BODE PLOT

Exact Frequency Analysis of a system

- Given T.F of a system

$$T(s)$$

S-1 : put $s = j\omega$ $\quad 0^+ < \omega < \infty$

S-2 : $T(j\omega) = |T(j\omega)|\, e^{j\angle T(j\omega)}$

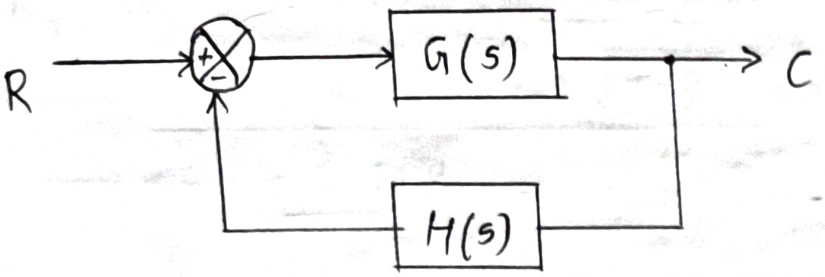
S-3 : $|T(j\omega)|$ vs $\omega \rightarrow$ Exact Magnitude plot

$\angle T(j\omega)$ vs $\omega \rightarrow$ Exact Phase plot

Note:

1. Exact ~~poles~~ plots are non-linear in shape. in general
2. Exact plots are drawn on normal graph paper.

Bode plot: APPROXIMATE FREQUENCY RESPONSE



$$T(s) = G(s)H(s) \xrightarrow{s=j\omega} T(j\omega) \begin{cases} |T(j\omega)| \text{ dB vs } \log_{10}\omega \\ \angle T(j\omega) \text{ dB vs } \log_{10}\omega \end{cases}$$

↓

OLTF

1. $T(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$

* **Stability of T(s)** → Time response will define stability based on pole location of T(s)

→ Can also be determined by plotting Bode plot of $G(s)H(s)$

* Bandwidth of T(s) → Exact frequency response analysis of T(s).

Bode Plot: Let OLTF $T(s) = G(s)H(s)$

CASE-1

S-1 Put $s = j\omega$ for +ve frequency line $(0 < \omega < \infty)$

$$T(j\omega) = G(j\omega)H(j\omega)$$

S-2 $$|T(j\omega)| = |G(j\omega)H(j\omega)|$$

$$20\log_{10}|T(j\omega)| = 20\log_{10}|G(j\omega)H(j\omega)|$$

$$\boxed{|T(j\omega)|_{dB} = 20\log_{10}|G(j\omega)H(j\omega)|}$$

$y = mx + c$

→ Make sure $|T(j\omega)|dB$ vs $\log_{10}\omega$ is Linear Graph.

Use Approximation if needed

$\angle T(j\omega)$ vs $\log_{10}\omega$

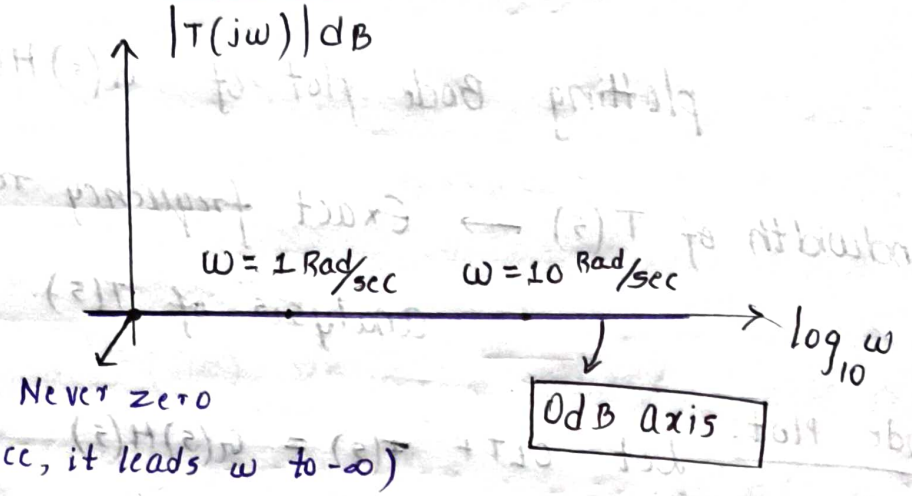
→ Exact Phase Plot

↓

Need not be Linear

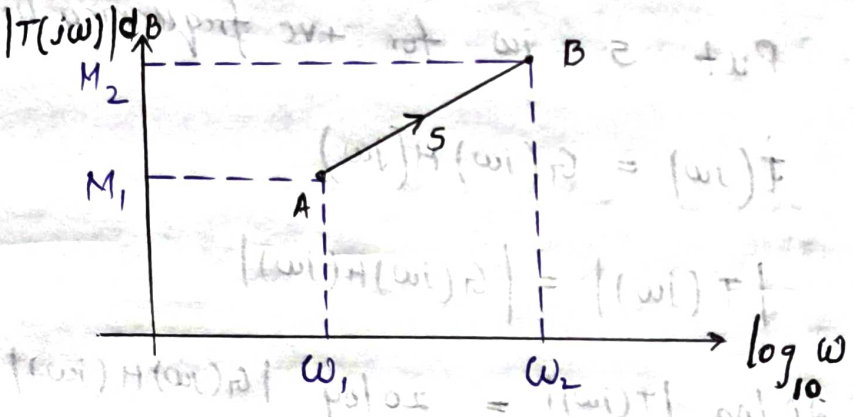
Important points :

1.



so, let $\omega = 0.001$ Rad/sec

2.

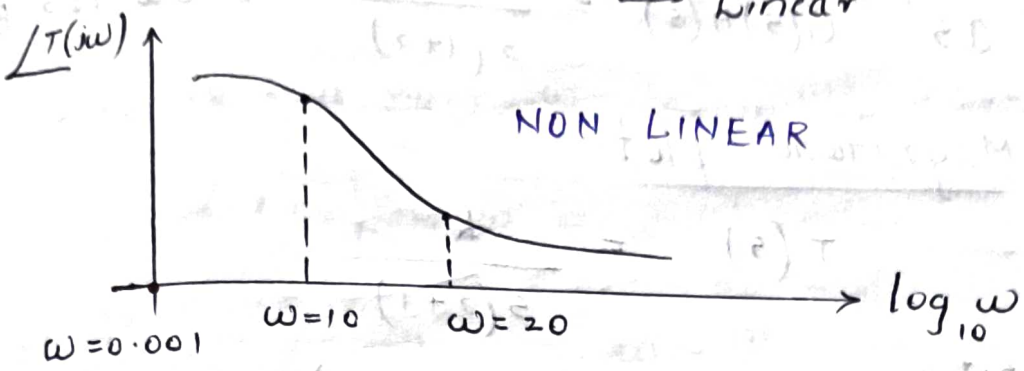


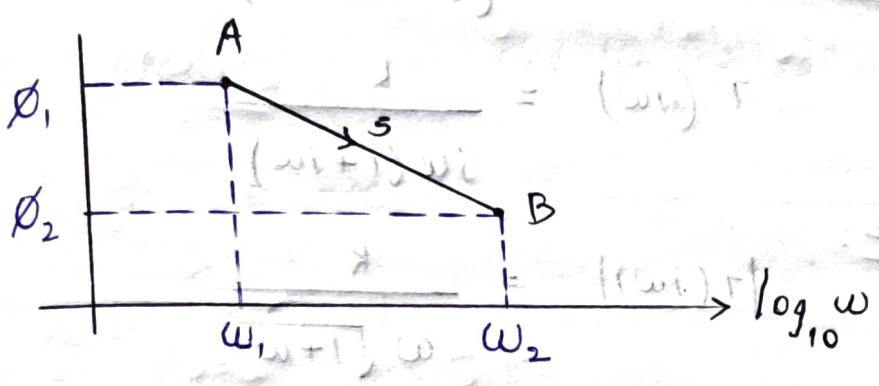
$$A(x_1, y_1) = A(\log_{10}\omega_1, M_1 dB)$$

$$B(x_2, y_2) = B(\log_{10}\omega_2, M_2 dB)$$

$$S = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(M_2 - M_1)}{\log_{10}\omega_2 - \log_{10}\omega_1}$$

3. $\angle T(j\omega)$ vs $\log_{10}\omega$ → Non-linear, Linear





$A(\log_{10}\omega_1, \phi_1)$ $B(\log_{10}\omega_2, \phi_2)$

$$S = \frac{\phi_2 - \phi_1}{\log_{10}\omega_2 - \log_{10}\omega_1}$$

Note:

1. Magnitude plot can be made linear by appropriate approximation, mathematically.
2. Phase plot can't be made linear by appropriate approximation, mathematically.
3. Phase plot will be made linear by using logical assumption.

→ **Question:** OLTF of a C.L.S is mentioned as $G(s)H(s) = \frac{k}{s(1+s)}$. Draw its BODE Plot

Magnitude plot

$$T(s) = \frac{k}{s(s+1)}$$

S-1: $s = j\omega \quad (0 < \omega < \infty)$

$$T(j\omega) = \frac{k}{j\omega(1+j\omega)}$$

S-2:

$$|T(j\omega)| = \frac{k}{\omega\sqrt{1+\omega^2}}$$

$$|T(j\omega)|\,dB = 20\log_{10} k - 20\log_{10}[\omega(1+\omega^2)^{3/2}]$$

$$|T(j\omega)|\,dB = 20\log_{10} k - 20\log_{10}\omega - 10\log_{10}(1+\omega^2)$$

$$\underbrace{|T(j\omega)|\,dB}_{y} = \underbrace{-20}_{m}\underbrace{\log_{10}\omega}_{x} + \left[20\log_{10} k - 10\log_{10}(1+\omega^2)\right]$$

$\downarrow \omega << 1$

$\omega^2 <<< 1$

$1+\omega^2 \approx 1$

$\log_{10} 1 = 0$

$$\boxed{\underbrace{|T(j\omega)|\,dB}_{y} = \underbrace{-20}_{m}\underbrace{\log_{10}\omega}_{x} + \underbrace{20\log_{10} k}_{c}}$$

↳ LINEAR

↓

BODE MAG. PLOT

S-3: $\angle T(j\omega) = -\frac{\pi}{2} - \tan^{-1}\omega$

↳ Non linear

since, $\tan^{-1}\omega = \omega - \frac{\omega^3}{3} + \frac{\omega^5}{5} - \ldots$

Let $\boxed{\tan^{-1}\omega \approx \omega}$

Therefore, $\angle T(j\omega) = -\omega - \frac{\pi}{2}$

We want $\angle T(j\omega$ vs $\log_{10}\omega$

$\log_{10}\omega = x$

$\angle T(j\omega) = y$

$\rightarrow y = -10^x - \frac{\pi}{2}$ → Non Linear

Numericals on BODE PLOT:

Question: $G(s)H(s) = 1 \rightarrow T(s) = 1$

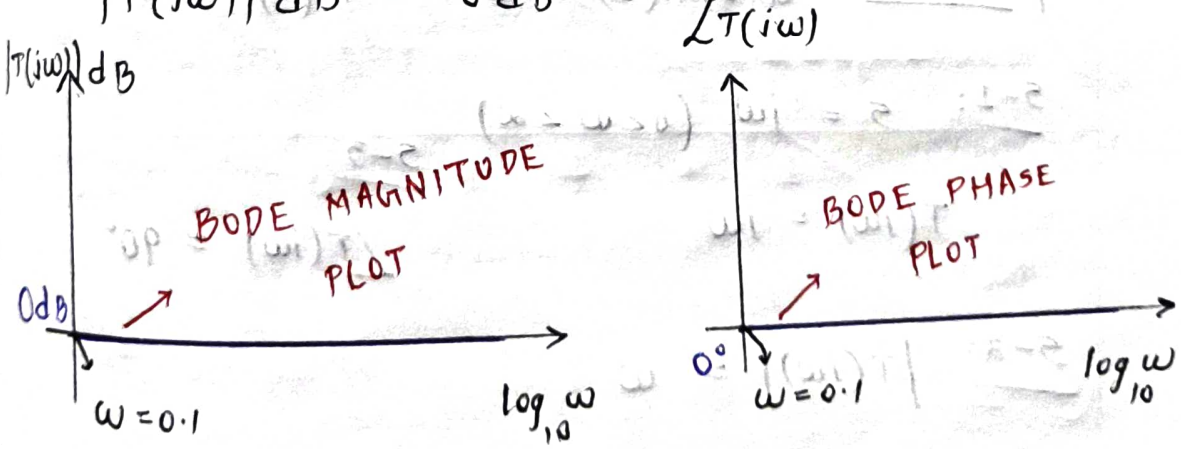
S-1: $s = j\omega \quad (0 < \omega < \infty)$

$T(j\omega) = 1$

S-2: $|T(j\omega)| = 1$

$|T(j\omega)|_{dB} = 0\,dB$

S-3: $\angle T(j\omega) = 0°$



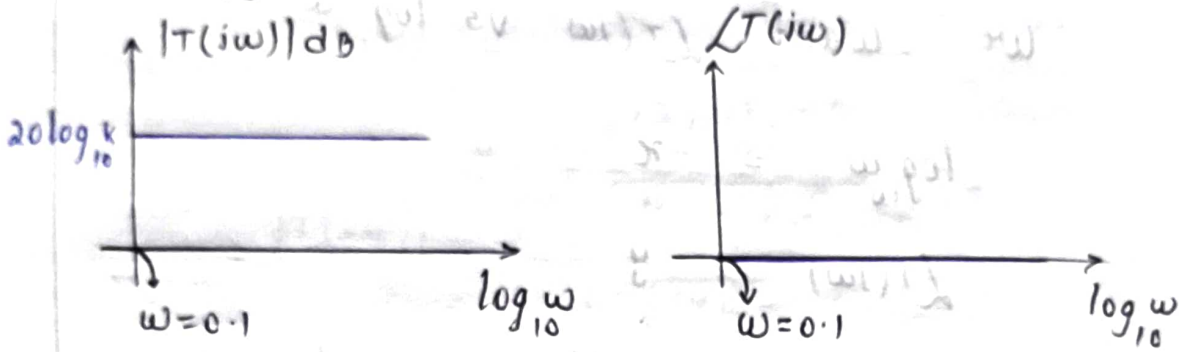
→ Question:

$G(s)H(s) = k \longrightarrow T(s) = k$

S-1 $T(j\omega) = k$, where $j\omega = s$, $0 \le \omega < \infty$

S-2 $|T(j\omega)|_{dB} = 20\log_{10} k$ — $0 < k < 1$: −ve in dB; $k > 1$: +ve in dB

S-3 $\angle T(j\omega) = 0°$



→ Note:

On multiplying a gain k to G(s)H(s)

(i) Bode magnitude plot shifts parallely by $(20\log_{10} k)$ dB upward or downward.

(ii) Bode plot remains unchanged.

→ Question: $G(s)H(s) = s \rightarrow T(s) = s$

S-1: $s = j\omega$ $(0 < \omega < \infty)$

$T(j\omega) = j\omega$

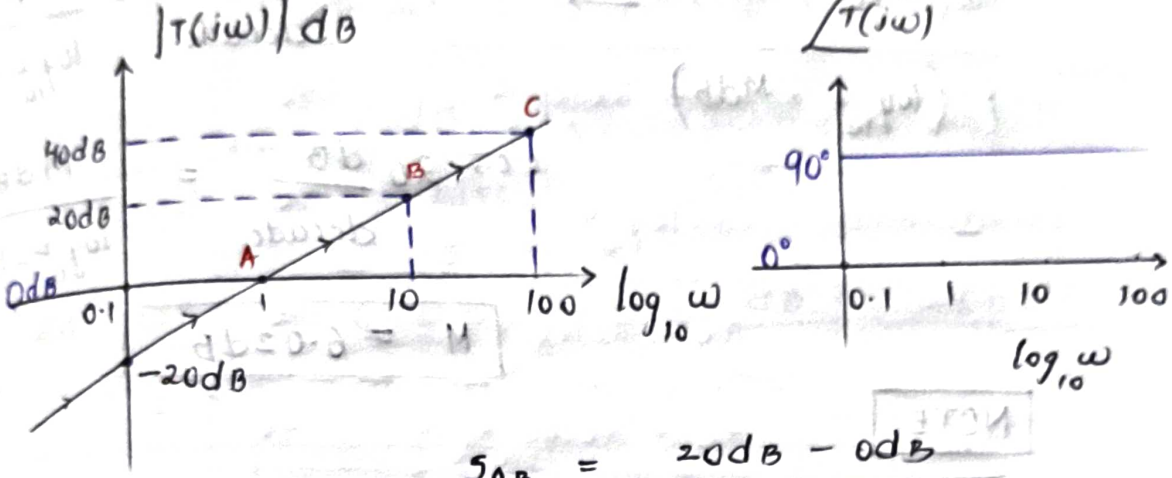
S-2: $|T(j\omega)| = \omega$

$|T(j\omega)|_{dB} = 20\log_{10} \omega$

$y = m \; x$

S-3: $\angle T(j\omega) = 90°$

ω	$\lvert T(j\omega)\rvert$ dB	$\angle T(j\omega)$
0.1	−20 dB	90°
1	0 dB	90°
10	20 dB	90°
100	40 dB	90°



$A(\log_{10}1, 0\,dB)$

$B(\log_{10}10, 20\,dB)$

$C(\log_{10}100, 40\,dB)$

$$S_{AB} = \frac{20\,dB - 0\,dB}{\log_{10}10 - \log_{10}1}$$

$$\rightarrow S_{AB} = \frac{20\,dB}{\log_{10}\left(\frac{10}{1}\right)} \rightarrow \text{decade}$$

$$S_{AB} = \frac{20\,dB}{\text{decade}}$$

$$S_{AC} = \frac{40\,dB - 0\,dB}{\log_{10}100 - \log_{10}1}$$

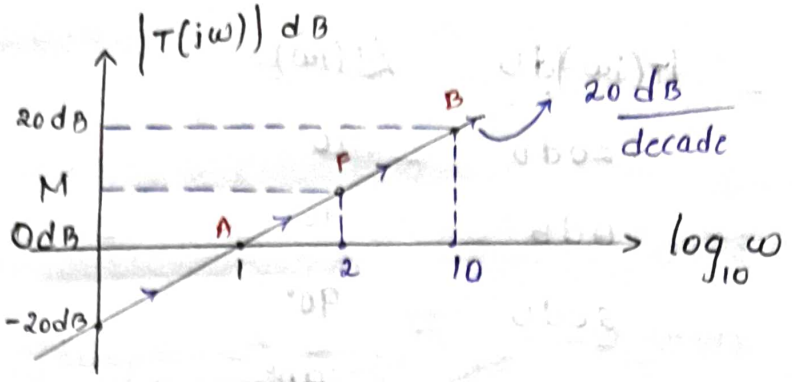
$$= \frac{40\,dB}{\log_{10}\left(\frac{100}{1}\right)}$$

$$= \frac{40\,dB}{\log\frac{10^2}{1}}$$

$$= \frac{40\,dB}{2\log_{10}\left(\frac{10}{1}\right)}$$

$$S_{AC} = \frac{20\,dB}{\log_{10}\left(\frac{10}{1}\right)}$$

$$S_{AC} = \frac{20\,dB}{\text{decade}} = S_{AB}$$



$A\left(\log_{10}1, 0dB\right) \longrightarrow S_{AF} = \dfrac{20\ dB}{decade} = \dfrac{MdB - 0dB}{\log_{10}2 - \log_{10}1}$

$F\left(\log_{10}2, MdB\right)$

i.e., $\dfrac{20\ dB}{decade} = \dfrac{M\,dB}{\log_{10}2 - \log_{10}1}$

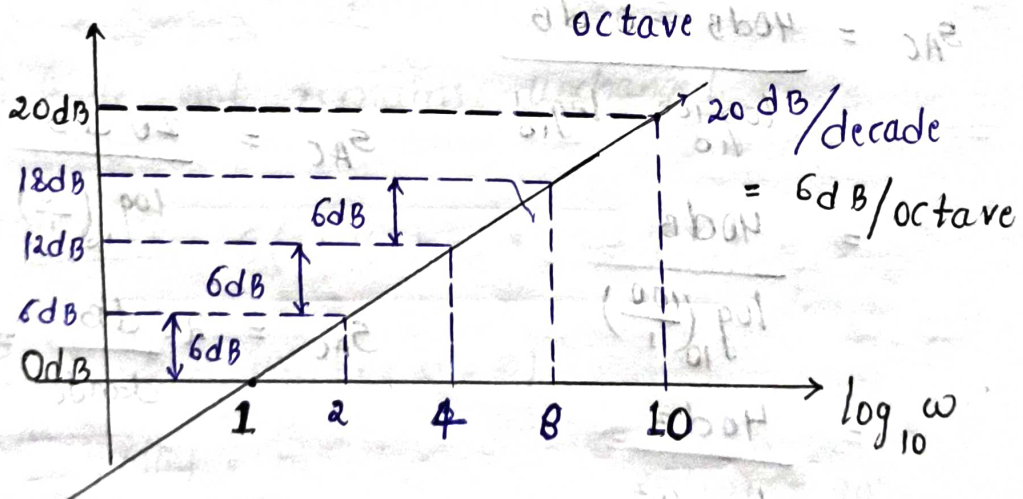
$$\boxed{M = 6.02\,dB}$$

NOTE

$$S_{AF} = \frac{20\ dB}{decade} = \frac{6.02dB - 0dB}{\log_{10}2 - \log_{10}1}$$

$$= \frac{6\,dB}{\log_{10}\left(\frac{2}{1}\right)} = \frac{6dB}{octave}$$

$\log_{10}\left(\frac{2}{1}\right)$ = octave



NOTE :

(1) $\dfrac{20\ dB}{decade}$ → As frequency ↑ by 10 times magnitude ↑ by 20 dB.

② $-20\ \frac{dB}{decade}$ → As frequency ↑ by 10 times, magnitude ↓ by 20 dB

③ $\pm 6\ \frac{dB}{octave}$ → As frequency ↑ by 2 times, magnitude ↑↓ by 6 dB

NOTE:

* $20\ \frac{dB}{decade} = 6\ \frac{dB}{octave}$	* $-20\ \frac{dB}{decade} = -6\ \frac{dB}{octave}$
* $20n\ \frac{dB}{decade} = 6n\ \frac{dB}{octave}$	* $-20n\ \frac{dB}{decade} = -6n\ \frac{dB}{octave}$
↓	↓
n = +ve integer	n = +ve integer

→ Question: $G(s)H(s) = ks$ → $T(s)$

S-1: $s = j\omega \quad (0 < \omega < \infty)$

$T(j\omega) = k\,j\omega$

S-2: $|T(j\omega)| = k\omega$

$|T(j\omega)|\,dB = 20\log_{10}\omega + 20\log_{10}k$

$y = m\,x + c$

S-3: $\angle T(j\omega) = 90°$

0dB Axis Intersection:

$|T(j\omega)|\,dB = 20\log_{10}\omega + 20\log_{10}k$

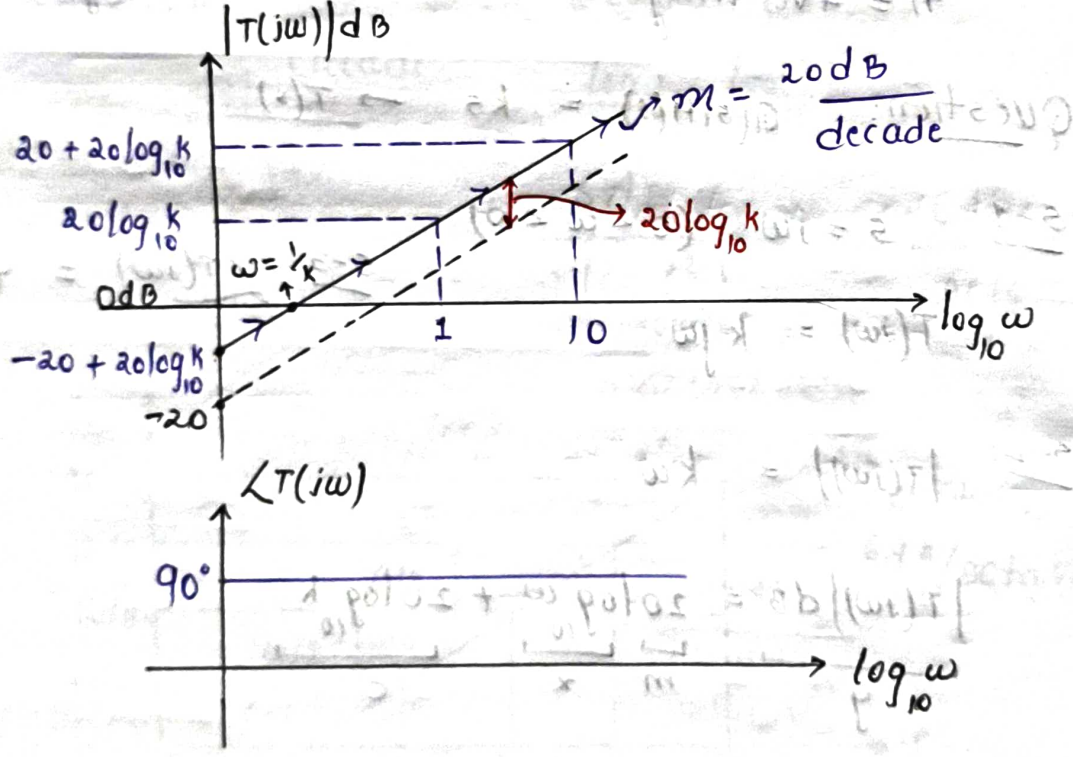
$|T(j\omega)|\,dB = 0\,dB$

$$20\log_{10}\omega + 20\log_{10}k = 0$$

$$\log_{10}\omega = -\log_{10}k$$

$$\boxed{\omega = \frac{1}{k}}$$

ω	$\lvert T(j\omega)\rvert$ dB	$\angle T(j\omega)$
0.1	$-20 + 20\log_{10}k$	90°
1	$20\log_{10}k$	90°
10	$20 + 20\log_{10}k$	90°
100	$40 + 20\log_{10}k$	90°



→ Question: $G(s)H(s) = ks^p \longrightarrow T(s)$

5-1: $s = j\omega$

$$T(j\omega) = k(j\omega)^p$$

5-2: $|T(j\omega)| = k\omega^p$

$$\underbrace{|T(j\omega)|_{dB}}_{y} = \underbrace{20p}_{m}\underbrace{\log_{10}\omega}_{x} + \underbrace{20\log_{10}k}_{c}$$

5-3: $\angle T(j\omega) = 90^\circ p$

ω	$\lvert T(j\omega)\rvert$ dB	$\angle T(j\omega)$
0.1	$-20p + 20\log_{10}k$	90°
1	$20\log_{10}k$	90°
10	$20p + 20\log_{10}k$	90°
100	$40p + 20\log_{10}k$	90°

Intersection with 0dB Axis

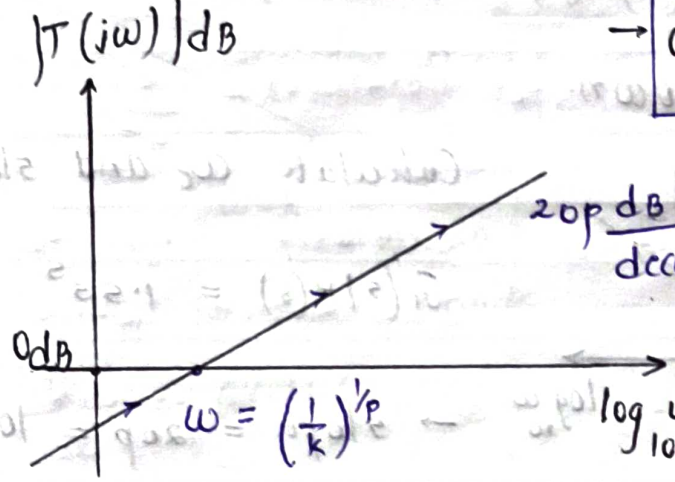
$|T(j\omega)|_{dB} = 0\,dB$

$20p\log_{10}\omega = -20\log_{10}k$

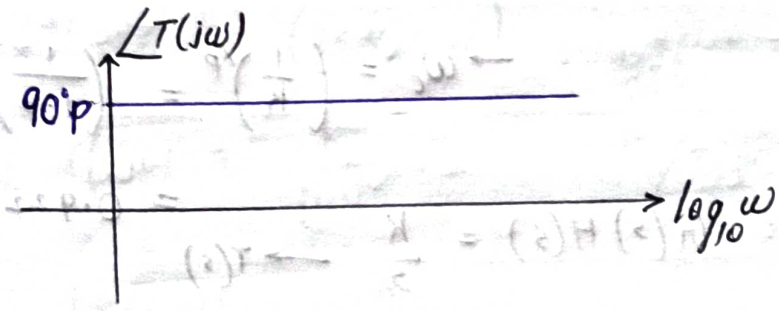
$\log\omega^p = \log_{10}k^{-1}$

$\omega^p = \frac{1}{k}$

$$\rightarrow \boxed{\omega = \left(\frac{1}{k}\right)^{\frac{1}{p}}}$$



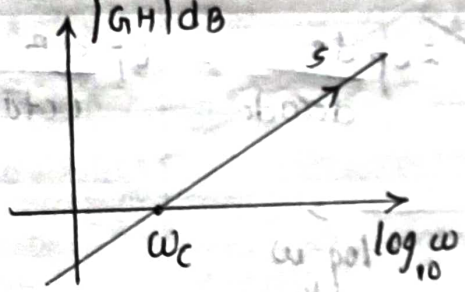
$20p\ \frac{dB}{decade} = 6p\ \frac{dB}{octave}$



Summary Table for $T(s) = Ks^p$

$G(s)H(s)$	Initial slope	0dB Axis Intersection	slope as $\omega \to \infty$	Phase
K	0dB/decade	NO	0dB/decade	$0°$
Ks	+20dB/decade	$\frac{1}{K}$	20dB/decade	$+90°$
Ks^2	+40dB/decade	$\frac{1}{\sqrt{K}}$	40dB/decade	$+180°$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
Ks^p	+20pdB/decade	$\frac{1}{\sqrt[p]{K}}$	20pdB/decade	$+90p°$

→ Question: $G(s)H(s) = 1.5s^5$ has Bode magnitude plot as shown.



Calculate ω_c and slope (s):

$G(s)H(s) = 1.5s^5$

→ slope = $20p = 100 \frac{dB}{decade}$

$$\rightarrow \omega_c = \left(\frac{1}{K}\right)^{1/p} = \left(\frac{1}{1.5}\right)^{1/5} = 0.922$$

→ Question: $G(s)H(s) = \frac{K}{s} \rightarrow T(s)$

5-1: $s = j\omega \quad (0 < \omega < \infty)$

$$T(j\omega) = \frac{K}{j\omega}$$

S-2 $|T(j\omega)| = \frac{k}{\omega}$

$|T(j\omega)|_{dB} = \underbrace{-20}_{m}\underbrace{\log_{10}\omega}_{x} + \underbrace{20\log_{10}k}_{c}$

y

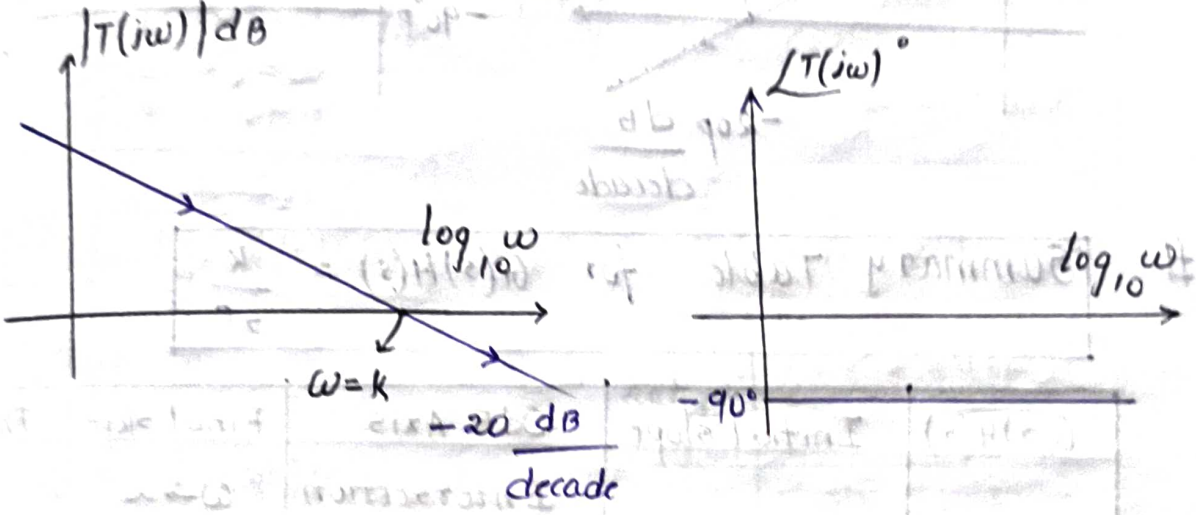
S-3 $\angle T(j\omega) = -90°$

Intersection with 0dB Axis:

$|T(j\omega)|_{dB} = -20\log_{10}\omega + 20\log_{10}k = 0\ dB$

$\rightarrow 20\log_{10}\omega = 20\log_{10}k$

$$\boxed{\omega = k}$$



→ **Question:** $G(s)H(s) = \frac{k}{s^p} \longrightarrow T(s)$

S-1: $s = j\omega \quad (0 < \omega < \infty)$

$T(j\omega) = \frac{k}{j\omega^p}$

S-3: $\angle T(j\omega) = -90°p$

S-2:

$|T(j\omega)| = \frac{k}{\omega^p}$

$\underbrace{|T(j\omega)|_{dB}}_{y} = \underbrace{-20p}_{m}\underbrace{\log_{10}\omega}_{x} + \underbrace{20\log_{10}k}_{c}$

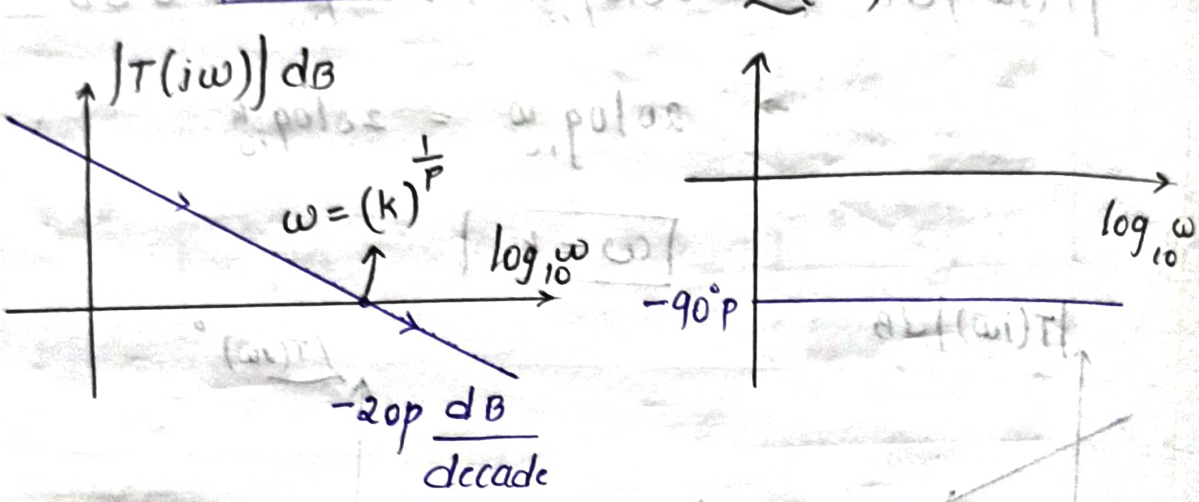
Intersection with 0dB Axis:

$$|T(i\omega)|_{dB} = -20p \log_{10}\omega + 20\log_{10} k = 0dB$$

$$20p \log_{10}\omega = 20\log_{10} k$$

$$\omega^p = k$$

$$\boxed{\omega = (k)^{1/p}}$$



Summary Table for $G(s)H(s) = \frac{k}{s^p}$

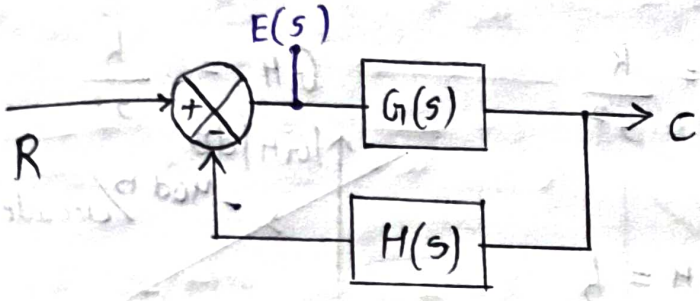
G(s)H(s)	Initial slope	0dB Axis Intersection	Final slope ω→∞	Phase
$\frac{k}{s}$	−20dB/dec	$\omega = k$	$-20\frac{dB}{dec}$	−90°
$\frac{k}{s^2}$	−40dB/dec	$\omega = \sqrt{k}$	−40dB/dec	−180°
$\frac{k}{s^3}$	−60dB/dec	$\omega = \sqrt[3]{k}$	−60dB/dec	−270°
⋮				
$\frac{k}{s^p}$	$-20p$dB/dec	$\omega = \sqrt[p]{k}$	$-20p$dB/dec	$-90p$

Important Observation

1.

Slope of Initial Line	Initial phase	Type
20p dB/dec	+90p°	0
0 dB/dec	0°	0
-20p dB/dec	-90p°	p

2.



Calculate the steady state error from Bode plot

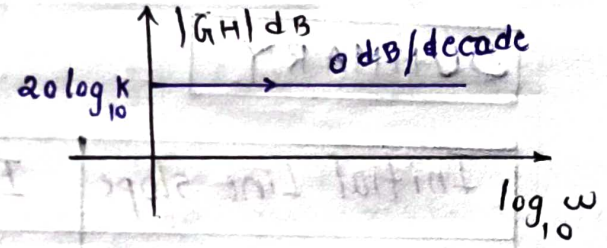
CASE : 1 $G(s)H(s) = k$

Type '0'

$$k_p = \lim_{s \to 0} G(s)H(s) = k$$

$$e_{ss} = \frac{A}{1+k}$$

$G(s)H(s) = k$



$20\log_{10} k = 20\log_{10} k_p$

$\rightarrow k_p$ --

$$e_{ss} = \frac{A}{1+k_p}$$

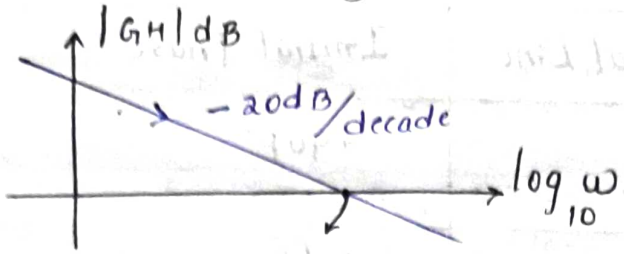
CASE : 2 $GH = \frac{k}{s}$

Type '1'

$$k_v = \lim_{s \to 0} sGH = k$$

$$e_{ss} = \frac{A}{k_v}$$

$$GH = \frac{k}{s}$$



$\omega = k$

$\downarrow$

$k_v = k \rightarrow$ **$e_{ss} = \frac{A}{k_v}$**

CASE : 3

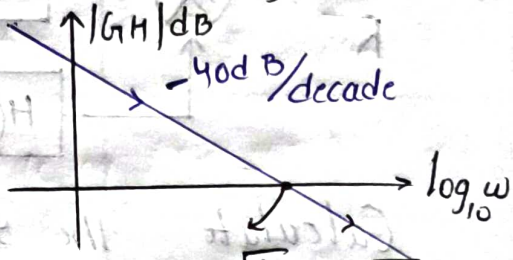
$$GH = \frac{k}{s^2}$$

Type '2'

$$k_a = \lim_{s \to 0} s^2 GH = k$$

$$e_{ss} = \frac{A}{k_a}$$

$$GH = \frac{k}{s^2}$$



$\omega = \sqrt{k} = \sqrt{k_a}$

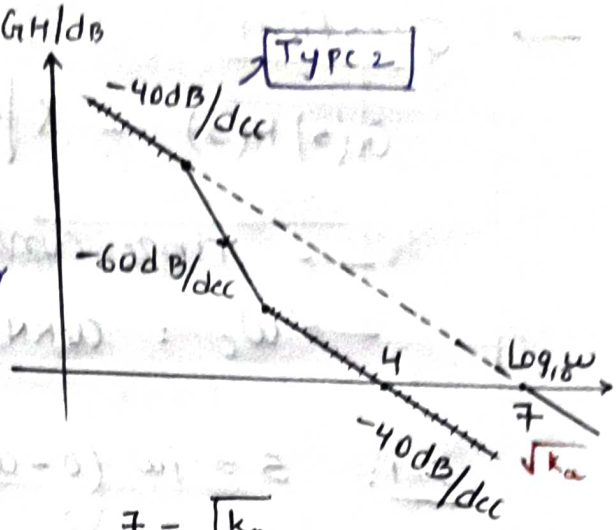
$e_{ss} = \frac{A}{k_a}$

SUMMARY

Initial Line slope	Information	e_{ss}
0dB/decade	Amplitude = $20 \log_{10} k_p$	$\frac{A}{1+k_p}$
-20dB/decade	0dB axis inter-section = k_v	$\frac{A}{k_v}$
-40dB/decade	0dB Axis Inter-section = $\sqrt{k_a}$	$\frac{A}{k_a}$

Question

Bode Magnitude plot



Calculate steady state error for

i/p $r(t) = 3t^2 u(t)$

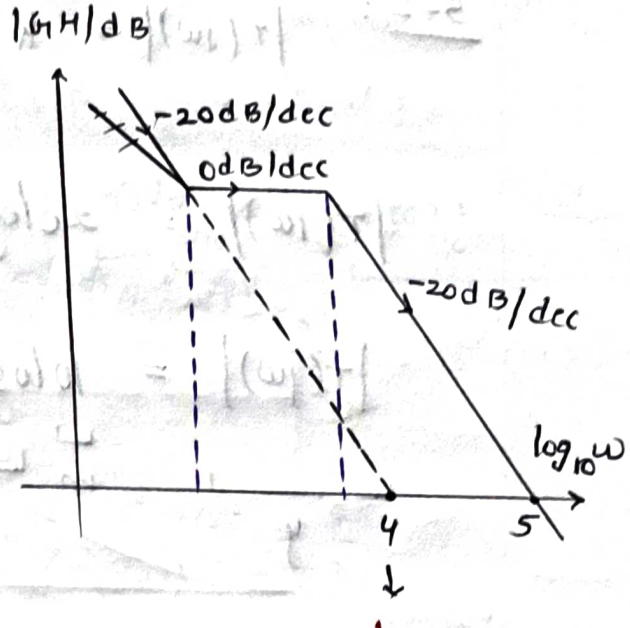
$$r(t) = 6\left(\frac{t^2}{2}u(t)\right)$$

$$e_{ss} = \frac{A}{k_a} = \frac{6}{49}$$

$$7 = \sqrt{k_a}$$

$$\rightarrow k_a = 49$$

Question



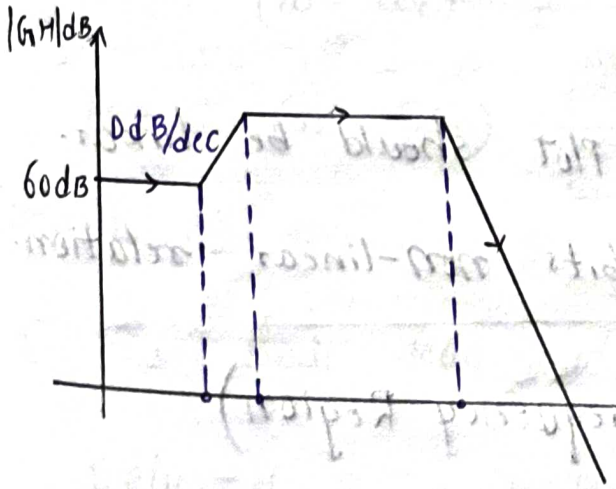
e_{ss} for $r(t) = 4tu(t)$?

→ Type 1

$$e_{ss} = \frac{A}{k_v} = \frac{4}{k_v} = \frac{4}{4}$$

$$= 1$$

Question



e_{ss} for $r(t) = 3u(t)$

$$e_{ss} = \frac{3}{1 + k_p}$$

$$= \frac{3}{1001}$$

$$\rightarrow 20\log_{10} k_p = 60$$

$$k_p = 1000$$

→ Question :

$$G(s)H(s) = K\left[\frac{s}{\omega_c} + 1\right]$$

→ Time constant Format

→ ω_c : CORNER FREQUENCY due to Zero.

S-1: $s = j\omega \quad (0 < \omega < \infty)$

$$T(j\omega) = k\left[1 + \frac{j\omega}{\omega_c}\right]$$

S-2:

$$|T(j\omega)| = k\sqrt{1 + \frac{\omega^2}{\omega_c^2}} = k\left(1 + \frac{\omega^2}{\omega_c^2}\right)^{1/2}$$

$$|T(j\omega)| = 20\log_{10} k + 10\log_{10}\left(1 + \frac{\omega^2}{\omega_c^2}\right)$$

$$|T(j\omega)| = 10\log_{10}\left(1 + \frac{\omega^2}{\omega_c^2}\right) + 20\log_{10} k$$

$y = m \cdot f(x) + C$

→ The plot of above cannot be called as BODE PLOT.

→ Bode Magnitude plot should be linear.

→ Above plot exhibits non-linear relation.

1. L.F.R - (Low Frequency Region)

$\omega << \omega_c$

$\frac{\omega}{\omega_c} << 1$ → $|T(j\omega)|_{dB} = 20\log_{10} k$

$\left(\frac{\omega}{\omega_c}\right)^2 <<< 1$

2. M·F·R (Mid Frequency Region)

$\omega = \omega_c$

$\rightarrow |T(j\omega)| = 20\log_{10} k + 3dB$

3. H·F·R (High Frequency Region)

$\omega >> \omega_c \quad |T(j\omega)|\ dB = 10\log_{10}\left(\frac{\omega}{\omega_c}\right)^2 + 20\log_{10} k$

$\frac{\omega}{\omega_c} >> 1 \quad \rightarrow \quad |T(j\omega)|\ dB = 20\log_{10}\left(\frac{\omega}{\omega_c}\right) + 20\log_{10} k$

$\left(\frac{\omega}{\omega_c}\right)^2 >>> 1$

$|T(j\omega)|dB = \underbrace{20}_{m}\underbrace{\log_{10}\omega}_{x} + \underbrace{[20\log_{10} k - 20\log_{10}\omega}_{c}$

y

S-3:

$\angle T(j\omega) = \tan^{-1}\frac{\omega}{\omega_c}$

L·F·R $(\omega << \omega_c) \longrightarrow \angle T(j\omega) = 0°$

M·F·R $(\omega = \omega_c) \longrightarrow \angle T(j\omega) = 45°$

H·F·R $(\omega >> \omega_c) \longrightarrow \angle T(j\omega) = 90°$

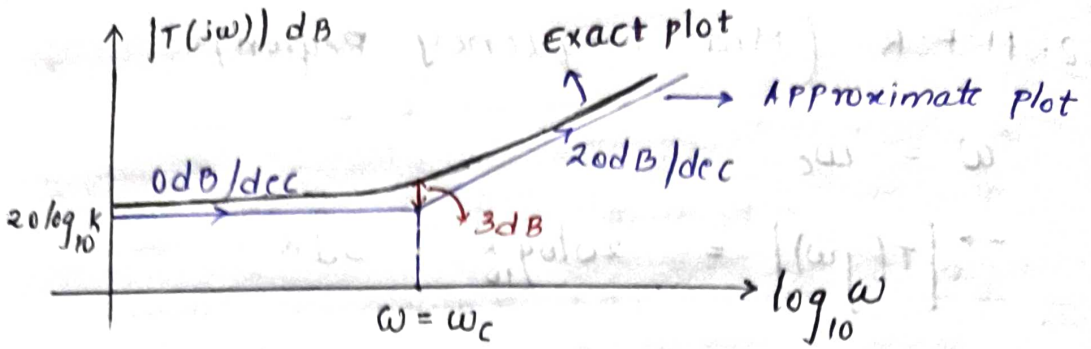
NOTE

1. For Bode Magnitude plot (Approximate plot)

Only LFR and HFR around ω_c considered.

2. For Exact magnitude plot, LFR; MFR; HFR are considered.

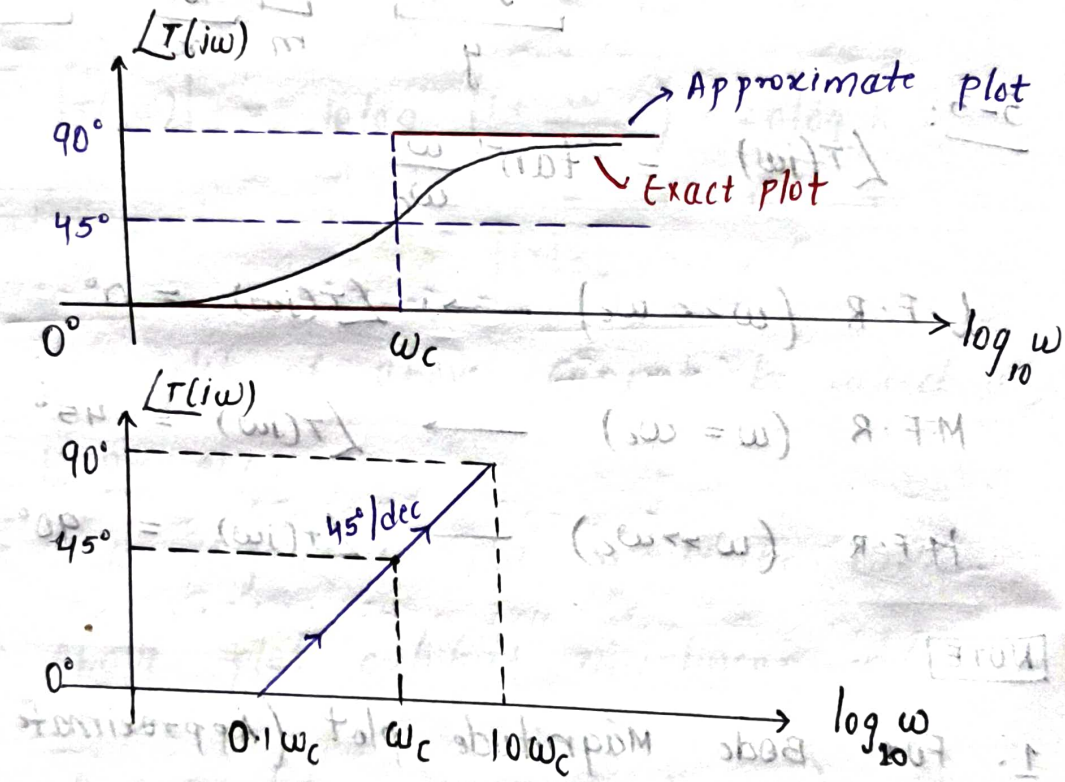
3. PHASE PLOT  Approximate, Exact, Linear



NOTE:

BODE PLOT → ASYMPTOTIC PLOT

* There is an Error of 3dB between Exact and Approximate plot at Corner frequency.



→ Question: $G(s)H(s) = k\left(1+\frac{s}{\omega_c}\right)^2$

S-1: $s = j\omega$; $T(j\omega) = k\left[1+\frac{j\omega}{\omega_c}\right]^2$

S-2:

$$|T(j\omega)| = k\left[1+\frac{\omega^2}{\omega_c^2}\right]$$

$$|T(j\omega)|_{dB} = 20\log_{10}\left(1 + \frac{\omega^2}{\omega_c^2}\right) + 20\log_{10} k$$

↳ NON-Linear

LFR ($\omega << \omega_c$): $|T(j\omega)|_{dB} = 20\log_{10} k$

MFR ($\omega = \omega_c$): $|T(j\omega)|_{dB} = 20\log_{10} k + 6dB$

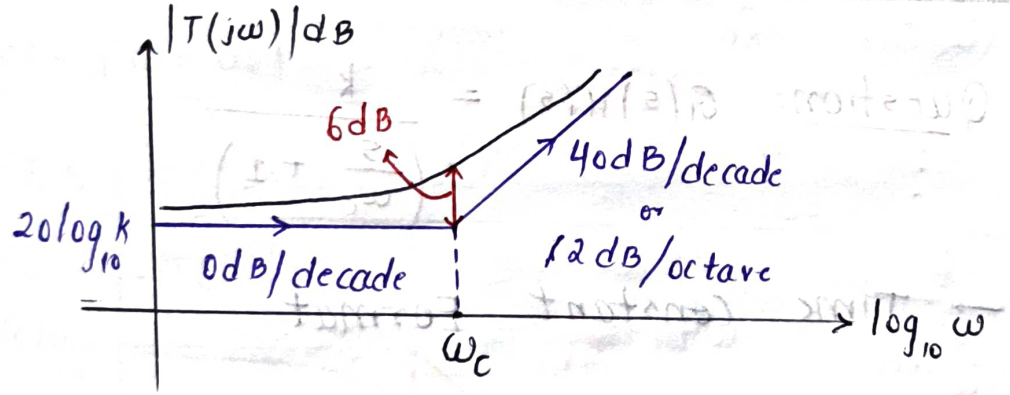
HFR ($\omega >> \omega_c$): $\underbrace{|T(j\omega)|_{dB}}_{y} = \underbrace{40}_{m}\underbrace{\log_{10}\omega}_{x} + (\underbrace{20\log_{10} k}_{} - \underbrace{40\log_{10}\omega_c}_{c})$

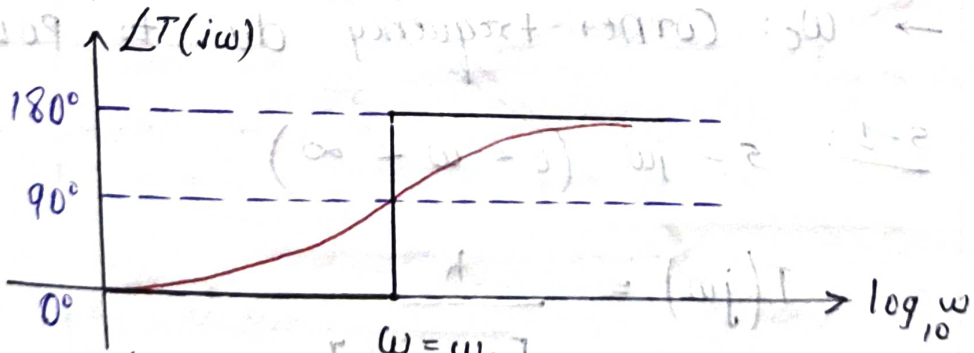
5-3: $\angle T(j\omega) = 2\tan^{-1}\frac{\omega}{\omega_c}$

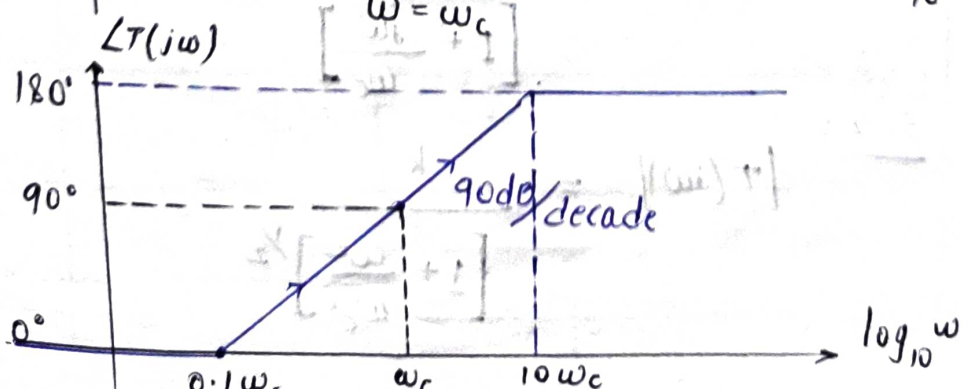
LFR : $\angle T(j\omega) = 0°$ $(\omega << \omega_c)$

MFR : $\angle T(j\omega) = 90°$ $(\omega = \omega_c)$

HFR : $\angle T(j\omega) = 180°$ $(\omega >> \omega_c)$







Generalization :

$G(s)H(s)$	Initial Line with slope	Change in slope at $\omega=\omega_c$	Error in Magnitude at $\omega=\omega_c$	slope at $\omega\to\infty$
$k\left(\frac{s}{\omega_c}+1\right)$	0dB/decade with magnitude $20\log_{10}k$	20dB/dec	+3dB	20dB/dec
$k\left(\frac{s}{\omega_c}+1\right)^2$	0dB/decade with magnitude $20\log_{10}k$	40dB/dec	6dB	40dB/dec
⋮	⋮	⋮	⋮	⋮
$k\left(\frac{s}{\omega_c}+\right)^p$	0dB/decade with magnitude $20\log_{10}k$	20p dB/dec	3p dB	20p dB/dec

→ Question: $G(s)H(s) = \dfrac{k}{\left(\frac{s}{\omega_c}+1\right)}$

→ Time constant Format

→ ω_c : Corner Frequency due to POLE.

S-1: $s = j\omega \quad (0 < \omega < \infty)$

$$T(j\omega) = \frac{k}{\left[1+\frac{j\omega}{\omega_c}\right]}$$

S-2:

$$|T(j\omega)| = \frac{k}{\left[1+\frac{\omega^2}{\omega_c^2}\right]^{1/2}}$$

$$|T(j\omega)|_{dB} = 20\log_{10} k - 10\log_{10}\left(1+\frac{\omega^2}{\omega_c^2}\right)$$

$$|T(j\omega)|_{dB} = -10\log_{10}\left(1+\frac{\omega^2}{\omega_c^2}\right) + 20\log_{10} k$$

↳ Non Linear

LFR ($\omega << \omega_c$): $|T(j\omega)|_{dB} = 20\log_{10} k$

MFR ($\omega = \omega_c$): $|T(j\omega)|_{dB} = 20\log_{10} k - 3dB$

HFR ($\omega >> \omega_c$): $|T(j\omega)|_{dB} = -20\log_{10}\omega + 20\log_{10} k - 20\log_{10}\omega_c$

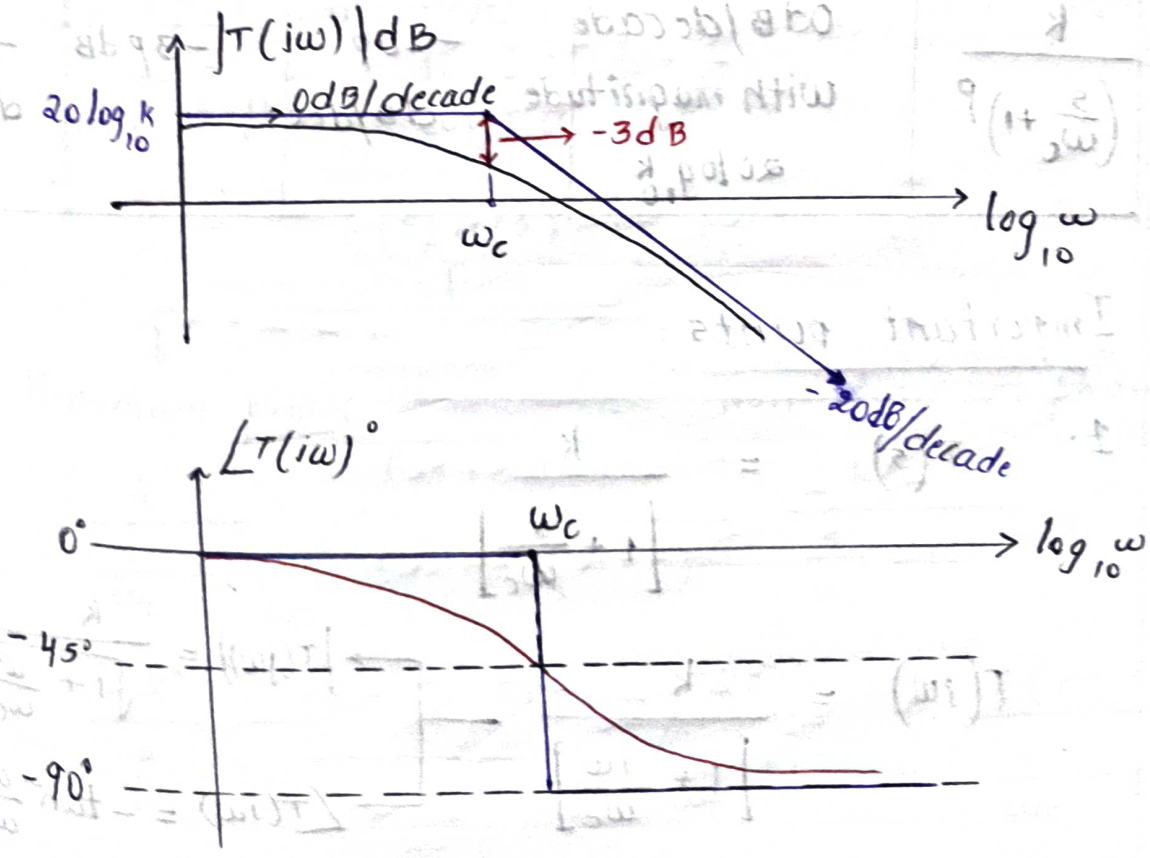
$y = mx + c$

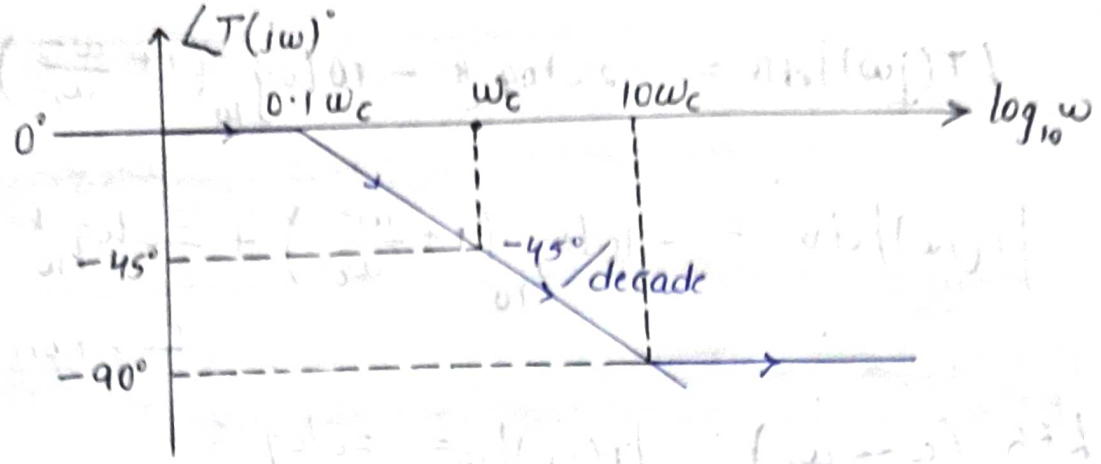
S-3: $\angle T(j\omega) = -\tan^{-1}\frac{\omega}{\omega_c}$

LFR ($\omega << \omega_c$): $\angle T(j\omega) = 0°$

MFR ($\omega = \omega_c$): $\angle T(j\omega) = -45°$

HFR ($\omega >> \omega_c$): $\angle T(j\omega) = -90°$





G(s)H(s)	Initial Line with slope	Change in Slope at $\omega=\omega_c$	Error in Magnitude at $\omega=\omega_c$	slope at $\omega\to\infty$
$\frac{k}{\left(\frac{s}{\omega_c}+1\right)^1}$	0dB/decade with magnitude $20\log_{10}k$	−20dB/dec	−3dB	−20dB/dec
$\frac{k}{\left(\frac{s}{\omega_c}+1\right)^2}$	0dB/decade with magnitude $20\log_{10}k$	−40dB/dec	−6dB	−40dB/dec
⋮	⋮	⋮	⋮	⋮
$\frac{k}{\left(\frac{s}{\omega_c}+1\right)^p}$	0dB/decade with magnitude $20\log_{10}k$	−20p dB/dec	−3p dB	−20p dB/dec

Important points:

1. $$T(s) = \frac{k}{\left[1+\frac{s}{\omega_c}\right]}$$

$$T(j\omega) = \frac{k}{\left[1+\frac{j\omega}{\omega_c}\right]}$$

$$\rightarrow |T(j\omega)| = \frac{k}{\sqrt{1+\frac{\omega^2}{\omega_c^2}}}$$

$$\rightarrow \angle T(j\omega) = -\tan^{-1}\frac{\omega}{\omega_c}$$

2. $T(s) = \dfrac{k}{\left[1 - \frac{s}{\omega_c}\right]}$

$T(j\omega) = \dfrac{k}{\left[1 - \frac{j\omega}{\omega_c}\right]}$ → $|T(j\omega)| = \dfrac{k}{\sqrt{1 + \frac{\omega^2}{\omega_c^2}}}$

→ $\angle T(j\omega) = \tan^{-1}\dfrac{\omega}{\omega_c}$

3. $T(s) = \dfrac{k e^{-s}}{\left[-1 - \frac{s}{\omega_c}\right]}$

$T(j\omega) = \dfrac{k e^{-i\omega}}{-1 - \frac{j\omega}{\omega_c}}$ → $|T(i\omega)| = \dfrac{k}{\sqrt{1 + \frac{\omega^2}{\omega_c^2}}}$

→ $\angle T(i\omega) = -\omega - \left\{180° + \tan^{-1}\dfrac{\omega}{\omega_c}\right\}$

1. If Bode plot (magnitude) is given, then recovered T.F is not unique.

2. If Bode magnitude plot and phase plot are given, then recovered T.F is unique.

→ $G(s)H(s)$

- Minimum phase
- Non-Minimum phase

$\dfrac{k}{\frac{s}{\omega_c} + 1}$ $\qquad$ $\dfrac{= k(\quad)}{\frac{s}{\omega_c} - 1}$

$\dfrac{k}{-\frac{s}{\omega_c} - 1}$ $\qquad$ $\dfrac{kc^{-s}}{\frac{s}{\omega_c} + 1} \approx \dfrac{k(1-s)}{\frac{s}{\omega_c} + 1}$

RECOVERY OF TRANSFER FUNCTION FROM BODE PLOT

Prerequisite :

1. Time constant Format and identification of corner frequency.

Example : 1 $T(s) = \frac{1}{s(s+5)}$

$\downarrow$

$$T(s) = \frac{1/5}{s\left(\frac{s}{5}+1\right)} = \frac{0.2}{s\left(\frac{s}{5}+1\right)^1}$$

$\hookrightarrow \omega_c = 5$ Rad/sec

Example - 2 $T(s) = \frac{4(s+1)}{(s+3)(s+5)}$

$$T(s) = \frac{4\left(\frac{s}{1}+1\right)}{3\left(\frac{s}{3}+1\right)5\left(\frac{s}{5}+1\right)} = \frac{\frac{4}{15}\left(\frac{s}{1}+1\right)}{\left(\frac{s}{3}+1\right)\left(\frac{s}{5}+1\right)}$$

$k = \frac{4}{15}$, $\omega_{c_1} = 1$, $\omega_{c_2} = 3$, $\omega_{c_3} = 5$

2 **Approximation of a T.F** *****

$$T(s) = \frac{5(s+20)(s+50)}{(s+10)(s+100)}$$

$\downarrow$

$$T(s) = \frac{5\left(\frac{s}{20}+1\right)\left(\frac{s}{50}+1\right)}{\left(\frac{s}{10}+1\right)\left(\frac{s}{100}+1\right)} \rightarrow \begin{matrix} \omega_{c_1} = 10 \\ \omega_{c_2} = 20 \\ \omega_{c_3} = 50 \end{matrix}$$

$\omega_{c_4} = 100, \quad k = 5$

1. $0 < \omega < 10$: $T(s) = \dfrac{5(1)(1)}{(1)(1)}$

2. $\omega = 10$: $T(s) = \dfrac{5(1)(1)}{(1)(1)} = \dfrac{5(1)(1)}{\left(\frac{s}{10}\right)\cdot 1}$

3. $10 < \omega < 20$: $T(s) = \dfrac{5(1)(1)}{\left(\frac{s}{10}\right)(1)} = \dfrac{50}{s}$

4. $\omega = 20$: $T(s) = \dfrac{5(1)(1)}{\left(\frac{s}{10}\right)(1)} = \dfrac{50}{s}$

5. $20 < \omega < 50$: $T(s) = \dfrac{5\left(\frac{s}{20}\right)\times 1}{\left(\frac{s}{10}\right)\times 1} = 2.5$

6. $\omega = 50$: $T(s) = \dfrac{5\left(\frac{s}{20}\right)}{\left(\frac{s}{10}\right)} = 2.5$

7. $50 < \omega < 100$: $T(s) = \dfrac{5\left(\frac{s}{20}\right)\left(\frac{s}{50}\right)}{\left(\frac{s}{10}\right)(1)} = \dfrac{s}{20}$

8. $\omega = 100$: $T(s) = \dfrac{5\left(\frac{s}{20}\right)\left(\frac{s}{50}\right)}{\left(\frac{s}{10}\right)\times 1} = \dfrac{s}{20}$

9. $\omega > 100$: $T(s) = \dfrac{5\left(\frac{s}{20}\right)\left(\frac{s}{50}\right)}{\left(\frac{s}{10}\right)\left(\frac{s}{100}\right)} = 5$

Numerical in Bode plot

Type-A

Given: BODE PLOT

Identify: T.F

95%

Type-B

Given: T.F

Identify: BODE PLOT

5%

Type-A:

Step-1: Identify Initial slope:

0 dB / decade $\longrightarrow$ k

$-20p$ dB / decade $\longrightarrow$ $\frac{k}{s^p}$

$+20p$ dB/decade $\longrightarrow$ ks^p

Step-2: Identify Corner frequency

Corner frequency: where slope changes

Change in slope around ω_c = Final slope − Initial slope

$\Delta s = +20p \longrightarrow \left(1+\frac{s}{\omega_c}\right)^p$

$\Delta s = -20p \longrightarrow \frac{1}{\left(1+\frac{s}{\omega_c}\right)^p}$

Step-3: For calculation of k

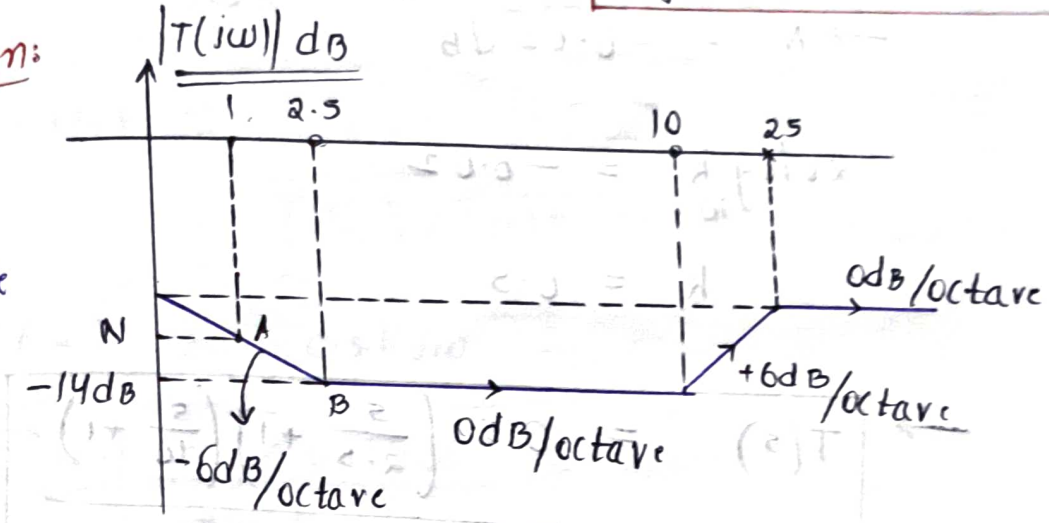
Method 1: Approximate T.F method.

