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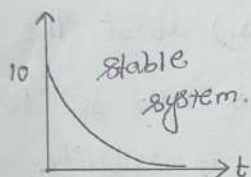
Stability Analysis In S-Domain (43-53) + 5 Graphs

(57-75)

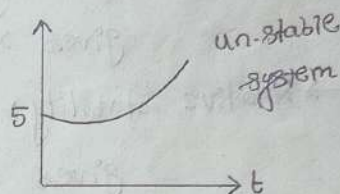
→ **Stability**:- Stability can be defined as A system is said to be stable, if a bounded (finite) output is obtained for an every bounded input.

→ For a stable system, the Impulse Response should approach to zero when $t \rightarrow \infty$.

Ex: $IR = 10e^{-3t}$



Ex: $5e^{3t}$



→ For a stable system, the Impulse Response must be absolutely integrable b/n the limits 0 & ∞ .

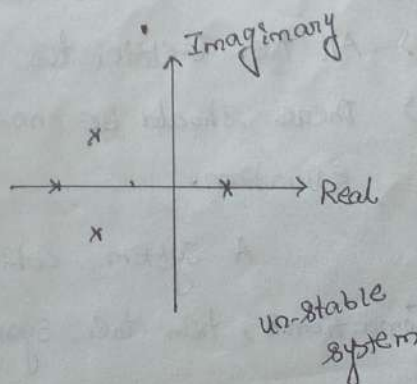
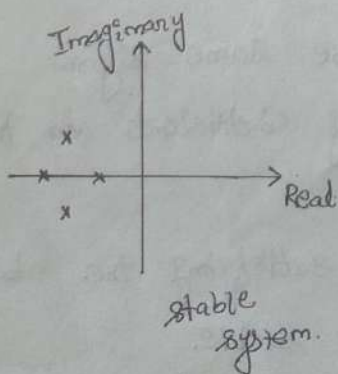
Ex: $IR = 10e^{-3t}$

$$\begin{aligned} &= \int_0^{\infty} (10e^{-3t}) dt = 10 \int_0^{\infty} e^{-3t} dt = 10 \left[\frac{e^{-3t}}{-3} \right]_0^{\infty} = \frac{10}{-3} \cdot [e^{-3t}]_0^{\infty} \\ &= \left[10 \left(\frac{1}{3} - \frac{1}{\infty} \right) \right]_0^{\infty} = 10 \left(\frac{1}{3} - \frac{1}{\infty} \right) = 10/3. \end{aligned}$$

$= \frac{10}{3} = 3.33$
 = Integrable Stable.

= Integrable stable.

→ In a system, the above two conditions are satisfied, a system is said to be stable if the poles of the system are lies in the left side of the s-plane.



→ Classification of stability :-

stability can be classified as follows.

→ Marginal stability :- A system is said to be Marginal stable, if non-repeatable poles are lies on the imaginary axis.

→ Asymptotic Stability :- A system is said to be Asymptotic stable, if the i/p made zero, the o/p should approach zero.

→ Conditional stability :- A system is said to be Conditional stable, for a certain value of K .

→ Absolute stability :- A system is said to be Absolute stable, it gives information (yes/no) about the stability.

→ Relative stability :- A system is said to be Relative stable, it gives strength/degree of stability.

Routh stability criterion :-

The necessary & sufficient condition for stability is that

(1) All the elements in the first column of the Routh array be positive. If this condition is not met, the system is unstable.

(2) the no. of sign changes in the elements of the first column of Routh array corresponds to the no. of roots of the characteristic equation in the right half of the s -plane.

Conditions for Routh Hurwitz Criterion :-

In a closed loop system $1 + G(s)H(s) = 0$ is called characteristic equation.

(1) All the Coefficients must be same sign.

(2) There should be no-missing coefficients in the characteristic equation.

A system, which is satisfying the above two conditions, then the system is stable.

→ mainly RH-criteria gives that, the no. of Right hand roots of the characteristic Equation without solving it. (58)

- (1) Using Routh criterion, determine the stability of the system represented by the characteristic Equation $s^4 + 8s^3 + 18s^2 + 16s + 5 = 0$? Comment on the location of the roots of characteristic Equation?

Given

The characteristic Equation is

$$s^4 + 8s^3 + 18s^2 + 16s + 5 = 0$$

This is 4th order Equation & so, it has 4-roots.

s^4	1	18	5
s^3	8	16	0
s^2	16	5	0
s^1	13.5	0	0
s^0	5	0	0

1st column

$$s^2: \frac{(8 \times 18) - (1 \times 16)}{8} = \frac{(8 \times 5) - 1(0)}{8}$$

$$16 \quad 5$$

$$s^1: \frac{(16 \times 16) - (8 \times 5)}{16}$$

$$13.5$$

$$s^0: \frac{13.5(5) - 0(16)}{13.5}$$

$$5$$

All the elements in the first column are positive & there is no-sign change. Hence all the roots are lying on the left half of s-plane & the system is stable.

- (2) By Routh stability criterion determine the stability of the system represented by the characteristic Equation $s^5 + s^4 + 6s^3 + 12s^2 + 18s + 6 = 0$. Comment on the location of roots of characteristic Equation.

Given

The characteristic Equation is

$$s^5 + s^4 + 6s^3 + 12s^2 + 18s + 6 = 0$$

This is 5th order Equation & so, it has 5-roots.

$$\begin{array}{c|ccc}
 s^5 & 1 & 6 & 18 \\
 s^4 & 1 & 12 & 6 \\
 s^3 & -6 & 12 & 0 \\
 s^2 & 14 & 6 & 0 \\
 s^1 & 14.57 & 0 & 0 \\
 s^0 & 6 & 0 & 0
 \end{array}$$

$$\begin{array}{c}
 s^3: \frac{(1 \times 6) - (12 \times 1)}{1} \quad \frac{(1 \times 18) - (1 \times 6)}{1} \\
 \quad \quad \quad -6 \quad 12 \\
 s^2: \frac{(-6 \times 12) - (1 \times 12)}{-6} \quad \frac{(-6 \times 6) - (1 \times 0)}{-6} \\
 \quad \quad \quad 14 \quad 6 \\
 s^1: \frac{(14 \times 12) - (6 \times -6)}{14} \\
 \quad \quad \quad 14.57 \\
 s^0: \frac{(14.57 \times 6) - (14 \times 0)}{14.57} \\
 \quad \quad \quad 6
 \end{array}$$

There are two sign changes in the 1st column of Routh array & so, these two roots are lying on right half side of s-plane. & remaining three roots are lying on left half of s-plane. Hence the system is un-stable.

By Routh stability criterion Determine the stability of the system.

represented by the characteristic Equation $s^5 + s^4 + 2s^3 + 2s^2 + 3s + 5 = 0$?

Comment on the location of the roots of characteristic Equation?

Given

The characteristic Equation is

$$s^5 + s^4 + 2s^3 + 2s^2 + 3s + 5 = 0$$

This is 5th order Equation, & so, it has 5-roots.

$$\begin{array}{c|ccc}
 s^5 & 1 & 2 & 3 \\
 s^4 & 1 & 2 & 5 \\
 s^3 & \epsilon & -2 & 0 \\
 s^2 & \frac{2\epsilon + 2}{\epsilon} & 5 & 0 \\
 s^1 & \frac{-(5\epsilon^2 + 4\epsilon + 4)}{2\epsilon + 2} & 0 & \\
 s^0 & 5 & 0 &
 \end{array}$$

$$\begin{array}{c}
 s^3: \frac{(1 \times 2) - (1 \times 2)}{1} \quad \frac{(1 \times 3) - (1 \times 5)}{1} \\
 \quad \quad \quad 0 \quad -2 \\
 \text{Replace: } \epsilon \text{ by } -2 \\
 s^3: \epsilon \quad -2 \\
 s^2: \frac{2\epsilon - (-2)}{\epsilon} \quad \frac{5\epsilon - 0}{\epsilon} \\
 \quad \quad \quad \frac{2\epsilon + 2}{\epsilon} \quad 5 \\
 s^1: \frac{\frac{2\epsilon + 2}{\epsilon}(-2) - 5\epsilon}{\frac{2\epsilon + 2}{\epsilon}} \\
 \quad \quad \quad \frac{-(5\epsilon^2 + 4\epsilon + 4)}{2\epsilon + 2}
 \end{array}$$

on letting $\epsilon \rightarrow 0$, we get

(59)

s^5	1	2	3
s^4	1	2	5
s^3	0	-2	0
s^2	∞	5	0
s^1	-2	0	0
s^0	5	0	0

There are two sign changes in the first column of the Routh criterion. Hence two roots are lying on the right half of s -plane. and the system is un-stable. The remaining three roots are lying on the left half of s -plane.

(4)

Construct Routh array & Determine the stability of the system whose characteristic Equation is $s^6 + 2s^5 + 8s^4 + 12s^3 + 20s^2 + 16s + 16 = 0$. And also give the comments also.

Given

The characteristic Equation is given by

$$s^6 + 2s^5 + 8s^4 + 12s^3 + 20s^2 + 16s + 16 = 0$$

This is the 6th order Equation & has 6-roots

s^6	1	8	20	16
s^5	2	12	16	0
s^4	2	12	16	0
s^3	0	0	0	0

$$s^4: \frac{16-12}{2} \quad \frac{40-16}{2} \quad \frac{32-0}{2}$$

$$2 \quad 12 \quad 16$$

$$s^3: \frac{24-24}{2} \quad \frac{32-32}{2}$$

$$0 \quad 0$$

Here s^3 row is called zeroth Equation. & Hence, the auxiliary Equation is given by

$$A = 2s^4 + 12s^2 + 16 \rightarrow (1)$$

Differentiate w.r.to. 's' we get

$$\frac{dA}{ds} = 8s^3 + 24s$$

The Co-efficients of $\frac{dA}{ds}$ are used to form s^3 row.

$$s^3: 4 \quad 12$$

s^6	1	8	20	16
s^5	2	12	16	0
s^4	2	12	16	0
s^3	4	12	0	0
s^2	6	16	0	0
s^1	1.33	0	0	0
s^0	16			

$$s^2: \frac{48-24}{4} \quad \frac{64-0}{4}$$

$$6 \quad 16$$

$$s^1: \frac{72-64}{6}$$

$$8/6 = 4/3 = 1.33$$

There are no sign changes in the 1st column of Routh's stability array. & the row with all zero's indicate the possibility of roots on imaginary axis. Hence the system is marginally stable.

From Equ (1) we have

$$A = 2s^4 + 12s^2 + 16 = 0$$

Let $s^2 = x$ then

$$2x^2 + 12x + 16 = 0$$

$$\Rightarrow x^2 + 6x + 8 = 0$$

$$(x+4)(x+2) = 0$$

$$x = -2 \text{ \& } x = -4.$$

But $s^2 = x$

$$\text{i.e., } s^2 = -2$$

$$s^2 = -4$$

$$\Rightarrow s = \pm j\sqrt{2}$$

$$\Rightarrow s = \pm j2$$

$\therefore$ The roots of auxiliary equation are also the roots of characteristic equation. Hence 4-roots are lying on imaginary axis & remaining 2-roots are lying on left half of s-plane. There are no-roots on the right half of s-plane.

(5) The characteristic equation of a system is

$$s^7 + 9s^6 + 24s^5 + 24s^4 + 24s^3 + 24s^2 + 23s + 15 = 0$$

Determine the location of roots on s-plane & hence the stability of the system.

