

- **System**:- The Combination of Components (elements) that acts together to perform a specific function (desired task) is called a "system".
- **Control System**:- In a system when the output quantity is controlled by varying the input quantity, then the system is known as "control system".

Here output quantity is called as "Controlled variable" and input quantity is called as "Command signal".

→ **Classification of Control systems**:-

- (1) Time variant & Time Invariant control systems
- (2) Continuous & Discrete control systems
- (3) Linear & Non-linear control systems
- (4) Lumped & Distributed control systems
- (5) open loop & closed loop control systems

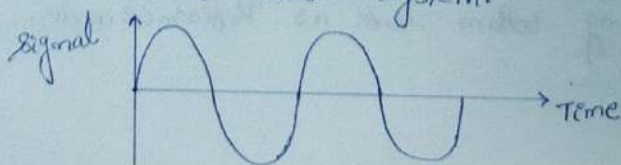
(1) **Time variant & Time Invariant control systems**:-

If the elements or parameters of the system vary with respect to time then this type of system is called as "Time variant control system".

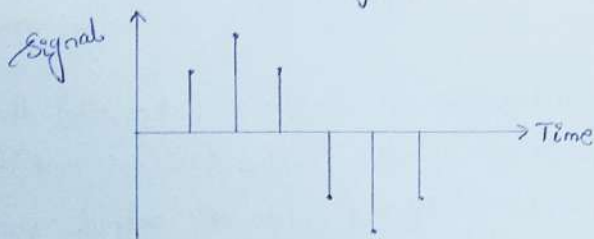
If the elements or parameters of the system does not change with respect to time then this type of system is called as "Time Invariant control system".

(2) **Continuous & Discrete control systems**:-

In a system, if the elements or parameters are continuous with respect to time then this type of systems are called as "Continuous control system".



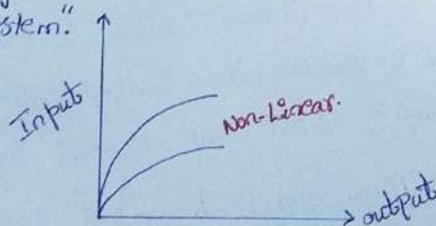
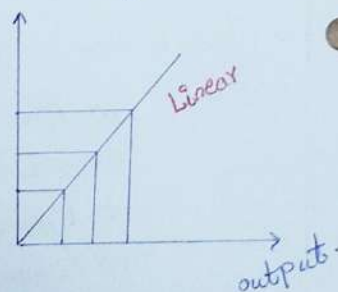
In a system, if the elements or parameters are not continuous with respect to time then this type of system are called as "Discrete Control system".



→ Linear & Non-Linear Control systems:-

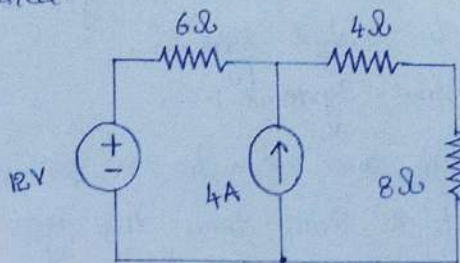
A system which is satisfying the principle of superposition, homogeneity. Then this type of system is called as "Linear Control system".

A system which is ^{not} satisfying the principle of superposition & homogeneity. Then this type of system is called as "non-Linear Control system".



Super position:- In a linear bi-lateral network containing N -sources each branch current is the algebraic sum of N -currents. which is determined by considering one source at a time and removing all other sources. In removing the sources, voltage sources are short circuited, while ideal current sources are open circuited.

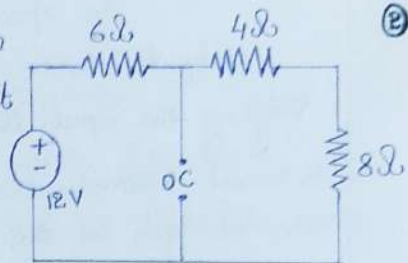
Example:-



Find the current flowing through 8Ω Resistor?

Assuming bottom node as Reference node.

First current source is open circuited. Then the circuit becomes a simple series circuit. It consists of 12V in series with the three resistances.



The current through

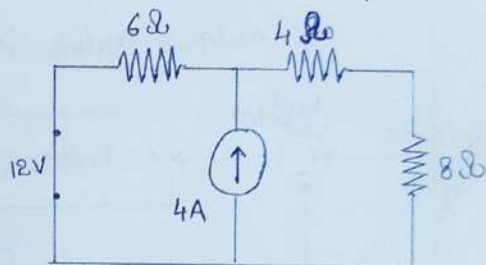
$$8\Omega \text{ Resistor is } = 12 / 6 + 4 + 8 = 0.667 \text{ A.}$$

$$R = \frac{V}{I}$$

Second the voltage source is short circuited. Then 4A current is distributed to two parallel branches. As per current Divider Rule

The current through

$$8\Omega \text{ Resistor is } = 4 \times \frac{6}{6+12} \\ = 1.333 \text{ A}$$



Hence the current through 8Ω Resistor is

$$= 0.667 + 1.333$$

$$= 2.0 \text{ A}$$

(4) Lumped & Distributed Control System:-

A system which can be described by an ordinary differential equation, then this type of system is called as "Lumped Control system".

A system which can be described by a partial differential equation, then this type of system is called as "Distributed Control system".

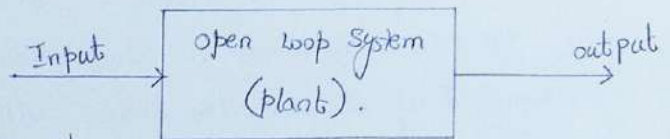
(5) open Loop & closed Loop Control system:-

Control system in which o/p quantity has no effect upon the i/p quantity are called "open loop control system". This means o/p is not feedback to the i/p for correction.

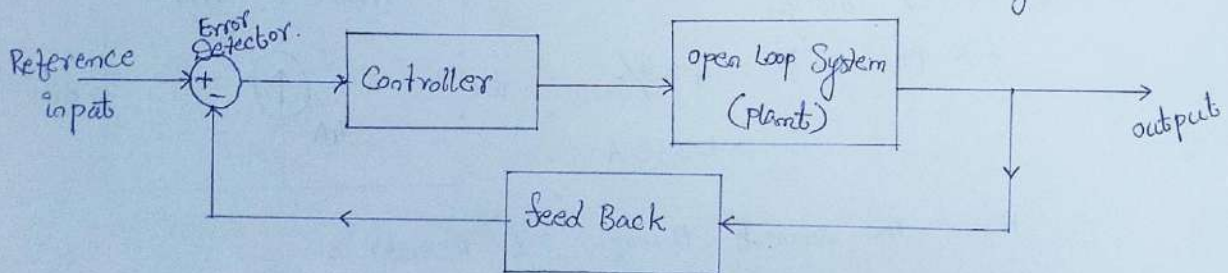
In open loop system

output can be varied by varying the input. But due to external disturbances the ^{system} o/p may change.

The changes in the output due to external disturbances are corrected by changing the i/p manually. So, the system is called not automatically open loop control system.



closed loop control system: The control system in which o/p quantity has effect upon the input quantity to maintain desired output value is called "closed loop control system".



open loop systems are modified as closed loop system by providing a feed back. Here feed back automatically corrects the changes in the o/p due to disturbances. Hence it is called as "Automatically control system".

Here feed back path elements convert o/p to a signal as that of reference signal. The feed back signal is proportional to o/p & it is fed to the error detector. The error signal is generated by error detector is difference b/n reference signal & feed back signal. The controller modifies and amplifies the error signal to produce better control action & is given to plant to correct its o/p.

open Loop Control Systems:

- Ex:
- (1) Traffic lights
 - (2) fans
 - (3) washing M/c's.

Closed Loop Systems:

- Ex:
- (1) A/c's
 - (2) Computers.

* Traffic Control System (OPCS):-

(3)

In case of Traffic lights, the sequence of control signals are based on a time slot given for each signal. The time slots are decided based on a traffic study. The system will not measure the density of the traffic before giving the signals. Since the time slot does not change according to traffic density, the system is open loop system.

→ Advantages & Dis-advantages of open loop System:-

Advantages:-

- (1) They are simple in construction.

- (2) Less Expensive.
- (4) Generally these are stable.

- (3) They are simple & Economical.

Dis-advantages:-

- (1) They are inaccurate.

- (2) The changes in the o/p due to external disturbances are not corrected automatically.

Advantages & Dis-advantages of closed loop systems:-

Advantages:-

- (1) They are accurate.

- (2) These are accurate even in the presence of non-linearities.

- (3) These are less affected by noise.

- (4) Fast operation.

Dis-advantages:-

- (1) They are complex in construction.

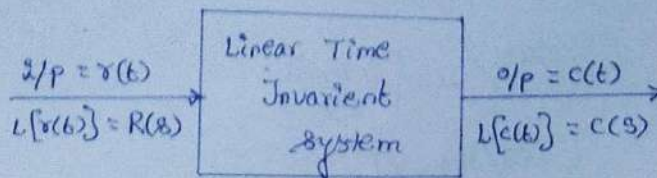
- (2) More Expensive.

- (3) Feed back reduces overall gain of the system.

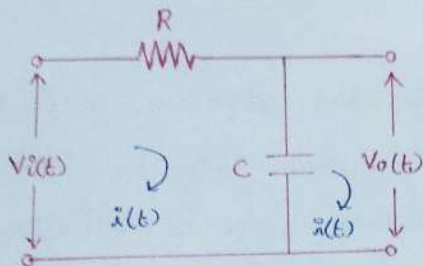
- (4) Stability is a major problem in closed loop system.

→ Transfer Function:- In a linear time invariant system the ratio b/w output response to input response at zero initial condition.

$$T.F = \left. \frac{C(s)}{R(s)} \right|_{\text{at zero initial condition}}$$



(1)



Find out the Transfer function?

Apply KVL for Loop (1)

$$V_i(t) = R i(t) + \frac{1}{C} \int i(t) dt$$

$$= R i(t) + \frac{1}{C} \int i(t) dt$$

Apply Laplace Transform

$$V_i(s) = R i(s) + \frac{1}{Cs} i(s) \rightarrow (1)$$

Apply KVL for Loop (2)

$$V_o(t) = \frac{1}{C} \int i(t) dt$$

Apply Laplace Transform

$$V_o(s) = \frac{1}{Cs} i(s)$$

$$\Rightarrow i(s) = Cs \cdot V_o(s) \rightarrow (2)$$

$$\text{But T.F} = \frac{V_o(s)}{V_i(s)} \rightarrow (3)$$

$$(1) \Rightarrow V_i(s) = R \cdot Cs \cdot V_o(s) + \frac{1}{Cs} [V_o(s) \cdot Cs]$$

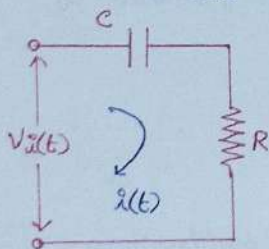
$$= V_o(s) [1 + RCs]$$

$$(3) \Rightarrow \text{T.F} = \frac{V_o(s)}{V_i(s)} \cdot [1 + RCs]$$

$$= \frac{1}{[1 + RCs]} = \frac{1}{[1 + s\tau]}$$

where $\tau \rightarrow$ Time constant of RC circuit.

(2)



Find out the Transfer function?

Apply KVL for Loop (1)

$$V_i(t) = \frac{1}{C} \int i(t) dt + R i(t)$$

$$\Rightarrow V_i(s) = \frac{1}{Cs} i(s) + R i(s)$$

$$= i(s) \left[R + \frac{1}{Cs} \right]$$

$$\Rightarrow \text{T.F} = \frac{i(s)}{V_i(s)}$$

$$= \frac{i(s)}{i(s) \left[\frac{RCs + 1}{Cs} \right]}$$

$$= \frac{Cs}{[1 + s\tau]}$$

Equating ④ & ③ we get

(26)

$$\Rightarrow K [E(s) - Y(s)] = (JS^2 + BS) Y(s)$$

$$K E(s) = (JS^2 + BS + K) Y(s)$$

$$\Rightarrow \frac{Y(s)}{E(s)} = K / (JS^2 + BS + K).$$

→ Signal Flow Graph:-

It is a pictorial. Representation of Transfer function. (And also it is a Graphical. representation of relationship between the variables & a set of linear algebraic Equations).

Node:- it represents a system variable & all incoming signals at the node.

