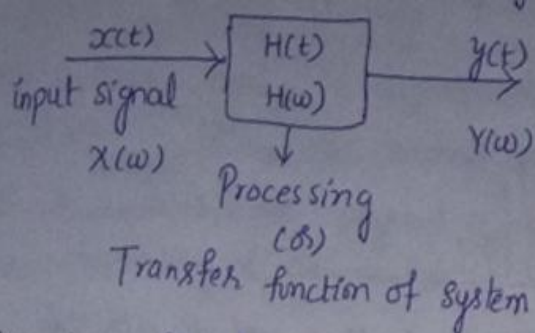


07/12/2021

DIGITAL SIGNAL PROCESSING

DSP :- Digital Signal processing



If the input signal is a digital signal passed through a any system then it is said to be a digital signal processing.

Signal :- Signal is a function that conveys information about a phenomenon.
(OR)
Combination of dependent and independent variables.

$$y = f(x_1, x_2, x_3, \dots, x_n)$$

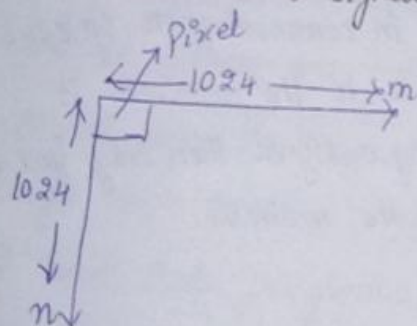
(dependent variable) (independent variables)

If one independent variable is there it is called one dimensional signal.

Ex :- $y = f(t)$ (Temperature)

Types of Signals :-

- ① $\rightarrow$ 1 dimensional signal (one independent variable) ex :- $y = f(x)$
- $\rightarrow$ 2 dimensional signal (two independent variable) ex :- Digital picture.



$y = f(m, n) \Rightarrow$ Smallest part of a digital picture is called a pixel. It depends on number of bits.

If bits = 8 $\rightarrow 2^8 - 1 = 255$

$0 \rightarrow 255$
 $\downarrow \quad \downarrow$
 Black white

Red Green Blue
42 56 78

For colour images there will be 3 values.

For black & white images there will be 1 value.

$\rightarrow$ Multidimensional signal (many independent variables)

② Natural / Synthetic signal :-

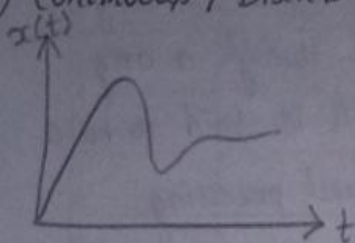
$\rightarrow$ Natural signals produce naturally we feel them.

ex :- These signals are abundant in nature and sound made by wind, water.

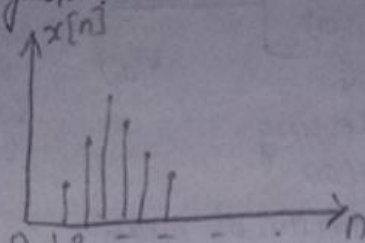
→ Synthetic signals are produced in laboratories.

ex :- Sine wave, ramp signal, unit step signal.

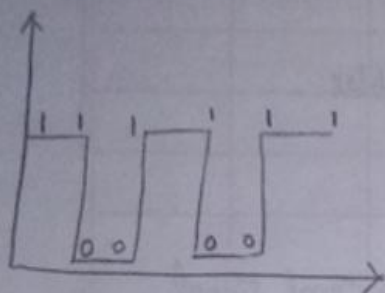
(3) Continuous / Discrete / Digital Signals :-



Continuous in both time and amplitude → Continuous signal



Continuous in time and discrete in amplitude → Discrete signal



Discrete in both time and amplitude → Digital signal.

Analog to Digital converters [ADC] are used to convert discrete signal to digital signal.

Why to study a signal?

Every signal will carry some information in them. This information has to be transmitted from a transmitter to receiver in communication system.

In order to transmit the information from a sender to the receiver it undergoes many processes. So when you study a signal then only you can perfectly transmit the data from sender to the receiver.

Signal processing :-

Signal processing is a processing of a signal. This signal could be analog, discrete and digital.

If signal will be analog — Analog Signal Processing
digital — Digital Signal Processing
discrete — Discrete Signal Processing

Why Digital Signal Processing?

These are certain advantages of having digital signal processing compare to Analog Signal Processing.

Analog Signal Processing :-

Any physical signal will be analog signal.

ex :- Audio signal, electrical signals like voltage and current signals, noise signal, Pressure, velocity, viscosity, Density.

Here, inputs and also outputs are analog signals

for example, Microphone converts physical signal to electrical signal. Microphone is transducer. i.e., physical signal is voice signal.

i.e., Audio signal converted to voltage and current signal.

* Analog signal processors are very bulky in nature and takes a lot of time to calculate.

Ex :- ISRO use analog computers.

* Analog computers are more accurate compare to digital processors bcoz, in digital they are approximate values.

History :-

* Initially, computers started with vacuum tubes and they also uses triodes for amplification in 1st generation computers.

* In 1947, they developed transistors. They obviously uses less power like $5V/12V$ with this everything change.

* In 1970s, OPAMP (Operational amplifier - ASP) and IC's are developed. These are both analog and digital.

* Digital processors are able to integrate large number of transistors into a single chip or IC's. DSP are not able to integrate analog components.

→ Small Scale Integration (SSI) - if the number of transistors ^{are} upto 10.

→ Medium Scale Integration (MSI) - transistors are 10-100.

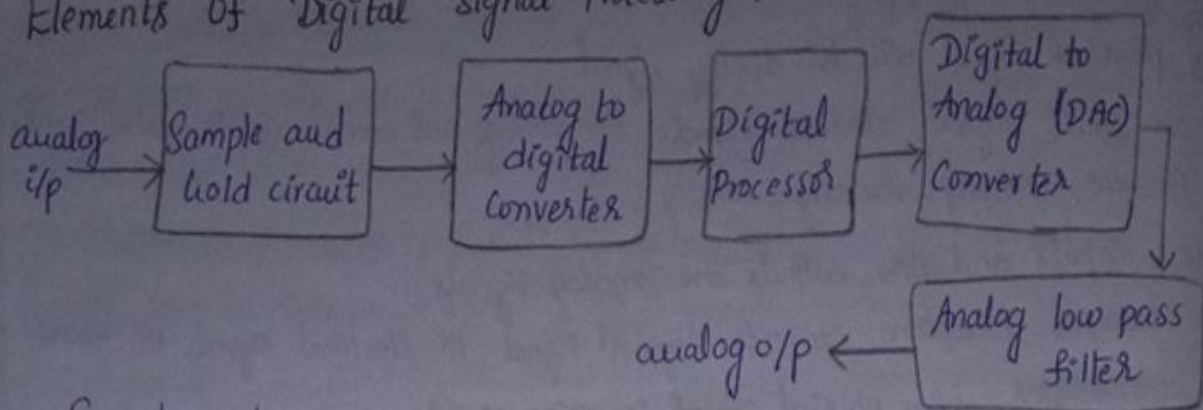
→ Large Scale Integration (LSI) - transistors are 100-1000

→ Very large Scale Integration (VLSI) - Range b/w 1000-10,000

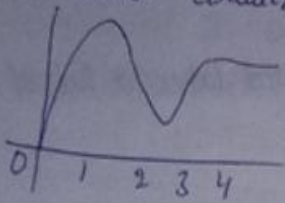
If the range increases, size decreases and amount of power will be very less.

This is why shifted from analog to digital signal processing.

Elements Of Digital Signal Processing :-



Sample and hold circuit is a specific circuit between analog electronic circuits.

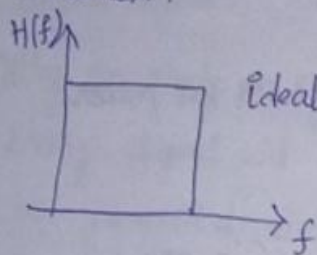


It take a sample and hold that value for a shorter period of time or till another value.

Sampling converts analog to discrete signal.

Analog to Digital converter converts analog to digital signal. But conversion takes place little bit of time. For that time it has to hold the value.

Digital processor perform some operation like $H(f)$. Now digital signal has to be converted to analog signal. For this we are using digital to analog converter.



ideal low pass filter.

It allow frequency within that range only, frequencies after that will not be allowed.

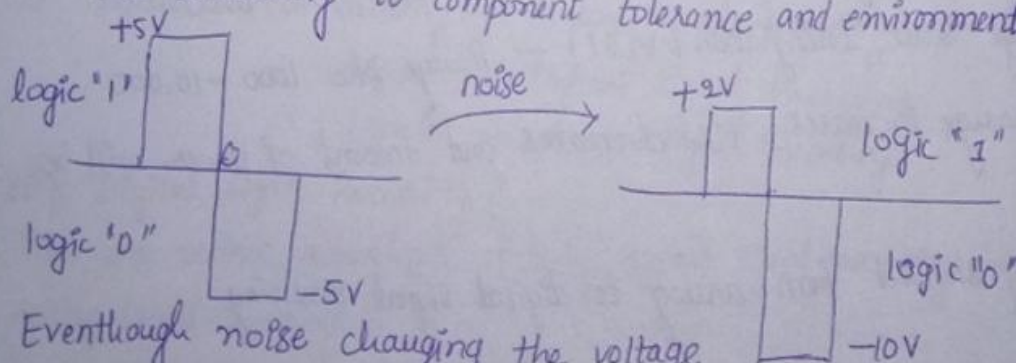
In order to remove high frequencies like noise we use analog low pass filter.

In complete DSP there are 3 operations:- filter.
 $\rightarrow$ adding, multiplication and delay $\rightarrow$ past values

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Advantages of DSP over ASP (made up of mosfets) (made up of opamps) :-

1. Less sensitivity to component tolerance and environmental changes.



Eventhough noise changing the voltage the logic will be same. i.e., less sensitive to noise.

2. Volume Production:-

The production compare to ASP DSP can produce in bulk bcoz the resources required for DSP is completely less compared to ASP.

3. Amendable for full integration.

In ASP transformer, inductor and other elements are present but in DSP transistors act as all other components. Because of the large size or bulky volume of transistor and inductor requires space to integrate. It takes high volume to integrate. With that space we can create 100% of chips in DSP. All the circuit elements in DSP can be integrated onto a single chip bcoz, they are small and we can make a chip out of it. We cannot make inductors and transformers into a single chip.