Method 2:

- If initial line is 0dB/dec
  → $20\log_{10} k = M$
- If initial line has slope
  - Magnitude at $\omega = 1$ → N

$20\log_{10} k = N$ ← Magnitude of inital line

Question:



Calculate T(s)?

$$T(s) = \frac{k\left(\frac{s}{2.5}+1\right)^{1}\left(\frac{s}{10}+1\right)}{s\left(\frac{s}{25}+1\right)}$$

Method-1:

$\omega = 2.5 \rightarrow |T(j\omega)|\,dB = -14\,dB$

$$\hookrightarrow T(s) = \frac{k\,(1)\,(1)}{s\,(1)}$$

$$T(j\omega) = \frac{k}{j\omega} \rightarrow |T(j\omega)| = \frac{k}{\omega}$$

$$|T(j\omega)|_{dB} = 20\log_{10} k - 20\log_{10}\omega$$

$$\rightarrow |T(j\omega)|\,dB = 20\log_{10} k - 20\log_{10} 2.5$$

$$= -14$$

$$\rightarrow \boxed{k = 0.5}$$

Method 2 — Initial line → slope ω = 1 → $|T(j\omega)|_{dB}$

A $(\log_{10}^{1}, N\,dB)$

B $(\log_{10}^{2.5}, -14)$

$$S_{AB} = -20 = \frac{N+14}{\log_{10}^{1} - \log_{10}^{2.5}}$$

$$\rightarrow N = -6.02\ dB$$

$$20\log_{10} k = -6.02$$

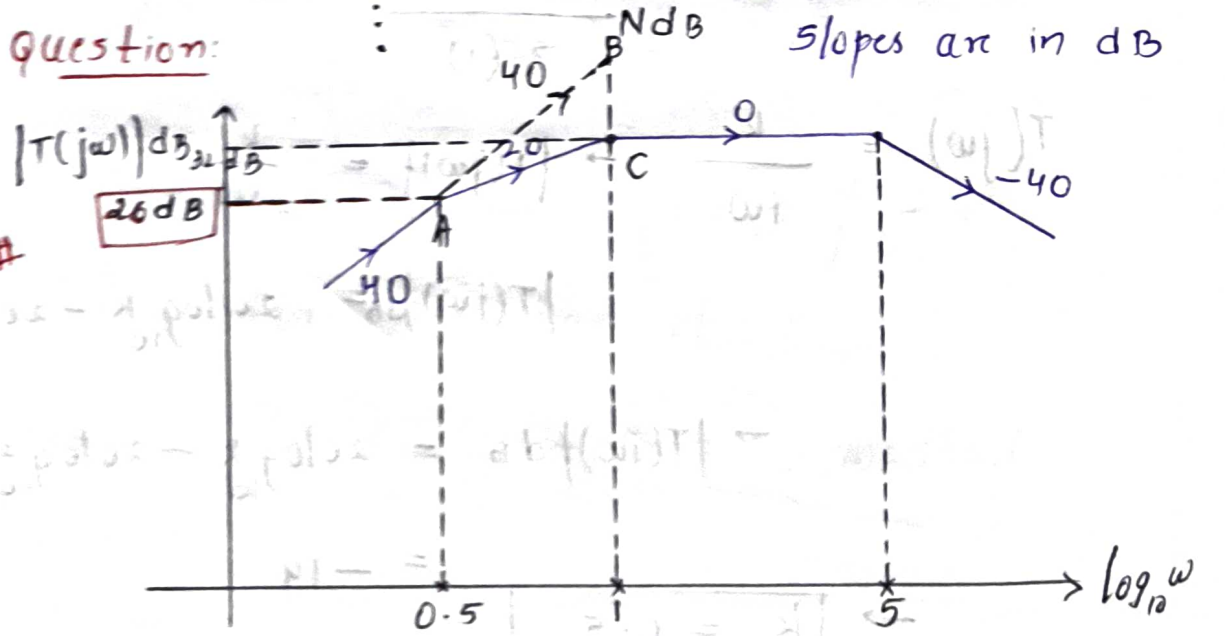
$$k = 0.5$$

$$\rightarrow \boxed{T(s) = \frac{0.5\left(\frac{s}{2.5}+1\right)\left(\frac{s}{10}+1\right)}{s\left(\frac{s}{25}+1\right)}}$$

(OR)

$$\boxed{T(s) = \frac{0.5\, e^{-s}\left(\frac{s}{2.5}+1\right)\left(\frac{s}{10}+1\right)}{s\left(\frac{s}{25}+1\right)}}$$

→ Question:



$$T(s) = \frac{ks^2}{\left(\frac{s}{0.5}+1\right)\left(\frac{s}{1}+1\right)\left(\frac{s}{5}+1\right)^2}$$

Calculation of k:

Method-1: $\omega = 1 \quad : \quad |T(j\omega)|_{dB} = 32\ dB$ → BODE PLOT IN

$$T(s) = \frac{ks^2}{\left(\frac{s}{0.5}\right)(1)(1)^2} = 0.5ks$$

$$T(j\omega) = 0.5kj\omega$$

$$|T(j\omega)| = 0.5k\omega$$

$$|T(j\omega)|_{dB} = 20\log_{10} 0.5 + 20\log_{10} k + 20\log_{10} \omega$$

$$\downarrow \omega = 1$$

$$20\log_{10} 0.5 + 20\log_{10} k + 0 = 32$$

$$\boxed{k = 79.62}$$

Method-2:

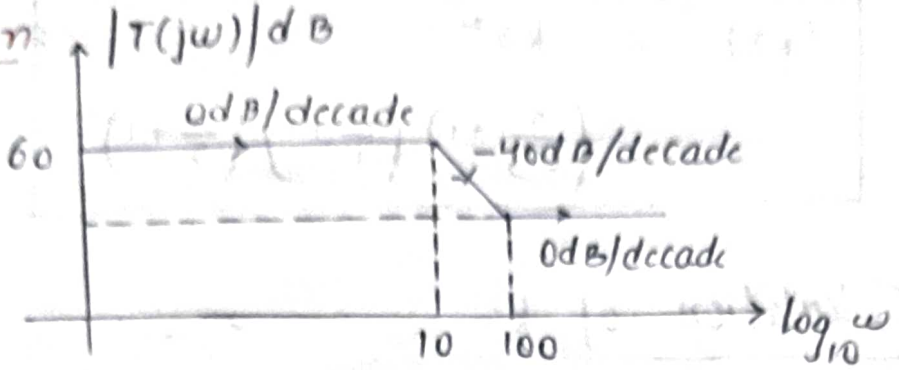
$A\left(\log_{10}^{0.5}, 26\,dB\right)$ $\quad \omega = 1 \rightarrow N\ dB$ ↓ Initial line

$B\left(\log_{10}^{1}, N\,dB\right)$

$$S_{AB} = 40 = \frac{26 - N}{\log_{10}^{0.5} - \log_{10}^{1}} \Rightarrow N = 38\,dB$$

$$20\log_{10} k = 38 \rightarrow \boxed{k = 79.62}$$

→ **Question:**



T(s) ?

$$T(s) = \frac{k\left[\frac{s}{100}+1\right]^2}{\left[\frac{s}{10}+1\right]^2}$$

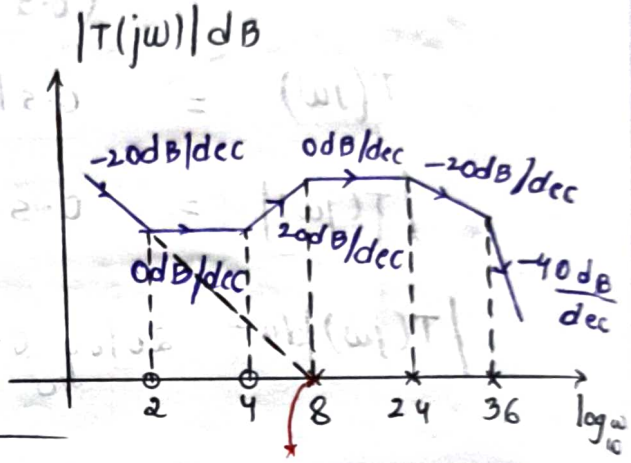
$$20\log_{10} k = 60$$

$$k = 1000$$

→ **Question:**

$k_v = k = 8$

$T(s) = ?$

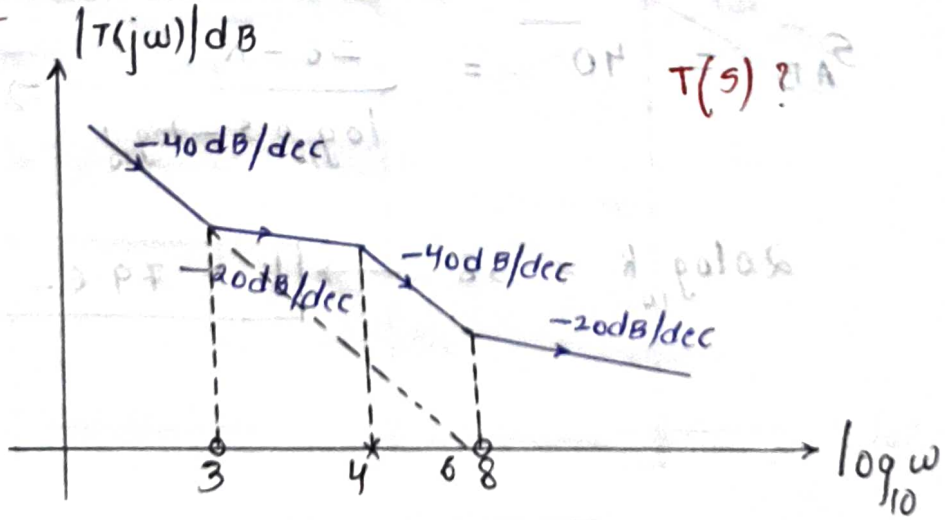


$$T(s) = \frac{k\left(\frac{s}{2}+1\right)\left(\frac{s}{4}+1\right)}{s\left(\frac{s}{8}+1\right)\left(\frac{s}{24}+1\right)\left(\frac{s}{36}+1\right)}$$

$k = 8$

$$T(s) = \frac{8\left(\frac{s}{2}+1\right)\left(\frac{s}{4}+1\right)}{s\left(\frac{s}{8}+1\right)\left(\frac{s}{24}+1\right)\left(\frac{s}{36}+1\right)}$$

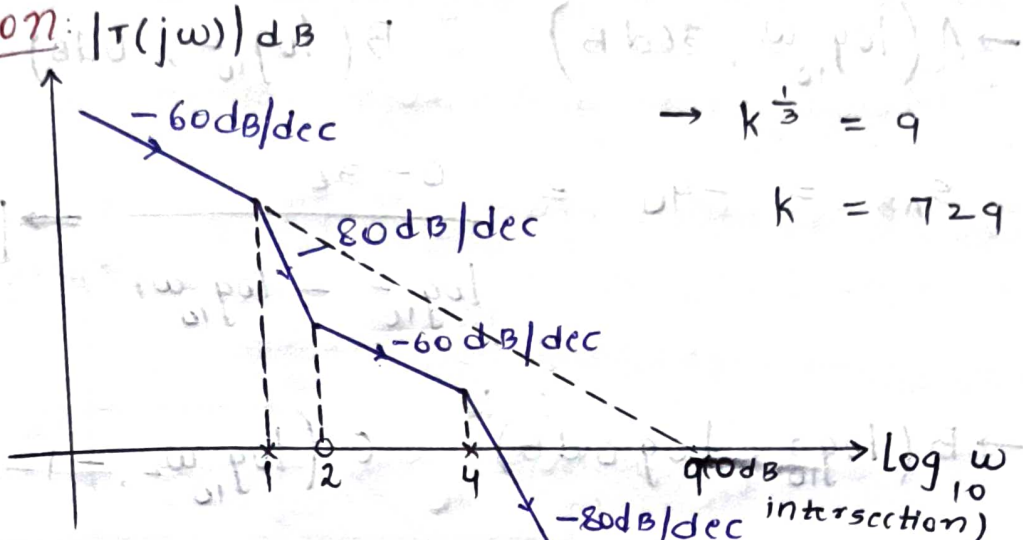
→ **Question:**



$$\sqrt{K} = \sqrt{k_a} = 6 \longrightarrow k = 36$$

$$T(s) = \frac{36\left(\frac{s}{3}+1\right)\left(\frac{s}{8}+1\right)}{s^2\left(\frac{s}{4}+1\right)}$$

Question:



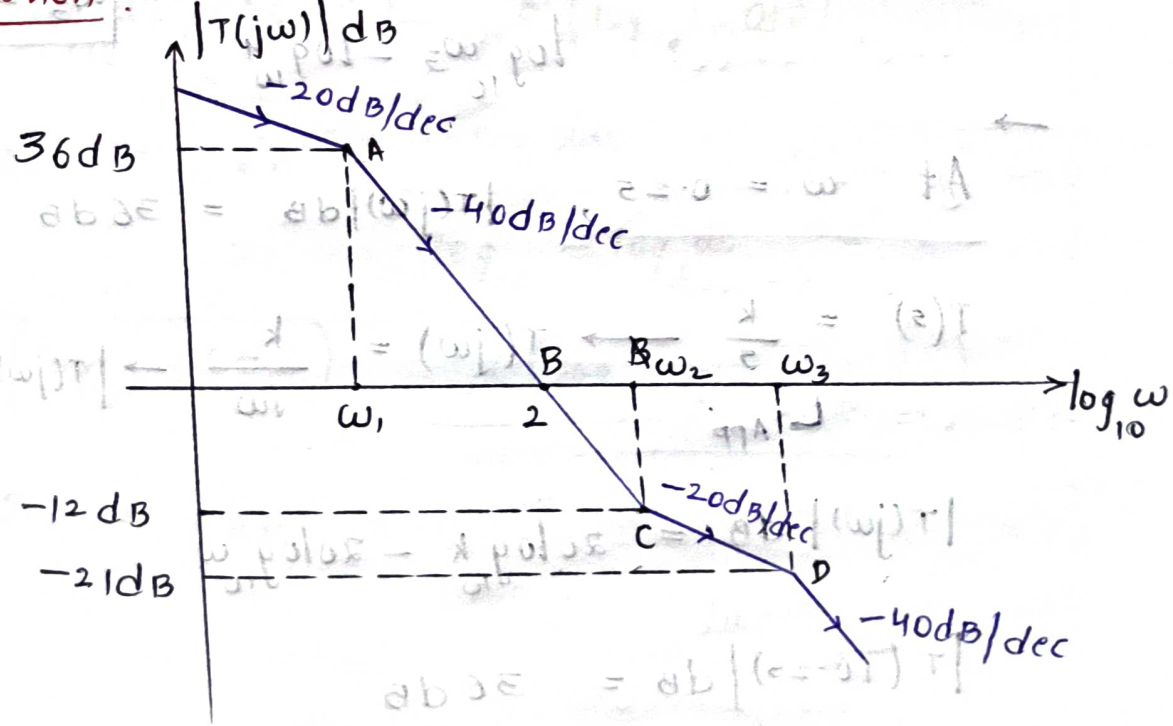
$\rightarrow k^{\frac{1}{3}} = 9$

$k = 729$

$$T(s) = \frac{k\left(\frac{s}{2}+1\right)}{s^3\left(\frac{s}{1}+1\right)\left(\frac{s}{4}+1\right)}$$

$$T(s) = \frac{729\left(\frac{s}{2}+1\right)}{s^3\left(\frac{s}{1}+1\right)\left(\frac{s}{4}+1\right)}$$

Question:



$T(s)$?

$$T(s) = \frac{k\left(\frac{s}{\omega_2}+1\right)}{s\left(\frac{s}{\omega_1}+1\right)\left(\frac{s}{\omega_3}+1\right)}$$

→ $A\left(\log_{10}\omega_1, 36dB\right)$ $B\left(\log_{10}2, 0dB\right)$

$$S_{AB} = -40 = \frac{0-36}{\log_{10}2 - \log_{10}\omega_1} \rightarrow \boxed{\omega_1 = 0.25}$$

→ $B\left(\log_{10}2, 0dB\right)$ $C\left(\log_{10}\omega_2, -12dB\right)$

$$S_{BC} = -40 = \frac{-12-0}{\log_{10}\omega_2 - \log_{10}2} \Rightarrow \boxed{\omega_2 = 4}$$

→ $C\left(\log_{10}4, -12dB\right)$ $D\left(\log_{10}\omega_3, -21dB\right)$

$$S_{CD} = -20 = \frac{(-21)-(-12)}{\log_{10}\omega_3 - \log_{10}4} \Rightarrow \boxed{\omega_3 = 8\sqrt{2}}$$

→

At $\omega = 0.25$, $|T(j\omega)|dB = 36dB$

$$T(s) = \frac{k}{s} \longrightarrow T(j\omega) = \frac{k}{j\omega} \rightarrow |T(j\omega)| = \frac{k}{\omega}$$

↳ App

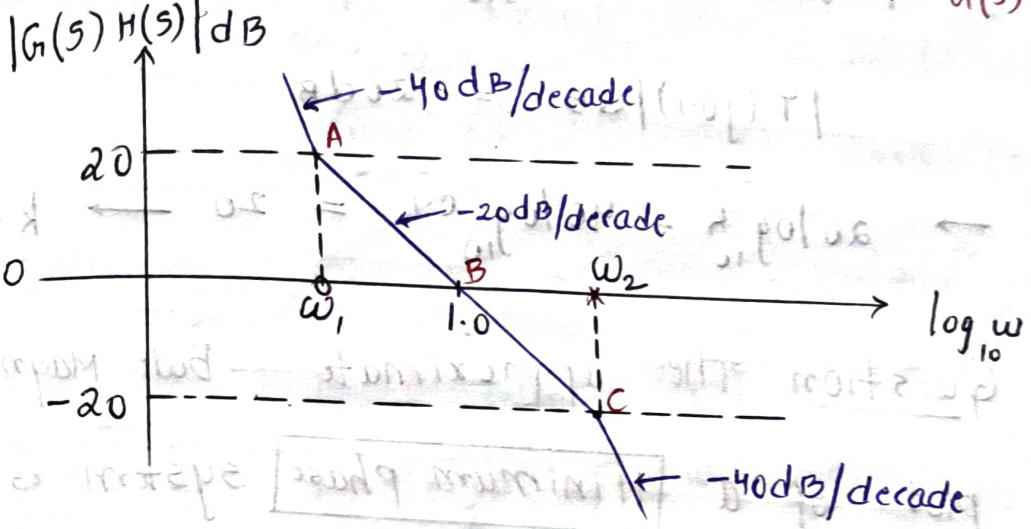
$$|T(j\omega)|dB = 20\log_{10}k - 20\log_{10}\omega$$

$$|T(j0.25)|dB = 36dB$$

$$20\log_{10}k - 20\log_{10}0.25 = 36 \rightarrow \boxed{k = 15.7}$$

$$\therefore T(s) = \frac{15.7\left(\frac{s}{4}+1\right)}{s\left(\frac{s}{0.25}+1\right)\left(\frac{s}{11.27}+1\right)}$$

Question: The asymptotic Bode plot of the minimum phase open-loop T.F $G(s)H(s)$ is as shown in figure. Obtain the T.F $G(s)H(s)$



$$G(s)H(s) = \frac{k\left(\frac{s}{\omega_1}+1\right)}{s^2\left(\frac{s}{\omega_2}+1\right)}$$

→ $A(\log_{10}\omega_1, 20\,dB)$ $B(\log_{10}1, 0\,dB)$

$$S_{AB} = -20 = \frac{0-20}{\log_{10}1 - \log_{10}\omega_1} \longrightarrow \omega_1 = 0.1$$

→ $B(\log_{10}1, 0\,dB)$ $C(\log_{10}\omega_2, -20\,dB)$

$$S_{BC} = -20 = \frac{-20-0}{\log_{10}\omega_2 - \log_{10}1} \longrightarrow \omega_2 = 10$$

At $\omega = 0.1$, $|T(i\omega)|_{dB} = 20dB$.

$T(s) \doteq \frac{k}{s^2}$ (After approximation)

$\downarrow$

$G(s)H(s) \xrightarrow{s=j\omega} \frac{-k}{\omega^2} \longrightarrow |T(i\omega)| = \frac{k}{\omega^2}$

$|T(j\omega)|_{dB} = 20\log_{10} k - 40\log_{10} \omega$

$|T(j0.1)|_{dB} = 20dB$

$\rightarrow 20\log_{10} k - 40\log_{10} 0.1 = 20 \longrightarrow k = 1000$

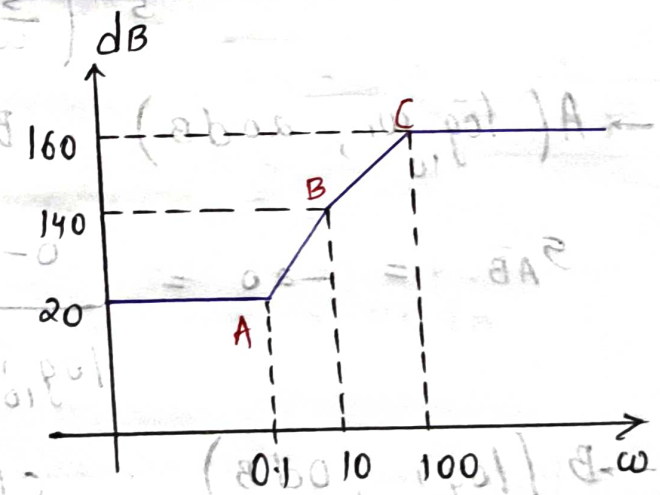
→ Question: The approximate Bode Magnitude plot of a **Minimum phase** system is shown in fig. The T.F of the system is

A) $\frac{10^8 (s+0.1)^3}{(s+10)^2 (s+100)}$

B) $\frac{10^8 (s+0.1)^3}{(s+10)(s+100)}$

C) $\frac{10^8 (s+0.1)^2}{(s+10)^2 (s+100)}$

D) $\frac{10^9 (s+0.1)^3}{(s+10)(s+100)^2}$



$S_{AB} = 60\ dB/dec$ $\quad S_{BC} = 20\ dB/dec$

$$T(s) = \frac{k\left(\frac{s}{0.1}+1\right)^3}{\left(\frac{s}{10}+1\right)^2\left(\frac{s}{100}+1\right)}$$

$$20\log_{10} k = 20 \longrightarrow k = 10$$

$$\rightarrow T(s) = \frac{10\,(s+0.1)^3}{(0.1)^3\,(s+10)^2(s+100)} = \frac{10^8\,(s+0.1)^3}{(s+10)^2(s+100)}$$

10^4

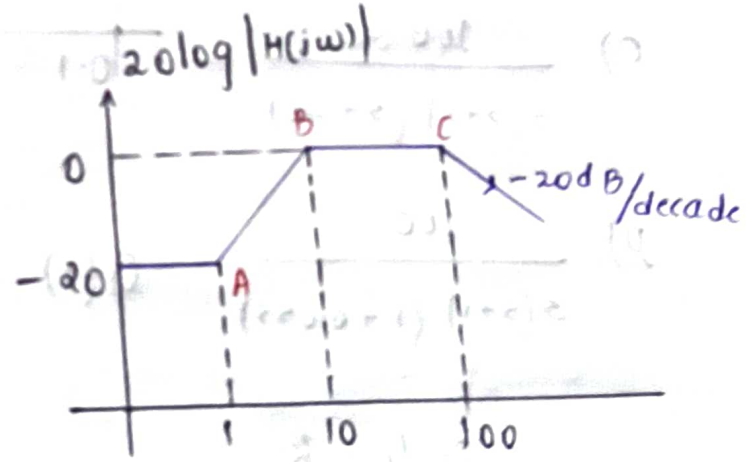
→ Question Consider the Bode Magnitude plot shown in Fig. The T.F H(s) is

A) $\frac{s+10}{(s+1)(s+100)}$

B) $\frac{10(s+10)}{(s+1)(s+100)}$

C) $\frac{10^2(s+1)}{(s+10)(s+100)}$

D) $\frac{10^3(s+100)}{(s+1)(s+10)}$



$S_{AB} = 20\ dB/decade$

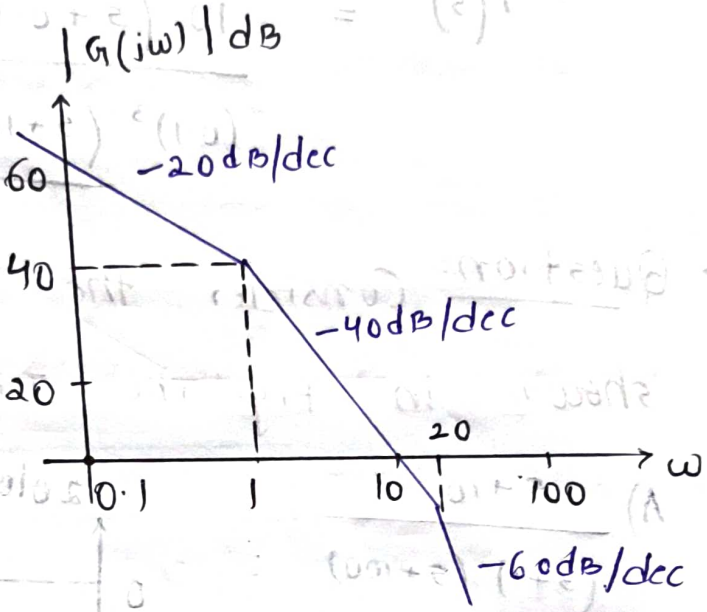
$$H(s) = \frac{k\left(\frac{s}{1}+1\right)}{\left(\frac{s}{10}+1\right)\left(\frac{s}{100}+1\right)}$$

$$20\log_{10} k = -20$$

$$k = 0.1$$

$$H(s) = \frac{0.1(s+1)}{\left(\frac{s+10}{10}\right)\left(\frac{s+100}{100}\right)} = \frac{10^2(s+1)}{(s+10)(s+100)}$$

→ Question: The asymptotic Bode plot of a T.F is as shown in the figure. The T.F G(s) corresponding to this Bode plot is



A) $\frac{1}{(s+1)(s+20)}$

B) $\frac{1}{s(s+1)(s+20)}$

C) $\frac{100}{s(s+1)(s+20)}$

D) $\frac{100}{s(s+1)(1+0.05s)}$

$$G(s) = \frac{k}{s\left(\frac{s}{1}+1\right)\left(\frac{s}{20}+1\right)}$$

$$20\log_{10} k = 40$$

$$k = 100$$

$$G(s) = \frac{100}{s(s+1)\left(\frac{s}{20}+1\right)}$$

$$G(s) = \frac{2000}{s(s+1)(s+20)} \qquad G(s) = \frac{100}{s(s+1)(0.05s+1)}$$

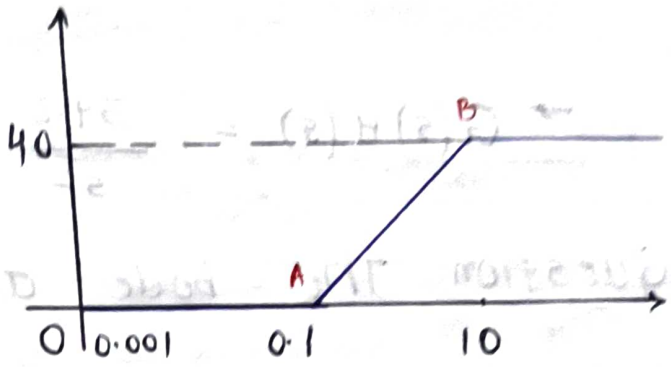
Question: For the asymptotic Bode magnitude plot shown below, the system T.F can be

A) $\frac{10s+1}{0.1s+1}$

B) $\frac{100s+1}{0.1s+1}$

C) $\frac{100s}{10s+1}$

D) $\frac{0.1s+1}{10s+1}$



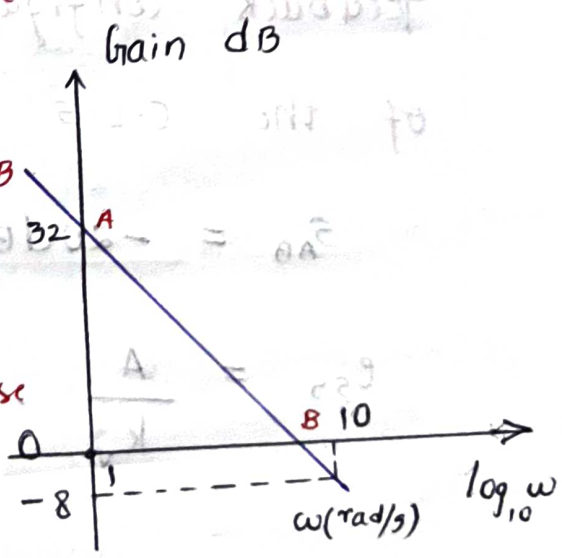
S_{AB} = 20 dB/decade

$$G(s)H(s) = \frac{k\left(\frac{s}{0.1}+1\right)}{\left(\frac{s}{10}+1\right)} = \frac{k(10s+1)}{(0.1s+1)}$$

$20\log_{10} k = 0$

$k = 1$

→ Question: The bode plot of a T.F G(s) is shown in the figure below.



The gain $20\log|G(s)|$ is 32dB and -8dB at 1 rad/s and 10 rad/s respectively. The phase is -ve for all ω. Then G(s) is

A) $\frac{39.8}{s}$ B) $\frac{39.8}{s^2}$ C) $\frac{32}{s}$ D) $\frac{32}{s^2}$

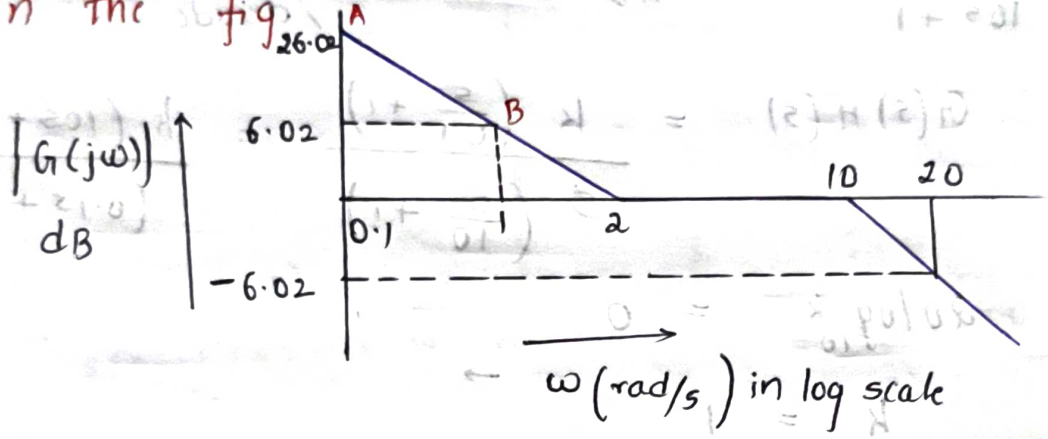
$S_{AB} = -40\ dB/decade$

$G(s)H(s) = \frac{k}{s^2}$; $20\log_{10} k = 32$

$k = 39.8$

$\rightarrow G(s)H(s) = \frac{39.8}{s^2}$

→ Question The bode asymptotic magnitude plot of a minimum phase system is shown in the fig.



If the system is connected in a unity negative feedback configuration, the steady state error of the C.L.S unit ramp i/p is,

$S_{AB} = -20\ dB/decade$

$e_{ss} = \frac{A}{k_v}$, Initial slope = −20

→ 1 pole at origin

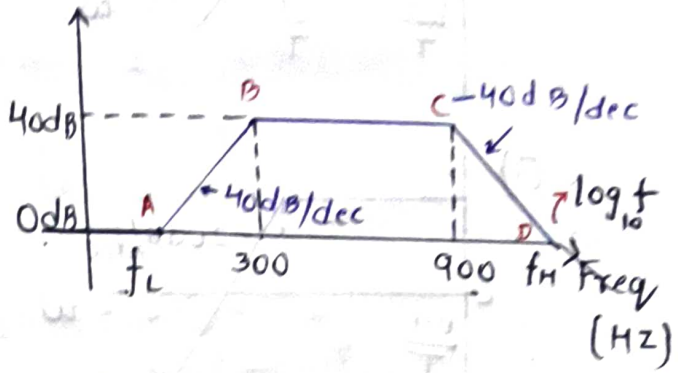
→ $k_p = \infty$, k_v = finite, $k_a = 0$

→ $k_v = k = 2$

$\therefore e_{ss} = \frac{1}{2} = 0.5$

Question: Consider the Bode plot shown in the figure. Assume that all poles and zeros are Real valued.

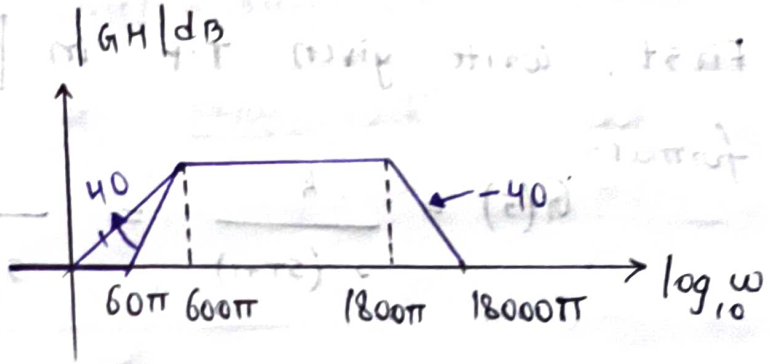
The value of $f_H - f_L$ (in Hz) is —



$f_L = 30\,Hz$, $f_H = 9000\,Hz$

$\therefore f_H - f_L = 9000 - 30 = \boxed{8970\,Hz}$

For T.F, T



$$GH = \frac{k\left(\frac{s}{60\pi}+1\right)^2\left(\frac{s}{18000\pi}+1\right)^2}{\left(\frac{s}{600\pi}+1\right)^2\left(\frac{s}{1800\pi}+1\right)^2}$$

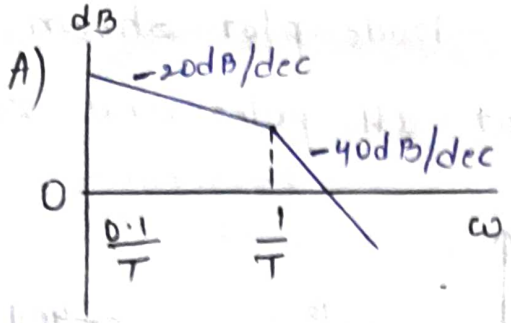
$$20\log_{10} k = 0$$

$$k = 1$$

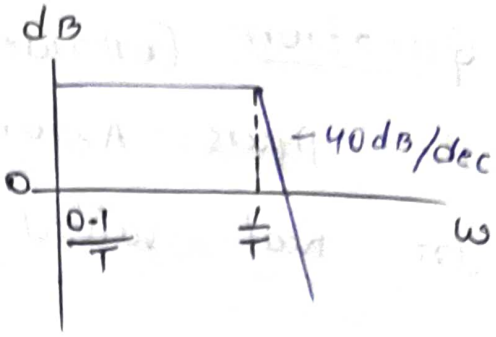
→ **Question:** The T.F of a system is given by

$G(s) = \frac{k}{s(sT+1)}$; $k < \frac{1}{T}$. The bode plot of this f^n is

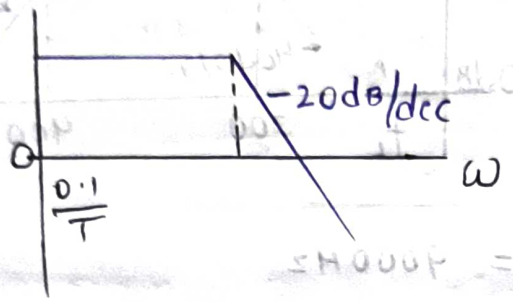
A)



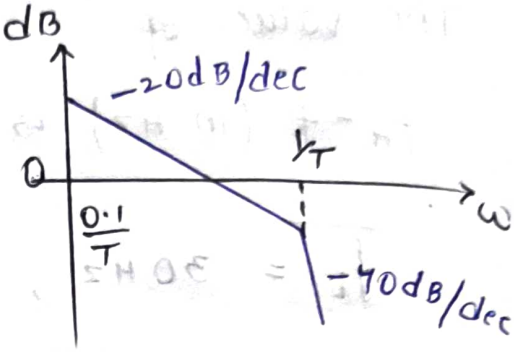
B)



C)



D)



$kT < 1$

First, write given T.F in **Time-constant** format.

$$G(s) = \frac{k}{s(sT+1)} = \frac{k}{s\left(\frac{s}{(1/T)}+1\right)}$$

#

$$\omega = 1/T \longrightarrow T(s) = \frac{k}{s}$$

$$T(j\omega) = \frac{k}{j\omega}$$

$$|T(j\omega)| = \frac{k}{\omega} = kT \quad \left(\because \omega = \frac{1}{T}\right)$$

$$|T(j\omega)|\,dB = 20\log_{10} kT \longrightarrow -ve\ (dB)$$

$$(\because kT < 1)$$

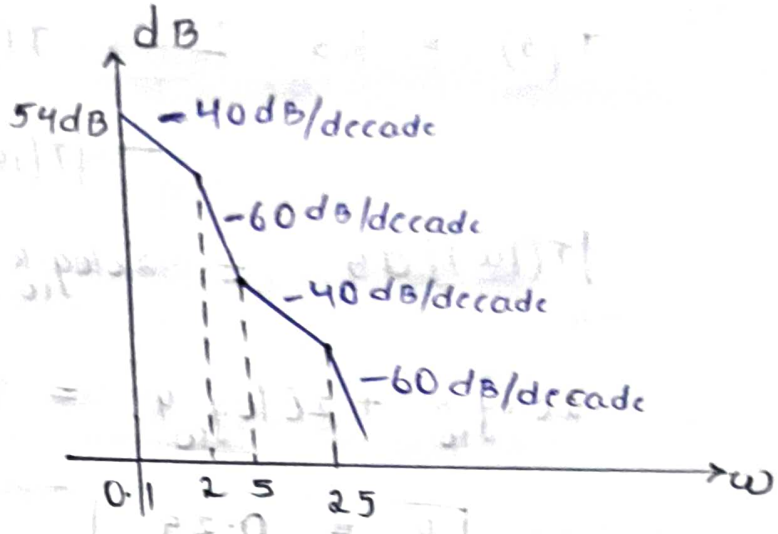
→ Question: The asymptotic approximation of the log-magnitude versus frequency plot of a certain system is shown below. It's T.F is

A) $\frac{50(s+5)}{s^2(s+2)(s+25)}$

B) $\frac{20(s+5)}{s^2(s+2)(s+25)}$

C) $\frac{10s^2(s+5)}{(s+2)(s+25)}$

D) $\frac{20(s+5)}{s(s+2)(s+25)}$



$$GH = \frac{k\left(\frac{s}{5}+1\right)}{s^2\left(\frac{s}{2}+1\right)\left(\frac{s}{25}+1\right)}$$

$\omega = 0.1$, $|GH| = 54dB$

$$GH = \frac{k}{s^2} \xrightarrow{s=j\omega} \frac{-k}{\omega^2} \rightarrow |GH| = \frac{k}{\omega^2}$$

$$20\log_{10}\frac{k}{0.1} - 40\log_{10}\frac{0.1}{0.1} = 54$$

$$k = 5$$

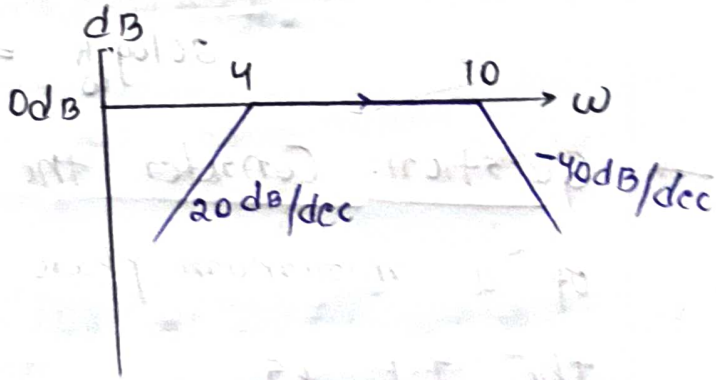
→ Question: For the Bode plot shown below, the T.F is

A) $\frac{100s}{(s+4)(s+10)^2}$

B) $\frac{100(s+4)}{s(s+10)^2}$

C) $\frac{100}{(s+4)(s+10)}$

D) $\frac{100}{s^2(s+4)(s+10)}$



$$T(s) = \frac{ks}{\left(\frac{s}{4}+1\right)\left(\frac{s}{10}+1\right)^2}$$

$\omega = 4$, $|T(j\omega)|_{dB} = 0\,dB$

$T(s) = ks \xrightarrow{s=j\omega} T(j\omega) = kj\omega$

$\rightarrow |T(j\omega)| = k\omega$

$|T(j\omega)|_{dB} = 20\log_{10} k + 20\log_{10} \omega$

$20\log_{10} k + 20\log_{10} 4 = 0$

$\boxed{k = 0.25}$

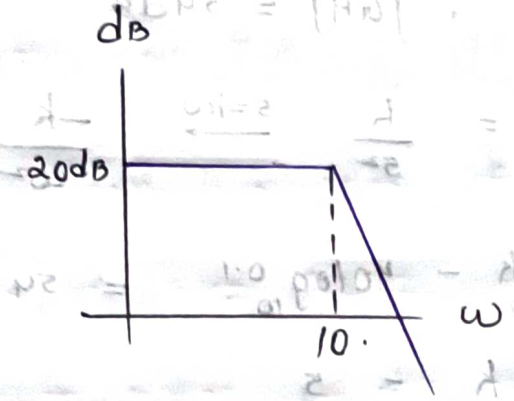
→ Question: Bode plot of a stable system is shown in figure. The OLTF of the ufb system is

A) $\frac{100}{s+10}$

B) $\frac{10}{s+10}$

C) $\frac{1}{s+10}$

D) None



$T(s) = \frac{k}{\left(\frac{s}{10}+1\right)}$

$20\log_{10} k = 20 \rightarrow k = 10$

→ Question: Consider the Asymptotic Bode plot of a minimum phase linear system in fig. The T.F is

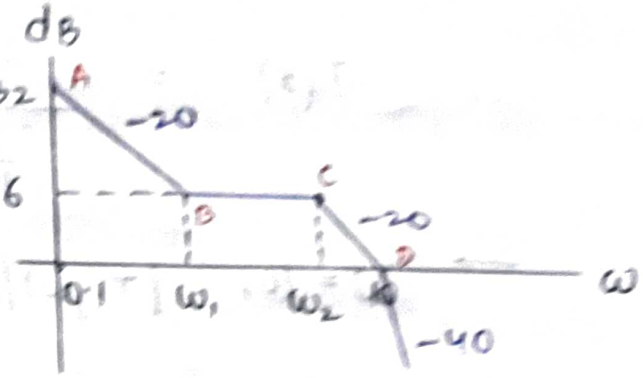
A) $\frac{85(s+2)}{(s+5)(s+10)}$

B) $\frac{4(s+5)}{(s+2)(s+10)}$

C) $\frac{4(s+2)}{s(s+5)(s+10)}$

D) $\frac{100(s+2)}{s(s+5)(s+10)}$

$T(s) \rightarrow$



$A(\log_{10}^{0.1}, 32dB)$

$B(\log_{10}^{\omega_1}, 6dB)$

$$S_{AB} = -20 = \frac{6-32}{\log_{10}^{\omega_1} - \log_{10}^{0.1}} \Rightarrow \omega_1 = 2$$

$C(\log_{10}^{\omega_2}, 6dB)$ $D(\log_{10}^{10}, 0dB)$

$$S_{CD} = -20 = \frac{0-6}{\log_{10}^{10} - \log_{10}^{\omega_2}} \Rightarrow \omega_2 = 5$$

$$T(s) = \frac{k\left(\frac{s}{2}+1\right)}{s\left(\frac{s}{5}+1\right)\left(\frac{s}{10}+1\right)}$$

$\rightarrow \omega = 0.1$, $|T(j\omega)|\,dB = 32dB$

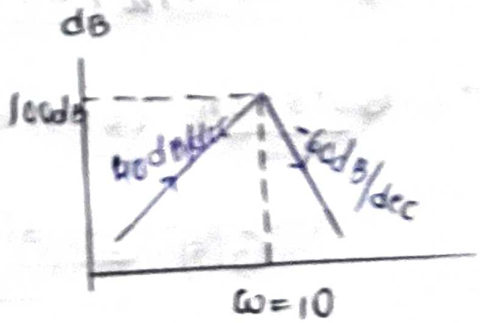
$$T(s) = \frac{k}{s} \xrightarrow{s=j\omega} T(j\omega) = \frac{k}{j\omega} \rightarrow |T(j\omega)| = \frac{k}{\omega}$$

$$|T(j\omega)|\,dB = 20\log_{10}^{k} - 20\log_{10}^{\omega}$$

$$20\log_{10}^{k} - 20\log_{10}^{0.1} = 32$$

$$k = 4$$

$\rightarrow$ Question: The bode plot shown below represent



A) $\frac{10^3 s^2}{(s+10)^5}$

C) $\frac{10^7 s^2}{(s+10)^5}$

B) $\frac{10^6 s^2}{(s+10)^5}$

D) $\frac{10^8 s^2}{(s+10)^5}$

$$T(s) = \frac{ks^2}{\left(\frac{s}{10}+1\right)^5}$$

→ $\omega = 10, \quad |T(j\omega)|dB = 100dB$ — ⊛

$$T(s) = ks^2 \xrightarrow{s=j\omega} T(j\omega) = -k\omega^2$$

$$\rightarrow |T(j\omega)| = k\omega^2$$

$$|T(j\omega)|dB = 20\log_{10} k + 40\log_{10}\omega$$

$$20\log_{10} k + 40\log_{10} 10 = 100$$

$$k = 1000$$

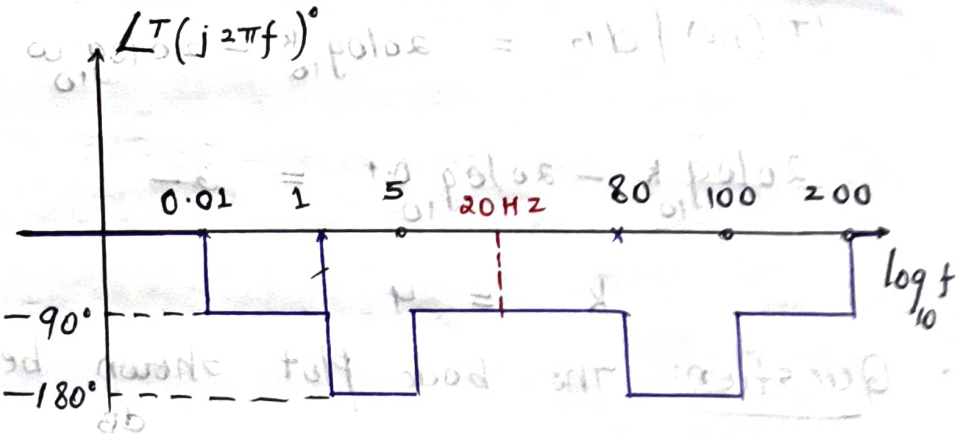
→ Question: A system has poles at 0.01HZ, 1HZ, 80HZ; zeroes at 5HZ, 100HZ and 200HZ. The approximate phase of the system response at 20HZ is

A) $-90°$

B) $90°$

C) $0°$

D) $-180°$



→ Question: Consider a stable system with T.F

$$G(s) = \frac{s^p + b_1 s^{p-1} + \cdots + b_p}{s^q + a_1 s^{q-1} + \cdots + a_q}$$

where $b_1, \ldots, b_p$ and $a_1, \ldots, a_q$ are real valued constants. The slope of the bode log magnitude curve of $G(s)$ converges to $-60dB$/decade as $\omega \to \infty$. A possible pair of values of p and q is

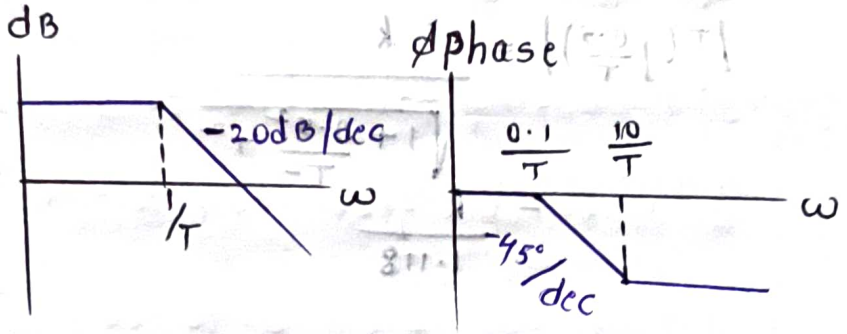
A) $p = 0$ and $q = 3$ $\quad q - p = 3$

B) $p = 1$ and $q = 7$

C) $p = 2$ and $q = 3$

D) $p = 3$ and $q = 5$

Question: The bode plot of the T.F $\frac{k}{1+sT}$ is given in the figure below. The error in gain at $\omega = \frac{0.5}{T}$ is



A) 1.25dB

B) 0.97dB

C) 1.93dB

D) 1.48dB

NOTE: $T(s) \xrightarrow{s = j\omega} T(j\omega)$

1. $|T(j\omega)|$ dB : FROM T.F $\longrightarrow M_{EXACT}$

$|T(j\omega)|$ dB : FROM BODE PLOT $\longrightarrow M_{approx}$

$$\text{Error} = M_{EXACT} - M_{APPROX}$$

2. $\angle T(j\omega)$: From T.F $\longrightarrow$ ϕ_{EXACT}

$\angle T(j\omega)$: From BODE PLOT $\longrightarrow$ ϕ_{approx}

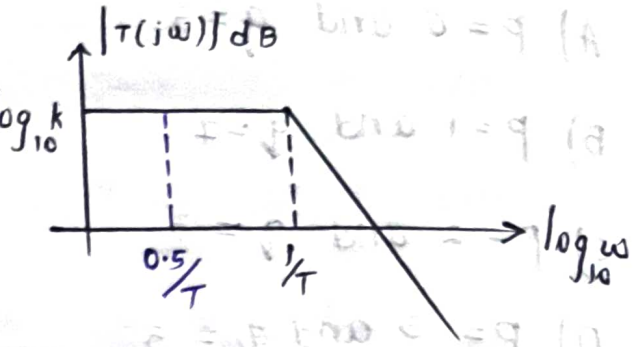
$\mathcal{E}_{TTOT} = \phi_{EXACT} - \phi_{approx}$

1. $T(s) = \dfrac{k}{\left(1 + \frac{s}{1/T}\right)}$

$\omega = \dfrac{0.5}{T}$

$T(j\omega) = \dfrac{k}{1 + \frac{j\omega}{1/T}}$

$|T(j\omega)| = \dfrac{k}{\sqrt{1 + \omega^2 T^2}}$



$\omega = \dfrac{0.5}{T}$

$\left|T\left(j\frac{0.5}{T}\right)\right| dB = 20\log_{10} k$

$M_{APPROX} = 20\log_{10} k$

$|T(j\frac{0.5}{T})| = \dfrac{k}{\sqrt{1 + \frac{0.5^2}{T^2} T^2}}$

$= \dfrac{k}{1.118}$

$|T(j\frac{0.5}{T})| = 20\log_{10} k - 20\log_{10} 1.118$

$M_{EXACT} = [20\log_{10} k - 0.97]\,dB$

ERROR $= -0.97$ dB

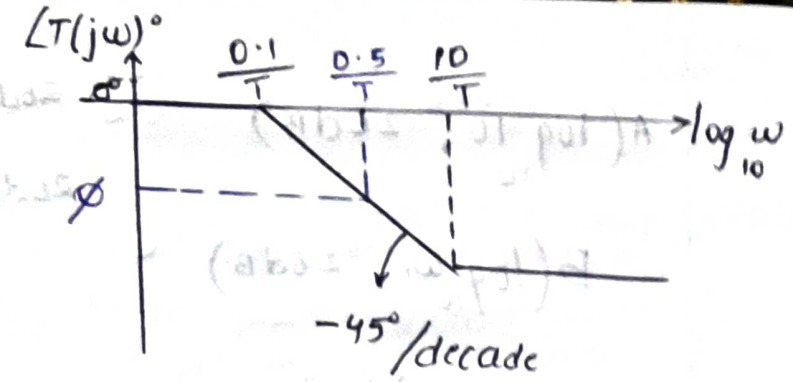
2. $T(s) = \dfrac{k}{1 + sT}$

$T(j\omega) = \dfrac{k}{1 + j\omega T}$

$\angle T(j\omega) = -\tan^{-1}\omega T$

$\angle T\left(j\frac{0.5}{T}\right) = -\tan^{-1} 0.5$

$\phi_{EXACT} = -26.56°$



$A\left(\log_{10}\frac{0.1}{T}, 0°\right)\ B\left(\log_{10}\frac{0.5}{T}, \phi\right)$

$S_{AB} = -45° = \frac{\phi - 0}{\log_{10}\frac{0.5}{T} - \log_{10}\frac{0.1}{T}}$

$\phi = \phi_{APPROX} = -31.45°$

$ERROR = \phi_{EXACT} - \phi_{APPROX}$

$= -26.56° - (-31.45°)$

$= 4.9°$

→ Question: A minimum phase system has 14 poles and 2 zeros. The shape of its highest frequency asymptote in its Bode plot is.

A) -240 dB/decade

B) -280 dB/decade

C) -320 dB/decade

D) -40 dB/decade

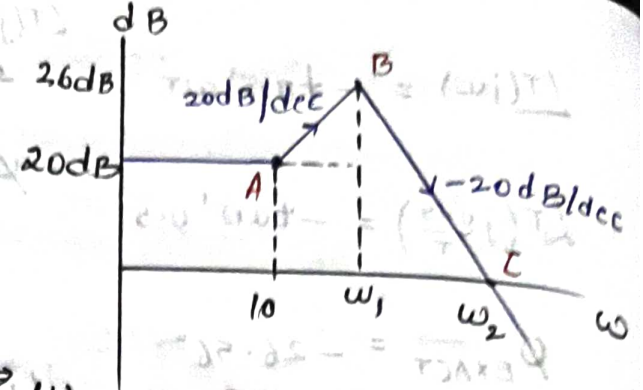
highest frequency asymptote

↳ $\omega \to \infty$

NET = 12 poles

$= 12 \times (-20) = -240$ dB/dec

→ Question: The magnitude-frequency response of a control system is given below. The value of ω_1 and ω_2 are respectively



$A(\log_{10} 10,\ 20dB)$

$B(\log_{10} \omega_1,\ 26dB)$

$$20 = \frac{26-20}{\log_{10}\omega_1 - \log_{10} 10} \Rightarrow \omega_1 = 20$$

$B(\log_{10} 20,\ 26dB)$ $C(\log_{10} \omega_2,\ 0dB)$

$$-20 = \frac{0-26}{\log_{10}\omega_2 - \log_{10} 20} \Rightarrow \omega_2 = 400$$

→ Question: The magnitude plot for a minimum phase f^n is shown in fig. The phase plot for this f^n will be monotonically,

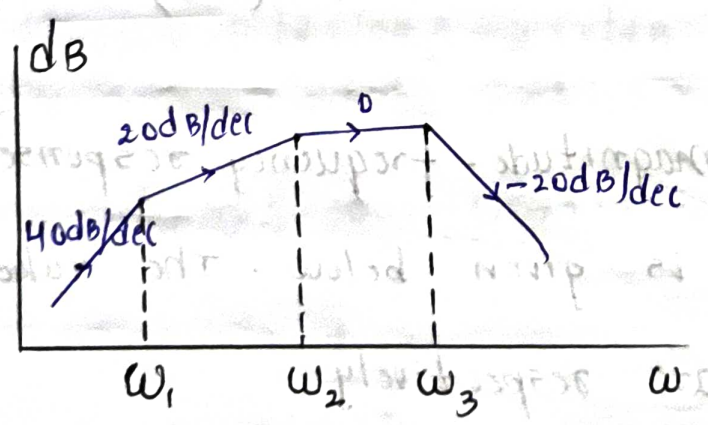
A) Decreasing from 180° to 0°

B) Decreasing from 180° to −90°

C) Increasing from 0° to 180°

D) Increasing from −90° to −180°

$$T(s) = \frac{ks^2}{\left(1+\frac{s}{\omega_1}\right)\left(1+\frac{s}{\omega_2}\right)\left(1+\frac{s}{\omega_3}\right)}$$



$\phi_{initial} = 180°$

$\phi_{final} = -90°$

$\omega \to \infty$, NET = 1 pole $\to$ $\phi_{FINAL} = -90°$

Initial Line slope = $+40dB/dec \to \phi_{initial} = 180°$.

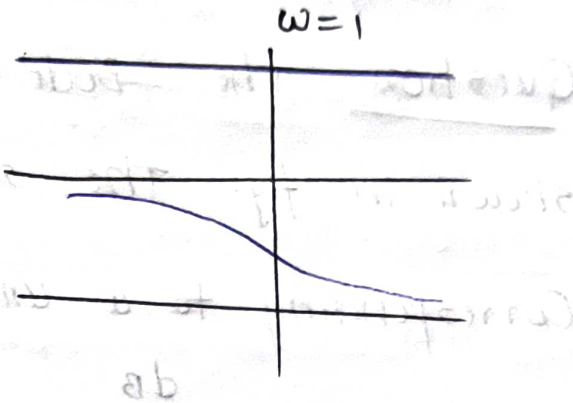
Question: For the Bode plot shown in the figure, the T.F is

A) $\frac{1-s}{1+s}$

B) $\frac{1}{(1+s)^2}$

C) $\frac{1}{s^2}$

D) $\frac{1}{s(s+1)}$



(A) $T(s) = \frac{1-s}{1+s}$

$T(j\omega) = \frac{1-j\omega}{1+j\omega}$

Both magnitude and phase plot are given.

$|T(j\omega)| = 1$, $|T(j\omega)|\,dB = 0dB$

$\angle T(j\omega) = -2\tan^{-1}\omega$

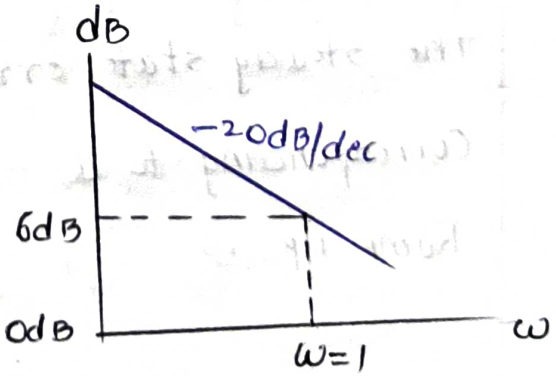
check for options

$\omega = 0$: $\angle T(j\omega) = 0°$

$\omega = \infty$: $\angle T(j\omega) = -180°$

Question: Consider the following statement regarding the low frequency magnitude curve of the forward T.F of ufb open-loop system given below.

1. This is type-0 system
2. The open loop gain $k = 2$
3. Position Error coefficient = 0



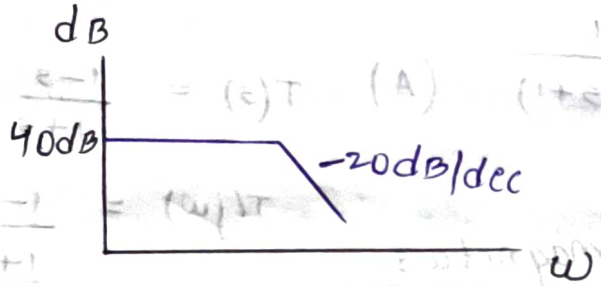
→ $G(s)H(s) = \frac{k}{s}$ → TYPE-1

→ $20\log_{10} k = 6$

$k = 2$

→ $k_p = \infty$

Question: The Bode plot for a T.F is shown in fig. The steady state error Corresponding to a unit step i/p is



A) $\frac{1}{100}$

B) $\frac{1}{101}$

C) $\frac{1}{41}$

D) $\frac{1}{40}$

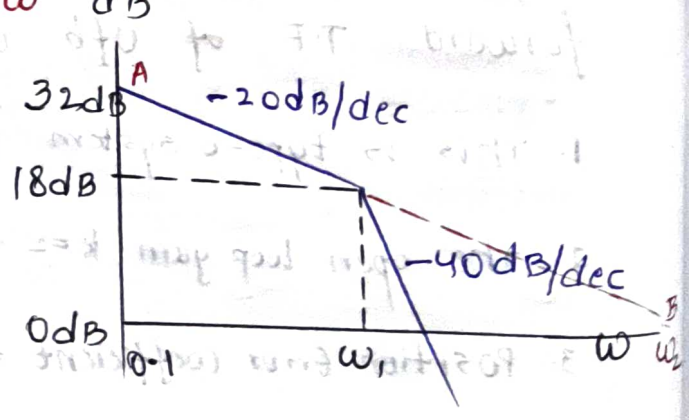
→ Type 0

$20\log_{10} k = 40$

$k = 100 = k_p$

Unit step i/p → $e_{ss} = \frac{A}{1+k_p} = \frac{1}{1+k_p} = \frac{1}{101}$

Question: Consider the Bode plot of a ufb system shown below



The steady state error Corresponding to a Ramp i/p is

A) 0.25 B) 0.2 C) 0 D) ∞

Ramp i/p → $e_{ss} = \frac{A}{k_v}$ $\omega_2 = k_v$

A($\log_{10} 0.1$, 32dB) B($\log_{10} \omega_2$, 0dB)

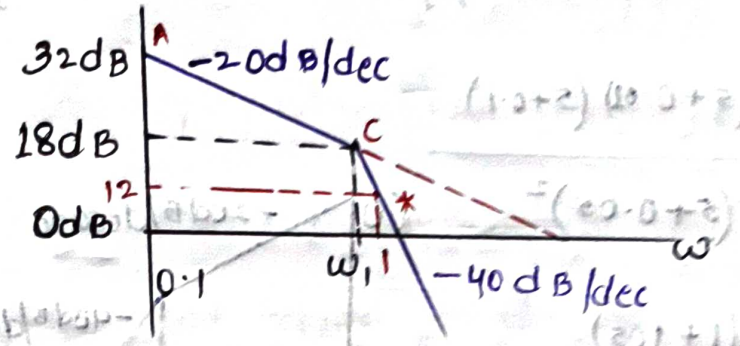
$$-20 = \frac{0-32}{\log_{10}\omega_2 - \log_{10} 0.1} \rightarrow \omega_2 = 4$$

$$\rightarrow k_v = 4$$

$$e_{ss} = \frac{1}{4} = 0.25$$

Question: The damping ratio for above Q?

A) 0.063
B) 0.179
C) 0.483
D) 0.639



$\omega_1 = 0.5$

$$T(s) = \frac{k}{s\left(\frac{s}{\omega_1}+1\right)} = \frac{k}{s(2s+1)}$$

$$T(s) = \frac{k/2}{s^2 + \frac{s}{2}} = \frac{k/2}{s\left(s+\frac{1}{2}\right)} \equiv \frac{k\omega_n^2}{s(s+2\xi\omega_n)}$$

$$2\xi\omega_n = \frac{1}{2} \qquad \omega_n^2 = \frac{1}{2}$$

$$\rightarrow \xi = \frac{1}{2 \cdot 2 \cdot \frac{1}{\sqrt{2}}} =$$

$$T(s) = \frac{k/2}{s\left(s+\frac{1}{2}\right)} \equiv \frac{\omega_n^2}{s(s+2\xi\omega_n)}$$

$20\log_{10} k = 12$

$\rightarrow k = 4$

$\rightarrow T(s) = \frac{2}{s\left(s+\frac{1}{2}\right)} = \frac{\omega_n^2}{s(s+2\xi\omega_n)}$

$2\xi\omega_n = \frac{1}{2}, \quad \omega_n^2 = 2$

$\rightarrow \xi = \frac{1}{4\sqrt{2}} = 0.179.$

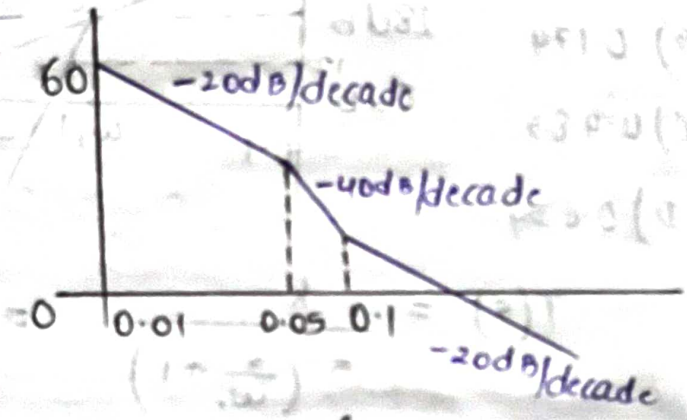
→ Question: The system having the Bode magnitude plot shown in fig has the Transfer fn

A) $\frac{60(s+0.01)(s+0.1)}{s^2(s+0.05)^2}$

B) $\frac{10(1+10s)}{s(1+20s)}$

C) $\frac{3(s+0.05)}{s(s+0.1)(s+1)}$

D) $\frac{5(s+0.1)}{s(s+0.05)}$



$T(s) = \frac{k\left(\frac{s}{0.1}+1\right)}{s\left(\frac{s}{0.05}+1\right)}$

$\omega = 0.01$

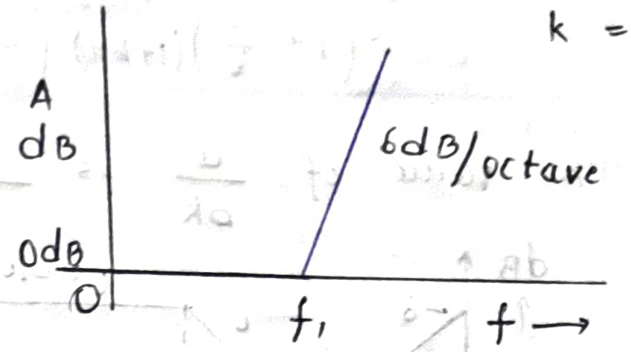
$T(s) = \frac{k}{s} \rightarrow |T(j\omega)| = \frac{k}{\omega}$

$|T(j\omega)|_{dB} = 20\log k - 20\log_{10}\omega$

$20\log_{10} k - 20\log_{10} 0.01 = 60$

$k = 10$

Question: The f^n corresponding to the Bode plot of fig. is:



$20\log_{10} k = 0$

$k = 1$

A) $A = jf/f_1$

B) $A = \dfrac{1}{1 - jf_1/f}$

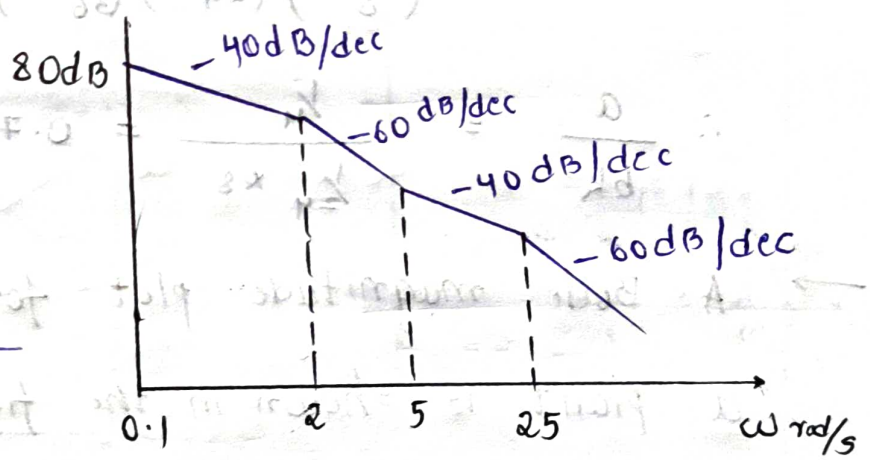
C) $A = \dfrac{1}{1 + jf_1/f}$

D) $A = 1 + jf/f_1$

$T(s) = k\left(1 + \dfrac{s}{2\pi f_1}\right)$

$T(s) = \left(1 + \dfrac{j\omega}{2\pi f_1}\right) = 1 + \dfrac{jf}{f_1}$ ($\omega \to 2\pi f$)

Question: The asymptotic approximation of the log-magnitude vs frequency plot of a system containing only real poles and zeros is shown.



Its T.F is

A) $\dfrac{10(s+5)}{s(s+2)(s+25)}$

B) $\dfrac{1000(s+5)}{s^2(s+2)(s+25)}$

C) $\dfrac{100(s+5)}{s(s+2)(s+25)}$

$T(s) = \dfrac{k\left(\frac{s}{5}+1\right)}{s^2\left(\frac{s}{2}+1\right)\left(\frac{s}{25}+1\right)}$

D) $\dfrac{80(s+5)}{s^2(s+2)(s+25)}$

$\omega = 0.1$:

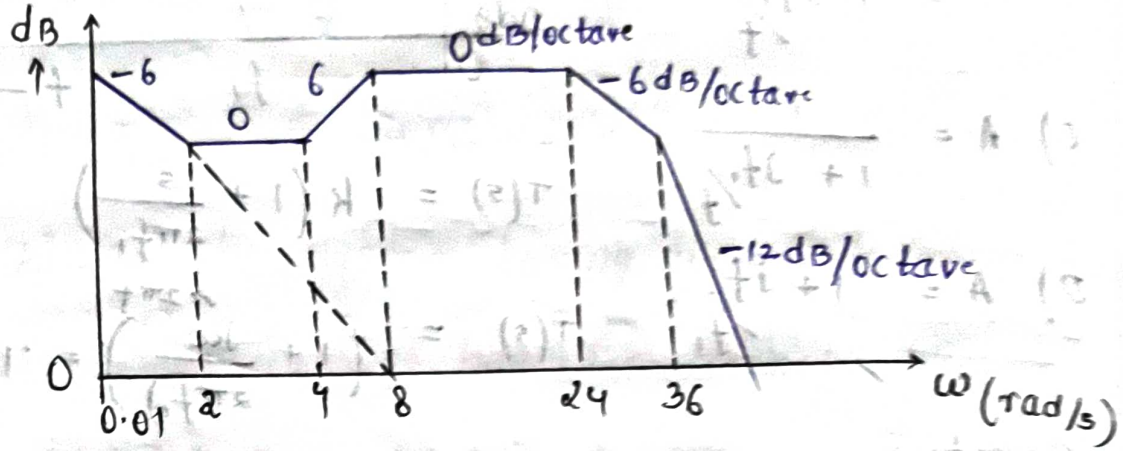
$20\log_{10} k - 40\log_{10} 0.1 = 80$

$\Rightarrow k = 100$

Question: The Bode magnitude plot of the T.F

$$G(s) = \frac{k(1+0.5s)(1+as)}{s\left(1+\frac{s}{8}\right)(1+bs)\left(1+\frac{s}{36}\right)}$$ is shown below

The value of $\frac{a}{bk}$ is ____



$\boxed{k = 8}$

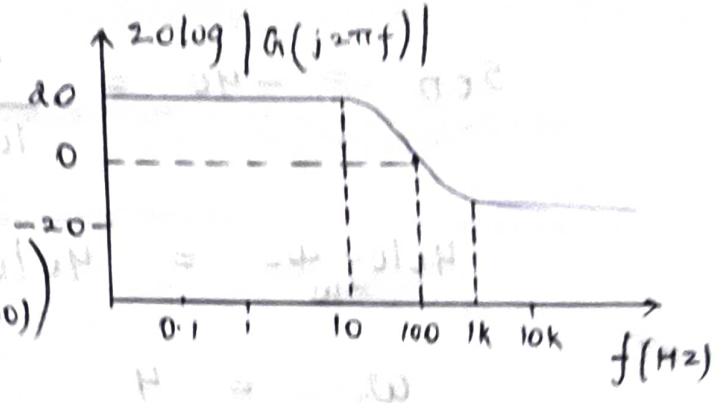
$$GH = \frac{k\left(\frac{s}{2}+1\right)\left(\frac{s}{4}+1\right)}{s\left(\frac{s}{8}+1\right)\left(\frac{s}{24}+1\right)\left(\frac{s}{36}+1\right)} \rightarrow a = \frac{1}{4},\ b = \frac{1}{24}$$

$$\therefore \frac{a}{bk} = \frac{1/4}{1/24 \times 8} = 0.75$$

A Bode magnitude plot for the T.F $G(s)$ of a plant is shown in the figure. Which one of the following transfer functions best describes the plant?

A) $\frac{1000(s+10)}{s+1000}$ B) $\frac{10(s+10)}{s(s+1000)}$

C) $\frac{s+1000}{10s(s+10)}$ D) $\frac{s+1000}{10(s+10)}$



$20\log_{10} k = 20$

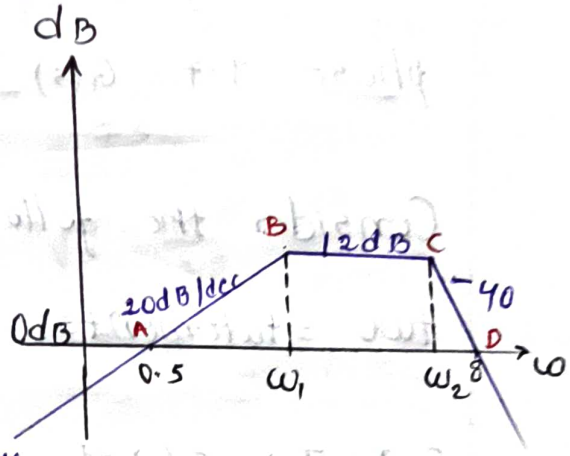
$\rightarrow k = 10$

$$G(s) = \frac{k\left(1+\frac{s}{2\pi(1000)}\right)}{\left(1+\frac{s}{2\pi(10)}\right)}$$

$$= \frac{2\pi \times 10}{2\pi \times 1000} \frac{k\left(s+20000\pi\right)}{s+20\pi}$$

$$= \frac{0.1\left(s+20000\pi\right)}{s+20\pi}$$

→ Question: Consider the following asymptotic bode magnitude plot (ω is in rad/s). Which one of the following Transfer fⁿs is best represented by the above bode magnitude plot.



A) $\frac{2s}{(1+0.5s)(1+0.25s)^2}$

B) $\frac{4(1+0.5s)}{s(1+0.25s)}$

C) $\frac{2s}{(1+2s)(1+4s)}$

D) $\frac{4s}{(1+2s)(1+4s)^2}$

$$S_{AB} = 20 = \frac{12-0}{\log_{10}\omega_1 - \log_{10} 0.5}$$

$$20\log_{10}\omega_1 = 12 + 20\log_{10} 0.5$$

$$\omega_1 = 2$$

$$\text{Slope} = -40 = \frac{0-12}{\log_{10} 8 - \log_{10} \omega_2}$$

$$40\log_{10}\omega_2 = 40\log_{10} 8 - 12$$

$$\omega_2 = 4$$

$$T(s) = \frac{ks}{\left(\frac{s}{2}+1\right)\left(\frac{s}{4}+1\right)^2}$$

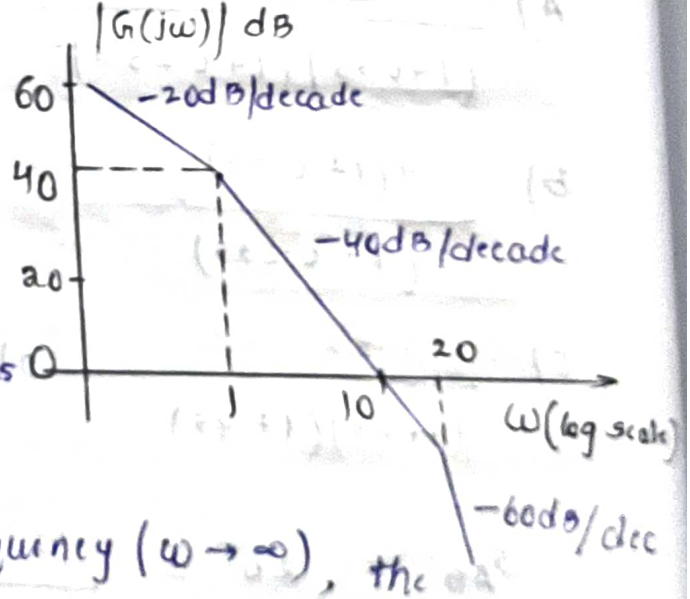
$\underline{\omega = 0.5}$:

$$T(s) = ks \rightarrow |T(j\omega)| = k\omega$$

$$20\log_{10} k + 20\log_{10} 0.5 = 0$$

$$k = 2$$

→ The asymptotic Bode magnitude of a min phase T.F G(s) is shown below.



