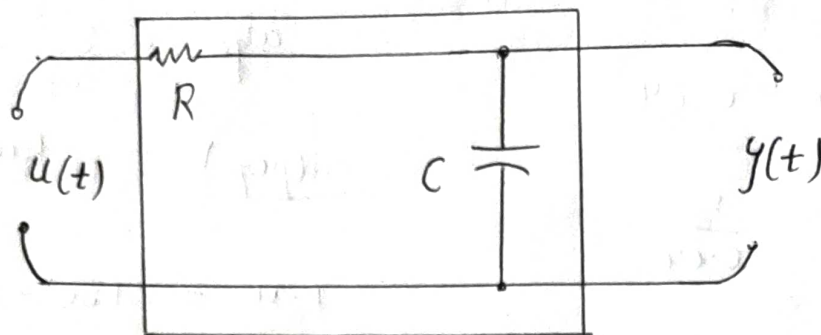


STATE - SPACE ANALYSIS

SINGLE-INPUT SINGLE-OUTPUT SYSTEM



System with physical components (R, C)

1. Mathematical Representation of Above system

$$H(s) = \frac{Y(s)}{U(s)} = \frac{1/RC}{s + 1/RC}$$

↳ when all initial conditions are Zero.

2. Mathematical Representation of above system if initial conditions are non-zero.

$$H(s) = \frac{Y(s)}{U(s)} = \frac{1/RC}{s + 1/RC}$$

$$\rightarrow sY(s) + \frac{1}{RC}Y(s) = \frac{1}{RC}U(s)$$

$$\boxed{\frac{dy(t)}{dt} + \frac{1}{RC}y(t) = \frac{1}{RC}u(t)} \rightarrow \text{Diff eqn}$$

MATH REP of RC LPF

Note: Taking ULT both sides

$$sY(s) - y(0^-) + \frac{1}{RC} Y(s) = \frac{1}{RC} X(s)$$

$$Y(s) = \frac{y(0^-)}{s + \frac{1}{RC}} + \frac{\frac{1}{RC} X(s)}{s + \frac{1}{RC}}$$

3. Mathematical Representation of above system in terms of state variable

$$\frac{dy(t)}{dt} + \frac{1}{RC} y(t) = \frac{1}{RC} u(t)$$

Let voltage across capacitor is state variable

$$y(t) = x(t)$$

$$\frac{dy(t)}{dt} = \dot{x}(t)$$

$$\dot{x}(t) = -\frac{1}{RC} x(t) + \frac{1}{RC} u(t)$$

STATE eqⁿ

$$y(t) = x(t)$$

OUTPUT eqⁿ

4. State model of above system

→ STATE VARIABLE REP.

$$\dot{x}(t) = -\frac{1}{RC} x(t) + \frac{1}{RC} u(t) \rightarrow \text{state eqⁿ}$$

$$y(t) = x(t) \rightarrow \text{output eqⁿ}$$

$$\begin{bmatrix} \dot{x}(t) \end{bmatrix}_{1 \times 1} = \begin{bmatrix} -\frac{1}{RC} \end{bmatrix}_{1 \times 1} \begin{bmatrix} x(t) \end{bmatrix}_{1 \times 1} + \begin{bmatrix} \frac{1}{RC} \end{bmatrix}_{1 \times 1} \begin{bmatrix} u(t) \end{bmatrix}_{1 \times 1}$$

$$\begin{bmatrix} y(t) \end{bmatrix}_{1 \times 1} = \begin{bmatrix} 1 \end{bmatrix}_{1 \times 1} \begin{bmatrix} x(t) \end{bmatrix}_{1 \times 1} + \begin{bmatrix} 0 \end{bmatrix}_{1 \times 1} \begin{bmatrix} u(t) \end{bmatrix}_{1 \times 1}$$



STATE MODEL REP. OF RC LPF

≡

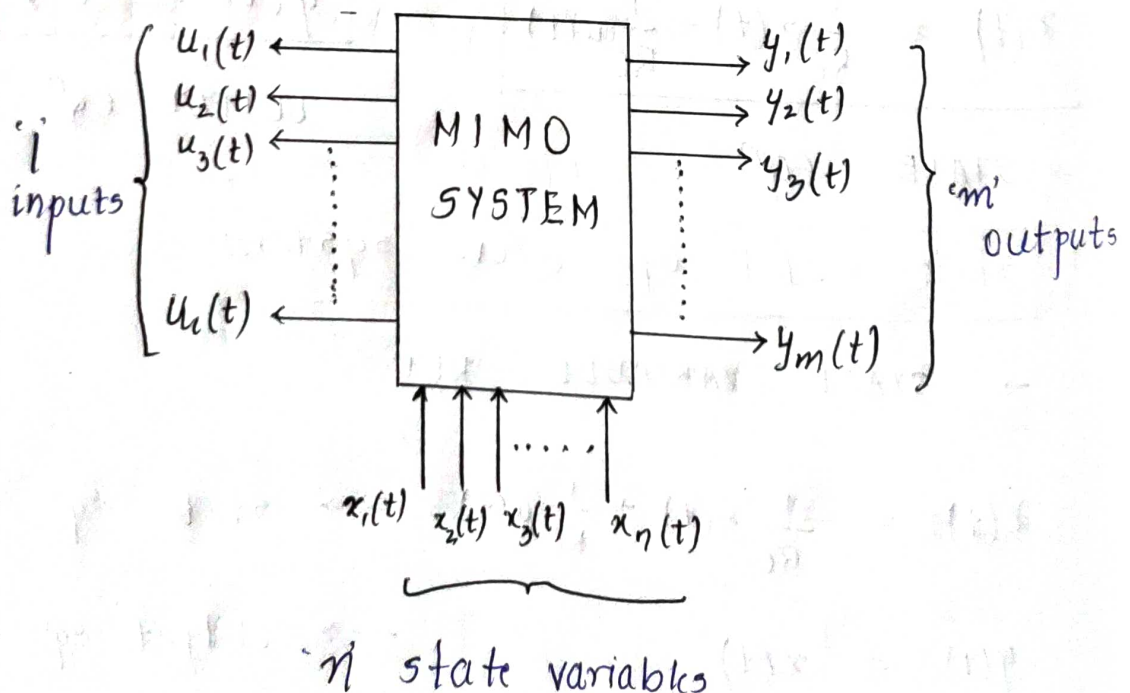
MATHEMATICAL REP. OF RC LPF

5. STATE MODEL REP. OF GENERAL SYSTEM

$$\begin{bmatrix} \dot{x}(t) \end{bmatrix} = \begin{bmatrix} A \end{bmatrix} \begin{bmatrix} x(t) \end{bmatrix} + \begin{bmatrix} B \end{bmatrix} \begin{bmatrix} u(t) \end{bmatrix} \rightarrow \text{state eqn}$$

$$\begin{bmatrix} y(t) \end{bmatrix} = \begin{bmatrix} C \end{bmatrix} \begin{bmatrix} x(t) \end{bmatrix} + \begin{bmatrix} D \end{bmatrix} \begin{bmatrix} u(t) \end{bmatrix} \rightarrow \text{output eqn}$$

→ Multiple Input - Multiple Output



→ For mathematical representation of above system, state model is used.

