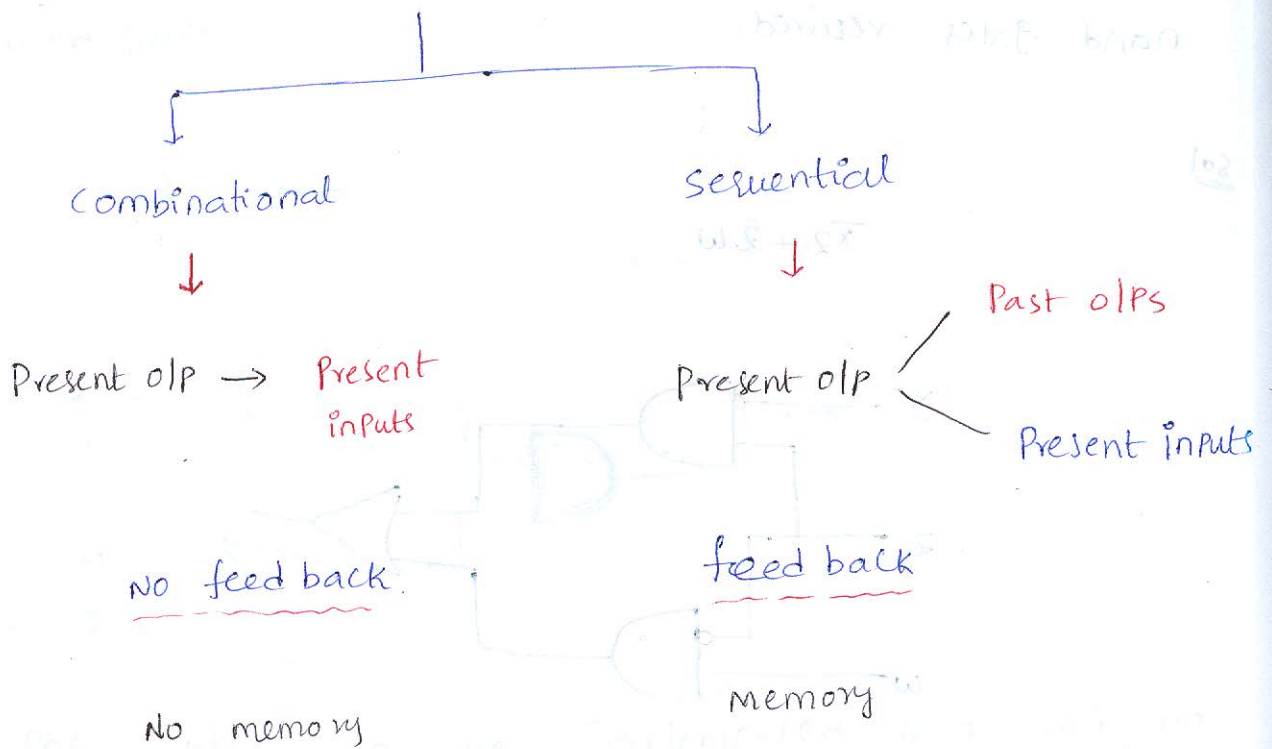


Digital CKts



Ex:- Half adder

Full adder

Mux

De mux

Ex:- Flip flops

Reg / counters

⇒ Combinational ckt design :-

- ① identify inputs and outputs
- ② construct truth table
- ③ write Logical Expression in SOP (or) POS form
- ④ minimize
- ⑤ implement logic ckt.

⇒ Arithmetic ckts:-

→



→ Truth Table :-

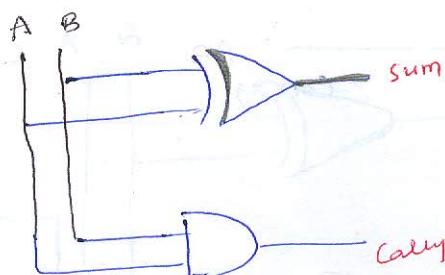
<u>A</u>	<u>B</u>	<u>Sum</u>	<u>Carry</u>
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

→ Logical Exp

$$\text{Sum} = \bar{A}B + A\bar{B} = A \oplus B$$

$$\text{Carry} = AB$$

→ Logic ckt :-



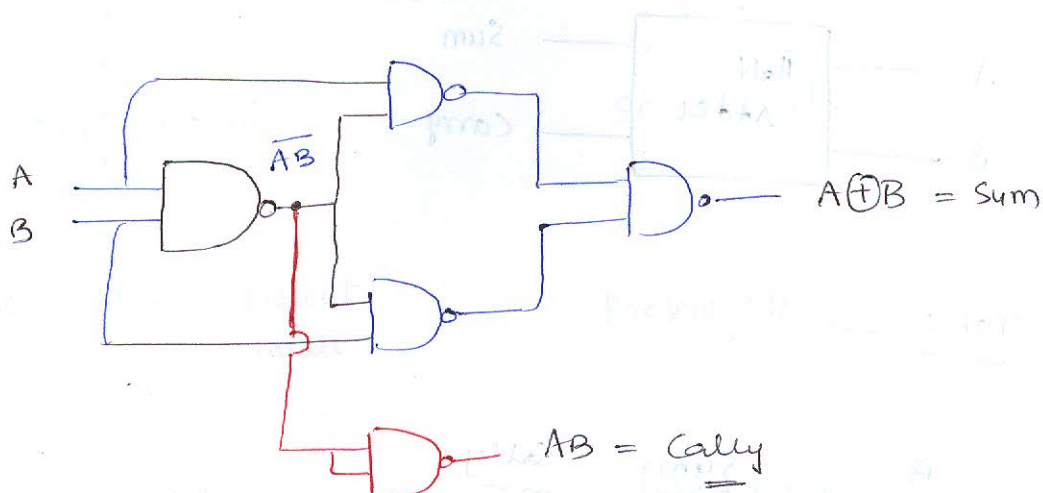
Q

No. of NAND 5

No. of NOR 5

⇒ Ex-OR gate using Nand :-

→



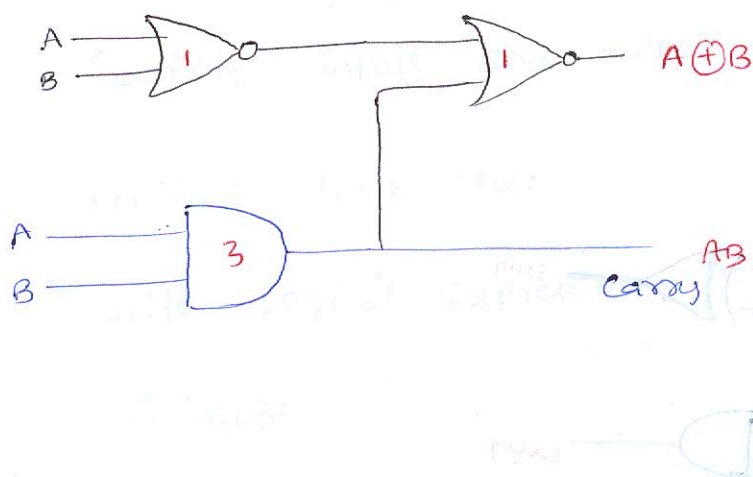
→

using NOR :-

$$A \oplus B = \overline{A \odot B}$$

$$= \overline{AB + \overline{A}\overline{B}}$$

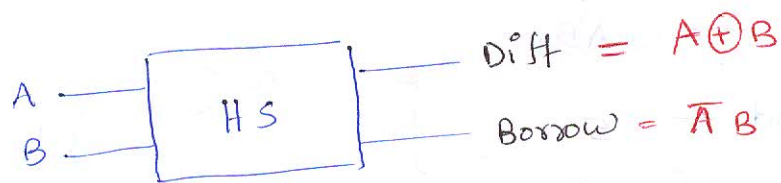
$$= \underbrace{\overline{AB}}_{\text{Carry}} + \underbrace{\overline{A+B}}_{\text{NOR}}$$



16/06/11.

