9/11/23

UNIT – V

BJT – Configurations & Multistage Amplifiers

↳ The Basic construction, characteristics and proper working along with different Biasing / stability approaches are examined / covered so far [in E.D.C]

We now begin to examine / learn Small Signal - ac Response of the 'BJT' and 'BJT' amplifiers, using different 'models' that are most frequently used.

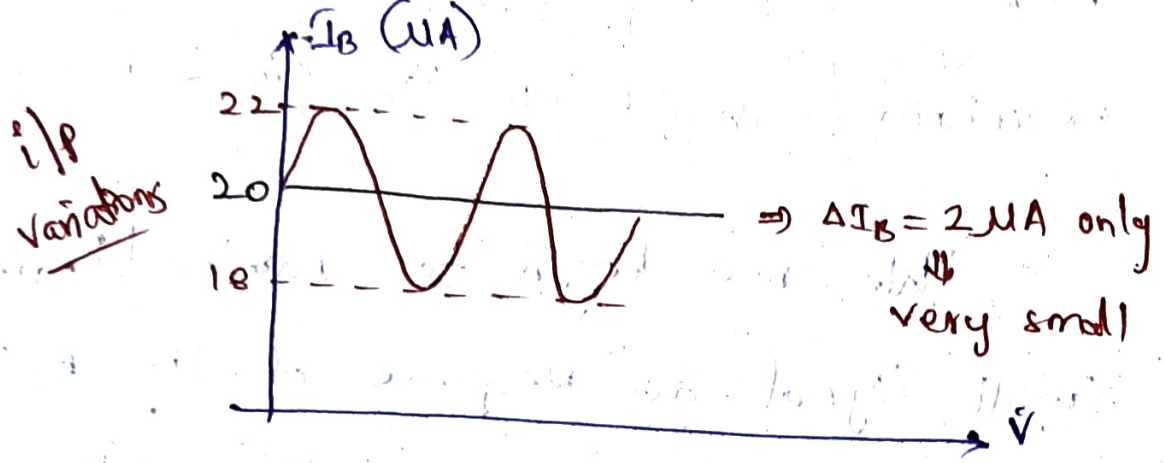
↳ Usually, the magnitute of input ac signal will determine whether 'Small-Signal' or 'Large Signals' techniques should apply.

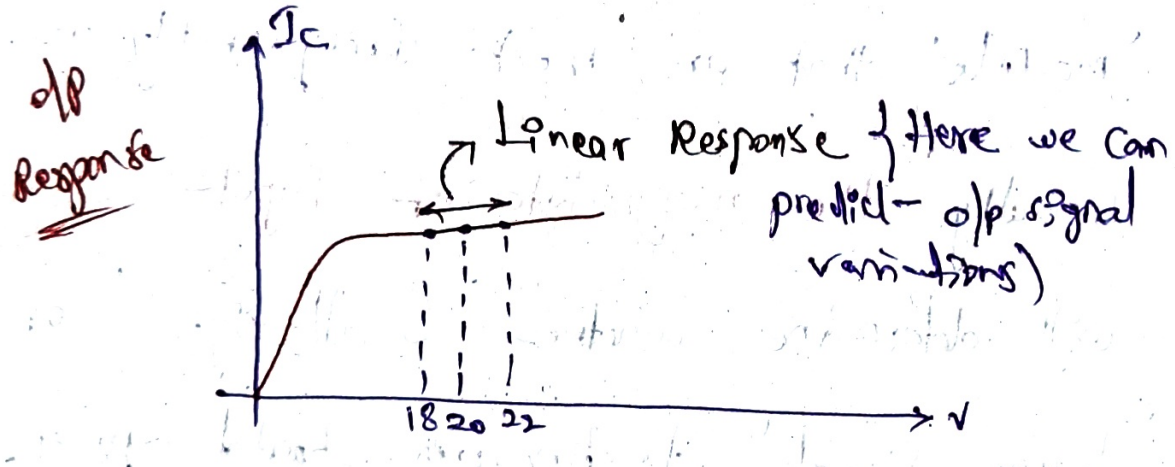
i.e. Analysis of amplifiers which predicts the Behaviour / Response we use following analysis methods based on 'signal variations' (i.e. i/p signal).

① Small-Signal Amplifiers / modelling

Here the signal variation is very small compared to DC-level

Ex: Voltage Amplifiers.



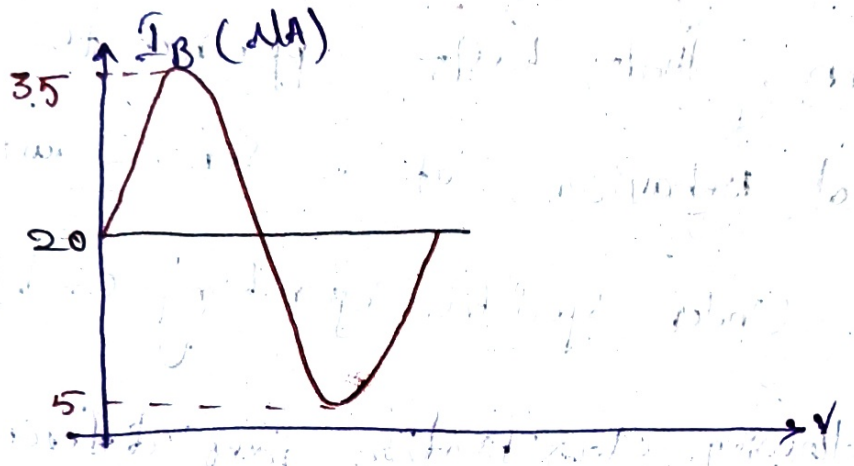


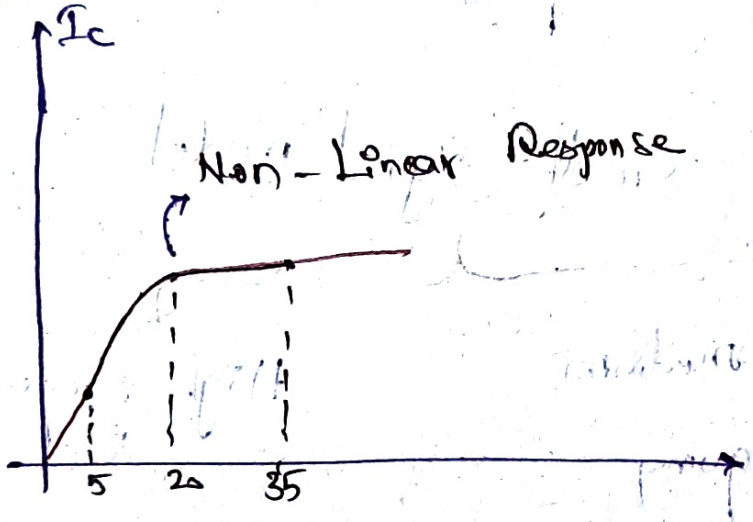
* As shown above, even though BJT is a non-linear device under small signal modelling it behaves like linear due to which we get distortion-less output.

② Large-signal Amplifiers / modelling

Here the i/p signal variations are very large

Ex: Power Amplifiers





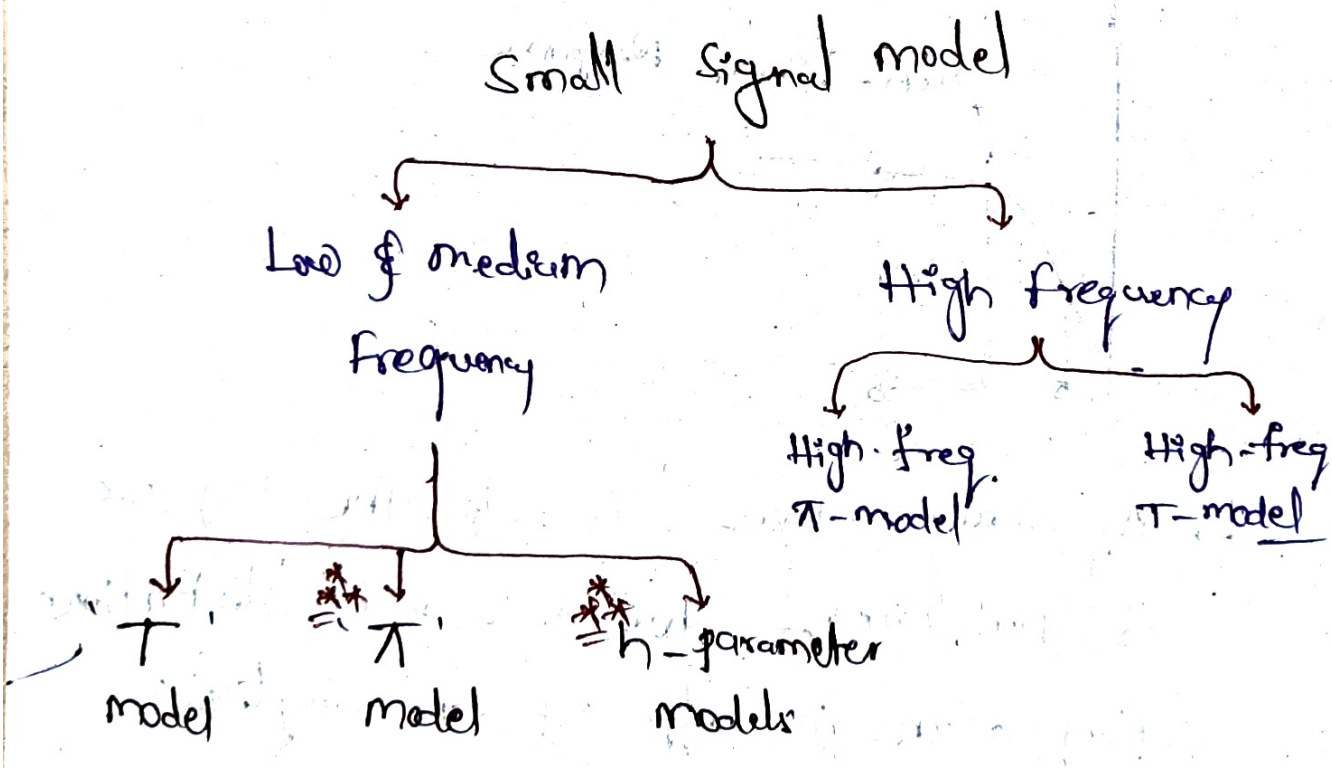
↳ As shown above, @ o/p the response is 'Non-Linear' due to which 'distortions' may occur in o/p of an amplifier

