow \sum$

2
 $\sum$



$$\boxed{\begin{matrix} \int \Rightarrow \sum \\ t \Rightarrow n \end{matrix}}$$

$$\left(\sum_{-2}^2 \quad \begin{array}{c} \text{---} \text{---} \text{---} \text{---} \text{---} \\ -2 \quad -1 \quad 0 \quad +1 \quad 2 \end{array} \right) \quad \boxed{2 - (-2) + 1}$$

Example Find whether the following signals are Energy/power signal?

(i) $x(t) = e^{-t} u(t)$

Soln Energy of $x(t) = E_{x(t)} = \lim_{T \rightarrow \infty} \int_{-T}^T |x(t)|^2 dt$

$$= \lim_{T \rightarrow \infty} \int_{-T}^T |e^{-t} u(t)|^2 dt$$

$$= \lim_{T \rightarrow \infty} \int_{-T}^T e^{-2t} \underbrace{u(t)} dt$$

$$= \lim_{T \rightarrow \infty} \int_{-T}^T e^{-2t} dt$$

$$= \lim_{T \rightarrow \infty} \left. \frac{e^{-2t}}{-2} \right|_0^T = \lim_{T \rightarrow \infty} \frac{e^{-2T} - e^0}{-2}$$

$$= \lim_{T \rightarrow \infty} \frac{1 - e^{-2T}}{2} = \frac{1 - 0}{2} = \frac{1}{2}$$

$u(t) = u(t)$
 $u^2(t) = u(t)$
 $u(t) = 1 \quad t > 0$
 $= 0 \quad t < 0$

$$\boxed{\lim_{T \rightarrow \infty} e^{-2T} = 0}$$

$t = 0 \Rightarrow e^{-2 \times 0} = e^0 = 1$
 $t = 1 \Rightarrow e^{-2 \times 1} = e^{-2} = \frac{1}{e^2} = 0.135$
 $t = 2 \Rightarrow e^{-2 \times 2} = e^{-4} = \frac{1}{e^4} = 0.0183$

t	e^{-2t}
0	1
1	0.135
2	0.0183

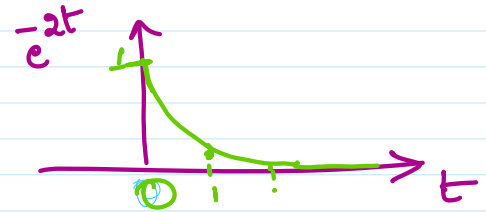
As $t \uparrow \Rightarrow e^{-2t} \downarrow$

As $t \rightarrow \infty$
 $\Rightarrow e^{-2t} \rightarrow 0$

-2t

$$\frac{1}{2} \mid 0.0183$$

$$\rightarrow e^{-2t} \rightarrow 0$$



$$\boxed{E_{x(t)} = \frac{1}{2}} \rightarrow \text{finite}$$

$$0 < E_{x(t)} < \infty$$

$$\checkmark 0 < \frac{1}{2} < \infty$$

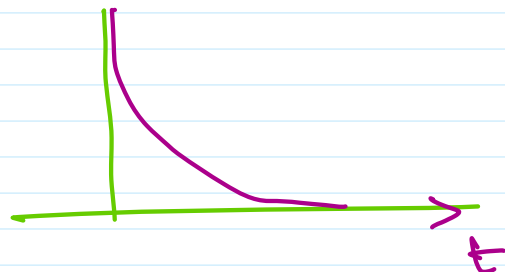
Avg power of $x(t) \Rightarrow P_{\text{avg}, x(t)} = \lim_{T \rightarrow \infty} \left(\frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt \right)$

$$* \boxed{P_{\text{avg}, x(t)} = \lim_{T \rightarrow \infty} \frac{E_{x(t)}}{T}}$$

$$= \lim_{T \rightarrow \infty} \frac{1/2}{T} = \frac{\text{Finite}}{\infty} = 0$$

$$\frac{2}{T}$$

T	2/T
0	∞
1	$2/1 = 2$
2	$2/2 = 1$
3	$2/3 = 0.66$
4	$2/4 = 0.5$



$$\checkmark E_{x(t)} = 1/2 \text{ finite}$$

$$P_{\text{avg}, x(t)} = 0$$

$\therefore$ Energy is finite & Avg power of $x(t)$ = Zero,
then the signal $x(t)$ is said to be

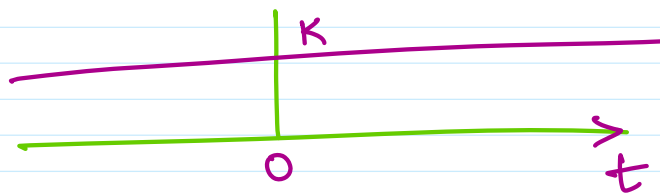
"ENERGY SIGNAL"

② $x(t) = K$ $K \Rightarrow \text{Constant}$

|K|

(2)

$$x(t) = K$$



$$E = \lim_{T \rightarrow \infty} \int_{-T}^T K^2 dt = \lim_{T \rightarrow \infty} K^2 \cdot 2T = \infty$$

$$P = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T}^T K^2 dt = \lim_{T \rightarrow \infty} \frac{1}{T} K^2 \cdot 2T = K^2 \quad (\text{finite})$$

"POWER SIGNAL"

Note A signal which maintains constant Amplitude over infinite time $\Rightarrow$ "POWER SIGNAL"

(3) $x_1(t) = e^{-2t} u(t)$ "ENERGY SIGNAL"

$$|x_1(t)|^2 = e^{-4t} \cdot u(t)$$

$$E_{x_1(t)} = \lim_{T \rightarrow \infty} \int_{-T}^T e^{-4t} u(t) dt = \lim_{T \rightarrow \infty} \int_0^T e^{-4t} dt$$

$$\lim_{T \rightarrow \infty} \left. \frac{e^{-4t}}{-4} \right|_0^T$$

$$\lim_{T \rightarrow \infty} \frac{e^{-4T} - e^{-4 \times 0}}{-4} = \lim_{T \rightarrow \infty} \frac{1 - e^{-4T}}{4} = \frac{1}{4} \quad (\text{finite})$$

$$E_{x_1(t)} = \frac{1}{4} \quad (\text{finite})$$

$$P_{x_1(t)} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T}^T |x_1(t)|^2 dt = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^{T/2} e^{-4t} u(t) dt$$

$$P_{x_1(t)} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x_1(t)|^2 dt = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} e^{-4t} u(t) dt$$

$$= \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^{T/2} e^{-4t} dt$$

$$\frac{1}{\infty} = 0$$

$$= \lim_{T \rightarrow \infty} \frac{1}{T} \left[\frac{e^{-4t}}{-4} \right]_0^{T/2} dt$$

$$= \lim_{T \rightarrow \infty} \frac{1}{T} \frac{e^{-4T/2} - 1}{-4} = \lim_{T \rightarrow \infty} \frac{1 - 0}{4T} = 0$$

$$P_{avg, x_1(t)} = 0$$

$$P_{avg, x_1(t)} = 0 \quad \sum \quad 0 < E_{x_1(t)} < \infty$$

ENERGY SIGNAL

(4) $x_2(t) = e^{j(2t + \pi/4)}$

$$|x_2(t)|^2 = 1$$

$$|x_2(t)|^2 = |e^{j(2t + \pi/4)}|^2 = 1$$

Magnitude

$$|e^{j(\cdot)}| = 1$$

$$E_{x_2(t)} = \lim_{T \rightarrow \infty} \int_{-T}^T |x_2(t)|^2 dt$$

$$= \int_{-\infty}^{\infty} |x_2(t)|^2 dt = \int_{-\infty}^{\infty} (1) dt = \infty$$

$$\lim_{T \rightarrow \infty} \int_{-T}^T |x_2(t)|^2 dt = \infty$$

$$P_{x_2(t)} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x_2(t)|^2 dt$$

$$= \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} 1 \cdot dt = \lim_{T \rightarrow \infty} \frac{1}{T} [T/2 - (-T/2)] = \lim_{T \rightarrow \infty} \frac{1}{T} \cdot T = 1 \text{ (finite)}$$

$E_{x_2(t)} = \infty$
 $0 < P_{x_2(t)} < \infty$ (finite) "POWER SIGNAL"

⑧ $x_5(t) = A \cos(t)$
 $|x_5(t)|^2 = A^2 \cos^2(t)$

$$E_{x_5(t)} = \int_{-\infty}^{\infty} A^2 \cos^2(t) dt$$

$$= \int_{-\infty}^{\infty} A^2 \left[\frac{1 + \cos(2t)}{2} \right] dt$$

$$A = 5$$

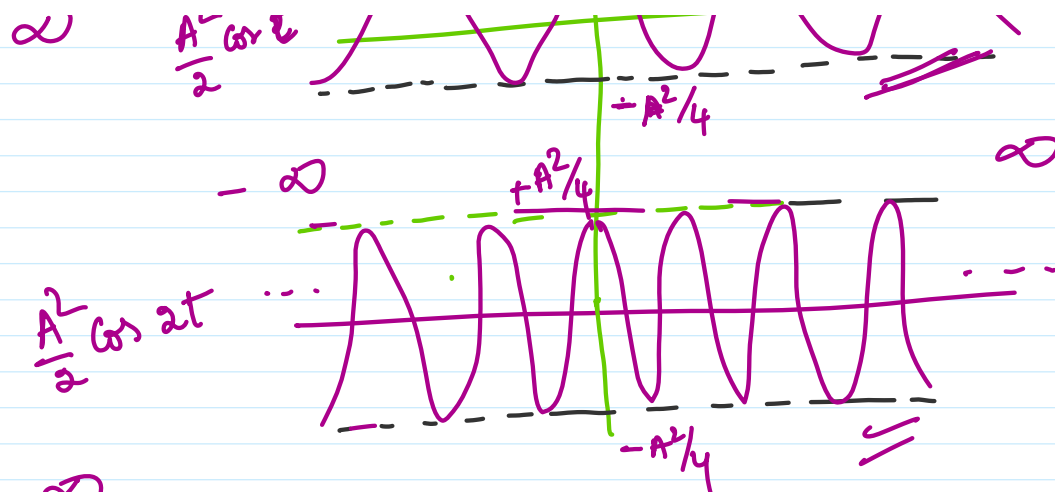
$$\frac{A^2}{2} = \frac{25}{2}$$

$$= \int_{-\infty}^{\infty} \frac{A^2}{2} dt + \int_{-\infty}^{\infty} \frac{A^2 \cos(2t)}{2} dt$$

$\Rightarrow \infty$
 $\Rightarrow \infty$

$$\frac{A^2}{4} - \left(-\frac{A^2}{4}\right) = \frac{A^2}{2}$$





$$E_{x_5(t)} = \infty + \infty = \infty$$

$$\begin{aligned} P_{x_5(t)} &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} A^2 \cos^2(t) dt \\ &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} \frac{A^2}{2} [1 + \cos 2t] dt \\ &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} \frac{A^2}{2} dt + \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} \frac{A^2}{2} \cos 2t dt \\ &\quad \underbrace{\hspace{10em}}_{= A^2/2} \quad \underbrace{\hspace{10em}}_{= 0} \end{aligned}$$

$$P_{x_5(t)} = \frac{A^2}{2} + 0 = \frac{A^2}{2} \text{ (finite)}$$

$$E_{x_5(t)} = \infty \quad \{ \quad 0 < P_{x_5(t)} < \infty \text{ (finite)} \}$$

"POWER SIGNAL"

⑥



$$x_6[n] = \left(\frac{1}{2}\right)^n u[n]$$

$$1, \dots, \frac{1}{2}, \frac{1}{4}, \dots, \frac{1}{2^n}, \dots$$

$$|| |x_6[n]|^2 = \left(\frac{1}{4}\right)^n u[n] ||$$

$$E_{x_6[n]} = \lim_{N \rightarrow \infty} \sum_{n=-N}^N |x_6[n]|^2 = \sum_{n=-\infty}^{\infty} |x_6[n]|^2$$

$$= \sum_{n=-\infty}^{\infty} \left(\frac{1}{4}\right)^n \underline{u[n]}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n = \frac{1}{1 - 1/4} = 4/3$$

$$\boxed{\sum_{n=0}^{\infty} (a)^n ; a < 1 = \frac{1}{1-a}}$$

$$P_{x_6[n]} = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^N |x_6[n]|^2$$

$$= \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=0}^N \left(\frac{1}{4}\right)^n$$

$$= \lim_{N \rightarrow \infty} \frac{1}{2N+1} \cdot \frac{1 - \left(\frac{1}{4}\right)^{N+1}}{1 - 1/4} = \frac{(\quad)}{\infty} = 0$$

$$\boxed{\sum_{n=0}^N (a)^n = \frac{1 - (a)^{N+1}}{1 - a}}$$

$$0 < E_{x_6[n]} < \infty \quad \sum p_{x_6[n]} = 0$$

$x_6[n] \Rightarrow$ "ENERGY SIGNAL"

$$\textcircled{7} \quad x_7[n] = e^{j[\frac{\pi}{2}n + \pi/8]} \Rightarrow \text{POWER}$$

$$\textcircled{8} \quad x_8[n] = \cos\left[\frac{\pi}{4}n\right] \Rightarrow \text{POWER}$$

$$\textcircled{9} \quad x_9(t) = A \sin t \quad -\infty < t < \infty \Rightarrow \text{POWER}$$