STATE EQUATION

$$\begin{bmatrix} \dot{x}(t) \end{bmatrix}_{n \times 1} = \begin{bmatrix} A \end{bmatrix}_{n \times n} \begin{bmatrix} x(t) \end{bmatrix}_{n \times 1} + \begin{bmatrix} B \end{bmatrix}_{n \times l} \begin{bmatrix} u(t) \end{bmatrix}_{l \times 1}$$

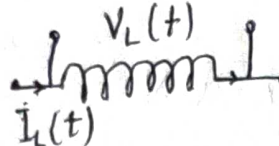
$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \vdots \\ \dot{x}_n(t) \end{bmatrix} = \begin{bmatrix} A \end{bmatrix}_{n \times n} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ \vdots \\ x_n(t) \end{bmatrix} + \begin{bmatrix} B \end{bmatrix}_{n \times l} \begin{bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_l(t) \end{bmatrix}_{l \times 1}$$

OUTPUT EQUATION

$$\begin{bmatrix} y(t) \end{bmatrix}_{m \times 1} = \begin{bmatrix} C \end{bmatrix}_{m \times n} \begin{bmatrix} x(t) \end{bmatrix}_{n \times 1} + \begin{bmatrix} D \end{bmatrix}_{m \times l} \begin{bmatrix} u(t) \end{bmatrix}_{l \times 1}$$

$$\begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_m(t) \end{bmatrix}_{m \times 1} = \begin{bmatrix} C \end{bmatrix}_{m \times n} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ \vdots \\ x_n(t) \end{bmatrix} + \begin{bmatrix} D \end{bmatrix}_{m \times l} \begin{bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_l(t) \end{bmatrix}_{l \times 1}$$

Important Observation

Case-1: Inductor 

(1) State model Representation:

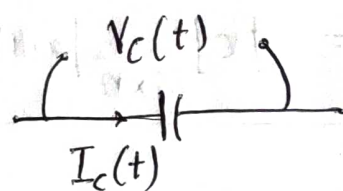
$$V_L(t) = L \frac{dI_L(t)}{dt}$$

$$\frac{dI_L(t)}{dt} = \frac{1}{L} V_L(t)$$

$$\dot{I}_L(t) = 0 I_L(t) + \frac{1}{L} V_L(t)$$

→ Current through inductor will behave as state variable.

→ Voltage across inductor will not behave as state variable.

Case-2: Capacitor 

(1) State model Representation:

$$I_C(t) = C \frac{dV_C(t)}{dt}$$

$$\dot{V}_C(t) = 0 V_C(t) + \frac{1}{C} I_C(t)$$

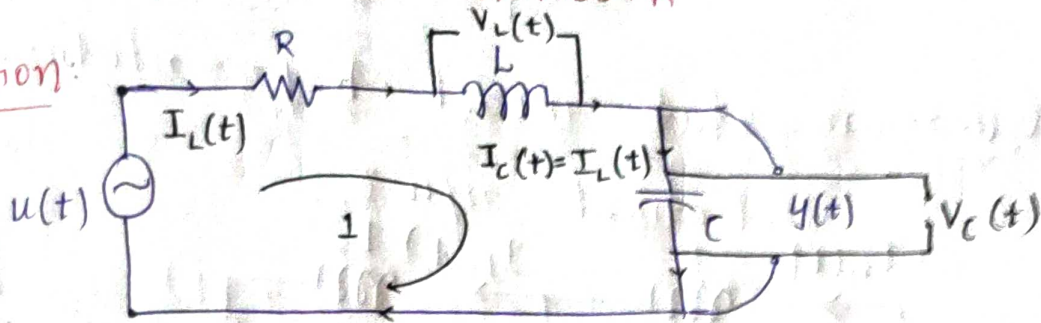
→ Voltage across capacitor is state variable.

→ Current through capacitor is not state variable.

MODEL - 1

"STATE MODEL" representation of electrical network

Question:



$$l = 1$$

$$n = 2$$

$$m = 1$$

KVL in loop 1:

$$R I_L(t) + L \frac{dI_L(t)}{dt} + V_C(t) = u(t)$$

$$\frac{dI_L(t)}{dt} = -\frac{R}{L} I_L(t) - \frac{1}{L} V_C(t) + \frac{1}{L} u(t)$$

$$\dot{I}_L(t) = -\frac{R}{L} I_L(t) - \frac{1}{L} V_C(t) + \frac{1}{L} u(t)$$

$$I_C(t) = I_L(t)$$

$$C \frac{dV_C(t)}{dt} = I_L(t)$$

$$\dot{V}_C(t) = \frac{1}{C} I_L(t) + 0 V_C(t) + 0 u(t)$$

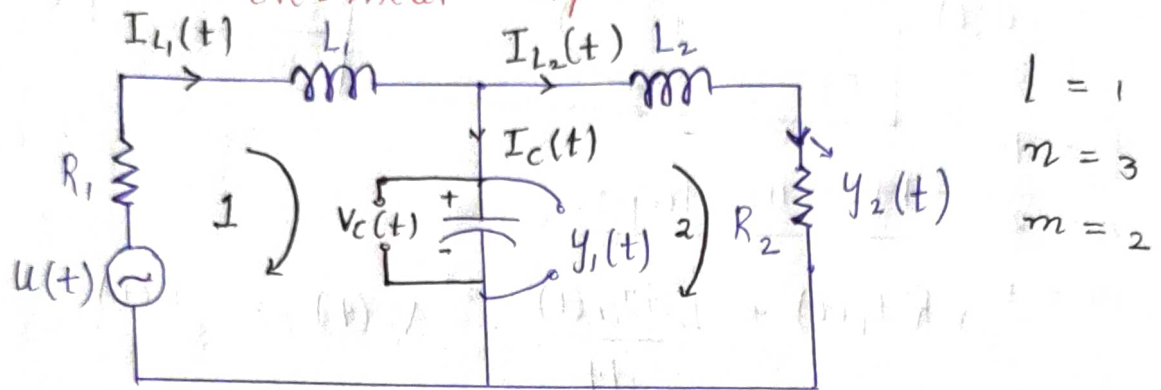
$$\begin{bmatrix} \dot{I}_L(t) \\ \dot{V}_C(t) \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & -\frac{1}{L} \\ \frac{1}{C} & 0 \end{bmatrix} \begin{bmatrix} I_L(t) \\ V_C(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} u(t)$$

$$y(t) = V_C(t)$$

$$y(t) = 0 I_L(t) + V_C(t) + 0 u(t)$$

$$\begin{bmatrix} y(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} I_L(t) \\ V_C(t) \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} u(t)$$

→ Question: state model Representation of electrical n/w?



KVL in loop 1:

$$L_1 \frac{dI_{L_1}(t)}{dt} + V_C(t) + R_1 I_{L_1}(t) = u(t)$$

$$\dot{I}_{L_1}(t) = -\frac{R_1}{L_1} I_{L_1}(t) - \frac{1}{L_1} V_C(t) + \frac{1}{L_1} u(t) \quad \text{--- (i)}$$

KVL in loop 2:

$$L_2 \frac{dI_{L_2}(t)}{dt} + R_2 I_{L_2}(t) - V_C(t) = 0$$

$$\dot{I}_{L_2}(t) = 0 I_{L_1}(t) - \frac{R_2}{L_2} I_{L_2}(t) + \frac{1}{L_2} V_C(t) \quad \text{--- (ii)}$$

KCL:

$$I_{L_1}(t) = I_C(t) + I_{L_2}(t)$$

$$C \frac{dV_C(t)}{dt} = I_{L_1}(t) - I_{L_2}(t)$$

$$\dot{V}_c(t) = \frac{1}{C} I_{L_1}(t) - \frac{1}{C} I_{L_2}(t) + 0V_c(t) + 0u(t) \quad \text{---(iii)}$$

STATE Eqⁿ:

$$\begin{bmatrix} \dot{I}_{L_1}(t) \\ \dot{I}_{L_2}(t) \\ \dot{V}_c(t) \end{bmatrix}_{3 \times 1} = \begin{bmatrix} -\frac{R_1}{L_1} & 0 & -\frac{1}{L_1} \\ 0 & -\frac{R_2}{L_2} & \frac{1}{L_2} \\ \frac{1}{C} & -\frac{1}{C} & 0 \end{bmatrix}_{3 \times 3} \begin{bmatrix} I_{L_1}(t) \\ I_{L_2}(t) \\ V_c(t) \end{bmatrix}_{3 \times 1} + \begin{bmatrix} \frac{1}{L_1} \\ 0 \\ 0 \end{bmatrix}_{3 \times 1} [u(t)]_{1 \times 1}$$

Output Eqⁿ:

$$y_1(t) = V_c(t)$$

$$y_2(t) = I_{L_2}(t)$$

$$y_1(t) = 0 I_{L_1}(t) + 0 I_{L_2}(t) + V_c(t) + 0u(t)$$

$$y_2(t) = 0 I_{L_1}(t) + I_{L_2}(t) + 0V_c(t) + 0u(t)$$

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix}_{2 \times 1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}_{2 \times 3} \begin{bmatrix} I_{L_1}(t) \\ I_{L_2}(t) \\ V_c(t) \end{bmatrix}_{3 \times 1} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}_{2 \times 1} [u(t)]_{1 \times 1}$$

MODEL - 2

State Model Representation from Differential Equation

→ Question: $\frac{d^3 y(t)}{dt^3} + 3 \frac{d^2 y(t)}{dt^2} + 6 \frac{dy(t)}{dt} + 7y(t) = 6u(t)$

Convert into state model Representation, where

$y \rightarrow o/p$, $u \rightarrow i/p$.