{just like Power-Amplifiers}

* BJT - Transistor Modelling :-

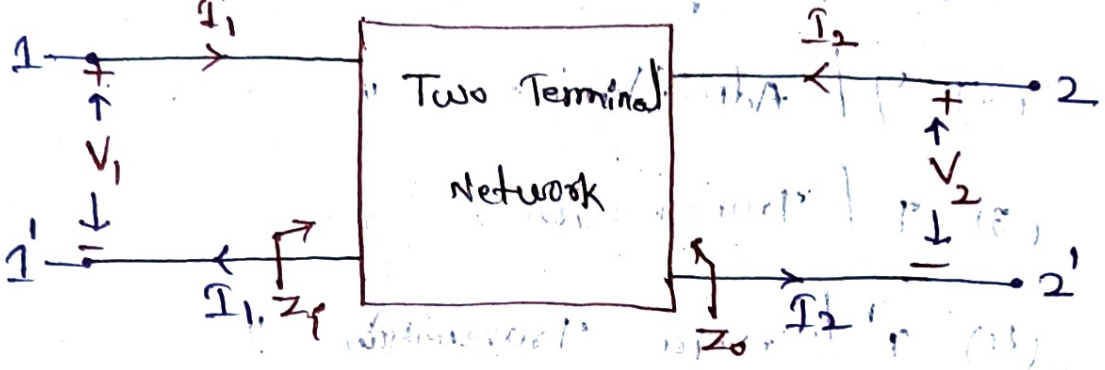
↳ A 'Model' is the combination of 'Circuit Elements' that are properly chosen, that best approximates the Actual Behaviour of a semiconductor device Under specific operating Conditions

↳ The following classification gives different modelling techniques under Small-Signal Analysis



* Before going in detail, let us recall the concepts of 'Two-Port Nlw's' where BJT can be analysed to calculate different parameters

* Two-Port Networks :-



↳ The above fig is a two port n/w with 1-1' as one port and 2-2' as another

↳ in each port current enters through one terminal and leaves through another terminal of same port

↳ There are four variables shown ie. 'V1' ; 'V2' ; 'I1' and 'I2'

↳ To analyse any two port n/w / device like 'BJT', the best practice is keeping Two variables as "Independent variables" and other left two as "Dependent variables"

↳ Hence out of '4'; 2 are dependent / Independent we get '$4C_2 = 6$' combinations which

called two port n/w parameters. They are

(1) Z / Impedance Parameters

(2) Y / Admittance "

(3) T / Transmission "

(4) T' / Inverse Transmission "

(5) g parameters

(6) h- Parameters

Hence, among above two port models choose the 'one' which best suits to analyse general characteristics of device.

We, here, for BJT we use 'hybrid parameter / h-parameter model'

* The Hybrid - Equivalent - model / h-model

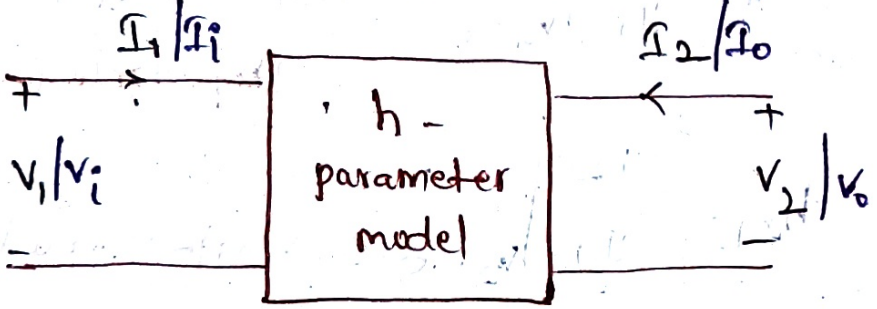
↳ Here, under this model we consider

V_1 & I_2 ⇒ Dependant variables

I_1 & V_2 ⇒ Independant "

↳ The coefficients of independant variables I_1 & V_2 are called 'h-parameters'.

↳ Consider following general - hybrid-equivalent model & set of equations which can be used to represent all 4-variables.



$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{——— (1)}$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{——— (2)}$$

↳ it can be also represented in matrix form

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

* Determining h-parameters :-

step ①: Short Circuit the output Port

↳ hence we get $V_2 = 0$ (∵ shorted)

∴ from eq ① and ②

$$V_1 = h_{11} I_1 \Rightarrow \boxed{h_{11} = \frac{V_1}{I_1}} \ (\Omega)$$

$$I_2 = h_{21} I_1 \Rightarrow \boxed{h_{21} = \frac{I_2}{I_1}} \ (\text{no units})$$

↳ i.e.

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} = \text{short circuited input impedance } (\Omega) \Rightarrow$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = \text{short circuited current gain}$$

step ②: Open Circuit Input Port

↳ here, $I_1 = 0$ (∵ opened)

∴ from eq ① and ②

$$V_1 = h_{12} V_2 \Rightarrow \boxed{h_{12} = \frac{V_1}{V_2}} \ (\text{No units})$$

$$I_2 = h_{22} V_2 \Rightarrow \boxed{h_{22} = \frac{I_2}{V_2}} \ (\mho)$$

i.e

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \text{Reverse voltage Gain}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \text{open circuited output-admittance (mhos - } \mho \text{)}$$

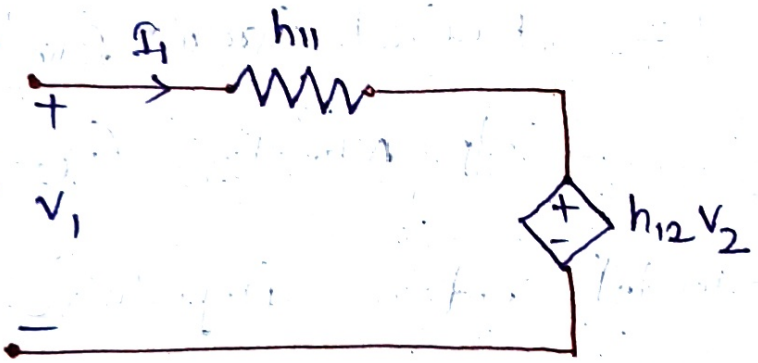
↓ seimen

* Now try to build equivalent h-model, based on above parameter definitions

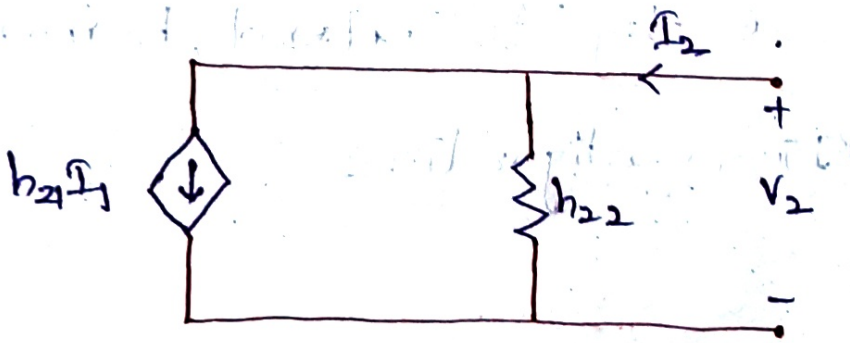
* h-parameter - Equivalent Two Port Network

↳ from eq. ① and ② & h-parameters above

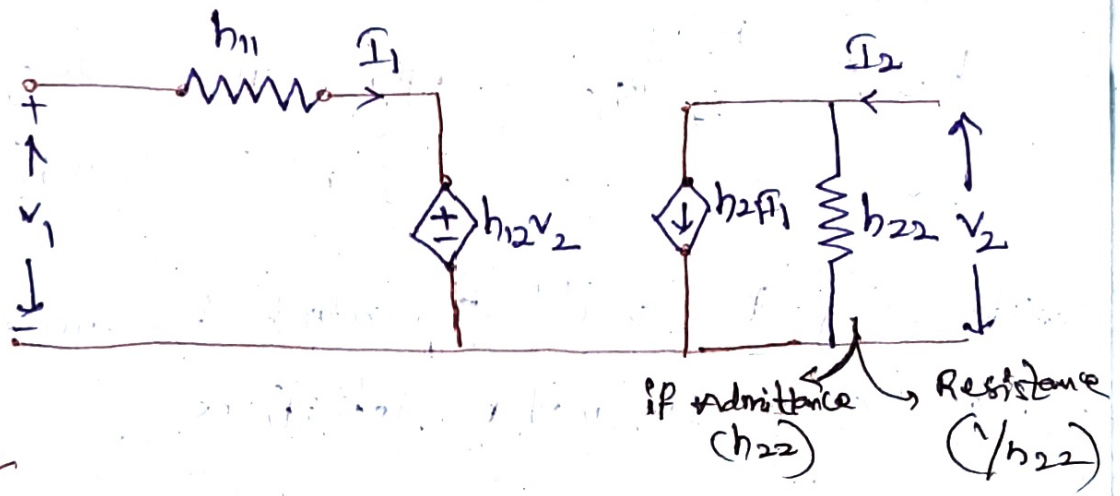
Eq ① can be modeled as follows



Eq ② can be modelled as follows



Together from eq ① and ②, the hybrid model is given as ..