211

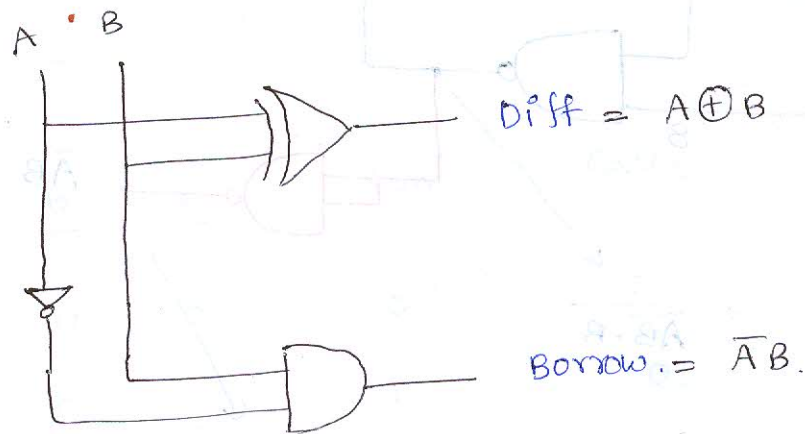
⇒ Half Subtractor :-



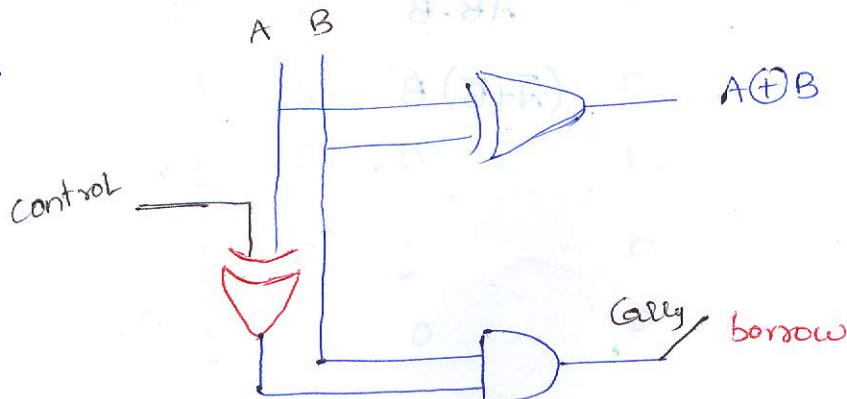
⇒ Truth Table :-

<u>A</u>	<u>B</u>	<u>Diff</u>	<u>Borrow</u>
0	0	0	0
0	1	1	1 ✓
1	0	1	0
1	1	0	0

→



⇒



→ Control = 0 → HA

→ Control = 1 → HS

HS

① → Diff — $A \oplus B$

② → Borrow — $\bar{A}B$

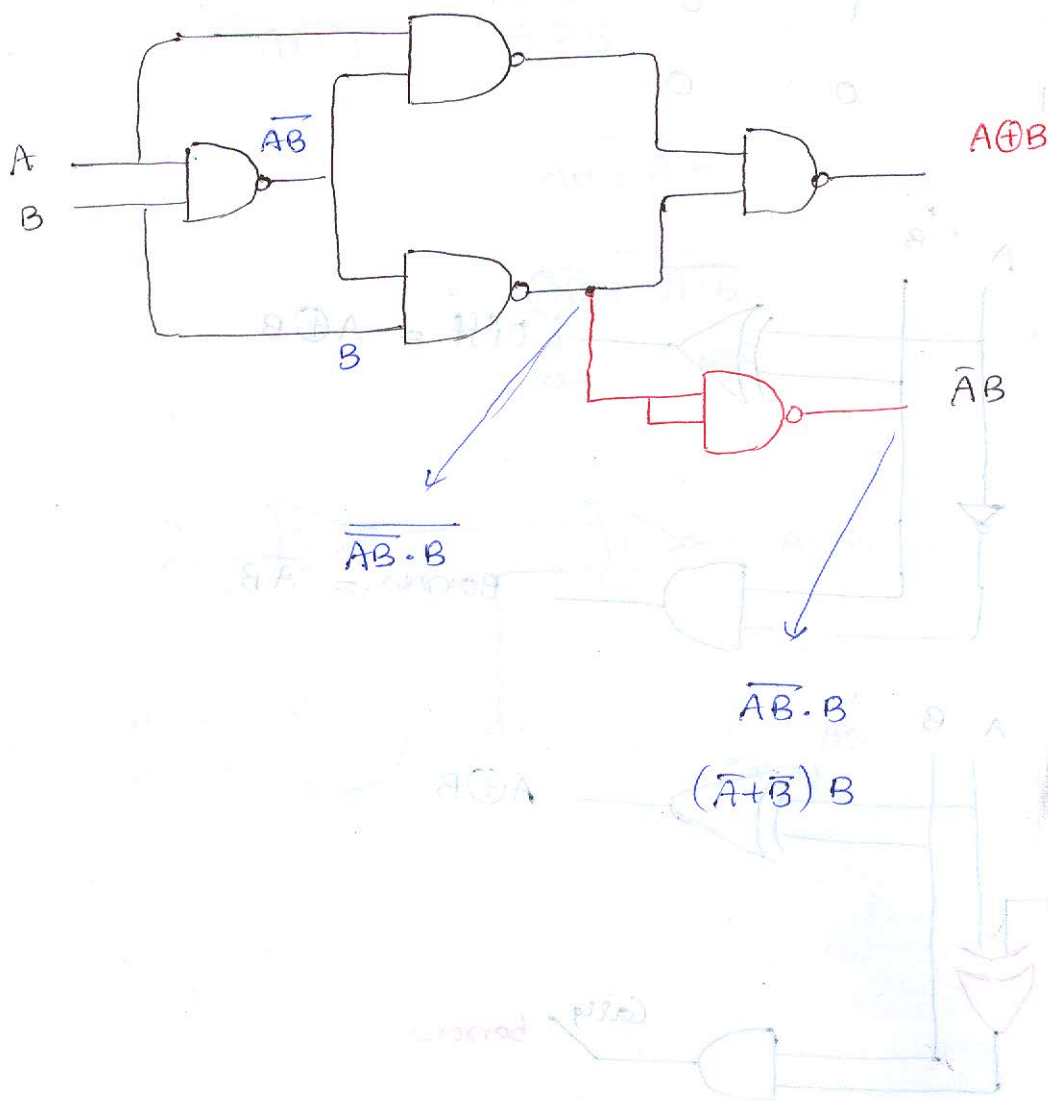
③ No. of Nand — 5

④ No. of Nor — 5

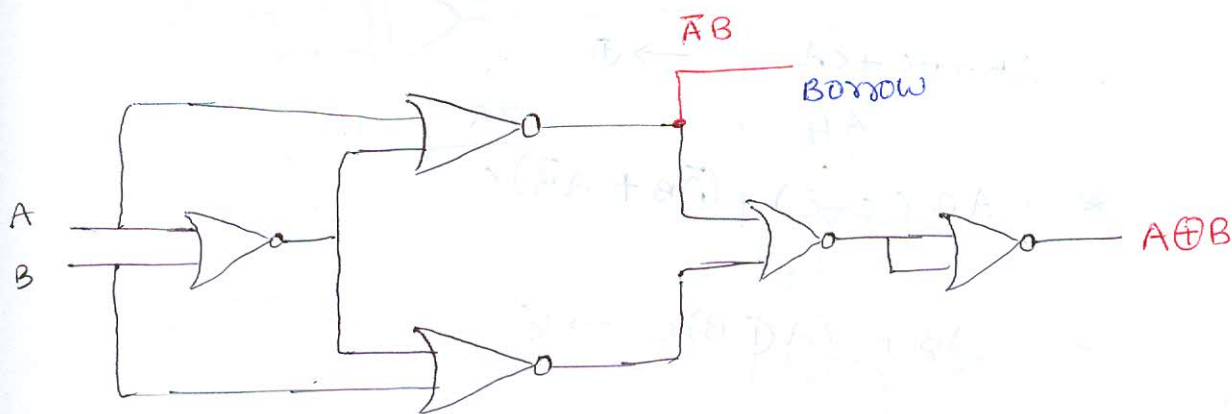
⑤ No. of mux

⑥ No. of Demux

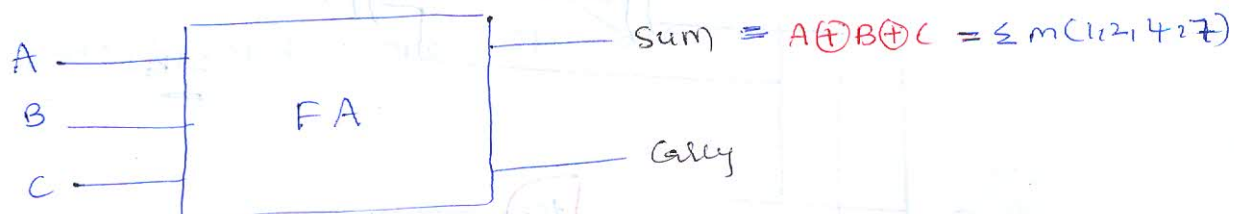
⇒ HS using Nand gates :-



⇒ HS using NOR gates :-



⇒ Full Adder :-



⇒ Truth table :-

<u>A</u>	<u>B</u>	<u>C</u>	Sum	Carry
0	0	0	0	0
0	0	<u>1</u>	<u>1</u>	0
0	<u>1</u>	0	<u>1</u>	0
0	<u>1</u>	<u>1</u>	0	1 ✓
<u>1</u>	0	0	<u>1</u>	0
<u>1</u>	0	<u>1</u>	0	1 ✓
<u>1</u>	<u>1</u>	0	0	1 ✓
<u>1</u>	<u>1</u>	<u>1</u>	<u>1</u> ✓	1 ✓

majority no. of
1's

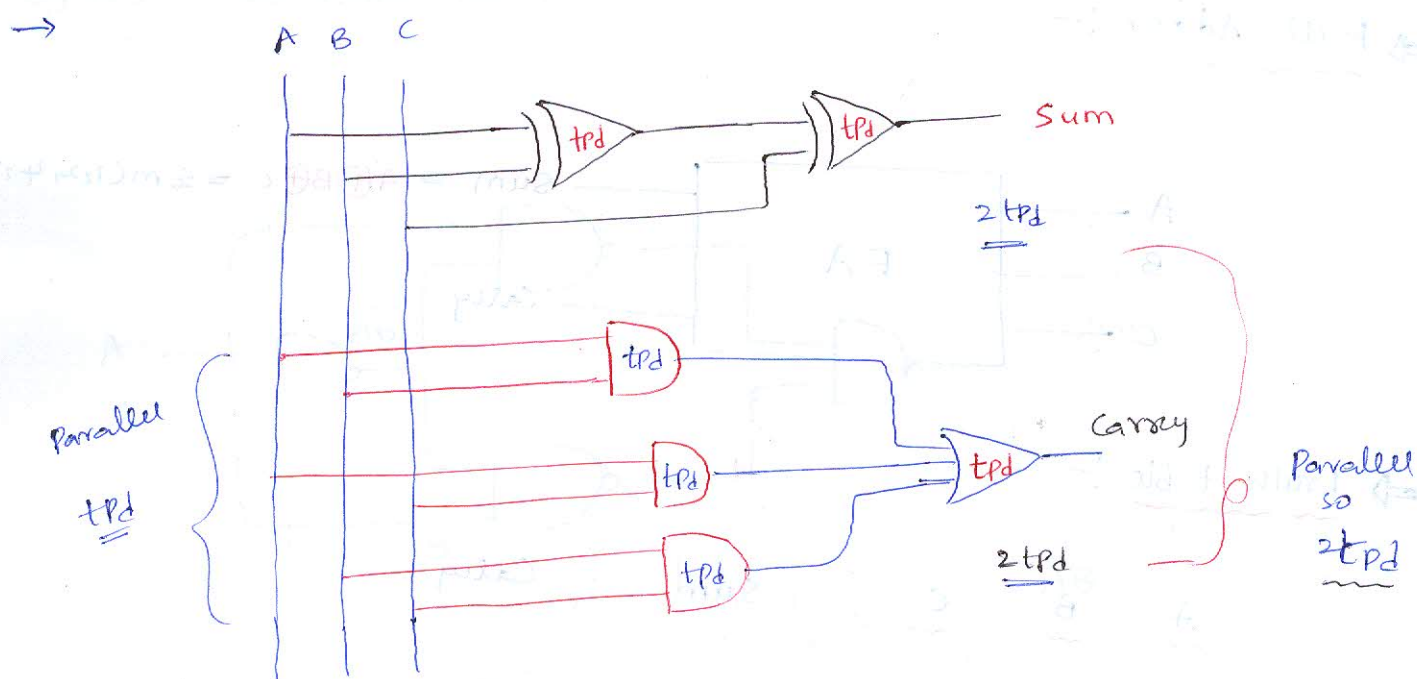
$$\rightarrow \text{Carry} = \overline{A}Bc + A\overline{B}c + AB\overline{C} + ABC$$