Ex: $R(s), C(s), E(s)$

Branch:- Branch is a direct line segments joining any two nodes.

Ex: $H(s), G(s), \dots$

Input/source Node:- it has only outgoing path.

Ex: $R(s)$

output/sink Node:- it has only incoming path/other than input nodes.

Ex: $C(s).$

forward path:- from i/p to o/p there is no-Repetition of nodes.

Loop:- it is the path which originates & terminates at the same node.

Mason's Gain formula:-

If a Signal flow graph is used to represent a control system, then a gain formula called Mason's Gain formula is used to obtain the Transfer function of the system.

If $R(s)$ is Laplace Transform of input signal & $C(s)$ is the Laplace transform of output signal then the transfer function of system is

$$T(s) = C(s)/R(s)$$

$$= \frac{1}{\Delta} \cdot \sum_{k=1}^n M_k \cdot \Delta_k.$$

where $T(s) \rightarrow$ Transfer function of the system.

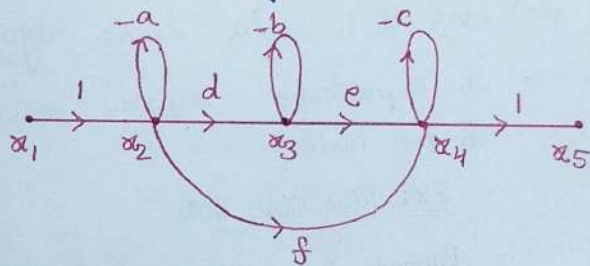
$M_k \rightarrow k^{\text{th}}$ forward path gain.

$n \rightarrow$ no. of forward paths.

$\Delta \rightarrow 1 - (\text{Sum of individual loop gains}) +$
 $(\text{Sum of Products of two non-touching loop gains}) -$
 $(\text{Sum of Products of three non-touching loop gains}) + \dots$

$\Delta_k \rightarrow$ The value of Δ non-touching M_k .

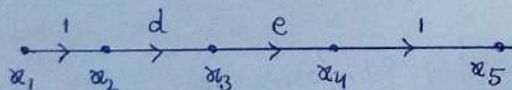
<1> obtain the Transfer function by using Mason's Gain formula for the above signal flow graph.



$\rightarrow$ Forward Path Gains

There are two forward paths. $\therefore k=2$

Let forward path Gains be M_1 & M_2 .



Forward path-①

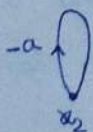


Forward path-②

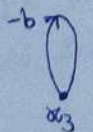
Gain of Forward path-1, $M_1 = d \cdot e$

Gain of Forward path-2, $M_2 = f$.

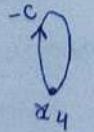
$\rightarrow$ Individual Loop Gains



Loop ①



Loop ②



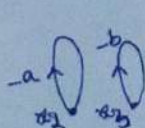
Loop ③

Loop Gain of Individual Loop ①, $M_{11} = -a$

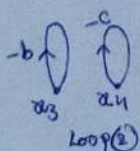
Loop Gain of Individual Loop ②, $M_{21} = -b$

Loop Gain of Individual Loop ③, $M_{31} = -c$

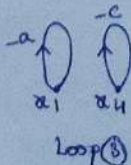
$\rightarrow$ Gain Products of Two Non-touching Loops



Combination of two non-touching Loop ①



Loop ②



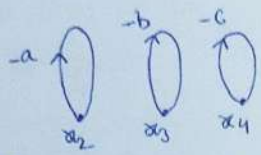
Loop ③

Gain product of 1st combination of Two Non-Touching loops $M_{12} = ab$

Gain product of 2nd combination of Two Non-Touching loops $M_{22} = bc$

Gain product of 3rd combination of Two Non-Touching loops $M_{32} = ac$

→ Gain Products of Three Non-Touching Loops.



Gain product of first combination of Three non-touching loops $M_{13} = -abc$

first combination of Three non-touching loops.

→ Calculation of Δ & Δ_k .

$$\begin{aligned}\Delta &= 1 - (M_{11} + M_{21} + M_{31}) + (M_{12} + M_{22}) - (M_{13} + M_{32}) \\ &= 1 - (-a - b - c) + (ab + bc + ac) - (-abc) \\ &= 1 + (a + b + c) + (ab + bc + ac) + (abc).\end{aligned}$$

$$\begin{aligned}\Delta_1 &= \text{Gain of Non-Touching } M_1 \\ &= 1 - (0) = 1.\end{aligned}$$

$$\begin{aligned}\Delta_2 &= 1 - \text{Gain of Non-Touching } M_2 \\ &= 1 - (-b) \\ &= 1 + b.\end{aligned}$$

→ Transfer function

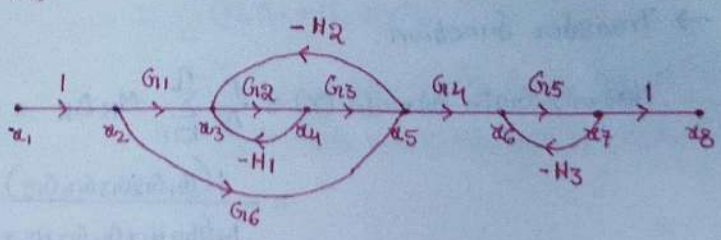
By Mason's Gain formula

$$\text{the transfer function} = \frac{1}{\Delta} \sum_{k=1}^N M_k \cdot \Delta_k$$

$$= \frac{1}{\Delta} (M_1 \Delta_1 + M_2 \Delta_2)$$

$$= \frac{dc(s) + f(1+b)}{1 + (a+b+c) + (ab+bc+ac) + (abc)}$$

(Q) Find the overall transfer function of the system whose signal flow graph is shown in figure.



→ Forward Path Gains

No. of Forward Paths $K=2$.

(1) $x_1 - x_2 - x_3 - x_4 - x_5 - x_6 - x_7 - x_8$

(2) $x_1 - x_2 - x_5 - x_6 - x_7 - x_8$

Gain of Forward Path-1 is $M_1 = G_1 G_2 G_3 G_4 G_5$

Gain of Forward Path-2 is $M_2 = G_4 G_5 G_6$

→ Individual Loop Gain

(1) $x_3 - x_4 - x_3$ Gain of Individual Loop-1 is $M_{11} = -G_2 H_1$

(2) $x_3 - x_4 - x_5 - x_3$ Gain of Individual Loop-2 is $M_{21} = -G_2 G_3 H_2$

(3) $x_6 - x_7 - x_6$ Gain of Individual Loop-3 is $M_{31} = -G_5 H_3$

→ Gain of Products of two non-Touching Loops

(1) $x_3 - x_4$ & $x_6 - x_7$

(2) $x_3 - x_5$ & $x_6 - x_7$

Gain Product of 1st Combination of

two non-touching Loops $M_{12} = (-G_2 H_1) (-G_5 H_3) = G_2 G_5 H_1 H_3$

Gain product of 2nd Combination of

two non-touching Loops $M_{22} = (-G_2 G_3 H_2) (-G_5 H_3) = G_2 G_3 G_5 H_2 H_3$

→ Calculation of Δ & Δ_K .

$$\Delta = 1 - (M_{11} + M_{21} + M_{31}) + (M_{12} + M_{22})$$

$$= 1 - (-G_2 H_1 - G_2 G_3 H_2 - G_5 H_3) + (G_2 G_5 H_1 H_3 + G_2 G_3 G_5 H_2 H_3)$$

$$= 1 + (G_2 H_1 + G_2 G_3 H_2 + G_5 H_3) + (G_2 G_5 H_1 H_3 + G_2 G_3 G_5 H_2 H_3).$$

$$\Delta_1 = 1 - (0) \quad \because M_1 \text{ is non-touching Loop Gain}$$

$$\Delta_2 = 1 - (-G_2 H_1)$$

$$= 1 + G_2 H_1.$$

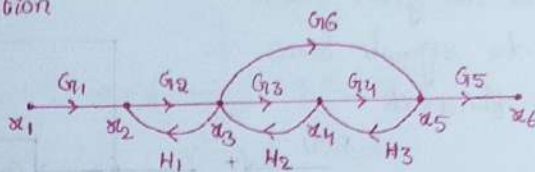
→ Transfer Function

$$\text{Mason's Gain Formula (T)} = \frac{1}{\Delta} \sum_{K=1}^n M_K \Delta_K = \frac{1}{\Delta} (M_1 \Delta_1 + M_2 \Delta_2)$$

$$= \frac{1(G_1 G_2 G_3 G_4 G_5) + (1 + G_2 H_1)(G_4 G_5 G_6)}{1 + (G_2 H_1 + G_2 G_3 H_2 + G_5 H_3) + (G_2 G_5 H_1 H_3 + G_2 G_3 G_5 H_2 H_3)}$$

(3)

Determine the transfer function for signal flow graph as shown in figure.



(28)

→ Forward Path Gains.

no. of Forward Paths $K = 2$.

(1) $x_1 - x_2 - x_3 - x_4 - x_5 - x_6$

(2) $x_1 - x_2 - x_3 - x_5 - x_6$

Gain of Forward Path-1 is $M_1 = G_1 G_2 G_3 G_4 G_5$

Gain of Forward Path-2 is $M_2 = G_1 G_2 G_6 G_5$

→ Individual Loop Gains.

(1) $x_2 - x_3 - x_2$ Loop Gain of individual Loop-1 is $M_{11} = G_2 H_1$

(2) $x_3 - x_4 - x_3$ Loop-2 is $M_{21} = G_3 H_2$

(3) $x_4 - x_5 - x_4$ Loop-3 is $M_{31} = G_4 H_3$

(4) $x_3 - x_4 - x_5 - x_3$ Loop-4 is $M_{41} = G_6 H_2 H_3$

→ Gain Products of Two Non-touching Loops

(1) $x_2 - x_3$ & $x_4 - x_5$

Gain product of first combination

of two non-touching Loops $M_{12} = (G_2 H_1) (G_4 H_3)$

$= G_2 G_4 H_1 H_3$.

→ calculation of Δ & Δ_K .

$\Delta = 1 - (M_{11} + M_{21} + M_{31} + M_{41}) + (M_{12})$

$= 1 - (G_2 H_1 + G_3 H_2 + G_4 H_3 + G_6 H_2 H_3) + (G_2 G_4 H_1 H_3)$

$\Delta_1 = 1 - (0)$

$\therefore$ Gain of non-Touching M_1 is "0".

$\Delta_2 = 1 - (0)$

$\therefore$ Gain of non-Touching M_2 is "0".

→ Transfer function

By Mason's formula $(Y) = \frac{1}{\Delta} \sum_{k=1}^n M_k \Delta_K$

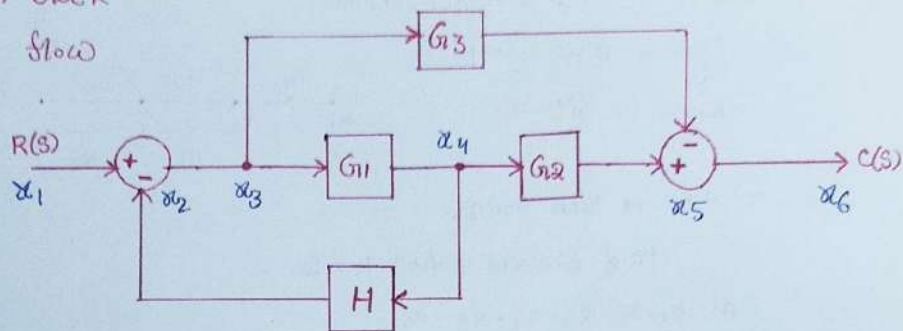
$= \frac{1}{\Delta} (M_1 \Delta_1 + M_2 \Delta_2)$

$= \frac{1(G_1 G_2 G_3 G_4 G_5) + (G_1 G_2 G_5 G_6)}{1 - (G_2 H_1 + G_3 H_2 + G_4 H_3 + G_6 H_2 H_3) + (G_2 G_4 H_1 H_3)}$

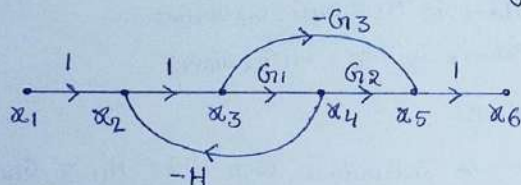
<4>

Convert the given block diagram to signal flow graph & find out

$$\frac{C(s)}{R(s)} ?$$



Signal flow graph of the above system is given below.



