

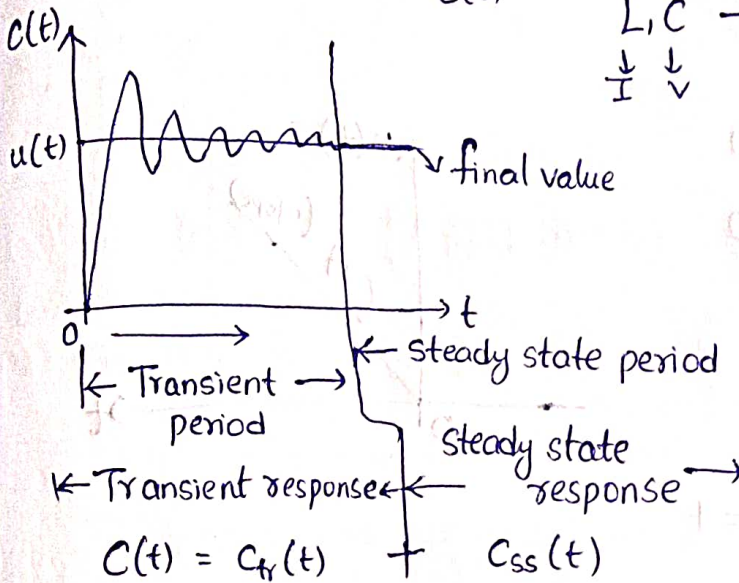
Time Response Analysis

$$C(s) = (T.F.) \cdot R(s)$$

Time Response $\rightarrow$ o/p $\rightarrow C(s) \xrightarrow{L.T} C(t)$ o/p w.r.t time

To observe
To evaluate
To check
To test

$L, C \rightarrow$ oppose sudden change
 $\downarrow \quad \downarrow$
 $I \quad V$

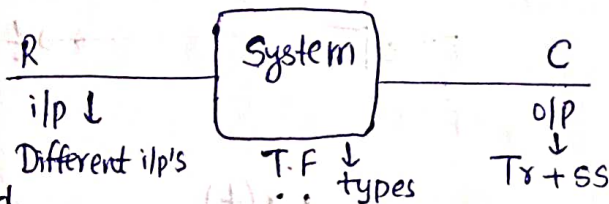


N/w Analysis

$\frac{V}{I}$ } finding

Total solution = homogeneous solution + non-homogeneous solution.

$i/p = 0$	$i/p \neq 0$
force free response	forced response
Source free response	Source fed response

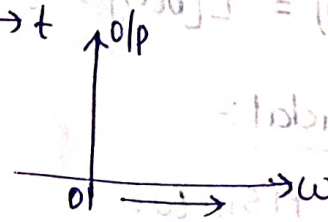
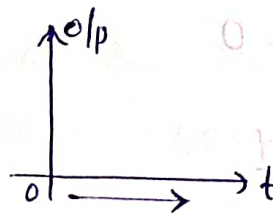


$$C(s) = (TF) R(s)$$

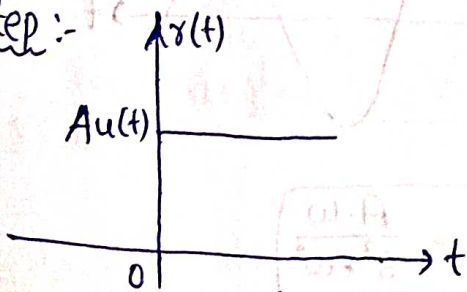
$$C = G \cdot R$$

Standard Test Signals (i/p's)

- 1) Step $\rightarrow$ Time response
- 2) Ramp $\rightarrow$ Time response
- 3) Parabolic $\rightarrow$ Time response
- 4) Impulse $\rightarrow$ Transient analysis
- 5) Sinusoidal $\rightarrow$ frequency response



Step:



$$x(t) = \begin{cases} 0, & t < 0 \\ A, & t \geq 0 \end{cases}$$

$$\rightarrow L[x(t)] = L[A \cdot u(t)] = \frac{A}{s}$$

$$R(s) = \frac{A}{s}$$

→ Unit step i/p : $A=1$

$$L[\gamma(t)] = L[u(t)] = 1/s$$

$$R(s) = \frac{1}{s}$$

② Ramp:

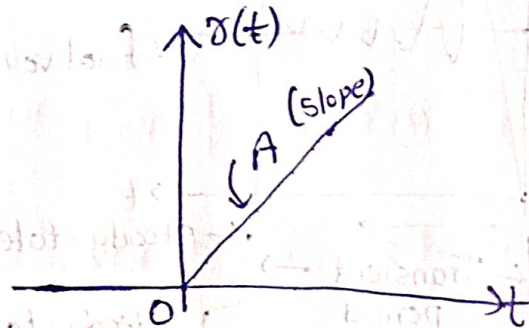
$$\rightarrow \gamma(t) = \begin{cases} 0 & , t < 0 \\ At & , t \geq 0 \end{cases}$$

$$\rightarrow L[\gamma(t)] = L[At] = \frac{A}{s^2}$$

$$R(s) = \frac{A}{s^2}$$

→ unit ramp i/p : $A=1$

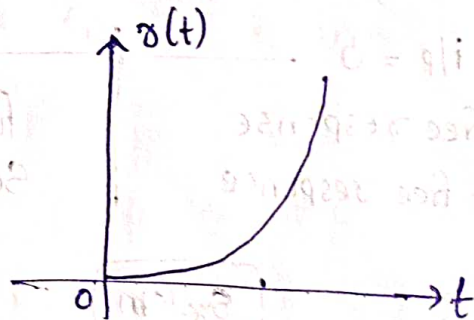
$$R(s) = \frac{1}{s^2}$$



③ Parabolic:

$$\rightarrow \gamma(t) = \begin{cases} 0 & t < 0 \\ \frac{A}{2}t^2 & t \geq 0 \end{cases}$$

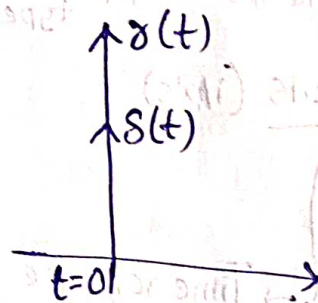
$$\rightarrow L[\gamma(t)] = L\left[\frac{A}{2}t^2\right] = \frac{A}{s^3}$$



④ Impulse:

$$\rightarrow \gamma(t) = \begin{cases} 0 & t \neq 0 \\ \delta(t) & t = 0 \end{cases}$$

$$\rightarrow L[\gamma(t)] = L[\delta(t)] = 1$$



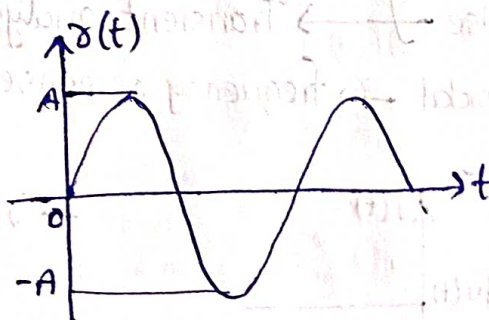
⑤ Sinusoidal:

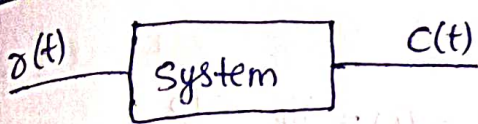
$$\gamma(t) = A \sin \omega t$$

$$\rightarrow \gamma(t) = \begin{cases} 0 & t < 0 \\ A \sin \omega t & t \geq 0 \end{cases}$$

$$L[\gamma(t)] = L[A \sin \omega t] = A \cdot \frac{\omega}{s^2 + \omega^2}$$

$$R(s) = \frac{A \cdot \omega}{s^2 + \omega^2}$$





- 1) first order system $\rightarrow RC, RL \rightarrow \tau = RC, \tau = L/R$
- 2) second order system $\rightarrow RLC$

Type of the system
Order of the system

Generalised T.F

$$\frac{C(s)}{R(s)} = T.F = \frac{K(1+ST_1)(1+ST_2) \dots \text{zeros}}{S^N(1+ST_a)(1+ST_b) \dots \text{poles}}$$

where k = Resultant gain

$(S+1) \rightarrow \text{roots} \rightarrow \text{Numerator} \rightarrow \text{zeros}$
 $(S+2)(S+3) \rightarrow \text{roots} \rightarrow \text{Denominator} \rightarrow \text{poles}$

$T \rightarrow$ Time constant $\rightarrow$ how much fast o/p is changing for change in i/p.

$T_1, T_2, \dots \rightarrow$ Time constants of zero's

$T_a, T_b, \dots \rightarrow$ Time constants of poles

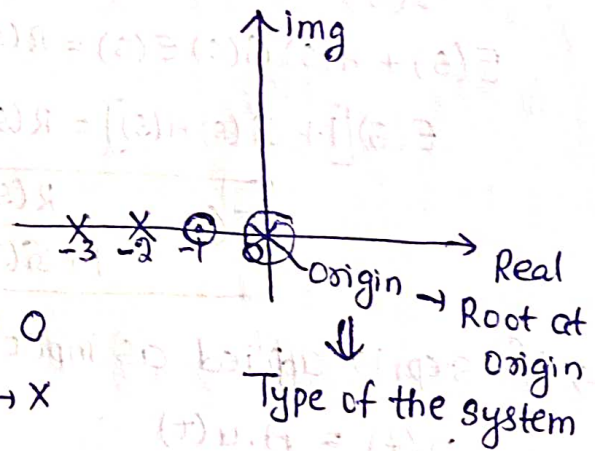
$N \rightarrow$ Type of the system

$N=0 \rightarrow$ Type-0 system

$N=1 \rightarrow$ Type-1 system

$N=2 \rightarrow$ Type-2 system

$(S+1) \rightarrow S=-1 \rightarrow \text{zero's} \rightarrow 0$
 $S(S+2)(S+3) \rightarrow S=0, -2, -3 \rightarrow \text{poles} \rightarrow X$



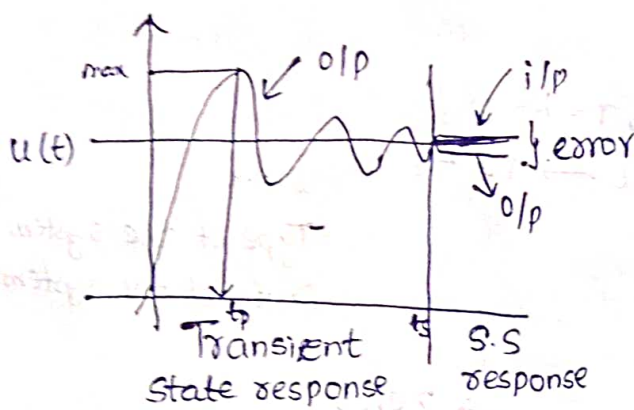
$\rightarrow$ Highest power of 'S' in Denominator is called $\rightarrow$ "Order"

3rd order system; $N=1$ Type-1 system.

$$T.F = k \cdot \frac{(s+z_1)(s+z_2)}{s^N(s+p_1)(s+p_2)}$$

1) $\frac{1}{s^0(s+1)}$ Type 0 order 1

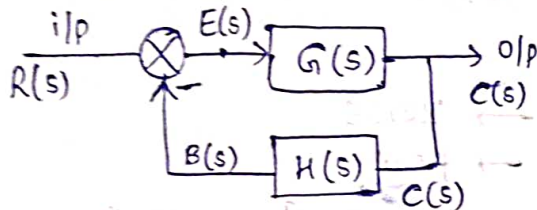
2) $\frac{s+1}{s^2(s+2)(s+4)}$ 2 4



$$e_{ss}(t) = i/p - o/p$$

S.S Analysis

Steady state error, $e_{ss}(t)$



$$R(s) \rightarrow G(s) \rightarrow C(s)$$

$$C(s) = G(s) \cdot R(s)$$

$$R(s) \leftarrow H(s) \leftarrow C(s)$$

$$B(s) = H(s) \cdot C(s)$$

$$\therefore E(s) = R(s) - B(s)$$

$$= R(s) - H(s) \cdot C(s)$$

$$= R(s) - H(s) G(s) R(s)$$

$$E(s) = R(s) - H(s) \cdot G(s) E(s)$$

$$E(s) + H(s) G(s) E(s) = R(s)$$

$$E(s) [1 + G(s) H(s)] = R(s)$$

$$E(s) = \frac{R(s)}{1 + G(s) H(s)}$$

$$C(s) = G(s) \cdot E(s)$$

in terms of error

in terms of error

→ if step is applied as input

$$r(t) = A \cdot u(t)$$

$$R(s) = \frac{A}{s}$$

$$e_{ss}(t) = \lim_{s \rightarrow 0} s \cdot E(s)$$

$$e_{ss}(t) = \lim_{s \rightarrow 0} s \cdot \frac{R(s)}{1 + G(s) H(s)}$$

→ time domain → error

$$e_{ss}(t) = \lim_{t \rightarrow \infty} e(t)$$

$$e_{ss}(t) = \lim_{s \rightarrow 0} s \cdot \left(\frac{A}{s} \right) \frac{1}{1 + G(s) H(s)}$$

→ final value theorem

$$\lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} s \cdot F(s)$$

$$e_{ss}(t) = \lim_{s \rightarrow 0} \frac{A}{1 + G(s) H(s)}$$

$$e_{ss}(t) = \frac{A}{1 + \lim_{s \rightarrow 0} G(s) H(s)} \rightarrow K_p$$

$$e_{ss}(t) = \lim_{t \rightarrow \infty} e(t) = \lim_{s \rightarrow 0} s E(s)$$

$$e_{ss}(t) = \frac{A}{1 + K_p}$$

where $K_p = \lim_{s \rightarrow 0} G(s)H(s)$

position error co-efficient

⇒ for Ramp input

$$r(t) = At \quad ; \quad R(s) = \frac{A}{s^2}$$

$$e_{ss}(t) = \lim_{s \rightarrow 0} s \cdot E(s) = \lim_{s \rightarrow 0} s \cdot \frac{R(s)}{1+G(s)H(s)}$$

$$= \lim_{s \rightarrow 0} s \cdot \frac{(A/s^2)}{1+G(s)H(s)} = \lim_{s \rightarrow 0} \frac{A}{s(1+G(s)H(s))}$$

$$= \lim_{s \rightarrow 0} \frac{A}{s + s \cdot G(s)H(s)} = \frac{A}{\lim_{s \rightarrow 0} s + \underbrace{\lim_{s \rightarrow 0} s \cdot G(s)H(s)}_{K_v}}$$

$$= \frac{A}{0 + K_v}$$

where $K_v = \lim_{s \rightarrow 0} s \cdot G(s)H(s)$

velocity error coefficient

$$e_{ss}(t) = \frac{A}{K_v}$$

⇒ for Parabolic input

$$r(t) = \frac{A}{2} t^2 \quad ; \quad R(s) = \frac{A}{s^3}$$

$$e_{ss}(t) = \lim_{s \rightarrow 0} s \cdot \frac{R(s)}{1+G(s)H(s)} = \lim_{s \rightarrow 0} s \cdot \frac{(A/s^3)}{1+G(s)H(s)}$$

$$= \lim_{s \rightarrow 0} \frac{A}{s^2(1+G(s)H(s))} = \lim_{s \rightarrow 0} \frac{A}{s^2 + s^2 G(s)H(s)}$$

$$= \frac{A}{\lim_{s \rightarrow 0} s^2 + \underbrace{\lim_{s \rightarrow 0} s^2 G(s)H(s)}_{K_a}} = \frac{A}{0 + K_a}$$

$$e_{ss}(t) = \frac{A}{K_a}$$

where $K_a = \lim_{s \rightarrow 0} s^2 G(s)H(s)$

Acceleration error coefficient

Input	$e_{ss}(t)$	
Step	$\frac{A}{1+K_p}$	$K_p = \lim_{s \rightarrow 0} G(s)H(s)$
Ramp	$\frac{A}{K_v}$	$K_v = \lim_{s \rightarrow 0} s \cdot G(s)H(s)$
Parabolic	$\frac{A}{K_a}$	$K_a = \lim_{s \rightarrow 0} s^2 \cdot G(s)H(s)$

(P8) $G(s) \cdot H(s) = \frac{s+1}{s(s+2)(s+3)}$ find $K_p, K_v, K_a, e_{ss}(t)$.

$$K_p = \lim_{s \rightarrow 0} G(s) H(s) = \lim_{s \rightarrow 0} \frac{(s+1)}{s(s+2)(s+3)} = \frac{s+1}{0} = \infty$$

$$K_v = \lim_{s \rightarrow 0} s \cdot G(s) H(s) = \lim_{s \rightarrow 0} s \cdot \frac{(s+1)}{s(s+2)(s+3)} = \lim_{s \rightarrow 0} \frac{s+1}{(s+2)(s+3)}$$

$$= \frac{1}{2+3} = \frac{1}{5}$$

$$K_a = \lim_{s \rightarrow 0} s^2 G(s) H(s) = \lim_{s \rightarrow 0} s^2 \cdot \frac{(s+1)}{s(s+2)(s+3)} = \lim_{s \rightarrow 0} \frac{s(s+1)}{(s+2)(s+3)} = 0$$