$$= AB + BC + CA \rightarrow \textcircled{1}$$

$$= AB(c + \overline{c}) + (\overline{A}B + A\overline{B})c$$

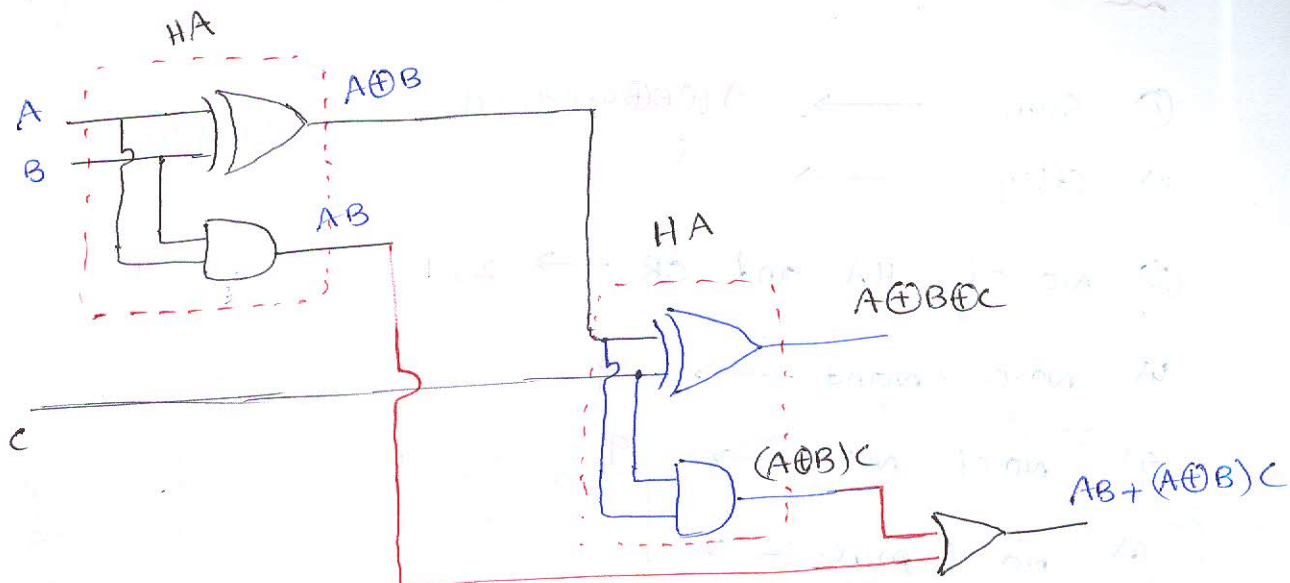
$$= AB + (A \oplus B)c \rightarrow \textcircled{2}$$

$$= \sum m(3, 5, 6, 7)$$



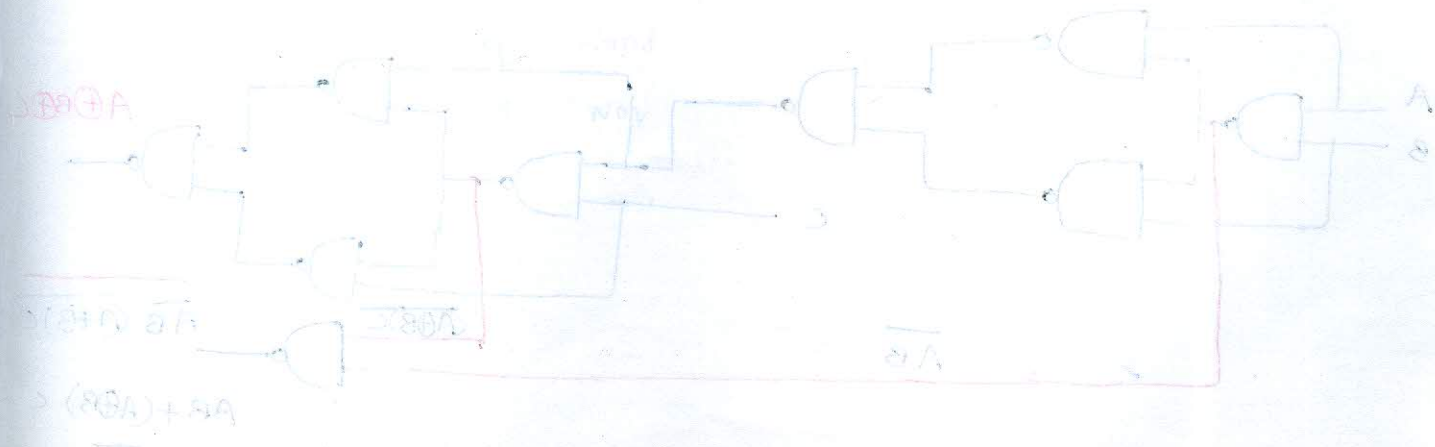
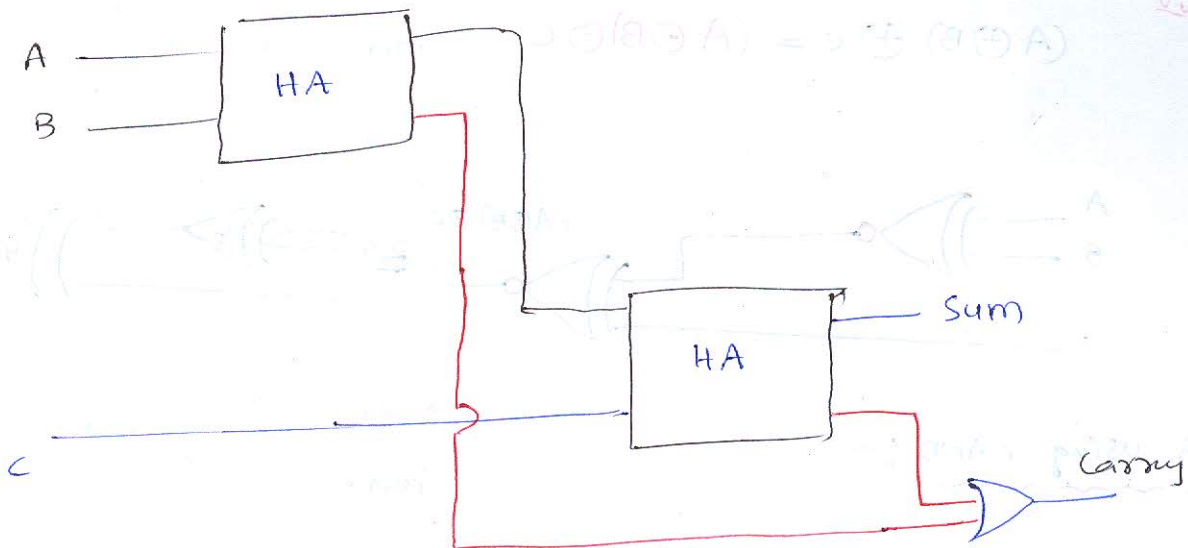
→ In full adder if each logic gate have propagation delay of t_{pd} to provide Sum (or) Carry output it require $2t_{pd}$ delay.

→



→ Full adder can be implemented with two Half adders and one OR gate.

→



⇒ FA

① Sum → $A \oplus B \oplus C$

2) Carry →

③ No. of HA and OR → 2, 1

4) No. of NAND → 9

5) No. of NOR → 9

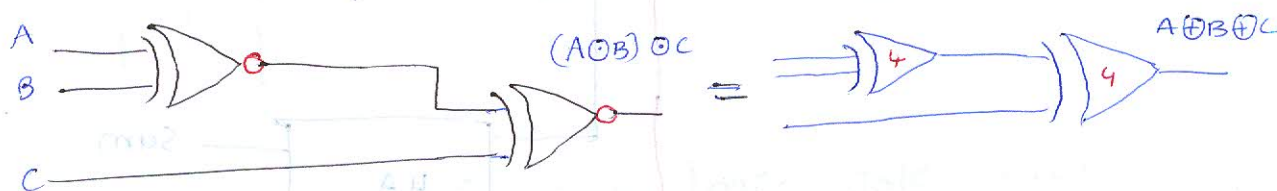
6) No. of MUX →

7) No. of Decoders →

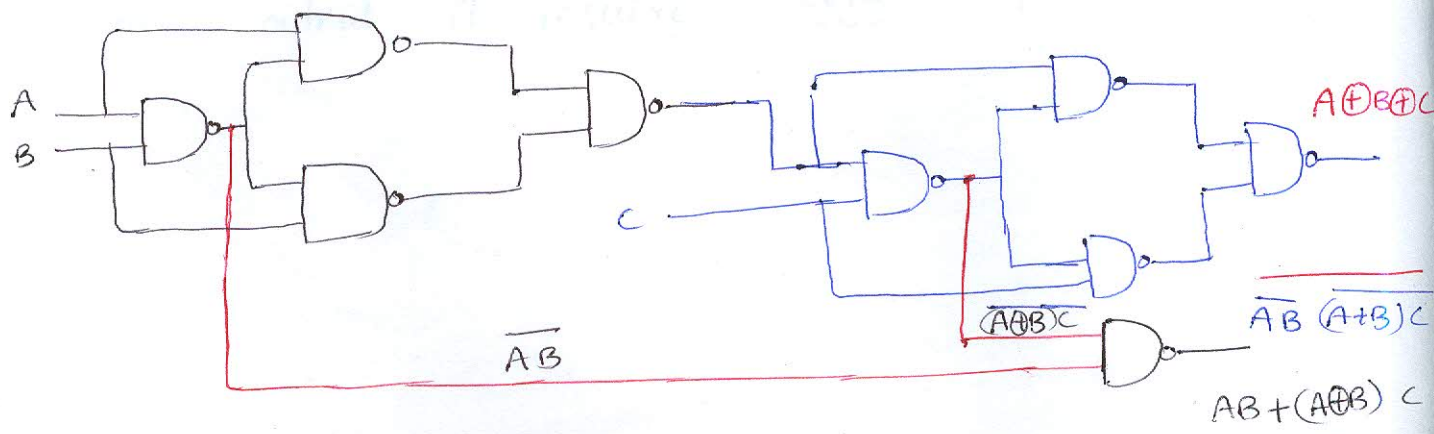
→ $(A \oplus B) \oplus C = \bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC = \sum m(1, 2, 4, 7)$

