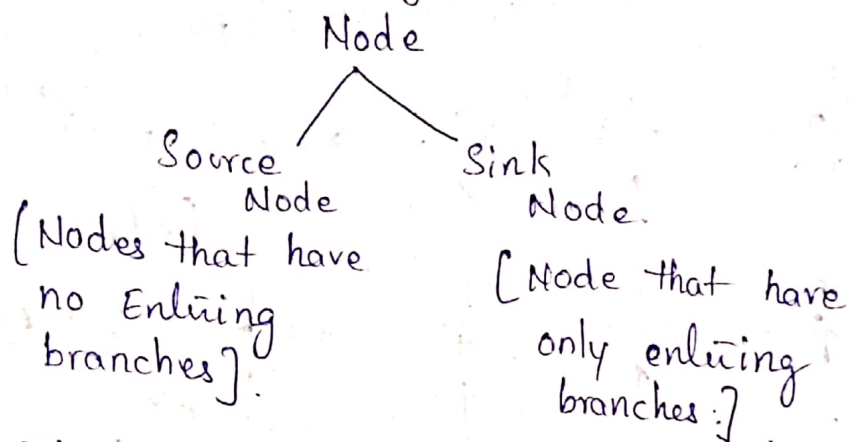


Signal Flowgraph:-

A signal flowgraph is a graphical representation of the relationships between the variables of a set of linear difference equations.

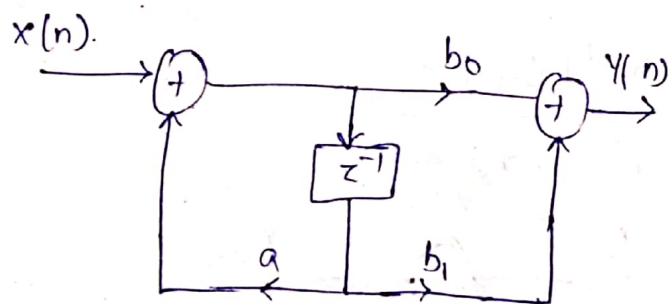
Its basic elements are branches & nodes.

Node:- A node represents a system variable, which is equal to the sum of incoming signals from all branches connecting to the node.

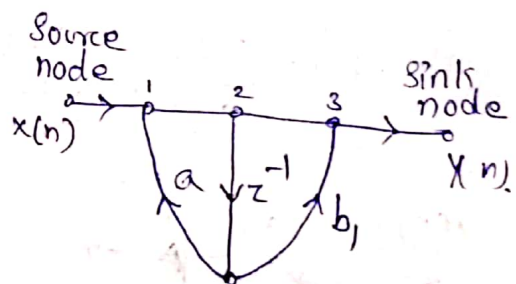


The signal out of a branch is equal to the branch gain times the signal into the branch.

The delay is indicated by the branch transmittance z^{-1} .



Block diagram representation of first order digital filter.



Signal flowgraph representation of first order filter.

Transposition theorem & transposed structure

The transpose of a structure is defined by the following operations

- (i) Reverse the direction of all branches in the signal flow graph.
- (ii) Interchange the inputs & outputs.
- (iii) Reverse the roles of all nodes in the flowgraph.
- (iv) Summing points become branching points.
- (v) Branching points become summing points.

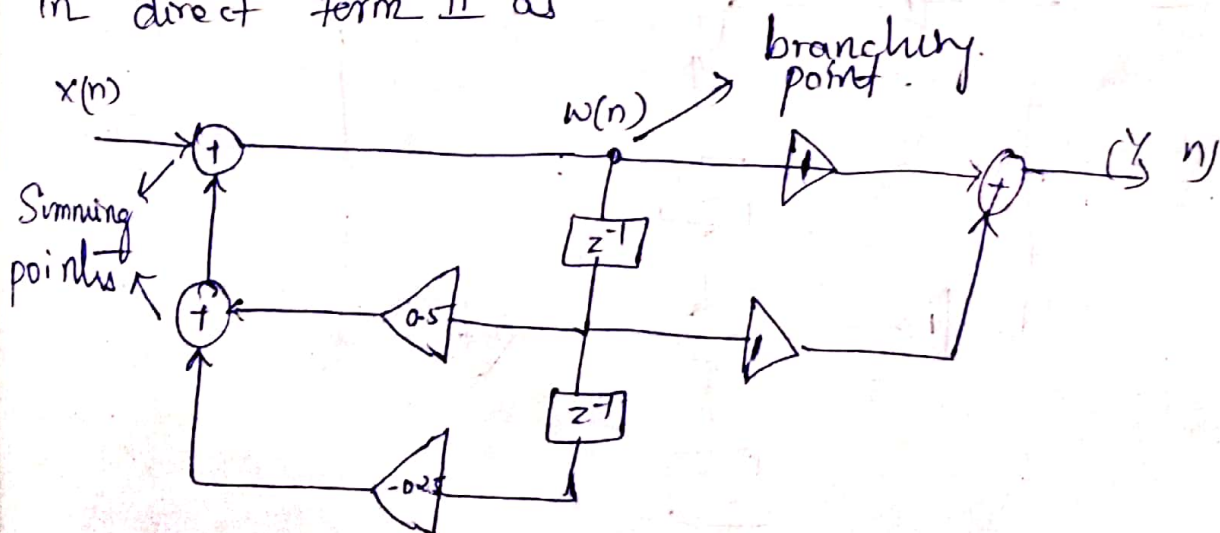
According to transposition theorem, the system transfer function remain unchanged by transposition.

→ Determine the direct form II & transposed direct form II for the given system

$$y(n) = \frac{1}{2}y(n-1) - \frac{1}{4}y(n-2) + x(n) + x(n-1)$$

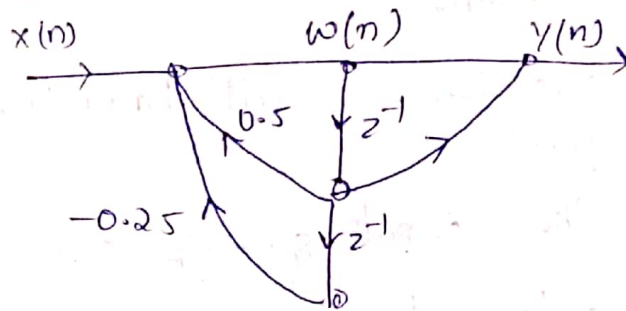
Sol:- $H(z) = \frac{Y(z)}{X(z)} = \frac{1+z^{-1}}{1-0.5z^{-1}+0.25z^{-2}}$

The above transfer function can be realized in direct form II as



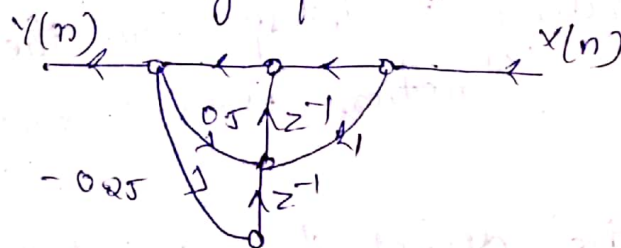
Transposed structure:-

Step 1:- Convert the block diagram into signal flow graph.