Where

$h_{11} = h_i$ = i/p impedance — Ω

$h_{12} = h_r$ = Reverse Voltage Gain —

$h_{21} = h_f$ = forward Current Gain/Transfer ratio

$h_{22} = h_o$ = o/p Admittance (℧/mho/seimen)

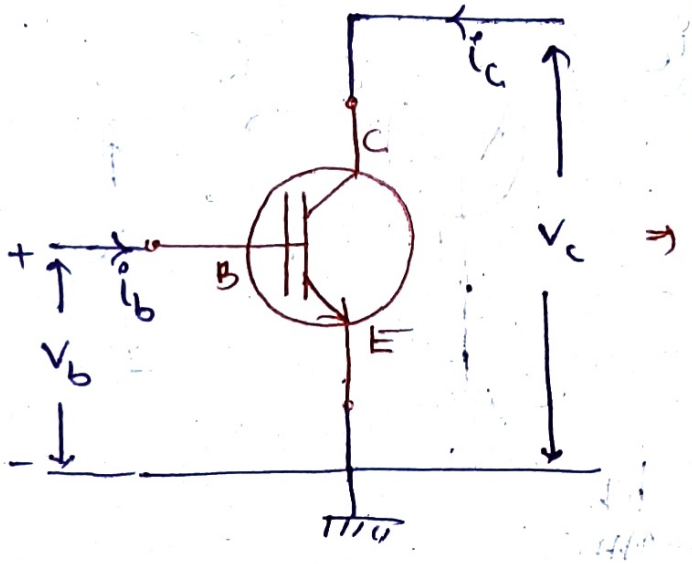
* As h-model contains impedance, admittance Av & Ai ... etc known as hybrid-model.

* Now, we try to extend, h-model concept to BJT-configurations.

* BJT Configurations & h-parameter Equivalent models :- (Simple Circuits)

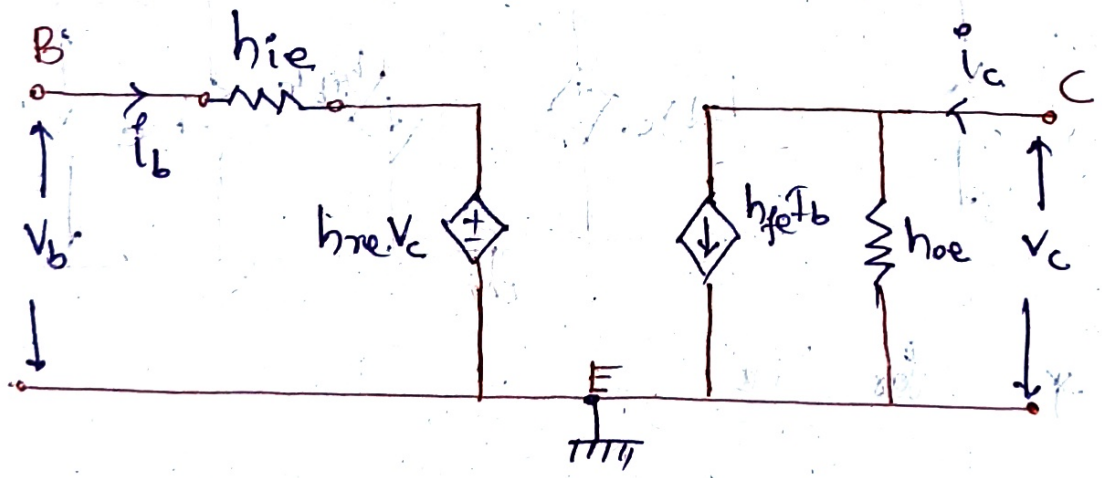
① CE - Configuration :-

*



i/p	Common	o/p
B	E	C

*



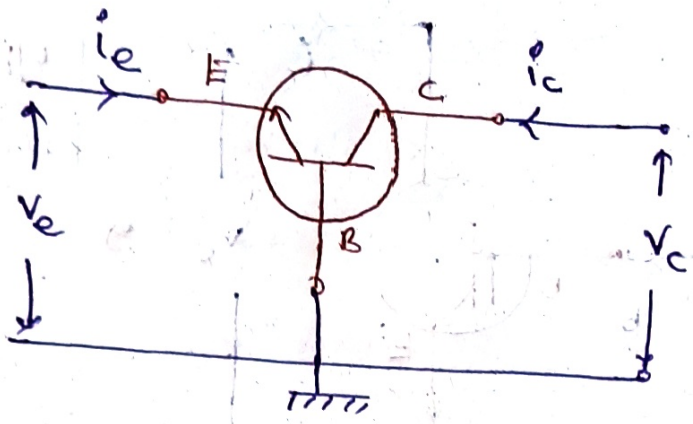
* The h-model equations for CE - are

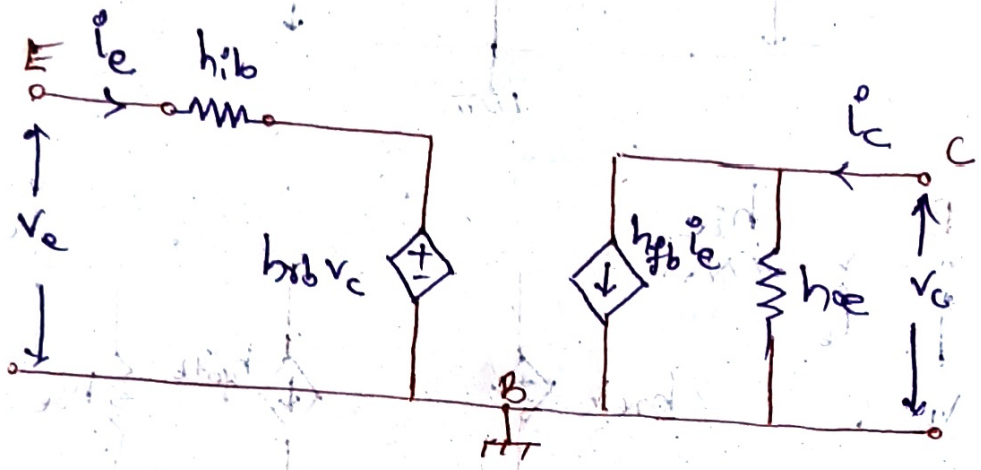
$$V_b = h_{ie} i_b + h_{re} V_c$$

$$i_c = h_{fe} i_b + h_{oe} V_c$$

② CB - Configuration :-

i/p	common	o/p
E	B	C





for 'CB'

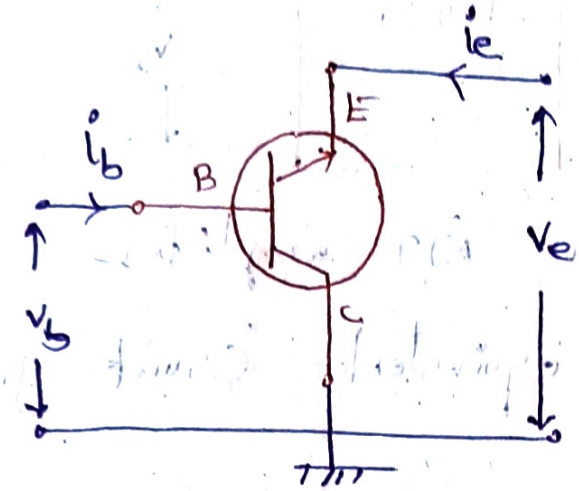
$$V_e = h_{ib} i_e + h_{rb} V_c$$

$$i_c = h_{fb} i_e + h_{oe} V_c$$

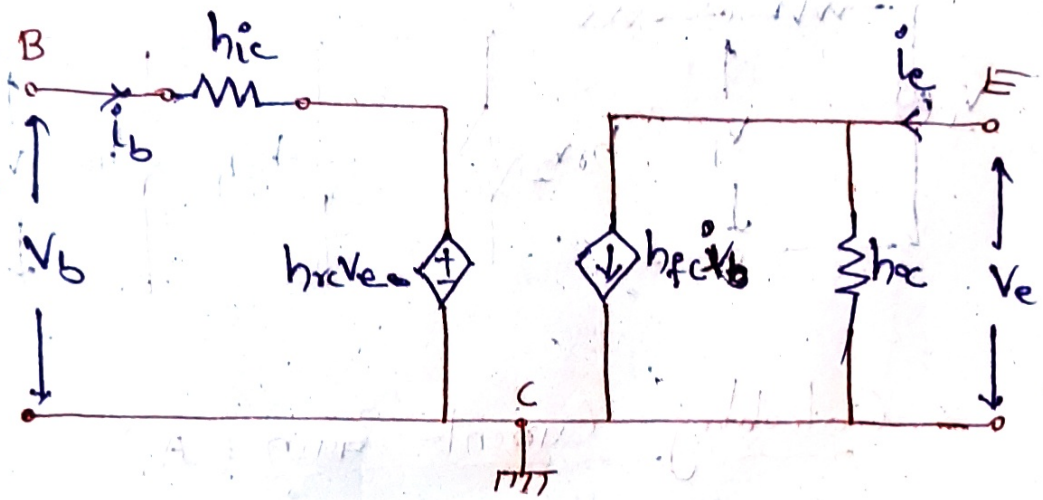
③ CC - configuration :

