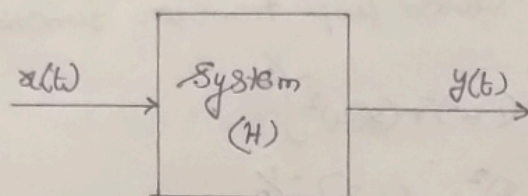


Frequency Response is the steady state Response (o/p) of a system when the input to a system is sinusoidal signal.

Consider a LTI system (H) is shown in figure. Let $x(t)$ be an input sinusoidal signal. The response (o/p) $y(t)$ is also a sinusoidal signal of same frequency but with different magnitude and phase angle.



$$x(t) = X \cdot \sin \omega t$$

$$y(t) = Y \cdot \sin(\omega t + \phi)$$

Hence the magnitude and phase relationship between the sinusoidal input and the steady state output of a system is termed as the frequency Response. In General frequency response of a system is normally obtained by varying the frequency of the i/p signal by keeping the magnitude of the input signal at a constant value. (or) In the system transfer function $T(s)$, if s is replaced by $j\omega$ then resulting transfer function $T(j\omega)$ is called sinusoidal transfer function. The frequency Response of a system can be directly obtained from the sinusoidal transfer function $T(j\omega)$.

→ Frequency Domain Specifications:-

→ Resonant Peak (M_r):- The maximum value of the magnitude of closed loop Transfer function is called "resonant peak".

Consider closed loop Transfer function of 2nd order system,

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

The sinusoidal T/F, $T(j\omega)$ is obtained by replace $s = j\omega$

$$\begin{aligned} T(j\omega) &= \frac{c(j\omega)}{R(j\omega)} = \frac{\omega_n^2}{(j\omega)^2 + 2\delta\omega_n(j\omega) + \omega_n^2} \\ &= \frac{\omega_n^2}{\omega_n^2 - \omega^2 + j2\delta\omega_n\omega} = \frac{\omega_n^2}{\omega_n^2} \left(1 - \frac{\omega^2}{\omega_n^2} + j2\delta\frac{\omega}{\omega_n} \right) \\ &= 1 / \left(1 - \left(\frac{\omega}{\omega_n}\right)^2 + j2\delta\left(\frac{\omega}{\omega_n}\right) \right) \end{aligned}$$

Let, normalized frequency $(u) = \omega/\omega_n$

$$\therefore T(j\omega) = 1 / (1 - u^2 + j2\delta u) = 1 / \sqrt{(1 - u^2)^2 + (2\delta u)^2} \cdot \angle \tan^{-1} \left(\frac{2\delta u}{1 - u^2} \right)$$

Let M = magnitude of closed loop transfer function

α = phase of closed loop transfer function.

$$\begin{aligned} M = |T(j\omega)| &= \left[1 / (1 - u^2)^2 + (2\delta u)^2 \right]^{1/2} \\ &= [(1 - u^2)^2 + 4\delta^2 u^2]^{-1/2} \end{aligned}$$

$$\alpha = \angle T(j\omega) = - \left[\tan^{-1} \left(\frac{2\delta u}{1 - u^2} \right) \right]$$

But Resonant peak is the maximum value of M . The condition for maximum value of M can be obtained by differentiating the Equation of M with respect to u and Evaluate to zero when $u = u_r$.
where $u_r = \omega_r/\omega_n$ = normalized resonant frequency.

$$\begin{aligned} \frac{dM}{du} &= \frac{d}{du} [(1 - u^2)^2 + 4\delta^2 u^2]^{-1/2} \\ &= \frac{-1}{2} [(1 - u^2)^2 + 4\delta^2 u^2]^{-3/2} [2(1 - u^2)(-2u) + 8\delta^2 u] \\ &= \frac{-1}{2} \left[\frac{-4u(1 - u^2) + 8\delta^2 u}{[(1 - u^2)^2 + 4\delta^2 u^2]^{3/2}} \right] = \frac{4u(1 - u^2) - 8\delta^2 u}{2[(1 - u^2)^2 + 4\delta^2 u^2]^{3/2}} \end{aligned}$$

Replace $u = u_r$ we get & Evaluate to zero.

$$\Rightarrow \frac{4u_r(1 - u_r^2) - 8\delta^2 u_r}{2[(1 - u_r^2)^2 + 4\delta^2 u_r^2]^{3/2}} = 0$$

$$\Rightarrow 4u_r(1 - u_r^2) - 8\delta^2 u_r = 0$$

$$\Rightarrow 4u_r - 4u_r^3 - 8\delta^2 u_r = 0$$

$$4 - 4u_r^2 - 8\delta^2 = 0$$

$$1 - u_r^2 - 2\delta^2 = 0$$

$$u_r^2 = 1 - 2\delta^2$$

$$\Rightarrow u_r = \sqrt{1 - 2\delta^2}$$

The resonant peak occurs when $u_r = \sqrt{1 - 2\delta^2}$
Put this condition in the equation for M and solve for M_r .

$$\therefore M_r = \left. \frac{1}{[(1-u^2)^2 + 4\delta^2 u^2]} \right|_{u=u_r}^{1/2}$$

$$= \frac{1}{[(1-u_r^2)^2 + 4\delta^2 u_r^2]}^{1/2}$$

$$= \frac{1}{[(1-(1-2\delta^2))^2 + 4\delta^2(1-2\delta^2)]}^{1/2}$$

$$= \frac{1}{[4\delta^4 + 4\delta^2 - 8\delta^4]}^{1/2} = \frac{1}{[4\delta^2(1-2\delta^2)]}^{1/2}$$

$$= \frac{1}{2\delta \sqrt{1-2\delta^2}}$$

$$\therefore \text{The resonant peak } (M_r) = \frac{1}{2\delta \sqrt{1-2\delta^2}}$$

Resonant frequency (ω_r): The frequency at which the resonant peak occurs is called "resonant frequency" (ω_r).

$$\text{The normalized resonant frequency } u_r = \frac{\omega_r}{\omega_n} = \sqrt{1 - 2\delta^2}$$

$$\text{The resonant frequency } (\omega_r) = \omega_n \sqrt{1 - 2\delta^2}$$

Band width (ω_b): The Bandwidth is the range of frequencies for which the system gain is more than -3db. (or) for feedback control systems, the range of frequencies over which M is equal to or greater than $(1/\sqrt{2})$ is defined as Band width.

