

18/9

Frequency Response Analysis

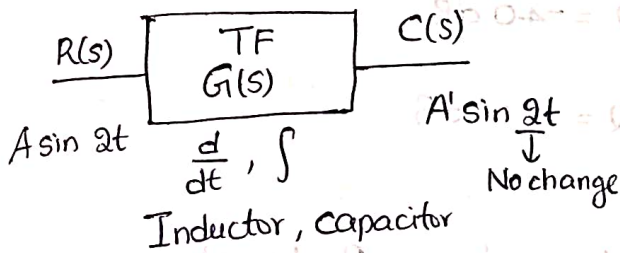
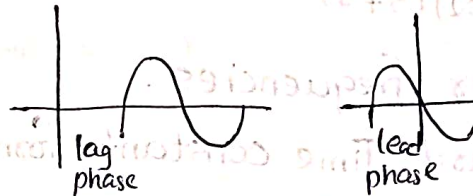
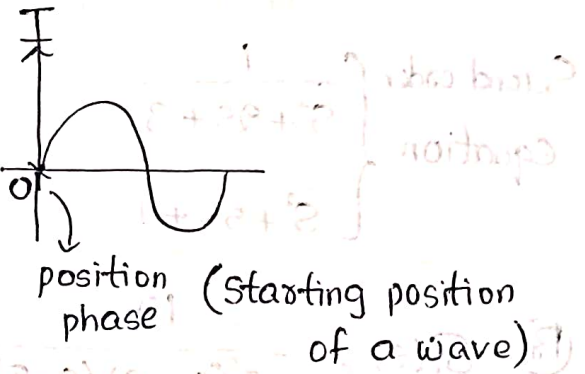
Time domain

Inputs $\rightarrow$ Step, ramp, parabola

frequency domain



Input - sinusoidal

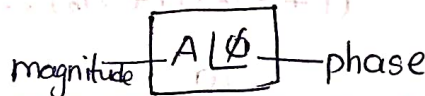
 $L \rightarrow I \rightarrow \text{lag}$ $C \rightarrow I \rightarrow \text{lead}$ 

Magnitude

$$A \sin(\omega t \pm \phi)$$

Constant $\rightarrow (+)C$
 $\rightarrow (-)I$

AC ckt analysis

Bode plot

A ϕ
magnitude phase

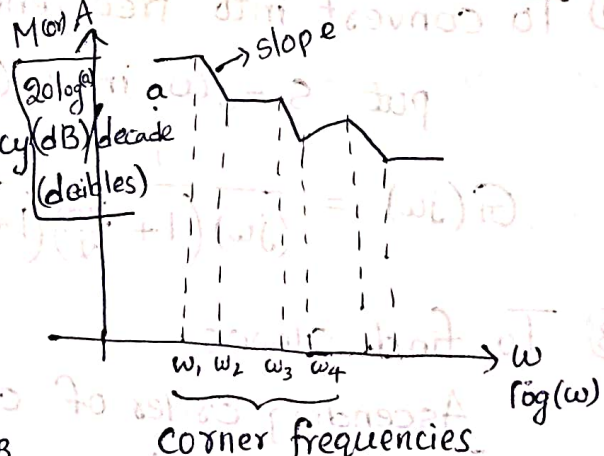
1) magnitude plot $\rightarrow$ Magnitude vs frequency ($\log(\omega)$)2) Phase plot $\rightarrow$ Phase vs frequency ($\log(\omega)$)

$\rightarrow$ The frequency at which the Slope changes $\Rightarrow$ corner frequency

 $N \rightarrow +$; $D \rightarrow -$

factor (n) term Slope

poles or zeros at origin	$\frac{1}{s}$	$-20 \times \text{order}$
		$-20 \times 1 = -20 \text{ dB}$
	$\frac{1}{s^2}$	$-20 \times 2 = -40 \text{ dB}$
	$\frac{1}{s^3}$	$-20 \times 3 = -60 \text{ dB}$
	$\frac{1}{s^2}$	$+20 \times 2 = 40 \text{ dB}$



	factor / term	Slope
first order equation	$\frac{1}{s+1}$	$-20 \times 1 = -20 \text{ dB}$
	$\frac{1}{(s+1)^2}$	$-20 \times 2 = -40 \text{ dB}$
	$s+1$	$+20 \times 1 = 20 \text{ dB}$
	$(s+1)^2$	$+20 \times 2 = 40 \text{ dB}$

Second order equation	$\frac{1}{s^2 + 2s + 3}$	$-20 \times 2 = -40 \text{ dB}$
	$\frac{1}{s^2 + 5s + 11}$	$+20 \times 2 = +40 \text{ dB}$

Q8) $G(s) = \frac{10}{s(s+2)(s+5)}$ sketch or draw Bode plot.

A: ① To find corner frequencies:

→ Write standard Time constant form of T.F ($G(s)$)

$$G(s) = \frac{(1 + sT_1)(1 + sT_2) \dots}{(1 + sT_a)(1 + sT_b) \dots}$$

$$G(s) = \frac{10}{s \cdot 2\left(\frac{s}{2} + 1\right) \cdot 5\left(\frac{s}{5} + 1\right)} = \frac{1}{s\left(1 + \frac{s}{2}\right)\left(1 + \frac{s}{5}\right)}$$

$$T_1 = \frac{1}{2} \Rightarrow \omega_{c1} = \frac{1}{T_1} = \frac{1}{\left(\frac{1}{2}\right)} = 2$$

$$T_2 = \frac{1}{5} \Rightarrow \omega_{c2} = \frac{1}{T_2} = \frac{1}{\left(\frac{1}{5}\right)} = 5$$

② To convert into frequency domain:

put $s = j\omega$ in $G(s)$

$$G(j\omega) = \frac{1}{(j\omega)\left(1 + \frac{j\omega}{2}\right)\left(1 + \frac{j\omega}{5}\right)}$$

③ To find slopes:

Ascending order of corner frequencies,

→ Write in factor form:-

$$\rightarrow G(j\omega) = \left(\frac{1}{j\omega}\right)\left(\frac{1}{1 + \frac{j\omega}{2}}\right)\left(\frac{1}{1 + \frac{j\omega}{5}}\right)$$

factor	corner frequency	Slope	Change in slope (or) Resultant slope (or) final slope
$\frac{1}{j\omega}$	—	$-20 \times 1 = -20 \text{ dB}$	—
$\frac{1}{1 + \frac{j\omega}{2}}$	$\omega_{c1} = 2$	$-20 \times 1 = -20 \text{ dB}$	$-20 \text{ dB} - 20 \text{ dB} = -40 \text{ dB}$
$\frac{1}{1 + \frac{j\omega}{5}}$	$\omega_{c2} = 5$	$-20 \times 1 = -20 \text{ dB}$	$-20 \text{ dB} - 40 \text{ dB} = -60 \text{ dB}$

④ Choose lower (ω_l) and higher (ω_h) frequencies:

$$\therefore \omega_l < \omega_{c1} ; \omega_l = 0.1 \text{ (or) } 0.5$$

$$\omega_h > \omega_{c2} ; \omega_h = 100 \text{ (or) } 50$$

$$\left. \begin{array}{l} \omega_l < \omega_{c1} \\ \omega_{c1} = 2 \\ \omega_{c2} = 5 \\ \omega_h > \omega_{c2} \end{array} \right\} \begin{array}{l} \text{same} \\ \text{procedure} \\ \text{same} \\ \text{procedure} \end{array}$$

⑤ To find magnitude (or) gain at ω_{c1} and ω_l :

$$A = 20 \log \left(\frac{1}{j\omega} \right)$$

$$A = 20 \log \left(\frac{1}{\omega} \right)$$

$$\text{At } \omega_l = 0.1, A = 20 \log \left(\frac{1}{0.1} \right) = 20 \text{ dB}$$

$$\omega_{c1} = 2, A = 20 \log \left(\frac{1}{2} \right) = -6.02 \approx -6 \text{ dB}$$

⑥ To find magnitude (or) gain at ω_{c2} and ω_h :

$$\text{At } \omega_{c2} = 5: A = \underbrace{\text{Slope from } \omega_{c1} \text{ to } \omega_{c2}}_{\text{Slope change at } \omega_{c1}} \times \log \left(\frac{\omega_{c2}}{\omega_{c1}} \right) + A \text{ at } \omega_{c1}$$

$$= -40 \times \log \left(\frac{5}{2} \right) + (-6 \text{ dB}) = -22.91 \approx -22 \text{ dB}$$

$$\text{At } \omega_h = 100:$$

$$A = \underbrace{\text{Slope from } \omega_{c2} \text{ to } \omega_h}_{\text{Slope change at } \omega_{c2}} \times \log \left(\frac{\omega_h}{\omega_{c2}} \right) + A \text{ at } \omega_{c2}$$

$$= -60 \times \log \left(\frac{100}{5} \right) + (-22 \text{ dB}) = -100.06 \approx -100 \text{ dB}$$

⑦ To Draw Phase plot (ϕ):

$$\frac{1}{s} \rightarrow -90^\circ$$

$$s \rightarrow +90^\circ$$

$$\frac{1}{1 + \frac{j\omega}{2}} \rightarrow \theta = -\tan^{-1} \left(\frac{\omega/2}{1} \right)$$

$$a + jb;$$

$$\text{magnitude} = \sqrt{a^2 + b^2}$$

$$\theta = \tan^{-1} \left(\frac{b}{a} \right)$$

$$\phi = -90 - \tan^{-1}\left(\frac{\omega/2}{1}\right) - \tan^{-1}\left(\frac{\omega/5}{1}\right)$$

$$\phi = -90 - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{5}\right)$$

	ω	ϕ	magnitude plot
$\omega_L = 0.1$	0.1	-9.4	$\omega_L = 0.1 \rightarrow A = 20 \text{ dB}$
$\omega_{c1} = 2$	2	-15.6	$\omega_{c1} = 2 \rightarrow A = -6 \text{ dB}$
$\omega_{c2} = 5$	5	-20.3	$\omega_{c2} = 5 \rightarrow A = -20 \text{ dB}$
$\omega_h = 100$	100	-26.5	$\omega_h = 100 \rightarrow A = -100 \text{ dB}$

