

Digital Filter Structures

Systems may be continuous-time systems & discrete-time systems

Discrete time Systems $\begin{cases} \text{FIR System} \\ \text{IIR System} \end{cases}$

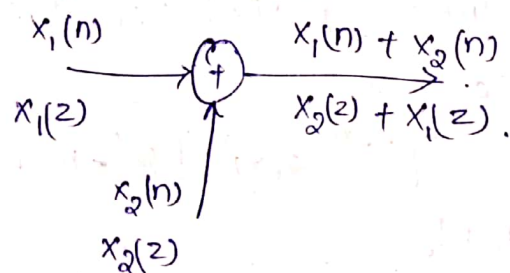
Realization of discrete time systems:-

- difference equation in time domain is converted into an algebraic equation in z-domain.
- Each term of that equation is to be represented by a suitable element.

For constructing the block diagram of a discrete-time system, the following basic elements are used.

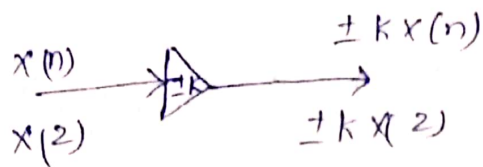
(i) Adder :- An adder is used to add two or more signals.

The output of the adder is equal to the sum of all incoming signals.

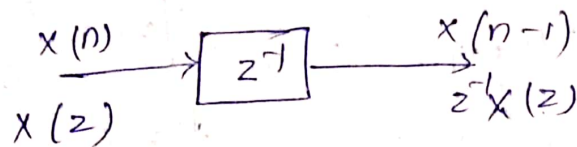


(ii) Constant Multiplier :- A constant multiplier is used to multiply the signals by a constant.

The output of the multiplier is equal to the product of the input signal & the constant of the multiplier.



(iii) Unit delay Element:- A unit delay Element is used to delay the signal passing through it by one sampling time.



→ Construct the block diagram for the discrete-time systems whose input output relations are described by the following difference equations.

(a) $y(n) = 0.7x(n) + 0.3x(n-1)$

Sol :-

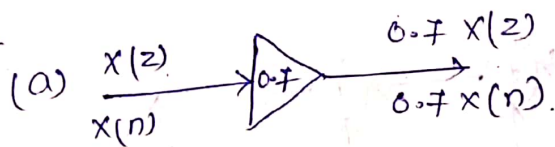


Fig (i)

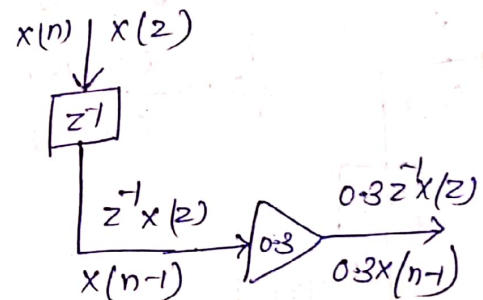
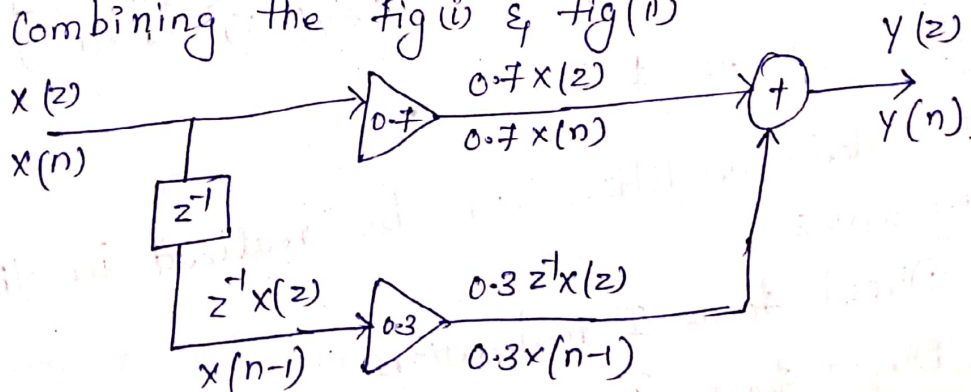