Let, Normalized band width, $(u_b) = \omega_b / \omega_n$
When $u = u_b$, the magnitude M, of the closed loop system is $1/\sqrt{2}$. Hence

put $u = u_b$ and equate to $1/\sqrt{2}$.

$$M = \frac{1}{[(1-u_b^2)^2 + 4\delta^2 u_b^2]^{\frac{1}{2}}} = \frac{1}{\sqrt{2}}$$

on squaring & cross multiplying

$$\Rightarrow (1-u_b^2)^2 + 4\delta^2 u_b^2 = 2$$

$$1 + u_b^4 - 2u_b^2 + 4\delta^2 u_b^2 = 2$$

$$u_b^4 - 2u_b^2 + 4\delta^2 u_b^2 = 1$$

$$u_b^4 - 2u_b^2(1-2\delta^2) - 1 = 0$$

Let $x = u_b^2$

$$\Rightarrow x^2 - 2x(1-2\delta^2) - 1 = 0$$

By taking only the +ve sign

$$x = 1 - 2\delta^2 + \sqrt{2 - 4\delta^2(1-\delta^2)}$$

$$\text{But } u_b = \sqrt{x} = [1 - 2\delta^2 + \sqrt{2 - 4\delta^2(1-\delta^2)}]^{\frac{1}{2}}$$

Also $u_b = \omega_b / \omega_n$

$$\therefore \text{Band Width } (\omega_b) = \omega_n \cdot u_b$$

$$= \omega_n [1 - 2\delta^2 + \sqrt{2 - 4\delta^2(1-\delta^2)}]^{\frac{1}{2}}$$

Gain Margin:-(Kg)

It is defined as the reciprocal of the magnitude of open loop transfer function, at phase cross-over frequency (ω_{pc}).

$$\Rightarrow \text{Gain Margin (Kg)} = 1 / |G(j\omega)|_{\omega=\omega_{pc}}$$

When expressed in decibels, it is given by, the -ve of db magnitude of $G(j\omega)$ at phase cross-over frequency.

$$\Rightarrow \text{Gain Margin in db} = 20 \log 1 / |G(j\omega)|_{\omega=\omega_{pc}}$$

$$= -20 \log |G(j\omega)|_{\omega=\omega_{pc}}$$

$$\begin{aligned} x &= \frac{2(1-2\delta^2) \pm \sqrt{4(1-2\delta^2)^2 + 4}}{2} \\ &= \frac{2(1-2\delta^2) \pm 2\sqrt{1+4\delta^4-4\delta^2+1}}{2} \\ &= (1-2\delta^2) \pm \sqrt{2-4\delta^2(1-\delta^2)} \end{aligned}$$

Phase Margin ϕ_m :- Phase Margin is that amount of additional phase lag at the gain cross over frequency required to bring the system to the verge of instability. (78)

$$\text{Phase Margin } (\gamma) = 180 + \phi_{gc}$$

where $\phi_{gc} \rightarrow$ phase angle of $G(j\omega)$ at $\omega = \omega_{gc}$.

$$= \angle G(j\omega)$$

The open Loop T/F of 2nd order system

$$G(s) = \omega_n^2 / s(s + 2\delta\omega_n)$$

The sinusoidal T/F of $G(j\omega)$ is obtained by $s = j\omega$.

$$\begin{aligned} G(j\omega) &= \omega_n^2 / (j\omega)(j\omega + 2\delta\omega_n) \\ &= \omega_n^2 / (-\omega^2 + j2\delta\omega\omega_n) = \omega_n^2 / \omega_n^2 \left(-\frac{\omega^2}{\omega_n^2} + j2\delta\frac{\omega}{\omega_n} \right) \end{aligned}$$

Let normalized frequency $u = \omega/\omega_n$

$$\begin{aligned} \Rightarrow G(j\omega) &= \omega_n^2 / \omega_n^2 (-u^2 + j2\delta u) \\ &= 1 / u(u + j2\delta) = 1 / -u^2 + j2\delta u \end{aligned}$$

Magnitude of $G(j\omega)$ is

$$\Rightarrow |G(j\omega)| = 1 / \sqrt{u^4 + 4\delta^2 u^2} \quad \text{Note: at } \omega_{gc}, |G(j\omega)| \text{ is unity.} \rightarrow (1)$$

Phase of $G(j\omega)$ is

$$\Rightarrow \angle G(j\omega) = -\tan^{-1} \left(\frac{2\delta u}{-u^2} \right) = -\tan^{-1} \left(-\frac{2\delta}{u} \right) = \tan^{-1} \left(\frac{2\delta}{u} \right) \rightarrow (2)$$

Let normalized gain cross over frequency $(u_{gc}) = \omega_{gc}/\omega_n$.

$\therefore$ at $u = u_{gc}$

$$|G(j\omega)| = 1 = 1 / \sqrt{u_{gc}^4 + 4\delta^2 u_{gc}^2}$$

$$\Rightarrow u_{gc}^4 + 4\delta^2 u_{gc}^2 = 1 \Rightarrow u_{gc}^4 + 4\delta^2 u_{gc}^2 - 1 = 0$$

$$\text{Let } x = u_{gc}^2$$

$$x^2 + 4\delta^2 x - 1 = 0$$

$$x = \frac{-4\delta^2 \pm \sqrt{16\delta^4 + 4}}{2} = \frac{-4\delta^2 \pm 2\sqrt{4\delta^4 + 1}}{2}$$

$$= -2\delta^2 \pm \sqrt{4\delta^4 + 1}$$

on taking only the positive sign

$$u_{gc} = \sqrt{x} = [-2\delta^2 + \sqrt{4\delta^4 + 1}]^{\frac{1}{2}}$$

$$\text{The phase Margin } (\gamma) = 180 + \left| G(j\omega) \right|_{\omega=\omega_{gc}, u=u_{gc}} \rightarrow (4)$$

Substitute (2) in (4) we get

$$\gamma = 180 + \tan^{-1}\left(\frac{2\delta}{u_{gc}}\right)$$

$$= 180 + \tan^{-1}\left\{2\delta / [-2\delta^2 + \sqrt{4\delta^4 + 1}]^{\frac{1}{2}}\right\}.$$

Note :- Gain Margin of 2nd order system is infinite.

→ **Frequency Response Plots :-**

(1) Bode plot

(2) Polar Plot (Nyquist plot)

(3) Nichols plot

(4) M & N Circles

These plots are used to determine the frequency domain specifications, to study the stability of the systems and to adjust the gain of the system to satisfy the desired specifications.

→ **Bode Plot :-** The Bode plot is a frequency response plot of the T/F of a system. It consists of two graphs. One is Magnitude Plot (Magnitude of a sinusoidal T/F vs $\log \omega$) & Phase plot (Phase angle of a sinusoidal T/F vs $\log \omega$). Bode plot can be drawn for both open loop & closed loop Transfer functions. usually the bode plot is drawn for open loop system.

Bode Plots:-

Bode plot of a Derivative factor (A zero at origin)

Ex: For open Loop T/F $G(s)H(s) = s$

Put $s = j\omega$

$$G(s)H(s) = 0 + j\omega$$

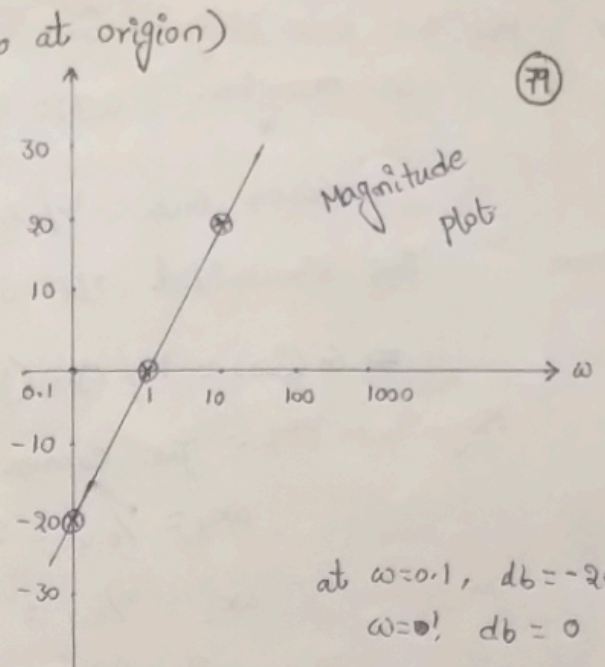
$$\text{Magnitude} = \sqrt{0^2 + \omega^2} = \omega$$