→ Forward Path Gains

no. of forward paths $k=2$

(1) $x_1 - x_2 - x_3 - x_4 - x_5 - x_6$

(2) $x_1 - x_2 - x_3 - x_5 - x_6$

Gain of forward path-1 is $M_1 = G_1 G_2$

Gain of forward path-2 is $M_2 = -G_3$

→ Individual Loop Gain

(1) $x_2 - x_3 - x_4 - x_2$

Loop gain of individual loop-1, is $M_{11} = -G_1 H$

→ Gain Products of Two Non-Touching Loops

There are no combination of non-touching loops.

→ calculation of ~~transfer function~~ Δ & Δ_k .

$$\Delta = 1 - (M_{11}) + 0$$

$$= 1 - (-G_1 H)$$

$$= 1 + G_1 H$$

$$\Delta_1 = 1 - (0) \quad \because \text{Gain of non-touching } M_1 \text{ is } 0$$

$$\Delta_2 = 1 - (0) \quad \because \text{Gain of non-touching } M_2 \text{ is } 0$$

→ Transfer function

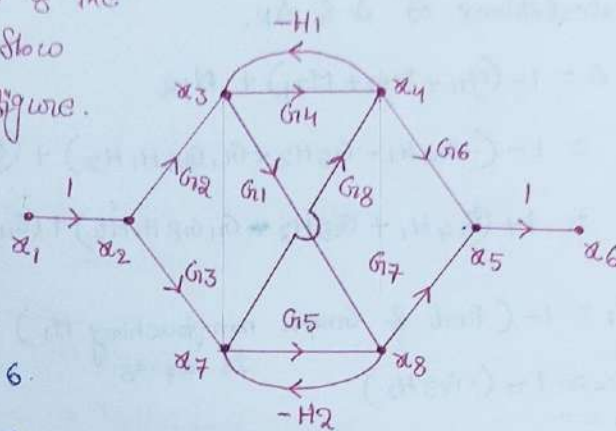
By Mason's formula $T(s) = \frac{1}{\Delta} \sum_{k=1}^n M_k \Delta_k$

$$= \frac{1}{\Delta} (M_1 \Delta_1 + M_2 \Delta_2)$$

$$= \frac{1(G_1 G_2) + 1(-G_3)}{1 + G_1 H}$$

(5)

Find the overall gain of the system whose signal flow graph is shown in figure.



(21)

→ Forward Path Gains

no. of Forward Paths $K = 6$

- (1) $x_1 - x_2 - x_3 - x_4 - x_5 - x_6$
- (2) $x_1 - x_2 - x_7 - x_8 - x_5 - x_6$
- (3) $x_1 - x_2 - x_3 - x_8 - x_5 - x_6$
- (4) $x_1 - x_2 - x_7 - x_4 - x_5 - x_6$
- (5) $x_1 - x_2 - x_7 - x_4 - x_3 - x_8 - x_5 - x_6$
- (6) $x_1 - x_2 - x_7 - x_8 - x_7 - x_4 - x_5 - x_6$

Gain of Forward Path-1 is $M_1 = G_2 G_4 G_6$

Gain of Forward Path-2 is $M_2 = G_3 G_5 G_7$

Gain of Forward Path-3 is $M_3 = G_1 G_2 G_7$

Gain of Forward Path-4 is $M_4 = G_3 G_8 G_6$

Gain of Forward Path-5 is $M_5 = -G_1 G_3 G_7 G_8 H_1$

Gain of Forward Path-6 is $M_6 = -G_1 G_2 G_6 G_8 H_2$

→ Individual Loop Gain

(1) $x_3 - x_4 - x_3$

(2) $x_7 - x_8 - x_7$

(3) $x_3 - x_8 - x_7 - x_4 - x_3$

Loop Gain of Individual Loop-1 is $M_{11} = -G_4 H_1$

Loop Gain of Individual Loop-2 is $M_{21} = -G_5 H_2$

Loop Gain of Individual Loop-3 is $M_{31} = G_1 G_8 H_1 H_2$

→ Gain Products of Two non-touching Loops

(1) $x_3 - x_4$ & $x_7 - x_8$

Gain Product of 1st combination of

two non-touching Loops

$$M_{12} = (-G_4 H_1) (-G_5 H_2) \\ = G_4 G_5 H_1 H_2$$

→ calculations of Δ & Δ_k .

$$\Delta = 1 - (M_{11} + M_{22} + M_{33}) + M_{12}$$

$$= 1 - (-G_{14}H_1 - G_{15}H_2 + G_{11}G_{18}H_1H_2) + (G_{14}G_{15}H_1H_2)$$

$$= 1 + (G_{14}H_1 + G_{15}H_2 - G_{11}G_{18}H_1H_2) + (G_{14}G_{15}H_1H_2)$$

$$\Delta_1 = 1 - (\text{Part of Graph non-touching } M_1)$$

$$= 1 - (-G_{15}H_2)$$

$$= 1 + G_{15}H_2$$

$$\Delta_2 = 1 - (\text{Part of Graph non-touching } M_2)$$

$$= 1 - (-G_{14}H_1)$$

$$= 1 + G_{14}H_1$$

There is no part of Graph non-touching with M_3, M_4, M_5 & M_6 .

$$\therefore \Delta_3 = \Delta_4 = \Delta_5 = \Delta_6 = 1$$

→ Transfer function

$$\text{By Mason's Gain formula} = \frac{1}{\Delta} \sum_{k=1}^n M_k \Delta_k$$

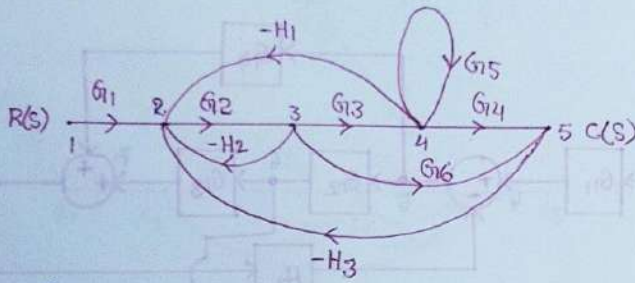
$$= \frac{1}{\Delta} (M_1 \Delta_1 + M_2 \Delta_2 + M_3 \Delta_3 + M_4 \Delta_4 + M_5 \Delta_5 + M_6 \Delta_6)$$

$$= \frac{G_{12}G_{14}G_{16}(1+G_{15}H_2) + G_{13}G_{15}G_{17}(1+G_{14}H_1) + G_{11}G_{12}G_{17} + G_{13}G_{16}G_{18} + (-G_{11}G_{13}G_{17}G_{18}H_1 - G_{11}G_{12}G_{16}G_{18}H_2)}{1 + G_{14}H_1 + G_{15}H_2 - G_{11}G_{18}H_1H_2 + G_{14}G_{15}H_1H_2}$$

$$1 + G_{14}H_1 + G_{15}H_2 - G_{11}G_{18}H_1H_2 + G_{14}G_{15}H_1H_2.$$

[P.T.O.]

[6]



→ Forward paths

(1) 1, 2, 3, 4, 5

(2) 1, 2, 3, 5

→ Individual Loops

(1) 2, 3, 4, 2

(2) 2, 3, 2

(3) 2, 3, 5, 2

(4) 2, 3, 4, 5, 2

(5) 4-4

→ Two non-Touching Loops

(1) 2-3-2 & 4-4

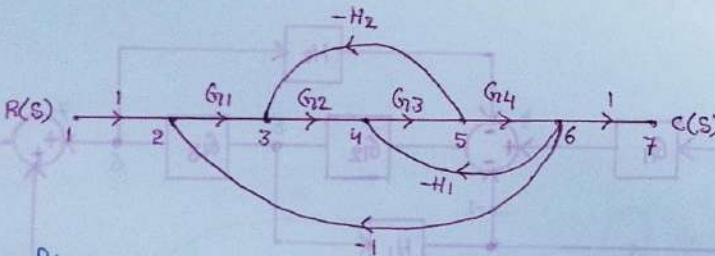
(2) 2-3-5-2 & 4-4

→ $\Delta_1 = 1$

$\Delta_2 = 1 - G_5$

$$\rightarrow T(s) = \frac{G_1 G_2 G_3 G_4 + G_1 G_2 G_6 - G_1 G_2 G_5 G_6}{1 + G_2 G_3 H_1 + H_2 G_2 + G_2 G_3 G_4 H_3 + G_2 G_6 H_3 - G_5 - G_2 H_2 G_5 - G_2 G_5 G_6 H_3}$$

[7]



→ Forward paths

(1) 1-2-3-4-5-6-7

→ $\Delta_1 = 1$

→ Individual Loops

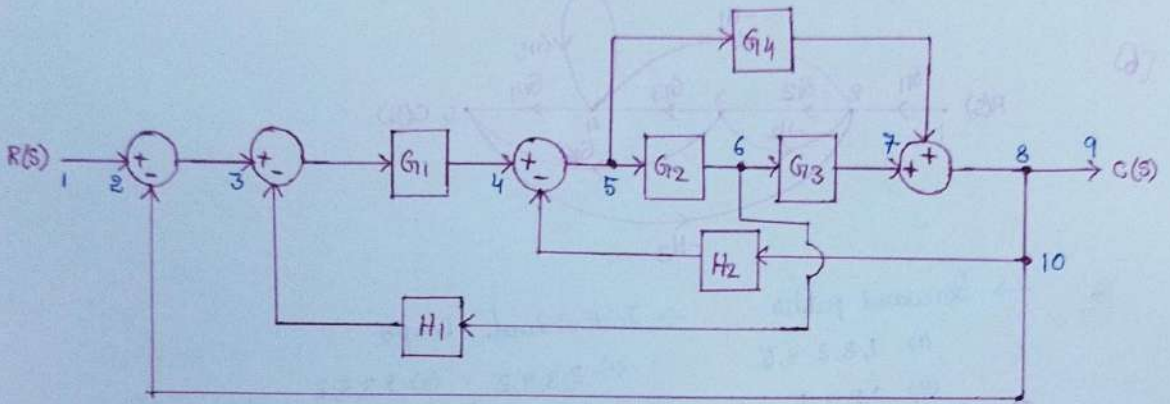
(1) 4-5-6-4

(3) 2-3-4-5-6-2

(2) 3-4-5-3

$$\rightarrow T(s) = \frac{G_1 G_2 G_3 G_4}{1 + G_3 G_4 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3 G_4}$$

[8]



→ Forward Paths

(1) 1-2-3-4-5-6-7-8-9

(2) 1-2-3-4-5-7-8-9

→ Individual Loops

(1) 2-3-4-5-6-7-8-10-2

(2) 3-4-5-6-3

(3) 4-5-6-7-8-10-4

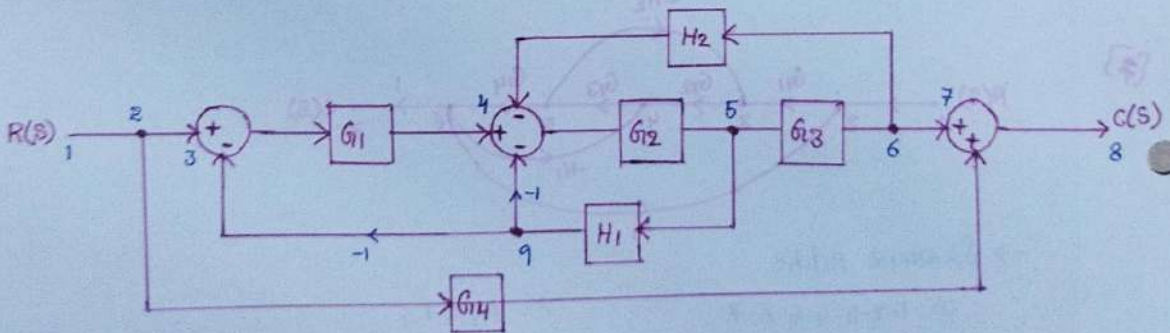
(4) 2-3-4-5-7-8-10-2

(5) 4-5-7-8-10-4

→ $\Delta_1 = \Delta_2 = 1$

$$\rightarrow T(s) = \frac{G_1 G_2 G_3 + G_1 G_4}{1 + G_1 G_2 G_3 + G_1 G_2 H_1 + G_2 G_3 H_2 + G_1 G_4 + G_4 H_2}$$

[9]