Type-1 system

Type-0 $\rightarrow$ Step

Type-1 $\rightarrow$ Ramp

Type-2 $\rightarrow$ Parabolic

Static error coefficients



$$\text{o/p} \leftarrow C = \underset{\substack{\downarrow \\ \text{T.F}}}{G} \cdot \underset{\substack{\downarrow \\ \text{i/p}}}{R}$$

$$\text{s.s } e(t) = \underset{\substack{\downarrow \\ K_p, K_v, K_a}}{\text{TF}} \cdot \underset{\substack{\text{order} \rightarrow \text{Transient state} \\ \text{Type} \rightarrow \text{S.S state}}}{\text{i/p}}$$

Generalised equation

$$G(s) \cdot H(s) = k \cdot \frac{(1+ST_1)(1+ST_2) \dots}{s^N (1+ST_a)(1+ST_b) \dots}$$

$\rightarrow$ zeros
 $\rightarrow$ poles

N = Type of the system

Type $\downarrow$ change $\rightarrow e_{ss}(t)$

$N=0$, Type 0 system

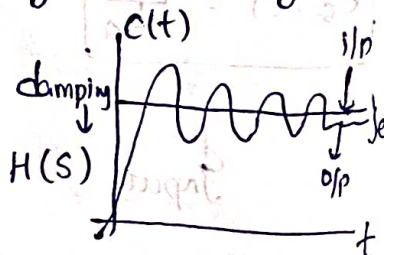
$N=1$, Type 1 system

$N=2$, Type 2 system

Steady State Error Analysis for type-0, type-1 and type-2 Systems with step input :-

step i/p :

$$e_{ss}(t) = \frac{A}{1+K_p}, \text{ where } K_p = \lim_{s \rightarrow 0} G(s) H(s)$$



1) Type-0 :

$$G(s) \cdot H(s) = k \frac{(1+ST_1)(1+ST_2) \dots}{(1+ST_a)(1+ST_b) \dots}$$

$$K_p = \lim_{s \rightarrow 0} \frac{k(1+ST_1)(1+ST_2) \dots}{(1+ST_a)(1+ST_b) \dots} = \frac{k(1+0)(1+0) \dots}{(1+0)(1+0) \dots} = k$$

$$\therefore e_{ss}(t) = \frac{A}{1+K_p} = \frac{A}{1+k} \rightarrow \text{Resultant gain}$$

1) Type-1: $G(s) \cdot H(s) = \frac{K(1+ST_1)(1+ST_2) \dots}{S^1(1+ST_a)(1+ST_b) \dots}$

$k_p = \lim_{s \rightarrow 0} \frac{K(1+ST_1)(1+ST_2) \dots}{S(1+ST_a)(1+ST_b) \dots} = \frac{1}{0} = \infty$

$e_{ss}(t) = \frac{A}{1+k_p} = \frac{A}{1+\infty} = \frac{A}{\infty} = 0 \Rightarrow \boxed{e_{ss}(t) = 0}$

2) Type-2: $G(s) \cdot H(s) = \frac{K(1+ST_1)(1+ST_2) \dots}{S^2(1+ST_a)(1+ST_b) \dots}$

$k_p = \lim_{s \rightarrow 0} \frac{K(1+ST_1)(1+ST_2) \dots}{S^2(1+ST_a)(1+ST_b) \dots} = \frac{1}{0} = \infty$

$e_{ss}(t) = \frac{A}{1+k_p} = \frac{A}{1+\infty} = \frac{A}{\infty} = 0 \Rightarrow \boxed{e_{ss}(t) = 0}$

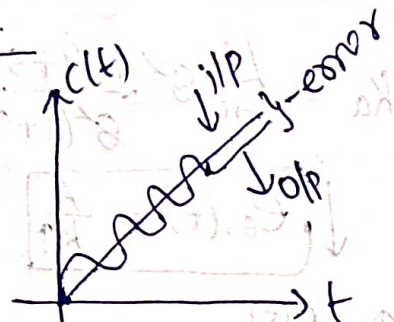
Steady State Error Analysis for type-0, type-1 and type-2 systems with ramp input:-

$e_{ss}(t) = \frac{A}{k_v}$, where $k_v = \lim_{s \rightarrow 0} s \cdot G(s) \cdot H(s)$

1) Type-0: $G(s) \cdot H(s) = \frac{K(1+ST_1)(1+ST_2) \dots}{(1+ST_a)(1+ST_b) \dots}$

$k_v = \lim_{s \rightarrow 0} s \cdot \frac{K(1+ST_1)(1+ST_2) \dots}{(1+ST_a)(1+ST_b) \dots} = 0$

$e_{ss}(t) = \frac{A}{k_v} = \frac{A}{0} = \infty \Rightarrow \boxed{e_{ss}(t) = \infty}$



2) Type-1: $G(s) \cdot H(s) = \frac{K(1+ST_1)(1+ST_2) \dots}{S(1+ST_a)(1+ST_b) \dots}$

$k_v = \lim_{s \rightarrow 0} s \cdot \frac{K(1+ST_1)(1+ST_2) \dots}{S(1+ST_a)(1+ST_b) \dots} = \frac{K(1+0)(1+0) \dots}{(1+0)(1+0) \dots} = K$

$\boxed{k_v = K}$ $e_{ss}(t) = \frac{A}{k_v} = \frac{A}{K} \Rightarrow \boxed{e_{ss}(t) = \frac{A}{K}}$

3) Type-2: $G(s) \cdot H(s) = \frac{K(1+ST_1)(1+ST_2) \dots}{S^2(1+ST_a)(1+ST_b) \dots}$

$k_v = \lim_{s \rightarrow 0} s \cdot \frac{K(1+ST_1)(1+ST_2) \dots}{S^2(1+ST_a)(1+ST_b) \dots} = \frac{1}{0} = \infty$

$e_{ss}(t) = \frac{A}{k_v} = \frac{A}{\infty} = 0 \Rightarrow \boxed{e_{ss}(t) = 0}$

we control error by decreasing K value.

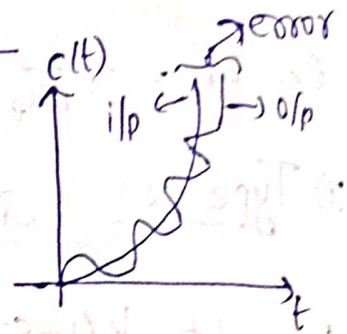
Steady State Error Analysis for type-0, type-1 and type-2 Systems with parabolic input:-

$$e_{ss}(t) = \frac{A}{K_a}, \text{ where } K_a = \lim_{s \rightarrow 0} s^2 G(s) H(s)$$

1) Type-0: $G(s) H(s) = \frac{K(1+ST_1)(1+ST_2) \dots}{(1+ST_a)(1+ST_b) \dots}$

$$K_a = \lim_{s \rightarrow 0} s^2 \frac{K(1+ST_1)(1+ST_2) \dots}{(1+ST_a)(1+ST_b) \dots} = 0$$

$$e_{ss}(t) = \frac{A}{K_a} = \frac{A}{0} = \infty \Rightarrow \boxed{e_{ss}(t) = \infty}$$



2) Type-1: $G(s) \cdot H(s) = \frac{K(1+ST_1)(1+ST_2) \dots}{S(1+ST_a)(1+ST_b) \dots}$

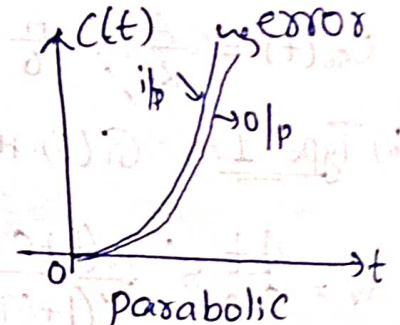
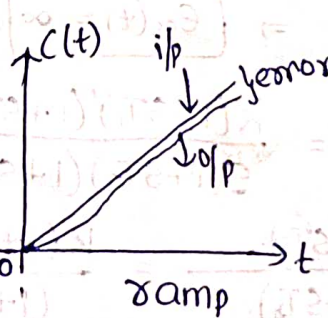
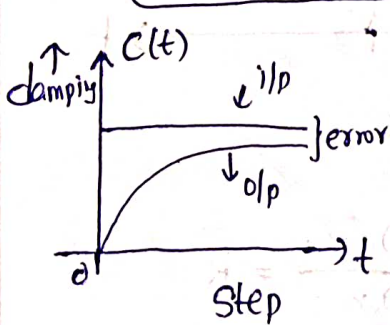
$$K_a = \lim_{s \rightarrow 0} s^2 \frac{K(1+ST_1)(1+ST_2) \dots}{S(1+ST_a)(1+ST_b) \dots} = 0$$

$$e_{ss}(t) = \frac{A}{K_a} = \frac{A}{0} = \infty \Rightarrow \boxed{e_{ss}(t) = \infty}$$

3) Type-2: $G(s) \cdot H(s) = \frac{K(1+ST_1)(1+ST_2) \dots}{S^2(1+ST_a)(1+ST_b) \dots}$

$$K_a = \lim_{s \rightarrow 0} s^2 \frac{K(1+ST_1)(1+ST_2) \dots}{S^2(1+ST_a)(1+ST_b) \dots} = \frac{K(1+0)(1+0) \dots}{(1+0)(1+0) \dots} = K$$

$$\downarrow \boxed{e_{ss}(t) = \frac{A}{K}} \text{ we can control the error}$$

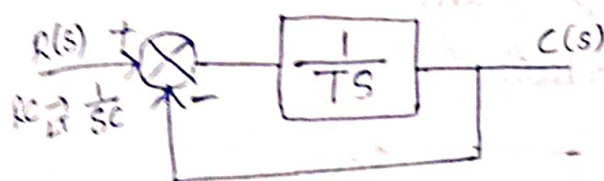


***	Static error coefficients (or) S.S error co-efficients			S.S Error		
	K_p	K_v	K_a	Step	Ramp	Parabolic
Type-0	K	0	0	$\frac{A}{1+K_p}$	∞	∞
Type-1	∞	K	0	0	$\frac{A}{K_v}$	∞
Type-2	∞	∞	K	0	0	$\frac{A}{K_a}$

Time Response Analysis of first order system :-

No. of storage elements = order of the system $\Rightarrow$ TF highest power of 's'

General System:



total no. of poles at origin

$$\frac{C(s)}{R(s)} = \frac{G}{1+GH} = \frac{(1/Ts)}{1+(1/Ts)} = \frac{\frac{1}{Ts}}{\frac{Ts+1}{Ts}} = \frac{1}{Ts+1} = \frac{1}{1+Ts}$$

$$\boxed{C(s) = \frac{R(s)}{1+Ts}}$$

$C(t)$

Unit step i/p: $A=1$; $R(s) = \frac{1}{s}$

$$C(s) = \frac{1/s}{1+Ts} = \frac{1}{s(1+Ts)} = \frac{1}{T \cdot s(s + \frac{1}{T})}$$

$$\frac{1}{Ts(s + \frac{1}{T})} = \frac{A}{s} + \frac{B}{s + \frac{1}{T}} = \frac{1}{s} - \frac{1}{s + \frac{1}{T}}$$

$$\frac{1}{T} = A(s + \frac{1}{T}) + Bs$$

$$s=0 \Rightarrow \frac{1}{T} = A(0 + \frac{1}{T}) + 0 \Rightarrow \frac{A}{T} = \frac{1}{T} \Rightarrow \boxed{A=1}$$

$$s = -\frac{1}{T} \Rightarrow \frac{1}{T} = A(0) + B(\frac{1}{T}) \Rightarrow \frac{1}{T} = \frac{B}{T} \Rightarrow \boxed{B=-1}$$

$$C(s) = \frac{1}{s} - \frac{1}{s + \frac{1}{T}}$$

Apply Inverse Laplace transform,

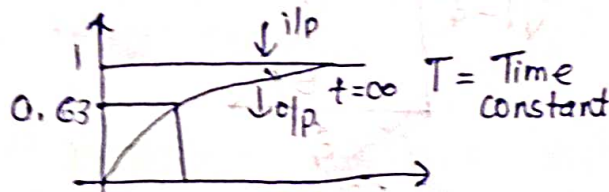
$$\left\{ \begin{array}{l} \because L(1) = \frac{1}{s} \\ L(e^{-at}) = \frac{1}{s+a} \end{array} \right.$$

$$\boxed{C(t) = 1 - e^{-t/T}}$$

$$t=0, C(0) = 1 - e^{-0} \Rightarrow 1 - 1 = 0$$

$$t=\infty, C(\infty) = 1 - e^{-\infty} \Rightarrow 1 - \frac{1}{\infty} = 1$$

$$t=T, C(T) = 1 - e^{-1} \Rightarrow 1 - e^{-1} = \boxed{0.63}$$

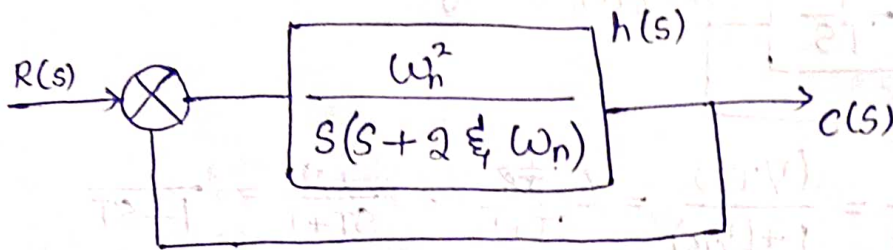


	order	type
$\frac{1}{s(s+1)(s+2)}$	3	1

Time Response Analysis of Second Order System:

Second order system $\rightarrow$ two storage elements
 $\rightarrow$ two roots, two poles

Standard second order closed loop system



$$\text{Closed loop TF} = \frac{C(s)}{R(s)} = \frac{G}{1+GH} = \frac{\left(\frac{\omega_n^2}{s(s+2\xi\omega_n)} \right)}{1 + \frac{\omega_n^2}{s(s+2\xi\omega_n)}}$$

$$= \frac{\omega_n^2 / s(s+2\xi\omega_n)}{\frac{s(s+2\xi\omega_n) + \omega_n^2}{s(s+2\xi\omega_n)}} = \frac{\omega_n^2}{s(s+2\xi\omega_n) + \omega_n^2}$$

$$= \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$\left\{ \begin{array}{l} \xi = \text{Zeta} \\ \xi \text{ or } \zeta \end{array} \right\}$

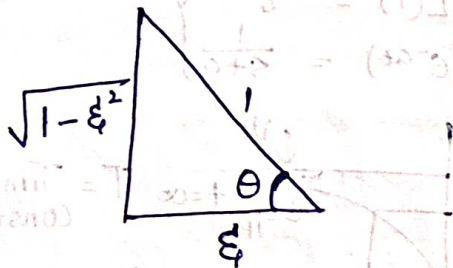
$$\boxed{\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}}$$

Standard second order system T.F

Where, ω_n = natural frequency of oscillations

ξ = Zeta = Damping ratio

Damping triangle, $\swarrow$ opposing nature



$$\sin \theta = \frac{\sqrt{1-\xi^2}}{1} = \sqrt{1-\xi^2}$$

$$\cos \theta = \frac{\xi}{1} = \xi$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\sqrt{1-\xi^2}}{\xi}$$

R, LC $\rightarrow$ oscillations

Power triangle $\omega = \frac{1}{\sqrt{LC}}$

Real P, Q, S

$P = S \cos \theta$

$Q = S \sin \theta$

→ $\xi = 0$, undamped system, only oscillatory response

→ $0 < \xi < 1$, underdamped system

→ (max) $\xi = 1$, critically damped system (no oscillations)

→ $\xi > 1$, overdamped system

Roots of second order system:

→ Standard form, $\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$ (N) → zeroes
(n) → poles (Storage)

Characteristic equation, (CE) $\Rightarrow [1 + G(s)H(s) = 0]$

$$CE \Rightarrow [s^2 + 2\xi\omega_n s + \omega_n^2 = 0]$$

$$\Rightarrow \xi \neq 0 \quad S_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = 1$$

$$b = 2\xi\omega_n$$

$$c = \omega_n^2$$

$$= \frac{-2\xi\omega_n \pm \sqrt{(2\xi\omega_n)^2 - 4 \times 1 \times \omega_n^2}}{2(1)}$$

$$= \frac{-2\xi\omega_n \pm \sqrt{4\xi^2\omega_n^2 - 4\omega_n^2}}{2}$$

$$= \frac{-2\xi\omega_n \pm 2\xi\omega_n\sqrt{\xi^2 - 1}}{2}$$

$$[S_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}]$$

→ $\xi = 0$ (undamped system)