Given

The characteristic equation is

$$s^7 + 9s^6 + 24s^5 + 24s^4 + 24s^3 + 24s^2 + 23s + 15 = 0$$

This is 7th order equation & it has 7 ^{roots.} ~~equations.~~

s^7	1	24	24	23
s^6	9	24	24	15
s^5	21.33	21.33	21.33	0
s^4	15	15	15	0
s^3	0	0	0	0

$$s^5: \frac{216-24}{9} \quad \frac{216-24}{9} \quad \frac{207-15}{9}$$

$$21.33 \quad 21.33 \quad 21.33$$

$$s^4: \frac{511.92-191.97}{21.33} \quad \frac{511.92-191.97}{21.33} \quad \frac{319.95-0}{21.33}$$

$$15 \quad 15 \quad 15$$

$$s^3: 0 \quad 0 \quad 0$$

Here s^3 -row is called 0th Equation. The Equation above 0th row is called auxiliary Equation, which is given by

$$A = 15s^4 + 15s^2 + 15 \rightarrow \textcircled{1}$$

Diff. w.r.to. 's' we get

$$\frac{dA}{ds} = 60s^3 + 30s$$

The Co-efficient of $\frac{dA}{ds}$ are used to form s^3 -row.

$$\therefore s^3: 60 \quad 30$$

s^7	1	24	24	23
s^6	9	24	24	15
s^5	21.33	21.33	21.33	0
s^4	15	15	15	0
s^3	60	30	0	0
s^2	7.5	15	0	0
s^1	-90	0	0	0
s^0	15	0	0	0

$$s^2: \frac{900-450}{60} \quad \frac{900-0}{60}$$

$$7.5 \quad 15$$

$$s^1: \frac{225-900}{7.5}$$

$$-90$$

$$s^0: \frac{15(-90)}{-90}$$

$$15$$

There are 2-sign changes in the 1st-Column of Routh's array. i.e, two roots are lying on the right half of s-plane. So, the system is un-stable.

The row of all zero indicates the possibility of roots on imaginary axis.

from Equⁿ ①

$$A = 15s^4 + 15s^2 + 15 = 0$$

Put $s^2 = x$ we get

$$\Rightarrow A = 15x^2 + 15x + 15 = 0$$

$$\Rightarrow A = s^2 + s + 1 = 0$$

$$\Rightarrow s = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm j\sqrt{3}}{2}$$

$$= -0.5 \pm j0.866 = 1/\underline{120^\circ} \text{ (or) } 1/\underline{-120^\circ}$$

$$\text{But } s^2 = s$$

$$\therefore s = \pm \sqrt{s} = \pm \sqrt{1/\underline{120^\circ}} \text{ or } \pm \sqrt{1/\underline{-120^\circ}}$$

$$= \pm \sqrt{1/\underline{120^\circ/2}} \text{ or } \pm \sqrt{1/\underline{-120^\circ/2}}$$

$$= \pm 1/\underline{60^\circ} \text{ or } \pm 1/\underline{-60^\circ}$$

$$= \pm (0.5 + j0.866) \text{ or } \pm (0.5 - j0.866)$$

The roots of Auxiliary Equation are the roots of the characteristic Equation. Two roots AE are lying on right half of the s-plane & Two-roots of AE are lying on left half of the s-plane. The roots of auxiliary Equation are also the roots of the characteristic Equation.

Hence the two roots lying on the right half of s-plane are indicated by two sign changes in the 1st column of Routh array. & the remaining five roots are lying on left half of the s-plane. No-roots lying on imaginary axis.

(6) The open loop transfer function of a unity feedback control system is given by $G(s) = K/s(1+sT_1)(1+sT_2)$. Apply RH-stability criterion determine the value of K in terms of T_1 & T_2 for the system to be stable?

Given

$$\text{Open Loop Transfer Function } G(s) = K/s(1+sT_1)(1+sT_2)$$

$$\text{unity feed back means } H(s) = 1.$$

$$\text{closed Loop Transfer Function } \frac{C(s)}{R(s)} = \frac{K/s(1+sT_1)(1+sT_2)}{1 + K/s(1+sT_1)(1+sT_2)}$$

$$= K/[K + s(1+sT_1)(1+sT_2)]$$

$$= K/[T_1T_2s^3 + (T_1+T_2)s^2 + s + K]$$

of the s-plane.

Remaining see
64 Page.

$$(7) \rightarrow 9s^5 - 20s^4 + 10s^3 - s^2 - 9s - 10 = 0$$

(61) 48

The given characteristic Equation is

$$9s^5 + 10s^3 - 20s^4 - s^2 - 9s - 10 = 0$$

The characteristic polynomial of above equation is $9s^5 - 20s^4 + 10s^3 - s^2 - 9s - 10 = 0$. Here some of the coefficients are negative and so, some roots will lie on the right half of the s-plane. Hence the system is un-stable.

The given characteristic equation has order-5. so, it has 5-roots. The Routh array can be constructed for the given characteristic equation as follows.

s^5	9	10	-9
s^4	-20	-1	-10
s^3	9.55	-13.5	0
s^2	-29.3	-10	0
s^1	-16.8	0	0
s^0	-10	0	0

↑ 1st column of Routh Array.

There are three sign changes in the first column of Routh array. so, three roots are lying on the right half of the s-plane. & Remaining two-roots are lying on the left half of the s-plane.

Result:

- The system is un-stable.
- Three roots are lying on the right half of the s-plane & Two-roots are lying on the left half of the s-plane.

$$(E) \rightarrow s^6 + s^5 + 3s^4 + 3s^3 + 3s^2 + 3s + 1 = 0$$

The above characteristic Equation has order of 6. So the system having 6-roots.

s^6	1	3	3	1
s^5	1	3	2	0
s^4	ϵ	1	1	0
s^3	$\frac{3\epsilon-1}{\epsilon}$	$\frac{2\epsilon-1}{\epsilon}$	0	0
s^2	$\frac{-2\epsilon+4\epsilon-1}{3\epsilon-1}$	1	0	0
s^1	$\frac{4\epsilon^2-\epsilon}{2\epsilon^2-4\epsilon+1}$	0	0	0
s^0	1	0	0	0

on letting $\epsilon \rightarrow 0$ we get

s^6	1	3	3	1
s^5	1	3	2	0
s^4	0	1	1	0
s^3	$-\infty$	$-\infty$	0	0
s^2	1	0	0	0
s^1	0	0	0	0
s^0	1	0	0	0

→ Auxiliary Equation.

→ Zero'th Equation

Here 0th row indicates that there is a possibility of roots on imaginary axis. The auxiliary Equation Polynomial is $s^2 + 1 = 0$.

The roots of the Auxiliary Polynomial are $= \pm \sqrt{-1}$

$$= \pm j1.$$

The roots of auxiliary polynomial are also roots of characteristic equation. Hence two roots are lying on imaginary axis.

Therefore divide the characteristic polynomial by auxiliary equation and construct the routh array for quotient polynomial. ~~to find the roots lying~~ The characteristic polynomial can be expressed as product of auxiliary polynomial & quotient polynomial.

$$\therefore s^6 + s^5 + 3s^4 + 3s^3 + 2s^2 + 2s + 1 = (s^2 + 1)(s^4 + s^3 + 2s^2 + 2s + 1)$$

The routh array for quotient polynomial is constructed as shown below.

s^4	1	2	1
s^3	1	2	0
s^2	ϵ	1	0
s^1	$\frac{2\epsilon - 1}{\epsilon}$	0	0
s^0	1	0	0

on letting $\epsilon \rightarrow 0$ we get

s^4	1	2	1
s^3	1	2	0
s^2	0	1	0
s^1	$-\infty$	0	0
s^0	1	0	0

1st Column of routh array.

There are two sign changes in 1st column of Routh array. Hence two roots are lying on the right half of s-plane. & the other two roots are lying on the left half of the s-plane.

The roots of characteristic equation are given by roots of auxiliary polynomial and quotient polynomial. Hence two roots are lying on imaginary axis, two roots are lying on right half of s-plane & the remaining two roots are lying on left half of s-plane. Hence the system is unstable.

Result:- <1> The system is unstable.

<2> Two roots are lying on imaginary axis, two roots are lying on right half of s-plane & two roots are lying on left half of s-plane.

(9) → Determine the range of K for stability of unity feedback system whose open loop transfer function is

$$G(s) = K/s(s+1)(s+2).$$

The closed loop transfer function

$$\begin{aligned} \frac{C(s)}{R(s)} &= \frac{G(s)}{1+G(s)} \\ &= \frac{K/s(s+1)(s+2)}{1 + K/s(s+1)(s+2)} = \frac{K/s(s+1)(s+2)}{1 + K/s(s+1)(s+2)}. \end{aligned}$$

The characteristic equation is

$$s(s+1)(s+2) + K = 0$$

$$s^3 + 3s^2 + 2s + K = 0$$

The Routh array is constructed as shown below.

s^3	1	2
s^2	3	K
s^1	$\frac{6-K}{3}$	0
s^0	K	0

1st Column of Routh Array

For the system to be stable there should not be any sign change in the elements of first column.

From s^0 row, for the system to be stable, $K > 0$

s^1 row, for the system to be stable, $\frac{6-K}{3} > 0$

$$\Rightarrow 6-K > 0$$

$$\Rightarrow 6 > K$$

$\therefore$ The range of K for the system to be stable is $0 < K < 6$.

Result:- The value of K is in the range $0 < K < 6$ for the system to be stable.

$$(10) \rightarrow s^7 + 5s^6 + 9s^5 + 9s^4 + 48s^3 + 70s^2 + 36s + 36 = 0$$

Result: (1) The system is un-stable.

(2) Two roots are lying on the right half of s -plane and five roots are lying on the left half of s -plane.

$$(11) \rightarrow s^5 + 48s^4 + 8s^3 + 8s^2 + 7s + 4 = 0$$

Result: (1) The system is limitedly/marginally stable.

(2) Two roots are lying on imaginary axis and three roots are lying on left half of s -plane.

(12)

→ A feed back system has open loop transfer function of $G(s) = K \cdot e^{-s} / s(s^2 + 5s + 9)$. Determine the maximum value of K for stability of closed loop system.

Hint: Generally control systems have very low bandwidth which implies that it has very low frequency range of operations. Hence for low frequency ranges the term e^{-s} can be replaced by $(1-s)$. {i.e. $e^{-sT} = 1 - sT$ }.

Result: For stability of the system 'K' is in the range $0 < K < 7.5$

Proof

$$0 = s^5 + s^4 + 5s^3 + 4s^2 + 9s + 1 \quad (1)$$

$$0 = s^4 + s^3 + 4s^2 + 5s + 1 \quad (2)$$

The characteristic Equation is

(64)

$$T_1 T_2 s^3 + (T_1 + T_2) s^2 + s + K = 0$$

By R-H criteria

s^3	$T_1 T_2$	1
s^2	$(T_1 + T_2)$	K
s^1	$1 - \frac{T_1 T_2 K}{(T_1 + T_2)}$	0
s^0	K	0

$$s^1: \frac{(T_1 + T_2) - K(T_1 T_2)}{(T_1 + T_2)}$$

$$1 - \frac{T_1 T_2 K}{T_1 + T_2}$$

$$s^0: \frac{\left(1 - \frac{T_1 T_2 K}{T_1 + T_2}\right) K - 0}{\left(1 - \frac{T_1 T_2 K}{T_1 + T_2}\right)}$$

For the system to be stable, there should not be any sign change in the elements of the first column.

Hence choose the elements in 1st column are positive.

i.e., $K > 0$ & $1 - \frac{T_1 T_2 K}{T_1 + T_2} > 0$

$$1 > \frac{T_1 T_2}{T_1 + T_2} (K) \Rightarrow K < \frac{T_1 + T_2}{T_1 T_2}$$

~~$$\frac{T_1 + T_2}{T_1 T_2} > K > 0$$~~

$\therefore$ The range of K for the system to be stable is

~~$$0 < K < \frac{T_1 + T_2}{T_1 T_2}$$~~

i.e., $0 < K < \frac{T_1 + T_2}{T_1 T_2}$

see more problems on RH-criteria in pg. 48, 49, 50 pages.

Root Locus:- Root Locus technique tells both absolute stability & also relative stability. where as RH-criteria tells about only the relative stability.

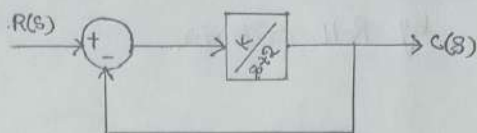
Root Locus can be defined as the path taken by the roots of the characteristic Equation when open loop gain(K) is varied from 0 to ∞ is called "Root Locus".