→ Forward Paths

(1) 1-2-3-4-5-6-7-8

(2) 1-2-7-8

→ Individual Loops

(1) 3-4-5-9-3

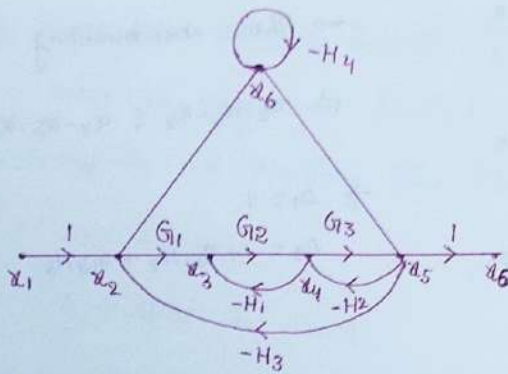
(2) 4-5-6-7

(3) 4-5-9-4

→ $\Delta_1 = 1$

$$\Delta_2 = 1 + G_2 G_1 H_1 + G_2 G_3 H_2 + G_2 H_1$$

$$\rightarrow T(s) = \frac{G_1 G_2 G_3 + G_4 + G_1 G_2 G_4 H_1 + G_2 G_3 G_4 H_2 + G_2 G_4 H_1}{1 + G_1 G_2 H_1 + G_2 G_3 H_2 + G_2 H_1}$$



→ Forward Paths

(1) $a_1 - a_2 - a_3 - a_4 - a_5$

(2) $a_1 - a_2 - a_6 - a_5$

→ Individual Loops

(1) $a_3 - a_4 - a_3$

(2) $a_4 - a_5 - a_4$

(3) $a_2 - a_3 - a_4 - a_5 - a_2$

(4) $a_6 - a_6$

(5) $a_2 - a_6 - a_5 - a_2$

→ Two non-touching loops

(1) $a_3 - a_4 - a_3$ & $a_6 - a_6$

(2) $a_3 - a_4 - a_3$ & $a_2 - a_6 - a_5 - a_2$

(3) $a_4 - a_5 - a_4$ & $a_6 - a_6$

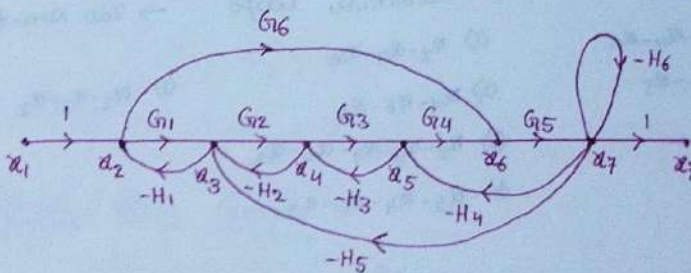
(4) $a_2 - a_3 - a_4 - a_5 - a_2$ & $a_6 - a_6$

→ $\Delta_1 = 1 + H_4$

$\Delta_2 = 1 + G_2 H_1$

→ $\Delta = 1 + G_2 H_1 + G_3 H_2 + G_1 G_2 G_3 H_3 + H_4 + G_4 G_5 H_3 + G_2 H_1 H_4 + G_2 G_4 G_5 H_1 H_3 + G_3 H_2 H_4 + G_1 G_2 G_3 H_3 H_4$

11



→ Forward paths

(1) $a_1 - a_2 - a_3 - a_4 - a_5 - a_6 - a_7$

(2) $a_1 - a_2 - a_6 - a_7$

→ Individual Loops

(1) $a_2 - a_3 - a_2$

(2) $a_3 - a_4 - a_3$

(3) $a_4 - a_5 - a_4$

(4) $a_5 - a_6 - a_5$

(5) $a_7 - a_7$

(6) $a_3 - a_4 - a_5 - a_6 - a_7 - a_3$

(7) $a_2 - a_6 - a_7 - a_5 - a_4 - a_3 - a_2$

(8) $a_2 - a_6 - a_7 - a_3 - a_2$

→ Two non-touching loops

(1) $x_2-x_3-x_2$ & $x_4-x_5-x_4$

(2) $x_2-x_3-x_2$ & $x_5-x_6-x_7-x_5$

(3) $x_2-x_3-x_2$ & x_7-x_7

(4) $x_3-x_4-x_3$ & $x_5-x_6-x_7-x_5$

(5) $x_3-x_4-x_3$ & x_7-x_7

(6) $x_4-x_5-x_4$ & x_7-x_7

(7) $x_4-x_5-x_4$ & $x_2-x_6-x_7-x_3-x_2$

→ Three non-touching loops

(1) $x_2-x_3-x_2$ & $x_4-x_5-x_4$ & x_7-x_7

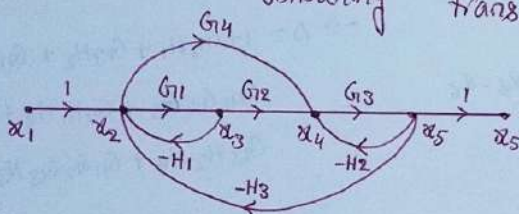
→ $\Delta_1 = 1$

$\Delta_2 = 1 + G_{12}H_2 + G_{13}H_3$

→ $\Delta = 1 - [-G_{11}H_1 - G_{12}H_2 - G_{13}H_3 - G_{44}G_{55}H_4 - H_6 - G_{12}G_{13}G_{44}G_{55}H_5 + G_{16}G_{15}H_4H_3H_2H_1 + G_{16}G_{15}H_5H_1] + [G_{11}H_1G_{13}H_3 + G_{11}H_1G_{44}G_{55}H_4 + G_{11}H_1H_6 + G_{12}H_2H_6 + G_{13}H_3H_6 + G_{13}H_3G_{16}G_{15}H_5H_1 + G_{12}H_2G_{44}G_{55}H_4] - [-G_{11}H_1G_{13}H_3H_6]$

[12]

Apply Gain formula to the signal flow graph shown in fig. to find the following transfer functions.



$\frac{x_5}{x_1}, \frac{x_4}{x_1}, \frac{x_2}{x_1}, \frac{x_5}{x_2}, \frac{x_4}{x_2}$

(a) To find $\frac{x_5}{x_1}$

→ Forward paths

(1) $x_1-x_2-x_3-x_4-x_5$

(2) $x_1-x_2-x_4-x_5$

→ Individual Loops

(1) $x_2-x_3-x_2$

(2) $x_4-x_5-x_4$

(3) $x_2-x_3-x_4-x_5-x_2$

(4) $x_2-x_4-x_5-x_2$

→ Two non-touching loops

(1) $x_2-x_3-x_2$ & $x_4-x_5-x_4$

→ $\Delta_1 = 1$

$\Delta_2 = 1$

$\Delta = 1 + G_{11}H_1 + G_{13}H_2 + G_{11}G_{12}G_{13}H_3 + G_{13}G_{44}H_3 + G_{11}G_{13}H_1H_2$

(b) To find $\frac{x_4}{x_1}$

There are two forward paths b/n x_1 & x_4 & there are no-loops which are not touching these forward paths.

(1) $x_1-x_2-x_3-x_4$

(2) $x_1-x_2-x_4$

$\Delta_1 = 1$

$\Delta_2 = 1$

$\Delta \rightarrow$ same as above.

(c) To find the transfer function $\frac{x_2}{x_1}$.

There is only one forward path between x_1 & x_2 . The loop $x_4-x_5-x_4$ is not touching this forward path.

$$\Delta_1 = 1 + G_3 H_2$$

$\Delta \rightarrow$ same in above.

(d) To find transfer function $\frac{x_5}{x_2}$.

$$\frac{x_5}{x_2} = \frac{x_5/x_1}{x_2/x_1} = \frac{G_1 G_2 G_3 + G_3 G_4}{1 + G_3 H_2}$$

(e) To find Transfer function $\frac{x_4}{x_2}$.

$$\frac{x_4}{x_2} = \frac{x_4/x_1}{x_2/x_1} = \frac{G_2 G_1 + G_4}{1 + G_3 H_2}$$

→ [13] Construct the signal flow graph for the given set of algebraic equations and find the overall gain using Mason's Gain Formula.

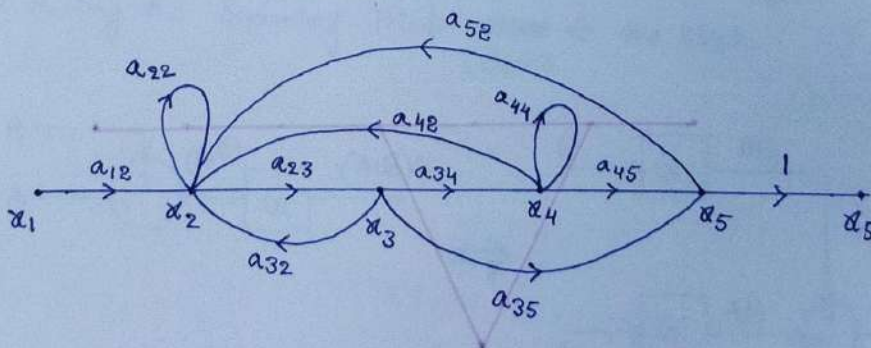
$$x_2 = a_{12} \cdot x_1 + a_{22} \cdot x_2 + a_{32} \cdot x_3 + a_{42} \cdot x_4 + a_{52} \cdot x_5$$

$$x_3 = a_{23} \cdot x_2$$

$$x_4 = a_{34} \cdot x_3 + a_{44} \cdot x_4$$

$$x_5 = a_{35} \cdot x_3 + a_{45} \cdot x_4$$

Signal Flow Graph



→ Total no. of Forward paths (K) = 2

$$M_1 = a_{12} \cdot a_{23} \cdot a_{34} \cdot a_{45}$$

$$M_2 = a_{12} \cdot a_{23} \cdot a_{35}$$

→ Total no. of Two non-touching Loops = 3

$$\text{Gain of Loop 1} = a_{22} \cdot a_{44}$$

$$\text{Loop 2} = a_{23} \cdot a_{32} \cdot a_{44}$$

$$\text{Loop 3} = a_{23} \cdot a_{35} \cdot a_{52} \cdot a_{44}$$

→ Total no. of Loops (Individual) = 6

$$\text{Gain of } L_1 = a_{22}$$

$$L_2 = a_{23} \cdot a_{32}$$

$$L_3 = a_{23} \cdot a_{34} \cdot a_{42}$$

$$L_4 = a_{23} \cdot a_{34} \cdot a_{45} \cdot a_{52}$$

$$L_5 = a_{44}$$

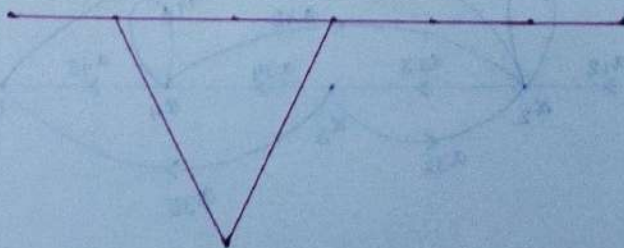
$$L_6 = a_{23} \cdot a_{35} \cdot a_{52}$$

$$\begin{aligned} \rightarrow \Delta_1 &= 1 - \text{Loop Gain of non-touching } M_1 \\ &= 1 - 0 = 1 \end{aligned}$$

$$\begin{aligned} \Delta_2 &= 1 - \text{Loop Gain of non-touching } M_2 \\ &= 1 - a_{44} \end{aligned}$$

→ By using Mason's Gain Formula

$$T(s) = \frac{(a_{12} \cdot a_{23} \cdot a_{34} \cdot a_{45}) + (a_{12} \cdot a_{23} \cdot a_{35})(1 - a_{44})}{1 - [a_{22} + a_{23} \cdot a_{32} + a_{23} \cdot a_{34} \cdot a_{42} + a_{23} \cdot a_{34} \cdot a_{45} \cdot a_{52} + a_{44} + a_{23} \cdot a_{35} \cdot a_{52}] + [a_{22} \cdot a_{44} + a_{23} \cdot a_{32} \cdot a_{44} + a_{23} \cdot a_{35} \cdot a_{52} \cdot a_{44}]}$$



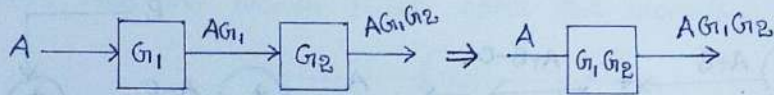
→ Block Diagram Reduction:

(33)

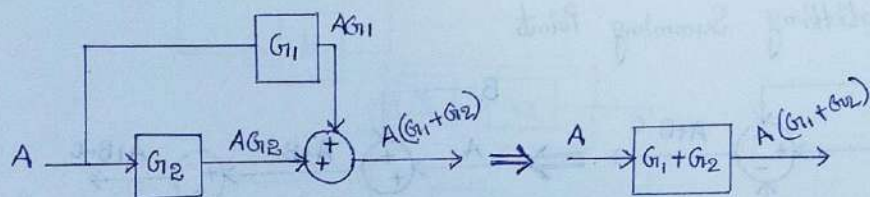
The block diagram can be reduced to find overall transfer function of the system.