Magnitude in db = $+20 \log(\omega)$

Phase angle $\phi = \tan^{-1}(y/x)$

$$= \tan^{-1}(\omega/0) = \tan^{-1}(\infty)$$

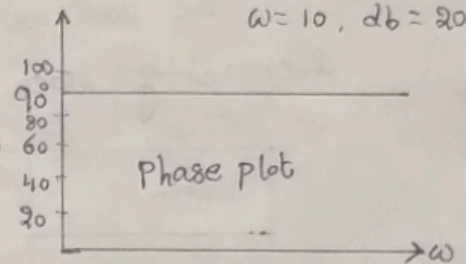
$$= 90^\circ$$



at $\omega = 0.1$, $db = -20$

$\omega = 1$, $db = 0$

$\omega = 10$, $db = 20$



Ex: Bode Plot of Integral factor (a pole at origin) For open Loop T/F $G(s)H(s) = 1/s$

Put $s = j\omega$

$$G(s)H(s) = 1/j\omega$$

$$= 0 - j\omega$$

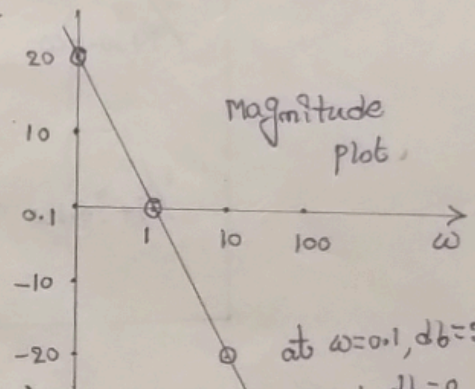
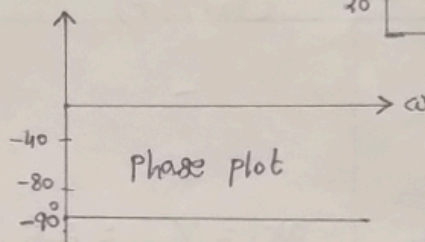
$$\text{Magnitude} = \sqrt{0^2 + \omega^2} = \omega$$

Magnitude in db = $-20 \log(\omega)$

Phase angle $\phi = \tan^{-1}(y/x)$

$$= \tan^{-1}(\omega/0) = -\tan^{-1}(\infty)$$

$$= -90^\circ$$



at $\omega = 0.1$, $db = 20$

$\omega = 1$, $db = 0$

$\omega = 10$, $db = -20$

Ex: For Multiple Derivative factors (Multiple Poles at origin) For open Loop T/F $G(s)H(s) = s^2$

Put $s = j\omega$

$$G(s)H(s) = 1/(j\omega)^2 = 1/(-\omega^2)$$

$$= \omega^2$$

$$\text{Magnitude} = \sqrt{0^2 + \omega^4} = \omega^2$$

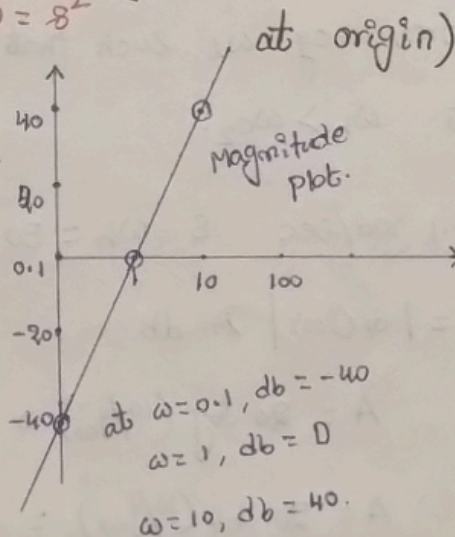
Magnitude in db = $20 \log(\omega^2)$

$$= 40 \log(\omega)$$

Phase angle (ϕ) = $\tan^{-1}(y/x)$

$$= \tan^{-1}(\omega^2/\omega^2)$$

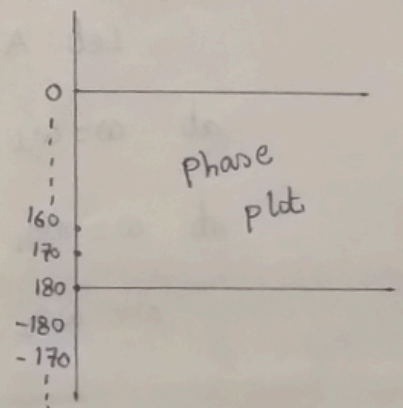
$$= 0$$



at $\omega = 0.1$, $db = -40$

$\omega = 1$, $db = 0$

$\omega = 10$, $db = 40$



(1) Plot the Bode Diagram for the following T/F & obtain the gain & phase margin. $G(s) = 10/s(1+0.4s)(1+0.1s)$

Given $G(s) = 10/s(1+0.4s)(1+0.1s)$

The sinusoidal T/F $G(j\omega)$ is obtained by replacing s by $j\omega$.

$$\Rightarrow G(j\omega) = 10/(j\omega)(1+0.4j\omega)(1+0.1j\omega)$$

Magnitude Plot:- The corner frequencies are

$$\omega_{c1} = 1/0.4 = 2.5 \text{ rad/sec}$$

$$\omega_{c2} = 1/0.1 = 10 \text{ rad/sec}$$

The various terms of $G(j\omega)$ are listed in table-1 in the increasing order of their corner frequencies.

Term	Corner frequency (rad/sec)	slope (db/dec)	change in slope (db/dec)
$10/j\omega$	—	-20	-20
$1/(1+j0.4\omega)$	$\omega_{c1} = 1/0.4 = 2.5$	-20	$-20-20 = -40$
$1/(1+j0.1\omega)$	$\omega_{c2} = 1/0.1 = 10$	-20	$-40-20 = -60$

choose low frequency ω_L such that $\omega_L < \omega_{c1}$ and high frequency ω_H such that $\omega_H > \omega_{c2}$.