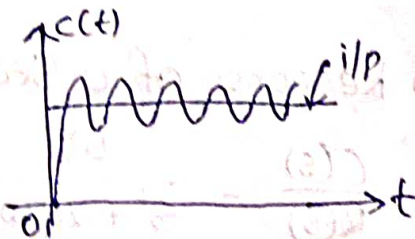
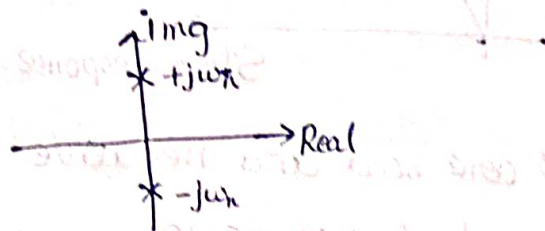
$$S_{1,2} = (0) \pm \omega_n\sqrt{0^2 - 1}$$

$$= 0 \pm \omega_n\sqrt{-1}$$

$$= \pm \omega_n\sqrt{-1} \quad (\because \sqrt{-1} = j)$$

$$[S_{1,2} = \pm j\omega_n]$$

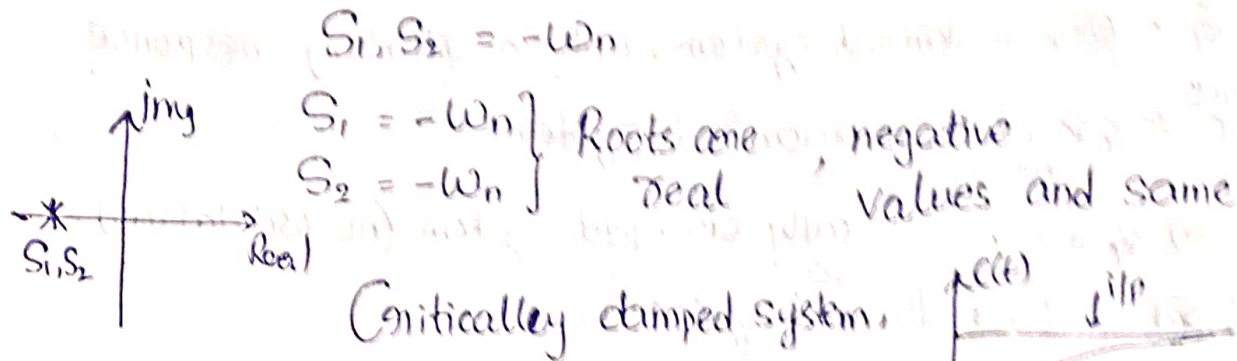
$$\left. \begin{array}{l} S_1 = +j\omega_n \\ S_2 = -j\omega_n \end{array} \right\} \text{Imaginary roots}$$



→ $\xi = 1$, $S_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$

$$S_{1,2} = -(1)\omega_n \pm \omega_n\sqrt{1-1}$$

$$= -\omega_n \pm \omega_n(0)$$



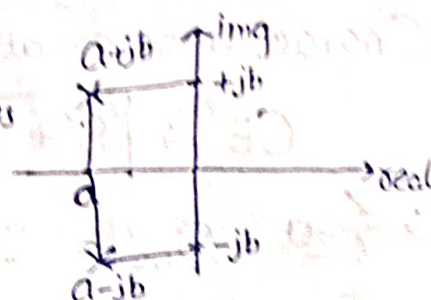
$\Rightarrow \xi < 1$ (under damped system)

roots $\rightarrow$ imag
 roots $\rightarrow$ real

Total roots = real + imag $\rightarrow$ complex values

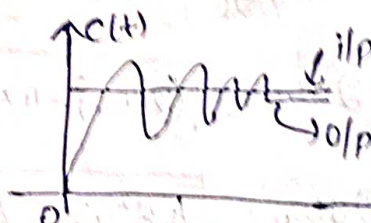
$S_1, S_2 = a \pm jb \Rightarrow a + jb$
 $a - jb$

real imag



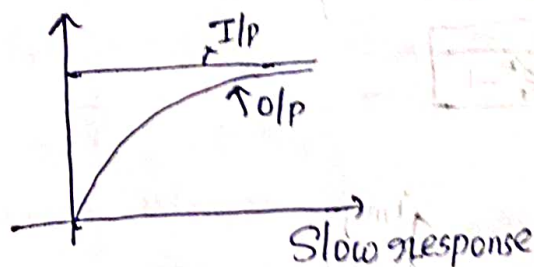
$0 < \xi < 1$

Start oscillation end no oscillation



$\Rightarrow$ under damped system

$\Rightarrow \xi > 1$ (over damped system)



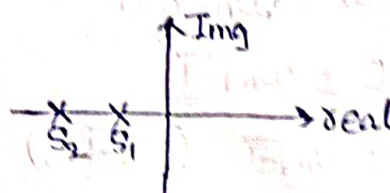
$S_1, S_2 = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$

real +ve, real

$S_1 = \text{real} + \text{real}$

$S_2 = \text{real} - \text{real}$

Roots are real and negative but not equal



Output (or) Response of a Second order system

$C(t) - ?$

$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$ $R(s) = \frac{1}{s}$ (step)

$C(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \times R(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \times \frac{1}{s}$

Inverse LT, $\rightarrow$ partial fractions

$$\frac{1}{s} \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} = \frac{A}{s} + \frac{Bs + C}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$\omega_n^2 = A(s^2 + 2\xi\omega_n s + \omega_n^2) + (Bs + C)s$$

$$\omega_n^2 = A(0 + 0 + \omega_n^2) + 0$$

$$\omega_n^2 = A\omega_n^2 \Rightarrow A = 1$$

$$\underline{s^2} \quad 0 = A + B \Rightarrow B = -A \Rightarrow B = -1$$

$$\underline{s} \quad 0 = 2\xi\omega_n + C \Rightarrow C = -2\xi\omega_n$$

$$C(s) = \frac{1}{s} + \frac{-s - 2\xi\omega_n}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$= \frac{1}{s} - \frac{s + 2\xi\omega_n}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$\frac{1}{s} = 1$$

$$\frac{1}{s+a} = e^{-at}$$

$$\sin^2 \omega t = \frac{\omega^2}{s^2 + \omega^2}$$

$$\cos^2 \omega t = \frac{s}{s^2 + \omega^2}$$

$$\begin{array}{l} a^2 = s^2 \\ a = s \end{array} \quad \left| \quad \begin{array}{l} 2\xi\omega_n s = 2ab \\ b = \xi\omega_n \end{array} \right.$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(s + \xi\omega_n)^2 = s^2 + 2\xi\omega_n s + (\xi\omega_n)^2$$

$$= \frac{1}{s} - \frac{s + 2\xi\omega_n}{s^2 + 2\xi\omega_n s + (\xi\omega_n)^2 + (\xi\omega_n)^2 + \omega_n^2}$$

$$= \frac{1}{s} - \frac{s + 2\xi\omega_n}{(s + \xi\omega_n)^2 - \xi^2\omega_n^2 + \omega_n^2}$$

$$= \frac{1}{s} - \frac{s + 2\xi\omega_n}{(s + \xi\omega_n)^2 + \omega_n^2(1 - \xi^2)}$$

$$= \frac{1}{s} - \frac{s + 2\xi\omega_n}{(s + \xi\omega_n)^2 + \omega_d^2} \quad \because \omega_d = \omega_n \sqrt{1 - \xi^2}$$

Damping of Oscillations

$$= \frac{1}{s} - \frac{(s + \xi\omega_n) + \xi\omega_n}{(s + \xi\omega_n)^2 + \omega_d^2}$$

$$= \frac{1}{s} - \frac{(s + \xi\omega_n)}{(s + \xi\omega_n)^2 + \omega_d^2} + \frac{\xi\omega_n}{(s + \xi\omega_n)^2 + \omega_d^2} \times \frac{\omega_d}{\omega_d}$$

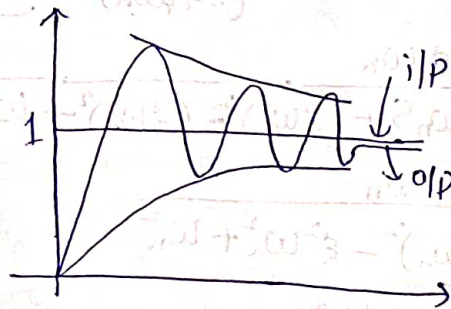
$$= \frac{1}{s} - \frac{s + \xi\omega_n}{(s + \xi\omega_n)^2 + \omega_d^2} + \frac{\omega_d}{(s + \xi\omega_n)^2 + \omega_d^2} \left(\frac{\xi\omega_n}{\omega_d} \right)$$

I.L.T

$$C(t) = 1 - e^{-\xi\omega_n t} \cos \omega_d t + e^{-\xi\omega_n t} \sin \omega_d t \times \xi \left(\frac{\omega_n}{\omega_d} \right)$$

$$\begin{aligned}
 C(t) &= 1 - \left\{ e^{-\xi \omega_n t} \cos \omega_d t + e^{-\xi \omega_n t} \sin \omega_d t \times \frac{\xi \omega_n}{\omega_n \sqrt{1-\xi^2}} \right\} \\
 &= 1 - \left\{ e^{-\xi \omega_n t} \cos \omega_d t + e^{-\xi \omega_n t} \sin \omega_d t \times \frac{\xi}{\sqrt{1-\xi^2}} \right\} \begin{matrix} \rightarrow \cos \theta \\ \rightarrow \sin \theta \end{matrix} \\
 &= 1 - \left\{ e^{-\xi \omega_n t} \cos \omega_d t + e^{-\xi \omega_n t} \sin \omega_d t \times \frac{\cos \theta}{\sin \theta} \right\} \\
 &= 1 - \frac{e^{-\xi \omega_n t}}{\sin \theta} \left\{ \cos \omega_d t \sin \theta + \sin \omega_d t \cos \theta \right\} \\
 &= 1 - \frac{e^{-\xi \omega_n t}}{\sin \theta} \left\{ \sin(\omega_d t + \theta) \right\} \quad \left[\because \sin(A+B) = \sin A \cos B + \cos A \sin B \right]
 \end{aligned}$$

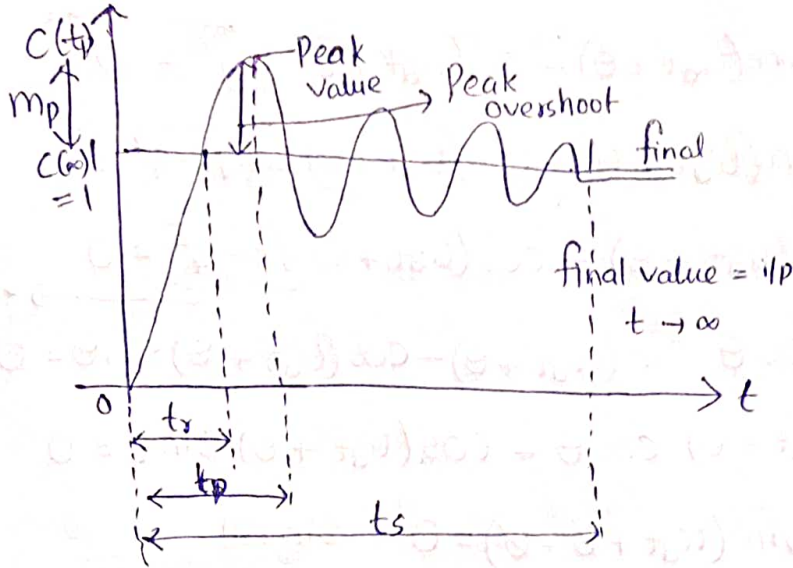
$$C(t) = 1 - \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta) \quad \text{where, } \theta = \tan^{-1} \left(\frac{\sqrt{1-\xi^2}}{\xi} \right)$$



Time (Transient) Response Specifications :

Time domain Specifications :

- 1) Rise time (t_r) $\rightarrow$ Time required by the response to reach its final value at first cycle
- 2) Peak time (t_p) $\rightarrow$ Time required by the response (o/p) to reach max value
- 3) Peak overshoot (m_p) $\rightarrow m_p = C(t_p) - C(\infty)$
 $\% m_p = \frac{C(t_p) - C(\infty)}{C(\infty)} \times 100 \Rightarrow \text{Representation}$
- 4) Settling time (t_s) $\rightarrow$ The time required by the response (o/p) to reach its final value.



$t_s \rightarrow$ time required for the oscillations become zero.

1) Rise time (t_r) :-

o/p equation for
second order
under damped system

$$C(t) = 1 - e^{-\xi \omega_n t} \sin(\omega_d t + \theta)$$

$$C(t) = 1 \quad \underline{t_r}$$

$$X = X - e^{-\xi \omega_n t} \sin(\omega_d t + \theta)$$

0 to 100% $\rightarrow$ under damped

1/p $\rightarrow$ 1 ; 0/p $\rightarrow$ 1

$$e^{-\xi \omega_n t} \sin(\omega_d t + \theta) = 0$$

0 to 95% $\rightarrow$ critically damped

0 to 90% $\rightarrow$ over damped

$$\sin(\omega_d t + \theta) = 0 = \sin n\pi$$

$$\omega_d t + \theta = n\pi \quad n=1,2,\dots$$

where, n = cycle number

$$\omega_d t = n\pi - \theta$$

for 1st cycle, $\boxed{t_r = \frac{\pi - \theta}{\omega_d}}; n=1$

$\boxed{t_r = t_s}$

$$\boxed{t_r = \frac{n\pi - \theta}{\omega_d}}$$

2) Peak time (t_p)

$$C(t) = 1 - \frac{e^{-\xi \omega_n t}}{\sqrt{1 - \xi^2}} \sin(\omega_d t + \theta)$$

$$\xi \omega_n \sin(\omega_d t + \theta) - \cos(\omega_d t + \theta) \omega_d = 0$$

$$\xi \omega_n \sin(\omega_d t + \theta) - \cos(\omega_d t + \theta) \omega_n \sqrt{1 - \xi^2} = 0$$

$$\xi \sin(\omega_d t + \theta) - \cos(\omega_d t + \theta) \sqrt{1 - \xi^2} = 0$$

$$\cos \theta \sin(\omega_d t + \theta) - \cos(\omega_d t + \theta) \sin \theta = 0$$

$$\sin(\omega_d t + \theta) \cos \theta - \cos(\omega_d t + \theta) \sin \theta = 0$$

$$\sin(\omega_d t + \theta - \theta) = 0 = \sin n\pi$$

$$\omega_d t = n\pi \quad n = \text{cycle number}$$

$$\boxed{t_p = \frac{n\pi}{\omega_d}} \quad n=1 \quad \text{first cycle} \quad \boxed{t_p = \frac{\pi}{\omega_d}}$$

3) Peak Overshoot

$$\% m_p = \frac{C(t_p) - C(\infty)}{C(\infty)} \times 100$$

$$C(t) = 1 - \frac{e^{-\xi \omega_n t}}{\sqrt{1 - \xi^2}} \sin(\omega_d t + \theta)$$

$$C(t_p) = 1 - \frac{e^{-\xi \omega_n t_p}}{\sqrt{1 - \xi^2}} \sin(\omega_d t_p + \theta)$$

$$C(t_p) - \underbrace{C(\infty)}_{=1} = 1 - \frac{e^{-\xi \omega_n t_p}}{\sqrt{1 - \xi^2}} \sin(\omega_d t_p + \theta) - 1 = -\frac{e^{-\xi \omega_n t_p}}{\sqrt{1 - \xi^2}} \sin(\omega_d t_p + \theta)$$

$$= -\frac{e^{-\xi \omega_n t_p}}{\sqrt{1 - \xi^2}} \sin\left(\omega_d \times \frac{n\pi}{\omega_d} + \theta\right) = -\frac{e^{-\xi \omega_n t_p}}{\sqrt{1 - \xi^2}} \sin(\pi + \theta)$$

$$= -\frac{e^{-\xi \omega_n t_p}}{\sqrt{1 - \xi^2}} (\sin \theta)$$

$$C(t_p) - C(\infty) = \frac{e^{-\xi \omega_n t_p}}{\underbrace{\sqrt{1 - \xi^2}}_{=\sin \theta}} \sin \theta = \frac{e^{-\xi \omega_n t_p}}{\sin \theta} \cdot \sin \theta$$

$$C(t_p) - C(\infty) = e^{-\xi \omega_n t_p}$$

$$\% m_p = \frac{C(t_p) - C(\infty)}{C(\infty)} \times 100 = \frac{e^{-\xi \omega_n t_p}}{1} \times 100$$

$$\% m_p = e^{-\xi \omega_n t_p} \times 100$$

i) Settling time (t_s):

$$c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \underbrace{\sin(\omega_d t + \theta)}_{\text{neglect}} = 1 - e^{-\xi\omega_n t}$$

$$0.98 = 1 - e^{-\xi\omega_n t}$$

$$e^{-\xi\omega_n t} = 1 - 0.98$$

$$e^{-\xi\omega_n t} = 0.02$$

$$-\xi\omega_n t = \ln(0.02)$$

$$\xi\omega_n t = 3.96$$

$$t_s = \frac{3.96}{\xi\omega_n}$$

$$\Rightarrow \boxed{t_s \approx \frac{4}{\xi\omega_n}} \text{ for } 2\% \text{ tolerance}$$

$$\Rightarrow \boxed{t_s \approx \frac{3}{\xi\omega_n}} \text{ for } 5\% \text{ tolerance}$$

Stability:

→ if o/p (response) is under control → stable (within given limits in i/p)

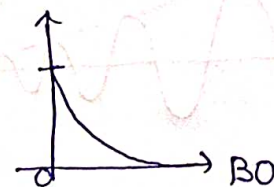
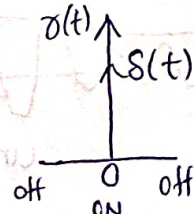
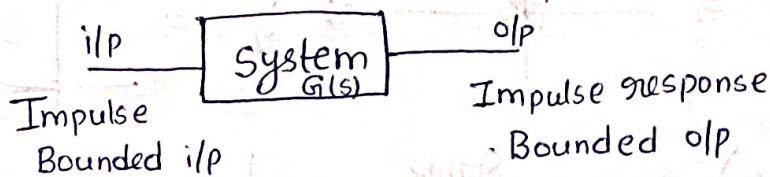
→ if o/p (response) is not under control → unstable

i) for any variation in i/p (or) in system parameter, the o/p is constant → stable
no change in o/p