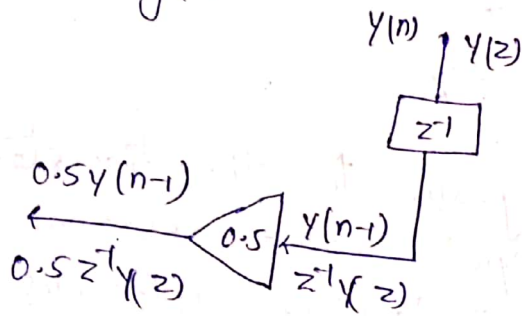
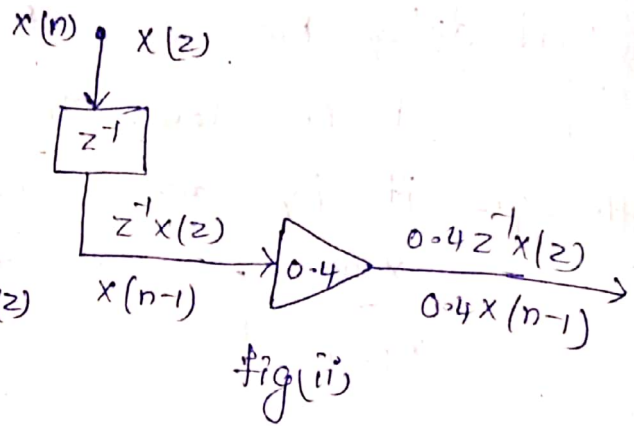
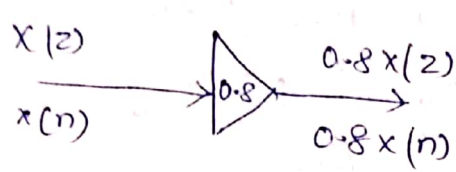
Fig (ii)

On Combining the Fig (i) & Fig (ii)

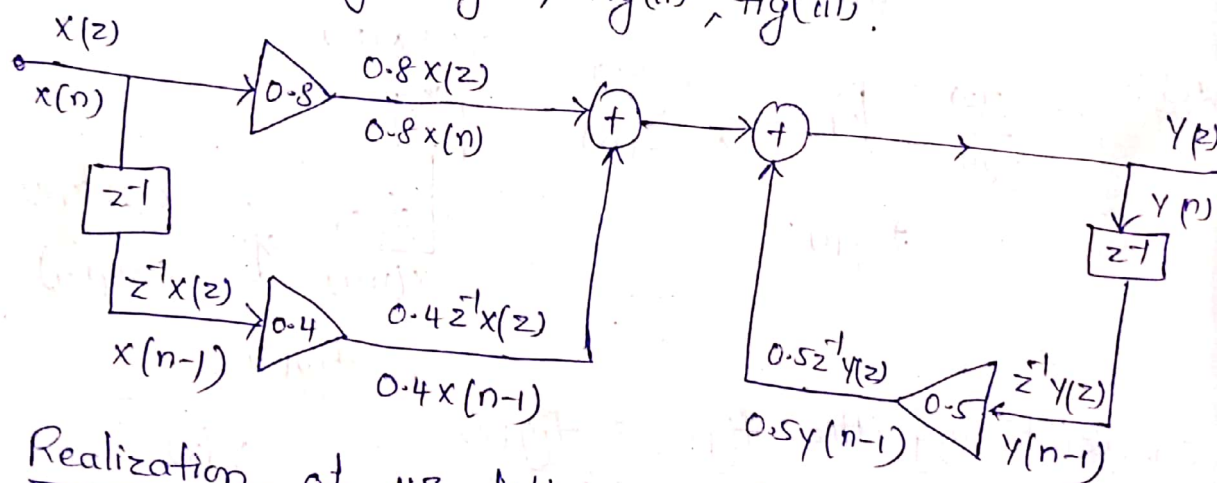


$$(b) \quad y(n) = 0.5y(n-1) + 0.8x(n) + 0.4x(n-1)$$

$$Y(z) = 0.5z^{-1}Y(z) + 0.8X(z) + 0.4z^{-1}X(z)$$



On Combining Fig (i), Fig (ii), Fig (iii):



Realization of IIR filters:-

IIR systems or filters can be realized in the following ways

1. Direct form-I realization
2. Direct form-II realization
3. Transposed direct form realization
4. Cascade form realization
5. Parallel form realization
6. Lattice-ladder structures

Direct form I realization

It is the direct implementation of the difference equation or transfer function.

It is the most simplest & straight forward realization structure available.

