UNIT - 1

System : A set of physical components connected to do some work (or) To perform some work (or) To get some objective

To control → to regulate (or) to command

Control System : A set of physical components connected to do desired work (or) to get desired output.

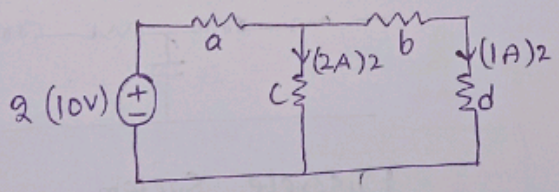
Types of Systems :

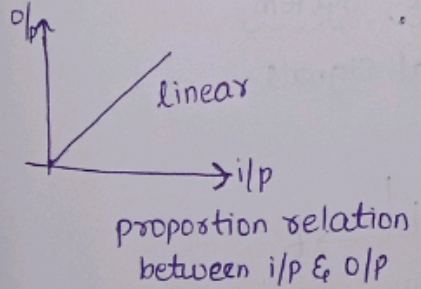
1) Linear and Non-linear
2) Time varient and Time Invarient
3) Unidirectional and Bidirectional
4) Lumped and Distributed
5) Continuous and Discrete
6) Open Loop and closed Loop

$V = iR$ R
$\psi = Li$ L
$q = CV$ C
} ohm's law formulae

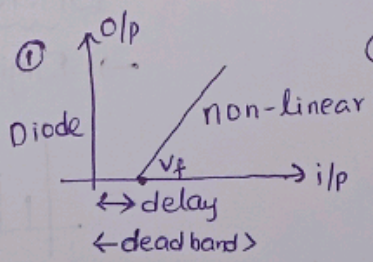
Linear :-

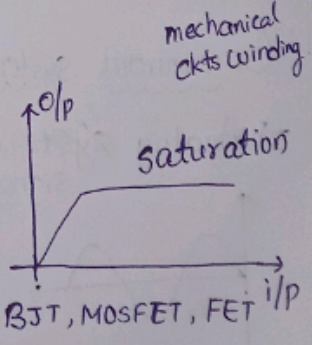
1) Should satisfy ohm's law
2) Output should follow input (slope is same)
3) Should contain slope when characteristics drawn b/w (i/p vs o/p)
4) Should satisfy superposition theorem (i/p_1 & $i/p_2 \rightarrow o/p_1 + o/p_2$)
5) Should satisfy homogenity principle





proportion relation between i/p & o/p

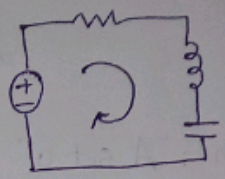




BJT, MOSFET, FET

Time Varient & Time Invarient

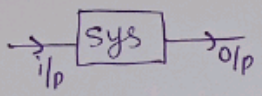
$$V = iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$



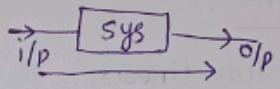
⇒ If the circuit parameters (R, L, C) have constant values or time independent then it is known as Time invarient system.

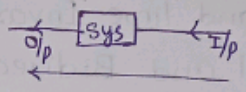
⇒ If the circuit parameters (R, L, C) are time dependent then it is known as Time varient system.

Uni directional



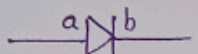
Bi directional





eg: R, L, C

Diode



$V_a > V_b \rightarrow$ S.C

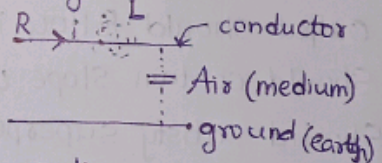
$V_a < V_b \rightarrow$ O.C

Lumped System

→ which can be separated physically are known as Lumped elements. eg: R, L, C, FET, diode,

Distributed System

→ which can't be separated physically

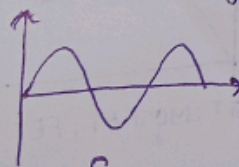


eg: Power lines



Continuous system

→ Analog ~~system~~ signals

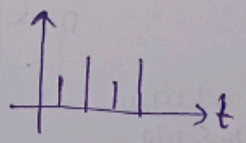


fourier series

Z-Transform

Discrete System

→ Digital signals



fourier transform

Transfer function

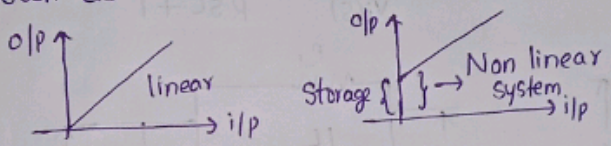
$T.F = \frac{L[o/p]}{L[i/p]}$ with initial conditions zero.

1) Electrical systems
2) Mechanical systems
3) Overall system → Block diagram reduction method
4) Signal flow graph
5) Mason's gain formula

Initial conditions → storage values

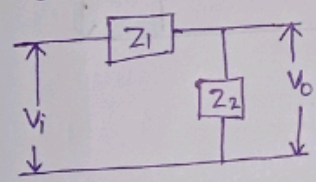
Storage elements → L, C

⇒ In a linear time invarient system, the ratio of laplace transform of output to laplace transform of input with initial conditions are zero is known as "Transfer Function".



Transfer function for electrical systems :

① find the TF for the given N/w

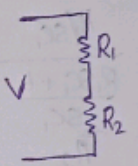


$T.F = \frac{V_o(s)}{V_i(s)}$

$V_o = \frac{Z_2}{Z_1 + Z_2} V_i$

$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{Z_2}{Z_1 + Z_2}}$$

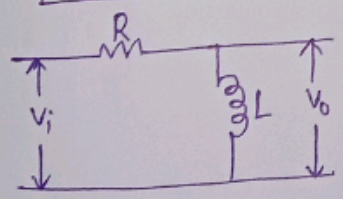
⇒ Voltage Division rule



$V_1 = \frac{R_1}{R_1 + R_2} V$

$V_2 = \frac{R_2}{R_1 + R_2} V$

② find $\frac{V_o(s)}{V_i(s)}$



$V_o(s) = \frac{SL}{R + SL} V_i(s)$

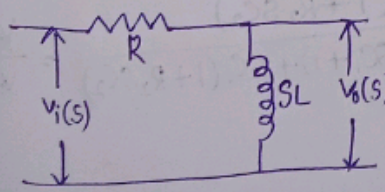
Apply laplace transform,

$V_i \rightarrow V_i(s)$, $V_o \rightarrow V_o(s)$

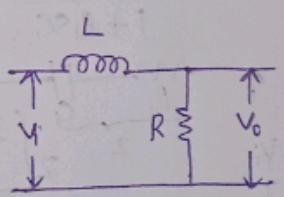
$R \rightarrow R$, $L \rightarrow SL$, $C \rightarrow \frac{1}{SC}$

$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{SL}{R + SL}}$$

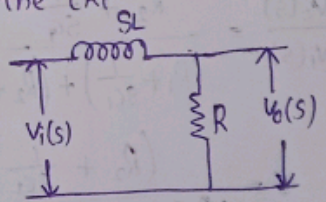
Redraw ckt with LT



③



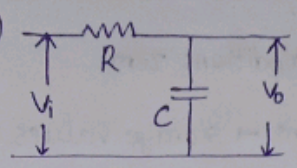
Apply LT and redraw the ckt



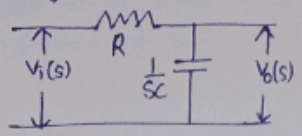
$V_o(s) = \frac{R}{R + SL} V_i(s)$

$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{R}{R + SL}}$$

④



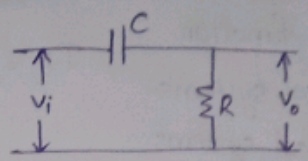
Apply LT and redraw



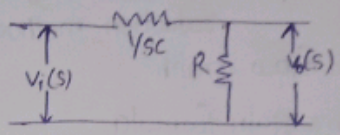
$$V_0(s) = \frac{1/sC}{R + \frac{1}{sC}} V_i(s) = \frac{\frac{1}{sC}}{\frac{RSC+1}{sC}} V_i(s)$$

$$\frac{V_0(s)}{V_i(s)} = \frac{1}{1+RSC}$$

⑤



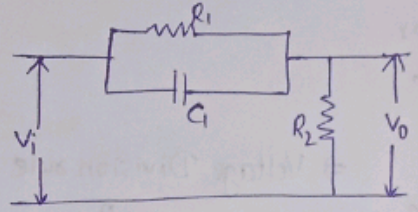
Apply LT and redraw



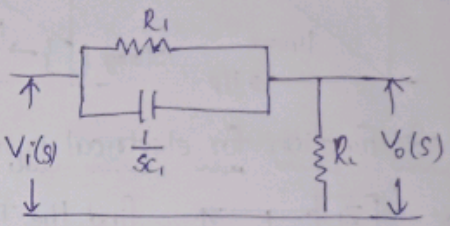
$$V_0(s) = \frac{R}{R + \frac{1}{sC}} V_i(s)$$

$$\frac{V_0(s)}{V_i(s)} = \frac{RSC}{RSC+1}$$

⑥



⇒

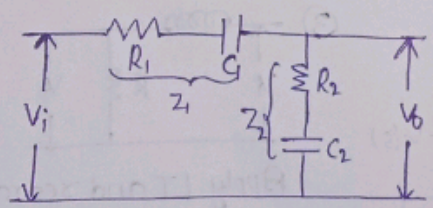


$$Z_1 = \frac{R_1 * \frac{1}{sC_1}}{R_1 + \frac{1}{sC_1}} = \frac{R_1/sC_1}{\frac{R_1sC_1+1}{sC_1}} = \frac{R_1}{1+R_1sC_1}$$

$$\frac{V_0(s)}{(s)} = \frac{Z_2}{Z_1+Z_2} = \frac{R_2}{R_2+Z_1} = \frac{R_2}{R_2+\left(\frac{R_1}{1+sR_1C_1}\right)}$$

$$= \frac{R_2}{\frac{R_2(1+sR_1C_1)+R_1}{1+R_1sC_1}} = \frac{R_2(1+R_1sC_1)}{R_2(1+sR_1C_1)+R_1}$$

⑦



$$\frac{V_0(s)}{V_i(s)} = \frac{R_2 + \frac{1}{sC_2}}{\left(R_1 + \frac{1}{sC_1}\right) + \left(R_2 + \frac{1}{sC_2}\right)}$$

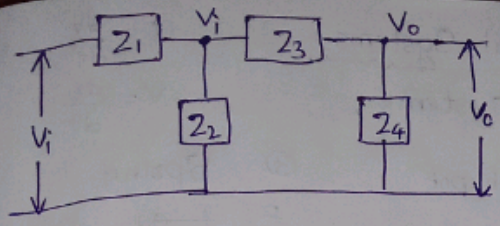
$$= \frac{\left(R_2 + \frac{1}{sC_2}\right)}{\left(\frac{R_1sC_1+1}{sC_1}\right) + \left(\frac{R_2sC_2+1}{sC_2}\right)}$$

$$= \frac{\left(\frac{R_2sC_2+1}{sC_2}\right)}{\left(\frac{R_1sC_1+1}{sC_1}\right) + \left(\frac{R_2sC_2+1}{sC_2}\right)}$$

$$\Rightarrow \frac{\frac{R_2sC_2+1}{sC_2}}{\frac{sC_2(R_1sC_1+1)+sC_1(1+R_2sC_2)}{sC_1 \cdot sC_2}}$$

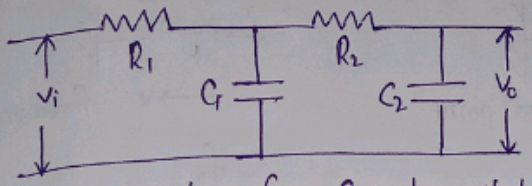
$$\Rightarrow \frac{sC_1(1+R_2sC_2)}{sC_2(R_1sC_1+1)+sC_1(1+R_2sC_2)} = \frac{V_0(s)}{V_i(s)}$$

II)

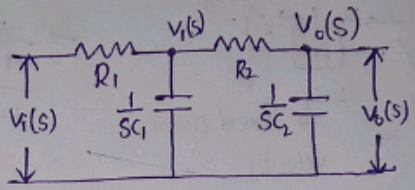


Apply mesh or nodal analysis

I)



$\Rightarrow$



Apply Laplace transform & redraw ckt

Apply nodal analysis

$$\frac{V_1(s)-V_i(s)}{R_1}+\frac{V_1(s)}{1/sC_1}+\frac{V_1(s)-V_o(s)}{R_2}=0 \rightarrow (1)$$

$$\frac{V_1(s)}{R_1}-\frac{V_i(s)}{R_1}+\frac{V_1(s)}{1/sC_1}+\frac{V_1(s)}{R_2}-\frac{V_o(s)}{R_2}=0$$

$$V_1(s)\left[\frac{1}{R_1}+sC_1+\frac{1}{R_2}\right]-\frac{V_o(s)}{R_2}=\frac{V_i(s)}{R_1}$$

$$V_1(s)\left[\frac{R_2+R_1R_2sC_1+R_1}{R_1R_2}\right]-\frac{V_o(s)}{R_2}=\frac{V_i(s)}{R_1} \rightarrow (2)$$

$$\frac{V_o(s)-V_1(s)}{R_2}+\frac{V_o(s)}{1/sC_2}=0 \rightarrow (3)$$

$$\frac{V_o(s)}{R_2}-\frac{V_1(s)}{R_2}+sC_2V_o(s)=0$$

$$-\frac{V_i(s)}{R_2}+V_o(s)\left[\frac{1}{R_2}+sC_2\right]=0$$

$$-\frac{V_i(s)}{R_2}+V_o(s)\left[\frac{1+R_2sC_2}{R_2}\right]=0 \rightarrow (4)$$

$$\frac{V_i(s)}{R_2}=V_o(s)\left[\frac{1+R_2sC_2}{R_2}\right]$$

$$V_i(s)=(1+R_2sC_2)V_o(s) \rightarrow (5)$$

Sub (5) in (2)

$$V_o(s)(1+sR_2C_2)\left[\frac{R_1+R_2+R_1R_2sC_2}{R_1R_2}\right]-\frac{V_o(s)}{R_2}=\frac{V_i(s)}{R_1}$$

$$V_o(s)\left[\frac{(1+sR_2C_2)(R_1+R_2+R_1R_2sC_2)}{R_1R_2}-\frac{1}{R_2}\right]=\frac{V_i(s)}{R_1}$$

$$V_o(s)\left[\frac{(1+sR_2C_2)(R_1+R_2+R_1R_2sC_2)-R_1}{R_1R_2}\right]=\frac{V_i(s)}{R_1}$$

$$\frac{V_o(s)}{V_i(s)}=\frac{R_2}{(1+sR_2C_2)(R_1+R_2+R_1R_2sC_2)-R_1}$$

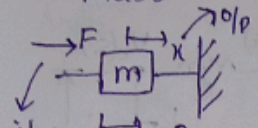
Transfer function for Mechanical Systems:

$a = \frac{dv}{dt}$

$v = \frac{dx}{dt}$

Basic elements of mechanical systems:

① Mass

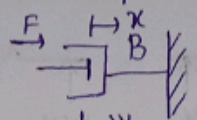


f_m → opposite to applied force

$f_m = m.a = m.\frac{dv}{dt}$

$= m\frac{d^2x}{dt^2}$

② Damper / Dash-pot

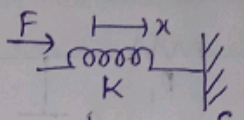


f_B ←

$f_B = B.v$

$= B.\frac{dx}{dt}$

③ Spring

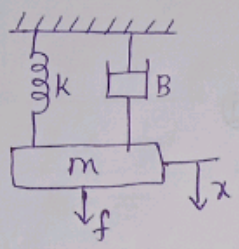


f_k ←

$f_k = k.x$

$= k\int v\,dt$

Q)



find TF ?