It is a graphical Method for finding out the roots of the characteristic Equations (or) poles of closed loop Transfer function.

Consider a closed loop system

Transfer function

$$\frac{C(s)}{R(s)} = \frac{k}{s+2+k}$$



The characteristic Equation is $s+2+k=0$

$$\Rightarrow k = -(s+2)$$

As we vary the value of k , we get different points (poles) on -ve. Real axis. By combining these points we get locus.

Rules for Constructing the root Locus:-

(1) Find out the open loop poles or the closed loop poles from their respective T/F.

(2) Identify the branches & Root locus.

Here (branches = poles).

(3) The root locus always starts at poles at $k=0$ & ends at zero's at $k=\infty$.

(4) Identify the root locus path.

(5) Find out the Centroid = $\frac{\sum P - \sum Z}{P - Z} = \frac{\text{sum of poles} - \text{sum of zero's}}{\text{no. of poles} - \text{no. of zero's}}$

(6) Consider a check points so that the sum of poles & zero's be odd. Then only that point will be in the root locus.

(7) Find out the no. of Asymptotes = $P - Z$

(8) Find out the Break away point $\frac{dk}{ds} = 0$.

(9) Find out Angle of Asymptotes = $\frac{(2q+1)\pi}{P-Z}$ where $q=0,1,2,\dots,(P-Z-1)$.

(10) Find point of intersection of root locus with imaginary axis.

Put $s=j\omega$ & Equate imaginary part to zero.

(11) Find Angle of departure & angle of Arrival.

Angle of departure = $180 - \phi$ ($\because$ Consider complex poles)

where $\phi = 180 - \angle P$

Angle of Arrival = $180 - \phi$ ($\because$ Consider complex zero's)

(12) Find out the value of gain "K" at any point ^(s-s_a) on root locus (65)

$$= \frac{\text{Product of vector lengths from open loop poles to the point } s_a}{\text{Product of vector lengths from open loop zeros to the point } s_a}$$

(1) Sketch the root locus of the system whose open loop transfer function is $G(s) = K/s(s+2)(s+4)$. Find the value of "K" so, that the damping ratio of the closed loop system is 0.5.

→ The poles are $= 0, -2, -4$ → no. of Poles $= 3$

→ no. zeros $= 0$

→ no. of branches $= 3 = \text{no. of Poles}$

→ no. of Asymptotes $= P - Z$

$$= 3 - 0 = 3$$

→ Centroid $= \frac{\sum P - \sum Z}{P - Z} = \frac{-6 - 0}{3} = -2$

→ Angle of Asymptotes $= \frac{(2q+1)\pi}{P-Z}$ where $q = 0, 1, 2$

$$= \pi/3, \pi, 5\pi/3 = 60^\circ, 180^\circ, 300^\circ$$

→ Break away & Break in points

$$\Rightarrow \frac{dk}{ds} = 0$$

The closed loop T/F $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$

$$= K/s(s+2)(s+4) + K$$

$$= K/s^3 + 6s^2 + 8s + K$$

The characteristic Equation is given by

$$s^3 + 6s^2 + 8s + K = 0$$

$$\Rightarrow K = -(s^3 + 6s^2 + 8s)$$

on diff. with respect to 's' & Equate to zero.

$$\frac{dk}{ds} = -(3s^2 + 12s + 8) = 0$$

$$s = -0.845 \text{ or } -3.154$$

check for k

when $s = -0.845$, the value of 'k' is given by

$$k = -[(-0.845)^3 + 6(-0.845)^2 + 8(-0.845)]$$

$$= 3.08$$

so,

which is +ve & real. So $s = -0.845$ is actual breakaway point.

when $s = -3.154$, the value of 'k' is given by

$$k = -[(-3.154)^3 + 6(-3.154)^2 + 8(-3.154)]$$

$$= -3.08$$

which is -ve for $s = -3.154$, this is not a actual breakaway point.

→ Angle of Departure

There are no complex pole or zero, so, we need not find angle of departure or arrival.

→ crossing point of imaginary axis

The Characteristic Equation is given by

$$\Rightarrow s^3 + 6s^2 + 8s + k = 0$$

$$\text{Put } s = j\omega$$

$$(j\omega)^3 + 6(j\omega)^2 + 8(j\omega) + k = 0$$

$$-j\omega^3 - 6\omega^2 + j8\omega + k = 0$$

Evaluating imaginary part to zero

$$-j\omega^3 + 8j\omega = 0$$

$$+j\omega^3 = +8j\omega$$

$$\omega^2 = 8$$

$$\Rightarrow \omega = \pm\sqrt{8}$$

$$= \pm 2.8$$

Evaluating real part to zero

$$-6\omega^2 + k = 0$$

$$k = 6\omega^2$$

$$= 6(8)$$

$$= 48.$$

check for k

when $s = -0.845$, the value of 'k' is given by

$$k = -[(-0.845)^3 + 6(-0.845)^2 + 8(-0.845)]$$
$$= 3.08$$

so,

which is +ve & real. So $s = -0.845$ is actual breakaway point.

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$$= -3.08$$

which is -ve for $s = -3.154$, this is not a actual breakaway point.

→ Angle of Departure

There are no complex pole or zero, so, we need not find angle of departure or arrival.

→ Crossing points of imaginary axis

The Characteristic Equation is given by

$$\Rightarrow s^3 + 6s^2 + 8s + k = 0$$

Put $s = j\omega$

$$(j\omega)^3 + 6(j\omega)^2 + 8(j\omega) + k = 0$$

$$-j\omega^3 - 6\omega^2 + j8\omega + k = 0$$

Evaluating imaginary part to zero

$$-j\omega^3 + 8j\omega = 0$$

$$+j\omega^3 = +8j\omega$$

$$\omega^2 = 8$$

$$\Rightarrow \omega = \pm \sqrt{8}$$

$$= \pm 2.8$$

Evaluating real part to zero

$$-6\omega^2 + k = 0$$

$$k = 6\omega^2$$

$$= 6(8)$$

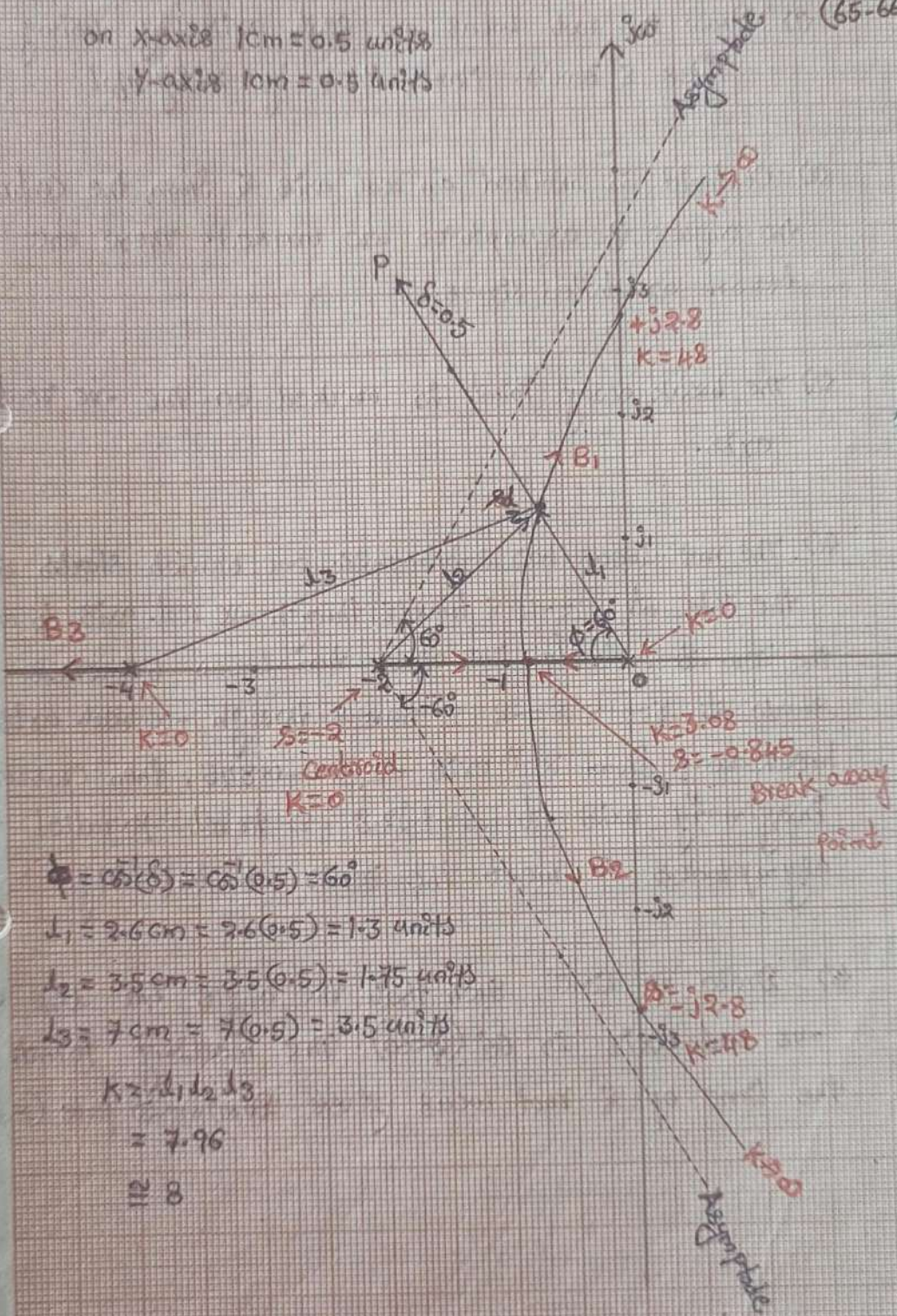
$$= 48.$$

Scale:

on x-axis 1cm = 0.5 units

y-axis 1cm = 0.5 units

(65-66)



$$\phi = \angle(s) = \angle(-0.5) = 60^\circ$$

$$L_1 = 2.6 \text{ cm} = 2.6(0.5) = 1.3 \text{ units}$$

$$L_2 = 3.5 \text{ cm} = 3.5(0.5) = 1.75 \text{ units}$$

$$L_3 = 7 \text{ cm} = 7(0.5) = 3.5 \text{ units}$$

$$K = L_1 L_2 L_3$$

$$= 7.96$$

$$\approx 8$$

(1) Draw the root locus on Real axis by using Poles & Zero's. Mark Poles by $\times$ -Symbol.

(2) Centroid is marked on real axis & from the Centroid the angles of asymptotes are marked. These are drawn as dotted lines.

(3) The break away Point is marked on the -ve real axis.

(4) The root locus has 3-branches. one branch starts at the Pole at $s = -4$ & travels through -ve real axis to meet the zero at infinity. The other two Locus branches starts at $s = 0$ & $s = -2$ & travels through -ve real axis break away from real axis at $s = -0.845$, then crosses imaginary axis at $s = \pm j2.8$ & travel parallel to asymptotes to meet the zero's at infinity.

(5) Draw op line, such that the angle b/w line op & -ve real axis is 60° ($\phi = 60$). The meeting point of the line op & root locus gives dominant pole (sd).

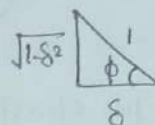
→ the value of "k" corresponding to $\delta = 0.5$

Given that $\delta = 0.5$

But $\cos \phi = \delta$

$$\phi = \cos^{-1}(\delta)$$

$$= \cos^{-1}(0.5) = 60^\circ$$



Draw a line op, such that the angle b/n line op & -ve real axis is 60° . The meeting point of the line op and root locus gives the dominant Pole (s_d).