Consider an LTI System described by the difference equation

$$y(n) = - \sum_{k=1}^N a_k y(n-k) + \sum_{k=0}^M b_k x(n-k)$$

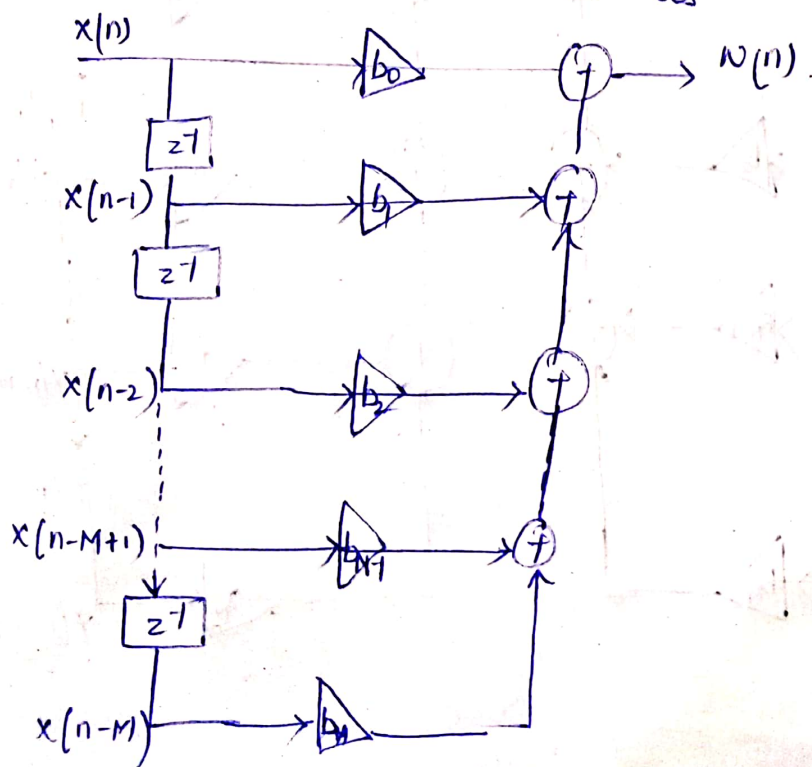
$$y(n) = -a_1 y(n-1) - a_2 y(n-2) \dots - a_{N-1} y(n-N+1) \\ - a_N y(n-N) + b_0 x(n) + b_1 x(n-1) + \dots + b_M x(n-M)$$

Let

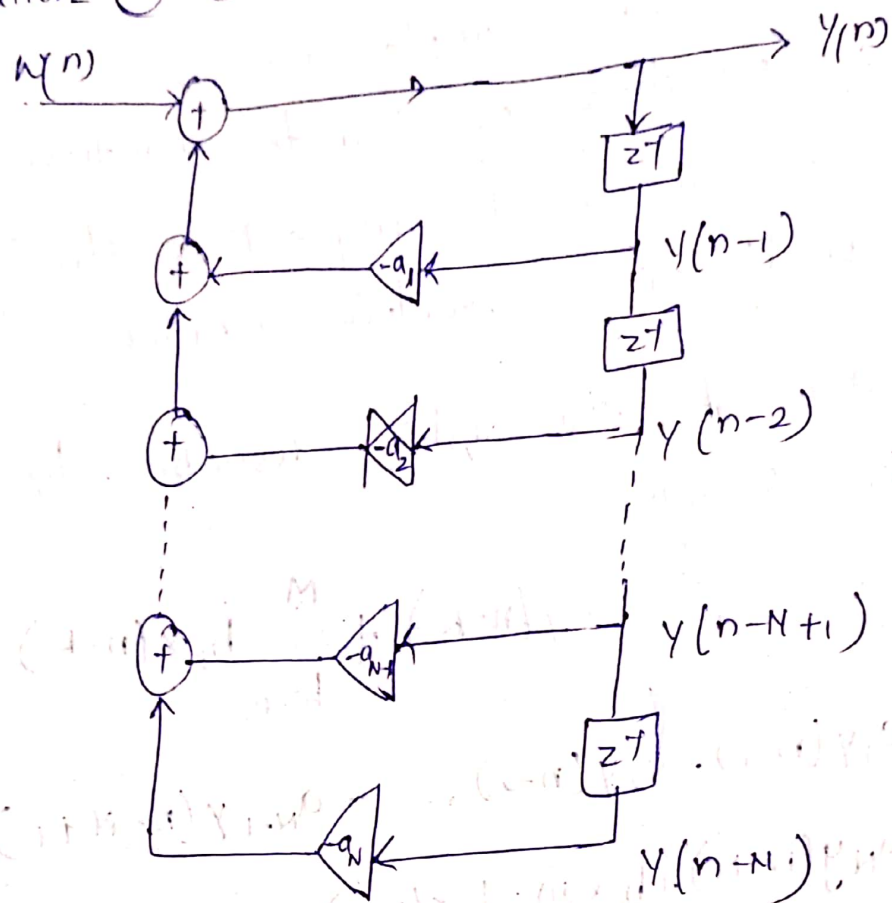
$$b_0 x(n) + b_1 x(n-1) + \dots + b_M x(n-M) = w(n) \rightarrow \textcircled{1}$$

Then $y(n) = -a_1 y(n-1) - a_2 y(n-2) + \dots - a_N y(n-N) + w(n) \rightarrow \textcircled{2}$