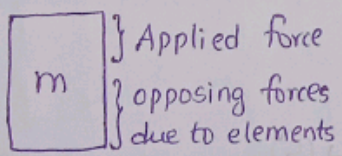
A:→ Nodal method:

→ No. of nodes = no. of mass elements

No. of differential equations = no. of mass elements = 1

→ free body diagram at 'm'

No. of free body diagrams = no. of mass elements = 1



→ Write force-balance equation, $\Sigma f = 0$

$\Sigma f_{applied} - \Sigma f_{opposing} = 0$

$$\boxed{\Sigma f_{applied} = \Sigma f_{opposing}}$$

$f = f_m + f_k + f_B$

→ Write differential equations

$$f = m.\frac{d^2x}{dt^2} + k.x + B\frac{dx}{dt} \rightarrow ①$$

Apply laplace transform

$f \rightarrow f(s)$ $\quad \frac{d^2}{dt^2} = s^2$

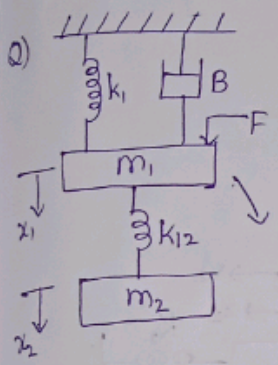
$x \rightarrow x(s)$ $\quad \frac{d}{dt} = s$

$\Rightarrow f(s) = m.s^2 X(s) + K.X(s) + B.s.X(s)$

$f(s) = X(s)[ms^2 + Bs + K]$

$$\boxed{\frac{X(s)}{f(s)} = \frac{1}{ms^2+Bs+K}}$$

2)

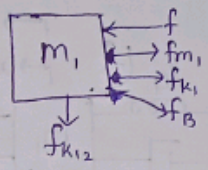


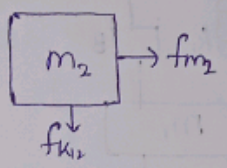
→ No. of nodes = No. of mass elements = 2

→ free-body diagrams = No. of mass elements = 2

→ free-body diagram at m_1 $\quad$ free-body diagram at m_2

$\frac{X_1(s)}{f(s)}$ =





→ force-balance equation,

$f_{applied} = f_{opposing}$

$f = f_{m_1} + f_{k1} + f_B + f_{k12}$

$0 = f_{m_2} + f_{k12}$

→ Write differential equations,

No. of differential equations = no. of nodes = no. of mass elements = 2

$$f = m_1 \frac{d^2x_1}{dt^2} + k_1 x_1 + B.\frac{dx_1}{dt} + k_{12}(x_1 - x_2) \rightarrow (1)$$

$$0 = m_2 \frac{d^2x_2}{dt^2} + k_{12}(x_2 - x_1) \rightarrow (2)$$

→ Apply laplace transform

$(1) \Rightarrow f(s) = m_1 s^2 X_1(s) + K_1 X_1(s) + B.s.X_1(s) + K_{12}X_1(s) - K_{12}X_2(s)$

$f(s) = X_1(s)[m_1 s^2 + K_1 + Bs + K_{12}] - K_{12}X_2(s) \rightarrow (3)$

$(2) \Rightarrow 0 = m_2 s^2 X_2(s) + K_{12}X_2(s) - K_{12}X_1(s)$

$0 = X_2(s)[m_2 s^2 + K_{12}] - K_{12}X_1(s) \rightarrow (4)$

$$X_2(s) = \frac{K_{12}X_1(s)}{(m_2 s^2 + K_{12})} \rightarrow (5)$$

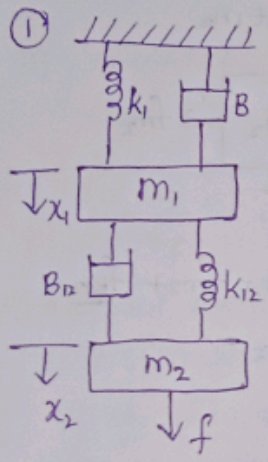
Substitute ⑤ in ③,

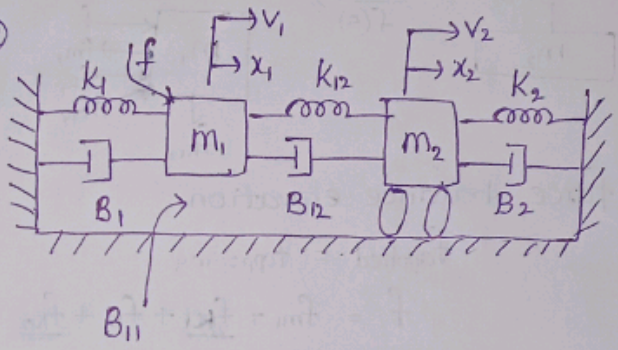
$$f(s) = X_1(s)\left[m_1 s^2 + Bs + K_1 + K_{12}\right] - K_{12}\frac{K_{12}}{m_2 s^2 + K_{12}} X_1(s)$$

$$= X_1(s)\left[(m_1 s^2 + Bs + K_1 + K_{12}) - \frac{K_{12}\,K_{12}}{m_2 s^2 + K_{12}}\right]$$

$$f(s) = X_1(s)\left[\frac{(m_1 s^2 + Bs + K_1 + K_{12})(m_2 s^2 + K_{12}) - K_{12}^2}{m_2 s^2 + K_{12}}\right]$$

$$\frac{X_1(s)}{f(s)} = \frac{m_2 s^2 + K_{12}}{(m_1 s^2 + Bs + K_1 + K_{12})(m_2 s^2 + K_{12}) - K_{12}^2}$$

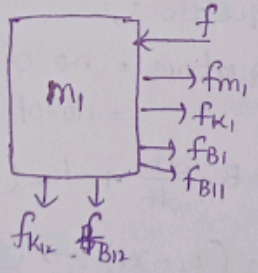
① 

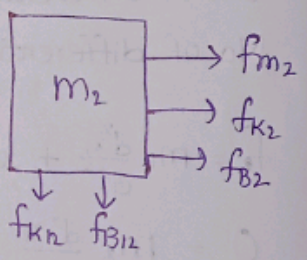
② 

→ Write differential eq'ns

① displacement

② velocity





$$f = f_{m_1} + f_{K_1} + f_{B_1} + f_{B_{11}} + f_{K_{12}} + f_{B_{12}}$$

$$0 = f_{m_2} + f_{K_2} + f_{B_2} + f_{K_{12}} + f_{B_{12}}$$

$$f = m_1 \frac{d^2x_1}{dt^2} + K_1 x_1 + B_1 \frac{dx_1}{dt} + B_{11} \frac{dx_1}{dt} + K_{12}(x_1 - x_2) + B_{12} \frac{d}{dt}(x_1 - x_2)$$

$$0 = m_2 \frac{d^2x_2}{dt^2} + K_2 x_2 + B_2 \frac{dx_2}{dt} + K_{12}(x_2 - x_1) + B_{12} \frac{d}{dt}(x_2 - x_1)$$

velocity

$$f = m_1 \frac{dv_1}{dt} + k_1 \int v_1\,dt + B_1 v_1 + B_{11} v_1 + k_{12} \int (v_1 - v_2)\,dt + B_{12}(v_1 - v_2)$$

$$0 = m_2 \frac{dv_2}{dt} + k_2 \int v_2\,dt + B_2 v_2 + k_{12} \int (v_2 - v_1)\,dt + B_{12}(v_2 - v_1)$$

Apply laplace transform,

$$\Rightarrow f(s) = m_1 s^2 X_1(s) + k_1 X_1(s) + B_1 s X_1(s) + B_{11} s X_1(s) + k_{12} X_1(s) - k_{12} X_2(s) + B_{12} s X_1(s) - B_{12} s X_2(s)$$

$$f(s) = X_1(s)\left[m_1 s^2 + k_1 + B_1 s + B_{11} s + k_{12} + B_{12} s\right] - X_2(s)\left[k_{12} + B_{12} s\right] \rightarrow (1)$$

$$\Rightarrow 0 = m_2 s^2 X_2(s) + k_2 X_2(s) + B_2 s X_2(s) + k_{12} X_2(s) - k_{12} X_1(s) + B_{12} s X_2(s) - B_{12} s X_1(s)$$

$$0 = X_2(s)\left[m_2 s^2 + k_2 + B_2 s + k_{12} + B_{12} s\right] - X_1(s)\left[k_{12} + B_{12} s\right] \rightarrow (2)$$

from eq (2),

$$X_2(s) = \frac{(k_{12} + B_{12} s)}{(m_2 s^2 + k_2 + B_2 s + k_{12} + B_{12} s)} X_1(s)$$

Substituting value of $X_2(s)$ in eq (1), we get

$$f(s) = X_1(s)\left[m_1 s^2 + k_1 + B_1 s + B_{11} s + k_{12} + B_{12} s\right] - \left[k_{12} + B_{12} s\right]\left[\frac{(k_{12} + B_{12} s)}{(m_2 s^2 + k_2 + B_2 s + k_{12} + B_{12} s)} X_1(s)\right]$$

$$f(s) = X_1(s)\left[\frac{(m_1 s^2 + B_1 s + B_{11} s + B_{12} s + k_1 + k_{12})(m_2 s^2 + k_2 + B_2 s + k_{12} + B_{12} s) - (k_{12} + B_{12} s)^2}{(m_2 s^2 + B_2 s + B_{12} s + k_2 + k_{12})}\right]$$

$$\frac{X_1(s)}{f(s)} = \frac{(m_2 s^2 + B_2 s + B_{12} s + k_2 + k_{12})}{(m_1 s^2 + B_1 s + B_{11} s + B_{12} s + k_1 + k_{12})(m_2 s^2 + B_2 s + B_{12} s + k_2 + k_{12}) - (k_{12} + B_{12} s)^2}$$

from eq (2),

$$X_1(s) = \frac{(m_2 s^2 + k_2 + B_2 s + k_{12} + B_{12} s)}{(k_{12} + B_{12} s)} X_2(s)$$

Substituting value of $x_1(s)$ in eq (1), we get

$$f(s) = \frac{(m_2 s^2 + k_2 + B_2 s + k_{12} + B_{12} s)}{(k_{12} + B_{12} s)}(m_1 s^2 + k_1 + B_1 s + B_{11} s + k_{12} + B_{12} s) X_2(s) - X_2(s)\left[k_{12} + B_{12} s\right]$$

$$f(s) = X_2(s)\left[\frac{(m_2 s^2 + k_2 + B_2 s + k_{12} + B_{12} s)(m_1 s^2 + k_1 + B_1 s + B_{11} s + k_{12} + B_{12} s) - (k_{12} + B_{12} s)^2}{(k_{12} + B_{12} s)}\right]$$

$$\frac{X_2(s)}{f(s)} = \frac{(k_{12} + B_{12} s)}{(m_2 s^2 + B_2 s + B_{12} s + k_2 + k_{12})(m_1 s^2 + k_1 + B_1 s + B_{11} s + k_{12} + B_{12} s) - (k_{12} + B_{12} s)^2}$$

Analogeous circuits

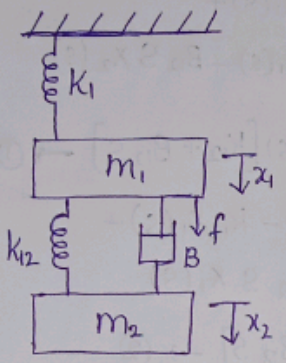
→ Electrical equivalent circuit of mechanical systems.

→ Mechanical ⟶ Electrical

→ 1) f - v [force - voltage analogeous ckt] → mesh ckt

2) f - i [force - current analogeous ckt] → nodal ckt

→



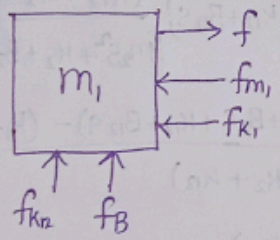
1) Write differential equations
2) find TF
3) Draw f-v, f-i ckts
4) Write mesh and nodal equations for the ckt.

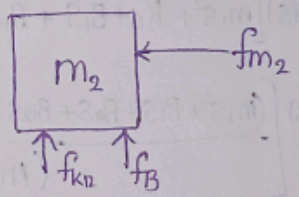
→ No. of nodes = No. of mass elements = 2

→ Draw free body diagram.