Consider the following two statements

S-I: T.F G(s) has 3 poles and 1 zero

S-II: At very high frequency ($\omega \to \infty$), the phase angle $\angle G(j\omega) = -\frac{3\pi}{2}$

I: 3 poles, 1 zero → NET = 2 POLES

→ −40dB/dec

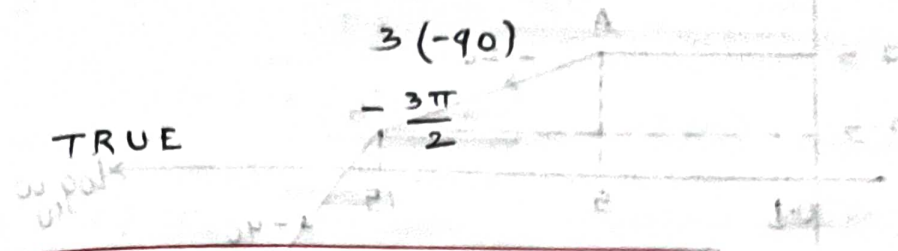
→ FALSE

II. $-60\,dB \rightarrow$ 3 POLES

$3(-90)$

$-\frac{3\pi}{2}$

TRUE



PLOTTING OF BODE PLOT

$$G(s)H(s) = \frac{(75)^2}{(s+5)(s+15)}$$

Time constant Format,

$$G(s)H(s) = \frac{75}{\left(\frac{s}{5}+1\right)\left(\frac{s}{15}+1\right)}$$

Initial Line : $0\,dB$ / decade of $20\log_{10}75 = 37.5\,dB$

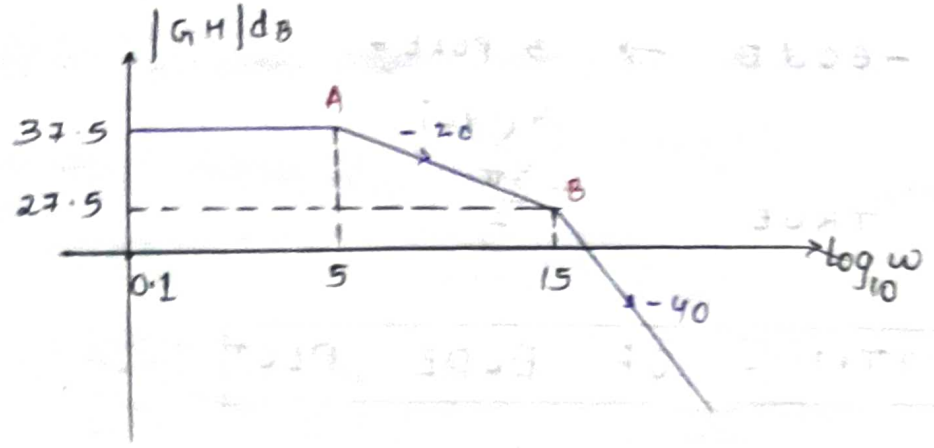
I. $\omega = 15 \rightarrow T(s) = \frac{75}{\left(\frac{s}{5}+1\right)} \rightarrow T(j\omega) = \frac{75}{1+\frac{j\omega}{5}}$

$$|T(j\omega)| = \frac{75}{\sqrt{1+\frac{\omega^2}{25}}} = \frac{75}{\sqrt{1+\frac{15^2}{25}}}$$

$$|J(j\omega)|\,dB = 27.5\,dB$$

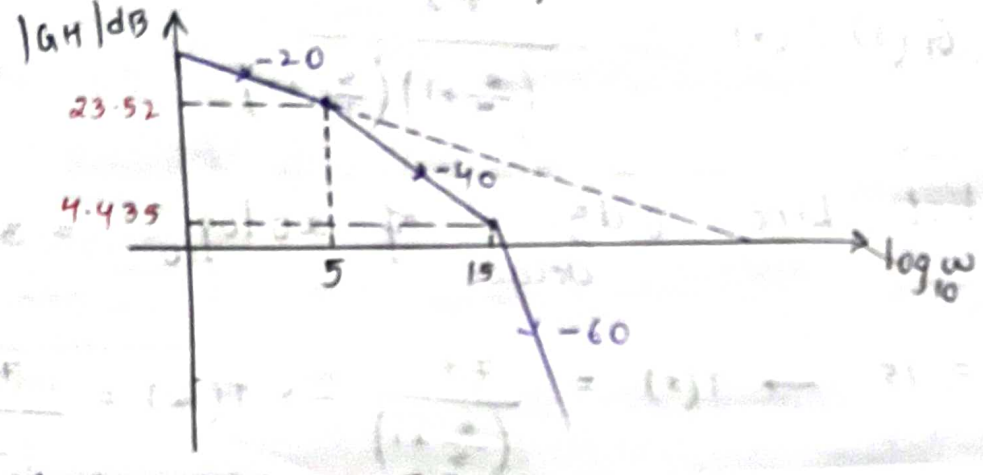
II. $A(\log_{10}5, 37.5dB) \quad B(\log_{10}15, M\,dB)$

$$S_{AB} = -20 = \frac{M-37.5}{\log_{10}15 - \log_{10}5} \rightarrow M = 27.5\,dB$$



$$G(s)H(s) = \frac{75}{s\left(\frac{s}{5}+1\right)\left(\frac{s}{15}+1\right)}$$

Initial Line : $-20\,dB/decade$



$\omega = 5$: $\quad T(s) = \frac{75}{s}$

$$|T(j\omega)|\,dB = 20\log_{10}75 - 20\log_{10}5 = 23.52\,dB$$

$A\left(\log_{10}5,\ 23.52\right) \qquad B\left(\log_{10}15,\ M\right)$

$$-40 = \frac{M - 23.52}{\log_{10}15 - \log_{10}5} \longrightarrow M = 4.435\,dB$$

Question:

$$G(s)H(s) = \frac{\omega_n^2}{s(s + 2\xi\omega_n)}$$

$$GH = \frac{\frac{\omega_n}{2\xi}}{s\left(\frac{s}{2\xi\omega_n}+1\right)} \longrightarrow \text{Initial Line} : -20\,dB/dec$$

$\rightarrow \omega = 2\xi\omega_n$

$$T(s) = \frac{\frac{\omega_n}{2\xi}}{s}$$

$$T(j\omega) = \frac{\omega_n}{2\xi\, j\omega}$$

$$T(j2\xi\omega_n) = \frac{\omega_n}{2\xi\, j2\xi\omega_n}$$

$$|T(j2\xi\omega_n)| = \frac{1}{4\xi^2}$$

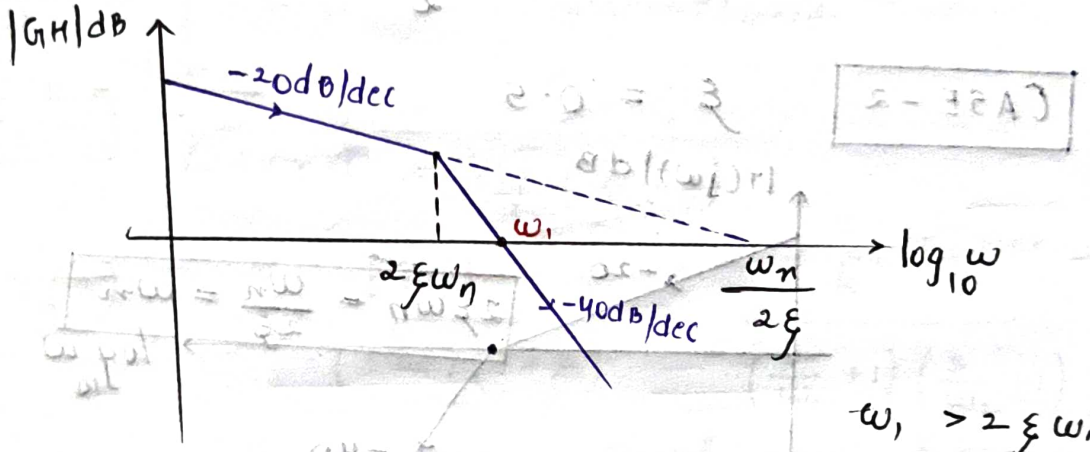
$$|T(j2\xi\omega_n)|\,dB = -40\log_{10} 2\xi$$

$\rightarrow 0 < \xi < 0.5 \longrightarrow +ve$

$\xi = 0.5 \longrightarrow 0\,dB$

$0.5 < \xi < \infty \rightarrow -ve$

CASE-1 : $0 < \xi < 0.5$



$\omega_1 > 2\xi\omega_n$

$$\frac{\omega_1}{2\xi\omega_n} > 1$$

$$T(s) = \frac{\frac{\omega_n}{2\xi}}{s\left(\frac{s}{2\xi\omega_n}+1\right)}$$

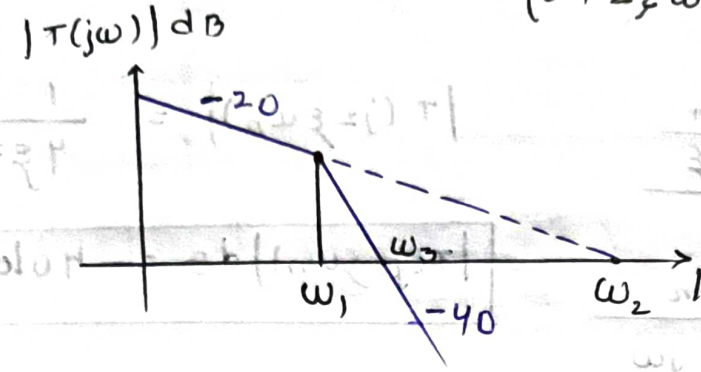
$\omega = \omega_1 : T(s) = \dfrac{\frac{\omega_n}{2\xi}}{s\left(\frac{s}{2\xi\omega_n}\right)} = \dfrac{\omega_n^2}{s^2}$

$$|T(j\omega)| = \frac{\omega_n^2}{\omega_1^2}$$

* $|T(j\omega)|\,dB = 20\log_{10}\left(\frac{\omega_n^2}{\omega_1^2}\right) = 0\,dB$

→ $\omega_1 = \omega_n$

SUMMARY: $GH = \dfrac{\omega_n^2}{s(s+2\xi\omega_n)}$, $0 < \xi < 0.5$



$\omega_1 = 2\xi\omega_n$

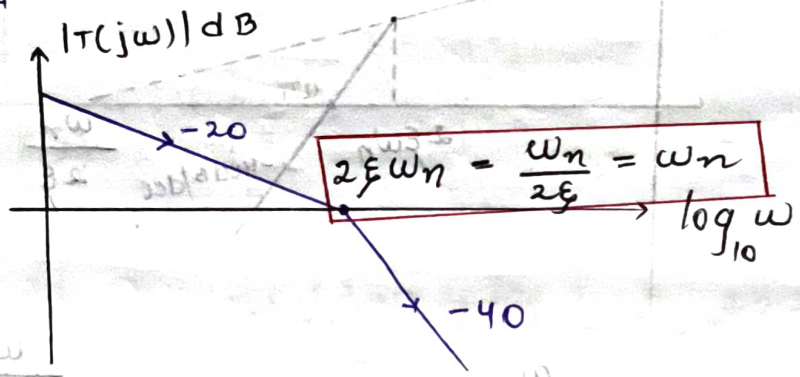
$\omega_3 = \omega_n$

$\omega_2 = \dfrac{\omega_n}{2\xi}$

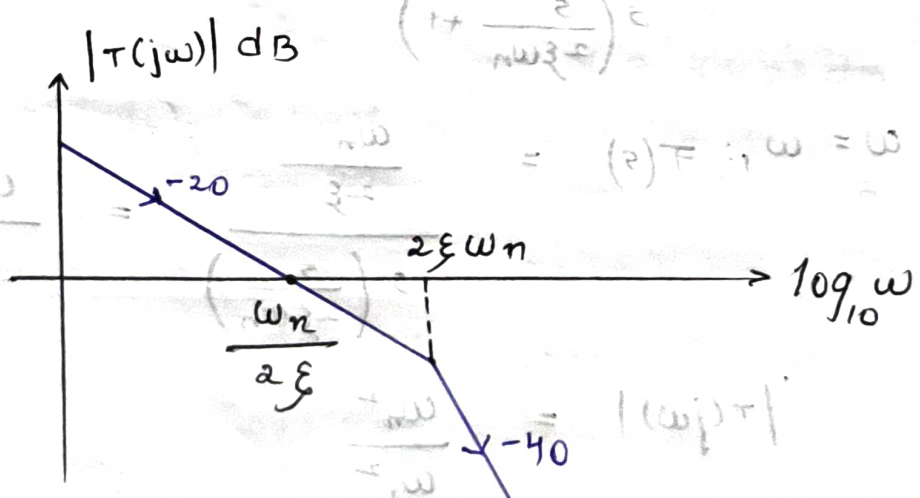
1. $\omega_3^2 = \omega_1 \omega_2$: **GM**

2. $\log_{10} \omega_3 = \dfrac{\log_{10}\omega_1 + \log_{10}\omega_2}{2}$: **A.M**

CASE - 2 $\xi = 0.5$



CASE - 3 $0.5 < \xi < \infty$

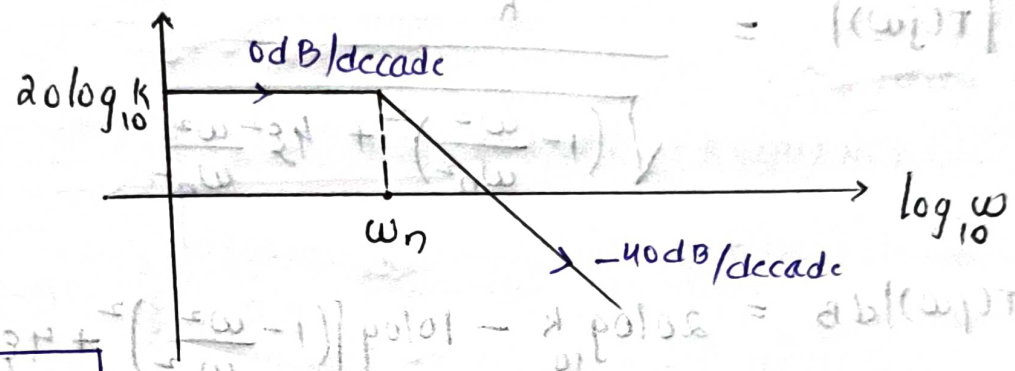


BODE PLOT OF $G(s)H(s) = \dfrac{k\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$

CASE-1 CRITICAL DAMPING ($\xi = 1$)

$$T(s) = \frac{k\omega_n^2}{(s+\omega_n)^2} = \frac{k}{\left(\frac{s}{\omega_n}+1\right)^2}$$

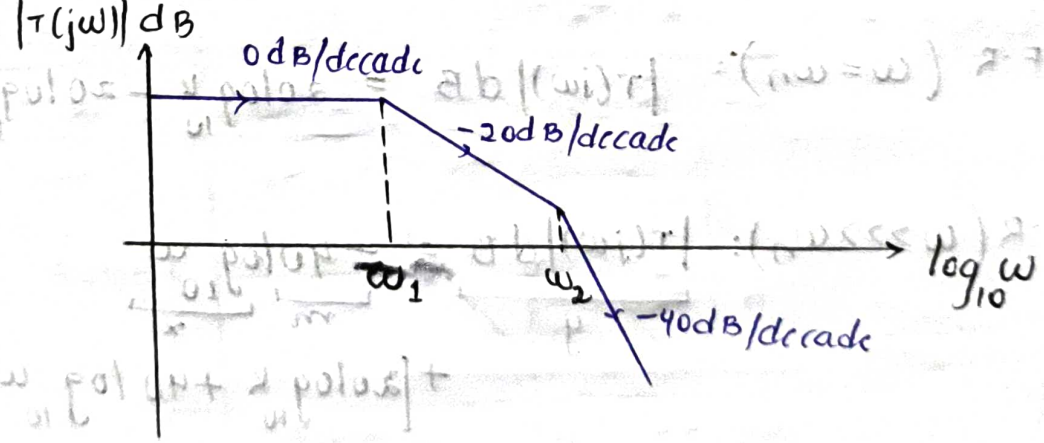
→ Initial Line 0dB/decade with $20\log_{10} k$



CASE-2 Over Damping ($\xi > 1$)

$$T(s) = \frac{k\omega_n^2}{(s+\alpha)(s+\beta)} = \frac{k}{\left(\frac{s}{\omega_1}+1\right)\left(\frac{s}{\omega_2}+1\right)}$$

→ Initial line: 0dB/decade with $20\log_{10} k$



CASE - 3 UNDERDAMP $(0 < \xi < 1)$

$$T(s) = \frac{k\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} = \frac{k}{\frac{s^2}{\omega_n^2} + \frac{2\xi s}{\omega_n} + 1}$$

$$T(j\omega) = \frac{k}{\left(1 - \frac{\omega^2}{\omega_n^2}\right) + j\frac{2\xi\omega}{\omega_n}}$$

$$|T(j\omega)| = \frac{k}{\sqrt{\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + 4\xi^2\frac{\omega^2}{\omega_n^2}}}$$

$$|T(j\omega)|dB = 20\log_{10} k - 10\log_{10}\left[\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + 4\xi^2\frac{\omega^2}{\omega_n^2}\right]$$

$$|T(j\omega)|dB = 20\log_{10} k - 10\log_{10}\left[1 + \frac{\omega^4}{\omega_n^4} - \frac{2\omega^2}{\omega_n^2} + 4\xi^2\frac{\omega^2}{\omega_n^2}\right]$$

L·F·R $(\omega \ll \omega_n)$: $|T(j\omega)|dB = 20\log_{10} k$

M·F·R $(\omega = \omega_n)$: $|T(j\omega)|dB = 20\log_{10}\frac{k}{2\xi}$

H·F·R $(\omega \gg \omega_n)$: $\underbrace{|T(j\omega)|dB}_{y} = \underbrace{-40}_{m}\underbrace{\log_{10}\omega}_{x} + \underbrace{\left[20\log_{10} k + 40\log_{10}\omega_n\right]}_{C}$

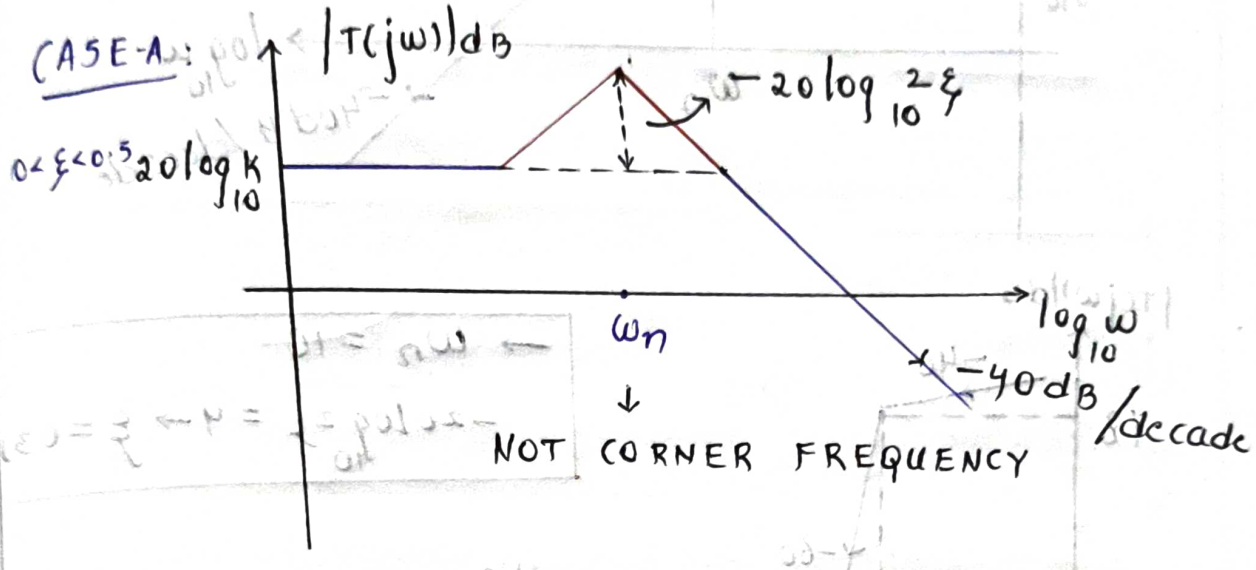
$-20\log_{10}^{2}\xi$

$0 < \xi < 0.5 \longrightarrow +ve$

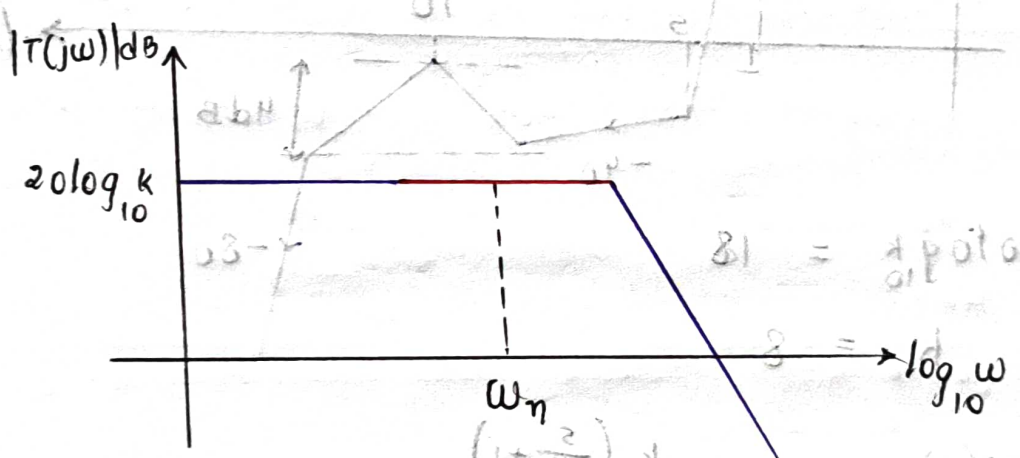
$\xi = 0.5 \longrightarrow 0$

$\xi > 0.5 \longrightarrow -ve$

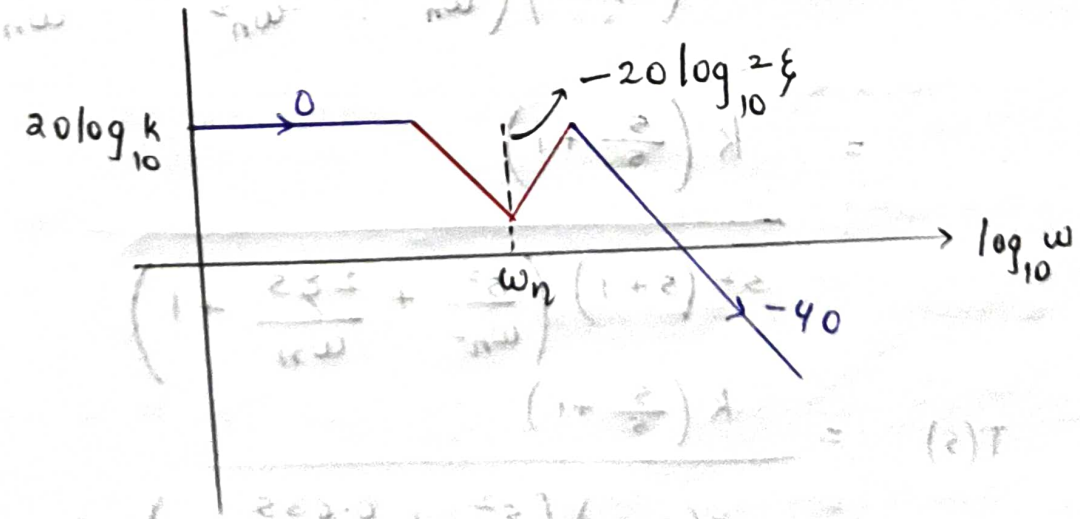
CASE-A:



CASE-B: $\xi = 0.5$

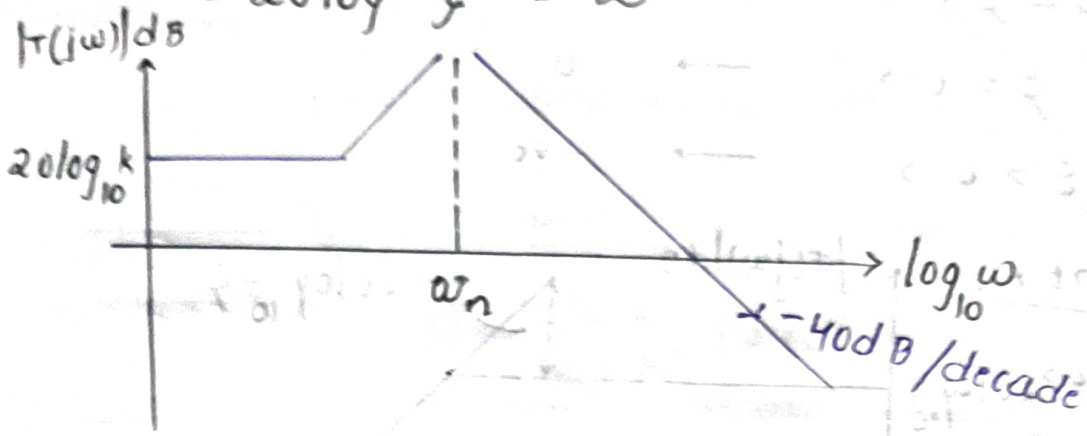


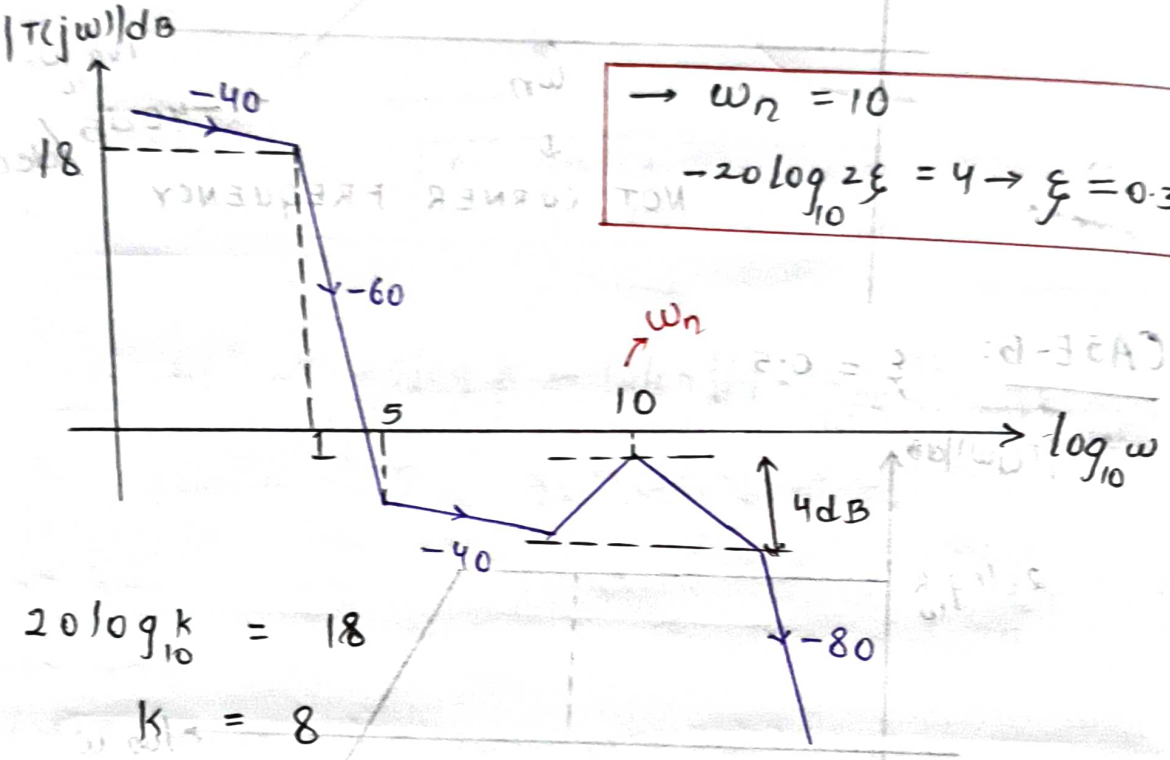
CASE-C: $\xi > 0.5$



CASE-4 UNDAMP SYSTEM ($\xi = 0$)

$-20\log 2\xi = \infty$





$\rightarrow \omega_n = 10$

$-20\log_{10} 2\xi = 4 \rightarrow \xi = 0.315$

$20\log_{10} k = 18$

$k = 8$

$$T(s) = \frac{k\left(\frac{s}{5}+1\right)}{s^2\left(\frac{s}{1}+1\right)\left(\frac{s^2}{\omega_n^2} + \frac{2\xi\omega_n s}{\omega_n^2} + \frac{\omega_n^2}{\omega_n^2}\right)}$$

$$= \frac{k\left(\frac{s}{5}+1\right)}{s^2(s+1)\left(\frac{s^2}{\omega_n^2} + \frac{2\xi s}{\omega_n} + 1\right)}$$

$$T(s) = \frac{k\left(\frac{s}{5}+1\right)}{s^2(s+1)\left(\frac{s^2}{100} + \frac{0.635}{10} + 1\right)}$$

DPP: Root Locus - III

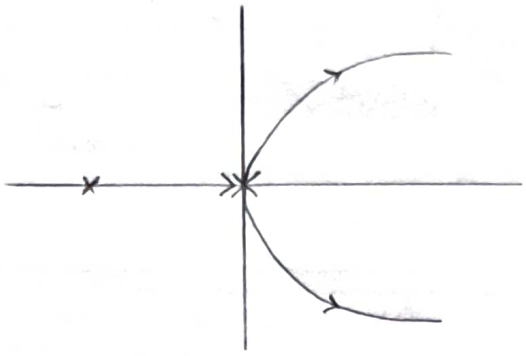
→ **Question1** The root locus of a system is given in the figure. The O.L.T.F is

A) $\frac{k}{s(s+T)^2}$

B) ~~$\frac{k}{s^2(s+T)}$~~

C) $\frac{k}{s(s+T)}$

D) $\frac{k}{s^3(s+1)}$



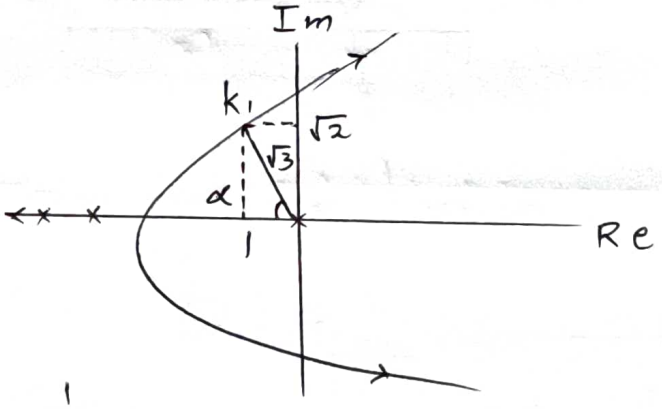
→ **Question2** The root locus plot is shown below The damping ratio for the value of $k = k_1$ is

A) 1.41

B) 1.73

C) 0.577

D) 0.707



Sol^n: $\xi = \cos\phi = \frac{1}{\sqrt{3}}$

→ **Question3** The C.E of a unity feedback C.S is given by $s^3 + k_1 s^2 + s + k_2 = 0$. Consider the following statements.

P. The CLS is stable only for $k_1 > k_2$

Q. The CLS is stable only for $k_1 < k_2$

P. The CLS is marginal stable only for the value of $k_1 = k_2$

Q. For a given value of k_2, all the root-locus branches will terminate at ∞ for variable k_1 in the +ve direction.

I → k_1 , → ·· k_2

R. For a given value of k_1, all the root locus branches will terminate at ∞ for variable k_2.

S. ·· k_2, only one root-locus branches will terminate at ∞ for variable k_1 in the +ve.

Soln:

s^3	1	1
s^2	k_1	k_2
s^1	$\frac{k_1 - k_2}{k_1}$	0
s^0	k_2	0

For stability

$k_1 > 0$; $k_2 > 0$

$k_1 - k_2 > 0 \rightarrow k_1 > k_2$

For marginal stable

$k_1 = k_2$

→ P, R are correct

For the given k_2 value

let k_2 = constant

$$s^3 + k_1 s^2 + s + k_2 = 0$$

$$1 + \frac{k_1 s^2}{s^3 + s + k_2} = 0$$

Open Loop: poles = 3, zeros = 2

→ only one root locus branch terminates at ∞.

→ V correct

For the given k_1,

Let k_1 = constant

$$T.F = 1 + \frac{k_2}{s^3 + k_1 s^2 + s}$$

open loop : poles = 3 , zeros = 0

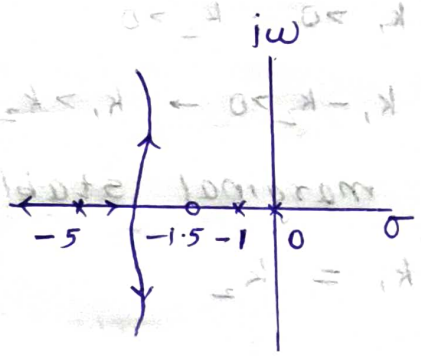
→ All branches terminate at ∞.

→ U correct.

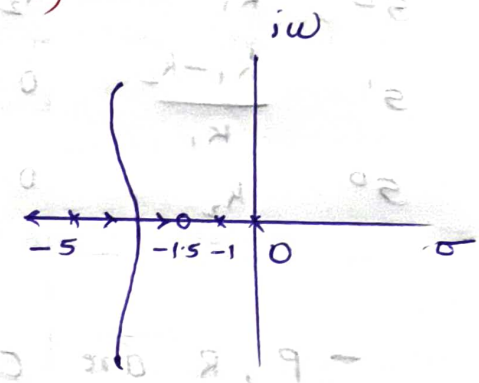
→ Question4 Which one of the following is the root locus of a system with O.L.T.F

$$G(s)H(s) = \frac{k(s+1.5)}{s(s+1)(s+5)}$$

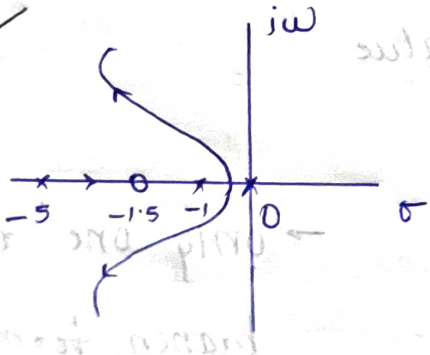
A)



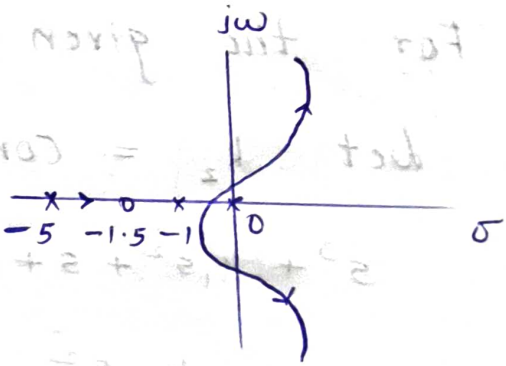
B)



C)



D)

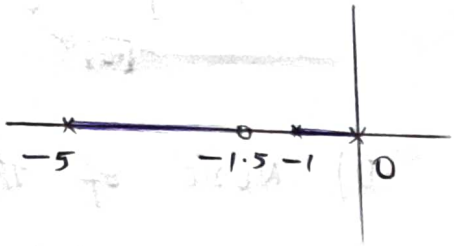


No. of Asymptotes = $p - z = 3 - 1 = 2$

Angle of asymptotes = 90°, 270°.

→ σ centroid $= \dfrac{0-1-5-(-1.5)}{3-1} = -2.25$

→ Real Axis root locus branches



2 Real axis root locus branches

→ Break points $\dfrac{dk}{ds} = 0$

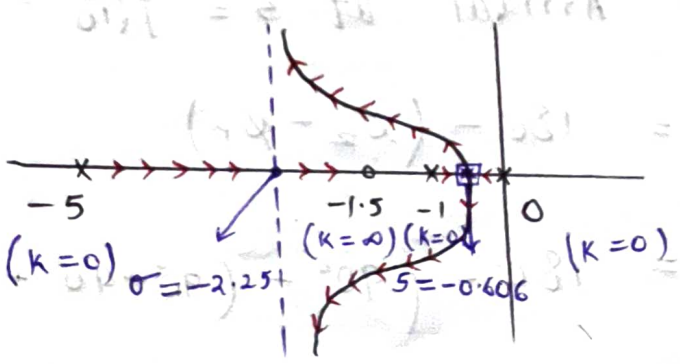
$$k = -\frac{s(s+1)(s+5)}{(s+1.5)} = -\frac{s^3+6s^2+5s}{s+1.5}$$

$$\frac{dk}{ds} = \frac{(s+1.5)(3s^2+12s+5) - (s^3+6s^2+5s)}{(s+1.5)^2} = 0$$

$$3s^3 + 16.5s^2 + 23s + 7.5 = s^3 + 6s^2 + 5s$$

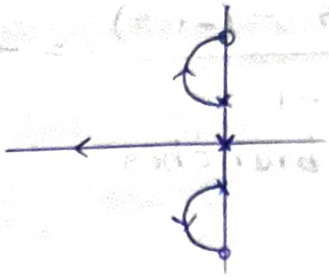
$$2s^3 + 10.5s^2 + 18s + 7.5 = 0$$

$s = -0.606$ is the valid B.P.

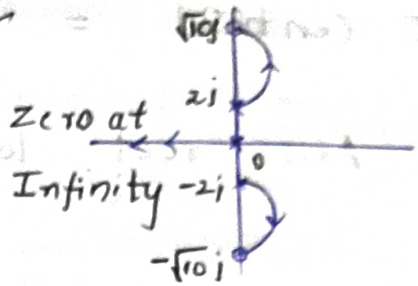


→ Questions OLTF of a ufb system is $\dfrac{k(s^2+10)}{s(s^2+4)}$. The RLD is

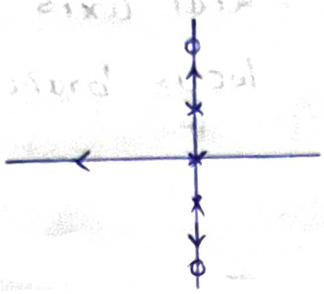
A)



B)



C)



D) None of the above

1n:

$$G(s)H(s) = \frac{k(s^2+10)}{s(s^2+4)}$$

Angle of departure at $s = 2j$

$$\theta_d = 180^\circ - (\phi_p - \phi_z)$$

$$= 180^\circ - (90^\circ + 90^\circ - (90 - 90^\circ))$$

$$= 0^\circ$$

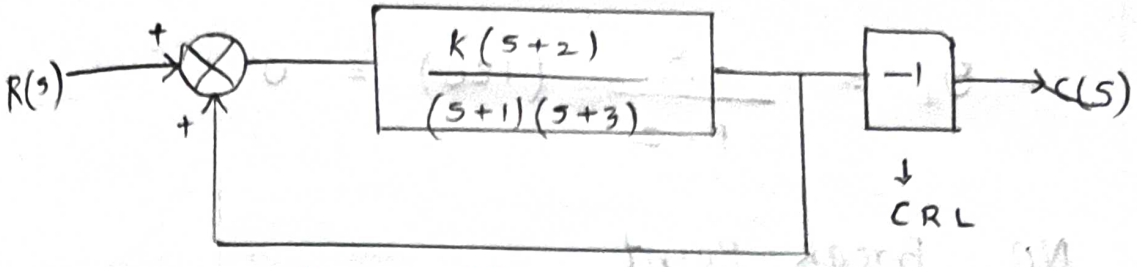
Angle of Arrival at $s = j\sqrt{10}$

$$\theta_a = 180^\circ - (\phi_z - \phi_p)$$

$$= 180^\circ - (90^\circ - (90^\circ + 90^\circ + 90^\circ))$$

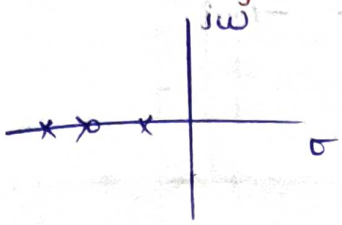
$$= 360^\circ$$

→ Questions Consider the feedback system shown below

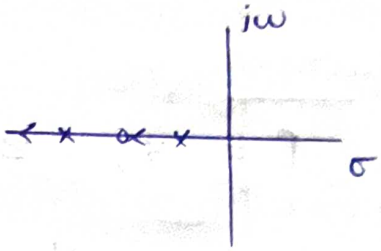


For this system, the root locus is

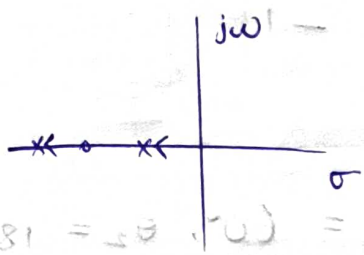
A)



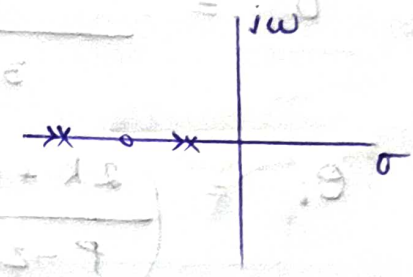
B)



C)

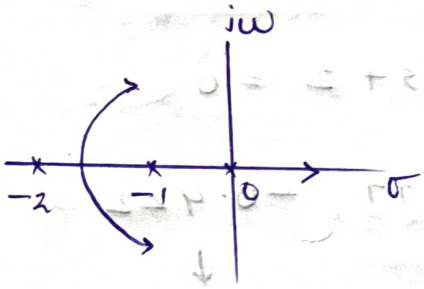


D)

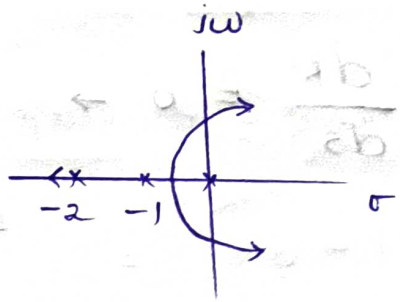


→ Question 7: The O.L.T.F of a system is given by $G(s)H(s) = \frac{k}{s(s+1)(s+2)}$. The RL plot of above system is

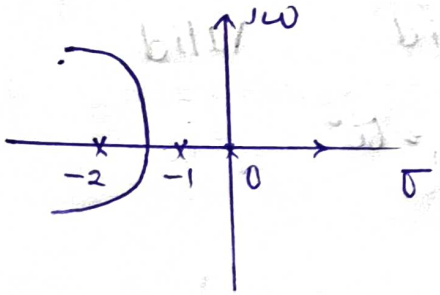
A)



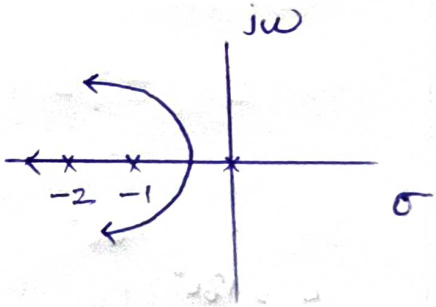
B)



C)



D)



Sol^n:

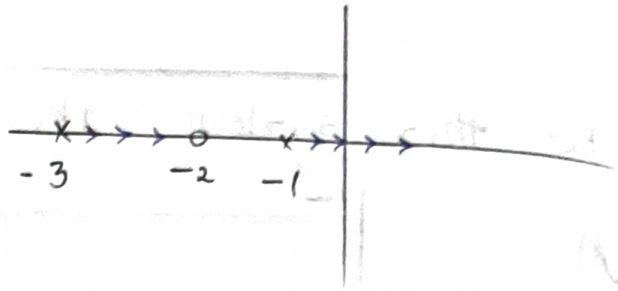
-1 term will make gain negative, so that it will form complementary Root Locus.

$P = 2 \quad Z = 1$

$P - Z = 1$

$$\theta_1 = \frac{2k}{P-Z}(180) = 0^\circ$$

No Break Point



Soln. $P = 3, \quad Z = 0$

$$\sigma = \frac{-1-2-0-0}{3-0} = -1$$

$$\theta_i = \left(\frac{2k+1}{P-Z}\right)180^\circ \rightarrow \theta_1 = 60^\circ, \ \theta_2 = 180^\circ, \ \theta_3 = 300^\circ.$$

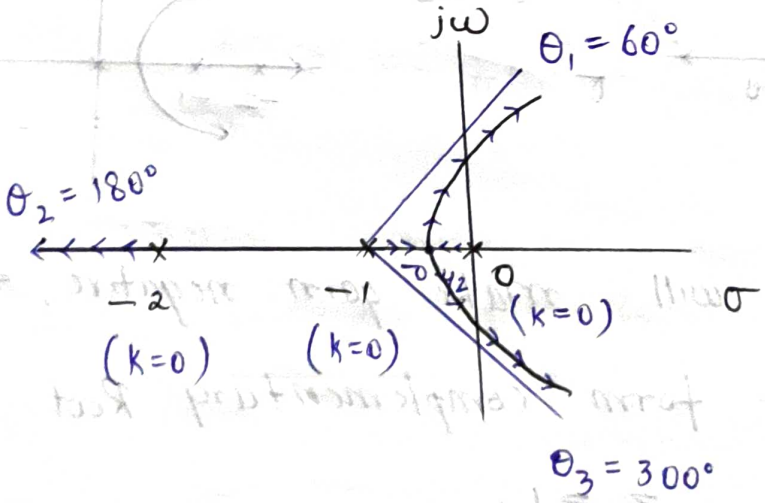
$i = 1, 2, 3, \ldots \qquad k = 0, 1, 2, \ldots$

$$K = -s(s+1)(s+2) = -(s^3 + 3s^2 + 2s)$$

$$\frac{dk}{ds} = 0 \rightarrow 3s^2 + 6s + 2 = 0$$

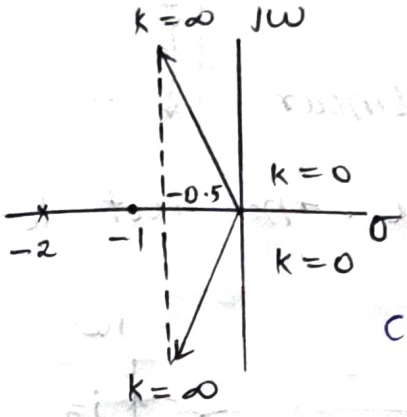
$$s = -1.577, \ -0.422$$

↓ Invalid ↓ Valid

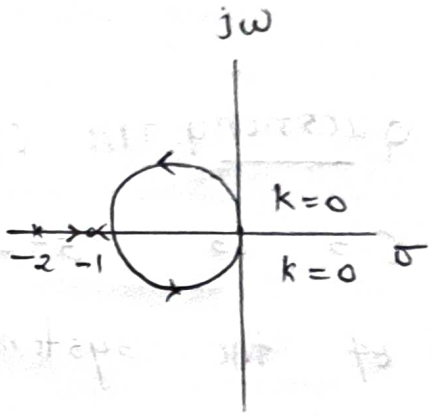


Question8: For the C.E $s^3 + 2s^2 + ks + k = 0$, the root locus of the system as k varies from 0 to ∞ is

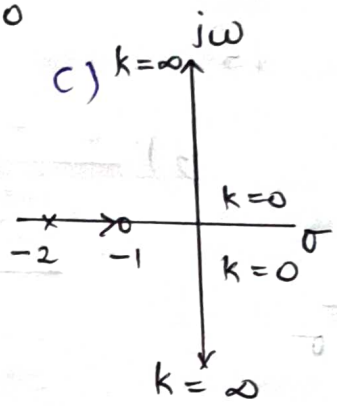
A)



B)



C)



D) None of these

$$1 + GH = C.E = 1 + \frac{k(s+1)}{s^2(s+2)} = 0$$

$P = 3, \quad Z = 1$

$P - Z = 2 \rightarrow \theta_1 = 90°, \quad \theta_2 = 270°$

$\sigma = -0.5$

B.A.P:

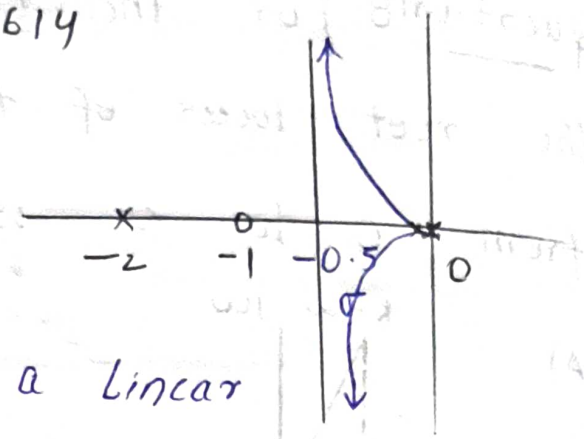
$$k = \frac{-s^2(s+2)}{s+1}$$

$$\frac{dk}{ds} = -\frac{[(s+1)(3s^2+4s) - (1)(s^3+2s^2)]}{(s+1)^2} = 0$$

$\rightarrow 3s^3 + 4s^2 + 3s^2 + 4s - s^3 - 2s^2 = 0$

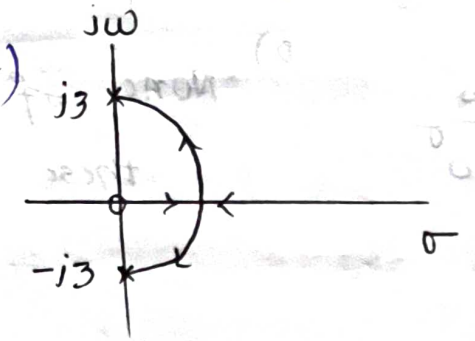
$\rightarrow 2s^3 + 5s^2 + 4s = 0$

$s = 0, \quad -1.25 \pm j0.6614$

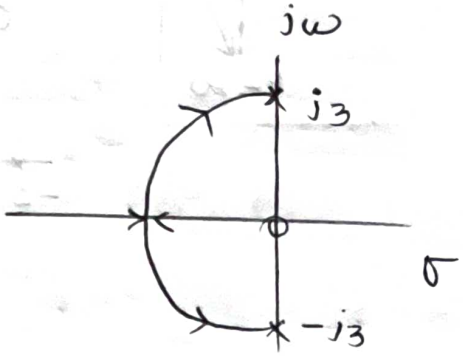


Question 9: The C.E of a Linear C.S is $s^2 + 5ks + 9 = 0$. The root loci of the system is

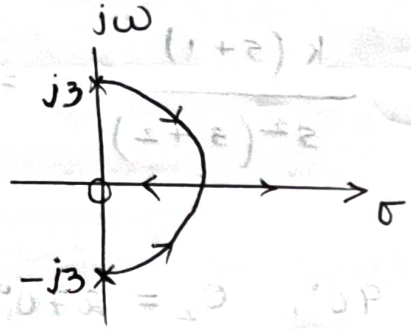
A)



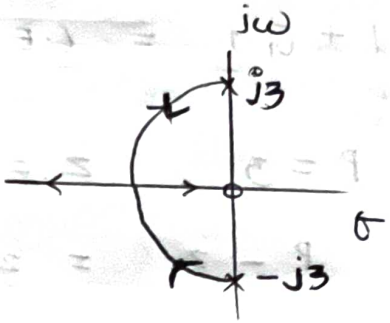
B)



C)



D)



$$1 + GH = 1 + \frac{5ks}{s^2+9}$$

$P = 2, \quad Z = 1 \rightarrow \theta_1 = 180°$

$\theta_D = 180°$

BAP/ BIP:

$$k = \frac{-(s^2+9)}{5s}$$

$$\frac{dk}{ds} = 0 \rightarrow 5s(2s) - 5(s^2+9) = 0$$

$\sigma s^2 - 45 = 0$

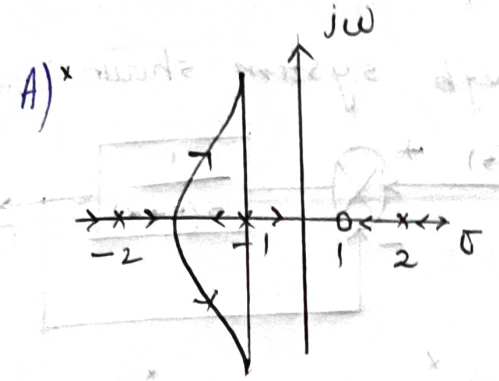
$s = -3, 3$

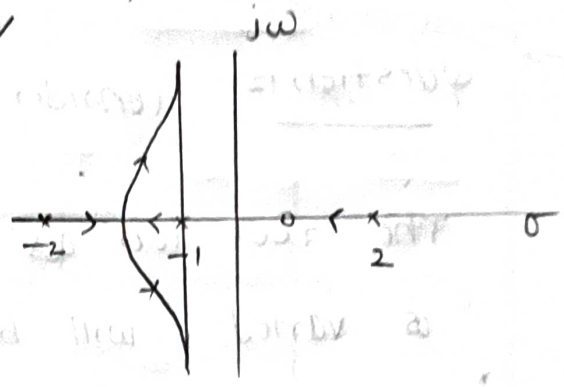
↓ Valid ↓ Invalid

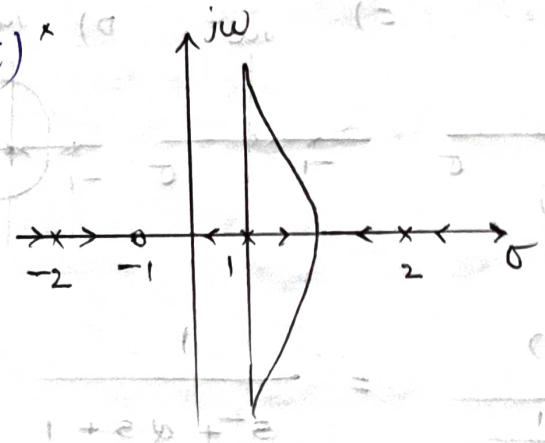
$s = -3$ ($k =$) +Ve

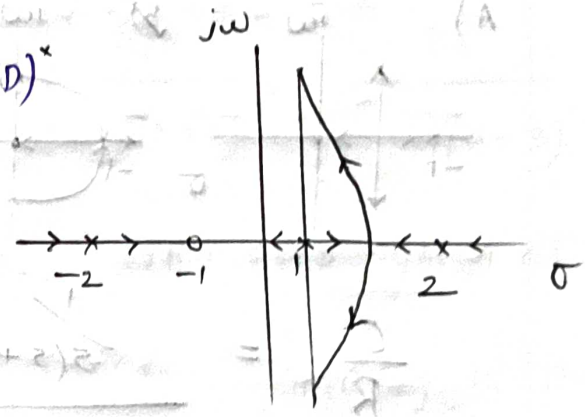
$s = 3$ ($k =$) -Ve

→ Question 10: A CLS has the C.E $(s^2-4)(s+1) + K(s-1) = 0$. Its root locus plot against

A) 

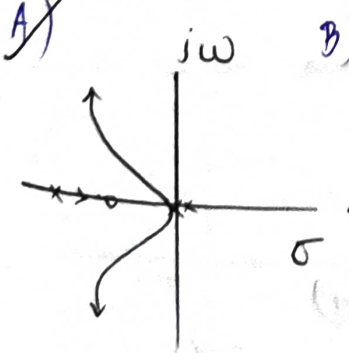
B) 

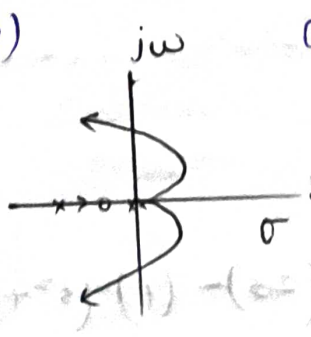
C) 

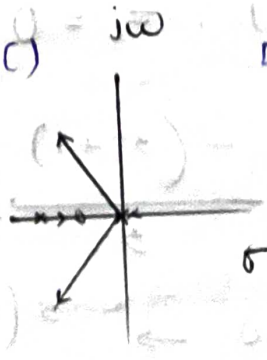
D) 

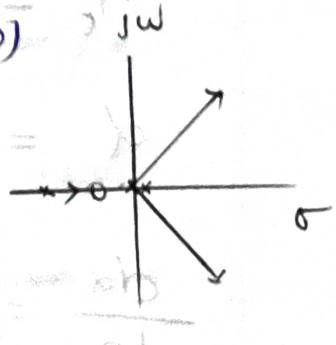
→ Question 11: A ufb system has OLTF,

$G(s) = \dfrac{K(s+\alpha)}{s^2(s+\beta)}$, $\beta > \alpha > 0$. The valid root locus for this system is

A) 

B) 

C) 

D) 

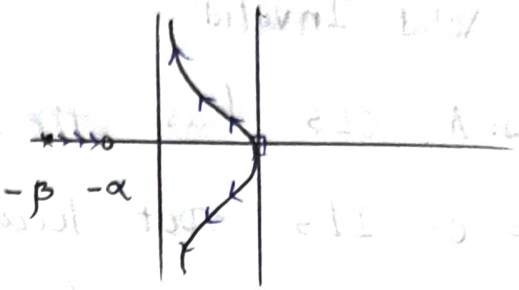
Sol?

$\beta > \alpha > 0, \quad P = 3, \quad Z = 1$

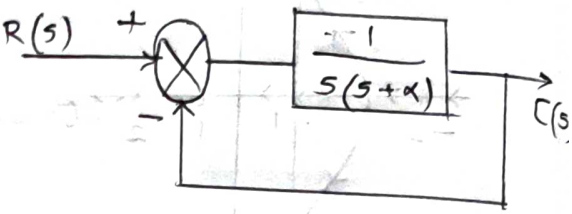
$$\sigma = \frac{-\beta - 0 - 0 + \alpha}{2} = \frac{\alpha - \beta}{2} < 0$$

$\theta_1 = 90°$

$\theta_2 = 270°$

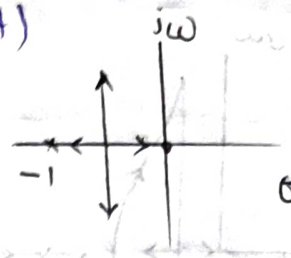


→ Question 12: Consider the ufb system shown below

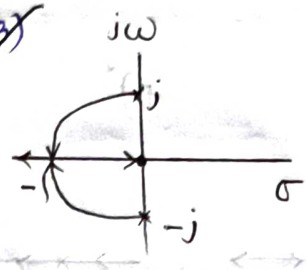


The root loci as α is varied will be

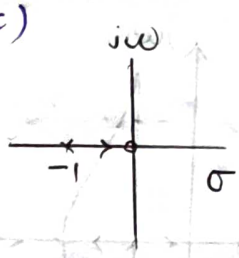
A)



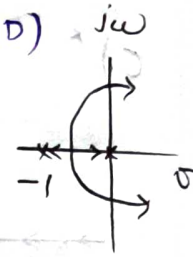
B)



C)



D)



$$\frac{C}{R} = \frac{\frac{1}{s(s+\alpha)}}{1 + \frac{1}{s(s+\alpha)}} = \frac{1}{s^2 + \alpha s + 1}$$

$$1 + GH = 1 + \frac{\alpha s}{s^2 + 1}$$

O.L: $P = 2, \quad Z = 1 \rightarrow \theta = 180°$

$P_c = \pm j, \quad Z_c = 0$

$$\alpha = \frac{-(s^2+1)}{s}$$

$$\frac{d\alpha}{ds} = 0 \rightarrow \frac{s(2s) - (1)(s^2+1)}{s^2} = 0$$

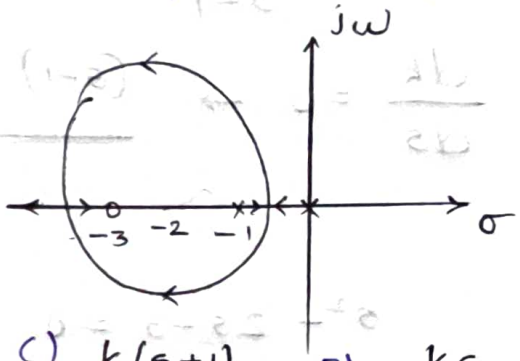
$s^2 - 1 = 0$

$s = \pm 1$ $\qquad \theta_D = 180°$

$s = +1$, $s = -1$

Invalid (α = -ve) $\qquad$ Valid (α = +ve)

→ Question 13: The root-locus of a ufb system is shown below

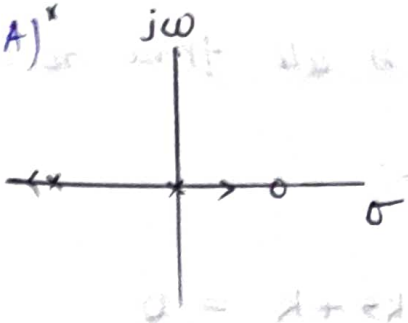


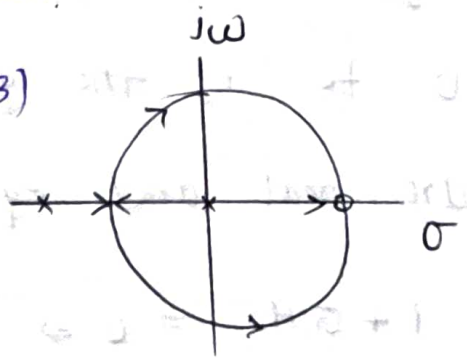
The O.L.T.F is

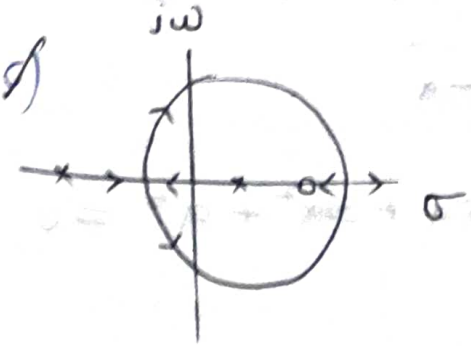
A) $\frac{K}{s(s+1)(s+3)}$ $\qquad$ B) $\frac{k(s+3)}{s(s+1)}$ $\qquad$ C) $\frac{k(s+1)}{s(s+3)}$ $\qquad$ D) $\frac{ks}{(s+1)(s+3)}$

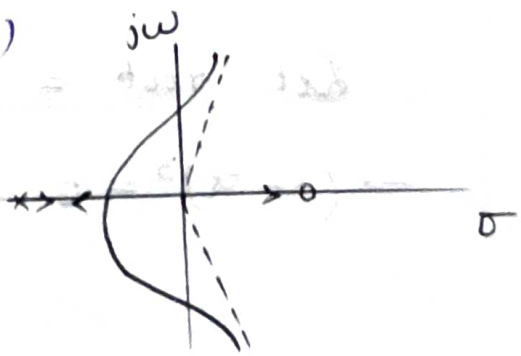
→ Question: A unity feedback system is given as

$G(s) = \frac{k(1-s)}{s(s+3)}$. Which of the following is the correct root locus diagram?

A) 

B) 

C) 

D) 

sol?

$$G(s)H(s) = \frac{-k(s-1)}{s(s+3)}$$

→ −ve k

→ Complementary Root Locus

A-O-A

$P_c = 0, -3, \quad Z_c = 1, \quad P - Z = 1 \rightarrow \theta = 180°$

$$K = \frac{s(s+3)}{s-1}$$

$$\frac{dk}{ds} = 0 \rightarrow \frac{(s-1)(2s+3) - (1)(s^2+3s)}{(s-1)^2} = 0$$

$$s^2 - 2s - 3 = 0 \rightarrow (s+1)(s-3) = 0$$

→ $s = -1, 3$ → Both valid BAP/BIP

→ Question15: The OLTF of a afb system is $G(s)H(s) = \frac{k(s+1)}{s^2(s+9)}$. In the root locus of the system, as parameter k is varied from 0 to ∞, the gain k when all three roots are real and equal is —

$$1 + GH = 0 \Rightarrow s^3 + 9s^2 + ks + k = 0$$

Let root $= -\alpha, -\alpha, -\alpha$

$$\rightarrow (s+\alpha)^3 = 0 \rightarrow s^3 + 3s^2\alpha + 3s\alpha^2 + \alpha^3 = 0$$

On comparing, $\alpha^3 = k$

$3\alpha = 9 \rightarrow \alpha = 3$

$k = 3^3 = 27$

NYQUIST

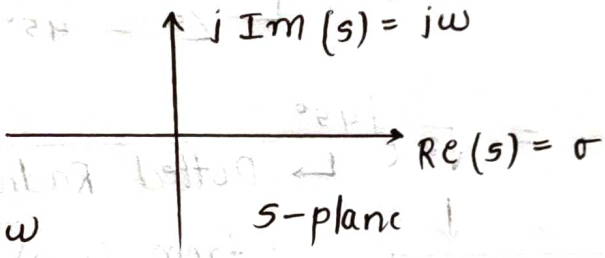
- Nyquist stability Criteria → 70-75%
- Nyquist Plot → 5%
- Gain Margin, Phase margin → 20%

Representation of Complex plane

1. Rectangular Complex plane
2. Polar complex plane

1. Rectangular Complex plane:

A. PLANE:

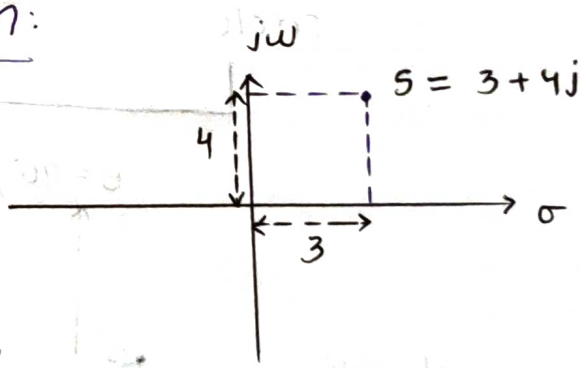


$s = \sigma + j\omega$

B. Point Representation:

$s = 3 + 4j$

$\equiv P(3, 4)$



Rectangular to Polar Conversion:

$s = a + jb$: Rectangular

$|s| = \sqrt{a^2 + b^2}$

$\angle s = \tan^{-1} \frac{b}{a}$

$$S = |S| e^{j\angle S} \quad : \text{POLAR}$$

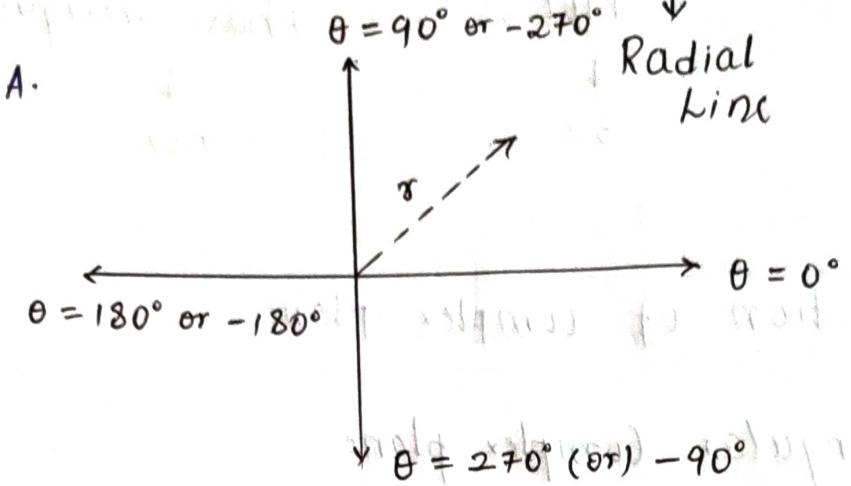
2. Polar Complex Plane:

Parameters

Radial Line

Angle

A.



B. Point Representation:

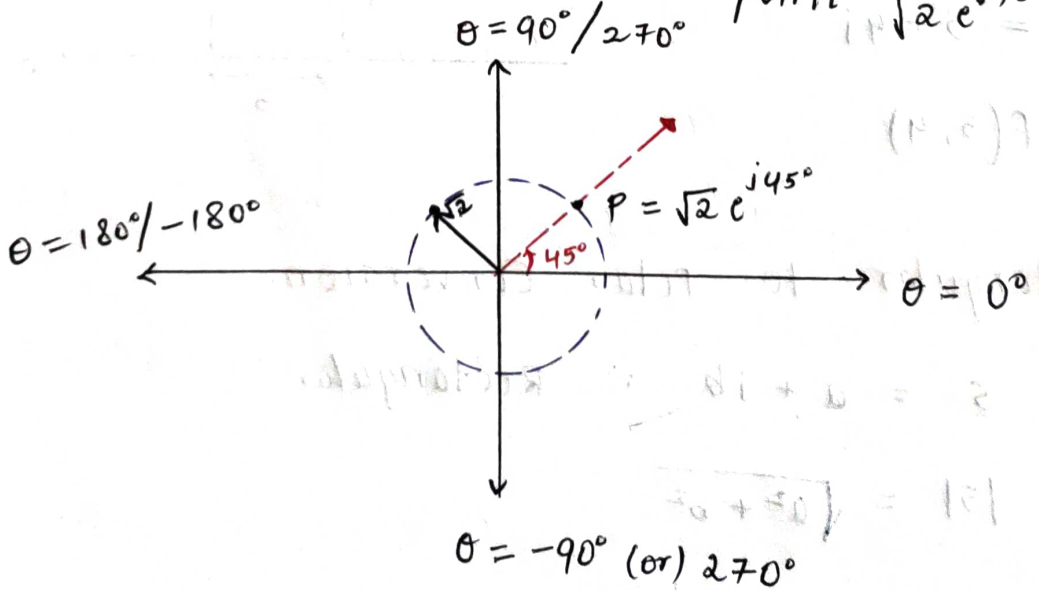
$S = 1 + j$ → $|S| = \sqrt{2}$, $\angle S = 45°$

$= \sqrt{2}\, e^{j45°}$

↳ Dotted Radial Line from (0, 0)

↓ Dotted circle

→ Intersection will define point $\sqrt{2}\, e^{j45°}$

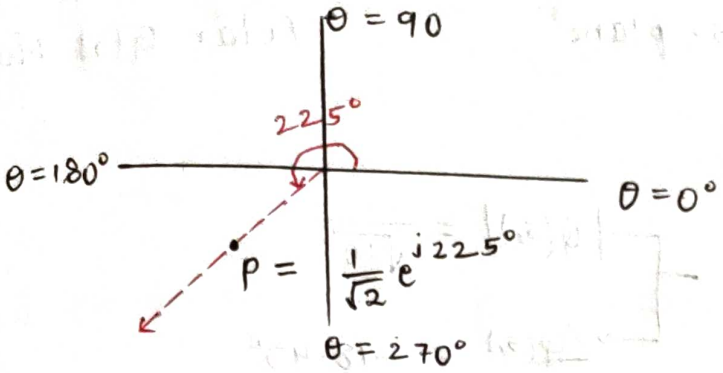


Q. $s = \frac{1}{-1+j}$ show in Complex Polar Plane.

$$s = \frac{1}{-1+j}$$

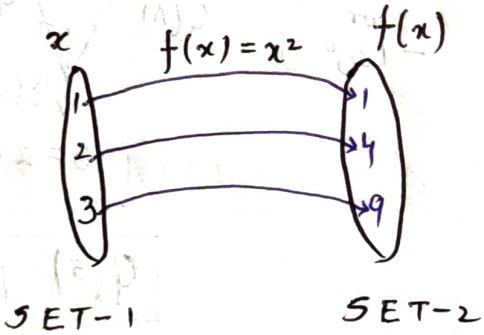
$$|s| = \frac{1}{\sqrt{2}}$$

$$\angle s = -135^\circ \,/\, 225^\circ$$



Function: Function performs mapping from one set to another set based on some predefined condition.