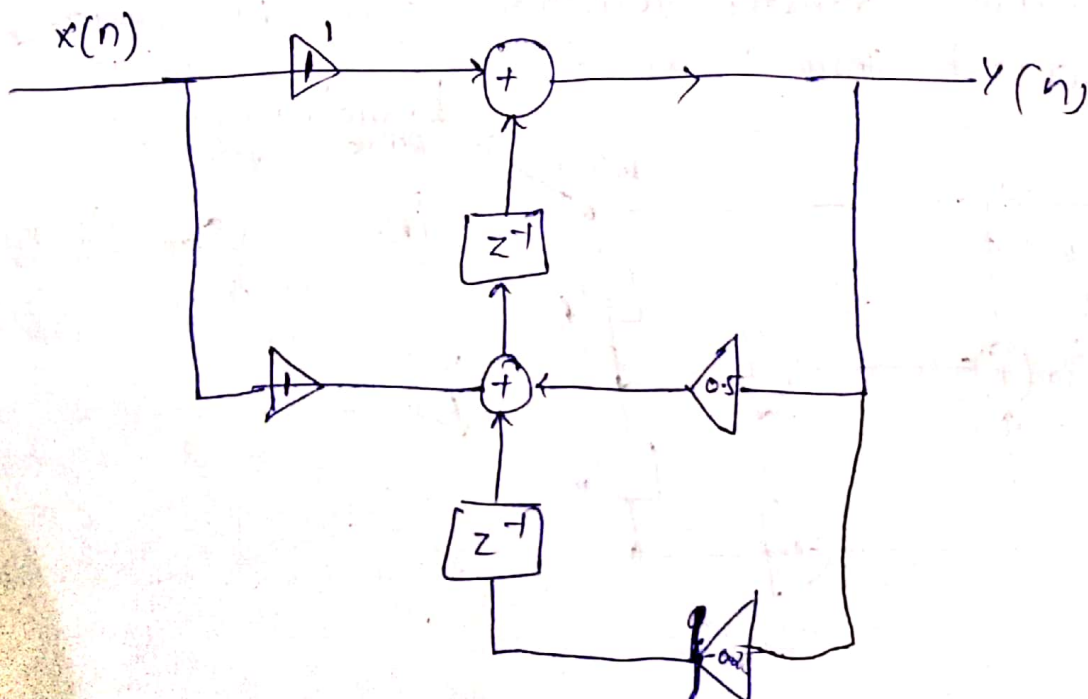
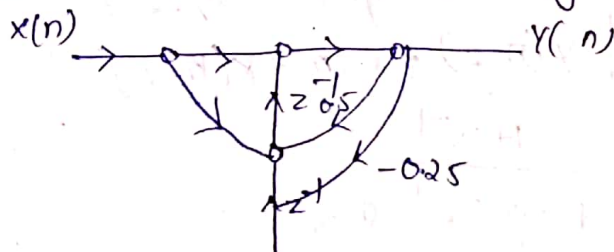


Step 2:- change the direction of all branches & interchange the i/p and o/p.

Step 3:- & change the summing point to branching point.



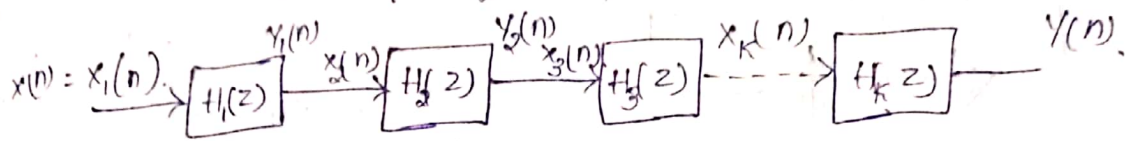
Step 4:- Re draw the ~~dis~~ signal flow graph



Cascade form :-

Consider an IIR System with system function

$$H(z) = H_1(z) H_2(z) \dots H_k(z).$$

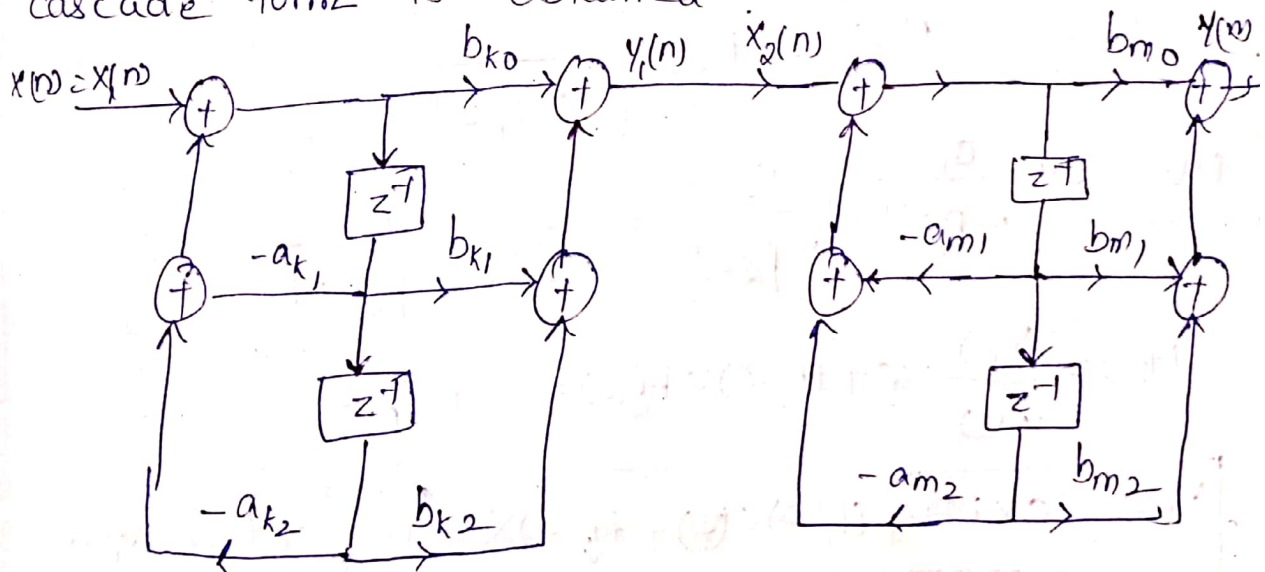


$$H(z) = \frac{(b_{k0} + b_{k1}z^{-1} + b_{k2}z^{-2}) (b_{m0} + b_{m1}z^{-1} + b_{m2}z^{-2})}{(1 + a_{k1}z^{-1} + a_{k2}z^{-2}) (1 + a_{m1}z^{-1} + a_{m2}z^{-2})}$$

$$= H_1(z) H_2(z)$$

Where $H_1(z) = \frac{b_{k0} + b_{k1}z^{-1} + b_{k2}z^{-2}}{1 + a_{k1}z^{-1} + a_{k2}z^{-2}}$ & $H_2(z) = \frac{b_{m0} + b_{m1}z^{-1} + b_{m2}z^{-2}}{1 + a_{m1}z^{-1} + a_{m2}z^{-2}}$