4. Accuracy.

In analog devices the voltage range from 20V to -20V. If 25V came it cannot show 25V as o/p for analog processor.

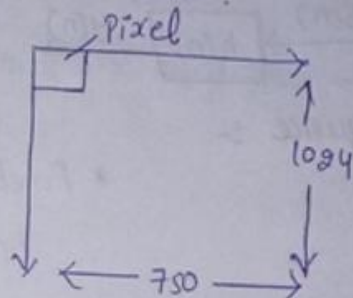
In DSP the voltage ranges from $-\infty$ to ∞ .

$$8 \text{ bits} \rightarrow 2^8 - 1$$

$$16 \rightarrow 2^{16} - 1$$

$$32 \rightarrow 2^{32} - 1$$

$$64 \rightarrow 2^{64} - 1$$



1024x750 resolution

Here bits are increasing and which in turn increases the accuracy. This bits are called word length.

If word length $\uparrow$ then Accuracy $\uparrow$ then $\rightarrow$ DSP.

5. Dynamic range

The dynamic range of DSP is more compared to ASP. dynamic range is maximum voltage to minimum voltage. We have floating point arithmetic in DSP but it is impossible in ASP.

6. Time Sharing.

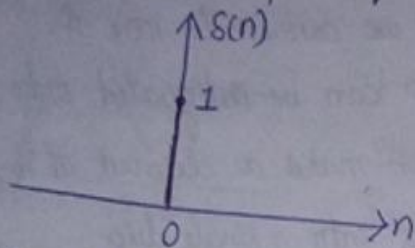
Time sharing possible in DSP. Time sharing is something called multiplexing. The word multiplexing is there will be a single channel but if you want 10 signals at the same time simultaneously, you can send 10 signals at the same time. Then this process is called Multiplexing.

We are gathering inputs at regular intervals and sending them along single channel and finally we are demultiplexing or distributing them to the respective receivers. This is called time sharing or multiplexing. This is possible only in DSP.

Time Domain Representation Of Discrete Time Signals :-

* Standard discrete time signals (sequences) :-

1) Unit Step Sample Sequence :-



* Functional Representation,

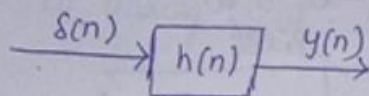
$$s(n) = \begin{cases} 1 & \text{for } n=0 \\ 0 & \text{elsewhere} \end{cases}$$

* Sequence Representation,

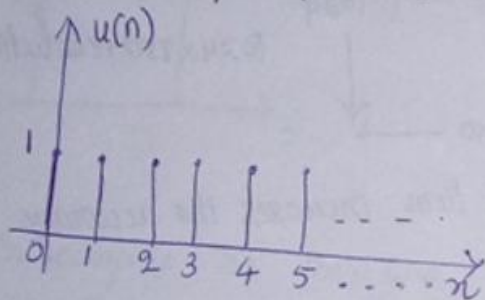
$$s(n) = \{ \underset{-\infty}{0}, \dots, 0, 0, \underset{\substack{\uparrow \\ \text{0th sample}}}{1}, 0, 0, \dots, \underset{\infty}{0} \}$$

* Graphical Representation

⇒ We use unit delta function for finding out the impulse response of a system.



2) Unit Step Sequence :-



* Functional Representation,

$$u(n) = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

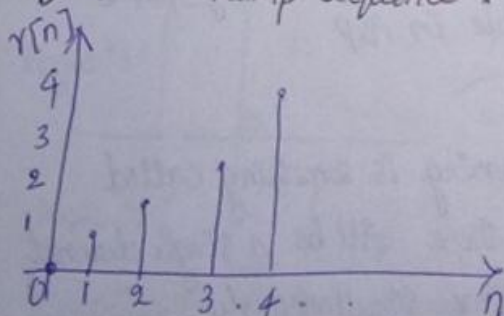
* Sequence Representation,

$$u(n) = \{ \dots, 0, 0, \underset{-1}{0}, \underset{\substack{\uparrow \\ \text{0th}}}{1}, 1, 1, 1, \dots \}$$

* Graphical representation

⇒ We use unit step response for finding out the step response of a system or dc input response.

3) Unit Ramp Sequence :-



* Functional Representation,

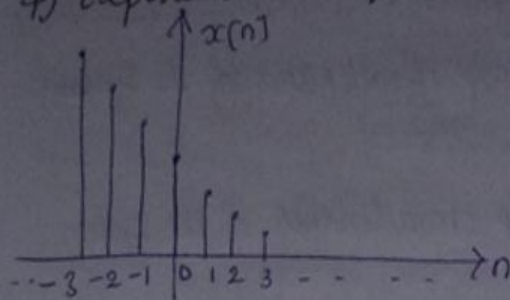
$$r(n) = \begin{cases} n & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

* Sequence Representation,

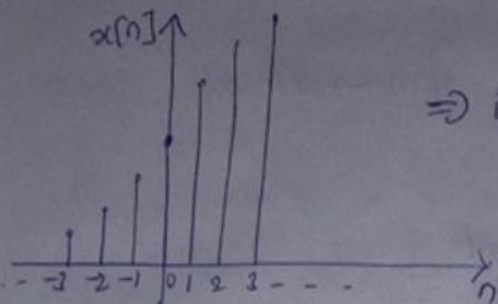
$$r(n) = \{ \underset{\substack{\uparrow \\ \text{0th}}}{0}, 1, 2, 3, \dots \}$$

* Graphical Representation

4) Exponential Sequence :-



$x[n] = a^n$
 $\Rightarrow$ if $0 < a < 1$
 n is -ve to +ve
 exponential decay



$\Rightarrow$ if $a > 1$
 exponential rise

if a is complex quantity

$$a = re^{j\theta}$$

$$= r(\cos\theta + j\sin\theta)$$

$$x[n] = a^n$$

$$= (re^{j\theta})^n$$

$$= r^n e^{jn\theta} \Rightarrow \text{magnitude} - r^n \text{ and phase} - n\theta$$

$$= r^n [\cos n\theta + j\sin n\theta]$$

$$x(n) = \underbrace{r^n \cos n\theta}_{\text{Re}\{x(n)\}} + j \underbrace{r^n \sin n\theta}_{\text{Im}\{x(n)\}}$$

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Classification Of Discrete Time Systems :-

1. Static and Dynamic Systems :-

$\rightarrow$ Static System:- When the o/p of a system depends only upon the present sample then it is said to be a static system.

$$\text{ex :- } y[n] = 10x[n]$$

$$\text{if } n=0 \quad y[0] = 10 \cdot x[0]$$

$\rightarrow$ Dynamic System:- When the o/p of a system depends upon present and past inputs then it is said to be a dynamic system.

$$\text{ex :- } y[n] = x[n] + x[n-1]$$

$$\text{if } n=0 \quad y[0] = \underset{\substack{\uparrow \\ \text{present o/p}}}{x[0]} + \underset{\substack{\uparrow \\ \text{present}}}{x[0]} + \underset{\substack{\uparrow \\ \text{past}}}{x[-1]}$$

2. Shift Invariant and Shift variant Systems :-

→ Shift Invariant System :- If i/p and o/p characteristics of the system doesn't change with shift of time.

→ Shift variant System :- If i/p and o/p characteristics of the system change with shift of time.

1) Shift the i/p by $(n-k) \rightarrow x[n-k] \quad y(n) \rightarrow x[n, k]$

2) Delay the o/p by $(n-k) \rightarrow y[n-k]$

If $y[n, k] = y[n-k] \rightarrow$ Shift invariant.

$y[n, k] \neq y[n-k] \rightarrow$ Shift variant.

ex 1 :- $y[n] = x[n] - x[n-1]$

1) $x[n-k]$

$$y[n, k] = x[n-k] - x[n-k-1]$$

2) $y[n-k] \quad n \rightarrow n-k$ (o/p)

$$y[n-k] = x[n-k] - x[n-k-1]$$

$y[n, k] = y[n-k] \rightarrow$ Shift invariant system

ex 2 :- $y[n] = n \cdot x[n]$

1) $x[n-k]$

$$y[n, k] = n \cdot x[n-k]$$

2) $y[n-k] \quad n \rightarrow n-k$

$$y[n-k] = (n-k)x[n-k]$$

$$y[n, k] \neq y[n-k]$$

$\therefore$ Shift variant system

ex 3 :- $y[n] = x[n] \cos \omega_0 n$

1) $x[n-k]$

$$y[n, k] = x[n-k] \cos \omega_0 n$$

2) $y[n-k] \quad n \rightarrow n-k$

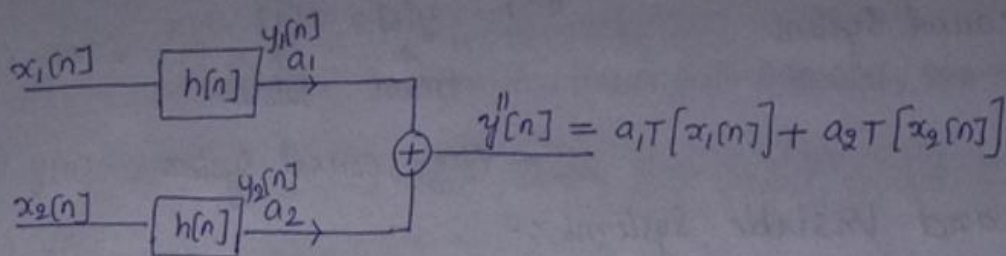
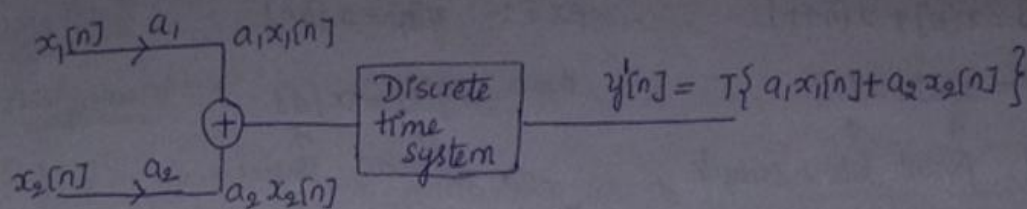
$$y[n-k] = x[n-k] \cos \omega_0 (n-k)$$

$y[n, k] \neq y[n-k] \rightarrow$ Shift variant system.