No. of free-body diagrams = No. of mass elements = 2

→ free-body diagram at m_1 free-body diagram at m_2





→ force - balance equation, $\Sigma f_{applied} = \Sigma f_{opposing}$

$f = fm_1 + f_{k1} + f_{k12} + f_B$ $\qquad 0 = fm_2 + f_{k12} + f_B$

→ Differential equations

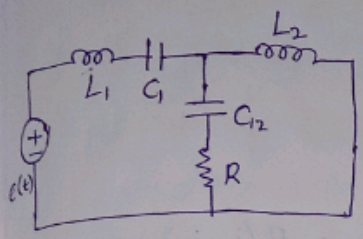
No. of equations = No. of nodes = No. of mass elements = 2

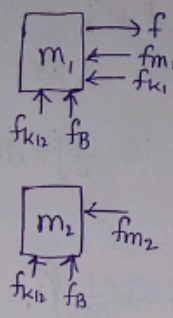
$$f = m_1 \frac{d^2x_1}{dt^2} + k_1x_1 + k_{12}(x_1 - x_2) + B\frac{d}{dt}(x_1 - x_2)$$

$$0 = m_2 \frac{d^2x_2}{dt^2} + k_{12}(x_2 - x_1) + B\frac{d}{dt}(x_2 - x_1)$$

f-v

① Observe free body diagram





f-v	f-i
B → R	→ R
M → L	→ C
K → C	→ L

Series ——

Parallel | ⇒ common

② Replace B, M, K with R, L, C

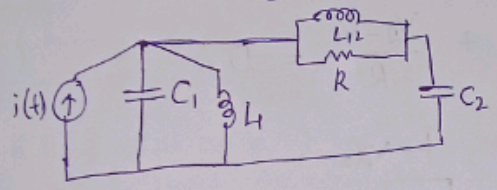
f-i

① observe f-v ckt

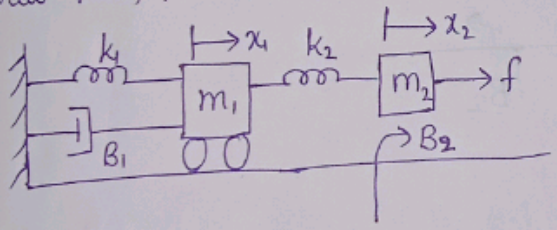
② Replace Series branch → parallel branch

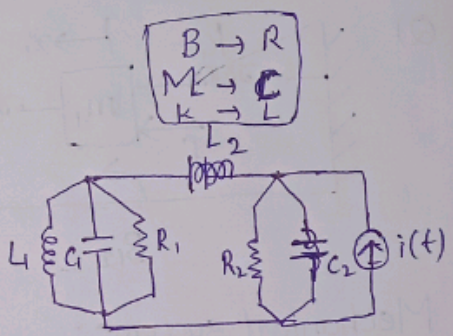
Series elements → parallel elements

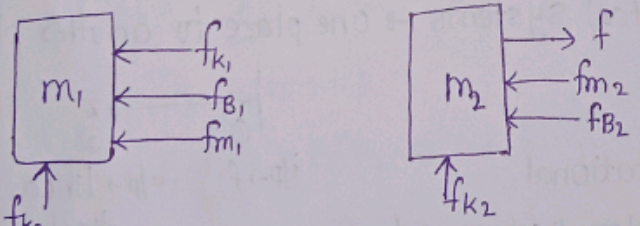
③ B → R ; M → C ; K → L

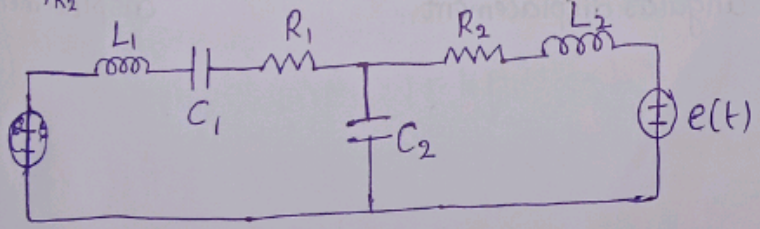


Draw f-i, f-v ckt's



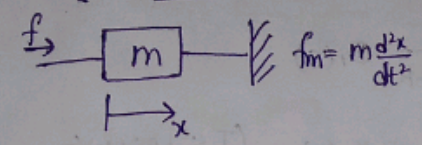




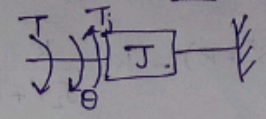


B → R
M → L
K → C

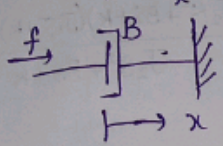
*** Translational $\frac{X(s)}{F(s)}$ Rotational $\frac{\theta(s)}{T(s)}$



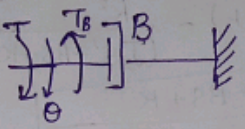
$f_m = m\frac{d^2x}{dt^2}$



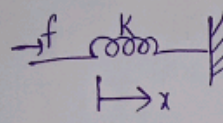
$T_j = J\frac{d^2\theta}{dt^2}$



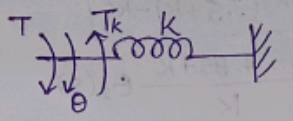
$f_B = B.\frac{dx}{dt}$



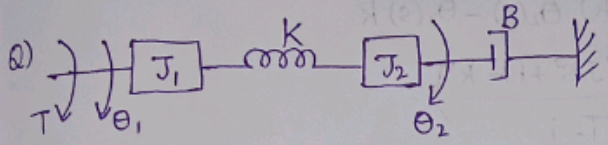
$T_B = B.\frac{d\theta}{dt}$



$f_k = K.x$



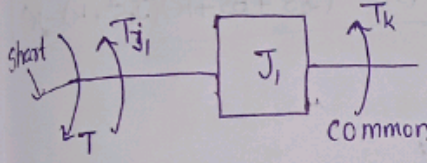
$T_k = K.\theta$

Q) 

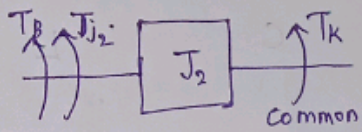
find T.F.

1) $\frac{\theta_1(s)}{T(s)}$ (or) 2) $\frac{\theta_2(s)}{T(s)}$

A: → free body diagram at J_1 (Shopt)



→ free body diagram at J_2



→ Torque-balance equations.

$\Rightarrow \sum T = 0$

$\sum T_{applied} = \sum T_{opposing}$

$T = T_{J_1} + T_k$

$T = J_1.\frac{d^2\theta_1}{dt^2} + k(\theta_1 - \theta_2) \rightarrow ①$

$\Rightarrow 0 = T_{J_2} + T_B + T_k$

$0 = J_2\frac{d^2\theta_2}{dt^2} + K(\theta_2 - \theta_1) + B.\frac{d\theta_2}{dt} \rightarrow ②$

→ Apply Laplace Transform

$T(s) = J_1 s^2 \theta_1(s) + K\theta_1(s) - K\theta_2(s) \rightarrow ③$

$0 = J_2 s^2 \theta_2(s) + K\theta_2(s) - K\theta_1(s) + B.s.\theta_2(s)$

$= \theta_2(s)[J_2 s^2 + K + B.s] - K\theta_1(s)$

$\theta_2(s) = \frac{K}{J_2 s^2 + K + B.s}\theta_1(s) \rightarrow ④$

$$T(s) = \theta_1(s)[J_1 s^2 + k] - \theta_2(s).K$$

$$= \theta_1(s)(J_1 s^2 + k) - K.\frac{K}{J_2 s^2 + Bs + k}\theta_1(s)$$

$$= \theta_1(s)\left((J_1 s^2 + k) - \frac{k^2}{(J_2 s^2 + Bs + k)}\right) \Rightarrow \theta_1(s) = \frac{(J_2 s^2 + Bs + k)(J_1 s^2 + k) - k^2}{J_2 s^2 + Bs + k}$$

$$\Rightarrow \boxed{\frac{\theta_1(s)}{T(s)} = \frac{J_2 s^2 + Bs + K}{(J_2 s^2 + Bs + k)(J_1 s^2 + k) - K^2}}$$

Now, $\theta_1(s) = \frac{J_2 s^2 + Bs + k}{k}\theta_2(s)$

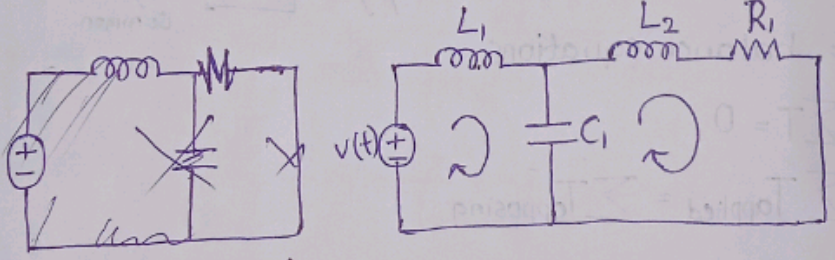
$$T(s) = \frac{J_2 s^2 + Bs + k}{k}.(J_1 s^2 + k).\theta_2(s) - \theta_2(s)K$$

$$= \theta_2(s)\left(\frac{(J_2 s^2 + Bs + k)(J_1 s^2 + k) - K^2}{k}\right)$$

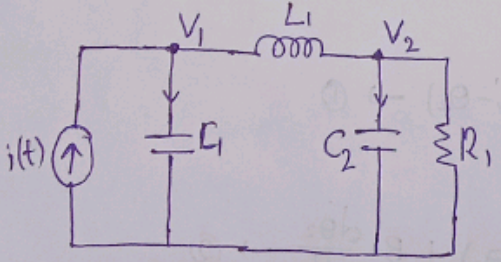
	T-v	T-i
B	R	R
J	L	C
k	C	L

$$\boxed{\frac{\theta_2(s)}{T(s)} = \frac{k}{(J_2 s^2 + Bs + k)(J_1 s^2 + k) - K^2}}$$

T-v



T-i



Mesh-1:

$$v(t) = L_1\frac{di_1}{dt} + \frac{1}{C_1}\int(i_1 - i_2)\,dt$$

Mesh-2:

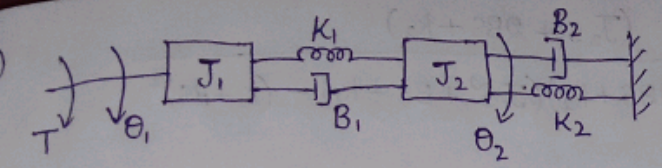
$$0 = L_2\frac{di_2}{dt} + i_2 R_1 + \frac{1}{C_1}\int(i_2 - i_1)\,dt$$

Nodal equations:

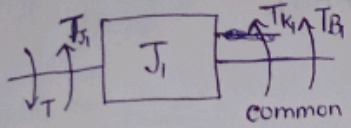
$$i(t) = C_1\frac{dV_1}{dt} + \frac{1}{L_1}\int(V_1 - V_2)\,dt$$

$$0 = C_2\frac{dV_2}{dt} + \frac{V_2}{R_1} + \frac{1}{L_1}\int(V_2 - V_1)\,dt$$

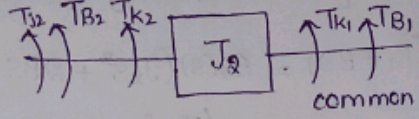
Q)



→ free body diagram at J_1



→ free body diagram at J_2



→ Torque balance equations, $\sum T_{applied} = \sum T_{opposing}$

$$T = T_{J1} + T_{K1} + T_{B1}$$

$$T = J_1 \frac{d^2\theta_1}{dt^2} + K_1(\theta_1 - \theta_2) + B \frac{d}{dt}(\theta_1 - \theta_2) \rightarrow (1)$$

$$0 = T_{J2} + T_{B2} + T_{K2} + T_{K1} + T_{B1}$$

$$0 = J_2 \frac{d^2\theta_2}{dt^2} + K_2\theta_2 + B \frac{d\theta_2}{dt} + K_1(\theta_2 - \theta_1) + B \frac{d}{dt}(\theta_2 - \theta_1) \rightarrow (2)$$

→ Apply Laplace Transform,

$$\Rightarrow T(S) = J_1 S^2 \theta_1(S) + K_1\theta_1(S) - K_1\theta_2(S) + B.S\,\theta_1(S) - B.S\,\theta_2(S)$$

$$= \theta_1(S)\left[J_1S^2 + K_1 + BS\right] - \theta_2(S)\left[K_1 + B.S\right] \rightarrow (1)$$

$$\Rightarrow 0 = J_2 S^2 \theta_2(S) + K_2\theta_2(S) + B.S\,\theta_2(S) + K_1\theta_2(S) - K_1\theta_1(S) + B.S.\theta_2(S) - B.S.\theta_1(S)$$

$$= \theta_2(S)\left[J_2S^2 + K_2 + B.S + B.S\right] - \theta_1(S)\left[K_1 + BS\right] \rightarrow (2)$$

from eq (2),

$$\theta_2(S)\left[J_2S^2 + K_2 + 2BS\right] = \theta_1(S)\left[K_1 + BS\right]$$

$$\theta_2(S) = \frac{(K_1 + BS)}{(J_2S^2 + 2BS + K_2)}\,\theta_1(S) \rightarrow (3)$$

Substituting eq (3) in eq (1), we get

$$T(S) = \theta_1(S)(J_1S^2 + BS + K_1) - \frac{(K_1 + BS)}{(J_2S^2 + 2BS + K_2)} \cdot (K_1 + BS) \cdot \theta_1(S)$$

$$= \theta_1(S)\left((J_1S^2 + BS + K_1) - \frac{(K_1 + BS)^2}{(J_2S^2 + 2BS + K_2)}\right)$$

$$T(S) = \theta_1(S)\left[\frac{(J_2S^2 + 2BS + K_2)(J_1S^2 + BS + K_1) - (K_1 + BS)^2}{(J_2S^2 + 2BS + K_2)}\right]$$

$$\frac{\theta_1(s)}{T(s)} = \frac{(J_2 s^2 + 2Bs + K_2)}{(J_2 s^2 + 2Bs + K_2)(J_1 s^2 + Bs + K_1) - (K_1 + Bs)^2}$$

from eq ②, we have

$$\Rightarrow 0 = \theta_2(s)\left[J_2 s^2 + 2Bs + K_2\right] - \theta_1(s)\left[K_1 + Bs\right]$$

$$\Rightarrow \theta_1(s)(K_1 + Bs) = \theta_2(s)(J_2 s^2 + 2Bs + K_2)$$

$$\theta_1(s) = \frac{(J_2 s^2 + 2Bs + K_2)}{(K_1 + Bs)} \cdot \theta_2(s) \rightarrow ④$$

Substituting eq ④ in eq ①, we get

$$T(s) = \theta_1(s)\left[J_1 s^2 + Bs + K_1\right] - \theta_2(s)\left[K_1 + Bs\right]$$

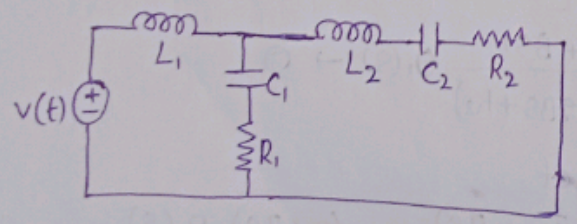
$$T(s) = \frac{(J_2 s^2 + 2Bs + K_2)}{(K_1 + Bs)} \cdot (J_2 s^2 + 2Bs + K_2)\,\theta_2(s) - \theta_2(s)(K_1 + Bs)$$

$$= \theta_2(s)\left[\frac{(J_2 s^2 + 2Bs + K_2)^2}{(K_1 + Bs)} - (K_1 + Bs)\right]$$

$$T(s) = \theta_2(s)\left[\frac{(J_2 s^2 + 2Bs + K_2)^2 - (K_1 + Bs)^2}{(K_1 + Bs)}\right]$$

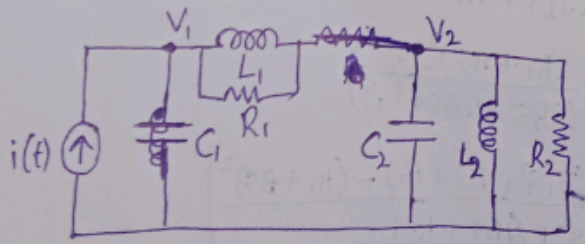
$$\frac{\theta_2(s)}{T(s)} = \frac{(K_1 + Bs)}{(J_2 s^2 + 2Bs + K_2)^2 - (K_1 + Bs)^2}$$

Torque - voltage circuit :-



	T-V
B	R
J	L
K	C

Torque - current circuit :-



	T-i
B	R
J	C
K	L

Mesh-1:

$$v(t) = L_1 \frac{di_1}{dt} + \frac{1}{C_1}\int (i_1 - i_2)\,dt + R_1(i_1 - i_2)$$