$H_1(z)$ & $H_2(z)$ can be realized in direct form II, cascade form is obtained



→ Realize the system with difference equation

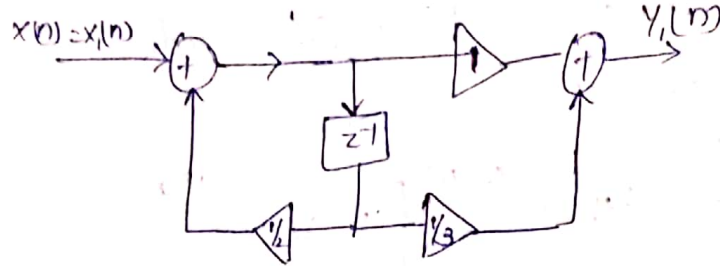
$$y(n) = \frac{3}{4} y(n-1) - \frac{1}{8} y(n-2) + x(n) + \frac{1}{3} x(n-1) \text{ in cascade form.}$$

Sol:- $H(z) = \frac{Y(z)}{X(z)} = \frac{1 + \frac{1}{3}z^{-1}}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}} = \frac{1 + \frac{1}{3}z^{-1}}{(1 - \frac{1}{2}z^{-1})(1 - \frac{1}{4}z^{-1})}$

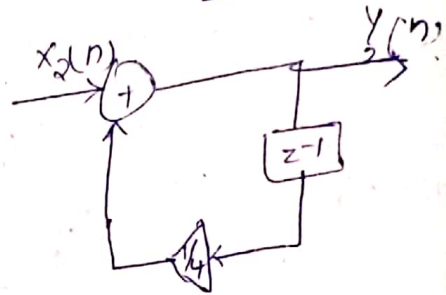
$$H_1(z) = \frac{1 + \frac{1}{3}z^{-1}}{1 - \frac{1}{2}z^{-1}}$$

$$H_2(z) = \frac{1}{1 - \frac{1}{4}z^{-1}}$$

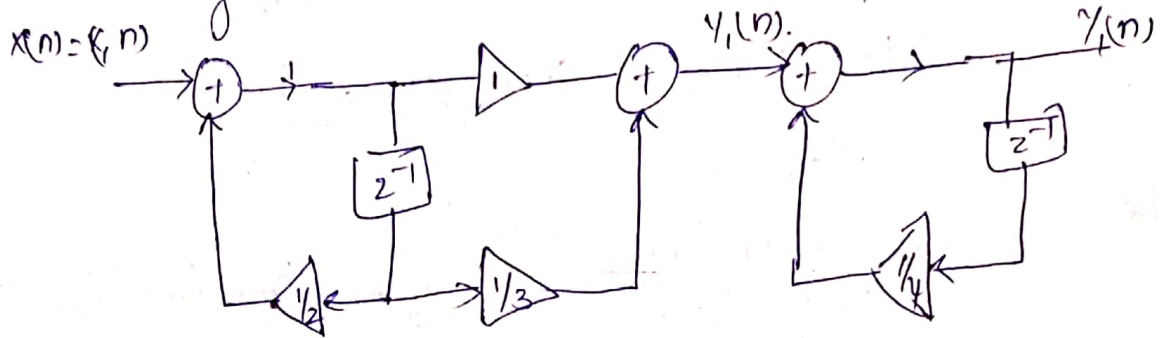
$H_1(z)$ in direct form II.



$H_2(z)$ in direct form II.



Cascading the realization of $H_1(z)$ & $H_2(z)$.



Parallel form structure:-

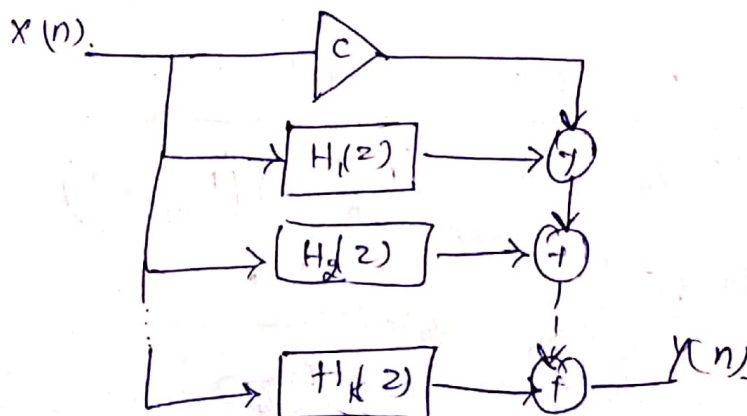
$$H(z) = C + \sum_{k=1}^N \frac{C_k}{1 - p_k z^{-1}}$$

$$H(z) = C + \frac{C_1}{1 - p_1 z^{-1}} + \frac{C_2}{1 - p_2 z^{-1}} + \dots + \frac{C_N}{1 - p_N z^{-1}}$$

$$H(z) = \frac{Y(z)}{X(z)} = C + H_1(z) + H_2(z) + \dots + H_N(z)$$

$$Y(z) = CX(z) + H_1(z)X(z) + H_2(z)X(z) + \dots + H_N(z)X(z)$$

The above Equation can be realized in parallel form as

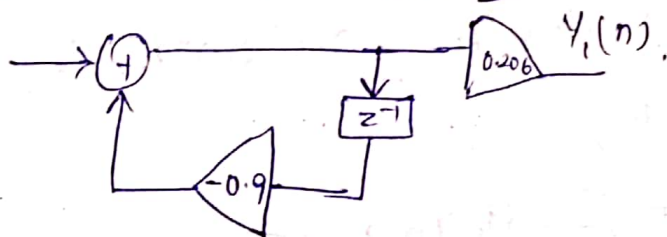


Realize the system given by difference equation in ~~cascade~~ parallel form.