3. Linear and Non-Linear Systems :-

→ Linear system :- A system is said to be linear if it satisfies the superposition and homogeneity principles.

→ Non-linear system :- A system is said to be non-linear if it is not linear system.

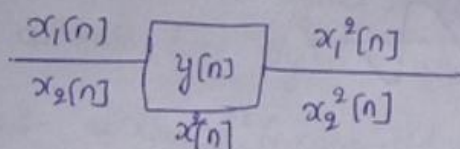


if $y'[n] = y''[n] \rightarrow$ linear system

$y'[n] \neq y''[n] \rightarrow$ Non-linear system.

ex1:- $y[n] = x^2[n]$

$$y'[n] = (a_1x_1[n] + a_2x_2[n])^2$$



$$y''[n] = a_1x_1^2[n] + a_2x_2^2[n]$$

$$y'[n] \neq y''[n]$$

$\therefore$ Non-linear system

ex2:- $y[n] = x[n^2]$

$$y'[n] = ?$$

$$a_1x_1[n] + a_2x_2[n] \rightarrow x[n^2]$$

$$y'[n] = a_1x_1[n^2] + a_2x_2[n^2]$$

$$y_1[n] = x_1[n^2]a_1$$

$$y_2[n] = x_2[n^2]a_2$$

$$y''[n] = a_1x_1[n^2] + a_2x_2[n^2]$$

$$y'[n] = y''[n]$$

$\therefore$ linear system

4. Causal and Non-causal system :-

$\rightarrow$ Causal system:- A system is said to be causal if the output depends on past and present inputs only.

$\rightarrow$ Non-causal system:- A system is said to be non-causal if the output depends on future inputs also.

ex1:- $y[n] = x[n] + x[n-1]$

$$n=0 \quad y[0] = x[0] + x[-1]$$

$\uparrow \quad \quad \uparrow \quad \quad \uparrow$
 Present Present Past

$$n=-1 \quad y[-1] = x[-1] + x[-2]$$

$\uparrow \quad \quad \uparrow \quad \quad \uparrow$
 Present Present Past sample

$$n=1 \quad y[1] = x[1] + x[0]$$

$\uparrow \quad \quad \uparrow \quad \quad \uparrow$
 Present Present Past

$\therefore$ Causal system.

$$\text{ex 2 :- } y[n] = x[n] + x[n+1]$$

$$n=0 \quad y[0] = x[0] + x[1]$$

$\uparrow \quad \uparrow \quad \uparrow$
 Present Present future sample

$\therefore$ Non-causal system

$$\text{ex 3 :- } y[n] = x[2n]$$

$$n=0 \quad y[0] = x[0]$$

$\uparrow \quad \uparrow$
 Present Present

$$n=1 \quad y[1] = x[2]$$

$\uparrow \quad \uparrow$
 Present future

$\therefore$ Non-causal system.

5. Stable and Unstable Systems :-

$\rightarrow$ Stable system :- For a bounded input we are getting a bounded output then it is said to be stable system (BIBO stable)

$\rightarrow$ Unstable system :- For a bounded input we are getting a unbounded output then it is said to be unstable system.

$$\text{ex 1 :- } y[n] = \cos[x[n]]$$

$\cos \theta$ always bounded because

$\cos$ values will be from -1 to $+1$.

So, For a bounded i/p we get bounded output.

$\therefore$ Stable system.

$$\text{ex 2 :- } y[n] = \sum_{k=-\infty}^{n+1} x[k]$$

for example, $x[k] = u[k]$

$$y[n] = \sum_{k=-\infty}^0 u[k] + \sum_{k=0}^{n+1} u[k]$$

$$y[n] = 0 + \sum_{k=0}^{n+1} u[k]$$

$$y[n] = \sum_{k=0}^{n+1} 1 = u[0] + u[1] + u[2] + \dots + u[n+1]$$

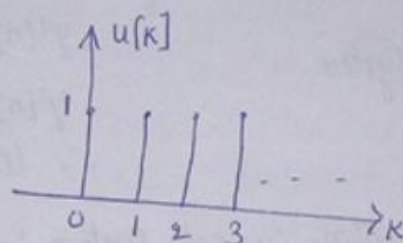
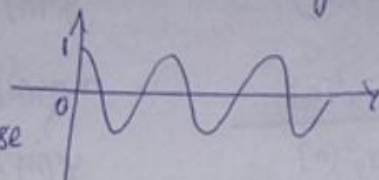
$$y[n] = 1 + 1 + 1 + \dots + 1 + 1 + n+1$$

$$y[3] = 1 + 1 + 1 + 1 + 1 = 5$$

$$y[4] = 1 + 1 + 1 + 1 + 1 + 1 = 6$$

$$\lim_{n \rightarrow \infty} y[n] = \lim_{n \rightarrow \infty} (1 + 1 + \dots + n+1)$$

$$= \infty \quad (\text{Unbounded o/p})$$



Unit step sequence (bounded input)

$$x[k] = u[k] = \begin{cases} 1 & k \geq 0 \\ 0 & \text{elsewhere} \end{cases}$$

$\therefore$ Unstable system.

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Assignment :-

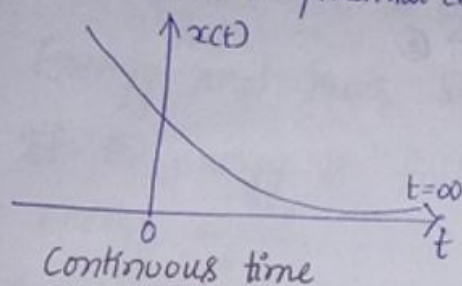
- 1) $y[n] = \cos[x[n]] \rightarrow$ Static, non-linear, shift invariant, causal, stable
- 2) $y[n] = \sum_{k=-\infty}^{n+1} x[k] \rightarrow$ Dynamic, linear, shift invariant, non-causal, unstable
- 3) $y[n] = x[n] \cos(\omega_0 n) \rightarrow$ Static, linear, shift variant, causal, stable
- 4) $y[n] = x[n] + nx[n+1] \rightarrow$ Dynamic, linear, shift variant, non-causal, Unstable
- 5) $y[n] = x[-n] \rightarrow$ Static, linear, shift invariant, non-causal, stable.

Classification Of Signals :-

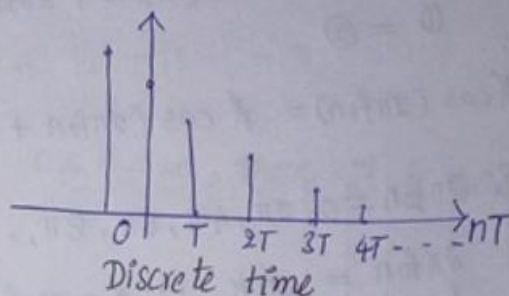
1. Continuous time and Discrete time signals :-

- $\rightarrow$ Continuous time signal :- Whenever the time is continuous
- $\rightarrow$ discrete time signal :- whenever the amplitude is continuous.

ex :- $x(t) = e^{-t}$ (exponential decay)

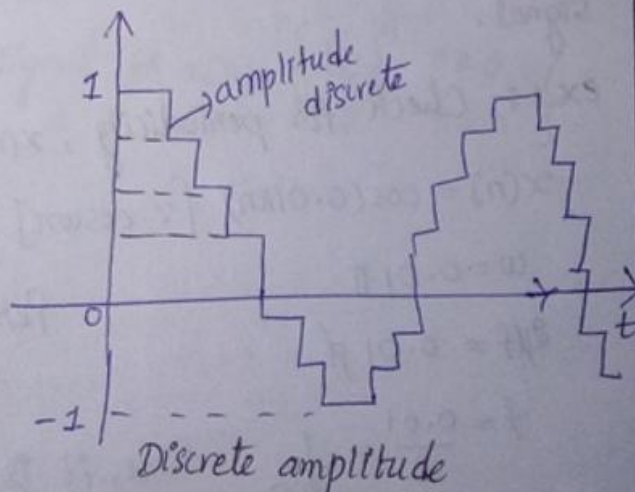
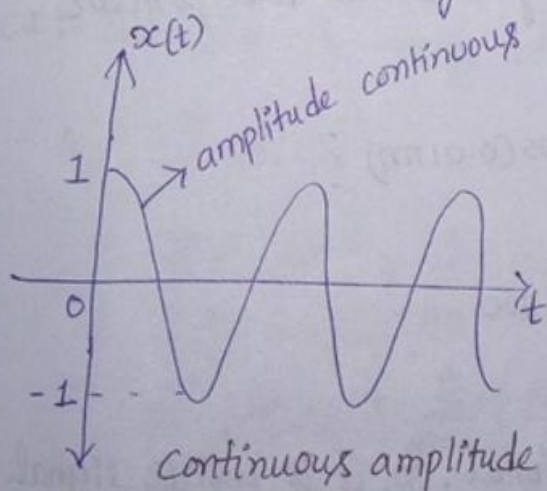


$$x[n] = e^{-nT}$$



2. Continuous amplitude and Discrete amplitude signals :-

- $\rightarrow$ When an amplitude varies continuously then it is said to be continuous amplitude signal.
- $\rightarrow$ When an amplitude takes discrete values then it is said to be discrete amplitude signal.



3. Analog and Digital Signals :-

→ When both amplitude and time is continuous then it is said to be analog signal.

→ When both amplitude and time is discrete then it is said to be digital signal.

4. Periodic and Non-periodic signal :-

→ A signal is said to be periodic if it repeats after a certain period.

$$x[n] = x[n+N] \text{ for all } N$$

→ A signal is said to be non-periodic if it is not a periodic signal.

$$x[n] \neq x[n+N]$$

ex :- $x[n] = A \cos(2\pi f_0 n) \rightarrow \textcircled{1}$

$$x[n+N] = A \cos(2\pi f_0 (n+N))$$

$$= A \cos(2\pi f_0 n + 2\pi f_0 N) \rightarrow \textcircled{2}$$

$$\textcircled{1} = \textcircled{2}$$

$$A \cos(2\pi f_0 n) = A \cos(2\pi f_0 n + 2\pi f_0 N)$$

$$\therefore 2\pi f_0 N = 0, 2\pi, 4\pi, 6\pi, 8\pi, \dots$$

$$2\pi f_0 N = 2\pi k \quad [\because k = 0, 1, 2, 3, \dots]$$

$$\boxed{f_0 = \frac{k}{n}} \text{ Condition of periodicity}$$

It is periodic only when $\frac{k}{n}$ value is a rational quantity.