Let $\omega_L = 0.1 \text{ rad/sec}$ & $\omega_H = 50 \text{ rad/sec}$

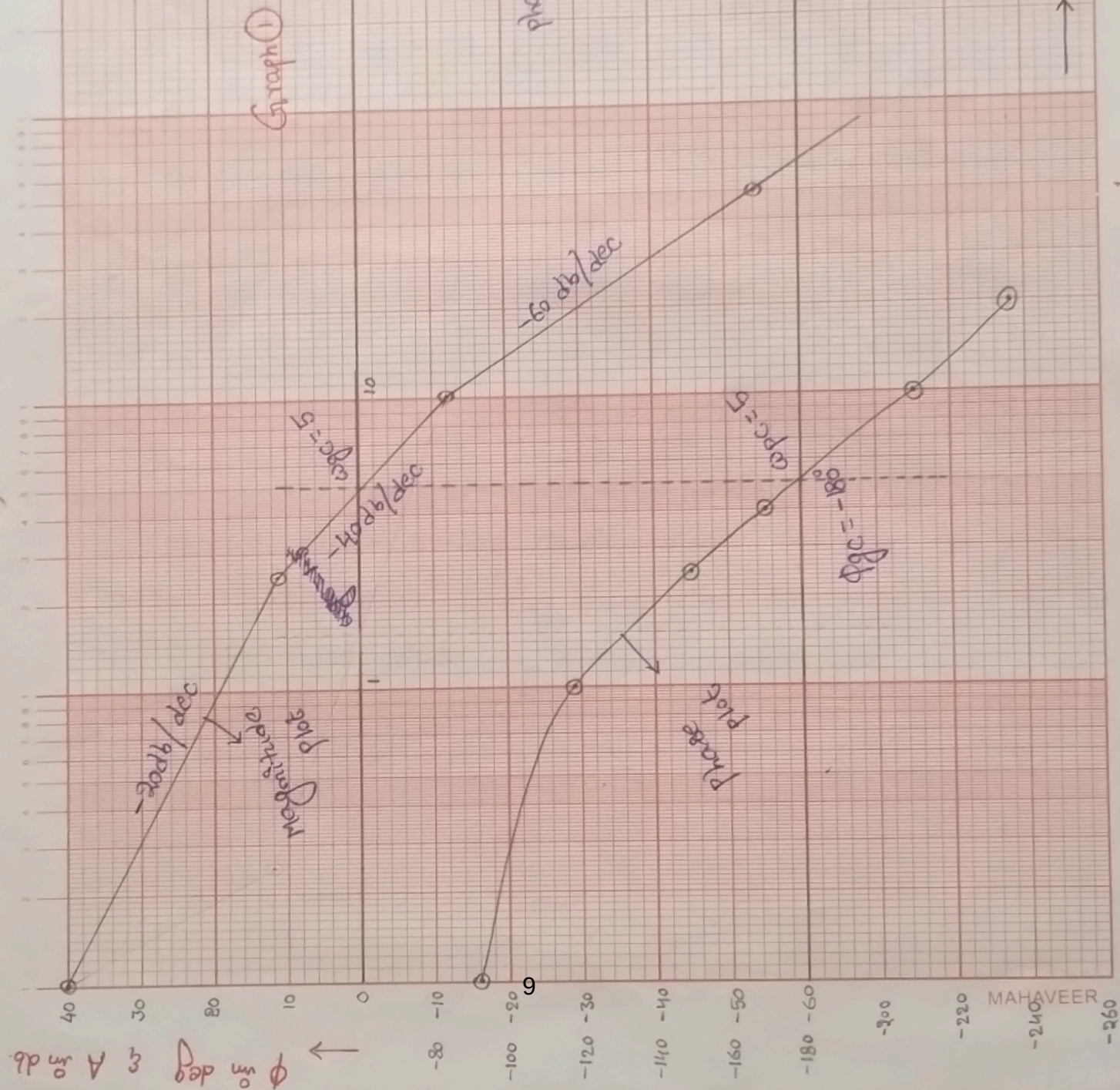
Let $A = |G(j\omega)|$ in db.

at $\omega = \omega_L$, $A = 20 \log(10/j\omega) = 20 \log(10/0.1) = 40 \text{ db}$.

at $\omega = \omega_{c1}$, $A = 20 \log(10/j\omega) = 20 \log(10/2.5) = 12 \text{ db}$.

$\omega = \omega_{c2}$, $A = \left[\text{slope from } \omega_{c1} \text{ to } \omega_{c2} \times \log \frac{\omega_{c2}}{\omega_{c1}} \right] + A \text{ at } (\omega = \omega_{c1})$
 $= -40 \times \log\left(\frac{10}{2.5}\right) + 12 = -12 \text{ db}$

At $\omega = \omega_{gc}$ $\Delta = 10 \log_{10} \dots$



$$\text{At } \omega = \omega_h, A = \left[\text{slope from } \omega_{c2} \text{ to } \omega_h \times \log \frac{\omega_h}{\omega_{c2}} \right] + A_{at}(\omega = \omega_{c2}) \quad (80)$$

$$= -60 \times \log \left(\frac{50}{10} \right) + (-12)$$

$$= -54 \text{ dB.}$$

Let the points a, b, c & d corresponding to frequencies $\omega_1, \omega_{c1}, \omega_{c2}$ & ω_h on magnitude plot. Join the points a, b, c & d on the semilog graph & join by a straight line & mark the slope in the respective region.

Phase plot :-

The phase angle of $G(j\omega)$ as a function of ω is given by

$$\Rightarrow \phi = -90 - \tan^{-1}(0.4\omega) - \tan^{-1}(0.1\omega)$$

ω (rad/sec)	$\phi = \angle G(j\omega)$ deg.
0.1	-92
1	-118
2.5	-150
4	-170
10	-210
20	-236

on the same semilog graph choose a scale on the y-axis on the right side of semilog sheet. Mark the phase angle on the graph & join these points by a smooth curve.

Graph ①

(2) Sketch Bode plot for the following transfer function & determine the system gain (K) if the gain cross over frequency to be 5 rad/sec.

$$G(s) = K s^2 / (1 + 0.2s)(1 + 0.002s)$$

The sinusoidal Transfer function $G(j\omega)$ is given by

$$\Rightarrow G(j\omega) = K(j\omega)^2 / (1 + 0.2j\omega)(1 + 0.002j\omega)$$

Let $K=1$; $G(j\omega) = (j\omega)^2 / (1 + 0.2j\omega)(1 + j0.02\omega)$

Magnitude Plot:-

The Corner frequencies are

$$\omega_{c1} = 1/0.2 = 5 \text{ rad/sec}$$

$$\omega_{c2} = 1/0.02 = 50 \text{ rad/sec.}$$

Let us draw a table in the increasing order of their corner frequency.

Term	Corner Frequency (rad/sec)	slope db/dec	change in slope db/dec.
$(j\omega)^2$	-	+40	+40
$1/(1+j0.2\omega)$	$\omega_{c1} = 1/0.2 = 5$	-20	$40-20=20$
$1/(1+j0.02\omega)$	$\omega_{c2} = 1/0.02 = 50$	-20	$20-20=0$

choose a low frequency ω_L such that $\omega_L < \omega_{c1}$ & choose a high frequency ω_H such that $\omega_H > \omega_{c2}$.