Some important results

$$\textcircled{1} \quad \sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \quad ; |a| < 1$$

$$1 + a^1 + a^2 + a^3 + \dots \infty = \frac{1}{1-a}$$

$$\textcircled{2} \quad a + a^2 + a^3 + a^4 + \dots \infty = \frac{a}{1-a}$$

$$\textcircled{3} \quad \sum_{n=0}^N a^n = \frac{1-a^{N+1}}{1-a}$$

$$\textcircled{4} \quad \sum_{n=0}^N (1)^n = N+1 \Rightarrow \sum_{n=0}^2 (1) = \underbrace{1+1+1}_{=3}$$

$$\textcircled{5} \quad \sum_{n=0}^N n = \frac{N(N+1)}{2}$$

$$\textcircled{6} \quad \sum_{n=0}^N n^2 = \frac{N(N+1)(2N+1)}{6}$$

$$\textcircled{7} \quad \sum_{n=0}^N n^3 = \left[\frac{N(N+1)}{2} \right]^2$$

$$\textcircled{8} \quad \sum_{n=a}^b 1 = (b-a) + 1$$

$$\textcircled{6} \sum_{n=0}^{\infty} n = \left[\frac{\infty}{2} \right]$$

$$\textcircled{7} \sum_{n=a}^{\infty} 1 = (\infty - a) \cdot 1$$

$$\Downarrow \sum_{n=-2}^2 1 = 1+1+1+1+1 = 5$$

Ex For an Energy signal $x(t)$ with Energy E_x

show that

(i) The energy of the signal $x_1(t) = -x(t)$

$$x_2(t) = x(-t)$$

$$x_3(t) = x(t-T) \text{ is } E_x$$

Soln

$$x(t) \xrightarrow{\text{Energy}} E_x$$

$$\left. \begin{aligned} x_1(t) = -x(t) &\rightarrow E_x \\ x_2(t) = x(-t) &\rightarrow E_x \\ x_3(t) = x(t-T) &\rightarrow E_x \end{aligned} \right\}$$

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt \quad \text{or} \quad E_x = \lim_{T \rightarrow \infty} \int_{-T}^T |x(t)|^2 dt$$

$$\textcircled{1} E_{x_1} = \int_{-\infty}^{\infty} |x_1(t)|^2 dt = \int_{-\infty}^{\infty} |-x(t)|^2 dt = \int_{-\infty}^{\infty} |x(t)|^2 dt = E_x$$

$$E_{x(t)} = E_{x_1(t)} = E_x$$

$$\textcircled{2} E_{x_2} = \int_{-\infty}^{\infty} |x_2(t)|^2 dt = \int_{-\infty}^{\infty} |x(-t)|^2 dt$$

$$-t = \tau$$

$$\begin{aligned}
 \textcircled{2} \quad E_{x_2} &= \int_{-\infty}^{\infty} |x_2(t)|^2 dt = \int_{t=-\infty}^{\infty} |x(-t)|^2 dt \\
 &\quad \text{let } -t = \tau \quad \text{As } t \rightarrow -\infty \Rightarrow \tau = +\infty \\
 &\quad \quad -dt = d\tau \quad \quad t \rightarrow +\infty \Rightarrow \tau = -\infty \\
 &\quad \quad dt = -d\tau \\
 &= \int_{\tau=-\infty}^{\tau=+\infty} |x(\tau)|^2 (-d\tau) \\
 &= - \int_{\tau=\infty}^{-\infty} |x(\tau)|^2 d\tau \\
 E_{x_2(t)} &= + \int_{\tau=-\infty}^{\infty} |x(\tau)|^2 d\tau \equiv E_x = \int_{t=-\infty}^{\infty} |x(t)|^2 dt \\
 &= E_x
 \end{aligned}$$

$$E_{x(t)} = E_{x_2(t)} = E_x$$

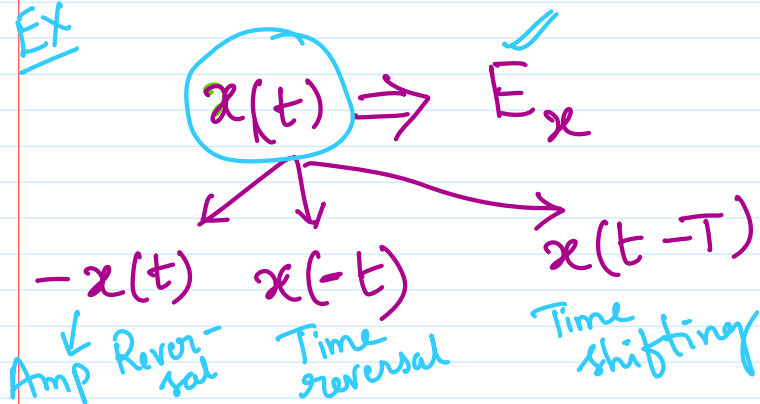
$$\begin{aligned}
 \textcircled{3} \quad x_3(t) &= x(t-T) \\
 E_{x_3} &= \int_{-\infty}^{\infty} |x_3(t)|^2 dt = \int_{t=-\infty}^{\infty} |x(\underbrace{t-T}_{\tau})|^2 dt \\
 &\quad \text{let } t-T = \tau \quad \text{As } t \rightarrow -\infty \Rightarrow \tau \rightarrow -\infty \\
 &\quad \quad \downarrow \text{shifting constant} \quad \text{As } t \rightarrow +\infty \Rightarrow \tau \rightarrow +\infty \\
 &\quad \quad dt = d\tau
 \end{aligned}$$

$$E_{x_3} = \int_{\tau=-\infty}^{\infty} |x(\tau)|^2 d\tau \equiv \int_{t=-\infty}^{\infty} |x(t)|^2 dt$$

$= E_x$

$$\therefore E_{x(t)} = E_{x_3(t)} = E_x$$

Ex



→ $x_4(t) = x(at)$

Given $E_{x(t)} = E_x$

$x_4(t) \Rightarrow E_{x_4} = ?$

$$E_{x_4} = \int_{-\infty}^{\infty} |x_4(t)|^2 dt = \int_{t=-\infty}^{\infty} |x(\underline{at})|^2 \underline{dt}$$

$$+at = \tau$$

$$a \cdot dt = d\tau \Rightarrow dt = \left(\frac{1}{a}\right) \cdot d\tau$$

As $t \rightarrow -\infty \Rightarrow \tau \rightarrow -\infty$

As $t \rightarrow +\infty \Rightarrow \tau \rightarrow +\infty$

$$= \int_{\tau=-\infty}^{\infty} |x(\tau)|^2 \cdot \left(\frac{1}{a}\right) d\tau$$

$$\begin{aligned}
 & \tau = -\infty \text{ to } \infty \quad (a) \\
 & = \left(\frac{1}{a}\right) \int_{-\infty}^{\infty} |x(\tau)|^2 d\tau = \left(\frac{1}{a}\right) \cdot E_x
 \end{aligned}$$

$$x_4(t) = x(at)$$

$$E_{x_4(t)} = \frac{E_x}{a} \quad | \quad E_x \Rightarrow E_x$$

$$\rightarrow x_5(t) = a x(t)$$

$$\begin{aligned}
 E_{x_5} &= \int_{-\infty}^{\infty} |a \cdot x(t)|^2 dt = a^2 \int_{-\infty}^{\infty} |x(t)|^2 dt \\
 &= a^2 \cdot E_x = E_x
 \end{aligned}$$

$$\rightarrow x_6(t) = x(at - b)$$

$$E_{x_6} = \int_{-\infty}^{\infty} |x(at - b)|^2 dt$$

$$\text{let } at - b = \tau$$

$$dt = (1/a) d\tau$$

$$\begin{aligned}
 \tau & \rightarrow -\infty \text{ as } t = -\infty \\
 \tau & \rightarrow +\infty \text{ as } t = +\infty
 \end{aligned}$$

$$E_{x_6} = \left(\frac{1}{a}\right) \int_{-\infty}^{\infty} |x(\tau)|^2 d\tau = E_x$$

Signal	Energy
$x(t)$	E_x
$x_4(t)$	E_x

$x(t)$	E_x
$-x(t)$	E_x
$x(-t)$	E_x
$x(at)$	E_x/a
$x(t-T)$	E_x
$x(at-b)$	E_x/a
$ax(t)$	$a^2 \cdot E_x$

Note - Time inversion and Time shifting do not affect signal Energy

- Time Compression of a signal $x(a)$ when $a > 1$ reduces the Energy

Ex $x(t) \Rightarrow E_x = 20 \text{ J}$

$x(at)$ $x(2t) \Rightarrow E_{x(2t)} = \frac{20}{2} = 10 \text{ J}$

when $a > 1$ $x(at) \Rightarrow$ Time Compressed signal

- Time Expansion of a signal $x(at)$ when $(0 < a < 1)$ increases the Energy

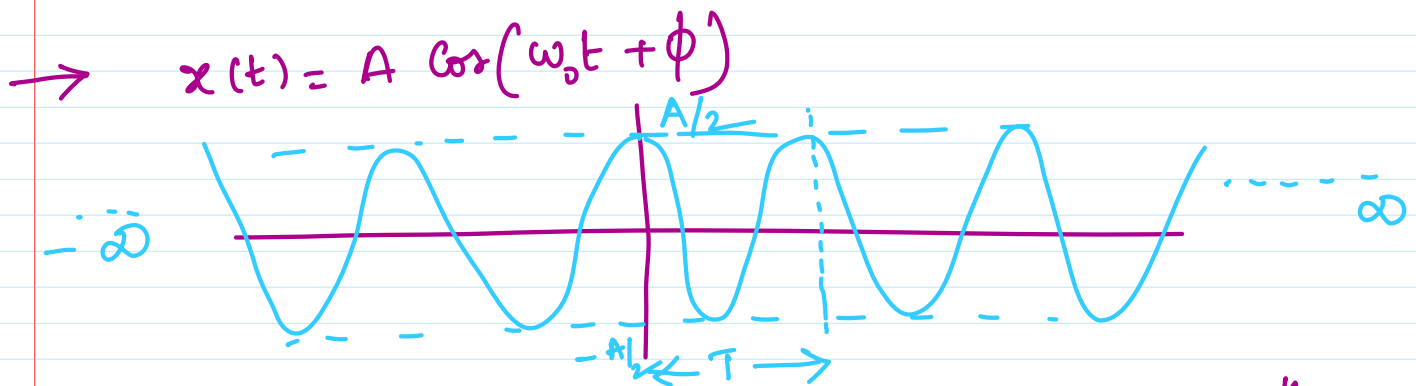
Ex $x(t) \Rightarrow E_x = 20 \text{ J}$

$x(at); 0 < a < 1 \Rightarrow$ Expanded signal
let $a = 0.5$

$x(at) \Rightarrow x\left(\frac{t}{2}\right)$

$x(at) \Rightarrow \frac{E_x}{a} = \frac{20}{(1/2)} = 40 \text{ J}$

$$x(at) \Rightarrow \frac{E_x}{a} = \frac{20}{(1/2)} = 40 \text{ J}$$



Note All periodic signals are "Power Signals"

$$P_{\text{avg}} = (\text{R.M.S value})^2 = \left(\frac{A}{\sqrt{2}}\right)^2$$

$$x(t) = 10 \cos(4\omega_0 t)$$

$$P_{\text{avg}} = (\text{R.M.S value})^2 = \left(\frac{10}{\sqrt{2}}\right)^2 = \frac{100}{2} = 50 \text{ W}$$

→ $x(t) = \underbrace{10}_{\text{Constant}} + \underbrace{5 \sin(3\pi t + \pi/4)}_{\text{periodic signal}}$

Constant
⇓
power

periodic signal
⇓
power

= POWER SIGNAL

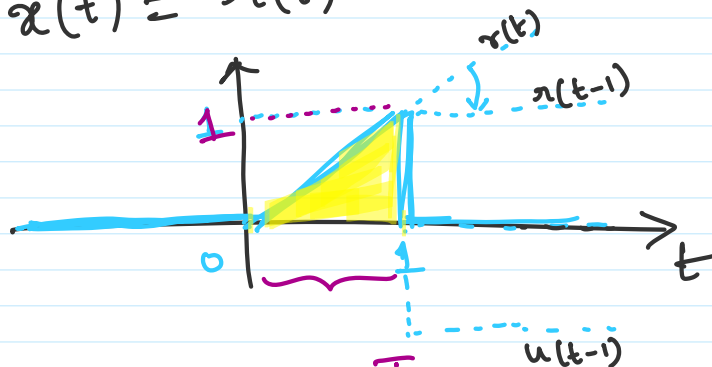
$$\text{Power} = 10^2 + \left(\frac{5}{\sqrt{2}}\right)^2 = 100 + 12.5 = 112.5 \text{ W}$$

→ $x(t) = (5 + 2 \sin 3t) \cos 5t$

$$\begin{aligned}
 \rightarrow x(t) &= (5 + 2 \sin 3t) \cos 5t \\
 &= 5 \cos 5t + 2 \sin 3t \cdot \cos 5t \\
 &= 5 \cos 5t + \left[\underbrace{1 \sin 8t}_{\substack{\text{Periodic signal} \\ \downarrow \\ \text{P.S}}} - \underbrace{1 \sin 2t}_{\substack{\text{Periodic signal} \\ \downarrow \\ \text{P.S}}} \right]
 \end{aligned}$$

$$\left(\frac{5}{\sqrt{2}} \right)^2 + \left(\frac{1}{\sqrt{2}} \right)^2 + \left(\frac{1}{\sqrt{2}} \right)^2 = \underline{\underline{13.5 \text{ W}}}$$

$$\rightarrow x(t) = x(t) - x(t-1) - u(t-1)$$



$$E_{x(t)} = \lim_{T \rightarrow \infty} \int_{-T}^T |x(t)|^2 dt = \int_0^1 t^2 dt$$

$$P_{\text{avg } x(t)} = \lim_{T \rightarrow \infty} \frac{\text{Energy}}{T} = \frac{\text{finite}}{\infty} = 0$$