→ $(A \odot B) \odot C = \sum m(1, 2, 4, 7)$

→ vv $(A \oplus B) \oplus C = (A \odot B) \odot C$



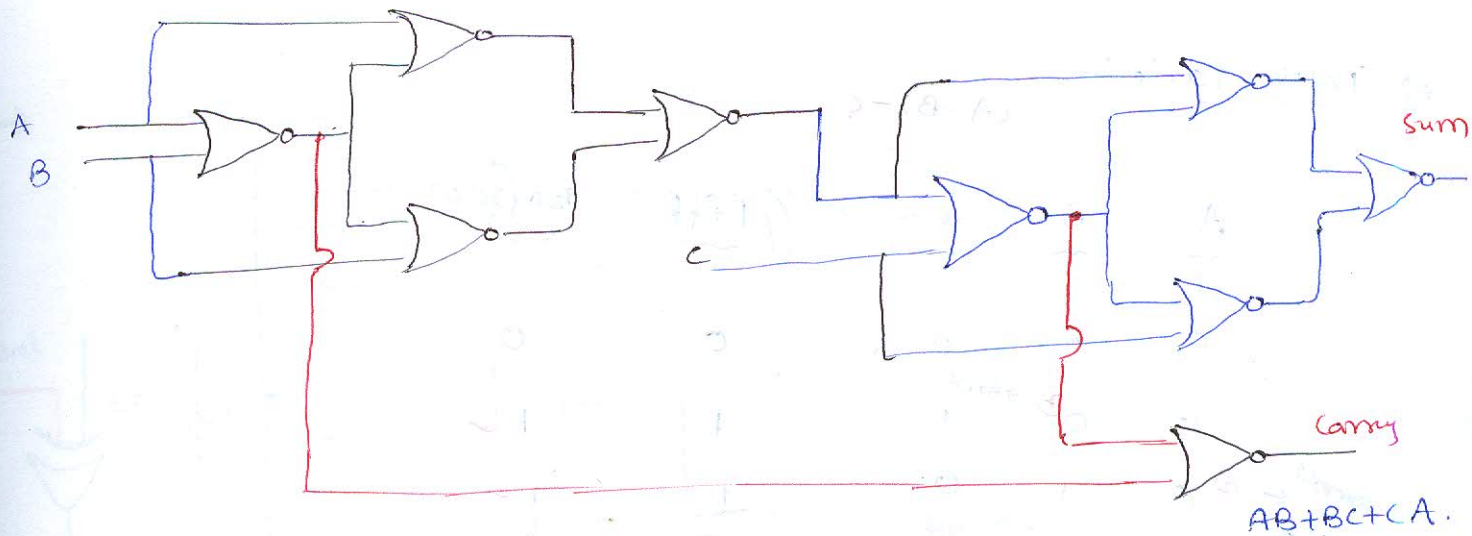
⇒ FA using NAND :-



⇒ FA using NOR :-

$$(A \oplus B) \oplus C = (A \odot B) \odot C$$

→



→ HA < 5 nand
5 NOR

→ HS < 5 nand
5 NOR

→ FA < 9 nand
9 NOR

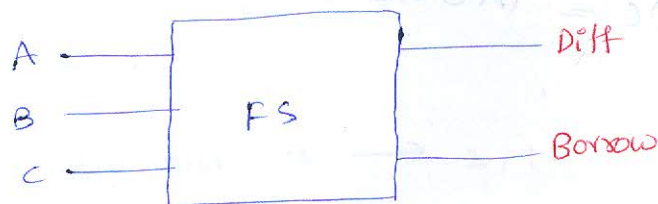
→ FS < 9 nand
9 NOR

DC

OC

C

⇒ Full Subtractor :-



⇒ Truth Table :-

$(A-B)-C$

<u>A</u>	<u>B</u>	<u>C</u>	<u>Diff</u>	<u>Borrow</u>
0	0	0	0	0
0	0	1	1	1 ✓
0	1	0	1	1 ✓
0	1	1	0	1 ✓
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1 ✓

Annotations:
 - Red circled '2' in the B column for rows 2, 3, and 4, with an arrow pointing to the Borrow output.
 - Red circled '2' in the A column for row 3, with an arrow pointing to the Borrow output.
 - Text: "Borrow for base 2".

→

$$\text{Diff} = A \oplus B \oplus C$$

$$= \sum m(1, 2, 4, 7)$$

→

$$\text{Borrow} = \sum m(1, 2, 3, 7)$$

$$= \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC + ABC$$

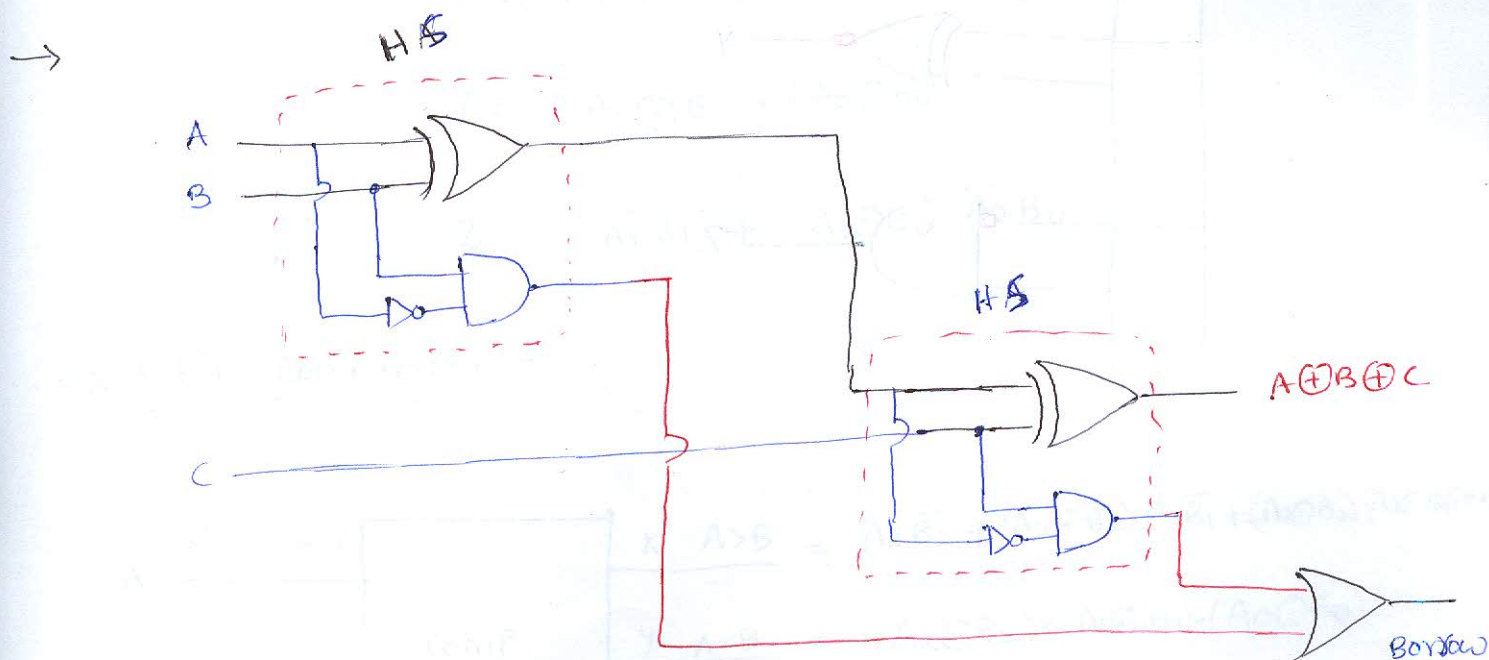
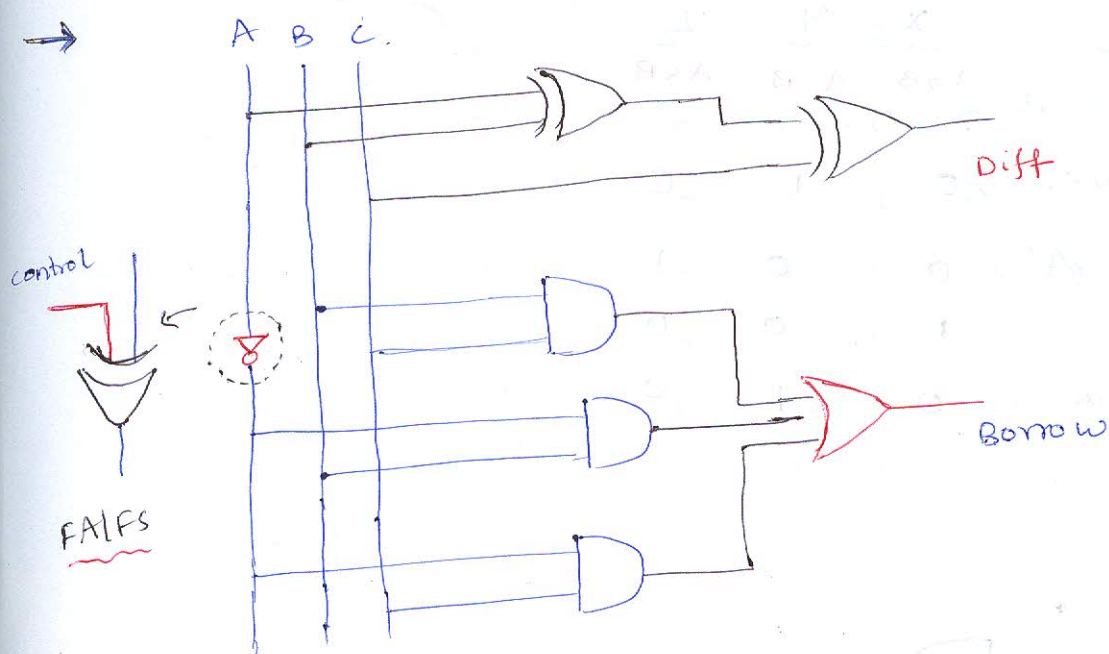
$\underbrace{\bar{A}\bar{B}C + \bar{A}B\bar{C}}_{\bar{A}C} + \underbrace{\bar{A}BC + ABC}_{BC}$

$$= \bar{A}B + BC + \bar{A}C \rightarrow \textcircled{1}$$

$$\rightarrow \text{Borrow} = \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC + ABC$$