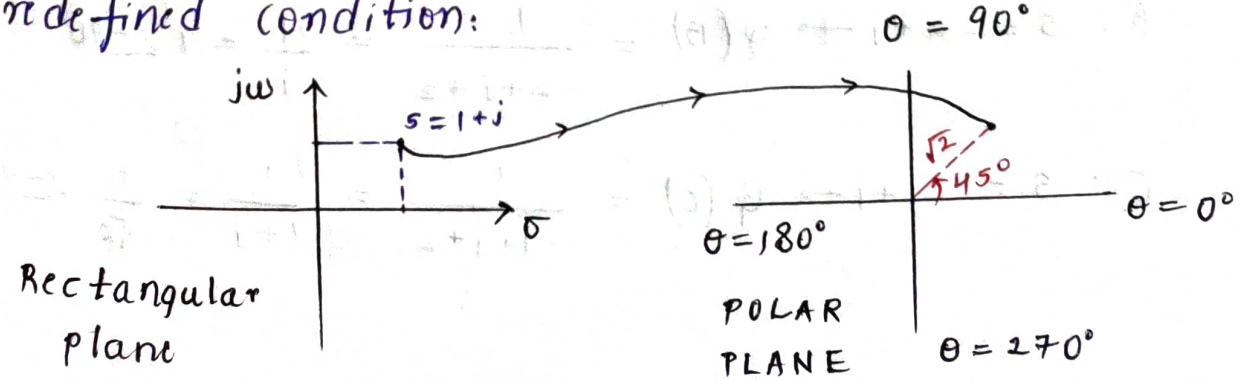
Ex: $f(x) = x^2$

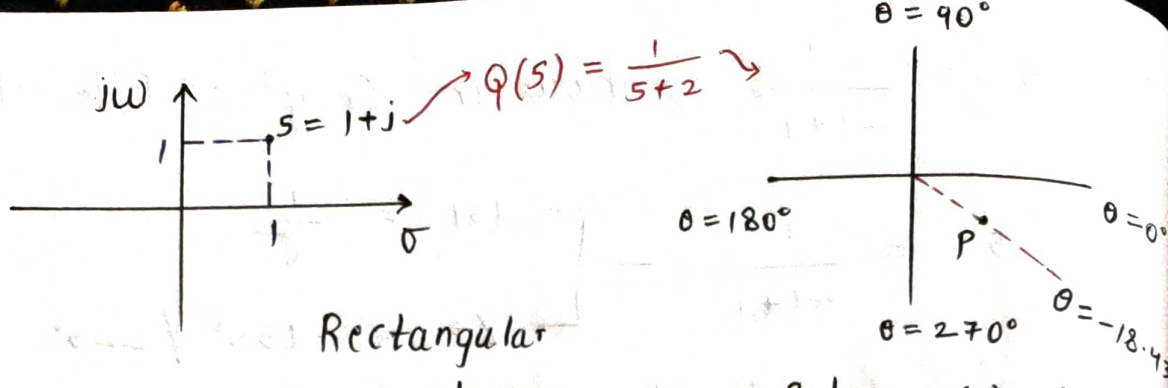


→ This is called as functional mapping.

→ As fn changes, mapping changes.

→ Mapping of point in Rectangular Complex Plane into Polar Complex Plane Under Some Predefined condition:





Rectangular s-plane

Polar Q(s) plane

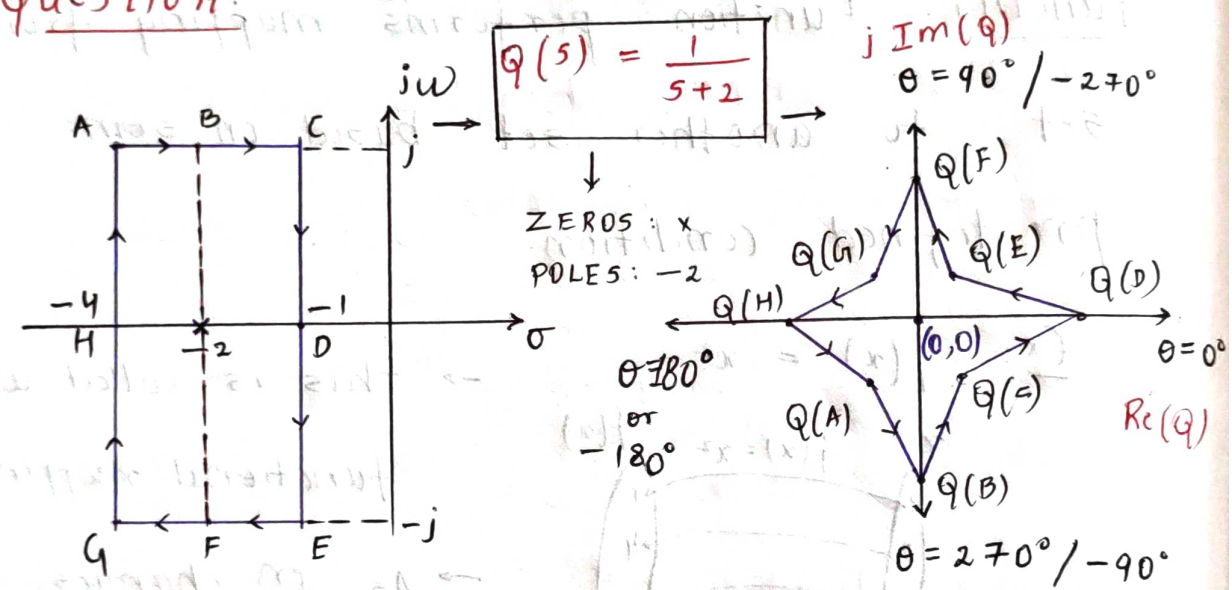
$s_1 = 1 + j$

$$Q(s) = \frac{1}{s+2}$$

$$Q(s_1) = \frac{1}{3+j} \rightarrow |Q(s_1)| = \frac{1}{\sqrt{10}}$$

$$\angle Q(s_1) = -18.43°$$

→ **Question:**



S PLANE CONTOUR
(Closed path in s-plane)

Q(s) plane

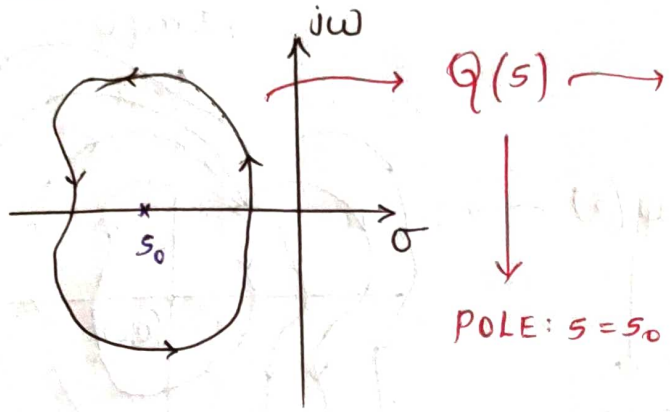
$$A: \; s = -4 + j \rightarrow Q(A) = \frac{1}{-4+j+2} = \frac{1}{-2+j} = 0.4\angle -153°$$

$$B: \; s = -2 + j \rightarrow Q(B) = \frac{1}{-2+j+2} = \frac{1}{j} = 1\angle -90°$$

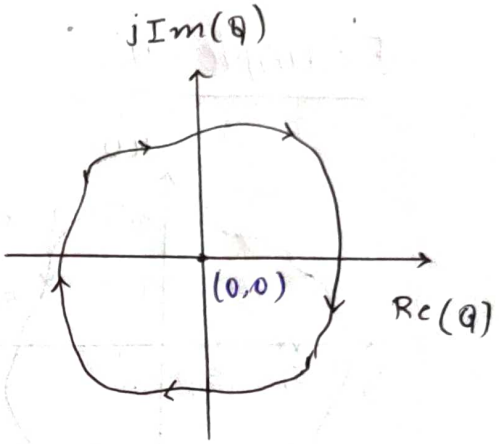
$$C: \; s = -1 + j \rightarrow Q(C) = \frac{1}{-1+j+2} = \frac{1}{1+j} = \frac{1}{\sqrt{2}}\angle -45°$$

Point	s	$Q(s) = \frac{1}{s+2}$
D	$-1+j0$	$1 \angle 0°$
E	$-1-j$	$0.7 \angle 45°$
F	$-2-j$	$1 \angle 90°$
G	$-4-j$	$0.4 \angle 153°$
H	$-4-j0$	$0.5 \angle 180°$

Example 1:

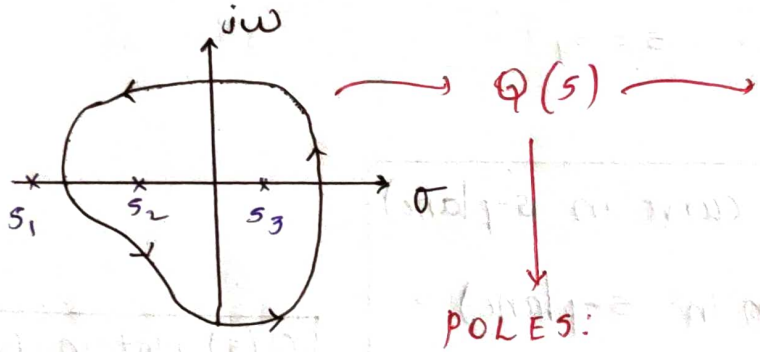


S-PLANE

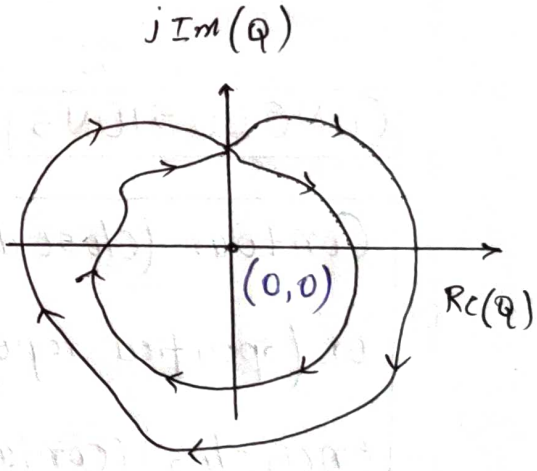


Q(s) PLANE

Example -2:

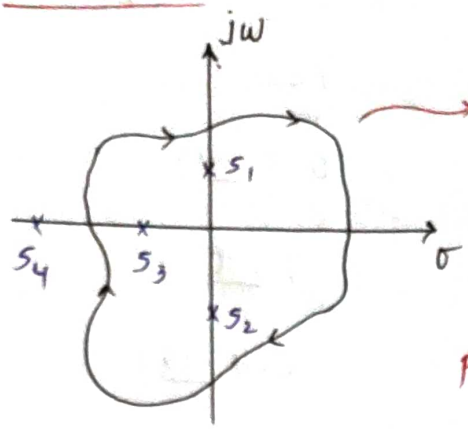


S-PLANE



Q(s) plane

Example : 3



S-PLANE

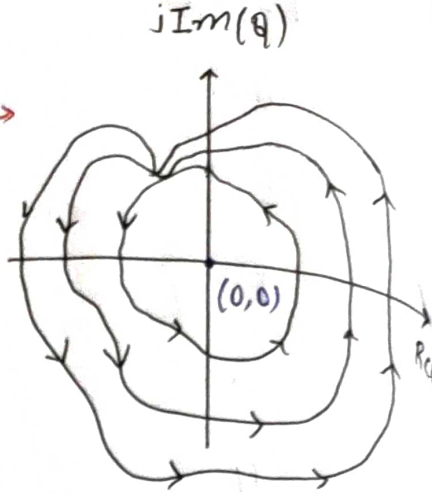
$\rightarrow Q(s) \rightarrow$

POLES:

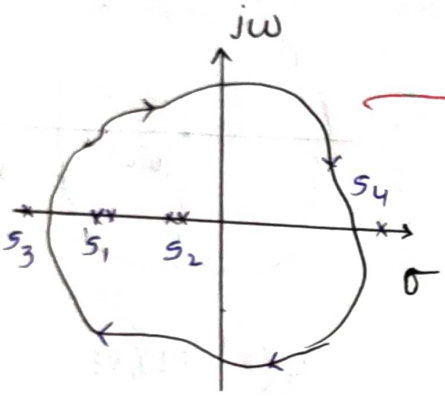
$s = s_1$ $s = s_4$

$s = s_2$

$s = s_3$



Example -4 :



S-PLANE

$\rightarrow Q(s) \rightarrow$

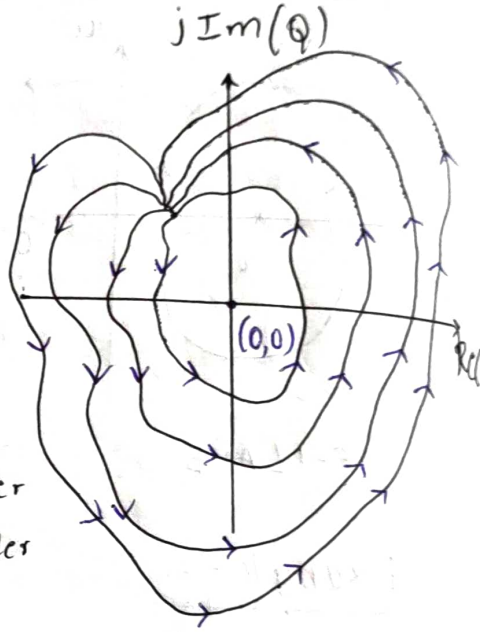
POLES:

$s = s_1$: II order

$s = s_2$: II order

$s = s_3$

$s = s_4$



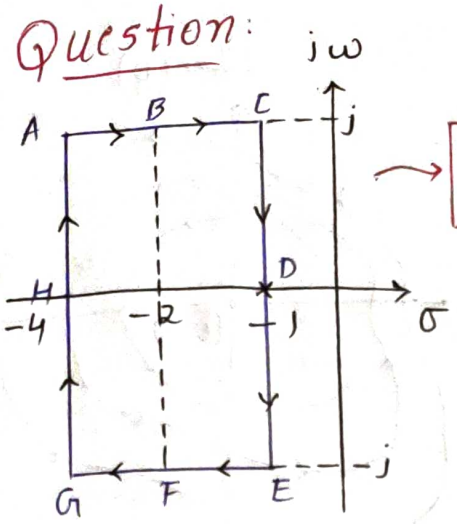
CONCLUSIONS

Contour (closed curve in s-plane) or (specified region in s-plane) encircles (contains) 'm' poles of Q(s) strictly inside it

Q(s) plot in Q(s) plane encircles origin (0,0) 'm' times

Opposite Direction

Question:

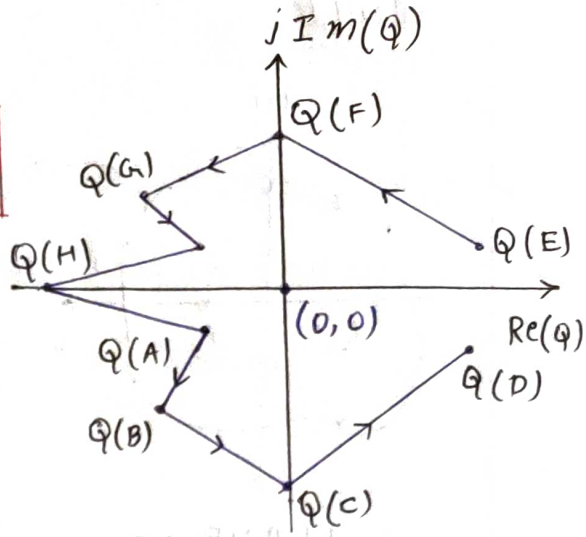


S-PLANE

$$Q(s) = \frac{1}{s+1}$$

Zeros: x

Poles: $s = -1$

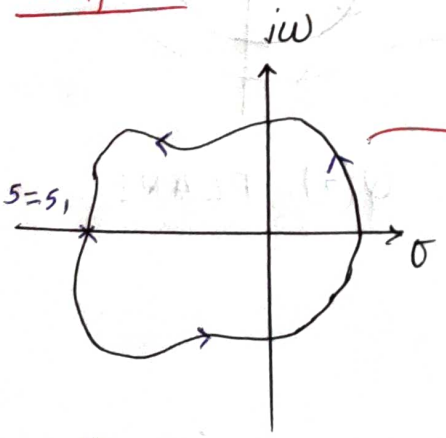


Q(s) PLANE

→ D: $s = -1$: $Q(s) = \frac{1}{s+1} \rightarrow Q(D) = \frac{1}{0} = \infty \angle 0^\circ$

Point	s	$Q(s) = \frac{1}{s+1}$
A	$-4+j$	$0.31 \angle -161.5^\circ$
B	$-2+j$	$0.707 \angle -135^\circ$
C	$-1+j$	$1 \angle -90^\circ$
D	-1	$\infty \angle 0^\circ$
E	$-1-j$	$1 \angle 90^\circ$
F	$-2-j$	$0.707 \angle 135^\circ$
G	$-4-j$	$0.31 \angle 161.5^\circ$
H	-4	$\frac{1}{3} \angle 180^\circ$

Example 1:

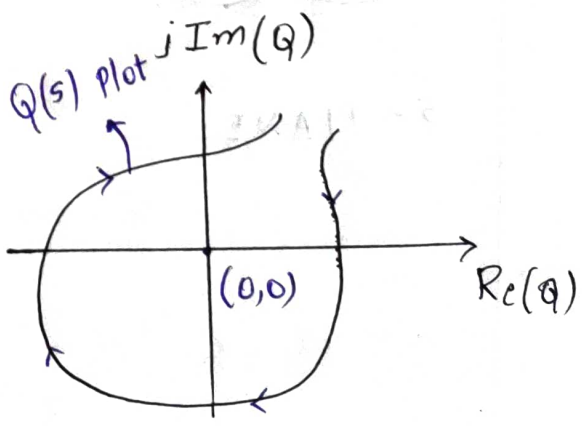


S-PLANE

$Q(s)$

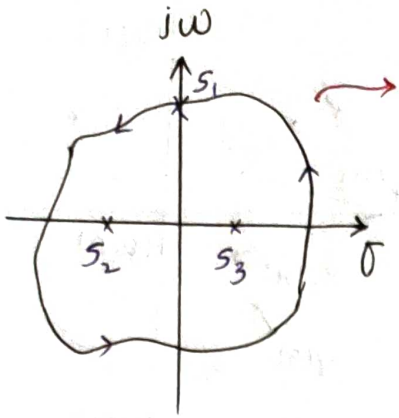
POLE S:

$s = s_1$



Q(s) PLANE

Example -2 :



S - PLANE

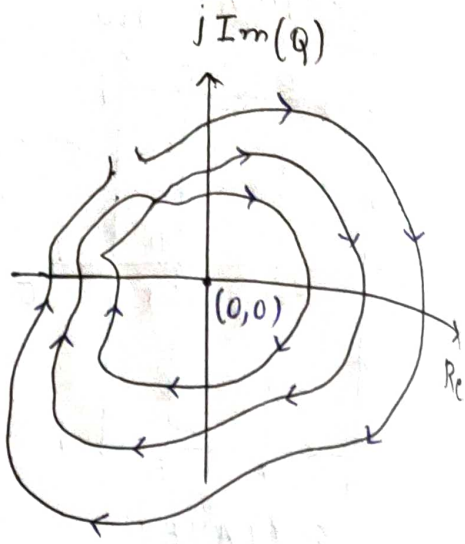
$\rightarrow Q(s) \rightarrow$

POLES:

$s = s_1$

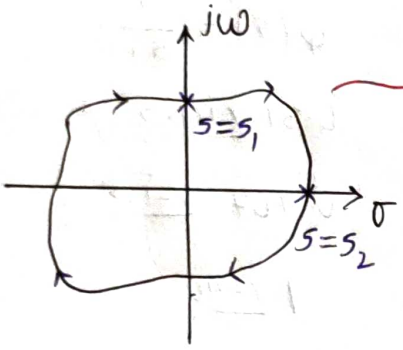
$s = s_2$

$s = s_3$



Q(s) PLANE

Example -3:

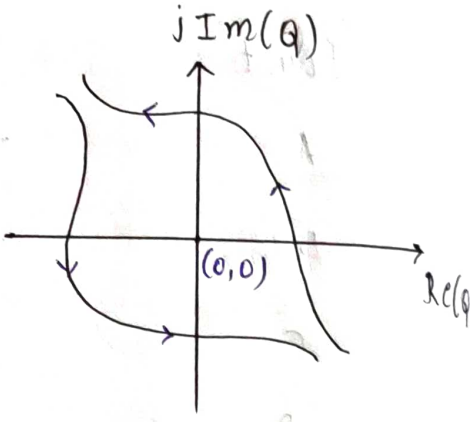


$\rightarrow Q(s) \rightarrow$

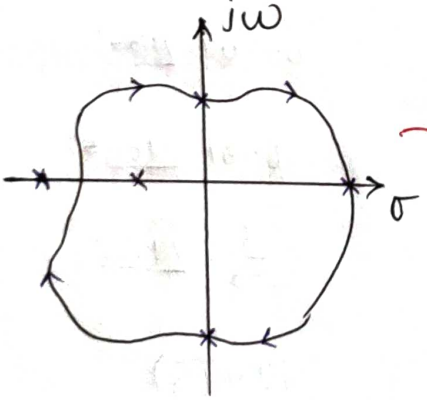
POLES:

$s = s_1$

$s = s_2$

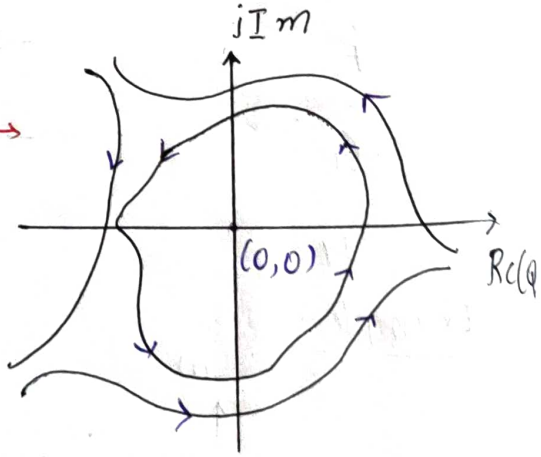


Example -4 :



S - PLANE

$\rightarrow Q(s) \rightarrow$

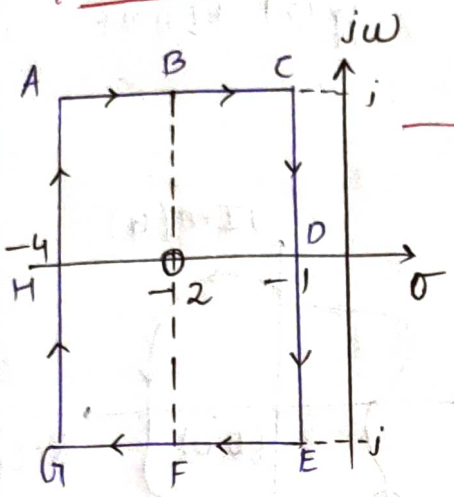


Q(s) PLANE

CONCLUSION

If contour in s-plane passes through one or more poles of Q(s) → Q(s) plot in Q(s) Plane remains open curve. Hence these poles do not contribute in encirclement of origin

Question:

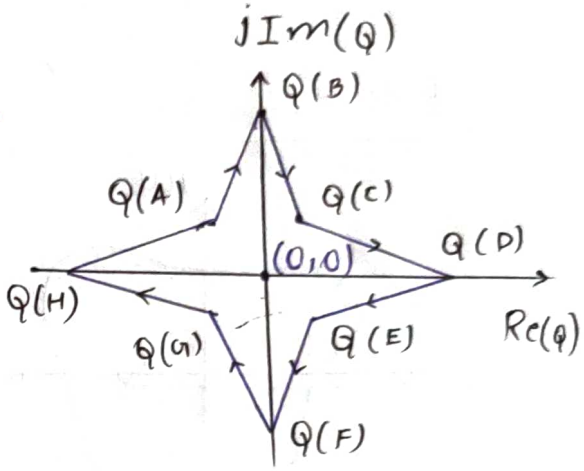


S-PLANE

$Q(s) = s + 2$

POLES: x

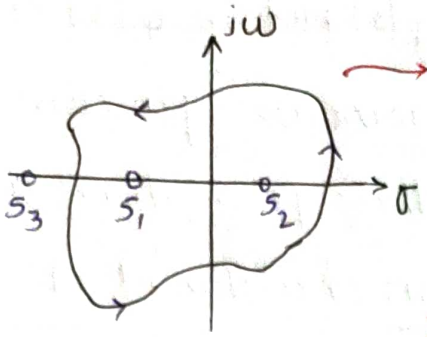
ZEROS: $s = -2$



Q(s) PLANE

Point	s	Q(s) = s + 2
A	$-4+j$	$2.2 \angle 153°$
B	$-2+j$	$1 \angle 90°$
C	$-1+j$	$\sqrt{2} \angle 45°$
D	-1	$1 \angle 0°$
E	$-1-j$	$\sqrt{2} \angle -45°$
F	$-2-j$	$1 \angle -90°$
G	$-4-j$	$2.2 \angle -153°$
H	-4	$2 \angle 180°$

Example -1:



→ $Q(s)$

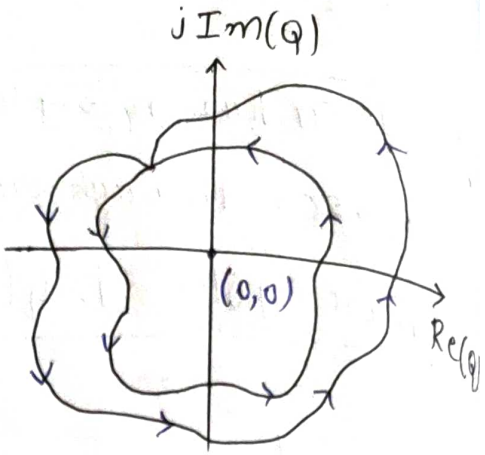
S - PLANE

Zeros:

$s = s_1$

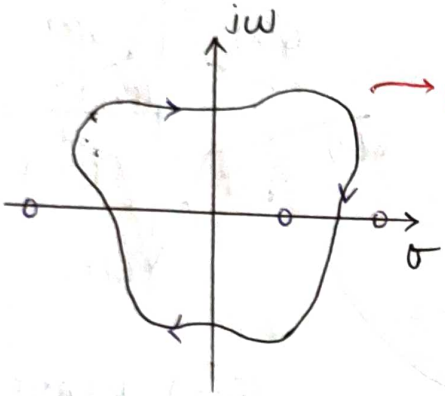
$s = s_2$

$s = s_3$



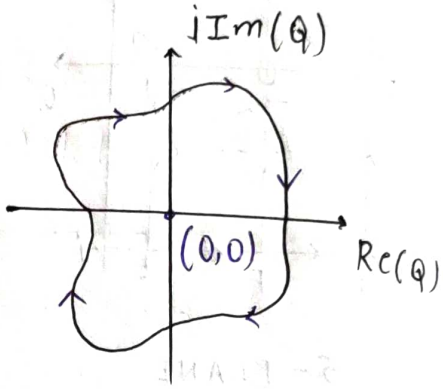
Q(s) PLANE

Example -2:



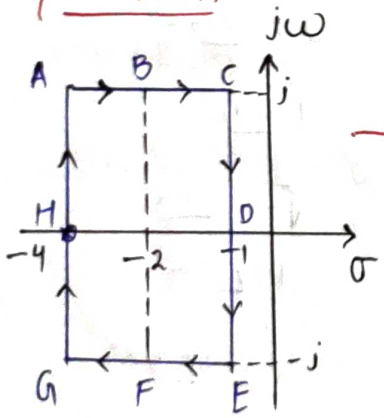
→ $Q(s)$ →

S PLANE



Q(s) PLANE

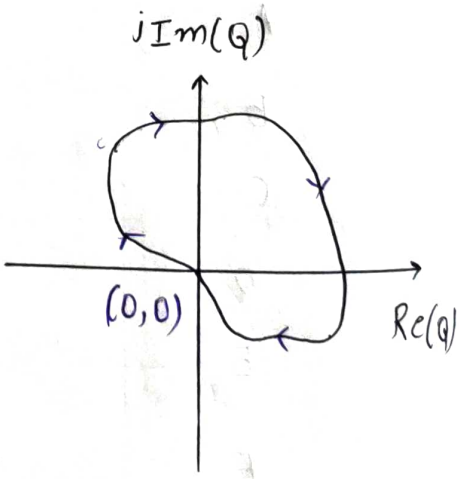
→ ## Question:



→ $Q(s) = s+4$ →

ZEROS: $s=-4$

POLES: x



S PLANE CONTOUR

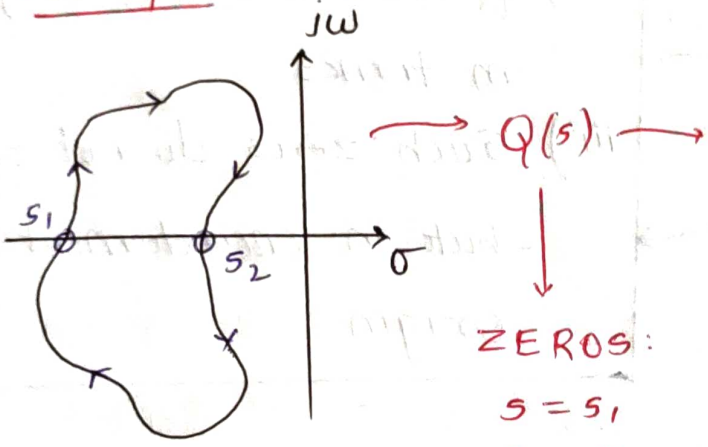
Q(s) PLANE

→ H : $s = -4$ $\quad Q(s) = s+4$

$Q(H) = 0\angle 0°$

point	s	$Q(s) = s+4$
A	$-4+j$	$1 \angle 90°$
B	$-2+j$	$2.23 \angle 26°$
C	$-1+j$	$3.16 \angle 18°$
D	-1	$3 \angle 0°$
E	$-1-j$	$3.16 \angle -18°$
F	$-2-j$	$2.23 \angle -26°$
G	$-4-j$	$1 \angle -90°$
H	-4	$0 \angle 0°$ → origin

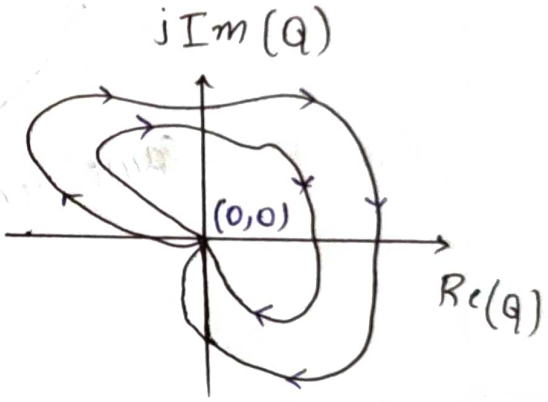
Example:



ZEROS:

$s = s_1$

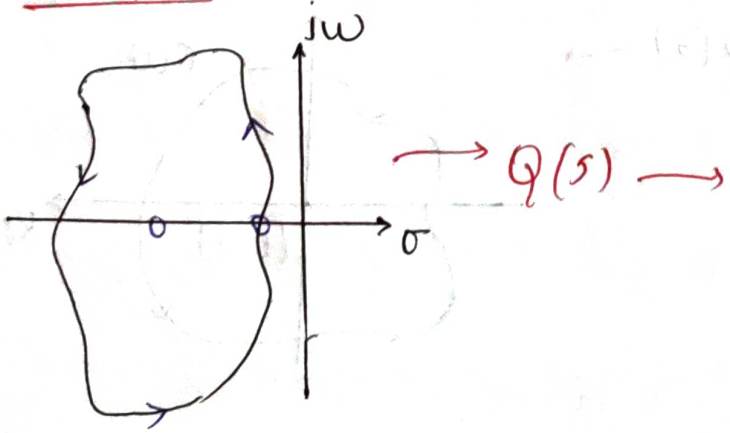
$s = s_2$

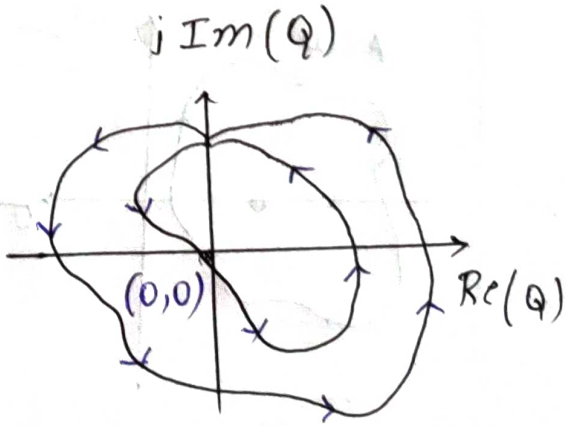


S PLANE

Q(S) PLANE

Example:





S PLANE

Q(S) PLANE

CONCLUSIONS

1. Contour in s-plane encircles 'm' zeros of Q(s) strictly inside it

Q(s) plot in Q(s) plane encircles the origin 'm' times

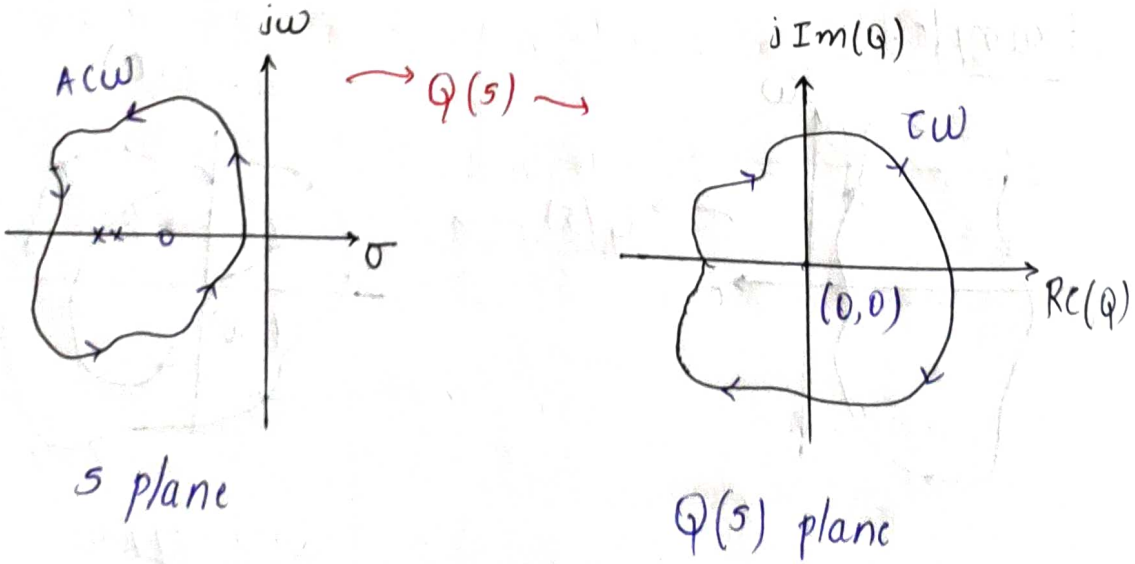
Same direction

2. Contour in s-plane has m zeros on the boundary.

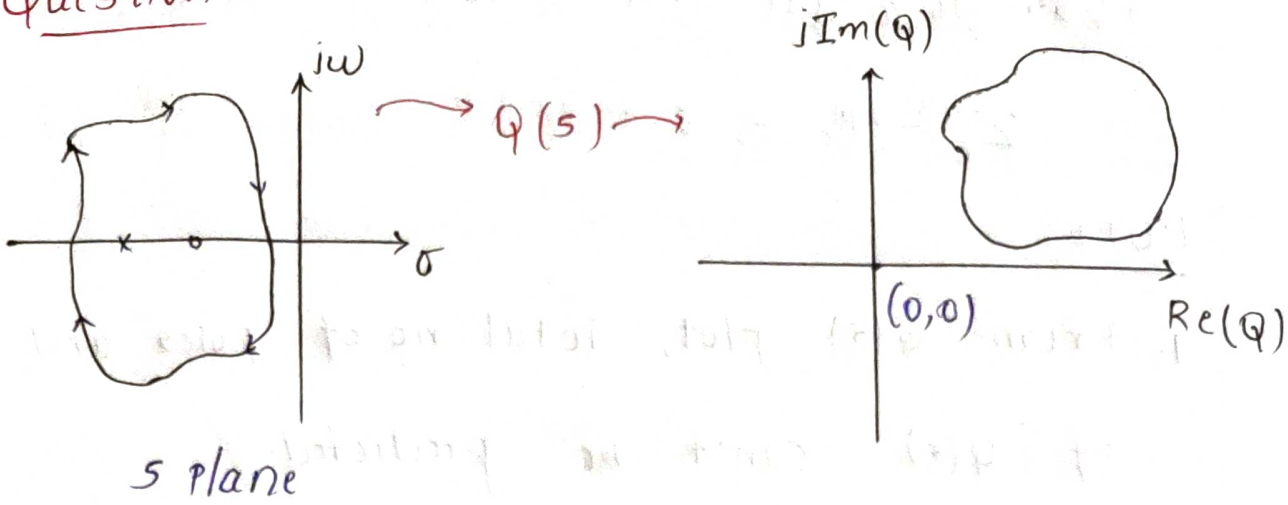
i) Q(s) plot in Q(s) plane is closed.
ii) Q(s) plot crosses origin m times
iii) Such zeros do not contribute in encirclement of origin

Same sense

→ **Question:**

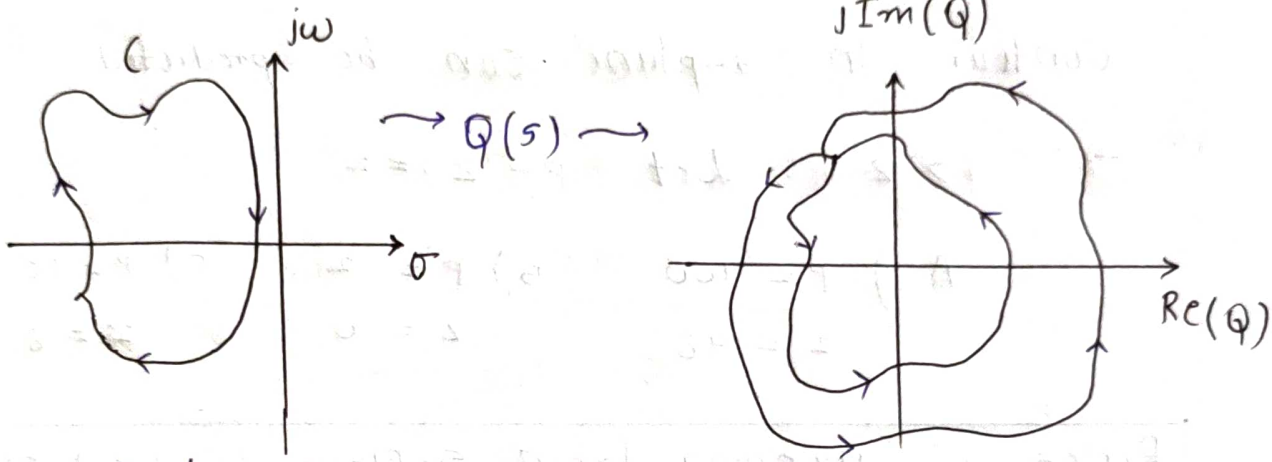


→ Question:



s plane

→ Question:



s-plane

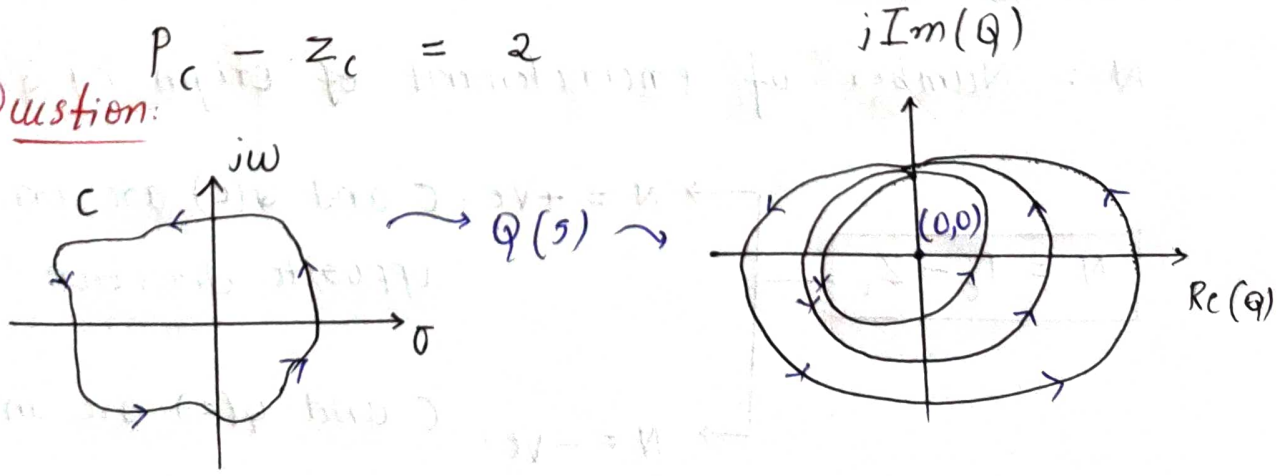
What are the possible pole, zero number present inside contour in s-plane

P_c = No. of poles of Q(s) inside C.

z_c = No. of zeros of Q(s) inside C.

$$P_c - z_c = 2$$

→ Question:



Possible pole and zeros inside C :

$$Z_c - P_c = 3$$

NOTE:

1. From Q(s) plot, total no. of poles and zeros of Q(s) can't be predicted.
2. From Q(s) plot, no. of poles and zeros inside Contour in s-plane can be predicted.

→ $P > Z$: Let $P - Z = 2$

A) $P = 100$, $Z = 98$ B) $P = 2$, $Z = 0$ C) $P = 10$, $Z = 8$ ...

→ **Rules of Mapping from S-Plane to Q(s) Plane**

$$Q(s) = \frac{N(s)}{D(s)}$$

C : Contour in s-plane

P_c : No. of poles of Q(s) present strictly inside C.

Z_c : No. of zeros of Q(s) present strictly inside C.

N : Number of encirclement of origin by Q(s) plot.

$N = P_c - Z_c$

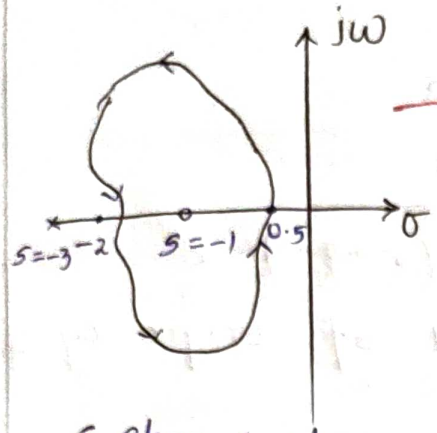
→ N = +ve. C and Q(c) are in opposite direction

→ N = -ve. C and Q(c) are in same direction.

Limitation of Above Rule:

1. If pole of $Q(s)$ lies on Boundary of C.
2. If zero of $Q(s)$ lies on Boundary of C.

Q.

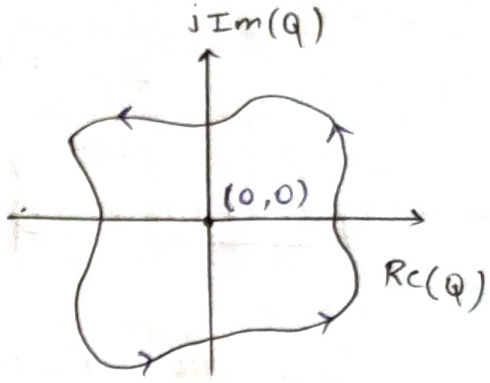


s Plane contour

$$\rightarrow Q(s) = \frac{s+1}{s+3} \rightarrow$$

POLES: $s = -3$

ZEROS: $s = -1$



$Q(s)$ plot in $Q(s)$ plane

Verify Rule of Mapping

$$N = P_c - Z_c$$

L.H.S: $N = -1$

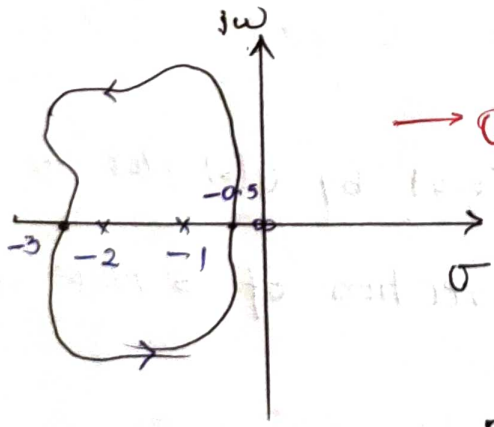
R.H.S: $P_c = 0$

$Z_c = 1$

$P_c - Z_c = 0 - 1 = -1$

LHS = RHS

Q.

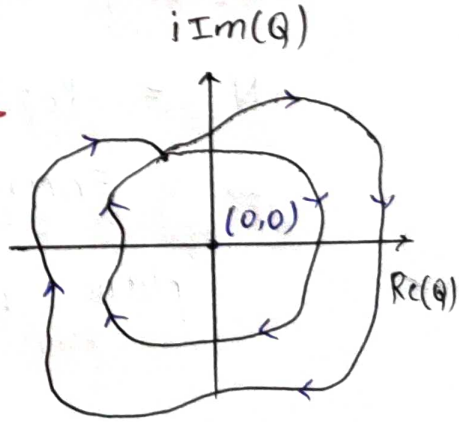


s PLANE

$$\rightarrow Q(s) = \frac{s^2}{(s+1)(s+2)} \rightarrow$$

POLES: $s = -1, s = -2$

ZEROS: $s = 0, 0$



$Q(s)$ plane

Verify Rule of Mapping.

Rule of Mapping: $N = P_c - Z_c$

L·H·S:

~~N ——~~ $N = 2$

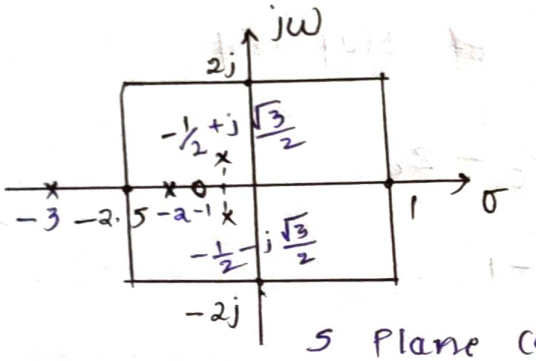
R·H·S:

$P_c = 2$ $\quad P_c - Z_c = 2$

$Z_c = 0$ $\quad L \cdot H \cdot S = R \cdot H \cdot S$

→ Question: $Q(s) = \dfrac{(s+1)}{(s+3)(s+2)(s^2+s+1)}$

Cal the no. of encirclement of origin by Q(s) plot in Q(s) plane for given contour in s plane



s Plane contour

Poles: $s = -3, -2, -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$

Zeros: $s = -1$

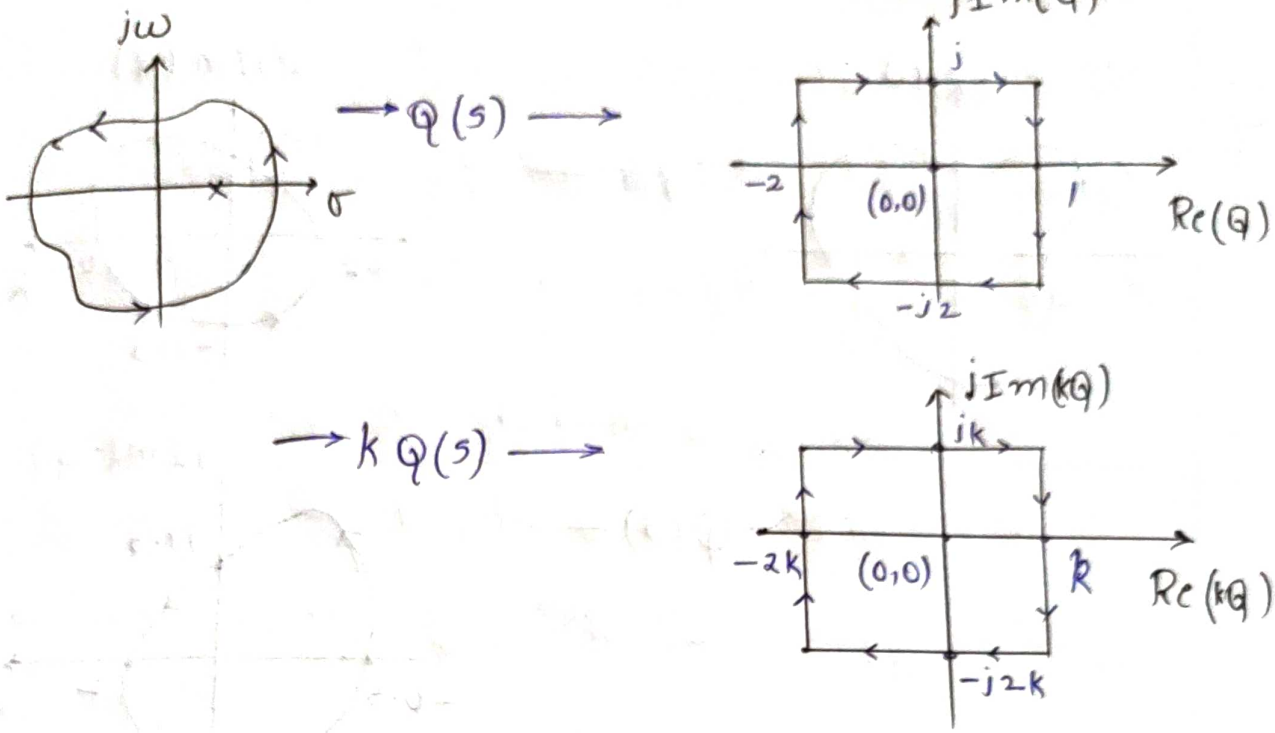
$N = P_c - Z_c = 3 - 1 = 2$

→ 2 Encirclement of (0,0) by Q(s) plot in Q(s) plane in opposite direction of s-plane contour

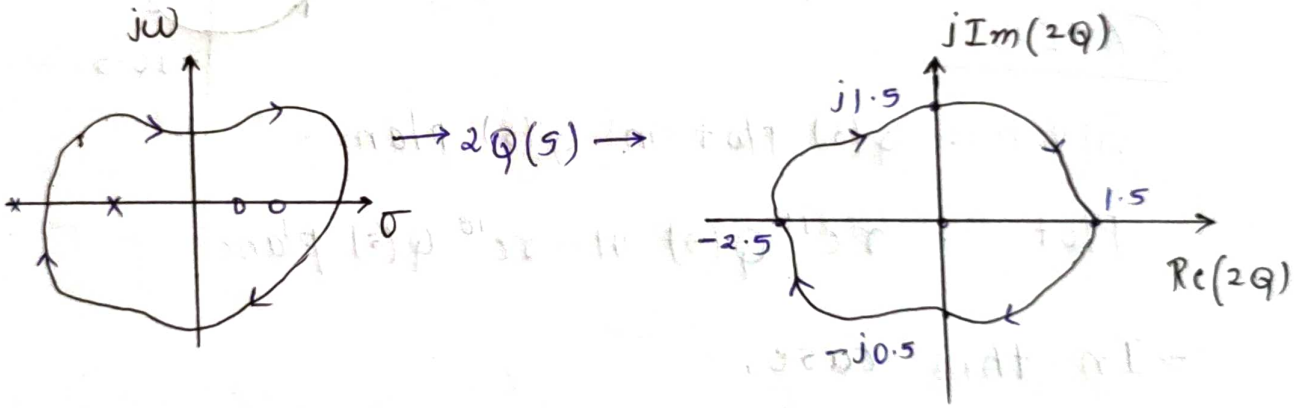
RELATION BETWEEN PLOTS IN Q(S) PLANE

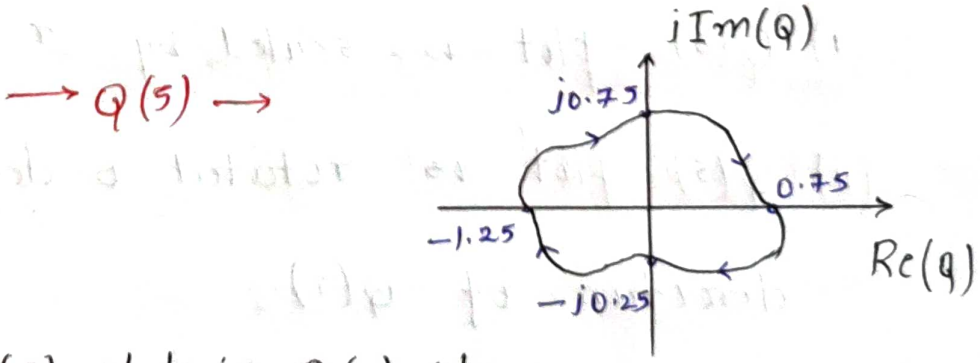
CASE-1: Given: Q(s) plot in Q(s) plane

Plot : kQ(s) plot in kQ(s) plane



Q.





CASE-2:

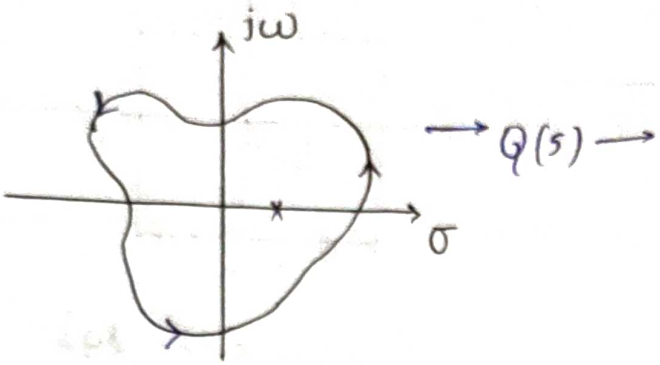
Given: Q(s) plot in Q(s) plane

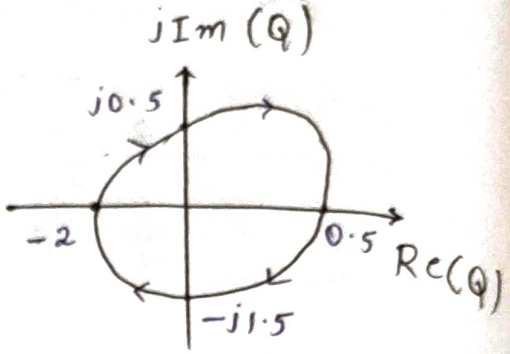
plot : -Q(s) plot in -Q(s) plane

Important point:

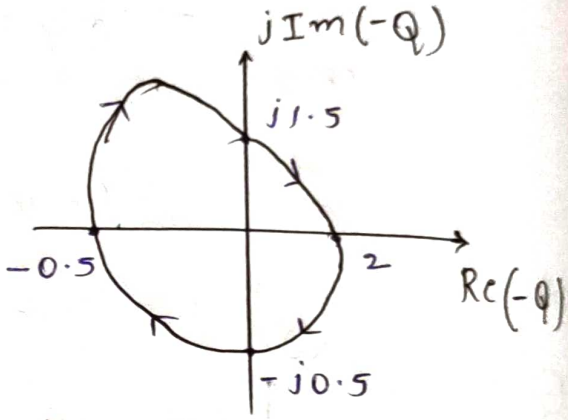
In this case, each and every point of $Q(s)$ is rotated $180°$ in the direction of $Q(s)$.

Q.









CASE-3:

Given : $Q(s)$ plot in $Q(s)$ plane.

Plot : $re^{j\theta}Q(s)$ in $re^{j\theta}Q(s)$ plane.

→ In this case,

(i) $Q(s)$ plot is scaled by r

(ii) $Q(s)$ plot is rotated θ degree in the direction of $Q(s)$.

CASE-4:

Given : $Q(s)$ plot in $Q(s)$ plane.

Plot : $\{Q(s)+1\}$ plot in $\{Q(s)+1\}$ plane.

→ In this case Q(s) plot is shifted 1 unit Rightwards.

CASE-5:

Given: $Q(s)$ plot in $Q(s)$ plane

Plot: $[Q(s)+m]$ plot in $[Q(s)+m]$ plane.

→ shift $Q(s)$ plot 'm' unit rightwards

Plot: $[Q(s)-m]$ plot in $[Q(s)-m]$ plane.

→ Shift $Q(s)$ plot 'm' unit leftwards.

D CASE-6: Given: $Q(s)$ plot in $Q(s)$ plane.

Plot: $[1-Q(s)]$ plot in $[1-Q(s)]$ plane.

S-1: $Q(s) \longrightarrow -Q(s)$

S-2: $-Q(s) \longrightarrow (-Q(s)) + 1$

CASE-7: Given: $Q(s)$ plot in $Q(s)$ plane.

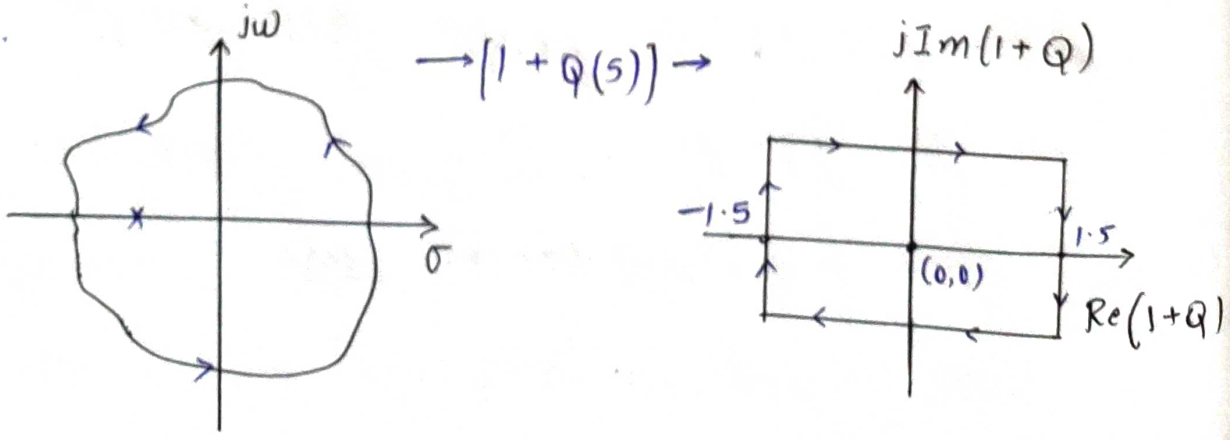
Plot: $A + BQ(s)$ plot in $A + BQ(s)$ plane.

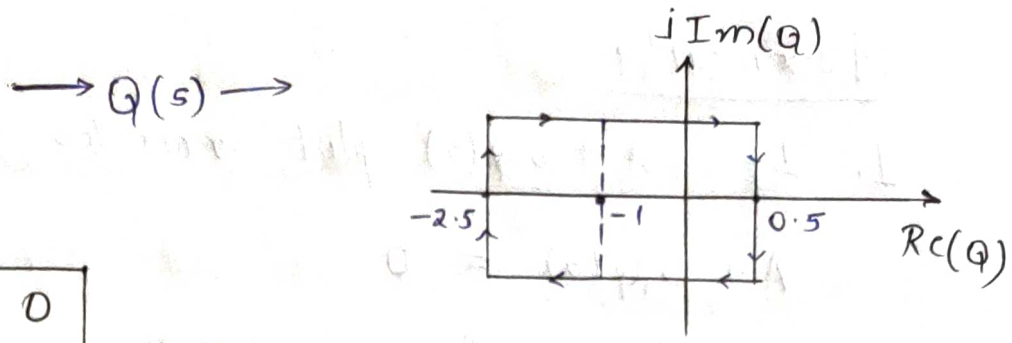
S-1: $Q(s) \longrightarrow BQ(s)$

S-2: $BQ(s) \longrightarrow BQ(s) + A$

Important observations

1.



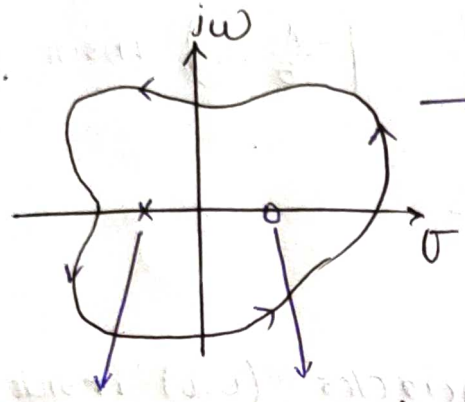


$1 + Q(s) = 0$

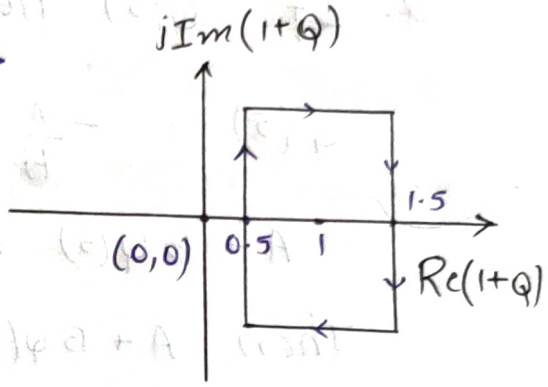
$Q(s) = -1$

→ If $1+q(s)$ encencircles its origin then $Q(s)$ will encircle $(-1, 0)$.

2.



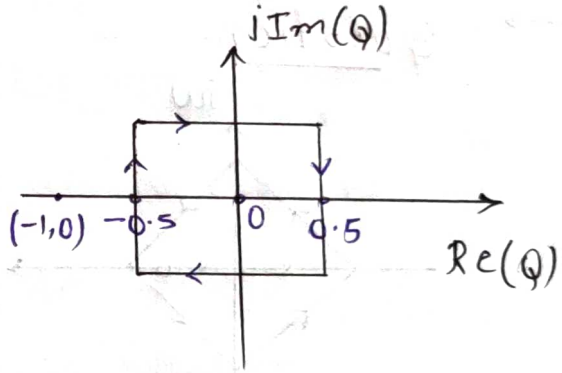
→ $[1+Q(s)]$ →



Pole of $1+Q(s)$

Zero of $1+Q(s)$

→ $Q(s)$ →



$1 + Q(s) = 0$

$Q(s) = -1$

→ If $1+Q(s)$ docsnot encircles its origin, then $Q(s)$ will not encircle $(-1, 0)$.

NOTE

$4 + 3Q(s) = 0$

$Q(s) = \frac{-4}{3}$

→ If $4 + 3Q(s)$ plot encircles origin, then $Q(s)$ plot will encircle $\left(\frac{-4}{3}, 0\right)$.

Statements:

1. If $A + BQ(s)$ plot encircles its origin, then

$$A + BQ(s) = 0$$

$$Q(s) = \frac{-A}{B}$$

$Q(s)$ plot will encircle $\left(\frac{-A}{B}, 0\right)$ in it.

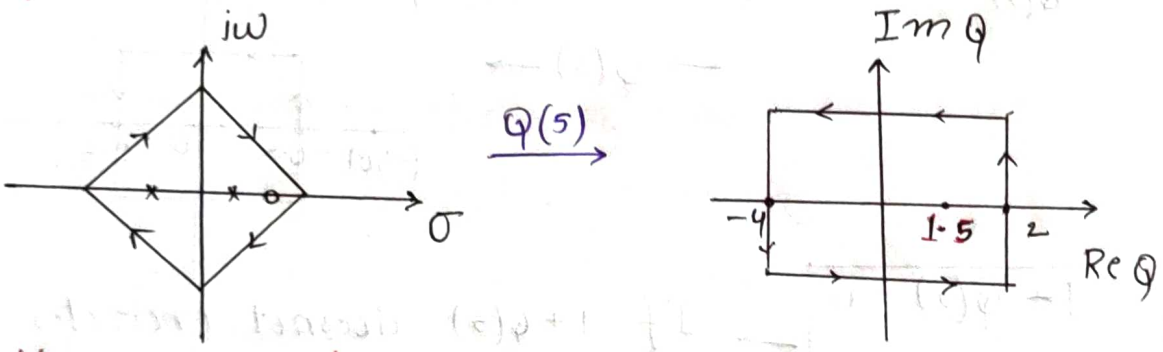
2. If $Q(s)$ plot encircle $\left(\frac{-A}{B}, 0\right)$ inside it.

$$Q(s) = -\frac{A}{B}$$

$$A + BQ(s) = 0$$

Then $A + BQ(s)$ plot encircles $(0,0)$ inside it.

→ **Question:**



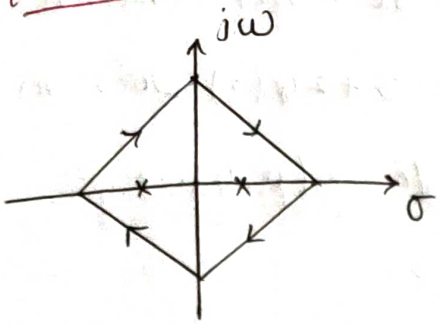
How many times $3 - 2Q(s)$ plot encircles its origin.

$$3 - 2Q(s) = 0$$

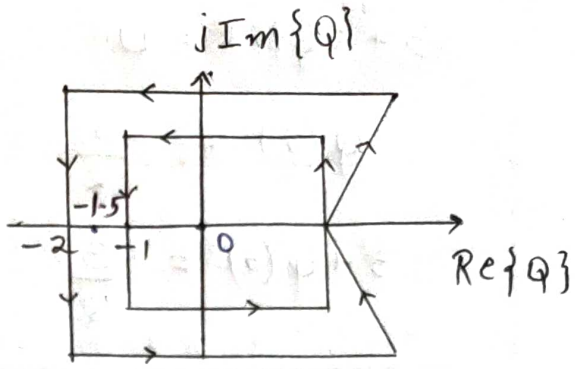
$$Q(s) = 1.5$$

Since, $Q(s)$ plot encircles $(+1.5, 0)$ once in ACW direction, $3 - 2Q(s)$ plot encircle its origin once in ACW direction.

→ Question:



$Q(s)$ →



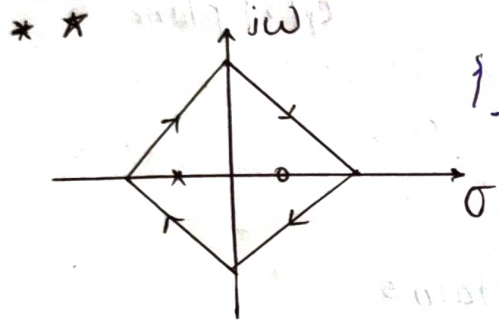
Number of encirclement of (0,0) by $3+2Q(s)$?

$$3+2Q(s)=0$$

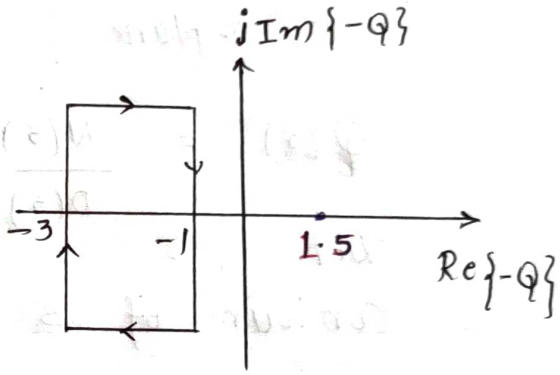
$$Q(s)=-1.5$$

→ Since, $Q(s)$ encircles $(-1.5,0)$ once in ACW direction, $3+2Q$ encircles $(0,0)$ once in ACW.

→ Question:



$\{-Q(s)\}$ →

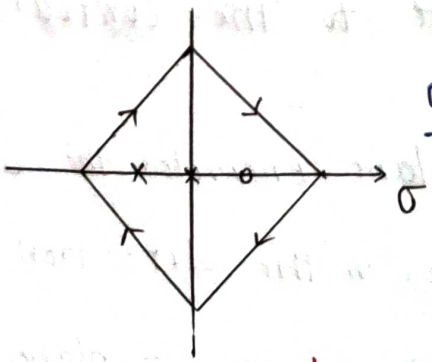


No. of encirclement of (0,0) by $3+2Q(s)$ plot?

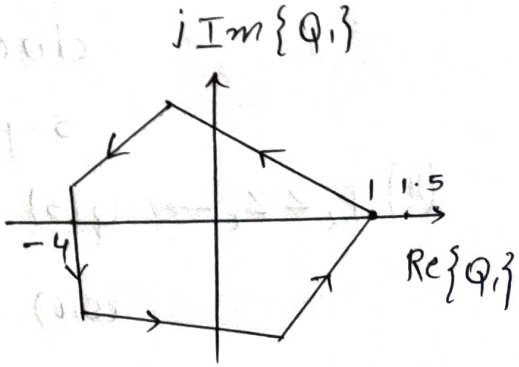
$3+2Q(s)=0$ → Since $-Q(s)$ plot does not encircle $(1.5,0)$, $3+2Q(s)$ does not encircle $(0,0)$.

$-Q(s)=1.5$

→ Question:



$Q_1(s) = 3+Q(s)$ →



How many times $3+2Q(s)$ encircles $(0,0)$

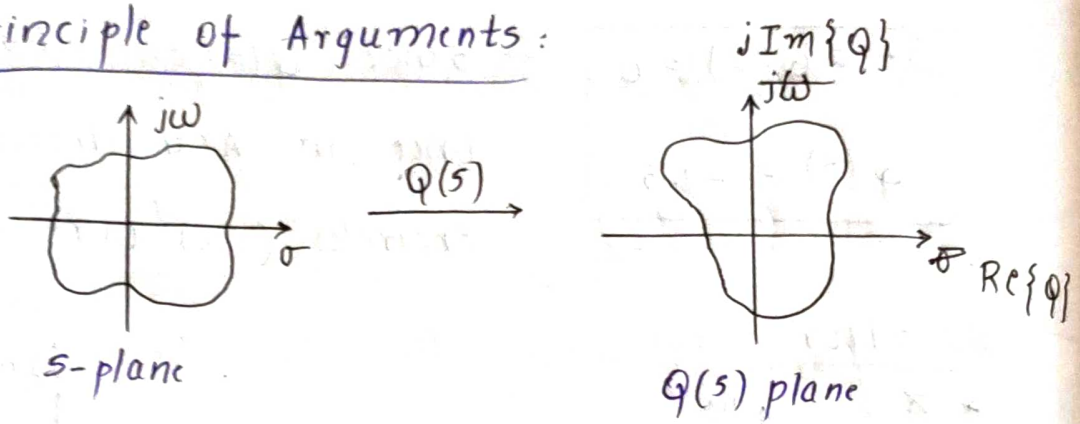
$3 + 2Q(s) = 0$ → $3 + Q(s)$ does not encircle $(1.5, 0)$

$Q(s) = -\frac{3}{2}$ Hence $3 + 2Q(s)$ does not

$3 + Q(s) = \frac{3}{2}$ encircle $(0, 0)$.

POINTS TO REMEMBER

1. Principle of Arguments:



s-plane Q(s) plane

$$Q(s) = \frac{N(s)}{D(s)}$$

Let

Contour of s plane contains

P_c number of poles and

Z_c number of zeros inside it.

If

(i) $P_c > Z_c$ → Q(s) plot in Q(s) plane encircles the origin $(0,0)$ $(P_c - Z_c)$ time in the direction opposite to the contour in s-plane.

(ii) $P_c < Z_c$ → Q(s) plot in Q(s) plane encircles the origin $(0,0)$ $(Z_c - P_c)$ times in the direction similar to the contour in s-plane.

(iii) $P_c = Z_c \rightarrow Q(s)$ plot in $Q(s)$ plane will not encircle the origin.

2. Rule of Mapping:

→ It is valid for Transfer function.

$$Q(s) = \frac{N(s)}{D(s)}$$

$$A + B\,Q(s) = A + B\frac{N(s)}{D(s)} = \frac{AD(s) + BN(s)}{D(s)}$$

(i) Let T.F is $Q(s) = \frac{N(s)}{D(s)}$

$$N = P_c - Z_c$$

→ P_c = No. of poles of $Q(s)$ lying inside Contour in s plane.

→ Z_c = No. of zeros of $Q(s)$ lying inside contour in s plane.

→ N = No. of encirclement of (0,0) by $Q(s)$ plot.

→ N = +ve if C and $Q(C)$ has opposite direction.

= −ve if C and $Q(C)$ has same direction.

(ii) Let T.F is $A + BQ(s)$

$$N = P_c - Z_c$$

→ P_c = No. of poles of $\{A + BQ(s)\}$ lying inside Contour in s plane.

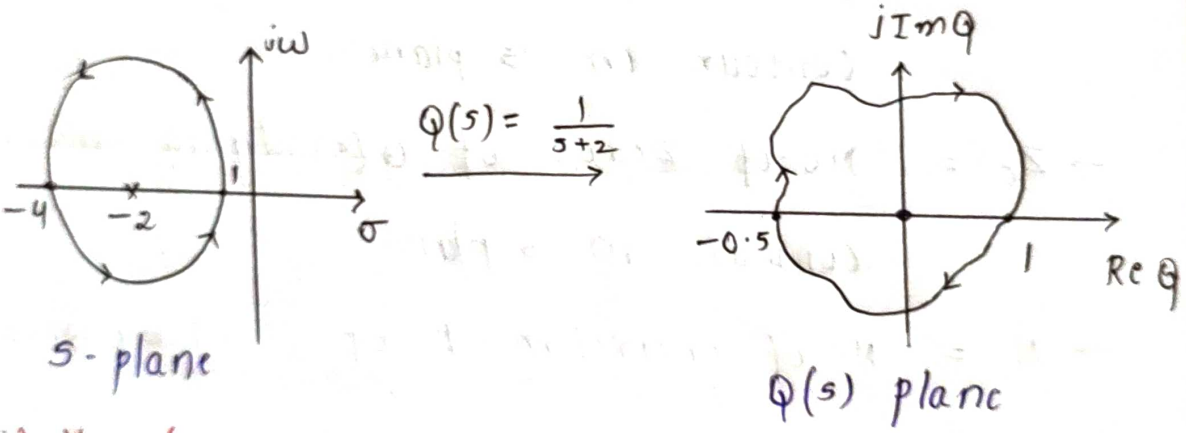
→ Z_c = No. of zeros of $\{A + BQ(s)\}$ lying inside contour in s plane.

→ N = No. of encirclement of (0,0) by $A + BQ(s)$ plot.

3. → If $A + BQ(s)$ plot encircle origin then $Q(s)$ plot will encircle $\left(-\frac{A}{B}, 0\right)$

→ If $Q(s)$ plot encircle $\left(-\frac{A}{B}, 0\right)$, then $A + BQ(s)$ plot will encircle (0,0).