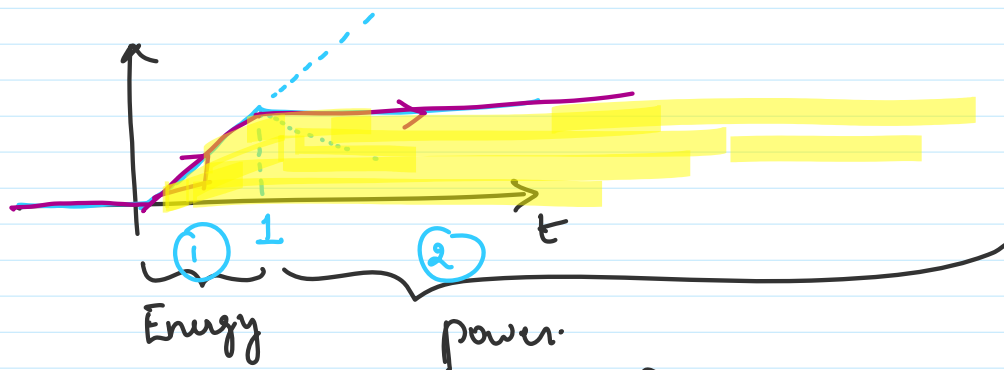
$E \Rightarrow \text{finite}$ $P \Rightarrow 0 \Rightarrow \text{"ENERGY"}$

Note

All Finite duration signals of finite Amplitude

No. All Finite duration signals of finite amplitude are "ENERGY SIGNALS"

$$\rightarrow y(t) = x(t) - x(t-1)$$



SUM OF ENERGY & power = POWER SIGNALS

$$P_{avg} = \lim_{T \rightarrow \infty} \left[\underbrace{\frac{1}{T} \int_0^1 t^2 dt}_{(1)} + \underbrace{\frac{1}{T} \int_1^T (1)^2 dt}_{(2)} \right]$$

$E \Rightarrow \text{Finite}$

$P \Rightarrow 0$

~~~~~

ENERGY

$E \Rightarrow \infty$

$P \Rightarrow \text{finite}$

~~~~~

POWER = POWER

$$\rightarrow A y(t) \Rightarrow \frac{\text{Energy}}{A^2 \cdot E_{y(t)}}$$

$$\underbrace{A}_{\text{Power signal}} + \underbrace{y(t)}_{E_{y(t)}} \Rightarrow \infty + E_{y(t)} = \infty$$

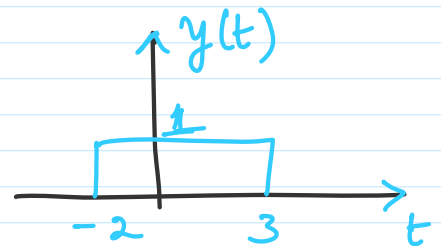
$$x(t) = \delta(t+2) - \delta(t-3)$$

$\rightarrow x(t) = \delta(t+2) - \delta(t-3)$
 $\hookrightarrow y(t) = \int_{-\infty}^t x(\tau) d\tau$ find Energy of $y(t) = \underline{\underline{5}}$

we know that

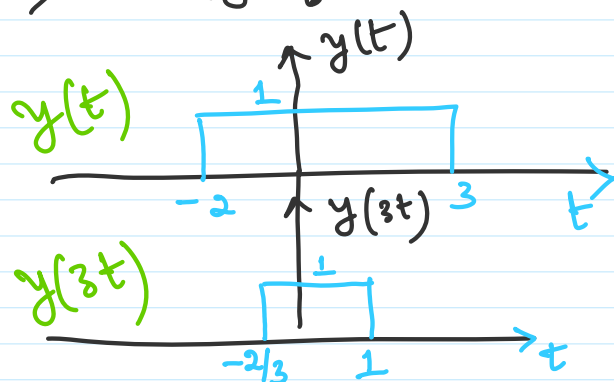
$$u(t) = \int_{-\infty}^t \delta(\tau) d\tau$$

$$y(t) = u(t+2) - u(t-3)$$

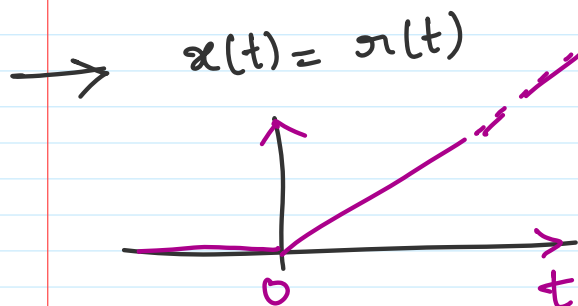


$$\text{Energy} = \int_{-2}^3 (1)^2 dt = 3 - (-2) = \underline{\underline{5 J}}$$

$y(3t) \Rightarrow \text{Energy ?}$



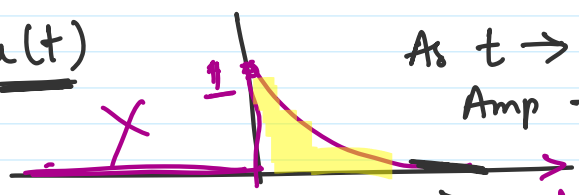
$$E_{y(3t)} = \int_{-2/3}^1 (1)^2 dt = \frac{5}{3} = \frac{E_{y(t)}}{3}$$



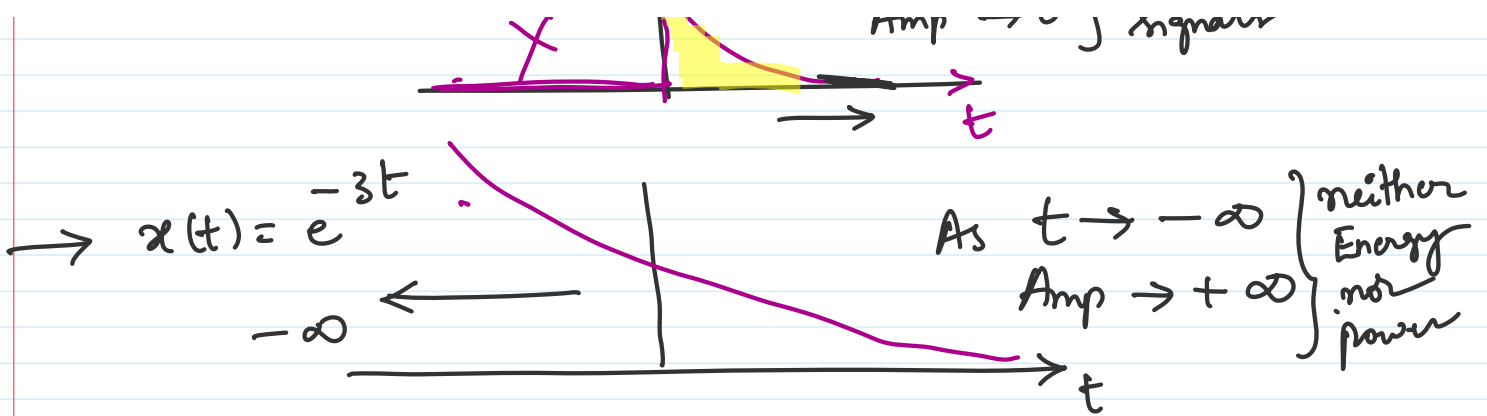
As $t \rightarrow \infty$
 Amp $\rightarrow \infty$

neither
 Energy
 nor
 power signals

$\rightarrow x(t) = e^{-3t} \underline{u(t)}$



As $t \rightarrow \infty$
 Amp $\rightarrow 0$ } Energy signals.

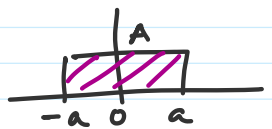


$x[n] = a^n u[n]$

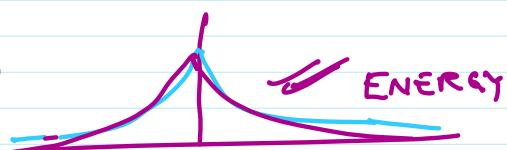
- ① $|a| < 1$ ENERGY
- ② $|a| = 1$ POWER ?
- ③ $a = -1$ POWER
- ④ $|a| > 1$ neither E NOR P

$x(t) = A \sin t \quad -\infty < t < \infty$

⑤ $x(t) = A [u(t+a) - u(t-a)]$

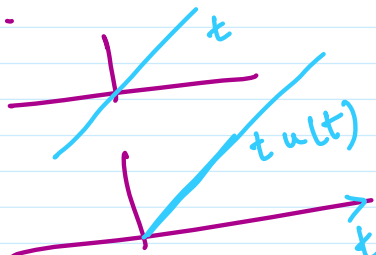


⑥ $x(t) = e^{-a|t|}; a > 0$

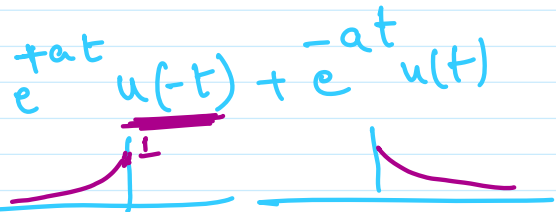


Neither E NOR P

⑦ $x(t) = t \cdot u(t)$

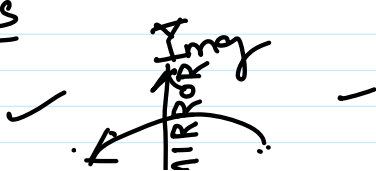


$a=1, t=0 \Rightarrow e = e^0 = 1$
 $a=1, t=-1 \Rightarrow e^{-1} = < 1$

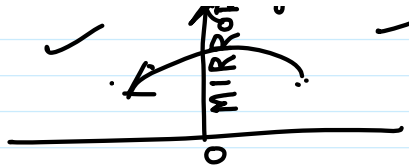


⑧ EVEN & ODD SIGNALS

EVEN $x(t)$ or $x[n]$

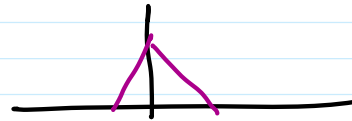
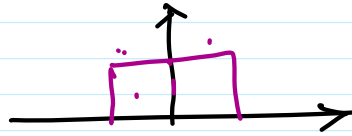


EVEN $x(t)$ or $x(n)$



CT $x(t) = x(-t)$
 DT $x[n] = x[-n]$ } - A signal is said to be even, if it is identical with its reflection about the origin

$x(t) \rightarrow x(-t)$
 EVEN



Even Signals are symmetric about vertical axis

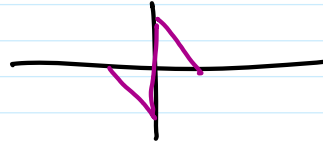
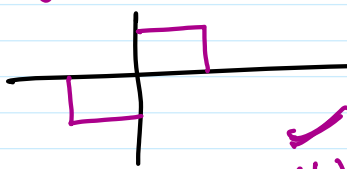
ODD

$$x(t) = -x(-t)$$

$$x[n] = -x[-n]$$

The value of an odd signal at the instant t (or n) is the negative of its value at the instant $-t$ (or $-n$)

- odd signal: It is AntiSymmetric about vertical axis.



$$x(t) \rightarrow x(-t) \rightarrow -x(-t)$$

Even & odd components of signal

C.T

$$x(t) = \sum \{x(t)\} + \ominus \{x(t)\}$$

$$x(t) = x_e(t) + x_o(t)$$

D.T

$$x[n] = x_e[n] + x_o[n]$$

Even part

$$\sum \{x(t)\} = x_e(t) = \frac{x(t) + x(-t)}{2}$$

$$\sum \{x[n]\} = x_e[n] = \frac{x[n] + x[-n]}{2}$$

Odd part

$$\circ \{x(t)\} = x_o(t) = \frac{x(t) - x(-t)}{2}$$

$$\circ \{x(n)\} = x_o(n) = \frac{x(n) - x(-n)}{2}$$

Properties

① let
 $x_1(t) \Rightarrow \text{odd}$ [if $x_1(t) = -x_1(-t)$
 $x_2(t) \Rightarrow \text{even}$ [if $x_2(t) = x_2(-t)$

② $x_3(t) = x_1(t) \cdot x_2(t)$
 $o = o \times e$

$x_3(t) \Rightarrow \text{odd signal}$

③ $x_4(t) = x_1(t) \cdot x_1(t)$
 $e = e \times e$

$x_4(t) \Rightarrow \text{even signal}$

④ $x_5(t) = x_2(t) \cdot x_2(t)$
 $e = e \times e$

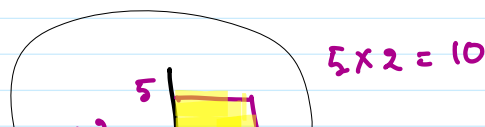
② If $x(t)$ is an even signal, then

$$\int_{-\infty}^{\infty} x(t) dt = 2 \int_0^{\infty} x(t) dt$$



Area $x(t) = \int_{-5}^5 2 \cdot dt$
 $2 \times \int_0^5 2 \cdot dt$

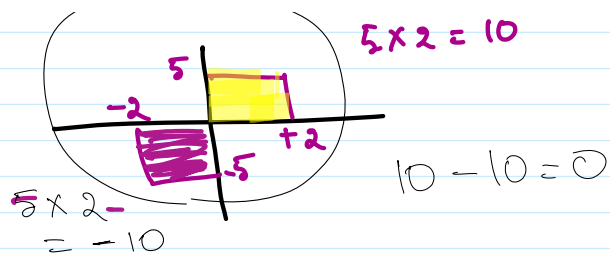
③ If $x(t)$ is odd signal, then



③ If $x(t)$ is odd signal, then

$$\int_{-\infty}^{\infty} x(t) dt = 0$$

Area



④ If $x(t)$ is an even signal, then its derivative is an odd signal

$$x(t) \Rightarrow \text{even}$$

$$x(t) = x(-t)$$

$$\frac{dx(t)}{dt} = -\frac{dx(-t)}{dt} \text{ (odd)}$$

⑤ If $x(t)$ is odd signal, then its derivative is an even signal.