Let $y(t) = x_1(t)$

$$\frac{dy(t)}{dt} = \dot{x}_1(t) = x_2(t)$$

$$\frac{d^2y(t)}{dt^2} = \dot{x}_2(t) = x_3(t)$$

$$\frac{d^3y(t)}{dt^3} = \dot{x}_3(t) = -3x_3(t) - 6x_2(t) - 7x_1(t) + 6u(t)$$

$$\dot{x}_1(t) = 0x_1(t) + x_2(t) + 0x_3(t) + 0u(t)$$

$$\dot{x}_2(t) = 0x_1(t) + 0x_2(t) + x_3(t) + 0u(t)$$

$$\dot{x}_3(t) = -7x_1(t) - 6x_2(t) - 3x_3(t) + 6u(t)$$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -7 & -6 & -3 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 6 \end{bmatrix} [u(t)]$$

$$y(t) = x_1(t) + 0x_2(t) + 0x_3(t) + 0u(t)$$

$$[y(t)] = [1 \ 0 \ 0] \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + [0][u(t)]$$

→ Question: $\frac{d^4y(t)}{dt^4} + 6\frac{d^3y(t)}{dt^3} - 5\frac{d^2y(t)}{dt^2} + 10y(t) = 2u(t)$

$$y(t) = x_1(t)$$

$$\frac{dy(t)}{dt} = \dot{x}_1(t) = x_2(t)$$

$$\frac{d^2y(t)}{dt^2} = \dot{x}_2(t) = x_3(t)$$

$$\frac{d^3y(t)}{dt^3} = \dot{x}_3(t) = x_4(t)$$

$$\frac{d^4y(t)}{dt^4} = \dot{x}_4(t) = -6x_4(t) + 5x_3(t) + 0x_2(t) - 10x_1(t) + 2u(t)$$

$$\dot{x}_1(t) = 0x_1(t) + x_2(t) + 0x_3(t) + 0x_4(t) + 0u(t)$$

$$\dot{x}_2(t) = 0x_1(t) + 0x_2(t) + x_3(t) + 0x_4(t) + 0u(t)$$

$$\dot{x}_3(t) = 0x_1(t) + 0x_2(t) + 0x_3(t) + x_4(t) + 0u(t)$$

$$\dot{x}_4(t) = -10x_1(t) + 0x_2(t) + 5x_3(t) - 6x_4(t) + 2u(t)$$

$$\begin{matrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{matrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -10 & 0 & 5 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 2 \end{bmatrix} [u(t)]$$

$$y(t) = x_1(t)$$

$$[y(t)] = [1 \ 0 \ 0 \ 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + [0] [u(t)]$$

MODEL - 3 State Model Representation from Transfer function

Let T.F is

$$T(s) = \frac{b(s^3 + c_2 s^2 + c_1 s + c_0)}{s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0} \rightarrow \text{SISO} \quad (l=1, m=1)$$

Highest power in $D^r \Rightarrow n = 4$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix}_{4 \times 1} = \begin{bmatrix} A \end{bmatrix}_{4 \times 4} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}_{4 \times 1} + \begin{bmatrix} B \end{bmatrix}_{4 \times 1} \begin{bmatrix} u \end{bmatrix}_{1 \times 1}$$

$$\begin{bmatrix} y \end{bmatrix}_{1 \times 1} = \begin{bmatrix} C \end{bmatrix}_{1 \times 4} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}_{4 \times 1} + \begin{bmatrix} 0 \end{bmatrix}_{1 \times 1} \begin{bmatrix} u \end{bmatrix}_{1 \times 1}$$

Case-1: Controllable Canonical Form (C.C.F)

$$\frac{Y(s)}{U(s)} = \frac{b(s^3 + c_2 s^2 + c_1 s + c_0)}{1 \cdot s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}$$

$\xleftarrow{+ve}$
 $\xleftarrow{-ve}$

$$[A]_{4 \times 4} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 & -a_3 \end{bmatrix}$$

$$[B] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ b \end{bmatrix}$$

$$[C] = [c_0 \ c_1 \ c_2 \ c_3]$$

$$[D] = [0]$$

Note:

1. No. of state variables = Highest order of D'
2. Co-efficient of Highest order of D' should be 1.

→ Question: $\frac{Y(s)}{U(s)} = \frac{2s+3}{s^3+3s^2+2s+1}$ Controllable Canonical form?

$\xleftarrow{+ve}$
 $\xleftarrow{-ve}$

$$[A] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} \quad [B] = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$[C] = [3 \ 2 \ 0] \quad [D] = [0]$$

→ Question: $\frac{Y(s)}{U(s)} = \frac{3(s^3+4s^2-5s-2)}{2s^4+6s^3-4s^2-8s-10}$

Controllable Canonical form?

$$\frac{Y(s)}{U(s)} = \frac{\frac{3}{2}(s^3+4s^2-5s-2)}{(s^4+3s^3-2s^2-4s-5)}$$

$\xleftarrow{+ve}$
 $\xleftarrow{-ve}$

$$[A] = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 5 & 4 & 2 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{3}{2} \end{bmatrix}$$

$$[C] = [-2 \ -5 \ 4 \ 1] \quad D = [0]$$

Case-2: Observable Canonical Form (O.C.F)

$$\dot{X} = AX + BU$$

$$Y = CX + DU$$

$$[A]_{OCF} = [A]_{CCF}^T \quad [C]_{OCF} = [B]_{CCF}^T$$

$$[B]_{OCF} = [C]_{CCF}^T$$

→ Question: $\frac{Y(s)}{U(s)} = \frac{3s^2 + 5s + 2}{s^3 + 2s^2 - s - 1}$

C.C.F:

$$[A] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

$$[B] = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$[C] = [2 \ 5 \ 3]$$

$$[D] = [0]$$

O.C.F:

$$[A] = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & -2 \end{bmatrix}$$

$$[B] = \begin{bmatrix} 2 \\ 5 \\ 3 \end{bmatrix}$$

$$[C] = [0 \ 0 \ 1]$$

$$[D] = [0]$$

Case-3: Diagonal Canonical Form

(only when all poles of $T(s)$ is different)

$$T(s) = \frac{b(s^3 + c_2 s^2 + c_1 s + c_0)}{s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}$$

$$= \frac{b_1}{(s+p_1)} + \frac{b_2}{(s+p_2)} + \frac{b_3}{(s+p_3)} + \frac{b_4}{(s+p_4)}$$

$$[A] = \begin{bmatrix} -p_1 & 0 & 0 & 0 \\ 0 & -p_2 & 0 & 0 \\ 0 & 0 & -p_3 & 0 \\ 0 & 0 & 0 & -p_4 \end{bmatrix} \quad [B] = \begin{bmatrix} l_1 \\ l_2 \\ l_3 \\ l_4 \end{bmatrix}$$

$$[C] = [m_1 \ m_2 \ m_3 \ m_4]$$

$$l_1 m_1 = b_1$$

$$l_2 m_2 = b_2$$

$$l_3 m_3 = b_3$$

$$l_4 m_4 = b_4$$

→ Question:

$$\frac{Y(s)}{U(s)} = \frac{-1}{(s+1)(s+2)(s+3)} \quad \text{state Model Representation.}$$