↳ Discrete Time Sinusoidal signals is periodic only if frequency f_0 is rational. if f_0 is rational then only it is said to be periodic signal.

ex 1 :- Check for periodicity, $x[n] = \cos(0.01\pi n)$?

$$x[n] = \cos(0.01\pi n) \quad [\because \cos \omega n]$$

$$\omega = 0.01\pi$$

$$\text{Period} = 200$$

$$2\pi f = 0.01\pi$$

$$f = \frac{0.01}{2} = \frac{1}{200}$$

Since, it is rational. So, it is periodic signal.

$$\text{ex 2 :- } x[n] = \sin 3n$$

$$\omega = 3$$

$$2\pi f = 3$$

$$f = \frac{3}{2\pi}$$

Since, f is irrational. So, it is non-periodic signal.

$$\text{ex 3 :- } x[n] = \cos\left(\frac{n}{8}\right) \cdot \cos\left(\frac{\pi n}{8}\right)$$

$$\omega_1 = \frac{1}{8} \quad \omega_2 = \frac{\pi}{8}$$

$$2\pi f_1 = \frac{1}{8} \quad 2\pi f_2 = \frac{\pi}{8}$$

$$f_1 = \frac{1}{2\pi(8)} \quad f_2 = \frac{\pi}{2\pi(8)}$$

$$f_1 = \frac{1}{16\pi} \quad f_2 = \frac{1}{16}$$

(irrational) (rational)

$$f_1 \cdot f_2 \text{ (irrational } \times \text{ rational)}$$

So, it is non-periodic signal.

5. Energy and Power Signals :-

→ If the energy is finite i.e., $0 < E < \infty$, then it is said to be energy signal.

$$\text{For calculating energy, } E = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

→ If the power is finite i.e., $0 < P < \infty$, then it is said to be power signal.

$$\text{For calculating power, } P = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^{+N} |x[n]|^2$$

ex :- Determine the energy of a signal for $x(n) = \begin{cases} (\frac{1}{4})^n & n \geq 0 \\ 2^n & n < 0 \end{cases}$

$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

$$= \sum_{n=-\infty}^{-1} |2^n|^2 + \sum_{n=0}^{\infty} \left| \left(\frac{1}{4}\right)^n \right|^2$$

$$= \sum_{n=-1}^{-\infty} |2^n|^2 + \sum_{n=0}^{\infty} \left| \left(\frac{1}{4}\right)^n \right|^2$$

$$= \sum_{n=-1}^{-\infty} |4^n| + \sum_{n=0}^{\infty} \left| \frac{1}{16} \right|^n$$

$$= (4^{-1} + 4^{-2} + 4^{-3} + \dots) + (1 + \frac{1}{16} + (\frac{1}{16})^2 + (\frac{1}{16})^3 + \dots)$$

$$[\because a + ar + ar^2 + \dots = \frac{a}{1-r}]$$

$$= \frac{\frac{1}{4}}{1 - \frac{1}{4}} + \frac{1}{1 - \frac{1}{16}}$$

$$= \frac{\frac{1}{4}(\frac{4}{3})}{(\frac{15}{16})}$$

$$= \frac{1}{3} + \frac{16}{15} = \frac{5+16}{15} = \frac{21}{15} = \frac{7}{5}$$

Energy of the signal is $\frac{7}{5}$

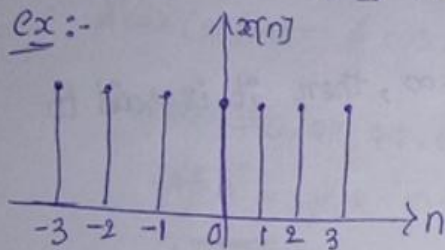
Since, energy is finite. It is a energy signal.

6. Even and Odd Signals:

→ If $x[n] = x[-n]$ then it is said to be even signal.

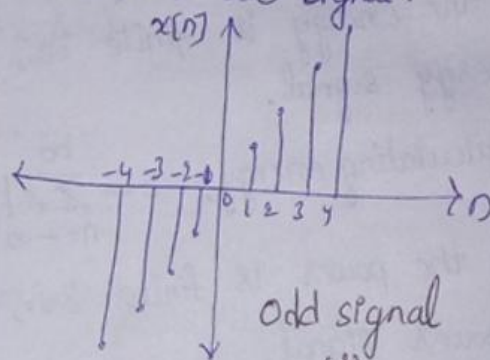
→ If $x[n] = -x[-n]$ then it is said to be odd signal.

ex:-



even signal

(S)
Symmetric signal



odd signal

(A)
Asymmetric signal

$$x[n] = x_e[n] + x_o[n] \rightarrow \textcircled{1}$$

$$x[-n] = x_e[-n] + x_o[-n]$$

$$x[-n] = x_e[n] - x_o[n] \rightarrow \textcircled{2}$$

adding $\textcircled{1}$ and $\textcircled{2}$, we get

$$x[n] + x[-n] = 2x_e[n]$$

$$x_e[n] = \frac{x[n] + x[-n]}{2} \text{ even part of a given sequence}$$

Subtract ① and ②, we get

$$x[n] - x[-n] = 2x_o[n]$$

$$x_o[n] = \frac{x[n] - x[-n]}{2}$$

Odd part of a given sequence.

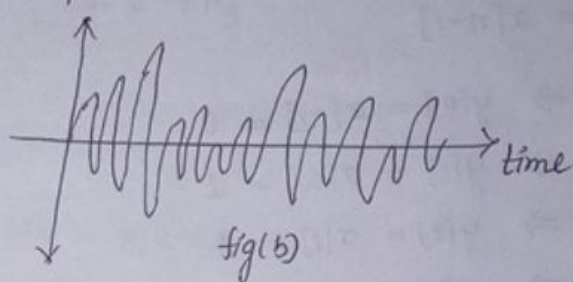
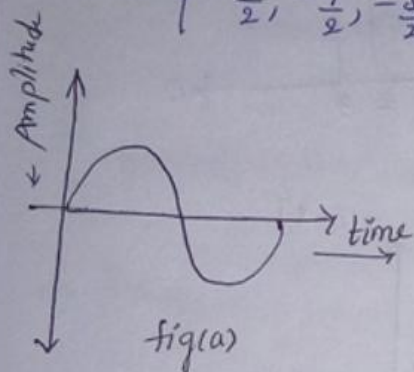
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Ex :- $x[n] = \{ 3, -4, 2, 0, 6, 3, 9, 5 \}$. Find out the odd part of a given sequence.

$$x_o[n] = \frac{x[n] - x[-n]}{2}$$

$n =$	-5	-4	-3	-2	-1	0	1	2	3	4	5
$x[n] =$	0	0	0	3	-4	2	0	6	3	9	5
$x[-n] =$	5	9	3	6	0	2	0	6	3	9	5
$x_o[n] =$	$-\frac{5}{2}$	$-\frac{9}{2}$	$-\frac{3}{2}$	$-\frac{3}{2}$	$-\frac{4^2}{2}$	0	$\frac{4^2}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{9}{2}$	$\frac{5}{2}$

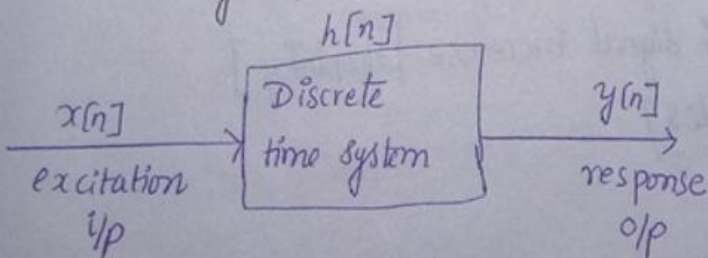
$$x_o[n] = \left\{ -\frac{5}{2}, -\frac{9}{2}, -\frac{3}{2}, -\frac{3}{2}, -2, 0, 2, \frac{3}{2}, \frac{3}{2}, \frac{9}{2}, \frac{5}{2} \right\}$$



In fig(a) we can see time, amplitude and frequency content clearly but it is not possible in fig(b).

By transforming a time domain signal to frequency domain signal we can clearly see the frequency content of a signal (or) magnitude (or) phase. And also it is easier for calculations by transforming a signal time to frequency domain. (like power, stability, energy)

Signal Processing Operations :-



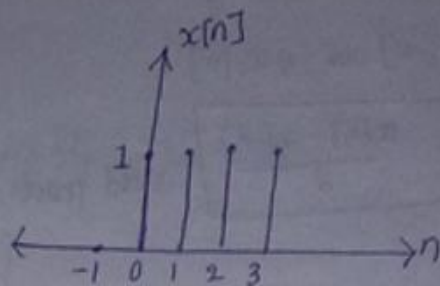
$$\Rightarrow y[n] = x[n] * h[n] \quad (\text{discrete time domain})$$

$$y(t) = x(t) * h(t) \quad (\text{continuous time domain})$$

Operations :-

1. folding [time reversal] :-

$$\text{Ex :- } x[n] = \begin{cases} 1 & n \geq 0 \\ 0 & \text{elsewhere} \end{cases}$$



$$y[n] = x[-n]$$

$$n=0 \Rightarrow y[0] = x[0] = 1$$

$$n=1 \Rightarrow y[1] = x[-1] = 0$$

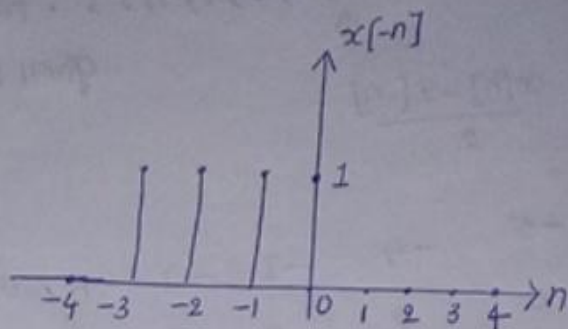
$$n=2 \Rightarrow y[2] = x[-2] = 0$$

$$n=-1 \Rightarrow y[-1] = x[1] = 1$$

$$n=-2 \Rightarrow y[-2] = x[2] = 1$$

$$n=-3 \Rightarrow y[-3] = x[3] = 1$$

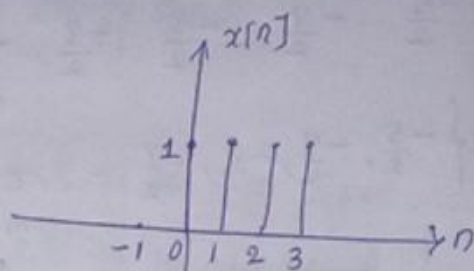
$$n=-4 \Rightarrow y[-4] = x[4] = 0$$



folding / mirror / reflecting

2. Delay :-

$$\text{Ex :- } x[n] = \begin{cases} 1 & n \geq 0 \\ 0 & \text{elsewhere} \end{cases}$$