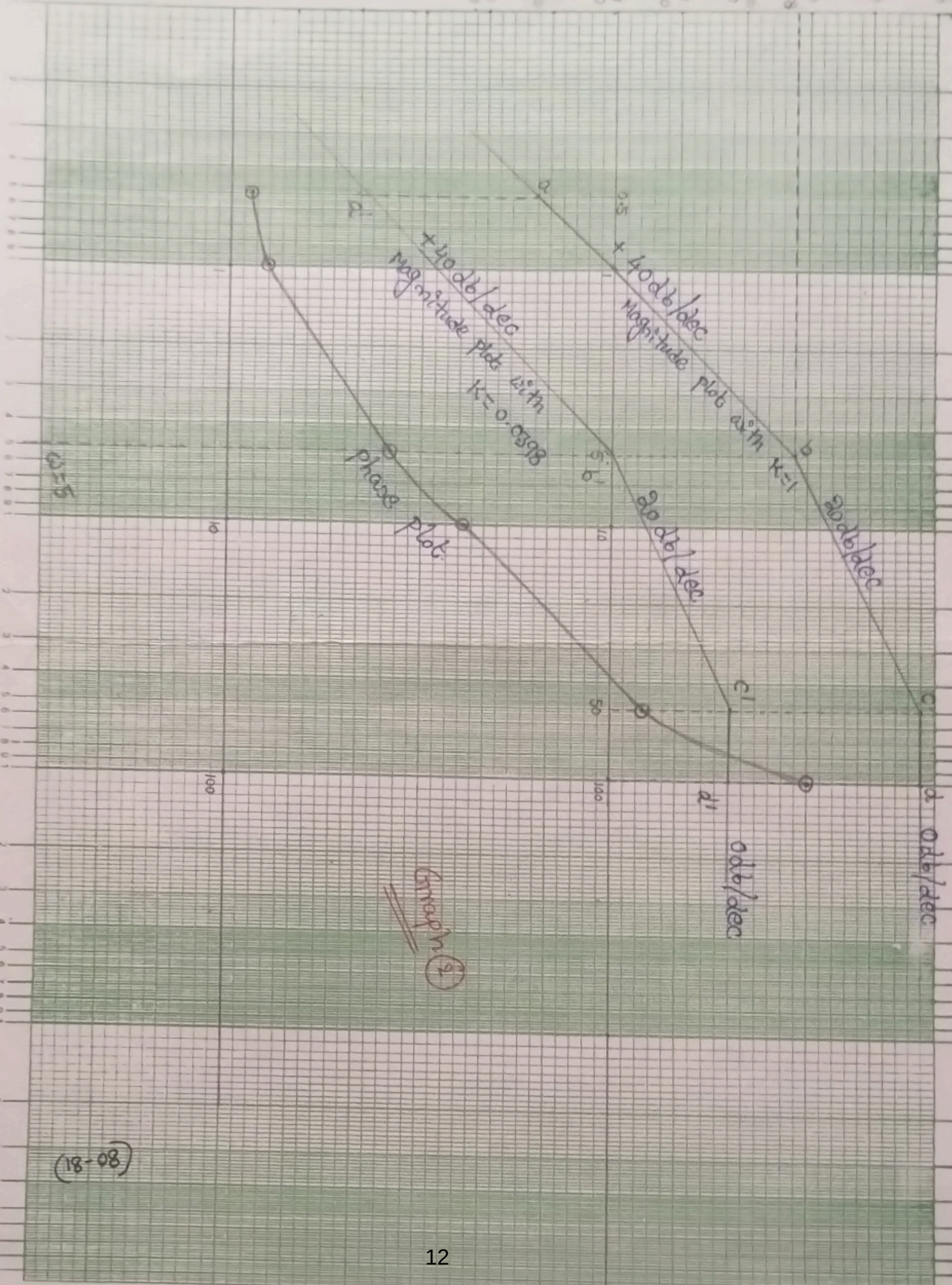
let $\omega_L = 0.5 \text{ rad/sec}$

$\omega_H = 100 \text{ rad/sec.}$

Let $A = |G(j\omega)|$ in db.

at $\omega = \omega_1$ $A = 0$ $\phi = 180^\circ$ $\rightarrow \phi$ in deg & A in db/dec

$\rightarrow \omega$ in rad/sec



Graph 2

(18-08)

at $\omega = \omega_1$, $A = 20 \log |(j\omega)^2| = 20 \log \omega^2 = 20 \log (0.5)^2 = -12 \text{ dB}$

at $\omega = \omega_{c1}$, $A = 20 \log |(j\omega)^2| = 20 \log \omega^2 = 20 \log (5)^2 = 28 \text{ dB}$

at $\omega = \omega_{c2}$, $A = \cancel{20 \log} \left[\text{slope from } \omega_{c1} \text{ to } \omega_{c2} \times \log \left(\frac{\omega_{c2}}{\omega_{c1}} \right) \right] + A_{(\text{at } \omega = \omega_{c1})}$
 $= 20 \log \left(\frac{50}{5} \right) + 28 = 48 \text{ dB}$

at $\omega = \omega_h$, $A = \left[\text{slope from } \omega_{c2} \text{ to } \omega_h \times \log \frac{\omega_h}{\omega_{c2}} \right] + A_{(\text{at } \omega = \omega_{c2})}$
 $= 0 \log \left(\frac{100}{50} \right) + 48 = 48 \text{ dB}.$

Let the points a, b, c & d are corresponding to frequencies ω_1 , ω_{c1} , ω_{c2} & ω_h on the magnitude plot. fix the points a, b, c & d and join these points by straight line & marked the slope on the respective region.

Phase plot :-

The phase angle of $G(j\omega)$ as a function of ω is given by

$$\Rightarrow \phi = \angle G(j\omega) = 180 - \tan^{-1}(0.2\omega) - \tan^{-1}(0.02\omega)$$

ω (rad/sec)	$\phi = \angle G(j\omega)$ deg
0.5	174
1	168
5	130
10	106
50	50
100	30

Graph (2)

on the semilog graph mark the phase angle & join these points by a smooth curve.

Calculation of K:-

Given that the gain crossover frequency is 5 rad/sec.

at $\omega = 5$ rad/sec the gain is 28db. If $\omega_{cg} = 5$ rad/sec then at that frequency the db gain should be zero. Hence Every point of magnitude plot a db gain of -28db should be added. The addition of -28db shifts the plot downwards. The corrected magnitude plot is obtained by shifting the plot with $K=1$ by 28 db downwards.

$$\therefore 20 \log K = -28 \text{ db}$$

$$\therefore \log K = \frac{-28}{20}$$

$$K = 10^{-28/20} = 0.0398.$$

(3) Draw the bode plot for the system having open loop T/F

$$G(s) = 100 / s(s+1)(s+2). \text{ find GM, PM, } \omega_{cg} \text{ \& } \omega_{cp} ?$$

The sinusoidal transfer function of $G(j\omega)$ is obtained by replacing s by $j\omega$.

$$\begin{aligned} G(j\omega) &= 100 / (j\omega)(1+j\omega)^2(1+0.5j\omega) \\ &= 50 / (j\omega)(1+j\omega)(1+0.5j\omega) \end{aligned}$$

