

Digital Electronics

→ Minimization of logical Exp

Boolean Algebra

Kmap

→ Logic gates

→ Combinational CKts

Arithmetic CKts

Halt Address

HS

FA

FS

Parallel adder

Carry look ahead adder

→ MUX

→ DEMUX

→ Decoder

→ Encoder

→ Comparators

→ Sequential CKts

FF

Registers

Counters

→ Semiconductor Memories

RAM

ROM

→ ADC/DAC

→ Logic families

→ Number Systems

codes

data representation

Reference

- * Digital logic design → M. Mano
- * Logic design → Roth [more Practice Questions]
- * Digital integrated Electronics → Taub & Shilling

13/06/11.

Boolean Algebra

1854 → G. Boole

⇒ Minimize methods:-

1) Boolean Algebra → 1, 2, 3 variables $\begin{matrix} 0 \\ 1 \end{matrix}$

2) Kmap → 2, 3, 4, 5 variables $\begin{matrix} 0 \\ 1 \\ X \end{matrix}$

3) Tabulation method. → any no. of variables

→ Boolean Algebra is used when no. of variables are less and o/p may be 0 (or) 1

→ $\overline{\overline{A}} = A$ } Not

→ $A \cdot A = A$

→ $A \cdot 0 = 0$

→ $A \cdot 1 = A$

→ $A \cdot \overline{A} = 0$

And

→ $A + A = A$

→ $A + 0 = A$

→ $A + 1 = 1$

→ $A + \overline{A} = 1$

OR

Q)

$$A + A + A + \dots + n = A$$

Q
 i.e.s 2009
 i.e.s 2003
 2004

$AB + A\bar{B}C + A\bar{B}\bar{C}$ min No. of And gates required.

- a) 0 b) 1 c) 2 d) 3

Sol
 $AB + A\bar{B}C + A\bar{B}\bar{C} = A$



⇒ advantage :-

- No. of logic gates ↓
- Speed ↑
- Power dissipation ↓
- No. of input variables may reduce $n = \bar{n}$
- ⇓
- Fan in
- $(A+B)(A+C)$

$$A \cdot A + AC + AB + BC$$

$$A + AC + AB + BC$$

$$A[1 + C + B] + BC$$

$$A + BC$$

Note :-

$1 + \text{Any thing} = 1$
 ⇓
 it is already high

$$\rightarrow (\bar{X}+Z)(\bar{X}+Y) = \bar{X}+YZ$$

$$\underline{Q1} \quad (A+B)(A+\bar{B})(\bar{A}+B)(\bar{A}+\bar{B})$$

$$(A+B\bar{B})(\bar{A}+B\bar{B})$$

$$(\underline{A})(\underline{\bar{A}}) = \underline{0}$$

$$\underline{Q2} \quad (\underline{A+B+C})(\underline{A+B+C})(\underline{A+B+C})$$

$$(\underline{A+C})(\underline{A+B+C})$$

$$= A+C(B+C)$$

$$= \underline{A+BC}$$

$$\rightarrow (A+B)(A+C) = A+BC \quad \text{Trans Position Theorem}$$

$$\rightarrow A+BC = (A+B)(A+C) \quad \text{Distribution Theorem}$$

① ② ③

①+② ①+③

$$\rightarrow A+\bar{A}B = (A+\bar{A})(A+B)$$

$$= \underline{A+B}$$

$$\rightarrow A+\bar{A}\bar{B} = \underline{A+\bar{B}}$$

$$\rightarrow \bar{A}+AB = \underline{\bar{A}+B}$$

$$\rightarrow \bar{A}+A\bar{B} = \underline{\bar{A}+\bar{B}}$$

$$Q) AB + A\bar{B} + \bar{A}\bar{B}$$

$$A(B + \bar{B}) + \bar{A}\bar{B}$$

$$A + \bar{A}\bar{B} = A + \bar{B}$$

$$Q) ABC + \bar{A}BC + AB\bar{C}$$

$$AB(C + \bar{C}) + \bar{A}BC$$

$$AB + \bar{A}BC$$

$$B(A + \bar{A})$$

$$B(A + C) = AB + BC$$

$$Q) \bar{x}yz + x\bar{y}\bar{z} + x\bar{y}z + x\bar{y}z$$

$$\bar{x}yz + x\bar{y}\bar{z} + x\bar{y}z + x\bar{y}z + \underline{x\bar{y}z + x\bar{y}z}$$

