

1) The open loop transfer function of a unity feedback system is given by $G(s) = \frac{1}{s(1+s)(1+2s)}$. Sketch the Polar plot and determine the gain margin and Phase margin.

Ans:-

Given that $G(s) = \frac{1}{s(1+s)(1+2s)}$

Put $s = j\omega$

$\therefore G(j\omega) = \frac{1}{j\omega(1+j\omega)(1+2j\omega)}$

C.F. = $\frac{a}{b}$

$a+ib$

mag = $\sqrt{a^2+b^2}$

Phase $\phi = \tan^{-1}(b/a)$

The corner frequencies are $\omega_{c1} = \frac{1}{(1+2j\omega)} \Rightarrow \frac{1}{2}$

$\omega_{c1} = 0.5 \text{ rad/sec}$

$\left[\frac{1}{1+sT} = \frac{1}{T} = \omega_c \right]$

$\omega_{c2} = \frac{1}{(1+j\omega)}$

$\omega_{c2} = \frac{1}{1} \Rightarrow 1 \text{ rad/sec}$

The magnitude and phase angle of $G(j\omega)$ are calculated for the corner frequencies and for frequencies around corner frequencies.

The Polar coordinates listed in table-1 are converted to rectangular coordinates and tabulated in table-2. The Polar plot using rectangular coordinates is sketched on an ordinary graph sheet as shown in fig (1).

$G(j\omega) = \frac{1}{(j\omega)(1+j\omega)(1+2j\omega)}$

$= \frac{1}{\omega \angle 90^\circ \sqrt{1+\omega^2} \angle \tan^{-1}\omega \sqrt{1+4\omega^2} \angle \tan^{-1}2\omega}$

$G(j\omega) = \frac{1}{\omega \sqrt{(1+\omega^2)(1+4\omega^2)}} \angle -90^\circ - \tan^{-1}\omega - \tan^{-1}2\omega$

$\therefore |G(j\omega)| = \frac{1}{\omega \sqrt{(1+\omega^2)(1+4\omega^2)}} \Rightarrow \frac{1}{\omega \sqrt{1+5\omega^2+4\omega^4}}$

$\angle G(j\omega) = -90^\circ - \tan^{-1}\omega_1 - \tan^{-1}2\omega$

$\frac{1}{\omega \sqrt{1+5\omega^2+4\omega^4}}$

Table-1 :- Magnitude and Phase of $G(j\omega)$ at various frequencies.

ω rad/sec	0.35	0.4	0.45	0.5	0.6	0.7	1.0
$ G(j\omega) $	2.2	1.8	1.5	1.2	0.9	0.7	0.3
$\angle G(j\omega)$ deg.	-144	-150	-156	-162	-171	-179.5 ≈ -180	-198

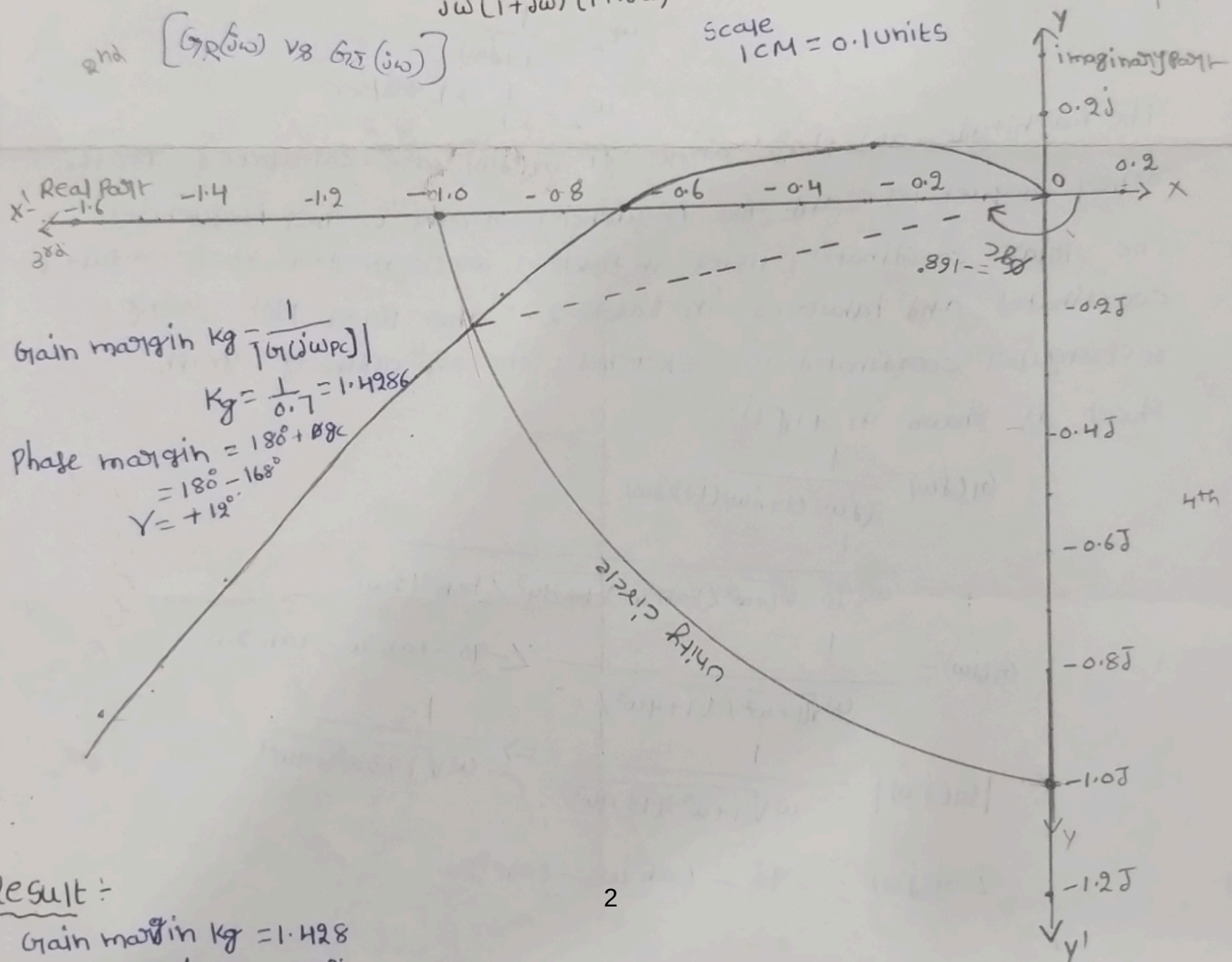
Table-2 :- Real and imaginary Part of $G(j\omega)$ at various frequencies.

ω rad/sec	0.35	0.4	0.45	0.5	0.6	0.7	1.0
$G_R(j\omega)$	-1.78	-1.56	-1.37	-1.14	-0.89	-0.7	-0.29
$G_I(j\omega)$	-1.29	-0.9	-0.61	-0.37	-0.14	0	0.09

Polar plot of $G(j\omega) = \frac{1}{j\omega(1+j\omega)(1+2j\omega)}$ (using rectangular coordinates).

2nd $[G_R(j\omega) \text{ vs } G_I(j\omega)]$

Scale
1CM = 0.1 units



Result :-

Gain margin $K_g = 1.428$
 Phase margin $\gamma = +12^\circ$