The value of k corresponding to the points ($s = s_d$) = $\frac{\text{Product of length of a vector from all poles to the point } (s = s_d)}{\text{Product of length of vectors from all zeros to the point } (s = s_d)}$

$$= \frac{d_1 \cdot d_2 \cdot d_3}{1}$$

$$= 1.3 \times 1.75 \times 3.5$$

$$= 7.96 \cong 8$$

(2)

A unity feedback control system has an open loop transfer function $G(s) = K / s(s^2 + 4s + 13)$. sketch the root locus.

The poles of open loop T/F are the roots of the equation

$$s(s^2 + 4s + 13) = 0$$

$$\text{The roots of the quadratic Eqn. are } s = \frac{-4 \pm \sqrt{16 - 52}}{2} = -2 \pm j3$$

→ The poles are $= 0, -2 + j3, -2 - j3$.

no. of poles = 3

no. of zeros = 0.

→ no. of branches = 3 = no. of poles

→ no. of Asymptotes = P - Z

$$= 3 - 0 = 3.$$

$$\rightarrow \text{Centroid} = \frac{\sum P - \sum Z}{P - Z} = \frac{-4}{3} = -1.33$$

$$\rightarrow \text{Angle of Asymptotes} = \frac{(2q+1)\pi}{P-Z} \quad \text{where } q = 0, 1, 2$$

$$= 60^\circ, 180^\circ, 300^\circ$$

→ Break away & Break in points

$$\Rightarrow \frac{dk}{ds} = 0$$

The closed loop T/F $\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$

$$= K / s(s^2 + 4s + 13) + K.$$

The characteristic Equation is given by

$$s^3 + 4s^2 + 13s + K = 0$$

$$\Rightarrow K = -(s^3 + 4s^2 + 13s)$$

& Evaluate to zero.

on differentiating Eqn of K w.r. to s we get

$$\frac{dk}{ds} = -(3s^2 + 8s + 13) = 0$$

$$\Rightarrow 3s^2 + 8s + 13 = 0$$

$$\Rightarrow s = \frac{-8 \pm \sqrt{64 - 156}}{6} = -1.33 \pm j1.6$$

check for K

when $s = (-1.33 + j1.6)$, the value of K is given by

$$\Rightarrow K = -[(-1.33 + j1.6)^3 + 4(-1.33 + j1.6)^2 + 13(-1.33 + j1.6)]$$

$\neq$ +ve & real.

when $s = (-1.33 - j1.6)$, the value of K is given by

$$K = -[(-1.33 - j1.6)^3 + 4(-1.33 - j1.6)^2 + 13(-1.33 - j1.6)]$$

$\neq$ +ve & real.

$\therefore$ The values of K for $s = (-1.33 \pm j1.6)$ are not real & +ve. Hence the root locus has neither breakaway nor break in points.

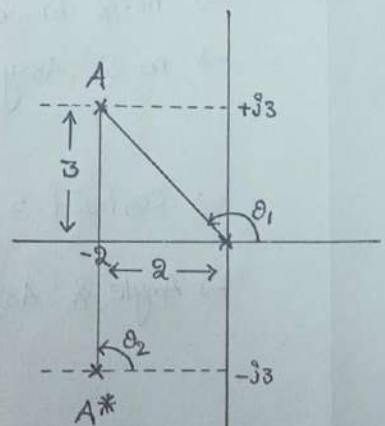
→ angle of Departure.

Let us consider the complex pole A shown in fig. Draw vectors from all other poles to the pole A shown in fig.

Let the angles of these vectors be θ_1 & θ_2 .

$$\theta_1 = 180 - \tan^{-1}(3/2) = 123.7^\circ$$

$$\theta_2 = 90^\circ$$



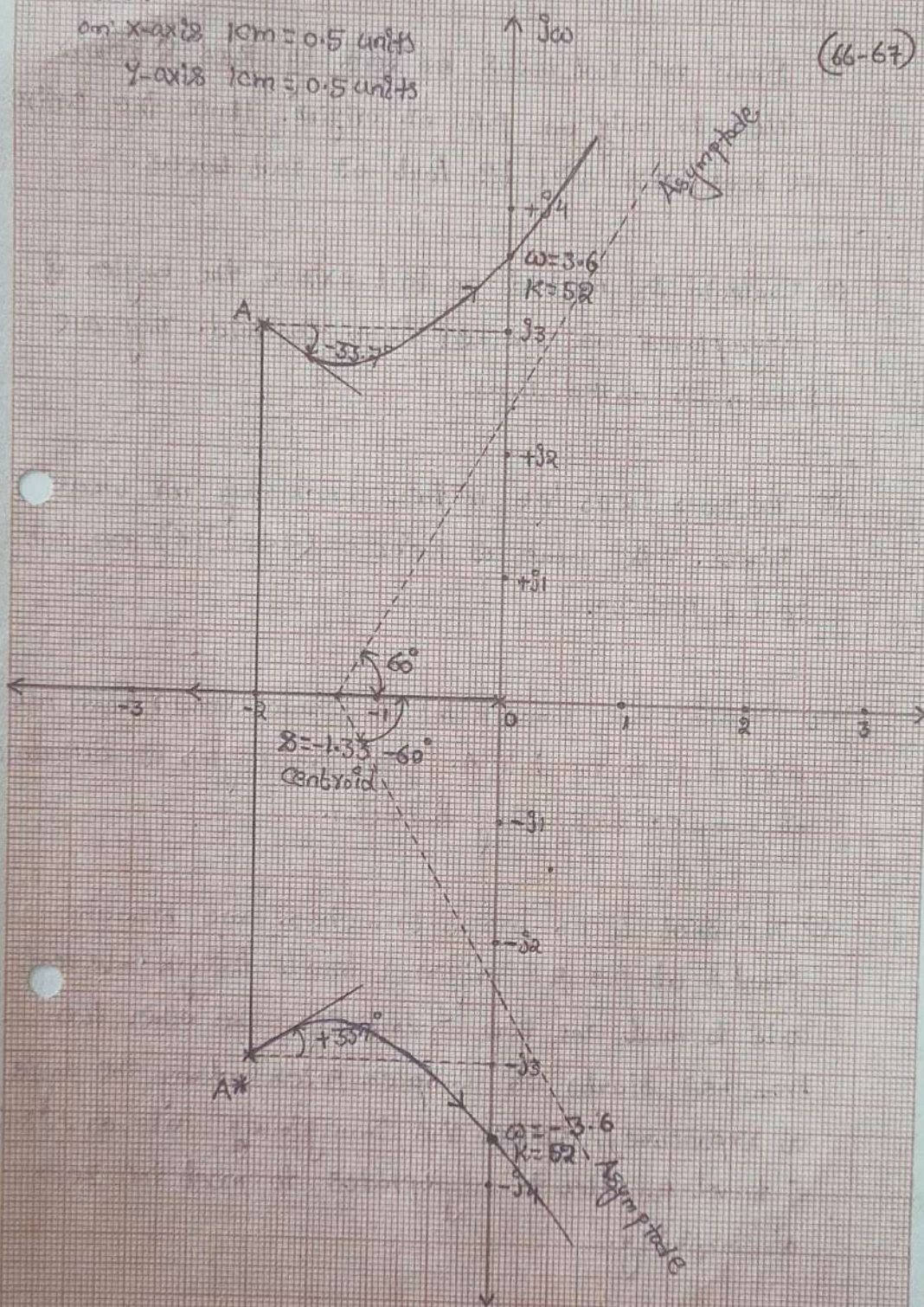
(49-50)

Scale:

on x-axis 1cm = 0.5 units

y-axis 1cm = 0.5 units

(66-67)



(1) Draw the root locus on real by poles & zero's.

There is only one pole at origin. Hence the entire -ve real axis will be part of root locus.

(2) Centroid is marked on real axis & the angles of asymptotes are marked from centroid. They are drawn as dotted lines.

(3) The roots Locus has neither breakaway nor break points. $\because$ the values of K for $s = (-1.33 \pm j1.6)$ are not real & positive.

(4) Consider the complex Pole A. Draw vectors from all other poles to the Pole A. Mark the angles of departure at complex poles using Protractor.

(5) The root Locus has 3-branches. one branch starts at the pole origin & travel through -ve real axis to meet the zero at infinity. The other two branches starts at complex poles (along the angle of departure), crosses the imaginary axis at $\pm j3.6$ & travel parallel to asymptotes to meet the zero at infinity.

Angle of Departure from the Complex pole (A) = $180 - (\theta_1 + \theta_2)$

(67)

$$= 180 - (123.7 + 90) = -33.7^\circ$$

The angle of Departure at Complex pole A^* is -ve of the angle of Departure at Complex pole A.

$$\therefore \text{Angle of Departure at pole } A^* = 33.7^\circ$$

→ Crossing points on imaginary axis

The characteristic Eqn. is given by

$$\Rightarrow s^3 + 4s^2 + 13s + K = 0$$

$$\text{Put } s = j\omega$$

$$(j\omega)^3 + 4(j\omega)^2 + 13(j\omega) + K = 0$$

$$\Rightarrow -j\omega^3 - 4\omega^2 + 13j\omega + K = 0$$

Equate imaginary parts to zero

$$-\omega^3 + 13\omega = 0$$

$$\omega^3 = 13\omega$$

$$\Rightarrow \omega^2 = 13$$

$$\omega = \pm \sqrt{13}$$

$$= \pm 3.6$$

Equate Real parts to zero

$$-4\omega^2 + K = 0$$

$$K = 4\omega^2$$

$$= 4(13)$$

$$= 52.$$

The crossing points of root locus is $\pm j3.6$. The value of K at this crossing point is $K=52$. (This is the limiting value of K for the stability of the system).

(3) Sketch the root locus for the unity feedback system whose open loop Transfer function is $G(s) = K/s(s^2 + 6s + 10)$.

The poles of open loop Transfer function are the roots of the Equation $s(s^2 + 6s + 10) = 0$.

$$\text{The roots of the quadratic are } s = \frac{-6 \pm \sqrt{36 - 40}}{2}$$

$$= -3 \pm j1.$$

→ The poles are $= 0, -3+j1, -3-j1$.

no. of poles = 3

no. of zeros = 0.

→ no. of branches = 3 = no. of Poles

→ no. of Asymptotes = $P - Z$
 $= 3 - 0 = 3$.

→ Centroid = $\frac{\sum P - \sum Z}{P - Z} = \frac{-3 + j1 - 3 - j1}{3} = \frac{-6}{3} = -2$

→ Angle of Asymptotes = $\frac{(2q+1)\pi}{P-Z}$ where $q = 0, 1, 2$
 $= 60^\circ, 180^\circ, 300^\circ$.