Mesh-2:

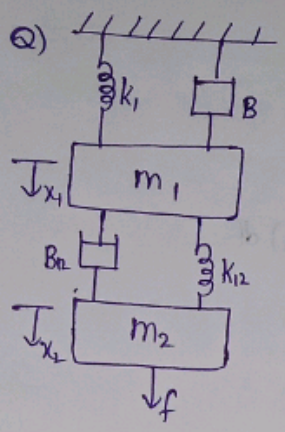
$$0 = L_2 \frac{di_2}{dt} + \frac{1}{C_2}\int i_2\,dt + R_2 i_2 + R_1(i_2 - i_1) + \frac{1}{C_2}\int (i_2 - i_1)\,dt$$

Nodal equations:

Applying lapace transform for velocity,

$f = m_1$

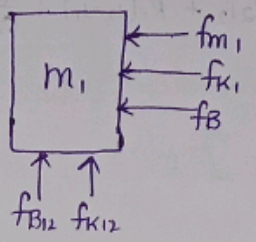
Q)



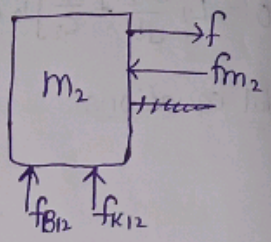
no. of nodes = no. of mass elements = 2

No. of free body diagrams = 2

m_1:



m_2:



⇒ force balance equations,

$f_{opposing} = f_{applied}$

$$0 = f_{m_1} + f_{K_1} + f_B + \underline{f_{B12} + f_{K12}}$$

$$f = f_{m_2} + \underline{f_{B12} + f_{K12}}$$

⇒ Differential equations,

$$0 = m_1\frac{d^2x_1}{dt^2} + K_1x_1 + B\frac{dx_1}{dt} + B_{12}\frac{d(x_1-x_2)}{dt} + K_{12}(x_1-x_2) \rightarrow (1)$$

$$f = m_2\frac{d^2x_2}{dt^2} + B_{12}\frac{d}{dt}(x_2-x_1) + K_{12}(x_2-x_1) \rightarrow (2)$$

⇒ Apply Laplace transform,

$$0 = m_1s^2X_1(s) + K_1X_1(s) + B_0sX_1(s) + B_{12}sX_1(s) - B_{12}sX_2(s) + K_{12}X_1(s) - K_{12}X_2(s)$$

$$0 = X_1(s)\left[m_1s^2 + K_1 + Bs + B_{12}s + K_{12}\right] - X_2(s)\left[B_{12}s + K_{12}\right] \rightarrow (3)$$

$$f(s) = m_2s^2X_2(s) + B_{12}sX_2(s) - B_{12}sX_1(s) + K_{12}X_2(s) - K_{12}X_1(s)$$

$$f(s) = X_2(s)\left[m_2s^2 + B_{12}s + K_{12}\right] - X_1(s)\left[B_{12}s + K_{12}\right] \rightarrow (4)$$

from eq (3), we have

$$X_1(s) = \frac{(B_{12}s + K_{12})}{(m_1s^2 + K_1 + Bs + B_{12}s + K_{12})}X_2(s)$$

Sub $X_1(s)$ in eq (4)

$$f(s) = X_2(s)\left[m_2s^2 + B_{12}s + K_{12}\right] - \frac{(B_{12}s + K_{12})(B_{12}s + K_{12})}{(m_1s^2 + K_1 + Bs + B_{12}s + K_{12})} \cdot X_2(s)$$

$$f(s) = X_2(s)\left[\frac{(m_2s^2 + B_{12}s + K_{12})(m_1s^2 + K_1 + Bs + B_{12}s + K_{12}) - (B_{12}s + K_{12})^2}{(m_1s^2 + K_1 + Bs + B_{12}s + K_{12})}\right]$$

$\frac{X_2(s)}{f(s)}$

from eq

Sub

$f(s) =$

$=$

$\frac{X_1(s)}{f(s)}$

$$\frac{X_2(s)}{f(s)} = \frac{(m_1s^2 + K_1 + Bs + B_{12}s + K_{12})}{(m_2s^2 + B_{12}s + K_{12})(m_1s^2 + K_1 + Bs + B_{12}s + K_{12}) - (B_{12}s + K_{12})^2}$$

from eq ③, we have

$$X_2(s) = \frac{(m_1s^2 + K_1 + Bs + B_{12}s + K_{12})}{(B_{12} + K_{12})} \cdot X_1(s)$$

Sub $X_2(s)$ in eq ④

$$f(s) = \frac{(m_1s^2 + K_1 + Bs + B_{12}s + K_{12})(m_2s^2 + B_{12}s + K_{12})}{(B_{12} + K_{12})} X_1(s) - X_1(s)(B_{12}s + K_{12})$$

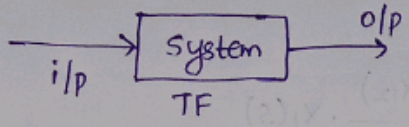
$$= X_1(s)\left[\frac{(m_1s^2 + K_1 + Bs + B_{12}s + K_{12})(m_2s^2 + B_{12}s + K_{12}) - (B_{12}s + K_{12})^2}{(B_{12} + K_{12})}\right]$$

$$\frac{X_1(s)}{f(s)} = \frac{(B_{12} + K_{12})}{(m_1s^2 + K_1 + Bs + B_{12}s + K_{12})(m_2s^2 + B_{12}s + K_{12}) - (B_{12}s + K_{12})^2}$$

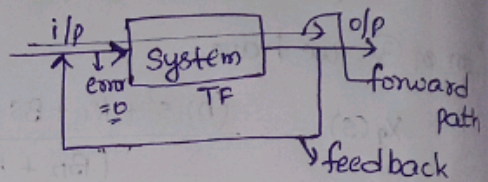
16/7

Imp

Open loop system



Closed loop system



feedback system

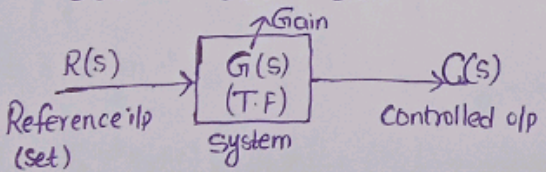
① No feedback
② No o/p is fed to i/p
③ manual control
④ Automatic with programmed timing
⑤ No sensors

① with feedback
② o/p is feedback to i/p
③ automatic control + with sensors

Block diagram

→ Pictorial representation of a control system ⇒ Block
→ To build system in computer based tools (MATLAB, PSIM)
→ Used in complex systems → multiple sub systems in one system
→ To find T.F of overall system

Block Diagram Representation

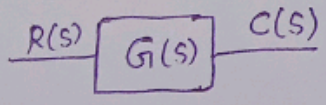


$$G(s) = \frac{L(o/p)}{L(i/p)} = T.F$$

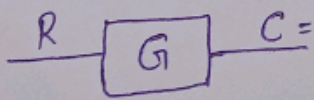
$$TF = \frac{C(s)}{R(s)}\ \begin{matrix}\text{voltage}\\ \text{voltage}\end{matrix} = \text{Gain}$$

Signal entering into the block → R(s) or X(s) → i/p
Signal going out from the block → C(s) or Y(s) → o/p

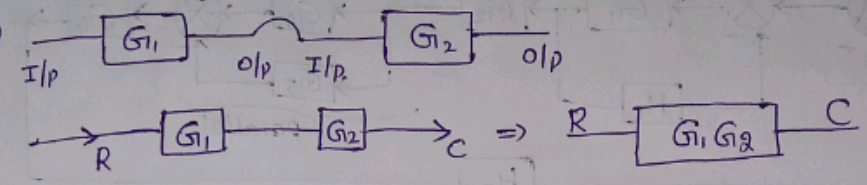
Block Diagram Reduction Techniques

→ 

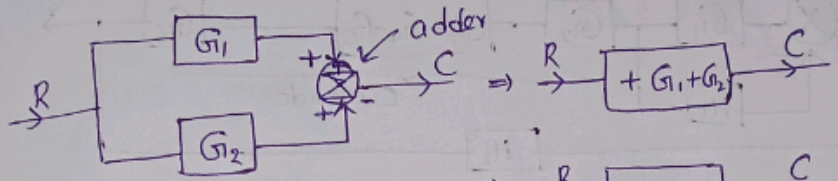
$$TF = G(s) = \frac{C(s)}{R(s)} \Rightarrow o/p = C(s) = G(s).R(s)$$

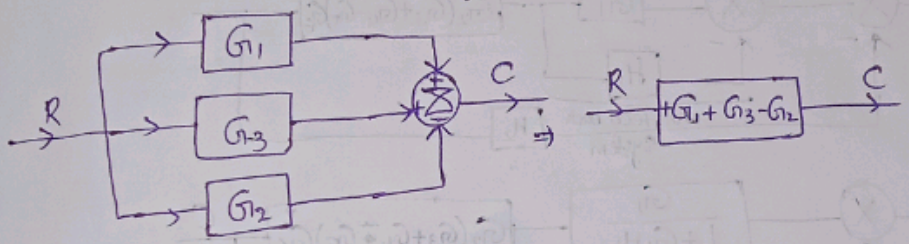


→ Two blocks are connected in cascade connection

① 

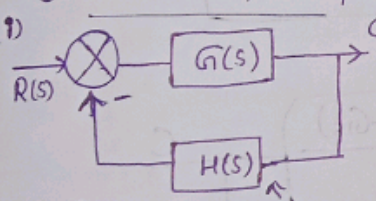
② Two blocks are connected in parallel





③ Closed loop

forward path gain(G)

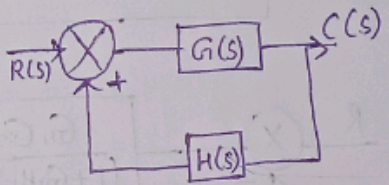
(i) 

Sensors / transducers

feedback path gain(H)

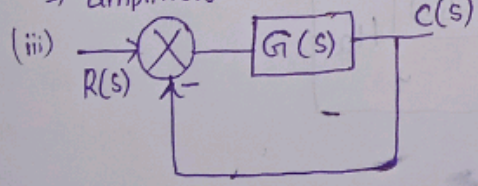
$$T.F = \frac{C(S)}{R(S)} = \frac{G(S)}{1 + G(S).H(S)}$$

⇒ -ve feed back system
⇒ amplifiers.

(ii) 

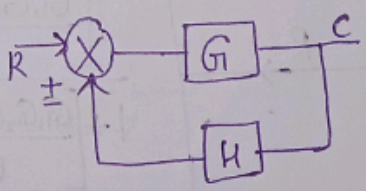
⇒ +ve feedback system
⇒ oscillators

$$T.F = \frac{C(S)}{R(S)} = \frac{G(S)}{1 - G(S)\,H(S)}$$

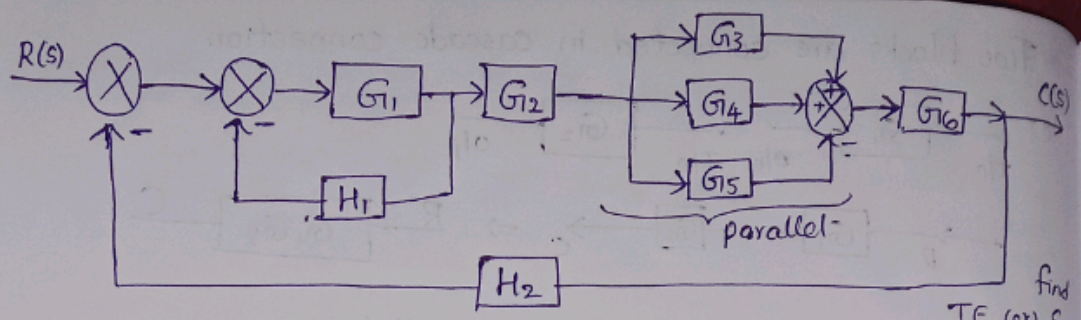
(iii) 

⇒ unity feedback system (H=1)

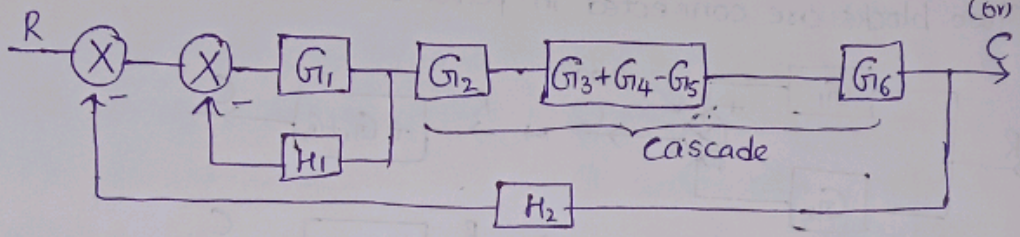
$$T.F = \frac{C(S)}{R(S)} = \frac{G(S)}{1 + G(S)} \quad \{\because H(S) = 1\}$$

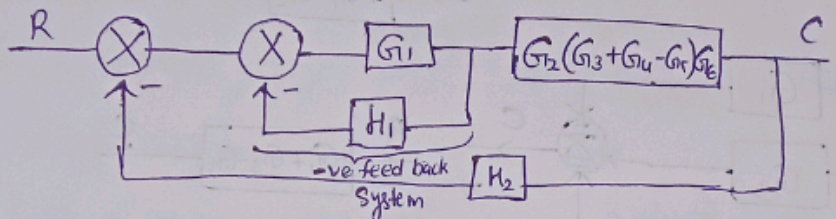
(iv) 

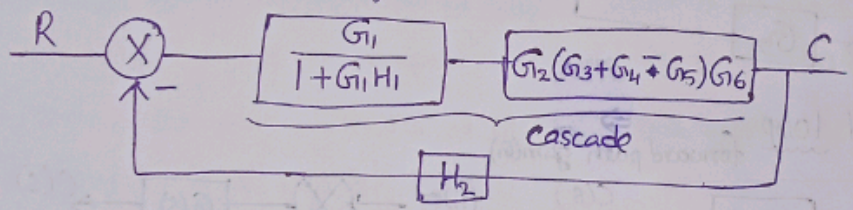
$$T.F = \frac{G}{1 \mp GH}$$

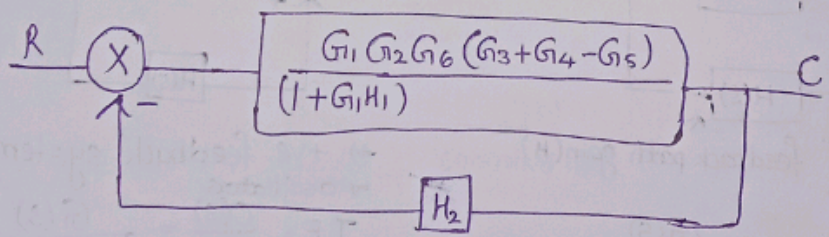


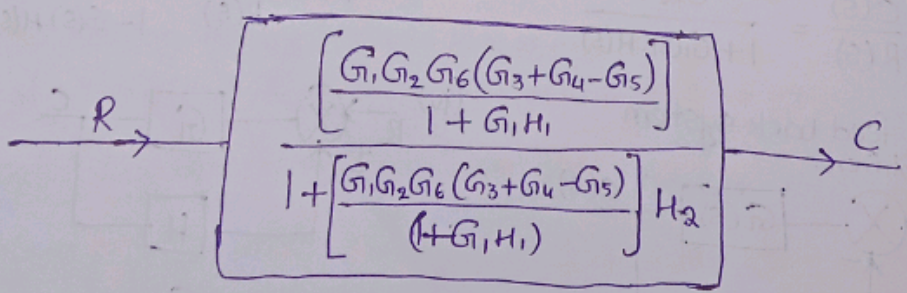
find TF (or) $\frac{C}{R}$ (or) $\frac{C(S)}{R(S)}$