Otherwise → unstable

iii) BIBO → Bounded input Bounded output

i/p	Bounded i/p
Step	Step → sudden change
Damp	impulse → sudden shock
Parabola	↳ high values within '0' time → stability
impulse	
Sinusoidal	



As $t \rightarrow \infty$, The impulse response should be zero

→ Steady State error

$$\text{step } e_{ss} = \frac{A}{1+K_p} = \frac{A}{1+\infty} = \frac{A}{\infty} = 0$$

$$\text{ramp } e_{ss} = \frac{A}{K_v} = \frac{A}{5} = \frac{A}{5} = \frac{4}{5}$$

$$\text{parabola } e_{ss} = \frac{A}{K_a} = \frac{A}{0} = \infty$$

② Open loop TF of a control system with unity feedback

$$\text{is } G(s) = \frac{18}{s(1+0.1s)}$$

Evaluate S.S error for $r(t) = 1 + 2t + t^2$

Soln $G(s) = \frac{18}{s(1+0.1s)}$

$\downarrow$ $\downarrow$ $\downarrow$
Step Ramp Parabola
 $A=1$ $A=2$ $A=\frac{t^2}{2}$

$$\rightarrow K_p = \lim_{s \rightarrow 0} s \cdot \frac{18}{s(1+0.1s)} = \frac{1}{0} = \infty$$

$$K_v = \lim_{s \rightarrow 0} s \cdot \frac{18}{s(1+0.1s)} = \frac{18}{1+0} = 18$$

$$K_a = \lim_{s \rightarrow 0} s^2 \cdot \frac{18}{s(1+0.1s)} = \frac{18 \times 0}{0} = 0$$

$$\left\{ \begin{array}{l} \frac{A}{2} t^2 = t^2 \\ \frac{A}{2} = 1 \\ A = 2 \end{array} \right.$$

Steady state error,

$$e_{ss} = \frac{A}{1+K_p} = \frac{A}{1+\infty} = \frac{A}{\infty} = 0$$

$$e_{ss} = \frac{A}{K_v} = \frac{A}{18} = \frac{2}{18} = \frac{1}{9}$$

$$e_{ss} = \frac{A}{K_a} = \frac{A}{0} = \infty$$

$$\rightarrow \text{S.S error for } r(t) = 1 + 2t + t^2 \text{ is}$$
$$= 0 + \frac{1}{9} + \infty$$

$\text{S.S error} = \infty$

13/9

$$\frac{C(s)}{R(s)} = \frac{25}{s^2 + 6s + 25}$$

model - II

Basic Second order system equation, $\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$

$$\rightarrow \omega_n^2 = 25 \Rightarrow \omega_n = \sqrt{25} = 5 \text{ rad/sec}$$

Natural frequency

$$\rightarrow 2\xi\omega_n = 6$$

$$\xi\omega_n = 3$$

$$\xi = \frac{3}{5} = 0.6$$

$$\rightarrow \omega_d = \omega_n \sqrt{1 - \xi^2} \quad \sqrt{1 - (0.6)^2} = 0.8$$

$$= 5 \times \sqrt{1 - (0.6)^2}$$

$$= 5 \times 0.8 = 4 \text{ rad/sec}$$

$$\rightarrow \theta = \tan^{-1} \left(\frac{\sqrt{1 - \xi^2}}{\xi} \right) = \tan^{-1} \left(\frac{0.8}{0.6} \right)$$

$$\theta = 53.13 \text{ in degrees}$$

$$\theta = 53.13 \times \frac{\pi}{180} = 0.92 \text{ radians}$$

$$\rightarrow t_r = \frac{\pi - \theta}{\omega_d} = \frac{3.14 - 0.92}{4} = 0.55 \text{ sec}$$

$$\rightarrow t_p = \frac{\pi}{\omega_d} = \frac{3.14}{4} = 0.755 \text{ sec}$$

$$\rightarrow t_s = \frac{4}{\xi\omega_n} = \frac{4}{0.6 \times 5} = 1.33 \text{ sec}$$

$$\rightarrow \% \text{ mp} = e^{-\xi\omega_n t_p} \times 100 = e^{-0.6 \times 5 \times 0.755} \times 100 =$$

To find $c(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1 - \xi^2}} \sin(\omega_d t + \theta)$

$$= 1 - \frac{e^{-0.6 \times 5 \times t}}{0.8} \sin(4t + 53.13^\circ)$$

$$C(t) = 1 - 1.2 e^{-3t} \sin(4t + 53.13^\circ)$$

Model-II:- $\frac{C(s)}{R(s)} = \frac{100}{s^2 + 6s + 25} = \frac{100}{25} \times \frac{25}{s^2 + 6s + 25}$

$$C(t) = \frac{100}{25} \left[1 - 1.2 e^{-3t} \sin(4t + 53.13^\circ) \right]$$

Model-III:- $G(s) = \frac{K}{s(s+10)}$ for step i/p having $\xi = 0.5$
unity feedback

find $K, t_p, t_r, t_s, x_{mp}, \omega_n, \xi, \omega_d, \theta$

Sol: Basic second order TF = $\frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$

To find K ,

$\therefore$ Write CE; $1 + G(s)H(s) = 0$

$$1 + \frac{K}{s(s+10)} \times 1 = 0$$

$$s(s+10) + K = 0$$

$$s^2 + 10s + K = 0$$

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0 \quad \leftarrow \text{compare}$$

$$\Rightarrow \begin{cases} 2\xi\omega_n = 10 \\ \xi\omega_n = 5 \\ \xi = \frac{5}{\omega_n} \end{cases} \quad \begin{cases} \omega_n^2 = K \\ \omega_n = \sqrt{K} \\ \omega_n = \sqrt{100} = 10 \text{ rad/sec} \end{cases}$$

$$\xi = \frac{5}{\sqrt{K}} \Rightarrow \sqrt{K} = \frac{5}{0.5}$$

$$\sqrt{K} = \frac{5}{0.5} = 10$$

$$K = (10)^2 = 100$$

$$\rightarrow \omega_d = \omega_n \sqrt{1 - \xi^2} = 10 \times 0.86 = 8.6 \text{ rad/sec}$$

$$\rightarrow \theta = \tan^{-1}\left(\frac{\sqrt{1 - \xi^2}}{\xi}\right) = \tan^{-1}\left(\frac{0.86}{0.5}\right) = 59.82^\circ$$

$$\rightarrow \theta_{\text{in radians}} = 59.82 \times \frac{\pi}{180} = 1.03 \text{ radians}$$

$$\rightarrow t_r = \frac{\pi - \theta}{\omega_d} = \frac{3.14 - 1.03}{8.6} = 0.244 \text{ sec}$$

$$\rightarrow t_p = \frac{\pi}{\omega_d} = \frac{3.14}{8.6} = 0.365 \text{ sec}$$

$$\rightarrow t_s = \frac{4}{\xi \omega_n} = \frac{4}{0.5 \times 10} = 0.8 \text{ sec}$$

$$\rightarrow \% \text{ mp} = e^{-\xi \omega_n t_p} \times 100 = e^{-0.5 \times 10 \times 0.365} \times 100 = 0.183 \times 100 = 18.3 \%$$

$$\text{To find } c(t) = 1 - \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)$$

$$C(t) = 1 - \frac{e^{-0.5 \times 10 t}}{0.86} \sin(8.6 t + 59.82^\circ)$$

$$T.F = \frac{\omega_n^2}{s^2 + 2\xi \omega_n s + \omega_n^2} = \frac{(10)^2}{s^2 + 2\xi(10)s + (10)^2} = \frac{100}{s^2 + 20\xi s + 100}$$

$$TF = \frac{100}{s^2 + 10s + 100}$$

Steady State

$$1) G(s) = \frac{10(s+2)}{s(s+1)(s+4)}$$

find 1) Type 2) order 3) static error coefficients

4) Steady state error for ramp i/p of magnitude 4.

Sol: $\rightarrow$ Type-1

$\rightarrow$ Order-3 (only poles) (Denominator highest power of s)

$\rightarrow$ Static error coefficients, $H(s)=1$ (Unity feedback)

$$K_p = \lim_{s \rightarrow 0} G(s)H(s) = \lim_{s \rightarrow 0} \frac{10(s+2)}{s(s+1)(s+4)} = \infty$$

$$K_v = \lim_{s \rightarrow 0} s \cdot G(s)H(s) = \lim_{s \rightarrow 0} s \cdot \frac{10(s+2)}{s(s+1)(s+4)} = \frac{10(0+2)}{(0+1)(0+4)} = \frac{10 \times 2}{4} = 5$$

$$K_a = \lim_{s \rightarrow 0} s^2 G(s)H(s) = \lim_{s \rightarrow 0} s^2 \cdot \frac{10(s+2)}{s(s+1)(s+4)} = 0$$

Stability $\rightarrow$ System $\rightarrow$ T.F

$$TF = \frac{K(1+ST_1)(1+ST_2) \dots \dots \dots}{S^N(1+ST_a)(1+ST_b) \dots \dots \dots}$$

$\rightarrow$ roots = zeros

$\rightarrow$ roots = poles

Stability depends on poles

$$T.F = \frac{1}{s+1}$$

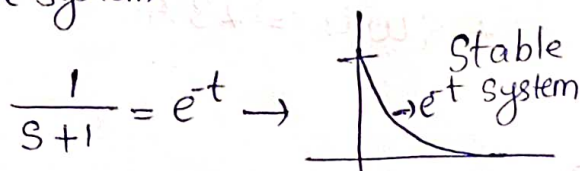
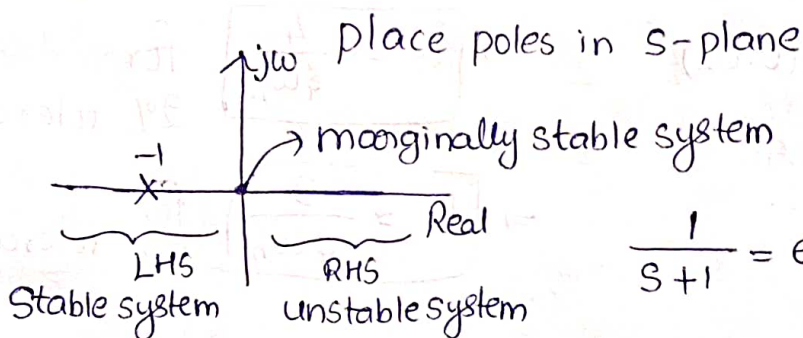
$$\Rightarrow O/P = (TF)(I/P) = \frac{1}{s+1} \times 1 = \frac{1}{s+1}$$

$$\delta(t) = \delta(t) \rightarrow \text{impulse}$$

$$\text{roots} = \text{poles} = 1$$

$$R(s) = 1$$

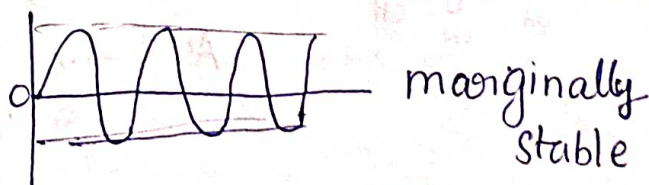
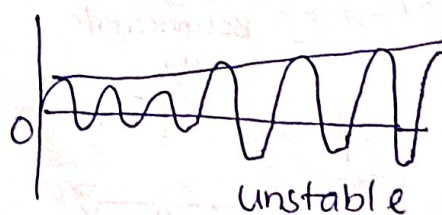
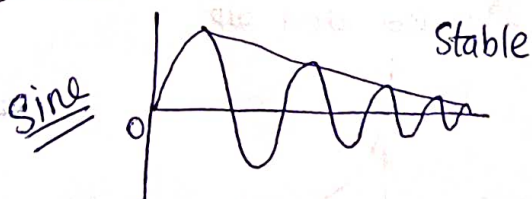
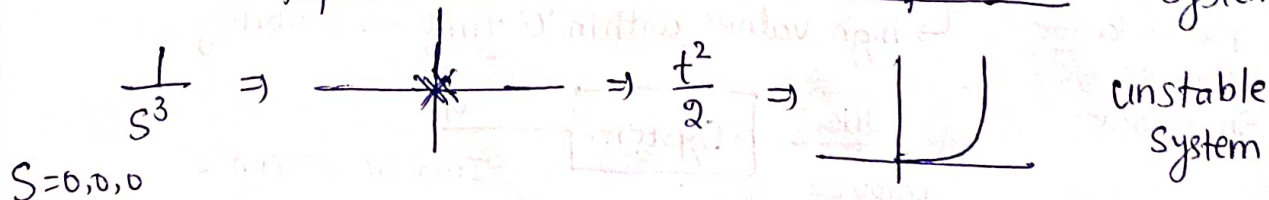
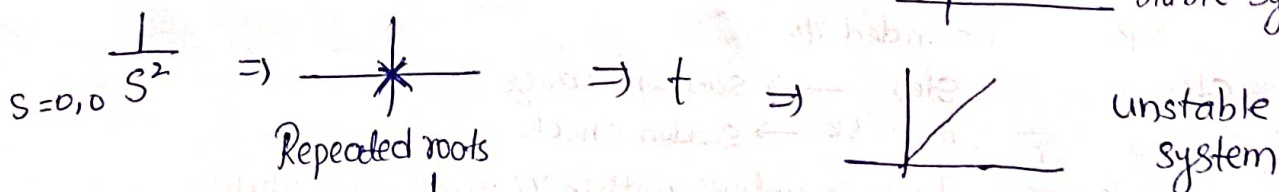
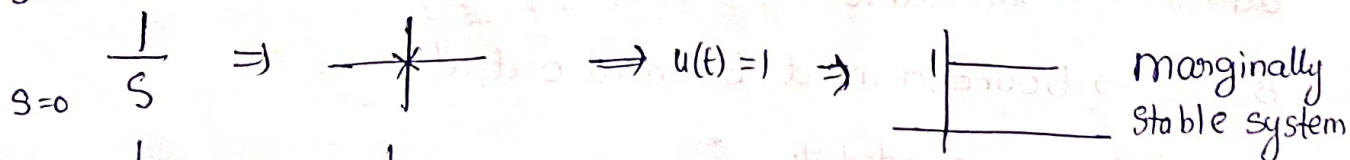
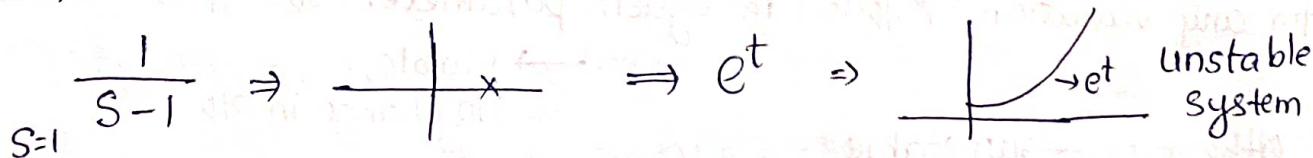
$$s = -1 \text{ (real plane)}$$



1) If poles are in LHS $\rightarrow$ Stable system

2) If poles are in RHS $\rightarrow$ Unstable system

3) If poles are on jw axis $\rightarrow$ marginally stable system.



Hurwitz Criteria (or Method)

→ given T.F

→ write C.E [characteristic equation]

$$CLTF = \frac{G(s)}{1+G(s)H(s)}$$

$$1+G(s)H(s) = 0 \rightarrow CE$$

$$+a_0s^3 + a_1s^2 + a_2s + a_3 = 0 \rightarrow CE$$

→ ① Check for sign → same sign → stable

→ if any sign change exist → unstable

② Check for power of 'S' ⇒

1) should be in ascending order (high to low) → stable

Otherwise → unstable

2) No missing terms of power → stable

missing terms → unstable

③ Check for coefficients ⇒ No missing terms → stable

a_0, a_1, a_2

otherwise → unstable

$$① s^3 + 2s^2 + s + 1 = 0$$

+ + + + ⇒ same sign → stable system

Roots are in LHS of s-plane

$$② s^4 + s^3 + 2s^2 - s + 1 = 0$$

+ + + - +
① ②

⇒ '2' sign changes → unstable system

No. of poles in RHS = no. of sign changes = 2

2-poles are in RHS

$$③ s^5 + 2s^4 - 3s^3 + 2s^2 - 2s + 1 = 0$$

unstable system

+ + - + - +
① ② ③ ④

⇒ no. of sign changes = 4

4 poles are in RHS of S-plane

Routh Method Criteria

Routh - Hurwitz Criteria (or) R-H Criteria

CE:

$$a_0 s^5 + a_1 s^4 + a_2 s^3 + a_3 s^2 + a_4 s + a_5 = 0$$

CLTF $\rightarrow$ stability

$$G(s)H(s)$$

$$1 + G(s)H(s) = 0$$

R-H Table:

s^5	a_0	a_2	a_4
s^4	a_1	a_3	a_5
s^3	b_1	b_2	b_3
s^2	c_1	c_2	
s^1			
s^0			

Stability Analysis - RH method:

- 1) Observe or check first column in RH Table
- 2) Check for sign $\Rightarrow$ Same $\rightarrow$ stable
Otherwise $\rightarrow$ unstable
- 3) No. of poles in RHS of s-plane = no. of sign changes

Ex $s^4 + 2s^3 + 4s^2 + 6s + 8 = 0$ check for stability using

RH criteria.