Method -1:

$$\frac{Y(s)}{U(s)} = \frac{\frac{1}{2}}{(s+1)} + \frac{(-1)}{(s+2)} + \frac{\frac{1}{2}}{(s+3)}$$

$$[A] = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \quad [B] = \begin{bmatrix} \frac{1}{2} \\ 1 \\ \frac{1}{2} \end{bmatrix}$$

$$[C] = [1 \ -1 \ 1]$$

Method-2

$$\frac{Y(s)}{U(s)} = \frac{\frac{1}{2}}{(s+1)} + \frac{(-1)}{(s+2)} + \frac{\frac{1}{2}}{(s+3)}$$

$$Y(s) = \frac{\frac{1}{2} U(s)}{(s+1)} + \frac{-1 U(s)}{s+2} + \frac{\frac{1}{2} U(s)}{s+3}$$

$$Y(s) = X_1(s) + X_2(s) + X_3(s)$$

$$X_1(s) = \frac{\frac{1}{2} U(s)}{s+1}$$

$$X_2(s) = \frac{-U(s)}{s+2}$$

$$X_3(s) = \frac{\frac{1}{2} U(s)}{s+3}$$

$$sX_1(s) + X_1(s) = \frac{1}{2} U(s)$$

$\updownarrow$ ILT

$$\dot{x}_1(t) = -x_1(t) + \frac{1}{2} u(t)$$

$$sX_2(s) + 2X_2(s) = -U(s)$$

$\updownarrow$ ILT

$$\dot{x}_2(t) = -2x_2(t) - u(t)$$

$$sX_3(s) + 3X_3(s) = \frac{1}{2} U(s)$$

$\updownarrow$ ILT

$$\dot{x}_3(t) = -3x_3(t) + \frac{1}{2} u(t)$$

$$\dot{x}_1(t) = -x_1(t) + 0x_2(t) + 0x_3(t) + \frac{1}{2} u(t)$$

$$\dot{x}_2(t) = 0x_1(t) - 2x_2(t) + 0x_3(t) - u(t)$$

$$\dot{x}_3(t) = 0x_1(t) + 0x_2(t) - 3x_3(t) + \frac{1}{2} u(t)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} \frac{1}{2} \\ -1 \\ \frac{1}{2} \end{bmatrix} [u]$$

$$y(t) = x_1(t) + x_2(t) + x_3(t)$$

$$[y] = [1 \ 1 \ 1] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + [0][u]$$

Case-4: JORDAN CANONICAL FORM

→ It is the extension of "diagonal canonical form" when poles of T.F are repeated.

Case-1 $\frac{Y(s)}{U(s)} = \frac{a_0}{(s+p_1)^3} + \frac{a_1}{(s+p_1)^2} + \frac{a_2}{s+p_1}$

$$[A] = \begin{bmatrix} -p_1 & 1 & 0 \\ 0 & -p_1 & 1 \\ 0 & 0 & -p_1 \end{bmatrix} \rightarrow \text{JORDAN BLOCK}$$

Case-2 $\frac{Y(s)}{U(s)} = \frac{a_0}{(s+p_1)^2} + \frac{a_1}{s+p_1} + \frac{a_2}{s+p_2}$

$$[A] = \begin{bmatrix} \boxed{\begin{matrix} -p_1 & 1 \\ 0 & -p_1 \end{matrix}} & 0 \\ 0 & 0 & \boxed{-p_2} \end{bmatrix} \rightarrow \begin{matrix} \text{JORDAN BLOCK} \\ \text{JORDAN BLOCK} \end{matrix}$$

Case-3

$$\frac{Y(s)}{U(s)} = \frac{a_0}{(s+p_1)^3} + \frac{a_1}{(s+p_1)^2} + \frac{a_2}{(s+p_1)} + \frac{a_3}{(s+p_2)^2} + \frac{a_4}{(s+p_2)}$$

$$[A] = \begin{bmatrix} \boxed{\begin{matrix} -p_1 & 1 & 0 \\ 0 & -p_1 & 1 \\ 0 & 0 & -p_1 \end{matrix}} & \begin{matrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{matrix} \\ \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} & \boxed{\begin{matrix} -p_2 & 1 \\ 0 & -p_2 \end{matrix}} \end{bmatrix}$$

JORDAN BLOCK

JORDAN BLOCK

Case-4:

$$\frac{Y(s)}{U(s)} = \frac{a_0}{(s+p_1)^3} + \frac{a_1}{(s+p_1)^2} + \frac{a_2}{(s+p_1)} + \frac{a_3}{(s+p_2)^3} + \frac{a_4}{(s+p_2)^2} + \frac{a_5}{(s+p_2)}$$

$$[A] = \begin{bmatrix} \boxed{\begin{matrix} -p_1 & 1 & 0 \\ 0 & -p_1 & 1 \\ 0 & 0 & -p_1 \end{matrix}} & \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} \\ \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} & \boxed{\begin{matrix} -p_2 & 1 & 0 \\ 0 & -p_2 & 1 \\ 0 & 0 & -p_2 \end{matrix}} \end{bmatrix}$$

JORDAN BLOCK

JORDAN BLOCK

→ Question: $\frac{Y(s)}{U(s)} = \frac{1}{(s+2)^2(s+3)}$, state model=?

$$\frac{Y(s)}{U(s)} = \frac{1}{(s+2)^2} + \frac{-1}{s+2} + \frac{1}{s+3}$$

$$Y(s) = \frac{U(s)}{(s+2)^2} + \frac{-U(s)}{s+2} + \frac{U(s)}{s+3}$$

$$Y(s) = X_1(s) + X_2(s) + X_3(s)$$

$$X_3(s) = \frac{U(s)}{s+3}$$

$$X_2(s) = \frac{-U(s)}{s+2}$$

$$sX_3(s) + 3X_3(s) = U(s) \quad sX_2(s) + 2X_2(s) = -U(s)$$

$$\boxed{\dot{x}_3(t) = -3x_3(t) + u(t)}$$

$$\boxed{\dot{x}_2(t) = -2x_2(t) - u(t)}$$

$$* \quad X_1(s) = \frac{U(s)}{(s+2)^2}$$

$$X_1(s) = \frac{-X_2(s)}{s+2}$$

$$sX_1(s) + 2X_1(s) = -X_2(s)$$

$$\boxed{\dot{x}_1(t) = -2x_1(t) - x_2(t)}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -2 & -1 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} [u]$$

$$y(t) = x_1(t) + x_2(t) + x_3(t)$$

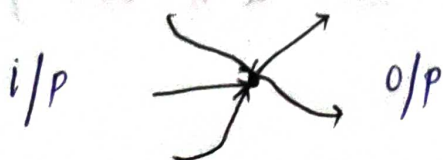
$$[y] = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + [0][u]$$

MODEL-4

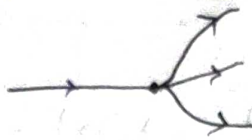
STATE MODEL REPRESENTATION FROM

SIGNAL FLOW GRAPH:

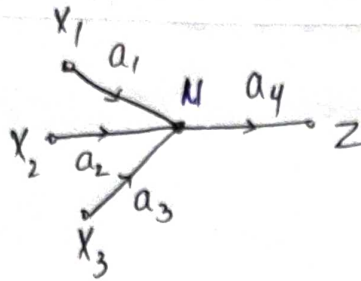
1. Summing Node: Multi-input



2. Take-off Node:



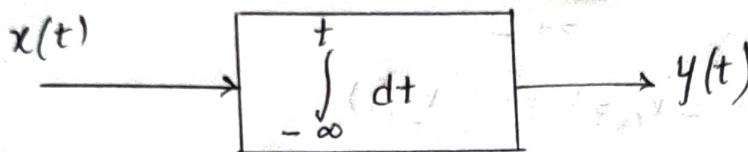
3. Potential of a Node:



$$N = a_1 x_1 + a_2 x_2 + a_3 x_3$$

$$z = a_4 N$$

→

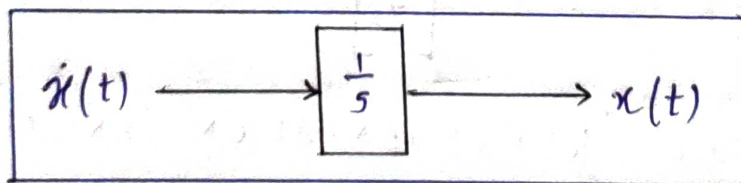
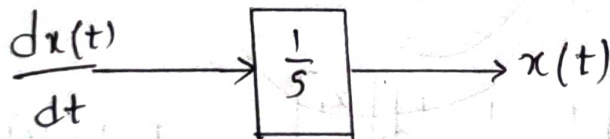


$$y(t) = \int_{-\infty}^t x(t) dt$$

$$Y(s) = \frac{X(s)}{s}$$

$$\frac{Y(s)}{X(s)} = \frac{1}{s}$$

→ Integrator T.F



4. Integrator Block:

→ o/p of an Integrator will always be a state variable.