$$x(t) = -x(-t)$$

$$\frac{dx(t)}{dt} = +\frac{dx(-t)}{dt} \text{ (even)}$$

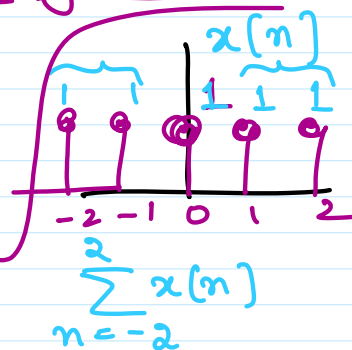
⑥ If $x(t)$ is an arbitrary signal with its even & odd parts x_e & x_o respectively, then

$$\int_{-\infty}^{\infty} x^2(t) dt = \int_{-\infty}^{\infty} x_e^2(t) dt + \int_{-\infty}^{\infty} x_o^2(t) dt$$

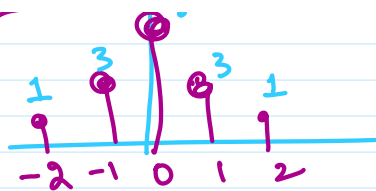
Properties of Discrete time even & odd signals

① If $x[n]$ is even signal, then

$$\sum_{n=-\infty}^{\infty} x[n] = x[0] + 2 \sum_{n=1}^{\infty} x[n]$$



$$x[0] + 2 \sum_{n=1}^{\infty} x[n]$$



$$x[n] = \{ \overset{-2}{1}, \overset{-1}{3}, \overset{0}{5}, \overset{1}{3}, \overset{2}{1} \}$$

$$= x[-2] + x[-1] + x[0] + x[1] + x[2]$$

$$= \sum_{n=-2}^2 x[n] = x[0] + 2 \sum_{n=1}^2 x[n]$$

② If $x[n]$ is odd signal,

$$\sum_{n=-\infty}^{\infty} x[n] = 0$$

③ $x[n] \Rightarrow$ arbitrary signal

$x_e[n] =$ even part of $x[n]$
 $x_o[n] =$ odd " " "

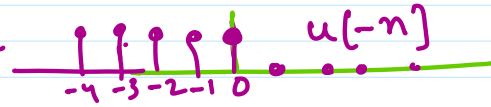
$$\sum_{n=-\infty}^{\infty} x^2[n] = \sum_{n=-\infty}^{\infty} x_e^2[n] + \sum_{n=-\infty}^{\infty} x_o^2[n]$$

Pb Determine the even and odd parts of unit step signal $u[n]$

$u[n] \rightarrow$ even part
 $\rightarrow$ odd part

$$\frac{u[n] + u[-n]}{2}$$

$$\frac{u[n] - u[-n]}{2}$$



n		-2	-1	0	1	2	
$u[n]$	... 0	0	0	1	1	1	1 ...
$u[-n]$	... 1	1	1	0	0	0	0 ...
$\Sigma \{u[n]\}$	$\dots \frac{1+0}{2} = \frac{1}{2}$	$\frac{1+0}{2} = \frac{1}{2}$	$\frac{1+0}{2} = \frac{1}{2}$	$\frac{1+1}{2} = 1$	$\frac{1+0}{2} = \frac{1}{2}$	$\frac{1+0}{2} = \frac{1}{2}$	$\frac{1}{2} \dots$
$O \{u[n]\}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{0-1}{2} = -\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2} \dots$

$$\left\{ \frac{1}{2} \quad n > 0 \right.$$

$$\sum \{u[n]\} = u_e[n] = \begin{cases} 1/2 & n > 0 \\ 1 & n = 0 \\ 1/2 & n < 0 \end{cases}$$

$$\bigcirc \{u[n]\} = u_o[n] = \begin{cases} -1/2 & n < 0 \\ 0 & n = 0 \\ 1/2 & n > 0 \end{cases}$$

Ex. $x(t) = e^{-2t} \cos(t)$

$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

$$= \frac{\cos(t) \left(\frac{e^{-2t} + e^{2t}}{2} \right)}{2}$$

$$x(t) = e^{-2t} \cos(t)$$

$$x(-t) = e^{+2t} \cos(-t)$$

$$= e^{2t} \cos(t)$$

$$\because \cos(-t) = \cos(t)$$

$$x_e(t) = \cos(t) \cdot \cosh(2t)$$

$$x_o(t) = \frac{x(t) - x(-t)}{2}$$

$$= \cos t \left[\frac{e^{-2t} - e^{2t}}{2} \right]$$

$$= -\cos t \cdot \left[\frac{e^{2t} - e^{-2t}}{2} \right] = -\cos t \cdot \sinh(2t)$$

Ex $x(t) = u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$

$$1 \Delta \cdot e^{-\alpha t} \quad t > 0$$

$$x_e(t) = 1/2 \quad \text{all } t \text{ except } t=0$$

$$x_o(t) = \begin{cases} 1/2 & t > 0 \\ -1/2 & t < 0 \end{cases}$$

Ex $x(t) = \begin{cases} A \cdot e^{-\alpha t} & t > 0 \\ 0 & t < 0 \end{cases}$

$$x_0(t) = \begin{cases} 1/2 & t > 0 \\ -1/2 & t < 0 \end{cases}$$

$$x_e(0) = 1/2$$

$$x_0(0) = 0$$

$$x_e(t) = \begin{cases} 1/2 A \cdot e^{-\alpha t}; & t \geq 0 \\ 1/2 A \cdot e^{\alpha t}; & t < 0 \end{cases}$$

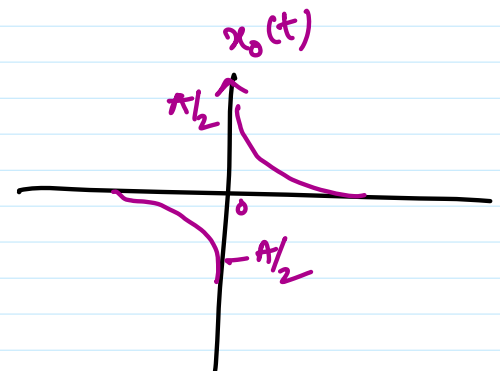
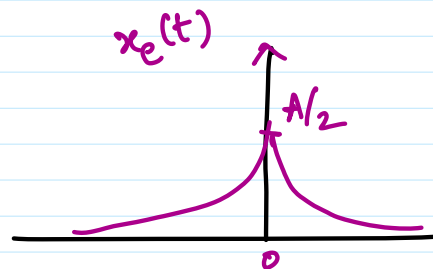
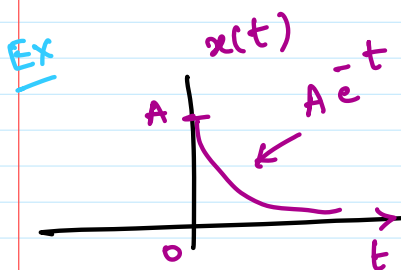
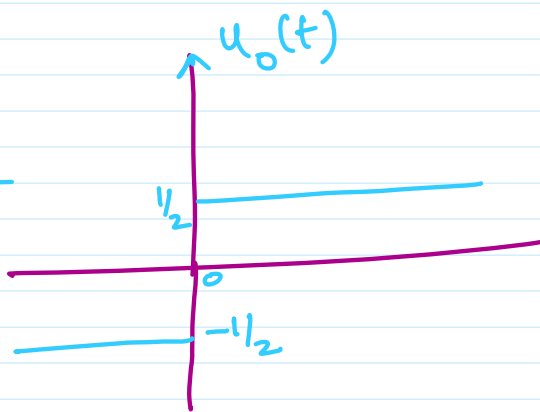
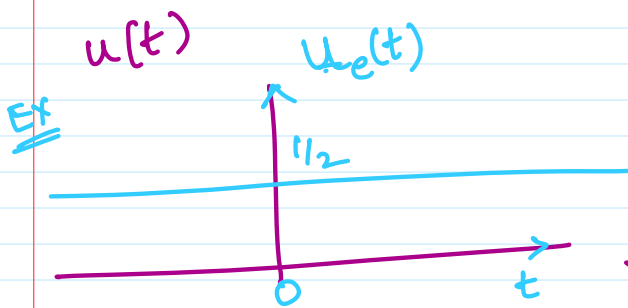
$$x_e(t) = \frac{1}{2} \cdot A \cdot e^{-\alpha|t|}$$

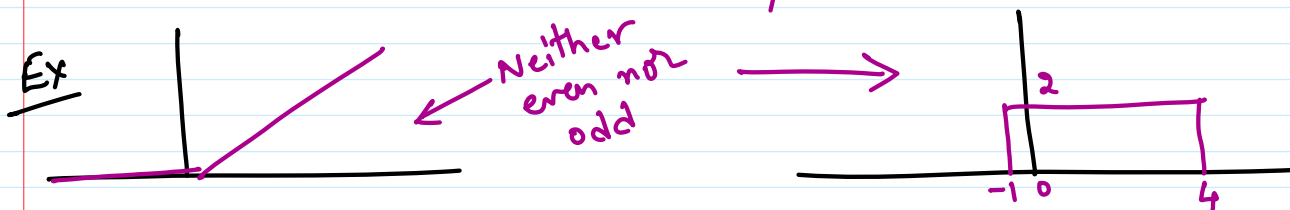
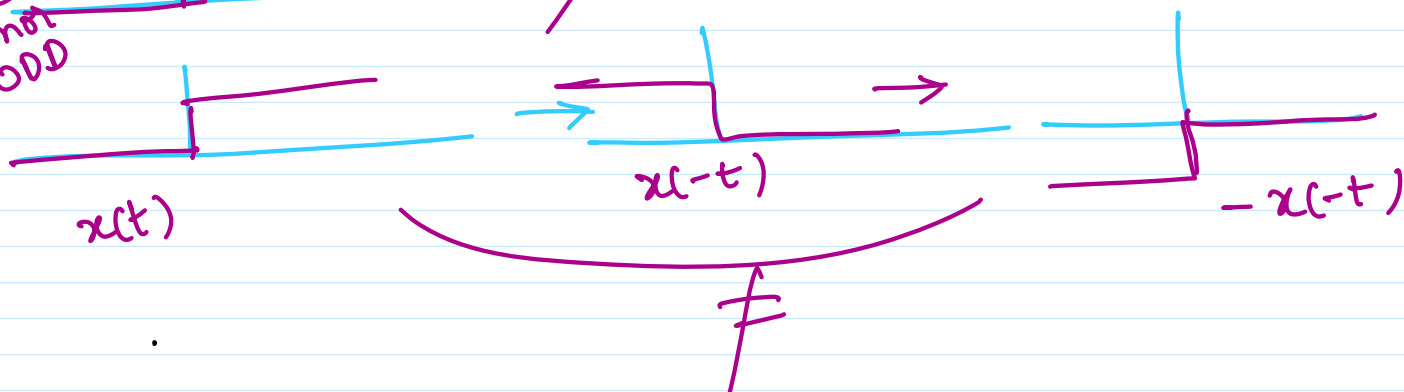
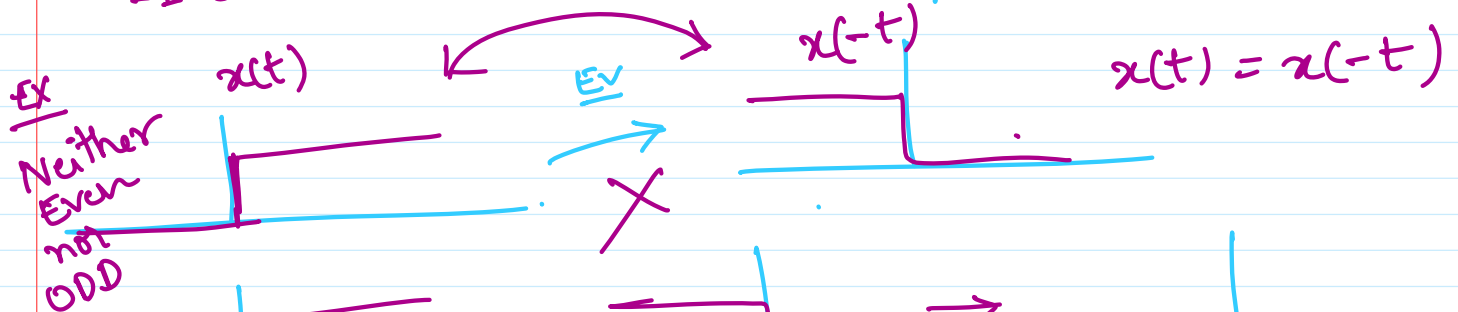
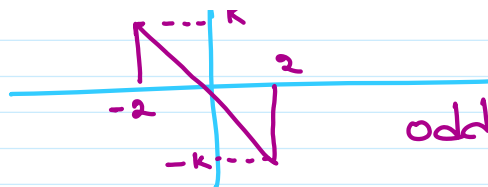
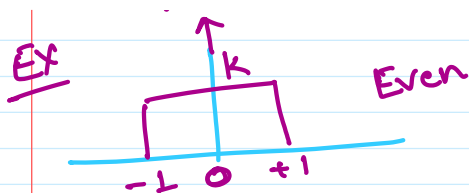
$$x_0(t) = \begin{cases} 1/2 A \cdot e^{-\alpha t}; & t \geq 0 \\ -1/2 A \cdot e^{\alpha t}; & t < 0 \end{cases}$$

$$e^{-\alpha|t|} = \begin{cases} e^{-\alpha(-t)} & t < 0 \\ e^{-\alpha(t)} & t \geq 0 \end{cases}$$

$$e^{-\alpha|t|} = e^{-\alpha(-t)} u(-t) + e^{-\alpha(t)} u(t)$$

$$e^{-\alpha|t|} = \underbrace{e^{+\alpha t} u(-t)}_{L.S.} + \underbrace{e^{-\alpha t} u(t)}_{R.S.}$$





Note It is not necessary that every signal be either even or odd, but every signal can be decomposed into sum of even and odd components.