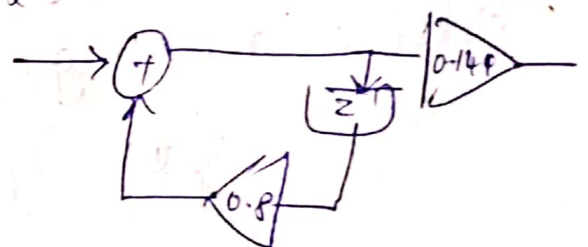
$$y(n) = -0.1y(n-1) + 0.72y(n-2) + 0.7x(n) - 0.252x(n-2)$$

$$\begin{aligned} H(z) &= \frac{0.7 - 0.252z^{-2}}{1 + 0.12z^{-1} - 0.72z^{-2}} \\ &= 0.35 + \frac{0.35 - 0.035z^{-1}}{1 + 0.12z^{-1} - 0.72z^{-2}} \\ &= 0.35 + \frac{0.35 - 0.035z^{-1}}{(1 + 0.9z^{-1})(1 - 0.8z^{-1})} \\ &= 0.35 + \frac{0.206}{1 + 0.9z^{-1}} + \frac{0.144}{1 - 0.8z^{-1}} \end{aligned}$$

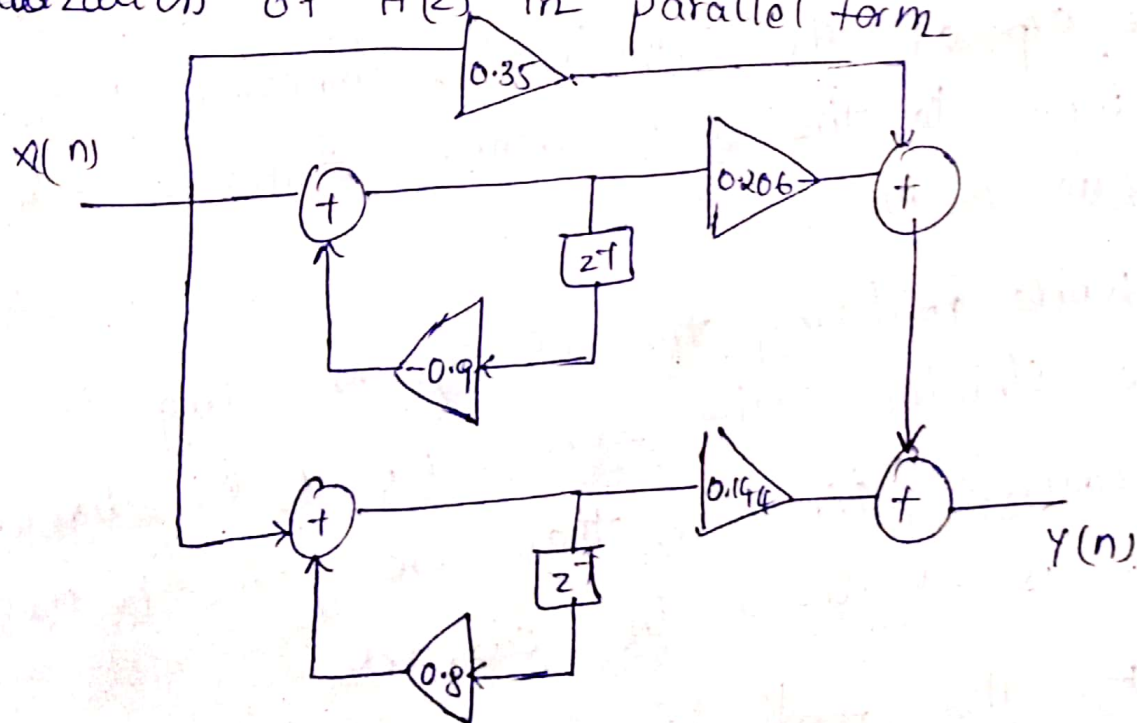
$H_1(z)$ in direct form-II



$H_2(z)$ in direct form-II



Realization of $H(z)$ in parallel form



Lattice structure Realization of IIR Systems:-

$$H(z) = \frac{\sum_{k=0}^M b_k z^{-k}}{1 + \sum_{k=1}^N a_k z^{-k}} = \left[\sum_{k=0}^M b_k z^{-k} \right] \left[\frac{1}{1 + \sum_{k=1}^N a_k z^{-k}} \right] = H_z(z) H_p(z).$$

$$H_z(z) = \sum_{k=0}^M b_k z^{-k} = \text{Zeros} \quad H_p(z) = \frac{1}{1 + \sum_{k=1}^N a_k z^{-k}} = \text{Poles}$$

The above equation for $H_p(z)$ can be rewritten as

$$H_p(z) = \frac{1}{1 + \sum_{k=1}^N a_k z^{-k}} = \frac{Y_p(z)}{X_p(z)}$$

$$X_p(z) = Y_p(z) + \sum_{k=1}^N a_k z^{-k} Y_p(z)$$

Taking the inverse z-transform on both the sides & rearranging it

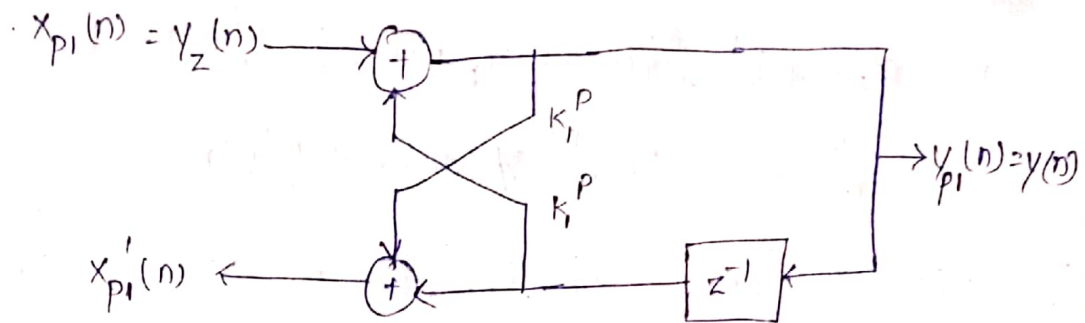
$$Y_p(n) = X_p(n) - \sum_{k=1}^N a_k Y_p(n-k)$$

The o/p of the sub-transfer function $H_z(z)$ is the input to the sub-transfer function $H_p(z)$.
So $Y_z(n) = X_p(n)$

While realizing the IIR system using the lattice structure, the zeros, i.e. $H_z(z)$ should be realized first & then the poles i.e. $H_p(z)$ which is realized in cascade

For the realization of the poles, a lattice structure consists of two paths, $X_p(n)$ & $X_p'(n)$ through which the input $X_p(n)$ or $Y_z(n)$ is processed. Those two paths are opposite in direction to each other.

A single lattice structure for a pole is shown in the figure below.