(P8) $G(s) = \frac{5}{s(1+2s)(1+4s)}$

A: ① To find corner frequencies,

→ Write standard time constant form, $(1+ST)$

$$G(s) = \frac{5}{s(1+2s)(1+4s)}$$

$$T_1 = 2, \omega_{c1} = \frac{1}{T_1} = \frac{1}{2} = 0.5$$

$$T_2 = 4, \omega_{c2} = \frac{1}{T_2} = \frac{1}{4} = 0.25$$

② To convert into frequency domain

→ put $s = j\omega$

$$G(j\omega) = \frac{5}{(j\omega)(1+j2\omega)(1+j4\omega)}$$

③ To find slopes

→ write in factor form

→ corner frequencies in ascending order

$$G(j\omega) = \left(\frac{5}{j\omega}\right) \left(\frac{1}{1+j4\omega}\right) \left(\frac{1}{1+j2\omega}\right)$$

factor	corner frequency	slope	change in slope
$\frac{5}{j\omega}$	—	$-20 \times 1 = -20 \text{ dB}$	—
$\frac{1}{1+j4\omega}$	0.25	$-20 \times 1 = -20 \text{ dB}$	$-20 - 20 = -40 \text{ dB}$
$\frac{1}{1+j2\omega}$	0.5	$-20 \times 1 = -20 \text{ dB}$	$-20 - 40 = -60 \text{ dB}$

④ To choose lower and higher frequencies (ω_e & ω_h),

$$\omega_e < \omega_{c1}, \quad \omega_e = 0.1$$

$$\omega_h > \omega_{c2}, \quad \omega_h = 100$$

⑤ To find magnitude (or) gain at ω_e and ω_{c1}

$$A = 20 \log\left(\frac{5}{j\omega}\right) = 20 \log\left(\frac{5}{\omega}\right)$$

$$\text{At } \omega_e = 0.1, A = 20 \log\left(\frac{5}{0.1}\right) = 33.9 \approx 34 \text{ dB}$$

$$\text{At } \omega_{c1} = 0.25, A = 20 \log\left(\frac{5}{0.25}\right) = 26.02 \approx 26 \text{ dB}$$

⑥ To find gain (or) magnitude at ω_{c2} and ω_h

$$\text{At } \omega_{c2} = 0.5, A = \underbrace{\text{slope from } \omega_{c1} \text{ to } \omega_{c2}}_{\text{Slope change at } \omega_{c1}} \times \log\left(\frac{\omega_{c2}}{\omega_{c1}}\right) + A \text{ at } \omega_{c1}$$

$$= -40 \times \log\left(\frac{0.5}{0.25}\right) + 26 \text{ dB} = 14 \text{ dB}$$

$$\text{At } \omega_h = 100, A = \underbrace{\text{slope from } \omega_{c2} \text{ to } \omega_h}_{\text{Slope change at } \omega_{c2}} \times \log\left(\frac{\omega_h}{\omega_{c2}}\right) + A \text{ at } \omega_{c2}$$

$$= -60 \times \log\left(\frac{100}{0.5}\right) + 14 \text{ dB} = -124 \text{ dB}$$

⑦ To find phase for phase plot

$$\phi = -90 - \tan^{-1}\left(\frac{2\omega}{1}\right) = \tan^{-1}\left(\frac{4\omega}{1}\right)$$

ω	ϕ	
→ 0.1	-123.6	$\omega_e = 0.1 \quad A = 34$
	-161	$\omega_{c1} = 0.25 \quad A = 26$
→ 0.25	-171	$\omega_{c2} = 0.5 \quad A = 14$
10.3	-198.4	$\omega_h = 100 \quad A = -124$
→ 0.5	-261.4	$\omega_{fe} = 10 \quad A = -64$
5	-265.4	
10	-269.4	
20		
→ 100		

$$(P_2) \quad G(s)H(s) = \frac{20(s+4)}{s(s+2)(s+10)}$$

$$(P_3) \quad G(s) = \frac{1}{s(s+1)(s^2+10s+100)}$$

$$\omega_n^2 = 100$$

$$\omega_n = 10$$

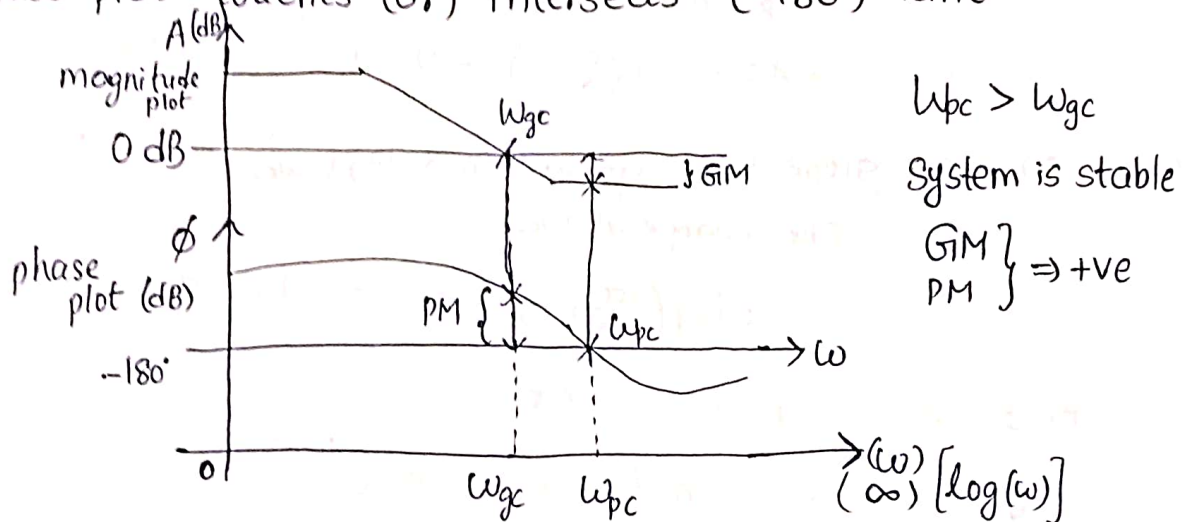
$$\omega_c = 10 \text{ rad/sec}$$

Bode plot

(i) Stability:

Gain cross over frequency (ω_{gc}): At which frequency the magnitude line touches (or) intersects 0dB line.

Phase cross over frequency (ω_{pc}): At which frequency the phase plot touches (or) intersects (-180°) line



$\therefore \textcircled{1}$ if $\omega_{pc} > \omega_{gc} \Rightarrow$ System is stable

$\Rightarrow$ Gain Margin (GM): The margin (or) dead line end (or) verge of the gain. After this system becomes unstable.

$\Rightarrow$ Acceptable gain (or) Allowable gain Value for stability,

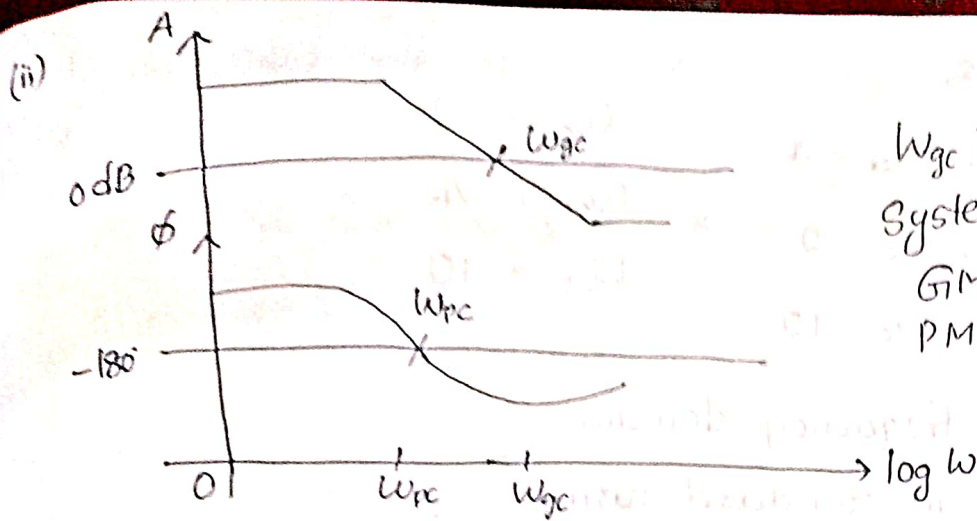
$$\therefore GM = 0 \text{ dB} - |G(j\omega)H(j\omega)| \Big|_{\omega=\omega_{gc}}$$

$\Rightarrow$ Phase Margin (PM):

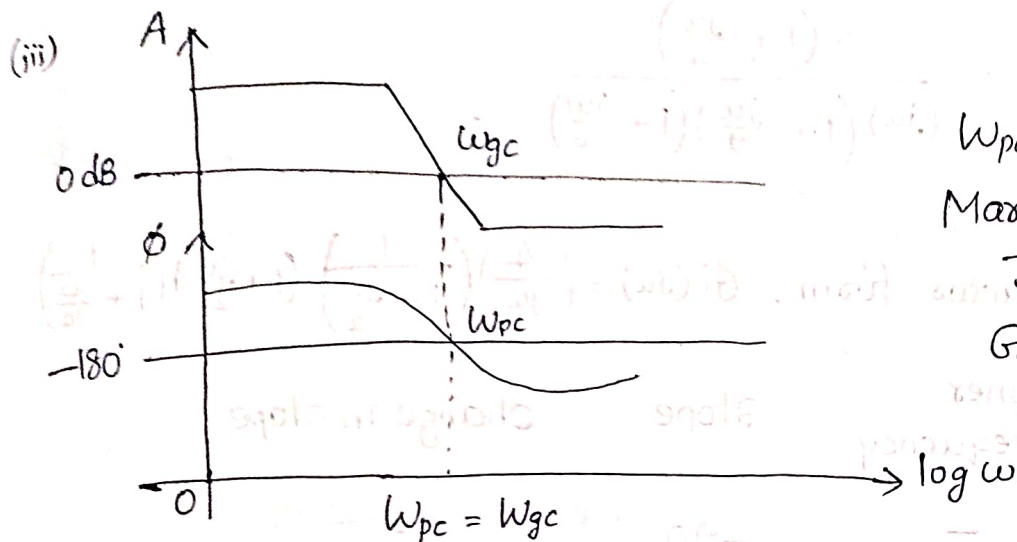
$$\therefore PM = 180^\circ + \angle G(j\omega)H(j\omega) \Big|_{\omega=\omega_{gc}}$$