$$y[n] = x[n-1]$$

$$n=0 \Rightarrow y[0] = x[-1] = 0$$

$$n=1 \Rightarrow y[1] = x[0] = 1$$

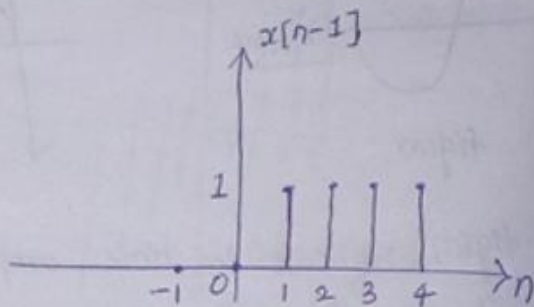
$$n=2 \Rightarrow y[2] = x[1] = 1$$

$$n=3 \Rightarrow y[3] = x[2] = 1$$

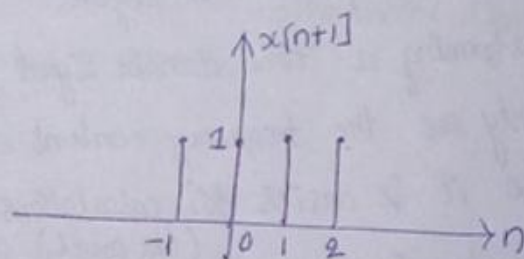
$$n=4 \Rightarrow y[4] = x[3] = 1$$

$$n=5 \Rightarrow y[5] = x[4] = 0$$

$$n=-1 \Rightarrow y[-1] = x[-2] = 0$$



Advance :- $x[n+1]$

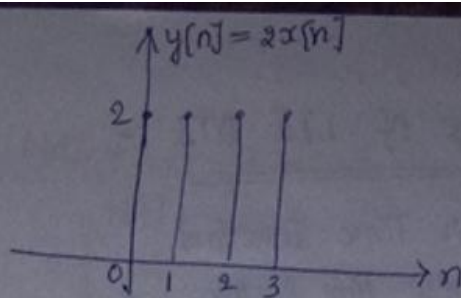
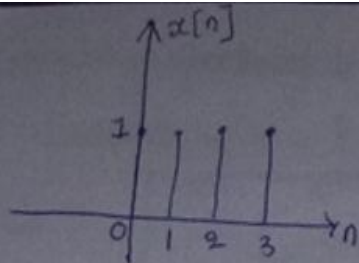


3. Scaling :-

$$\text{Ex :- } y[n] = ax[n]$$

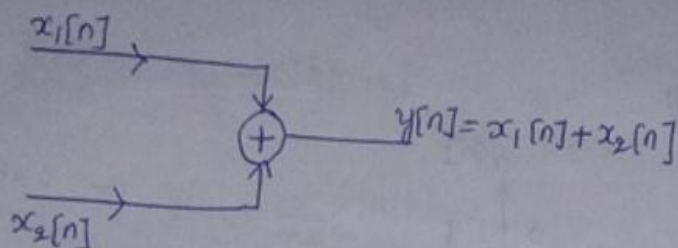
if $a > 1 \Rightarrow$ amplitude of signal increase [Amplify]

$a < 1 \Rightarrow$ [Attenuates]

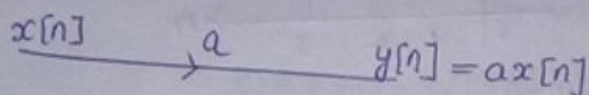


Representation :-

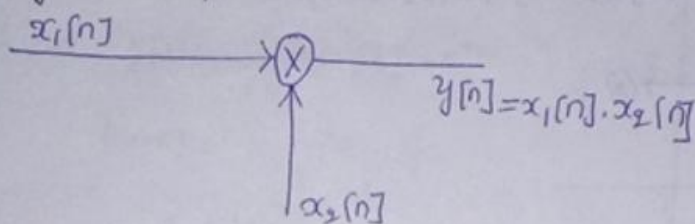
1. Adder :-



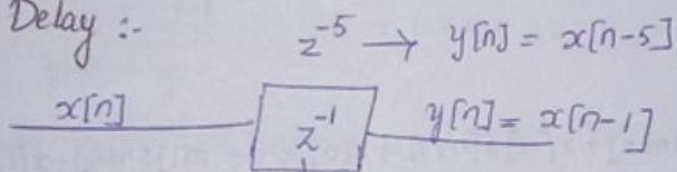
2. Multiplier / Scaling :-



3. Signal Multipliers :-

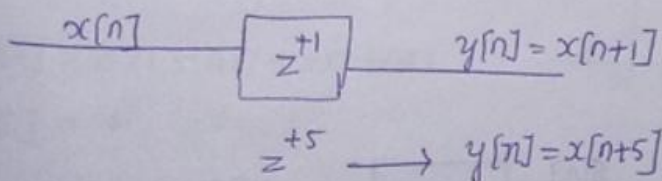


4. Delay :-



it symbolizes z -transform. but in time domain it represents $x[n-1]$

5. Advance :-



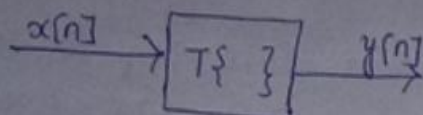
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Characteristics of LTI DTS :-

LTI - Linear Time Invariant

DTS - Discrete Time System

Convolution :-



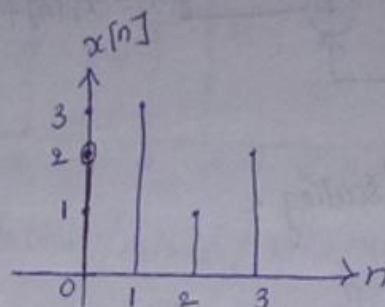
$$y[n] = T\{x[n]\} \rightarrow (1)$$

$$2 \times \delta(0)$$

$$3 \times \delta(n-1)$$

$$1 \times \delta(n-2)$$

$$2 \times \delta(n-3)$$



$$2 \times \delta(0) + 3 \times \delta(n-1) + 1 \times \delta(n-2) + 2 \times \delta(n-3)$$

$$x[0] \cdot \delta[n] + x[1] \delta[n-1] + x[2] \delta[n-2] + x[3] \delta[n-3]$$

Step 1:-

$$x[n] = \sum_{\forall k} x[k] \delta[n-k] \rightarrow (2)$$

Sub eq(2) in eq(1)

$$y[n] = T\left\{ \sum_{\forall k} x[k] \delta[n-k] \right\}$$

Step 2:-

$$y[n] = T\left\{ \dots + x[-2] \delta[n+2] + x[-1] \delta[n+1] + x[0] \delta[n] + x[1] \delta[n-1] + x[2] \delta[n-2] + \dots \right\}$$

$$= \dots + x[-1] T\{\delta[n+1]\} + x[0] T\{\delta[n]\} + x[1] T\{\delta[n-1]\} + \dots$$

$$[\because T\{a x[n]\} = a T\{x[n]\}]$$

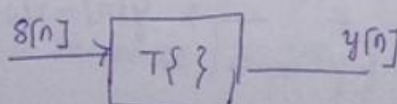
$$y[n] = \sum_{k=-\infty}^{\infty} x[k] T\{\delta[n-k]\}$$

Step 3:-

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k]$$



response of the LTI system (or) linear convolution of LTI DTS

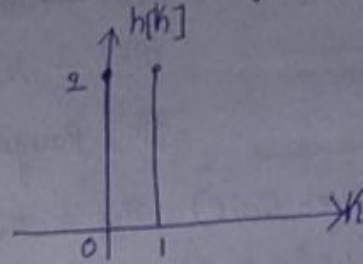
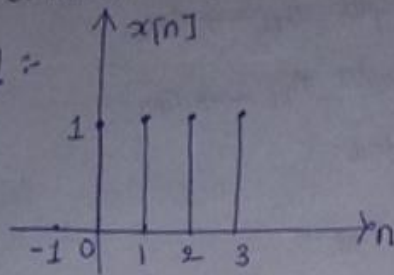


$T\{\delta[n]\} = h[n]$ (impulse response of a system)