Ex

$$x(t) = 1 + 2t + 3t^2$$

$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

$$= \frac{1 + 2t + 3t^2 + 1 - 2t + 3t^2}{2} = 1 + 3t^2$$

$$x(t) = 1 + 2t + 3t^2$$

$$x(-t) = 1 + 2(-t) + 3(-t)^2$$

$$x(-t) = 1 - 2t + 3t^2$$

$$\frac{2 + 6t^2}{2} = 1 + 3t^2$$

$$x_o(t) = \frac{x(t) - x(-t)}{2}$$

$$= \frac{1 + 2t + 3t^2 - (1 - 2t + 3t^2)}{2} = 2t$$

$$x_0(t) = \frac{\dots}{2}$$

$$= \frac{1+2t+3t^2 - (1-2t+3t^2)}{2} = \frac{1+2t+3t^2 - 1 + 2t - 3t^2}{2} = 2t$$

Ex $x[n] = \{-2, 1, 2, -4, 2\}$

$$x_e[n] = \{0, -1.5, 2, -1.5, 0\}$$

$$x_e[n] = \frac{x[n] + x[-n]}{2}$$

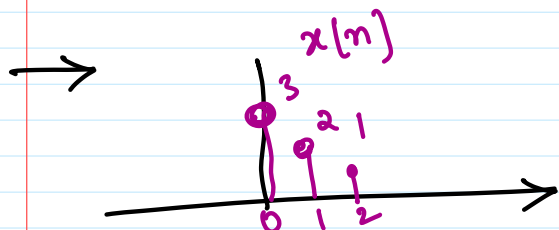
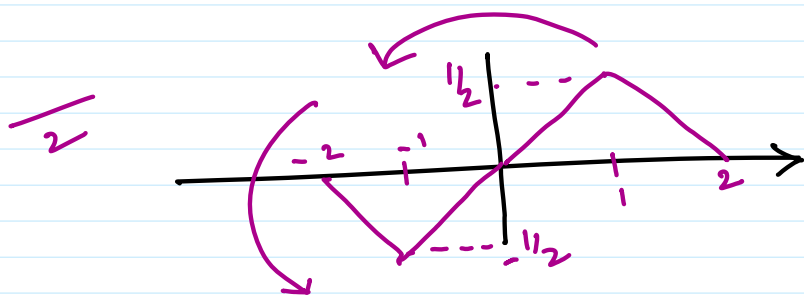
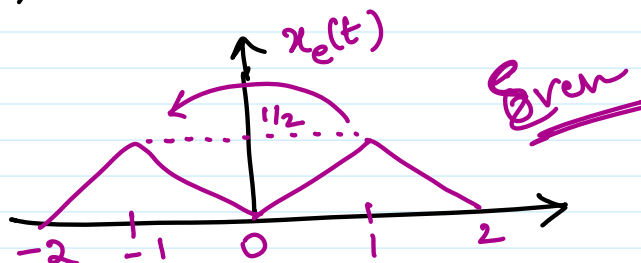
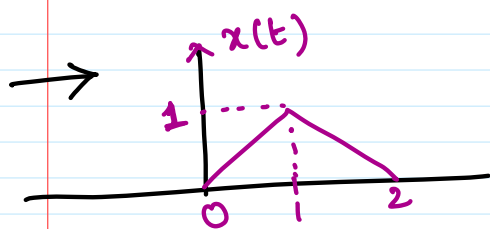
$$x[n] = \{-2, 1, 2, -4, 2\}$$

$$x[-n] = \{2, -4, 2, 1, -2\}$$

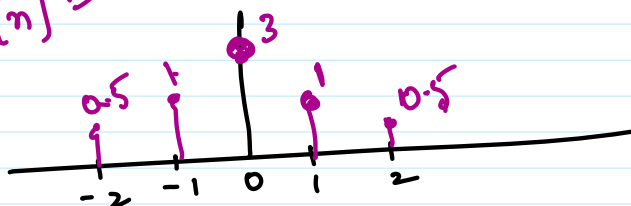
$$x_o[n] = \{-2, 2.5, 0, -2.5, 2\}$$

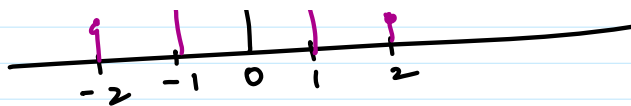
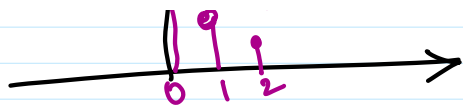
$$x_e[n] = \{0, -1.5, 2, -1.5, 0\}$$

$$x_o[n] = \{-2, 2.5, 0, -2.5, 2\}$$

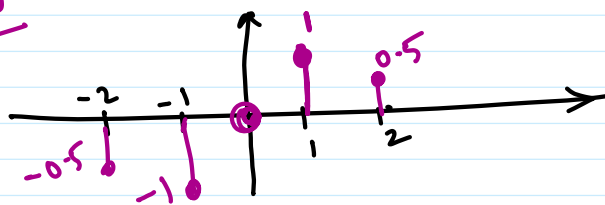


$$x_e[n] = \frac{x[n] + x[-n]}{2}$$





$$x_0[n] = \frac{x[n] \oplus x[-n]}{2}$$



For complex signal

Even
conjugate

$$x(t) = x^*(-t)$$

odd
conjugate

$$x(t) = -x^*(-t)$$

Ex
E. conj

$$x(t) = t^2 \cdot e^{j\omega t}$$

$$x(-t) = (-t)^2 \cdot e^{-j\omega t}$$

$$x^*(-t) = t^2 \cdot e^{-(j)\omega t}$$

$$x^*(-t) = t^2 \cdot e^{j\omega t}$$

Ex
O. conj

$$x(t) = t \cdot e^{j\omega t}$$

$$x(-t) = -t \cdot e^{j\omega(-t)}$$

$$x^*(-t) = -t \cdot e^{+j\omega t}$$

$$-x^*(-t) = +t \cdot e^{j\omega t}$$

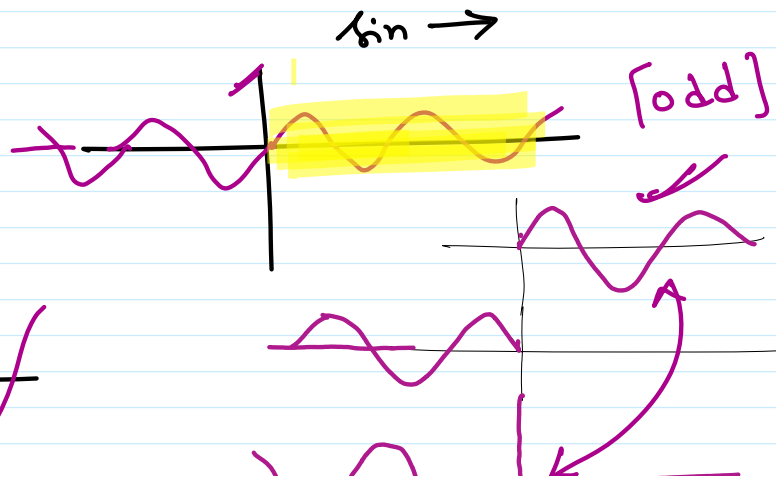
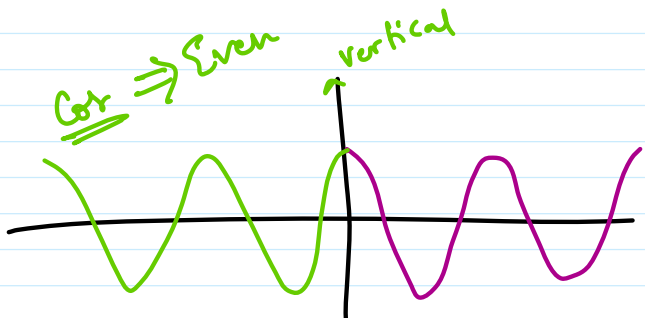
Ex

$$x(t) = e^{jt}$$

Even Conjugate

Ex

$$x(t) =$$





$\epsilon/0?$

Ex $x(t) = \sin t \cdot \sin t$
 $\downarrow \quad \downarrow \quad \downarrow$
 $\epsilon = 0 \times 0$

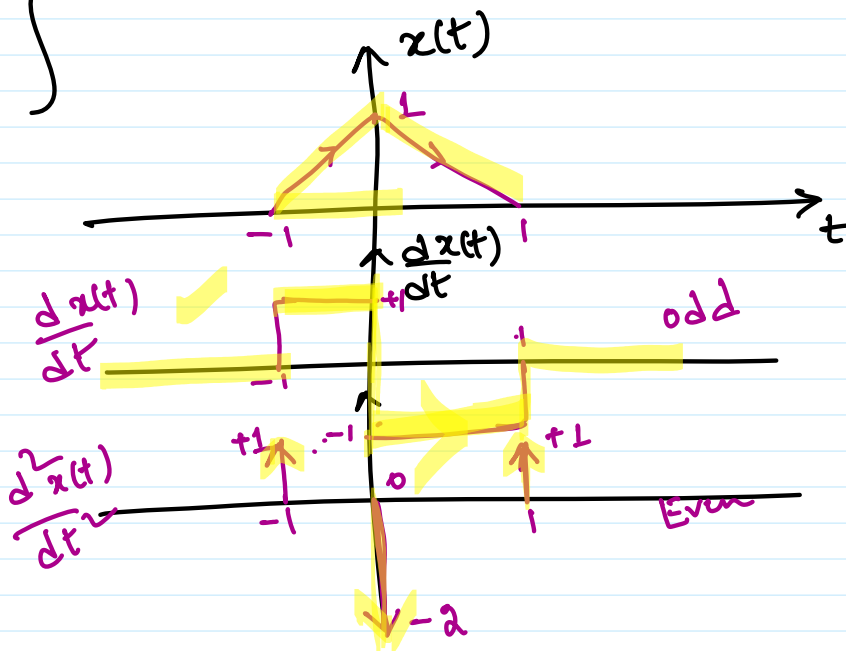
Ex $x(t) = \sin t \cdot \cos t$
 $\downarrow \quad \downarrow \quad \downarrow$
 $0 = 0 \times \epsilon$

Ex $x(t) = \cos t \cdot \cos t$
 $\downarrow \quad \downarrow \quad \downarrow$
 $\epsilon = \epsilon \times \epsilon$

Ex show that $\frac{dx(t)}{dt}$ and $\frac{d^2x(t)}{dt^2}$ of an even signal are odd & even respectively

$x(t) \Rightarrow \text{even}$

$\frac{dx(t)}{dt} \Rightarrow \text{odd}$
 $\frac{d^2x(t)}{dt^2} \Rightarrow \text{even}$ } show that



Ex Find even part $x_e(t)$

Ex Find even part
of $x(t) = \sin t \cdot u(t)$

→ PERIODIC / APERIODIC SIGNALS

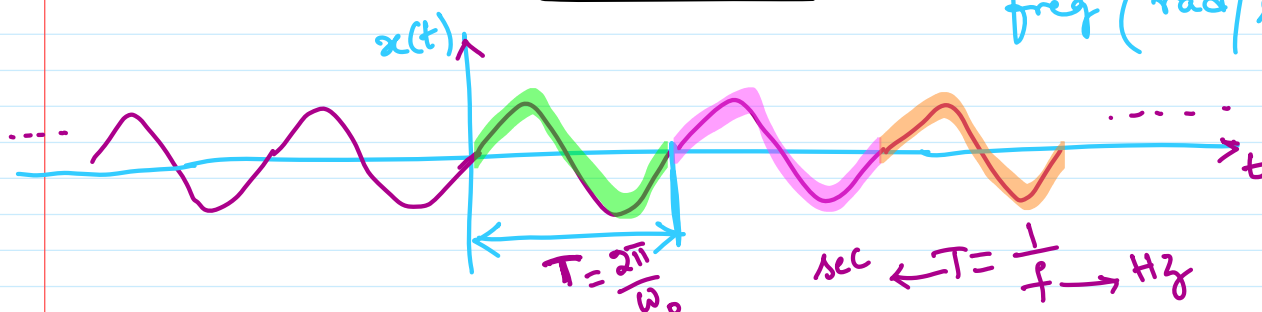
A signal is periodic if

- ① It must exist for $-\infty \leq t \leq \infty$
- ② It must repeat itself after some constant amount of time ' T '

T = fundamental Time period

$$T = \frac{2\pi}{\omega_0}$$

→ fundamental
freq (rad/sec)



Periodic
signal

$$x(t) = x(t+T) \quad \forall 't'$$

$$x[n] = x[n+N] \quad \forall 'n'$$

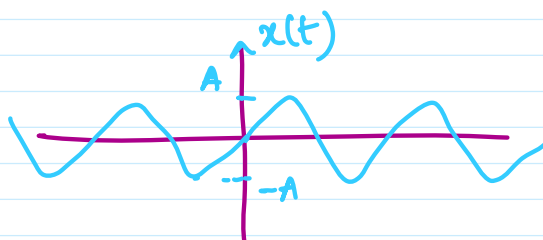
General Representation

$$x(t) = A \sin(\omega_0 t + \phi)$$

Amplitude

Phase
angle

Phase
shift



→ $x(t) = 4 \sin \overset{= \omega_0}{300\pi} t$

Comparing with standard representation

$$\omega_0 = 300\pi \text{ rad/sec}$$

$$\omega_0 = 300\pi \text{ rad/sec}$$

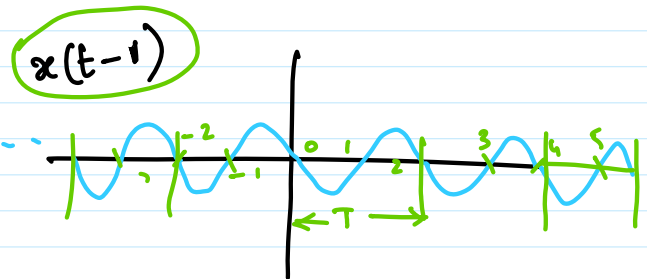
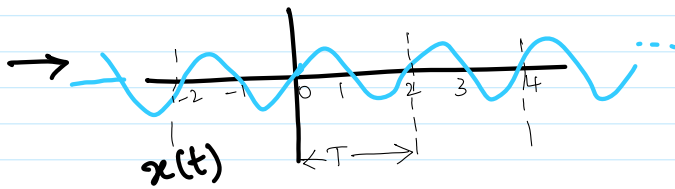
$$\therefore T = \frac{2\pi}{\omega_0} = \frac{2\pi}{300\pi} = \frac{1}{150} \text{ sec}$$

$$\rightarrow x(t) = 5 \cos(20\pi t + \pi/4)$$

$$\omega_0 = 20\pi$$

$$\therefore T = \frac{2\pi}{20\pi} = \frac{1}{10} = 0.1 \text{ sec}$$

$$x(t) = x(t \pm T)$$



$$\text{C.T} \quad x(t) = x(t + mT)$$

$m \Rightarrow$ an integer

$$T = T_0, 2T_0, 3T_0, \dots$$

$\downarrow$
fundamental period

$$f = \frac{1}{T} \text{ Hz}$$

$$\text{D.T} \quad x[n] = x[n+N]$$

$N \Rightarrow$ period of the signal & +ve integer

The smallest value of 'N' for which above condition is satisfied is called as "fundamental period"

$\swarrow$
defined as minimum no. of samples taken by signal to repeat itself

$$N = \frac{2\pi}{\omega_0} m$$

Combination of periodic signal

will be periodic if ratio of individual time periods is a rational number.