$\Rightarrow$ Acceptable phase angle (or) Allowable phase value for stability

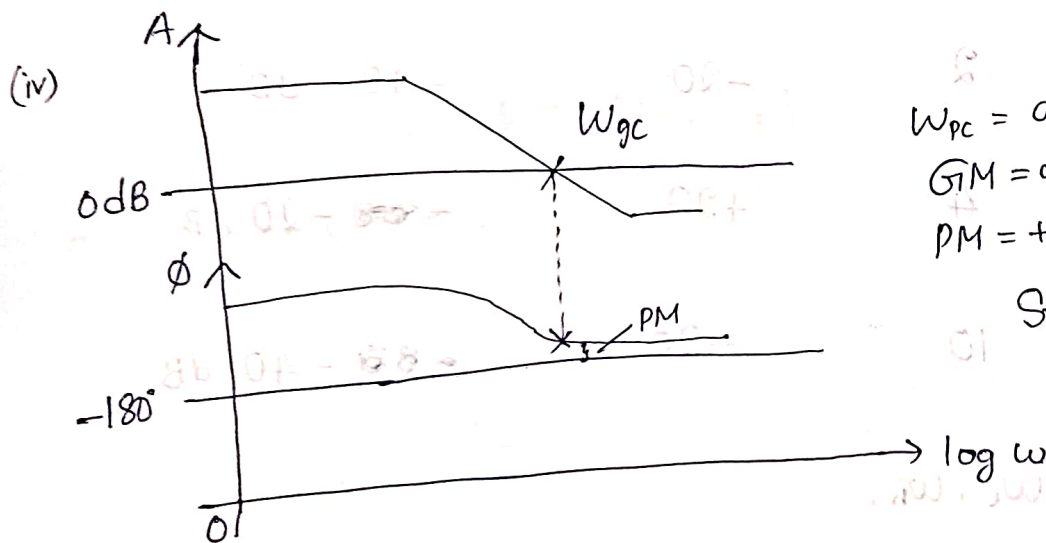
$\Rightarrow$ The margin (or) dead end (or) verge of the ~~gain~~ ^{phase} After this system becomes unstable $\Rightarrow$ Phase Margin.



$W_{gc} > W_{pc}$
System is unstable
 $\left. \begin{matrix} GM \\ PM \end{matrix} \right\} \Rightarrow -ve$



$W_{pc} = W_{gc}$
Marginal stable (or)
Just stable
 $GM = 0$; $PM = 0$



$W_{pc} = \infty$
 $GM = \infty$
 $PM = +ve$

Stable system

(P0) $G(s)H(s) = \frac{20(s+4)}{s(s+2)(s+10)}$

A: ① To find corner frequencies,

→ Write in standard Time constant form

$$G(s) = \frac{20 \times 4 \left(1 + \frac{s}{4}\right)}{s \cdot 2 \left(1 + \frac{s}{2}\right) \cdot 10 \left(1 + \frac{s}{10}\right)}$$

$$G(s) = \frac{4 \left(1 + \frac{s}{4}\right)}{s \left(1 + \frac{s}{2}\right) \left(1 + \frac{s}{10}\right)}$$

Corner frequencies,

$$T = \frac{1}{4} ; \omega_c = 4$$

$$\omega_{c1} = 2$$

$$T = \frac{1}{2} ; \omega_c = 2$$

$$\omega_{c2} = 4$$

$$T = \frac{1}{10} ; \omega_c = 10$$

$$\omega_{c3} = 10$$

② To convert into frequency domain,
put $s = j\omega$ in standard form

$$G(j\omega) = \frac{4(1 + \frac{j\omega}{4})}{(j\omega)(1 + \frac{j\omega}{2})(1 + \frac{j\omega}{10})}$$

③ To find slopes,

→ write in factor form, $G(j\omega) = \left(\frac{4}{j\omega}\right) \left(\frac{1}{1 + \frac{j\omega}{2}}\right) \left(1 + \frac{j\omega}{4}\right) \left(\frac{1}{1 + \frac{j\omega}{10}}\right)$

factor	corner frequency	slope	change in slope
$\frac{4}{j\omega}$	—	-20	—
$\frac{1}{1 + \frac{j\omega}{2}}$	2	-20	-40 dB
$1 + \frac{j\omega}{4}$	4	+20	~~-20~~ -20 dB
$\frac{1}{1 + \frac{j\omega}{10}}$	10	-20	~~-20~~ -40 dB

④ To choose ω_e, ω_h ;

$$\omega_e < \omega_{c1} \quad \omega_e = 0.1$$

$$\omega_h > \omega_{c3} \quad \omega_h = 50$$

⑤

⑥ To find magnitude (or) gain of at $\omega_{c2}, \omega_{c3}, \omega_h$,
 At $\omega = \omega_{c2} = 4$

$$A = \text{slope from } \omega_{c1} \text{ to } \omega_{c2} \times \log\left(\frac{\omega_{c2}}{\omega_{c1}}\right) + A \text{ at } \omega_{c1}$$

$$A = -40 \times \log\left(\frac{4}{2}\right) + 6 = -6 \text{ dB}$$

At $\omega = \omega_{c3} = 10$

$$A = \text{slope from } \omega_{c2} \text{ to } \omega_{c3} \times \log\left(\frac{\omega_{c3}}{\omega_{c2}}\right) + A \text{ at } \omega_{c2}$$

$$= -20 \times \log\left(\frac{10}{4}\right) + (-6) = -14 \text{ dB}$$

At $\omega = \omega_h = 50$

$$A = \text{slope from } \omega_{c3} \text{ to } \omega_h \times \log\left(\frac{\omega_h}{\omega_{c3}}\right) + A \text{ at } \omega_{c3}$$

$$= -40 \times \log\left(\frac{50}{10}\right) +$$

⑦ To find phase for phase plot,

$$G(j\omega) = \frac{4(1 + \frac{j\omega}{4})}{(j\omega)(1 + \frac{j\omega}{2})(1 + \frac{j\omega}{10})}$$

$$\phi = -90 - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{10}\right) + \tan^{-1}\left(\frac{\omega}{4}\right)$$

$\omega \quad \phi$

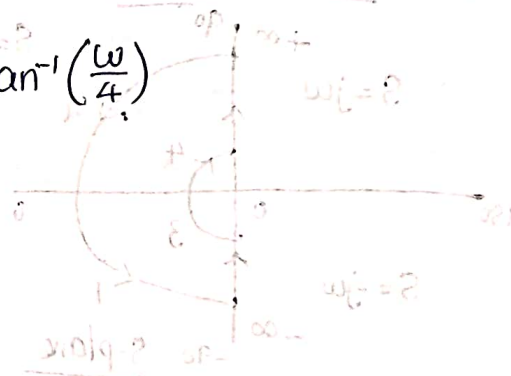
→ 0.1 -92

→ 2 -119.7

→ 4 -130.2

→ 10 -145.5

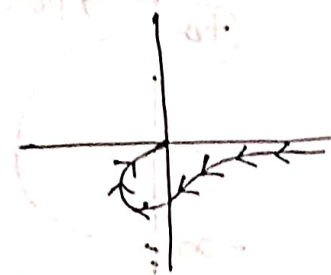
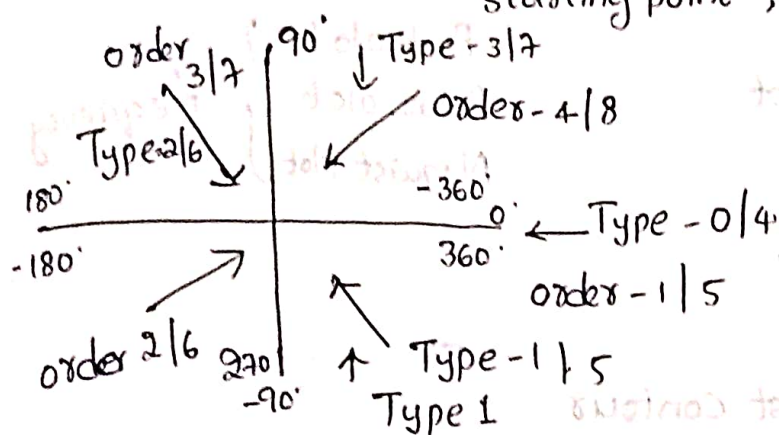
→ 50 -171



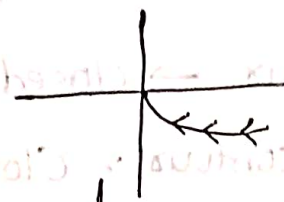
Polar Plot

$$G(s) = \frac{1}{(s+1)(s+2)}; \text{ Type} = 0, \text{ order} = 2$$

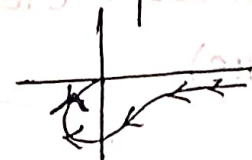
Starting point, ending point



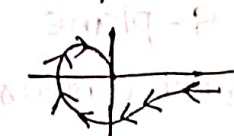
$$\Rightarrow G(s) = \frac{1}{s+1}; \text{ Type} = 0; \text{ order} = 1$$



$$\Rightarrow G(s) = \frac{1}{(s+1)(s+2)}; \text{ Type} = 0; \text{ order} = 2$$



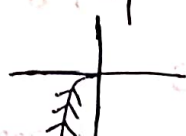
$$\Rightarrow G(s) = \frac{1}{(s+1)(s+2)(s+3)}; \text{ Type} = 0; \text{ order} = 3$$



$$\Rightarrow G(s) = \frac{1}{(s+1)(s+2)(s+3)(s+4)}; \text{ Type} = 0; \text{ order} = 4$$



$$\Rightarrow G(s) = \frac{1}{s(s+1)}; \text{ Type} = 1; \text{ order} = 2$$



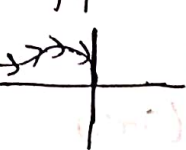
$$\Rightarrow G(s) = \frac{1}{s(s+1)(s+2)}; \text{ Type} = 1; \text{ order} = 3$$