$$x\bar{y} + \bar{y}z + 2x$$

$$Q) AB + \bar{B}C + A\bar{B}$$

$$A(B + \bar{B}) + \bar{B}C$$

$$A + \bar{B}C$$

V.V. Imp
Q)

$$AB + \bar{A}C + BC$$

$$(A + C)B + \bar{A}C$$

$$\rightarrow AB(C + \bar{C}) + \bar{A}C(B + \bar{B}) + BC(A + \bar{A})$$

$$ABC + AB\bar{C} + \bar{A}BC + \bar{A}\bar{B}C + \underline{ABC} + \underline{\bar{A}BC}$$

$$AB + \bar{A}BC + \bar{A}\bar{B}C + ABC$$

$$AB + BC + \bar{A}\bar{B}C =$$

Q) $AB + \bar{A}C + BC \rightarrow$ redundant term.

$$AB + \bar{A}C + BC \underline{(A + \bar{A})}$$

$$AB + \bar{A}C + \overbrace{ABC + \bar{A}BC}$$

$$AB(1+C) + \bar{A}C(1+B)$$

$$AB + \bar{A}C$$

$\Rightarrow$ consensus Theorem :-

$$\rightarrow AB + \bar{A}C + BC = AB + \bar{A}C$$

$\rightarrow$ Three variables

$\rightarrow$ twice are used

$\rightarrow$ only one variable is complemented / un complemented

$$\rightarrow \check{A}\check{B} + \check{B}\check{C} + \check{A}C = AB + \bar{B}C$$

$$\rightarrow \check{A}\check{B} + \check{B}\check{C} + \check{A}C = A\bar{B} + BC$$

$$\rightarrow \check{A}\check{B} + \check{B}\check{C} + \check{A}C = \bar{B}\bar{C} + AC$$

$\Rightarrow$ Note :- The term is eliminated that term is known as

Redundant term

$$Q) (\check{A} + \check{B})(\check{A} + \check{C})(B + C) = (A + B)(\bar{A} + C)$$

$$Q) \check{A}\check{B} + \check{B}\check{C} + \check{A}C = A\bar{B} + \bar{A}C$$

$$Q) (\check{A} + \check{B})(\check{B} + \check{C})(\check{A} + \check{C}) = (\bar{A} + \bar{B})(B + \bar{C})$$

$$b) \bar{A}B + \underline{BC} + \underline{AB}$$

$$BC(\bar{A}+A) + BC$$

$$B+BC$$

$$(B+B)B(1+C) = \underline{B}$$

⇒ Demorgan's Theorem :-

$$\rightarrow \overline{ABC} = \bar{A} + \bar{B} + \bar{C}$$

$$\rightarrow \overline{A+B+C} = \bar{A} \cdot \bar{B} \cdot \bar{C}$$

⇒ Boolean Algebra :-

① minimize & simplify

② SOP < minimal
canonical

③ POS < minimal
canonical

④ Dual

⑤ Complements

⑥ Truth Tables

⑦ Venn diagrams

⑧ Switching ckt

⑨ Statements.

$$Q) \frac{xy}{A} + \frac{\overline{xy}wz}{B}$$

$$A + \overline{A}B = A + B$$

$$= \underline{xy + wz}$$

Q) Let $f(A, B) = \overline{A} + B$ Then the value of $f(f(x+y, y), z)$

Sol

$$f(x+y, y) = \overline{x+y} + y$$

$$= \overline{x} \cdot \overline{y} + y$$

$$= \underline{\overline{x} + y}$$

$$f(\overline{x+y}, z) = (\overline{\overline{x+y}}) + z$$

$$= \underline{xy + z}$$

Q) Let $x * y = \underline{\overline{x} + y}$ and $z = \underline{x * y}$ then value of

$$z * x$$

Sol:-

$$\downarrow$$

$$z = \overline{x} + y$$

$$z * x = \overline{z} + x$$

$$= \overline{\overline{x} + y} + x$$

$$= xy + x = x$$

⇒ SOP :-

→ SOP form is used when output of logic circuit is logic 1.