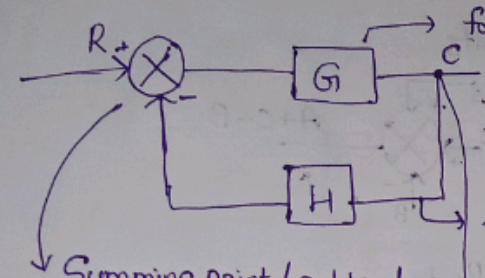












forward path gain
(i/p to o/p)

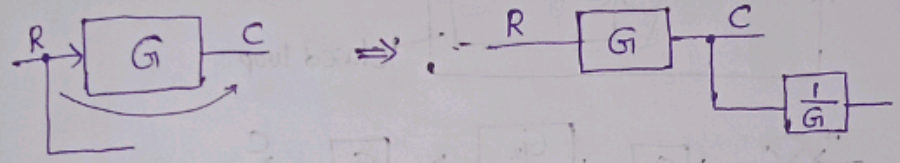
Automatic + sensors

feedback path
(o/p to i/p)

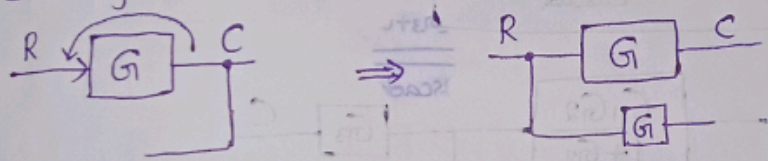
Summing point / adder / Summing Junction
→ Dynamic
→ Signals addition

Take-off point / pick-off point / Branch point
→ Collecting signals.

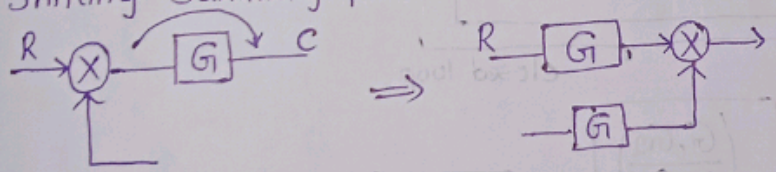
④ Shifting Take-off point after the block :-



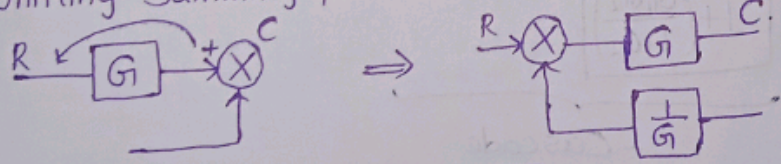
⑤ Shifting Take-off point before the block :-



⑥ Shifting Summing point after the block :-



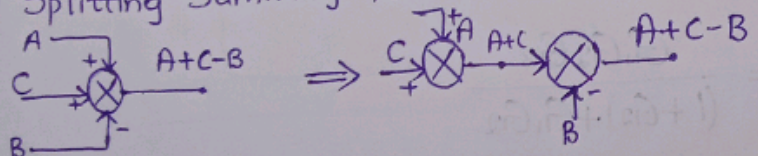
⑦ Shifting Summing point before the block :-



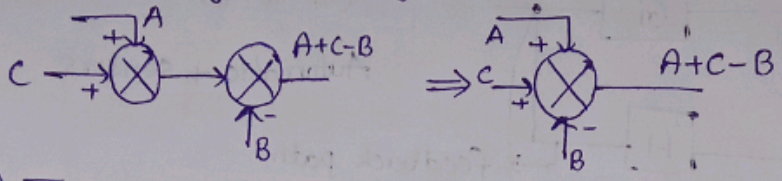
⇒ moving to original point → $\frac{1}{G}$

⇒ moving to opposite → G

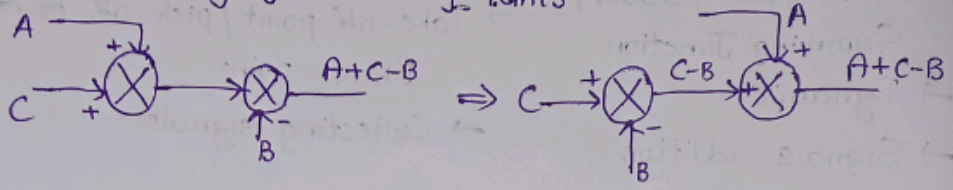
⑧ Splitting Summing Point :-



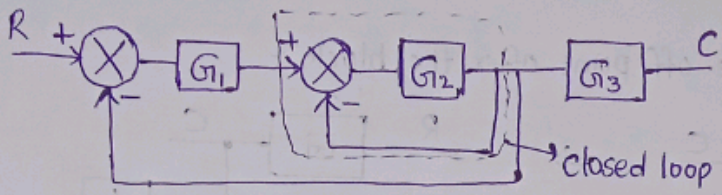
⑨ Combining Summing Points



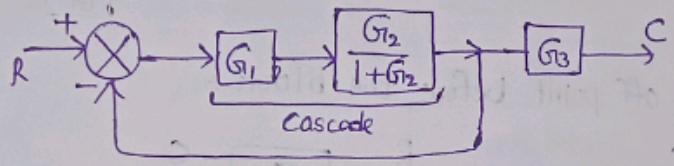
⑩ Interchanging Summing Points



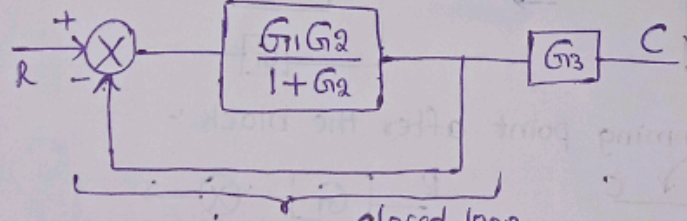
Pr



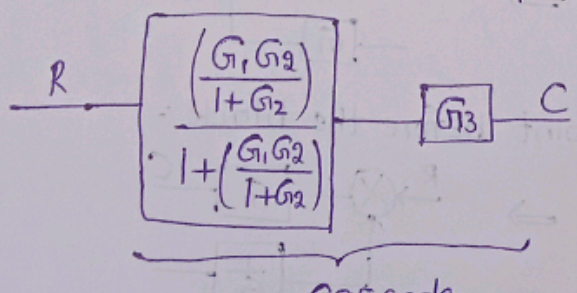
⇒



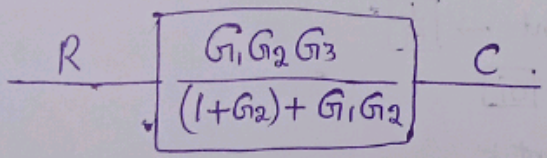
⇒



⇒

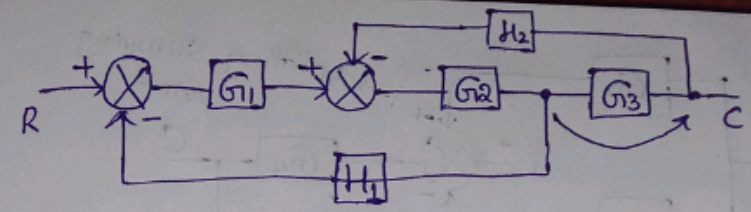


⇒



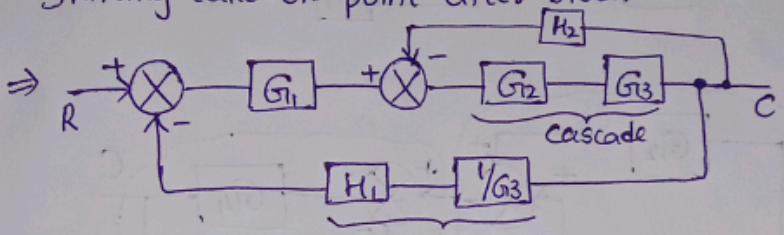
$$T.F = \frac{G_1 G_2 G_3}{(1+G_2)+G_1 G_2}$$

(Pr)

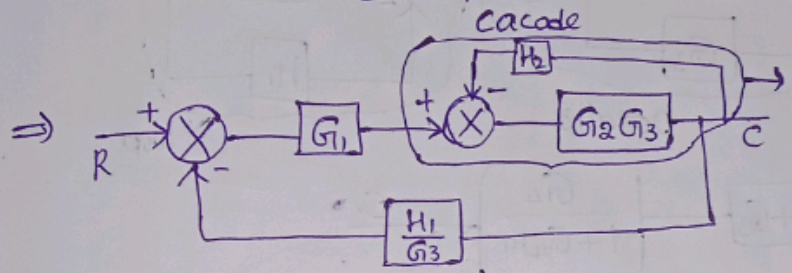


take-off point prefer.

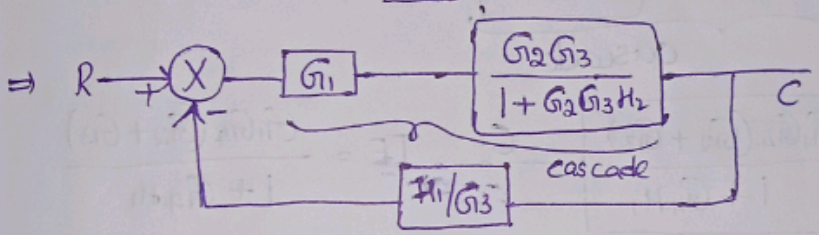
Shifting take-off point after block

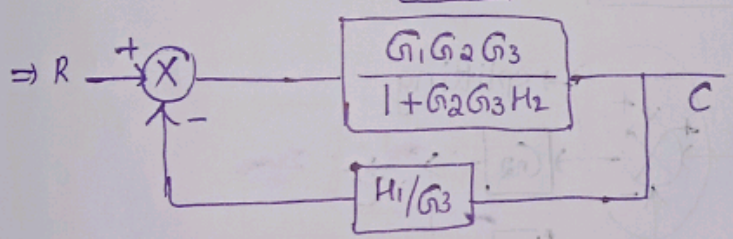


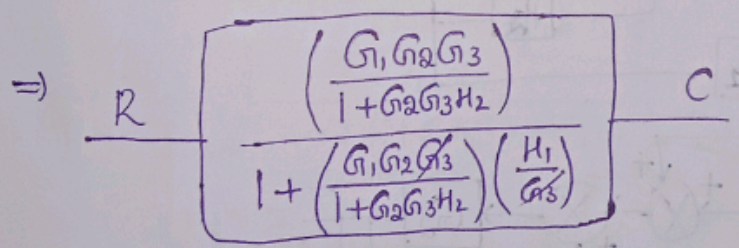
cacade



→ closed loop

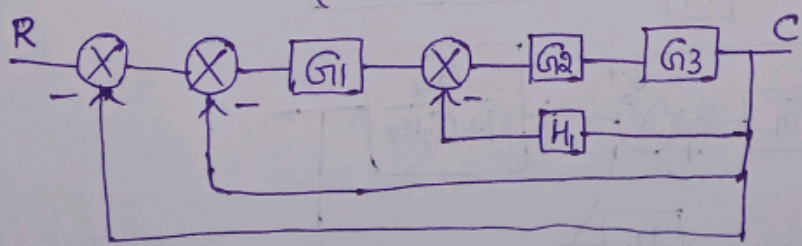




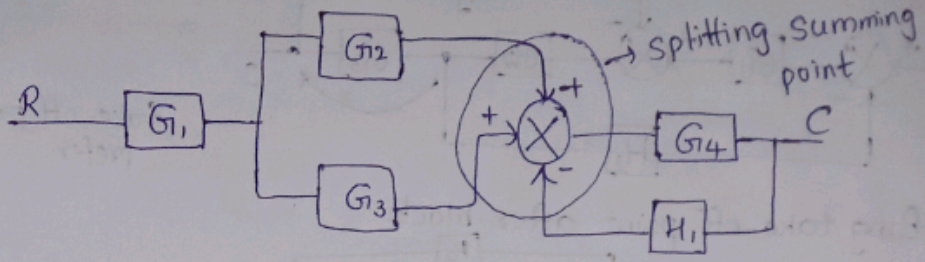


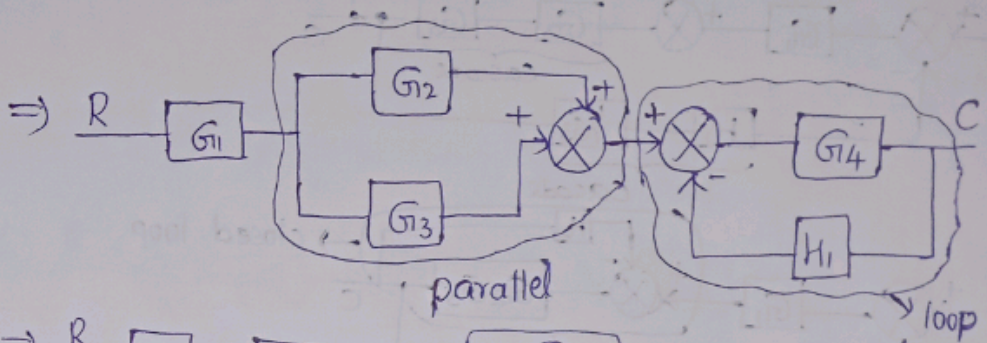
$$T.F = \frac{C}{R} = \frac{G_1 G_2 G_3}{(1 + G_2 G_3 H_2) + G_1 G_2 H_1}$$

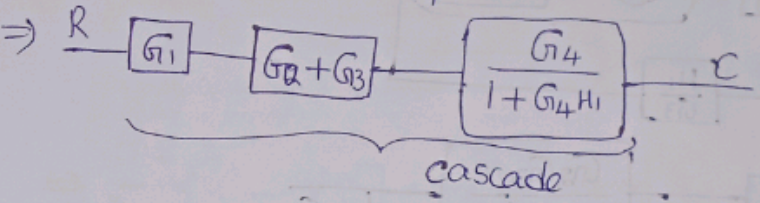
(Pr)