→ Question:



(i) Verify Rule of mapping for T.F $Q(s)$

(ii) Verify Rule of mapping for T.F $1 + Q(s)$

(iii) Verify Rule of mapping for T.F $4 + Q(s)$

→ (i) L.H.S N = 1

R.H.S $P_c = 1$

$Z_c = 0$

$P_c - Z_c = 1$

LHS = RHS

(ii) $N = P_c - Z_c$

LHS : $N = 0 \rightarrow 1 + Q(s) = 0$

$Q(s) = -1$

RHS : $P_c = 1$, $Z_c = 1 \rightarrow Q(s) = \frac{1}{s+2}$

$1 + Q(s) = \frac{s+3}{s+2}$

$P_c - Z_c = 0$

LHS = RHS

(iii) LHS $N = 0 \rightarrow 4 + Q(s) = 0$

$Q(s) = -4$

RHS $P_c = 1$, $Z_c = 1 \rightarrow 4 + \frac{1}{s+2} = 4 + Q(s)$

$\frac{4s+9}{s+2} = 4 + Q(s)$

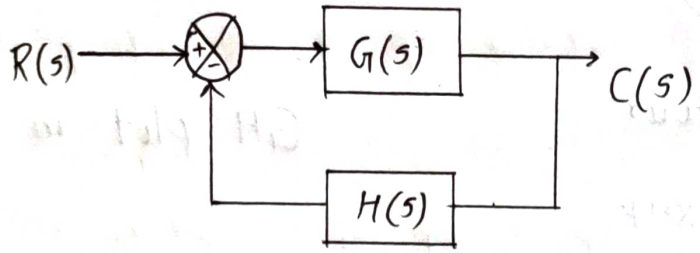
$P_c - Z_c = 0$

LHS = RHS

POLE : $s = -2$

ZERO : $s = -2.25$

→ # NYQUIST #



★ Types of Transfer Function :

1. $G(s)H(s) = \frac{N(s)}{D(s)}$

2. $1 + G(s)H(s) = \frac{N(s) + D(s)}{D(s)}$

3. $\frac{G(s)}{1 + G(s)H(s)} = \frac{P_1(s)}{P_2(s)}$

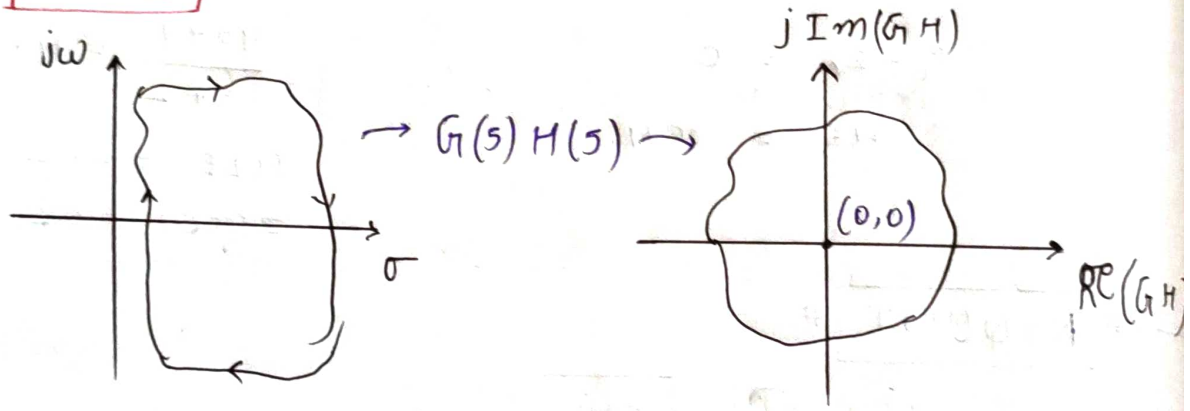
$$\rightarrow A + B\,G(s)H(s) = \frac{A\,D(s) + B\,N(s)}{D(s)}$$

Note: Poles of the T.F $A + BG(s)H(s)$ is same as the T.F $G(s)H(s)$.

* Zeros of $(1 + G(s)H(s))$
  $= $ Roots of $(1 + G(s)H(s))$
  $= $ Poles of C.L.S

* Poles of $1 + G(s)H(s)$ = Poles of $G(s)H(s)$

CASE 1:



- S-plane contour
- CW, Entire RHP

GH plot in GH plane

MAPPING RULE in T.F $G(s)H(s)$

$$N = P_c - Z_c$$

N = No. of encirclement of (0,0) by $G(s)H(s)$ plot in $G(s)H(s)$ plane.

N = +ve, Encirclement of (0,0) by $G(s)H(s)$ plot in $G(s)H(s)$ plane in ACW.

N = −ve, Encirclement of (0,0) by $G(s)H(s)$ plot in $G(s)H(s)$ plane in CW.

P_c = No. of poles of $G(s)H(s)$ inside C

(or)

No. of poles of $G(s)H(s)$ lying in R.H.P.

Z_c = No. of zeros of $G(s)H(s)$ inside C

(or)

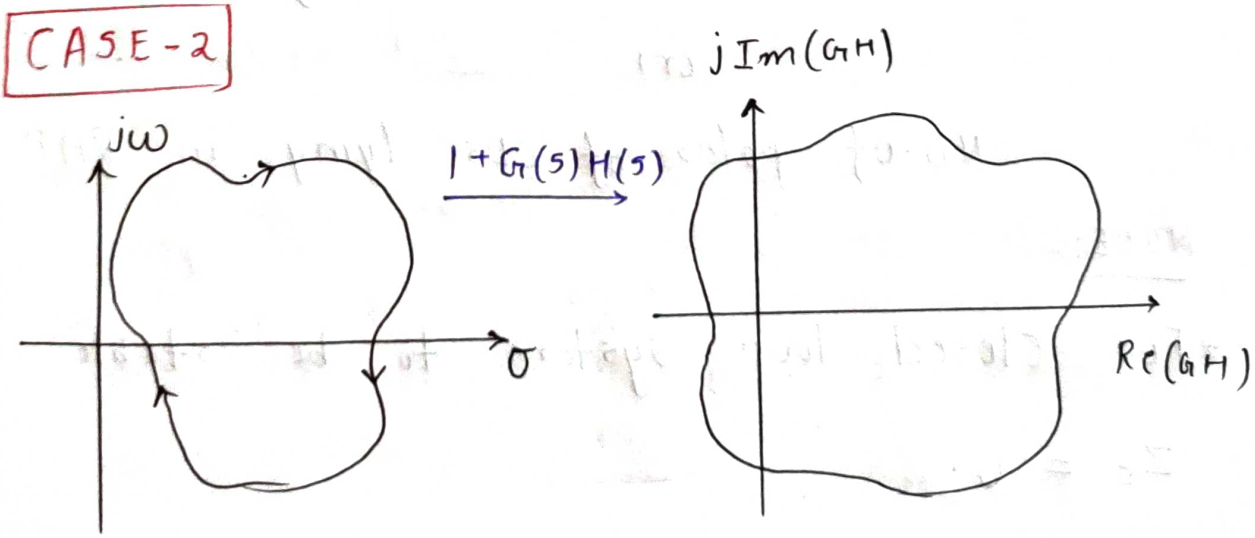
No. of zeros of $G(s)H(s)$ lying in R.H.P.

NOTE:

* For open loop system to be stable,

$P_c = 0$.

CASE-2



RULE of Mapping in 1+GH

$$N = P_c - Z_c$$

$$GH = \frac{N}{D}$$

$$1+GH = \frac{N+D}{D}$$

N = Encirclement of (0,0) by 1+GH plot in 1+GH plane (or)

Encirclement of (-1,0) by GH plot in GH plane

N = +ve : GH plot ACW

N = -ve : GH plot CW

P_c = No. of poles of 1+GH lying in RHP (c)

(or)

No. of poles of GH lying in RHP (c)

Z_c = No. of zeros of 1+GH lying in RHP (or C)

(or)

No. of poles of CLS lying in RHP.

NOTE:

* For closed loop system to be stable

$Z_c = 0$.

Rule of Mapping (Given by Cauchy)

Mapping Rule : $N = P_c - Z_c$

Nyquist Stability Criteria (Given by Nyquist)

Nyquist stability Criteria : $N = P_+ - Z_+$

NOTE:

1. s plane contour : Entire R.H.P
2. Plot of $G(s)H(s)$ is needed only

→ Stability of all the transfer functions of type $A + B\,G(s)H(s)$ can be determined.

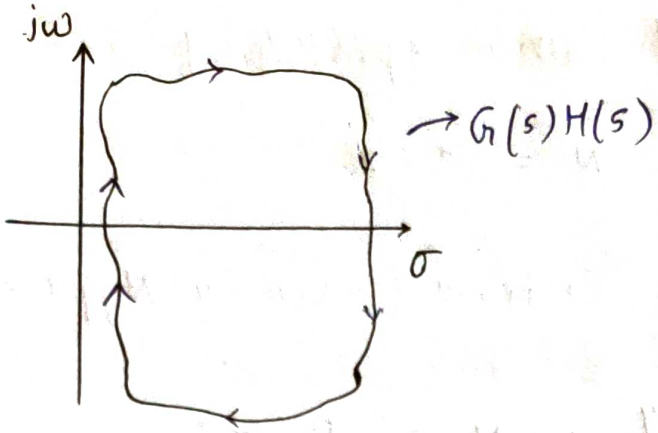
→ Also stability of closed loop system can be determined by applying Nyquist stability Criteria in $1 \pm G(s)H(s)$.

→ s plane contour → Entire R.H.P

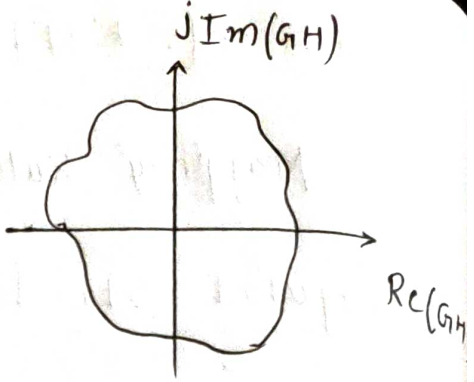
→ NYQUIST CONTOUR

NYQUIST CONTOUR : 'Contour containing entire R.H.P'

#



NYQUIST CONTOUR

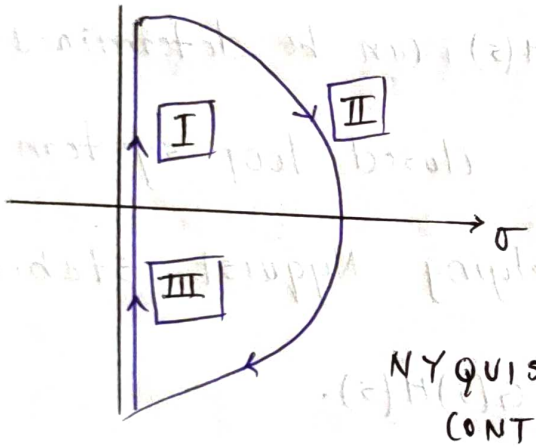


G(s)H(s) plot

NYQUIST PLOT

Types of Nyquist contour:

1. $G(s)H(s) = \frac{N(s)}{D(s)}$ → No pole or zero on jω axis or origin



NYQUIST CONTOUR

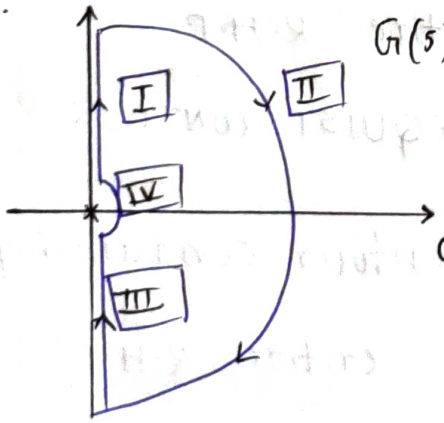
I $s = j\omega$: $0 < \omega < \infty$

II $s = Re^{j\theta}$: $R \to \infty$

θ: $\frac{\pi}{2}$ to $-\frac{\pi}{2}$ (C.W)

III $s = j\omega$: $-\infty < \omega < 0$

2. $G(s)H(s) = \frac{N(s)}{D(s)}$ → Pole or zero on origin



NYQUIST CONTOUR

I $s = j\omega$ $(0 < \omega < \infty)$

II $s = Re^{j\theta}$ $R \to \infty$

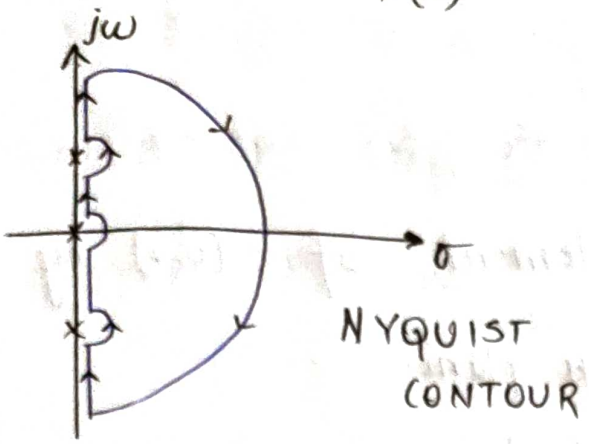
$\theta = \frac{\pi}{2}$ to $-\frac{\pi}{2}$ (C.W)

III $s = j\omega$ $(-\infty < \omega < 0)$

IV $s = Re^{j\theta}$ $R \to 0$

$\theta = -\frac{\pi}{2}$ to $\frac{\pi}{2}$ (ACW)

3. $G(s)H(s) = \frac{N(s)}{D(s)}$ → Poles and zeros on jω axis



Nyquist stability criteria for various tranfer fns:

Let Nyquist contour is clockwise

CASE-1: Nyquist stability criteria for

T.F $G(s)H(s)$

$$\boxed{N = P_c - Z_c}$$

N = No. of encirclement of (0,0) by $G(s)H(s)$ plot in $G(s)H(s)$ plane.

P_c = No. of poles of $G(s)H(s)$ lying inside C.

Z_c = No. of zeros of $G(s)H(s)$ lying inside C.

NOTE:

→ If C is entire RHP

For $G(s)H(s)$ to be stable : $P_c = 0$.

CASE-2: Nyquist stability Criteria for T.F

$1 \pm G(s)H(s)$

$$\boxed{N = P_C - Z_C}$$

N = No. of encirclement of (0,0) by $1 \pm GH$ plot in $1 \pm GH$ plane.

(or)

No. of encirclement of $(\mp 1, 0)$ by GH plot in GH plane.

P_C = No. of poles of $1 \pm GH$ lying inside C.

(or)

No. of poles of GH lying inside C.

Z_C = No. of zeros of $1 \pm GH$ lying inside C.

(or)

No. of poles of T.F / CLS of type $\frac{N(s)}{1 \pm GH}$ lying inside C.

NOTE:

→ If C is entire RHP

For $1 \pm GH$ to be stable: $P_C = 0$

For CLS $\left(\frac{G}{1 \pm GH}\right)$ to be stable: $Z_C = 0$.

CASE-3: Nyquist stability Criteria for T.F $A + BGH$:

$$N = P_C - Z_C$$

N = No. of encirclements of $(0,0)$ by $A + BGH$ plot in $A + BGH$ plane.

(or)

No. of encirclement of $\left(\frac{-A}{B}, 0\right)$ by GH plot in GH plane.

P_C = No. of poles of $A + BGH$ lying inside C

(or)

No. of poles of GH lying inside C.

Z_C = No. of zeros of $A + BGH$ lying inside C.

(or)

No. of poles of TF of type $\frac{N(s)}{A + BGH}$ lying inside C.

Problem Solving Approach:

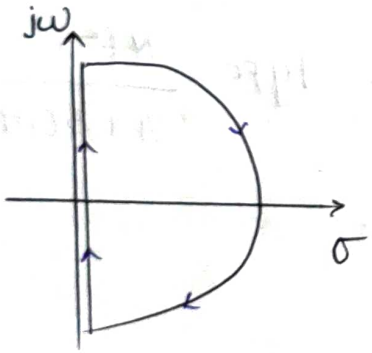
FLOW CHART

↳

S Plane Contour

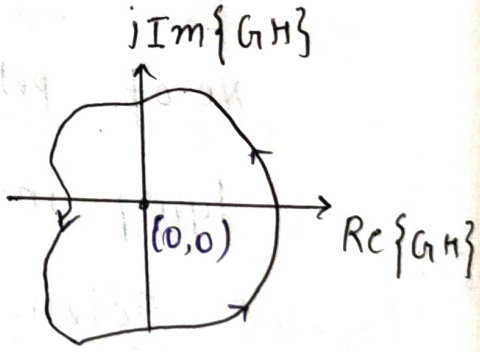
- Clockwise
  - Let ACW direction in G(s)H(s) plot is +ve. → $N = P_C - Z_C$
  - Let CW direction in G(s)H(s) plot is +ve. → $N = Z_C - P_C$
- Anti-clockwise
  - Let ACW direction in G(s)H(s) plot is +ve → $N = P_C - Z_C$
  - Let CW direction in G(s)H(s) plot is +ve → $N = P_C - Z_C$

→ Question



Nyquist Contour

G(s)H(s)



Nyquist Plot

Nyquist stability criteria is

A) $N = P_+ - Z_+$ B) $N = Z_+ - P_+$

C) $N = P - Z$ D) $N = Z - P$

S-1: S-plane contour : Clockwise (R·H·P)

S-2: Ref +ve direction in GH plot



ACW (+ve)

$N = P_c - Z_c$

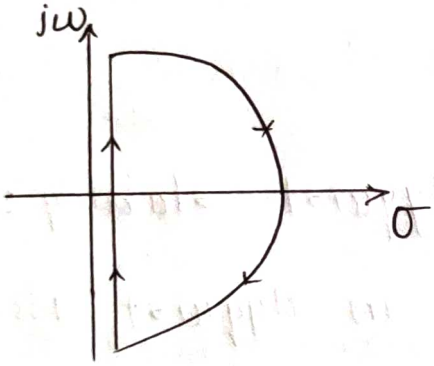
$\boxed{N = P_+ - Z_+}$

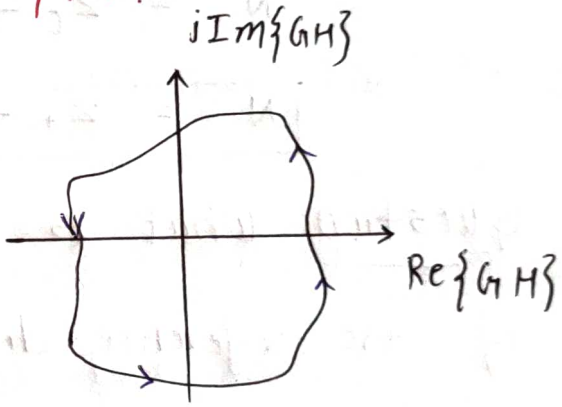
CW (+ve)

$N = Z_c - P_c$

$\boxed{N = Z_+ - P_+}$

→ Question: What will be Nyquist stability criteria if reference direction of CW is considered to be +ve in G(s)H(s) plot?





A) $N = P_+ - Z_+$

B) $N = Z_+ - P_+$

C) $N = P - Z$

D) $N = Z - P$

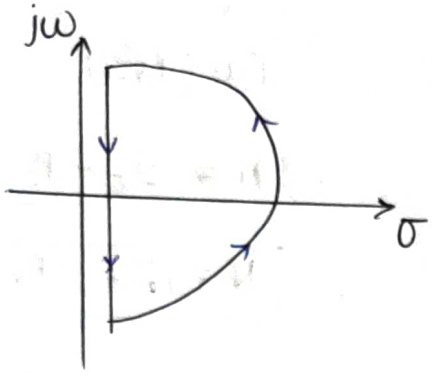
S-1: S-plane contour (CW) : RHP

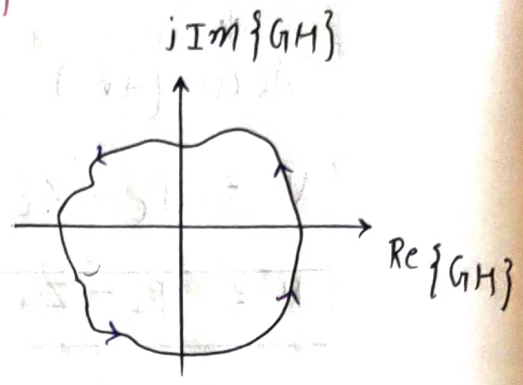
S-2: REF +ve direction in GH plot: CW

$N = Z_c - P_c$

$N = Z_+ - P_+$

→ Question: What is Nyquist stability criteria, if reference direction of ACW is considered to be +ve in G(s)H(s) plot?





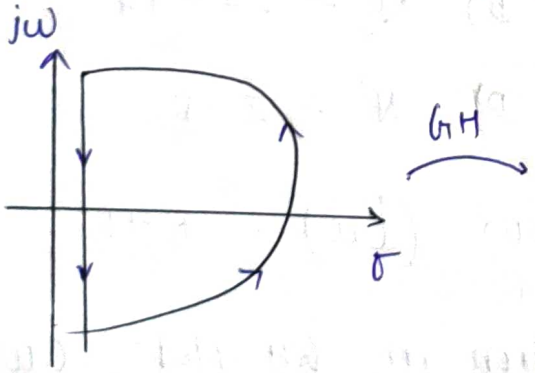
5-1: s-plane contour (ACW) : RHP

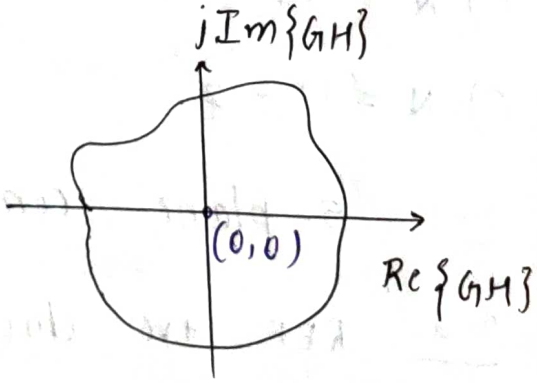
5-2: REF +ve direction : ACW

$$N = Z_C - P_C$$

$$\boxed{N = Z_+ - P_+}$$

→ Question: what is the Nyquist stability criteria, if +ve reference direction in Nyquist plot is considered as clockwise?





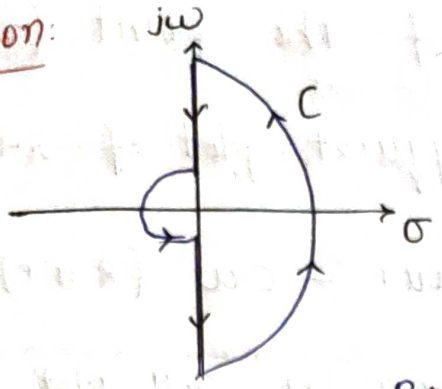
5-1: s-plane contour (A.C.W) : RHP

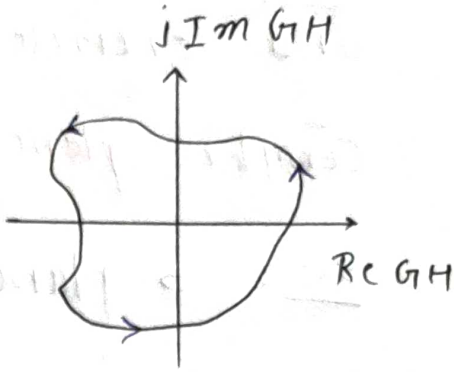
5-2: REF +ve direction in GH plot : C.W

$$N = P_c - Z_c$$

$$\boxed{N = P_+ - Z_+}$$

→ Question:





REF +ve DIRECTION : ACW

A) $N = Z_+ - P_+$

B) $N = P_+ - Z_+$

C) $N = Z_c - P_c$ ✓

D) $N = P_c - Z_c$

5-1: s plane contour ACW (Not RHP)

5-2: REF +ve direction in GH plot : ACW

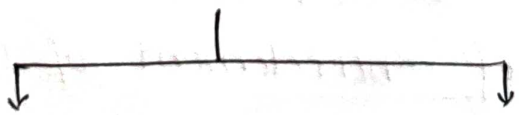
$$\boxed{N = Z_c - P_c}$$

STEPS

Step 1: Direction of s-plane

Step 2: Reference +ve direction of s-plane

Step 3: T.F in which NSC to be applied.



$N = P_c - Z_c$ $\qquad$ $N = Z_c - P_c$

→ **Question:** The nyquist contour in s-plane is in clockwise direction. Calculate the direction and encirclement of the point $(-1+j0)$ in Complex plane by nyquist plot of $GH = \frac{1-s}{4+2s}$

S-1: s-plane contour – C.W (R.H.P)

S-2: REF +ve direction in GH plot is ACW.

S-3: NSC in $GH = -1$

i.e., NSC in $1+GH$

$$N = P_c - Z_c$$

$N = ?$

$P_c = 0$

$Z_c = 0$

$$1+GH = \frac{5+s}{4+2s}$$

$\therefore N = 0$

i.e., 1. $1+GH$ plot never encircles $(0,0)$ in $1+GH$ plane.

2. GH plot never encircles $(-1,0) \rightarrow (-1+j0)$ in GH plane.

→ **Question:** Let Nyquist contour in s-plane is C.W. Calculate i) No. of encirclement of (1,0) by the Nyquist plot of $G(s)H(s)$

ii) No. of encirclement of $(-4,0)$ by the Nyquist plot of $3-G(s)H(s)$

where $G(s)H(s) = \frac{1-s}{4+3s}$

i)

S-1: s-plane contour : C.W (R.H.P)

S-2: REF +ve direction for GH plot : ACW

S-3: NSC in GH = 1

NSC in 1-GH

$N = P_c - Z_c$

N = ?

$P_c = 0$

$Z_c = 0$

$\therefore N = 0$

$1+(GH) = \frac{3+3s}{4+3s}$

ii)

NSC in 3-GH = -4

NSC in 7-GH

$N = P_c - Z_c$

N = ?

$P_c = 0$

$Z_c = 0$

$N = 0$

$7-GH = \frac{225+27}{4+3s}$

→ Question: The OLTF of the system is

$G(s)H(s) = \frac{k(s+1)}{(s+0.5)(s-2)}$, where $k > \frac{3}{2}$.

i) Calculate the no. of times $1+G(s)H(s)$ plot encircles origin

ii) Calculate the no. of times $G(s)H(s)$ plot encircles origin.

i) S-1: S-plane contour : C.W (C) RHP

S-2: REF +ve direction in GH plot: A.C.W

S-3: NSC in 1+GH =

$$N = P_C - Z_C$$

$N = ?$

$P_C = 1$

$Z_C = 0$

$\therefore N = P_C - Z_C$

$\boxed{N = 1}$

$$1+GH = \frac{s^2 - 1.5s - 1 + ks + k}{(s+0.5)(s-2)}$$

$$= \frac{s^2 + s(k-1.5) + k - 1}{(s+0.5)(s-2)}$$

(+ve, +ve) $k > 1.5$

$Z_C = 0$

↳ 1 + GH encircles (0,0) once in ACW.

ii) NSC in GH

$$N = P_C - Z_C$$

$N = ?$

$P_C = 1$

$Z_C = 0$

$\boxed{N = 1}$

↳ GH encircles (0,0) once in ACW direction.

→ Question: The s-plane contour is ACW, where $G(s)H(s) = \frac{k(s+1)}{(s+0.5)(s-2)}$ where $k<1$. Calculate

i) No. of encirclement of (-1,0) by GH plot.

ii) No. of encirclement of (0,0) by G(s)H(s) plot.

i) 5-1: s-plane contour : ACW (R·H·P)

5-2: REF +ve direction in GH plot: A·C·W

5-3: NSC in GH = −1

i.e., in 1 + GH

$N = -P_c + Z_c$

$1 + GH = \dfrac{s^2 - 1.5s - 1 + ks + k}{(s+0.5)(s-2)}$

N = ?

$P_c = 1$

$Z_c = 1$ →

$\boxed{N = Z_c - P_c = 0}$

R·H

+	s^2	1
−	s^1	$k - 1.5$
−	s^0	$k - 1$

ii) 5-1: "

5-2:

5-3: NSC in GH

$N = Z_c - P_c$

N = ?

$P_c = 1$

$Z_c = 0$

$\boxed{N = 0 - 1 = -1}$ → GH plot encircles (0,0) once in C·W direction.

→ Question: $G(s)H(s) = \dfrac{k(s+1)}{s-1}$; $k > 1$

i) No. of encirclement of (0,0) by GH plot

ii) (0,0) by 1+GH plot

iii) (0,0) by 1−GH plot

iv) (−3,0) by 3−4GH plot

Consider Nyquist contour to be in C·W direction.

i) S-1: S plane contour : C.W (R.H.P)

S-2: REF +ve direction in GH plot: A.C.W

S-3: NSC in GH

$$N = P_c - Z_c$$

$P_c = 1$

$Z_c = 0$

$\boxed{N = 1}$ → GH plot encircles (0,0) once in ACW

ii) NSC in $1 + GH$

$$N = P_c - Z_c$$

$P_c = 1$

$Z_c = 0$ ↳

$\boxed{N = 1}$ → $1+GH$ plot encircles (0,0) once in ACW direction

$$1 + GH = \frac{s-1+ks+k}{s-1}$$

$$= \frac{s(1+k)+(k-1)}{s-1} \quad \downarrow Z_c = 0$$

R.H:

s^1	$k+1$
s^0	$k-1$

$k > 1$

iii) NSC in $1 - GH$

$$N = P_c - Z_c$$

$P_c = 1$

$Z_c = 0$ ↳

$\boxed{N = 1}$

$$1 - GH = 1 - \frac{k(s+1)}{s-1}$$

$$= \frac{s-1-ks-k}{s-1}$$

$$= \frac{s(1-k)-1-k}{s-1} \quad \rightarrow \text{R.H} \downarrow Z_c = 0$$

I:

-ve s^1 $1-k$

-ve s^0 $-1-k$

II: $1 - GH = 0$

(or) $\rightarrow s = \frac{1+k}{1-k} = -ve$

$(\because k > 1)$

$\hookrightarrow z_c = 0$

$\therefore N = 1 \rightarrow 1 - GH$ encircles $(0,0)$ once in A·C·W direction

iv) NSC in $3 - 4GH = -3$

NSC in $6 - 4GH$

$N = P_c - z_c$

$P_c = 1$

z_c

$6 - 4GH = \frac{s(6-4k) - 4k - 6}{(s-1)}$

$6 - 4GH \xrightarrow{\text{zeros}} 6 - 4GH = 0$

$s = \frac{4k+6}{6-4k}$

$6-4k>0$: $k < 1.5$: $s = +ve$: R·H·P $\hookrightarrow z_c = 1$

$6-4k<0$: $k > 1.5$: $s = -ve$: L·H·P $\hookrightarrow z_c = 0$

$\rightarrow$

$1 < k < 1.5 \rightarrow N = P_c - z_c = 1 - 1 = 0$

$1.5 < k < \infty \rightarrow N = P_c - z_c = 1 - 0 = 1$

$\rightarrow$ Question: The unity -ve feedback system has OLTF

$$G(s) = \frac{1}{(s-1)(s+2)(s+5)}$$

i) How many times Nyquist plot of $G(s)$ encircles $(0,0)$

ii) How many times Nyquist plot of $1+GS$ encir... (0,0) (or)

How many times Nyquist plot of $G(s)$ encircles $(-1,0)$.

iii) How many times Nyquist plot of $-1+G(s)$ encircles $(0,0)$. (or)

How many times Nyquist plot of $G(s)$ encircles $(1,0)$.

Note: Let Nyquist contour is in clockwise direction.

→ i) 5-1: s-plane contour : CW (R·H·P) → C

5-2: REF +ve direction for $G(s)H(s)$ plot : ACW.

5-3: NSC in GH

$$N = P_c - Z_c$$

$P_c = 1$

$Z_c = 0$

$N = 1 \rightarrow$ GH plot encircles (0,0) once in ACW direction.

ii) NSC in $1+GH$

i.e., $N = P_c - Z_c$

$P_c = 1$

$Z_c = 1$

$$1+GH = \frac{\overbrace{s^3+6s^2+3s-9}^{R.H}}{(s-1)(s+2)(s+5)}$$

→

+	s^3	1	3
+	s^2	2	−3
+	s^1	9/2	0
−	s^0	−3	

$\rightarrow Z_c = 1$

$\therefore N = 0$

↳ 1+GH does not encircle (0,0).

iii) NSC in GH = 1

i.e., NSC in −1+GH

$N = P_c - Z_c$

$P_c = 1$

$Z_c = 1$

$\therefore N = 0$

↳ −1+GH plot doesn't encircle (0,0)

$$-1+GH = \underbrace{\frac{-s^3-6s^2-3s+11}{(s-1)(s+2)(s+5)}}_{R.H}$$

−	s^3	−1	−3
−	s^2	−6	11
−	s^1	−29/6	0
+	s^0	11	

$\rightarrow Z_c = 1$

Question: $G(s)H(s) = \dfrac{(s+0.5)}{s^2(s-0.5)(s+1)}$

i) Calculate encirclement of (0,0) by GH plot

ii) Calculate encirclement of (−1,0) by GH plot.

iii) Calculate encirclement of (−2,0) by GH plot.

iv) Calculate encirclement of (−5,0) by GH plot.

→ 5-1: s-plane contour : CW (RHP) → C

5-2: REF +ve direction for G(s)H(s) plot: ACW

#5-3: NSC in M+GH

$N = P_c - Z_c$

$P_c = 1$

$$M + GH = M + \frac{s + 0.5}{s^2(s-0.5)(s+1)}$$

$$M + GH = \frac{\overbrace{Ms^4 + 0.5Ms^3 - 0.5Ms^2 + s + 0.5}^{RH}}{s^2(s-0.5)(s+1)}$$

s^4	M	$-0.5M$	0.5
s^3	$0.5M$	1	0
s^2	$\frac{-(0.25M+1)}{0.5}$	0.5	0
s^1	$\frac{0.375M+1}{0.25M+1}$		
s^0	0.5		

i) $M = 0$

$N = P_c - Z_c$

$\boxed{N = 1 - 0 = 1}$ → GH encircles (0,0) once in Acw direction.

ii) $M = 1$

$\boxed{N = 1 - 2 = -1}$ → GH plot encircles (-1,0) once in c.w direction

iii) $M = 2$

$\boxed{N = 1 - 2 = -1}$ → GH encircles (-2,0) once in c.w direction

iv) $M = 5$

$\boxed{N = 1 - 2 = -1}$ → GH plot encircles (-5,0) once in c.w.

Question: For a feedback system $G(s) = \frac{s^2+0.25+100}{s^2-0.25+100}$ and $H(s) = \frac{1}{2}$, with −ve feedback. The no. of clockwise encirclements of point (−1, 0) in Nyquist plot of T.F $G(s)H(s)$ is —

Consider Nyquist contour to be clockwise.

S-1: s plane contour : CW (RHP) → C

S-2: REF +ve direction for $G(s)H(s)$ plot : ACW

S-3: NSC in $GH = -1$

i.e., NSC in $1 + GH$

$$N = P_C - Z_C$$

$P_C = 2$ → $GH = \left(\frac{s^2+0.25+100}{\underbrace{s^2-0.25+100}_{RH}}\right)\frac{1}{2}$

$Z_C = 2$

$s^2 - 0.25 + 100$

+	s^2	1	100
−	s^1	−0.2	
+	s^0	100	

↓

$1+GH = \frac{3s^2-0.25+300}{2s^2-0.25+100}$ (R.H)

+	s^2	3	300
−	s^1	−0.2	
+	s^0	300	

→ $P_C = 2$

→ $Z_C = 2$

$\boxed{N = 0}$ → GH does not encircle (−1, 0).

→ * Question: Nyquist contour in s-plane is CW, where $G(s)H(s) = \frac{k}{(s+2)(s^2+2s+2)}$

The Nyquist plot of T.F

A) Does not encircle the point (-1,0) for k = 10

B) Encircle the point (-1,0) for k = 100

Then, CLS with unity -ve feedback is

	k = 10	k = 100
(i)	stable	Unstable
(ii)	stable	stable
(iii)	Unstable	stable
(iv)	Unstable	Unstable

METHOD - I: CLS : $\frac{C}{R} = \frac{G}{1+GH}$

RH in $1+GH$ $\quad 1+GH = \frac{s^3+4s^2+6s+4+k}{(s+2)(s^2+2s+2)}$

RH in $s^3+4s^2+6s+(4+k)$

s^3	1	6	
s^2	4	4+k	
s^1	$\frac{20-k}{4}$		
s^0	4+k		

For CLS to be stable,

$\boxed{-4 < k < 20}$

→ k = 10: CLS stable

k = 100: CLS Unstable

METHOD - II :

S-1 : s plane contour : CW (RHP) → C

S-2 : REF +ve direction for G(s)H(s) plot : ACW

S-3 : NSC in 1+GH

$$N = P_c - Z_c$$

CASE-1 $K = 10$: $N = 0$

$$P_c = 0$$

$$N = P_c - Z_c$$

→ $\boxed{Z_c = 0}$ → 0 poles of CLS in RHP

↓

STABLE CLS

CASE-2 : $K = 100$, $N \neq 0$,

$$P_c = 0$$

$$N = P_c - Z_c$$

$$\boxed{Z_c = -N}$$

$N = -1$: GH encircles (-1,0) once in C.W → $Z_c = 1$ → 1 pole of CLS in R.H.P

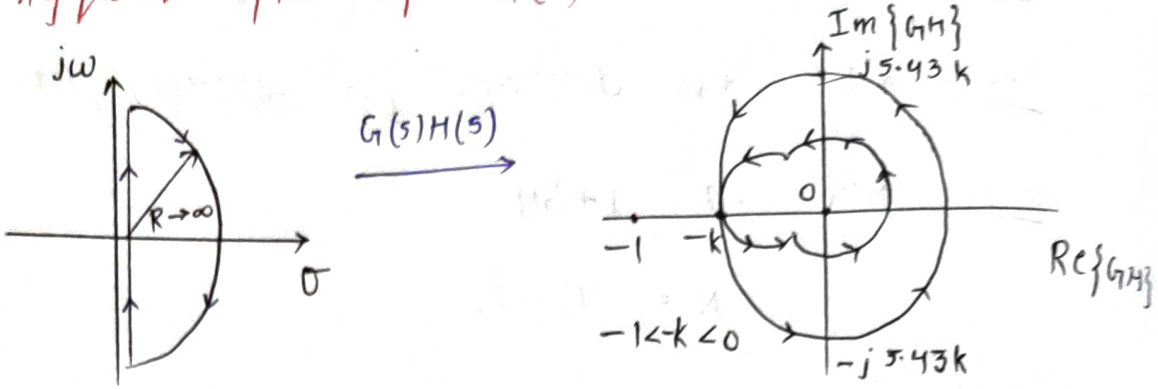
$N = -2$: GH encircles (-1,0) twice in C.W → $Z_c = 2$ → 2 poles of CLS in R.H.P

↳ UNSTABLE

→ Question: A ufb system is characterized by the OLTF

$$G(s)H(s) = \frac{10k(s+2)}{s^3 + 3s^2 + 10}$$

The Nyquist path and the corresponding nyquist plot of $G(s)$ are shown in fig.



For $0<k<1$, the no. of poles of CLTF that lies on the right half side of s-plane.

Method 1: Roots of $1+GH$ lying in R.H.P $(0<k<1)$

$$1+GH = 1+\frac{10k(s+2)}{s^3+3s^2+10} = \frac{s^3+3s^2+10+10ks+20k}{s^3+3s^2+10}$$

RH in $s^3+3s^2+10ks+(10+20k)$

$+s^3$	1	$10k$
$+s^2$	3	$10+20k$
$-s^1$	$\frac{10}{3}(k-1)$	0
$+s^0$	$10+20k$	

→ 2 roots of $1+GH$ or 2 poles of CLS in RHP

Method 2:

S-1: S plane contour : CW (RHP) → C

S-2: REF +ve direction for $G(s)H(s)$ plot : ACW

S-3: NSC in $1+GH$

$$N = P_c - Z_c$$

$N = 0$

$P_C = 2$

$0 = P_C - Z_C$

$\boxed{Z_C = 2 - 0 = 2}$

↳ 2 zeros of $1+GH$ or 2 poles of CLS in RHP

↓

$$GH = \frac{10k(s+2)}{\underbrace{s^3+3s^2+10}_{RH}}$$

$+ s^3$	1	0
$+ s^2$	3	10
$- s^1$	$\frac{-10}{3}$	$\rightarrow P_C = 2$
$+ s^0$	10	

Method-3:

→ s plane contour : CW (RHP)

→ REF +ve direction in GH : ACW

→ NSC in $1+GH$

$N = P_C - Z_C$

$\| \quad \overset{?}{\longrightarrow}$ NSC in GH

0

$\therefore Z_C = P_C - N$

$= 2 - 0$

$\boxed{Z_C = 2}$

$N = P_C - Z_C$

$N = 2$

$P_C = ?$

$Z_C = 0$

$\rightarrow \boxed{P_C = 2}$

* Question: Consider a -ve unity feedback system with forward path T.F $G(s) = \frac{k}{(s+a)(s-b)(s+c)}$,

where k, a, b, c are +ve real numbers. For a

Nyquist path enclosing entire imaginary axis and right half of the s-plane in the cw direction; the Nyquist plot of $1+G(s)$ encircles the origin of $1+G(s)$ plane once in the cw direction and never passes through this origin for a certain value of k. Then the no. of poles of $\frac{G(s)}{1+G(s)}$ lying in the open right half of the s-plane is -

→ s-plane contour : cw [(R.H.P) + jw] → C

→ REF +ve direction in GH : ACW

→ NSC in $1+GH$

$$N = P_c - Z_c$$

$$-1 = 1 - Z_c$$

$$\boxed{Z_c = 2}$$

→ Question: Taking N as +ve for cw encirclement, otherwise -ve, the no. of encirclements N of (-1,0) in the Nyquist plot of $G(s) = \frac{3}{s-1}$ is -

S-1: s plane contour : cw (RHP) ↓ $P_c = 1$

S-2: REF +ve direction : cw

S-3: NSC in $1+GH$

$$N = Z_c - P_c$$

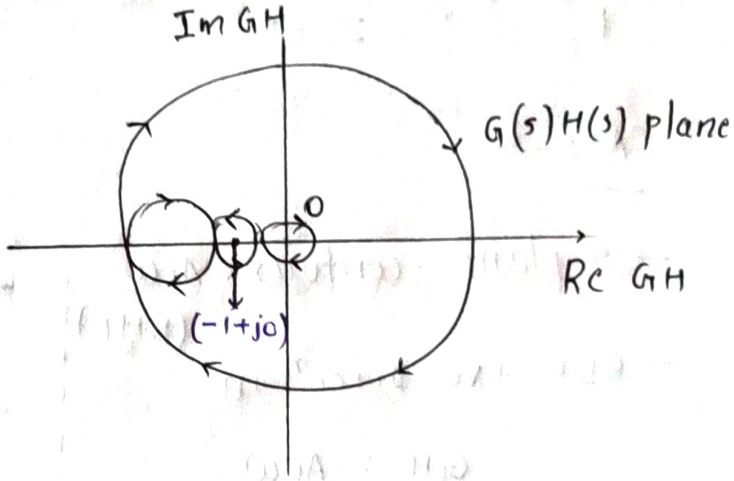
$N = 0 - 1$

$1 + GH = \frac{s+2}{s-1}$

$\hookrightarrow Z_c = 0$

$N = -1$ → GH encircles (-1,0) once in ACW direction

→ * **Question:** The complete Nyquist plot of the open-loop T.F $G(s)H(s)$ of a feedback C.S is shown in the figure. If $G(s)H(s)$ has one zero in the right-half of the s-plane, the no. of poles that the CLS will have in the right-half of the s-plane is.



A) 4

B) 0

C) 3

D) 1

CASE 1:

→ s-plane contour : CW (RHP)

→ REF +ve direction in GH : ACW

→ NSC in $1+GH$

$N = P_c - Z_c$

$N = 0$ → $P_c = ?$, $Z_c = ?$ Since one CW, one ACW rotation of GH plot around (-1,0)

→ NSC in GH

$N = P_c - Z_c$

$N = -2$

$Z_c = 1$ (Given)

$-2 = P_c - 1$

$P_c = -1$

↳ Invalid

CASE-2 :

→ S plane contour : CW (RHP)

→ REF +ve direction : CW in GH

→ NSC in 1 + GH

$N = Z_c - P_c$

$N = 0$

$P_c = ?$ $Z_c = ?$

→ NSC in GH

$N = Z_c - P_c$

$N = 2$

$Z_c = 1$

→ $P_c = -1$

↓

Invalid

CASE-3 :

→ S-plane contour : ACW (RHP)

→ REF +ve Direcⁿ in GH : ACW

→ NSC in 1 + GH

$N = Z_c - P_c$

$N = 0$

$P_c = ?$

↳ → $0 = Z_c - 3$

→ $\boxed{Z_c = 3}$

→ NSC in GH

$N = Z_c - P_c$

$N = -2$

$Z_c = 1$

$-2 = 1 - P_c$

$\boxed{P_c = 3}$

↵

CASE 4 :

→ S plane contour : ACW (RHP)

→ REF +ve direⁿ in GH : CW

→ NSC in 1 + GH

$N = P_c - Z_c$

→ NSC in GH

$N = P_c - Z_c$

$N = 2$

$Z_c = 1$

$2 = P_c - 1$

$\boxed{P_c = 3}$

↵

$N = 0$

$P_C = 3$

$\rightarrow \boxed{Z_C = 3} \rightarrow$ 1+GH plot will have 3 roots / 3 poles in CLS R.H.P

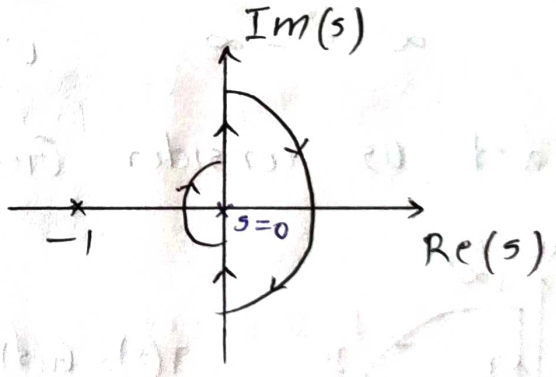
Question: The open loop T.F of a unity gain -ve feedback system is given as

$$G(s) = \frac{1}{s(s+1)}$$

The Nyquist contour in the s-plane encloses the entire right half plane and a small neighbourhood around the origin in the left half plane as shown. The no. of encirclements of point $(-1+j0)$ by the Nyquist plot of $G(s)$. Corresponding to the Nyquist contour is denoted as N then N equals

A) 0 B) 1

C) 2 D) 3



S-1: s plane contour : CW

S-2: REF +ve direction : ACW

S-3: NSC in 1+GH

$$1+GH = \frac{\overbrace{s^2+s+1}^{Z_C = 0}}{s(s+1)}$$

$N = P_C - Z_C$

$P_C = \emptyset 1$

$Z_C = 0$

$\boxed{\therefore N = 1} \rightarrow$ GH plot encircles (-1, 0) once in ACW.

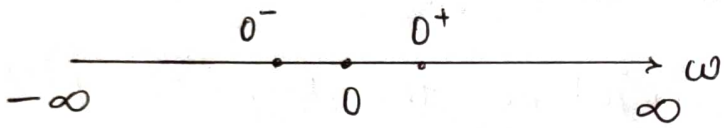
Plotting the Nyquist plot

Assumptions:

1. Nyquist contour is clockwise.
2. Nyquist contour does not contain pole or zero on jω axis.
3. Mapping of CW Nyquist contour is to be performed on $G(s)H(s)$ plane.

Prerequisite:

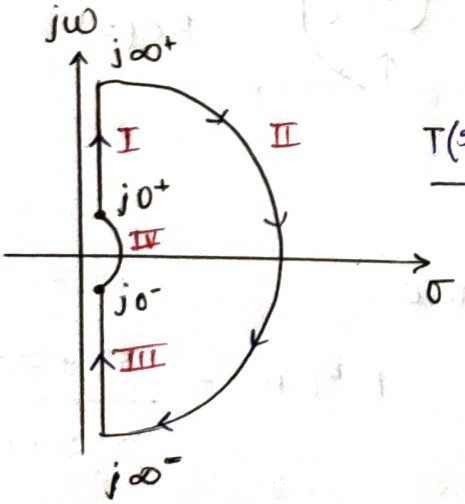
1. Range of ω: $-\infty < \omega < \infty$



$\infty^+ = +\infty$ $\quad 0^+ = +0$

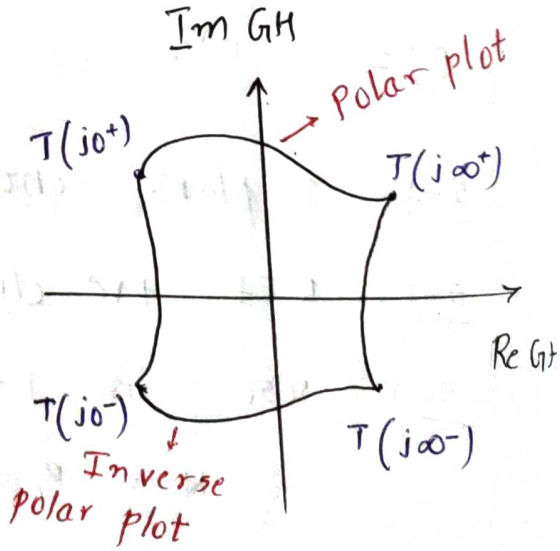
$\infty^- = -\infty$ $\quad 0^- = -0$

2. Let us consider Generised Nyquist Contour



NYQUIST CONTOUR

$T(s) = G(s)H(s)$



NYQUIST PLOT

2. Closing of Nyquist plot:

Nyquist contour	Nyquist Plot

1. $s = j\omega \rightarrow 0^+ < \omega < \infty^+ \longrightarrow T(s)$: $T(j0^+)$ to $T(j\infty^+)$

2. $s = Re^{j\theta} \rightarrow \infty^+ < \omega < \infty^- \longrightarrow T(s)$: $T(j\infty^+)$ to $T(j\infty^-)$

3. $s = j\omega \rightarrow \infty^- < \omega < 0^- \longrightarrow T(s)$: $T(j\infty^-)$ to $T(j0^-)$

4. $s = Re^{j\theta} \rightarrow 0^- < \omega < 0^+ \longrightarrow T(s)$: $T(j0^-)$ to $T(j0^+)$

3. Segment I $\quad s = j\omega \quad 0 < \omega < \infty$

Segment II $\quad s = Re^{j\theta} \quad R \rightarrow \infty$

$\theta = \frac{\pi}{2}$ to $\frac{-\pi}{2}$ C.W

segment III $\quad s = j\omega \quad -\infty < \omega < 0$

Segment IV $\quad s = Re^{j\theta} \quad R \rightarrow 0$

$\theta = -\pi/2$ to $\pi/2$ A.C.W

4.* $\theta \longrightarrow \frac{\pi}{2}$ to $\frac{-\pi}{2}$: CW

$-\theta \longrightarrow \frac{-\pi}{2}$ to $\frac{\pi}{2}$: ACW

5. $\rightarrow T(s) = \frac{1}{s+1}$

Segment 1 $\quad s = j\omega \quad (0 < \omega < \infty)$

$$T(j\omega) = \frac{1}{1+j\omega}$$

Method - I:

$$|T(j\omega)| = \frac{1}{\sqrt{1+\omega^2}}$$

$$\angle T(j\omega) = -\tan^{-1}\omega$$

$$\boxed{T(j\omega) = \frac{1}{\sqrt{1+\omega^2}} \angle -\tan^{-1}\omega}$$

→ $T(j0) = 1 \angle 0°$

$T(j\infty) = 0 \angle -90°$

Method - II:

$$T(s) = \frac{1}{1+s} \rightarrow T(j\omega) = \frac{1}{1+j\omega}$$

$\omega = 0$: $T(j0) = \frac{1}{1+j0} = 1 = 1 \angle 0°$

$\omega = \infty$: $T(j\infty) = \frac{1}{1+j\infty} = 0 = 0 \angle -90°$

6.

1. Mapping of segment I on G(s)H(s) plane

   ↳ Polar plot

2. Mapping of segment III on G(s)H(s) plane

   ↳ Inverse polar plot.

NOTE:

Inverse Polar plot is mirror image of polar plot w.r.t horizontal axis by keeping direction of flow same.

Steps to Draw Polar Plot:

- Polar plot of $T(s)$

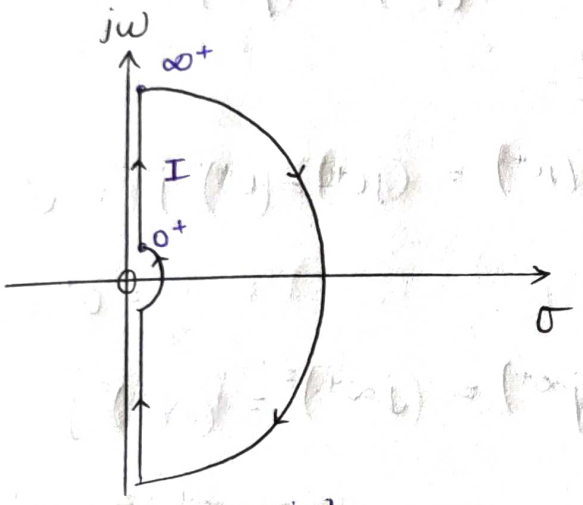
step 1: Put $s = j\omega$: $T(j\omega)$ $\quad 0^+ < \omega < \infty^+$

step 2: $T(j0^+) = M_1 \angle \phi_1$

$T(j\infty^+) = M_2 \angle \phi_2$

step 3: Rationalize $T(j\omega) = Re\{T\} + j\,Im\{T\}$

→ **Question:** $T(s) = s$. Draw Polar plot.



Nyquist contour

S-1: $s = j\omega \quad (0^+ < \omega < \infty^+)$

$T(j\omega) = j\omega$

S-2:

$T(j0^+) = j0^+ = 0^+ \angle 90^\circ$

$T(j\infty^+) = j\infty^+ = \infty^+ \angle 90^\circ$

↓

Rationalize

S-3: $T(j\omega) = 0 + j\omega \rightarrow Re(T) = 0$

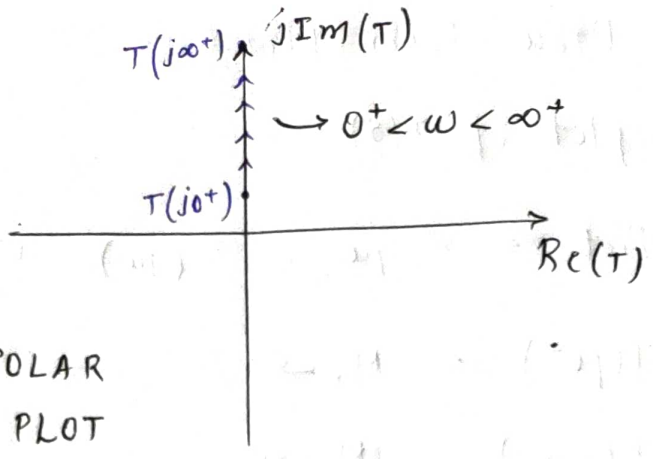
NOTE: $T(j\infty^+) = 0 + j\infty^+ \quad Im(T) = \omega$

As $0^+ < \omega < \infty^+$ $Re(T) = 0$

$Im(T) =$ positi

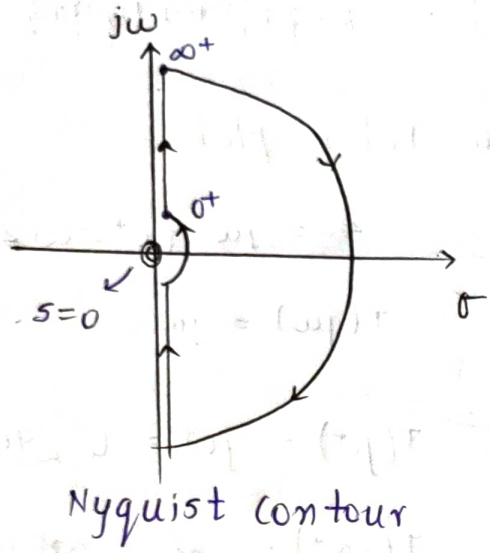
↓

Polar plot always lies on +ve Imaginary Axis of the $T(s)$ plane.



POLAR PLOT

→ Question: $T(s) = s^2$. Draw polar plot.



Nyquist contour

S-1: $s = j\omega \quad (0^+ < \omega < \infty^+)$

$T(j\omega) = (j\omega)^2$

S-2:

$T(j0^+) = (j0^+)^2 = (0^+)^2 j^2 = 0^+ j^2$

$= 0^+ \angle 180^\circ$

$T(j\infty^+) = (j\infty^+)^2 = (\infty^+)^2 j^2$

$= \infty^+ j^2 = \infty^+ \angle 180^\circ$

↓ Rationalize

S-3:

$T(j\omega) = -\omega^2 + j0$

NOTE: $T(j\infty^+) = -\infty^+ + j0$

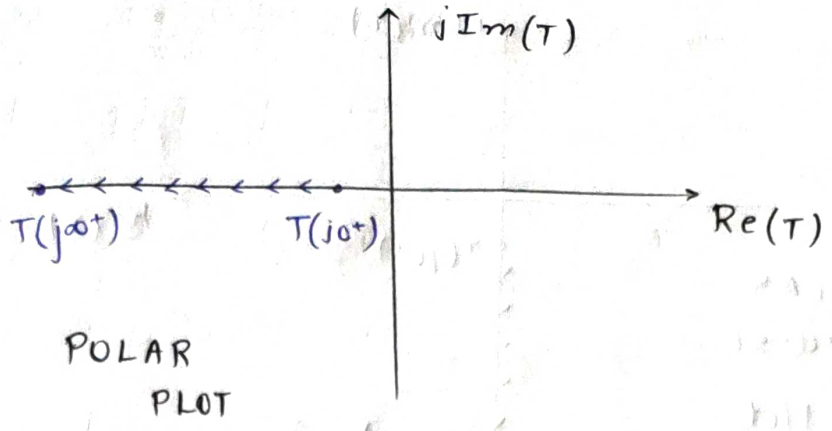
$Re(T) = -\omega^2$ → As $0^+ < \omega < \infty^+$

$Im(T) = 0$ $\quad Re(T) = -ve$

$Im(T) = 0$

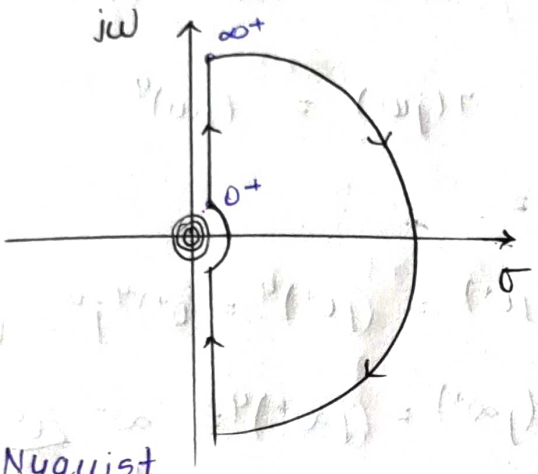
↓

Polar plot always remains on −ve Real Axis of T(s) plane.



POLAR PLOT

→ **Question:** $T(s) = s^3$. Draw Polar Plot



Nyquist Contour

S-1: $s = j\omega \quad (0^+ < \omega < \infty^+)$

$T(s) = (j\omega)^3$

S-2:

$T(j0^+) = (j0^+)^3 = (0^+)^3 j^3$

$= 0^+ \angle 270°$

$T(j\infty^+) = (j\infty^+)^3 = (\infty^+)^3 j^3$

$= \infty^+ \angle 270°$

↓ Rationalise

S-3:

$T(j\omega) = 0 - j\omega^3$

NOTE: $T(j\infty^+) = 0 - j\infty^+$

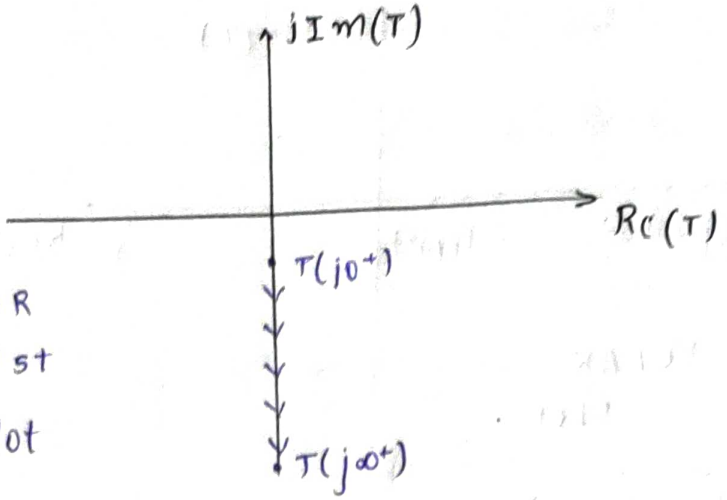
$Re(T) = 0$ → As $0^+ < \omega < \infty^+$

$Im(T) = -\omega^3$ $\quad Re(T) = 0$

$Im(T) = -ve$

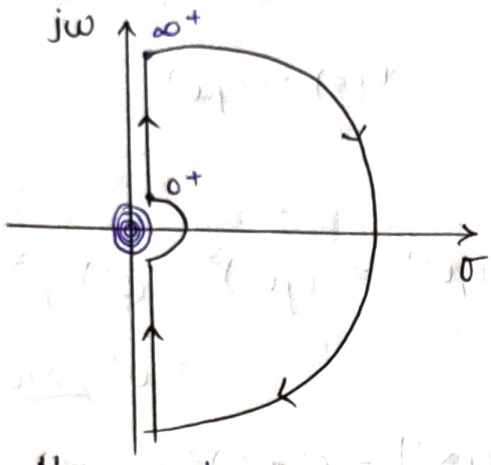
↓

Polar plot always remains on -ve Imaginary Axis of T(s) plane.



POLAR Nyquist Plot

→ **Question:** $T(s) = s^4$. Draw Polar Plot



Nyquist contour

S-1: $T(j\omega) = (j\omega)^4$

S-2:

$T(j0^+) = (j0^+)^4 = (0^+)^4 j^4 = 0^+ \angle 0°$

$T(j\infty^+) = (j\infty^+)^4 = \infty^+ \angle 0°$

↓

Rationalise

S-3: $T(j\omega) = \omega^4 + j0$

NOTE: $T(j\infty^+) = \infty^+ + j0$

→ $Re(T) = \omega^4$

$Im(T) = 0$

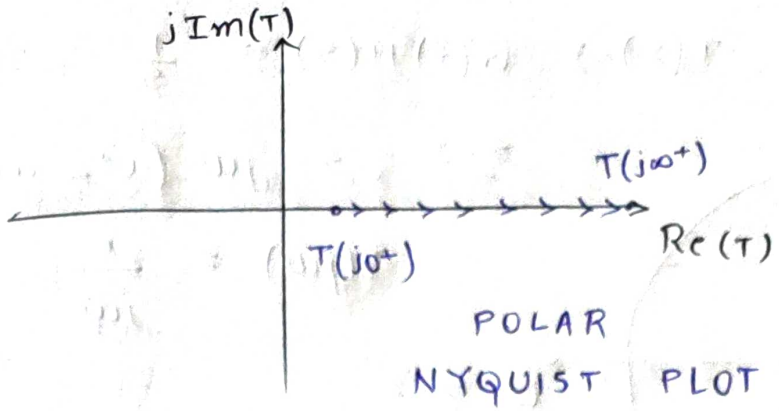
↓

As $0^+ < \omega < \infty^+$ $Re(T) = +ve$

$Im(T) = 0$

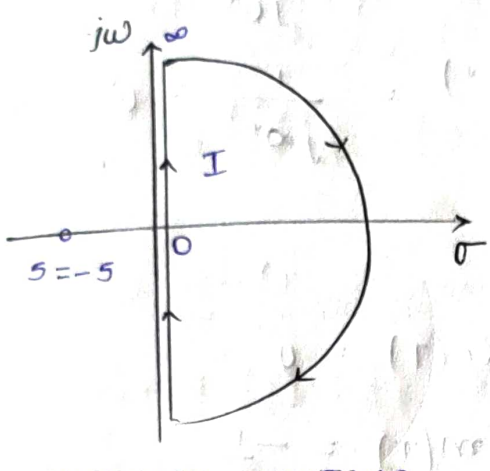
↓

Polar plot always remains on +ve Real Axis of T(s) plane.



POLAR
NYQUIST PLOT

→ Question: $T(s) = s + 5$. Draw Polar Plot.



NYQUIST CONTOUR

S-1: $T(j\omega) = j\omega + 5$

S-2: $T(j0^+) = 5 + j0^+ = 5 \angle 0^\circ$

$T(j\infty^+) = 5 + j\infty^+ = j\infty^+$

$= \infty^+ \angle 90^\circ$

S-3: $T(j\omega) = 5 + j\omega$

NOTE: $T(j\infty^+) = 5 + j\infty^+$

$Re(T) = 5$

$Im(T) = \omega$

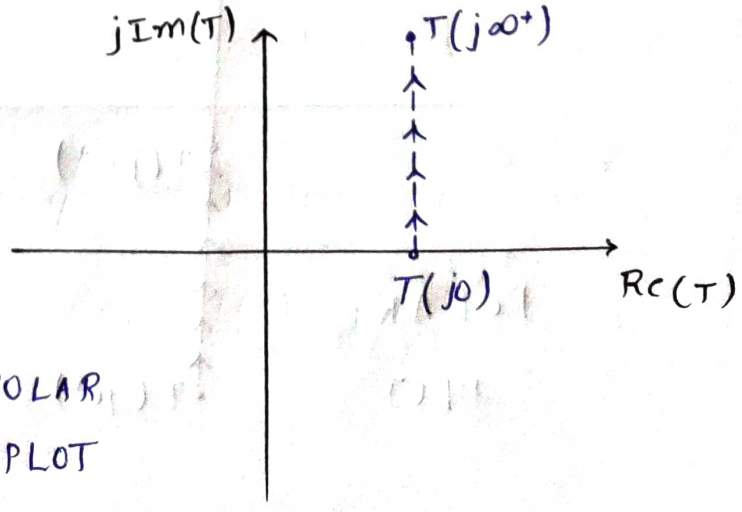
As $0^+ < \omega < \infty^+$

$Re(T) = 5 = +ve$

$Im(T) = \omega = +ve$

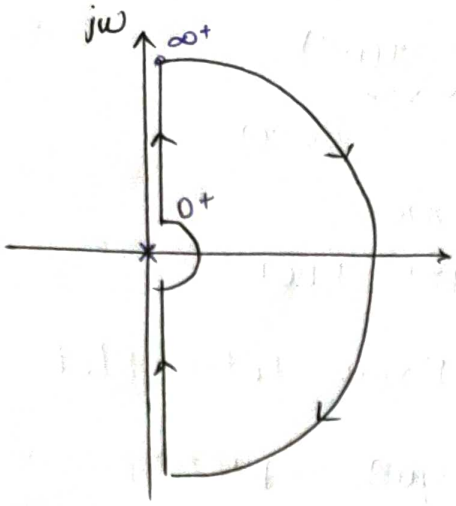
↓

Polar plot will always remains in I-Q on a line ||el to +ve Imaginary Axis in T(s) plane.



POLAR
PLOT

Question: $T(s) = G(s)H(s) = \frac{1}{s}$



NYQUIST CONTOUR

S-1: $s = j\omega \quad (0^+ < \omega < \infty^+)$

$$T(j\omega) = \frac{1}{j\omega}$$

S-2:

Rationalise ↑

$$T(j0^+) = \frac{1}{j0^+} = \infty^+ \angle -90^\circ$$

$$T(j\infty^+) = \frac{1}{j\infty^+} = 0^+ \angle -90^\circ$$

S-3:

$$T(j\omega) = 0 + j\left(\frac{-1}{\omega}\right) \rightarrow Re(T) = 0$$

NOTE:

$$T(j0^+) = 0 + j(-\infty^+) \qquad Im(T) = -\frac{1}{\omega}$$

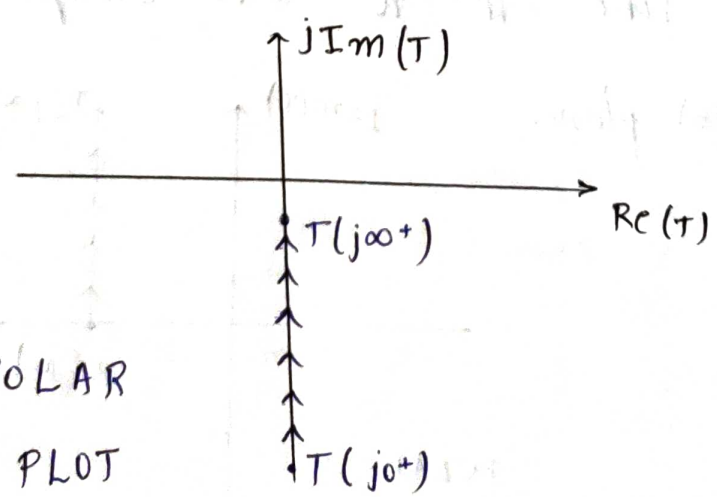
↓

As $0^+ < \omega < \infty^+$, $Re(T) = 0$

$Im(T) = -Ve$

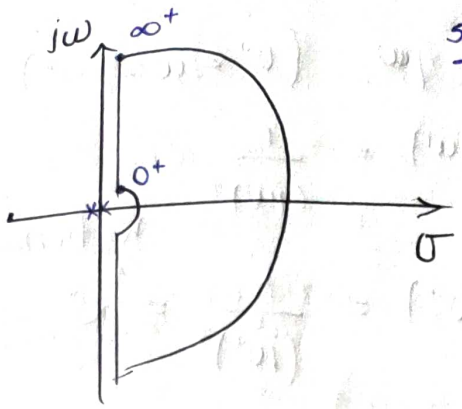
↓

Polar plot always remains on −ve Imaginary Axis of T(s) plane.



POLAR PLOT

Question : $T(s) = \frac{1}{s^2}$



NYQUIST CONTOUR

S-1 : $s = j\omega \quad (0^+ < \omega < \infty^+)$

$$T(j\omega) = \frac{1}{(j\omega)^2}$$

S-2 :

$$T(j0^+) = \frac{1}{(j0^+)^2} = \frac{1}{0^+ j^2}$$

$$= \infty^+ \angle -180^\circ$$

Rationalise

$$T(j\infty^+) = \frac{1}{(j\infty^+)^2} = \frac{1}{\infty^+ j^2}$$

S-3 :

$$T(j\omega) = -\frac{1}{\omega^2} + j0 \qquad = 0^+ \angle -180^\circ$$

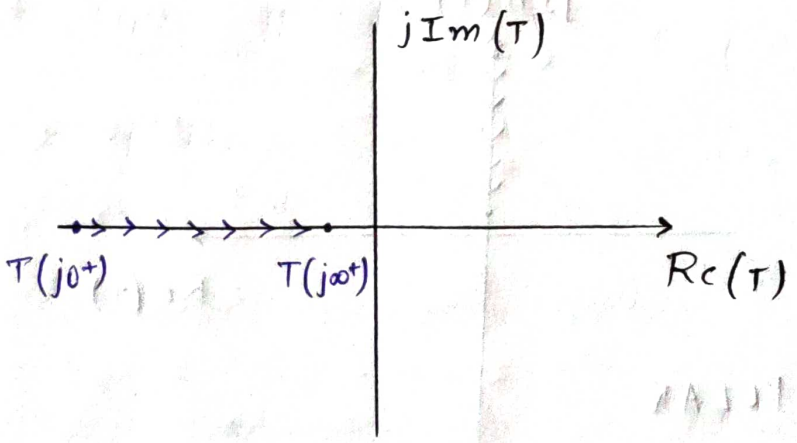
NOTE: $T(j0^+) = -\infty + j0$

$Re(T) = -\frac{1}{\omega^2}$ → As $0^+ < \omega < \infty^+$, $Re(T) = -ve$

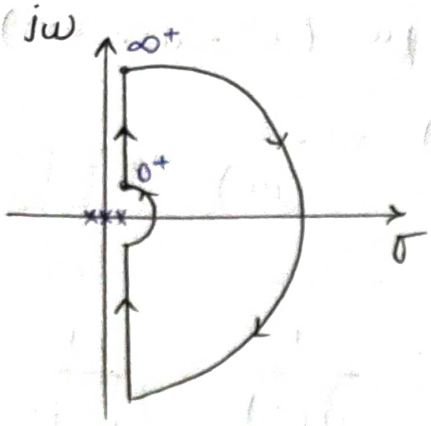
$Im(T) = 0$ $\qquad Im(T) = 0$

↓

Polar plot always remains on -ve Real Axis of T(s) plane.



→ **Question :** $T(s) = \frac{1}{s^3}$



NYQUIST CONTOUR

S-1: $s = j\omega \quad (0^+ < \omega < \infty^+)$

$$T(j\omega) = \frac{1}{(j\omega)^3}$$

Rationalise

S-2:

$$T(j0^+) = \frac{1}{(j0^+)^3} = \infty^+ \angle -270^\circ$$

$$T(j\infty^+) = \frac{1}{(j\infty^+)^3} = 0^+ \angle -270^\circ$$

S-3: $T(j\omega) = 0 + j\left(\frac{1}{\omega^3}\right) \rightarrow Re(T) = 0$

NOTE: $T(j0^+) = 0 + j\infty^+ \qquad Im(T) = \frac{1}{\omega^3}$

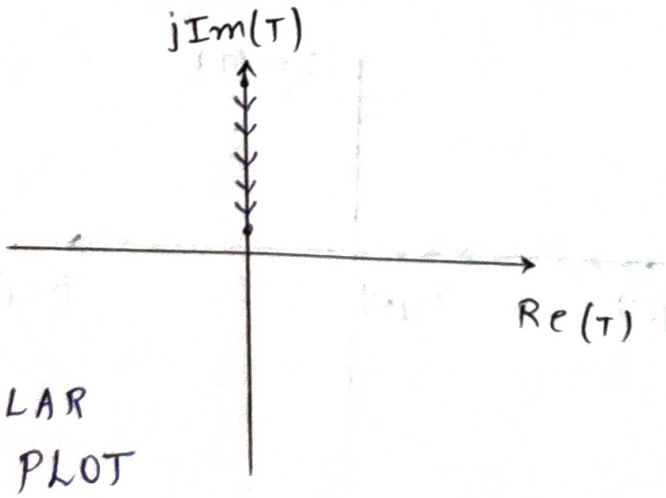
↓

As $0^+ < \omega < \infty^+$, $Re(T) = 0$

$Im(T) = +ve$

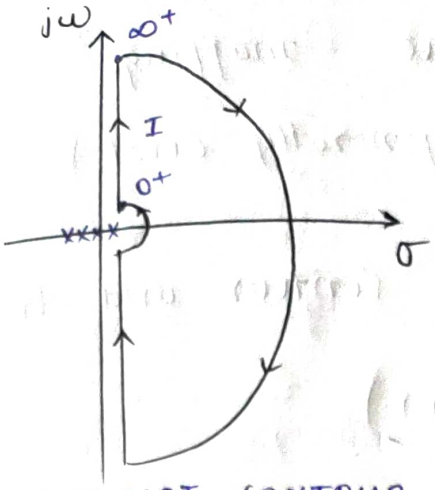
↓

Polar plot always remains on +ve Imaginary Axis of $T(s)$ plane.