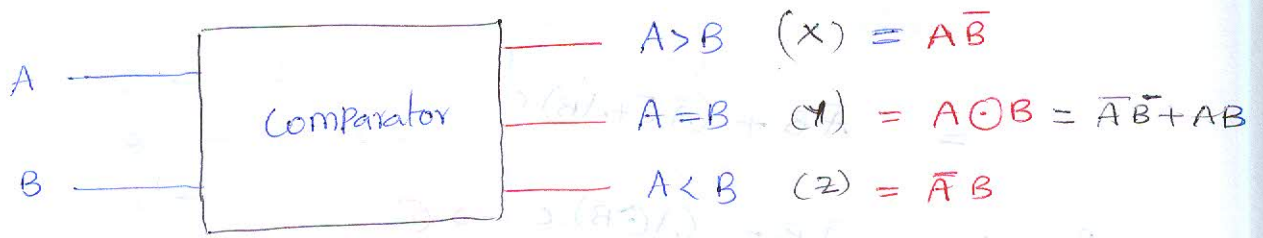
$$= \bar{A}B + (\bar{A}\bar{B} + AB)C$$

$$\text{Borrow} = \bar{A}B + (A \odot B)C \rightarrow \textcircled{2}$$



→ Full Subtractor is implemented by 2 Half Subtractors and one OR gate.

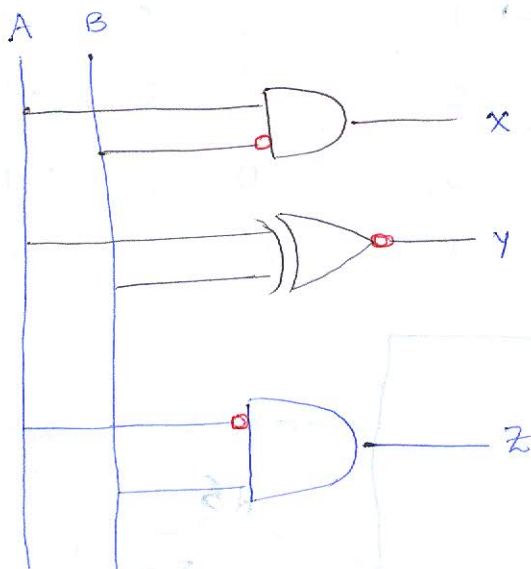
⇒ Comparator :- $100\bar{A}\bar{B} + 010\bar{A}B + 001A\bar{B} + 000AB = \text{output}$



→

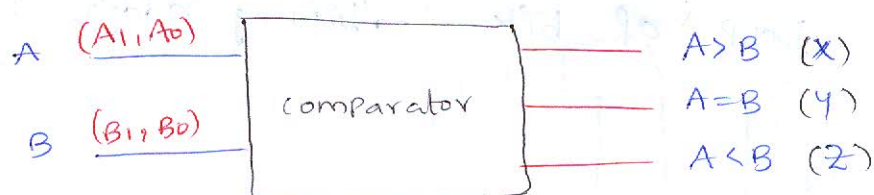
<u>A</u>	<u>B</u>	<u>X</u> $A > B$	<u>Y</u> $A = B$	<u>Z</u> $A < B$
0	0	0	1	0
0	1	0	0	1
1	0	1	0	0
1	1	0	1	0

→



Truth Table for $A > B$ is $A\bar{B}$ and $A < B$ is $\bar{A}B$.
Logic Circuit for $A > B$ is $A\bar{B}$ and $A < B$ is $\bar{A}B$.

⇒ 2 bit comparator :-



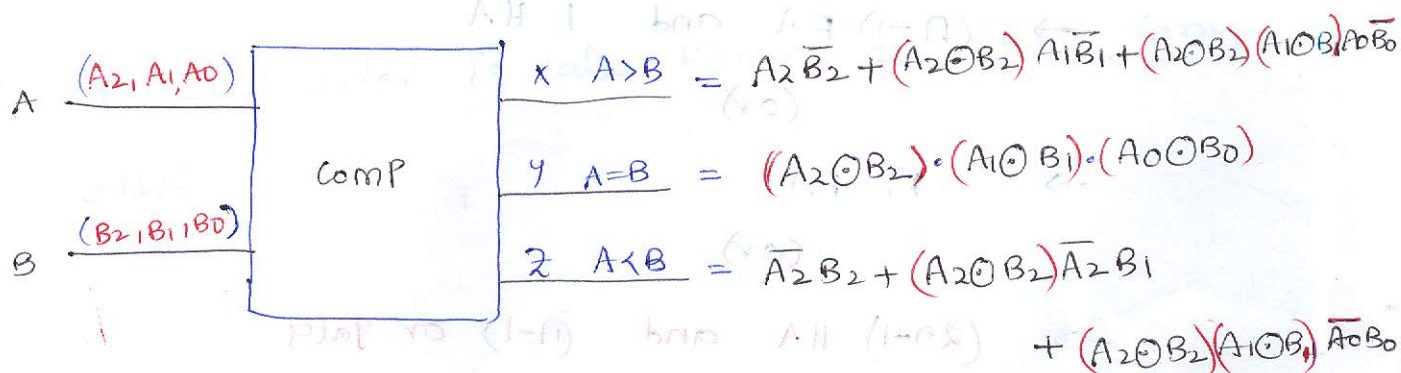
A_1	A_0	B_1	B_0	<u>X</u>	<u>Y</u>	<u>Z</u>	
0	0	0	0	0	1	0	→ $A_1=B_1$ and $A_0=B_0$
0	0	0	1	0	0	1	→ $A_1=B_1$ and $B_0 > A_0$
0	0	1	0	0	0	1	→ $A_1 > B_1$
0	0	1	1	0	0	1	
0	1	0	0	1	0	0	→ $A_1=B_1$ and $A_0 > B_0$
0	1	0	1	0	1	0	
0	1	1	0				

$$X = A_1 \bar{B}_1 + (A_1 \odot B_1) A_0 \bar{B}_0$$

$$Y = (A_1 \odot B_1) \cdot (A_0 \odot B_0)$$

$$Z = \bar{A}_1 B_1 + (A_1 \odot B_1) \bar{A}_0 B_0$$

⇒ 3 bit comparator :-



$$X \ A > B = A_2 \bar{B}_2 + (A_2 \odot B_2) A_1 \bar{B}_1 + (A_2 \odot B_2) (A_1 \odot B_1) A_0 \bar{B}_0$$

$$Y \ A = B = (A_2 \odot B_2) \cdot (A_1 \odot B_1) \cdot (A_0 \odot B_0)$$

$$Z \ A < B = \bar{A}_2 B_2 + (A_2 \odot B_2) \bar{A}_2 B_1 + (A_2 \odot B_2) (A_1 \odot B_1) \bar{A}_0 B_0$$

⇒ Note:- Half adder and Full adder are only one-bit adders

→ To add group of bits following adder ckt's are used

- 1) Serial adder
- 2) Parallel adder
- 3) Look ahead carry adder.

⇒ Serial adder:-

⇒ In serial adder to add n bit nos only one full adder is used

⇒ for n bit addition n clock pulses are required.

⇒ It is slowest adder

⇒ Parallel adder:-

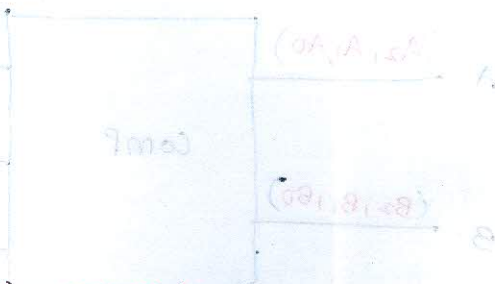
→ In parallel adder to add n-bit numbers it require

→ (n-1) FA and 1 HA
(or)

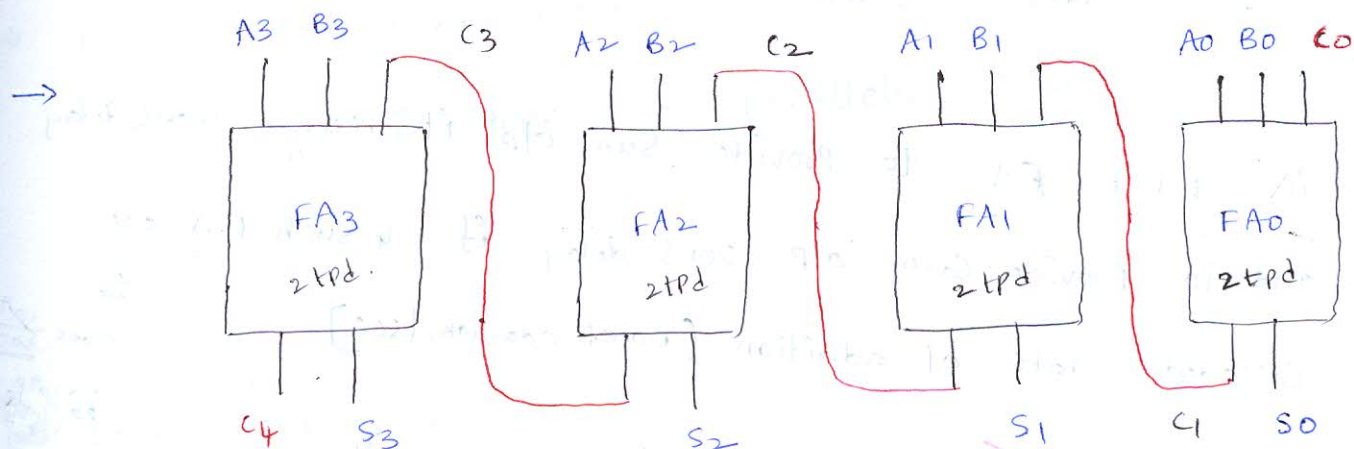
→ n FA

(or)

→ (2n-1) HA and (n-1) or gates



⇒ 4 bit Parallel adder :-



C_4 S_3 S_2 S_1 S_0
 ↓
 Carry

→ Total delay $8tp_d$
 $2 \times 4tp_d$

→ Parallel adder is faster than the serial adder but in this carry propagation delay present from i/p stage to o/p stage.