Graphical method :-

1. Convolve $x[n] = \{1, 1, 1, 1\}$ $h[n] = \{2, 2\}$ graphically

Sol :-



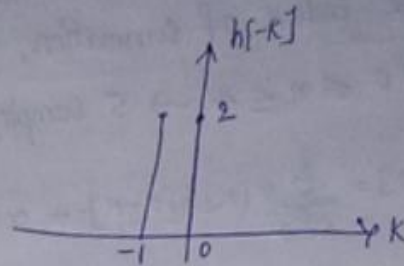
$$y[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k]$$

$$n=0 \Rightarrow y[0] = \sum_{\forall k} x[k] h[0-k]$$

$$= \sum_{\forall k} x[k] \underset{\substack{\downarrow \\ \text{folding (time reversal)}}}{h[-k]}$$

$$= 2x[0] + 2x[1] + 0x[2] + 0x[3] = 2$$

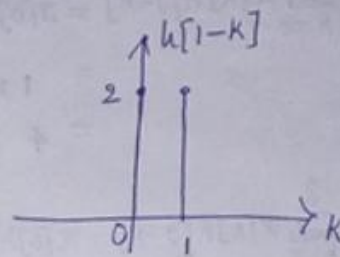
$$y[0] = 2$$



$$n=1 \Rightarrow y[1] = \sum_{\forall k} x[k] h[1-k]$$

$$h[1-k] = h[-(k-1)]$$

1. folding 2. delay



$$y[1] = 1x[2] + 1x[1] + 0x[0] + 0x[3] = 4$$

$$y[1] = 4$$

$$n=2 \Rightarrow y[2] = \sum_{\forall k} x[k] h[2-k]$$

$$y[2] = 0x[1] + 0x[0] + 1x[2] + 1x[3] = 4$$

$$y[2] = 4$$

$$n=3 \Rightarrow y[3] = \sum_{\forall k} x[k] h[3-k]$$

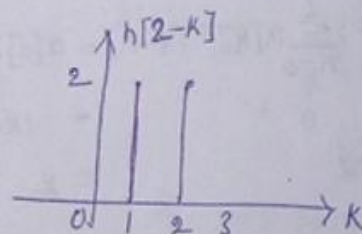
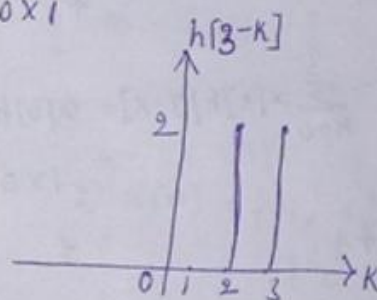
$$y[3] = 0x[1] + 2x[0] + 2x[1] + 0x[2] = 4$$

$$y[3] = 4$$

$$n=4 \Rightarrow y[4] = \sum_{\forall k} x[k] h[4-k] = 2$$

$$y[4] = 2$$

$$y[n] = \{2, 4, 4, 4, 2\}$$



$$x[n] \rightarrow M \text{ samples} = 4$$

$$h[n] \rightarrow N \text{ samples} = 2$$

$$y[n] \rightarrow M+N-1 = 4+2-1=5$$

2. $x[n] = \begin{Bmatrix} 1, 1, 1, 1 \\ 0, 1, 2, 3 \end{Bmatrix}$ $h[n] = \begin{Bmatrix} 2, 2 \\ 0, 1 \end{Bmatrix}$ using convolution equation.

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$x[k] \rightarrow M \text{ samples } n_{xL} \rightarrow n_{xH}$$

$$h[k] \rightarrow N \text{ samples } n_{hL} \rightarrow n_{hH}$$

$y[n]$ Ranges from

$$(\text{lower limit}) \quad n_{yL} = n_{xL} + n_{hL}$$

$$(\text{higher limit}) \quad n_{yH} = n_{xH} + n_{hH}$$

Determine the index of summation,

$$y[n] \rightarrow 0 \leq n \leq 4 \rightarrow 5 \text{ samples}$$

$$\begin{aligned} n=0 \Rightarrow y[0] &= \sum_{k=0}^3 x[k]h[0-k] = x[0]h[0] + x[1]h[-1] + x[2]h[-2] + x[3]h[-3] \\ &= 1 \times 2 + 1 \times 0 + 1 \times 0 + 1 \times 0 \\ &= 2 \end{aligned}$$

$$y[0] = 2$$

$$\begin{aligned} n=1 \Rightarrow y[1] &= \sum_{k=0}^3 x[k]h[1-k] = x[0]h[1] + x[1]h[0] + x[2]h[-1] + x[3]h[-2] \\ &= 1 \times 2 + 1 \times 2 + 1 \times 0 + 1 \times 0 \\ &= 4 \end{aligned}$$

$$y[1] = 4$$

$$\begin{aligned} n=2 \Rightarrow y[2] &= \sum_{k=0}^3 x[k]h[2-k] = x[0]h[2] + x[1]h[1] + x[2]h[0] + x[3]h[-1] \\ &= 1 \times 0 + 1 \times 2 + 1 \times 2 + 1 \times 0 \\ &= 4 \end{aligned}$$

$$y[2] = 4$$

$$\begin{aligned} n=3 \Rightarrow y[3] &= \sum_{k=0}^3 x[k]h[3-k] = x[0]h[3] + x[1]h[2] + x[2]h[1] + x[3]h[0] \\ &= 1 \times 0 + 1 \times 0 + 1 \times 2 + 1 \times 2 \\ &= 4 \end{aligned}$$

$$y[3] = 4$$

$$\begin{aligned} n=4 \Rightarrow y[4] &= \sum_{k=0}^3 x[k]h[4-k] = x[0]h[4] + x[1]h[3] + x[2]h[2] + x[3]h[1] \\ &= 1 \times 0 + 1 \times 0 + 1 \times 0 + 1 \times 2 \\ &= 2 \end{aligned}$$

$$y[4] = 2$$

$$y[n] = \{ 2, 4, 4, 4, 2 \}$$

Shortcut method :-

$$x[n] = \{1, 1, 1, 1\} \quad h[n] = \{2, 2\}$$

	$x[n]$			
	1	1	1	1
$h[n]$	2	2	2	2
	2	2	2	2

$$y[n] = \{2, 4, 4, 2\}$$

$$x[n] = \{4, 3, 2, 1\} \quad h[n] = \{3, 2, 1\}$$

	4	3	2	1
3	12	9	6	3
2	8	6	4	2
1	4	3	2	1

$$y[n] = \{12, 17, 16, 10, 4, 1\}$$

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Impulse and Step Response:-

Linear convolution:-

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$1) \quad x[n] = \{1, 2, 3, 4\}$$

$$h[n] = \{1, 2, 3\}$$

$$L \text{ in } x[n] = 4$$

$$M \text{ in } h[n] = 3$$

$$N = L + M - 1 = 4 + 3 - 1 = 6$$

$$N \text{ in } y[n] = 6$$

Ranges:-

$$n \rightarrow x_1 + h_1 = 0 + 0 = 0$$

$$x_n + h_n = 3 + 2 = 5$$

$$n = 0 \text{ to } 5$$

$$k \rightarrow \min\{x[n], h[n]\} \rightarrow 0$$

$$\max\{x[n], h[n]\} \rightarrow \{3, 2\} = 3$$

$$k = 0 \text{ to } 3$$

$$2) \quad x[n] = \{1, -1, 3\}$$

$$h[n] = \{1, 2\}$$

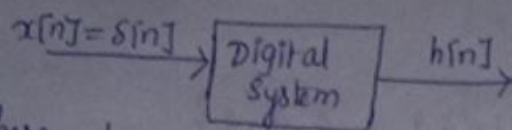
$$k: -1 \text{ to } 1$$

$$\{-1, -1\} \quad \{1, 0\}$$

$$k: -1 \leq k \leq 1$$

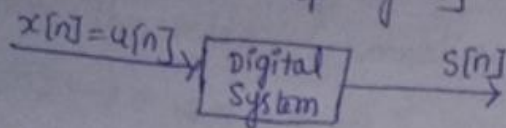
Impulse and Step response :-

Consider a signal, $x[n] = \delta[n]$ [unit impulse signal]



where, $h[n]$ is unit impulse response

$x[n] = u[n]$ [unit step signal]



where, $s[n]$ is unit step response

ex:- 1) $y[n] = \sum_{l=-\infty}^n x[l]$ find out the impulse response of a system.

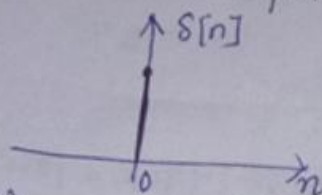
$$h[n] = \sum_{l=-\infty}^n \delta[l]$$

$$h[n] = \sum_{l=-\infty}^{-1} \delta[l] + \sum_{l=0}^n \delta[l]$$

$$h[n] = 0 + \sum_{l=0}^n \delta[l]$$

$$h[n] = \sum_{l=0}^n \delta[l]$$

$$h[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$



$$n \geq 0$$

$$n=0 \Rightarrow h[0] = s[0] = 1$$

$$n=1 \Rightarrow h[1] = \sum_{l=0}^1 \delta[l] = \delta[0] + \delta[1] = 0 + 1 = 1$$

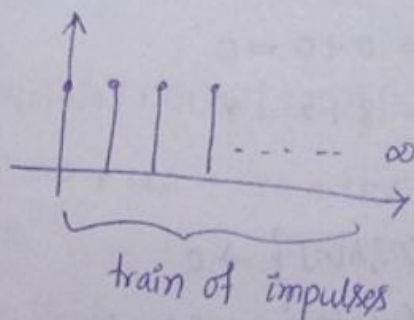
2) $y[n] = \sum_{k=0}^{\infty} x[n-k]$. Calculate the impulse response of a system.

Consider, $x[n-k] \rightarrow \delta[n-k]$

$$h[n] = \sum_{k=0}^{\infty} \delta[n-k]$$

$$= u[n]$$

$$h[n] = u[n]$$



$$* y[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k]$$

$h[n]$ is causal system then $h[n] = 0, n < 0$ (LTI and also causal system)

$$y[n] = \sum_{k=-\infty}^n x[k] h[n-k] \rightarrow \text{when } h[n] \text{ is causal}$$

$$y[n] = \sum_{k=0}^n x[k] h[n-k] \rightarrow \text{when } x[k] \text{ and } h[n] \text{ is causal}$$

Properties Of Convolution :-

1. Commutative property :-

$$x[n] * h[n] = h[n] * x[n]$$

$$\downarrow \qquad \qquad \qquad \downarrow$$

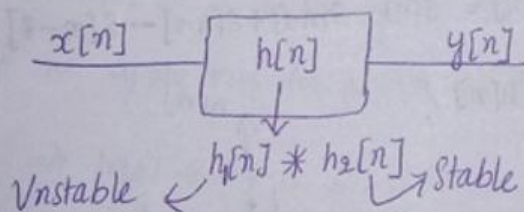
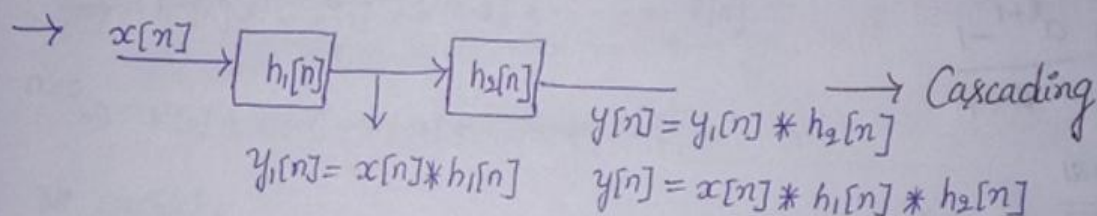
$$y[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k] \quad y[n] = \sum_{k=-\infty}^{\infty} h[k] x[n-k]$$

2. Associative property :-

$$x_1 * x_2 * x_3 = (x_1 * x_2) * x_3 = x_1 * (x_2 * x_3)$$

3. Distributive property :-

$$x_1 * (x_2 + x_3) = (x_1 * x_2) + (x_1 * x_3)$$

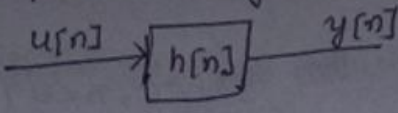


When unstable system is convolved with stable system obviously output is unstable system.