Ex $x(t) = 4 \underbrace{\cos 2\pi t}_{T_1} + 3 \underbrace{\sin 6t}_{T_2} + 5 \underbrace{\cos 60\pi t}_{T_3}$

$$\frac{T_1}{T_2} = \frac{T_2}{T_3} = \frac{T_1}{T_3} \Rightarrow \text{rational number}$$

Then, period of resultant signal

period of $x(t)$ $\leftarrow T = \text{L.C.M.} [T_1, T_2, T_3]$

$$\omega_0 = \text{G.C.D} / \text{H.C.F of } [\omega_0, \omega_02, \omega_03] \dots$$

$$\frac{T_1}{T_2} = \frac{10}{3} = 3.3333 \dots \text{rational}$$

$$\frac{T_1}{T_2} = \frac{10}{4} = 2.5 \text{ rational}$$

$$\pi = \frac{T_1}{T_2} = \frac{22}{7} = 3.1428 \dots \neq \text{rational}$$

$e \Rightarrow \text{irrational}$

$$\rightarrow x(t) = 5 \cos \pi t$$

$$\omega_0 = \pi$$

$$\therefore T = \frac{2\pi}{\omega_0} = \frac{2\pi}{\pi} = 2 \checkmark$$

$$\dots \cos t + 3 \sin 2\pi t + 2 \sin 3\pi t$$

$$\rightarrow x(t) = 4 \cos t + 3 \sin 2\pi t + 2 \sin 3\pi t$$

$T_1 \quad T_2 \quad T_3$

$$\frac{T_1}{T_2} = \frac{T_3}{T_1} \Rightarrow \text{rational number}$$

$$T = \text{L.C.M} [T_1, T_2, T_3]$$

$$\omega_0 = \text{H.C.F} [\omega_{01}, \omega_{02}, \omega_{03}]$$

$$\text{L.C.M of rational number} = \frac{\text{L.C.M of numerator}}{\text{H.C.F of Denominator}}$$

$$\text{H.C.F of Rational number} = \frac{\text{H.C.F of numerator}}{\text{L.C.M of denominator}}$$

Ex L.C.M of $\left\{ \frac{1}{2}, \frac{3}{6}, \frac{5}{4} \right\}$

$\begin{matrix} \text{L.C.M} \\ \text{of num} \end{matrix} \rightarrow \frac{3 \times 5}{\begin{matrix} \text{H.C.F} \\ \text{of Den} \end{matrix} 2} = 7.5$

$$\rightarrow x(t) = 4 \cos t + 3 \sin 2\pi t + 2 \sin 3\pi t$$

$T_1 \quad T_2 \quad T_3$

$$\begin{aligned} \omega_{01} &= 1 \\ T_1 &= \frac{2\pi}{\omega_{01}} = \frac{2\pi}{1} = 2\pi \\ \omega_{02} &= 2\pi \\ T_2 &= \frac{2\pi}{\omega_{02}} = \frac{2\pi}{2\pi} = 1 \\ \omega_{03} &= 3\pi \\ T_3 &= \frac{2\pi}{\omega_{03}} = \frac{2\pi}{3\pi} = \frac{2}{3} \end{aligned}$$

$$\frac{T_1}{T_2} = \frac{2\pi}{1} \neq \text{rational}$$

rational

$$\frac{T_3}{T_1} = \frac{1}{3\pi} \neq \text{rational}$$

= irrational

$$\frac{1}{T_2} = \frac{1}{1}$$

$$\frac{T_2}{T_3} = \frac{1}{2/3} = \frac{3}{2} = \text{rational}$$

$x(t) \Rightarrow$ APERIODIC SIGNAL

$$\rightarrow x(t) = \sin\left[\frac{2\pi}{3}t\right] \cos\left[\frac{4\pi}{5}t\right]$$

$$x(t) = \frac{1}{2} \left[\sin\left[\frac{2\pi}{3} + \frac{4\pi}{5}\right]t + \sin\left[\frac{2\pi}{3} - \frac{4\pi}{5}\right]t \right]$$

$$x(t) = \frac{1}{2} \left[\underbrace{\sin \frac{22\pi}{15}t}_{T_1} + \underbrace{\left[-\sin \frac{2\pi}{15}t\right]}_{T_2} \right]$$

$$T_1 = \frac{2\pi}{22\pi/15} = \frac{15}{11}$$

$$T_2 = \frac{2\pi}{2\pi/15} = 15$$

$$\frac{T_1}{T_2} = \frac{15/11}{15} = \frac{1}{11} \quad \begin{matrix} \text{(rational)} \\ \text{(periodic)} \end{matrix}$$

$$T = \text{L.C.M} \times T_1 = 11 \cdot \frac{15}{11} = 15 \text{ sec}$$

$$\omega_{01} = \frac{22\pi}{15} = 11 \left[\frac{2\pi}{15} \right]$$

$$\omega_{02} = \frac{2\pi}{15} = 1 \cdot \left[\frac{2\pi}{15} \right]$$

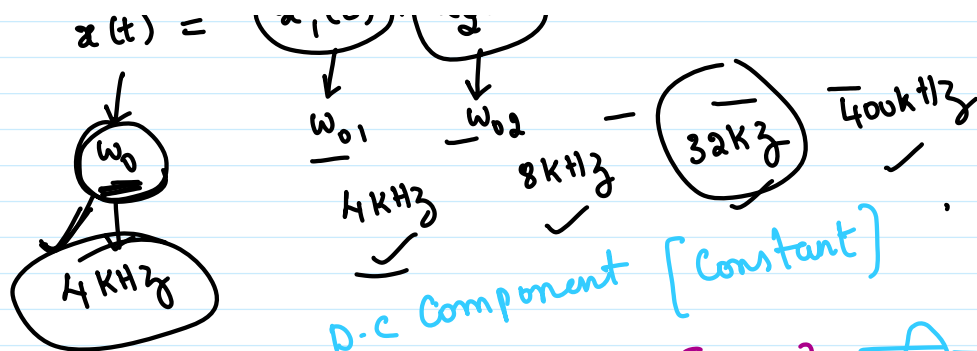
$$\frac{2\pi}{15} = \omega_0$$

$$\omega_0 = \frac{2\pi}{15} \rightarrow \frac{2\pi}{T}$$

$$\boxed{T=15}$$

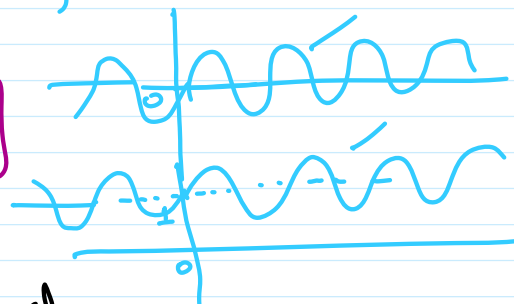
$$x(t) = x_1(t) + x_2(t) + \dots$$

Time shift



→ $x(t) = 1 + \sin\left[\frac{2\pi}{3}t\right] \cos\left[\frac{4\pi}{5}t\right]$

D-c Component [Constant]



Note Addition of D-c component does not affect the time period of the signal

→ $x(t) = 14 + 50 \cos 60t$

$T = \frac{2\pi}{60} = \frac{\pi}{30} \text{ sec}$

$\cos 60t = \cos \omega_0 t$
 $\omega_0 = 60$

→ $x(t) = 10 \sin 5t - 4 \cos 7t$

$T_1 = \frac{2\pi}{5}$ $T_2 = \frac{2\pi}{7}$

$T = \text{L.C.M} \left[\frac{2\pi}{5}, \frac{2\pi}{7} \right] = \frac{\text{L.C.M of } (2\pi, 2\pi)}{\text{H.C.F of } (5, 7)}$

$T = \frac{2\pi}{1} = 2\pi$

→ $x(t) = \cos(1 \cdot 2\pi t) + \cos(2\pi t) + \cos(2 \cdot 8\pi t)$

$\cos(\omega_0 t)$ $\omega = 2\pi f$

$$\cos(\omega_0 t) \quad \omega = 2\pi f$$

$$1 \cdot 2\pi = 2\pi f_1$$

$$f_1 = 0.6 \text{ Hz}$$

$$2\pi f_2 = 2\pi$$

$$f_2 = 1$$

$$2\pi f_3 = 2 \cdot 8\pi$$

$$f_3 = 1.4$$

$\omega \Rightarrow \text{rad/sec}$
 $f \Rightarrow \text{Hz}$

$$f = \text{G.C.D.}(f_1, f_2, f_3)$$

$$= \text{G.C.D.}(0.6, 1, 1.4)$$

$$f = \underline{\underline{0.2 \text{ Hz}}}$$

$$\rightarrow x(t) = 2 \sin(2\pi t) + 3 \sin(3\pi t)$$

$$T_1 = \frac{2\pi}{2\pi} = 1$$

$$T_2 = \frac{2\pi}{3\pi} = \frac{2}{3}$$

$$\frac{T_1}{T_2} = \frac{1}{2/3} = \frac{3}{2}$$

$$T = \text{L.C.M} \left[1, \frac{2}{3} \right] = \frac{\text{L.C.M}(1, 2)}{\text{H.C.F}(1, 3)} = \frac{2}{1}$$

$$\rightarrow x(t) = [1 + 0.5 \cos 60\pi t] \cdot \cos 300\pi t$$

$$= \cos 300\pi t + \underbrace{0.5 \cos 60\pi t \cdot \cos 300\pi t}_{\cos A \cos B}$$

$$f = \underline{\underline{\quad}} \text{ Hz} \quad T = \underline{\underline{\quad}}$$

$$\rightarrow x(t) = \cos \left(\frac{\pi}{4} (t-1) \right)$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\pi/4}$$

$\rightarrow x(t) = \cos\left(\frac{11}{4}(t-1)\right)$ $\omega = \frac{11}{4}$
 shifting doesn't affect the periodicity $T = 8 \text{ sec}$

$\rightarrow x(t) = e^{j20t}$ Complex Exp $\Rightarrow$ periodic

$T = \frac{2\pi}{20} = \frac{\pi}{10}$ (periodic)

$\rightarrow x(t) = e^{-(2+j)t}$ $T = \frac{2\pi}{2+j}$ not rational (Aperiodic)

$\rightarrow x[n] = 4 - 3 \sin\left[\frac{7n\pi}{4}\right]$

$N = \frac{2\pi}{\omega_0} \cdot m$

$N = \frac{2\pi}{(7\pi/4)} \cdot m = \frac{8}{7} m$

$N = \frac{8}{7} m$

if $m=7$ $N = \frac{8}{7} \times 7 = 8$

Period $N=8$

$\rightarrow x[n] = 2 \cos\left[\frac{n\pi}{4}\right] + \cos^2\left[\frac{n\pi}{4}\right]$