→ In SOP form Each term is known as min term.

$$\rightarrow \textcircled{A\bar{B}C} + AB\bar{C} + A\bar{B}\bar{C}$$

↑
Product
↓
min term.

Q) write min term for 5.

→ 5 101

$A\bar{B}C$ (min term)

Q) for the given truth table minimized SOP expression is

<u>min term</u>	A	B	Y
$\bar{A}\bar{B}$ ←	0	0	1 ✓
	0	1	0
$A\bar{B}$ ←	1	0	1 ✓
AB ←	1	1	1 ✓

$$Y = \bar{A}\bar{B} + A\bar{B} + AB = \underline{A+B}$$

$$\rightarrow y(A, B) = \sum m(0, 2, 3)$$

00 10 11

minimize expression for $y(A, B) = \sum m(0, 1, 3)$

↓ ↓ ↓
00 01 11

Sol:-

$$y = \bar{A}\bar{B} + \bar{A}B + AB$$

$$= \bar{A} + B$$

→ SOP < ^{minimal} canonical (or) standard form.

→ minimal form :- minimizing the expression $A + \bar{A}B$

→ canonical SOP form :- in canonical SOP form Each min term will contain all the variable.

Ex:- $A + \bar{A}B \rightarrow$ minimal form.

$A(B + \bar{B})(C + \bar{C}) + \bar{A}B(C + \bar{C}) \rightarrow$ Canonical form.

Ies 2010

Q) NO. of min terms present in Canonical SOP form

for $A + \bar{B}C$ a) 3 b) 4 c) 5 d) 6.

Sol:-

$$\begin{aligned} & A(B + \bar{B})(C + \bar{C}) + (A + \bar{A})\bar{B}C \\ &= A(BC + B\bar{C} + \bar{B}C + \bar{B}\bar{C}) + A\bar{B}C + \bar{A}\bar{B}C \\ &= ABC + \underline{AB\bar{C}} + \bar{A}\bar{B}C + A\bar{B}\bar{C} + \underline{\bar{A}\bar{B}C} + \bar{A}\bar{B}C \end{aligned}$$

⇒ Pos :-

(0, 1, 0) m.e. = (0, 1, 0) = 0

→ Pos form is used when output of logic circuit is logic 0

→ In pos form Each term is known as max term

→ $(A + \bar{B} + C) (A + B + C) (\bar{A} + \bar{B} + C)$
 $\Downarrow$
 Sum term
 $\Downarrow$
max term

Q) max term for 5

Q

$\bar{A} + \bar{B} + \bar{C}$

Q) for the given truth table minimize pos form

	<u>A</u>	<u>B</u>	<u>Y</u>
	0	0	1
→ $A + \bar{B}$	0	1	0 ✓
	1	0	1
→ $\bar{A} + \bar{B}$	1	1	0 ✓

$$Y = (A + \bar{B}) (\bar{A} + \bar{B})$$

$$Y = \bar{B}$$

$$\rightarrow Y(A, B) = \prod M(1, 3)$$

$\rightarrow$ SOP

$$Y = \overline{A}\overline{B} + A\overline{B} = \left\{ \sum m(0, 2) = \prod M(1, 3) \right\}$$

$$= \overline{B} \checkmark$$

$$\rightarrow \sum m(0, 1, 3, 7) = \prod M(2, 4, 5, 6)$$

$\rightarrow$ with 'n' variables max possible min terms are (or)

$$\text{max term} \rightarrow \underline{2^n}$$

$\rightarrow$ with 'n' variables maximum possible Logical Expressions

are

$$\underline{2^{2^n}}$$

$$\frac{2^n}{2} \text{ — number of variables} \\ \text{— (SOP/POS)}$$

$$\rightarrow n=2 \rightarrow 2^{2^2} = 2^4 = 16$$

$$\rightarrow n=3 \rightarrow 2^{2^3} = 2^8 =$$

$$\rightarrow n=4 \rightarrow 2^{2^4} = 2^{16} \Rightarrow 2^6 \times 2^{10}$$

$$= \underline{64 \text{ K}}$$

$\rightarrow$

$$(A+B)(\overline{A}+C) = AC + \overline{A}B \quad (\text{Transposition theorem II})$$

14/06/11.