Magnitude plot:-

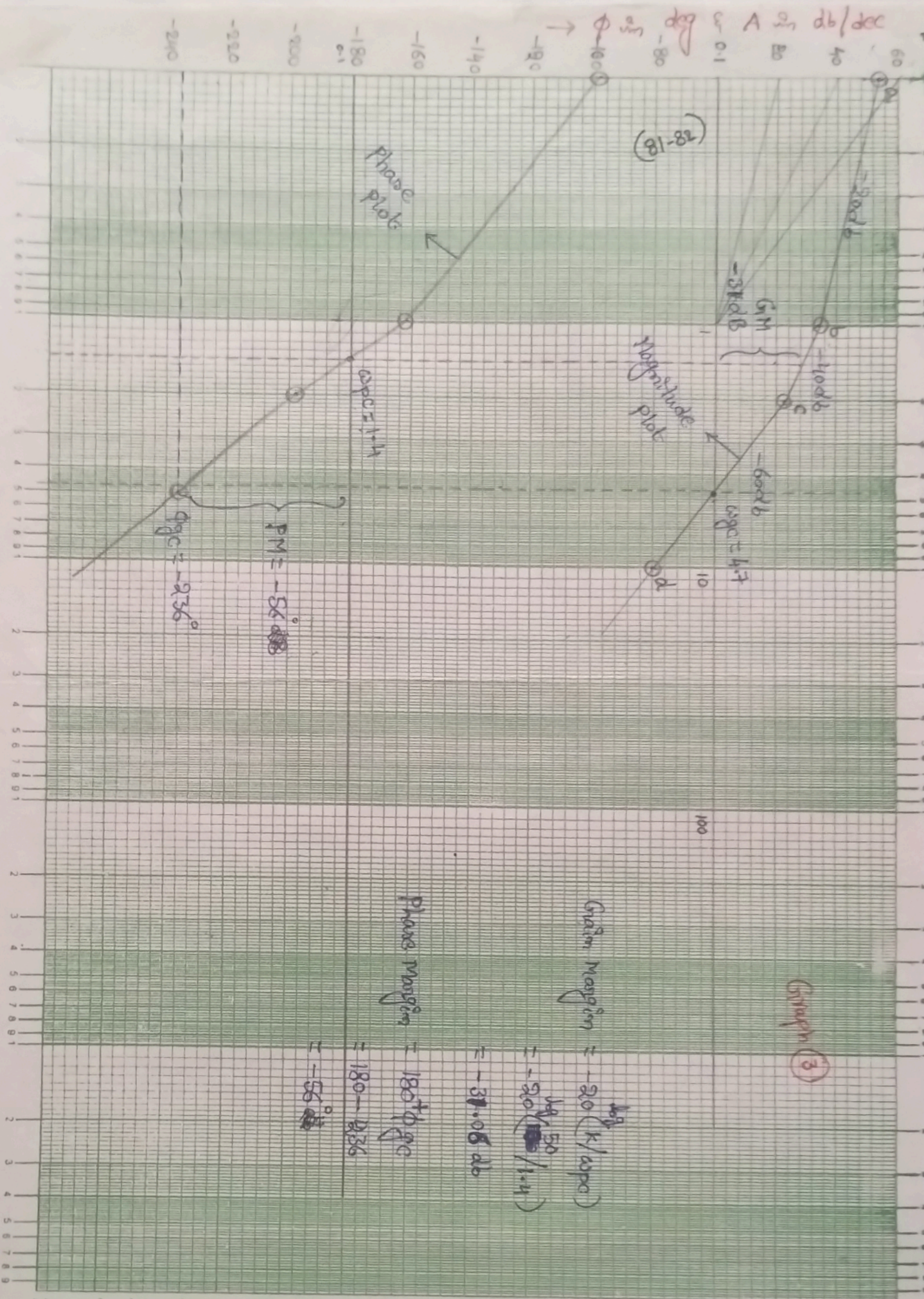
The corner frequencies are

$$\omega_{C1} = 1/1 = 1 \text{ rad/sec.}$$

$$\omega_{C2} = 1/0.5 = 2 \text{ rad/sec.}$$

Term	corner frequency (rad/sec)	slope (db/dec)	change in slope (db/dec)
$50/j\omega$	-	-20	-20
$1/(1+j\omega)$	$\omega_{C1} = 1/1 = 1$	-20	$-20 - 20 = -40$
$1/(1+0.5j\omega)$	$\omega_{C2} = 1/0.5 = 2$	-20	$-40 - 20 = -60$

→ $\omega \text{ in rad/sec}$



Phase margin = $180^\circ \phi_{gc}$

$$= 180 - 236$$

$$= -56.48^\circ$$

Gain Margin = $-20 \log(K/\omega_{gc})$

$$= -20 \log(50/1.4)$$

$$= -37.06 \text{ dB}$$

Choose a low frequency ω_l such that $\omega_l < \omega_{c1}$ and high frequency ω_h such that $\omega_h > \omega_{c2}$. (2)

Let $\omega_l = 0.1 \text{ rad/sec}$

$\omega_h = 10 \text{ rad/sec}$.

Graph (3)

Let $A = |G(j\omega)|$ in db.

at $\omega = \omega_l$, $A = 20 \log\left(\frac{50}{j\omega}\right) = 20 \log\left(\frac{50}{0.1}\right) = 53.98 \approx 54 \text{ db}$

at $\omega = \omega_{c1}$, $A = 20 \log\left(\frac{50}{j\omega}\right) = 20 \log\left(\frac{50}{1}\right) = 33.98 \approx 34 \text{ db}$

at $\omega = \omega_{c2}$, $A = \left[\text{slope from } \omega_{c1} \text{ to } \omega_{c2} \times \log\left(\frac{\omega_{c2}}{\omega_{c1}}\right) \right] + A(\text{at } \omega = \omega_{c1})$

$= -40 \log\left(\frac{2}{1}\right) + 34 = 21.96 \approx 22 \text{ db}$

at $\omega = \omega_h$, $A = \left[\text{slope from } \omega_{c2} \text{ to } \omega_h \times \log\left(\frac{\omega_h}{\omega_{c2}}\right) \right] + A(\text{at } \omega = \omega_{c2})$

$= -60 \log\left(\frac{10}{2}\right) + 22 = -19.94 \approx -20 \text{ db}.$

Let the points a, b, c & d are corresponding to frequencies ω_l , ω_{c1} , ω_{c2} & ω_h - on magnitude plot. Fix the points a, b, c & d on graph & join these points by a straight line & mark the slope in the respective region.

Phase Plot :-

The phase angle of $G(j\omega)$ as a function of ω is given by

$\phi = -90 - \tan^{-1}(0.5\omega) - \tan^{-1}\omega$

ω (rad/sec)	$\phi = \angle G(j\omega)$ deg.
0.1	-98.57
1	-161.56
2	-198.43
4	-229.40
5	-236.89
10	-252.98
20	-261.43
50	-266.56

Mark the phase angle on the graph & join these points by a smooth curve.

(4) Sketch the bode plots for a system $G(s) = 15(s+5)/s(s^2+16s+100)$ & also determine GM, PM, ω_{gc} , ω_{pc} & determine the stability of the system.

Given $G(s) = 15(s+5)/s(s^2+16s+100)$

on comparing the quadratic factor of $G(s)$ with standard form of quadratic factor.

$$s^2 + 16s + 100 = s^2 + 2\delta\omega_n s + \omega_n^2$$

$$\begin{array}{l|l} \omega_n^2 = 100 & 2\delta\omega_n = 16 \\ \Rightarrow \omega_n = 10 & \Rightarrow \delta = 16/2(10) = 0.8 \end{array}$$

$$G(s) = 15(5)(1+0.2s)/s(100)(1+0.01s^2+0.16s)$$

The sinusoidal transfer function $G(j\omega)$ is obtained by replacing s by $j\omega$.

$$\begin{aligned} G(j\omega) &= 75(1+j0.2\omega)/100(j\omega)(1+0.01\omega^2+j0.16\omega) \\ &= 0.75(1+j0.2\omega)/(j\omega)(1-0.01\omega^2+j0.16\omega) \end{aligned}$$