5. Integrator SFG:



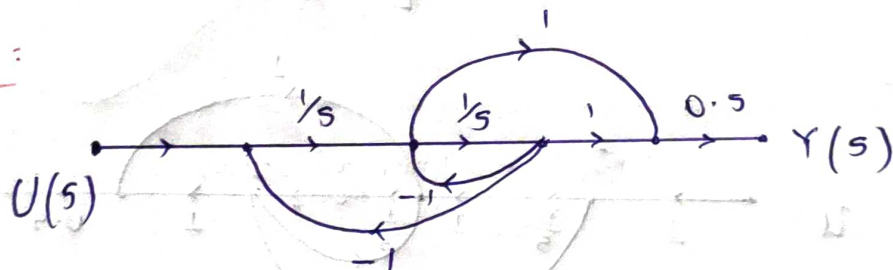
NOTE:

- O/P of an INTEGRATOR is state variable and state variable transfers to next node only if next node is "TAKE OFF NODE".

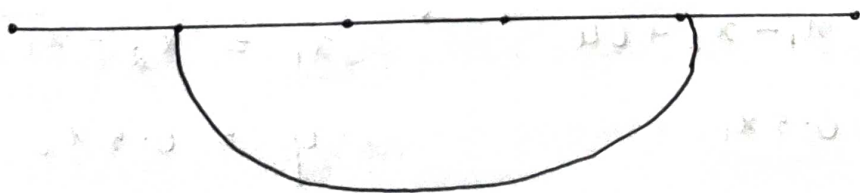
Problem solving approach:

1. No. of integrators = No. of state variables
2. Use concept of dummy variable to assign state variable.

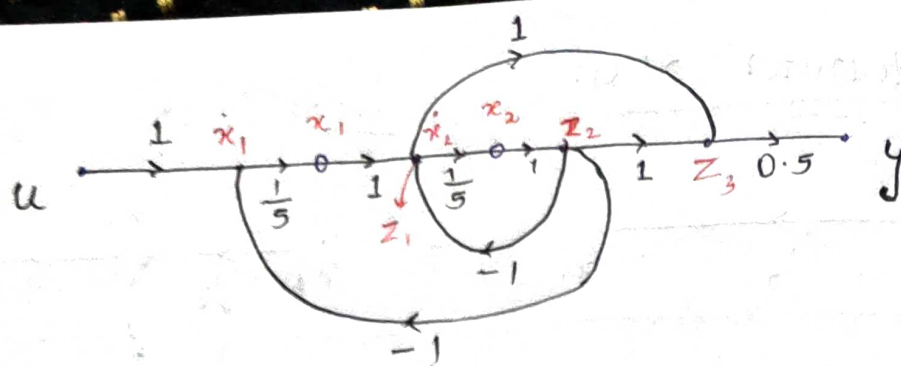
→ Question:



Obtain state model of above SFG.



I:



$$\dot{x}_1 = u - x_2$$

$$\dot{x}_2 = x_1 - x_2$$

$$y = 0.5 x_2$$

$$z_2 = x_2$$

$$\dot{x}_2 = x_1 - x_2$$

$$y = 0.5 (z_2 + \dot{x}_2)$$

$$\dot{x}_1 = u - x_2$$

$$y = 0.5 (x_2 + x_1 - x_2)$$

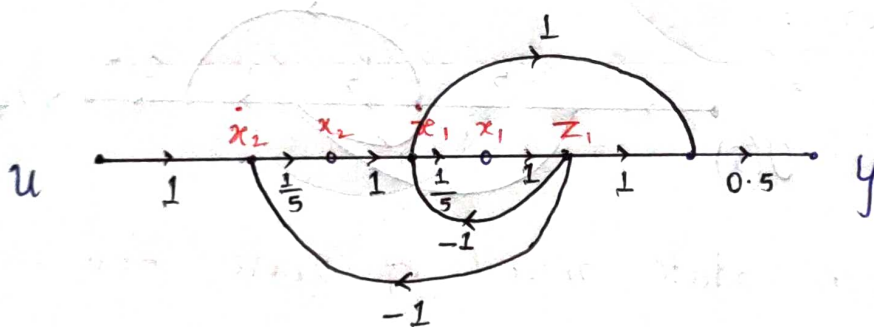
$$\dot{x}_1 = 0x_1 - x_2 + u$$

$$y = 0.5 x_1$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} [u]$$

$$[y] = [0.5 \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [0] [u]$$

II:



$$\dot{x}_1 = 0x_1 - x_2 + u$$

$$\dot{x}_2 = 0x_2 - x_1 + u$$

$$\dot{x}_2 = x_1 - x_2 + 0u$$

$$\dot{x}_1 = x_2 - x_1 + 0u$$

$$y = 0.5 x_1$$

$$y = 0.5 x_2$$

$$\dot{x}_1 = -x_1 + x_2 + 0u$$

$$\dot{x}_2 = 0x_1 - x_1 + 0x_2 + u$$

$$y = 0x_1 + 0.5x_2$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} [u]$$

$$[y] = \begin{bmatrix} 0 & 0.5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [0][u]$$

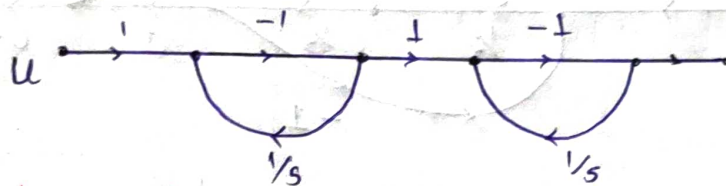
NOTE: If $x_1 \leftrightarrow x_2$

$$[A] \xrightarrow{\text{Row swap}} \xrightarrow{\text{Column swap}} [A]$$

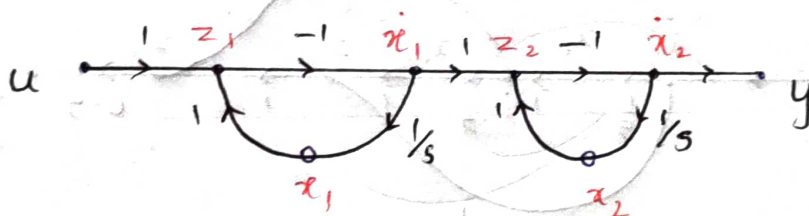
$$[B] \xrightarrow{\text{Row swap}} [B]$$

$$[C] \xrightarrow{\text{column swap}} [C]$$

Question:



Obtain state model of above system



$$\dot{x}_1 = -z_1$$

$$\dot{x}_2 = -z_2$$

$$z_1 = u + x_1$$

$$z_2 = \dot{x}_1 + x_2$$

$$y = \dot{x}_2$$

$$\dot{x}_1 = -x_1 - u$$

$$z_2 = -x_1 - u + x_2$$

$$y = x_1 - x_2 + u$$

$$\dot{x}_2 = -(x_1 + x_2 - u)$$

$$\dot{x}_2 = x_1 - x_2 + u$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} -1 \\ 1 \end{bmatrix} [u]$$