⇒ Dual :-

→ Dual is used to convert +ve Positive logic into negative logic (or) vice versa.

→ +ve Logic

→ Logic 0 → 0V

→ Logic 1 → +5V

→ -ve Logic

→ Logic 0 → +5V

→ Logic 1 → 0V

→

ECL

→ Logic 0 → -1.7V

→ Logic 1 → -0.85V

} +ve logic

→

AND $\xleftrightarrow{\text{Dual}}$ OR

$AB \xleftrightarrow{\text{Dual}} A+B$

→ +ve Logic AND $\equiv$ -ve Logic OR

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

A	B	Y
1	1	1
1	0	0
0	1	0
0	0	0

+ve Logic OR

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

→ 1) Convert And operation to OR (or) OR to AND

→ 2) $1 \xleftrightarrow{\text{Dual}} 0$

→ 3) Keep variables as it is.

Ex write a dual exp $A\bar{B}C + AB\bar{C} + A\bar{B}\bar{C}$
 $\Downarrow$ Dual

Sol $(A + \bar{B} + C)(A + B + \bar{C})(A + \bar{B} + \bar{C})$

→ For any logical expression if two times dual is used resulting same expression

Q) $A + \bar{A}B = AB$

→ Dual

$$A \cdot (\bar{A} + B)$$

$$AB$$

$\Downarrow$

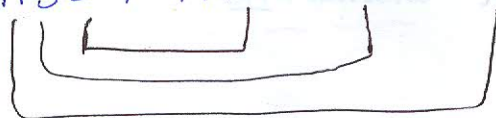
→ Dual

$$A + B$$

Q) $(A + B + C)(\bar{A} + B + C)(A + \bar{B} + C)(A + B + \bar{C})$

Dual

$$ABC + \bar{A}BC + A\bar{B}C + AB\bar{C}$$



$$AB + BC + AC$$

Dual

$$(A + B)(B + C)(A + C)$$

⇒ Self Dual :-

→ In Self dual expressions if one time Dual is used result in Same Expression

Ex $AB + BC + AC$

↓ Dual

$(A+B)(B+C)(A+C)$

$(B+AC)(A+C)$

$AB + BC + AC$

⇒

$(A+B)(B+C)(A+C) = AB + BC + AC$

→ In 'n'-variables maximum possible Self

Dual expressions are $2^{2^{n-1}}$

→ $A \cdot 1 \rightarrow A + 0$

$\bar{A} \rightarrow \bar{A}$

$n=1 \rightarrow 2^{\text{self. dual}}$

→ $A \rightarrow A$

$\bar{A} \rightarrow \bar{A}$

$B \rightarrow B$

$\bar{B} \rightarrow \bar{B}$

$n=2 \rightarrow 4^{\text{self dual}}$

→ Inverse.
⇒ Complement :-

1) And $\leftrightarrow$ OR

2) 1 $\leftrightarrow$ 0

3) Complement each literal ✓
Variable.

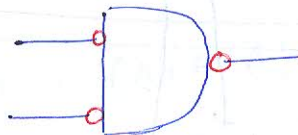
$$\rightarrow Y = \underbrace{A\bar{B}C}_{\substack{\Downarrow \\ \text{3 Literals}}} + \underbrace{A\bar{B}\bar{C}}_{\substack{\Downarrow \\ \text{3 literals}}} + \underbrace{ABC}_{\substack{\Downarrow \\ \text{3 Literals}}}$$

→ Total 9 Literals 3 variables

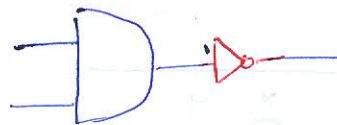
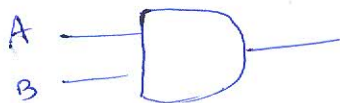
→ Ex:

$$Y = A\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC$$

$$\bar{Y} = (\bar{A}+B+\bar{C})(\underline{\bar{A}+B+C})(\underline{\bar{A}+B+C})$$