→ Rules for Block Diagram Reduction:

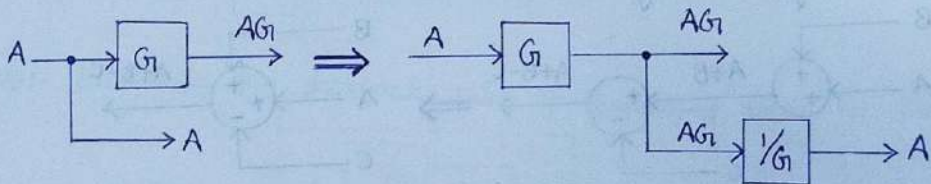
(1) Combining the blocks in cascade



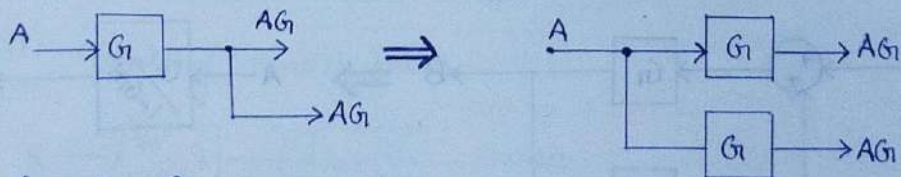
(2) Combining Parallel blocks



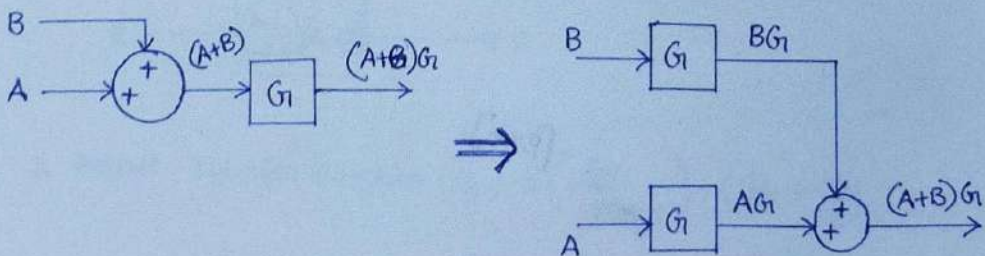
(3) Moving the branch point ~~ahead~~ of the block (take-off) (after)



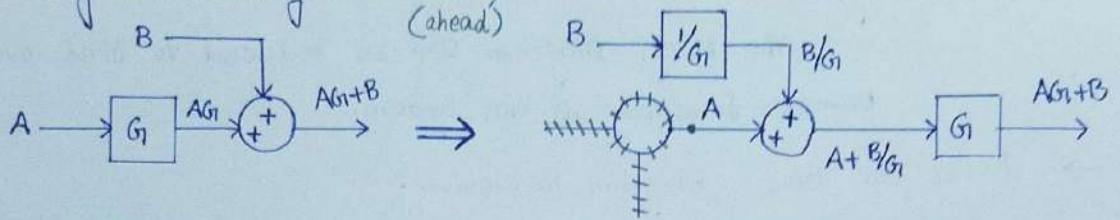
(4) Moving the branch point before/ the block (take-off) (ahead)



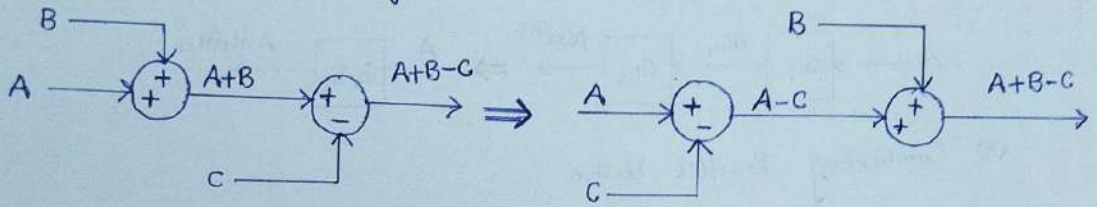
(5) Moving the Summing Point ~~ahead~~ of the block. (after)



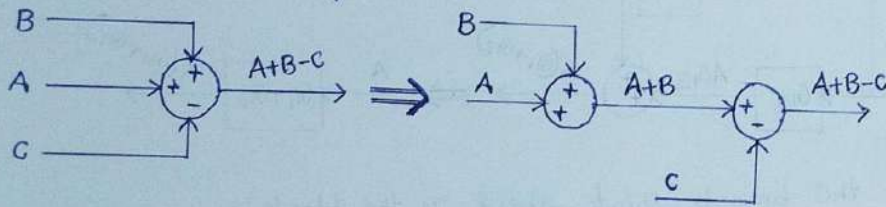
(6) Moving the Summing Point before/ (ahead) the block



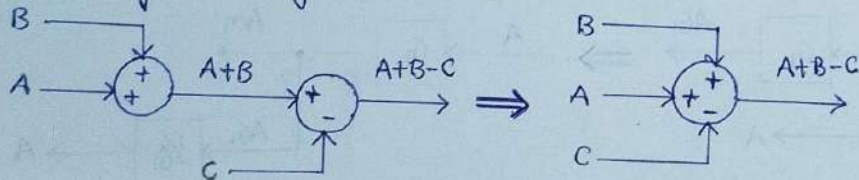
(7) Interchanging Summing Points



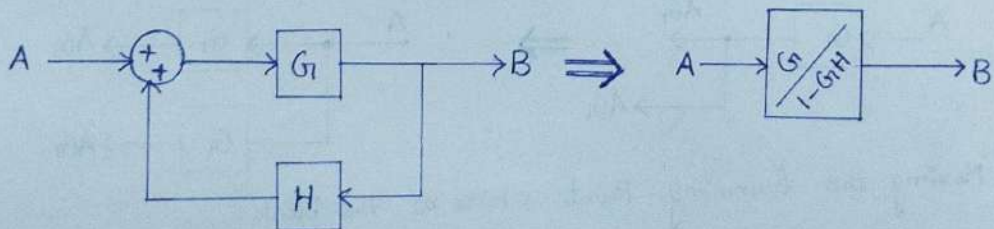
(8) Splitting Summing Point



(9) Combining Summing Points



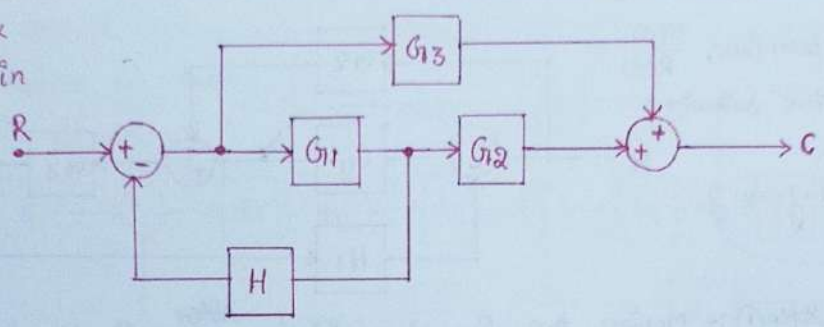
(10) Elimination of feed back loop.



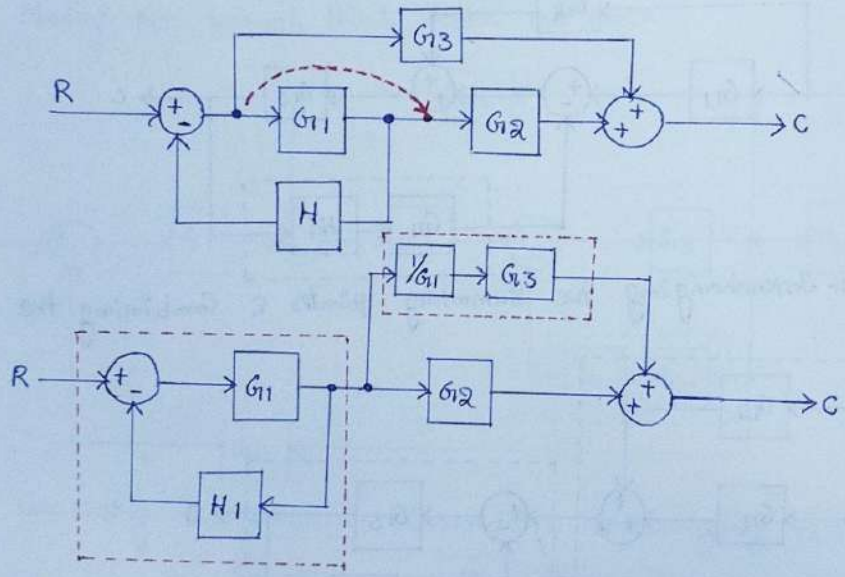
[P.T.O.]

(1)

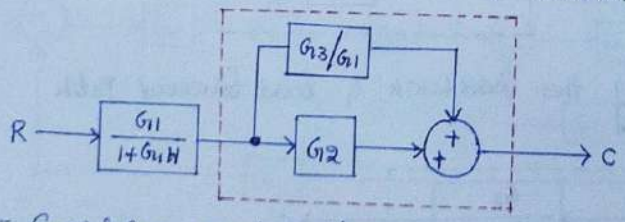
Reduce the block diagram shown in figure & find C/R.



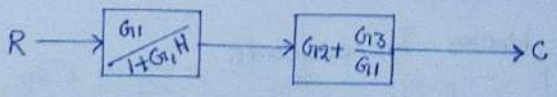
step ①: Move the branch point after the block (G_1)



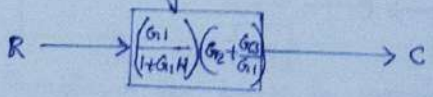
step ②:- Eliminate the feedback path & combining blocks in cascade



step ③:- Combining parallel blocks

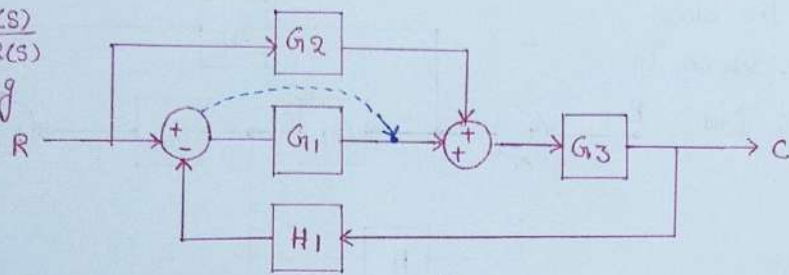


step ④:- Combining blocks in cascade

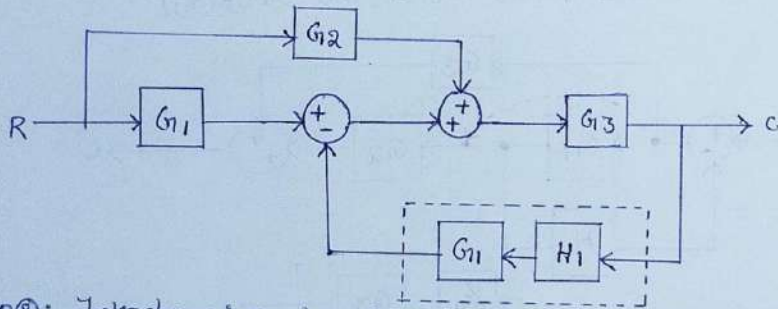


$\therefore$ overall Transfer function $\left(\frac{C}{R}\right) = \left(\frac{G_1}{1+G_1H}\right) \cdot \left(G_2 + \frac{G_3}{G_1}\right)$

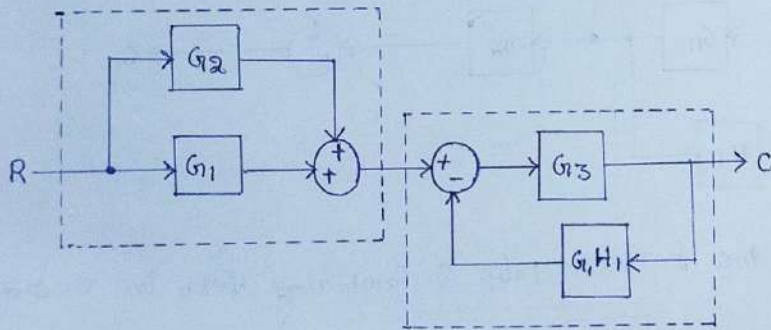
(2) Determine $\frac{C(s)}{R(s)}$ for the following block diagram?