POLAR PLOT

Question: $T(s) = \frac{1}{s^4}$



NYQUIST CONTOUR

5-1: $s = j\omega \quad (0^+ < \omega < \infty^+)$

$$T(j\omega) = \frac{1}{(j\omega)^4}$$

5-2: $$T(j0^+) = \frac{1}{(j0^+)^4} = \infty^+ \angle 0^\circ$$

$$T(j\infty^+) = \frac{1}{(j\infty^+)^4} = 0^+ \angle 0^\circ$$

5-3: $$T(j\omega) = \frac{1}{\omega^4} + j0$$

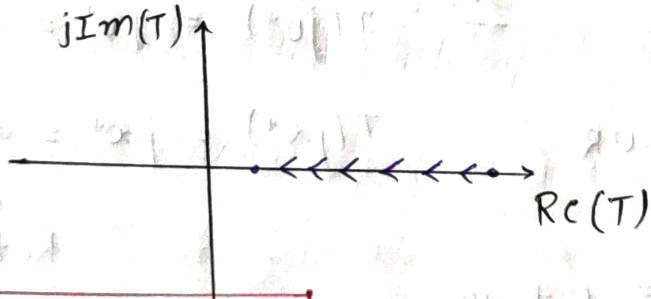
$$T(j0^+) = \infty^+ + j0$$

$Re(T) = \frac{1}{\omega^4}$ → As $0^+ < \omega < \infty^+$, $Re(T) = +ve$

$Im(T) = 0$ $\qquad$ $Im(T) = 0$

↓

Polar plot always remains on +ve Real Axis of T(s) plane.



NYQUIST PLOT

Step 1: Draw Pole-Zero diagram in s-plane and select proper contour.

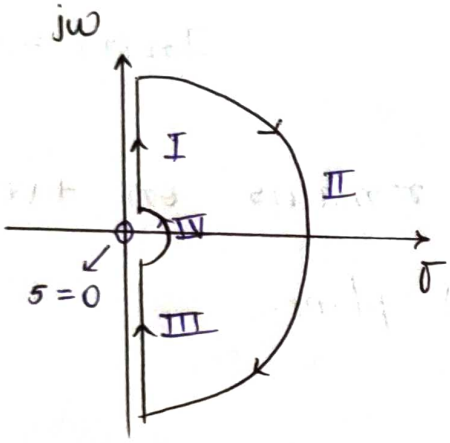
Step 2: Map segment (I) of contour and draw Polar Plot.

Step 3: Map segment (II) of contour and draw the respective mapping. (generally circle)

Step 4: Map segment (III) of contour and draw Inverse Polar plot. (IPP).

Step 5: Map segment (IV) of contour and draw respective mapping (generally circle).

→ Question: Draw Nyquist plot of $T(s) = s$.



NYQUIST CONTOUR

Segment - I

5-1: $s = j\omega \quad (0^+ < \omega < \infty^+)$

$T(j\omega) = j\omega$

5-2: $T(j0^+) = j0^+ = 0^+ \angle 90°$

$T(j\infty^+) = j\infty^+ = \infty^+ \angle 90°$

↓ Rationalise

5-3: $T(j\omega) = 0 + j\omega$ ⟶

NOTE: $T(j\infty^+) = 0 + j\infty^+$

$Re(T) = 0$

$Im(T) = \omega$

↓

As $0^+ < \omega < \infty^+$ $Re(T) = 0$

↓ $Im(T) = +ve$

Polar plot always remains on +ve Imaginary Axis of the T(s) plane.

Segment - II

$S = Re^{j\theta}$, $R \to \infty$

$\theta = \frac{\pi}{2}$ to $-\frac{\pi}{2}$ C.W

$$s = \lim_{R\to\infty} Re^{j\theta}$$

$$T(s) = s = \lim_{R\to\infty} Re^{j\theta} = \infty e^{j\theta}$$

↓

Circle of ∞ Radius

with θ : $\frac{\pi}{2}$ to $-\frac{\pi}{2}$ C.W

Segment - III

I.P.P

Segment - IV

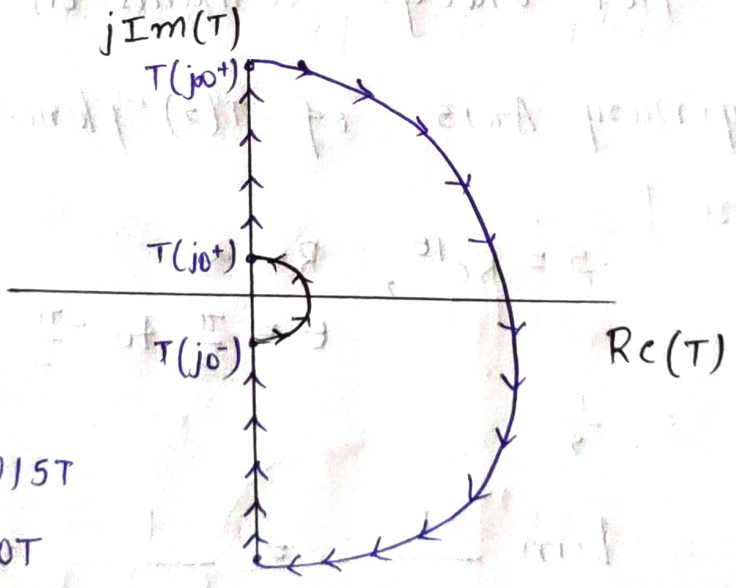
$S = Re^{j\theta}$, $R \to 0$

$\theta = -\frac{\pi}{2}$ to $\frac{\pi}{2}$ A.C.W

$T(s) = s$

$$T(s) = \lim_{R\to 0} Re^{j\theta} = 0e^{j\theta} \to 0^{+} \text{ Radius circle}$$

$\theta = -\frac{\pi}{2}$ to $\frac{\pi}{2}$ A.C.W



NYQUIST PLOT

NSC in $T(s) = s$:

$N = P_c - Z_c$

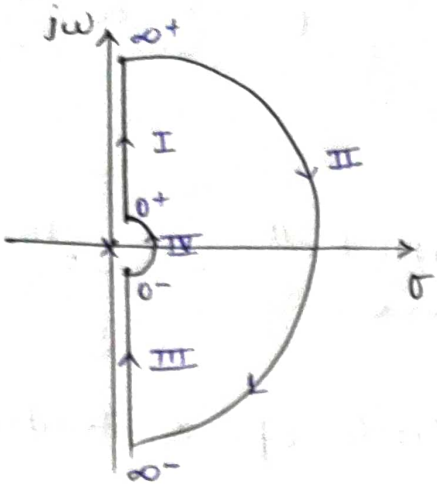
L.H.S $N = 0$

R.H.S $P_c = 0$

$Z_c = 0$

L.H.S = R.H.S

Question: $T(s) = \frac{1}{s}$



Segment-I

5-1: $s = j\omega \quad (0^+ < \omega < \infty^+)$

$$T(s) = \frac{1}{j\omega}$$

5-2:

$$T(j0^+) = \frac{1}{j0^+} = \infty^+ \angle -90^\circ$$

(Rationalise)

$$T(j\infty^+) = \frac{1}{j\infty^+} = 0^+ \angle -90^\circ$$

5-3: $T(j\omega) = 0 + j\left(\frac{-1}{\omega}\right) \rightarrow Re(T) = 0$

NOTE:

$T(j0^+) = 0 - j\infty$

$Im(T) = -\frac{1}{\omega}$

↓

As $0^+ < \omega < \infty^+$, $Re(T) = 0$

$Im(T) = -ve$

↓

Polar plot always remains on −ve Imaginary Axis of $T(s)$ plane.

Segment - II

$s = Re^{j\theta}, \quad R \rightarrow \infty$

$\theta = \frac{\pi}{2}$ to $\frac{-\pi}{2}$ C.W.

$$T(s) = \frac{1}{s}$$

$$T(s) = \lim_{R\to\infty} \frac{1}{Re^{j\theta}} = \frac{1}{\infty e^{j\theta}} = 0^+ e^{-j\theta}$$

→ 0 Radius circle with

$-\theta$: $\frac{-\pi}{2}$ to $\frac{\pi}{2}$ A.C.W

Segment - III IPP

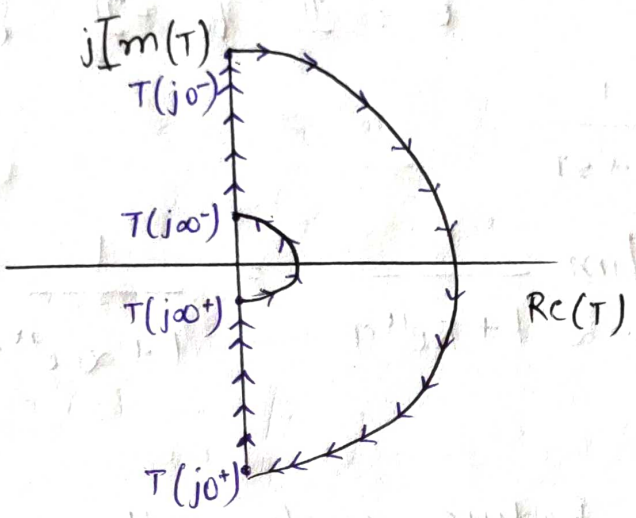
Segment - IV $s = Re^{j\theta}$, $R \to 0$

θ : $-\frac{\pi}{2}$ to $\frac{\pi}{2}$ A·C·W

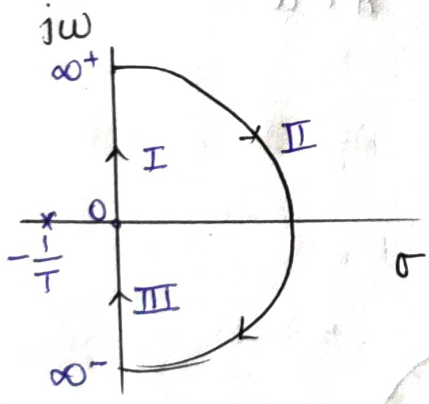
$$T(s) = \lim_{R \to 0} \frac{1}{Re^{j\theta}} = \frac{1}{0e^{j\theta}} = \infty e^{j\theta}$$

→ ∞ Radius circle with

$-\theta$: $\frac{\pi}{2}$ to $-\frac{\pi}{2}$ C·W·



→ Question: $T(s) = \frac{1}{1+sT}$. Draw Nyquist Plot



NYQUIST CONTOUR

Segment I

S-1: $s = j\omega$ $(0 < \omega < \infty)$

$$T(s) = \frac{1}{1+j\omega T}$$

S-2:

$$T(j0) = \frac{1}{1+j0T} = 1\angle 0^\circ$$

$$T(j\infty^+) = \frac{1}{1+j\infty^+ T} = \frac{1}{j\infty^+}$$

$$= 0^+ \angle -90^\circ$$

S-3 : $T(j\omega) = \frac{1}{1+\omega^2T^2} + j\left[\frac{-\omega T}{1+\omega^2T^2}\right]$

$Re(T) = \frac{1}{1+\omega^2T^2}$ As $0 < \omega < \infty^+$,

$Im(T) = \frac{-\omega T}{1+\omega^2T^2}$ $Re(T) = +ve$

$Im(T) = -ve$

↓

IV QUAD

Segment-II

$s = Re^{j\theta}$, $R \to \infty$

$\theta : \frac{\pi}{2}$ to $\frac{-\pi}{2}$ c·w.

$T(s) = \frac{1}{1+sT}$

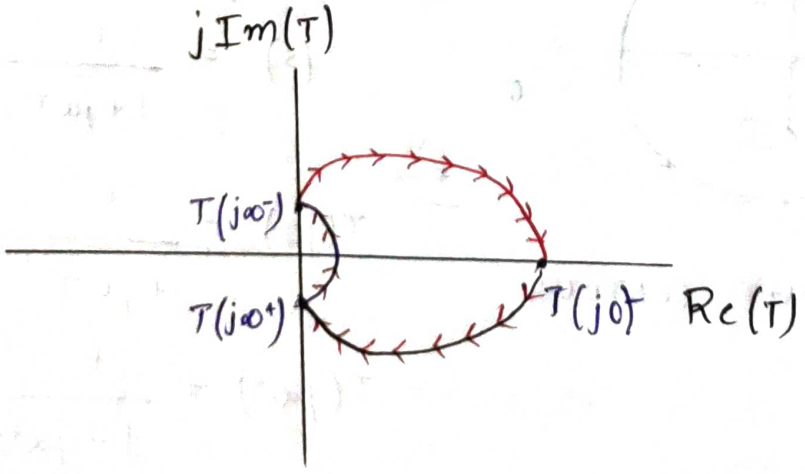
$T(s) = \lim_{R\to\infty} \frac{1}{1+Re^{j\theta}T} = \frac{1}{1+\infty e^{j\theta}T} = \frac{1}{\infty e^{j\theta}}$

$= 0^+ e^{-j\theta}$

→ 0 Radius circle with

$-\theta : -\frac{\pi}{2}$ to $\frac{\pi}{2}$ A·C·w

Segment-III IPP



NYQUIST PLOT

NSC in $GH = \frac{1}{1+sT}$:

$N = P_c - Z_c$

L·H·S

$N = 0$

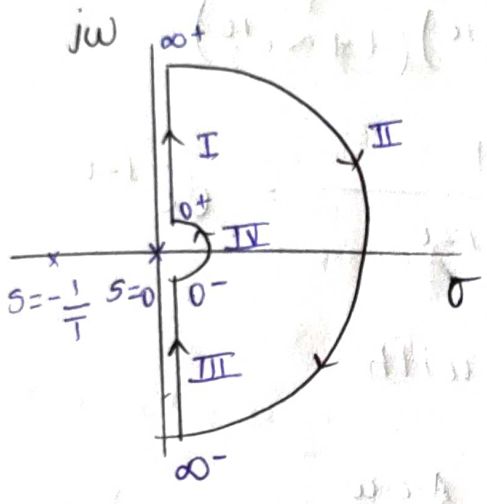
R·H·S

$P_c = 0$

$Z_c = 0$

$L \cdot H \cdot S = R \cdot H \cdot S$

→ Question: $T(s) = \frac{1}{s(1+sT)}$



Segment-I

S-1: $s = j\omega \quad (0^+ < \omega < \infty^+)$

$$T(s) = \frac{1}{j\omega(1+j\omega T)}$$

S-2: (Rationalise ↑)

$$T(j0^+) = \frac{1}{j0^+(1+j0^+)} = \frac{1}{j0^+} = \infty^+ \angle -90^\circ$$

S-3:

$$T(j\infty^+) = \frac{1}{j\infty^+(1+j\infty^+)} = \frac{1}{j^2\infty^2} = 0^+ \angle -180^\circ$$

$$T(j\omega) = \frac{1}{j\omega(1+j\omega T)} \cdot \frac{(1-j\omega T)}{(1-j\omega T)}$$

$$T(j\omega) = \frac{-j(1-j\omega T)}{\omega(1+\omega^2T^2)}$$

$$T(j\omega) = \frac{-\omega T}{\omega(1+\omega^2T^2)} + j\left(\frac{-1}{\omega(1+\omega^2T^2)}\right)$$

NOTE: $T(j0^+) = -T + j(-\infty)$

$$Re(T) = \frac{-T}{1+\omega^2T^2}$$

As $0^+ < \omega < \infty^+$,

$Re(T)$: -ve

$Im(T)$: -ve

$\rightarrow$ III QUAD

$$Im(T) = \frac{-1}{\omega(1+\omega^2T^2)}$$

Segment - II $s = Re^{j\theta}$, $R \rightarrow \infty$

$\theta : \frac{\pi}{2}$ to $\frac{-\pi}{2}$ C.W.

$$T(s) = \frac{1}{s(1+sT)} = \frac{1}{(\infty e^{j\theta})(1+\infty e^{j\theta})}$$

$$= \frac{1}{\infty^2 e^{j2\theta}} = 0^+ e^{-j2\theta}$$

$\rightarrow$ 0 Radius Circle with

-2θ : -180° to 180° A.C.W

Segment - III IPP

Segment - IV $s = Re^{j\theta}$, $R \rightarrow 0$

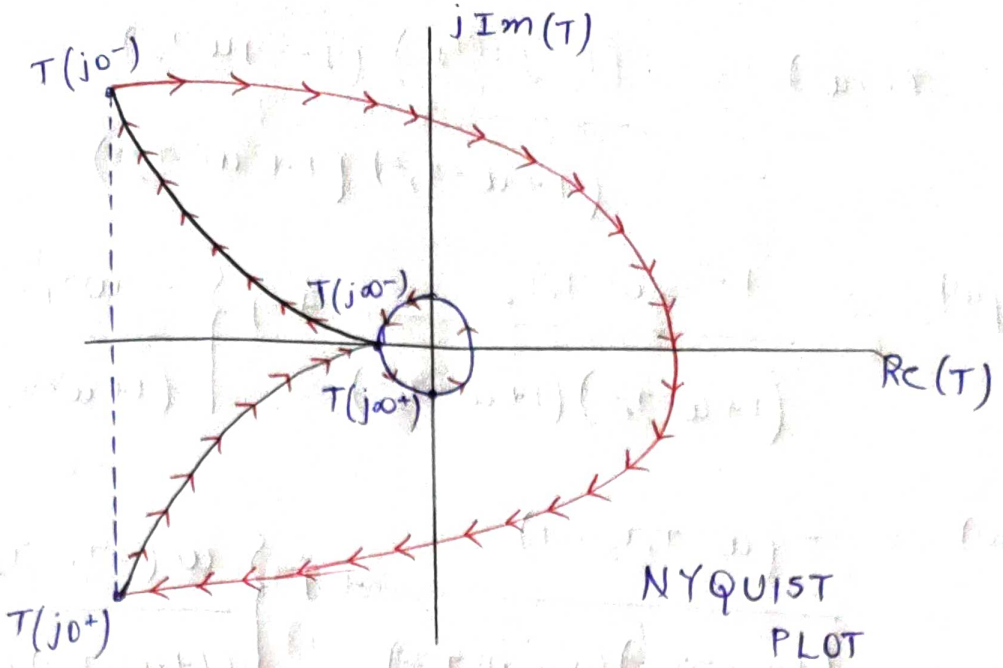
$\theta : \frac{-\pi}{2}$ to $\frac{\pi}{2}$ A.C.W

$$T(s) = \frac{1}{s(1+sT)} = \frac{1}{0e^{j\theta}(1+0e^{j\theta})} = \frac{1}{0e^{j\theta}}$$

$$= \infty e^{-j\theta}$$

$\rightarrow \infty$ Radius circle with

$-\theta$: $\frac{\pi}{2}$ to $\frac{-\pi}{2}$ C.W.



NYQUIST PLOT

NSC in GH $= \frac{1}{s(1+sT)}$:

$$N = P_c - Z_c$$

L·H·S :

$N = 0$

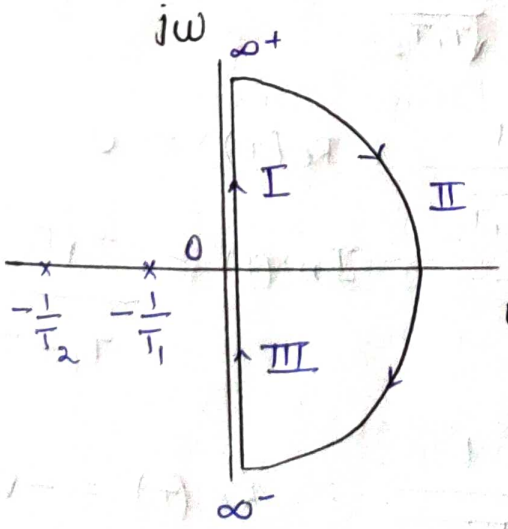
R·H·S :

$P_c = 0$

$Z_c = 0$

L·H·S = R·H·S

→ **Question:** $T(s) = \frac{1}{(1+sT_1)(1+sT_2)}$



Segment - I

S-1: $s = j\omega \quad (0 < \omega < \infty)$

$$T(j\omega) = \frac{1}{(1+j\omega T_1)(1+j\omega T_2)}$$

S-2:

$T(j0) = 1 \angle 0°$

$T(j\infty^+) = \frac{1}{j^2\omega^2} = 0^+ \angle -180°$

5-3: $T(j\omega) = \dfrac{(1-j\omega T_1)(1-j\omega T_2)}{(1+\omega^2 T_1^2)(1+\omega^2 T_2^2)}$

$$T(j\omega) = \frac{1-\omega^2 T_1 T_2}{(1+\omega^2 T_1^2)(1+\omega^2 T_2^2)} + j\left\{\frac{-\omega T_1 - \omega T_2}{(1+\omega^2 T_1^2)(1+\omega^2 T_2^2)}\right\}$$

$$T(j\omega) = \frac{-(\omega^2 T_1 T_2 - 1)}{(1+\omega^2 T_1^2)(1+\omega^2 T_2^2)} + j\left\{\frac{\omega(-T_1 - T_2)}{(1+\omega^2 T_1^2)(1+\omega^2 T_2^2)}\right\}$$

$$Re(T) = \frac{-(\omega^2 T_1 T_2 - 1)}{(1+\omega^2 T_1^2)(1+\omega^2 T_2^2)}$$

$$Im(T) = \frac{\omega(-T_1 - T_2)}{(1+\omega^2 T_1^2)(1+\omega^2 T_2^2)} = \frac{-\omega(T_1+T_2)}{(1+\omega^2 T_1^2)(1+\omega^2 T_2^2)}$$

→ As $0 < \omega < \infty^+$,

$\omega^2 T_1 T_2 - 1 < 0 \rightarrow \omega < \dfrac{1}{\sqrt{T_1 T_2}}$: $Re(T) = +ve$

$\omega^2 T_1 T_2 - 1 = 0 \rightarrow \omega = \dfrac{1}{\sqrt{T_1 T_2}}$: $Re(T) = 0$

$$Im(T) = \frac{-\sqrt{T_1 T_2}}{T_1 + T_2}$$

$\omega^2 T_1 T_2 - 1 > 0 \rightarrow \omega > \dfrac{1}{\sqrt{T_1 T_2}}$: $Re(T) = -ve$

$Im(T) = -ve$

ω	$Re(T)$	$Im(T)$	
$0 < \omega < \frac{1}{\sqrt{T_1 T_2}}$	$+ve$	$-ve$	→ IV QUAD
$\omega = \frac{1}{\sqrt{T_1 T_2}}$	0	$\frac{-\sqrt{T_1 T_2}}{T_1 + T_2}$	→ Cuts -ve jIm Axis
$\omega > \frac{1}{\sqrt{T_1 T_2}}$	$-ve$	$-ve$	→ III Quad

Segment - II $\quad s = \infty e^{j\theta}$, $\theta = 90^\circ$ to -90° C.W.

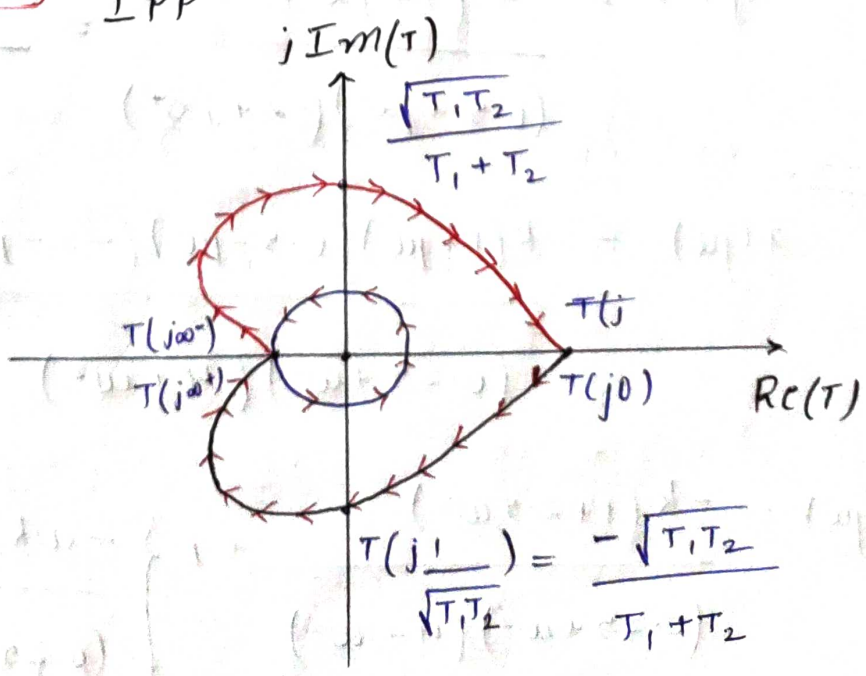
$$T(s) = \frac{1}{(1+\infty e^{j\theta} T_1)(1+\infty e^{j\theta} T_2)} = \frac{1}{\infty^2 e^{j2\theta}}$$

$$= 0^+ e^{-j2\theta}$$

→ 0 Radius Circle with

-2θ : -180° to 180° A.C.W

Segment - III IPP



NSC in $GH = \dfrac{1}{(1+sT_1)(1+sT_2)}$

$N = P_c - Z_c$

L·H·S:

$N = 0$

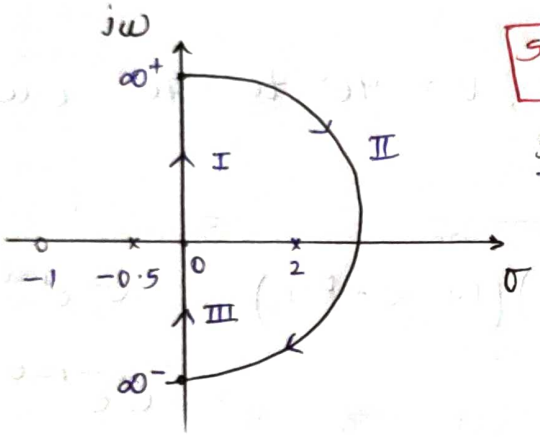
$L \cdot H \cdot S = R \cdot H \cdot S$

R·H·S

$P_c = 0$

$Z_c = 0$

→ Question: $G(s)H(s) = \dfrac{k(s+1)}{(s+0.5)(s-2)}$: $0 < k < \infty$



Segment - I

S-I: $s = j\omega \quad (0 < \omega < \infty^+)$

$$T(j\omega) = \frac{k(1+j\omega)}{(0.5+j\omega)(-2+j\omega)}$$

S-2:

$$T(j0) = \frac{k(1+j0)}{(0.5+j0)(-2+j0)} = k\angle 180^\circ$$

$$T(j\infty^+) = \frac{k(1+j\infty^+)}{(0.5+j\infty^+)(-2+j\infty^+)} = \frac{k}{j\infty^+} = 0^+\angle -90^\circ$$

S-3:

$$T(j\omega) = \frac{k(1+j\omega)(0.5-j\omega)(-2-j\omega)}{(0.25+\omega^2)(4+\omega^2)}$$

$$T(j\omega) = \frac{-k(1+2.5\omega^2)}{(0.25+\omega^2)(4+\omega^2)} + j\left\{\frac{-\omega k(\omega^2-0.5)}{(0.25+\omega^2)(4+\omega^2)}\right\}$$

As $0 < \omega < \infty^+$,

$$Re(T) = \frac{-k(1+2.5\omega^2)}{(0.25+\omega^2)(4+\omega^2)} \longrightarrow -ve$$

$$Im(T) = j\left\{\frac{-\omega k(\omega^2-0.5)}{(0.25+\omega^2)(4+\omega^2)}\right\}$$

$\omega < \sqrt{0.5} \longrightarrow Im(T) = +ve$

$\omega = \sqrt{0.5} \longrightarrow Im(T) = 0, \; Re(T) = \frac{-2k}{3}$

$\omega > \sqrt{0.5} \longrightarrow Im(T) = -ve$

ω	$Re(T)$	$Im(T)$	
$0 < \omega < \sqrt{0.5}$	$-ve$	$+ve$	→ II QUAD
$\omega = \sqrt{0.5}$	$-\frac{2}{3}k$	0	→ Cuts -ve Real Axis
$\omega > \sqrt{0.5}$	$-ve$	$-ve$	→ III QUAD

Segment-II

$s = \infty e^{j\theta}$, θ: 90° to -90° c.w.

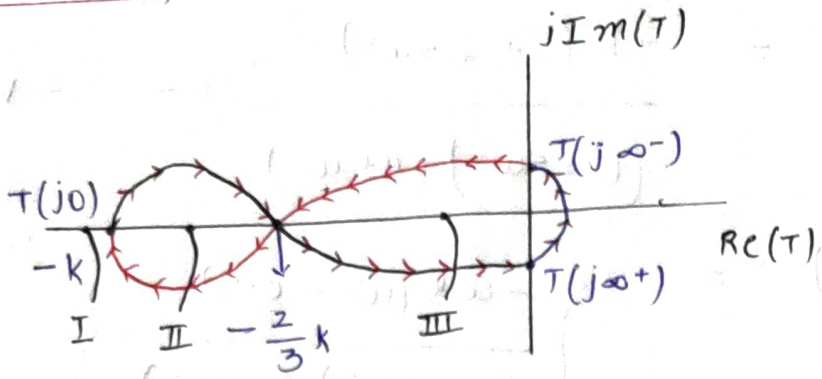
$$T(s) = \frac{k(s+1)}{(s+0.5)(s-2)} = \frac{k(\infty e^{j\theta}+1)}{(\infty e^{j\theta}+0.5)(\infty e^{j\theta}-2)}$$

$$= \frac{k\infty e^{j\theta}}{(\infty e^{j\theta})^2}$$

→ 0 Radius circle with

$-\theta$: -90° to 90° A.c.w $= 0^+ e^{-j\theta}$

Segment - III I·P·P



NYQUIST PLOT

→ NSC in GH = $\frac{k(s+1)}{(s+0.5)(s-2)}$:

$N = P_c - Z_c$

L·H·S : $N = 1$ L·H·S = R·H·S

R·H·S : $P_c = 1$

$Z_c = 0$

→ NSC in 1 + GH :

$N = P_c - Z_c$

I, II, III - Possible positions of (-1,0).

I: $K < 1$ $N = 0$

$P_c = 1$

$0 = 1 - Z_c \rightarrow \boxed{Z_c = 1} \rightarrow$ CLS UNSTABLE

II: $(K > 1) \cap \left(\frac{2}{3}K < 1\right)$ $N = -1$

$P_c = 1$

$1 < K < 1.5$

$-1 = 1 - Z_c$

$\boxed{Z_c = 2}$

↳ CLS UNSTABLE

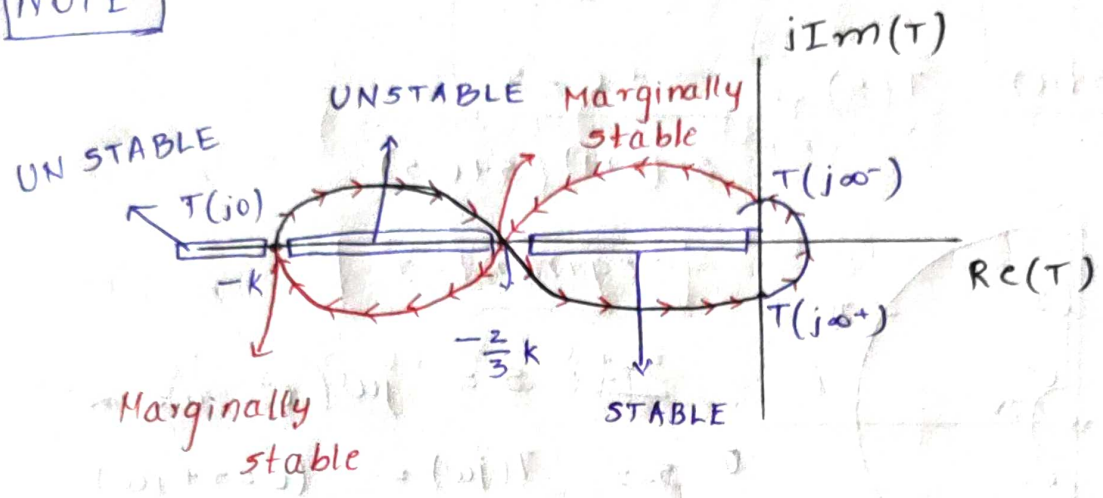
III $\frac{2}{3}K > 1$ $\quad N = 1$

$K > 1.5$ $\quad P_c = 1$

$1 = 1 - Z_c$

$\boxed{Z_c = 0} \rightarrow$ CLS STABLE

NOTE

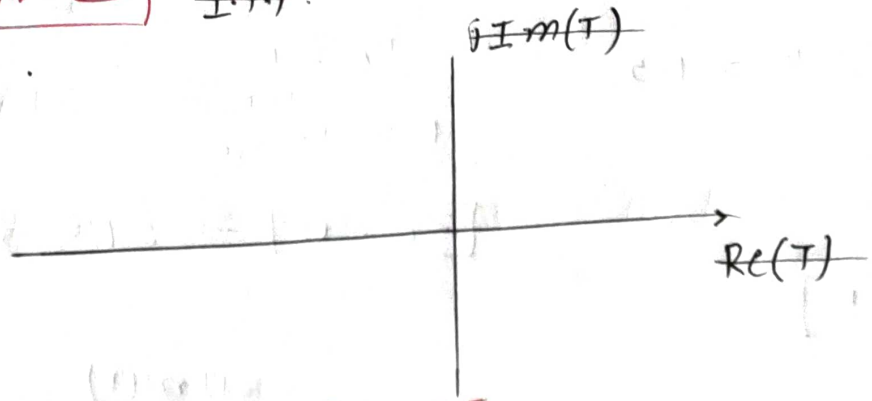


NOTE :

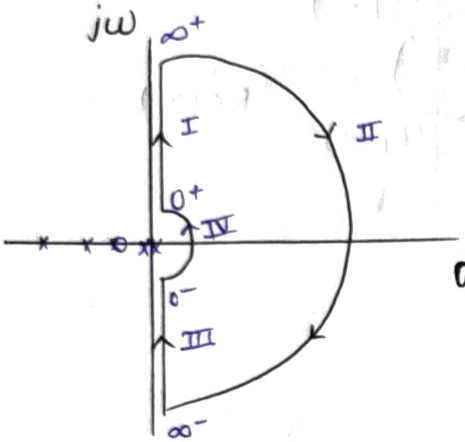
If $G(s)H(s) = kf(s)$; $0 < k < \infty$ and Nyquist Plot needs to be drawn

1. Coordinates on $G(s)H(s)$ plane will be f^n of k.
2. As k varies, (-1, 0) point varies.
3. For different placing of (-1,0) point, different ranges of k ax obtained.
4. Each Range belongs to stable / Unstable Region of C.L.S
5. Nature of stability at TRANSITION POINT can also be identified.

~~Segment - III~~ ~~I.P.P~~



→ Question: $T(s) = \dfrac{s + 0.25}{s^2(s+1)(s+0.5)}$



NYQUIST CONTOUR

Segment - I

5-1: $s = j\omega \quad (0^+ < \omega < \infty^+)$

5-2: $T(j\omega) = \dfrac{(0.25 + j\omega)}{(j\omega)^2(1+j\omega)(0.5+j\omega)}$

$$T(j0^+) = \frac{(0.25 + j0^+)}{(j0^+)^2(1+j0^+)(0.5+j0^+)}$$

Rational-ise $= \infty^+ \angle -180^\circ$

5-3: $T(j\infty^+) = \dfrac{1}{j^3\infty^{+3}} = 0^+ \angle -270^\circ$ or $0^+ \angle 90^\circ$

$$T(j\omega) = \frac{(0.25+j\omega)(1-j\omega)(0.5-j\omega)}{-\omega^2(1+j\omega)(0.5+j\omega)(1-j\omega)(0.5-j\omega)}$$

$$= \frac{(0.25+j\omega)(1-j\omega)(0.5-j\omega)}{-\omega^2(1+\omega^2)(0.25+\omega^2)}$$

$$T(j\omega) = \frac{-(0.125 + 0.125\omega^2)}{\omega^2(1+\omega^2)(0.25+\omega^2)} + j\left\{\frac{(\omega^2 - 0.125)}{\omega(1+\omega^2)(0.25+\omega^2)}\right\}$$

NOTE: $T(j0^+) = -\infty + j(-\infty)$

As $0^+ < \omega < \infty^+$,

$Re(T) = -ve$

$\omega^2 - 0.125 < 0 \rightarrow \omega < \sqrt{0.125}$, $Im(T) = -ve$

$\omega^2 - 0.125 = 0 \rightarrow \omega = \sqrt{0.125}$, $Im(T) = 0$ & $Re(T) = -5.33$

$\omega^2 - 0.125 > 0 \rightarrow \omega > \sqrt{0.125}$, $Im(T) = +ve$

ω	$Re(T)$	$Im(T)$	
$0^+ < \omega < \sqrt{0.125}$	$-ve$	$-ve$	→ III QUAD
$\omega = \sqrt{0.125}$	-5.33	0	→ Cuts $-ve$ Real Axis
$\omega > \sqrt{0.125}$	$-ve$	$+ve$	→ II QUAD

Segment-II $\quad s = \infty e^{j\theta}$, $\theta = 90^\circ$ to -90° C.W

$$T(s) = \frac{(\infty e^{j\theta} + 0.25)}{(\infty e^{j\theta})^2 (\infty e^{j\theta} + 1)(\infty e^{j\theta} + 0.5)} = \frac{\infty e^{j\theta}}{(\infty e^{j\theta})^4}$$

→ 0 Radius circle with $= 0^+ e^{-j3\theta}$

-3θ : -270° to 270° A.C.W

$\delta = 540^\circ = 360^\circ + 180^\circ$

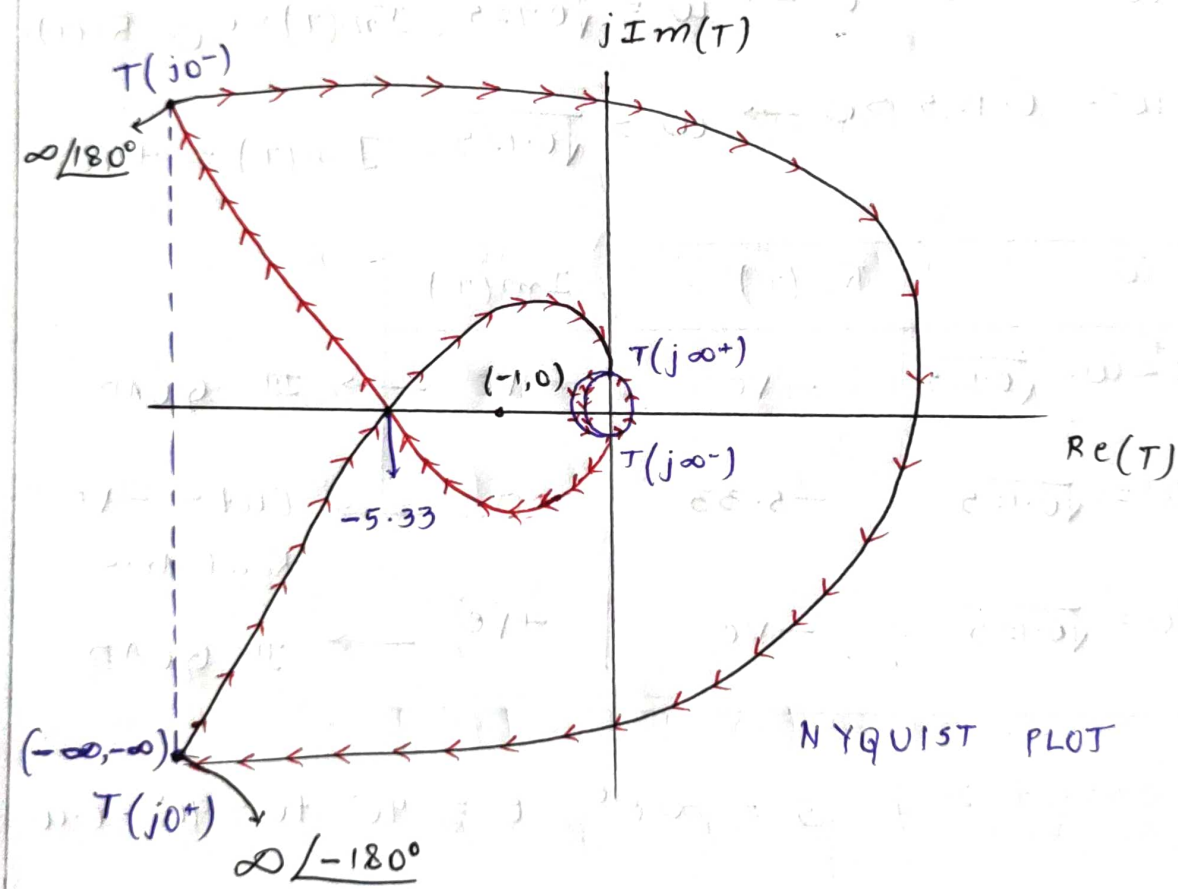
Segment - III I.P.P

Segment - IV $s = 0e^{j\theta}$, $\theta : -90^\circ$ to 90° A.C.W

$$T(s) = \frac{0e^{j\theta} + 0.25}{(0e^{j\theta})^2 (0e^{j\theta} + 1)(0e^{j\theta} + 0.5)}$$

$= \infty e^{-j2\theta} \rightarrow \infty$ Radius circle with

$-2\theta : +180^\circ$ to -180° C.W.



NYQUIST PLOT

NSC in GH : $N = P_c - Z_c$

L.H.S : $N = 0$

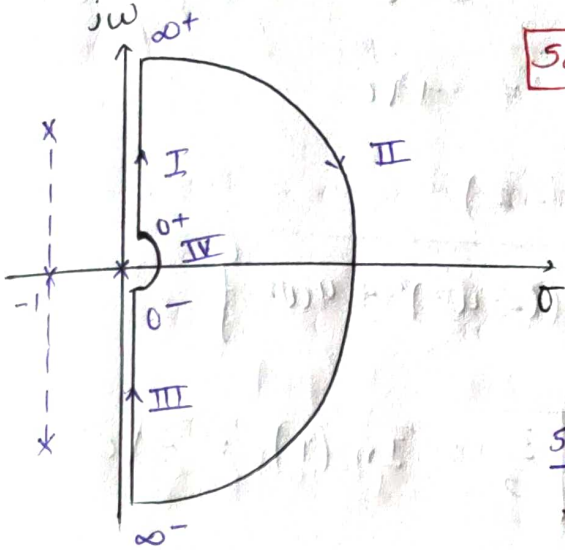
R.H.S : $P_c = 0$, $Z_c = 0$

NSC in $1+GH$: $N = P_c - Z_c$

$N = -2$ $\quad -2 = 0 - Z_c$

$P_c = 0$ $\quad Z_c = 2 \rightarrow$ CLS UNSTABLE

Q. $GH = \dfrac{2 \cdot 2}{5(s+1)(s^2+2s+2)}$



NYQUIST CONTOUR

Segment-I

S-1: $s = j\omega \quad (0^+ < \omega < j\omega)$

$$T(j\omega) = \frac{2 \cdot 2}{(j\omega)(j\omega+1)((j\omega)^2+2(j\omega)+2)}$$

S-2:

$$T(j0^+) = \frac{2 \cdot 2}{2(j0^+)} = \infty^+ \angle -90^\circ$$

$$T(j\infty^+) = \frac{2 \cdot 2}{(j\infty^+)^4} = 0^+ \angle 0^\circ$$

S-3:

$$T(j\omega) = \frac{2 \cdot 2 \times (1-j\omega)(2-\omega^2-j2\omega)}{(j\omega)(j\omega+1)(2-\omega^2+j2\omega)(1-j\omega)(2-\omega^2-j2\omega)}$$

$$T(j\omega) = \frac{2 \cdot 2\ (1-j\omega)(2-\omega^2-j2\omega)}{j\omega(1+\omega^2)((2-\omega^2)^2+4\omega^2)}$$

$$T(j\omega) = \frac{2 \cdot 2(\omega^2-4)}{(1+\omega^2)((2-\omega^2)^2+4\omega^2)} + j\left\{\frac{-2 \cdot 2(2-3\omega^2)}{\omega(1+\omega^2)((2-\omega^2)^2+4\omega^2)}\right\}$$

NOTE: $T(j0^+) = -2 \cdot 2 - j\infty$

As $0^+ < \omega < \infty$

$$Re(T) = \frac{2 \cdot 2(\omega^2 - 4)}{(1+\omega^2)((2-\omega^2)^2 + 4\omega^2)}$$

$\omega < 2 \longrightarrow Re(T) = -ve$

$\omega = 2 \longrightarrow Re(T) = 0$ & $Im(T) = 0.11$

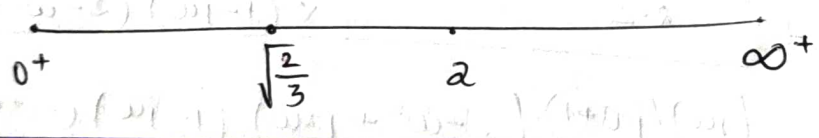
$\omega > 2 \longrightarrow Re(T) = +ve$

$$Im(T) = \left\{ \frac{2 \cdot 2(3\omega^2 - 2)}{\omega(1+\omega^2)((2-\omega^2)^2 + 4\omega^2} \right\}$$

$3\omega^2 - 2 < 0 \rightarrow \omega < \sqrt{\frac{2}{3}}$: $Im(T) = -ve$

$3\omega^2 - 2 = 0 \rightarrow \omega = \sqrt{\frac{2}{3}}$ $Im(T) = 0$, $Re(T) = -0.99$

$3\omega^2 - 2 > 0 \rightarrow \omega > \sqrt{\frac{2}{3}}$ $Im(T) = +ve$



ω	$Re(T)$	$Im(T)$	
$0^+ < \omega < \sqrt{\frac{2}{3}}$	$-ve$	$-ve$	→ III QUAD
$\omega = \sqrt{\frac{2}{3}}$	-0.99	0	→ Cuts -ve Real Axis
$\sqrt{\frac{2}{3}} < \omega < 2$	$-ve$	$+ve$	→ II QUAD
$\omega = 2$	0	0.11	→ Cuts +ve Imaginary Axis
$\omega > 2$	$+ve$	$+ve$	→ I QUAD

Segment - II $s = \infty R e^{j\theta}$: $\theta = 90^\circ$ to -90° C.W.

$$GH = \frac{2 \cdot 2}{\infty^4 e^{j4\theta}} = 0^+ e^{-j4\theta}$$

→ 0 Radius circle with $\delta = 720^\circ$

-4θ : -360° to 360° A.C.W

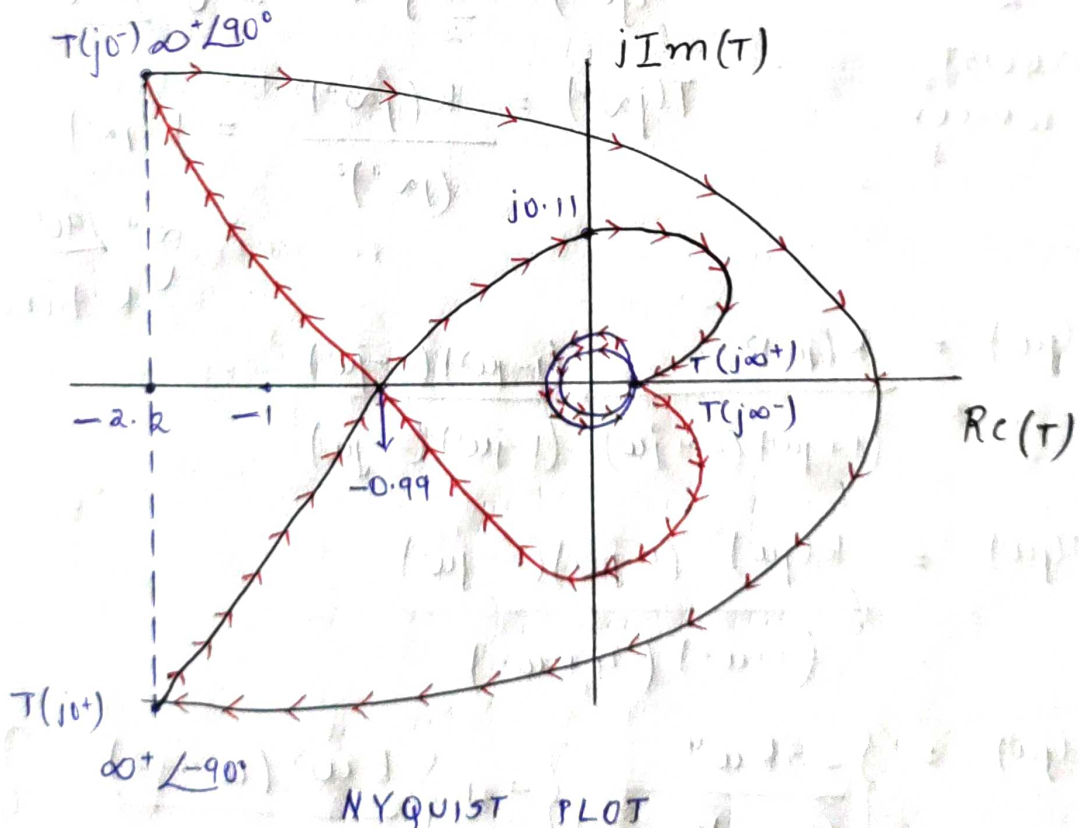
Segment - III I.P.P

Segment - IV $s = 0^+ e^{j\theta}$, θ : -90° to 90° A.C.W

$$GH = \frac{2 \cdot 2}{0^+ e^{j\theta}} = \infty^+ e^{-j\theta}$$

→ ∞ Radius circle with

$-\theta$: 90° to -90° C.W.



NYQUIST PLOT

NSC in GH

$N = P_c - Z_c$

L.H.S: $N = 0$

R.H.S: $P_c = 0$

$Z_c = 0$

NSC in 1+GH

$N = P_c - Z_c$

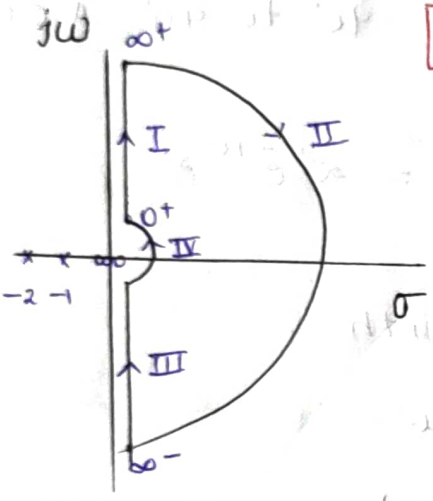
$N = 0$

$P_c = 0$

$Z_c = 0 \rightarrow$ CLS STABLE

→ Question: $T(s) = \dfrac{ks^3}{(s+1)(s+2)}$ · Draw Nyquist Plot

$0 < k < \infty$.



NYQUIST CONTOUR

Segment-I

S-1: $S = j\omega \quad (0^+ < \omega < \infty^+)$

$$T(j\omega) = \frac{k(j\omega)^3}{(1+j\omega)(2+j\omega)}$$

S-2:

$$T(j0^+) = \frac{k}{2} j^3 0^{+3} = 0^+ \angle 270^\circ$$

$$T(j\infty^+) = \frac{k(j\infty^+)^3}{(j\infty^+)^2} = -k(j\infty^+) = \infty^+ \angle 90^\circ$$

S-3:

$$T(j\omega) = \frac{k(j\omega)^3}{(1+j\omega)(2+j\omega)} \times \frac{(1-j\omega)(2-j\omega)}{(1-j\omega)(2-j\omega)}$$

↳ Rationalise

$$T(j\omega) = \frac{k(j\omega)^3 (1-j\omega)(2-j\omega)}{(1+\omega^2)(4+\omega^2)}$$

$$T(j\omega) = \left\{\frac{-3k\omega^4}{(1+\omega^2)(4+\omega^2)}\right\} + j\left\{\frac{k\omega^3(\omega^2-2)}{(1+\omega^2)(4+\omega^2)}\right\}$$

NOTE: $T(j\omega) = \lim_{\omega \to \infty^+} \left\{ \frac{-3k}{\left(\frac{1}{\omega^2}+1\right)\left(\frac{4}{\omega^2}+1\right)} \right\} + j\infty$

$= -3k + j\infty$

As $0^+ < \omega < \infty$,

$Re(T) = \frac{-3k\omega^4}{(1+\omega^2)(4+\omega^2)}$ → Always -ve

$Im(T) = \frac{k\omega^3(\omega^2-2)}{(1+\omega^2)(4+\omega^2)}$

$\omega < \sqrt{2}$, $Im(T) = -ve$

$\omega = \sqrt{2}$, $Im(T) = 0$, $Re(T) = -\frac{2}{3}k$

$\omega > \sqrt{2}$, $Im(T) = +ve$

ω	$Re(T)$	$Im(T)$	
$0^+ < \omega < \sqrt{2}$	-ve	-ve	→ III QUAD
$\omega = \sqrt{2}$	$-\frac{2}{3}k$	0	→ cuts -ve Real Axis
$\omega > \sqrt{2}$	-ve	+ve	→ II QUAD

Segment - II $s = \infty e^{j\theta}$, $\theta = 90^\circ$ to -90° c.w.

$T(s) = \frac{k(\infty e^{j\theta})^3}{(\infty e^{j\theta})^2} = \infty e^{j\theta}$

↓

∞ Radius circle with

$\theta = 90^\circ$ to -90° c.w.

Segment - III I.P.P

Segment - IV $s = 0e^{j\theta}$ $\theta = -90^\circ$ to 90° A.C.W

$$T(s) = \frac{k\,(0e^{j\theta})^3}{2} = 0^+ e^{j3\theta}$$

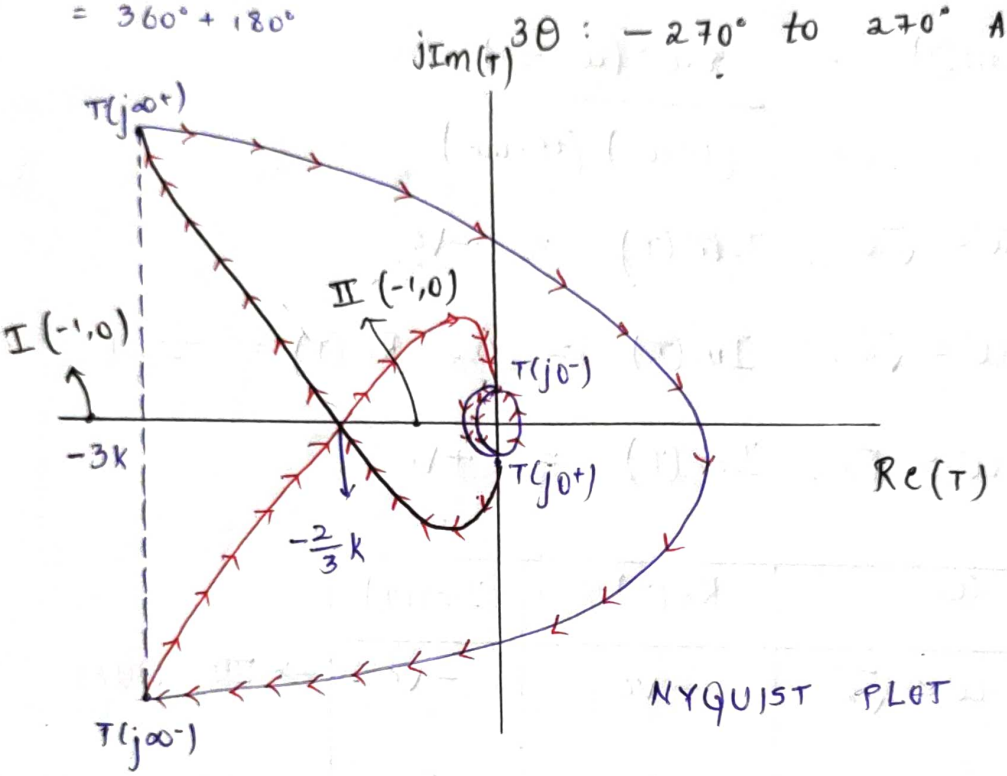
↓

0 Radius circle with

$3\theta : -270^\circ$ to 270° A.C.W

$\delta = 540^\circ$

$= 360^\circ + 180^\circ$



NYQUIST PLOT

NSC in GH

$N = P_c - Z_c$

L.H.S: $N = 0$

R.H.S: $P_c = 0$

$Z_c = 0$

L.H.S = R.H.S

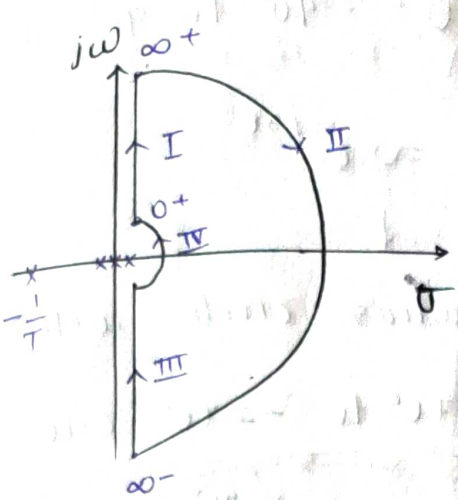
NSC in 1+GH

$N = P_c - Z_c$

$\frac{2}{3}k < 1$	$\frac{2}{3}k > 1$
$N = 0$	$N = -2$
$P_c = 0$	$P_c = 0$
$Z_c = 0$	$Z_c = 2$
↓	↓
CLS STABLE	CLS UNSTABLE

Question: $G(s)H(s) = \dfrac{k}{s^3(1+sT)}$. Draw Nyquist Plot

$0 < k < \infty$



NYQUIST CONTOUR

Segment - I

S-1: $s = j\omega \quad (0^+ < \omega < \infty^+)$

$$T(j\omega) = \frac{k}{(j\omega)^3(1+j\omega T)}$$

S-2:

$$T(j0^+) = \frac{k}{(j0^+)^3(1+j0^+)} \quad \text{(Rationalise)} = \infty^+ \angle -270^\circ$$

$$T(j\infty^+) = \frac{k}{(j\infty^+)^3(j\infty^+)} = 0^+ \angle 0^\circ$$

S-3:

$$T(j\omega) = \frac{k(1-j\omega T)}{(j\omega)^3(1+j\omega T)(1-j\omega T)} = \frac{k(1-j\omega T)}{-j\omega^3(1+\omega^2T^2)}$$

$$T(j\omega) = \frac{kT}{\omega^2(1+\omega^2T^2)} + \frac{jk}{\omega^2(1+\omega^2T^2)}$$

NOTE: $T(j0^+) = \infty^+ + j\infty^+$

As $0^+ < \omega < \infty^+$

$Re(T) = +ve$

$Im(T) = +ve$

$\rightarrow$ I QUAD

Segment - II $s = \infty e^{j\theta}, \quad \theta = 90^\circ$ to -90° C.W

$$T(s) = \frac{k}{\infty^4 e^{j4\theta}} = 0^+ e^{-j4\theta}$$

$\downarrow$

0 Radius circle with

$-\theta$: -360° to 360° A.C.W

Segment - III I·P·P

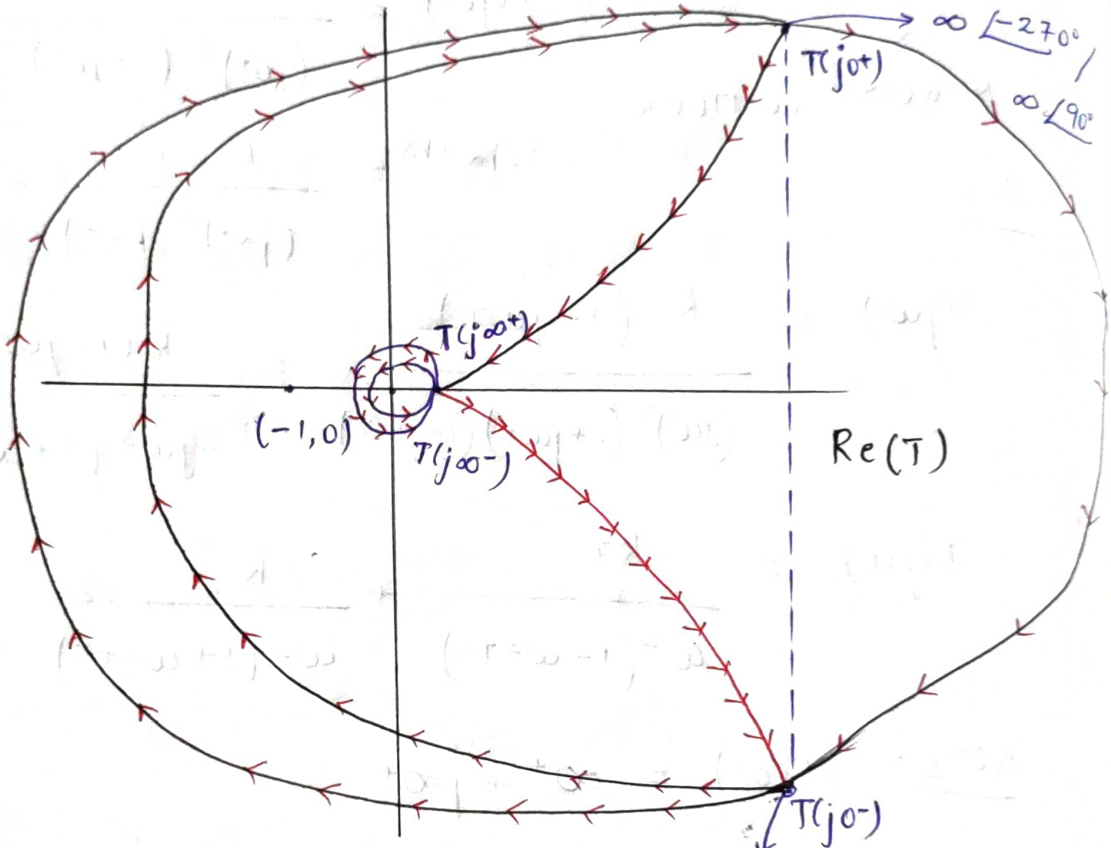
Segment - IV $s = 0^{+} e^{j\theta}$: $\theta = -90^{\circ}$ to 90° A·C·W

$$T(s) = \frac{k}{0^{+3} e^{j3\theta}} = \infty e^{-j3\theta}$$

$\downarrow$

∞ Radius circle with

-3θ : 270° to -270° C·W



NSC in GH

$N = P_c - Z_c$

L·H·S : $N = 0$

R·H·S : $P_c = 0$

$Z_c = 0$

L·H·S = R·H·S

NSC in $1 + GH$

$N = P_c - Z_c$

$N = -2$

$P_c = 0$

$\boxed{Z_c = 2}$

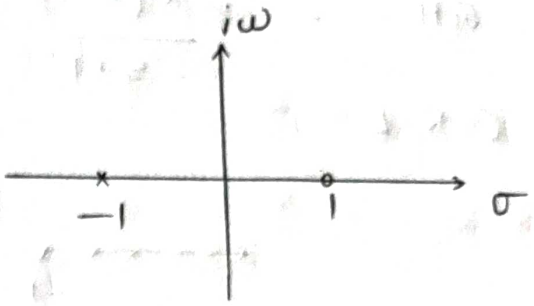
CLS UNSTABLE

ALL PASS SYSTEM / FILTER

→ Poles and Zeros are at mirror image w.r.t jω axis.

Example 1:

$$T(s) = \frac{1-s}{1+s}$$

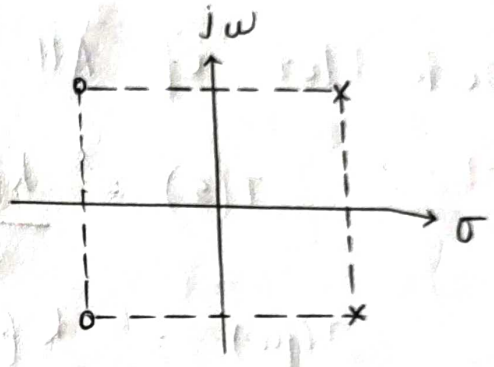


Example 2:

$$T(s) = \frac{s^2+s+1}{s^2-s+1}$$

Poles : $\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$

Zeros : $-\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$



NOTE AND Observation:

$$T(s) = \frac{k(1-s)}{1+s}$$

$$T(j\omega) = \frac{k(1-j\omega)}{1+j\omega}$$

$$|T(j\omega)| = k$$

↓

Independent of ω

$$T(s) = \frac{k(s^2+s+1)}{s^2-s+1}$$

$$T(j\omega) = \frac{k(1-\omega^2+j\omega)}{(1-\omega^2-j\omega)}$$

$$|T(j\omega)| = k$$

↓

Independent of ω

1. Root Locus of All Pass Filter:

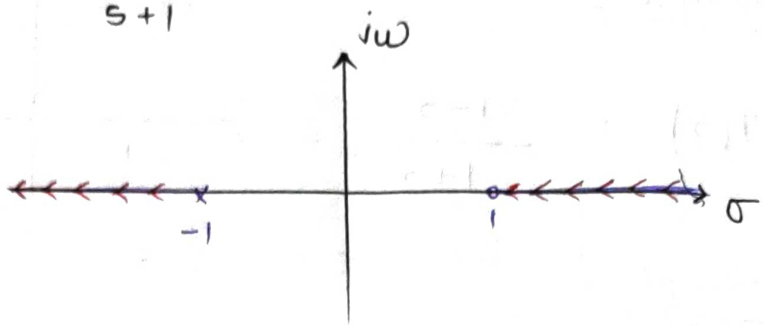
$$G(s)H(s) = \frac{k(1-s)}{1+s} \qquad 0 < k < \infty$$

$-ve$ Feedback

$$GH = -\frac{k(s-1)}{s+1}$$

1. C.R.L

2.



2. Bode Plot of All Pass Filter:

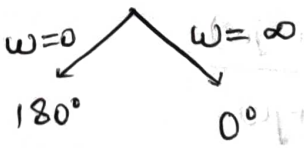
CASE - I:

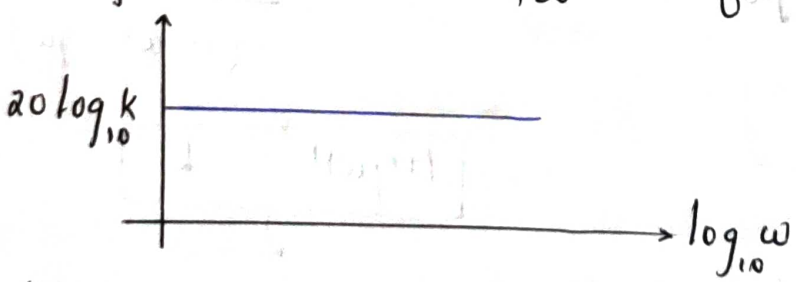
$$T(s) = \frac{k(s-1)}{s+1}$$

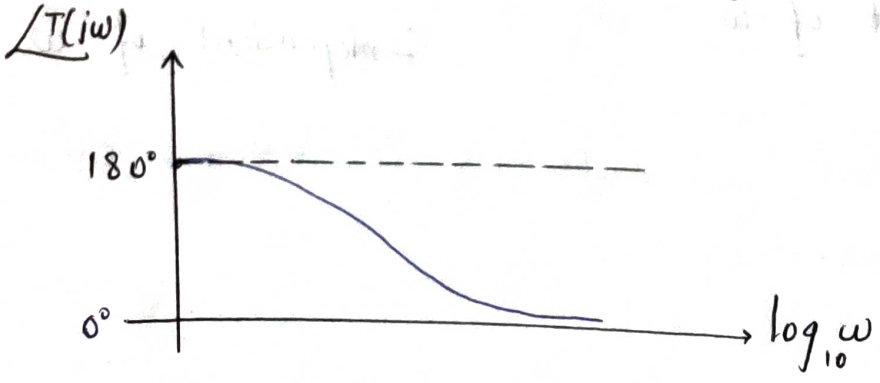
$$T(j\omega) = \frac{k(-1+j\omega)}{1+j\omega}$$

$$|T(j\omega)|\,dB = 20\log_{10} k$$

$$\angle T(j\omega) = 180^\circ - 2\tan^{-1}\omega$$





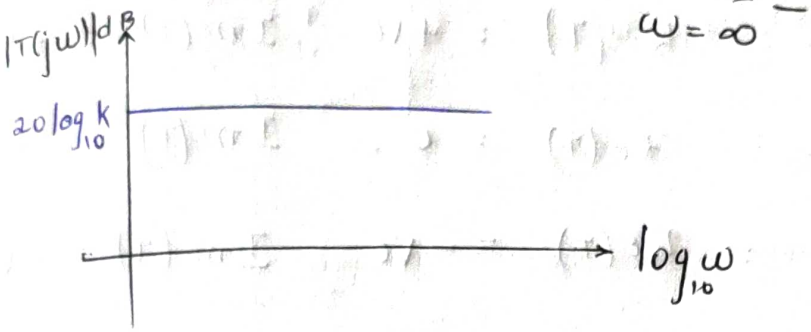


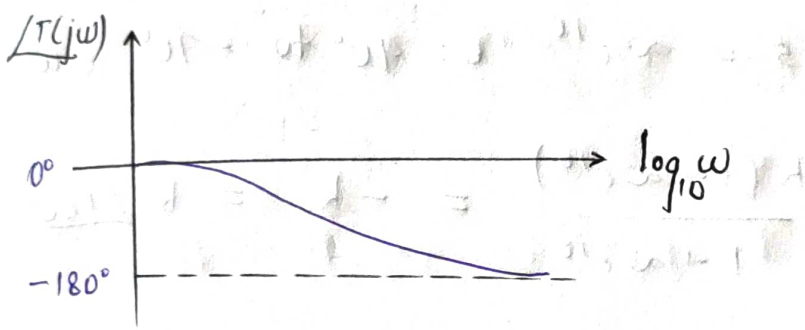
CASE - II: $T(s) = \dfrac{k(1-s)}{1+s}$

$$T(j\omega) = \frac{k(1-j\omega)}{1+j\omega}$$

$$|T(j\omega)| \text{ dB} = 20\log_{10} k$$

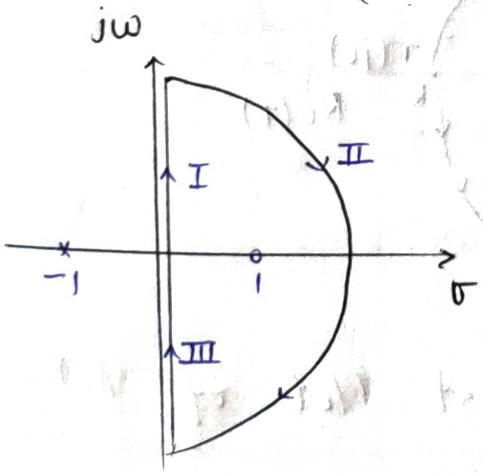
$$\angle T(j\omega) = -2\tan^{-1}\omega \quad \begin{matrix} \omega=0 \rightarrow 0^\circ \\ \omega=\infty \rightarrow -180^\circ \end{matrix}$$





3. NYQUIST Plot of All Pass Filter:

$$G(s)H(s) = \frac{k(1-s)}{(1+s)}$$



Nyquist Contour

Segment - I

S-1: $s = j\omega \quad (0 < \omega < \infty^+)$

$$T(j\omega) = \frac{k(1-j\omega)}{1+j\omega}$$

S-2:

$$T(j0) = \frac{k(1-j0)}{1+j0} = k\angle 0^\circ$$

$$T(j\infty^+) = \frac{k(1-j\infty^+)}{1+j\infty^+} = -k = k\angle 180^\circ$$

S-3: $T(j\omega) = \frac{k(1-j\omega)}{(1+j\omega)} \times \frac{(1-j\omega)}{(1-j\omega)}$

$$T(j\omega) = \frac{k(1-\omega^2)}{(1+\omega^2)} + j\left\{\frac{(-2k\omega)}{(1+\omega^2)}\right\}$$

As $0 < \omega < \infty^+$,

$0 < \omega < 1$: $Re(T) = +ve$, $Im(T) = -ve$ → IV

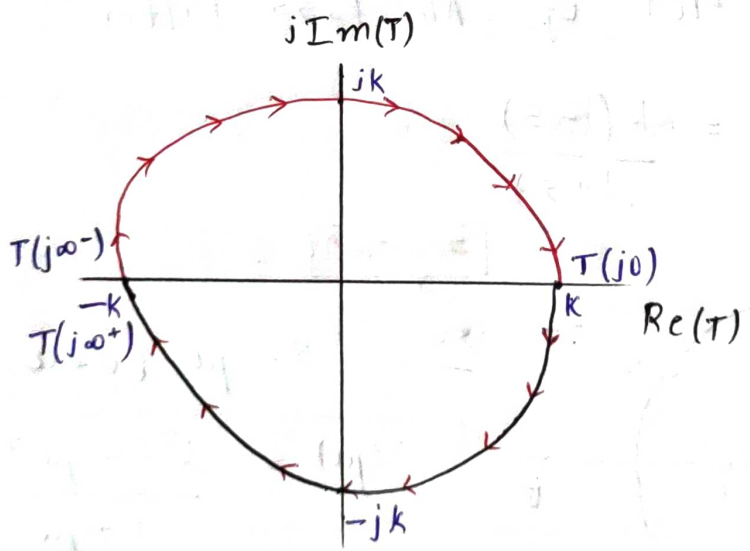
$\omega = 1$: $Re(T) = 0$, $Im(T) = -k$

$\omega > 1$: $Re(T) = -ve$ $Im(T) = -ve$ → III

Segment - II $s = \infty e^{j\theta}$ θ: 90° to −90° C.W

$$T(s) = \frac{k(1-\infty e^{j\theta})}{1+\infty e^{j\theta}} = -k = k\angle 180°$$

Segment - III IPP



Exact shape of Nyquist Plot of APF:

$$T(s) = \frac{k(1-s)}{1+s} \longrightarrow T(j\omega) = \frac{k(1-j\omega)}{1+j\omega}$$

$$T(j\omega) = \underbrace{T_R(j\omega)}_{x} + j\underbrace{T_I(j\omega)}_{y}$$

$$\rightarrow |T(j\omega)| = k = \sqrt{x^2+y^2}$$

$$\rightarrow x^2 + y^2 = k^2$$

$$\downarrow$$

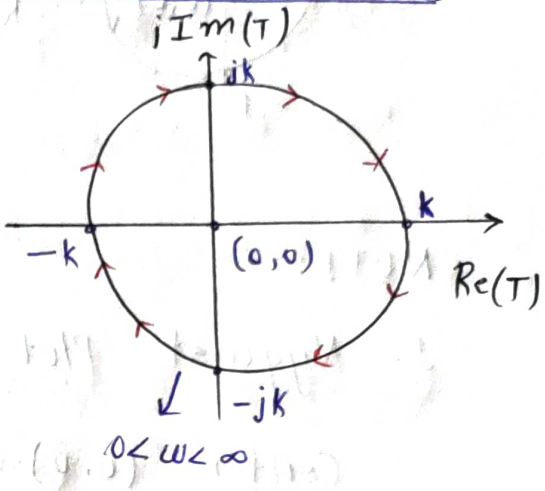
Circle with center (0,0), Radius k

$$\rightarrow \angle T(j\omega) = -2\tan^{-1}\omega$$

$\omega = 0$: $\angle T(j\omega) = 0°$

$\omega = 1$: $\angle T(j\omega) = -90°$

$\omega = \infty$: $\angle T(j\omega) = -180°$



$$T(s) = \frac{k(1+s)}{1-s}$$ Nyquist plot ?