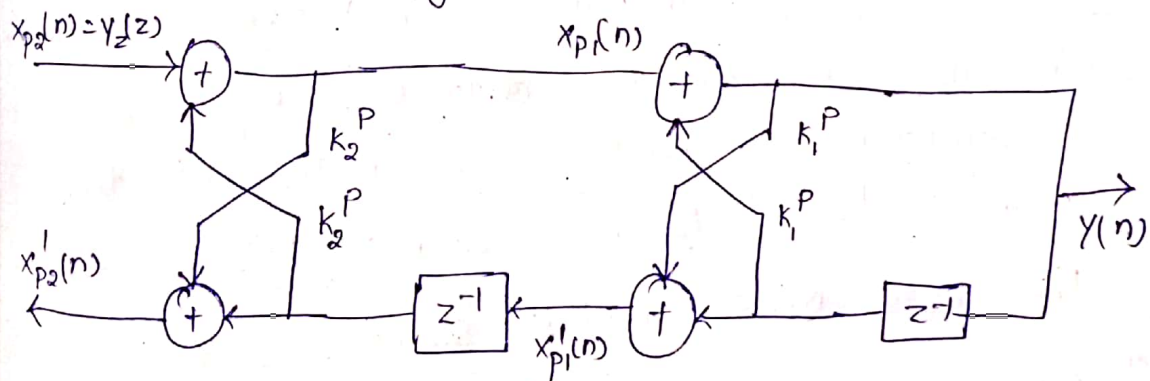
The output from the single stage lattice structure is

$$y(n) = x_{p1}(n) + k_1^P y(n-1)$$

& the feedback response is

$$x'_{p1}(n) = k_1^P y(n) + y(n-1)$$

Similarly, the two stage lattice structure for a pole is shown in figure below.



The intermediate o/p of the two-stage lattice structure is

$$x_{p1}(n) = x_{p2}(n) + k_2^P x'_{p1}(n-1)$$

Substituting the value of $x'_{p1}(n)$ in $x_{p1}(n)$

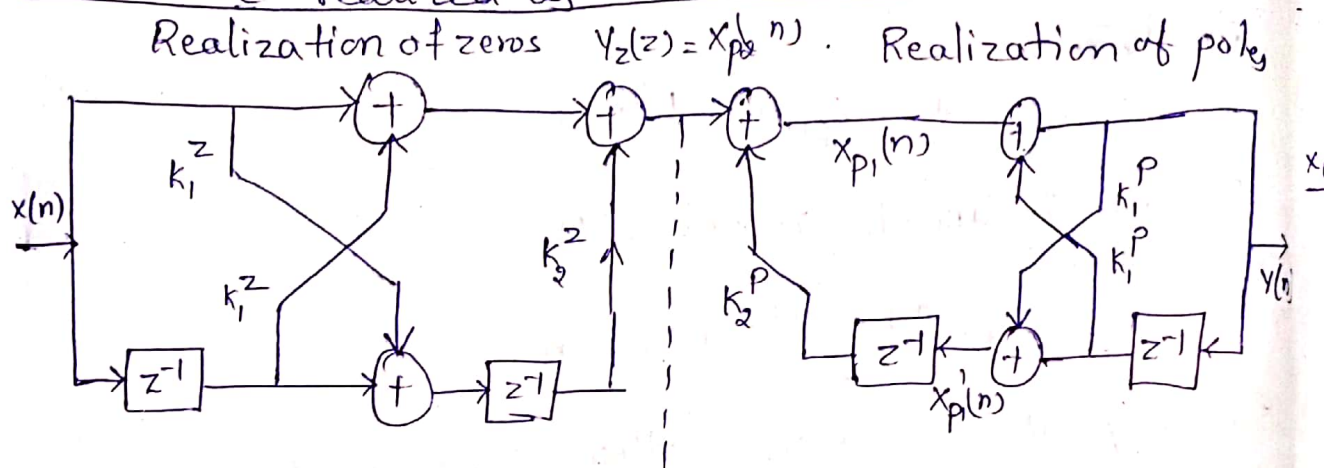
$$\begin{aligned} x_{p1}(n) &= x_{p2}(n) + k_2^P [k_1^P y(n-1) + y(n-2)] \\ &= x_{p2}(n) + k_1^P k_2^P y(n-1) + k_2^P y(n-2) \end{aligned}$$

Substituting this value of $x_{p1}(n)$ in the expression for $y(n)$

$$y(n) = x_{p2}(n) + k_1^P k_2^P y(n-1) + k_2^P y(n-2) + k_1^P y(n-1)$$

$$= x_{p2}(n) + k_1^P [1 + k_2^P] y(n-1) + k_2^P y(n-2)$$

The overall lattice structure for a second order IIR system (both poles & zeros are of 2nd order) can be realized as



Procedure to realize the lattice structure of IIR System

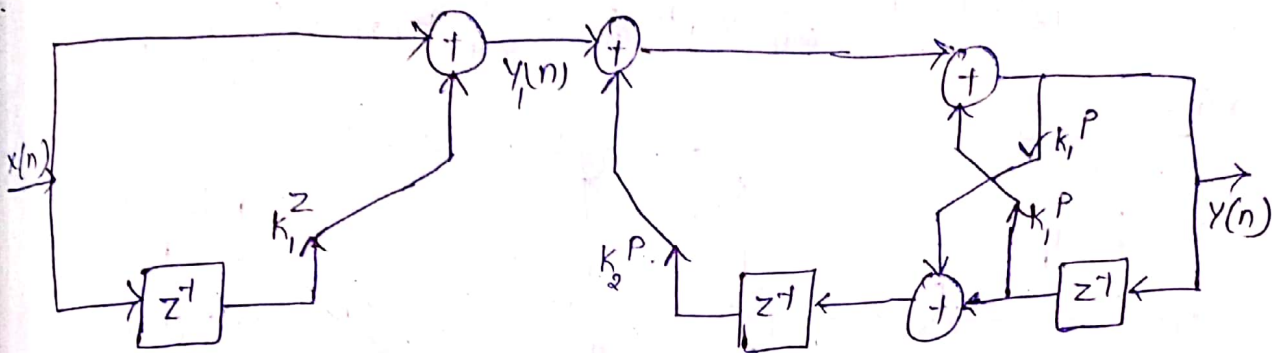
1. Find the order of the difference equation & compare the coefficients with reflection coefficients $k_1^P, k_2^P, k_3^P \dots$ and k_1^Z, k_2^Z, k_3^Z of the same order lattice structure o/p.
2. Assign the calculated values of the reflection coefficients k_1^P, k_2^P, k_3^P & k_1^Z, k_2^Z, k_3^Z & construct the structures.
3. Cascade the structures of both zeros & poles.

Determine the lattice coefficients corresponding to an IIR filter described by

$$y(n) - \frac{2}{5}y(n-1) + \frac{1}{5}y(n-2) = x(n) + \frac{1}{4}x(n-1]$$

realize it.

Sol:- The difference equation has first order zeros & second order poles.
The proposed lattice structure is.