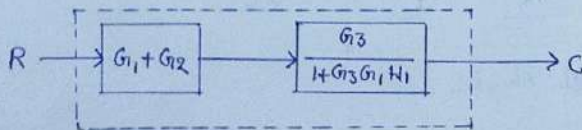
Step ①:- Moving the Summing point ~~before~~ ^{After} of the block



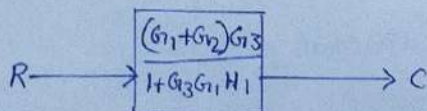
Step ②:- Interchanging the summing points & Combining the blocks in cascade.



Step ③:- Eliminating the feedback & feed forward path



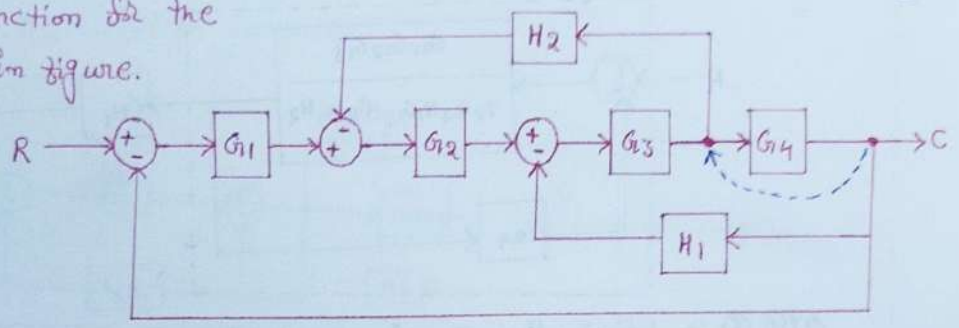
Step ④:- Combining the blocks in cascade



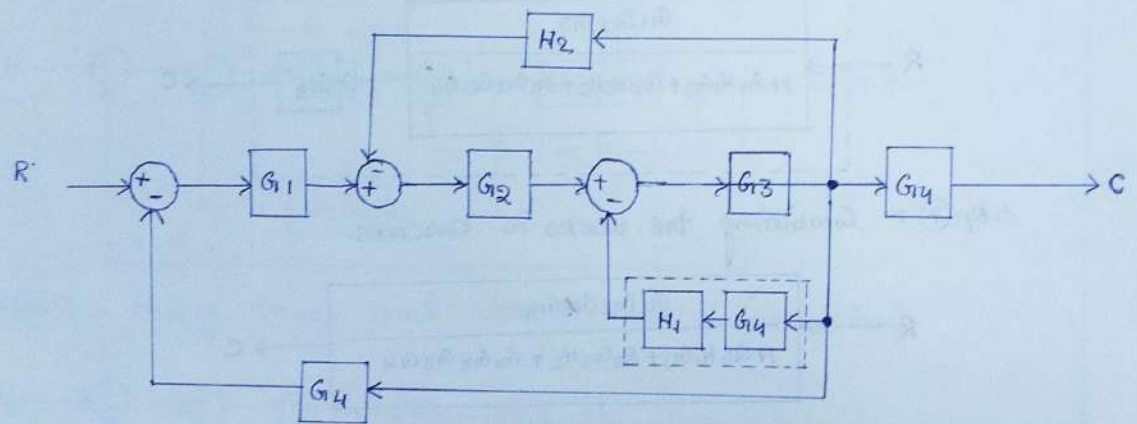
$\therefore$ The Required Transfer function = $\frac{(G_1 + G_2)G_3}{1 + G_3 G_1 H_1}$

(3) Determine the overall Transfer function for the system shown in figure.

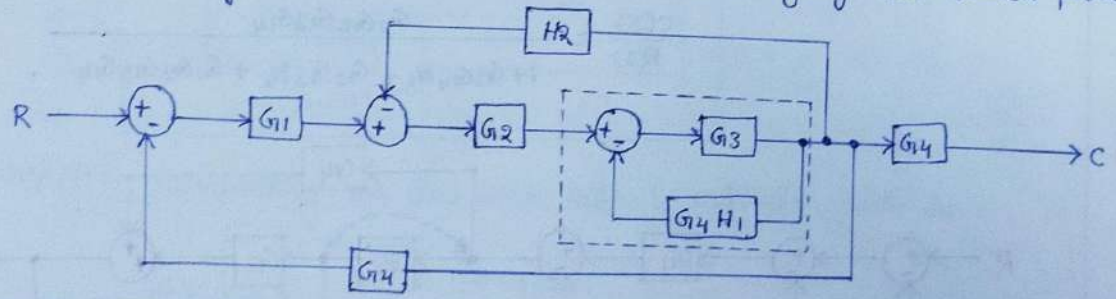
(35)



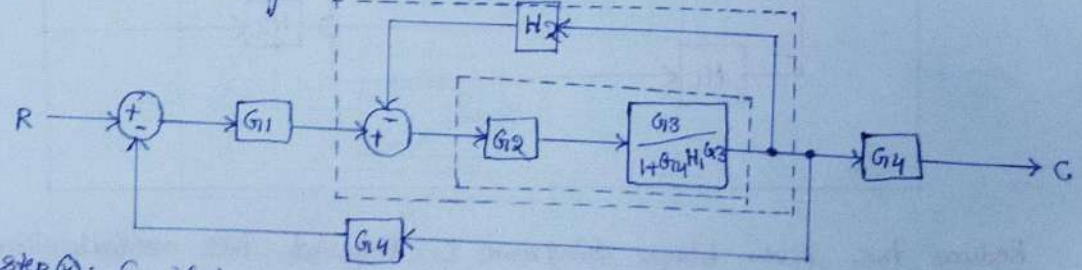
Step ①:- Moving the branch points before the block.



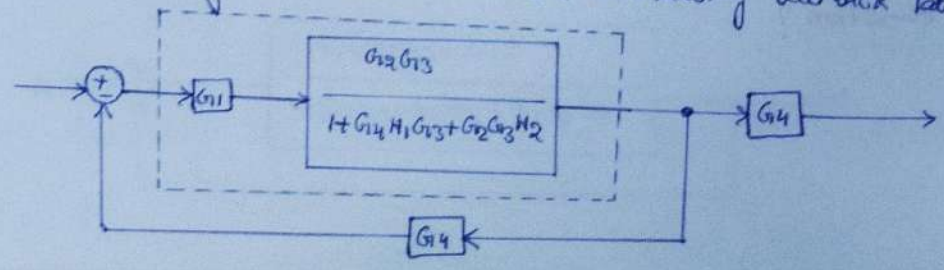
Step ②:- Combining the blocks in Cascade & rearranging the branch points



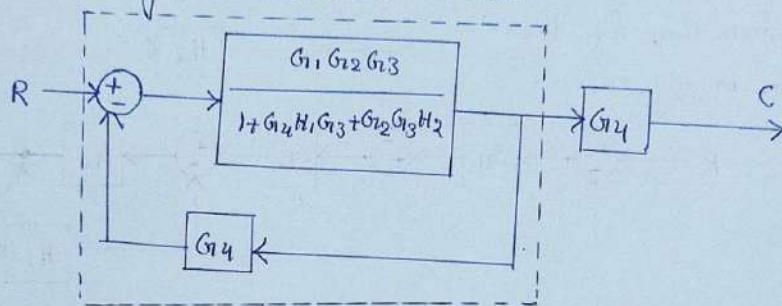
Step ③:- Eliminating the feed back path



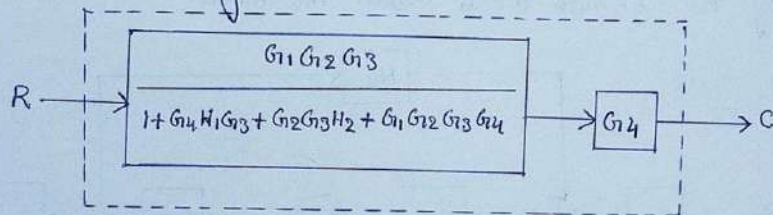
Step ④:- Combining the blocks in cascade & eliminating feed back path.



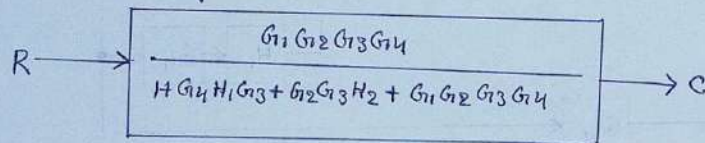
step ⑤ :- Combining the blocks in cascade



step ⑥ :- Eliminating the feedback path



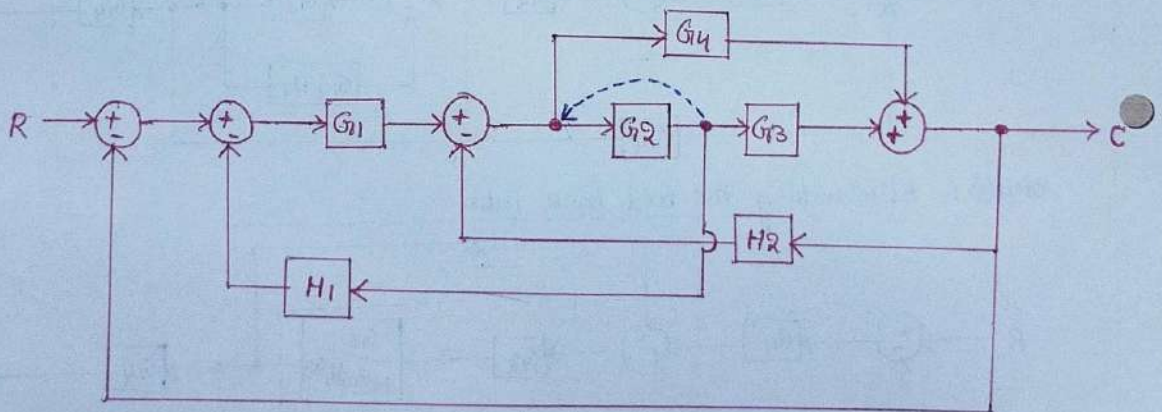
step ⑦ :- Combining the blocks in cascade.



∴ Transfer function

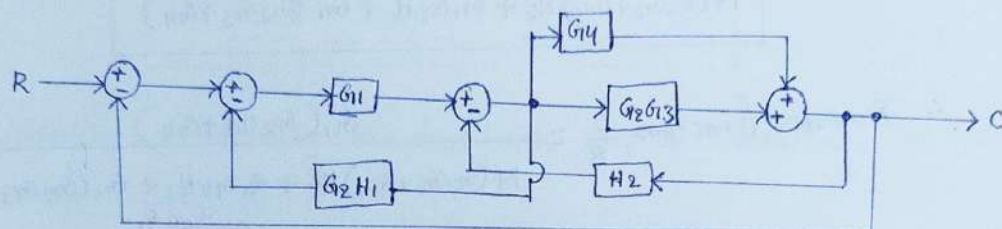
$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 G_4}{1 + G_4 H_1 G_3 + G_2 G_3 H_2 + G_1 G_2 G_3 G_4}$$

<4>

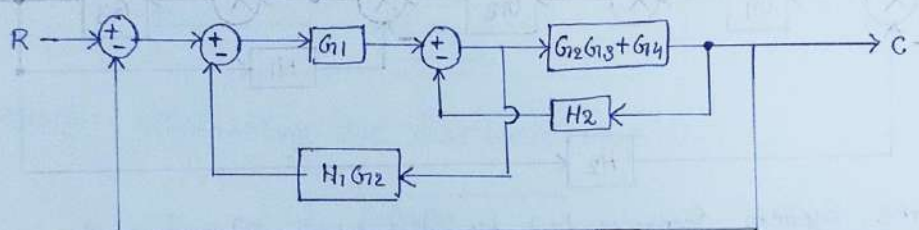


Reduce the given block diagram & find out the overall Transfer function?