→ Breakaway & Break-in Points

The closed Loop Transfer Function

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$
$$= \frac{K}{s(s^2 + 6s + 10) + K}$$

The characteristic Eqn. is $s(s^2 + 6s + 10) + K = 0$

$$\therefore K = -(s^3 + 6s^2 + 10s)$$

on diff. Eqn. of K w.r. to s and set it equal to zero.

$$\frac{dK}{ds} = -(3s^2 + 12s + 10) = 0$$

$$\Rightarrow 3s^2 + 12s + 10 = 0$$

$$\Rightarrow s = \frac{-12 \pm \sqrt{144 - 120}}{6} = -1.18 \text{ or } -2.82$$

check for K

when $s = -1.18$, $K = -\left[(-1.18)^3 + 6(-1.18)^2 + 10(-1.18)\right] = 5.09$

$s = -2.82$, $K = -\left[(-2.82)^3 + 6(-2.82)^2 + 10(-2.82)\right] = 2.91$

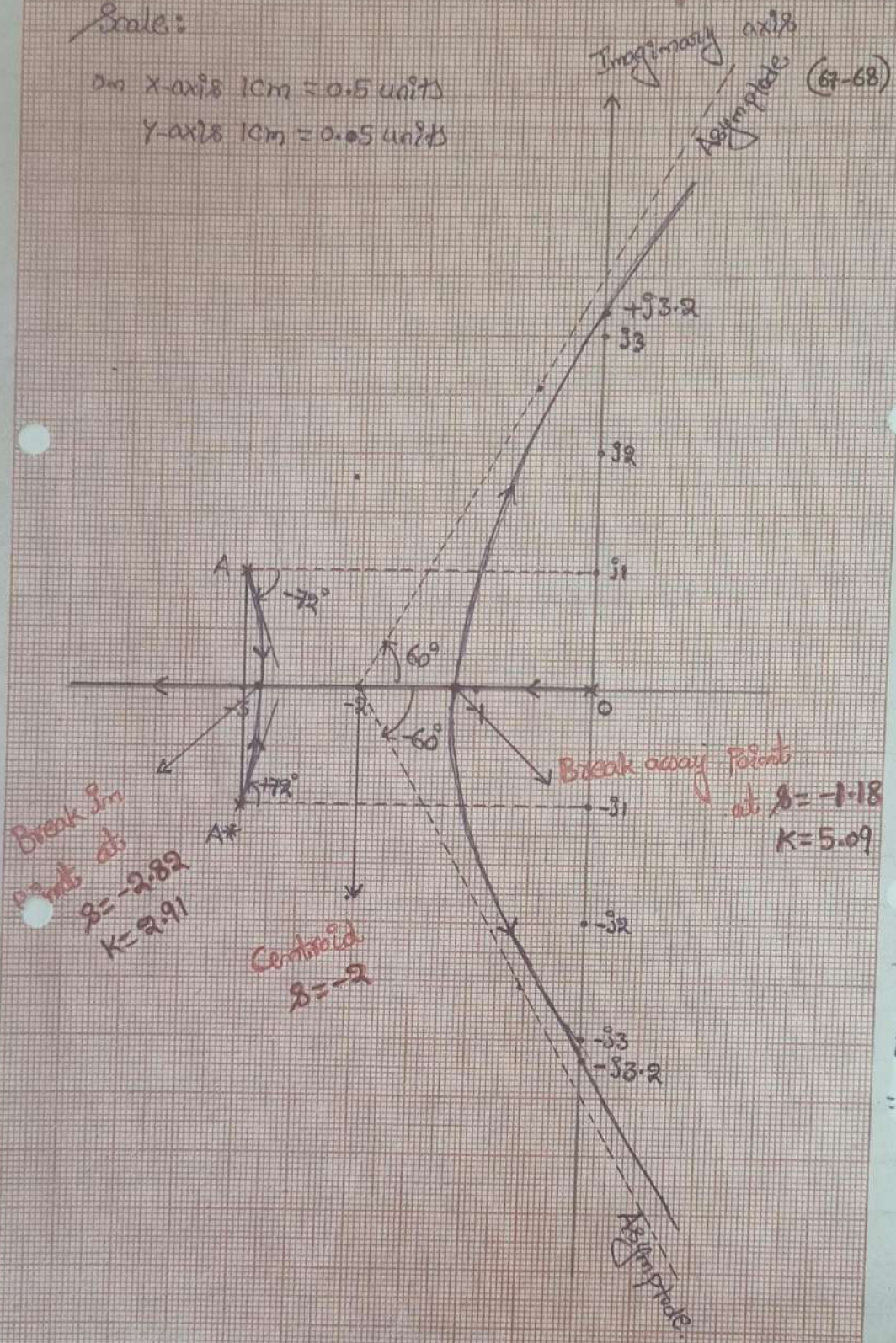
$\therefore$ the values of K for $s = -1.18$ & -2.82 are +ve & real, so, both the points are actual breakaway / break in points. It can be proved that $s = -2.82$ is a break in point & $s = -1.18$ is a breakaway point.

(3)

Scale:

on X-axis 1cm = 0.5 units

Y-axis 1cm = 0.05 units



$K = 0$
 $= 6\omega^2$

→ Angle of Departure.

(68)

Consider the Complex pole A. Draw vectors from all other poles to pole A. And the angle of these vectors be θ_1 & θ_2 .

Here $\theta_1 = 180 - \tan^{-1}(1/3)$

$$= 161.6^\circ$$

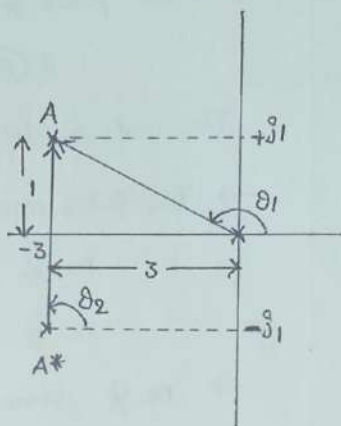
$$\theta_2 = 90^\circ$$

Angle of Departure from the Complex pole A

$$\phi_A = 180 - (\theta_1 + \theta_2)$$

$$= 180 - (161.6 + 90)$$

$$= -71.6^\circ \approx -72^\circ$$



The angle of Departure at Complex pole A^* is -ve of the angle of Departure at Complex pole A.

$$\therefore \text{Angle of Departure at pole } A^* = +72^\circ.$$

→ Crossing points on imaginary axis

The characteristic Eqn. is given by

$$\Rightarrow s(s^2 + 6s + 10) + K = 0$$

$$\Rightarrow s^3 + 6s^2 + 10s + K = 0$$

Put $s = j\omega$

$$\Rightarrow (j\omega)^3 + 6(j\omega)^2 + 10(j\omega) + K = 0$$

$$\Rightarrow -j\omega^3 - 6\omega^2 + j10\omega + K = 0$$

on Equate imaginary part to zero

$$-\omega^3 + 10\omega = 0$$

$$\omega^3 = 10\omega$$

$$\Rightarrow \omega^2 = 10$$

$$\omega = \pm\sqrt{10}$$

$$= \pm 3.16$$

$$\approx 3.2$$

Equate Real part to zero

$$-6\omega^2 + K = 0$$

$$K = 6\omega^2$$

$$= 6(10)$$

$$= 60.$$

$\therefore$ The root locus crosses imaginary axis at $\pm j3.2$ & the gain K corresponding to this point is 60. This is the limiting value of K for stable system.

(4) The open Loop Transfer function of a unity feedback system is given by $G(s) = K(s+9) / s(s^2+4s+11)$. Sketch the root locus of the system.

The poles of open Loop transfer function are the roots of the Eqn.
 $s(s^2+4s+11)=0$.

The roots of the quadratic are $s = \frac{-4 \pm \sqrt{16-44}}{2} = -2 \pm j2.64$.

→ The poles are $= 0, -2+j2.64, -2-j2.64$

no. of poles $= 3$

no. of zero's $= 1$ i.e., -9 .

→ no. of branches $= 3 =$ no. of poles

→ no. of Asymptotes $= P-Z$

$$= 3-1 = 2.$$

→ Angle of Asymptotes $= \frac{(2v+1)\pi}{P-Z}$ where $v=0,1,2$

$$= 90^\circ, 270^\circ$$

→ Centroid $= \frac{\sum P - \sum Z}{P-Z} = \frac{0-2+j2.64-2-j2.64+9}{2} = 5/2 = 2.5$

→ Break away & break in points

From the location of poles & zero's & from the root locus diagram, it can be concluded that there is no possibility of break away or break in points.

→ Angle of departure

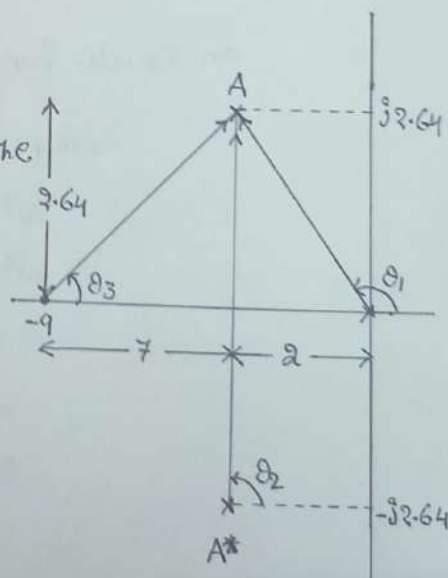
Let us consider the complex pole A.

Draw vectors from all other poles & zero to the pole A. Let the angles of these vectors be θ_1, θ_2 & θ_3 .

Here $\theta_3 = \tan^{-1}\left(\frac{2.64}{7}\right) = 20.7^\circ$

$$\theta_2 = 90^\circ$$

$$\theta_1 = 180 - \tan^{-1}\left(\frac{2.64}{2}\right) = 127.1^\circ$$



[4]

(57-58)

(68-69)

Scale:

(cm)

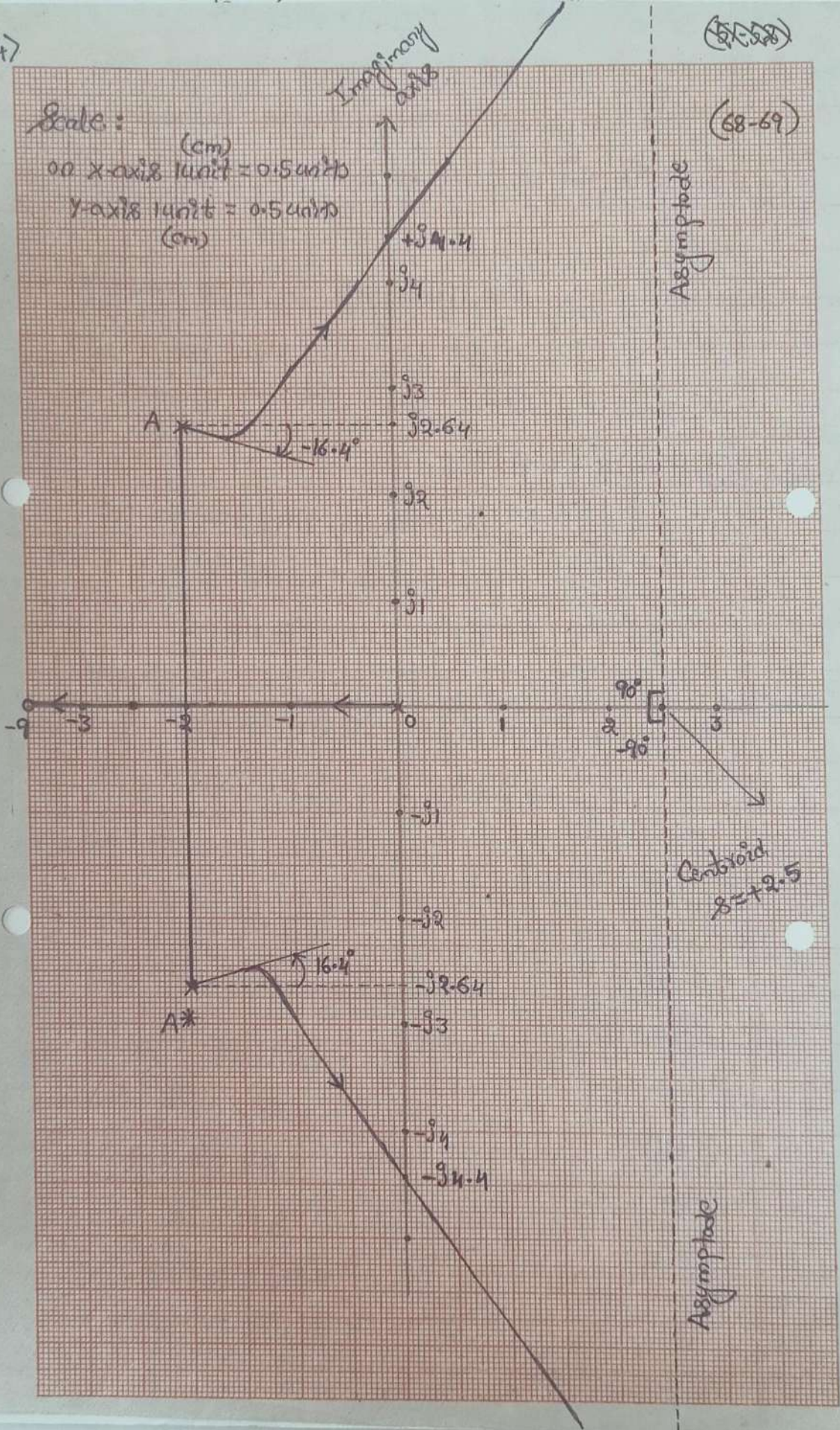
00 X-axis 1 unit = 0.5 units

Y-axis 1 unit = 0.5 units

(cm)