↳ All pass Filter

$$T(j\omega) = \frac{k(1+j\omega)}{1-j\omega}$$

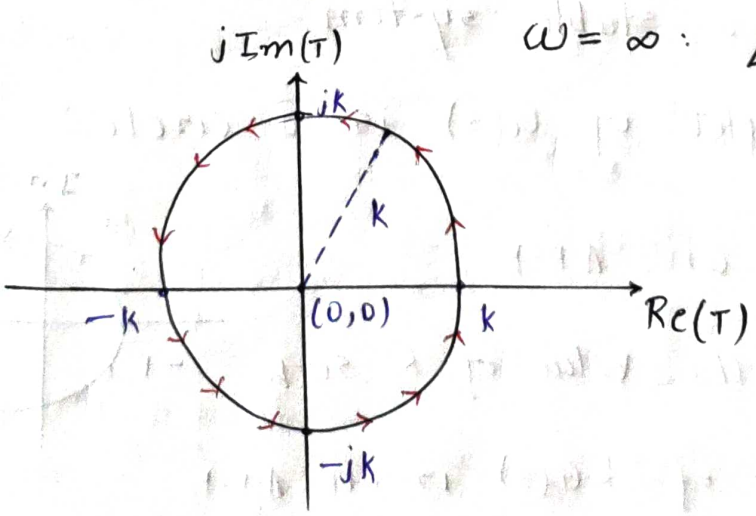
$$|T(j\omega)| = k$$

$$\angle T(j\omega) = +2\tan^{-1}\omega$$

$\omega = 0$: $\angle T = 0°$

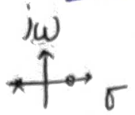
$\omega = 1$: $\angle T = 90°$

$\omega = \infty$: $\angle T = 180°$

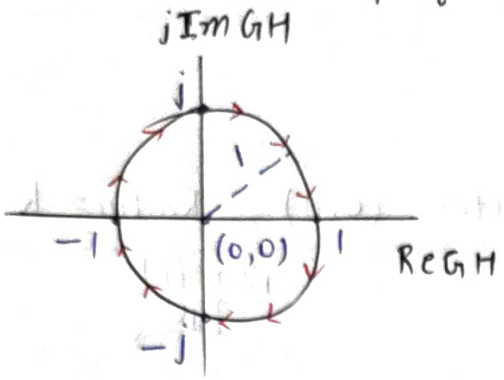


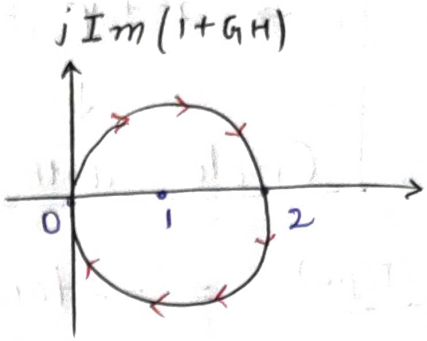
→ Question: $G(s)H(s) = \frac{1-s}{1+s}$

A·P·F



$1 + GH = \frac{2}{1+s}$ → NOT A·P·F

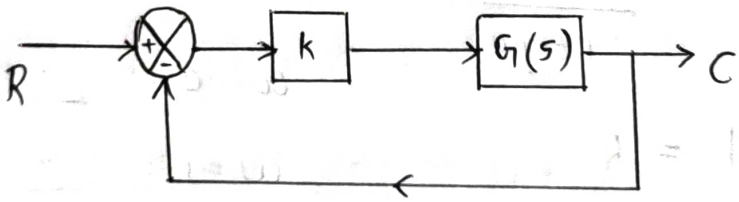




NOTE:

1. Nyquist Plot of APF is circle with Center (0,0).
2. If Nyquist plot is circle, then $G(s)H(s)$ may or may not be All Pass Filter.

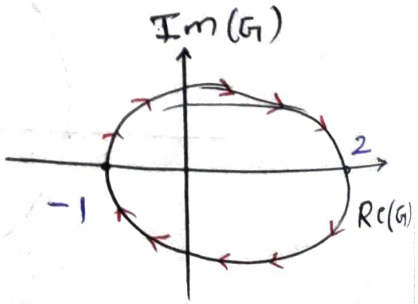
→ * Question:



i) $G(s)$ is a stable system

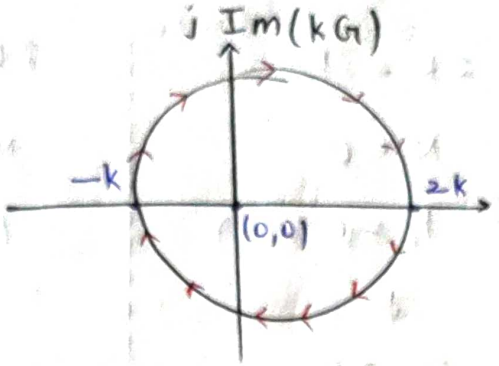
ii) Nyquist plot of $G(s)$ is a circle

1. Is $G(s)$ an A·P·F
2. Calculate the value of k such that plot of $kG(s)$ is an A·P·F

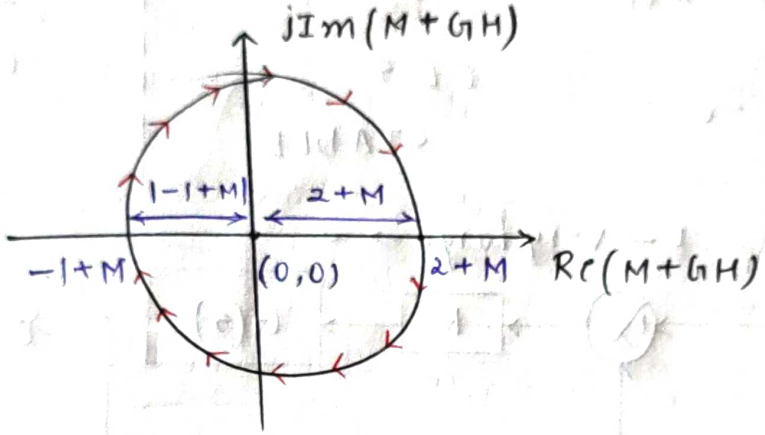


3. Calculate the value of M such that $M + G(s)$ is an A.P.F

1. NO (since, center is not (0,0))

2. 

No such k exists

3. 

$|-1+M| = 2+M$

I:

$-1+M = 2+M$

↓

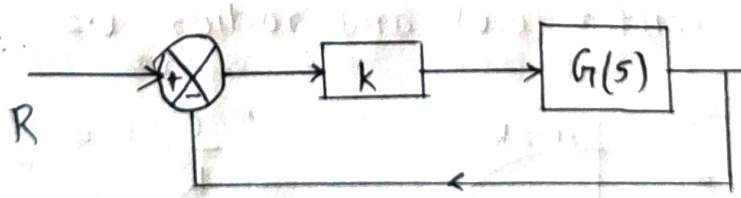
Inconsistent

II:

$1-M = 2+M$

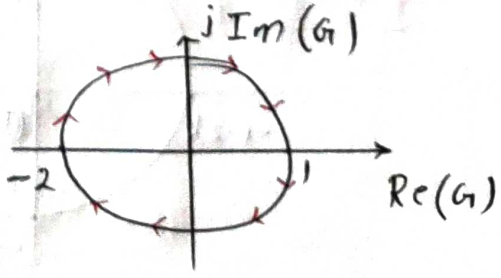
$M = -0.5$

→ Question:

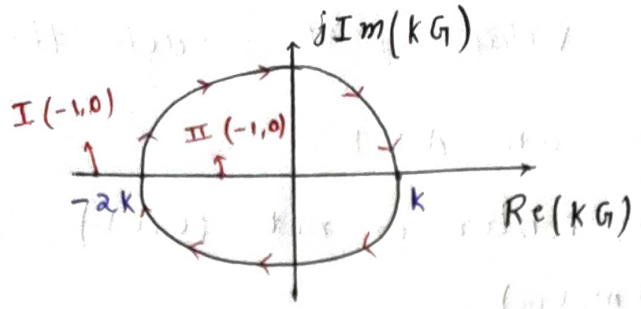


i) G(s) is stable system ↗ $P_c = 0$

ii) Nyquist plot of G(s) is



Comment on stability of Feedback system



$$\frac{C}{R} = \frac{kG}{1+kG}$$

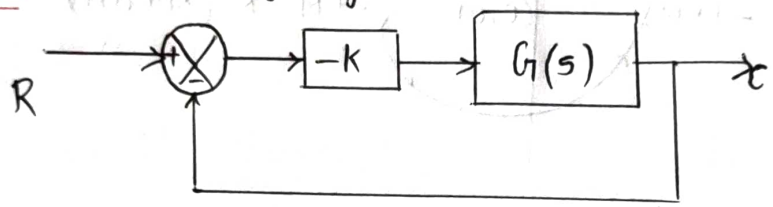
STABILITY of $\frac{C}{R}$

NSC in $1+kG$

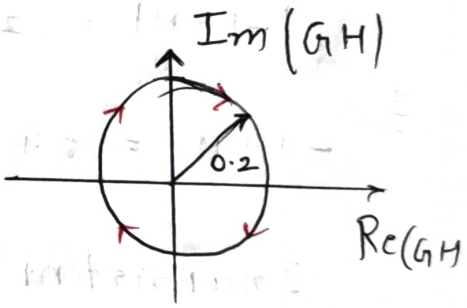
$N = P_c - Z_c$

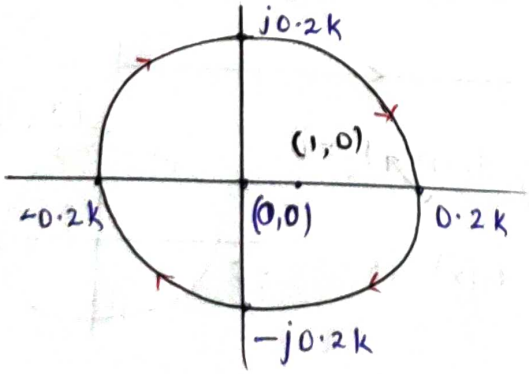
$2k < 1$	$2k > 1$
$N = 0$	$N = -1$
$P_c = 0$	$P_c = 0$
$\downarrow$	$\downarrow$
$Z_c = 0$	$Z_c = 1$
CLS	CLS
STABLE	UNSTABLE

→ Question: $k \to$ very Large $\to \infty$



Comment on the stability of feedback system, if Nyquist plot of $G(s)$ is given as a circle with center (0,0) and radius 0.2.





$$\frac{C}{R} = \frac{-kG}{1-kG}$$

NSC in $1-kG$

$N = P_c - Z_c$

$N = -1$

$P_c = ?$

$Z_c = ?$

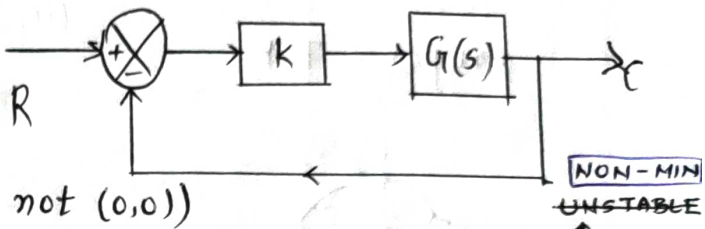
$-1 = P_c - Z_c$

$Z_c = 1 + P_c \rightarrow$

$P_c = 0$:	$Z_c = 1$
$P_c = 1$:	$Z_c = 2$
$P_c = 2$:	$Z_c = 3$

→ CLS UNSTABLE

→ **Question:** Consider the feedback system shown in the figure. The Nyquist plot of $G(s)$ is also shown. Which one of the following Conclusions is correct?



A) $G(s)$ is an A.P.F (Center is not (0,0))

B) $G(s)$ is strictly proper T.F

$N = P_c - Z_c$

$-1 = 0 - Z_c$

$Z_c = 1$

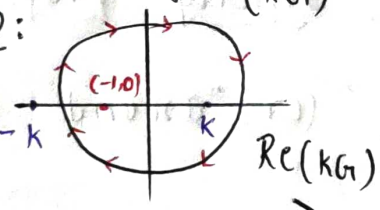
NON-MIN ~~UNSTABLE~~

C) $G(s)$ is a stable and minimum phase T.F

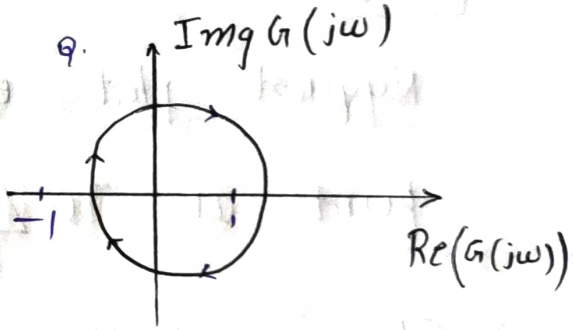
$P_c = 0$

D) The C.L.S is stable for sufficiently large and +ve k.

D:



Q.



B: NOTE

STRICT PROPER T.F

$D^r > N^r$

$-1 = P_c - Z_c$

$Z_c = 1 + P_c \rightarrow$ CLS UNSTABLE

Examples:

$T(s) = \frac{1}{s}$

$T(j\omega) = \frac{1}{j\omega}$

$T(j0) = \infty \angle -90°$

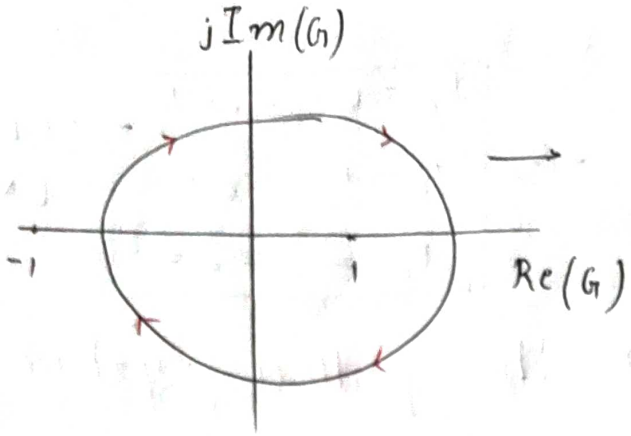
$T(j\infty) = 0 \angle -90°$

$T(s) = \frac{s+1}{s^2}$

$T(j\omega) = \frac{1+j\omega}{(j\omega)^2}$

$T(j0) = \infty \angle -180°$

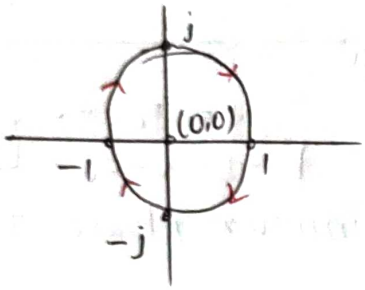
$T(j\infty) = 0 \angle -90°$

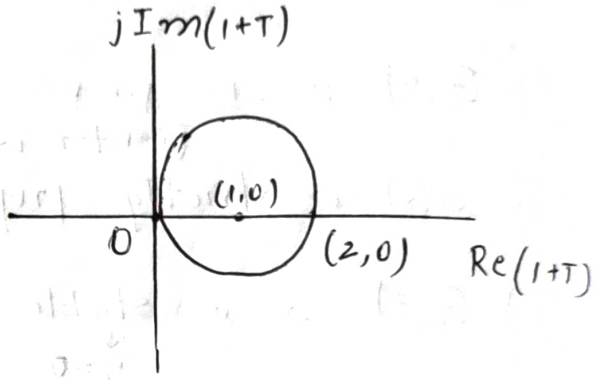


→ NOT Nyquist plot of strictly proper T·F

→ $T(s) = \frac{1-s}{1+s}$ ↓ Not strictly proper T·F

$1+T(s) = \frac{2}{s+1}$ ↓ strictly proper T·F





NOTE

Nyquist plot of strictly proper T·F has point at "$0\angle\theta$" (or) "around $0\angle\theta$"

→ **Closing of Nyquist Plot from Polar Plot:**

CASE 1: O·L·T·F contains 'n' poles at origin.

GH : Type 'n'

→ In this case $G(\infty^+)$ and $G(\infty^-)$ will be connected by 0^+ radius circle

→ short circuit

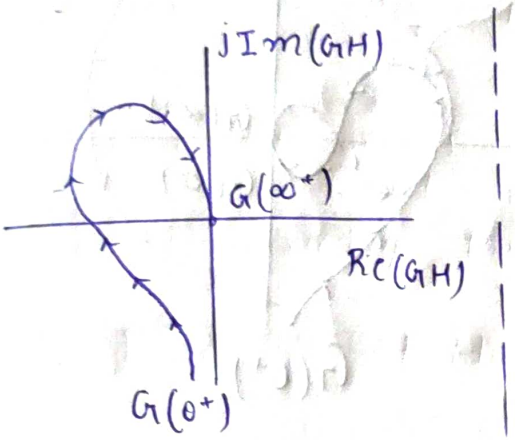
→ $G(0^+)$ and $G(0^-)$ will be open circuited.

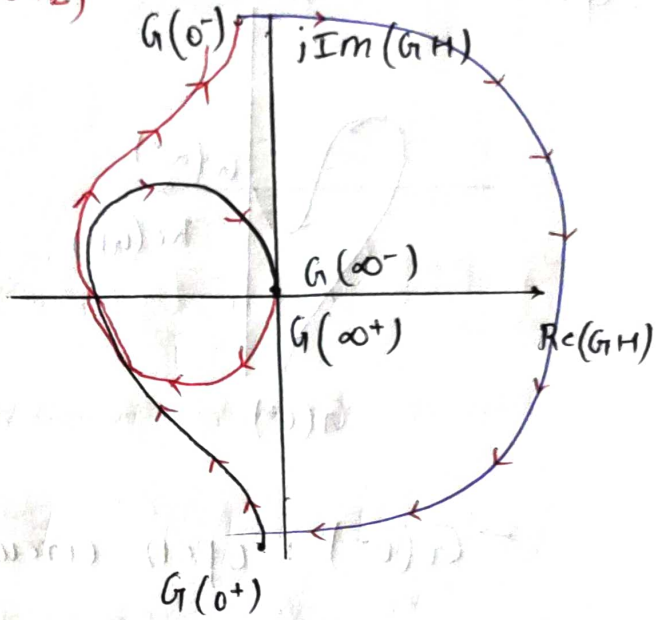
→ To close this, $n\pi$ clockwise encirclement is performed from $G(0^-)$ to $G(0^+)$.

Question: **CASE-1.1** T.F and polar plot both given

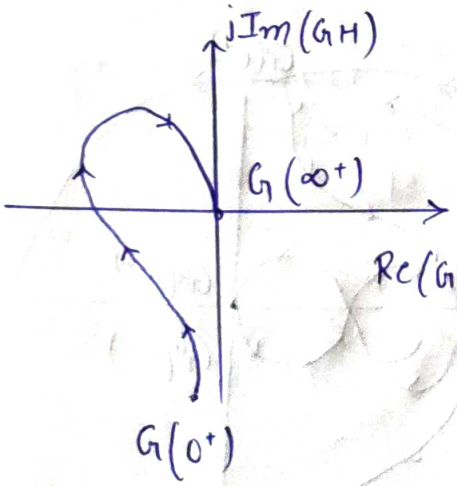
$$G(s)H(s) = \frac{1}{s(1+sT_1)(1+sT_2)}$$

→ Type 1 → $n=1$





CASE-1.2 Only Polar Plot is given



1. $G(0^+)$: open circuited

↓

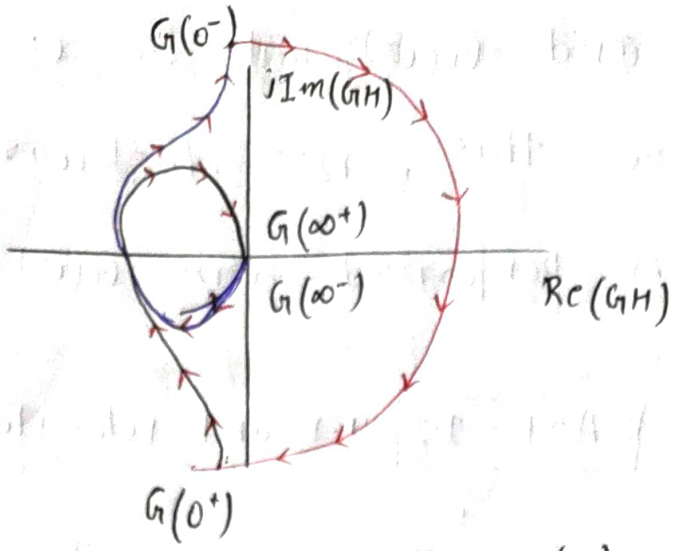
(Type: NON-ZERO)

2. Initial Angle = $G\angle 0^+$

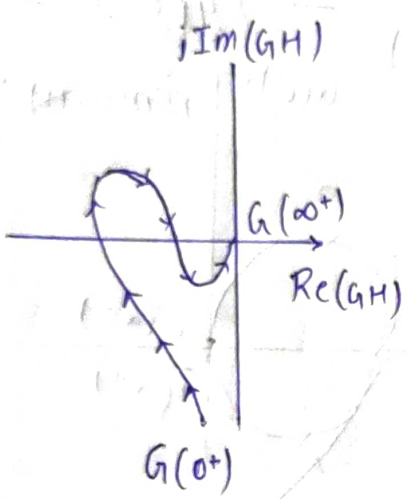
$= -90°$

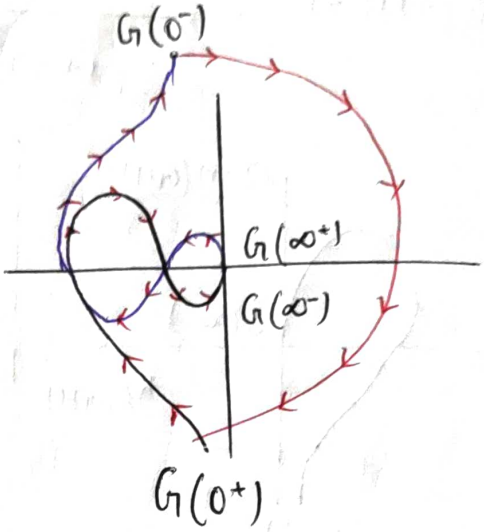
→ T-I

3. I.P.P and closing



Q.



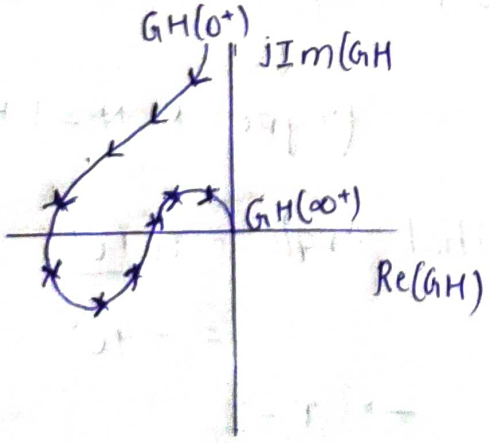


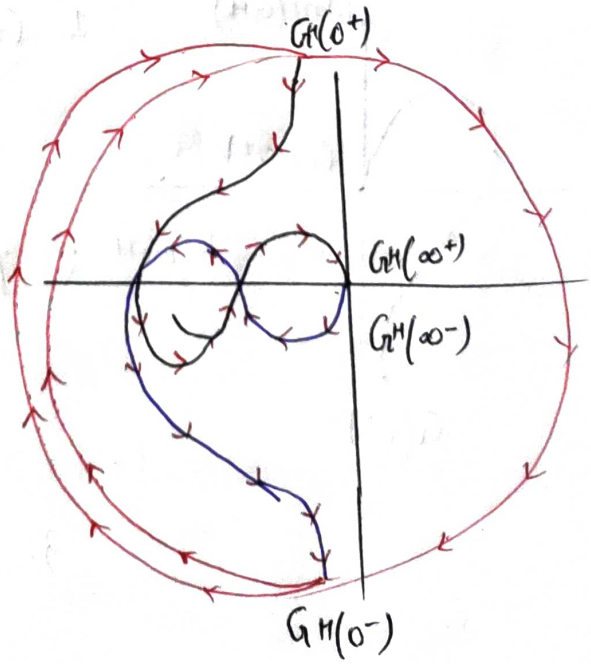
→ $G(0^+)$: open circuited (Type: Non zero)

→ $\angle G(0^+) = -90°$: T-I

→ IPP & closing

Q.





→ $G(0^+)$: open circuited (Type: NON ZERO)

→ $\angle G(0^+) = -270^\circ \rightarrow T = 3$

→ IPP & closing: 3π clockwise

(From $T(j0^-)$ to $T(j0^+)$)

CASE-2: If type of O.L.T.F is zero and order of zero is greater than order of pole.

→ In this case $G(0^-)$ and $G(0^+)$ will be connected by 0 radius circle → (short circuit)

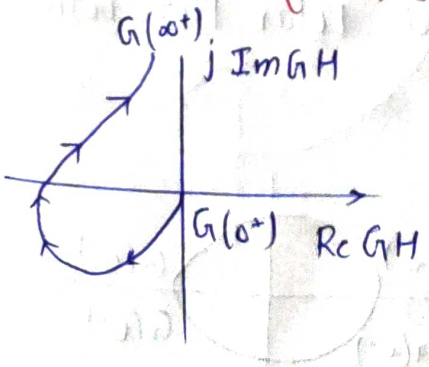
→ In this case $G(\infty^+)$ and $G(\infty^-)$ will be at open circuited.

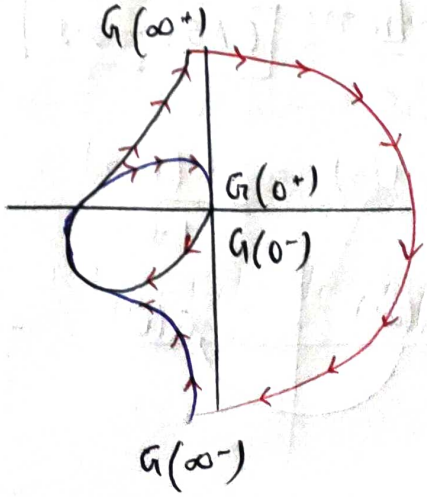
→ $(m-n)\pi$ clockwise encirclement from $G(\infty^+)$ to $G(\infty^-)$ is performed.

Question: **CASE-2.1** T.F and Polar Plot both are given

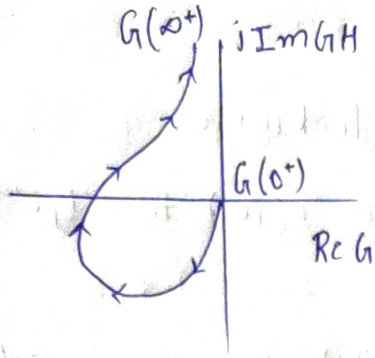
$$G(s)H(s) = \frac{ks^3}{(s+1)(s+2)} \rightarrow \text{Type} : 0$$

$z = 3 = m$, $p = 2 = n$ → $(m-n)\pi = \pi$



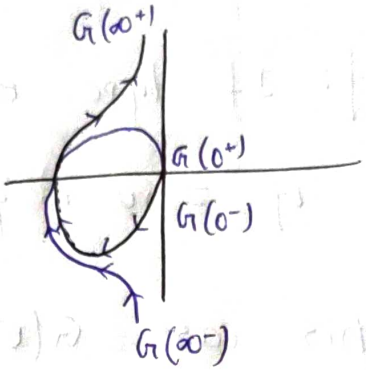


CASE-2.2 Only Polar Plot is given



→ Not possible to get info of m-n

→ we can't draw Nyquist Plot



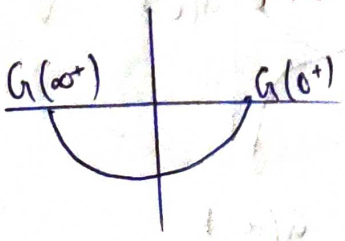
CASE-3: Type of OLTF is zero and order of zero is either less or equal to order of pole.

→ In this case, $G(0^+)$ and $G(0^-)$ will be connected by radius of 0^+ circle → (short ckt)

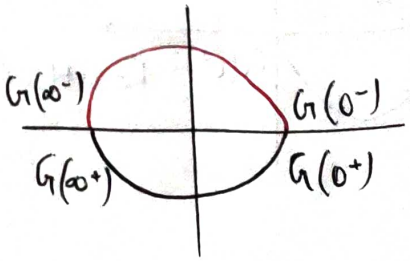
→ $G(\infty^+)$ and $G(\infty^-)$ will be connected by radius of 0^+ circle → (short ckt).

→ Question: **CASE-3.1** T.F and Polar Plot both are given

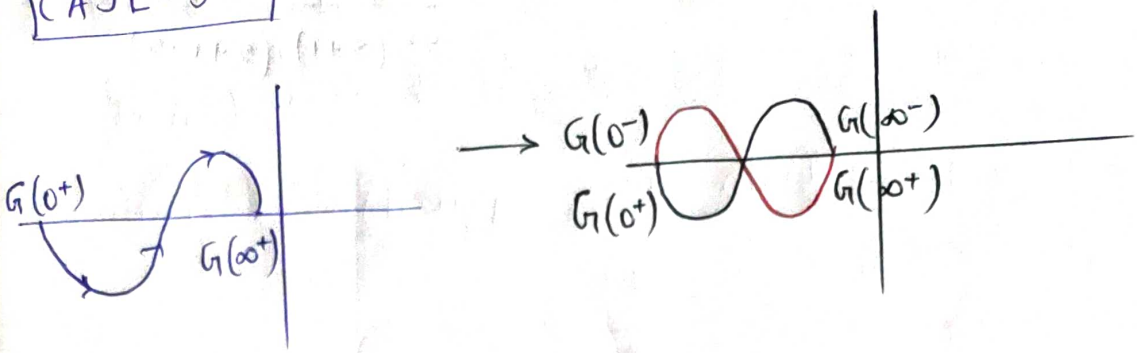
$$G(s)H(s) = \frac{k(1-s)}{1+s}$$



→



CASE-3.2



SUMMARY:

Type ≠ 0 : G(0⁺) and G(0⁻) → open circuited

Type = 0 → G(0⁻) and G(0⁺) → short circuit
↓ Z > P
G(0⁻) and G(0⁺) G(∞⁺), G(∞⁻) O·C
↓
S·C
↓ Z ≤ P
G(∞⁺), G(∞⁻) S·C

Correlation Between Encirclement & Enclosement

$|T(j\omega)|$

$0^+ < \omega < \infty$

$|T(j\omega)| \rightarrow \infty$

↓

Encirclement and Enclosement are correlated

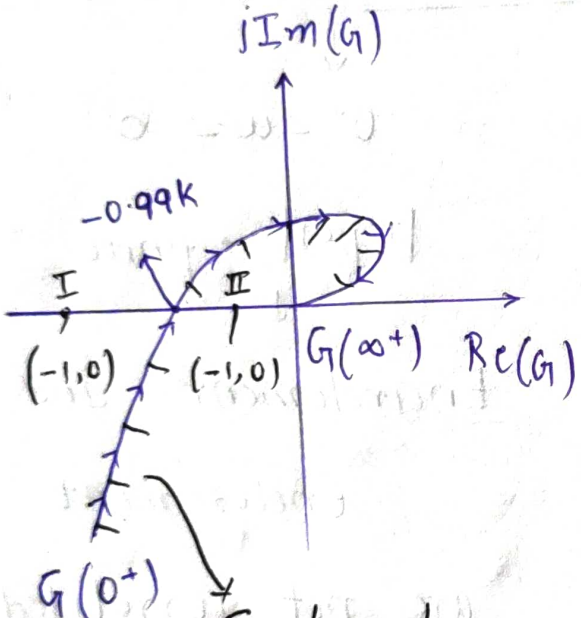
$0^+ < \omega < \infty$

$|T(j\omega)| \rightarrow$ finite

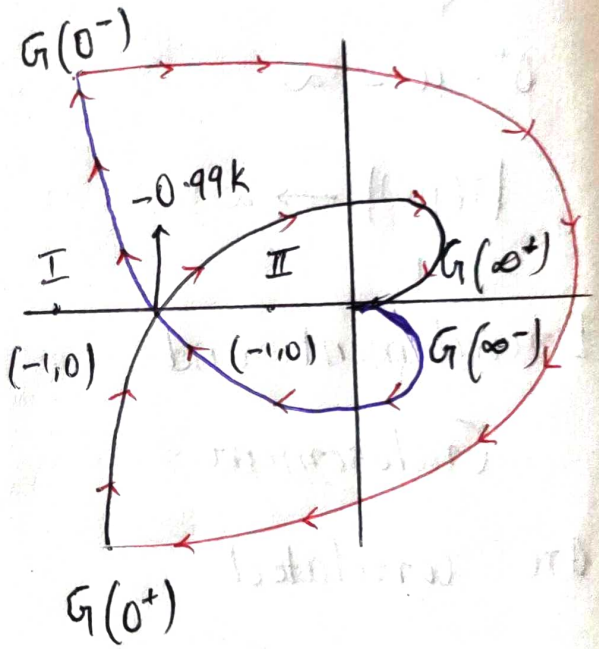
↓

Encirclement and Enclosement are not correlated

→ Example-2: $GH = \dfrac{2 \cdot 2k}{5(s+1)(s^2+2s+2)}$

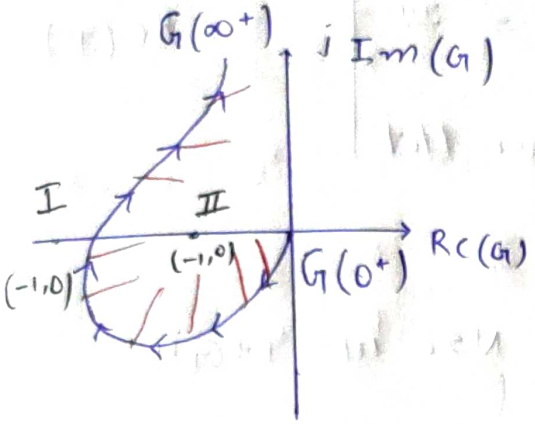


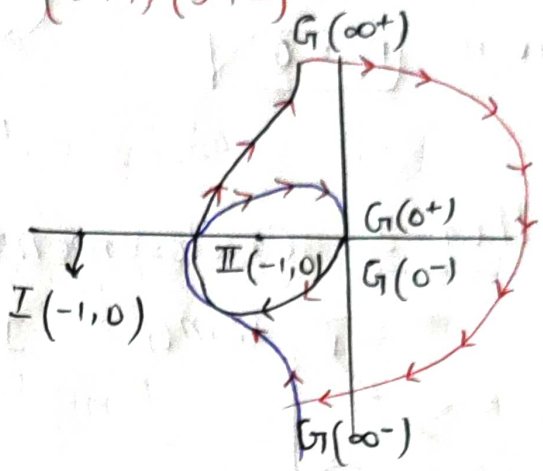
Enclosed Region



Region	Polar Plot	Nyquist plot
I	N = 0	N = 0
II	N ≠ 0	N = −2

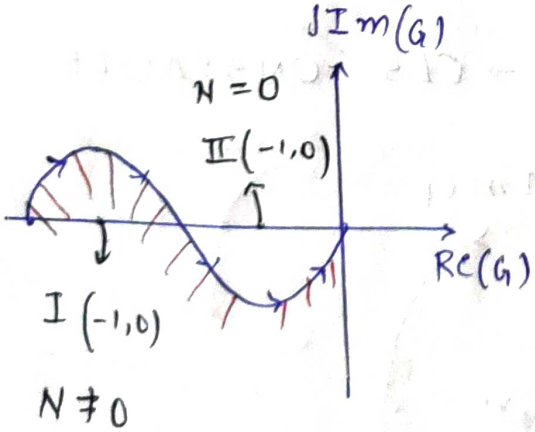
→ Example -3 : $GH = \frac{ks^3}{(s+1)(s+2)}$

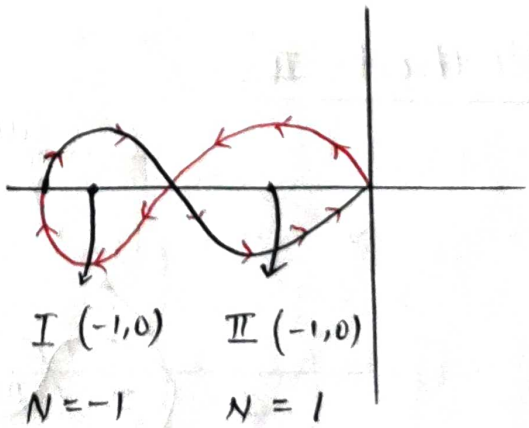




Region	Polar Plot	Nyquist Plot
I	N = 0	N = 0
II	N ≠ 0	N = −2

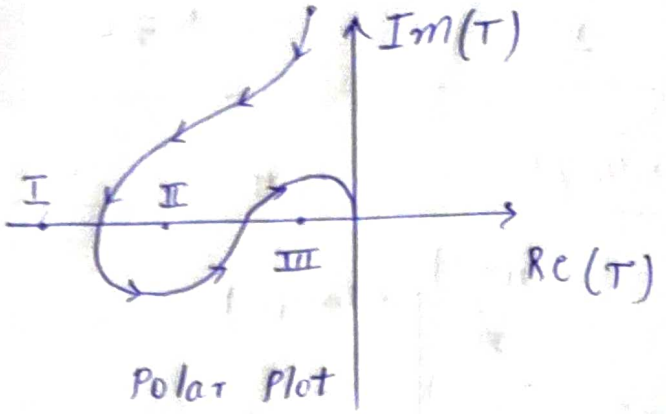
→ Example -4 : $GH = \frac{k(s+1)}{(s+0.5)(s-2)}$





→ Enclosrment and Encirclement are not Coroclated.

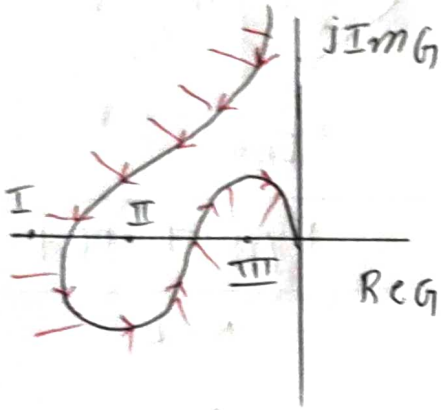
→ Question: If open loop system is stable, Comment on stability of C.L.S at position (I), (II), III.



Polar Plot

Given $P_c = 0$

Method-I: Enclosement



I. NSC in $1+GH$

$N = P_c - Z_c$

$N \neq 0$, $P_c = 0$

$Z_c = -N \neq 0$

→ CLS UNSTABLE

II. $N = 0$, $P_c = 0$

$Z_c = 0$

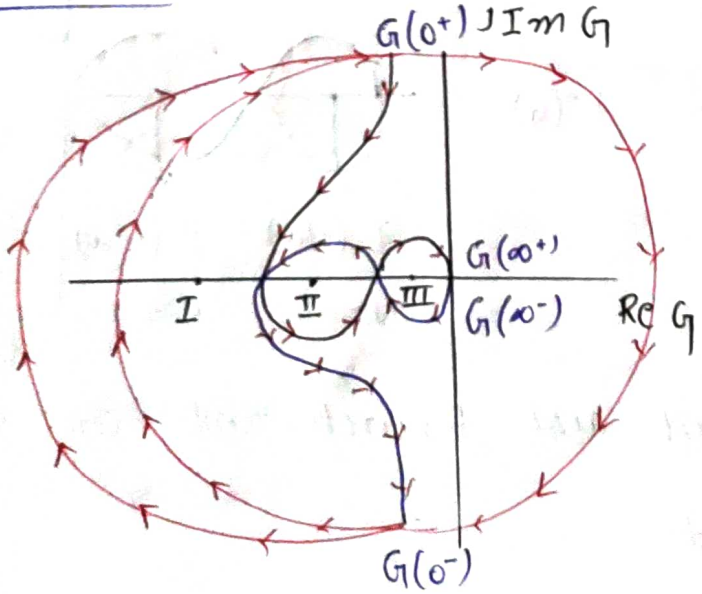
→ CLS STABLE

III. $N \neq 0$, $P_c = 0$

$Z_c = -N \neq 0$

→ CLS UNSTABLE

Method - II



I. NSC in $1+GH=0$

$N = P_c - Z_c$

$-2 = 0 - Z_c$

$Z_c = 2$

↳ CLS UNSTABLE

II. NSC in $1+GH$

$N = P_c - Z_c$

$0 = 0 - Z_c$

$Z_c = 0$

↓

CLS STABLE

III. NSC in $1+GH$

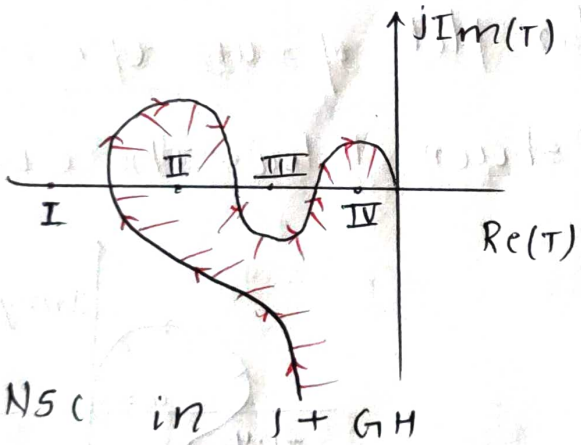
$N = P_c - Z_c$

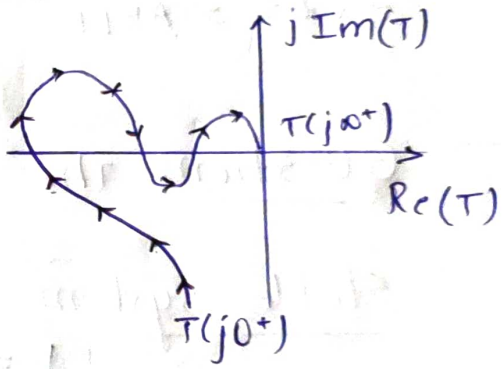
$-2 = 0 - Z_c$

$Z_c = 2$ → CLS UNSTABLE

Question: If open Loop system is stable, comment on stability of C.L.S at position I, II, III, IV.

Method-I: Enclosement





NSC in $1+GH$

$N = P_c - Z_c$

I. $N = 0 \rightarrow Z_c = 0$

$P_c = 0$ CLS STABLE

II. $N \neq 0 \rightarrow Z_c = -N \neq 0$

$P_c = 0$ CLS UNSTABLE

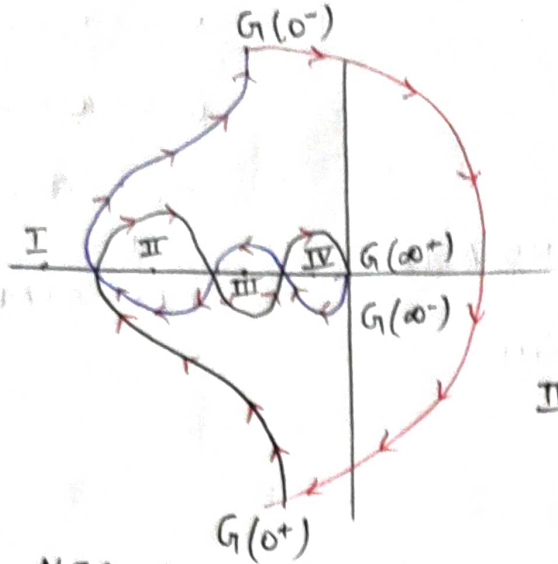
III. $N = 0 \rightarrow Z_c = 0$

$P_c = 0$ CLS STABLE

IV. $N \neq 0 \rightarrow Z_c = -N \neq 0$

$P_c = 0$ CLS UNSTABLE

Method-2 → Encirclement



$\angle G(0^+) = -90^\circ$ → T-I

NSC in $1+GH$

$N = P_c - Z_c$

I. $N = 0$, $P_c = 0$ → $Z_c = 0$ → CLS STABLE

II. $N = -2$, $P_c = 0$ → $Z_c = 2$ → CLS UNSTABLE

III. $N = 0$, $P_c = 0$ → $Z_c = 0$ → CLS STABLE

IV. $N = -2$, $P_c = 0$ → $Z_c = 2$ → CLS UNSTABLE

→ Question: The polar plot of an open loop stable system is shown below. The C.L.S is

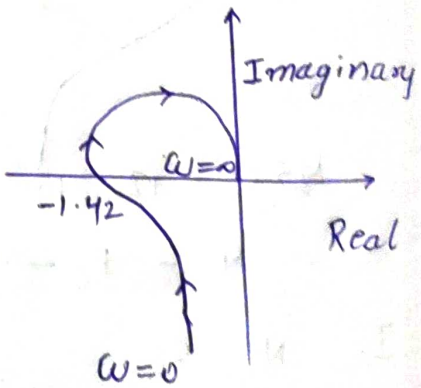
$P_c = 0$

A) Always stable

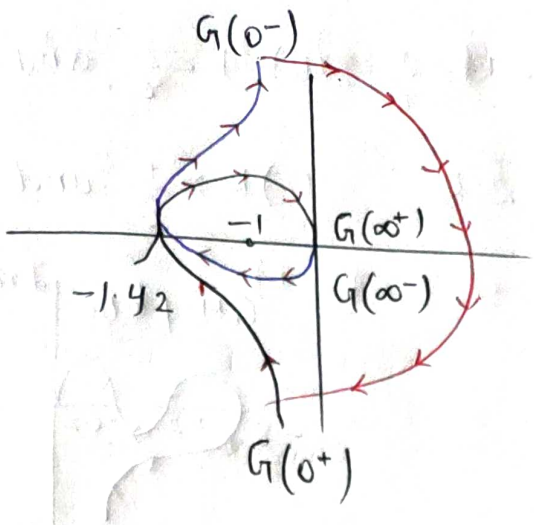
B) Marginally stable

C) Unstable with one pole on the RH s-plane

D) Unstable with two poles on the RH s-plane.



Given $P_c = 0$



$\angle G(0^+) = -90^\circ$

↳ Type: 1

↳ $n\pi = \pi$

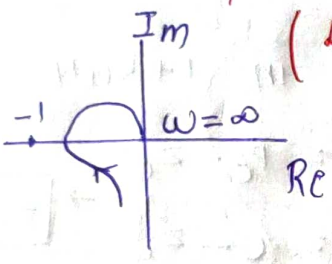
NSC in $1+GH$

$N = -2 \rightarrow Z_c = 2$

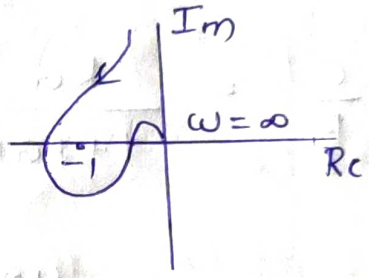
$P_c = 0$

Question: Consider the following Nyquist plots of open loop T.F's over $\omega = 0$ to $\omega = \infty$. Which of these plots represent a stable C.L.S?

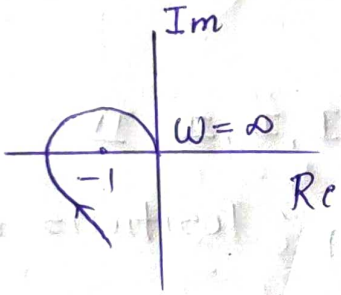
1. (Let OLS is stable)



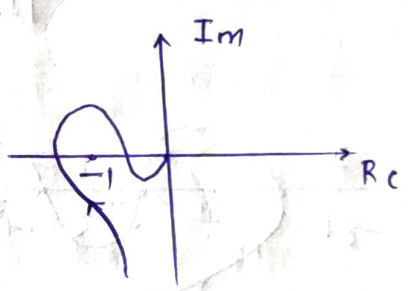
2.



3.



4. ✓



A) 1 only B) All except 1 C) All except 3 D) 1 & 2 only

1. $N = 0 \rightarrow Z_c = 0$

2. $N = 0 \rightarrow Z_c = 0$

3. $N \neq 0 \rightarrow Z_c = -N \neq 0$

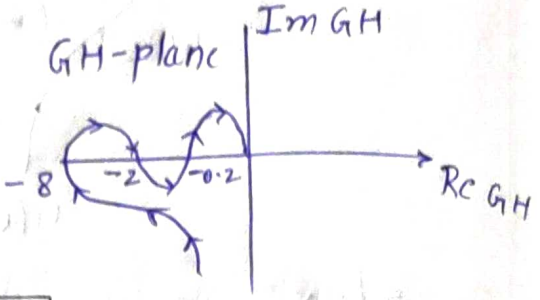
4. $N \neq 0 \rightarrow Z_c = -N \neq 0$

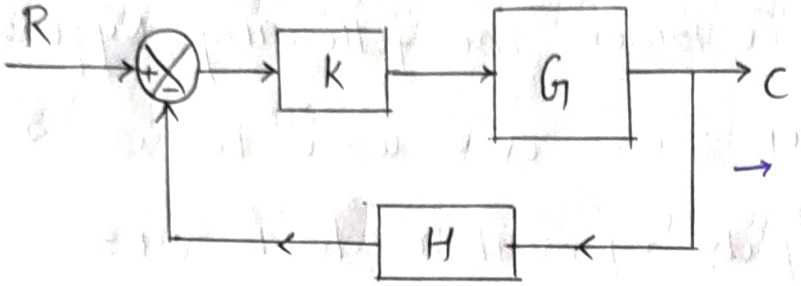
Question: The polar diagram of a Conditionally stable system for open loop gain $k = 1$ shown in the figure. The OLTF of the system is known to be stable. The CLS is stable for

A) $k<5$ and $\frac{1}{2}<k<\frac{1}{8}$ B) $k<\frac{1}{8}$ and $\frac{1}{2}<k<5$

C) $k<\frac{1}{8}$ and $5<k$ D) $k>\frac{1}{8}$ and $5>k$

Given $P_c = 0$



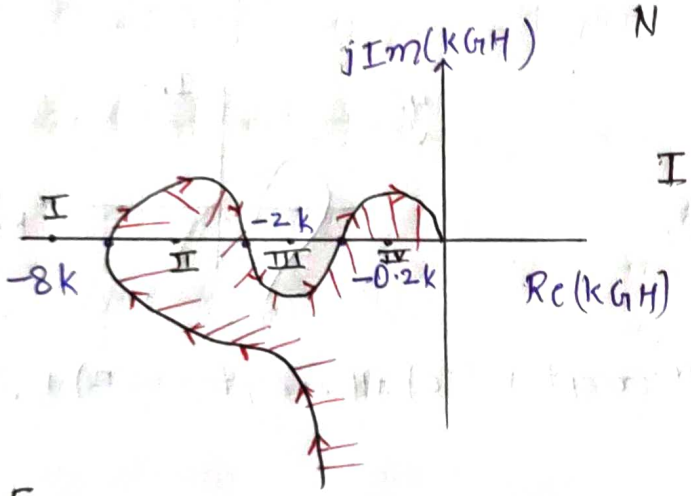


→ CONDITIONAL SYSTEM (on k)

$$\frac{C}{R} = \frac{kG}{1+kGH} \rightarrow \text{NSC in } 1+kGH$$

$$N = P_c - Z_c \rightarrow \boxed{Z_c = -N}$$

$$N = 0 - Z_c$$



I, II, III, IV : Positions of (-1,0)

For CLS to be stable : $Z_c = 0 \rightarrow N = 0$

i.e., $(-1,0)$ should be in I or III

I ∪ III

$$(1>8k) \cup \{(2k>1) \cap (0.2k<1)\}$$

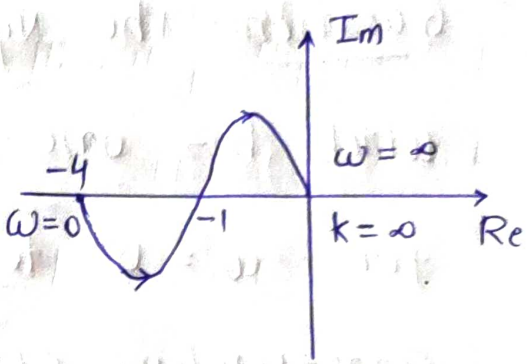
$$\left(k<\frac{1}{8}\right) \cup \left(\frac{1}{2}<k<5\right)$$

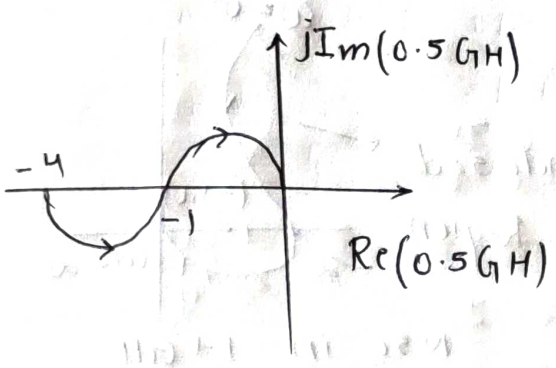
Question: The polar diagram for the open loop gain $k = 0.5$ of a conditionally stable system is shown below. What should be the condition of k for the stability of a CLS, if the OLTF has one pole in the right half of s-plane?

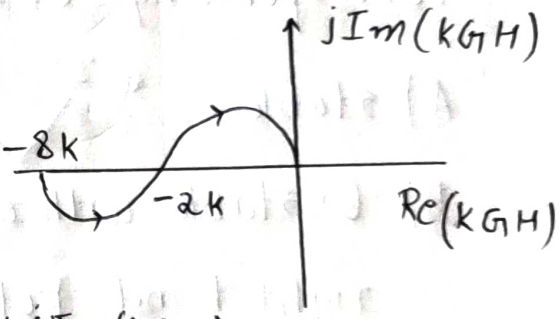
↳ $P_c = 1$

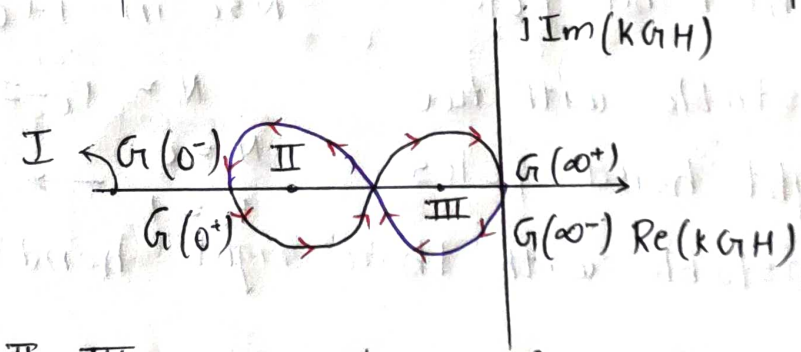
A) $k < \frac{1}{2}$ B) $k < \frac{1}{8}$

C) $\frac{1}{8} < k < \frac{1}{2}$ D) $\frac{1}{4} < k < 1$









I, II, III → positions of $(-1, 0)$.

$$\frac{C}{R} = \frac{kG}{1 + kGH}$$

NSC in $1 + kGH$

$$N = P_c - Z_c$$

I. $N = 0$, $P_c = 1$ → $Z_c = 1$ → CLS UNSTABLE

II. $N = 1$, $P_c = 1$ → $Z_c = 0$ → CLS STABLE

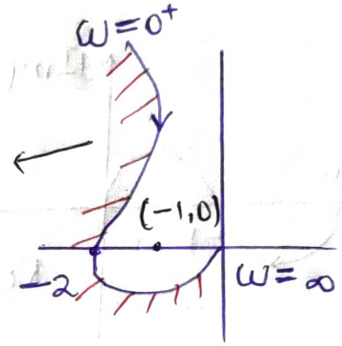
III. $N = -1$, $P_c = +1$ → $Z_c = 2$ → CLS UNSTABLE

For CLS to be stable (-1,0) should be in II.

→ $(8k > 1) \cap (2k < 1)$

→ $\frac{1}{8} < k < \frac{1}{2}$

→ Question: The Nyquist plot of a type 3 system which has one right half of s-plane pole, for $\omega = 0^+$ to $\omega = \infty^+$ is given below. The corresponding CLS is



A) stable

B) Unstable with one right hand pole

C) Unstable with two right hand poles

D) Unstable with three right hand poles

NSC in $1+GH$

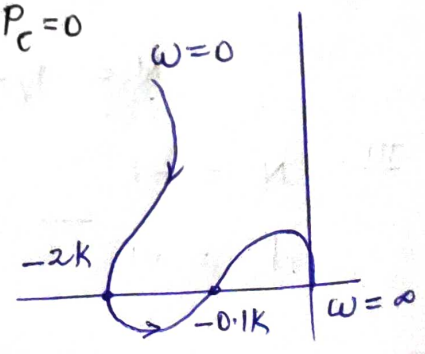
$N = P_c - Z_c$

$N = 0$

$P_c = 1$ → $Z_c = 1$

→ Question: The polar plot of a system is given below. The range of k for stability of System given that OLS is stable.

$P_c = 0$



A) $0 < k < 20$

B) $k < 20$

C) $\frac{1}{2} < k < 10$

D) $-20 < k < \frac{1}{2}$

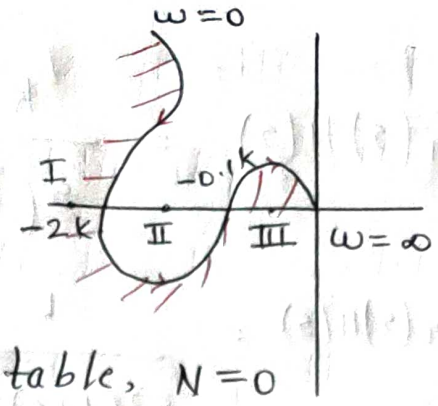
NSC in $1+GH$

$N = P_c - Z_c$

$Z_c = -N$

↳ For CLS to be stable, $N = 0$

↳ i.e., $-N$ $(-1,0)$ should be in II



$2k > 1 > 0.1k$

$\frac{1}{2} < K < 10$

NOTE

→ Location of Tail (start) of Polar Plot

$= -90° \times$ (Number of poles at origin in $G(s)H(s)$)

→ Location of Head (end) of Polar Plot

$= -90° \times$ (Total Number of poles in $G(s)H(s)$)

$+90°$ (Total Number of Zeros in $G(s)H(s)$)

* Above observation is Valid, if all the Co-efficient of s is +ve in $G(s)H(s)$.

Shortcut to Draw Polar Plot:

1. $G(s)H(s)$ should not have any zeros.
2. $G(s)H(s)$ should have poles only.
3. All the co-eff of s in $G(s)H(s)$ should be +ve.

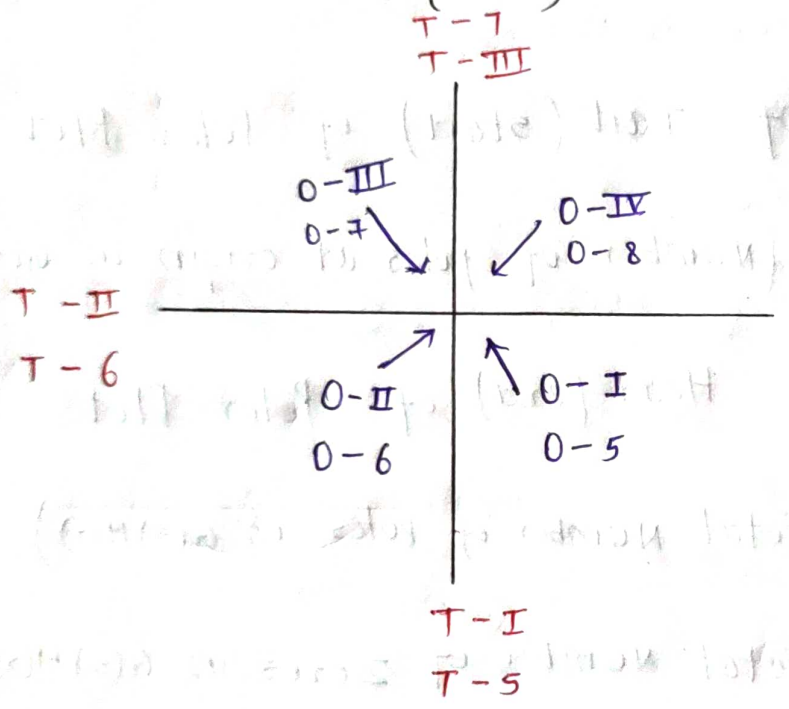
$$G(s)H(s) = \frac{K}{s(1+s)} \rightarrow \text{Type} - 1, \quad \text{Order} - 2$$

→ SHORTCUT VALID

$$G(s)H(s) = \frac{K}{s^3(1+s)(1+2s)} \rightarrow \text{Type} - 3, \quad \text{Order} - 5$$

$$G(s)H(s) = \frac{K}{s(1-s)} \rightarrow \text{SHORTCUT Not VALID}$$

$$G(s)H(s) = \frac{K}{s^3(-1-s)}$$

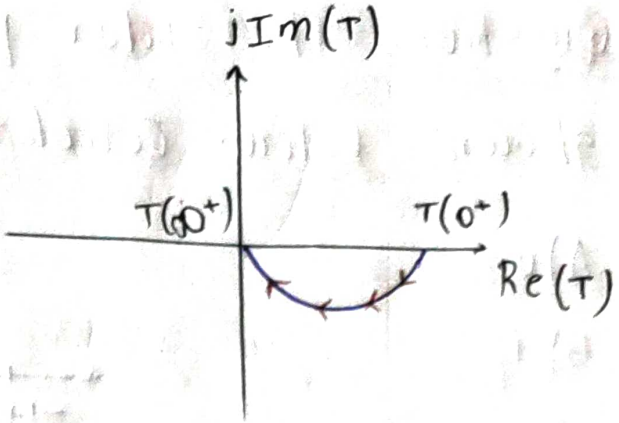


(i) Such polar plots starts from TYPE and ends at ORDER by moving in clockwise direction.

(ii) Starting point is not on axis, it is little bit shifted, if pole is present at origin.

Example - 1:

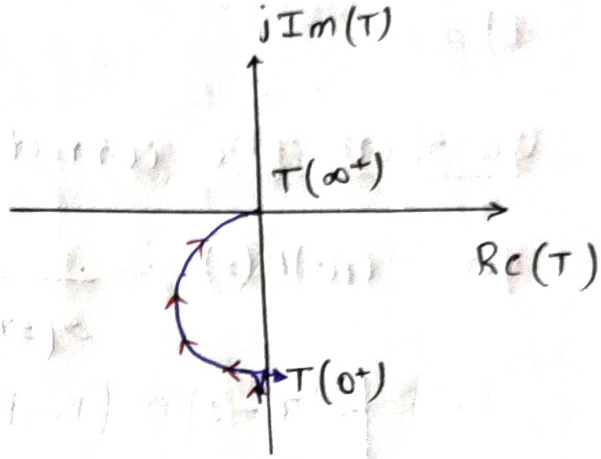
$$T(s) = \frac{1}{1+s}$$



→ T - 0
→ 0 - 1

Example - 2:

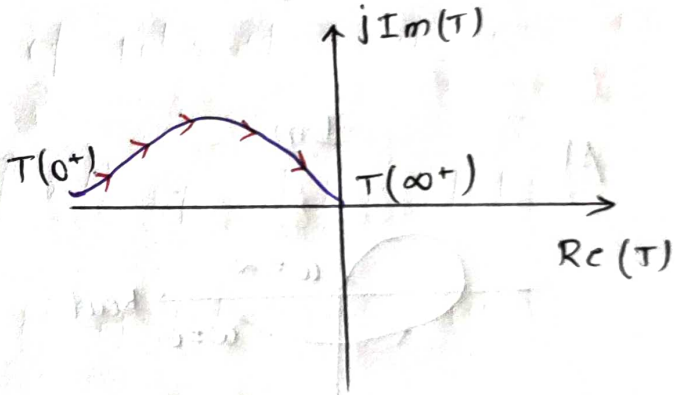
$$T(s) = \frac{1}{s(1+s)}$$



→ T - 1
→ 0 - 2

Example - 3:

$$T(s) = \frac{1}{s^2(1+s)}$$



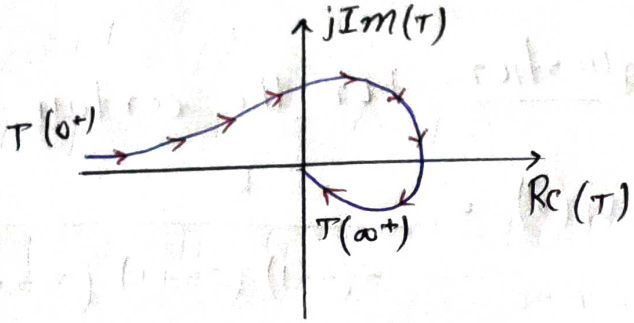
→ T - 2
→ 0 - 3

→ Question: Consider a C.S having OLTF

$$G(s)H(s) = \frac{1}{s^2(s+a)(s+b)(s+c)}$$

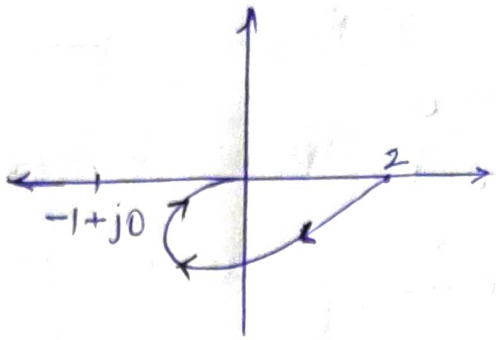
If all the finite poles has +ve real part, then the polar plot of the system will be

G H → T - 2
→ 0 - 5



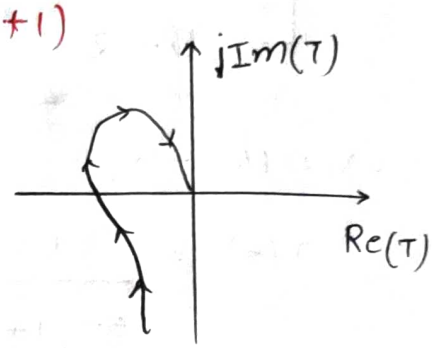
→ Question: The polar plot of the system is shown below. Calculate the order of the system

A) 0
B) 1
C) 2
D) 3

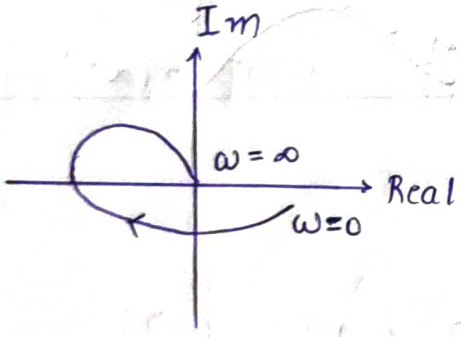


→ Question: The correct shape of the polar plot for $G(s)H(s) = \frac{1}{s(sT_1+1)(sT_2+1)}$ is

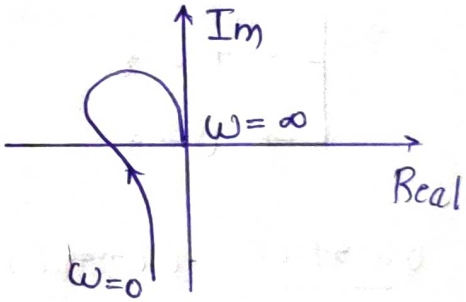
↳ T - 1
↳ σ - 3



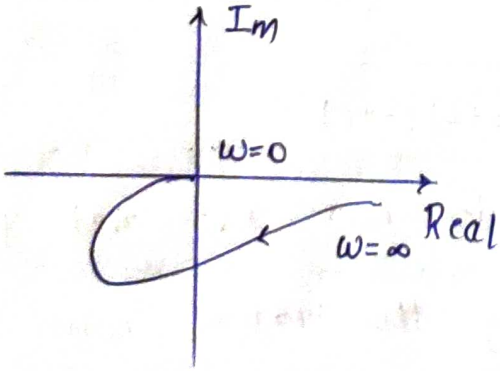
A)



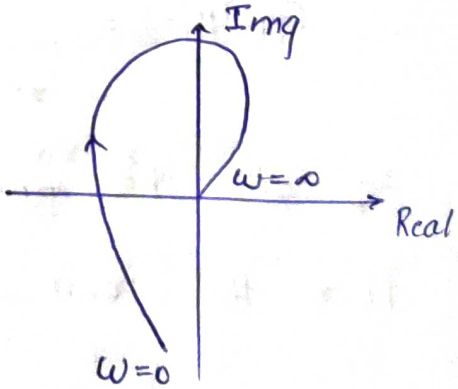
B)



C)

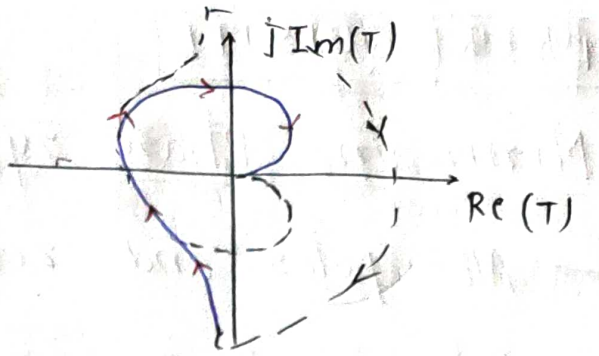


D)



→ Question: For the certain unity feedback system $G(s) = \frac{k}{s(s+1)(2s+1)(3s+1)}$. The Nyquist plot is

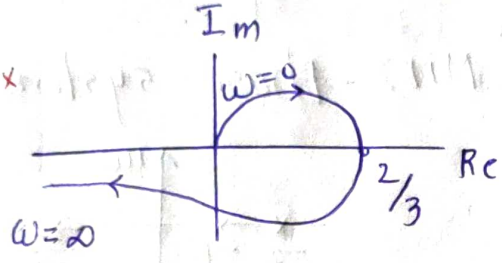
↳ T-1
↳ 0-4

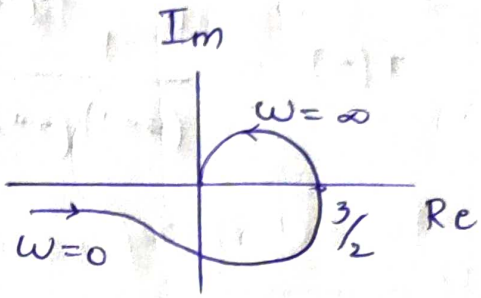


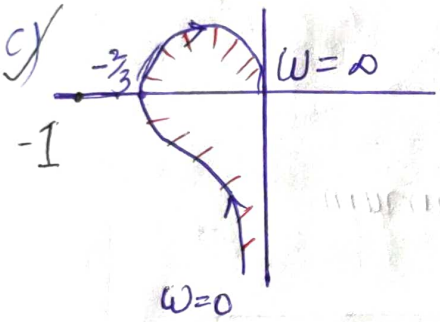
Question: A unity feedback system has O.L.T.F

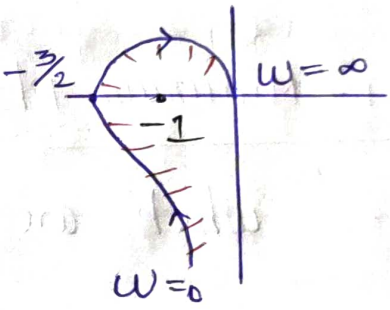
$$G(s) = \frac{1}{5(2s+1)(s+1)}$$

. The Nyquist plot for the system is

A)


B)


C)


D)


→ Enclosement concept

NSC in 1+GH

$N = 0$
$P_c = 0$ → $Z_c = 0$

NSC in 1+GH = 0

$N \neq 0$
$P_c = 0$ → $Z_c = -N \neq 0$

$$1 + G(s)H(s) = 2s^3 + 3s^2 + 5 + 1 = 0$$

EP: $2s^3$ IP: $3s^2 + 5 + 1$

IP > EP → CLS STABLE

↳ $Z_c = 0$

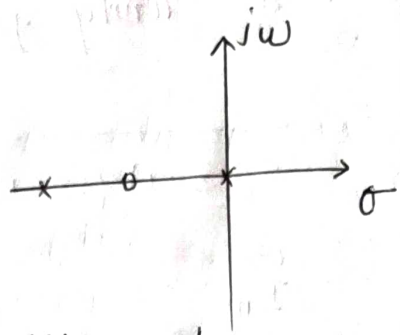
↳ C

PREREQUISITE

Minimum Phase System:

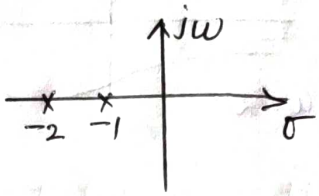
→ All poles and zeros must be on L·H·P

→ Poles and zeros at origin or jω axis are allowed.