i/p	Common	o/p
B	C	E

*



*



* for CC

$$V_b = h_{ic} i_b + h_{rc} V_e$$

$$i_e = h_{fc} i_b + h_{oc} V_e$$

* Analysis of BJT - Amplifier using h-model

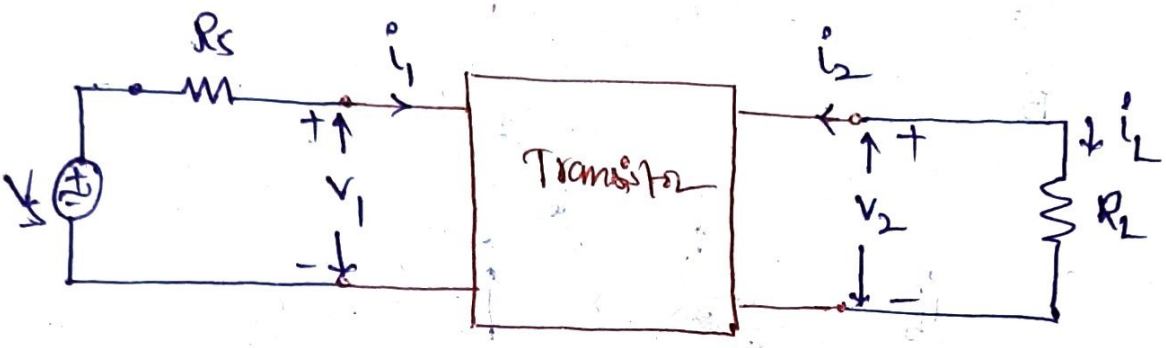
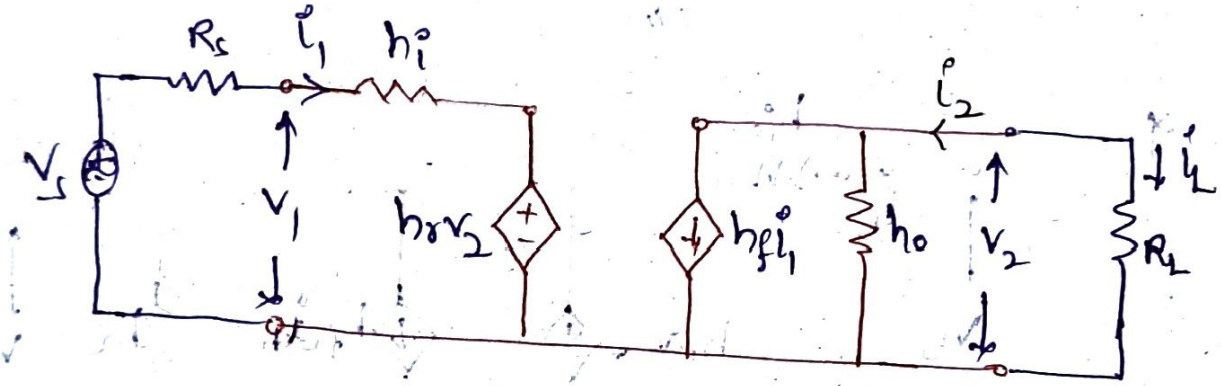


Fig: Basic BJT amplifier

* The h-model equivalent circuit given as



* Calculating Current Gain (A_I)

we have, $A_I = \frac{i_L}{i_1}$

$$= \frac{-I_2}{i_1} \quad \text{---(1)}$$

Now, from above circuit,

$$i_2 = h_f i_1 + h_o V_2 \quad \text{---(2)}$$

but $V_2 = i_L R_L = -i_2 R_L$ ---(3)

substitute (3) in (2)

$$\therefore i_2 = h_f i_1 + h_o(-i_2 R_L)$$

$$\therefore i_2(1 + h_o R_L) = h_f i_1$$

$$\therefore \frac{i_2}{i_1} = \frac{h_f}{1 + h_o R_L}$$

But from ①

$$A_I = \frac{-i_2}{i_i} = \frac{-h_f}{1 + h_o R_L}$$

* Calculating i/p impedance (Z_i)

We have, $Z_i = \frac{v_i}{i_i} = \frac{v_1}{i_1}$ —①

But, from above circuit—

$$v_1 = h_i i_1 + h_r v_2$$

$\therefore$ from ①,

$$Z_i = \frac{h_i i_1 + h_r v_2}{i_1}$$

$$Z_i = h_i + \frac{h_r v_2}{i_1} \quad \text{—②}$$

But $V_2 = -i_2 R_L = A_I \cdot i_1 \cdot R_L$

$(\because -i_2 = A_I i_1)$

$\therefore$ from (2),

$$Z_i = h_i + \frac{h_r (A_I \cdot i_1 \cdot R_L)}{i_1}$$

$$\boxed{Z_i = h_i + h_r \cdot A_I \cdot R_L} \quad \text{(or)}$$

substitute 'A_I' value,

$$Z_i = h_i + h_r \cdot R_L \cdot \left(\frac{-h_f}{1 + h_o R_L}\right)$$

$$\boxed{Z_i = h_i - \frac{h_f \cdot h_r}{\left(\frac{1}{R_L}\right) + h_o}} \quad \text{(or)}$$

$$\boxed{Z_i = h_i - \frac{h_f h_r}{Y_L + h_o}}$$

where $Y_L = \frac{1}{R_L}$ = admittance.

* Calculating Voltage Gain / Amplification factor

we have, $A_V = \frac{V_2}{V_1}$

but $V_2 = -i_2 R_L = A_I \cdot i_1 \cdot R_L$

$\therefore A_V = \frac{A_I \cdot i_1 \cdot R_L}{V_1}$

$$\boxed{A_V = \frac{A_I \cdot R_L}{Z_i}}$$

* Calculating output Admittance (Y_o)

we have, $Y_o = \left.\frac{i_2}{V_2}\right|_{V_S=0 \;;\; R_L=\infty}$

from fig

$$i_2 = h_f i_1 + h_o V_2$$

devide above eq. by V_2

$$\therefore \frac{i_2}{V_2} = h_f \frac{i_1}{V_2} + h_o \quad \text{---(1)}$$

But with $V_S = 0$, by KVL @ i/p circuit

$$R_S i_1 + h_i i_1 + h_r V_2 = 0$$

$$\therefore (R_S + h_i) i_1 = -h_r V_2$$

$$\therefore \frac{i_i}{V_2} = \frac{-h_r}{R_s + h_i} \quad \text{— (2)}$$

$\therefore$ from (1) and (2),

$$\frac{i_2}{V_2} = h_f\left(\frac{-h_r}{R_s + h_i}\right) + h_o$$

$$\therefore \boxed{Y_o = h_o - \frac{h_f h_r}{R_s + h_i}}$$

Note: for CE

1) $A_I = \dfrac{-h_{fe}}{1 + h_{oe} R_L}$

2) $Z_i = h_{ie} - \dfrac{h_{fe} h_{re}}{Y_L + h_{oe}}$

3) $A_V = \dfrac{A_I \cdot R_L}{Z_i}$

4) $Y_o = h_{oe} - \dfrac{h_{fe} h_{re}}{R_s + h_{ie}}$

(5) $h_{fe} = \dfrac{i_c}{i_b} = \beta$ (or) $i_c = h_{fe} i_b = \beta i_b$

* Approximated h-model :-

↳ Since $h_r = \frac{V_1}{V}$ is relatively very small

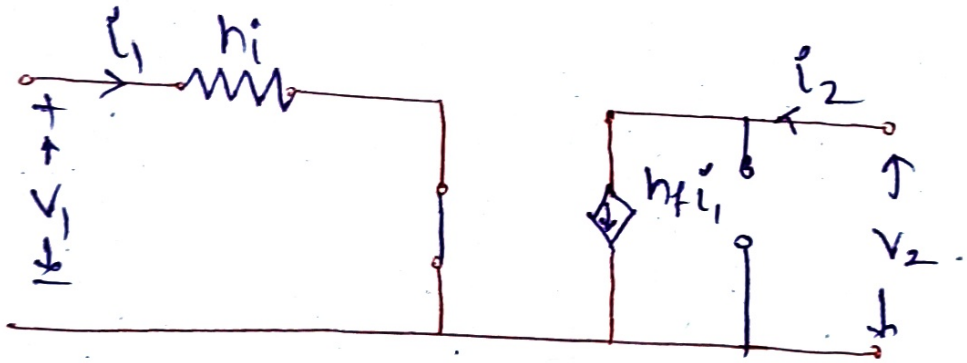
i.e. $h_r \cong 0 \Rightarrow h_r V_2 \cong 0 \Rightarrow$ shorted.