If pole of Unstable system is cancelled with zero of stable system there will be a chance of output with stable system.

$$\frac{s+1}{s-1} \times \frac{s-1}{s^2+2s+1}$$

Ex:- Solve for the response of a system if the excited is unit step.

$$h[n] = a^n u[n] \quad |a| < 1$$


$$y[n] = u[n] * h[n]$$

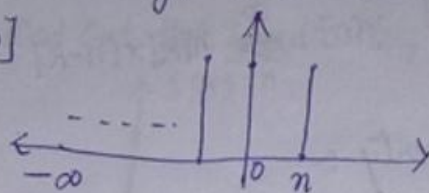
$$= \sum_{k=-\infty}^{+\infty} u[k] h[n-k]$$

$$= \sum_{k=-\infty}^{\infty} u[k] a^{n-k} \cdot u[n-k] \quad \bigg/ \quad \sum_{k=-\infty}^{\infty} u[n-k] a^k \cdot u[k]$$

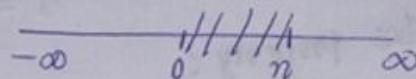
$u[k] \rightarrow k$ ranges from 0 to ∞

$u[n-k] \rightarrow k$ ranges from $-\infty$ to n .

$$u[-(k-n)]$$



$$= \sum_{k=0}^n a^k$$



$$\sum_{k=0}^N a^k = \frac{a^{N+1} - 1}{a - 1}$$

formula for geometric progression

$$y[n] = \frac{a^{n+1} - 1}{a - 1}$$

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1. Given $x[n] = \begin{cases} 1 & n = -2, 0, 1 \\ 2 & n = -1 \\ 0 & \text{elsewhere} \end{cases}$ $h[n] = \delta[n] - \delta[n-1] + \delta[n-2] - \delta[n-3]$

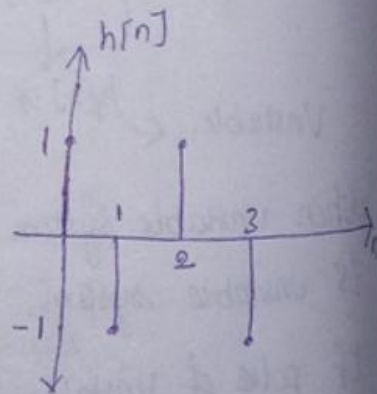
Find out linear convolution of $x[n]$ and $h[n]$?

Sol:-

$$y[n] = x[n] * h[n]$$

$$x[n] = \{1, 2, 1, 1\}$$

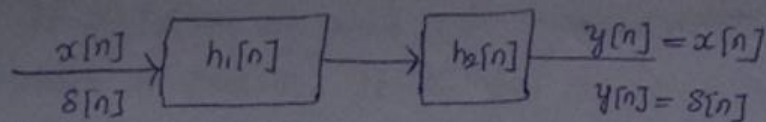
$$h[n] = \{1, -1, 1, -1\}$$



$x[n]$	1	2	1	1
$h[n]$	1	2	1	1
-1	-1	-2	-1	-1
1	1	2	1	1
-1	-1	-2	-1	-1

$$y[n] = \{1, 1, 0, 1, -2, 0, -1\}$$

Inverse system :-



When a system is said to be an inverse system. $h_1[n]$ is a system, $h_2[n]$ is an inverse system when both are said to be complementary when given i/p $x[n]$ and still after passing to two systems we get an $x[n]$. then $h_2[n]$ is an inverse system.

$$s[n] = h_1[n] * h_2[n]$$

ex:- $y[n] = \sum_{k=0}^{\infty} x[n-k]$

$$x[n-k] \rightarrow s[n-k]$$

$$y[n] \rightarrow h[n]$$

$$h[n] = \sum_{k=0}^{\infty} s[n-k]$$

if inverse system exists, $h'[n]$

According to formula, $h'[n] * \sum_{k=0}^{\infty} s[n-k] = s[n]$

$$\sum_{k=0}^{\infty} h'[n-k] = s[n]$$

$$\Rightarrow h'[n] + h'[n-1] + h'[n-2] + \dots = s[n]$$

$n=0$
 $\Rightarrow h'[0] + h'[-1] + h'[-2] + \dots = s[0]$

We consider,

$h[n]$ is a causal system. So, $h[n] = 0, n < 0$

if we apply this for above equation.

$$\Rightarrow h'[0] + 0 + 0 + \dots = s[0]$$

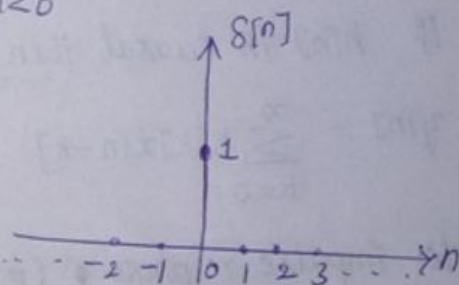
$$\Rightarrow h'[0] = 1$$

$n=1$
 $\Rightarrow h'[1] + h'[0] + h'[-1] + \dots = s[1]$

$$\Rightarrow h'[1] + h'[0] + 0 + 0 + \dots = s[1]$$

$$\Rightarrow h'[1] + 1 = 0$$

$$h'[1] = -1$$

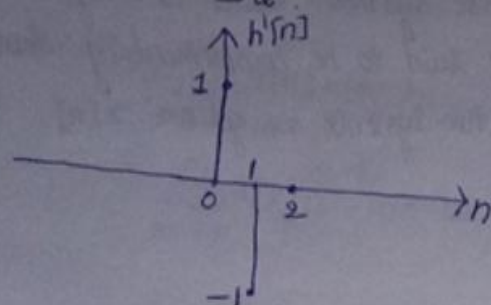


$$n=2 \Rightarrow h'[2] + h'[1] + h'[0] + h'[-1] + \dots = \delta[2]$$

$$\Rightarrow h'[2] - 1 + 1 + 0 + \dots = 0$$

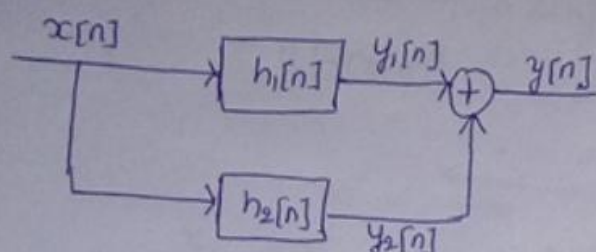
$$h'[2] = 0$$

$$h'[n] = 0, n \geq 2$$



$$h'[n] = \delta[n] - \delta[n-1]$$

→ Parallel Connection :



$$y_1[n] = x[n] * h_1[n]$$

$$y_2[n] = x[n] * h_2[n]$$

$$y[n] = y_1[n] + y_2[n]$$

$$y[n] = x[n] * h_1[n] + x[n] * h_2[n]$$

$$y[n] = x[n] [h_1[n] + h_2[n]]$$

* Parallel connection will increase speed of a system and also very important in Digital Signal processing → Advantage of parallel connection

Linear constant coefficient difference equation : LTI

$$y[n] = \sum_{k=-\infty}^{\infty} h[k] x[n-k]$$

if $h[n]$ is causal then $h[n] = 0, n < 0$. So, equation we get

$$y[n] = \sum_{k=0}^{\infty} h[k] x[n-k]$$

Finite impulse response • [FIR] system :-

If a system for which unit sample response $h[n]$ has finite number of terms then it is said to be finite impulse response.

$$\text{Ex :- } h[n] = \underbrace{\{1, 1, 3, 7, 2, 1\}}_{\text{finite terms}} \leftarrow \text{FIR.}$$

Infinite impulse response [IIR] systems :-

If a system for which unit impulse response $h[n]$ has infinite number of terms then it is said to be infinite impulse response.

$$\text{Ex :- } h[n] = a^n u[n] \quad u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & \text{elsewhere} \end{cases}$$

$$h[n] = \cos \omega n \cdot u[n]$$

Non recursive systems :-

When the output of a system depends upon the present and past inputs then it is said to be Non recursive system.

$$\text{Ex :- } y[n] = F\{x[n], x[n-1]\}$$

$$y[0] = F\{ \underset{\substack{| \\ \text{Present}}}{x[0]}, \underset{\substack{| \\ \text{Past}}}{x[-1]} \}$$

Recursive systems :-

When the output of a system depends upon the present and past inputs as well as past outputs then it is said to be Recursive systems.

$$\text{Ex :- } \underbrace{y[n]}_{\text{Present o/p}} - K \underbrace{y[n-1]}_{\text{Past o/p}} = x[n] \rightarrow \text{recursive system.}$$

$$y[n] = x[n] + x[n-1] + x[n-2] \rightarrow \text{Non-recursive system}$$

28/12/2021

Representation of Discrete Time Systems via Difference equations :-

Generalized equation for any discrete time system,

$$y[n] = - \sum_{k=1}^N a_k y[n-k] + \sum_{k=0}^M b_k x[n-k]$$

where, N is order of difference equation.

Solution for this divide into two parts :-

1. Natural response
(zero i/p response)
2. Forced response
(zero state response)

Natural response :-

This is o/p of a system when i/p is zero

Forced response :- o/p of a system for a i/p with zero initial conditions.