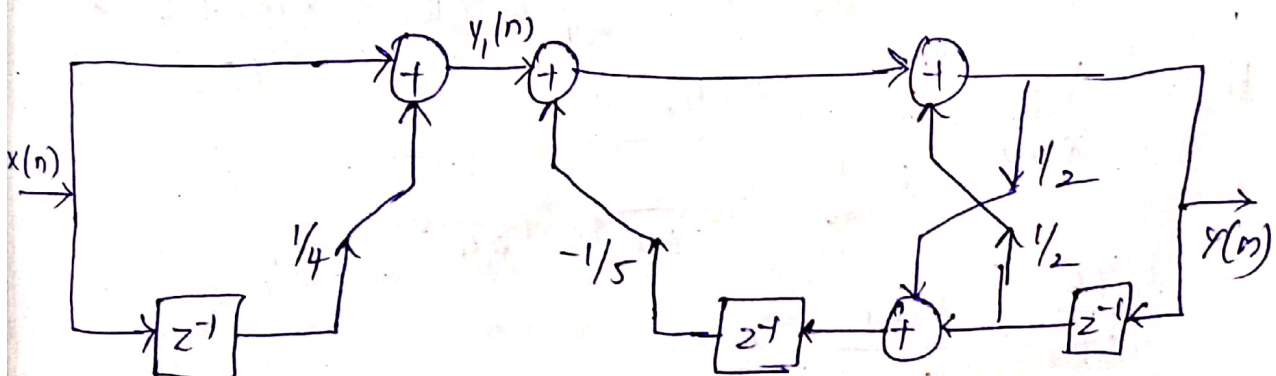
From the above figure, the o/p $y(n)$ is

$$y(n) = y_1(n) + k_1^P [1 + k_2^P] y(n-1) + k_2^P y(n-2)$$

$$y(n) = x(n) + k_1^z x(n-1) + k_1^P [1 + k_2^P] y(n-1) + k_2^P y(n-2)$$

Comparing the equation with the given difference equation $k_1^z = \frac{1}{4}$, $k_2^P = \frac{-1}{5}$

$$k_1^P [1 + k_2^P] = \frac{2}{5} \Rightarrow k_1^P = \frac{2/5}{1 - 1/5} = \frac{1}{2}$$



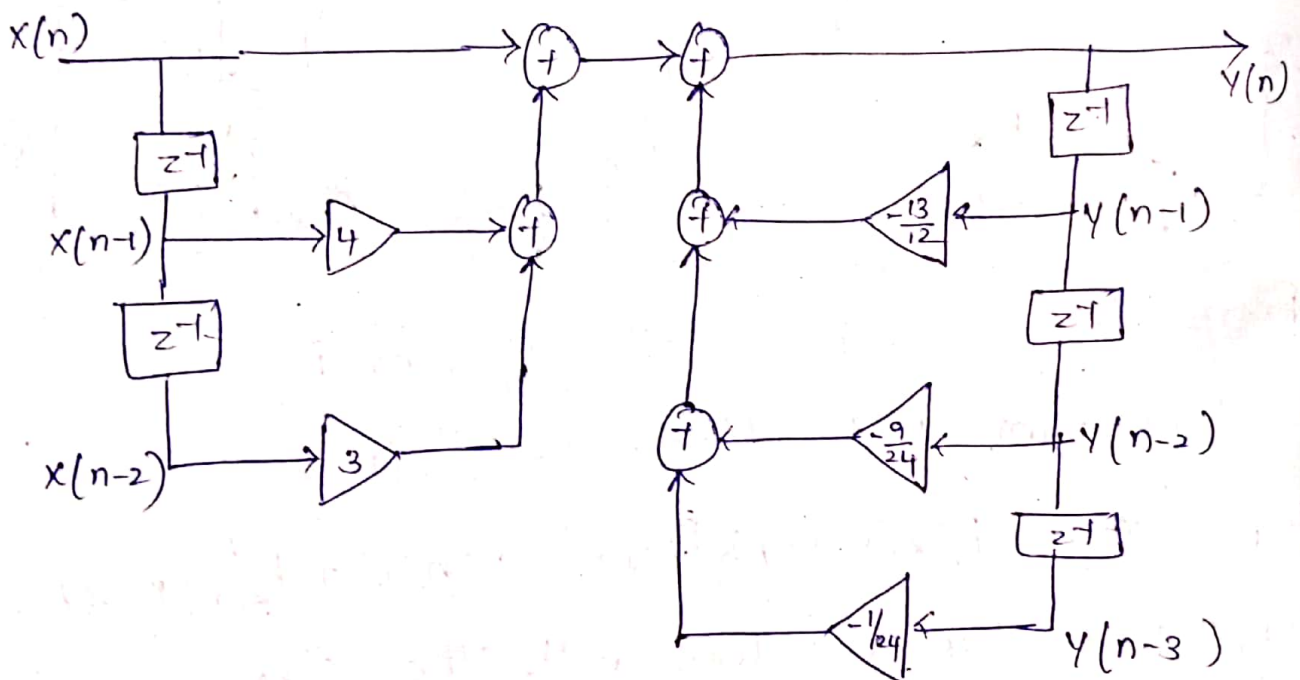
→ Obtain the direct form-I, direct form-II, cascade & parallel form realizations for the LTI system governed by the equation

$$Y(n) = -\frac{13}{12}Y(n-1) - \frac{9}{24}Y(n-2) - \frac{1}{24}Y(n-3) + X(n) + 4X(n-1) + 3X(n-2)$$

Direct form-I :-

$$X(n) + 4X(n-1) + 3X(n-2) = W(n)$$

$$Y(n) = -\frac{13}{12}Y(n-1) - \frac{9}{24}Y(n-2) - \frac{1}{24}Y(n-3) + W(n)$$



Direct form-II :-

On applying the z-transform to the difference equation

$$Y(z) = -\frac{13}{12}z^{-1}Y(z) - \frac{9}{24}z^{-2}Y(z) - \frac{1}{24}z^{-3}Y(z) + X(z) + 4z^{-1}X(z) + 3z^{-2}X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + 4z^{-1} + 3z^{-2}}{1 + \frac{13}{12}z^{-1} + \frac{9}{24}z^{-2} + \frac{1}{24}z^{-3}}$$

$$\frac{Y(z)}{X(z)} = \frac{Y(z)}{W(z)} \cdot \frac{W(z)}{X(z)}$$

$$\frac{W(z)}{X(z)} = \frac{1}{1 + \frac{13}{12}z^{-1} + \frac{9}{24}z^{-2} + \frac{1}{24}z^{-3}}$$