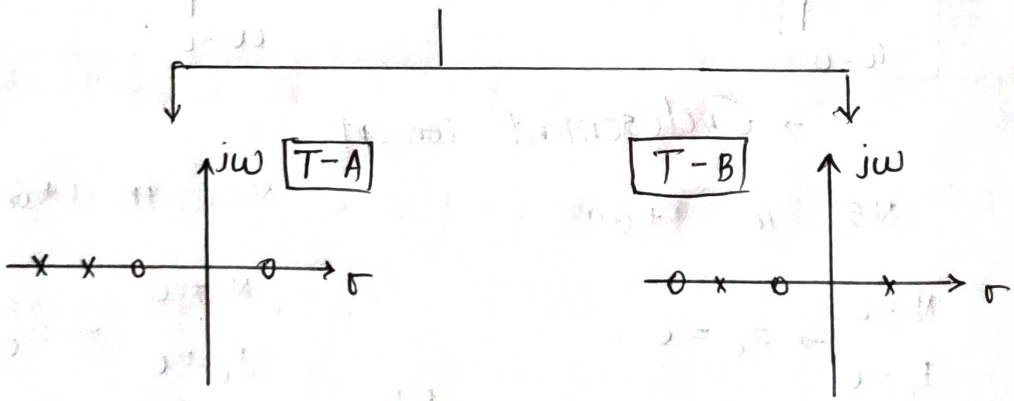
Min-Phase system ←

$$T(s) = \frac{1}{(s+1)(s+2)}$$ → Min-phase system



Non-Minimum phase System:

which are not minimum



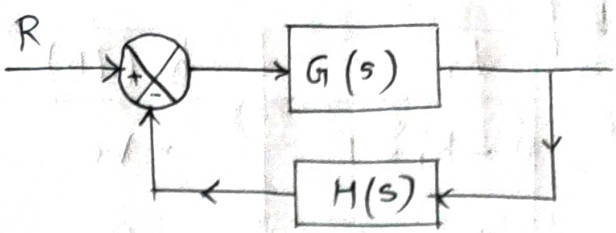
→ All poles in L·H·P

→ Few zeros in R·H·P

→ All zeros in L·H·P

→ Few poles in R·H·P

Gain Margin



$$C.L.T.F = \frac{G(s)}{1 + G(s)H(s)}$$

$$O.L.T.F = G(s)H(s)$$

1. Gain Margin is a factor used to determine the stability of C.L.S.
2. It is the amount of gain (k_1) that is needed to multiplied in O.L.T.F $G(s)H(s)$ such that corresponding C.L.S becomes Marginally Stable.
3. It is the amount of gain (in dB) that is needed to be added in O.L.T.F $G(s)H(s)$ such that corresponding C.L.S becomes Marginal Stable (on the verge of instability).

$T = k_1 GH \rightarrow$ k_1 is G.M in Ratio

$T(dB) = 20\log_{10} k_1 + 20\log_{10} GH$

$\hookrightarrow 20\log_{10} k_1$ is the G.M in dB.

NOTE

k_1	k_1 (dB)
$0 < k_1 < 1$	-ve
$k_1 = 1$	0
$k_1 > 1$	+ve

* G.M in Ratio (k_1) is always +ve ($0 < k_1 < \infty$)

* G.M in dB → +ve (dB)
→ -ve (dB)
→ 0dB

Phase Margin

1. It is a factor used to determine the stability of C.L.S.
2. It is the amount of phase angle that is needed to be added in phase of $G(s)H(s)$ such that corresponding C.L.S becomes Marginal stable.

* P.M is always measured in degree → -ve in degree
→ 0°
→ +ve in degree

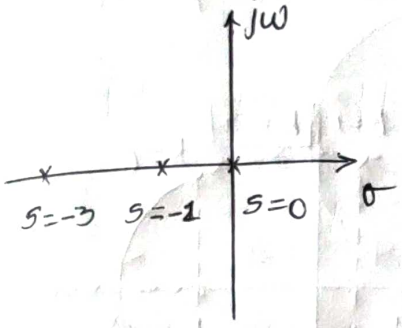
Important Observations:

CASE-1:

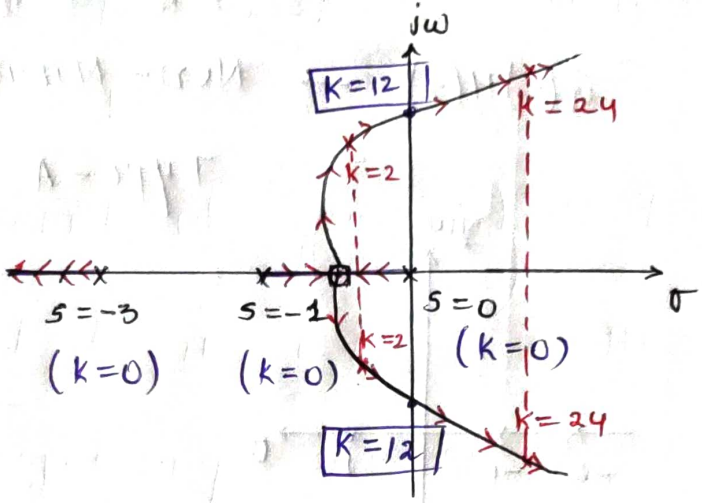
O.L.T.F of C.L.S is given

$$G(s)H(s) = \frac{k}{s(s+1)(s+3)}$$

Comment on Gain Margin, when $0 < k < \infty$, -ve feedback.



Minimum phase system



ROOT LOCUS

$\longrightarrow$

$$G(s)H(s) = \frac{2}{s(s+1)(s+3)} \xrightarrow{G.M=6}$$

G.M = 6 > 1

G.M (dB) = +ve

$$G(s)H(s) = \frac{12}{s(s+1)(s+3)} \xrightarrow{G.M=1}$$

G.M = 1

G.M (dB) = 0dB

$$G(s)H(s) = \frac{24}{s(s+1)(s+3)} \xrightarrow{G.M=0.5}$$

G.M = 0.5

G.M (dB) = -ve

CASE-2:

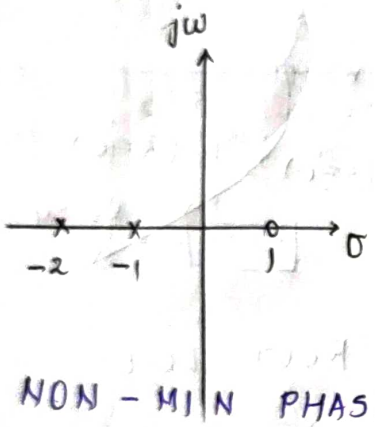
O·L·T·F of C·L·S is given

$$G(s)H(s) = \frac{k(1-s)}{(s+1)(s+2)} = \frac{-k(s-1)}{(s+1)(s+2)}$$

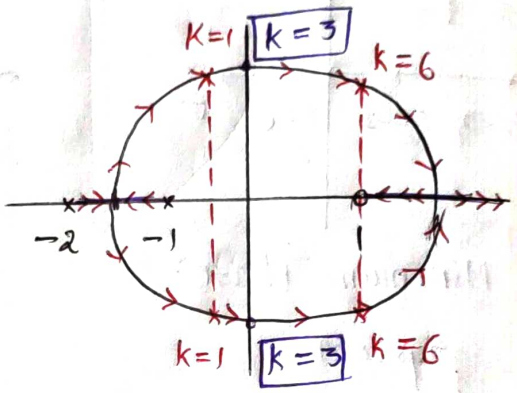
Comment on ~~stability of~~ Gain Margin, when $0 < k < \infty$, -ve feedback.

$G(s)H(s)$ → Non-Minimum Phase system

Type-A



NON-MIN PHASE SYSTEM



ROOT LOCUS

→

$$GH = \frac{-(s-1)}{(s+1)(s+2)} \xrightarrow{GM=3}$$

G·M = 3 > 1

G·M(dB) = +ve

$$GH = \frac{-3(s-1)}{(s+1)(s+2)} \xrightarrow{GM=1}$$

G·M = 1

G·M(dB) = 0

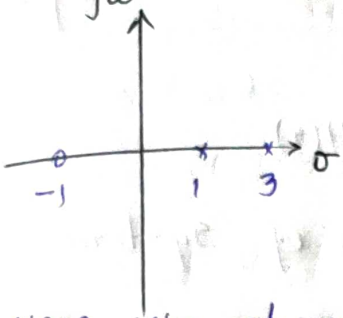
$$GH = \frac{-6(s-1)}{(s+1)(s+2)} \xrightarrow{G·M=0.5}$$

G·M = 0.5

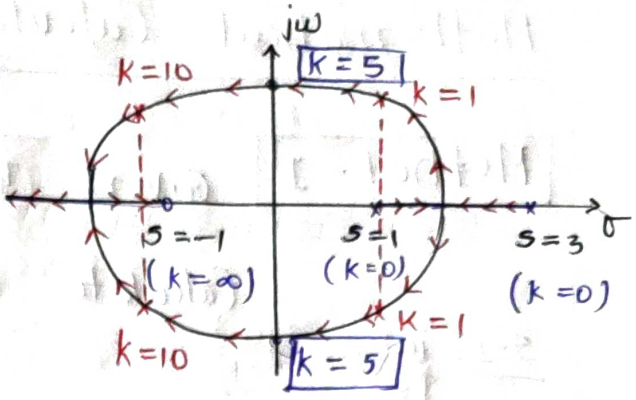
G·M(dB) = -ve

CASE-3 $G(s)H(s) = \dfrac{k(s+1)}{(s-1)(s-3)}$ $0 < k < \infty$, -ve feedback

Open loop System → Non-minimum Phase, Type B.



Non-Min phase system, T-B



ROOT LOCUS

→

$GH = \dfrac{s+1}{(s-1)(s-3)}$ $\xrightarrow{G.M = 5}$ $G.M > 1$

$G.M(dB) = +ve$

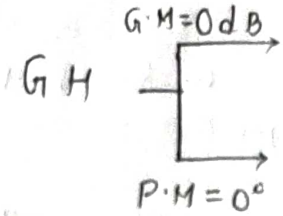
$GH = \dfrac{5(s+1)}{(s-1)(s-3)}$ $\xrightarrow{G.M = 1}$ $G.M = 1$

$G.M = 0\,dB$

$GH = \dfrac{10(s+1)}{(s-1)(s-3)}$ $\xrightarrow{G.M = 0.5}$ $G.M < 1$

$G.M(dB) = -ve$

O.L.T.F	C.L.T.F	GM (dB)	P.M in degree
Minimum phase System	STABLE	+ve	+ve
	UNSTABLE	-ve	-ve
Non-Minimum phase system (T-A)	STABLE	+ve	+ve
	UNSTABLE	-ve	-ve
Non-Minimum Phase system (T-B)	STABLE	-ve	-ve
	UNSTABLE	+ve	+ve



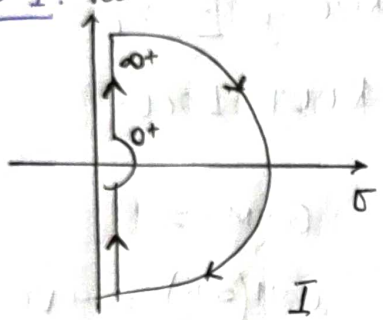
→ MARGINAL STABLE

Mathematical Calculation of Gain Margin

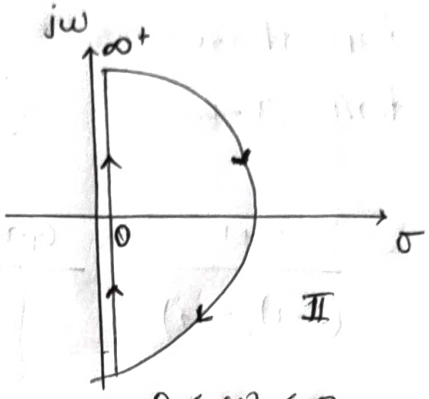
Method - I Given $T(s) = G(s)H(s)$

Calculate Gain Margin of System

S-1: jω



$0 < \omega < \infty$



$0 \leq \omega < \infty$

S-2:

Put $s = j\omega$ and determine range of ω

$$T(s)\big|_{s=j\omega} = T(j\omega)$$

Contour I : $0 < \omega < \infty$; Contour II : $0 \leq \omega < \infty$

S-3: $\angle(T(j\omega)) = -180°$.

→ Solve and calculate $\omega \rightarrow \omega_{pc}$

→ Possible Phase cross over frequency.

S-4: Validation of ω

i) ω is Real and +ve

ii) $\angle T(j\omega) = -180°$

If ω valid : $\omega = \omega_{pc}$ = PHASE CROSSOVER FREQUENCY

S-5: At $\omega = \omega_{pc}$, $|T(j\omega_{pc})| = M$

$$\text{Gain Margin} = \frac{1}{M} \text{ (in Ratio)}$$

$$G.M \text{ (dB)} = 20 \log_{10} GM \text{ (in Ratio)}$$

Note:

1. If ω is not valid ω_{pc}, then gain margin will be either $+\infty$ dB or $-\infty$ dB depending upon.
   i) Nature of Open Loop system
   ii) Absolute stability of closed Loop System
2. G.M = $+\infty$ dB or $-\infty$ dB represents absolute stable / unstable nature.

Invalid ω_{pc}:

O.L.S	C.L.S	G.M
Min Phase system	STABLE	$+\infty$ dB
	UNSTABLE	$-\infty$ dB
Non-Minimum Phase system (T-A)	STABLE	$+\infty$ dB
	UNSTABLE	$-\infty$ dB
Non-Minimum Phase System (T-B)	STABLE	$-\infty$ dB
	UNSTABLE	∞ dB

→ **Question:** O.L.T.F or C.L.S is given by

$$G(s)\,H(s) = \frac{1}{(s+1)(s+2)}$$

Calculate Gain Margin of system.

S-1: $$T(j\omega) = \frac{1}{(1+j\omega)(2+j\omega)} \qquad 0 \le \omega < \infty^+$$

S-2: $$\angle T(j\omega) = -180^\circ$$

$$\rightarrow -\tan^{-1}\omega - \tan^{-1}\frac{\omega}{2} = -180^\circ$$

$$\tan^{-1}\left\{\frac{\omega + \frac{\omega}{2}}{1 - \frac{\omega^2}{2}}\right\} = 180^\circ$$

$$\frac{3\omega}{2-\omega^2} = \tan 180^\circ$$

$$\frac{3\omega}{2-\omega^2} = 0 \quad \begin{cases} \omega = 0 \\ \omega = \infty \end{cases} \rightarrow \text{Possible } \omega_{pc}$$

S-3 Validity $\omega = \infty$ is invalid

$\omega = 0$

1. Real and Positive

2. $$\left(-\tan^{-1}\omega - \tan^{-1}\frac{\omega}{2}\right)_{\omega=0} = -180^\circ$$

$$0 \neq -180^\circ$$

↳ $\omega = 0$: Invalid ω_{pc}

5-4: O.L.S: $GH = \frac{1}{(s+1)(s+2)}$ → Min-phase system

C.L.S: $1 + GH = 0$

$s^3 + 3s + 3 = 0$

All Roots L.H.P

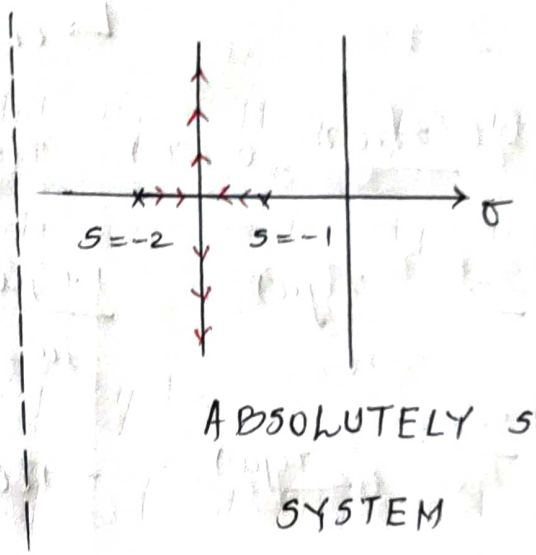
↳ CLS STABLE → G.M = $+\infty$ dB

NOTE

$GH = \frac{1}{(s+1)(s+2)}$

↳ $G_1H_1 = \frac{k}{(s+1)(s+2)}$

ROOT LOCUS $(0 < k < \infty)$



ABSOLUTELY STABLE SYSTEM

→ Question: O.L.T.F or C.L.S is given by

$G(s)H(s) = \frac{1}{s(s+1)}$

Calculate Gain Margin of system.

5-1: $T(j\omega) = \frac{1}{j\omega(j\omega+1)}$ $0 < \omega < \infty$

5-2: $\angle T(j\omega) = -180°$

→ $-90° - \tan^{-1}\omega = -180°$

$\tan^{-1}\omega = 90°$

$\omega = \infty$ → Invalid

S-3: ω_{pc} → Invalid

S-4: O·L·T·F : $GH = \frac{1}{s(s+1)}$ Min-Phase

C·L·T·F : $1 + GH = 0$

$s^2 + s + 1 = 0$

All Roots LHP

↳ CLS STABLE → GM = $+\infty$ dB

→ Question: O·L·T·F $T(s) = \frac{1+s}{s}$. Cal G·M of S/S

S-1: $T(j\omega) = \frac{1+j\omega}{j\omega}$; $0 < \omega < \infty$

S-2: $\angle T(j\omega) = -180°$

→ $\tan^{-1}\omega - 90° = -180°$

$\tan^{-1}\omega = -90°$

$\omega = -\infty$

S-3: ω_{pc} invalid

S-4: OLTF : $GH = \frac{s+1}{s}$: Minimum Phase

C·L·T·F : $1 + GH = 0$

$2s + 1 = 0$

↳ CLS STABLE → G·M = $+\infty$ dB

Question: O.L.T.F $G(s)H(s) = \frac{s+1}{s^2}$. Cal G.M

S-1: $T(j\omega) = \frac{1+j\omega}{(j\omega)^2}$ $\quad 0 < \omega < \infty$

S-2: $\angle T(j\omega) = -180°$

$\tan^{-1}\omega - 180° = -180°$ (or) $\tan^{-1}\omega + 180° = -180°$

$\omega = 0$

$\omega = 0 \rightarrow$ Invalid $(\because 0 < \omega < \infty)$

S-3: Invalid ω_{pc}

S-4: O.L.T.F : $GH = \frac{s+1}{s^2}$ → Minimum Phase

C.L.T.F : $1 + GH = 0$

$s^2 + s + 1 = 0$

C.L.S STABLE → G.M = $+\infty$ dB

Mathematical Calculation of PHASE MARGIN

Given $T(s)$

step-1: $s = j\omega$ and range of ω.

step-2: $|T(j\omega)| = 1$ → Possible gain crossover frequency (ω_{gc})

Step-3: Validity: Real and +ve.

step-4: Phase Margin = $\angle T(j\omega_{gc}) + 180°$

NOTE:

* If ω_{gc} becomes invalid, then Phase Margin will be $+\infty^\circ$ or $-\infty^\circ$ depending on nature of open Loop system and absolute stability of C.L.S.

Invalid ω_{gc}:

OLS	CLS	PM
Min phase system	STABLE	$+\infty^\circ$
	UNSTABLE	$-\infty^\circ$
Non-Min phase system (T-A)	STABLE	$+\infty^\circ$
	UNSTABLE	$-\infty^\circ$
Non-Min Phase system (T-B)	STABLE	$-\infty^\circ$
	UNSTABLE	$+\infty^\circ$

→ Question: $G(s)H(s) = \frac{10}{s(s+1)}$. Calculate Phase Margin of the system.

S-1: $T(j\omega) = \frac{10}{j\omega(j\omega+1)} \quad 0 < \omega < \infty$

S-2

$|T(j\omega)| = 1$

$$\frac{10}{\omega\sqrt{1+\omega^2}} = 1 \rightarrow 10 = \omega\sqrt{1+\omega^2}$$

$$100 = \omega^2(1+\omega^2)$$

Let $\omega^2 = x$

$$x^2 + x - 100 = 0$$

$\rightarrow$ $x = 9.51 = \omega^2 \rightarrow \omega = 3.082$

$x = -10.5 \rightarrow$ Invalid

5-3: $\omega_{gc} = 3.082$ (Real and Positive)

5-4: $\angle T(j\omega_{gc}) + 180^\circ = P.M$

$$P.M = [-90^\circ - \tan^{-1}\omega]_{\omega=\omega_{gc}} + 180^\circ$$

$$P.M = -162^\circ + 180^\circ = 18^\circ$$

$\rightarrow$ Question:

$$G(s)H(s) = \frac{10}{s(1+s)(2+s)}$$

Calculate Gain Margin and Phase Margin.

Gain Margin:

5-1: $$T(j\omega) = \frac{10}{j\omega(1+j\omega)(2+j\omega)} \qquad 0 < \omega < \infty$$

5-2: $$\angle T(j\omega) = -180^\circ$$

$$-90^\circ - \tan^{-1}\omega - \tan^{-1}\frac{\omega}{2} = -180^\circ$$

$$\tan^{-1}\omega - \tan^{-1}\frac{\omega}{2} = 90^\circ$$

$$\tan^{-1}\left\{\frac{\omega + \frac{\omega}{2}}{1 - \frac{\omega^2}{2}}\right\} = 90^\circ$$

$$\frac{3\omega}{2-\omega^2} = \tan 90^\circ = \infty = \frac{1}{0}$$

$$2 - \omega^2 = 0$$

$$\omega = \sqrt{2} \longrightarrow \text{Possible } \omega_{pc}$$

5-3: $\omega = \sqrt{2}$: Real and Positive

$$\omega_{pc} = \sqrt{2}$$

5-4:

$$|T(j\omega_{pc})| = \left\{\frac{10}{\omega\sqrt{1+\omega^2}\sqrt{4+\omega^2}}\right\}_{\omega=\sqrt{2}} = \frac{10}{6}$$

$$G.M = \frac{1}{|T(j\omega_{pc})|} = \frac{6}{10} = 0.6$$

$$G.M\,(dB) = 20\log_{10} 0.6 = -4.43\ dB$$

Phase Margin:

5.1:

$$T(j\omega) = \frac{10}{j\omega(1+j\omega)(2+j\omega)} \qquad 0 < \omega < \infty$$

5.2:

$$|T(j\omega)| = 1$$

$$\frac{10}{\omega\sqrt{1+\omega^2}\sqrt{2+\omega^2}} = 1$$

$$\omega^2(1+\omega^2)(2+\omega^2) = 100$$

$\omega^2 = x$, $\quad x(1+x)(2+x) = 100$

$\rightarrow \quad x = 3.24 = \omega^2$

$$\boxed{\omega = 1.8}$$

S-3: $\omega = 1.8$ (Real and positive) : valid ω_{gc}

$\omega_{gc} = 1.8$

S-4: $P.M = 180^\circ + \angle T(j\omega_{gc})$

$$= 180^\circ + \left[-90^\circ - \tan^{-1}\omega - \tan^{-1}\frac{\omega}{2}\right]_{\omega = 1.8}$$

$P.M = -13^\circ$

→ Question: $T(s) = \dfrac{10(s+3)}{s(s+1)(s+2)}$

Calculate Gain Margin and Phase Margin

Gain Margin:

S-1: $T(j\omega) = \dfrac{10(3+j\omega)}{j\omega(1+j\omega)(2+j\omega)}$, $0 < \omega < \infty$

S-2:

$\angle T(j\omega) = -180^\circ$

$$\tan^{-1}\frac{\omega}{3} - 90^\circ - \tan^{-1}\omega - \tan^{-1}\frac{\omega}{2} = -180^\circ$$

$$\tan^{-1}\left\{\frac{\frac{\omega}{3} - \omega}{1 + \frac{\omega^2}{3}}\right\} - \tan^{-1}\frac{\omega}{2} = -90^\circ$$

$$\tan^{-1}\left(\frac{-2\omega}{3+\omega^2}\right) - \tan^{-1}\frac{\omega}{2} = -90^\circ$$

$$\tan^{-1}\left\{\frac{\frac{-2\omega}{3+\omega^2} - \frac{\omega}{2}}{1 - \frac{2\omega^2}{2(3+\omega^2)}}\right\} = -90^\circ$$

$$\frac{-4\omega - 3\omega + \omega^3}{6 + 2\omega^2 - 2\omega^2} = \tan(-90^\circ) = -\infty$$

$$= \frac{-1}{0}$$

$0 = -6 \rightarrow$ No valid ω_{pc}

5-3: Invalid ω_{pc}

5-4: OLTF : Min - phase

CLTF : $1 + GH = 0$

$$s^3 + 3s^2 + 12s + 30 = 0$$

EP: $s^3 + 12s$; IP: $3s^2 + 30$

IP > EP

C.L.S STABLE $\rightarrow$ G.M $= +\infty$ dB

Phase Margin:

5-1: $$T(j\omega) = \frac{10(3+j\omega)}{j\omega(1+j\omega)(2+j\omega)} \qquad 0 < \omega < \infty$$

5-2: $|T(j\omega)| = 1$

$$\frac{10\sqrt{9+\omega^2}}{\omega\sqrt{1+\omega^2}\sqrt{2+\omega^2}} = 1$$

$$100(9+\omega^2) = \omega^2(1+\omega^2)(4+\omega^2)$$

Let $\omega^2 = x$

$$100(9+x) = x(1+x)(4+x)$$

$$x = 11.05 = \omega^2$$

$$\omega = 3.32$$

S-3: $\omega = 3.32$: Real and +ve : Valid ω_{gc}

S-4: $P.M = 180^\circ + \angle T(j\omega_{gc})$

$$= 180^\circ + \left\{ \tan^{-1}\frac{\omega}{3} - 90^\circ - \tan^{-1}\omega - \tan^{-1}\frac{\omega}{2} \right\}_{\omega=3.32}$$

$$P.M = 180^\circ - 174.2^\circ = 5.8^\circ$$

→ Question: $G(s)H(s) = \dfrac{1-s}{(s+1)(s+2)}$

Calculate Gain Margin and Phase Margin.

Gain Margin:

S-1: $T(j\omega) = \dfrac{1-j\omega}{(1+j\omega)(2+j\omega)}$ $0 \le \omega < \infty$

S-2: $\angle T(j\omega) = -180^\circ$

$$-\tan^{-1}\omega - \tan^{-1}\omega - \tan^{-1}\frac{\omega}{2} = -180^\circ$$

$$\tan^{-1}\omega + \tan^{-1}\omega + \tan^{-1}\frac{\omega}{2} = 180^\circ$$

$$\tan^{-1}\left\{\frac{2\omega}{1+\omega^2}\right\} + \tan^{-1}\frac{\omega}{2} = 180^\circ$$

$$\tan^{-1}\left\{\frac{\frac{2\omega}{1-\omega^2}+\frac{\omega}{2}}{1-\frac{2\omega^2}{2(1-\omega^2)}}\right\} = 180^\circ$$

$$\frac{4\omega+\omega-\omega^3}{2(1-\omega^2)-2\omega^2} = \tan 180^\circ = 0$$

$$5\omega-\omega^3 = 0$$

$$\omega = 0, \quad \omega = \sqrt{5}$$

5-3 $\omega = 0 \rightarrow 0^\circ \neq -180^\circ \rightarrow$ Invalid ω_{pc}

$\omega = \sqrt{5}$: Real and Positive : valid ω_{pc}.

5-4 : $$|T(j\omega)|_{\omega=\sqrt{5}} = \frac{\sqrt{1+\omega^2}}{\sqrt{1+\omega^2}\sqrt{1+\omega^2}} = \frac{1}{\sqrt{9}} = \frac{1}{3}$$

$$G.M = \frac{1}{|T(j\omega)|_{\omega=\omega_{pc}}} = 3$$

$$G.M\,(dB) = 20\log_{10}^{3}$$

Phase Margin:

5-1 : $$T(j\omega) = \frac{1-j\omega}{(1+j\omega)(2+j\omega)} \qquad 0 \leq \omega < \infty$$

5-2 : $$|T(j\omega)| = 1$$

$$\frac{\sqrt{1+\omega^2}}{\sqrt{1+\omega^2}\ \sqrt{4+\omega^2}} = 1$$

$4 + \omega^2 = 1$

$\omega^2 = -3$

$\omega = \pm j3 \rightarrow$ Purely Imaginary : Invalid ω_{pc}

S-3: O.L.T.F : Non minimum phase system T-A

C.L.T.F $1 + GH = 0$

$s^3 + 2s + 3 = 0$

5 stable C.L.S $\rightarrow$ P.M $= +\infty°$

Question: $G(s)H(s) = \dfrac{1}{(1+s)^3}$

Calculate Gain Margin and Phase Margin

Gain Margin :

S-1: $T(j\omega) = \dfrac{1}{(1+j\omega)^3}$ $\quad 0 \leq \omega < \infty$

S-2: $\angle T(j\omega) = -180°$

$-3\tan^{-1}\omega = -180°$

$\tan^{-1}\omega = 60°$

$\omega = \sqrt{3} \rightarrow$ Real and positive

S-3: $\rightarrow$ Valid ω_{pc}

$$|T(j\omega)|_{\omega=\omega_{pc}} = \frac{1}{(\sqrt{1+\omega^2})^3}\Bigg|_{\omega=\sqrt{3}} = \frac{1}{8}$$

$$G.M = 8$$

$$G.M\,(dB) = 20\log_{10} 8$$

Phase Margin:

5-1: $$T(j\omega) = \frac{1}{(1+j\omega)^3} \qquad 0 \le \omega < \infty$$

5-2:

$$|T(j\omega)| = 1$$

$$\frac{1}{\left(\sqrt{1+\omega^3}\right)^3} = 1$$

$$1 + \omega^3 = 1$$

$\omega = 0 \rightarrow$ Real and Positive : Valid ω_{gc}

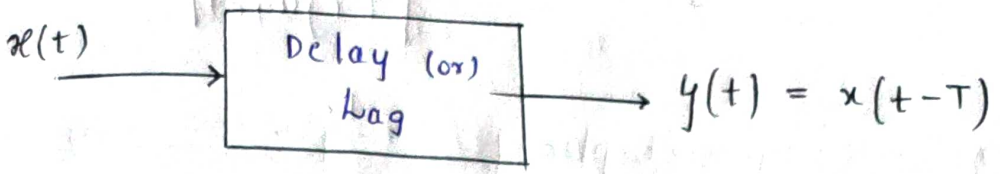
5-3:

$$P.M = 180^\circ + \left(\angle T(j\omega)\right)_{\omega=\omega_{gc}}$$

$$P.M = 180^\circ + \left(-3\tan^{-}\omega\right)_{\omega=0}$$

$$P.M = 180^\circ$$

TRANSPORTATION LAG

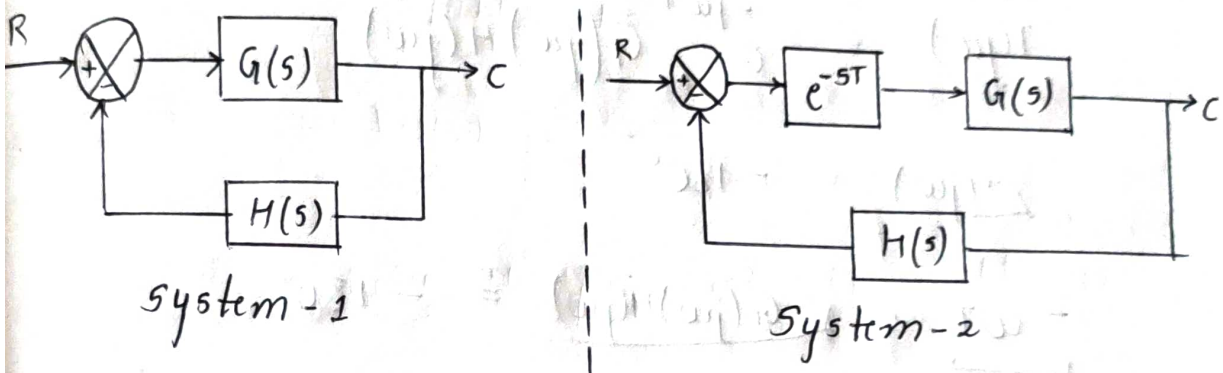


$$Y(s) = e^{-sT} X(s)$$

$$H(s) = e^{-sT}$$

T : Delay Time or Dead Time

Transportation Lag in Feedback System:



OLTF: $T(s) = G(s)H(s)$

OLTF: $T(s) = e^{-sT} G(s)H(s)$

System-1

Open Loop Transfer f^n

$$T(s) = G(s)H(s)$$

Gain Margin:

i) Phase Crossover Frequency

$$T(j\omega) = G(j\omega)H(j\omega)$$

$$\angle T(j\omega) = -180^\circ$$

$$\angle G(j\omega)H(j\omega) = -180^\circ$$

$$\hookrightarrow \omega_{pc} = \omega_1$$

ii) Gain Margin $= \dfrac{1}{|T(j\omega_{pc})|}$

$= \dfrac{1}{|G(j\omega_{pc})H(j\omega_{pc})|}$, $\omega_{pc} = \omega_1$

System - 2

i) Phase Crossover Frequency

$$T(j\omega) = e^{-j\omega T} G(j\omega)H(j\omega)$$

$$\angle T(j\omega) = -180^\circ$$

$$\underbrace{-\omega T}_{\text{RADIAN}} + \angle G(j\omega)H(j\omega) = -180^\circ$$

$\hookrightarrow \omega_{pc} = \omega_2$

ii)

$$G.M = \frac{1}{|T(j\omega_{pc})|}$$

$$= \frac{1}{|G(j\omega_{pc})H(j\omega_{pc})|}, \quad \omega_{pc} = \omega_2$$

Introduction of Transportation Lag in OLTF (Forward path)

i) changes ω_{pc}

ii) changes Gain Margin

System - 1 O.L.T.F

$$T(s) = G(s)H(s)$$

Phase Margin:

i) Gain Crossover Frequency

$$|T(j\omega)| = |G(j\omega)H(j\omega)| = 1$$

$$\hookrightarrow \omega_{gc} = \omega_1$$

ii) $$P.M = \angle T(j\omega_{gc}) + 180^\circ = \angle G(j\omega_{gc})H(j\omega_{gc}) + 180^\circ$$

System - 2 $$O.L.T.F = T(s) = e^{-sT}G(s)H(s)$$

i) Gain Crossover Frequency

$$|T(j\omega)| = 1$$

$$\rightarrow |G(j\omega)H(j\omega)| = 1 \quad (\because |e^{-j\omega T}| = 1)$$

$$\hookrightarrow \omega_{gc} = \omega_1$$

ii) $$P.M = \angle T(j\omega_{gc}) + 180^\circ$$

$$P.M = \left\{ -\omega T + \underline{\angle G(j\omega)H(j\omega)} \right\}_{\omega_{gc}} + 180^\circ$$

With Introduction of Transportation Lag:

i) ω_{gc} remains same.

ii) Phase margin changes.

* iii) Phase margin decreases, so overall stability of C.L.S ↓.

→ Question: $G(s)H(s) = \frac{e^{-0.1s}}{s}$.

Calculate Gain Margin and phase Margin.

Gain Margin:

S-1: $T(j\omega) = \frac{e^{-j0.1\omega}}{j\omega}$, $0 < \omega < \infty$

S-2: $\angle T(j\omega) = -180°$

$\underbrace{-0.1\omega}_{\text{Radian}} - \underbrace{90°}_{\text{degree}} = \underbrace{-180°}_{\text{degree}}$

$-0.1\omega - \frac{\pi}{2} = -\pi$

$0.1\omega = \frac{\pi}{2}$

$\omega = 5\pi$ → Real and +ve : Valid ω_{gc}

S-3:

$|T(j\omega)|_{\omega=5\pi} = \left(\frac{1}{\omega}\right)_{\omega=5\pi} = \frac{1}{5\pi}$

$G.M = 5\pi$

$$G.M(dB) = 20\log_{10} 5\pi$$

Phase Margin:

5-1: $$T(j\omega) = \frac{e^{-j0.1\omega}}{j\omega}, \quad 0 < \omega < \infty$$

5-2: $$|T(j\omega)| = 1$$

$$\frac{1}{\omega} = 1 \rightarrow \omega_{gc} = 1$$

5-3: $$P.M = 180^\circ + \angle T(j\omega)\Big|_{\omega=\omega_{gc}}$$

$$P.M = 180^\circ + [-0.1\omega - 90^\circ]_{\omega_{gc}=1}$$

$$= \pi + \left[-0.1 - \frac{\pi}{2}\right]$$

$$P.M = \left(\frac{\pi}{2} - 0.1\right) \text{ RADIAN} = 1.47 \text{ RAD}$$

$$P.M = 90^\circ - 0.1 \times \frac{180^\circ}{\pi} = 84.27^\circ$$

→ Question: $$G(s)H(s) = \frac{3e^{-2s}}{s(s+2)}$$

Calculate Gain Margin and Phase Margin

Gain Margin:

5-1: $$T(j\omega) = \frac{3e^{-2j\omega}}{j\omega(2+j\omega)} \qquad 0 < \omega < \infty$$

5-2:

$$\angle T(j\omega) = -180^\circ$$

$$-2\omega - \frac{\pi}{2} - \tan^{-1}\frac{\omega}{2} = -\pi$$

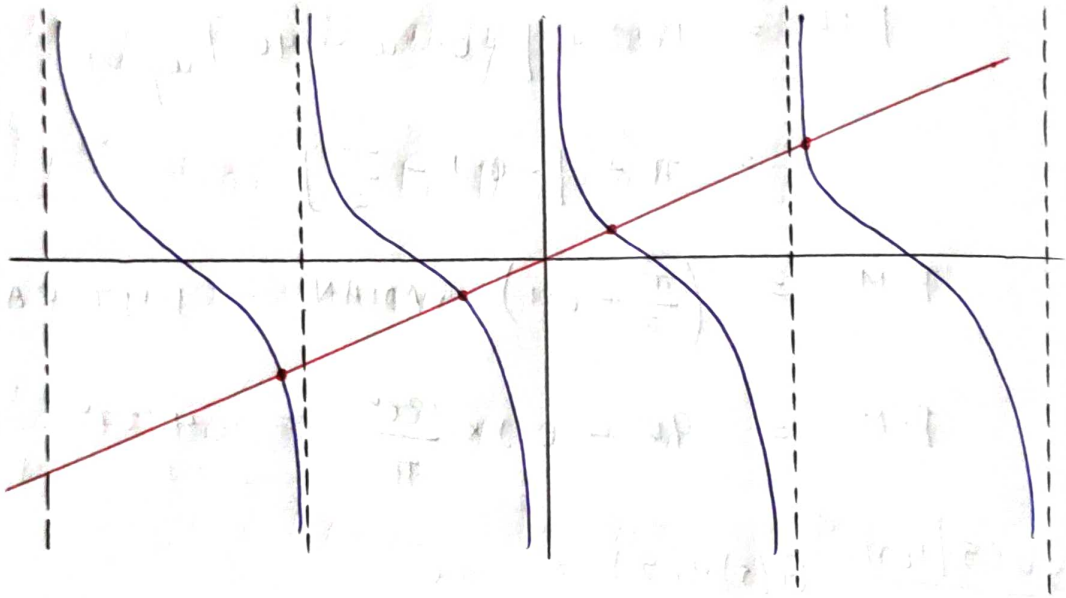
$$2\omega + \tan^{-1}\frac{\omega}{2} = \frac{\pi}{2}$$

METHOD - 1

$$\tan^{-1}\frac{\omega}{2} = \frac{\pi}{2} - 2\omega$$

$$\frac{\omega}{2} = \tan\left(\frac{\pi}{2} - 2\omega\right)$$

$$\cot 2\omega = \omega/2$$



$\cot 2\omega = \frac{\omega}{2}$ → Infinite possible values of ω.

METHOD - 2

NOTE

$$2\omega + \tan^{-1}\frac{\omega}{2} = \frac{\pi}{2} \qquad \tan^{-1}x = x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots$$

$$2\omega + \left\{\left(\frac{\omega}{2}\right) - \frac{\left(\frac{\omega}{2}\right)^3}{3} + \frac{\left(\frac{\omega}{2}\right)^5}{5} - \cdots\right\} = \frac{\pi}{2}$$

↓ Linear Approximation

$$2\omega + \frac{\omega}{2} = \frac{\pi}{2}$$

$$\omega = \frac{\pi}{5} \longrightarrow \boxed{\omega_{pc} = \frac{\pi}{5}}$$

S-3

$$|G(j\omega)H(j\omega)|_{\omega=\omega_{pc}} = \left\{ \frac{3}{\omega\sqrt{4+\omega^2}} \right\}_{\omega=\frac{\pi}{5}} = M$$

$$G \cdot M = \frac{1}{M} = 0.439$$

$$G \cdot M(dB) = 20\log_{10}(0.439) = -7.33\ dB$$

Phase Margin:

S-1:

$$T(j\omega) = \frac{3e^{-2j\omega}}{j\omega(2+j\omega)} \qquad 0 < \omega < \infty$$

S-2:

$$|T(j\omega)| = 1$$

$$\frac{3}{\omega\sqrt{4+\omega^2}} = 1$$

$$\omega^2(4+\omega^2) = 9$$

$$\omega^2 = x \rightarrow x^2 + 4x - 9 = 0$$

$$\longrightarrow \omega = 1.267 = \omega_{gc}$$

S-3:

$$P.M = 180^\circ + \angle T(j\omega)\Big|_{\omega=\omega_{gc}}$$

$$P.M = 180^\circ + \left[\underbrace{-2\omega}_{RAD} - 90^\circ - \tan^{-1}\frac{\omega}{2} \right]_{\omega=1.267}$$

$$P.M = 180^\circ + \left[-2 \times 1.267 \times \frac{180^\circ}{\pi} - 90^\circ - \tan^{-1}\frac{(1.267)}{2} \right]$$

$P.M = -86.74^\circ$

Shortcut of Gain Margin

- Given : $T(s) = G(s)H(s)$
- Let gain k_1 is multiplied in $G(s)H(s)$

$$k_1 G(s) H(s) \qquad 0 < k_1 < \infty$$

So that roots of $1 + k_1 G(s) H(s)$ represents Marginal stability

Step-1: Form Routh Table for

$$1 + k_1 G(s) H(s) = 0$$

Step-2:

ODD Row	Last Row	Gain Margin
Invalid	Invalid	$+\infty$ or $-\infty$
Invalid	Valid	Finite
Valid	Invalid	Finite
Valid	Valid	Absurd case

Not possible

Step-3: VALID ODD ROW

- Odd Row = 0
- k = +ve

- Auxiliary Eqn Roots → Non Repeated on jω Axis

Then, k_1 = Gain Margin in Ratio

and Roots of Auxiliary Eqn → ω_{pc}

Step-4: VALID LAST ROW

Last Row = 0

k = +ve → $0 < k < \infty$

→ Question: $G(s)H(s) = \dfrac{s-2}{(s+1)(s+3)(s+4)}$ Cal G.M.

$$k_1 GH = \frac{k_1(s-2)}{(s+1)(s+3)(s+4)}$$

$$1 + k_1 GH = 0$$

$$s^3 + 8s^2 + (19+k_1)s + (12-2k_1) = 0$$

s^3	1	$19+k_1$	→ ODD ROW = 0
s^2	8	$12-2k_1$	$k_1 = -14$ → Invalid
s^1	$\frac{10k_1+140}{8}$	0	→ Last Row = 0
s^0	$12-2k_1$		$k_1 = 6$ $(0 < k_1 < \infty)$

G.M = 6

$$G.M(dB) = 20\log_{10} 6$$

→ **Question:** $G(s)H(s) = \frac{10}{s(s+1)(s+2)}$ · Cal G.M

$$k_1 GH = \frac{10k_1}{s(s+1)(s+2)}$$

$$1 + k_1 GH = 0$$

$$s^3 + 3s^2 + 2s + 10k_1 = 0$$

s^3	1	2
s^2	3	$10k_1$
s^1	$\frac{6-10k_1}{3}$	0
s^0	$10k_1$	

→ ODD Row = 0

$$6 - 10k_1 = 0$$

$$\boxed{k_1 = 0.6}$$ ↘ G.M

→ LAST Row = 0

$k_1 = 0$ → Invalid

→ A.E : $3s^2 + 10k_1 = 0$

$$3s^2 + 6 = 0$$

$$s = \pm j\sqrt{2}$$

↓

ω_{pc}

→ **Question:** $G(s)H(s) = \frac{1-s}{(s+1)(s+2)}$ · Cal G.M

$$k_1 GH = \frac{k_1(1-s)}{(s+1)(s+2)}$$

$$1 + k_1 GH = 0$$

$$s^2 + s(3 - k_1) + 2 + k_1 = 0$$

s^2	1	$2+k_1$
s^1	$3-k_1$	0
s^0	$2+k_1$	

→ ODD ROW = 0

$K_1 = 3$

→ A.E: $s^2 + (2+k_1) = 0$

$s = \pm j\sqrt{5}$

$\omega_{pc} = \sqrt{5}$

G.M = 3

→ LAST ROW = 0

$k_1 = -2$

↳ Invalid

Question: $T(s) = \frac{1}{(s+1)^3}$ · Cal G.M

$k_1 T = \frac{k_1}{(1+s)^3}$

$1 + k_1 T = 0$

$s^3 + 3s^2 + 3s + 1 + k_1 = 0$

s^3	1	3
s^2	3	$1+k_1$
s^1	$\frac{8-k_1}{3}$	0
s^0	$1+k_1$	

→ ODD ROW = 0

$k_1 = 8$

→ A.E: $3s^2 + 1 + k_1 = 0$

$3s^2 + 9 = 0$

$s = \pm j\sqrt{3}$

$\omega_{pc} = \sqrt{3}$

$k_1 = 8 = G.M$

→ LAST ROW = 0

$k_1 = -1$

↳ Invalid

Question: $G(s)H(s) = \frac{1}{(s+1)(s+2)}$ · Cal G.M

$$k_1 GH = \frac{k_1}{s^2 + 3s + 2}$$

$$1 + k_1 GH = 0$$

$$s^2 + 3s + (2 + k_1) = 0$$

(Invalid)

s^2	1	$2 + k_1$	→ ODD ROW: Non zero
s^1	3		→ LAST ROW = 0
s^0	$2 + k_1$		$k_1 = -2$ (Invalid)

O.L.S → O.L.T.F : Min-phase

C.L.S : STABLE

} G.M = $+\infty$ dB

→ **Question:** $G(s)H(s) = \frac{s+1}{s^2}$

$$k_1 GH = \frac{k_1 (s+1)}{s^2}$$

$$1 + k_1 GH = 0$$

s^2	1	k_1
s^1	k_1	
s^0	k_1	

→ $s^2 + k_1 s + k_1 = 0$ → ODD ROW = 0

$k_1 = 0$ → Invalid

O.L.S: Min phase → LAST ROW = 0

$k_1 = 0$ → Invalid

C.L.S : STABLE

G.M = $+\infty$ dB

→ **Question:** $G(s)H(s) = \frac{10(s+3)}{s(1+s)(s+2)}$

$$k_1 GH = \frac{10 k_1 (s+3)}{s(1+s)(2+s)}$$

$1 + k_1 GH = 0$

$s^3 + 3s^2 + (2 + 10k_1)s + 30k_1 = 0$

s^3	1	$2 + 10k_1$	→ ODD ROW: NON ZERO
s^2	3	$30k_1$	→ LAST ROW = 0
s^1	2	0	$k_1 = 0$
s^0	$30k_1$		↳ Invalid

O.L.S : MIN PHASE

C.L.S : STABLE

G.M : $+\infty$ dB

→ Question: $G(s)H(s) = \dfrac{10}{s^2(1+s)}$

$k_1 GH = \dfrac{10k_1}{s^2(1+s)}$

$1 + k_1 GH = 0$

$s^3 + s^2 + 10k_1 = 0$

s^3	1	0
s^2	1	$10k_1$
s^1	$-10k_1$	
s^0	$10k_1$	

→ ODD ROW = 0

$k_1 = 0$ (Invalid)

→ LAST ROW = 0

$k_1 = 0$ (Invalid)

O.L.S : min phase

C.L.S : Unstable

G.M = $-\infty$ dB

Gain Margin and Phase Margin from Nyquist Plot:

Gain Margin:

1. ω_{pc}

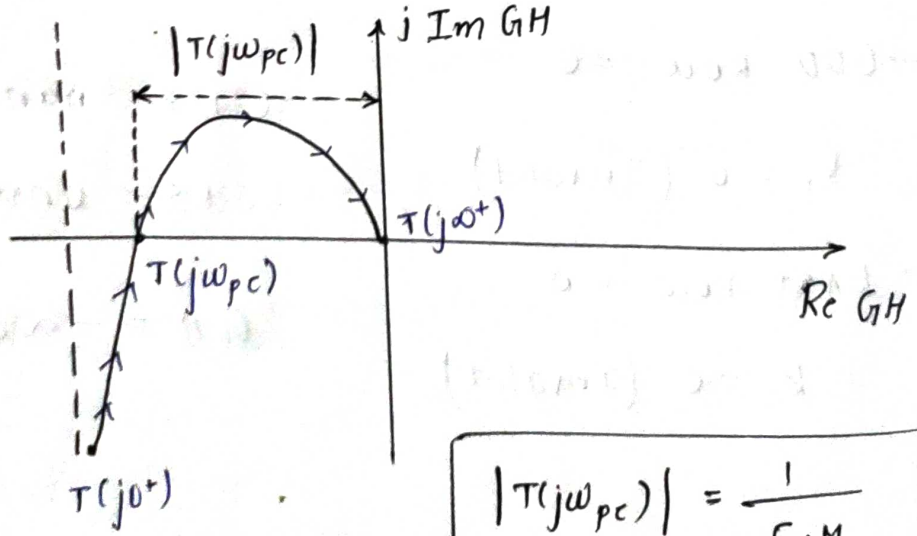
$\angle T(j\omega) = -180°$

→ Nyquist plot must cross or must intersect -ve Real Axis $T(j\omega_{pc})$

2. $G.M = \dfrac{1}{|T(j\omega_{pc})|}$

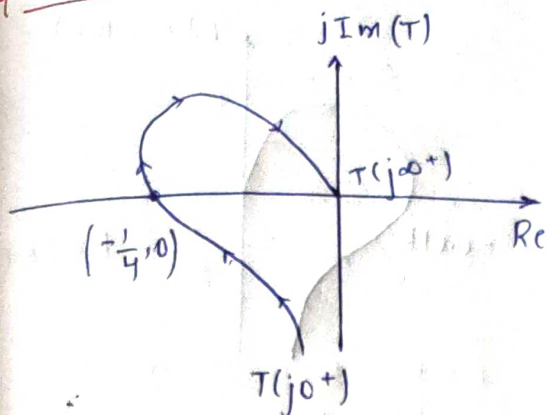
$$G.M = \frac{1}{|\text{Length on -ve Real Axis till } \omega_{pc}|}$$

Example - 1: $G(s)H(s) = \dfrac{1}{s(1+sT_1)(1+sT_2)} = T(s)$



$$|T(j\omega_{pc})| = \frac{1}{G.M}$$

→ Question:



Polar plot of $T(s) = G(s)H(s)$ is given.

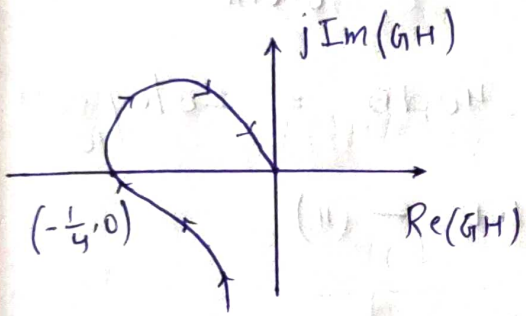
Calculate Gain Margin

$$|T(j\omega_{pc})| = \frac{1}{4}$$

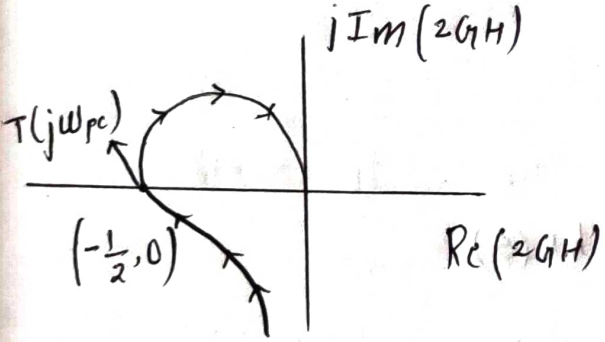
$$G.M = 4$$

→ Question:

Polar plot of $G(s)H(s)$ is mentioned



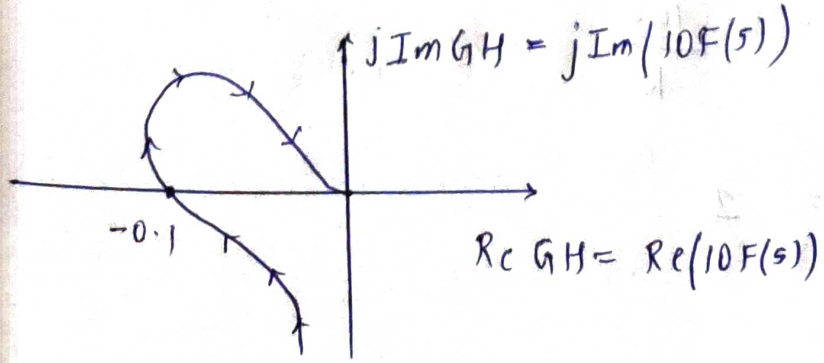
Calculate Gain Margin of $2G(s)H(s)$



$$G.M = 2$$

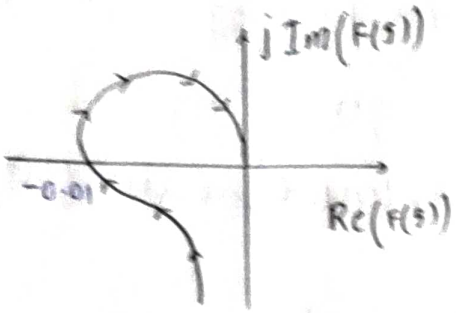
→ Question: $G(s)H(s) = kF(s)$

Polar Plot of $G(s)H(s)$ for $k = 10$ is given.

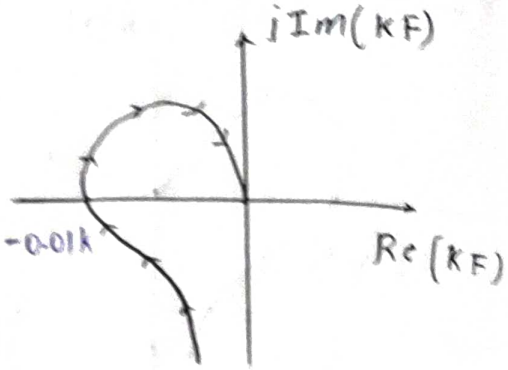


Calculate value of k for which $G.M = 40dB$

METHOD - 1



$k = 1$



$k = k$

$G.M\,(k=1) = 100$

FOR $kF(s)$

$$G.M = \frac{1}{0.01k} = \frac{100}{k} \quad -(i)$$

Given $G.M\,(dB) = 40dB = 20\log_{10} G.M$

$$\rightarrow G.M = 100 \quad -(ii)$$

$$\frac{100}{k} = 100$$

$$k = 1$$

METHOD - 2:

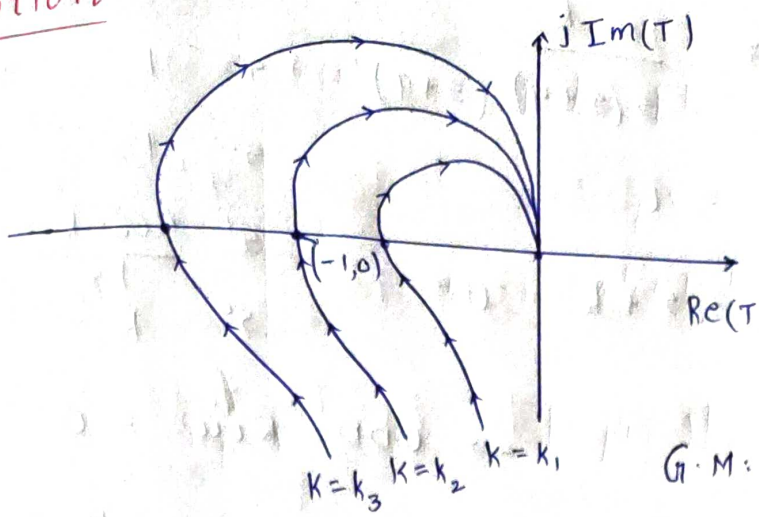
$GH = kF(s)$

$$G.M = \frac{G.M\,(k=1)}{k_{[desired]}} = \frac{k_{[MS]}}{k_{[desired]}}$$

$$100 = \frac{100}{k}$$

$$k = 1$$

Question:



Polar plot of

$G(s)H(s) = kF(s)$

$G.M : G.M_1 > G.M_2 > G.M_3$

Question: $G(s)H(s) = \frac{k}{s(s+4)(s+9)}$. If $G.M = 33.4dB$ then value of k is ___.

METHOD-1 :

$$k_1 GH = \frac{k_1 k}{s(s+4)(s+9)} \qquad k_1 = G.M$$

$$1 + k_1 GH = 0$$

$$s^3 + 13s^2 + 36s + kk_1 = 0$$

s^3	1	36	→ ODD ROW = 0
s^2	13	kk_1	
s^1	$\frac{468 - kk_1}{13}$		
s^0	kk_1		

$$k_1 = \frac{468}{k} = G.M$$

Given $G.M(dB) = 33.4$

$$20\log_{10} G.M = 33.4$$

$$G.M = 46.77$$

→ $\frac{468}{k} = 46.77$ → $k = 10$

METHOD-2:

$$G.M = \frac{k\,[MS]}{k\,[desired]}$$

$$GH = \frac{k}{s(s+4)(s+9)}$$

$$1 + GH = 0$$

$$s^3 + 13s^2 + 36s + k = 0$$

s^3	1	36
s^2	13	k
s^1	$\frac{468-k}{13}$	
s^0	k	

→ ODD Row = 0

$$k_{[MS]} = 468$$

Given $G.M\ (dB) = 33.4 = 20 \log_{10} G.M$

$$G.M = 46.77$$

$$G.M = \frac{k_{[MS]}}{k_{[desired]}} \rightarrow 46.77 = \frac{468}{k_{[desired]}}$$

$$k_{[desired]} = 10$$

→ Question: $G(s)H(s) = \frac{k}{s(0.2s+1)(0.05s+1)}$. Calculate k such that G.M = 20dB

METHOD-1: Let $GM = k_1$

$$k_1 GH = \frac{k k_1}{s(0.2s+1)(0.05s+1)}$$

$$1 + k_1 GH = 0$$

$$s(0.2s+1)(0.05s+1) + kk_1 = 0$$

* $(0.25)(s+5)(0.05)(s+20) + kk_1 = 0$

$$s(s+5)(s+20) + \frac{kk_1}{(0.2)(0.05)} = 0$$

$$s^3 + 25s^2 + 100s + 100kk_1 = 0$$

$$25 \times 100 = 100kk_1$$

$$k = \frac{25}{k_1} = \frac{25}{10} = 2.5$$

METHOD-2:

$$G.M = \frac{k_{[MS]}}{k_{[desired]}}$$

$$10 = \frac{25}{k_{[desired]}} \longrightarrow k_1 = 1$$

$$\therefore k_{[desired]} = 2.5$$

$$GH = \frac{k}{s(0.25+1)(0.05s+1)}$$

$$1 + GH = 0$$

$$s^3 + 25s^2 + 100s + 100k = 0$$

$$25 \times 100 = 100\, k_{[MS]}$$

$$k_{[MS]} = 25$$

→ Question: $G(s)H(s) = \frac{k}{s(s+4)(s+5)}$

$k = 1 \quad G.M = 45dB \rightarrow G.M = 10^{\frac{45}{20}} = k_{[MS]}$

$k = k_1 \quad G.M = 18dB$

Factor by which k needs to be changed.

$$G.M = \frac{k_{[MS]}}{k_1} \rightarrow 10^{\frac{18}{20}} = \frac{10^{\frac{45}{20}}}{k_1}$$

$$k_1 = 22.67$$

PHASE MARGIN

(i) ω_{gc} → Nyquist plot intersects (crosses) circle of unity Radius.

↳ $|T(j\omega)| = 1$

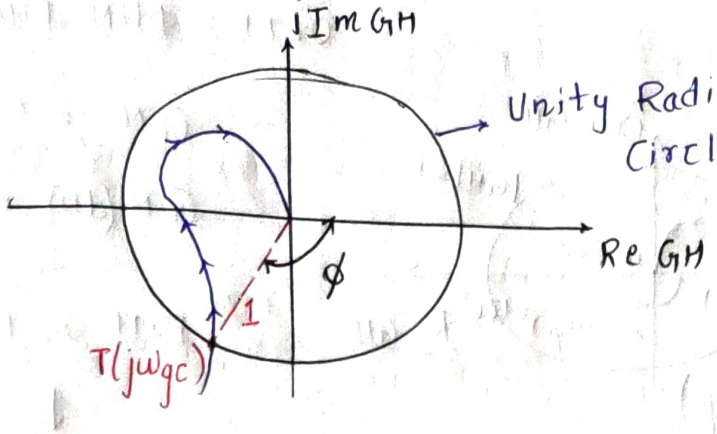
- If Nyquist plot intersects unity radius circle, then ω_{gc} exists.

(ii) $P.M = \angle T(j\omega_{gc}) + 180°$

$\angle T(j\omega_{gc})$ → From Positive Real Axis

Clockwise (-ve)

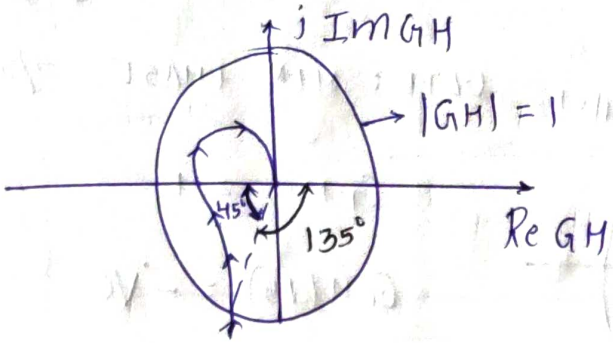
Anti-clockwise (+ve)



$$P.M = \angle T(j\omega_{gc}) + 180^\circ$$

$$\boxed{P.M = 180^\circ - \phi}$$

Q.

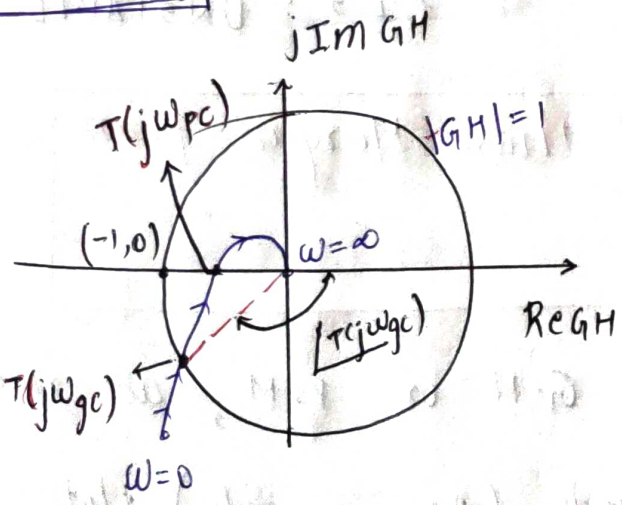


P.M ?

$$P.M = 180^\circ - 135^\circ = 45^\circ$$

Important Points

CASE - 1



OLTF: min phase sys

$G.M > 1$

$G.M(dB) = +ve$

$P.M = +ve$

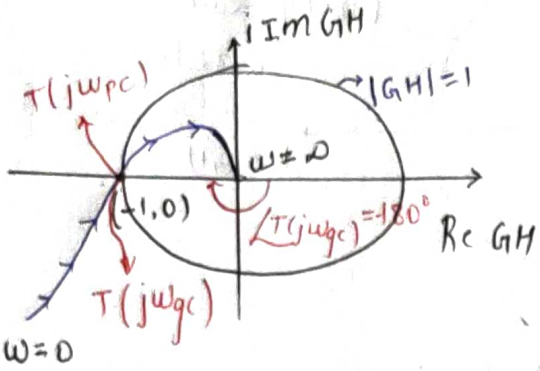
↓

$$\boxed{\omega_{pc} > \omega_{gc}}$$

↓

C.L.S : STABLE

CASE-2



O.L.T.F : MIN PHASE SYSTEM

G.M = 1

G.M (dB) = 0 dB

P.M = 180° - 180° = 0°

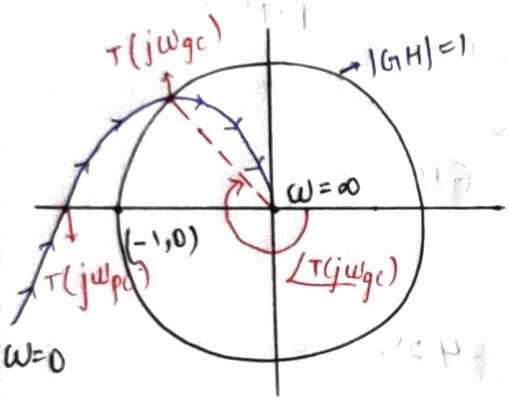
↓

$\omega_{pc} = \omega_{gc}$

↓

CLS → MARGINAL STABLE

CASE-3



O.L.T.F : MIN PHASE S/S

G.M < 1

G.M (dB) = -Ve

P.M = -Ve

↓

$\omega_{pc} < \omega_{gc}$

↓

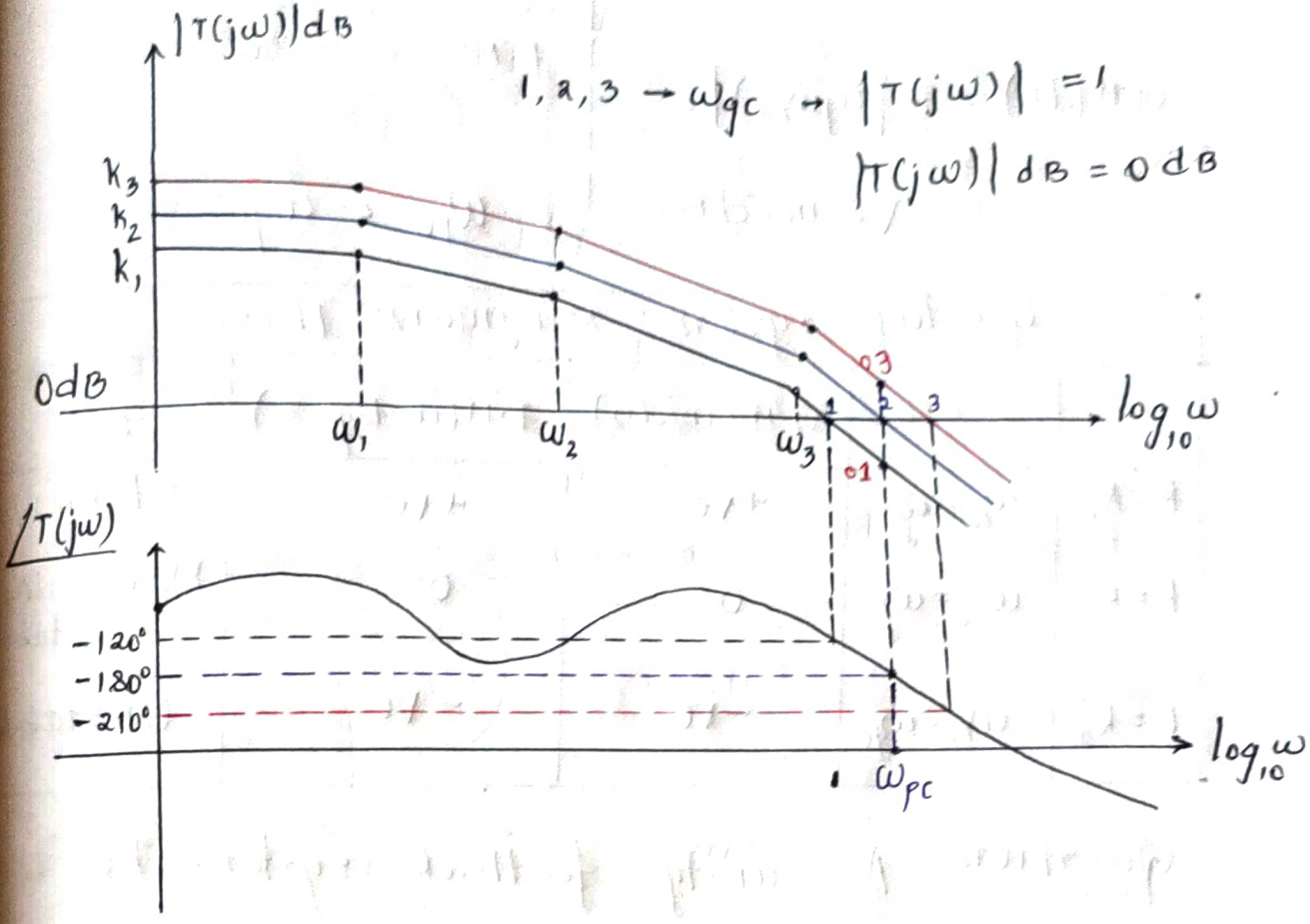
C.L.S : UNSTABLE

Gain Margin and Phase Margin from Bode Plot

→ To calculate G.M or P.M, we need

- Bode magnitude (Approximate) plot.
- Exact phase plot.

$$T(s) = \frac{k}{s\left(\frac{s}{\omega_1}+1\right)\left(\frac{s}{\omega_2}+1\right)\left(\frac{s}{\omega_3}+1\right)}$$



CASE-1 $k = k_1$

$|T(j\omega_{pc})| = -ve$ in dB

G.M (dB) $= -|T(j\omega_{pc})|$ dB

$= +ve$ in dB

$\angle T(j\omega_{gc}) = -120°$

P.M $= \angle T(j\omega_{gc}) + 180°$

P.M $= +60°$

$\omega_{pc} > \omega_{gc}$

CASE-2 $k = k_2$

$|T(j\omega_{pc})|$ dB $= 0$ dB

G.M (dB) $= -|T(j\omega)|$ dB

$= 0$ dB

$\angle T(j\omega_{gc}) = -180°$

P.M $= \angle T(j\omega_{gc}) + 180° = 0°$

$\omega_{pc} = \omega_{gc}$

CASE-3 $k = k_3$

$|T(j\omega_{pc})| = +Ve$ in dB

$G.M\,(dB) = -|T(j\omega_{pc})|$

$= -Ve$ in dB

$\angle T(j\omega_{gc}) = -210°$

$P.M = -30°$

$\omega_{pc} < \omega_{gc}$

open loop system : Minimum phase				
		G.M (in dB)	P.M (in degree)	C.L.S
$K = k_1$	$\omega_{pc} > \omega_{gc}$	+ve	+ve	Stable
$k = k_2$	$\omega_{pc} = \omega_{gc}$	0	0°	Marginally Stable
$k = k_3$	$\omega_{pc} < \omega_{gc}$	-ve	-ve	Unstable

→ Question: A unity feedback system has the following open loop frequency response. The Gain and Phase Margin of the system are

ω (rad/sec)	$\lvert G(j\omega)\rvert$	$\angle G(j\omega)$
2	7.5	-118°
3	4.8	-130°
4	3.15	-140°
5	2.25	-150°
6	1.70	-157°
8	1.00	-170°
10	0.64	-180°

A) 0dB, -180°

B) 3.88dB, -170°

C) 0dB, 10°

D) 3.88dB, 10°

Gain Margin

$\angle G(j\omega_{pc}) = -180^\circ$

$\omega_{pc} = 10$

$|G(j\omega_{pc})| = 0.64$

$G.M = \frac{1}{0.64}$

$G.M(dB) = 20\log_{10}\frac{1}{0.64}$

$G.M(dB) = 3.87\ dB$

Phase Margin

$|G(j\omega_{gc})| = 1$

$\omega_{gc} = 8$

$\angle G(j\omega_{gc}) = -170^\circ$

$P.M = 180^\circ - 170^\circ = 10^\circ$

→ Question: The frequency response of a linear system $G(j\omega)$ is provided in the tabular form below

$\lvert G(j\omega)\rvert$	1.3	1.2	1.0	0.8	0.5	0.3
$\angle G(j\omega)$	-130°	-140°	-158	-160°	-180°	-200°

The gain margin and phase margin of the system are

A) 6dB, 30° B) 6dB, -30° C) -6dB, 30° D) -6dB, -30°

Gain Margin

$G.M = 2$

$G.M(dB) = 20\log_{10}2$

$= 6dB$

Phase Margin

$P.M = 30^\circ$