When the i/p is zero equation will be

$$y[n] = -\sum_{k=1}^N a_k y[n-k]$$

1. Calculate zero input response for the following system

$$y[n] - \frac{1}{4}y[n-1] - \frac{1}{8}y[n-2] = x[n] + x[n-1] \text{ with } y[-1]=0 \text{ \& } y[-2]=1$$

$$y[n] - \frac{1}{4}y[n-1] - \frac{1}{8}y[n-2] = x[n] + x[n-1] \rightarrow \textcircled{1}$$

$$\Rightarrow y[n] - \frac{1}{4}y[n-1] - \frac{1}{8}y[n-2] = 0 \rightarrow \textcircled{2} \quad [\because \text{when there is no i/p}]$$

This is called characteristic equation.

Replace $y[n] = \lambda^n$

$$y[n-1] = \lambda^{n-1}$$

$$y[n-2] = \lambda^{n-2}$$

$$\lambda^n - \frac{1}{4}\lambda^{n-1} - \frac{1}{8}\lambda^{n-2} = 0$$

$$\lambda^n \left(1 - \frac{1}{4}\lambda^{-1} - \frac{1}{8}\lambda^{-2}\right) = 0$$

$$\lambda^n \left(1 - \frac{1}{4\lambda} - \frac{1}{8\lambda^2}\right) = 0$$

$$1 - \frac{1}{4\lambda} - \frac{1}{8\lambda^2} = 0$$

$$\lambda^2 - \frac{\lambda}{4} - \frac{1}{8} = 0$$

$$\lambda^2 - \frac{1}{2}\lambda + \frac{1}{4}\lambda - \frac{1}{8} = 0$$

$$\lambda \left[\lambda - \frac{1}{2}\right] + \frac{1}{4} \left[\lambda - \frac{1}{2}\right] = 0$$

$$\left(\lambda + \frac{1}{4}\right) \left(\lambda - \frac{1}{2}\right) = 0$$

$$\lambda = -\frac{1}{4} \text{ \& } \lambda = \frac{1}{2}$$

Natural response, $y^n(n) = c_1(\lambda_1)^n + c_2(\lambda_2)^n$ for different roots.

$$y^n(n) = c_1 \left(-\frac{1}{4}\right)^n + c_2 \left(\frac{1}{2}\right)^n$$

$$y^n(0) = c_1 + c_2 \rightarrow \textcircled{2}$$

$$y^n(1) = -\frac{c_1}{4} + \frac{c_2}{2} \rightarrow \textcircled{3}$$

$$y[n] - \frac{1}{4}y[n-1] - \frac{1}{8}y[n-2] = 0$$

$$y[0] - \frac{1}{4}y[-1] + \frac{1}{8}y[-2]$$

$$y[0] = \frac{1}{4}(0) + \frac{1}{8}(1)$$

$$y[0] = \frac{1}{8}$$

$$y[1] = \frac{1}{4}y[0] + \frac{1}{8}y[-1]$$

$$y[1] = \frac{1}{4}\left(\frac{1}{8}\right) + \frac{1}{8}(0)$$

$$y[1] = \frac{1}{32}$$

Substitute $y[0]$ and $y[1]$ in eq(2) and eq(3)

$$y^n[0] = c_1 + c_2$$

$$\frac{1}{8} = c_1 + c_2$$

$$y^n[1] = -\frac{c_1}{4} + \frac{c_2}{2}$$

$$\frac{1}{32} = -\frac{c_1}{4} + \frac{c_2}{2}$$

$$\Rightarrow -c_1 + 2c_2 = \frac{1}{8}$$

$$c_1 + c_2 = \frac{1}{8}$$

$$-c_1 + 2c_2 = \frac{1}{8}$$

$$3c_2 = \frac{2}{84}$$

$$3c_2 = \frac{1}{4}$$

$$c_2 = \frac{1}{12}$$

$$c_1 = \frac{1}{24}$$

$$c_1 = \frac{1}{8} - c_2 = \frac{1}{8} - \frac{1}{12} = \frac{12-8}{96} = \frac{4}{96} = \frac{1}{24}$$

Natural Response,

$$y^n(n) = \frac{1}{24}\left(-\frac{1}{4}\right)^n + \frac{1}{12}\left(\frac{1}{2}\right)^n$$

SI. No

input $x[n]$

Particular solution $y^p(n)$

1.

1

k

2.

α^n

$k\alpha^n$

3.

$\cos(-\omega n + \phi)$

$k_1 \cos -\omega n + k_2 \sin -\omega n$

4.

$\alpha^n \cos(-\omega n + \phi)$

$\alpha^n [k_1 \cos -\omega n + k_2 \sin -\omega n]$

5.

n

$k_0 + k_1 n$

6.

n^p

$k_0 + k_1 n + k_2 n^2 + \dots + k_p n^p$

7.

$n\alpha^n$

$\alpha^n (k_0 + k_1 n)$

Forced response :-

2. $y[n] - \frac{1}{4}y[n-1] - \frac{1}{8}y[n-2] = x[n] + x[n-1]$ for this function find out Particular solution / forced response? $x[n] = (\frac{1}{8})^n u[n]$ with zero initial conditions.

Sol :- 1> Similar to $y^n(n)$
2> particular solution $y^p(n)$

$$y[n] - \frac{1}{4}y[n-1] - \frac{1}{8}y[n-2] = x[n] + x[n-1]$$

$$y[n] - \frac{1}{4}y[n-1] - \frac{1}{8}y[n-2] = 0$$

1. Natural response,

$$y^n(n) = c_1(\frac{1}{2})^n + c_2(-\frac{1}{4})^n$$

2. For particular solution :-

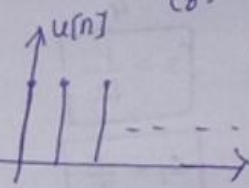
$$\text{Here i/p is } x[n] = (\frac{1}{8})^n u[n]$$

$$y^p(n) = K(\frac{1}{8})^n u[n]$$

$$3. K(\frac{1}{8})^n u[n] - \frac{1}{4}K(\frac{1}{8})^{n-1} - \frac{1}{8}K(\frac{1}{8})^{n-2} = (\frac{1}{8})^n u[n] + (\frac{1}{8})^{n-1} u[n-1]$$

$$n \geq 2,$$

$$K(\frac{1}{8})^n - \frac{1}{4}K(\frac{1}{8})^{n-1} - \frac{1}{8}K(\frac{1}{8})^{n-2} = (\frac{1}{8})^n + (\frac{1}{8})^{n-1}$$



$$n \geq 0 - u[n]$$

$$n \geq 1 - u[n-1]$$

$$n \geq 2 - u[n-2]$$

(This is common for all)

$$(\frac{1}{8})^n \left[K - \frac{1}{4}K(\frac{1}{8})^{-1} - \frac{1}{8}K(\frac{1}{8})^{-2} \right] = (\frac{1}{8})^n \left[1 + (\frac{1}{8})^{-1} \right]$$

$$K - \frac{1}{4}K(\frac{1}{8})^{-1} - \frac{1}{8}K(\frac{1}{8})^{-2} = 1 + (\frac{1}{8})^{-1}$$

$$K \left(1 - \frac{1}{4}(\frac{1}{8})^{-1} - \frac{1}{8}(\frac{1}{8})^{-2} \right) = 1 + (\frac{1}{8})^{-1}$$

$$K \left(1 - \frac{1}{4}(8)^2 - \frac{1}{8}(8^2) \right) = 1 + 8$$

$$K(1 - 2 - 8) = 9$$

$$K = \frac{9}{-9} = -1$$

$$\boxed{K = -1}$$

Forced response particular integral,

$$\boxed{y^p(n) = -1 \left(\frac{1}{8} \right)^n}$$

$$y^f[n] = c_1 \left(\frac{1}{2}\right)^n + c_2 \left(-\frac{1}{4}\right)^n - \left(\frac{1}{8}\right)^n \rightarrow \text{forced Response}$$

$$n=0$$

$$y^f[0] = c_1 + c_2 - 1 \rightarrow \textcircled{1}$$

$$n=1$$

$$y^f[1] = c_1 \left(\frac{1}{2}\right) - \frac{c_2}{4} - \frac{1}{8} \rightarrow \textcircled{2}$$

$$y[0] - \frac{1}{4}y[-1] - \frac{1}{8}y[-2] = x[0] + x[-1]$$

initial conditions are zero.

$$\text{So, } y[-1] = y[-2] = 0.$$

$$y[0] - \frac{1}{4}(0) - \frac{1}{8}(0) = 1 + 0$$

$$\underline{y[0] = 1}$$

$$y[1] - \frac{1}{4}y[0] - \frac{1}{8}y[-1] = x[1] + x[0]$$

$$y[1] - \frac{1}{4}(1) - \frac{1}{8}(0) = \frac{1}{8} + 1$$

$$y[1] = 1 + \frac{1}{4} + \frac{1}{8}$$

$$y[1] = \frac{8 + 36}{4 \times 8} = \frac{44}{32} = \frac{11}{8}$$

$$\underline{y[1] = \frac{11}{8}}$$

Substitute $y[0]$ and $y[1]$ in eq① and eq②

$$1 = c_1 + c_2 - 1$$

$$\Rightarrow c_1 + c_2 = 2$$

$$2c_1 - c_2 = 6$$

$$c_1 + c_2 = 2$$

$$\underline{2c_1 = 8}$$

$$\boxed{c_1 = \frac{8}{3}}$$

$$\boxed{c_2 = -\frac{2}{3}}$$

$$\frac{11}{8} = \frac{c_1}{2} - \frac{c_2}{4} - \frac{1}{8}$$

$$\frac{c_1}{2} - \frac{c_2}{4} = \frac{11}{8} + \frac{1}{8}$$

$$2c_1 - c_2 = \frac{12}{8} \times 4$$

$$2c_1 - c_2 = 6$$

$$\frac{8}{3} + c_2 = 2 \Rightarrow c_2 = 2 - \frac{8}{3} = -\frac{2}{3}$$

$$x[n] = \left(\frac{1}{8}\right)^n u[n]$$

$$x[0] = \left(\frac{1}{8}\right)^0 u[0] = 1$$

$$x[-1] = \left[\frac{1}{8}\right]^{-1} u[-1] = \left[\frac{1}{8}\right]^{-1}(0) = 0$$

$$x[1] = \left[\frac{1}{8}\right]^1 u[1] = \frac{1}{8}$$

$$\boxed{y^f[n] = \frac{8}{3} \left(\frac{1}{2}\right)^n - \frac{2}{3} \left(-\frac{1}{4}\right)^n - \left(\frac{1}{8}\right)^n} \quad \text{Complete solution.}$$