$\omega_1 = \frac{2\pi}{8} m$

$\cos^2 \frac{n\pi}{4}$

$$N_1 = \frac{2\pi}{\omega_0} m$$

$$= \frac{2\pi}{\pi/4} \cdot m = 8m$$

$$N_1 = 8 \cdot m$$

$$\text{if } m=1, N_1=8$$

$$\cos^2 \frac{n\pi}{4}$$

$$= \frac{1 + \cos \frac{2n\pi}{4}}{2} = \frac{1 + \cos \frac{n\pi}{2}}{2}$$

$$N_2 = \frac{2\pi}{\pi/2} \cdot m = \frac{1}{2} + \frac{1}{2} \cos \frac{n\pi}{2}$$

$$N_2 = 4 \cdot m$$

$$\text{if } m=1; N_2=4$$

$$N = \text{LCM}(8, 4) = 8$$

$$\rightarrow y(n) = (j)^{n/2} \quad j^2 = -1$$

$$0 + j1 = e^{j\pi/2}$$

$$y(n) = (j)^{n/2} = \left(e^{j\pi/2} \right)^{n/2} = e^{jn\pi/4}$$

$$N = \frac{2\pi}{\omega_0} \cdot m = \frac{2\pi}{\pi/4} m$$

$$N = 8m$$

$$\text{if } m=1$$

$$N = 8 \times 1$$

$$N = 8$$

$$\rightarrow x(n) = \sin \left[\frac{n}{3} - \frac{\pi}{2} \right]$$

$$N = \frac{2\pi}{\omega_0} m = 6\pi \cdot m$$

$$N = \frac{2\pi m}{1/3} = 6\pi \cdot m$$

$$N = 6\pi m \Rightarrow \text{not rational}$$

$\therefore$ Aperiodic signal

$$\rightarrow x[n] = \sin \frac{\pi}{3} n + \cos \frac{\pi}{4} n$$

$$N_1 = \frac{2\pi m_1}{\pi/3}$$

$$N_2 = \frac{2\pi}{\pi/4} m_2$$

$$N_1 = 6m_1$$

$$N_2 = 8m_2$$

$$\text{if } m_1 = 1$$

$$N_1 = 6$$

$$\text{if } m_2 = 1$$

$$N_2 = 8$$

$$N = \text{L.C.M}(N_1, N_2) = \underline{\underline{24}}$$

$\rightarrow$

$$\sin(-t) \Rightarrow \omega$$

$$\sin(-\pi t) \Rightarrow f$$

$$\rightarrow x(t) = 30 \sin 100t + 10 \cos 300t + 6 \sin(500t + \pi/4)$$

Find the fundamental freq

$$\omega_1 = 100$$

$$\omega_2 = 300$$

$$\omega_3 = 500$$

$$\therefore \dots \dots \dots \text{L.C.M}(100, 300, 500)$$

$$\omega_1 = 100$$

2

$$G.C.D(\omega_1, \omega_2, \omega_3) = G.C.D(100, 300, 500)$$

$$\omega = 100 \text{ rad/sec}$$

$$\rightarrow x(t) = e^{j10t}$$

$$T = \frac{2\pi}{10} = \pi/5 \text{ sec}$$

$$\rightarrow x(t) = j e^{j10t}$$

$$T = \frac{2\pi}{10} = \pi/5 \text{ sec}$$

additional
phase angle
 $\therefore$ it will not

change
periodicity

$$x(t) = j e^{j10t} = e^{j(10t + \pi/2)} = e^{j\pi/2} \cdot e^{j10t} = j \cdot e^{j10t}$$

due to 'j'

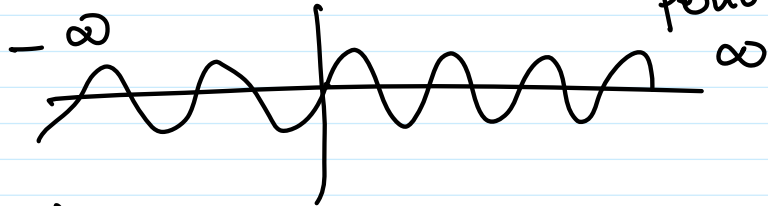
$$\omega_0 = 10 \Rightarrow T = \frac{2\pi}{10} = \pi/5$$

Ex

$$x(t + \pi/5) = j e^{j10(t + \pi/5)} = j e^{j10t} \cdot e^{j10 \cdot \frac{\pi}{5}} = j e^{j10t} \cdot e^{j2\pi} = j e^{j10t} \cdot 1 = j e^{j10t}$$

$$x(t + \pi/5) = x(t)$$

$$x(t + \pi/5) = x(t)$$

→ $y(t) = \sin(t)$  Periodic

$y(t) = \sin(t) \cdot u(t)$
Aperiodic 

→ $y(t) = \sum_{\text{Even}} \left\{ \cos 3\pi t \cdot u(t) \right\}$  $\frac{x(t) + x(-t)}{2}$

$$\begin{aligned} \sum_{\text{Even}} \left\{ \cos 3\pi t \cdot u(t) \right\} &= \frac{\cos 3\pi t \cdot u(t) + \cos 3\pi(-t) \cdot u(-t)}{2} \\ &= \frac{\cos 3\pi t}{2} \left\{ u(t) + u(-t) \right\} = \frac{\cos 3\pi t}{2} \cdot 1 \\ &= \frac{\cos 3\pi t}{2} \end{aligned}$$

$$\begin{aligned} \omega &= 3\pi \\ T &= \frac{2\pi}{3\pi} = \underline{\underline{2/3}} \end{aligned}$$

→ $e^{-j10t} + e^{j15t}$
 $\omega_1 = 10 \Rightarrow 2(5)$
 $\omega_2 = 15 \Rightarrow 3(5)$

$$\begin{aligned} \omega_0 &= 5 = \frac{2\pi}{T} \\ T &= \underline{\underline{2\pi/5}} \end{aligned}$$

Q. a. L

$$\rightarrow \cos 6t + \sin 8t + e^{j2t}$$

$$\omega_1 = 6 \Rightarrow 3(2)$$

$$\omega_2 = 8 \Rightarrow 4(2)$$

$$\omega_3 = 2 \Rightarrow 1(2)$$

$$\omega_0 = 2 \Rightarrow \frac{2\pi}{T}$$

$$T = \frac{2\pi}{2} = \pi$$

$$\rightarrow \sin \frac{\pi}{6}t + \sin \frac{\pi}{3}t$$

$$\omega_1 = \frac{\pi}{6} = 1\left(\frac{\pi}{6}\right)$$

$$\omega_2 = \pi/3 = 2\left(\frac{\pi}{6}\right)$$

$$\omega_0 = \frac{\pi}{6} = \frac{2\pi}{T}$$

$$T = \underline{\underline{12}}$$

$$\rightarrow e^{j\frac{5\pi}{7}n}$$

$$\frac{\omega_0}{2\pi} = \frac{5\pi/7}{2\pi} = \frac{5}{14} = \frac{3}{2}$$

$$\therefore N = 14$$

HINT For finding fundamental period of discrete sinusoid

(or) Complex sinusoids always use the equation $\frac{\omega_0}{2\pi}$ ratio is a rational number

$$\boxed{\frac{\omega_0}{2\pi} = \frac{m}{N}}$$

$\xrightarrow{+ve \text{ integer}}$
 $\xrightarrow{\text{period}}$

Assignment whether the given signals are periodic/Aperiodic?

① $x_1(t) = 2e^{j(t+\pi/4)} \cdot u(t)$

$$x_1[n] = u[n] \quad \dots \quad u[n]$$

$$(2) \quad x_2[n] = u[n] + u[-n]$$

$$(3) \quad x_3[n] = \sum_{k=-\infty}^{\infty} [g[n-4k] - g[n-1-4k]]$$

$$(4) \quad x_4[n] = (-1)^n \quad (5) \quad x_5[n] = (-1)^{n^2}$$

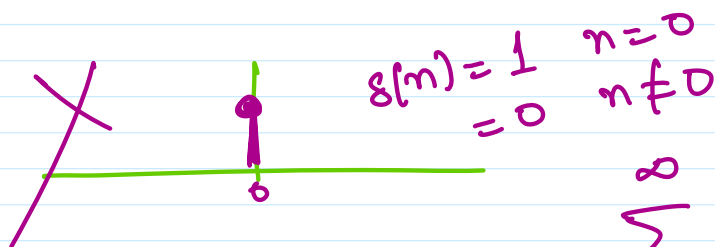
$$(6) \quad x(t) = \sum_{k=-\infty}^{\infty} (-1)^k \cdot g(t-2k)$$

[2] Find and sketch the even & odd ~~parts~~ components of the following

$$(1) \quad u(t) \quad (2) \quad x(t) = t \cdot u(t)$$

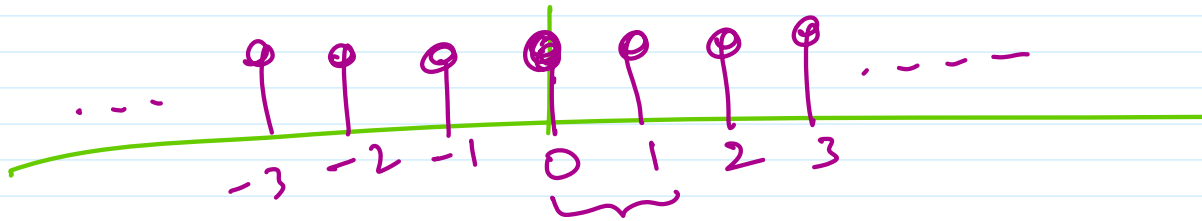
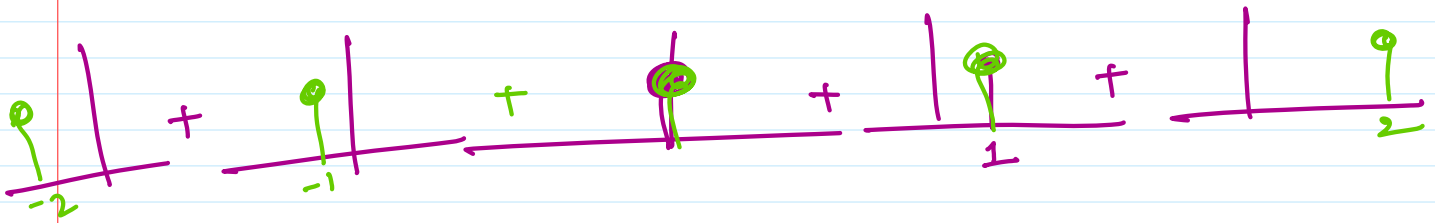
$$(3) \quad \sin(\omega_0 t) \cdot u(t) \quad (4) \quad \cos(\omega_0 t) \cdot u(t)$$

[3] An Aperiodic signal is defined as $x(t) = \sin(\pi t) u(t)$, where $u(t)$ is the continuous time step function. Is the odd portion of the signal, $x_o(t)$ periodic? Justify your answer.



$$\sum_{k=-\infty}^{\infty} g[n-k]$$

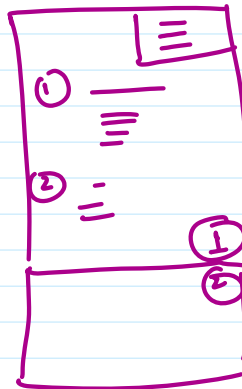
$$= \dots + g[n+2] + g[n+1] + g[n-0] + g[n-1] + g[n-2] + \dots$$



$$T = 1$$

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N192001 - BATCH 3
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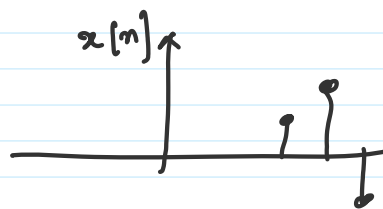
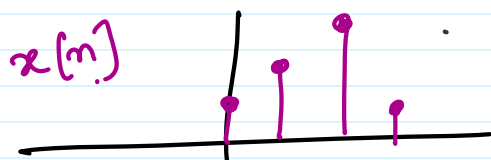
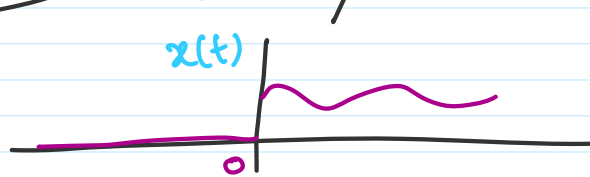


4) Causal/Anti causal/Non Causal signals

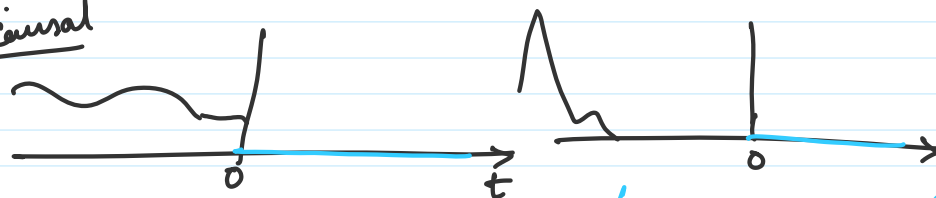
	C.T	D.T	Condition
Causal	$x(t) = 0$	$x[n] = 0$	for $t < 0 / n < 0$ Causal
Anti Causal	$x(t) = 0$	$x[n] = 0$	for $t > 0 / n > 0$

Anti Causal $x(t)=0$ | $x[n]=0$ | for $t > 0 / n > 0$
non-Causal $x(t) \neq 0$ | $x[n] \neq 0$ | for $t < 0 / n < 0$

Causal $x(t)=0$ / $x[n]=0$ for $t < 0 / n < 0$



anti causal



$x(t)=0$ for $t > 0$ / $x[n]=0$ for $n > 0$

non-causal

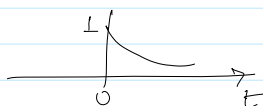
$x(t) \neq 0$ for $t < 0$ $x[n] \neq 0$ for $n < 0$



5 Deterministic/Random Signals

If the value of a signal can be predicted for all time $[t \text{ or } n]$ in advance without any error.

Ex: $x(t) = e^{-t} u(t)$ $x(t) = \text{rect}(t)$



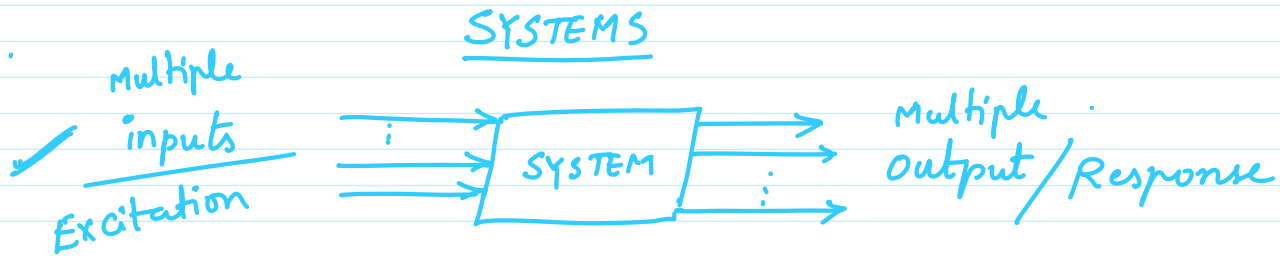
Random signals

: Whose value can't be predicted with complete accuracy for all time.