Magnitude plot:-

The corner frequency are $\omega_{c1} = 1/0.2 = 5 \text{ rad/sec}$

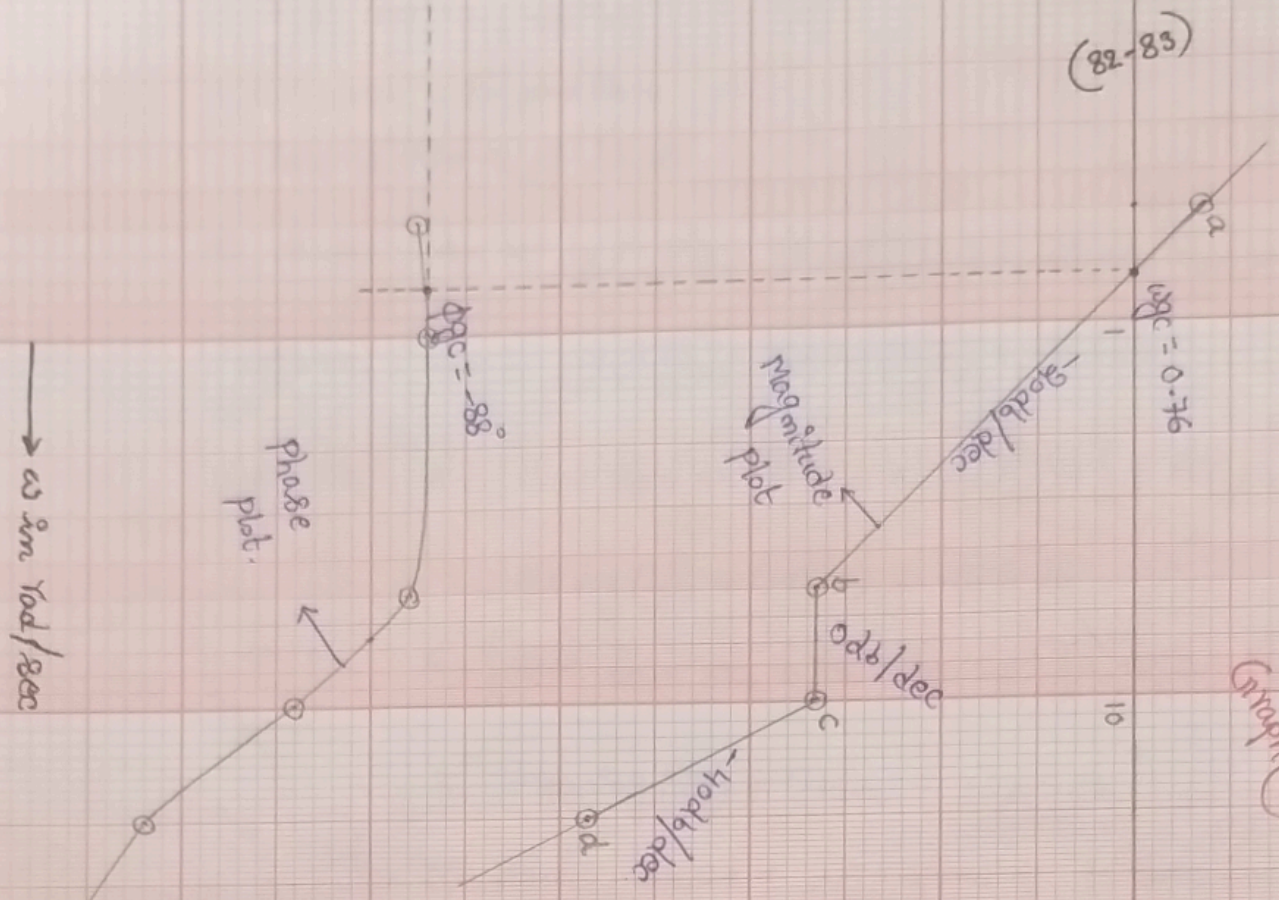
$\omega_{c2} = \omega_n = 10 \text{ rad/sec}$. $\therefore$ from quadratic factor.

Term	Corner frequency rad/sec.	slope db/dec	change in slope db/dec.
$0.75/j\omega$	-	-20	-20
$1+0.2j\omega$	$1/0.2 = 5 = \omega_{c1}$	+20	$-20+20 = 0$
$1/(1-0.01\omega^2+j0.16\omega)$	$\omega_{c2} = \omega_n = 10$	-40	$0-40 = -40$

A in db
 ϕ in deg

ϕ in deg & A in db/dec

Graph 4



Scale:-

Y-axis unit = 5 db

Y-axis unit = 90 deg.

Phase Margin (PM) = $180 + \phi_{gc}$

= $180 - 88$

= 92°

Hence Gain margin = $+\infty$

∴ The phase plot crosses -180° at infinity.

The $|a(\omega)|$ at infinity is $-\infty$ db. Hence

Gain margin is $+\infty$.

Gain Crossover frequency (ω_{gc}) = 0.76

Phase Crossover frequency (ω_{pc}) = ∞ .

∴ The system is stable because

both Gain margin & Phase margin are +ve.

choose a low frequency ω_l such that $\omega_l < \omega_{c1}$ and choose a high frequency ω_h such that $\omega_h > \omega_{c2}$.

(83)

Let $\omega_l = 0.5 \text{ rad/sec}$

$\omega_h = 20 \text{ rad/sec}$

Graph (4)

Let $A = |G(j\omega)| \text{ in db.}$

At $\omega = \omega_l$, $A = 20 \log \left(\frac{0.75}{j\omega} \right) = 20 \log \left(\frac{0.75}{0.5} \right) = 3.5 \text{ db.}$

$\omega = \omega_{c1}$, $A = 20 \log \left(\frac{0.75}{j\omega} \right) = 20 \log \left(\frac{0.75}{5} \right) = -16.5 \text{ db.}$

$\omega = \omega_{c2}$, $A = \left[\text{slope from } \omega_{c1} \text{ to } \omega_{c2} \times \log \left(\frac{\omega_{c2}}{\omega_{c1}} \right) \right] + A(\text{at } \omega = \omega_{c1})$
 $= 0 \times \log \left(\frac{10}{5} \right) + (-16.5) = -16.5 \text{ db}$

$\omega = \omega_h$, $A = \left[\text{slope from } \omega_{c2} \text{ to } \omega_h \times \log \left(\frac{\omega_h}{\omega_{c2}} \right) \right] + A(\text{at } \omega = \omega_{c2})$
 $= -40 \log \left(\frac{20}{10} \right) + (-16.5) = -28.5 \text{ db}$

Let the points a, b, c & d corresponding to frequencies ω_l , ω_{c1} , ω_{c2} & ω_h respectively. on the magnitude plot. fix the points a, b, c & d on the graph & join the points by straight line & mark the slope on the respective region.

Phase plot :-

The phase angle of $G(j\omega)$ as a function of ω is given by

$\phi = \tan^{-1}(0.2\omega) - 90^\circ - \tan^{-1} \left(\frac{0.16\omega}{1 - 0.01\omega^2} \right)$ for $\omega \leq \omega_n$
 $= \tan^{-1}(0.2\omega) - 90^\circ - \left(\tan^{-1} \frac{0.16\omega}{1 - 0.01\omega^2} + 180^\circ \right)$ for $\omega > \omega_n$.

ω (rad/sec)	$\tan^{-1}(0.2\omega)$	$\tan^{-1} \left(\frac{0.16\omega}{1 - 0.01\omega^2} \right)$	$\phi = \angle G(j\omega)$
0.1	1.14	0.916	$\nearrow -90$
0.5	5.7	4.6	$\nearrow -88$
1	11.3	9.2	$\nearrow -92$
5	45	46.8	$\nearrow -116$
10	63.4	90	$\nearrow -148$
20	75.9	$-46.8 + 180 = 133.2$	$\nearrow$
50	84.3	$-18.4 + 180 = 161.6$	-168
100	87.1	$-9.2 + 180 = 170.8$	-174

Note: In quadratic factors the angle varies from 0° to 180° . But the calculator calculates $\tan^{-1}$ only b/n 0° to 90° . Hence a correction factor of 180° should be added to the phase angle after corner frequency.

(5) Plot the Bodeplot for the function $G(s) = 5(1+2s)/(1+4s)(1+0.25s)$.