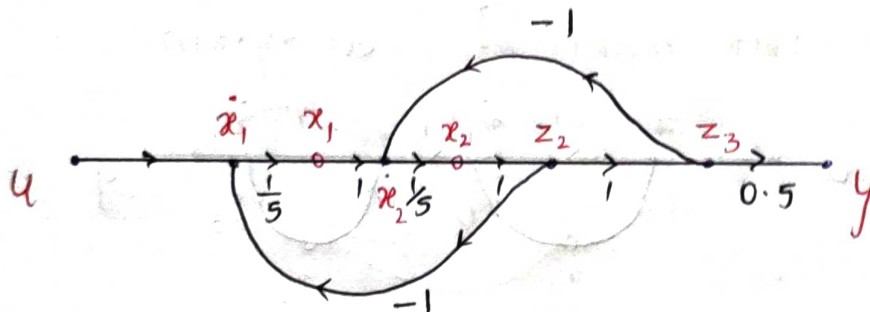
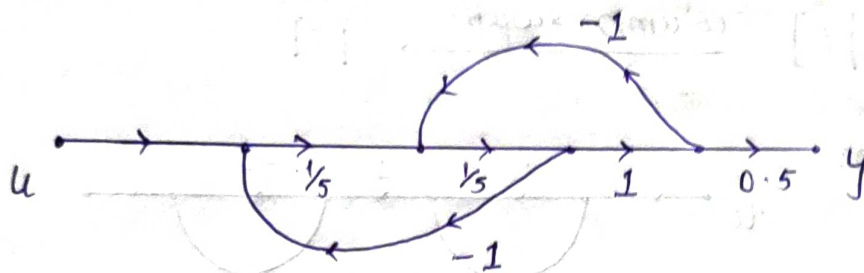
$$[y] = \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [1][u]$$

(OT)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} -1 \\ -1 \end{bmatrix} [u]$$

$$[y] = \begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [1][u]$$

Q.



$$\dot{x}_1 = u - z_2$$

$$\dot{x}_2 = x_1 - z_3$$

$$z_2 = x_2$$

$$z_3 = z_2$$

$$y = 0.5 z_3$$

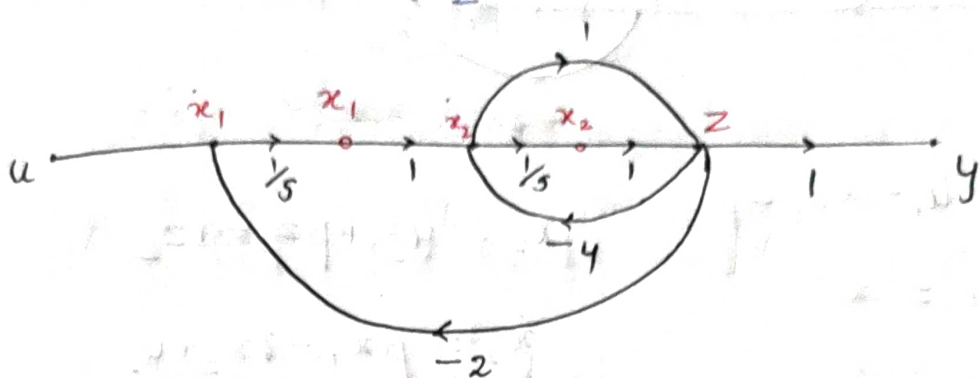
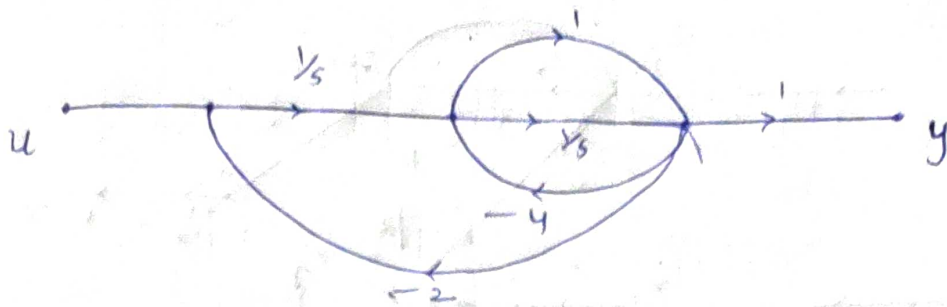
$$\dot{x}_1 = u - x_2$$

$$z_3 = x_2$$

$$y = 0.5 x_2$$

$$\dot{x}_1 = -x_2 + u$$

$$\dot{x}_2 = x_1 - x_2$$



$$\dot{x}_1 = u - 2z$$

$$z = x_2 + \dot{x}_2$$

$$\dot{x}_2 = x_1 - 4z$$

$$\dot{x}_1 = u - 2x_2$$

$$\dot{x}_2 = x_1 - \frac{4x_1}{5} - \frac{4x_2}{5}$$

$$\dot{x}_2 = x_1 - 4z$$

$$\dot{x}_2 = \frac{x_1}{5} - \frac{4x_2}{5}$$

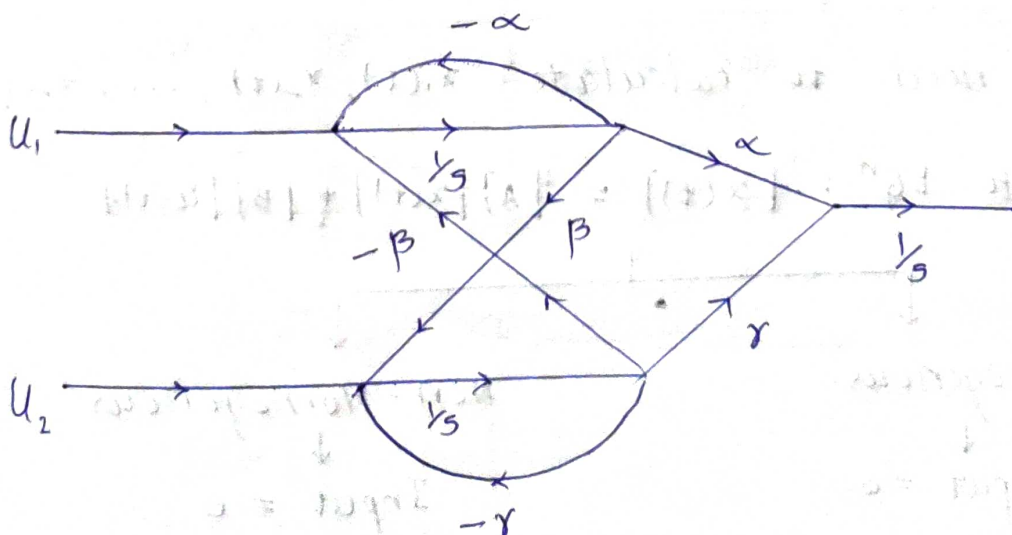
$$z = x_2 + x_1 - 4z$$

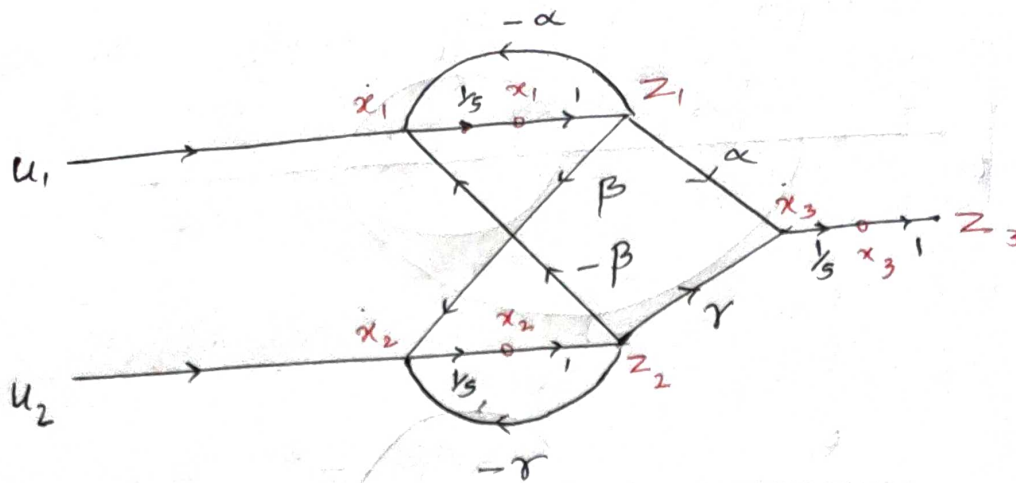
$$z = \frac{x_1}{5} + \frac{x_2}{5}$$

$$y = z$$

$$y = \frac{x_1}{5} + \frac{x_2}{5}$$

$$\dot{x}_1 = -\frac{2x_1}{5} - \frac{2x_2}{5} + u$$





$$\dot{x}_1 = u_1 - \alpha z_1 - \beta z_2 \quad \dot{x}_2 = u_2 + \beta z_1 - \gamma z_2$$

$$z_1 = x_1$$

$$z_2 = x_2$$

$$\dot{x}_2 = \beta x_1 - \gamma x_2 + u_2$$

$$\dot{x}_1 = u_1 - \alpha x_1 - \beta x_2$$

$$\dot{x}_3 = \alpha z_1 + \gamma z_2$$

$$\dot{x}_3 = \alpha x_1 + \gamma x_2$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix}_{3 \times 1} = \begin{bmatrix} -\alpha & -\beta & 0 \\ \beta & -\gamma & 0 \\ \alpha & \gamma & 0 \end{bmatrix}_{3 \times 3} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}_{3 \times 1} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}_{3 \times 2} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}_{2 \times 1}$$