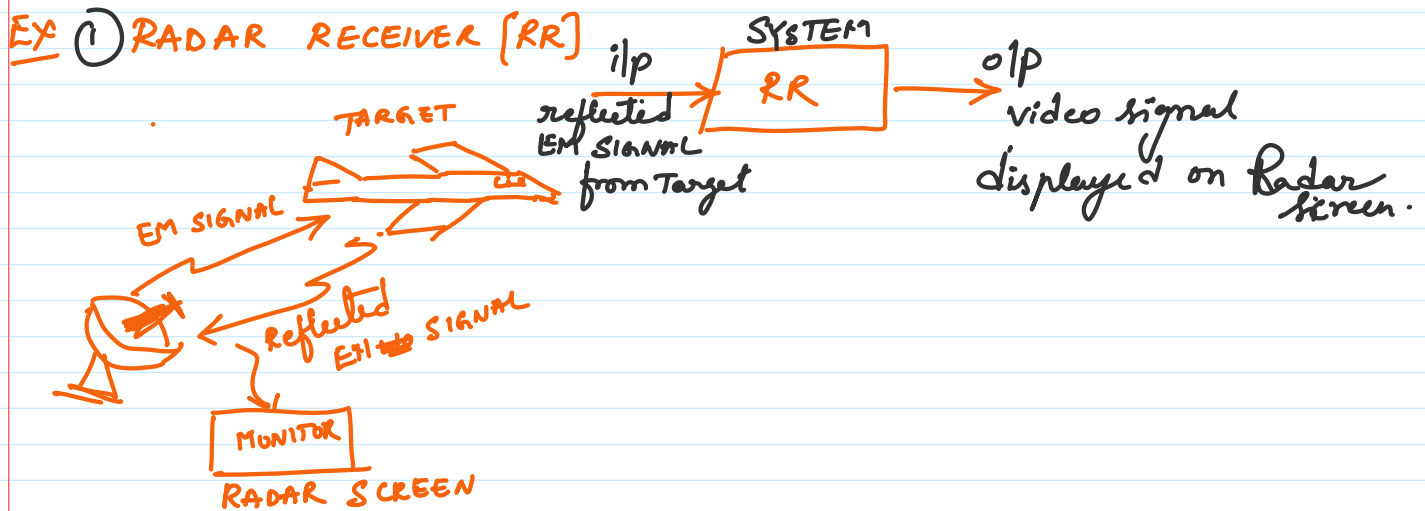
- These signals are characterised by mean, mean square value etc.

These signals are characterised by mean, mean square value etc.

Ex Noise

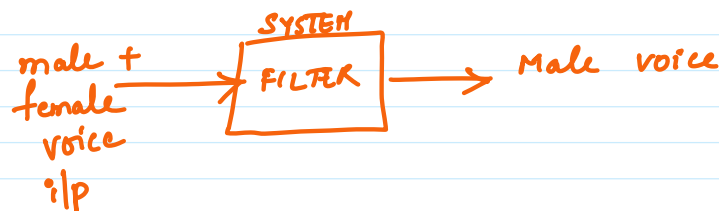


Physical system is characterized by its ability to accept i/p's such as V , I , force, pressure, displacement etc and to produce o/p's in response to these i/p's.

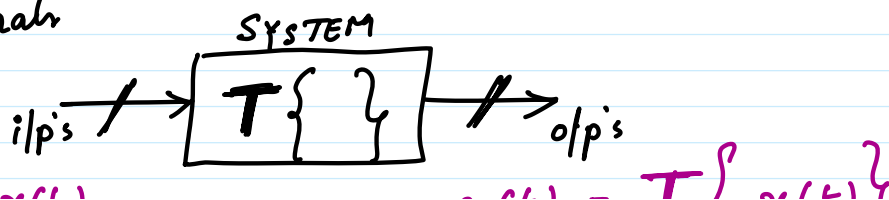


② ROBOT i/p $\Rightarrow$ Electric Control Signal
 o/p $\Rightarrow$ motion/action on the part of the robot

③ FILTER



"System" is a process that results in transforming i/p signals into o/p signals



i/p's $\left[\begin{array}{|c|c|c|} \hline & & \\ \hline \end{array} \right]$ o/p's

$$x(t) \quad y(t) = T \{ x(t) \}$$

$$x[n] \quad y[n] = T \{ x[n] \}$$

- Classification of systems are based on how systems interact with the i/p signal
- This interaction, which defines the model for the system.
 - ① Linear / Non-linear
 - ② Time invariant / Time variant
or shift invariant or shift variant
 - ③ Memoryless / with memory
 - ④ Static / Dynamic
 - ⑤ Causal / Non causal systems
 - ⑥ Stable / Unstable.

① Linear & Non-linear Systems

"SUPERPOSITION" Principle

RGUKT $x_1(t) \rightarrow [S] \rightarrow y_1(t)$ III T

NUZVID $x_2(t) \rightarrow [S] \rightarrow y_2(t)$ A.P

RGUKT NUZVID $x_1(t) + x_2(t) \rightarrow [S] \rightarrow y_1(t) + y_2(t)$ III T AP "ADDITIVE PROPERTY"

RGUKT $x(t) \rightarrow [S] \rightarrow y(t)$ III T

RGUKT $a \cdot x(t) \rightarrow [S] \rightarrow a \cdot y(t)$ "SCALING" (or) "HOMOGENEITY PROPERTY" III T

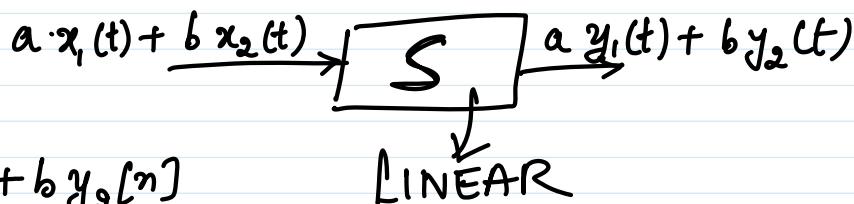
ADDITIVITY + SCALING (or Homogeneity) = SUPERPOSITION PRINCIPLE $\Rightarrow$ LINEAR SYSTEM

C.T :-

$$a x_1(t) + b x_2(t) \rightarrow a y_1(t) + b y_2(t)$$

$a x_1(t) + b x_2(t) \rightarrow [S] \rightarrow a y_1(t) + b y_2(t)$

$$a x_1(t) + b x_2(t) \rightarrow a y_1(t) + b y_2(t)$$



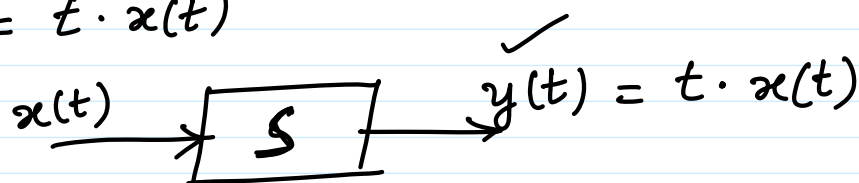
$$\text{D.T} \quad a x_1[n] + b x_2[n] \rightarrow a y_1[n] + b y_2[n]$$

$a \text{ \& } b \Rightarrow$ Complex constant

Note When a system violates either of the two properties $\Rightarrow$ NON-LINEAR

Ex For each of the following i/p-o/p relationships, determine whether the corresponding system is linear.

① $y(t) = t \cdot x(t)$



t	x(t)	y(t) = t · x(t)
0	x(0) = 2	y(0) = 0 · x(0) = 0 × 2 = 0
1	x(1) = 5	y(1) = 1 · x(1) = 1 × 5 = 5
2	x(2) = -1	y(2) = 2 · x(2) = 2 × -1 = -2
3	x(3) = 4	y(3) = 3 · x(3) = 3 × 4 = 12
4	x(4) = 1	y(4) = 4 · x(4) = 4 × 1 = 4

$$y(t) = t \cdot x(t)$$

let $x_1(t) \rightarrow y_1(t) = t \cdot x_1(t)$

let $x_2(t) \rightarrow y_2(t) = t \cdot x_2(t)$

$$x_3(t) \rightarrow y_3(t) = t \cdot x_3(t)$$

Assume that $x_3(t)$ is the linear combination of $x_1(t)$ & $x_2(t)$

$$x_3(t) = a x_1(t) + b x_2(t)$$

TL the system is linear, then

$$x_3(t) = a x_1(t) + b x_2(t)$$

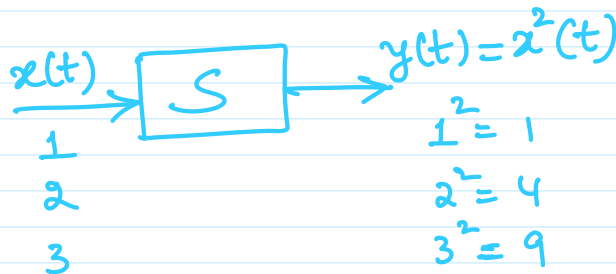
If the system is linear, then

$$\text{LHS } y_3(t) = a \cdot \underbrace{y_1(t)}_{\text{RHS}} + b \underbrace{y_2(t)}_{\text{RHS}}$$

$$\begin{aligned} x_3(t) &\xrightarrow{\text{L.H.S}} y_3(t) = t \cdot x_3(t) \\ &= t \cdot [a x_1(t) + b x_2(t)] \\ &= a \cdot \underbrace{t \cdot x_1(t)}_{= y_1(t)} + b \cdot \underbrace{t \cdot x_2(t)}_{= y_2(t)} \end{aligned}$$

$$y_3(t) = a y_1(t) + b y_2(t) \quad \leftarrow \text{RHS LINEAR}$$

② $y(t) = x^2(t)$



$$x_1(t) \rightarrow y_1(t) = x_1^2(t)$$

$$x_2(t) \rightarrow y_2(t) = x_2^2(t)$$

$$x_3(t) \rightarrow y_3(t) = x_3^2(t)$$

let $x_3(t) = a x_1(t) + b x_2(t)$

$$\text{L.H.S } y_3(t) = \underbrace{a \cdot y_1(t) + b \cdot y_2(t)}_{\text{RHS}}$$

$$\text{L.H.S } y_3(t) = x_3^2(t)$$

$$= [a x_1(t) + b x_2(t)]^2$$

$$= a^2 x_1^2(t) + b^2 x_2^2(t) + 2 \cdot a \cdot b x_1(t) x_2(t)$$

$$\begin{aligned}
 &= a^2 \underbrace{x_1^2(t)}_{y_1(t)} + b^2 \underbrace{x_2^2(t)}_{y_2(t)} + 2 \cdot a \cdot b \cdot x_1(t) \cdot x_2(t) \\
 &= a^2 y_1(t) + b^2 y_2(t) + 2ab x_1(t) \cdot x_2(t) \\
 &\neq \text{R.H.S} \quad \text{"NON-LINEAR"}
 \end{aligned}$$

③ $y[n] = 2x[n] - 3$

$x_1[n] \rightarrow y_1[n] = 2 \cdot x_1[n] - 3$

$x_2[n] \rightarrow y_2[n] = 2 \cdot x_2[n] - 3$

$x_3[n] \rightarrow y_3[n] = 2 \cdot x_3[n] - 3$ $x_3[n] = a x_1[n] + b x_2[n]$

$y_3[n] = a y_1[n] + b y_2[n]$

LHS $y_3[n] = 2 \cdot x_3[n] - 3$

$$\begin{aligned}
 &= 2 [a \cdot x_1[n] + b x_2[n]] - 3 \\
 &= a \cdot 2 \cdot x_1[n] + b \cdot 2 x_2[n] - 3 \\
 &\neq \text{R.H.S} \quad \text{"NON-LINEAR"}
 \end{aligned}$$

Note

Zero-i/p $x[n] = 0$

$x[n] \rightarrow \boxed{S} \rightarrow y[n] = 2 \cdot x[n] - 3 \quad (NL)$

$y[n] = 2 \times 0 - 3$

$y[n] = -3$

Violates the Zero i/p / Zero o/p property
 $\Rightarrow$ So NON-LINEAR

if $x[n] = 0 \Rightarrow y[n] = 0$ LINEAR

$x[n] \neq 0 \Rightarrow y[n] \neq 0$ NON-LINEAR

$$(4) \quad y(t) = x(t) \cdot \cos(5t)$$

$$x_1(t) \rightarrow y_1(t) = x_1(t) \cos(5t)$$

$$x_2(t) \rightarrow y_2(t) = x_2(t) \cos(5t)$$

$$x_3(t) \rightarrow y_3(t) = x_3(t) \cos(5t)$$

$$\text{let } x_3(t) = a \cdot x_1(t) + b \cdot x_2(t)$$

$$y_3(t) = a \cdot y_1(t) + b \cdot y_2(t)$$

$$\begin{aligned} \text{L.H.S } y_3(t) &= x_3(t) \cos(5t) \\ &= [a x_1(t) + b x_2(t)] \cdot \cos(5t) \\ &= a \cdot \underbrace{x_1(t) \cos(5t)}_{= y_1(t)} + b \cdot \underbrace{x_2(t) \cos(5t)}_{= y_2(t)} \\ &= a y_1(t) + b y_2(t) \\ &= \text{R.H.S} \quad \text{"LINEAR"} \end{aligned}$$

Note: Non-linear Operators

$\sin, \cos, \tan, \sin^{-1}, \cos^{-1}, \tan^{-1}, \exp, \log, \text{modulus},$
 $\text{median, square, cube, square root, } u[\], \text{sgn}[\],$
 $\text{Sa}(\), \text{sinc}(\),$ either of 'x' or 'y' makes
 system "Non-linear"