↳ And @ o/p

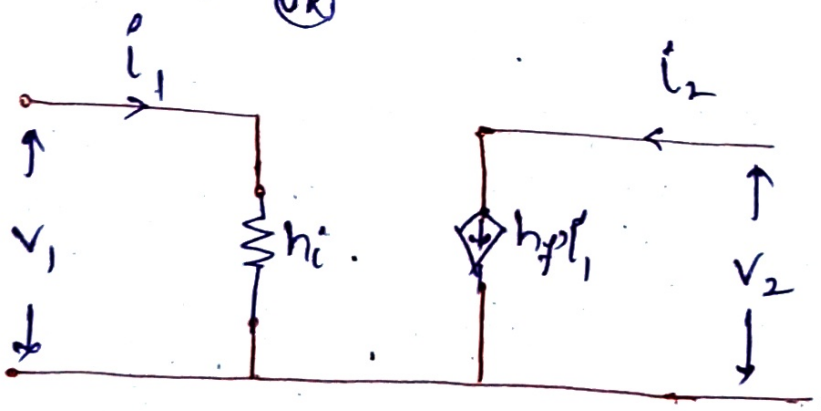
h_o = admittance, comparatively small, but resistance $= \frac{1}{h_o}$ which is very large i.e

$R = \frac{1}{h_o} \cong \infty \Rightarrow$ opened //

↳ Hence the approximated h-model is



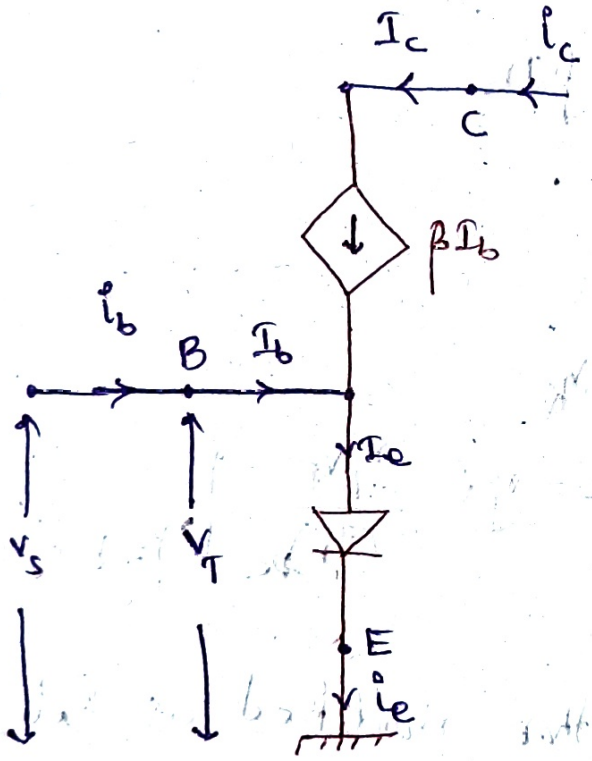
(OR)



Other Different models used for BJT small signal Analysis :-

* r_e model / T-model :-

↳ at first it has given diode equivalent model which shows below (consider CE)



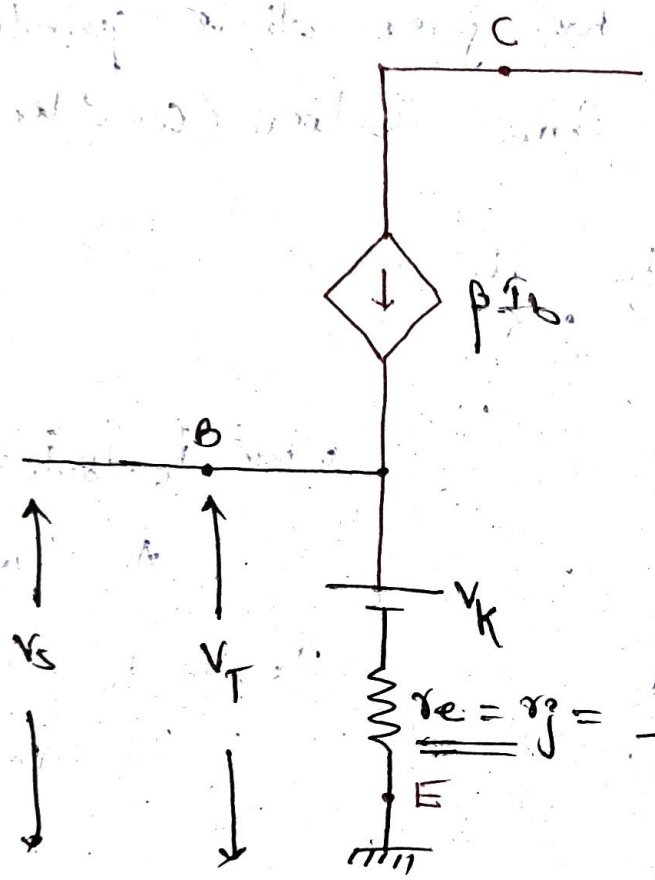
where i_c, i_b, i_c = pure AC signals

I_E, I_B, I_C = 'DC' in AC signals (Avg)

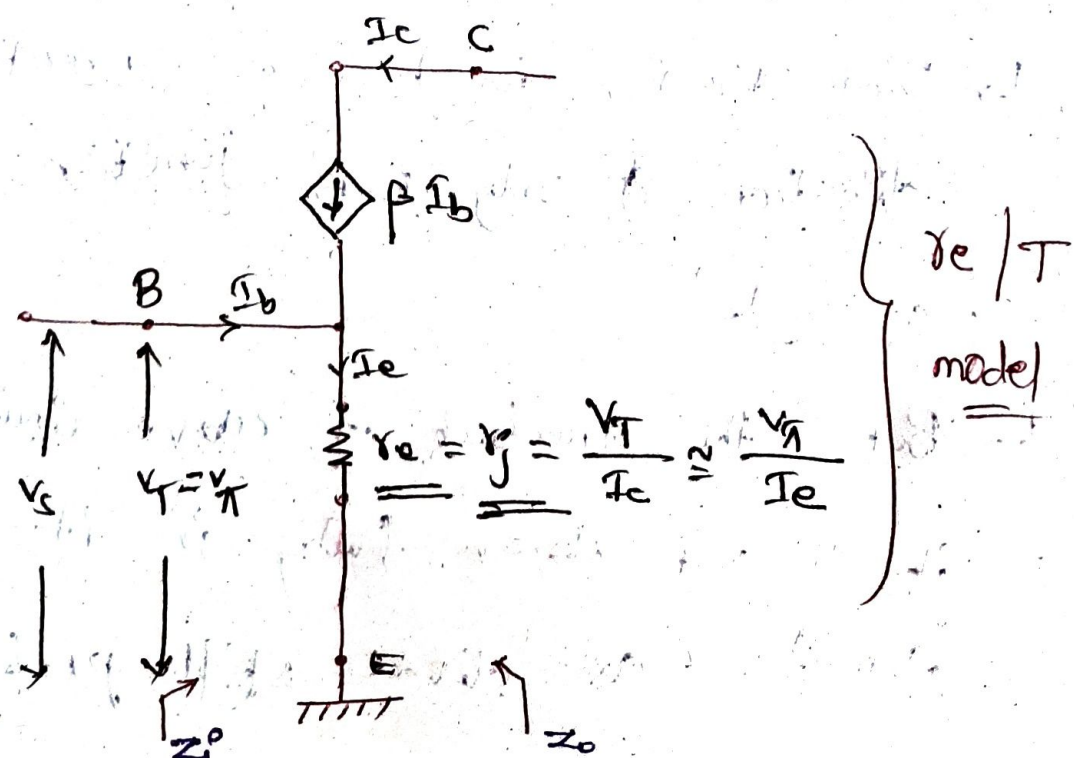
↳ Diode direction is shown as conventional direction of only 'E-B' junction can be a 'diode'

↳ But, the drawback of above circuit is it is not shown fully in its equivalent elements (as diode still present)

↳ The solution is, simply draw the diode equivalent circuit to make it more linear / Lumped n/w. i.e.



↳ This can be further modified as below



↳ The above model is known as "T-model" or "re- model" of BJT

↳ Here, $\boxed{r_e = \frac{1}{g_m}}$ and for more linearity $g_m = \frac{1}{26}\ (A/V)$

$g_m = \frac{I_C}{V_T}$

$r_e = \frac{1}{g_m} = \frac{V_T}{I_C}$

Note: By miller

* Above 're' model consists overlapped Emitter and collector. i.e. 're' is common for both 'i/p' and 'o/p' & need to be seperated and need to be placed at i/p side & op side