MODEL - 5

Solution of State Variable

→ We need to calculate $x_1(t), x_2(t), \dots, x_n(t)$

$$\text{State Eq}^n: [\dot{x}(t)] = [A][x(t)] + [B][u(t)]$$

Homogenous

↓
Input = 0

Non-Homogenous

↓
Input $\neq 0$

$$[\dot{x}(t)] = [A][x(t)]$$

$$[\dot{x}(t)] = [A][x(t)] + [B][u(t)]$$

Generalised Process to solve State Variable:

$$[\dot{x}(t)] = [A][x(t)] + [B][u(t)]$$

Laplace on both sides

$$[sX(s) - x(0^-)] = [A][X(s)] + [B][U(s)]$$

$$[sX(s)] - [x(0^-)] = [A][X(s)] + [B][U(s)]$$

$$[sX(s)] - [A][X(s)] = [x(0^-)] + [B][U(s)]$$

$$[X(s)][sI - A] = [x(0^-)] + [B][U(s)]$$

$$[X(s)] = [sI - A]^{-1}[x(0^-)] + [sI - A]^{-1}[B][U(s)]$$

$$\begin{matrix} n=1 \\ l=1 \end{matrix} \quad [X(s)] = [sI - A]^{-1}[x(0^-)] + [sI - A]^{-1}[B][U(s)]$$

$$\begin{matrix} n=2 \\ l=2 \end{matrix} \quad \begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} = [sI - A]^{-1} \begin{bmatrix} x_1(0^-) \\ x_2(0^-) \end{bmatrix} + [sI - A]^{-1}[B] \begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix}$$

SOLUTION OF STATE VARIABLE:

$$[X(s)] = [sI - A]^{-1}[x(0^-)] + [sI - A]^{-1}[B][U(s)]$$

↓
solution of state
variable due to

initial condition

↓
solution of state
variable due to
input

Question: State Model of a system is given

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} [u]$$

Input is unit step. Calculate state variables

Solⁿ: $\rightarrow$ Non Homogeneous state Eqⁿ

$$[X(s)] = [sI - A]^{-1} [X(0^-)] + [sI - A]^{-1} [B] [U(s)]$$

$\downarrow$
[0]

$$[X(s)] = [sI - A]^{-1} [B] [U(s)]$$

$$[sI - A] = \begin{bmatrix} s-1 & 0 \\ 0 & s-2 \end{bmatrix}$$

$$[sI - A]^{-1} = \begin{bmatrix} \frac{1}{s-1} & 0 \\ 0 & \frac{1}{s-2} \end{bmatrix}$$

$$[X(s)] = \begin{bmatrix} \frac{1}{s-1} & 0 \\ 0 & \frac{1}{s-2} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \frac{1}{s}$$

$$[X(s)] = \begin{bmatrix} \frac{1}{s(s-1)} \\ \frac{1}{s(s-2)} \end{bmatrix} = \begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix}$$

$$X_1(s) = \frac{1}{s(s-1)} = \frac{1}{s-1} - \frac{1}{s} \Leftrightarrow$$

$$x_1(t) = [-1 + e^t] u(t)$$

$$X_2(s) = \frac{1}{s(s-2)} = \frac{-1}{2s} + \frac{1}{2(s-2)}$$

$$\leftrightarrow x_2(t) = \left(-\frac{1}{2} + \frac{1}{2} e^{2t} \right) u(t)$$

Question:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

Calculate $x_1(t)$ and $x_2(t)$ where $x_1(0^-) = 1$, $x_2(0^-) = 0$

→ Homogenous state Eqn

$$[X(s)] = [sI - A]^{-1} [x(0^-)] + \underbrace{[sI - A]^{-1} [B] [U(s)]}_{\downarrow 0}$$

$$[X(s)] = [sI - A]^{-1} [x(0^-)]$$

$$[sI - A] = \begin{bmatrix} s-1 & -2 \\ -3 & s+1 \end{bmatrix}$$

$$\rightarrow [sI - A]^{-1} = \frac{1}{s^2 - 1 - 6} \begin{bmatrix} s+1 & 2 \\ 3 & s-1 \end{bmatrix}$$

$$[X(s)] = \frac{1}{s^2 - 7} \begin{bmatrix} s+1 & 2 \\ 3 & s-1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \frac{1}{s^2 - 7} \begin{bmatrix} s+1 \\ 3 \end{bmatrix}$$

$$X_1(s) = \frac{s+1}{s^2 - 7} \longleftrightarrow$$

$$X_2(s) = \frac{3}{s^2 - 7} \longleftrightarrow$$

State Transmission

State Transition Matrix (S.T.M)

1. S.T.M in s-domain: $[sI - A]^{-1} \rightarrow n \times n$ SQUARE MATRIX
2. S.T.M in time domain: $[\phi(t)] = [e^{At}] \rightarrow$

$$[\phi(t)]_{n \times n} \xleftrightarrow[\text{I.L.T}]{\text{L.T}} [sI - A]^{-1}$$

Q. $\dot{X} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} X + \begin{bmatrix} 0 \\ 1 \end{bmatrix} U$ S.T.M = ?

$$[sI - A] = \begin{bmatrix} s-1 & 0 \\ 0 & s-2 \end{bmatrix}$$

$$[sI - A]^{-1} = \begin{bmatrix} \frac{1}{s-1} & 0 \\ 0 & \frac{1}{s-2} \end{bmatrix}$$

$$[\phi(t)] = [e^{At}] = \begin{bmatrix} e^t & 0 \\ 0 & e^{2t} \end{bmatrix}$$

$$[\phi(t)]_{t=0} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_n$$

$$\left[\frac{d\phi(t)}{dt} \right]_{t=0} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

Properties of S.T.M:

$$1. \left[(e^{At})_{t=0} \right] = \left[\phi(t)_{t=0} \right] = [I_n]$$

$$2. \left[\left(\frac{d e^{At}}{dt} \right)_{t=0} \right] = \left[\left(\frac{d \phi(t)}{dt} \right)_{t=0} \right] = [A]$$

$$3. [\phi(-t)] = [\phi^{-1}(t)]$$

$$4. [\phi(t_1 + t_2)] = [\phi(t_1) \phi(t_2)]$$

$$5. [\phi^k(t)] = [\phi(kt)]$$

$$e^{A(-t)} = (e^{At})^{-1}$$

$$e^{A(t_1 + t_2)} = e^{At_1} e^{At_2}$$

$$e^{Akt} = (e^{At})^k$$

Homogenous State Equation

$$\dot{x}(t) = [A] x(t)$$

$$\text{SOLUTION: } [X(s)] = [sI - A]^{-1} [x(0^-)]$$

$$\updownarrow$$
$$[x(t)] = [\phi(t)] [x(0^-)]$$

$$\hookrightarrow [\phi(t)] = \text{ILT}([sI - A]^{-1})$$

Dependent on A only

$$[x_1(t)] = [\phi(t)] [x_1(0^-)]$$

$$[x_2(t)] = [\phi(t)] [x_2(0^-)]$$

$$[M x_1(t) + N x_2(t)] = [\phi(t)] [M x_1(0^-) + N x_2(0^-)]$$

Question: $\dot{x}(t) = Ax(t)$ and $x(0) = x_0$

Given: $x_0 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \rightarrow x(t) = \begin{pmatrix} e^{-t} \\ -e^{-t} \end{pmatrix}$