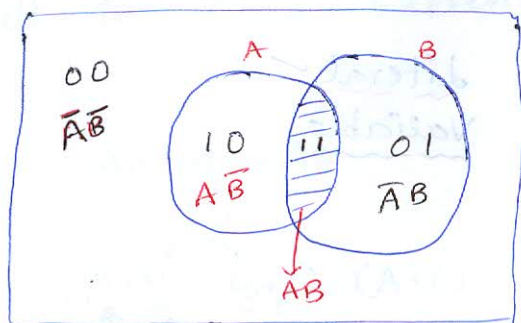
Dual



Complement

⇒ venn diagram :-

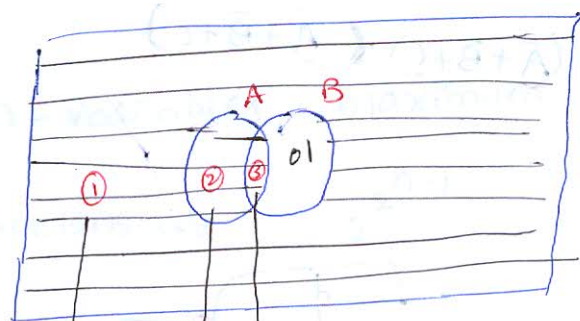
→



$n \rightarrow \cdot$

$U \rightarrow +$

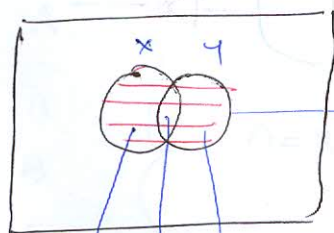
Q) In the venn diagram minimize the logical expression in this shaded area.



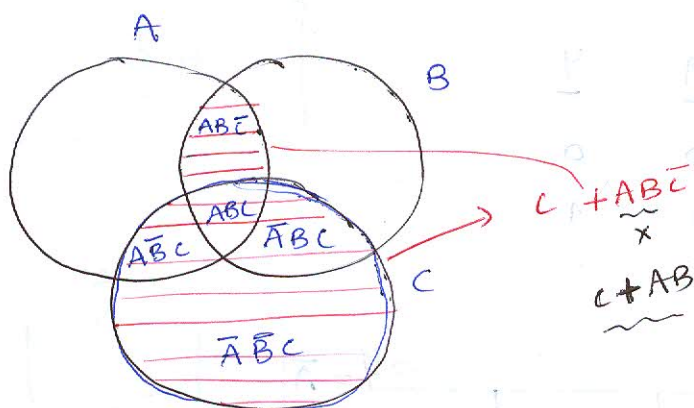
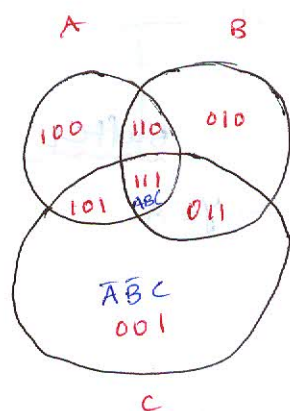
$$\bar{A}\bar{B} + \bar{A}B + AB$$

$$A + \bar{A}\bar{B} = A + \bar{B}$$

Q3



$$\bar{x}\bar{y} + x\bar{y} + x\bar{y} = x + y$$



$$\rightarrow AB\bar{C} + AB\bar{C} + A\bar{B}C + \bar{A}B\bar{C} + \bar{A}\bar{B}C$$

$$AB + BC + CA + \bar{A}\bar{B}\bar{C}$$

$$AB + C[A + B + \bar{A}\bar{B}]$$

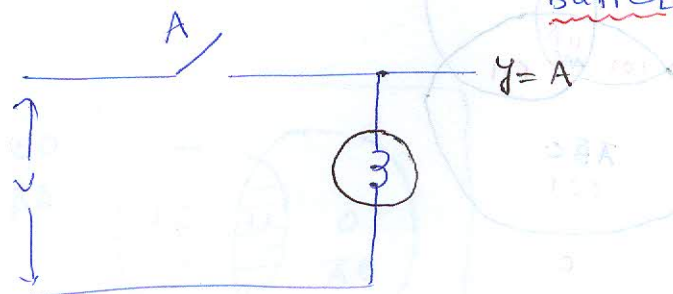
$$AB + C[\underline{A} + B + \underline{\bar{A}}]$$