→ In parallel adder each full adder will provide minimum two logic gate delays hence in n-bit parallel adder to provide result minimum

$2n tp_d$ delay is required.

→ Parallel adder is also known as Ripple Carry adder.

→ To overcome Carry Propagation delay Look ahead

Carry adder is used in Arithmetic CKts

ies 2005

Q)

in 1-bit FA to provide Sum o/p it receive 40ns delay and to provide Carry o/p 20ns delay if 4 such F.A are cascaded rate of addition [no. of additions/sec]

a) 10^5

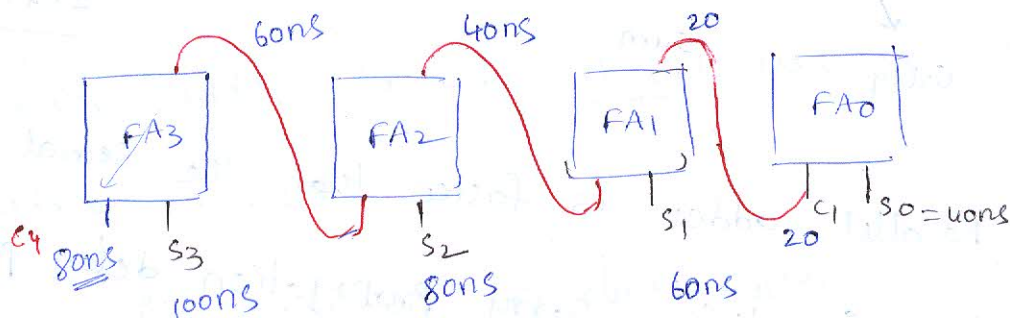
b) 10^7

c) 6.25×10^7

d) 1.25×10^9

$$\frac{1}{180} \times \frac{10^9}{10^3} = \frac{10^6}{180} = \frac{10^5}{18}$$

Sol:-



C ₄	S ₃	S ₂	S ₁	S ₀
80ns	100ns	80	60	40

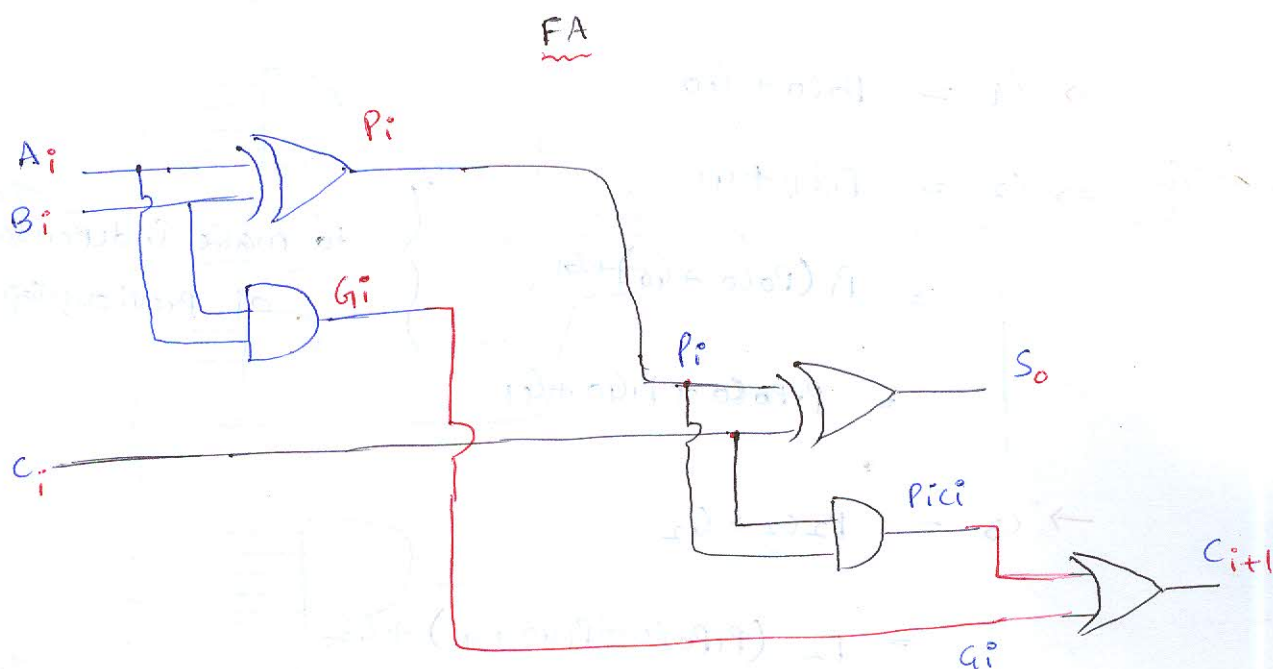
min delay S₀

$$\text{No. of additions} = \frac{1}{100 \times 10^{-9}} = 10^7 \text{ additions}$$

⇒ Look ahead Carry adder :-

→ look ahead carry adder is the fastest adder in this carry is generated parallelly using Look ahead Carry Circuit.

⇒ Look ahead Carry ckt :-



→ G_i - Carry generator

→ P_i = Propagation term

→ $P_i = A_i \oplus B_i$ ✓

$G_i = A_i B_i$ ✓

$S_i = P_i \oplus C_i$ ✓

$C_{i+1} = P_i C_i + G_i$ ✓

→

$$P_0 = A_0 \oplus B_0$$

$$P_1 = A_1 \oplus B_1$$

$$P_2 = A_2 \oplus B_2$$

$$P_3 = A_3 \oplus B_3$$

$$G_0 = A_0 B_0$$

$$G_1 = A_1 B_1$$

$$G_2 = A_2 B_2$$

$$G_3 = A_3 B_3$$

$$S_0 = P_0 \oplus C_0$$

$$S_1 = P_1 \oplus C_1$$

$$S_2 = P_2 \oplus C_2$$

$$S_3 = P_3 \oplus C_3$$

→ $C_0 \rightarrow$ input carry

$$\rightarrow C_1 = P_0 C_0 + G_0$$

$$\rightarrow C_2 = P_1 C_1 + G_1$$

$$= P_1 (P_0 C_0 + G_0) + G_1$$

$$= P_1 P_0 C_0 + P_1 G_0 + G_1$$

to make independent
of previous carry

$$\rightarrow C_3 = P_2 C_2 + G_2$$

$$= P_2 (P_1 P_0 C_0 + P_1 G_0 + G_1) + G_2$$

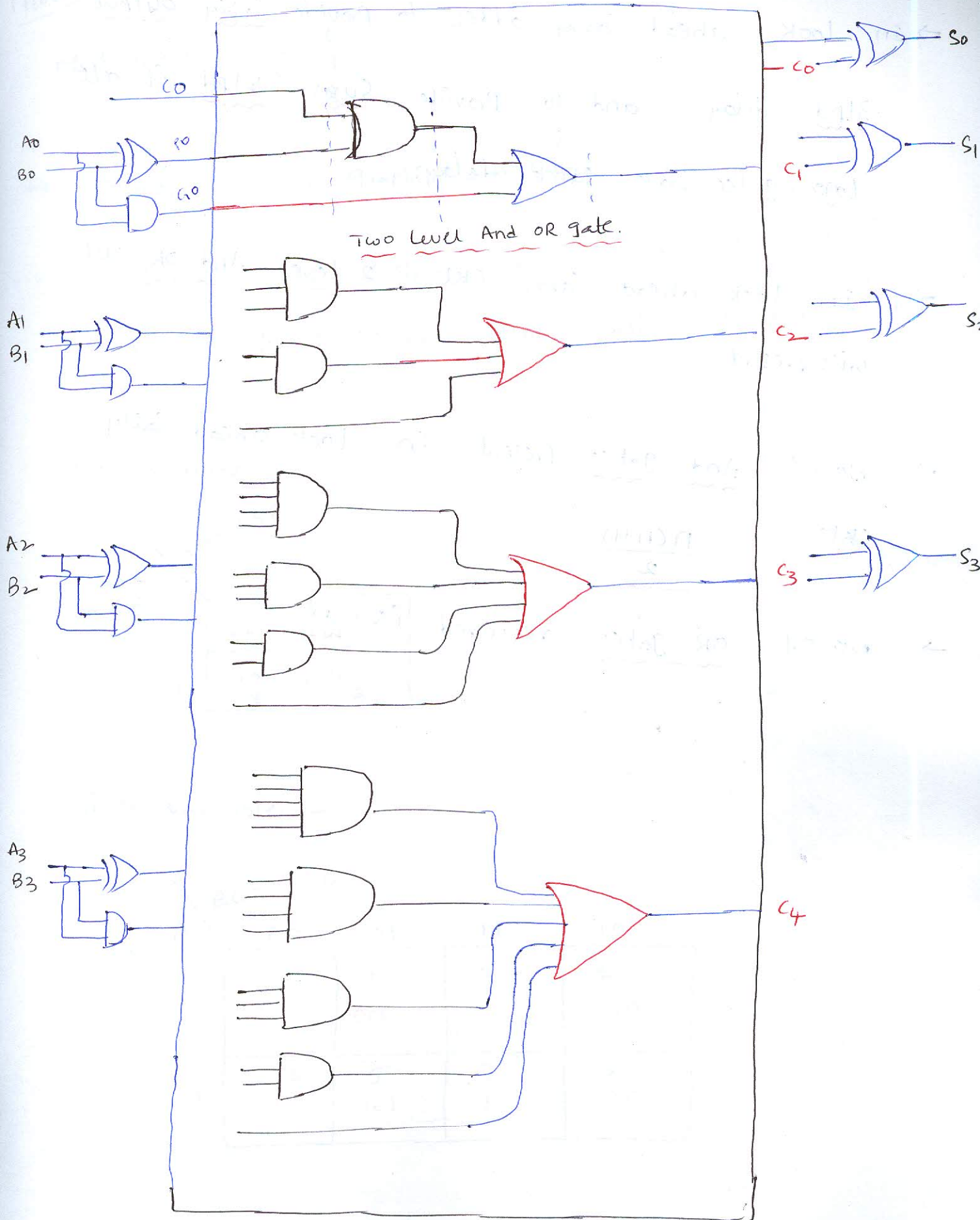
$$= P_2 P_1 P_0 C_0 + P_2 P_1 G_0 + P_2 G_1 + G_2$$

$$\rightarrow C_4 = P_3 C_3 + G_3 \quad \text{— (final carry)}$$

$$= P_3 (P_2 P_1 P_0 C_0 + P_2 P_1 G_0 + P_2 G_1 + G_2) + G_3$$

$$= P_3 P_2 P_1 P_0 C_0 + P_3 P_2 P_1 G_0 + P_3 P_2 G_1 + P_3 G_2 + G_3$$

Look ahead Carry ckt.