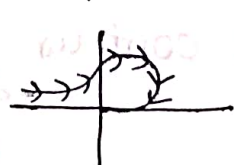
$$\Rightarrow G(s) = \frac{1}{s(s+1)(s+2)(s+3)}; \text{ Type} = 1; \text{ order} = 4$$



$$\Rightarrow G(s) = \frac{1}{s^2(s+1)}; \text{ Type} = 2; \text{ order} = 3$$



$$\Rightarrow G(s) = \frac{1}{s^2(s+1)(s+2)}; \text{ Type} = 2; \text{ order} = 4$$



$$\Rightarrow G(s) = \frac{1}{s^3(s+1)}; \text{ Type} = 3; \text{ order} = 4$$



Nyquist Plot

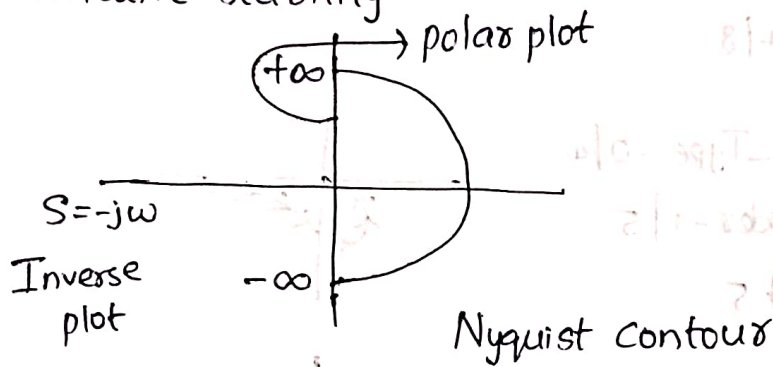
Stability

- 1) Absolute stability
- 2) Relative stability

Stability

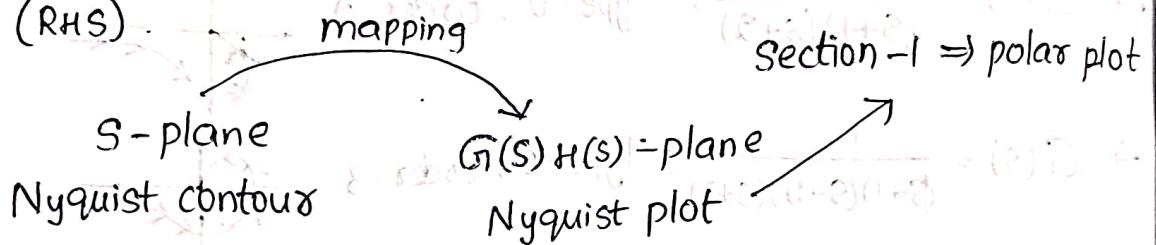
RH } Time
RL }

Bode plot }
Polar plot } frequency
Nyquist plot }



Contour $\rightarrow$ closed path

Nyquist contour :- Closed path in s-plane at nyquist specified region (RHS).



Section -1
 $S = jw$
 $w = 0 \rightarrow \infty$ } mapping $\rightarrow$ polar plot

$\rightarrow$ Section -3

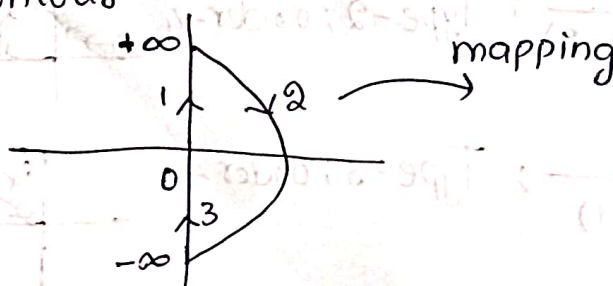
$S = -jw$
 $w \rightarrow -\infty$ to 0 } mapping $\rightarrow$ Inverse polar plot.

$\rightarrow$ Section -2

$S = Re^{j\theta}$
 $R \rightarrow \infty$
 $\theta = +90$ to -90

① $G(s) = \frac{1}{(s+1)(s+2)}$ Draw Nyquist plot

$\Rightarrow$ Nyquist contour



→ To map section-1

$$\rightarrow s = j\omega$$

$$\rightarrow \omega \rightarrow 0 \text{ to } \infty$$

→ polar plot

$$G(s) = \frac{1}{(s+1)(s+2)}$$

$$s = j\omega$$

$$G(j\omega) = \frac{1}{(j\omega+1)(j\omega+2)}$$

$$|G(j\omega)| = \frac{1}{\sqrt{\omega^2+1} \sqrt{\omega^2+2^2}}$$

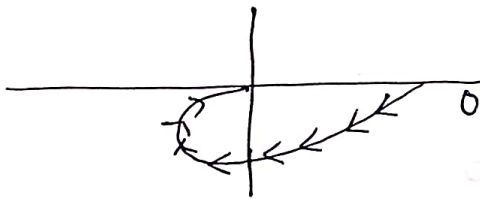
$$\angle G(j\omega) H(j\omega) = -\tan^{-1}\left(\frac{\omega}{1}\right) - \tan^{-1}\left(\frac{\omega}{2}\right)$$

$$\omega=0 \quad |G(0)H(0)| = \frac{1}{\sqrt{0^2+1} \sqrt{0^2+4}} = \frac{1}{\sqrt{4}} = \frac{1}{2} = 0.5$$

$$\angle G(0)H(0) = -\tan^{-1}(0) - \tan^{-1}(0) = 0$$

$$\omega=\infty \quad |G(\infty)H(\infty)| = \frac{1}{\sqrt{\infty^2+1} \sqrt{\infty^2+4}} = \frac{1}{\infty} = 0$$

$$\angle G(\infty)H(\infty) = -\tan^{-1}(\infty) - \tan^{-1}(\infty) = -90 - 90 = -180$$



To map section-3:

$$s = -j\omega$$

$$\omega \rightarrow -\infty \text{ to } 0$$

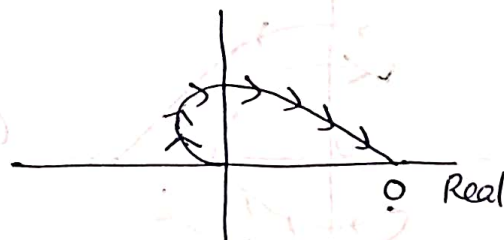
Inverse polar plot

Mirror image of section-1 w.r.t real axis

To map section-2:

$$s = Re^{j\theta}; R \rightarrow \infty$$

$$\boxed{s = \infty e^{j\theta}} \quad \theta = +90 \text{ to } -90$$



$$G(\infty e^{j\theta}) = \frac{1}{(s+1)(s+2)}$$

$$= \frac{1}{(s+1)} \approx \frac{1}{s} ; \frac{1}{(s+2)} \approx \frac{1}{s}$$

$$G(\infty e^{j\theta}) = \frac{1}{s \cdot s} = \frac{1}{s^2}$$

$$= \frac{1}{(\infty e^{j\theta})^2} = \frac{1}{\infty^2} \cdot \frac{1}{e^{j2\theta}} = 0 \cdot e^{-j2\theta}$$

$$s = \infty e^{j\theta}$$

$$R \rightarrow \infty$$

$$\theta = +90 \text{ to } +90$$

mapping $\rightarrow$

$$0 e^{-j2\theta}$$

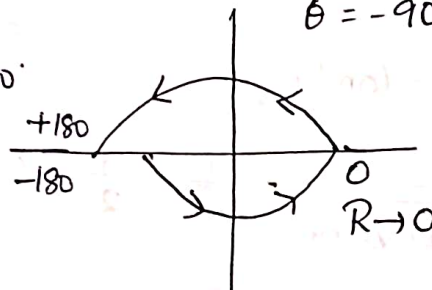
$$R \rightarrow 0$$

$$\theta = 90 \rightarrow -2\theta = -2 \times 90^\circ = -180^\circ$$

$$\theta = -90 \rightarrow -2\theta = -2 \times -90 = 180^\circ$$

$$-180 \text{ to } 0$$

$$0 \text{ to } 180^\circ$$

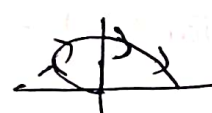


$\Rightarrow$ Total Nyquist plot

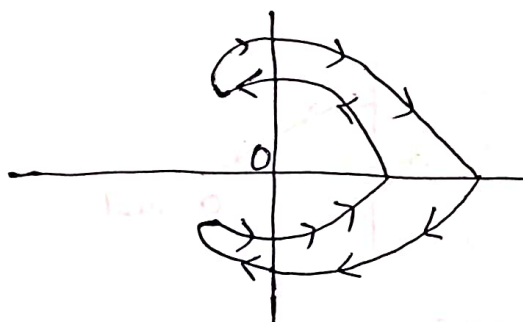
Section-1 $\uparrow 1$



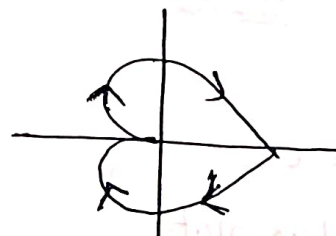
Section-3 $\uparrow 3$



Section-2 $\uparrow 1$



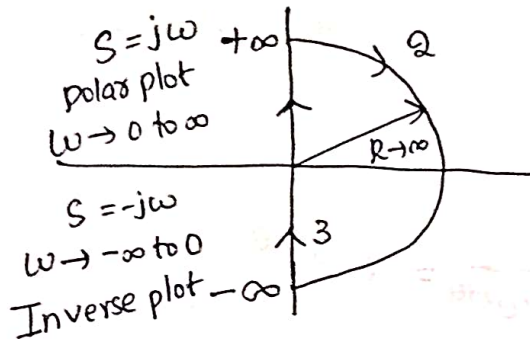
(or)



Nyquist plot.

② $G(s) = \frac{1}{(s+1)(s+2)(s+3)}$ Draw Nyquist plot.