(8)

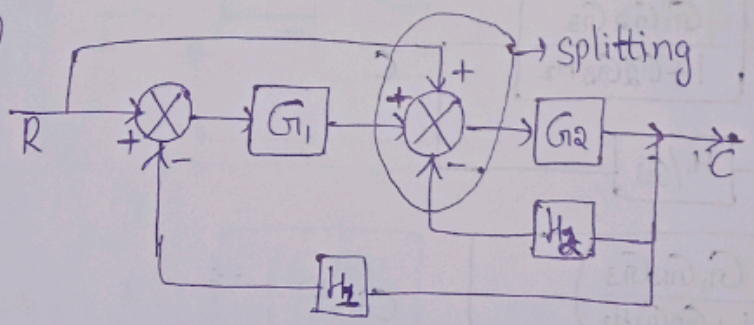


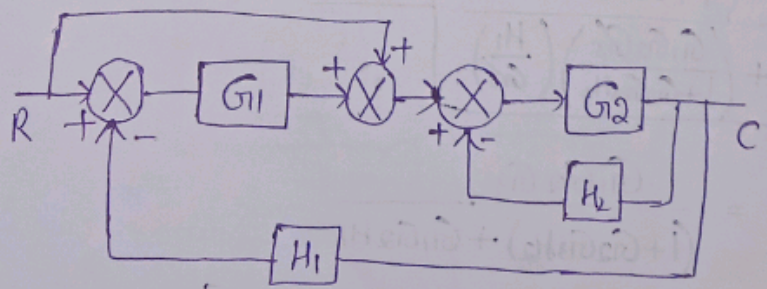


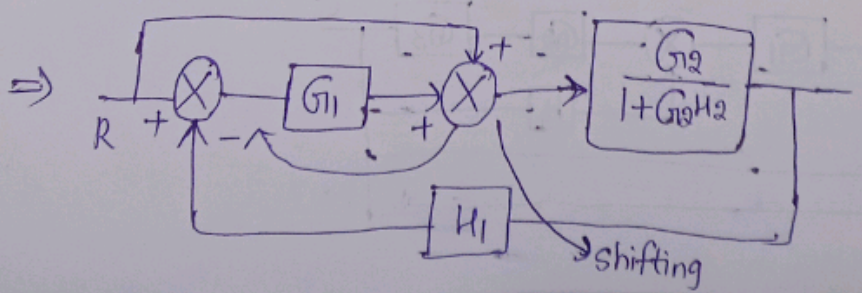


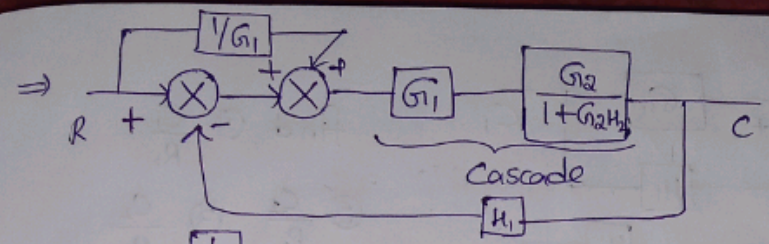
$$\Rightarrow \quad R \;\; \frac{G_1 G_4 (G_2 + G_3)}{1 + G_4 H_1} \;\; C \qquad TF = \frac{G_1 G_4 (G_2 + G_3)}{1 + G_4 H_1}$$

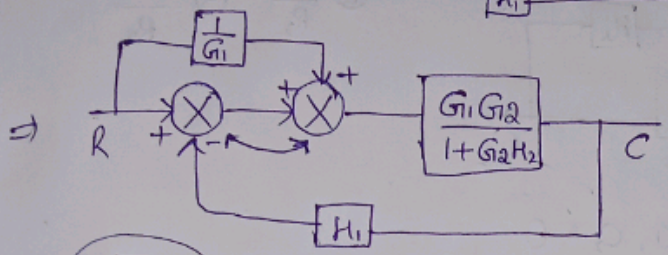
(9)

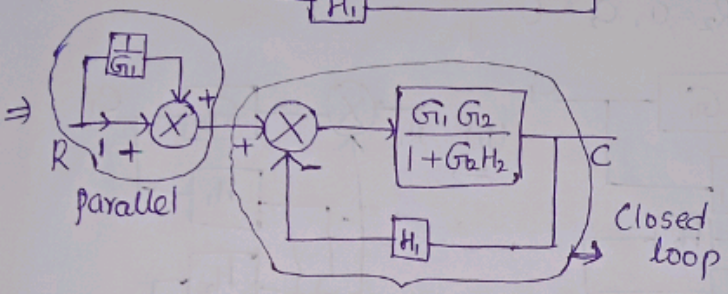


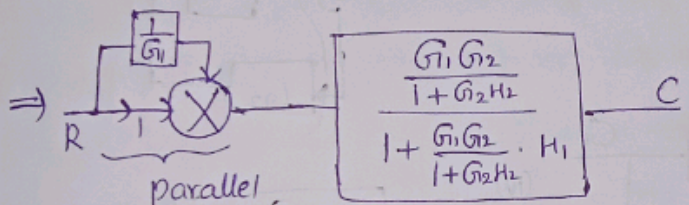


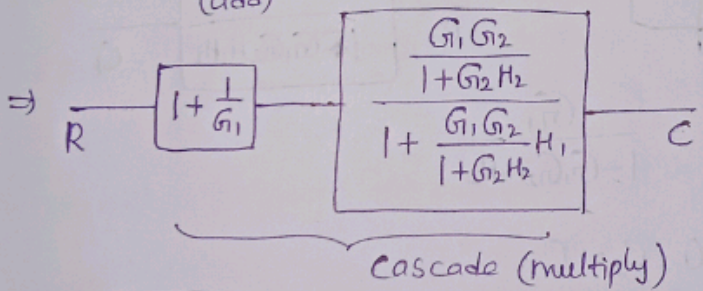


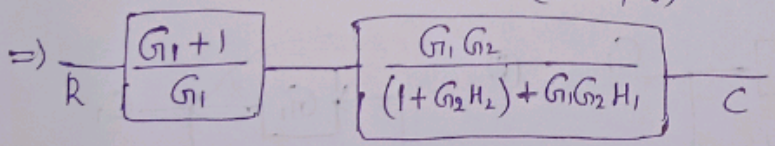


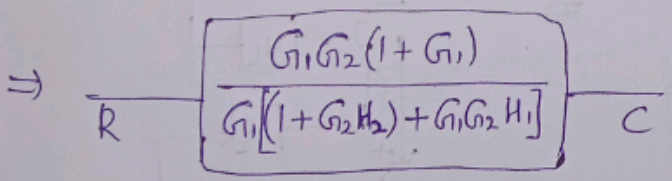






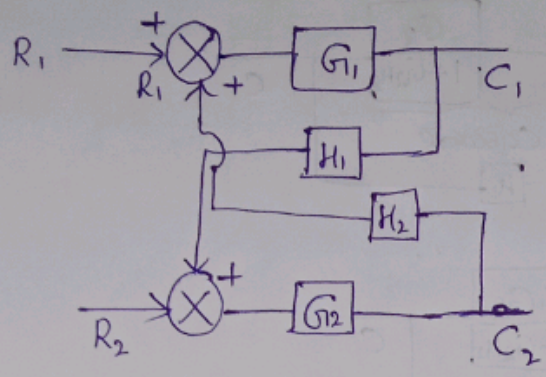






$$TF = \frac{G_2(1+G_1)}{(1+G_2H_2)+G_1G_2H_1} = \frac{C}{R}$$

(8)

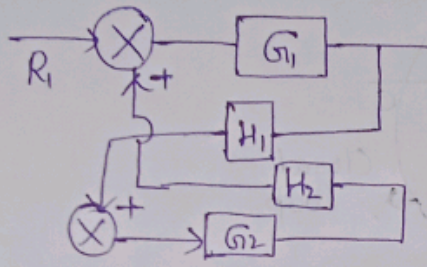


find ① $\frac{C_1}{R_1}$

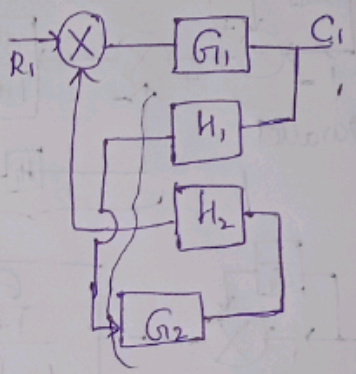
② $\frac{C_1}{R_2}$ ③ $\frac{C_2}{R_2}$

① To find $\frac{C_1}{R_1}$; $R_2 = 0, C_2 = 0$

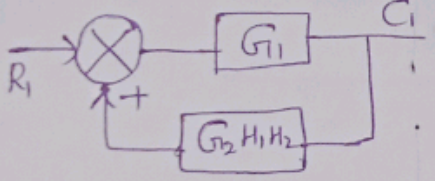
⇒ (i)



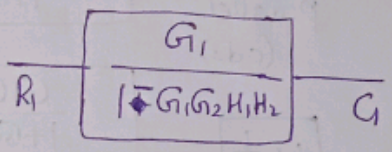
(ii) ⇒



⇒ (iii)



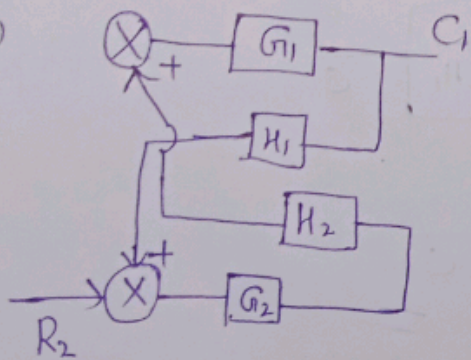
(iv) ⇒



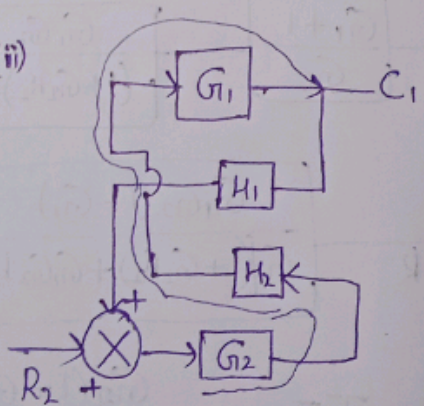
$$TF = \frac{C_1}{R_1} = \frac{-G_1}{1 - G_1G_2H_1H_2}$$

② To find $\frac{C_1}{R_2}$; $R_1 = 0, C_2 = 0$

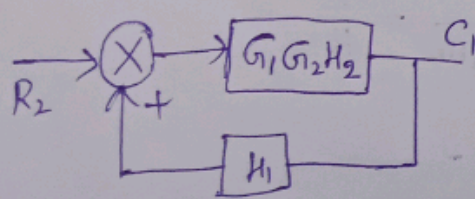
⇒ (i)



(ii) ⇒



⇒ (iii)



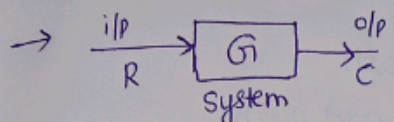
$$T.F = \frac{G_1G_2H_2}{1 - G_1G_2H_1H_2} = \frac{C_1}{R_2}$$

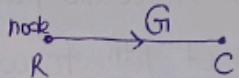
Signal flow graph

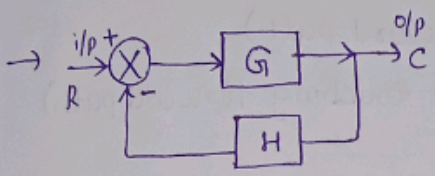
→ Graphical representation of control system

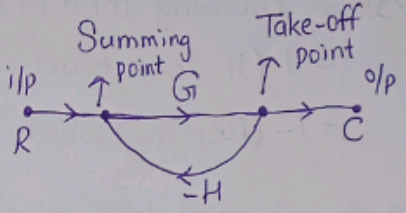
T.F → mathematical representation

To find overall TF of a control system { Block diagram → pictorial representation
Signal flow graph → graphical representation

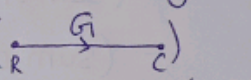




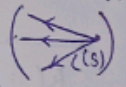




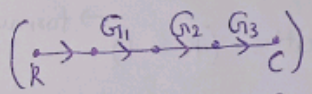
① Node : Connecting point which represents a variable or signal.

② Branch : A line which connects two nodes 

③ Source / Input node : which contains only outgoing signals 

④ Sink / Output node : which contains only incoming signals 

⑤ Forward path : The path which travels from input to output.

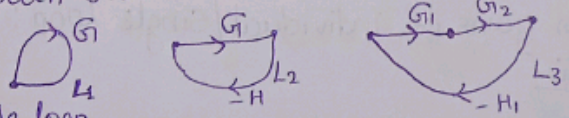


⑥ Forward path gain : The product of gains in forward path.

$$P = G_1 G_2 G_3$$

Individual

⑦ Loop : A path starts at one node and ends at the same node.



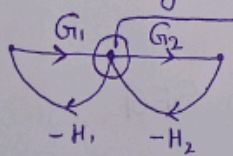
Simple loop

Individual

⑧ Loop gains: Product of gains of a loop.

$L_1 = G$, $L_2 = -GH$, $L_3 = -G_1 G_2 H_1$

⑨ Non-Touching loops : The loops which are not sharing a node.



→ Touching loops (sharing a node)

$L_1 = -G_1 H_1$; $L_2 = -G_2 H_2$

Mason's Gain formula :-

$$T = \frac{1}{\Delta} \sum_k P_k \Delta_k$$

Where, T or $T(s)$ = overall transfer function of a control system

P = forward path gain

k = no. of forward paths

P_k = gain of k^{th} forward path

Δ_k = missing loop of a k^{th} forward path

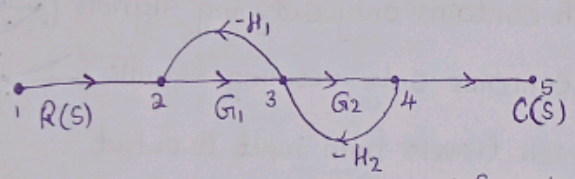
= 1 (if loop touches forward path)

= 1 − (loop gain which is not touching forward path)

Δ = 1 − (sum of individual loop gains)

+ (sum of gain products of two non-touching loops)

− (sum of gain products of three non-touching loops).....

① 

find overall TF of a SFG.

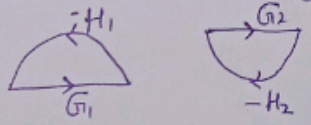
→ formula

→ No. of forward paths ($k = 1$)



P_1 = forward path gain = G_1G_2

→ Individual loops or gains of individual/single loop :-



$L_1 = -G_1H_1$ $\quad L_2 = -G_2H_2$

→ Gains of two non-touching loops = 0.