$$\cancel{AB + BC} AB + C[1 + B]$$

$$\underline{AB + C}$$

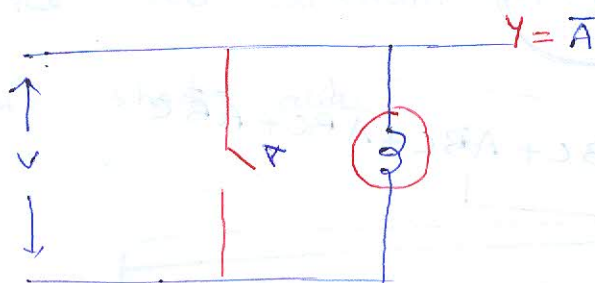
⇒ Switching ckts :-

→



<u>A</u>	<u>Y</u>
0	0
1	1

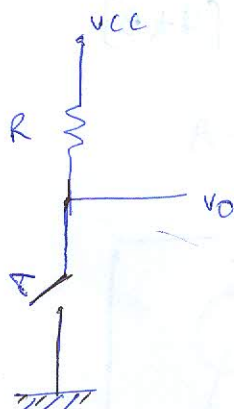
→



<u>A</u>	<u>Y</u>
0	1
1	0

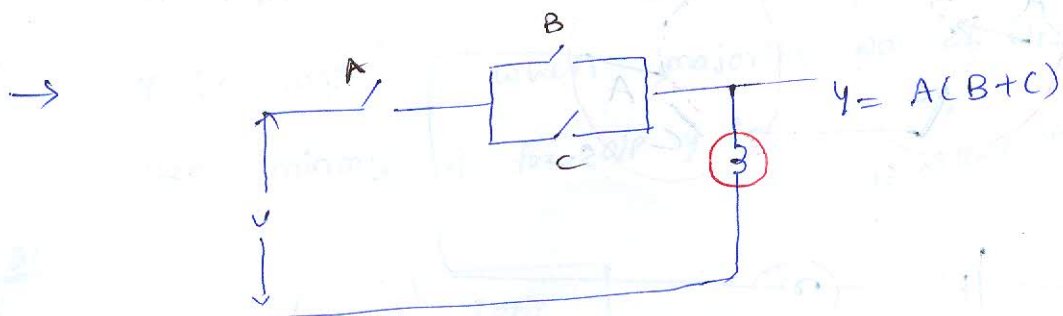
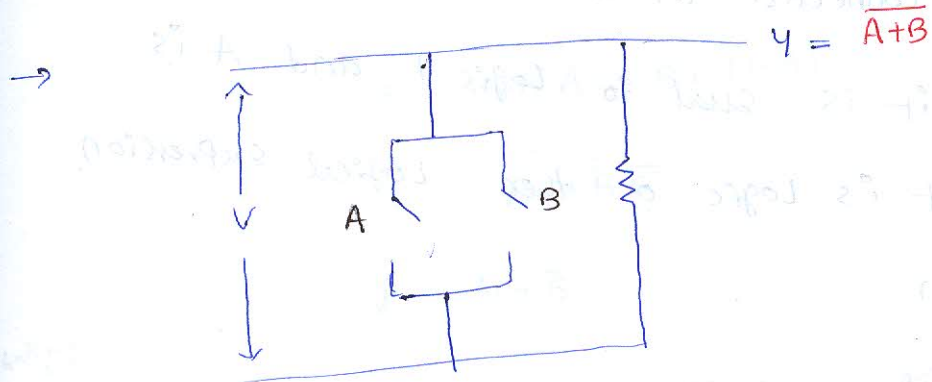
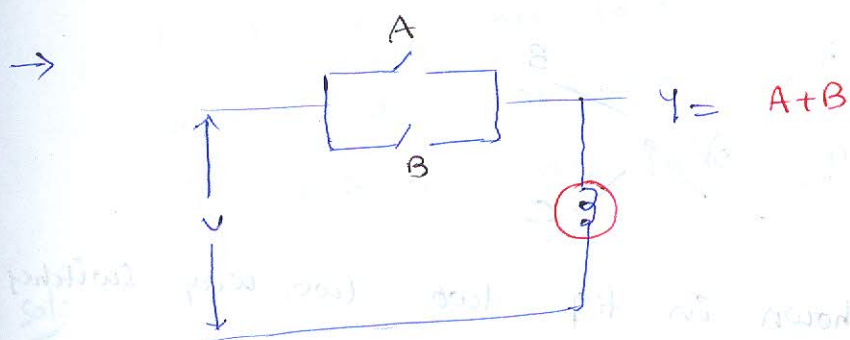
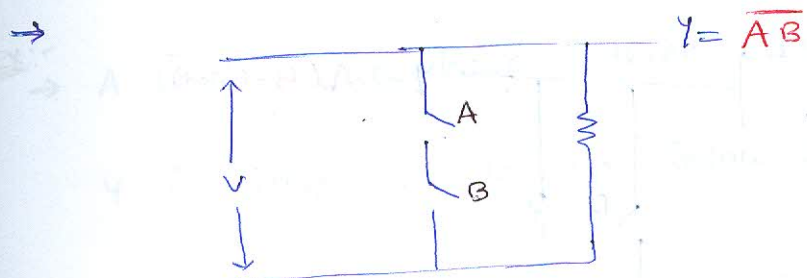
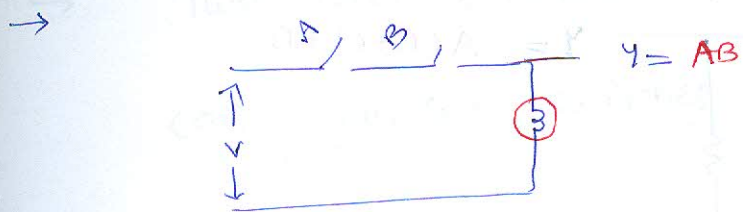
→