→ Nyquist contour :



$$s = Re^{j\theta}$$

$$R \rightarrow \infty$$

$$\theta = +90^\circ \text{ to } -90^\circ$$

→ To map section - 1

$$s = j\omega, \omega \rightarrow 0 \text{ to } \infty \rightarrow \text{polar plot}$$

$$G(s) \cdot H(s) = \frac{1}{(s+1)(s+2)(s+3)}$$

$$G(j\omega) \cdot H(j\omega) = \frac{1}{(j\omega+1)(j\omega+2)(j\omega+3)}$$

$$|G(j\omega)H(j\omega)| = \frac{1}{\sqrt{\omega^2+1} \sqrt{\omega^2+2^2} \sqrt{\omega^2+3^2}}$$

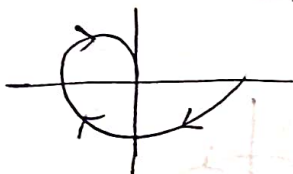
$$\angle G(j\omega)H(j\omega) = -\tan^{-1}\left(\frac{\omega}{1}\right) - \tan^{-1}\left(\frac{\omega}{2}\right) - \tan^{-1}\left(\frac{\omega}{3}\right)$$

$$\omega=0 \quad |G(0)H(0)| = \frac{1}{\sqrt{0+1} \sqrt{0+4} \sqrt{0+9}} = \frac{1}{\sqrt{4}\sqrt{9}} = \frac{1}{2 \times 3} = \frac{1}{6}$$

$$\omega=\infty \quad |G(\infty)H(\infty)| = \frac{1}{\sqrt{\infty^2+1} \sqrt{\infty^2+2^2} \sqrt{\infty^2+3^2}} = \frac{1}{\infty} = 0$$

$$\omega=0 \quad \angle G(0)H(0) = -\tan^{-1}(0) - \tan^{-1}(0) - \tan^{-1}(0) = 0$$

$$\omega=\infty \quad \angle G(\infty)H(\infty) = -\tan^{-1}(\infty) - \tan^{-1}(\infty) - \tan^{-1}(\infty) \\ = -90 - 90 - 90 = -270$$



Type - 0

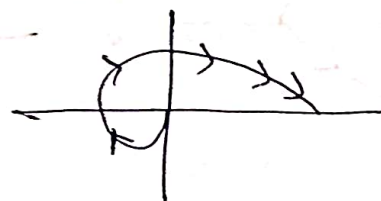
order - 3

→ To map section - 3 :

$$s = -j\omega$$

$$\omega \rightarrow -\infty \text{ to } 0$$

Inverse polar plot



To map section-2

$$S = Re^{j\theta}, R \rightarrow \infty$$

$$\theta = +90 \text{ to } -90$$

$$\therefore G(s)H(s) = \frac{1}{s \cdot s \cdot s} ; \frac{1}{s+a} \approx \frac{1}{s} \quad s \gg a$$

$$= \frac{1}{s^3}$$

$$G(\infty e^{j\theta})H(\infty e^{j\theta}) = \frac{1}{(\infty e^{j\theta})^3} = \frac{1}{\infty^3 e^{j3\theta}} = 0 e^{-j3\theta}$$

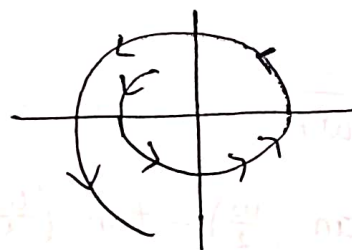
Nyquist contour :-

$$S = Re^{j\theta}$$

$$= \infty e^{j\theta} \xrightarrow{\text{map}} 0 e^{-j3\theta}$$

$$\theta = +90, \longrightarrow -3\theta = -3 \times 90 = -270$$

$$\theta = -90, \longrightarrow -3\theta = -3 \times -90 = +270$$



Nyquist contour

Nyquist plot

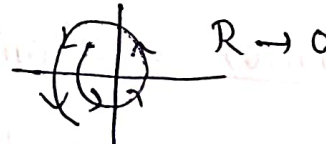
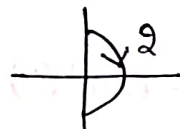
Section - 1



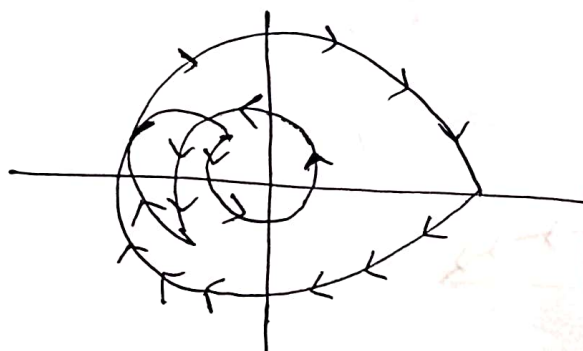
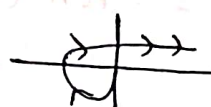
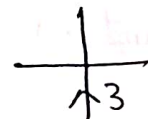
mapping



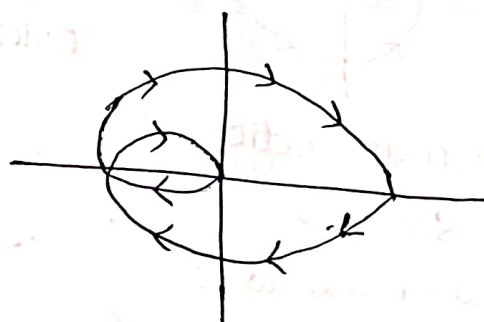
Section - 2



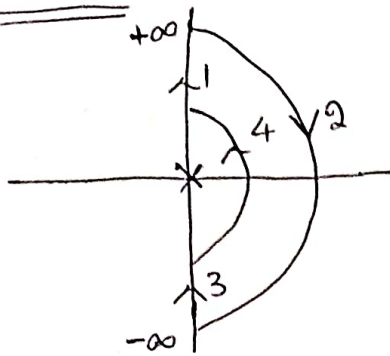
Section - 3



(or)

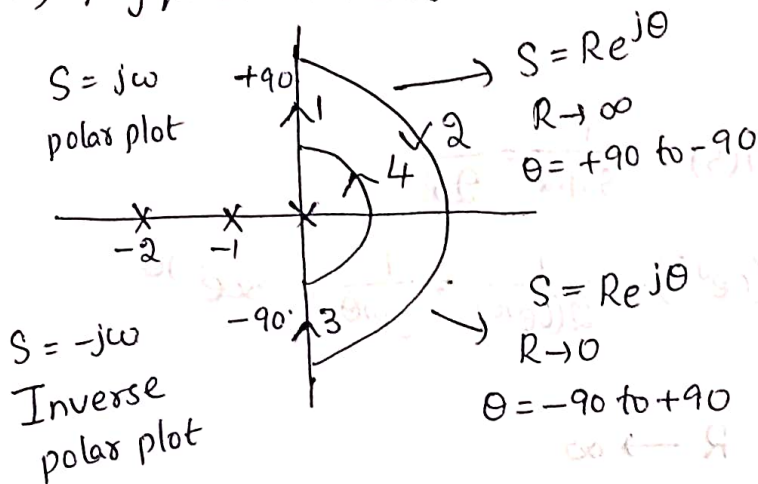


Model-2 :-



$$\Rightarrow G(s) = \frac{1}{s(s+1)(s+2)}$$

$\Rightarrow$ Nyquist contour



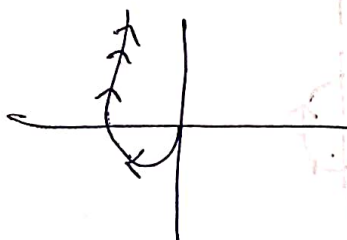
To map section-1 :-

$S = jw$; $w \rightarrow 0$ to ∞ , polar plot



Section-3:-

$S = -jw$, $w \rightarrow -\infty$ to 0



Section-2 :-

$$S = Re^{j\theta}$$

$$S = \infty e^{j\theta}$$

$$G(s) = \frac{1}{s(s+1)(s+2)} = \frac{1}{s \cdot s \cdot s} = \frac{1}{s^3}$$

$$G(\infty e^{j\theta}) = \frac{1}{(\infty e^{j\theta})^3} = 0 e^{-j3\theta}$$

Nyquist contour

$R \rightarrow \infty$

map

Nyquist plot

$R \rightarrow 0$

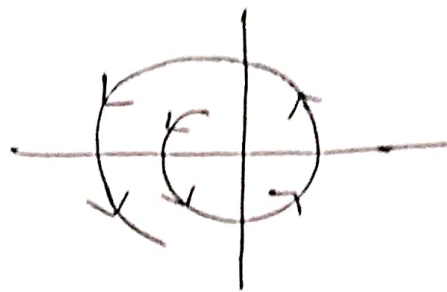
$$-270 \rightarrow 0$$

$$0 \rightarrow 270$$

$$\theta = +90 \text{ to } -90$$

$$\theta = +90 \rightarrow -3\theta = -3 \times 90 = -270$$

$$\theta = -90 \rightarrow -3\theta = -3 \times -90 = 270$$



Section-4 :-

$$S = Re^{j\theta} = \infty e^{j\theta}$$

$$R \rightarrow \infty$$

$$G(s) = \frac{1}{s(s+1)(s+2)}$$

$$\frac{1}{s+a} \approx \frac{1}{a} \quad S \ll a$$

$$\frac{1}{s+1} = \frac{1}{1} = 1 \quad \left| \quad G(s) = \frac{1}{s \cdot 1 \cdot 2} = \frac{1}{2s}$$

$$\frac{1}{s+2} = \frac{1}{2} = 0.5 \quad \left| \quad G(\infty e^{j\theta}) = \frac{1}{2(\infty e^{j\theta})} = \frac{1}{\infty e^{j\theta}} = \infty e^{-j\theta}$$

$$R \rightarrow 0 \xrightarrow{\text{mapping}} R \rightarrow \infty$$

$$\theta = -90 \text{ to } +90$$

$$\theta = -90 \longrightarrow -\theta = -(-90) = 90$$

$$\theta = 90 \longrightarrow -\theta = -(90) = -90$$

$$90 \text{ to } 0$$

$$0 \text{ to } -90$$

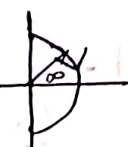


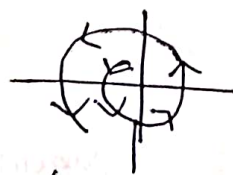
Nyquist contour

Nyquist plot

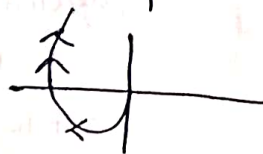
Section-1  $\xrightarrow{\text{mapping}}$