→ Gains of three non-touching loops

There are no three non-touching loops in SFG

→ To calculate Δ_k

k = no. of forward paths

$k = 1$

$\Delta_1 = 1$

→ To calculate Δ

$\Delta = 1 - (\text{sum of gains of single loops})$
$+ (\text{sum of products of two non-touching loops})$
$- (\text{sum of product of three non-touching loops}) + \ldots$

$\Rightarrow \Delta = 1 - [L_1 + L_2] + 0 + 0$

$\Rightarrow \Delta = 1 - [L_1 + L_2]$

→ Mason's Gain formula

Overall gain / overall TF $= \frac{C(S)}{R(S)}$ (or) $\frac{C}{R} = \frac{1}{\Delta}\sum_{k} P_k \Delta_k$

k=1 $\quad TF = \frac{P_1 \Delta_1}{\Delta}$

$TF = \frac{G_1 G_2 \times 1}{1 + G_1H_1 + G_2H_2}$

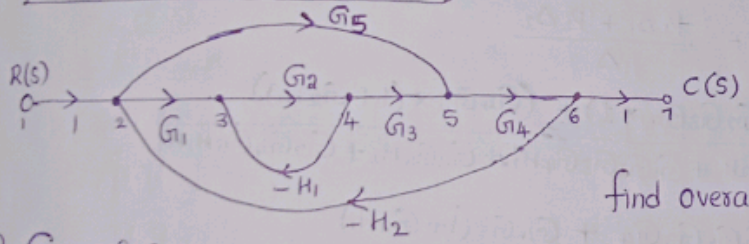
$\Delta = 1 - [L_1 + L_2]$

$\Delta = 1 - [-G_1H_1 - G_2H_2]$

$\boxed{\Delta = 1 + G_1H_1 + G_2H_2}$

$\boxed{T.F = \frac{C}{R} = \frac{G_1 G_2}{1 + G_1H_1 + G_2H_2}}$

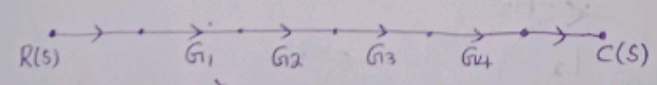
②



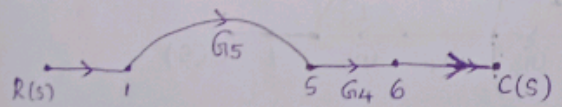
find overall gain of SFG

i) Gains of forward paths

No. of forward paths, k = 2

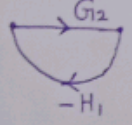


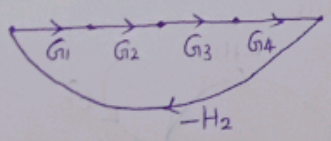
$P_1 = G_1 G_2 G_3 G_4$

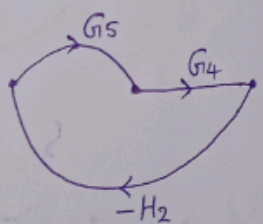


$P_2 = G_4 G_5$

ii) Gains of single loops







$L_1 = -G_2H_1$ $\quad L_2 = -G_1G_2G_3G_4H_2$ $\quad L_3 = -G_4G_5H_2$

iii) Gains of two non-touching loops

L_1, L_2 → Touching $\quad L_2$, L_3 → Touching

L_1, L_3 → Non-touching

L_1 , L_3 are non - touching loops

iv) Gains of three non-touching loops

There are no three non-touching loops

v) To calculate Δ_k , $k = 2$

$$\Delta_1 = 1$$

$$\Delta_2 = 1 - [-G_2H_1] = 1 + G_2H_1$$

vi) To calculate Δ

$$\Delta = 1 - [L_1 + L_2 + L_3] + [L_{13}] + [0]$$

$$= 1 - [-G_2H_1 - G_1G_2G_3G_4H_2 - G_5G_4H_2] + G_2G_4G_5H_1H_2$$

$$\Delta = 1 + G_2H_1 + G_1G_2G_3G_4H_2 + G_4G_5H_2 + G_2G_4G_5H_1H_2$$

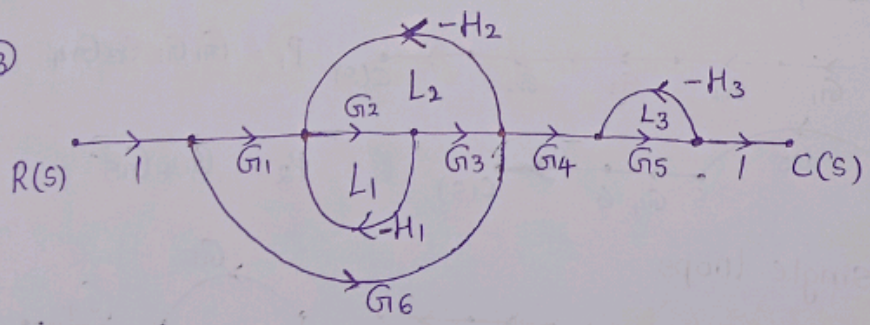
vii) $$T.F = \frac{C}{R} = \frac{1}{\Delta}\sum_k P_k \Delta_k$$

$k = 2 \Rightarrow$ $$T.F = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

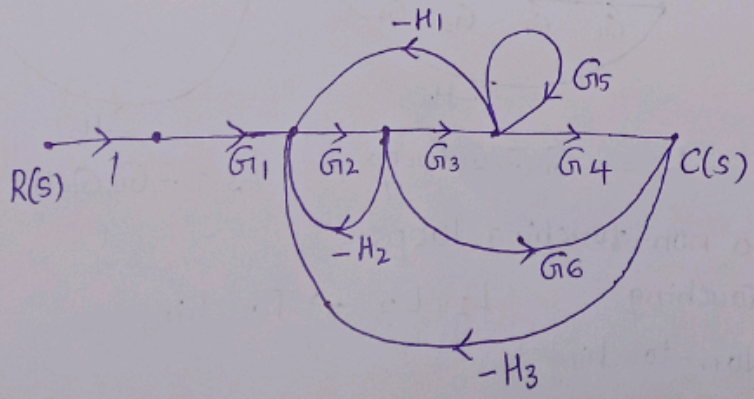
$$T.F = \frac{(G_1G_2G_3G_4 \times 1) + (G_4G_5 \times (1 + G_2H_1))}{1 + G_2H_1 + G_1G_2G_3G_4H_2 + G_4G_5H_2 + G_2G_4G_5H_1H_2}$$

$$T.F = \frac{G_1G_2G_3G_4 + G_4G_5(1 + G_2H_1)}{1 + G_2H_1 + G_1G_2G_3G_4H_2 + G_4G_5H_2 + G_2G_4G_5H_1H_2}$$

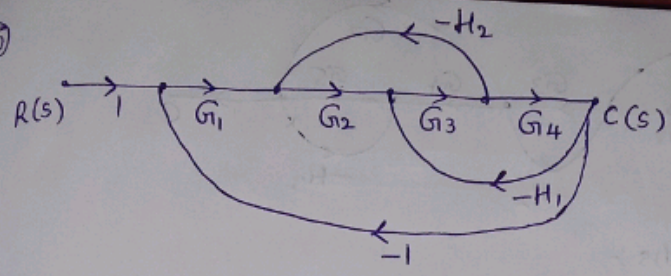
③



④



(5)

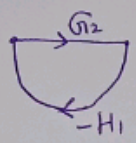


3) Ans: No. of forward paths = 2 = k

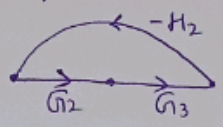
No. of single loops = 3

$P_1 = G_1 G_2 G_3 G_4 G_5$

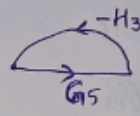
$P_2 = G_6 G_4 G_5$



$L_1 = -G_2 H_1$



$L_2 = -G_2 G_3 H_2$



$L_3 = -G_5 H_3$

Gains of two non-touching loops

$L_1, L_3 \rightarrow$ non-touching ✓

$L_2, L_3 \rightarrow$ ~~non~~ touching ✓

There are no three non-touching loops

$\Rightarrow$ To calculate Δ_k, $k = 2$

$\Delta_1 = 1$

$\Delta_2 = 1 - [L_1 + \cancel{L_2}] = 1 - [-G_2 H_1] - \cancel{[G_2 G_3 H_2]}$

$= 1 + G_2 H_1 + \cancel{G_2 G_3 H_2} - G_2 G_3 H_2$

$\Rightarrow \Delta = 1 - [L_1 + L_2 + L_3] + [L_{13} + L_{23}] + [0]$

$L_{23} = L_2 \times L_3$

$= -G_2 G_3 H_2 \times -G_5 H_3$

$= G_2 G_3 G_5 H_2 H_3$

$L_{13} = L_1 \times L_3$

$= -G_2 H_1 \times -G_5 H_3$

$= G_2 G_5 H_1 H_3$

$\Delta = 1 - [-G_2 H_1 - \cancel{G_2 G_5 H_2} - G_5 H_3] + G_2 G_5 H_1 H_3 + G_2 G_3 G_5 H_2 H_3$

$\Delta = 1 + G_2 H_1 + \cancel{G_2 G_3 H_2} + G_5 H_3 + G_2 G_5 H_1 H_3 + G_2 G_3 G_5 H_2 H_3$

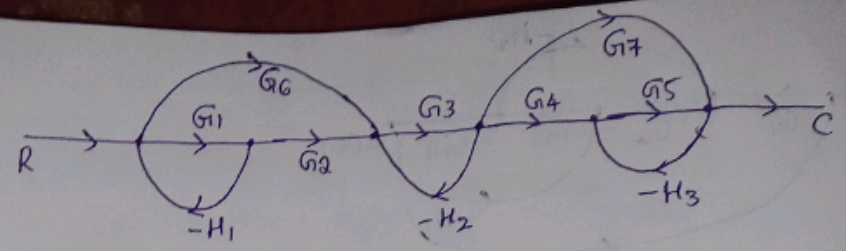
$\Rightarrow T.F = \frac{C}{R} = \frac{1}{\Delta} \sum_k P_k \Delta_k$

$k = 2 \Rightarrow T.F = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$

$= \frac{(G_1 G_2 G_3 G_4 G_5 \times 1) + (G_4 G_5 G_6 * (1 + G_2 H_1) \cancel{G_2 G_3 H_2})}{1 + G_2 H_1 + G_2 G_3 H_2 + G_5 H_3 + G_2 G_5 H_1 H_3 + G_2 G_3 G_5 H_2 H_3}$

$T.F = \frac{G_1 G_2 G_3 G_4 G_5 + G_4 G_5 G_6 (1 + G_2 H_1) \cancel{G_2 G_3 H_2}}{1 + G_2 H_1 + G_2 G_3 H_2 + G_5 H_3 + G_2 G_5 H_1 H_3 + G_2 G_3 G_5 H_2 H_3}$

Q)



① Gains of forward paths :-

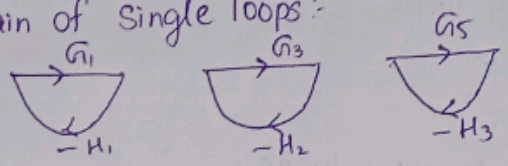
$P_1 = G_1G_2G_3G_4G_5$

$P_2 = G_3G_4G_5G_6$

$P_3 = G_1G_2G_3G_7$

$P_4 = G_3G_6G_7$

② Gain of single loops :-



$L_1 = -G_1H_1 \quad L_2 = -G_3H_2 \quad L_3 = -G_5H_3$

③ Gains of two non-touching loops :-

$L_1L_2 = L_{12} = L_1 \times L_2 = (-G_1H_1)(-G_3H_2) = G_1G_3H_1H_2$

$L_1L_3 = L_{13} = L_1 \times L_3 = (-G_1H_1)(-G_5H_3) = G_1G_5H_1H_3$

$L_2L_3 = L_{23} = L_2 \times L_3 = (-G_3H_2)(-G_5H_3) = G_3G_5H_2H_3$

④ Gains of three non-touching loops

$L_{123} = L_1 \times L_2 \times L_3 = (-G_1H_1)(-G_3H_2)(-G_5H_3)$

$= -G_1G_3G_5H_1H_2H_3$

⑤ Calculation of Δ_k

no. of forward paths, $k = 4$

$\Delta_1 = 1 \; ; \; \Delta_2 = 1 \; ; \; \Delta_3 = 1 \; ; \; \Delta_4 = 1$

⑥ Calculation of Δ :-

$\Delta = 1 - [L_1 + L_2 + L_3] + [L_{12} + L_{13} + L_{23}] - [L_{123}]$

$= 1 - [-G_1H_1 - G_3H_2 - G_5H_3] + [G_1G_3H_1H_2 + G_1G_5H_1H_3 + G_3G_5H_2H_3] - [-G_1G_3G_5H_1H_2H_3]$

$= 1 + G_1H_1 + G_3H_2 + G_5H_3 + G_1G_3H_1H_2 + G_1G_5H_1H_3 + G_3G_5H_2H_3 + G_1G_3G_5H_1H_2H_3$

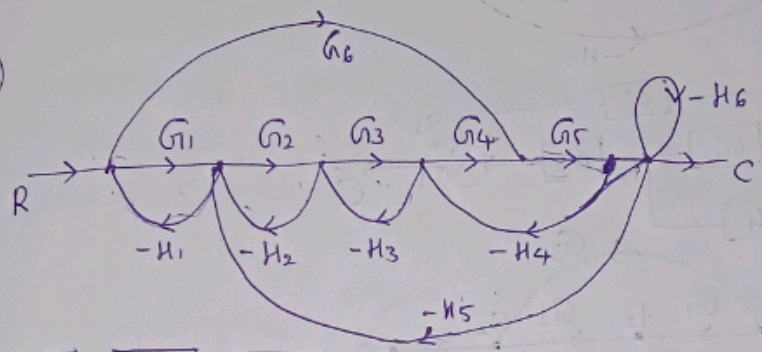
⑦ Mason's Gain formula :-

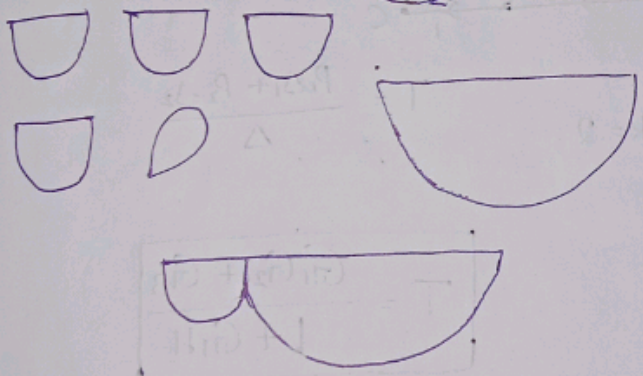
$$T = \frac{1}{\Delta} \sum_k P_k \Delta_k \qquad (k = 4)$$

$$T = \frac{1}{\Delta} [P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4]$$

$$T = \frac{G_1G_2G_3G_4G_5 + G_3G_4G_5G_6 + G_1G_2G_3G_7 + G_3G_6G_7}{1 + G_1H_1 + G_3H_2 + G_5H_3 + G_1G_3H_1H_2 + G_1G_5H_1H_3 + G_3G_5H_2H_3 + G_1G_3G_5H_1H_2H_3}$$