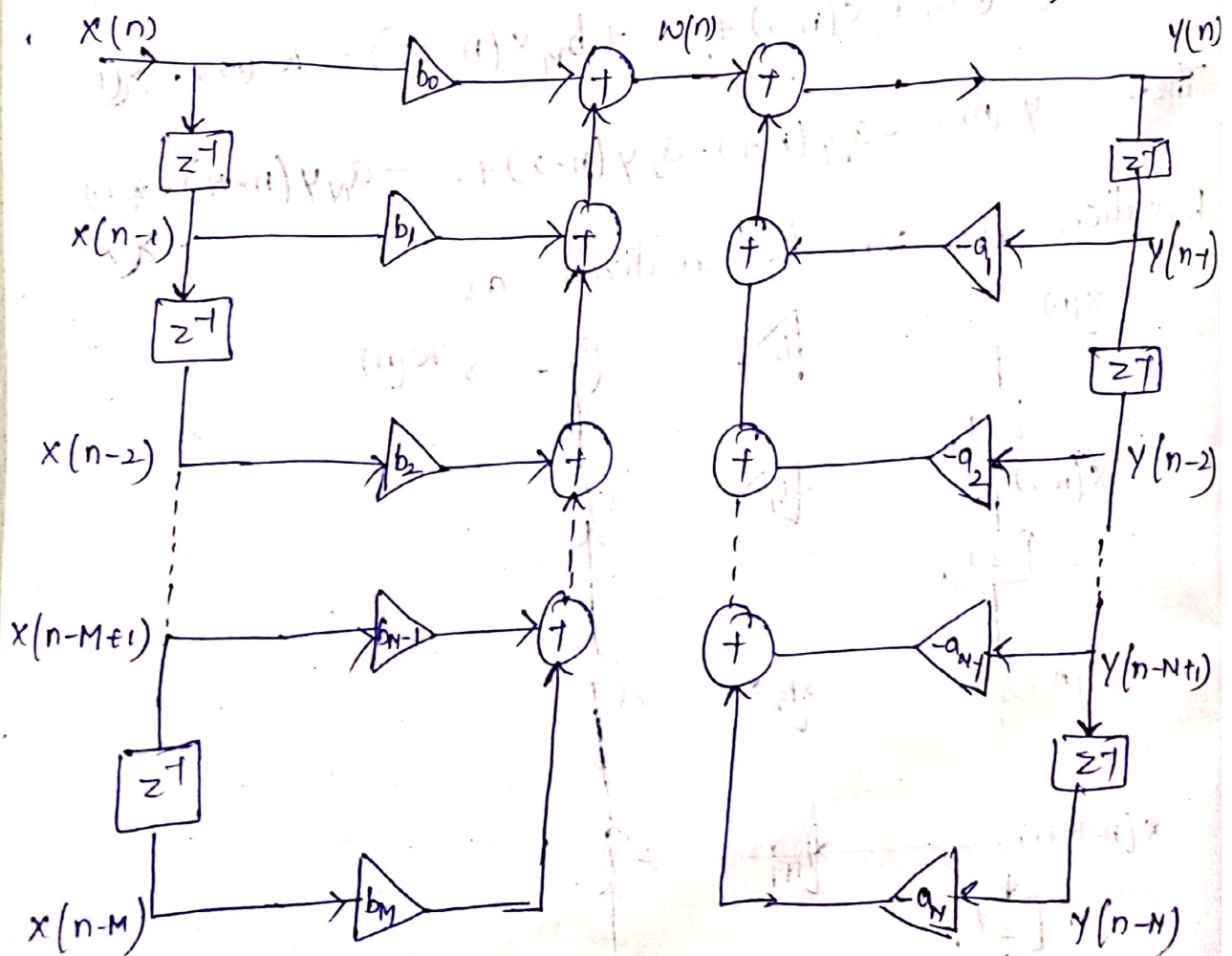
Equation $\textcircled{1}$ can be realized as