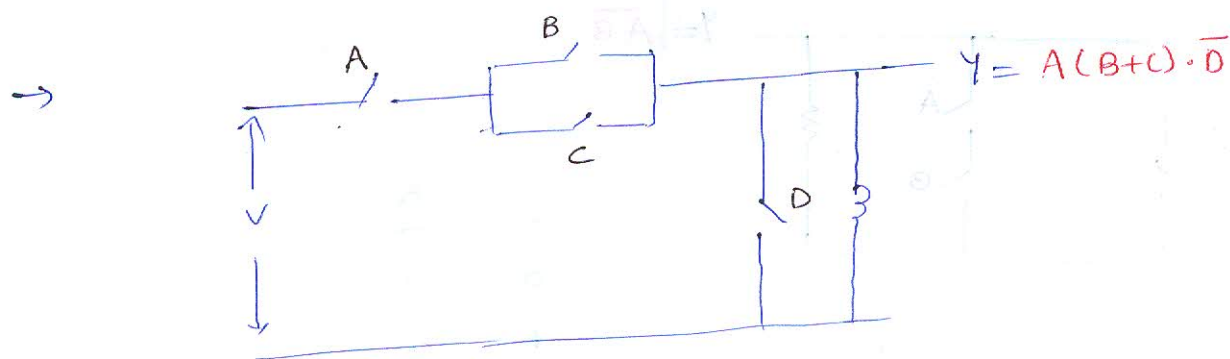
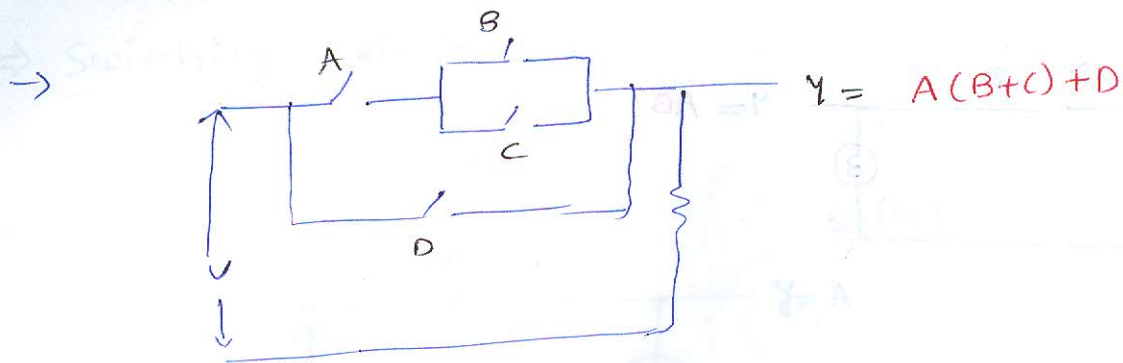
not