Hence, we have

$$Z_i = \frac{V_s}{I_b} = \frac{I_e \cdot r_e}{I_b}$$

$$= \frac{(I_B + I_c) \cdot r_e}{I_b}$$

$$= \frac{I_b\left(1 + \frac{I_c}{I_b}\right) \cdot r_e}{I_b}$$

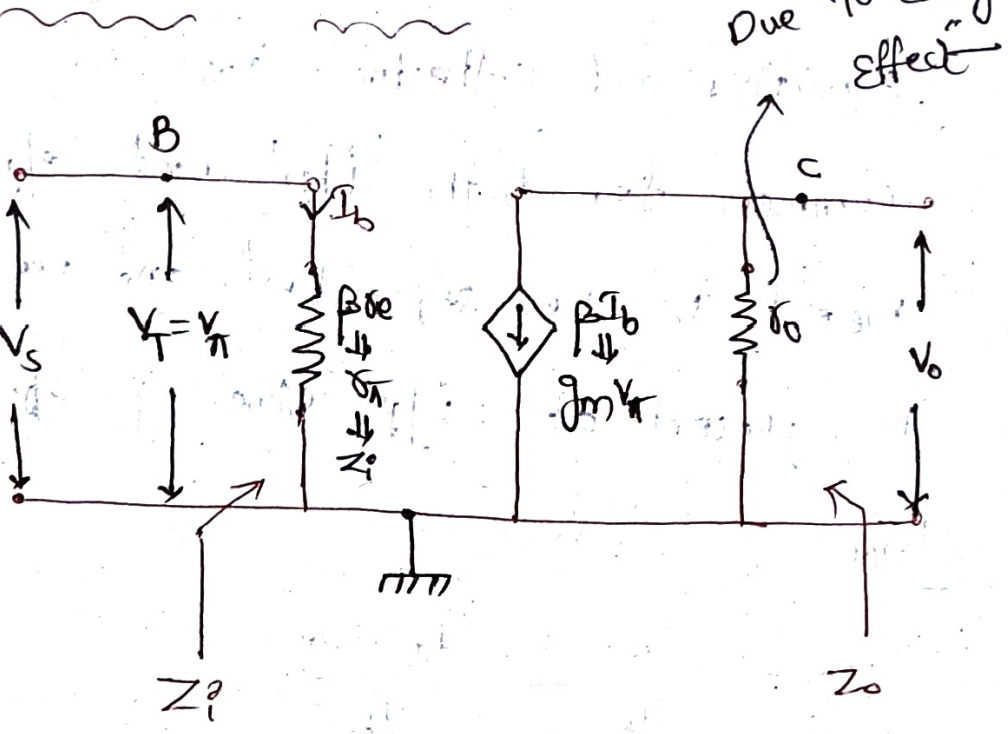
$$\boxed{Z_i = (1+\beta)\, r_e \cong \beta r_e}$$

* From above,

$$\boxed{Z_i = r_\pi = \beta r_e}$$

And the equivalent model is given & named as "r_π - model", which is given below

* r_π - model of BJT



↳ Here under 'r_π - model' 'BJT' is represented in its equivalent model completely

↳ $g_m = \frac{I_c}{V_T} = \frac{\beta I_b}{V_T}$

$\therefore \boxed{g_m V_T = \beta I_b}$ (or) $\boxed{g_m V_\pi = \beta I_b}$

↳ $Z_i = r_\pi = \beta r_e$

↳ Voltage Gain (A_V)

$$A_V = \frac{V_o}{V_i} = \frac{-\beta I_b \cdot Z_o}{I_b Z_i}$$

$$= \frac{-\beta r_o}{\beta r_e}$$

$$\boxed{A_V = \frac{-r_o}{r_e}}$$

↳ Current Gain (A_I)

$$A_I = \frac{I_o}{I_i} = \frac{I_c}{I_b} = \frac{\beta I_b}{I_b}$$

$$\therefore \boxed{A_I = \beta}$$

* $Z_o = r_o$

* $r_e = \frac{V_\pi}{I_e} = \frac{V_T}{I_e} = \frac{26mV}{I_e}$

where '$-$' indicates 180° - Phase shift for 'CE'. (As above analysis is for CE)

Note:

↳ here i/p impedance (Z_i) = Low

o/p " (Z_o) = High

hence it is not advisible for Voltage amplification & good for 'Current' amplification.

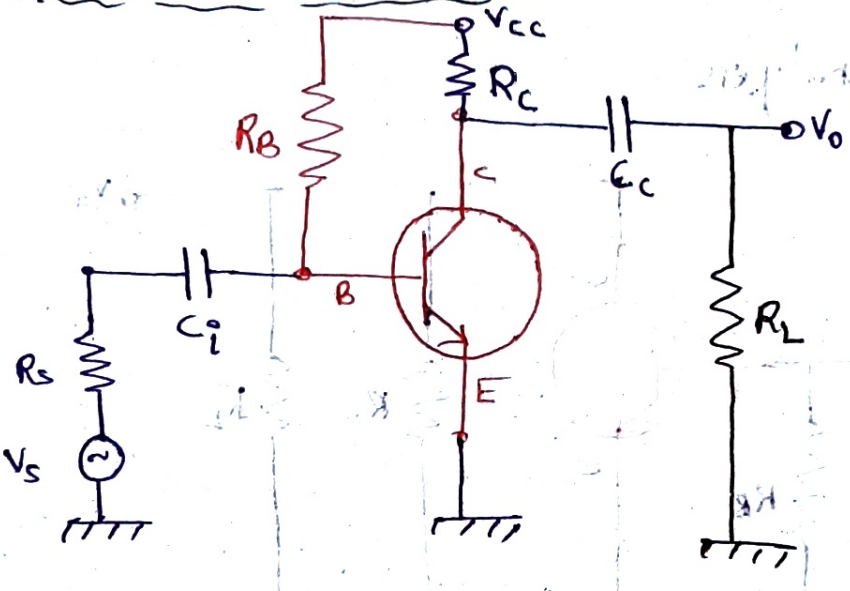
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* BJT - Small signal Analysis of stability of Biasing Techniques :-

① Small Signal - Low frequency - RC coupling single stage - Fixed Bias / Base Bias CE - Amplifier :-

(OR)

Fixed Bias Circuit -



* AC - Equivalent - model

in order to draw AC - equivalent model

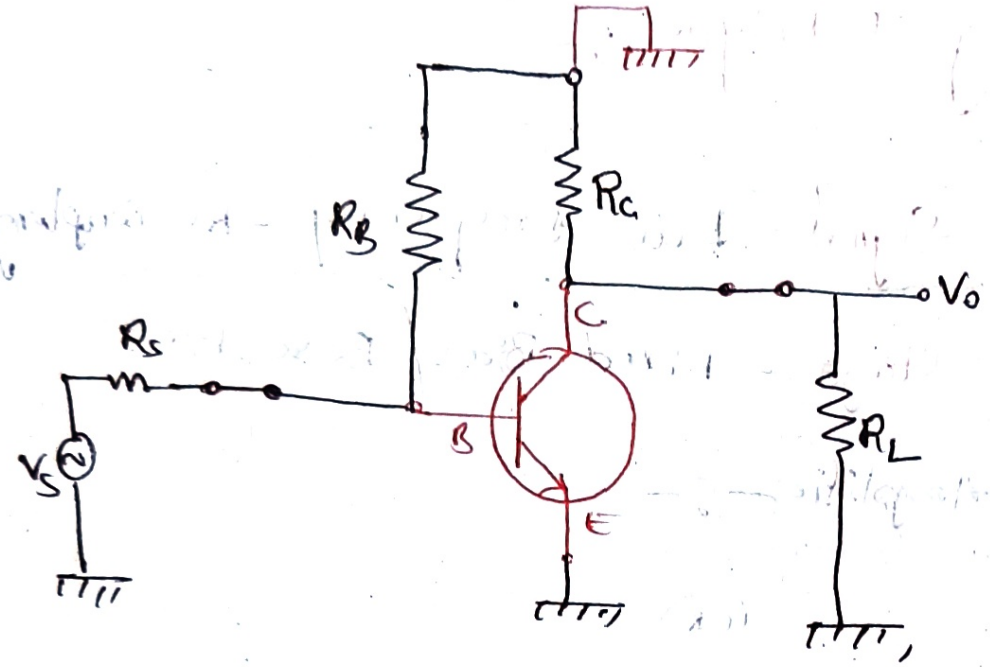
↳ All the DC - sources ⇒ made in active i.e

DC Voltage source = shorted

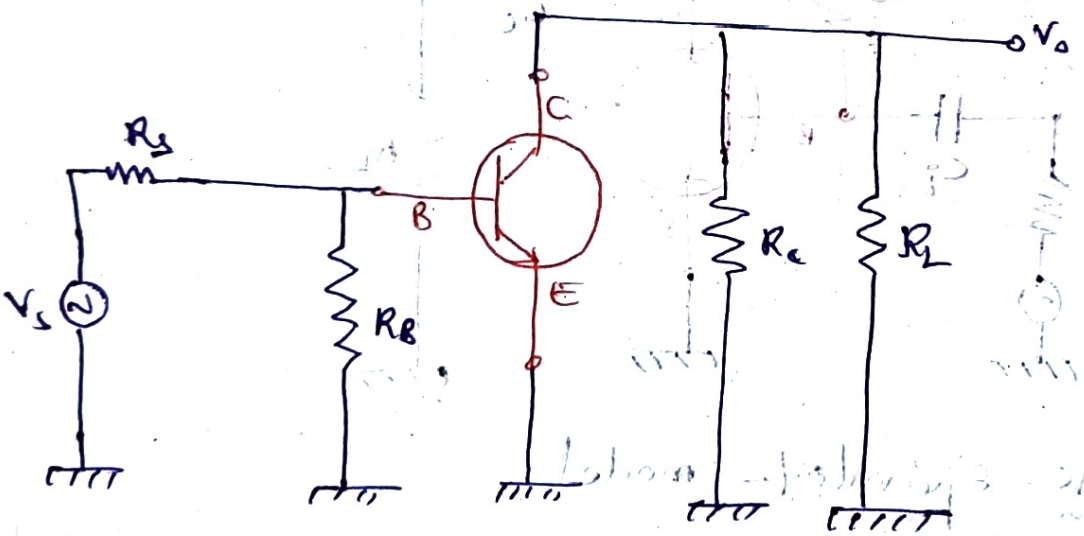
DC current source = opened

↳ All the Capacitors ⇒ shorted

hence the circuit will be;



↳ Now re-draw above circuit like below for better analysis



Now,

draw the small signal - r_π - equivalent model for BJT

i.e.