→ In look ahead Carry adder to provide Carry output

3tpd delay and to provide Sum 4tpd if all

logic gates have same delay.

→ In look ahead Carry ckt 2 level And-OR ckt will present

→ No. of And gates present in look ahead Carry

ckt

$$\frac{n(n+1)}{2}$$

→ No. of OR gates required is n.

17/06/11

⇒ K-map :- 

→ K-map is an graphical method.

→ In K-map the graph is represented with gray code to provide to successive terms differ by only one bit.

Two variables :-

Diagram for Two variables K-map:

MSB ← A, B ← LSB

	0	1
0	00 0	01 1
1	10 2	11 3

Three variables :-

Diagram for Three variables K-map:

MSB ← A, BC ← LSB

	00	01	11	10
0	0 000	1 001	3 011	2 010
1	4 100	5 101	7 111	6 110

Four variables :-

AB \ CD				
	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

$\Rightarrow$ SOP :-

$\Rightarrow$ Two variable :-

Q) $\bar{A}\bar{B} + AB + \bar{A}B$

A \ B	0	1
0	1	1
1		1

Pair

$\bar{A}$ B

$\bar{A} + B$

Q) $f(A, B) = \sum m(0, 1, 2)$

A \ B	0	1
0	1	1
1	1	0

$\bar{A} + \bar{B}$

Q5

$$\Sigma m(0,1,2,3)$$

A \ B	0	1
0	1	1
1	1	1

$$= 1$$

$$(\sim 0)5 + (811)10 = (811A)1$$

Q6

$$f(A,B) = \Sigma m(0,3) + \Sigma d(1)$$

A \ B	0	1
0	1	X
1		1

$$\bar{A} + B$$

⇒ Note :- Here dont care is considered with logic 1

Q7

$$f(A,B) = \Sigma m(1,3) + \Sigma d(2)$$

A \ B	0	1
0		
1	X	1

not received
minimize

B → minimized expression $\Sigma m(1,3) + \Sigma d(2)$

B - no need of any gate

$$B \rightarrow Y$$

⇒ Note :- Here dont care value is 0

Q) $f(A,B) = \sum m(1,3) + \sum d(0,2)$

	B	0	1
A	0	X	1
	1	X	1

= 1

	B	0	1
A	0	X	X
	1	X	1

= 1

	B	0	1
A	0	1	1
	1	1	1

→ uv imp.

	B	0	1
A	0	X	X
	1	X	X

= X ✓

⇒ Three Variables :-

Q) $f(A,B,C) = \sum m(0,1,4,5,7)$

	BC	00	01	11	10
A	0	1	1	0	1
	1	1	0	1	0

Annotations:
 - $\bar{A}\bar{C}$ (circled 1 at 00, 0)
 - $\bar{A}B$ (circled 1 at 01, 0)
 - $\bar{B}\bar{C}$ (circled 1 at 00, 1)
 - ABC (circled 1 at 11, 1)

8 → octal

4 → Quadrant

2 → Pair.

1 -

$\bar{A}\bar{B} + \bar{B}\bar{C} + ABC + \bar{A}C$

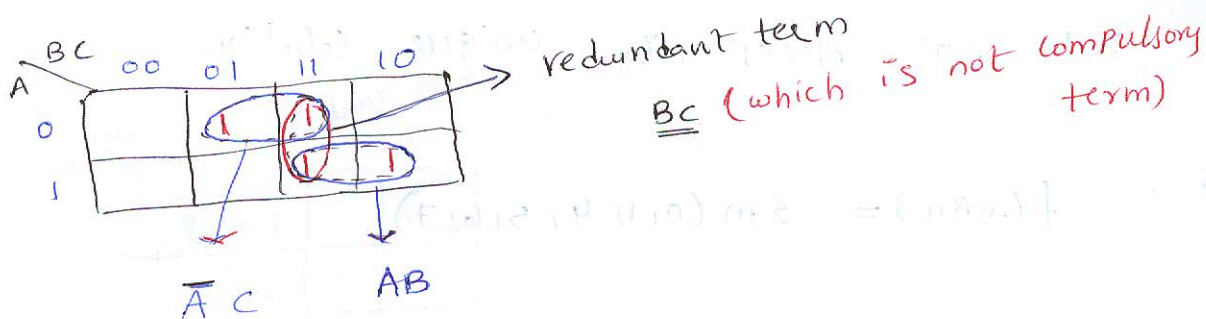
	B	0	1
A	0	1	1
	1	1	X

Q) $f(A,B,C) = \sum m(0,1,3,5,7)$

	BC	00	01	11	10
A	0	1	1	1	
	1		1	1	

$C + \bar{A}\bar{B} = \bar{A}\bar{B} + C$

Q) $f(A,B,C) = \sum m(1,3,6,7)$

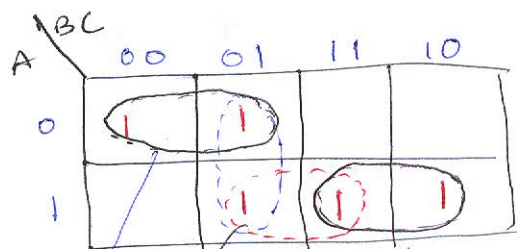


$\underline{AB + \bar{A}C}$

⇒ Procedure:-

- ① Octet (it eliminates three variables)
- ② Quad (it eliminates two variables)
- ③ Pairs (it eliminates one variable)
- ④ Single terms (no variable)
- ⑤ Remove Redundant terms

Q) $f(A,B,C) = \sum m(0,1,5,6,7)$



$\bar{A}\bar{B}$

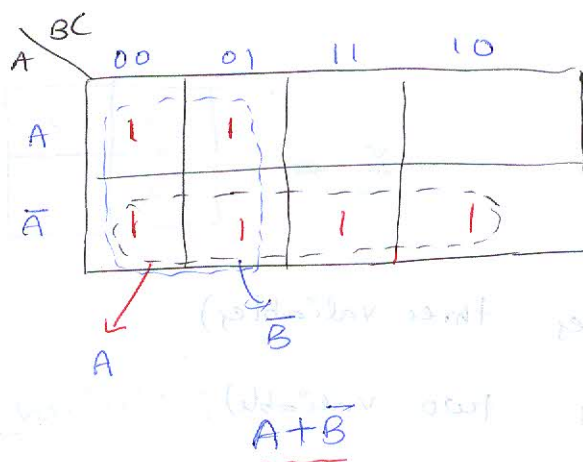
AB

$\bar{A}\bar{B} + AB + \underline{\bar{B}C}$

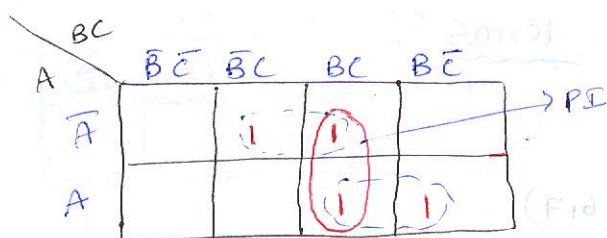
$\bar{A}\bar{B} + AB + \underline{AC}$

→ K-map always provides minimized expression but not necessarily unique solution.

Q) $f(A, B, C) = \sum m(0, 1, 4, 5, 6, 7)$



Q) $f(A, B, C) = \sum m(1, 3, 6, 7)$



$PI = \bar{A}C + BC + AB$
 $EPS = \underline{2}$

⇒ Implicant :- min terms - 4

⇒ Prime implicant :- (PI)

Prime implicant is a Product term which is obtained by combining maximum possible adjacent cells.

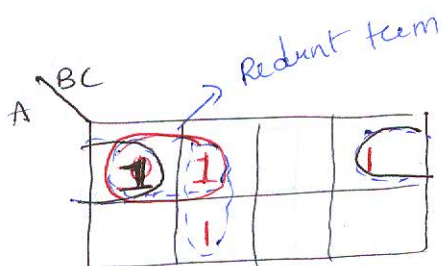
⇒ Essential Prime Implicant :- (EPS)

It is an prime implicant which is obtained by combining only one way and no alternative way.