Q)





Construction of SFG :-

① from electrical circuit

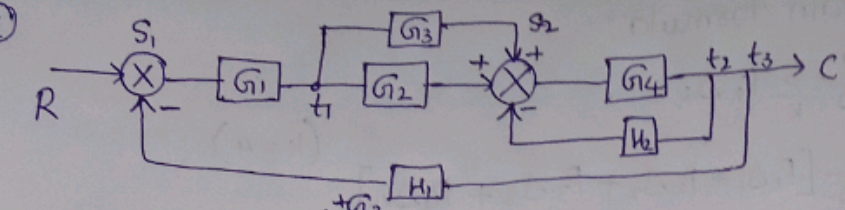
② from block diagram

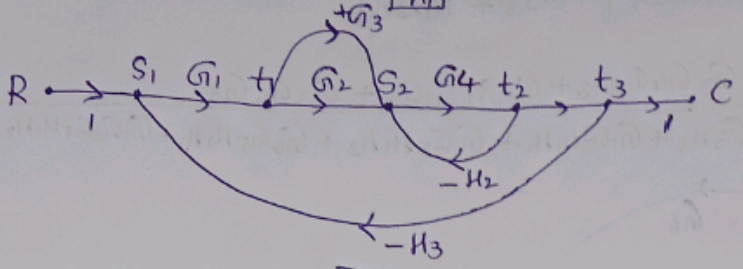
Nodes → i/p , o/p
Summing point
Take off point

$_2H_3]$

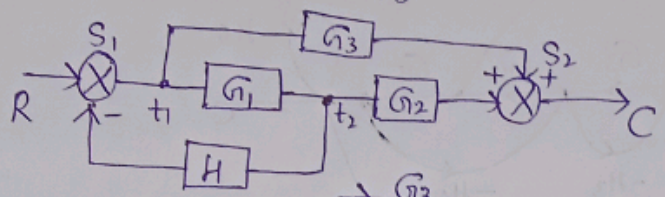
$_2H_3$

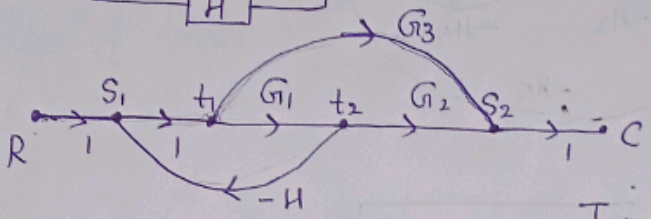
②





③





$$T = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

No. of forward paths, $k = 2$

$P_1 = G_1G_2$

$P_2 = G_3$

Single loops,

$L_1 = -G_1H$;

$$T = \frac{G_1G_2 + G_3}{1 + G_1H}$$

No. of two non-touching loops = 0

No. of three non-touching loops = 0

$\Delta_1 = 1$

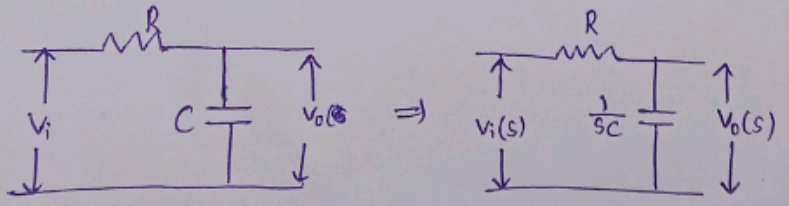
$\Delta_2 = 1$

$\Delta = 1 - [L_1] = 1 + G_1H$

Series branch – current

parallel branch – voltage

④



mesh & nodal at a time

1) i/p , o/p

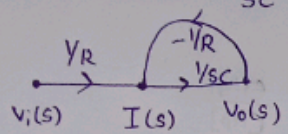
3) current node

4) voltage node

nodes

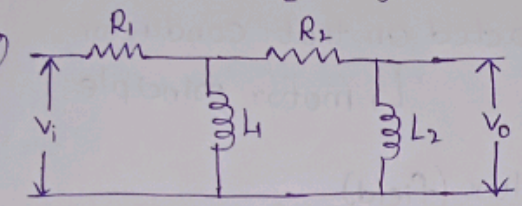
equations: $I(s) = \frac{V_i(s) - V_o(s)}{R} = \frac{V_i(s)}{R} - \frac{V_o(s)}{R}$

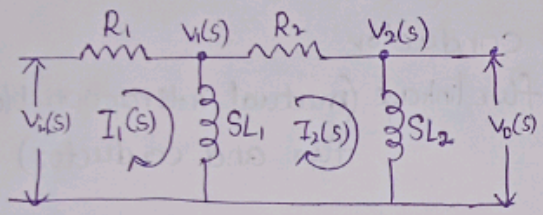
$V_o(s) = \frac{1}{sC} I(s) = \frac{I(s)}{sC}$



$$T.F = \frac{P_1\Delta_1}{\Delta} = \frac{\frac{1}{R}\,\frac{1}{sC} \times 1}{1 - \left[-\frac{1}{R} \times \frac{1}{sC}\right]} = \frac{1/RSC}{1 + \frac{1}{RSC}} = \frac{1/RSC}{\frac{RSC+1}{RSC}} = \boxed{\frac{1}{1+RSC} = TF}$$

⑤





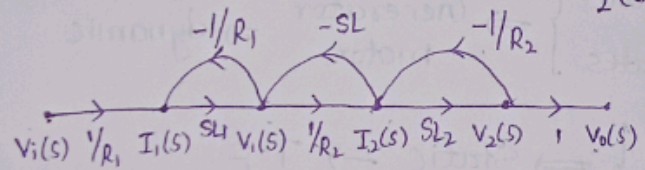
$I_1(s) = \frac{V_i(s) - V_1(s)}{R_1} = \frac{V_i(s)}{R_1} - \frac{V_1(s)}{R_1}$

$V_1(s) = [I_1(s) - I_2(s)]\, SL_1$

$= SL_1 I_1(s) - SL_1 I_2(s)$

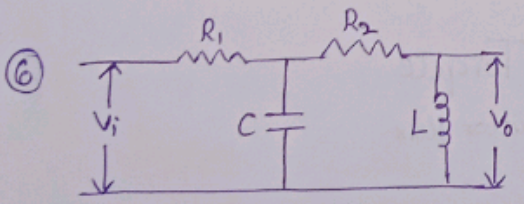
$I_2(s) = \frac{V_2(s) - V_2(s)}{R_2} = \frac{V_1(s)}{R_2} - \frac{V_2(s)}{R_2}$

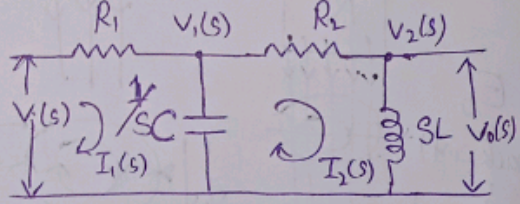
$V_2(s) = I_2(s)\, SL_2$



$$TF = \frac{1/R_1 \times SL \times 1/R_2 \times SL_2 \times 1}{1 - [L_1 + L_2 + L_3] + L_{13}}$$

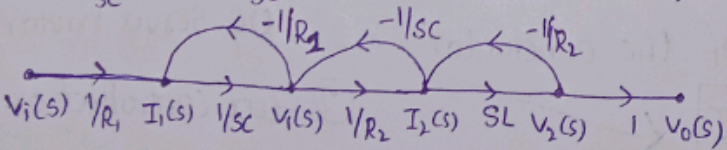
⑥





$I_1(s) = \frac{V_i(s)}{R_1} - \frac{V_1(s)}{R_2}$ $\qquad$ $I_2(s) = \frac{V_1(s)}{R_2} - \frac{V_2(s)}{R_2}$

$V_1(s) = \frac{1}{sC} I_1(s) - \frac{1}{sC} I_2(s)$ $\qquad$ $V_2(s) = SL\, I_2(s)$



$$T.F = \frac{1/R_1 \times \frac{1}{sC} \times 1/R_2 \times SL \times 1}{1 - [L_1 + L_2 + L_3] + L_{13}}$$

DC Servo motors

motor + servo mechanism = servo motor

↓ discrete rotation

Continuous Step by step rotation

precise control → Angle (θ) position $\frac{\theta(s)}{C(ip)}$

If a current carrying conductor is placed in a magnetic field some force will be acted on that conductor

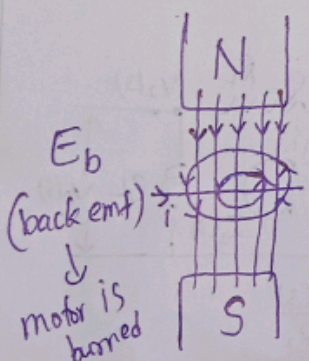
↳ motor principle

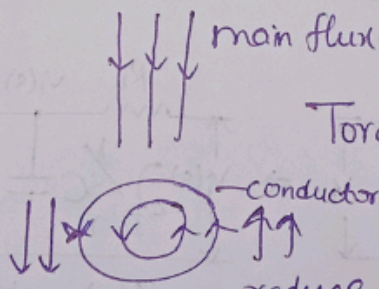
Faraday's law

1) Emf - production
   - ① flux (field)
   - ② conductor
   - ③ flux linkage (mutual interaction b/w flux and conductor)
2) Len'z law

1) flux (stator) = constant, Conductor (rotor) = rotates } ⇒ Generator motor ⇒ dynamic

2) flux → rotate, conductor → constant } ⇒ Static → T.F





$E_b \propto$ speed of the conductor

$T \propto \phi I$ ↓ constant

① Armature controlled DC servo motor

② flux controlled or field controlled DC servo motor

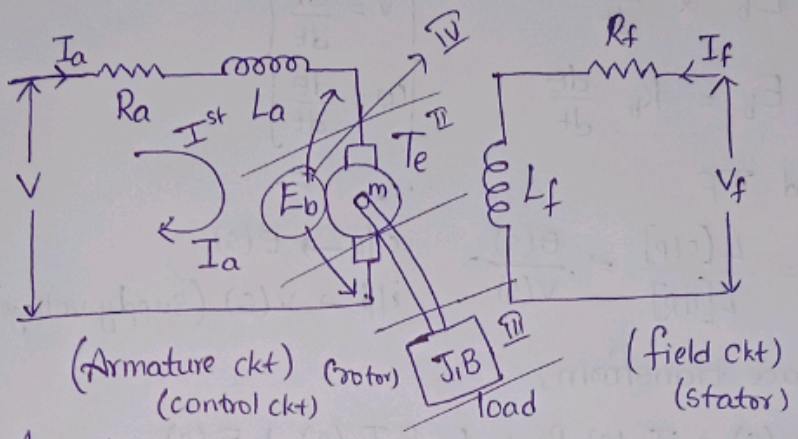
$T \propto \phi I$ ↓ vary, ↓ constant

$V = I_a R_a + E_b$

$\frac{V - E_b}{R_a} = I_a$

$I_a = \frac{V}{R_a} = \frac{V}{1\Omega} = \frac{200\ DC}{1\Omega} = 200\ Amp$

Stattor → to control the Starting current of the motor.



(i) Armature controlled DC servo motor

ϕ = constant

$V_f \longrightarrow I_f \longrightarrow \phi_f \Rightarrow$ constant

$\boxed{T \propto I_a}$

① control ckt eqn
② Torque equation
③ mechanical load equation

① Control ckt equation :-

$-V + I_a R_a + L_a \frac{di_a}{dt} + E_b = 0$

$V = I_a R_a + L_a \frac{di_a}{dt} + E_b \longrightarrow ①$

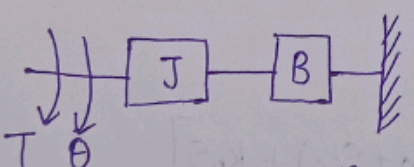
② Torque developed equation :-

$I_a \uparrow, T_a \uparrow$ & $I_a \downarrow, T_a \downarrow$

$\boxed{T \propto I_a}$

$\boxed{T = K_t \cdot I_a} \longrightarrow ②$

③ Mechanical load equation :-



$T = J \frac{d^2\theta}{dt^2} + B \frac{d\theta}{dt} \longrightarrow ③$

④ Back-emf equation :

$E_b \propto$ Speed of conductor

$E_b \propto \omega$

$E_b \propto \frac{d\theta}{dt}$ $\quad \left\{ V = \frac{dx}{dt} \right\}$ $\quad \left\{ \omega = \frac{d\theta}{dt} \right\}$

$E_b = K_b \frac{d\theta}{dt}$

⑤ To find T.F :

$T.F = \frac{L[o/p]}{L[i/p]} = \frac{\theta(s)}{V(s)}$ $\quad o/p \rightarrow \theta(s)$ $\quad i/p \rightarrow V(s)$ (supply voltage)

Apply laplace transform,

① $\Rightarrow V(s) = I_a(s) \cdot R_a + L_a \cdot S I_a(s) + E_b(s)$

$= I_a(s)[R_a + SL_a] + E_b(s) \longrightarrow ⑤$

② $\Rightarrow T_e(s) = K_t \cdot I_a(s) \longrightarrow ⑥$

③ $\Rightarrow T_m(s) = JS^2\theta(s) + BS\theta(s)$

$= \theta(s)[JS^2 + BS] \longrightarrow ⑦$

④ $\Rightarrow E_b(s) = K_b \cdot S \cdot \theta(s) \longrightarrow ⑧$

Sub eq ⑧ in eq ⑤ $\Rightarrow V(s) = I_a(s)[R_a + SL_a] + K_b S\theta(s) \rightarrow ⑨$

$\Longrightarrow T_e(s) = T_m(s)$

$K_t I_a(s) = \theta(s)[JS^2 + BS]$

$I_a(s) = \theta(s)\left[\frac{JS^2 + BS}{K_t}\right] \rightarrow ⑩$

⑨ $\Rightarrow V(s) = \underbrace{\theta(s)\left[\frac{JS^2 + BS}{K_t}\right]}_{I_a(s)}[R_a + SL_a] + SK_b\theta(s)$

$= \theta(s)\left[\left(\frac{JS^2 + BS}{K_t}\right)(R_a + SL_a) + K_b S\right]$

$$V(S) = \Theta(S)\left[\frac{(JS^2+BS)(Ra+SLa) + K_t K_b \cdot S}{K_t}\right]$$

$$\boxed{\frac{\Theta(S)}{V(S)} = \frac{K_t}{\underbrace{(JS^2+BS)(Ra+SLa)}_{K_t} + K_t K_b \cdot S}} = T.F$$

Block diagram

$$= \frac{K_t}{(JS^2+BS)(Ra+SLa)\left[1 + \frac{K_t K_b \cdot S}{(JS^2+BS)(Ra+SLa)}\right]}$$