$x_0 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow x(t) = \begin{pmatrix} e^{-t} - e^{-2t} \\ e^{-t} + e^{-2t} \end{pmatrix}$

Calculate $x(t)$ for $x_0 = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$

METHOD - 1:

Sol: $\rightarrow$ Homogenous state eqn

$$[X(s)] = [sI - A]^{-1} [x(0^-)]$$

$$[x(t)] = [\phi(t)] [x(0^-)]$$

$$\begin{bmatrix} e^{-t} \\ -e^{-t} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$A - B = e^{-t} - 1$$

$$C - D = -e^{-t} - 2$$

$$\begin{bmatrix} e^{-t} - e^{-2t} \\ e^{-t} + e^{-2t} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$B = e^{-t} - e^{-2t} \rightarrow A = 2e^{-t} - e^{-2t}$$

$$D = e^{-t} + e^{-2t} \rightarrow C = e^{-2t}$$

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 2e^{-t} - e^{-2t} & e^{-t} - e^{-2t} \\ e^{-2t} & e^{-t} + e^{-2t} \end{bmatrix} \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$x_1(t) = 6e^{-t} - 3e^{-2t} + 5e^{-t} - 5e^{-2t} = 11e^{-t} - 8e^{-2t}$$

$$x_2(t) = 3e^{-2t} + 5e^{-t} + 5e^{-2t} = 5e^{-t} + 8e^{-2t}$$

METHOD - II :

$$\begin{bmatrix} M \\ -M \end{bmatrix} + \begin{bmatrix} 0 \\ N \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$M = 3, N = 8$$

$$3 \begin{pmatrix} e^{-t} \\ -e^{-t} \end{pmatrix} + 5 \begin{pmatrix} e^{-t} - e^{-2t} \\ e^{-t} + e^{-2t} \end{pmatrix} = \phi(t) \left\{ 3 \begin{pmatrix} 1 \\ -1 \end{pmatrix} + 5 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right\}$$

$$\begin{pmatrix} 11e^{-t} - 8e^{-2t} \\ 5e^{-t} + 8e^{-2t} \end{pmatrix} = \phi(t) \begin{pmatrix} 3 \\ 5 \end{pmatrix}$$

NON - HOMOGENOUS STATE EQⁿ

SOL:

$$[X(s)] = [sI - A]^{-1} [x(0^-)] + [sI - A]^{-1} [B] [U(s)]$$

$$[X(s)] \xrightleftharpoons[L.T]{I.L.T} [x(t)]$$

STATE MODEL

$$\dot{X} = AX + BU$$

SOLUTION OF : $[X(s)] = [sI - A]^{-1} [x(0^-)] + [sI - A]^{-1} [B] [U(s)]$

STATE VARIABLE

STATE MODEL:

$$\dot{X} = A^T X + (B^T U)$$

SOLUTION OF : $[X(s)] = [sI - A^T]^{-1} [x(0^-)] + [sI - A^T]^{-1} [B^T] [U(s)]$

STATE VARIABLE

Question: state eqⁿ

$$\dot{x}_1(t) = -x_1(t) + x_2(t)$$

$$\dot{x}_2(t) = x_1(t) - x_2(t)$$

$$x_1(0^-) = x_2(0^-) < \infty$$

$$x_{1f} = \lim_{t \rightarrow \infty} x_1(t) \quad x_{2f} = \lim_{t \rightarrow \infty} x_2(t)$$

Relation b/w x_{1f} and x_{2f}

Solⁿ

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \rightarrow \text{Homogeneous}$$

$$[X(s)] = [sI - A]^{-1} [x(0^-)]$$

$$[sI - A] = \begin{bmatrix} s+1 & -1 \\ -1 & s+1 \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{1}{s(s+2)} \begin{bmatrix} s+1 & 1 \\ 1 & s+1 \end{bmatrix}$$

$$\begin{bmatrix} x_1(s) \\ x_2(s) \end{bmatrix} = \frac{1}{s(s+2)} \begin{bmatrix} s+1 & 1 \\ 1 & s+1 \end{bmatrix} \begin{bmatrix} x_1(0^-) \\ x_2(0^-) \end{bmatrix}$$

$$X_1(s) = \frac{(s+1)x_1(0^-) + x_2(0^-)}{s(s+2)}$$

$$x_{1f} = \lim_{t \rightarrow \infty} x_1(t) = \lim_{s \rightarrow 0} s X_1(s)$$

$$x_{1f} = \frac{x_1(0^-) + x_2(0^-)}{2}$$

$$X_2(s) = \frac{x_1(0^-) + (s+1)x_2(0^-)}{s(s+2)}$$

$$x_{2f} = \lim_{t \rightarrow \infty} x_2(t) = \lim_{s \rightarrow 0} s X_2(s)$$

$$x_{2f} = \frac{x_1(0^-) + x_2(0^-)}{2}$$

$$x_{1f} = x_{2f}$$

Question:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -9 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{pmatrix} 0 \\ 45 \end{pmatrix} u(t) \quad \text{step i/p}$$

$$\begin{bmatrix} x_1(0^-) \\ x_2(0^-) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \text{calculate } \lim_{t \rightarrow \infty} \sqrt{x_1^2(t) + x_2^2(t)}$$

sol:

$$[X(s)] = [sI - A][x(0^-)] + [sI - A]^{-1}[B][U(s)]$$

$$[X(s)] = [sI - A]^{-1}[B][U(s)]$$

$$[sI - A] = \begin{bmatrix} s & 0 \\ 0 & s+9 \end{bmatrix} \rightarrow [sI - A]^{-1} = \frac{1}{s(s+9)} \begin{bmatrix} s+9 & 0 \\ 0 & s \end{bmatrix}$$

$$X(s) = \frac{1}{s(s+9)} \begin{bmatrix} s+9 & 0 \\ 0 & s \end{bmatrix} \begin{bmatrix} 0 \\ 45 \end{bmatrix} \begin{bmatrix} 1 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} x_1(s) \\ x_2(s) \end{bmatrix} = \frac{1}{s(s+9)} \begin{bmatrix} 0 \\ 45s \end{bmatrix} \frac{1}{s} = \begin{bmatrix} 0 \\ \frac{45}{s(s+9)} \end{bmatrix}$$

$$\begin{aligned} \lim_{t \rightarrow \infty} \sqrt{x_1^2(t) + x_2^2(t)} &= \sqrt{x_1^2(\infty) + x_2^2(\infty)} \\ &= \sqrt{0 + 5^2} = 5 \end{aligned}$$

Question: $\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} u(t)$

$\begin{pmatrix} x_1(0^-) \\ x_2(0^-) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$. Cal $x_1(t)$ and $x_2(t)$?

Solⁿ:

$$[sI - A] = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} s-1 & 0 \\ -1 & s-1 \end{bmatrix}$$

$$[sI - A]^{-1} = \frac{1}{(s-1)^2} \begin{bmatrix} s-1 & 0 \\ 1 & s-1 \end{bmatrix}$$

$$[X(s)] = [sI - A]^{-1} [x(0^-)] + [sI - A]^{-1} [B] [U(s)]$$

$$[X(s)] = \frac{1}{(s-1)^2} \begin{bmatrix} s-1 & 0 \\ 1 & s-1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \frac{1}{(s-1)^2} \begin{bmatrix} s-1 & 0 \\ 1 & s-1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \frac{1}{s}$$

$$[X(s)] = \frac{1}{(s-1)^2} \begin{bmatrix} s-1 \\ 1 \end{bmatrix} + \frac{1}{(s-1)^2} \begin{bmatrix} s-1 \\ s \end{bmatrix} \frac{1}{s}$$

$$[X(s)] = \frac{1}{(s-1)^2} \begin{bmatrix} s-1 + \frac{s-1}{s} \\ -1 + 1 \end{bmatrix} = \frac{1}{(s-1)^2} \begin{bmatrix} \frac{(s-1)(s+1)}{s} \\ 0 \end{bmatrix}$$

$$X_1(s) = \frac{s+1}{s(s-1)^2} \longleftrightarrow x_1(t) = (-1 + 2e^t) u(t)$$

$$X_2(s) = \frac{2}{(s-1)^2} \longleftrightarrow x_2(t) = 2te^t u(t)$$