Imaginary axis

Asymptote



Angle of Departure from the complex pole A is

$$= 180 - (\theta_1 + \theta_2) + \theta_3$$

$$= 180 - (127.1 + 90) + 20.7 = -16.4^\circ$$

The angle of Departure at the complex pole A^* is negative of the angle of Departure at Complex pole A.

$$\therefore \text{Angle of Departure at Complex pole } A^* = -(-16.4) \\ = 16.4^\circ$$

→ Crossing points of imaginary axis

The closed Loop

$$\text{Transfer Function } \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

$$= K(s+9) / s(s^2+4s+11) + K(s+9)$$

The characteristic Equation is

$$s(s^2+4s+11) + K(s+9) = 0$$

$$(s^3+4s^2+11s) + Ks+9K=0$$

Put $s=j\omega$

$$(j\omega)^3 + 4(j\omega)^2 + 11(j\omega) + K(j\omega) + 9K = 0$$

$$\Rightarrow -j\omega^3 - 4\omega^2 + 11j\omega + jK\omega + 9K = 0$$

Equate imaginary part to zero

$$-j\omega^3 + 11j\omega + jK\omega = 0$$

$$j\omega^3 = j(11\omega + K\omega)$$

$$\omega^3 = \omega(11+K)$$

$$\Rightarrow \omega^2 = 11+K$$

$$\text{Put } K = 8.8$$

$$\omega^2 = 19.8$$

$$\Rightarrow \omega = \pm \sqrt{19.8}$$

$$= \pm 4.4$$

Equate Real part to zero

$$-4\omega^2 + 9K = 0$$

$$9K = 4\omega^2$$

$$\Rightarrow \omega^2 = \frac{9}{4}K$$

$$\text{Put } \omega^2 = 11+K$$

$$\therefore 4(11+K) = 9K$$

$$9K - 4K = 44$$

$$5K = 44$$

$$K = \frac{44}{5} = 8.8$$

The crossing point of root locus is $\pm j4.4$. The value of K at this crossing point is $K=8.8$. (This is the limiting value of K for the stability of the system).

(5) Sketch the root locus plot of a unity feedback system whose open loop T/F is $G(s) = s / (s^2 + 4)(s + 2)$.

The poles of open loop Transfer function are the roots of the Equation.

$$(s^2 + 4)(s + 2) = 0$$

Roots of the quadratic Eqn. are $s = \pm j2$

→ The poles are $= -2, +j2, -j2$

no. of poles $= 3$

The zero's are $= 0$

no. of zero's $= 1$

→ no. of branches $= 3 =$ no. of poles

→ no. of Asymptotes $= P - Z$

$$= 3 - 1 = 2.$$

→ Angle of Asymptotes $= \frac{(2q+1)\pi}{P-Z}$ where $q=0, 1$,

$$= 90^\circ, 270^\circ.$$

→ Centroid $= \frac{\sum P - \sum Z}{P-Z}$

$$= \frac{-2 + j2 - j2 - 0}{2} = -1.$$

→ Break away & Break in points

From the location of poles & zero & from the root locus diagram, it can be concluded that there is no possibility of breakaway & break in points.

(57)

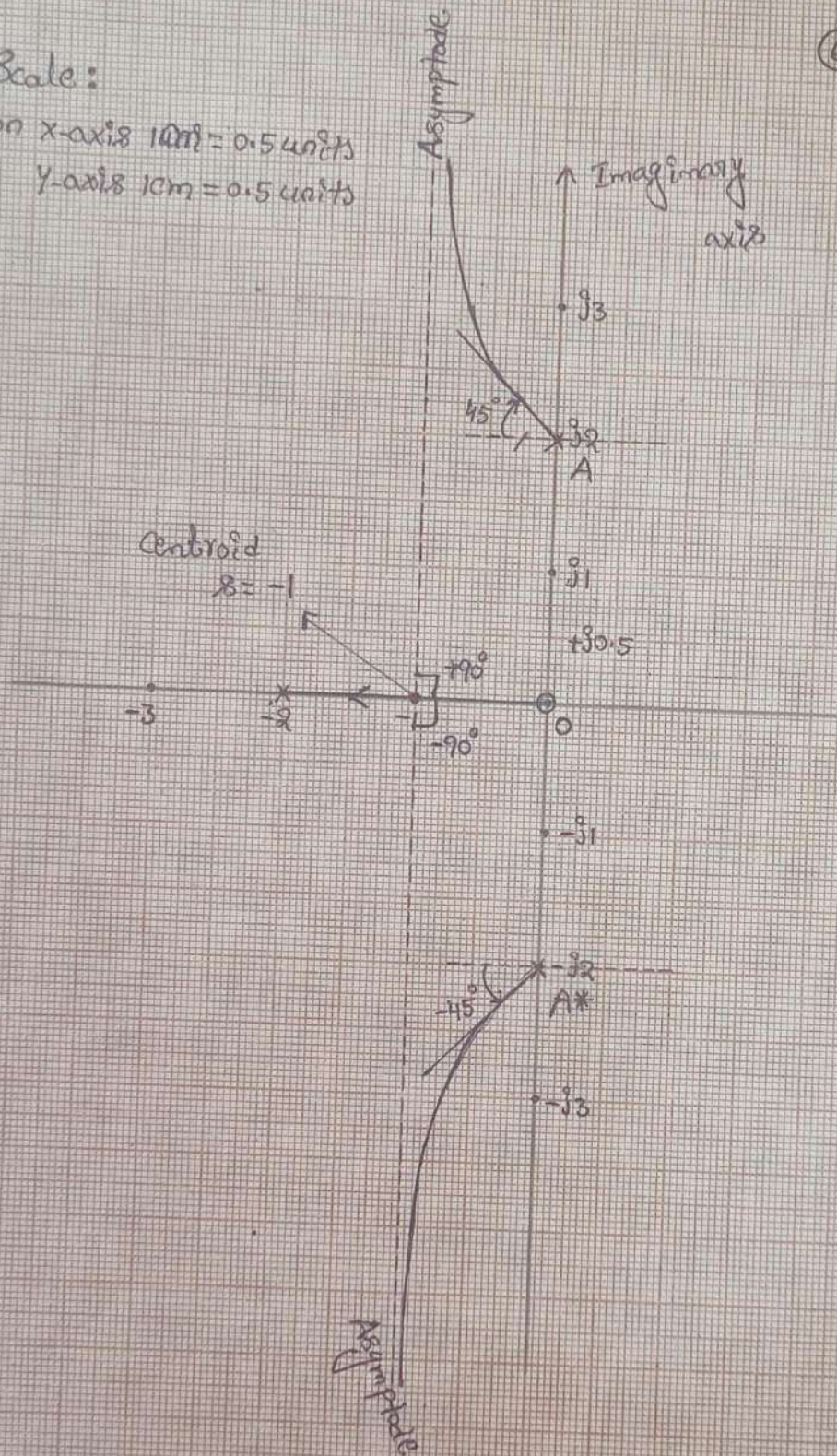
(52-53)

Scale:

on x-axis 1cm = 0.5 units

y-axis 1cm = 0.5 units

(69-70)

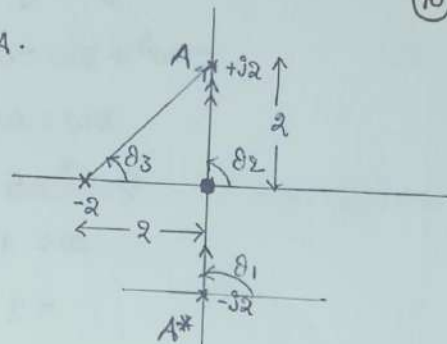


→ Angle of Departure

(70)

Let us consider the Complex pole A.

Draw vectors from all other poles & zero's to the pole A. Let the angles of these vectors to be θ_1, θ_2 & θ_3 .



$$\theta_1 = 90^\circ$$

$$\theta_2 = 90^\circ$$

$$\begin{aligned}\theta_3 &= 180 - \tan^{-1}(2/2) \\ &= 135^\circ\end{aligned}$$

Angle of Departure from the Complex pole A is

$$= 180 - (\theta_1 + \theta_3) + \theta_2$$

$$= 180 - (90 + 135) + 90$$

$$= 45^\circ$$

The angle of departure at the complex pole A^* is negative of the angle of departure at complex pole A.

$$\begin{aligned}\therefore \text{Angle of Departure at Complex pole } A^* &= -(45^\circ) \\ &= -45^\circ\end{aligned}$$

→ Crossing point on imaginary axis

The closed Loop

$$\begin{aligned}\text{Transfer function } \frac{C(s)}{R(s)} &= \frac{s}{(s^2+4)(s+2)+8} \\ &= \frac{s}{s^3+2s^2+4s+8+8} \\ &= \frac{s}{(s^3+2s^2+5s+8)}\end{aligned}$$

The characteristic Equation is

$$s^3+2s^2+5s+8=0$$

Put $s=j\omega$

$$(j\omega)^3+2(j\omega)^2+5(j\omega)+8=0$$

$$\Rightarrow -j\omega^3-2\omega^2+j5\omega+8=0$$

This is not necessary.

bcz already the poles are crossing on imaginary axis.

Evaluate Imaginary Part to zero.

$$-\omega^3 + 5\omega = 0$$

$$5\omega = \omega^3$$

$$\Rightarrow \omega^2 = 5$$

$$\omega = \pm \sqrt{5}$$

$$= \pm 2.24$$

→ Extra Problems:-

$$\langle 1 \rangle G(s) \cdot H(s) = K / s(s+3)(s+1)$$

$$\langle 2 \rangle G(s) \cdot H(s) = K / s(s+2)(s^2+2s+5)$$

$$\langle 3 \rangle G(s) \cdot H(s) = K / s(s+4)(s^2+4s+20)$$

$$\langle 4 \rangle G(s) \cdot H(s) = K / s(s+2)(s^2+6s+25)$$

$$\langle 5 \rangle G(s) \cdot H(s) = K / s(s+6)(s^2+4s+13)$$

$\langle 6 \rangle$ The output of a Control System is related to its input by

$$[s^4 + 5s^3 + 3s^2 + (3+K)s + K] C(s) = K(s+2) R(s)$$

where "K" Represents the Positive gain of the amplifier.

(a) with $K=8$, & a step input, will the o/p response be stable? (No)

(b) Determine the limiting Positive value that K-Can have for a stable output response? $[0 < K < 2]$

* $\langle 7 \rangle$ By using Routh Stability Criterion Determine the Location of roots on S-plane & hence the stability of the System represented by the characteristic Equation

$$s^5 + 4s^4 + 8s^3 + 8s^2 + 7s + 4 = 0$$

* $\langle 8 \rangle$ what is BAP? A unity Feed Back System has an open Loop transfer function $U(s) = K / s(s^2+6s+10)$. Find its Break away and Break in points?

— END —

(6) → Sketch the root Locus of the open Loop Transfer function given below.

$$G(s)H(s) = K / s(s+2)(s^2+2s+5).$$

(71)

Given,

The open Loop Transfer function $G(s)H(s) = K / s(s+2)(s^2+2s+5)$

→ The poles are $0, -2, -1+j2, -1-j2$

$$\text{no. of poles (P)} = 4$$

$$\text{no. of zeros (Z)} = 0$$

→ no. of Branches = no. of Poles = 4

$$\rightarrow \text{Centroid } (\sigma) = \frac{\sum P - \sum Z}{P - Z} = \frac{0 - 2 - 1 + j2 - 1 - j2}{4} = -1$$

$$\rightarrow \text{no. of Asymptotes} = P - Z = 4.$$

$$\rightarrow \text{Angle of Asymptotes} = \frac{(2q+1)\pi}{P-Z} \text{ where } q=0, 1, \dots, (P-Z-1)$$

Here $P-Z-1=3$ so, $q=0, 1, 2, 3$.

$$= \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}.$$

$$= 45^\circ, 135^\circ, 225^\circ, 315^\circ.$$

→ Break away / Break in points.