Equation (2) can be realized as



On combining the above two structures,



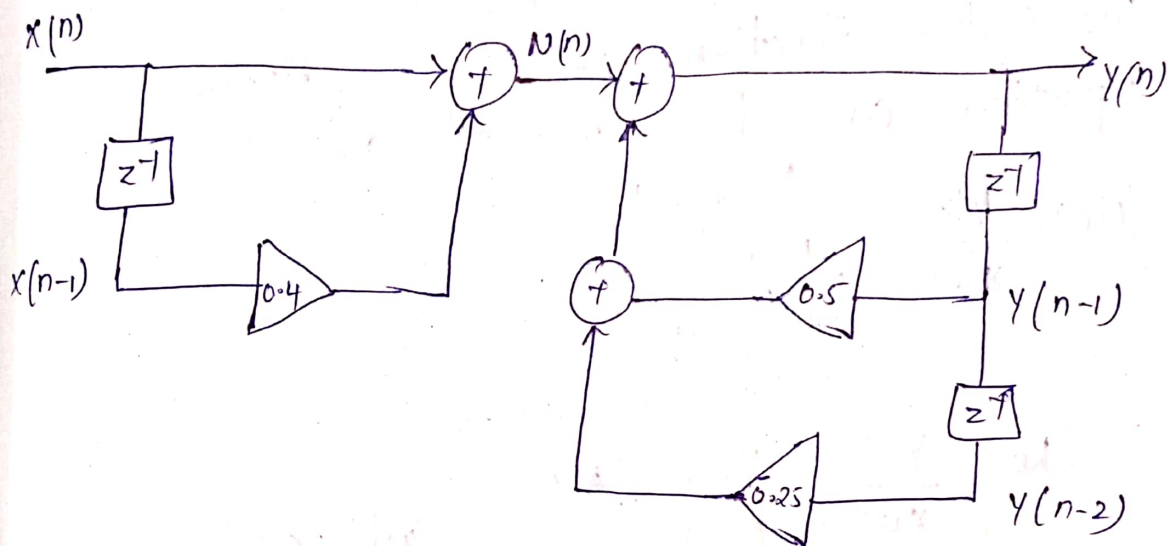
Problems:-

Obtain the direct form-I realization for the system described by the difference equation (i) $y(n) = 0.5y(n-1) - 0.25y(n-2) + x(n) + 0.4x(n-1)$

let

$$x(n) + 0.4x(n-1) = w(n)$$

$$y(n) = 0.5y(n-1) - 0.25y(n-2) + w(n)$$



Limitations of direct form-I

- Since the no. of delay elements used is more than (double) the order of difference equation it is not effective.
- It lacks hardware flexibility.
- There are chances of instability due to quantization noise.

Direct form-II Structure:-

Direct form-II structure is more advantageous than direct form-I structure, because it uses less no. of delay elements than direct form-I structure.

Consider the difference equation of the form

$$y(n) = -\sum_{k=1}^N a_k y(n-k) + \sum_{k=0}^M b_k x(n-k).$$

The system's function of above difference equation can be expressed as

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^M b_k z^{-k}}{1 + \sum_{k=1}^N a_k z^{-k}}.$$

$$\text{Let } \frac{Y(z)}{X(z)} = \frac{Y(z)}{W(z)} \cdot \frac{W(z)}{X(z)} \quad \text{where}$$

$$\frac{W(z)}{X(z)} = \frac{1}{1 + \sum_{k=1}^N a_k z^{-k}} \quad \text{and} \quad \frac{Y(z)}{W(z)} = \sum_{k=0}^M b_k z^{-k}.$$

$$\text{Consider } \frac{W(z)}{X(z)} = \frac{1}{1 + \sum_{k=1}^N a_k z^{-k}}.$$

$$W(z) + \sum_{k=1}^N a_k z^{-k} W(z) = X(z).$$

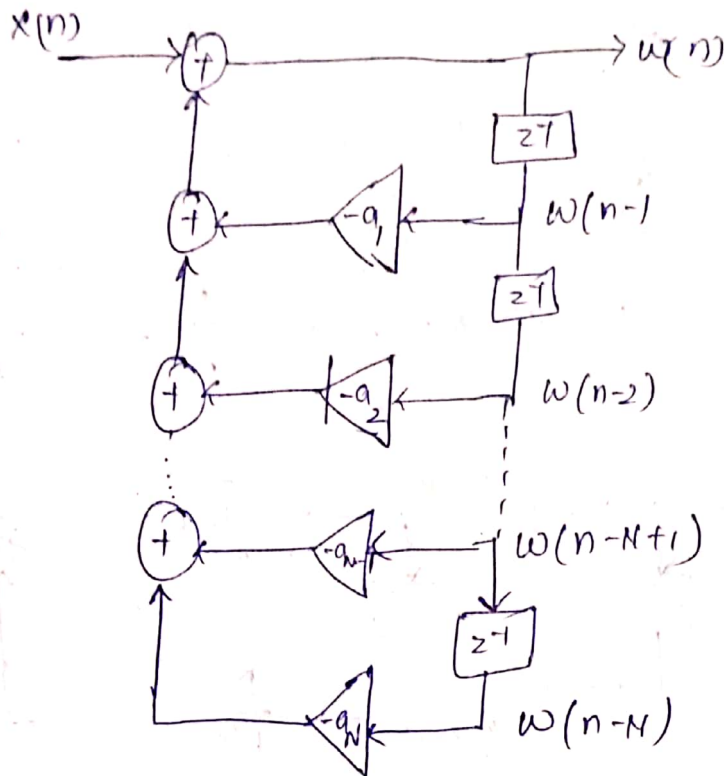
$$W(z) = X(z) - \sum_{k=1}^N a_k z^{-k} W(z).$$