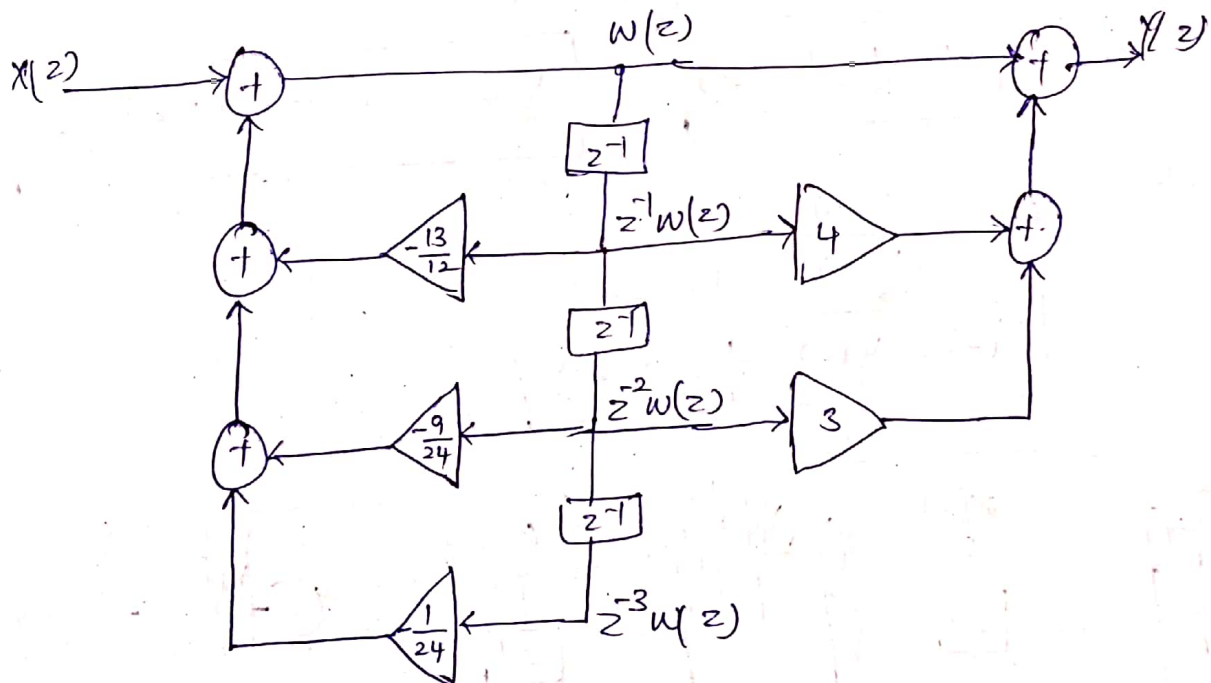
$$W(z) \left[1 + \frac{13}{12}z^{-1} + \frac{9}{24}z^{-2} + \frac{1}{24}z^{-3} \right] = X(z)$$

$$W(z) = X(z) - \frac{13}{12}z^{-1}W(z) - \frac{9}{24}z^{-2}W(z) - \frac{1}{24}z^{-3}W(z)$$

$$\frac{Y(z)}{W(z)} = 1 + 4z^{-1} + 3z^{-2}$$

$$Y(z) = [1 + 4z^{-1} + 3z^{-2}]W(z)$$

$$Y(z) = W(z) + 4z^{-1}W(z) + 3z^{-2}W(z)$$



Cascade form:-

$$H(z) = \frac{Y(z)}{X(z)} = \frac{(1+z^{-1})(1+3z^{-1})}{\left(1+\frac{1}{2}z^{-1}\right)\left(1+\frac{1}{3}z^{-1}\right)\left(1+\frac{1}{4}z^{-1}\right)}$$

$$H(z) = H_1(z) H_2(z) H_3(z)$$

$$H_1(z) = \frac{1+z^{-1}}{1+\frac{1}{2}z^{-1}}, \quad H_2(z) = \frac{1+3z^{-1}}{1+\frac{1}{3}z^{-1}}, \quad H_3(z) = \frac{1}{1+\frac{1}{4}z^{-1}}$$

$H_1(z)$:-

$$H_1(z) = \frac{Y_1(z)}{X(z)} = \frac{Y_1(z)}{W_1(z)} \cdot \frac{W_1(z)}{X(z)} = \frac{1+z^{-1}}{1+\frac{1}{2}z^{-1}}$$

$$\frac{W_1(z)}{X(z)} = \frac{1}{1+\frac{1}{2}z^{-1}} \quad \& \quad \frac{Y_1(z)}{W_1(z)} = 1+z^{-1}$$

$$W_1(z) = X(z) - \frac{1}{2}z^{-1}W_1(z)$$

$$Y_1(z) = W_1(z) + z^{-1}W_1(z)$$

$$\underline{H_2(z)} = \frac{Y_2(z)}{Y_1(z)} = \frac{Y_2(z)}{W_2(z)} \cdot \frac{W_2(z)}{Y_1(z)} = \frac{1+3z^{-1}}{1+\frac{1}{3}z^{-1}}$$

$$\frac{W_2(z)}{Y_1(z)} = \frac{1}{1+\frac{1}{3}z^{-1}} \quad \& \quad \frac{Y_2(z)}{W_2(z)} = 1+3z^{-1}$$

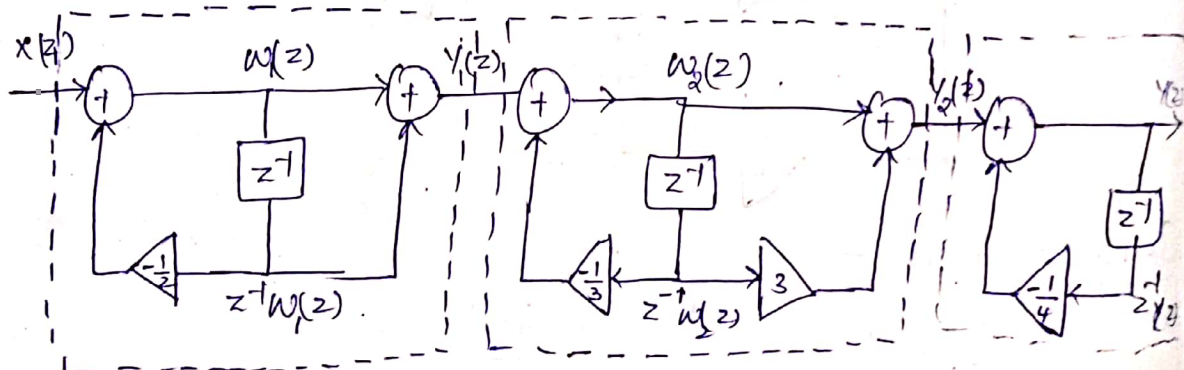
$$W_2(z) = Y_1(z) - \frac{1}{3}z^{-1}W_2(z)$$

$$Y_2(z) = W_2(z) + 3z^{-1}W_2(z)$$

$H_3(z)$:-

$$H_3(z) = \frac{Y(z)}{Y_2(z)} = \frac{1}{1+\frac{1}{4}z^{-1}}$$

$$Y(z) = Y_2(z) - \frac{1}{4}z^{-1}Y(z)$$



Parallel form:-

Consider the equation $H(z) = \frac{(1+z^{-1})(1+3z^{-1})}{(1+\frac{1}{2}z^{-1})(1+\frac{1}{3}z^{-1})(1+\frac{1}{4}z^{-1})}$

$$H(z) = \frac{A}{1+\frac{1}{2}z^{-1}} + \frac{B}{1+\frac{1}{3}z^{-1}} + \frac{C}{1+\frac{1}{4}z^{-1}}$$