Ans RH Table:

s^4	1	4	8
s^3	2	6	0
s^2	1	8	
s^1	-10	0	
s^0	8		

$$\frac{2 \times 4 - 1 \times 6}{2} = \frac{8-6}{2} = 1$$

$$\frac{1 \times 6 - 2 \times 8}{1} = -10$$

$$\frac{2 \times 8 - 1 \times 0}{2} = \frac{16}{2} = 8$$

$$\frac{2 \times 0 - 1 \times 0}{1} = 0$$

$$\frac{-10 \times 8 - 1 \times 0}{-10} = 8$$

Stability checking:

$\rightarrow$ Check first column in RH Table

Check for sign changes:-

Sign changes exist $\rightarrow$ unstable system

No. of poles in RHS of S-plane = no. of sign changes = 2

$$s^4 + 2s^3 + 3s^2 + 4s + 5 = 0$$

RH Table:

s^4	1	3	5
s^3	2	4	0
s^2	+1	5	
s^1	-6	0	
s^0	5		

$$\begin{array}{cc} 1 & 3 \\ 2 & 4 \end{array} \Rightarrow \frac{6-4}{2} = -1$$

$$\begin{array}{cc} 1 & 5 \\ 2 & 0 \end{array} \Rightarrow \frac{10}{2} = 5$$

$$\begin{array}{cc} 2 & 4 \\ +1 & 5 \end{array} \Rightarrow \frac{+4-10}{+1} = -6$$

$$\begin{array}{cc} +1 & 5 \\ -6 & 0 \end{array} \Rightarrow \frac{-30}{-6} = 5$$

Sign changes exist $\rightarrow$ unstable system

No. of poles in RHS of S-plane = no. of sign changes = 2

$$s^5 + 4s^4 + 3s^3 + 4s^2 + 7s + 1 = 0$$

$$s^3 + 5s^2 + 7s + 8 = 0$$

RH Table:

s^3	1	7
s^2	5	8
s^1	$\frac{27}{5}$	0
s^0	8	

$$\begin{array}{cc} 1 & 7 \\ 5 & 8 \end{array} \Rightarrow \frac{35-8}{5} = \frac{27}{5}$$

$$\begin{array}{cc} 5 & 8 \\ \frac{27}{5} & 0 \end{array} \Rightarrow \frac{\frac{27}{5} \times 8 - 0}{\frac{27}{5}} = 8$$

All are same sign

System is stable

All poles are in LHS of S-plane

$$s^5 + 4s^4 + 3s^3 + 4s^2 + 7s + 1 = 0$$

s^5	1	3	7	2	$\frac{27}{4}$
s^4	4	4	1	$-\frac{19}{2}$	1
s^3	2	$\frac{27}{4}$	0		
s^2	$-\frac{19}{2}$	1	0		
s^1	$\frac{5 \times 27}{16}$	0			
s^0	1				

No. of sign changes = 2

No. of poles in RHS of S-plane = 2

unstable system

$$\begin{array}{cc} 1 & 3 \\ 4 & 4 \end{array} \Rightarrow \frac{12-4}{4} = \frac{8}{4} = 2$$

$$\begin{array}{cc} 1 & 7 \\ 4 & 1 \end{array} \Rightarrow \frac{28-1}{4} = \frac{27}{4}$$

$$\begin{array}{cc} 4 & 4 \\ 2 & \frac{27}{4} \end{array} \Rightarrow \frac{8-27}{2} = -\frac{19}{2}$$

$$\begin{array}{cc} 4 & 1 \\ 2 & 0 \end{array} \Rightarrow \frac{2}{2} = 1$$

II) If any element in first column of RH Table is zero

→ Construct RH-Table

→ Identify the zero element in first column

→ $s = \frac{1}{z}$ in CE

→ Rearrange 'Z' powers in descending order

→ Again construct RH-Table

→ Check for sign in first column.

→ Same $\Rightarrow$ stable ; otherwise $\Rightarrow$ unstable.

(P8) $s^5 + 2s^4 + 3s^3 + 6s^2 + 2s + 1 = 0$ check for Stability using RH-method.

A: RH-Table:-

s^5	1	3	2
s^4	2	6	1
s^3	$\boxed{0}$	$\frac{3}{2}$	
s^2			

element in first column is zero

→ $s = \frac{1}{z}$ in CE

$$CE \Rightarrow \left(\frac{1}{z}\right)^5 + 2\left(\frac{1}{z}\right)^4 + 3\left(\frac{1}{z}\right)^3 + 6\left(\frac{1}{z}\right)^2 + 2\left(\frac{1}{z}\right) + 1 = 0$$

$$\frac{1}{z^5} + 2\frac{1}{z^4} + 3\frac{1}{z^3} + 6\frac{1}{z^2} + 2\frac{1}{z} + 1 = 0$$

$$1 + 2z + 3z^2 + 6z^3 + 2z^4 + z^5 = 0$$

$$z^5 + 2z^4 + 6z^3 + 3z^2 + 2z + 1 = 0$$

System is unstable

No. of sign changes = 2

2 poles are in RHS of S-plane.

Again RH Table for z

z^5	1	6	2
z^4	2	3	1
z^3	$\frac{9}{2}$	$\frac{3}{2}$	0
z^2	$\frac{21}{4} - \frac{7}{3}$	1	
z^1	$-\frac{3}{2}$	0	
z^0	1		

Q) $s^5 + s^4 + 2s^3 + 2s^2 + 3s + 5 = 0$ Again RH Table for Z

$$\begin{array}{c|ccc} s^5 & 1 & 2 & 3 \\ s^4 & 1 & 2 & 5 \\ s^3 & \cancel{1} & -2 & \end{array}$$

element in first column is zero

$\rightarrow s = \frac{1}{2}$ in CE

$\Rightarrow \left(\frac{1}{2}\right)^5 + \left(\frac{1}{2}\right)^4 + 2\left(\frac{1}{2}\right)^3 + 2\left(\frac{1}{2}\right)^2 + 3\left(\frac{1}{2}\right) + 5 = 0$

$\Rightarrow \frac{1}{2^5} + \frac{1}{2^4} + 2\frac{1}{2^3} + 2\frac{1}{2^2} + 3\frac{1}{2} + 5 = 0$

$\Rightarrow 1 + Z + 2Z^2 + 2Z^3 + 3Z^4 + 5Z^5 = 0$

$\Rightarrow 5Z^5 + 3Z^4 + 2Z^3 + 2Z^2 + Z + 1 = 0$

System is unstable

No. of sign changes = 2

2 poles are in RHS of S-plane

III) If All the elements in a row are zero.

Q) $s^6 + 2s^5 + 8s^4 + 12s^3 + 20s^2 + 16s + 16 = 0$

RH-Table:

$$\begin{array}{c|cccc} s^6 & 1 & 8 & 20 & 16 \\ s^5 & 2 & 12 & 16 & \\ s^4 & 2 & 12 & 16 & \\ s^3 & \boxed{0} & \boxed{0} & \boxed{0} & \end{array} \rightarrow$$

marginally stable

$\rightarrow$ equate $A(s) = 0$

$2s^4 + 12s^2 + 16 = 0$

$s^4 + 6s^2 + 8 = 0$

$\rightarrow$ Write Auxiliary equation

$A(s) = 2s^4 + 12s^2 + 16$

$\rightarrow$ do $\frac{dA(s)}{ds} = 2 \times 4s^3 + 12 \times 2s + 0$
 $= 8s^3 + 24s$

$\rightarrow$ RH-Table:-

$$\begin{array}{c|cccc} s^6 & 1 & 8 & 20 & 16 \\ s^5 & 2 & 12 & 16 & \\ s^4 & 2 & 12 & 16 & \\ s^3 & 8 & 24 & 0 & \\ s^2 & 6 & 16 & & \\ s^1 & \frac{16}{6} & 0 & & \\ s^0 & 16 & & & \end{array}$$

18) $2s^5 + s^4 + 4s^3 + 2s^2 + 2s + 1 = 0$

$s^2 = x \Rightarrow x^2 + 6x + 8 = 0$

$x = -2, -4$

$s = \pm\sqrt{-2} \quad s = \pm\sqrt{-4}$

$s = \pm\sqrt{2}j \quad s = \pm 2j$

marginally stable.



RH-Method - stability

I) Sign changes $\rightarrow$ same sign - stable
 Sign change - unstable

II) If any element in first column is zero

$s = \frac{1}{z}$ proceed same I model

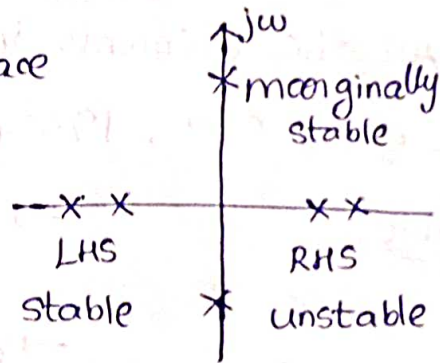
III) Total row elements zero

$\rightarrow A(s)$

$\rightarrow \frac{dA(s)}{ds}$

Co efficient replace

IV) To find K value.



$\Rightarrow$ Conditionally stable

Limitations

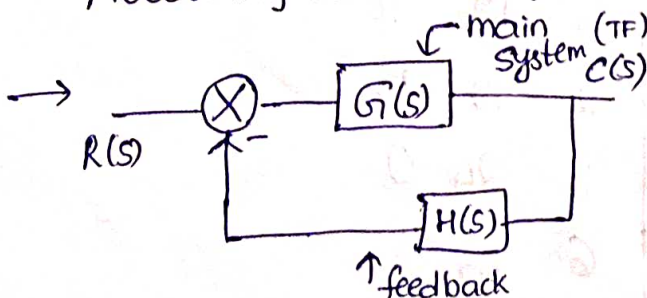
Conditions

rules

$K \rightarrow$ Control variable

According to our requirement stability can be changed

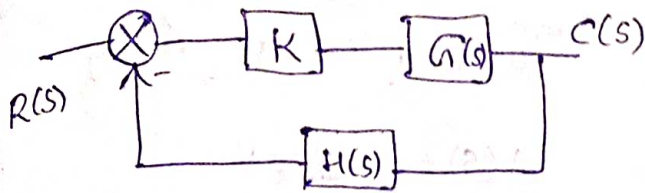
Standard TF form



$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$

$G(s)H(s) = (K) \frac{(1+ST_1)(1+ST_2) \dots}{(1+ST_a)(1+ST_b) \dots}$

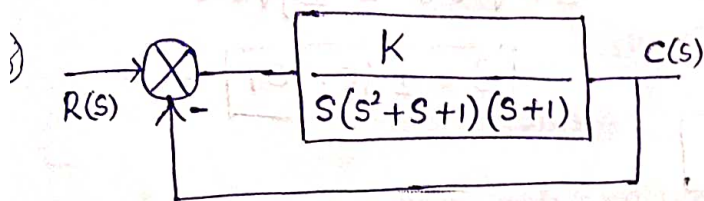
Resultant gain or
 Gain control variable



Model - IV :- To find 'K' value

- 1) find the range of K for system to be always stable.
- 2) find the marginal value of K and frequency of sustained oscillations at K_{marg}

< 0 (-ve) unstable
 > 0 (+ve) stable
 $= 0$ marginally



1) Write CE

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K}{s(s^2 + s + 1)(s + 1)} = 0$$

$$\frac{s(s^2 + s + 1)(s + 1) + K}{s(s^2 + s + 1)(s + 1)} = 0$$

$$s(s^2 + s + 1)(s + 1) + K = 0$$

$$s[s^3 + s^2 + s^2 + s + s + 1] + K = 0$$

$$s^4 + s^3 + s^3 + s^2 + s^2 + s + K = 0$$

$$s^4 + 2s^3 + 2s^2 + s + K = 0$$

RH-Table :-

s^4	1	2	K
s^3	2	1	0
s^2	$3/2$	K	
s^1	$\frac{(\frac{3}{2} - 2K)}{3/2}$	0	
s^0	K		

To find the range of 'K' (To get always stable)

> 0 (first column) (+ve)

$$\frac{\frac{3}{2} - 2K}{3/2} > 0 \quad \left| \quad K > 0 \right.$$

$$\frac{3}{2} - 2K > 0$$

$$0 < K < 0.75$$

$$\frac{3}{2} > 2K$$

$$\frac{3}{4} > K$$

$$0.75 > K$$

2) To find marginal K,

$$\frac{\left(\frac{3}{2} - 2k\right)}{3/2} = 0$$

$$\frac{3}{2} - 2k = 0$$

$$\frac{3}{2} = 2k$$

$$\frac{3}{4} = k$$

$$K_{\text{marg}} = 0.75$$

$$S = \pm j 0.707$$

$$S = j\omega$$

$$\omega = 0.707 \text{ rad/sec}$$

To find 'w' at K_{marg}

$$A(s) = \frac{3}{2}s^2 + K$$

$$\text{At } K_{\text{marg}}, A(s) = \frac{3}{2}s^2 + K_{\text{marg}} = 0$$

$$\frac{3}{2}s^2 + K_{\text{marg}} = 0$$

$$\frac{3}{2}s^2 + 0.75 = 0$$

$$\frac{3}{2}s^2 = -0.75$$

$$s^2 = -0.75 \times \frac{2}{3} = -0.5$$

$$s = \sqrt{-0.5} = \pm j\sqrt{0.5}$$

$$s = \pm j 0.707$$

P8

$$G(s)H(s) = \frac{k}{s(s^2+s+1)(s+2)}$$

$$1 + G(s)H(s) = 0$$

$$1 + \frac{k}{s(s^2+s+1)(s+2)} = 0$$

$$\frac{s(s^2+s+1)(s+2)+k}{s(s^2+s+1)(s+2)} = 0$$

$$s(s^2+s+1)(s+2)+k = 0$$

$$s[s^3+s^2+s+2s^2+2s+2]+k = 0$$

$$s^4+s^3+s^2+2s^3+2s^2+2s+k=0$$

$$s^4+3s^3+3s^2+2s+k=0$$

$$\begin{array}{cc} 1 & 3 \\ 3 & 2 \end{array} = \frac{1-2}{3} = \frac{1}{3}$$

$$\begin{array}{cc} 1 & k \\ 3 & 0 \end{array} = \frac{3k}{3} = k$$

$$\begin{array}{cc} 3 & 2 \\ 1/3 & k \end{array} = \frac{14/3 - 3k}{1/3}$$

RH-Table

$$\begin{array}{l|lll} s^4 & 1 & 3 & k \\ s^3 & 3 & 2 & 0 \\ s^2 & 1/3 & k & \\ s^1 & \frac{14/3 - 3k}{1/3} & 0 & \\ s^0 & k & & \end{array}$$

$$\frac{14/3 - 3k}{1/3} > 0 \quad k > 0$$

$$\frac{14}{3} > 3k$$

$$\frac{14}{9} > k$$

$$1.56 > k$$

$$0 < k < 1.56$$

1) To find marginal K

$$\frac{14}{3} - 3k = 0$$

$$\frac{14}{3} = 3k$$

$$\frac{14}{9} = K_{\text{marg}} = 1.56$$

To find 'w' at K_{marg}

$$A(s) = \frac{7}{3}s^2 + K$$

At K_{marg} , $A(s) = \frac{7}{3}s^2 + K_{\text{marg}} = 0$

$$\frac{7}{3}s^2 + 1.56 = 0$$

$$\frac{7}{3}s^2 = -1.56$$

$$s^2 = -1.56 \times \frac{3}{7}$$

$$s^2 = -0.668$$

$$s = \sqrt{-0.668} = \pm j\sqrt{0.668}$$

$$s = \pm j0.8173$$

$$s = jw$$

$$\boxed{w = 0.8173 \text{ rad/sec}}$$

$$\left. \begin{aligned} \text{Q) } s^4 + 2s^3 + 4s^2 + 3s + K &= 0 \\ \text{R) } s^4 + 22s^3 + 10s^2 + s + K &= 0 \end{aligned} \right\} \begin{aligned} &\text{find marginal value of 'K'} \\ &\text{and 'w' at } K_{\text{marg}} \end{aligned}$$

Q) $s^3 + 8s^2 + 2s + 4K = 0$

Q) $G(s)H(s) = \frac{K}{(s+1)(s+3)(s+5)(s+7)}$

R) $s^3 + 7s^2 + 6s + K = 0$

Q) $G(s)H(s) = \frac{K(1-s)}{s(s^2+5s+9)}$

find the range of 'K' for the system to be always stable.

Model-III

Q) $s^6 + s^5 + 5s^4 + 3s^3 + 2s^2 - 4s - 8 = 0$

→ unstable

Q) $s^6 + 3s^5 + 4s^4 + 6s^3 + 5s^2 + 3s + 2 = 0$

Time response analysis:

1) Transient state + ss

$\downarrow$
 t_p, t_r, t_s, m_p $\downarrow$
error

3) stability $\left\{ \begin{array}{l} \text{RH method} \rightarrow \text{mathematical analysis of stability} \\ \text{RL method} \rightarrow \text{graphical analysis of stability} \end{array} \right.$ time response analysis

$\rightarrow$ frequency response analysis

① $\rightarrow$ stability $\left\{ \begin{array}{l} \text{Bode plot} \\ \text{polar plot} \\ \text{Nyquist plot} \end{array} \right.$

$K \rightarrow 0 \text{ to } \infty$
poles $\rightarrow$ change (move)

Roots $\rightarrow$ poles zeros
X O

X $\rightarrow$ Starting (K=0) $\rightarrow$ zero end (K=∞)

Root locus:-

Locus: path travelled by the particle.