$$W(z) = X(z) - a_1 z^{-1} W(z) - a_2 z^{-2} W(z) - \dots - a_N z^{-N} W(z)$$

$$w(n) = x(n) - a_1 w(n-1) - a_2 w(n-2) - \dots - a_N w(n-N)$$

Realization of the Equation

$$w(n) = x(n) - a_1 w(n-1) - a_2 w(n-2) \dots - a_N w(n-N)$$



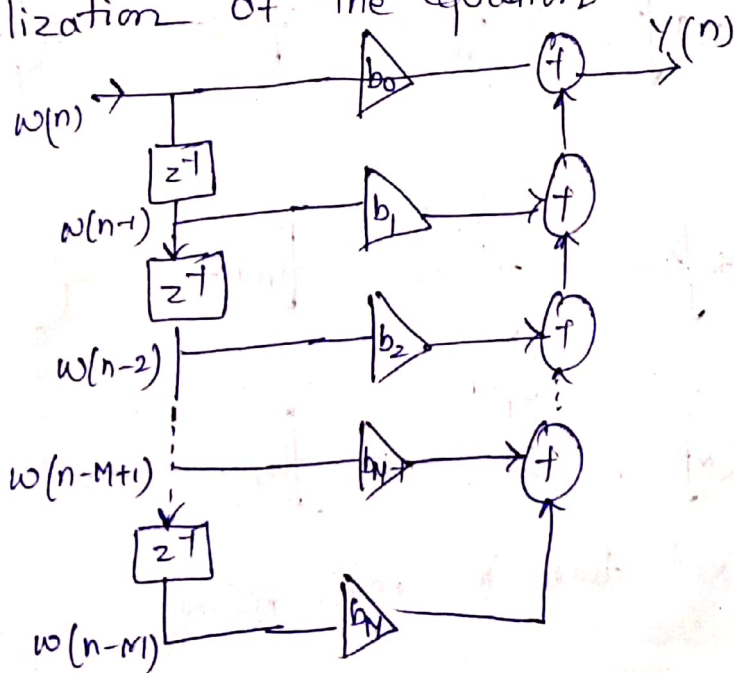
Consider the Equation $\frac{Y(z)}{W(z)} = \sum_{k=0}^M b_k z^{-k}$