The characteristic Equation is given by

$$1 + G(s)H(s) = 0$$

$$\Rightarrow 1 + \frac{K}{s(s+2)(s^2+2s+5)} = 0$$

$$\Rightarrow s(s+2)(s^2+2s+5) + K = 0$$

$$\Rightarrow K = -(s^4 + 4s^3 + 9s^2 + 10s) \rightarrow \text{①}$$

& Evaluate to zero.

Diff. the above Equation w.r. to s we get

$$\frac{dK}{ds} = 0 \Rightarrow -(4s^3 + 12s^2 + 18s + 10) = 0$$

$$4s^3 + 12s^2 + 18s + 10 = 0$$

By solving the above Equation we get

$$s = -1, -1 \pm j1.225$$

When $s = -1$, The value of 'K' is given by

$$K = -[(-1)^4 + 4(-1)^3 + 9(-1)^2 + 10(-1)]$$

$$= -(1 - 4 + 9 - 10) = +4.$$

= +ve & real. So, $s = -1$ is the actual Break away point.

When $s = -1 + j1.225$, the value of 'K' is given by

$$K = -[(1 + j1.225)^4 + 4(-1 + j1.225)^3 + 9(-1 + j1.225)^2 + 10(-1 + j1.225)]$$

$\neq$ positive & ^{Complex} real value. So, $s = -1 + j1.225$ is not a BAP.

When $s = -1 - j1.225$, the value of 'K' is given by

$$K = -[(1 - j1.225)^4 + 4(-1 - j1.225)^3 + 9(-1 - j1.225)^2 + 10(-1 - j1.225)]$$

$\neq$ +ve & ^{Complex} real value. So, $s = -1 - j1.225$ is not a BAP.

→ Angle of Departure / angle of Arrival

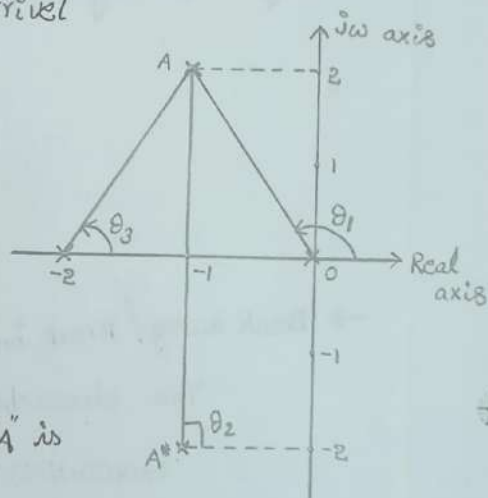
$$\theta_1 = 180 - \tan^{-1}\left(\frac{p}{1}\right)$$

$$= 116.5651$$

$$\theta_2 = 90^\circ$$

$$\theta_3 = \tan^{-1}\left(\frac{p}{1}\right)$$

$$= 63.4349$$



Angle of Departure at Complex pole 'A' is

$$\text{given as } = 180 - (\theta_1 + \theta_2 + \theta_3)$$

$$= -90^\circ$$

Angle of Departure at Complex Conjugate pole A^* is given by

= -ve sign of Angle of Departure at Complex pole A.

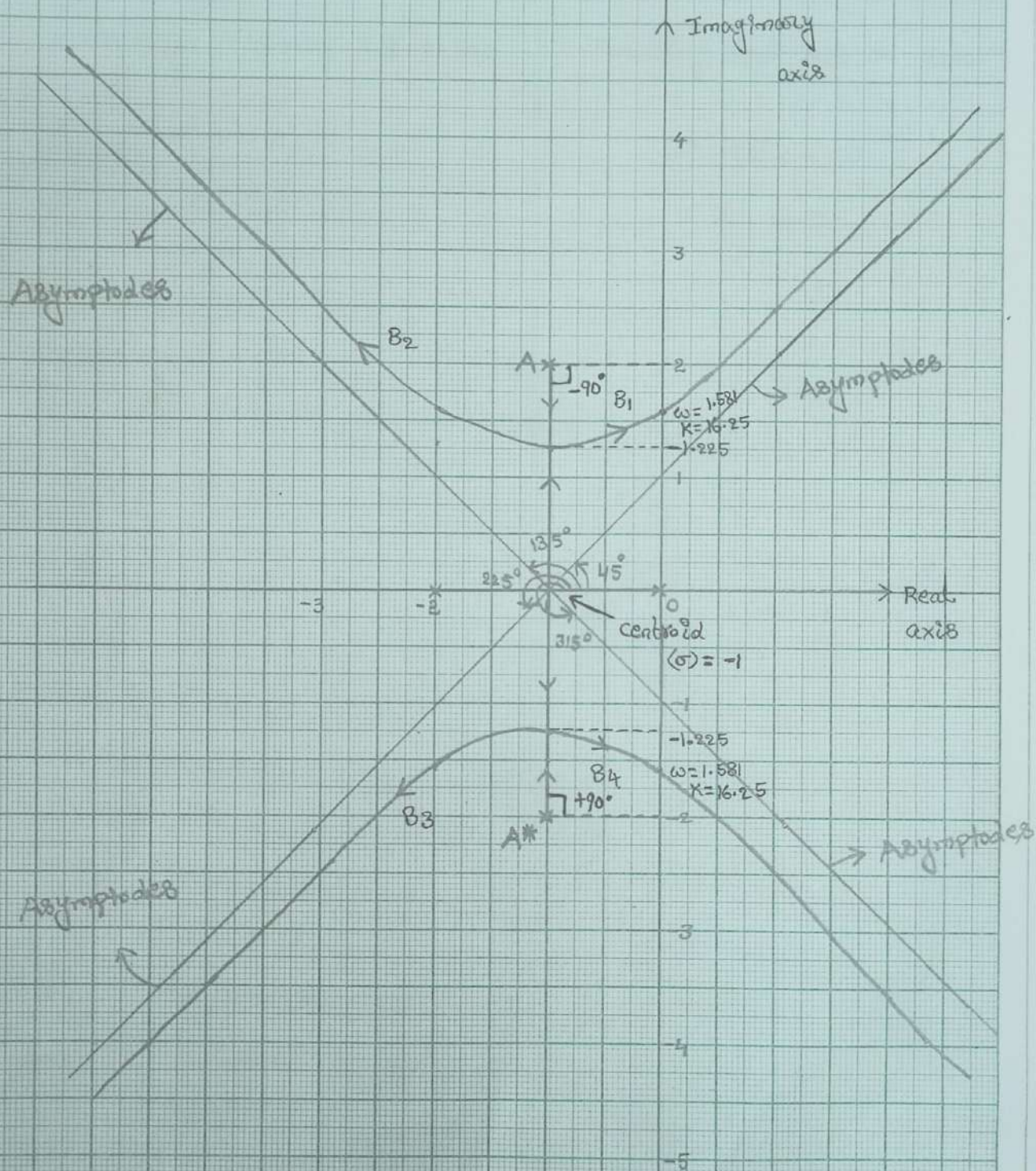
$$= +90^\circ.$$

→ Crossing points on imaginary axis

The characteristic Equation is given by

$$s^4 + 4s^3 + 9s^2 + 10s + K = 0$$

(71-72)



Put $s=j\omega$ we get

(72)

$$\Rightarrow (j\omega)^4 + 4(j\omega)^3 + 9(j\omega)^2 + 10(j\omega) + K = 0$$

$$\omega^4 + (-4j\omega^3) - 9\omega^2 + 10j\omega + K = 0$$

$$\Rightarrow \omega^4 - 4j\omega^3 - 9\omega^2 + 10j\omega + K = 0$$

Evaluating Imaginary part to zero

$$-4\omega^3 + 10\omega = 0$$

$$4\omega^2 = 10$$

$$\omega^2 = \frac{10}{4}$$

$$= 2.5$$

$$\Rightarrow \omega = \pm 1.581$$

Evaluating Real part to zero.

$$\omega^4 - 9\omega^2 + K = 0$$

$$K = 16.25$$

NO

→ Sketch the Complete root locus of the open loop Transfer function given below.

$$G(s) \cdot H(s) = K / s(s+2)(s^2+6s+25)$$

Given the open loop Transfer function is

$$G(s) \cdot H(s) = K / s(s+2)(s^2+6s+25)$$

→ The poles are $0, -2, -3+j4, -3-j4$

$$\text{no. of poles } (P) = 4$$

$$\text{no. of zeros } (Z) = 0$$

→ Centroid is given by

$$\sigma = \frac{\sum P - \sum Z}{P - Z} = \frac{0 - 2 - 3 + j4 - 3 - j4}{4} = \frac{-8}{4} = -2$$

$$\rightarrow \text{No. of Asymptotes} = \text{no. of poles} \\ = 4$$

$$\rightarrow \text{no. of Branches} = P - Z \\ = 4 - 0 = 4$$

$$\rightarrow \text{Angle of Asymptotes} = \frac{(2q+1)\pi}{P-Z} \quad \text{where } q=0, 1, 2, \dots, (P-Z-1)$$

Here $q=0, 1, 2, 3$.

$$= \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

$$= 45^\circ, 135^\circ, 225^\circ, 315^\circ$$

→ Break away / Break in points

The characteristic Equation is

$$1 + G(s) \cdot H(s) = 0$$

$$1 + \frac{K}{s(s+2)(s^2+6s+25)} = 0$$

$$\Rightarrow s(s+2)(s^2+6s+25) + K = 0$$

$$\Rightarrow s^4 + 8s^3 + 37s^2 + 50s + K = 0$$

$$\Rightarrow K = -(s^4 + 8s^3 + 37s^2 + 50s) \rightarrow (1)$$

Diff. the above Equation w.r. to 's' & Equate to zero we get

$$\frac{dk}{ds} = 0 \Rightarrow -(4s^3 + 24s^2 + 74s + 50) = 0$$

$$\Rightarrow 4s^3 + 24s^2 + 74s + 50 = 0$$

$$s = -0.898, -2.55 + 2.722i, -2.55 - 2.722i$$

when $s = -0.898$, then the value of 'K' is given by

$$K = -[(-0.898)^4 + 8(-0.898)^3 + 37(-0.898)^2 + 50(-0.898)]$$

= +ve & ^{Real}~~Complex~~ value. So, $s = -0.898$ is the actual BAP.

when $K = -2.55 \pm 2.722i$ then the value of K is given by

$$K = -[(-2.55 \pm 2.722i)^4 + 8(-2.55 \pm 2.722i)^3 + 37(-2.55 \pm 2.722i)^2 + 50(-2.55 \pm 2.722i)]$$

= +ve & Complex value. So, $s = -2.55 \pm 2.722i$ is the actual BAP.