Q)

$$f(A,B,C) = \sum m(0,1,2,5)$$

$$(F1, 2, 5, 10) m3 = (0, 8, 1A) + \dots$$



$$I = 4$$

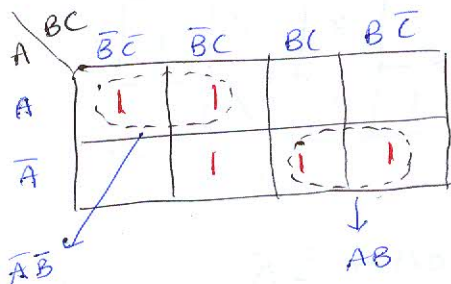
$$PI = 3 \quad \bar{A}\bar{B} + \bar{A}\bar{C} + \bar{B}C$$

$$EPI = 2 \quad \bar{A}C + \bar{B}C$$

Q)

$$f(A,B,C) = \sum m(0,1,5,6,7)$$

$$\bar{A} + \bar{B} + \bar{C}$$



$$(2, 10, 11, 10) m3 = (0, 8, 1A) + \dots$$



$$I = 5$$

$$PI = 4$$

$$EPI = 2$$

$$\bar{A}\bar{B} + \bar{A}B + \bar{B}C$$

$$\bar{A}\bar{B} + \bar{A}B + \bar{A}C$$

EPI

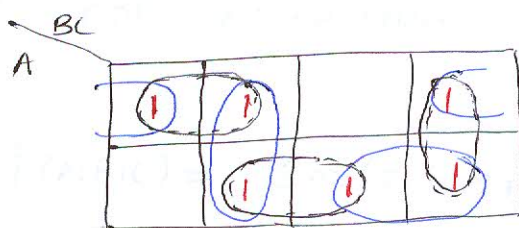
$$2 = 1$$

$$8 = 19$$

$$2 = 193$$

$$3A + \bar{B}$$

Q) $f(A,B,C) = \sum m(0,1,2,5,6,7)$



Two Sols

$I = 6$

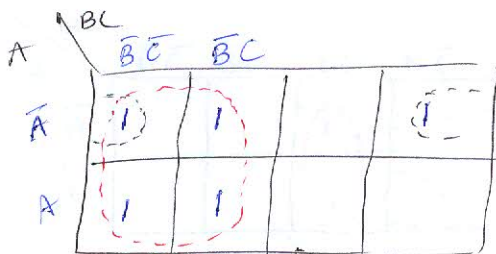
$PI = 6$

$EPI = 0$

$\bar{A}\bar{B} + AC + BC$

$\bar{B}C + AB + \bar{A}\bar{C}$

Q) $f(A,B,C) = \sum m(0,1,2,4,5)$



$I = 5$

$PI = 2$

$EPI = 2$

$\bar{B} + \bar{A}\bar{C}$

* Q)

$$f(A,B,C) = \sum m(0,1,3,5,7) + \sum d(2)$$

A \ BC					
		00	01	11	10
0		1	1	1	X
1			1	1	

$\bar{A} + C$

Q1) $f(A,B,C) = \sum m(0,1,6,7) + \sum d(3,5)$

A \ BC					
		00	01	11	10
0		1	1	X	
1			X	1	1

$\bar{A}\bar{B} + AB$

Q2) $f(A,B,C) = \sum m(0,1,6,7) + \sum d(3,4,5)$

A \ BC					
		00	01	11	10
0		1	1	X	
1		X	X	1	1

$$\bar{B} + AB = A + \bar{B}$$

Q3) $f(A,B,C) = \sum m(0,1,6,7) + \sum d(2,3,4,5)$

$$= 1$$

four variables :-

Q> $f(A,B,C,D) = \sum m(0, 1, 3, 5, 7, 8, 9, 11, 13, 15)$

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	$C\bar{D}$	CD
$\bar{A}\bar{B}$	1	1	1	
$\bar{A}B$		1	1	
$A\bar{B}$	1	1	1	
AB				

$D + \bar{B}\bar{C}$

Q> $f(A,B,C,D) = \sum m(0, 1, 4, 5, 8, 9, 13, 15)$

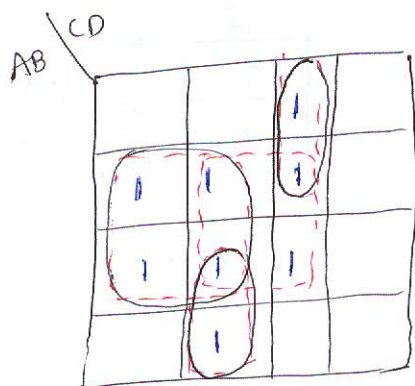
AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	$C\bar{D}$	CD
$\bar{A}\bar{B}$	1	1		
$\bar{A}B$	1	1		
$A\bar{B}$			1	1
AB	1	1		

$\bar{A}\bar{C} + \bar{B}\bar{C} + ABD$

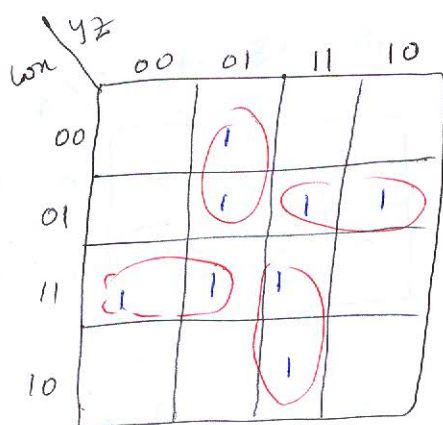
Q> $f(A,B,C,D) = \sum m(1, 5, 6, 7, 11, 13, 15)$

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	$C\bar{D}$	CD
$\bar{A}\bar{B}$	0	1		
$\bar{A}B$			1	1
$A\bar{B}$		1		
AB			1	1

Q) $f(A, B, C, D) = \sum m(3, 4, 5, 7, 9, 12, 13, 15)$



Q) IES 2005
Gate 2009



$$\overline{w} \overline{y} z + \overline{w} x y + w x \overline{y} + y z \underline{w}$$

⇒ Pos :-

Q1) $f(A|B) = \pi M(0, 1|3)$

		B	$\bar{B}$
A	0	0	1
$\bar{A}$	1	0	1

$A \cdot \bar{B}$

0	1	0	1
1	0	1	0
0	1	0	1
1	0	1	0

Q2) $f(A|B) = \pi M(1, 2|3)$

		B	$\bar{B}$
A	1	0	1
$\bar{A}$	2	0	1

$\bar{A} \bar{B}$

0	1	0	1
1	0	1	0
0	1	0	1
1	0	1	0

Q3) $f(A|B) = \pi M(0, 1|2, 3)$

0	0
0	0

= 0 ✓

Q4) $f(A|B) = \pi M(1|3) + \pi d(2)$

		B	$\bar{B}$
A	1	0	1
$\bar{A}$	X	0	1

Here X has value 1. $\bar{B}$

Q3) $f(A, B) = \Pi M(1, 2) + \Pi d(3)$

	B	$\bar{B}$
A		0
$\bar{A}$	0	X

$\bar{A}\bar{B}$

⇒ Three variables:-

Q4) $f(A, B, C) = \Pi M(0, 1, 5, 7)$

	BC	$B+C$	$B+\bar{C}$	$\bar{B}+\bar{C}$	$\bar{B}+C$
		00	01	11	10
A	0	0	0		
$\bar{A}$	1		0	0	

$A+B$ $\bar{A}+\bar{C}$

$(A+B)(\bar{A}+\bar{C})$

Q5) $f(A, B, C) = \Pi M(0, 1, 3, 5, 6, 7)$

	BC	$B+C$	$B+\bar{C}$	$\bar{B}+\bar{C}$	$\bar{B}+C$
		00	01	11	10
A	0	0	0	0	
$\bar{A}$	1	0	0	0	0

$(\bar{C})(\bar{A}+\bar{B})(A+B)$

19

A	BC			
	B+C	B+C̄	B̄+C	B̄+C̄
	00	01	11	10
0	0	1	X	0
1	X	1	0	X

in pos form

$$\underline{\underline{B \cdot C}}$$

Four variables :-

Q)

AB	CD			
	C+D	C+D̄	C̄+D	C̄+D̄
A+B	0	1	X	0
A+B̄	1	X	0	1
Ā+B	0	0	X	0
Ā+B̄	0	X	1	0

in pos form

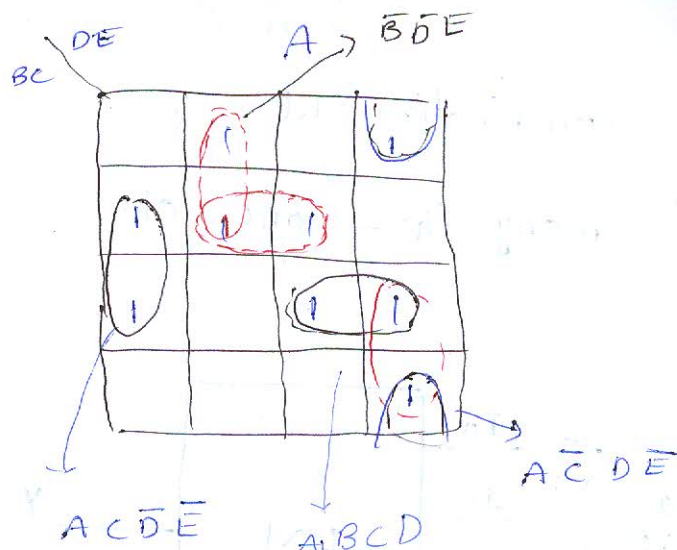
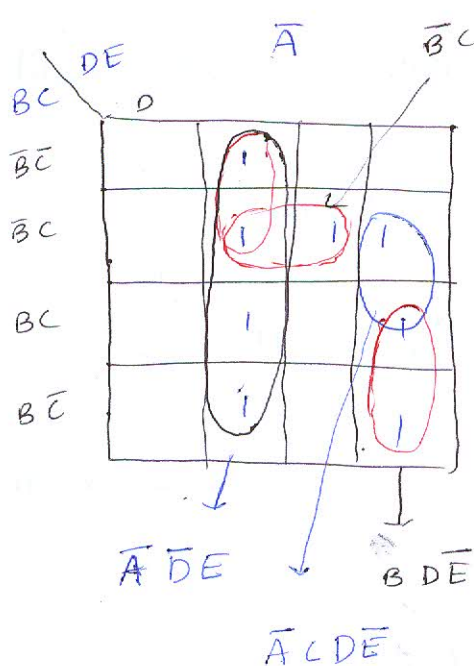
$$(D+B) (\bar{A}+\bar{B}) (\bar{D}+\bar{B})$$

⇒ 5 variables :-

		$\bar{A}$			
BC	DE	00	01	11	10
		0	1	3	2
00		0	1	3	2
01		4	5	7	6
11		12	13	15	14
10		8	9	11	10

		A			
BC	DE	00	01	11	10
		16	17	19	18
00		16	17	19	18
01		20	21	23	22
11		28	29	31	30
10		24	25	27	26

Q) minimized SOP expression is



$3I_0 + 3I_1$