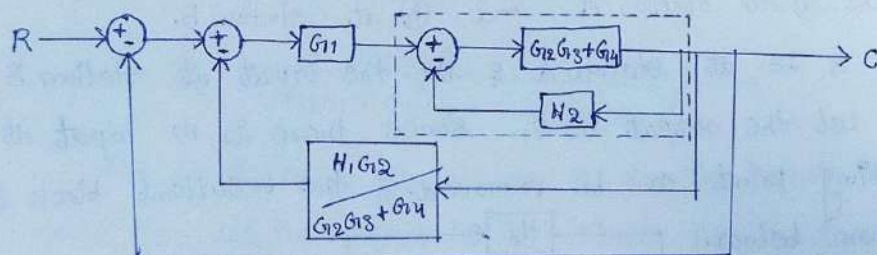
Step ①:- Moving ~~the~~ ^{branch} point ^{before} of the block " G_2 ". & Combining the blocks in cascade.



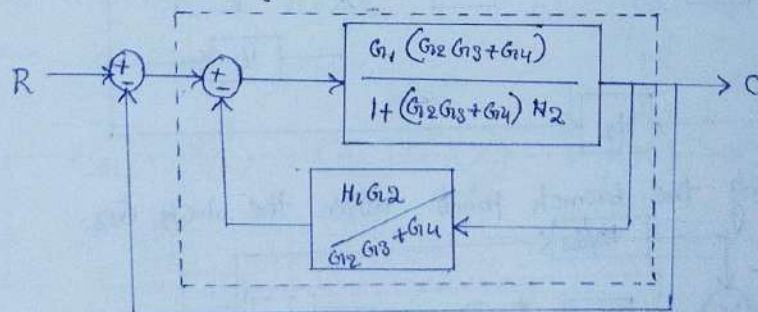
Step ②:- Combining blocks in parallel.



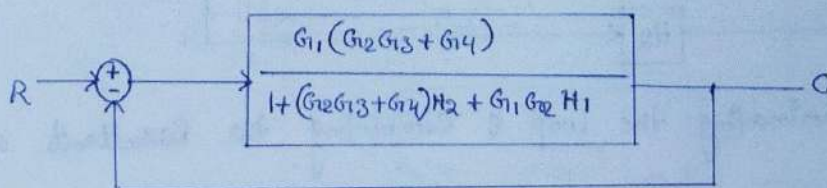
Step ③:- Moving branch point after of block of $(G_2 G_3 + G_4)$



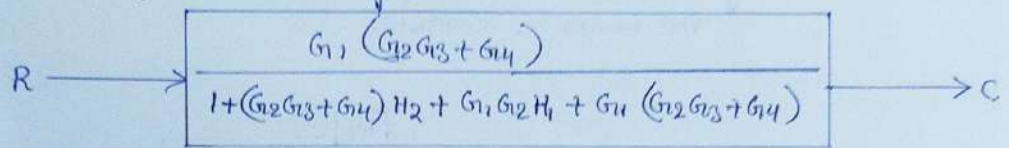
Step ④:- Eliminating the feed back path & cascade with G_1 .



Step ⑤:- Eliminating the feed back path

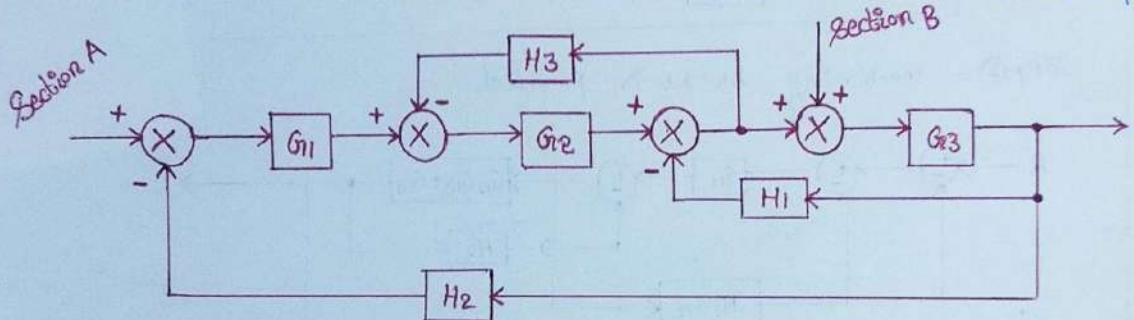


Step ⑥:- Again eliminating the feedback path



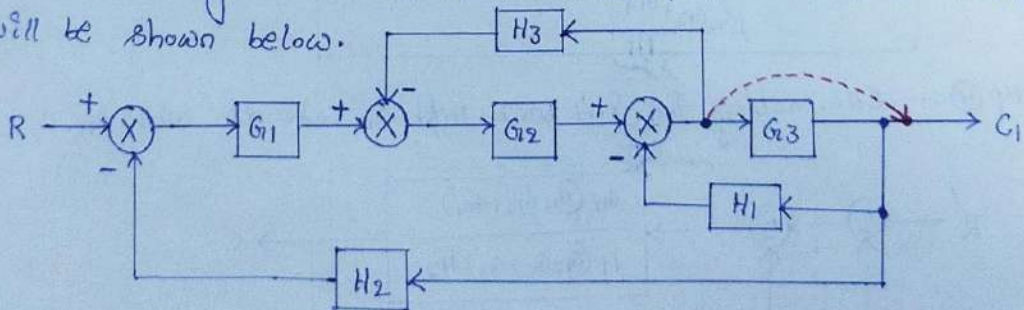
$\therefore$ Transfer Function $\frac{C}{R} = \frac{G_1(G_2G_3+G_4)}{1+(G_2G_3+G_4)H_2+G_1G_2H_1+G_1(G_2G_3+G_4)}$

(5)

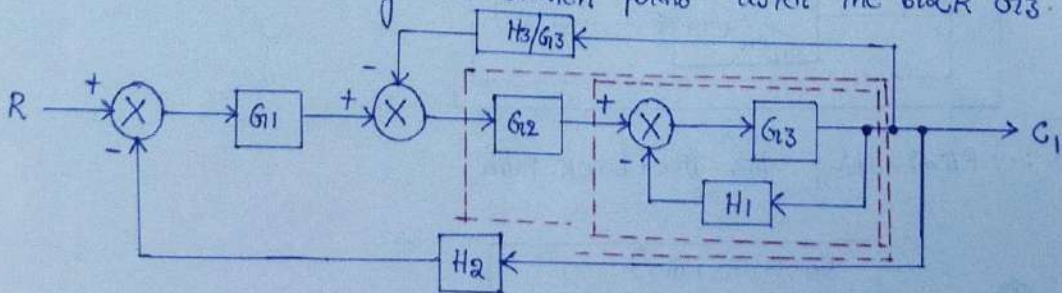


For the system represented by the block diagram shown in above figure. Evaluate closed-loop transfer function when the input R is (a) at station A and (b) at station B.

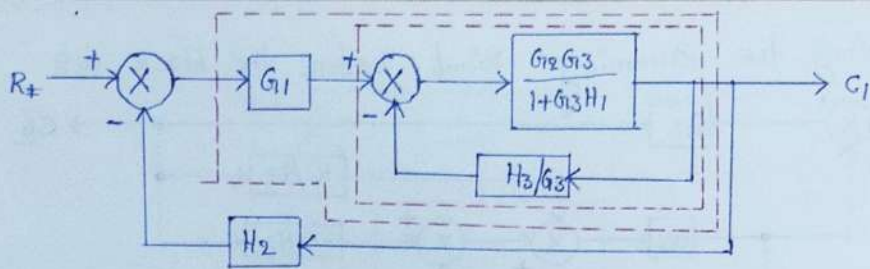
(a) The input R is at station A & so, the input at station B is made zero. Let the output be C_1 . Since there is no input at station B, that summing point can be removed & the resultant block diagram will be shown below.



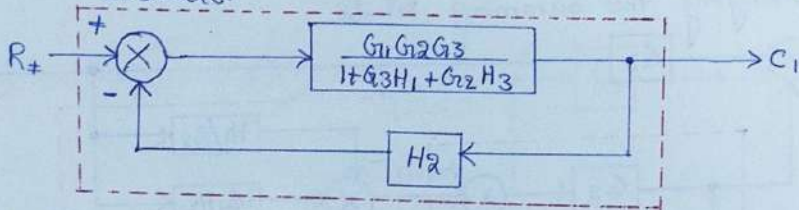
Step ①:- Moving the branch point after the block G_{13} .



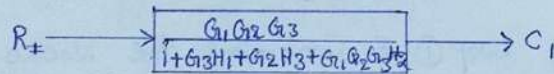
Step ②:- Eliminating the loop & combining the Resultant with G_{12} .



Step ③:- Eliminating the feed back loop & combining the blocks in cascade.



Step ④:- Eliminating the feed back loop

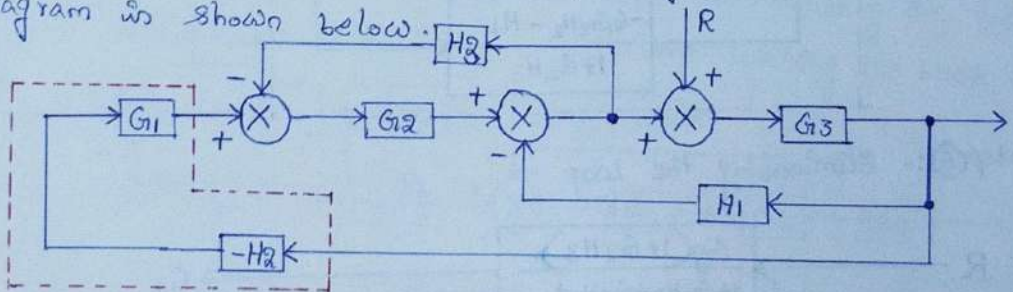


Step ⑤:- after simplification the closed loop Transfer function is

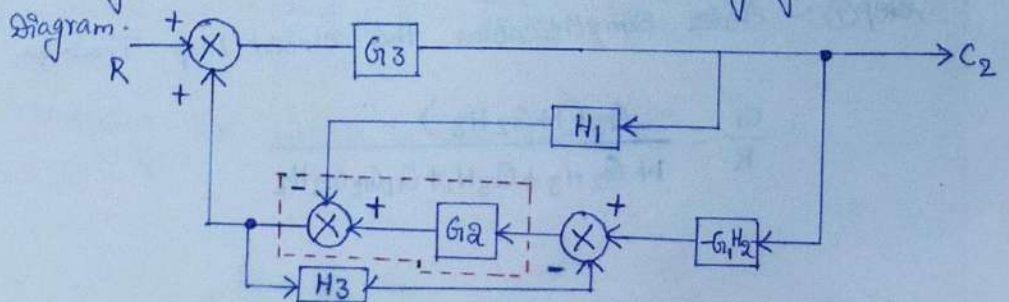
$$\frac{C_1}{R_+} = \frac{G_1 G_2 G_3}{1 + G_1 G_3 H_1 + G_1 G_2 H_3 + G_1 G_2 G_3 H_2} //$$

(b)

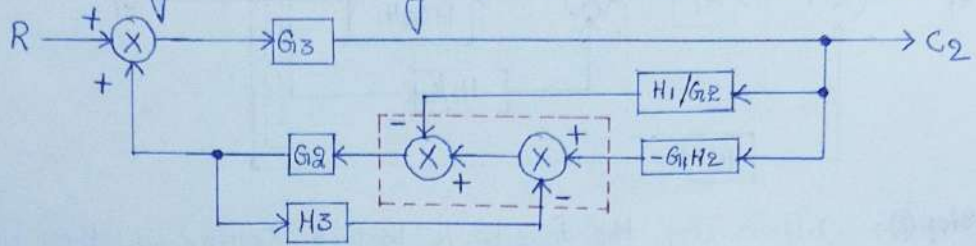
The input 'R' is at station B. So, the input at station A is made zero. Let the output be 'C2'. Since the input at station A is zero, the corresponding summing point can be removed & a -ve sign can be attached to the feed back path gain H2. The resultant block diagram is shown below.