→ Angle of Departure / Angle of Arrival.

$$\theta_1 = 180 - \tan^{-1}\left(\frac{4}{3}\right)$$

$$= 126.87^\circ$$

$$\theta_2 = 180 - \tan^{-1}\left(\frac{4}{1}\right)$$

$$= 104.04^\circ$$

$$\theta_3 = 90^\circ$$

Angle of Departure at Complex pole 'A'

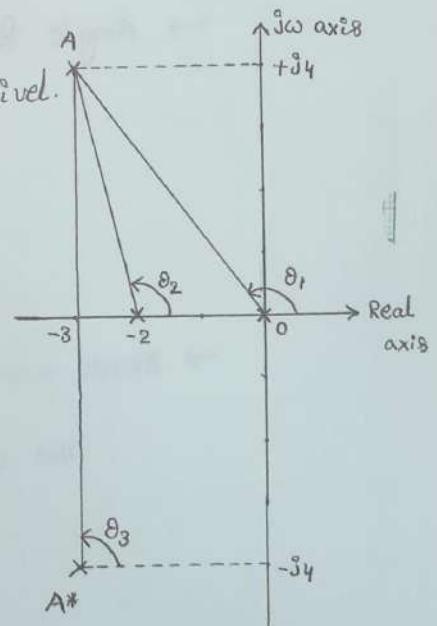
$$\angle = 180 - (\theta_1 + \theta_2 + \theta_3)$$

$$= 180 - (126.87 + 104.04 + 90)$$

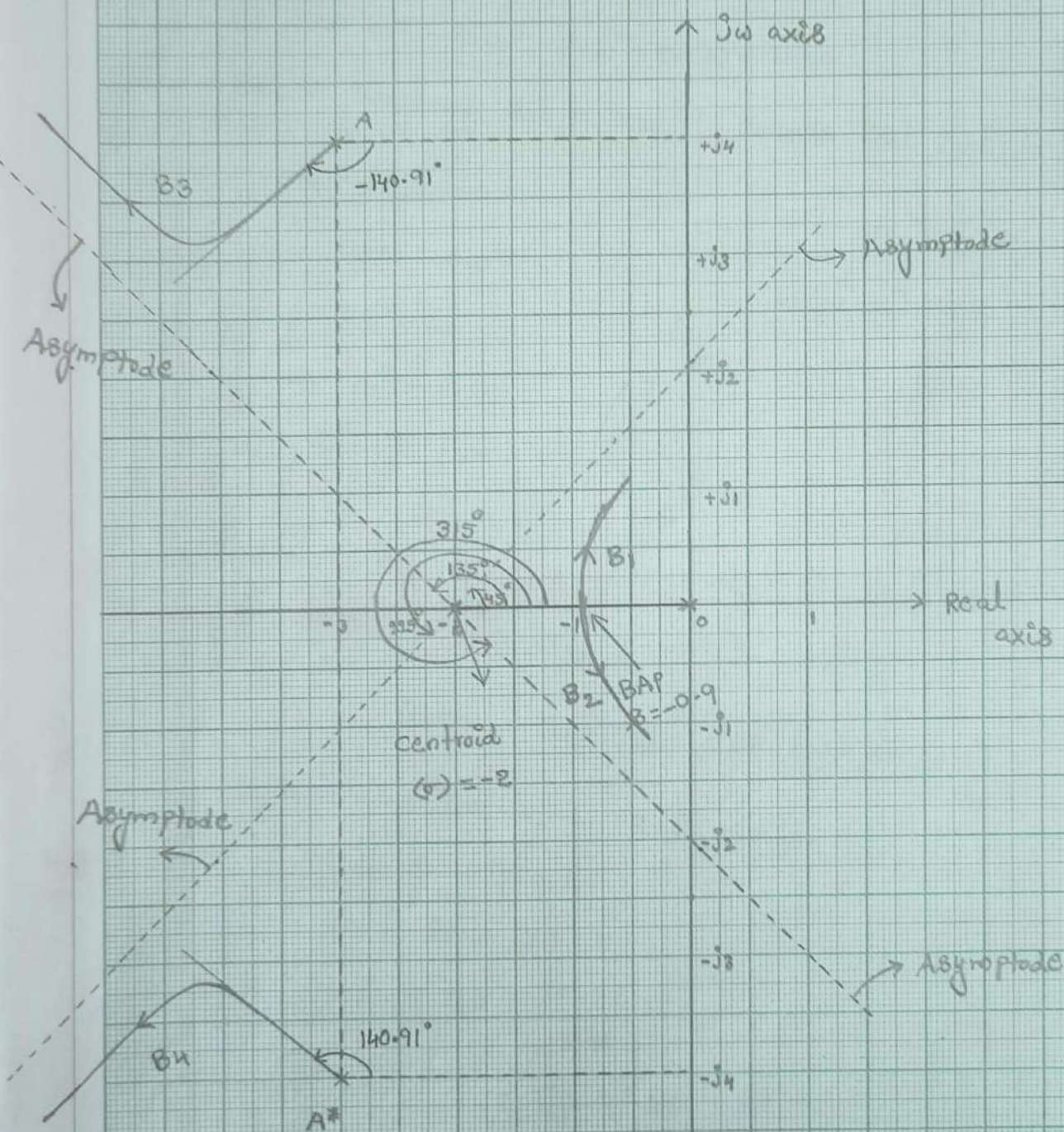
$$= -140.91^\circ$$

Angle of Departure at Complex Conjugate pole A^* is

$$= -ve \text{ of angle of Departure at } A = +140.91^\circ$$



(7374)



→ Crossing points on imaginary axis

(74)

The characteristic Equation is

$$s^4 + 8s^3 + 37s^2 + 50s + K = 0$$

Put $s = j\omega$

$$\Rightarrow (j\omega)^4 + 8(j\omega)^3 + 37(j\omega)^2 + 50(j\omega) + K = 0$$

$$\Rightarrow \omega^4 - 8j\omega^3 - 37\omega^2 + 50j\omega + K = 0$$

Evaluate Imaginary Part to zero

$$-8\omega^3 + 50\omega = 0$$

$$50\omega = 8\omega^3$$

$$8\omega^2 = 50$$

$$\Rightarrow \omega^2 = 6.25$$

$$\Rightarrow \omega = \pm 2.5$$

Evaluate Real Part to zero

$$\omega^4 - 37\omega^2 + K = 0$$

$$\omega^2(\omega^2 - 37) + K = 0$$

$$\Rightarrow K = 192.19$$

(8) → Construct the Complete root Locus for the system with
 $G(s)H(s) = K/s(s+6)(s^2+4s+13)$.

Given $G(s)H(s) = K/s(s+6)(s^2+4s+13)$.

→ The poles are 0, -6, -2+j3, -2-j3

no. of poles (p) = 4

no. of zeros (z) = 0.

→ Centroid (σ) = $\frac{\sum p - \sum z}{p - z}$

$$= \frac{-10}{4} = -2.5$$

→ no. of Asymptotes = $p - z = 4 - 0 = 4$.

A unity feedback Control System is characterised by the open-loop transfer function

$$G(s) = \frac{K(s+3)}{s(s+3)(s+7)}$$

using the Routh criterion.

(a) calculate the range of values of "K" for the system to be stable.

(b) what is the marginal value of 'K' for stability? Determine the frequency of oscillations if any.

(c) check if for $K=1$, all the roots of the characteristic equation of the above system have the damping factor greater than 0.5.

(a) The characteristic equation of the system is

$$1 + G(s) \cdot H(s) = 0$$

$$1 + \frac{K(s+3)}{s(s+3)(s+7)} = 0$$

$$s(s+3)(s+7) + K(s+3) = 0$$

$$s^3 + 10s^2 + (21+K)s + 13K = 0$$

The Routh table is formulated as follows:

s^3	1	$21+K$
s^2	10	$13K$
s^1	$\frac{210-3K}{10}$	
s^0	$13K$	

For the system to be stable, all the elements in the first column of the Routh array must be +ve.

$$\text{i.e. } 13K > 0 \quad \& \quad \frac{210-3K}{10} > 0$$

$$K > 0.$$

$$210-3K > 0$$

$$3K < 210$$

$$K < 70.$$

Hence, the range of values of 'K' for stability of the system is $0 < K < 70$.

(b). The marginal value of 'K' for stability is $K = 70$.

The frequency of Sustained oscillations is given by the solution of the auxiliary equation formed by using the coefficients of the row just above the row from which the marginal value of 'K' is determined.

$$10s^2 + 13K = 0$$

$$10s^2 + 13(70) = 0 \Rightarrow s^2 = -91$$

$$\Rightarrow s = \pm j9.53$$

$\therefore$ The frequency of Sustained oscillations is

$$\omega = 9.53 \text{ rad/sec.}$$

(c). Putting $K=1$ in the characteristic equation

$$\text{we get, } \Rightarrow s^3 + 10s^2 + 22s + 13 = 0$$

$$\Rightarrow (s+1)(s^2 + 9s + 13) = 0$$

$$\Rightarrow (s+1)[s + (4.5 + j5.38)] * [s + (4.5 - j5.38)] = 0$$

$\therefore$ The roots of the characteristic equation are

$$s = -1, s = -4.5 - j5.38 \quad \& \quad s = -4.5 + j5.38.$$

Now for root at $s = -1$, $\theta = 0^\circ$, and so, the damping factor $\delta = \cos \theta = \cos 0^\circ = 1$. for root at $s = -4.5 \pm j5.38$,

$$\theta = \tan^{-1} \left(\frac{5.38}{4.5} \right)$$

$$= 50.08^\circ$$

$\therefore$ The damping factor $\delta = \cos \theta = \cos(50.08) = 0.64$.

Thus the damping factors for the roots are 1 & 0.64, both greater than 0.5. That means, all the roots of the characteristic equation of the above system have damping factor greater than 0.5.

The damping factor associated with $s^2 + 9s + 13$ can be determined by comparing it with the standard equation $s^2 + 2\delta\omega_n s + \omega_n^2 = 0$. Therefore,

$$\omega_n^2 = 13$$

$$\Rightarrow \omega_n = \sqrt{13} = 3.6$$

$$2\delta\omega_n = 9$$

$$\Rightarrow \delta = 9/2(3.6)$$

$$= 1.25.$$

The output of Control System is related to its input by

$$[s^4 + 5s^3 + 3s^2 + (3+K)s + K] C(s) = K(s+2) R(s).$$

where K - represents the positive gain of the Amplifier.

(a) with $K=8$, and a step input, will the output response be stable?

(b) Determine the limiting positive value that K - can have for a stable output response.

(a) The closed loop Transfer function is

$$\frac{C(s)}{R(s)} = \frac{K(s+2)}{s^4 + 5s^3 + 3s^2 + (3+K)s + K}$$

The characteristic Equation is

$$s^4 + 5s^3 + 3s^2 + (3+K)s + K = 0$$

$\therefore$ with $K=8$, the characteristic Equation is

$$s^4 + 5s^3 + 3s^2 + 11s + 8 = 0$$

The Routh array is formulated as follows:

s^4	1	3	8
s^3	5	11	
s^2	0.8	8	
s^1	-39	0	
s^0	8		

There are two sign changes in the elements of the first column of Routh array, which indicates that there are two roots of the characteristic Equation for

$K=8$ in the right half of the s-plane. So, the output response of the system will be un-stable.

(b) The characteristic Equation for the given system is

$$s^4 + 5s^3 + 3s^2 + (3+K)s + K = 0$$

To determine the limiting positive value that K can have for a stable output response, the Routh table for the given characteristic Equation is formulated as follows:

s^4	1	3	K
s^3	5	$3+K$	
s^2	$\frac{12-K}{5}$	K	
s^1	$\frac{36-16K-K^2}{12-K}$		
s^0	K		

For stable output response, all the elements in the first column of the Routh table must be positive. Therefore, for stability

$$K > 0.$$

$$\frac{36-16K-K^2}{12-K} > 0$$

$$K < 12.$$

$$\& \quad \frac{12-K}{5} > 0$$

$$K < 12.$$

Therefore, the range of values of K for stability is $0 < K < 12$. The limiting positive value that K can have for a stable output response is $K=12$.

$K=8$ in the right half of the s-plane. So, the output response of the system will be un-stable.

(b) The characteristic Equation for the given system is

$$s^4 + 5s^3 + 3s^2 + (3+K)s + K = 0$$

To determine the limiting positive value that K can have for a stable output response, the Routh table for the given characteristic Equation is formulated as follows:

s^4	1	3	K
s^3	5	$3+K$	
s^2	$\frac{12-K}{5}$	K	
s^1	$\frac{36-16K-K^2}{12-K}$		
s^0	K		

For stable output response, all the elements in the first column of the Routh table must be positive. Therefore, for stability

$$K > 0.$$

$$\frac{36-16K-K^2}{12-K} > 0 \quad \& \quad \frac{12-K}{5} > 0$$

$$K < 2.$$

$$K < 12.$$

Therefore, the range of values of K for stability is $0 < K < 2$. The limiting positive value that K can have for a stable output response is $K=2$.