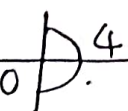


Section-2  $\xrightarrow{\text{mapping}}$

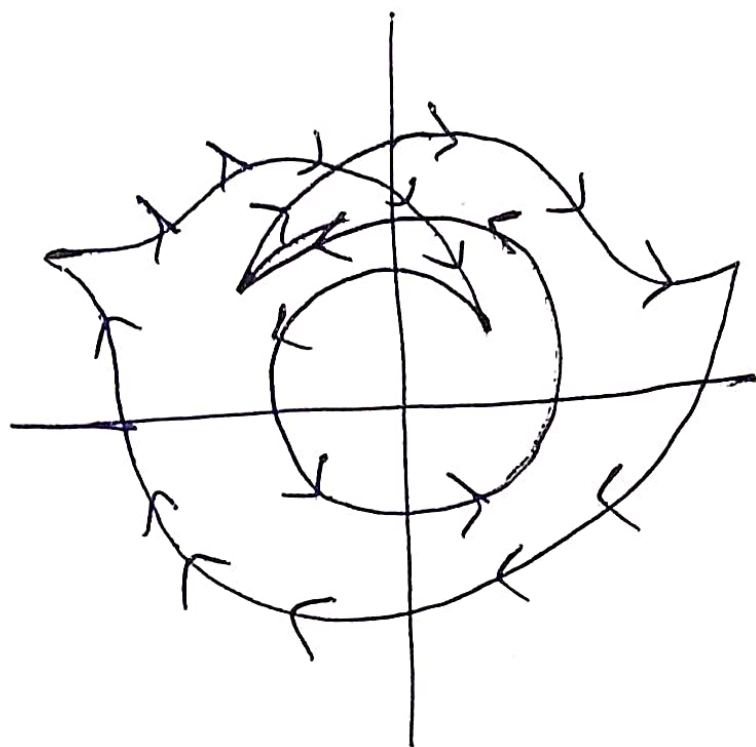


Section-3  $\xrightarrow{\text{mapping}}$

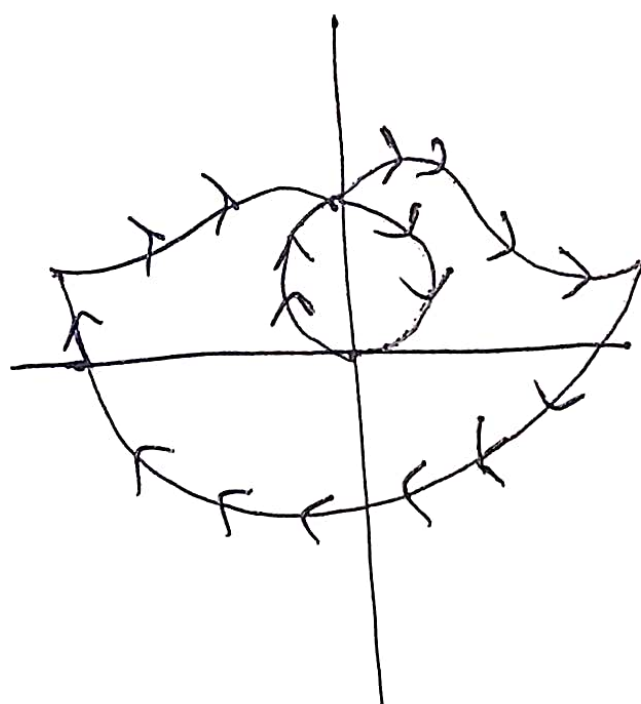


Section-4  $\xrightarrow{\text{mapping}}$





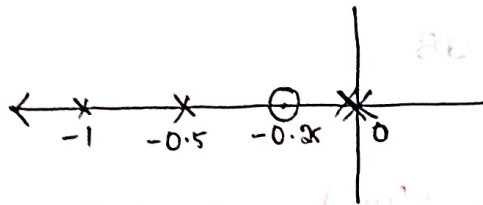
(20)



$$Q) G_H = \frac{(s+0.25)}{s^2(s+0.5)(s+1)}$$

Draw Nyquist plot

$$s = -0.5, -1, 0, 0 ; s = -0.25$$



finite pole, finite zero
near to origin

finite zero condition

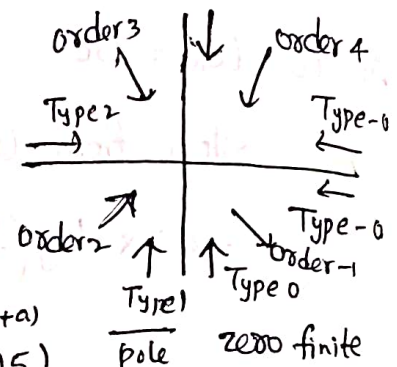
Type - starting point - General - 2

Order - ending point - General - 4

Type - starting point changes

$$\text{order - decrease} = \frac{(s+0.25)}{s^2(s+0.5)(s+1)} - 3$$

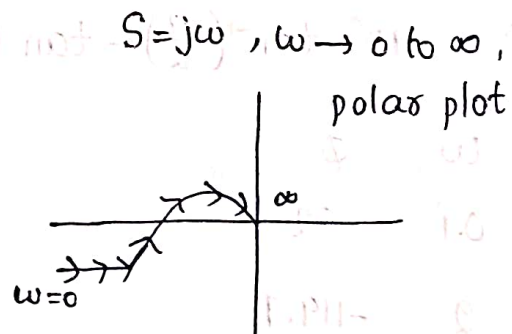
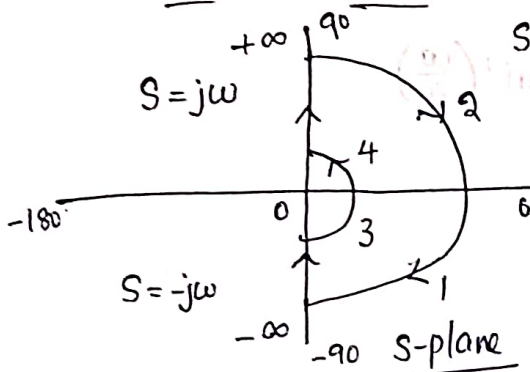
Write finite zero → Type
Type - starting point changes
order - decrease



Nyquist Contour :-

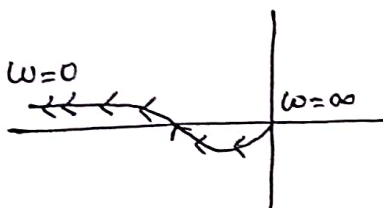
Type-2

To map section-1:-



To map section-3:-

To map section-2:-



$$S = Re^{j\theta}, R \rightarrow \infty, \theta = -90^\circ \text{ to } +90^\circ$$

$$R \rightarrow \infty$$

$$G_H = \frac{1}{s^2 \cdot s \cdot s} = \frac{1}{s^3}$$

$$S = \infty e^{j\theta}$$

$$G_H = \frac{1}{(\infty e^{j\theta})^3} = \frac{1}{\infty e^{j3\theta}} = 0 e^{-j3\theta}$$

$$\frac{1}{s+a} \approx \frac{1}{s} ; s \gg a.$$

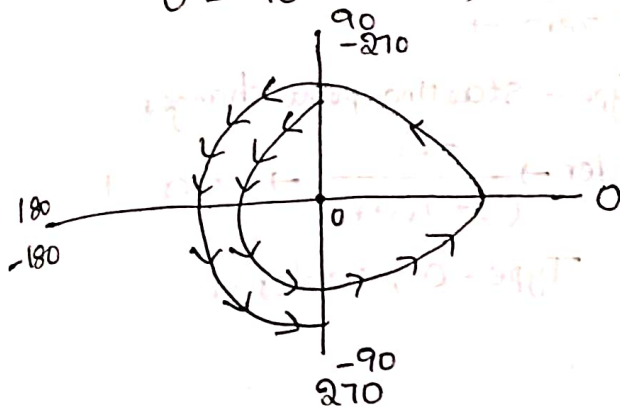
$s+a \approx s$ write only 's' terms

$$s = \infty e^{j\theta} \longrightarrow 0 e^{-j3\theta}$$

$$\theta \longrightarrow -3\theta$$

$$R \rightarrow \infty \longrightarrow R \rightarrow 0$$

$$\left. \begin{array}{l} \theta = 90 \longrightarrow -3\theta = -3(90) = -270 \\ \theta = -90 \longrightarrow -3\theta = -3(-90) = 270 \end{array} \right\} \begin{array}{l} -270 \text{ to } 0 \\ 0 \text{ to } 270 \end{array}$$



$$\frac{1}{s+a} \approx \frac{1}{s}$$

$s+a \approx s$ neglect 's' terms

To map section - 4

$$GH = \frac{0.25}{s^2(0.5)(1)} = \frac{0.25}{0.5} \cdot \frac{1}{s^2}$$

$$s = 0 e^{j\theta}$$

$$GH = \frac{0.25}{0.5} \frac{1}{(0 \cdot e^{j\theta})^2} = \infty e^{-j2\theta}$$

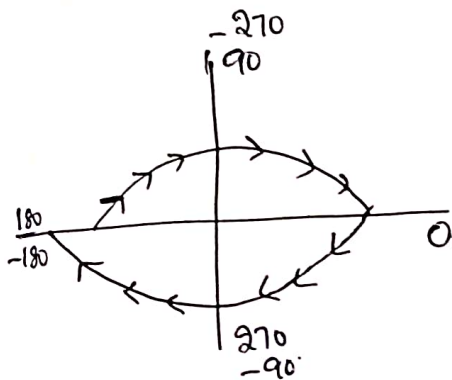
$$s = 0 e^{j\theta} \longrightarrow \infty e^{-j2\theta}$$

$$0 \longrightarrow \infty$$

$$\theta \longrightarrow -2\theta$$

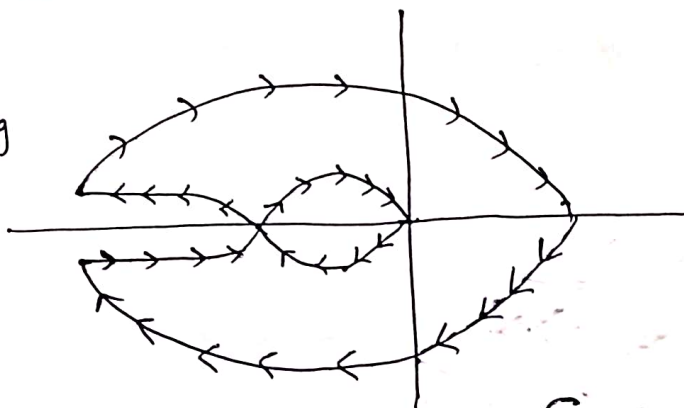
$$\theta = -90 \longrightarrow -2\theta = -2(90) = 180$$

$$\theta = 90 \longrightarrow -2\theta = -2(90) = -180$$



Nyquist Plot:-

mapping

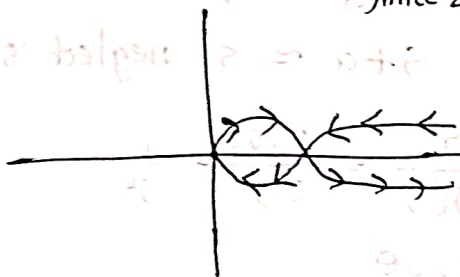
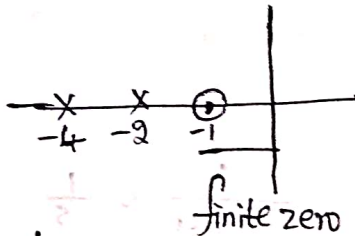
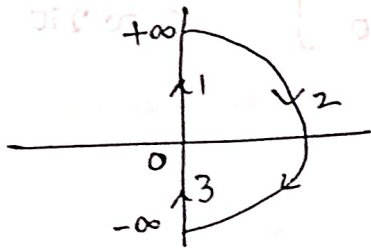


GH plane

$G(s)H(s)$ plane

Q) $G_H = \frac{(s+1)}{(s+2)(s+4)}$

Nyquist contour



Nyquist plot

Section-1 :-

Type-0, order-2 (General)

finite zero $\rightarrow$

Type - starting point changes

order $\rightarrow \frac{s+1}{(s+2)(s+4)} \rightarrow \text{order} = 1$

Type-0, order-1



State Space Analysis

→ Block diagram → s.s

→ ** Solution of the state equation

Homogenous
Solution

Non-Homogenous
Solution

Definitions

- 1) State
- 2) State variable
- 3) State model
- 4) STM and properties

→ State Transition Matrix (STM)
and properties

→ Diagonalization

→ *** Controllability and Observability.