$$Y(z) = \sum_{k=0}^M b_k z^{-k} W(z)$$

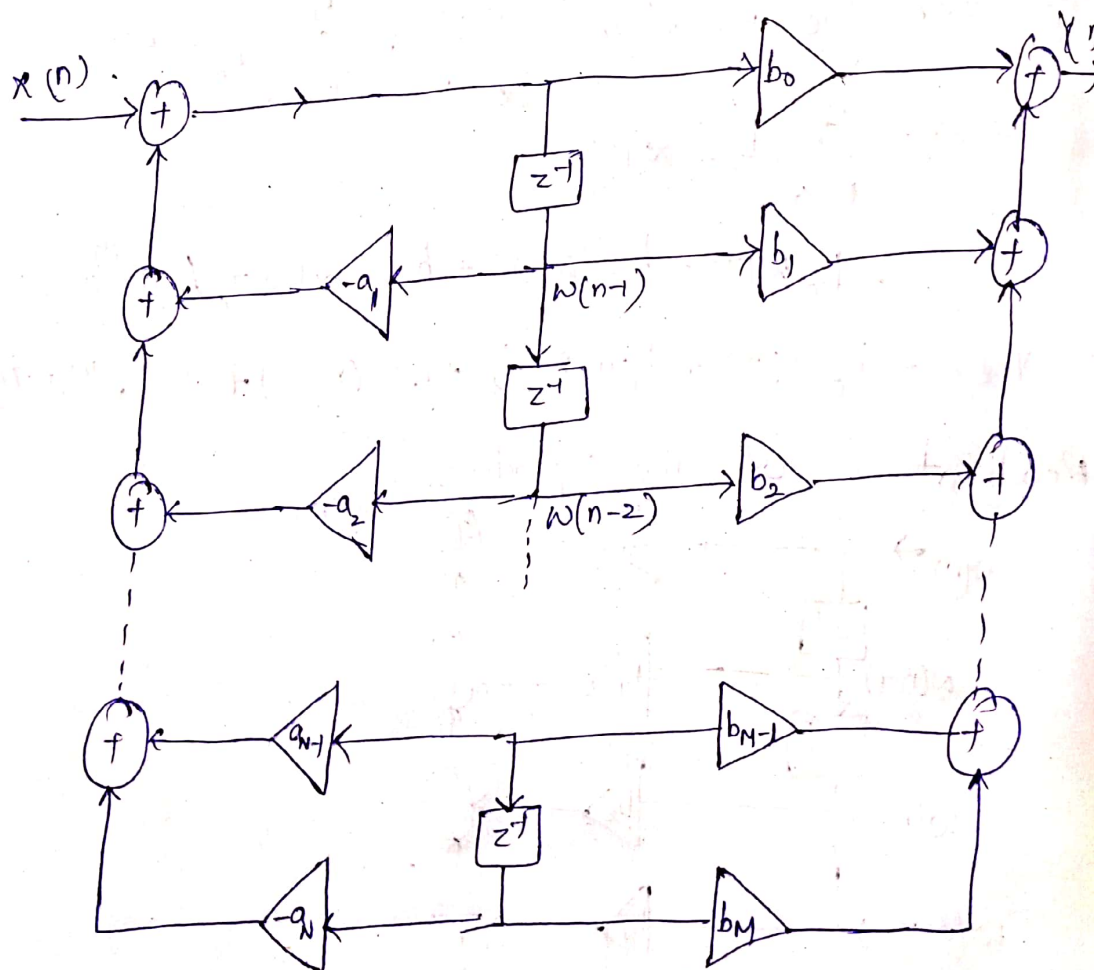
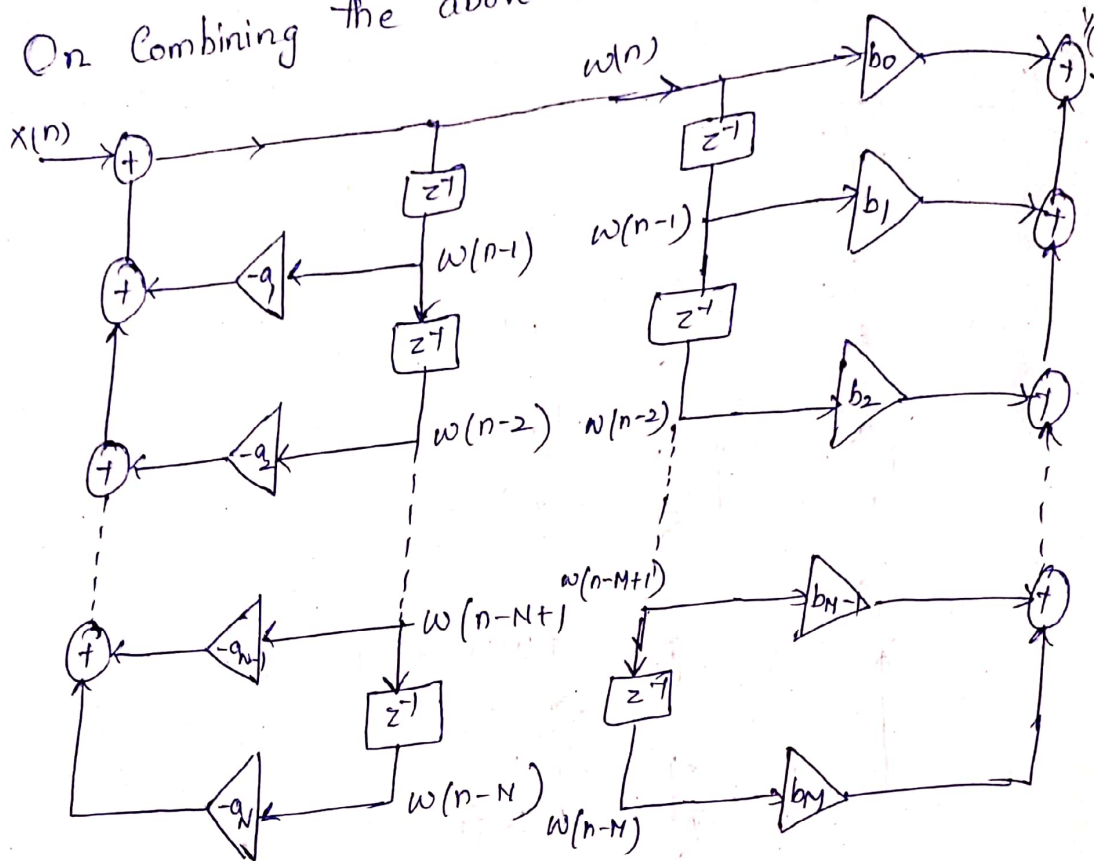
$$= b_0 w(z) + b_1 z^{-1} w(z) + b_2 z^{-2} w(z) + \dots + b_M z^{-M} w(z)$$