The sinusoidal T/F is given by

$$\Rightarrow G(j\omega) = 5(1+j2\omega)/(1+j4\omega)(1+j0.25\omega)$$

Magnitude plot:-

The corner frequencies are

$$\omega_{c1} = 1/4 = 0.25 \text{ rad/sec} \quad \omega_{c2} = 1/2 = 0.5 \text{ rad/sec}$$

$$\omega_{c3} = 1/0.25 = 4 \text{ rad/sec}$$

Term	Corner frequency rad/sec	Slope db/dec	Change in slope db/deg
5	-	0	0
$1/(1+j4\omega)$	$\omega_{c1} = 1/4 = 0.25$	-20	$0 - 20 = -20$
$1/(1+j2\omega)$	$\omega_{c2} = 1/2 = 0.5$	+20	$-20 + 20 = 0$
$1/(1+j0.25\omega)$	$\omega_{c3} = 1/0.25 = 4$	-20	$0 - 20 = -20$

Choose a low frequency ω_L such that $\omega_L < \omega_{c1}$ & high frequency ω_H such that $\omega_H > \omega_{c3}$.

$$\text{Let } \omega_L = 0.1 \text{ rad/sec} \quad \& \quad \omega_H = 10 \text{ rad/sec}$$

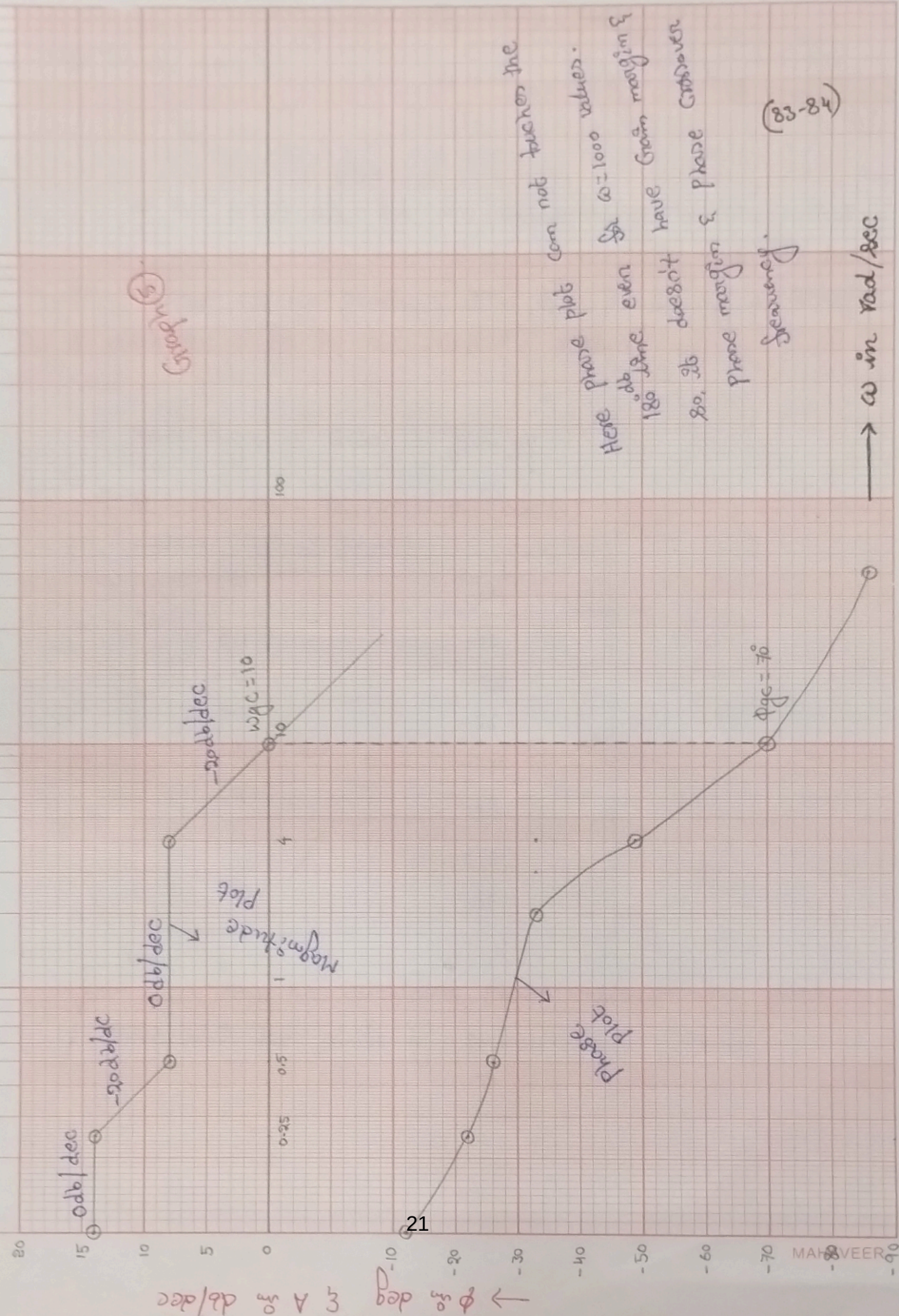
$$\text{Let } A = |G(j\omega)| \text{ in db}$$

$$\text{at } \omega = \omega_L, A = 20 \log(5) = +14 \text{ db}$$

$$\omega = \omega_{c1}, A = 20 \log(5) = +14 \text{ db}$$

20

$$\begin{aligned} \omega = \omega_{c2}, A &= \left(\text{slope from } \omega_{c1} \text{ to } \omega_{c2} \times \log \frac{\omega_{c2}}{\omega_{c1}} \right) + A(\text{at } \omega = \omega_{c1}) \\ &= -20 \log \left(\frac{0.5}{0.25} \right) + 14 = +8 \text{ db} \end{aligned}$$



Graph 5

Here phase plot can not be taken the for $\omega = 1000$ values. 180 dB line even 80, 36 doesn't have Gain margin & phase margin & phase crossover frequency. (83-84)

$\rightarrow \omega$ in rad/sec

$\rightarrow \phi$ in deg & A in dB/dec

$$\text{at } \omega = \omega_{c3}, A = \left[\text{slope from } \omega_{c2} \text{ to } \omega_{c3} \times \log\left(\frac{\omega_{c3}}{\omega_{c2}}\right) \right] + A(\text{at } \omega = \omega_{c2}) \quad (84)$$

$$= 0 \log(4/0.5) + 8 = 8 \text{ db}$$

$$\text{at } \omega = \omega_h, A = \left[\text{slope from } \omega_h \text{ to } \omega_{c3} \times \log\left(\frac{\omega_h}{\omega_{c3}}\right) \right] + A(\text{at } \omega = \omega_{c3})$$

$$= -20 \log\left(\frac{10}{4}\right) + 8 = 0 \text{ db.}$$

Let the points a, b, c & d are correspondings to frequencies $\omega_1, \omega_{c1}, \omega_{c2}, \omega_{c3}$ & ω_h respectively on magnitude plot. Fix the points a, b, c & d on the graph & join the points by a straight line & mark the slope in the respective region.

Phase Plot :-

The phase angle of $G(j\omega)$ is given by

$$\Rightarrow \phi = \tan^{-1}(2\omega) - \tan^{-1}(4\omega) - \tan^{-1}(0.25\omega)$$

ω	$\phi = \angle G(j\omega)$
0.1	-12
0.25	-22
0.5	-26
2	-33
4	-49
10	-70
50	-86

Graph (5)

Mark the calculated phase angle on the graph sheet. Join the points by smooth curve.