$Y = \bar{A}$

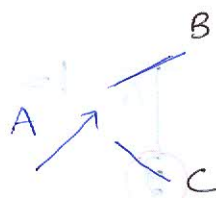
<u>A</u>	<u>Y</u>
0	1
1	0



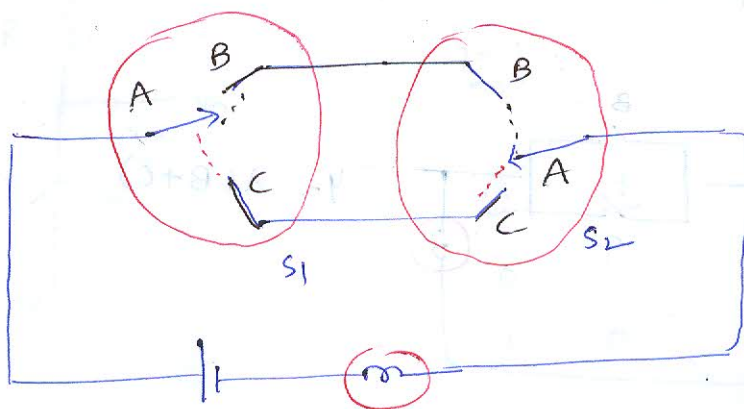


→

Two way Switch



Q) In the ckt shown in fig two two way switches S_1 and S_2 are connected in which if A is connected to B it is equal to Logic 1 and A is connected to C it is Logic 0 then Logical expression for bulb to be on.



a) $S_1 S_2$

b) $S_1 + S_2$

c) $S_1 S_2 + \bar{S}_1 \bar{S}_2$

d) $S_1 \bar{S}_2 + \bar{S}_1 S_2$

→ Two way Switches is known as XOR Switches
(or) Stair Case Switches.

⇒ Statements :-

Ex:- → A logic ckt have three i/p's A, B and C o/p is Y.
Y is logic 1 for the following combinations.

I> A and C are true AC

II> B and C are false $\bar{B}\bar{C}$

III> A, B and C are true ABC

IV> A, B and C are false minimized SOP for Y is $\bar{A}\bar{B}\bar{C}$

Sol:-

$$Y = AC + \bar{B}\bar{C} + ABC + \bar{A}\bar{B}\bar{C}$$

$$= (1+B)AC + \bar{B}\bar{C}(1+\bar{A})$$

$$= AC + \bar{B}\bar{C}$$

$$\underline{Y = AC + \bar{B}\bar{C}}$$

hate 2000
2005

(Q)

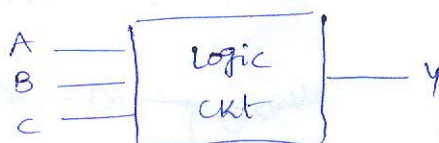
A logic ckt have three i/p's A, B and C o/p is Y

Y is logic 1 when majority no. of i/p's are logic 1.

then minimize SOP for o/p Y is

↓
atleast two i/p's are equal to 1.

Sol



→

<u>A</u>	<u>B</u>	<u>C</u>	<u>Y</u>
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

→ $\bar{A}BC$

→ $A\bar{B}C$

→ $AB\bar{C}$

→ ABC

$$Y = \bar{A}BC + A\bar{B}C + AB\bar{C} + ABC$$

$$Y = AB + BC + AC$$

→ 3 variable majority fun $Y = \underline{AB + BC + CA}$

→ 3 variable minority func $\bar{Y} = (\bar{A} + \bar{B})(\bar{B} + \bar{C})(\bar{C} + \bar{A})$

$$\bar{Y} = \underline{\bar{A}\bar{B} + \bar{B}\bar{C} + \bar{C}\bar{A}}$$