$$y(n) = b_0 w(n) + b_1 w(n-1) + b_2 w(n-2) + \dots + b_M w(n-M)$$

Realization of the Equation



On Combining the above two realizations



Direct form II realization

Determine the direct form II realization for the following system

$$y(n] = -0.1y(n-1) + 0.72y(n-2) + 0.7x(n) - 0.252x(n-2)$$

Sol:- $\frac{Y(z)}{X(z)} = \frac{0.7 - 0.252z^{-2}}{1 + 0.1z^{-1} - 0.72z^{-2}}$

Let $\frac{Y(z)}{W(z)} = 0.7 - 0.252z^{-2}$

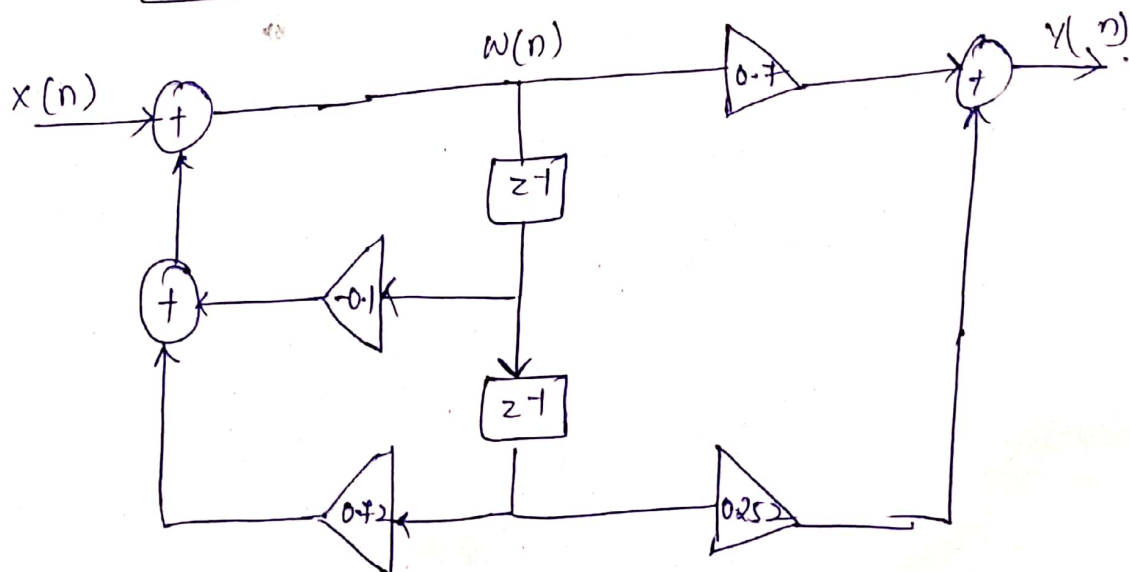
$$Y(z) = 0.7W(z) - 0.252z^{-2}W(z)$$

$$Y(n) = 0.7w(n) - 0.252w(n-2)$$

Let $\frac{W(z)}{X(z)} = \frac{1}{1 + 0.1z^{-1} - 0.72z^{-2}}$

$$W(z) = X(z) + 0.1z^{-1}W(z) - 0.72z^{-2}W(z)$$

$$W(n) = X(n) - 0.1w(n-1) + 0.72w(n-2)$$



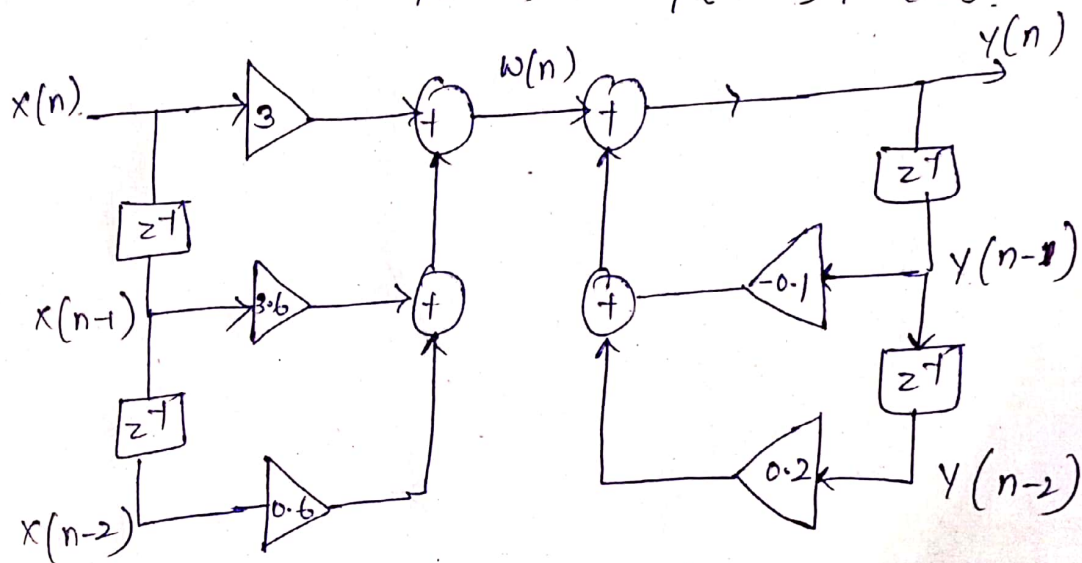
Obtain the direct form-I, direct form-II realization for the following system

$$y(n) = -0.1y(n-1) + 0.2y(n-2) + 3x(n) + 3.6x(n-1) + 0.6x(n-2)$$

Sol:- Direct form I structure

$$\text{Let } 3x(n) + 3.6x(n-1) + 0.6x(n-2) = w(n)$$

$$y(n) = -0.1y(n-1) + 0.2y(n-2) + w(n)$$



Direct form II structure

$$H(z) = \frac{Y(z)}{X(z)} = \frac{3 + 3.6z^{-1} + 0.6z^{-2}}{1 + 0.1z^{-1} - 0.2z^{-2}}$$