$$H(z) = \frac{30}{1 + \frac{1}{2}z^{-1}} - \frac{128}{1 + \frac{1}{3}z^{-1}} + \frac{99}{1 + \frac{1}{4}z^{-1}}$$

$$Y(z) = \frac{30}{1 + \frac{1}{2}z^{-1}} X(z) - \frac{128}{1 + \frac{1}{3}z^{-1}} X(z) + \frac{99}{1 + \frac{1}{4}z^{-1}} X(z)$$

$$Y(z) = Y_1(z) + Y_2(z) + Y_3(z)$$

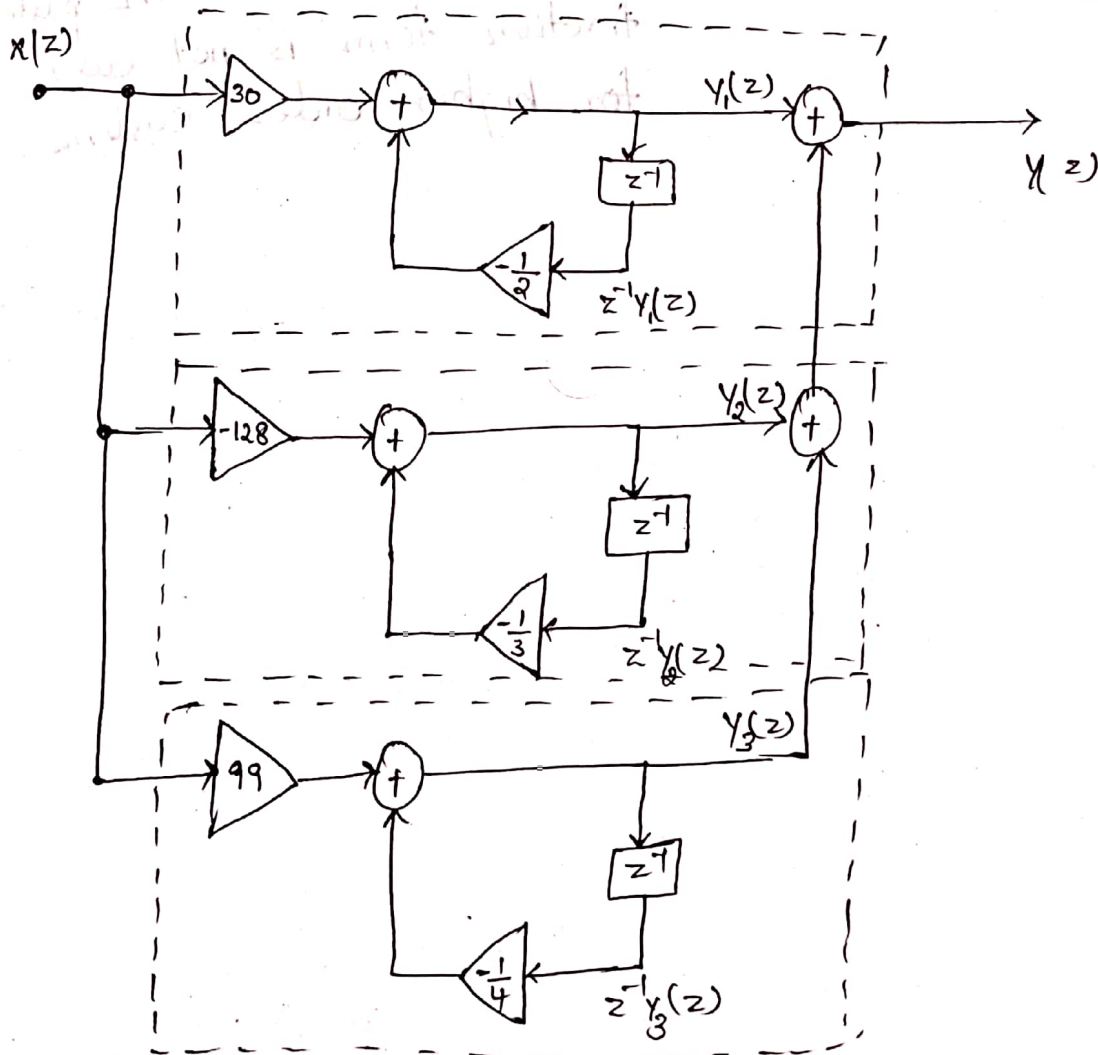
$$Y_1(z) = \frac{30}{1 + \frac{1}{2}z^{-1}} X(z), \quad Y_2(z) = \frac{-128}{1 + \frac{1}{3}z^{-1}} X(z), \quad Y_3(z) = \frac{99}{1 + \frac{1}{4}z^{-1}} X(z)$$

$$H_1(z) = \frac{Y_1(z)}{X(z)} = \frac{30}{1 + \frac{1}{2}z^{-1}}; \quad H_2(z) = \frac{Y_2(z)}{X(z)} = \frac{-128}{1 + \frac{1}{3}z^{-1}}; \quad H_3(z) = \frac{Y_3(z)}{X(z)} = \frac{99}{1 + \frac{1}{4}z^{-1}}$$

$$Y_1(z) = 30X(z) - \frac{1}{2}z^{-1}Y_1(z)$$

$$Y_2(z) = -128X(z) - \frac{1}{3}z^{-1}Y_2(z)$$

$$Y_3(z) = 99X(z) - \frac{1}{4}z^{-1}Y_3(z)$$



Comparison of Direct form-I & direct form-II structure

Direct form-I structure	Direct form-II structure
1. It uses separate delays (memory) for both i/p & o/p signal samples. 2. It is also called non-canonical, because it requires more no. of delays. 3. It is not efficient in terms of memory requirements compared to direct form-II. 4. For the $(M-1)^{th}$ & $(N-1)^{th}$ order IIR Systems, it requires $M+N-1$ multipliers $M+N-2$ adders $M+N-2$ delays 5. It can be viewed as two LTI systems in cascade. The first one is non-recursive & second one recursive	1. It uses a single delay (memory) for both the i/p & o/p signal samples. 2. It is also called canonical, because it requires a minimum no. of delays. 3. It is more efficient in terms of memory requirements 4. For the $(M-1)^{th}$ or $(N-1)^{th}$ order IIR Systems, it requires $M+N-1$ multipliers $M+N-2$ Adders $\max[(M-1), (N-1)]$ delays 5. It can also be viewed as two LTI systems in cascade. The first one is recursive & the second one non-recursive.

The difficulties in cascade structure

1. Decision of pairing poles & zeros.
2. Deciding the order of cascading the first & second order sections.
3. Scaling multipliers should be provided between individual sections to prevent the filter variables from becoming too large or too small.

Parallel form structure is used for high speed filtering application.

The difficulty is in expressing the transfer function in partial fraction form is not easy for higher order systems.