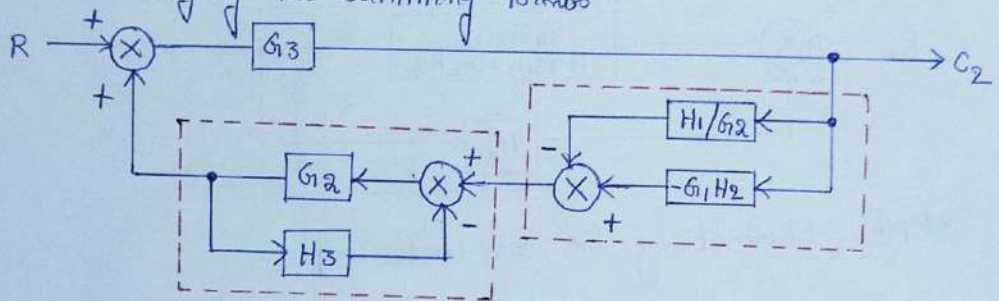
Step ①:- Combining the blocks in cascade & re-arranging the block diagram.



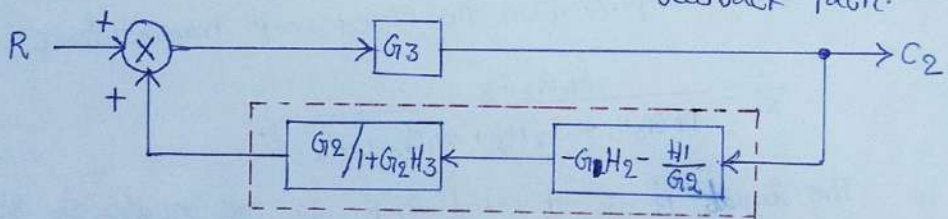
Step ② :- Moving the summing point before the block G_2 .



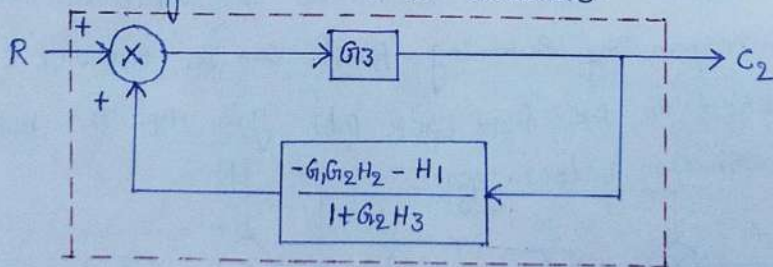
Step ③ :- Interchanging the summing points



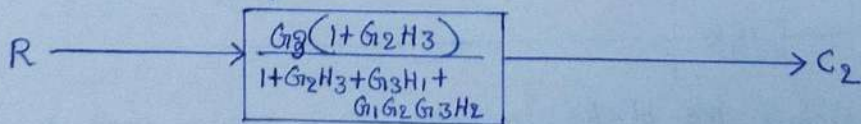
Step ④ :- Eliminating loop ① & Combining the blocks in parallel in feedback path.



Step ⑤ :- Combining the blocks in cascade.



Step ⑥ :- Eliminating the loop

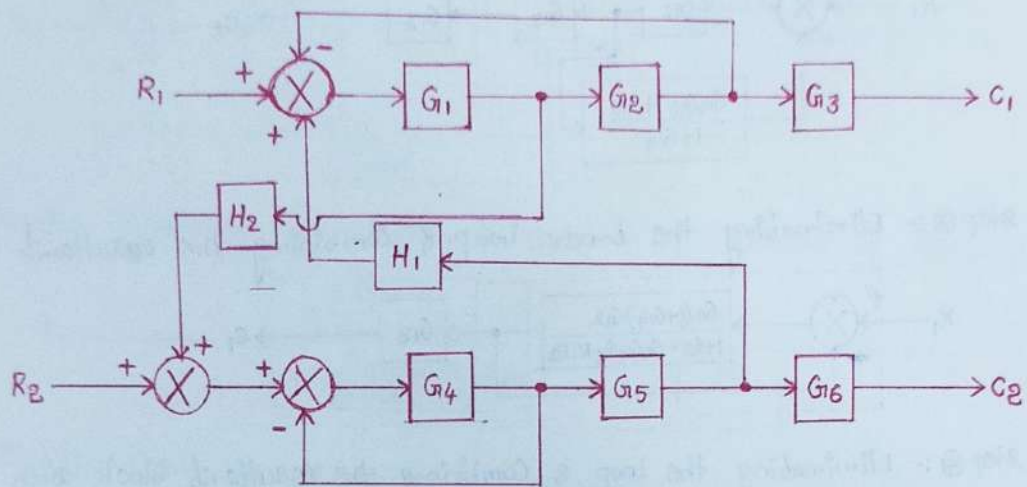


Step ⑦ :- After simplification, the closed loop Transfer function is

$$\frac{C_1}{R} = \frac{G_3(1+G_2H_3)}{1+G_2H_3+G_3H_1+G_1G_2G_3H_2}$$

(6)

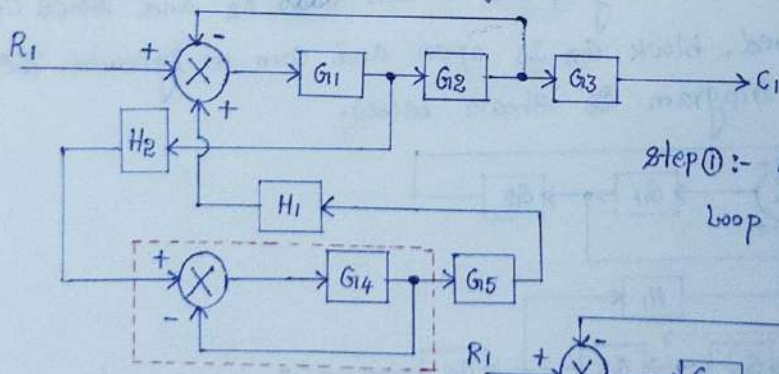
38



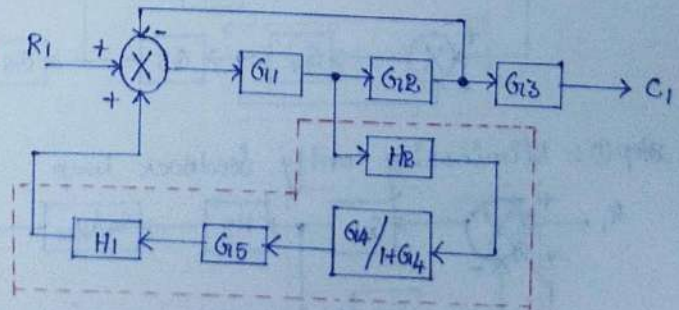
For the system represented by the block diagram shown in figure. Determine (a) C_1/R_1 and (b) C_2/R_1 .

(a) Determination of C_1/R_1 :-

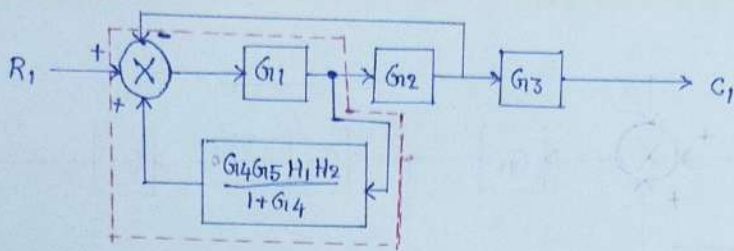
To determine C_1/R_1 , set $R_2=0$ and consider only one output C_1 and ignore C_2 . Hence the summing point which adds R_2 can be removed and there is no need to consider block G_{16} . With these modifications, the block diagram will be shown below.



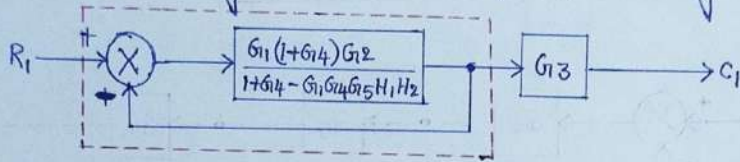
Step ①:- Eliminating the feedback loop involving the block G_{14} .



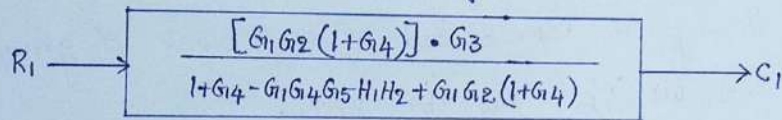
Step ②:- Combining the blocks in cascade.



Step ③:- Eliminating the inner loop & combining the resultant block & G_{12} .



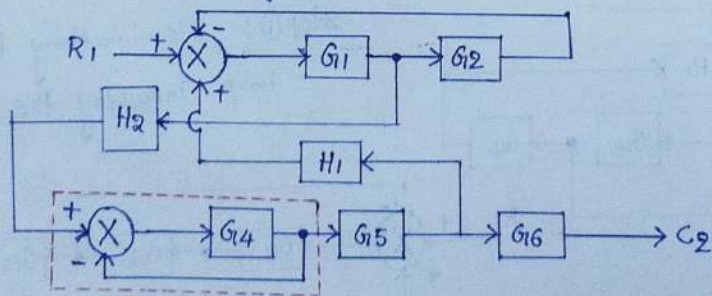
Step ④:- Eliminating the loop & combining the resultant block with G_{13} .



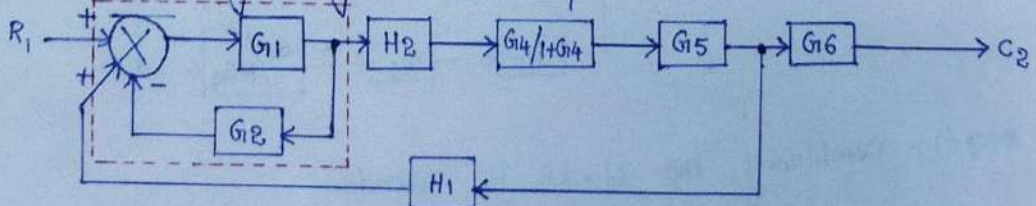
Hence the closed loop transfer function is

$$\frac{C_1}{R_1} = \frac{G_{11}G_{12}G_{13}(1+G_{14})}{1+G_{14}-G_{11}G_{4}G_{5}H_1H_2+G_{11}G_{12}(1+G_{14})} //$$

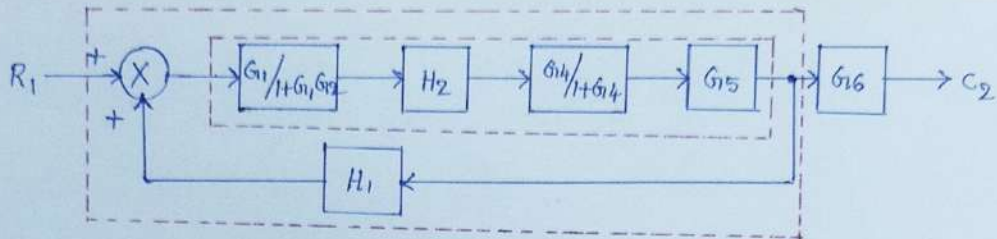
(b) Determination of C_2/R_1 :- In this case, $R_2=0$ and C_1 is ignored. Hence remove the summing point which adds R_2 and since C_1 is not considered, block G_{13} is open and can be ignored. The resultant block diagram is shown below.



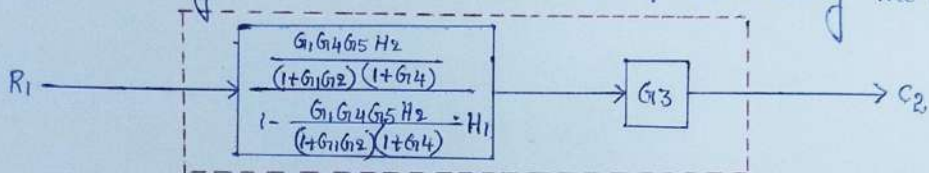
Step ①:- Eliminating unity feedback loop



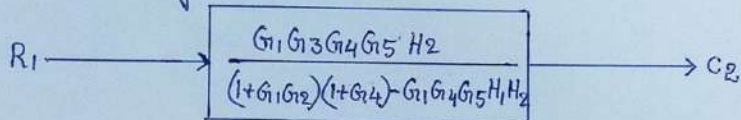
Step ②:- Eliminating the inner loop



Step ③:- Combining the blocks in cascade & eliminating the loop.



Step ④:- Combining the blocks in cascade.



Hence the closed loop transfer function is

$$\frac{C_2}{R_1} = \frac{G_1 G_3 G_4 G_5 H_2}{(1 + G_1 G_2)(1 + G_4) - G_1 G_4 G_5 H_1 H_2} //$$