Controllability and Observability:-

Controllability:-

A system is controllable, if it is possible to transfer the state (position) to any desired state (or) position within a specified time using control variable $u(t)$.

⇒ Kalman's test:-

Controllability matrix, $Q_c = [B \ AB \ A^2B \ A^3B \ \dots \ A^{n-1}B]$

$$A \rightarrow 2 \times 2 \rightarrow Q_c = [B \ AB]$$

$$A \rightarrow 3 \times 3 \rightarrow Q_c = [B \ AB \ A^2B]$$

$|Q_c| \neq 0$, system is controllable

$|Q_c| = 0$, system is not controllable.

Observability:-

A system is said to be observable, if every state completely identified or measured in the output $y(t)$.

⇒ Kalman's test:-

Observability matrix, $Q_o = [C^T \ A^T C^T \ (A^T)^2 C^T \ \dots \ (A^T)^{n-1} C^T]$

$$A \rightarrow 2 \times 2 \rightarrow Q_0 = [C^T \ A^T C^T]$$

$$A \rightarrow 3 \times 3 \rightarrow Q_0 = [C^T \ A^T C^T \ (A^T)^2 C^T]$$

$|Q_0| \neq 0$ System is observable

$|Q_0| = 0$ System is not observable

$$(P_3) \quad \dot{x} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} x + \begin{bmatrix} -1 \\ 1 \end{bmatrix} u$$

$$y = [1 \quad 1] x \quad \text{check for controllability and observability.}$$

Sol: State model, $\dot{x} = Ax + Bu$; $y = Cx + Du$

On comparing, $A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$; $B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$; $C = [1 \quad 1]$; $D = 0$

Controllability matrix, $Q_c = [B \ AB \ A^2 B \ A^3 B \ \dots \ A^{n-1} B]$

for $A \rightarrow 2 \times 2 \rightarrow Q_c = [B \ AB]$

$$A \cdot B = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \times 1 + 1 \times (-1) \\ -1 \times 1 + (-2) \times (-1) \end{bmatrix} = \begin{bmatrix} -1 \\ -1 + 2 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$Q_c = [B \ AB] = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$|Q_c| = (1 \times 1) - [(-1) \times (-1)] = 1 - 1 = 0$$

System is not controllable.

Observability, $Q_0 = [C^T \ A^T C^T]$

$$C^T = \begin{bmatrix} 1 \\ 1 \end{bmatrix}; \quad A^T = \begin{bmatrix} 0 & -1 \\ 1 & -2 \end{bmatrix}$$

$$A^T \cdot C^T = \begin{bmatrix} 0 & -1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \times 1 + (-1) \times (1) \\ 1 \times 1 + (-2) \times (1) \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

$$Q_0 = [C^T \ A^T C^T] = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$$

$$|Q_0| = 1 \times (-1) - 1 \times (-1) = -1 + 1 = 0$$

$$|Q_0| = 0$$

System is not observable.

$$\textcircled{P_0} \quad \dot{x} = \begin{bmatrix} 0 & 1 \\ -20 & -9 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = [1 \ 0] x$$

Check for controllability and observability.

Sol: State model, $\dot{x} = Ax + Bu$; $y = Cx + du$

on comparing, $A = \begin{bmatrix} 0 & 1 \\ -20 & -9 \end{bmatrix}$; $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$; $C = [1 \ 0]$; $D = 0$

Controllability matrix, $Q_c = [B \ AB \ A^2B \ A^3B \ \dots \ A^{n-1}B]$

for $A \rightarrow 2 \times 2 \rightarrow Q_c = [B \ AB]$

$$A \cdot B = \begin{bmatrix} 0 & 1 \\ -20 & -9 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -9 \end{bmatrix}$$

$$Q_c = [B \ AB] = \begin{bmatrix} 0 & 1 \\ 1 & -9 \end{bmatrix}$$

$$|Q_c| = 0 - 1 = -1$$

System is controllable.

Observability,

for $A \rightarrow 2 \times 2 \rightarrow Q_0 = [C^T \ A^T C^T]$

$$C^T = \begin{bmatrix} 1 \\ 0 \end{bmatrix}; \quad A^T = \begin{bmatrix} 0 & -20 \\ 1 & -9 \end{bmatrix}; \quad A^T C^T = \begin{bmatrix} 0 & -20 \\ 1 & -9 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$Q_0 = [C^T \ A^T C^T] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$|Q_0| = 1 - 0 = 1$$

$$|Q_0| \neq 0$$

System is observable.

SM Solution of the State equation (s.s) $\rightarrow x(t)$

$\downarrow$ o/p $\rightarrow$ Time domain

Response of the state equation

State equation, $\dot{x} = A \underset{\substack{\downarrow \\ \text{state}}}{x} + B \underset{\substack{\downarrow \\ \text{input}}}{u}$

Apply laplace transform,

$$s \cdot x(s) - x(0) = A \cdot x(s) + B \cdot U(s)$$

$$s \cdot x(s) - A x(s) = x(0) + B U(s)$$

$$x(s) [sI - A] = x(0) + B U(s)$$

$$x(s) = [sI - A]^{-1} [x(0) + B U(s)]$$

$$x(s) = [sI - A]^{-1} x(0) + [sI - A]^{-1} B \cdot U(s)$$

Apply inverse laplace transform, $\uparrow$ initial state

$$x(t) = \underbrace{L^{-1} [(sI - A)^{-1} x(0)]}_{\text{Tr. state solution}} + \underbrace{L^{-1} [(sI - A)^{-1} B \cdot U(s)]}_{\text{S.S. solution}}$$

Total solution (or)

Total response (or)

time response (or)

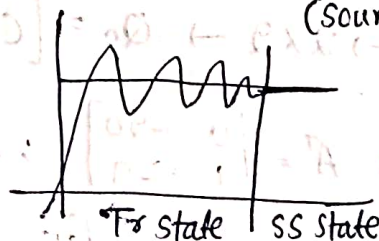
Complete response

$$\Rightarrow x(t) =$$

Source free response + Source feed response

Natural response + forced response

$$\dot{x} = A \underset{\substack{\downarrow \\ \text{state}}}{x} + B \underset{\substack{\downarrow \\ \text{input}}}{u}$$



$$\text{Zero Input Response (ZIR)} = L^{-1} [(sI - A)^{-1} x(0)]$$

$$\text{Zero State Response (ZSR)} = L^{-1} [(sI - A)^{-1} B \cdot U(s)]$$

Transient state
Sudden change

$\overline{\text{ON}}$
 $\overline{\text{off}}$

$\downarrow$
L, C $\rightarrow$ storage elements
discharge

S.S $\rightarrow$ initial values

Total solution

Homogeneous
- Solution

$$I/p = 0$$

$$U(s) = 0$$

$$x(t) = L^{-1}[(sI-A)^{-1} x(0)]$$

$$x(t) = ZIR$$

Non-Homogeneous
Solution

$$I/p \neq 0$$

$$U(s) \neq 0$$

$$x(t) = L^{-1}[(sI-A)^{-1} x(0)] + L^{-1}[(sI-A)^{-1} B U(s)]$$

$$x(t) = ZIR + ZSR$$

Ⓑ Obtain the time response of the following system, where $u(t)$ is the unit step function.

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t)$$

$$x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ (or)}$$

$$x_1(0) = 1 ; x_2(0) = 0$$

Ans time response,

$$x(t) = L^{-1}[(sI-A)^{-1} x(0)] + L^{-1}[(sI-A)^{-1} B U(s)]$$

$$x(t) = ZIR + ZSR$$

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} ; B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

To find ZIR:

$$x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$ZIR = L^{-1}[(sI-A)^{-1} x(0)]$$

$$\begin{aligned} sI-A &= s \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \\ &= \begin{bmatrix} s-0 & 0-1 \\ 0-(-2) & s-(-3) \end{bmatrix} = \begin{bmatrix} s & -1 \\ 2 & s+3 \end{bmatrix} \end{aligned}$$

$$(sI-A)^{-1} = \frac{1}{s(s+3)+2} \begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix}$$

$$(sI-A)^{-1} x(0) = \frac{1}{s^2+3s+2} \begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \frac{1}{s^2+3s+2} \begin{bmatrix} (s+3) \times 1 + (1 \times 0) \\ -2 \times 1 + s \times 0 \end{bmatrix} = \frac{1}{s^2+3s+2} \begin{bmatrix} s+3 \\ -2 \end{bmatrix}$$

$$L^{-1}[(SI-A)^{-1}x(0)] = L^{-1}\begin{bmatrix} \frac{s+3}{s^2+3s+2} \\ \frac{-2}{s^2+3s+2} \end{bmatrix} = L^{-1}\begin{bmatrix} \frac{s+3}{(s+1)(s+2)} \\ \frac{-2}{(s+1)(s+2)} \end{bmatrix}$$

$$\frac{s+3}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2} \quad \frac{-2}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

$$s+3 = A(s+2) + B(s+1) \quad -2 = A(s+2) + B(s+1)$$

if $s = -2$:

$$-2+3 = B(-2+1)$$

$$+1 = B(-1)$$

$$B = -1$$

if $s = -1$:

$$-2 = B(-1+1)$$

$$-2 = B(0)$$

$$B = 2$$

if $s = -1$:

$$-1+3 = A(-1+2)$$

$$2 = 1A$$

$$A = 2$$

if $s = -1$:

$$-2 = A(-1+2)$$

$$-2 = A$$

$$A = -2$$

$$\frac{s+3}{(s+1)(s+2)} = \frac{2}{s+1} - \frac{1}{s+2} \quad \frac{-2}{(s+1)(s+2)} = \frac{-2}{s+1} + \frac{2}{s+2}$$

$$ZIR = L^{-1}[(SI-A)^{-1}x(0)] = \begin{bmatrix} \frac{2}{s+1} - \frac{1}{s+2} \\ \frac{-2}{s+1} + \frac{2}{s+2} \end{bmatrix} = \begin{bmatrix} 2e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} \end{bmatrix}$$