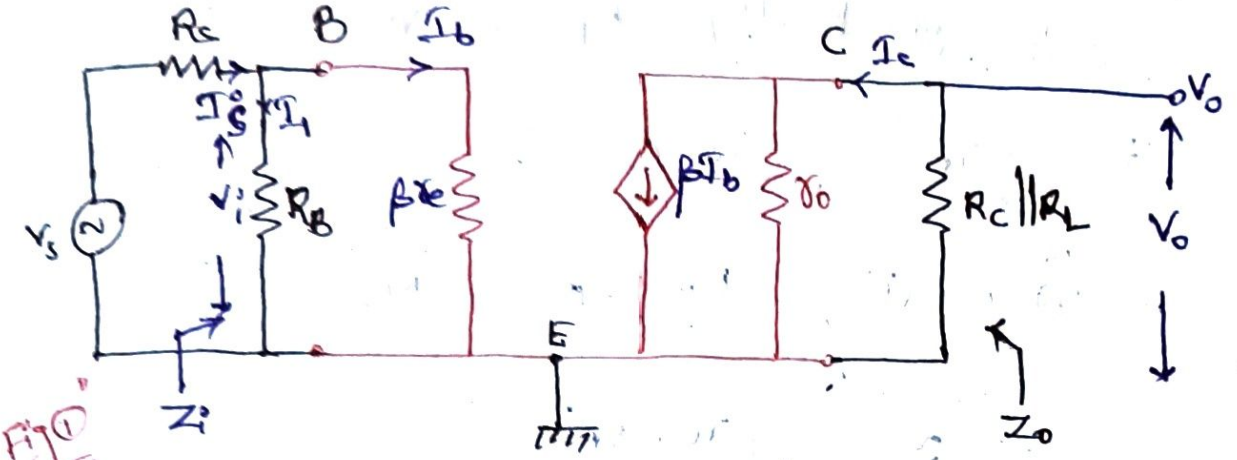


Fig ①

Now, analyse to calculate different parameters:

* (alculating Input Impedance (Z_i):

we have, $Z_i = \frac{V_i}{I_i}$ — ①

↳ But from above circuit-

$$\boxed{Z_i = R_B \,||\, \beta r_e \cong \beta r_e}$$

↳ (OR)

we have, $V_i = (I_b)(\beta r_e)$ — ②

but $I_b = I_i \times \frac{R_B}{R_B + \beta r_e}$ — ③ } current division

substitute eq. ③ in ②, we get

$$V_i = I_i \cdot \left(\frac{R_B}{R_B + \beta r_e}\right) \cdot (\beta r_e)$$

$$\therefore \boxed{\frac{V_i}{I_i} = Z_i = \frac{\beta r_e \cdot R_B}{R_B + \beta r_e}} \quad — ④$$

But practically, R_B is very large compared to 'βr_e', hence

from ④, $R_B + \beta r_e \cong R_B$

$\therefore Z_i \cong \dfrac{\beta r_e \cdot R_B}{R_B}$

$\therefore \boxed{Z_i = \beta r_e}$

$r_e = \dfrac{V_T}{I_C} = \dfrac{V_T}{I_E} = \dfrac{26mV}{I_E}$

* Calculating output Impedance (Z_o / R_o)

↳ from fig ①, above

$Z_o = r_o \,||\, R_C \,||\, R_L$

$\boxed{Z_o \cong R_C \,||\, R_L}$ ($\because r_o$ is high)

↳ (OR)

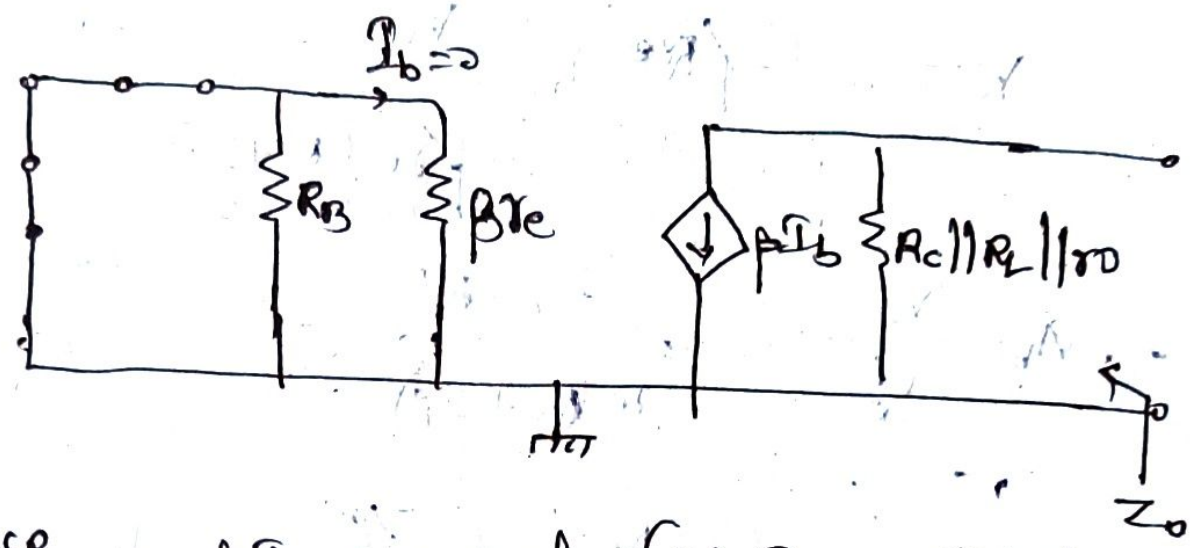
calculate 'Z_o' as thevenins equivalent impedance, hence

to calculate 'Z_o' make all independent sources 'inative' i.e.

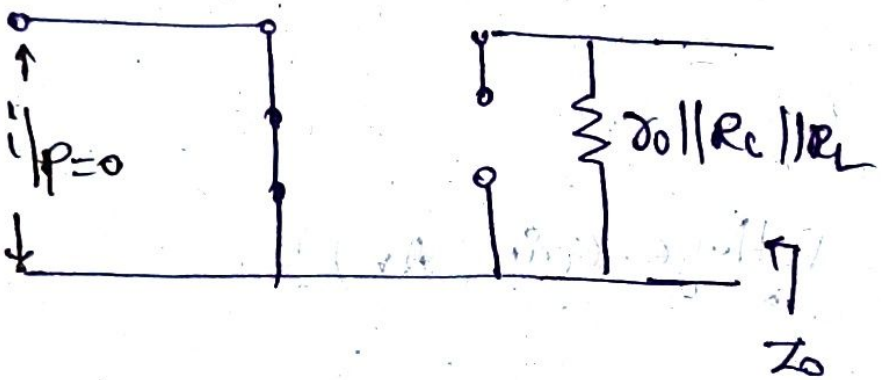
voltage sources ⇒ shorted

current sources ⇒ opened

↳ hence from fig ①



Hence, βI_b = opened ($\because I_b = 0$ @ i/p side)



$\therefore \quad Z_o = r_o \| R_c \| R_L$

$\cong R_c \| R_L$

$$\boxed{Z_o \cong R_c} \quad (\because R_L >> R_c)$$

Calculating voltage Gain (Av) :-

↳ From fig ①,

$$A_{VS} = \frac{V_o}{V_S}$$

$$= \frac{V_o}{V_i'} \times \frac{V_i'}{V_S} \quad \text{— ①}$$

But, $V_o = -I_C \{(r_o \| R_C \| R_L)\}$ i.e. $-I_C Z_o$

$$\cong -I_C . R_C$$

$$\therefore V_o = (-\beta I_b) R_C \quad (\because I_c = \beta I_b)$$

— ②

But from input side,

$$I_b = \frac{V_i}{\beta r_e}$$

∴ from eq. , (2)

$$V_o = (-\beta R_c)\left(\frac{V_i}{\beta r_e}\right)$$

$$\therefore \boxed{\frac{V_o}{V_i} = A_v = \frac{-R_c}{r_e}} \quad \rightarrow (3)$$

⇒ Voltage Gain without "R_s"

Now

$$V_i = V_s \left\{\frac{Z_i}{Z_i + R_s}\right\} \quad (\because \text{from eq } (1))$$

(voltage division)

$$\therefore \frac{V_i}{V_s} = \frac{Z_i}{Z_i + R_s} \quad \rightarrow (4)$$

Substitute (3) and (4) ~~from~~ in (1)

$$\therefore \boxed{A_{vs} = \frac{-R_c}{r_e}\left(\frac{Z_i}{Z_i + R_s}\right)}$$

180° + Phase shift

* Calculating Current-Gain (A_I):

we have $A_I = \frac{I_o}{I_s} \simeq \frac{I_L}{I_s}$ —— (1) $= \frac{I_L}{I_i} \times \frac{I_i}{I_s}$

Now,

$I_L = I_o = I_c \cdot (r_o \| R_c) / \{(r_o \| R_c) + R_L\}$ Current dividing

$= (\beta I_b) \cdot (r_o \| R_c) / (r_o \| R_c + R_L)$

$\therefore \frac{I_o}{I_b} = \frac{\beta (r_o \| R_c)}{r_o \| R_c + R_L}$ —— (2)