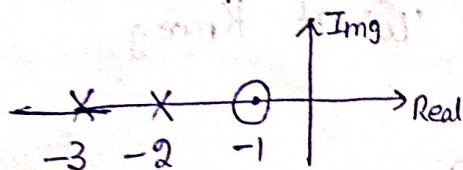
Root locus: The path travelled by the roots of CE when 'K' is varied from 0 to ∞ .

Properties of Root locus (or)

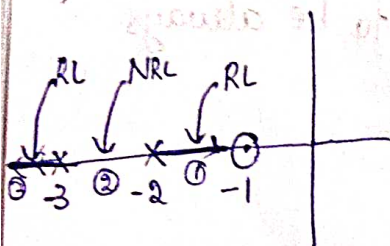
Construction rules of root locus:-

① $G(s) \cdot H(s) = \frac{S+1}{(S+2)(S+3)}$ poles, $S = -2, -3$
zeros, $S = -1$

1) To locate poles and zeros in s-plane



2) To find root locus branch on real axis



① $\Rightarrow$ RHS $\Rightarrow$ sum of (poles + zeros) = odd (RL exist)
 $= -0 + 1 = 1$ (odd) $\rightarrow$ RL

② $\Rightarrow$ RHS $\Rightarrow 1 + 1 = 2$ (even) $\rightarrow$ NRL

③ $\Rightarrow$ RHS $\Rightarrow 2 + 1 = 3$ (odd) $\rightarrow$ RL

3) Angles of Asymptotes:-

$\rightarrow$ Straight lines which are travelling towards ∞ .

$\rightarrow$ Root locus travels parallel to this asymptotes towards ∞

3) T.F given

No. of asymptotes, $A = P - Z$

$P \rightarrow$ no. of poles

$Z \rightarrow$ no. of zeroes

$$\Rightarrow \theta = \frac{(2q+1)}{|P-Z|} \times 180^\circ$$

where, q depends on A ,

$A=2 ; q=0,1$

if $A=3 ; q=0,1,2$

if $A=2 , q=0,1$

$$\theta_0 = \frac{2 \times 0 + 1}{2} \times 180^\circ = 90^\circ$$

$$\theta_1 = \frac{2 \times 1 + 1}{2} \times 180^\circ = 3 \times 90^\circ = 270^\circ$$

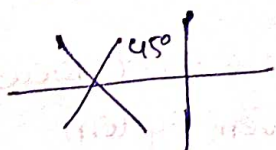
if $A=3, q=0,1,2$

$$\theta_0 = \left(\frac{2 \times 0 + 1}{3} \right) \times 180^\circ = \frac{1}{3} \times 180 = 60^\circ$$

$$\theta_1 = \left(\frac{2 \times 1 + 1}{3} \right) \times 180^\circ = \frac{3}{3} \times 180 = 180^\circ$$

$$\theta_2 = \left(\frac{2 \times 2 + 1}{3} \right) \times 180^\circ = \frac{5}{3} \times 180 = 300^\circ$$

if $A=4 ;$



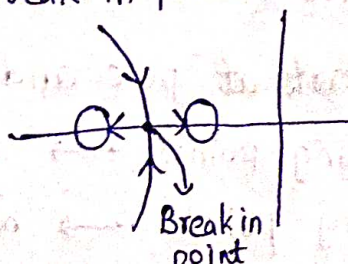
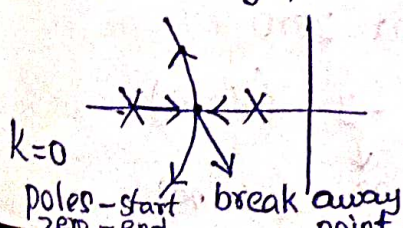
4) Centroid : centre of angles of asymptotes in a real axis of (s) plane.

$$\sigma = \frac{\text{Sum of poles} - \text{sum of zeros}}{\text{no. of poles} - \text{no. of zeros}} = \frac{\sum P - \sum Z}{P - Z}$$

5) Break away point (or) Break in point

Break away point

Break in point



from TF: $\rightarrow \frac{dK}{ds} = 0$

$\downarrow$ CE $\rightarrow 1 + G(s)H(s) = 0$

$s^3 + s^2 + s + K = 0 \rightarrow$ CE

$K = -[s^3 + s^2 + s]$

$\frac{dK}{ds} = 0$

roots $\rightarrow$ break away point / break in point

⑥ Angle of departure / Angle of arrival :-

roots $\rightarrow$ complex conjugate

poles $\nearrow$ zeros

real + img = roots

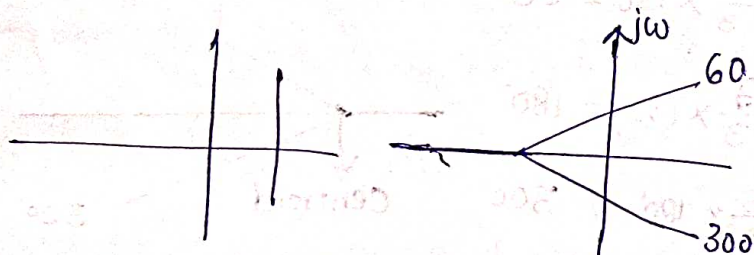
$S = -1 \pm j$

$S = -2 \pm j\sqrt{3}$

⑦ Intersection point of RL on jw axis :-

$\rightarrow$ RH Criteria

$\rightarrow$ 4th model, Kmax, ω



⑧ $G(s)H(s) = \frac{K}{s(s+1)(s+3)}$ Draw the root locus for the given system

Model-I

only poles (Real values)

① To identify or locate poles and zeros :-

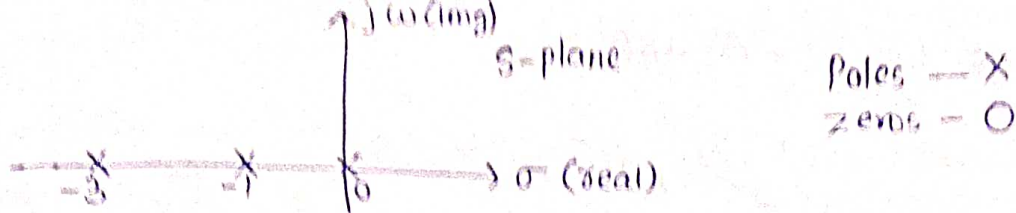
poles, $S = 0, -1, -3$, $P = \text{no. of poles} = 3$

zeros, No zeros, $Z = \text{no. of zeros} = 0$

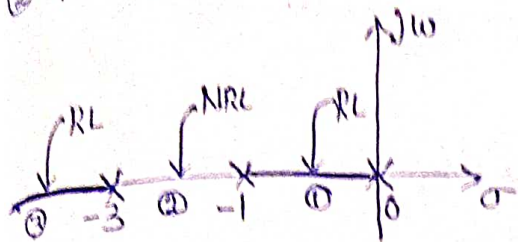
$\rightarrow$ Root locus starts at pole and ends at zero.

Root locus starting points $\rightarrow 0, -1, -3$ ($K=0$) :

ending points $\rightarrow \infty, \infty, \infty$ ($K=\infty$)



② To find existence of root locus on real axis :



$$\textcircled{1} \Rightarrow R_{115} = \text{no. of poles} + \text{no. of zeros} = 1 + 0 = 1 \text{ (odd)} \rightarrow \text{RL}$$

$$\textcircled{2} \Rightarrow R_{116} = 2 + 0 = 2 \text{ (even)} \rightarrow \text{NRL}$$

$$\textcircled{3} \Rightarrow R_{115} = 3 + 0 = 3 \text{ (odd)} \rightarrow \text{RL}$$

③ To find centroid (σ) :

$$\sigma = \frac{\text{Sum of poles} - \text{Sum of zeros}}{\text{No. of poles} - \text{No. of zeros}} = \frac{\sum p - \sum z}{p - z} = \frac{(0 - 1 - 3) - (0)}{3 - 0} = \frac{-4}{3}$$

$$\boxed{\sigma = -1.33}$$

④ To find Angles of asymptotes :

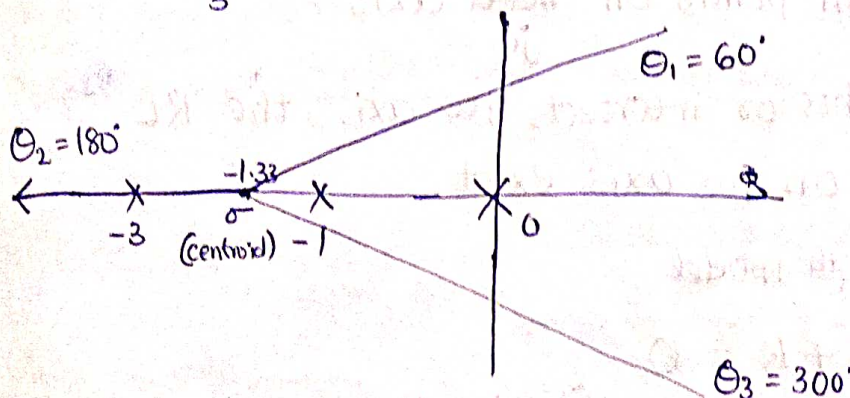
$$\text{No. of asymptotes, } A = p - z = 3 - 0 = 3$$

$$\theta = \frac{2q+1}{p-z} \times 180^\circ ; \text{ where } q = 0, 1, 2$$

$$\theta_0 = \frac{2 \times 0 + 1}{3} \times 180^\circ = \frac{1}{3} \times 180^\circ = 60^\circ \rightarrow \text{for } q = 0$$

$$\theta_1 = \frac{2 \times 1 + 1}{3} \times 180^\circ = \frac{3}{3} \times 180^\circ = 180^\circ \rightarrow \text{for } q = 1$$

$$\theta_2 = \frac{2 \times 2 + 1}{3} \times 180^\circ = \frac{5}{3} \times 180^\circ = 300^\circ \rightarrow \text{for } q = 2$$



⑤ To find Break-away point / Break-in point

Between two poles

Between two zeros

$$\rightarrow \boxed{\text{Condition } \frac{dK}{ds} = 0}$$

→ Write CE, $1 + G(s)H(s) = 0$

$$1 + \frac{K}{s(s+1)(s+3)} = 0$$

$$s(s+1)(s+3) + K = 0$$

$$(s^2 + s)(s+3) + K = 0$$

$$s^3 + 3s^2 + s^2 + 3s + K = 0$$

$$s^3 + 4s^2 + 3s + K = 0 \rightarrow \text{CE}$$

$$\textcircled{2} \Rightarrow K = -(s^3 + 4s^2 + 3s)$$

$$\textcircled{3} \Rightarrow \frac{dK}{ds} = 0$$

$$\frac{dK}{ds} = -(3s^2 + 8s + 3) = 0$$

$$\Rightarrow \boxed{3s^2 + 8s + 3 = 0}$$

$$\textcircled{4} \Rightarrow \text{roots} \rightarrow -0.45, -2.21 \rightarrow \begin{matrix} \downarrow \\ \text{valid} \end{matrix} \text{invalid}$$

$$\text{Break-away point, } \boxed{S = -0.45}$$

⑥ To find Angle of departure/arrival:-

only for complex roots

poles, zeros

No angle of departure/arrival

⑦ To find intersection points on ~~real~~ $j\omega$ axis:-

If asymptotes touches or intersects $j\omega$ axis, the RL intersection points on $j\omega$ axis exist.

→ Use RH-method - 4th model

→ CE, $s^3 + 4s^2 + 3s + K = 0$

RH-Table

s^3	1	3
s^2	4	K
s^1	$\frac{12-K}{4}$	0
s^0	K	

To find K_{marg} :-

$$\frac{12-K}{4} = 0$$

$$12-K = 0$$

$$\boxed{K_{\text{marg}} = 12}$$

To find 'w' at K_{marg} :-

$$A(s) = 4s^2 + K_{\text{marg}} = 0$$

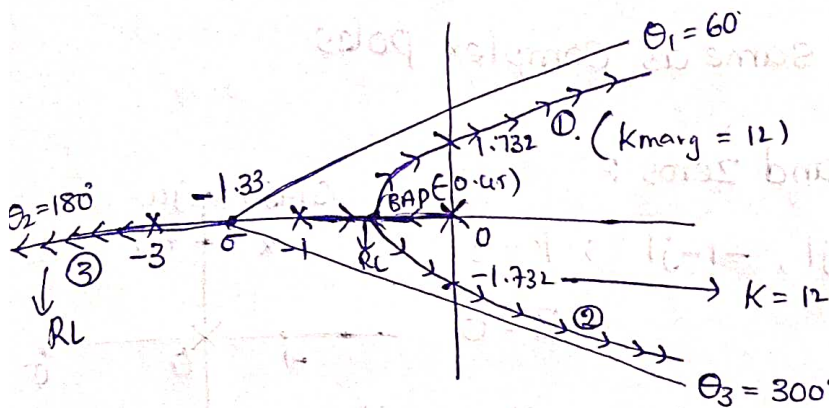
$$4s^2 + 12 = 0$$

$$4s^2 = -12$$

$$s^2 = -3$$

$$s = \pm j\sqrt{3} = \pm j1.732$$

$$\omega = 1.732 \text{ rad/sec}$$



$K < K_{\text{marg}} \rightarrow \text{stable}$

$K > K_{\text{marg}} \rightarrow \text{unstable}$

Poles $\rightarrow$ LHS $\rightarrow$ stable

$K \rightarrow 0 \text{ to } \infty$ $K < 12 \rightarrow$ stable

jw axis $\rightarrow K_{\text{marg}} = 12 \rightarrow$ marginally stable

poles RHS $\rightarrow$ unstable, $K > 12 \uparrow$

$\rightarrow$ centroid, $\sigma = -1.33$

$\rightarrow$ Angles of asymptotes, $\theta = 60^\circ, 180^\circ, 300^\circ$

$\rightarrow$ BAP = -0.45

$\rightarrow j\omega = \pm j1.732$
 $\downarrow$
 RL

method (ii) Complex poles

$$a + jb \quad \theta = \tan^{-1}\left(\frac{b}{a}\right)$$

Q8) $G(s)H(s) = \frac{K}{s(s^2 + 2s + 2)}$ sketch root locus diagram

Angle of departure :-

The angle at which the root locus

is departing (moving away) at complex pole

Angle of Arrival :-

The angle at which the root locus is

(incoming) arriving at complex zero

angle of departure / angle of arrival
 $\downarrow \quad \downarrow$
 at poles at zeros

$X \rightarrow$ away departure out going
 $\rightarrow O$ Arrival incoming

Complex

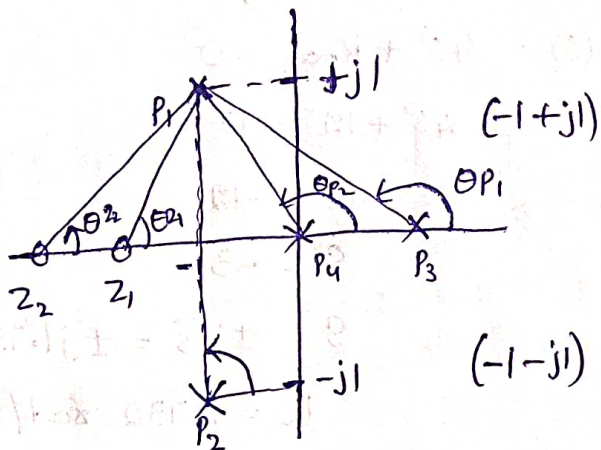
At pole

$$\phi_d = 180 + \phi$$

where $\phi = \sum \theta_z - \sum \theta_p$

$$\sum \theta_p = \theta_{p_2} + \theta_{p_3} + \theta_{p_4}$$

$$\sum \theta_z = \theta_{z_1} + \theta_{z_2}$$



$\Rightarrow$ Complex zeros $\rightarrow$ same as Complex poles

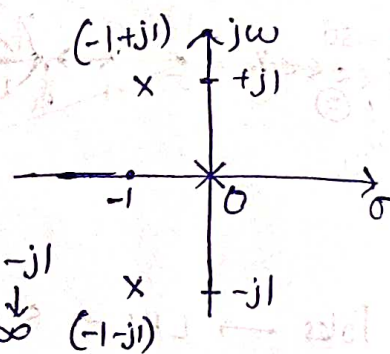
① To locate poles and zeros:

Poles, $S = 0, -1 + j1, -1 - j1$; $P = 3$

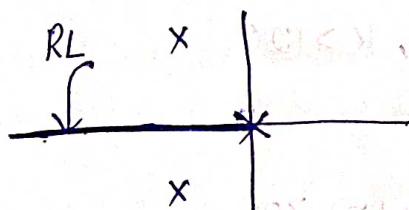
Zeros, no zeroes

Starting points, $\rightarrow 0, -1 + j1, -1 - j1$

ending points $\rightarrow \infty, \infty, \infty$



② To find existence of RL on Real axis:



③ To find centroid:

$$\sigma = \frac{\sum P - \sum Z}{P - Z} = \frac{(0 - 1 + j1 - 1 - j1) - (0)}{3 - 0} = \frac{-2}{3} = -0.66$$

$$\sigma = -0.66$$

④ To find angles of Asymptotes:

No. of Asymptotes, $A = P - Z = 3 - 0 = 3$

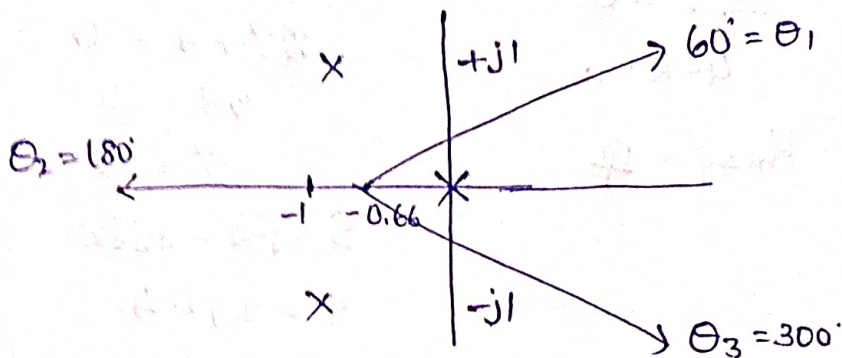
$$q = 0, 1, 2$$

$$\theta = \frac{2q+1}{P-Z} \times 180^\circ$$

for $q=0$, $\theta_1 = \frac{2 \times 0 + 1}{3} \times 180 = 60^\circ$

for $q=1$, $\theta_2 = \frac{2 \times 1 + 1}{3} \times 180 = \frac{3}{3} \times 180 = 180^\circ$

for $q=2$, $\theta_3 = \frac{2 \times 2 + 1}{3} \times 180 = \frac{5}{3} \times 180 = 300^\circ$



⑤ To find Break away point / Break in point :-

Between two poles $\rightarrow$ Break away point

No two poles on Real axis

Hence no break away point

⑥ To find angle of departure / angle of arrival :-

$$\phi_d = 180 + \phi \quad \text{where } \phi = \sum \theta_z - \sum \theta_p$$

$$\therefore \theta_{p1} = 180 - \tan^{-1}\left(\frac{1}{1}\right)$$

$$= 180 - 45^\circ = 135^\circ$$

$$\theta_{p3} = 90^\circ$$

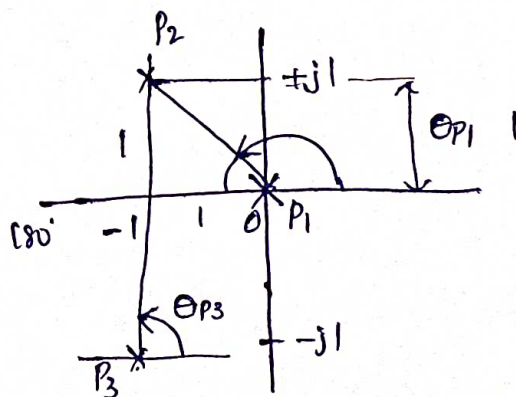
$$\therefore \sum \theta_p = \theta_{p1} + \theta_{p3}$$

$$= 135 + 90 = 225^\circ$$

$$\sum \theta_z = 0$$

$$\phi = 0 - 225^\circ$$

$$\boxed{\phi = -225^\circ}$$



$$\phi_{d1} = 180 + \phi = 180 - 225^\circ$$

$$= -45^\circ$$

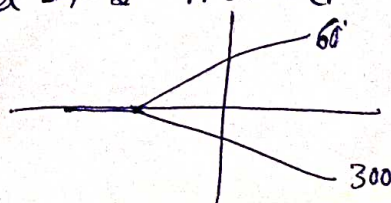
$$\phi_{d2} = -(\phi_{d1}) = 45^\circ$$

⑦ To find intersection points of RL on $j\omega$ axis

use RH-method $\rightarrow$ 4th method $\rightarrow$ 2nd method

CE $1 + G(s)H(s) = 0$

$$1 + \frac{K}{s(s^2 + 2s + 2)} = 0$$



$$S(S^2 + 2S + 2) + K = 0$$

$$S^3 + 2S^2 + 2S + K = 0 \rightarrow CE$$

→ RH-Table

S^3	1	2
S^2	2	K
S^1	$\frac{4-K}{2}$	0
S^0	K	

To find K_{marg}

$$\frac{4-K}{2} = 0$$

$$4-K=0$$

$$K_{marg} = 4$$

To find 'w'

$$A(s) = 2s^2 + K = 0$$

$$2s^2 + 4 = 0$$

$$2s^2 = -4$$

$$s^2 = -2$$

$$s = \sqrt{-2} = \pm j\sqrt{2}$$

$$s = \pm j1.4$$

$K = 4 \rightarrow$ marginally stable

$K < 4 \rightarrow$ Stable

$K > 4 \rightarrow$ unstable



Q8) $G(s)H(s) = \frac{K}{S(s+4)(s^2+2s+2)} \rightarrow S = -1 \pm j$

Draw the root locus and comment on stability

A: 1) To identify (or) locate poles and zeros

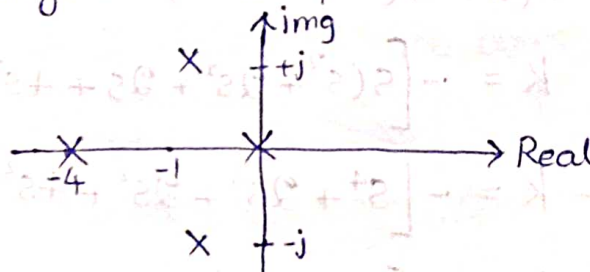
poles, $S = 0, -4, -1+j, -1-j$; $P = 4$

zeros, No zeros $Z = 0$

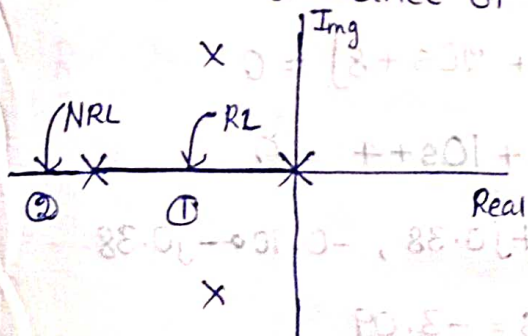
Starting points of RL $\rightarrow$ poles $\rightarrow 0, -4, -1+j, -1-j$

ending points of RL $\rightarrow$ zeros $\rightarrow \infty, \infty, \infty, \infty$

$\therefore$ Root locus always start at pole and ends at zero



2) To find existence of RL on Real axis :-



① $\Rightarrow$ RHS $\Rightarrow P+2 = 1+0 = 1$ (odd)

② $\Rightarrow$ RHS $\Rightarrow 2+0 = 2$ (even)

Odd $\rightarrow$ RL exist

even $\rightarrow$ RL do not exist

3) To find centroid :-

$$\text{Centroid} = \frac{\sum P - \sum Z}{P - Z} = \frac{[0 - 4 - 1 + j - 1 - j] - [0]}{4 - 0} = \frac{-6}{4} = -\frac{3}{2}$$

Centroid, $\sigma = -1.5$

4) To find Angles of Asymptotes :-

No. of asymptotes, $A = P - Z = 4 - 0 = 4$

$\therefore \theta = \frac{(2q+1)}{P-Z} \times 180^\circ$ where $q = 0, 1, 2, 3$

for $q = 0$; $\theta_1 = \frac{2 \times 0 + 1}{4} \times 180^\circ = \frac{1}{4} \times 180^\circ = 45^\circ$

for $q = 1$; $\theta_2 = \frac{2 \times 1 + 1}{4} \times 180^\circ = \frac{3}{4} \times 180^\circ = 135^\circ$

for $q = 2$; $\theta_3 = \frac{2 \times 2 + 1}{4} \times 180 = \frac{5}{4} \times 180 = 225^\circ$

for $q = 3$; $\theta_4 = \frac{2 \times 3 + 1}{4} \times 180 = \frac{7}{4} \times 180 = 315^\circ$

5) To find breakaway / breakin point :-

Condition, $\frac{dk}{ds} = 0$

→ write CE ; $1 + G(s)H(s) = 0$

→ $K = f(s) \quad 1 + \frac{K}{s(s+4)(s^2+2s+2)} = 0$

→ $\frac{dk}{ds} = 0$

$s(s+4)(s^2+2s+2) + K = 0 \rightarrow CE$

$K = -[s(s^3+2s^2+2s+4s^2+8s+8)]$

$K = -[s^4+2s^3+2s^2+4s^3+8s^2+8s]$

$K = -[s^4+6s^3+10s^2+8s]$

$\frac{dk}{ds} = -[4s^3+18s^2+20s+8] = 0$

$2s^3+9s^2+10s+4 = 0$

$s = -3.09, -0.70 + j0.38, -0.70 - j0.38$

Break away point, $s = -3.09$

Roots $\begin{cases} \rightarrow \text{valid (exist on RL path)} \\ \rightarrow \text{invalid (not exist on RL path)} \end{cases}$

6) To find Angle of departure / Angle of Arrival :-

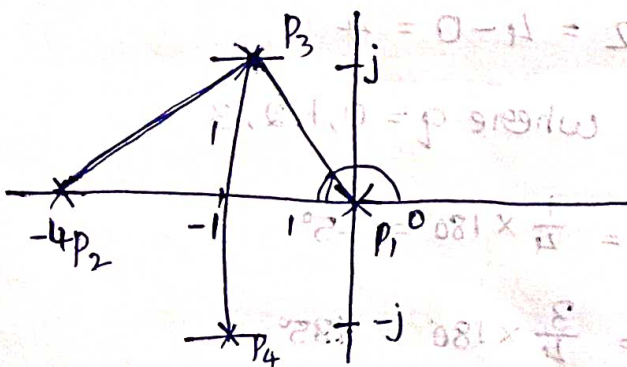
Complex poles ✓

Complex poles → departure

Complex zeros → arrival

$\phi_D \text{ or } \phi_A = 180 \pm \phi$
 $\rightarrow \text{departure}$
 $\rightarrow \text{arrival}$

$\phi = \sum \theta_z - \sum \theta_p$



$$\theta_{p1} = 180 - \tan^{-1}\left(\frac{1}{1}\right) \\ = 180 - 45^\circ = 135^\circ$$

$$\theta_{p2} = 90^\circ$$

$$\theta_{p2} = \tan^{-1}\left(\frac{1}{3}\right) = 18.43^\circ$$

$$\sum \theta_p = \theta_{p1} + \theta_{p2} + \theta_{p4} = 135^\circ + 90^\circ + 18.4^\circ = 243.4^\circ$$

$$\sum \theta_z = 0$$

$$\phi = 0 - 243.4^\circ = -243.4^\circ$$

$$\phi_{D1} = 180 + \phi = 180 + (-243.4^\circ) = -63.4^\circ$$

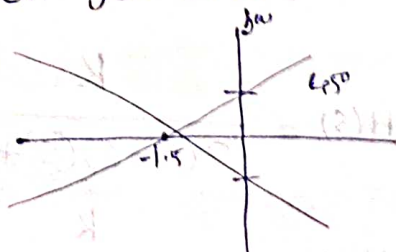
$$\phi_{D2} = -\phi_{D1} = 63.4^\circ$$

7) To find intersection points on jw axis;

RH method $\rightarrow$ 4th model

$\rightarrow$ 2nd method

$$K_{\text{marg}} = ? , \omega = ?$$



RH Table :-

$$CE \Rightarrow s^4 + 6s^3 + 10s^2 + 8s + K = 0$$

s^4	1	10	K
s^3	6	8	0
s^2	8.66	K	
s^1	$\frac{8 \times 8.66 - 6K}{8.66}$	0	
s^0	K		

To find K_{marg}

$$\frac{8 \times 8.66 - 6K}{8.66} = 0$$

$$8 \times 8.66 = 6K$$

$$K = \frac{8 \times 8.66}{6}$$

$$K_{\text{marg}} = 11.54$$

To find ω

$$A(s) = 8.66s^2 + K_{\text{marg}} = 0$$

$$8.66s^2 + 11.54 = 0$$

$$8.66s^2 = -11.54$$

$$s^2 = \frac{-11.54}{8.66}$$

$$s^2 = -1.33$$

$$s = \pm j\sqrt{1.33}$$

$$s = \pm j1.15$$

$$\boxed{\omega = \pm 1.15}$$

To plot graph:

$$S = 0, -4, -1+j, -1-j$$

$$\sigma = -1.5$$

$$\theta_1 = 45^\circ, \theta_2 = 135^\circ, \theta_3 = 225^\circ, \theta_4 = 315^\circ$$

$$BAP = -3.09$$

$$\phi_{01} = -63.4^\circ ; \phi_{02} = 63.4^\circ$$

Stability: $K = K_{\text{marg}} = 11.54 \rightarrow$ system is marginally stable

$K < K_{\text{marg}} \rightarrow$ stable

$K > K_{\text{marg}} \rightarrow$ system is unstable

Q) *** $G(s)H(s) = \frac{K}{s(s+6)(s^2+4s+13)}$

Q) $G(s)H(s) = \frac{K}{s(s+1)^2(s+2)}$

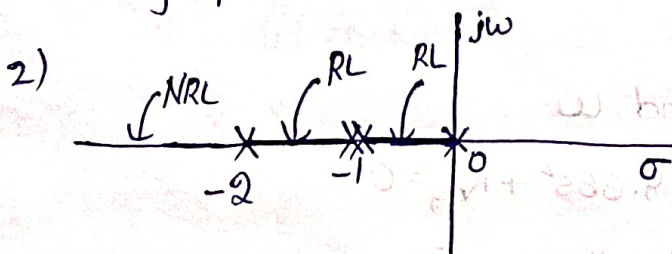
A: 1) To identify or locate poles and zeros :-

poles, $s = 0, -1, -1, -2 ; P = 4$

zeros, No zeros ; $Z = 0$

Starting points of RL $\rightarrow$ poles $\rightarrow 0, -1, -1, -2$

ending points of RL $\rightarrow$ zeros $\rightarrow \infty, \infty, \infty, \infty$



3) Centroid, $\frac{0-1-1-2}{4} = -1$

4) $A = P - Z = 4 - 0 = 4$

$$\theta_1 = 45^\circ ; \theta_2 = 135^\circ$$

$$\theta_3 = 225^\circ ; \theta_4 = 315^\circ$$

$$5) 1 + G(s)H(s) = 0$$

$$1 + \frac{K}{s(s+1)^2(s+2)} = 0$$

$$s(s+1)^2(s+2) + K = 0$$

$$s[s^2 + 2s + 1](s+2) + K = 0$$

$$K = -[s^4 + 4s^3 + 5s^2 + 2s]$$

$$\frac{dK}{ds} = 0 \Rightarrow -[4s^3 + 12s^2 + 10s + 2] = 0$$

$$4s^3 + 12s^2 + 10s + 2 = 0$$

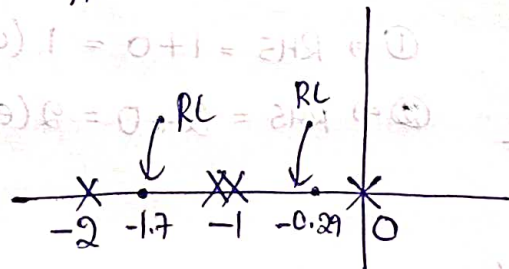
$$2s^3 + 6s^2 + 5s + 1 = 0$$

$$\text{Roots, } s = \underset{\times}{-1}, \underset{\checkmark}{-1.70}, \underset{\checkmark}{-0.291}$$

6) No complex roots

$$\phi_o \text{ or } \phi_A = 0$$

7) jw axis



$$s^4 + 4s^3 + 5s^2 + 2s + K = 0 \rightarrow \text{CE}$$

RH Table:

s^4	1	5	K
s^3	4	2	0
s^2	4.5	K	
s^1	$\frac{4.5 \times 2 - 4K}{4.5}$	0	
s^0	K		

To find K_{max}

$$4.5 \times 2 - 4K = 0$$

$$4.5 \times 2 = 4K$$

$$\frac{4.5 \times 2}{4} = K$$

$$K_{max} = \frac{9}{4} = 2.25$$

To find ω

$$A(s) = 4.5s^2 + K_{max} = 0$$

$$4.5s^2 + 2.25 = 0$$

$$s = \pm j0.70$$

$$\omega = \pm 0.70$$

$$\rightarrow S = 0, -1, -2$$

$$\rightarrow \sigma = -1$$

$$\rightarrow \theta_1 = 45^\circ, \theta_2 = 135^\circ, \theta_3 = 225^\circ, \theta_4 = 315^\circ$$

$$\rightarrow BAP = -0.29, -1.70$$

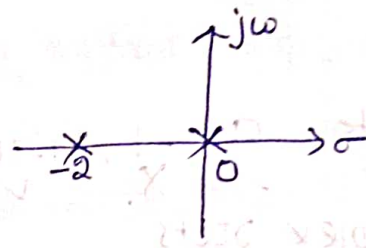
$$\rightarrow j\omega = \pm j0.70$$

$$\textcircled{B} \quad G(s)H(s) = \frac{K}{S(S+2)}$$

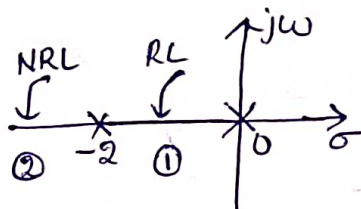
Sol: 1) To locate poles and zeros,

poles, $S = 0, -2$; $P = 2$

zeros, No zeros ; $Z = 0$



2) To find existence of RL on Real axis,



$$\textcircled{1} \Rightarrow RHS = 1 + 0 = 1 \text{ (odd)} \checkmark$$

$$\textcircled{2} \Rightarrow RHS = 2 + 0 = 2 \text{ (even)} \times$$

3) To find centroid,

$$\sigma = \frac{\sum P - \sum Z}{P - Z} = \frac{(0 - 2) - (0)}{2 - 0} = \frac{-2}{2} = -1$$