To find ZSR:

$$ZSR = L^{-1}[(SI-A)^{-1}BU(s)]$$

$$(SI-A)^{-1} = \frac{1}{s^2+3s+2} \begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix}$$

$$(SI-A)^{-1}B = \frac{1}{s^2+3s+2} \begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \frac{1}{s^2+3s+2} \begin{bmatrix} (s+3) \times 0 + 1 \times 1 \\ -2 \times 0 + s \times 1 \end{bmatrix} = \frac{1}{s^2+3s+2} \begin{bmatrix} 1 \\ s \end{bmatrix}$$

$$(sI - A)^{-1} B U(s) = \frac{1}{s^2 + 3s + 2} \begin{bmatrix} 1 \\ s \end{bmatrix} \begin{bmatrix} 1/s \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{s^2 + 3s + 2} \\ \frac{s}{s^2 + 3s + 2} \end{bmatrix} \begin{bmatrix} 1/s \end{bmatrix} = \begin{bmatrix} \frac{1}{s(s^2 + 3s + 2)} \\ \frac{s}{s(s^2 + 3s + 2)} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{s(s^2 + 3s + 2)} \\ \frac{1}{s^2 + 3s + 2} \end{bmatrix} \quad \begin{array}{l} \text{for unit step} \\ u(t) = 1 \\ U(s) = 1/s \end{array}$$

$$ZSR = L^{-1} \left[(sI - A)^{-1} B \cdot U(s) \right] = L^{-1} \begin{bmatrix} \frac{1}{s(s^2 + 3s + 2)} \\ \frac{1}{s^2 + 3s + 2} \end{bmatrix}$$

$$= L^{-1} \begin{bmatrix} -\frac{0.5}{s} + \frac{1}{s+1} - \frac{0.5}{s+2} \\ \frac{1}{s+1} - \frac{1}{s+2} \end{bmatrix}$$

$$= \begin{bmatrix} -0.5 + e^{-t} - 0.5e^{-2t} \\ e^{-t} - e^{-2t} \end{bmatrix}$$

$$\text{Time response} = ZIR + ZSR$$

$$= \begin{bmatrix} 2e^{-t} - e^{-2t} \\ 2e^{-t} - 2e^{-2t} \end{bmatrix} + \begin{bmatrix} -0.5 + e^{-t} - 0.5e^{-2t} \\ e^{-t} - e^{-2t} \end{bmatrix}$$

$$= \begin{bmatrix} 2e^{-t} - e^{-2t} - 0.5 + e^{-t} - 0.5e^{-2t} \\ 2e^{-t} - 2e^{-2t} + e^{-t} - e^{-2t} \end{bmatrix}$$

$$x(t) = \begin{bmatrix} -0.5 + 3e^{-t} - 1.5e^{-2t} \\ 3e^{-t} - 3e^{-2t} \end{bmatrix}$$

(P8) $\dot{x}(t) = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$, compute the homogeneous solution for the given system, if $x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

(P9) Obtain the time response of the system

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u(t)$$

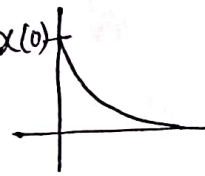
$$y = [1 \quad 1] x(t) \text{ for } x(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

STM (State Transition Matrix) and its properties

Homogeneous solution $\rightarrow i/p = 0$

$$ZIR = L^{-1}[(sI - A)^{-1} \cdot x(0)]$$

$$= \underbrace{L^{-1}[(sI - A)^{-1}]}_{\text{State transition matrix}} \cdot x(0)$$



$\therefore$ State transition matrix, $\phi(t) = L^{-1}[(sI - A)^{-1}] = e^{At}$, $A < 0$

Properties:

$$1) \phi(0) = I$$

$$\phi(t) = e^{At}$$

$$\phi(0) = e^{A(0)} = e^0 = I$$

$$3) [\phi(t)]^{-1} = \phi(-t)$$

$$\phi(t) = e^{At}$$

$$[\phi(t)]^{-1} = (e^{At})^{-1} = e^{-At}$$

$$2) [\phi(t)]^k = \phi(kt)$$

$$= e^{A(-t)}$$

$$\phi(t) = e^{At}$$

$$[\phi(t)] = \phi(-t)$$

$$[\phi(t)]^k = (e^{At})^k = e^{Atk}$$

$$= e^{A(kt)}$$

$$[\phi(t)]^k = \phi(kt)$$

$$\textcircled{4} \quad \phi(t_2+t_1) = \phi(t_2) \cdot \phi(t_1)$$

$$\phi(t) = e^{At}$$

$$\phi(t_2+t_1) = e^{A(t_2+t_1)} = e^{At_2} \cdot e^{At_1}$$

$$\phi(t_2+t_1) = \phi(t_2) \cdot \phi(t_1)$$

$$\textcircled{5} \quad \phi(t_1-t_0) \cdot \phi(t_0-t_2) = \phi(t_1-t_2)$$

$$\phi(t) = e^{At}$$

$$\phi(t) = e^{At}$$

$$\phi(t_1-t_0) = e^{A(t_1-t_0)} = e^{At_1} \cdot e^{-At_0}$$

$$\phi(t_0-t_2) = e^{A(t_0-t_2)} = e^{At_0} \cdot e^{-At_2}$$

$$\phi(t_1-t_0) \cdot \phi(t_0-t_2) = e^{At_1} e^{-At_0} e^{At_0} e^{-At_2}$$

$$= e^{At_1 - At_0 + At_0 - At_2}$$

$$= e^{At_1 - At_2} = e^{A(t_1-t_2)}$$

$$\phi(t_1-t_0) \cdot \phi(t_0-t_2) = \phi(t_1-t_2)$$

$$\textcircled{P8} \quad \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$y = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad \text{Determine the state transition matrix?}$$

Sol: State Transition Matrix, $\phi(t) = L^{-1} [sI - A]^{-1}$

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}; \quad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}; \quad C = \begin{bmatrix} 1 & 1 \end{bmatrix}; \quad D = 0$$

$$(sI - A) = s \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (s-1) & 0-0 \\ 0-1 & (s-1) \end{bmatrix} = \begin{bmatrix} s-1 & 0 \\ -1 & s-1 \end{bmatrix}$$

$$\Rightarrow [sI - A]^{-1} = \frac{1}{(s-1)(s-1) - 0} \begin{bmatrix} (s-1) & 0 \\ +1 & (s-1) \end{bmatrix}$$

$$= \frac{1}{(s-1)^2} \begin{bmatrix} (s-1) & 0 \\ +1 & (s-1) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{(s-1)}{(s-1)^2} & 0 \\ \frac{1}{(s-1)^2} & \frac{(s-1)}{(s-1)^2} \end{bmatrix} = \begin{bmatrix} \frac{1}{(s-1)} & 0 \\ \frac{1}{(s-1)^2} & \frac{1}{(s-1)} \end{bmatrix}$$

$$L^{-1}[sI - A]^{-1} = L^{-1} \begin{bmatrix} \frac{1}{(s-1)} & 0 \\ \frac{1}{(s-1)^2} & \frac{1}{s-1} \end{bmatrix} = \begin{bmatrix} e^{+t} & 0 \\ te^{+t} & e^t \end{bmatrix}$$

$$\therefore \text{STM} = \phi(t) = L^{-1}[sI - A]^{-1} = \begin{bmatrix} e^t & 0 \\ te^t & e^t \end{bmatrix}$$

Q6) $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ Determine STM?

Sol: $A = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix}$

$$(sI - A) = s \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} (s-0) & (0-1) \\ 0-0 & s-(-2) \end{bmatrix} = \begin{bmatrix} s & -1 \\ 0 & s+2 \end{bmatrix}$$

$$\Rightarrow [sI - A]^{-1} = \frac{1}{s(s+2) + 0} \begin{bmatrix} s+2 & +1 \\ 0 & s+2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{(s+2)}{s(s+2)} & \frac{1}{s(s+2)} \\ 0 & \frac{s}{s(s+2)} \end{bmatrix} = \begin{bmatrix} \frac{1}{s} & \frac{1}{s(s+2)} \\ 0 & \frac{1}{s+2} \end{bmatrix}$$

$$\frac{1}{s(s+2)} = \frac{A}{s} + \frac{B}{s+2}$$

$$1 = A(s+2) + Bs$$

$$1 = (A+B)s + 2A$$

$$2A = 1$$

$$A = \frac{1}{2}$$

$$A+B = 0$$

$$B = -A$$

$$B = -\frac{1}{2}$$

$$\frac{1}{s(s+2)} = \frac{1/2}{s} - \frac{1/2}{s+2}$$

$$= \frac{1}{2s} - \frac{1}{2(s+2)}$$

$$\Rightarrow (sI - A)^{-1} = \begin{bmatrix} \frac{1}{s} & \frac{1}{2s} - \frac{1}{2(s+2)} \\ 0 & \frac{1}{s+2} \end{bmatrix}$$

$$\mathcal{L}^{-1}(sI - A)^{-1} = \begin{bmatrix} 1 & \frac{1}{2} - \frac{1}{2}e^{-2t} \\ 0 & e^{-2t} \end{bmatrix}$$

$$\textcircled{P8} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t)$$

$$y = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad \text{Determine STM ?}$$

$$\textcircled{P9} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad \text{Determine STM ?}$$