4) To find angles of asymptotes

$$\text{No. of asymptotes, } A = P - Z = 2 - 0 = 2$$

$$\theta = \frac{(2q+1)}{P-Z} \times 180^\circ; q = 0, 1$$

$$\theta_1 = \frac{2 \times 0 + 1}{2} \times 180 = \frac{1}{2} \times 180 = 90^\circ$$

$$\theta_2 = \frac{2 \times 1 + 1}{2} \times 180 = \frac{3}{2} \times 180 = 270^\circ$$

$$5) 1 + G(s)H(s) = 0 \quad K = -[S^2 + 2S]$$

$$1 + \frac{K}{S(S+2)} = 0 \quad \frac{dK}{dS} = -[2S + 2] = 0$$

$$S(S+2) + K = 0 \quad 2S + 2 = 0$$

$$S^2 + 2S + K = 0$$

$$S + 1 = 0$$

$$S = -1 \checkmark$$

$$\boxed{BAP = -1}$$

6) No complex roots

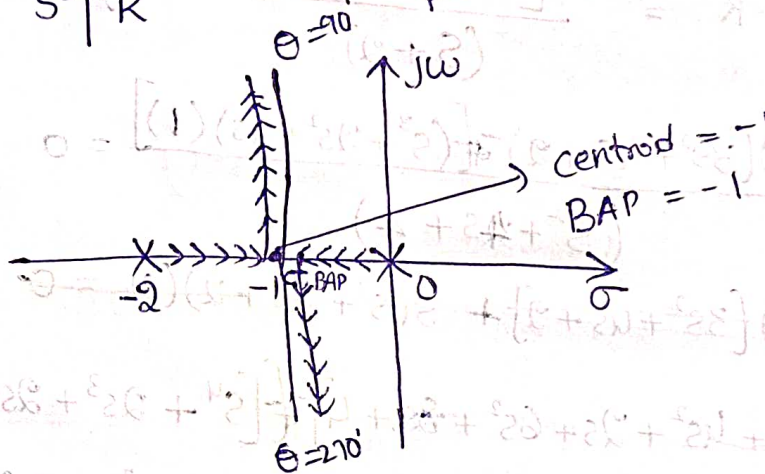
$$\phi_0 \text{ or } \phi_0 = 0$$

$$7) s^2 + 2s + k = 0 \rightarrow CE$$

RH Table:

s^2	1	k
s^1	2	0
s^0	k	

No intersection points

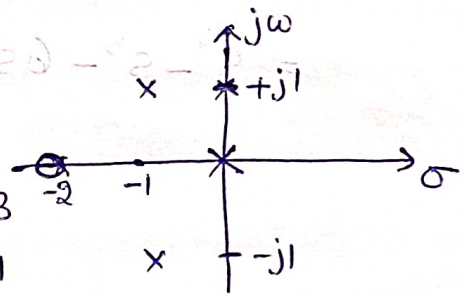


$$\textcircled{B} G(s) H(s) = \frac{k(s+2)}{s(s^2+2s+2)}$$

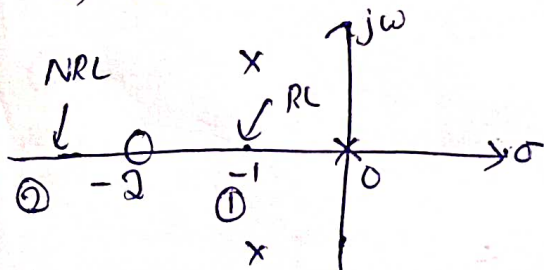
Sol: 1) To locate poles and zeros,

poles, $s = 0, -1+j, -1-j$; $P = 3$

zeros, $s = -2$; $Z = 1$



2) To find existence of RL on Real axis,



3) Centroid,

$$\sigma = \frac{\sum P - \sum Z}{P - Z} = \frac{[-1+j, -1-j] - (-2)}{3-1} = \frac{-2+2}{2} = 0$$

$$4) A = P - Z = 3 - 1 = 2 ; q = 0, 1$$

$$\theta_1 = \frac{2 \times 0 + 1}{2} \times 180 = \frac{1}{2} \times 180 = 90^\circ$$

$$\theta_2 = \frac{2 \times 1 + 1}{2} \times 180 = \frac{3}{2} \times 180 = 270^\circ$$

⇒ Controllers

→ Dynamic error coefficients:-

$$\Rightarrow \text{Error, } E(s) = \frac{1}{1 + G(s)H(s)} R(s) \longrightarrow \textcircled{1}$$

$$e(t) = C_0 \delta(t) + \frac{C_1}{1!} \delta'(t) + \frac{C_2}{2!} \delta''(t) + \dots + \frac{C_n}{n!} \delta^n(t)$$

$$\text{LT} \Rightarrow E(s) = C_0 R(s) + \frac{C_1}{1!} s \cdot R(s) + \frac{C_2}{2!} s^2 R(s) + \dots + \frac{C_n}{n!} s^n R(s)$$

$$E(s) = \left[C_0 + \frac{C_1}{1!} s + \frac{C_2}{2!} s^2 + \dots + \frac{C_n}{n!} s^n \right] R(s) \longrightarrow \textcircled{2}$$

Compare equation's ① and ②,

$$\frac{1}{1 + G(s)H(s)} = C_0 + \frac{C_1}{1!} s + \frac{C_2}{2!} s^2 + \dots + \frac{C_n}{n!} s^n \longrightarrow \textcircled{3}$$

Apply $\lim_{s \rightarrow 0}$ on both sides,

$$\lim_{s \rightarrow 0} \frac{1}{1 + G(s)H(s)} = \lim_{s \rightarrow 0} C_0 + \lim_{s \rightarrow 0} \frac{C_1}{1!} s + \lim_{s \rightarrow 0} \frac{C_2}{2!} s^2 + \dots + \lim_{s \rightarrow 0} \frac{C_n}{n!} s^n$$

$$\lim_{s \rightarrow 0} \frac{1}{1 + G(s)H(s)} = C_0$$

To find C_1 :-

Differentiate equation ③ w.r.t 's',

$$\frac{d}{ds} \left[\frac{1}{1 + G(s)H(s)} \right] = 0 + \frac{C_1}{1!} + \frac{C_2}{2!} \times 2s + \dots + \frac{C_n}{n!} \cdot n \times s^{n-1}$$

Take $\lim_{s \rightarrow 0}$ on both sides,

$$\lim_{s \rightarrow 0} \frac{d}{ds} \left[\frac{1}{1 + G(s)H(s)} \right] = \lim_{s \rightarrow 0} \frac{C_1}{1!} + \lim_{s \rightarrow 0} \frac{C_2}{2!} \times 2s + \dots + \lim_{s \rightarrow 0} \frac{C_n}{n!} \cdot n \cdot s^{n-1}$$

$$= C_1 + 0 + 0 + 0 \dots$$

$$C_1 = \lim_{s \rightarrow 0} \left\{ \frac{d}{ds} \left[\frac{1}{1 + G(s)H(s)} \right] \right\}$$

$$C_0 = K_p$$

$$C_1 = K_v$$

$$C_2 = K_a$$

General dynamic error coefficients,

$$C_n = \frac{1}{n!} \lim_{s \rightarrow 0} \frac{d^n}{ds^n} \left[\frac{1}{1 + G(s)H(s)} \right]$$

⇒ Controllers :-

- * Control → To command or regulate output of the system.
- * Controller → Which commands or regulates the output of the system to get desired or required output.

1) Proportional Controller (P-controller)

2) Integral Controller (I-controller)

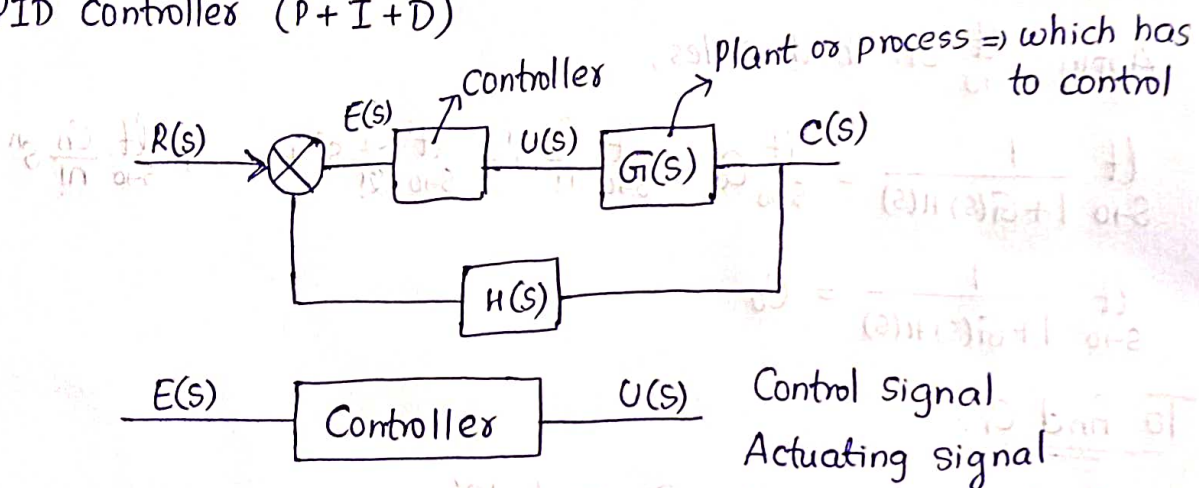
3) Differential Controller (D-controller)

* Based on mode of operation

1) PI controller (P + I)

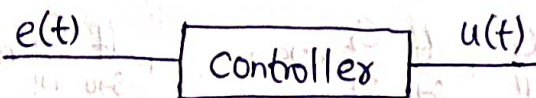
2) PD controller (P + D)

3) PID Controller (P + I + D)



P-controller :-

Def :- Actuating signal or control signal is directly proportional to error signal.

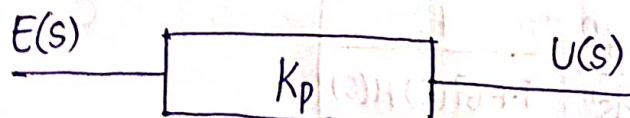


$$\therefore u(t) \propto e(t)$$

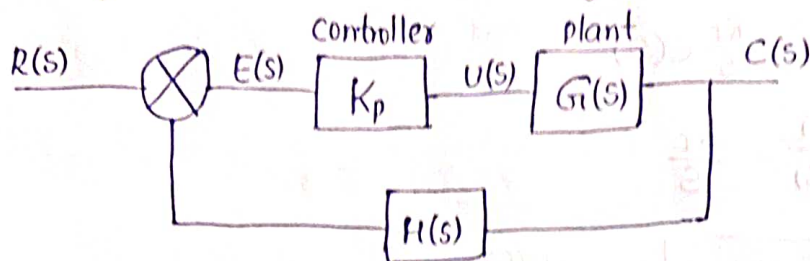
$$u(t) = K_p e(t)$$

$$TF = \frac{L[u(t)]}{L[e(t)]} = \frac{U(s)}{E(s)}$$

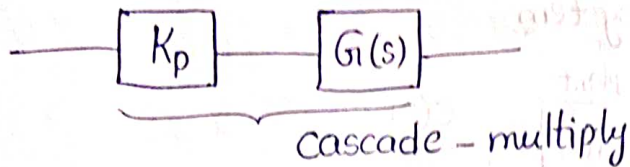
$$LT, U(s) = K_p E(s) \Rightarrow \frac{U(s)}{E(s)} = K_p$$



Block diagram $\rightarrow$ closed loop system



Effect:- with controller



without controller

$$G(s) = \frac{1}{s(s+8)}$$

$$C(s) = K_p G(s) = K_p \frac{1}{s(s+8)}$$

$$CE, 1 + G(s)H(s) = 0$$

$$1 + \frac{1}{s(s+8)} \Rightarrow s(s+8) + 1 = 0$$

$$s^2 + 8s + 1 = 0$$

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

$$\begin{array}{l|l} \omega_n^2 = 1 & 2\xi\omega_n = 8 \\ \omega_n = 1 & 2\xi = 8 \\ & \xi = 4 \end{array}$$

$$\xi = \frac{4}{\sqrt{K_p}}$$

$K_p \uparrow, \xi \downarrow, \text{Oscillations} \uparrow$

Opposing nature

transient state poor performance.

effect:-

$\rightarrow \xi = \frac{1}{\sqrt{K_p}}$ As $K_p \uparrow, \xi \downarrow, \text{Oscillations} \uparrow$
poor transient response.

$\rightarrow$ error decreases, $ess = \frac{A}{1+K_p \uparrow}$; Steady state error decreases
good steady state performance.

I-controller:-

Def:- Actuating signal or control signal is directly proportional to the integral of error signal.

$$u(t) \propto \int e(t)$$

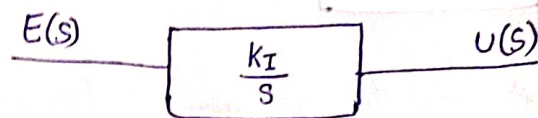
$$u(t) = K_I \int e(t)$$

TF.

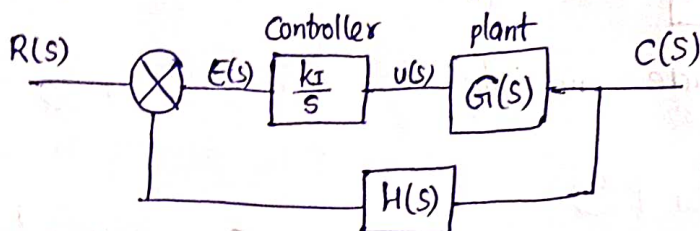
$$U(s) = K_I \frac{1}{s} E(s)$$

$$U(s) = \frac{K_I}{s} E(s)$$

$$\frac{U(s)}{E(s)} = \frac{K_I}{s}$$



Block diagram $\rightarrow$ closed loop system

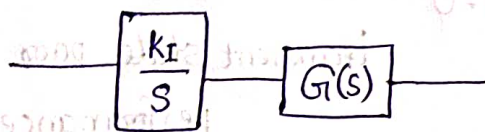


Pole is added in the system

Effect: Without controller

$$G(s) = \frac{1}{s(s+8)} \quad \text{Type-1, order-2}$$

With controller



$$\text{net } G(s) = \frac{K_I}{s} \times \frac{1}{s(s+8)}$$

$$= \frac{K_I}{s^2(s+8)} \quad \text{Type-2, order-3}$$

* Pole is added

* Type and order of the system increases

* As type increases error decreases

$$\text{Type} \propto \frac{1}{\text{error}}$$

* $\xi \rightarrow$ To state $\rightarrow$ order $\uparrow$

unstable, oscillations $\uparrow$, $\xi \downarrow$

$$\left(\xi \propto \frac{1}{\text{order}} \propto \frac{1}{\text{oscillations}} \right)$$

D-controller:

Def :- Actuating signal or control signal is directly proportional to the derivative of the error signal.

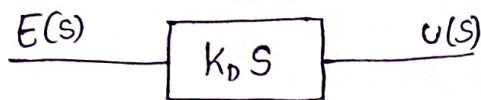
$$u(t) \propto \frac{d}{dt}[e(t)]$$

$$u(t) = K_D \frac{d}{dt}[e(t)]$$

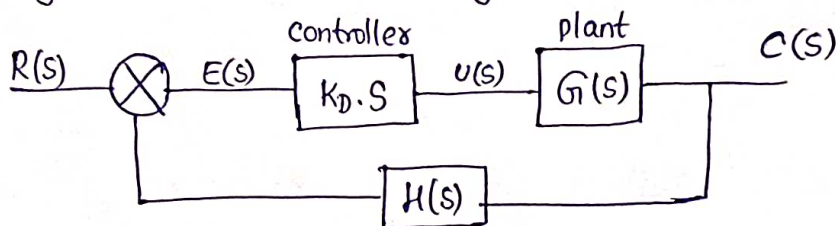
T.F $\Rightarrow$ Apply laplace transform,

$$U(s) = K_D \cdot S \cdot E(s)$$

$$\frac{U(s)}{E(s)} = K_D \cdot S$$



Block diagram $\rightarrow$ closed loop system



Effect:- without controller

$$G(s) = \frac{1}{s(s+8)} \quad \text{Type -1, order -2.}$$

with controller

$$\text{net } G(s) = \frac{K_D \cdot s}{s(s+8)} = \frac{K_D}{s+8} \quad \text{Type = 0, order -1}$$

* Zero is added

* Type and order of the system decreases

* $\xi \rightarrow T_r \text{ state} \rightarrow \text{order} \downarrow$, stability $\uparrow$, Oscillations $\downarrow$, $\xi \uparrow$

Good Transient state performance and T_r state is improved.

* Error, As type decreases, error increases
poor steady state response.