But $I_b = I_i \left(\frac{R_B}{R_B + \beta r_e}\right)$ —— (3)

Substitute eq. (3) in (2)

$\therefore I_o = \frac{\beta (r_o \| R_c)}{(r_o \| R_c + R_L)} \left\{ I_i \frac{R_B}{R_B + \beta r_e} \right\}$

$\therefore \frac{I_o}{I_i} = A_{I_1} = \frac{\beta R_B (r_o \| R_c)}{(R_B + \beta r_e) \cdot (r_o \| R_c + R_L)}$

$$A_{I_1} = \frac{\beta \cdot R_B (r_o \| R_c)}{(R_B + \beta r_e)(r_o \| R_c \| R_L)}$$

(But $r_o > 10 R_c$ & $R_B > 10 \cdot \beta r_e$)

i.e. r_o & R_B are very large

$$\therefore A_{I_1} = \beta \cdot \left[\frac{R_B}{R_B + \beta r_e}\right]\left[\frac{r_o \| R_c}{r_o \| R_c + R_L}\right]$$

$$\simeq \beta \cdot \left[\frac{r_o}{r_o + R_L}\right] \simeq \beta$$

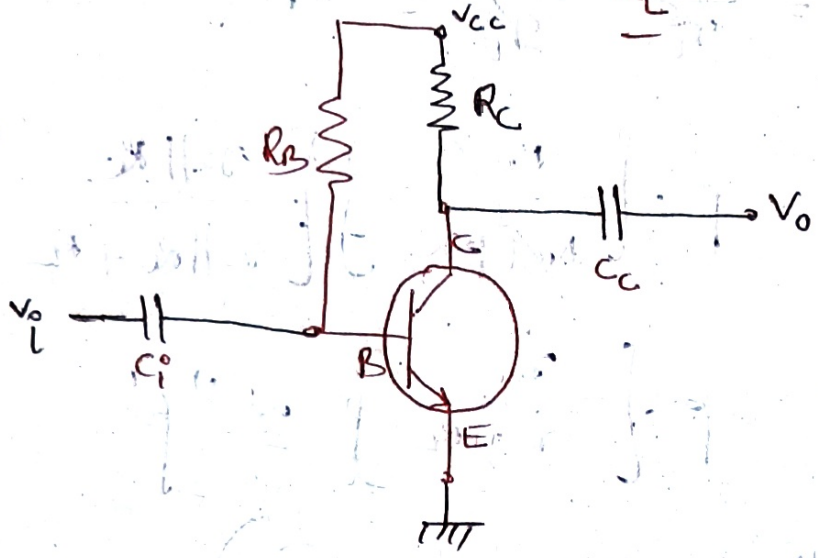
$\therefore$ over all A_I (i.e. from (1))

$$A_I = A_{I_1} \times \left(\frac{R_S}{R_S + Z_i}\right)$$

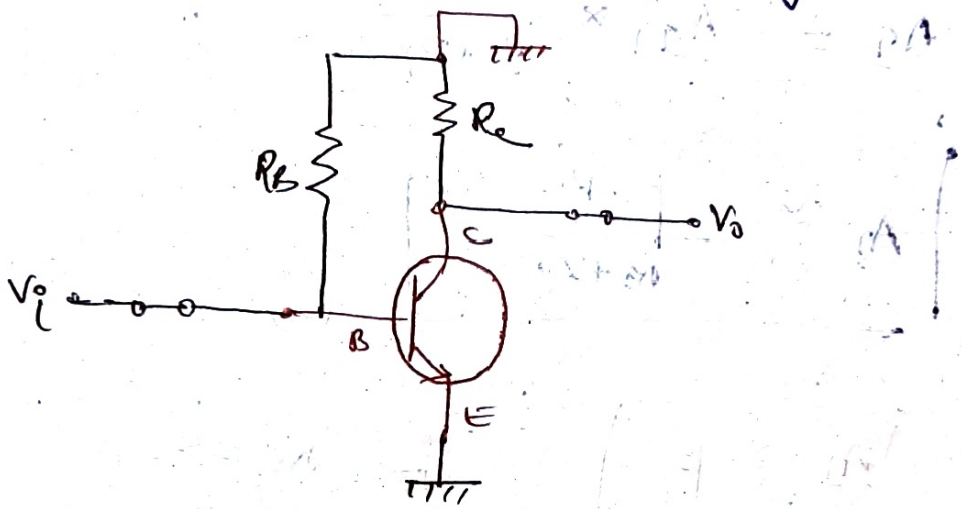
$$\boxed{A_I \simeq \frac{\beta \cdot R_S}{R_S + Z_i}} \quad \text{(or)}$$

$$\boxed{A_I \simeq \beta} \quad (\because R_S >> Z_i)$$

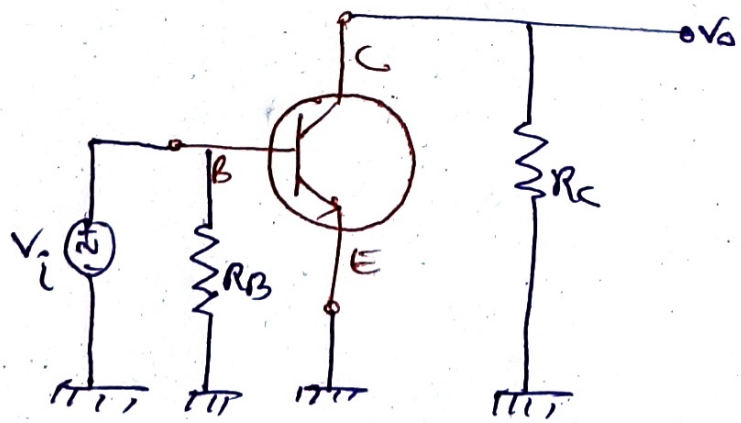
* Fixed Bias without "R_s" & "R_L" (simple Circuit)



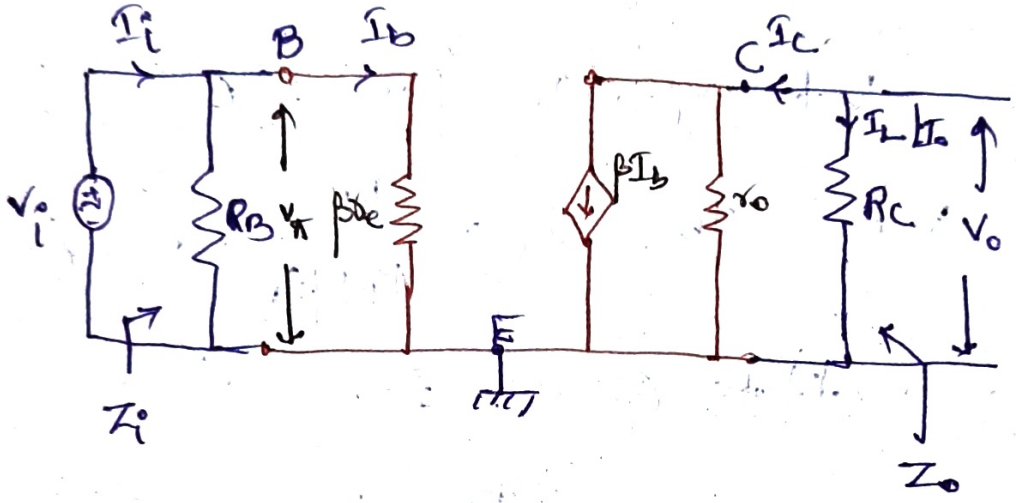
↳ AC equivalent model is given below.



The above circuit can be re-drawn as



* Now, draw the small signal equivalent model for BJT



* Calculating "Z_i"

By the observation, $Z_i = R_B \| \beta r_e$

$$Z_i \cong \beta r_e \Omega \quad (\because R_B > 10.\beta r_e)$$

(OR)

* $Z_i = \dfrac{V_i}{I_i}$ — (1) $\Rightarrow \dfrac{V_\pi}{I_i}$

but $V_i = (I_b)(\beta r_e)$ — (2) (from circuit)

and $I_b = I_i \cdot \left(\dfrac{R_B}{R_B + \beta r_e}\right)$ ($\because$ Current division)

$I_b \cong I_i$ ($\because R_B > 10\beta r_e$, $R_B + \beta r_e \cong R_B$)

$\therefore$ from (2) $V_i = I_i(\beta r_e)$

$\therefore$ $\boxed{Z_i = \dfrac{V_i}{I_i} = \beta r_e}$

(OR)

* $Z_i = \frac{V_i}{I_i}$ — ①

but $V_i = (I_b)\beta r_e$ — ②

but, $I_b = I_i \times \frac{R_B}{R_B + \beta r_e}$ — ③

∴ Substitute ③ in ②

$\therefore V_i = \left\{ I_i \cdot \frac{R_B}{R_B + \beta r_e} \right\} \cdot \beta r_e$

$\therefore \boxed{Z_i = \frac{V_i}{I_i} = \frac{\beta \cdot r_e \cdot R_B}{R_B + \beta r_e}}$

if $R_B >> \beta r_e$, then

$Z_i \cong \frac{\beta \cdot r_e \cdot R_B}{R_B}$

$\therefore \boxed{Z_i \cong \beta r_e}$

* Calculating "Z_o" :-

By observation, from circuit

$$Z_o = R_c \| r_o \Omega$$

$$\therefore \boxed{Z_o \cong R_c \Omega} \left(\because r_o >>> R_c\right)$$

(i.e $r_o = 10 \cdot R_c$)

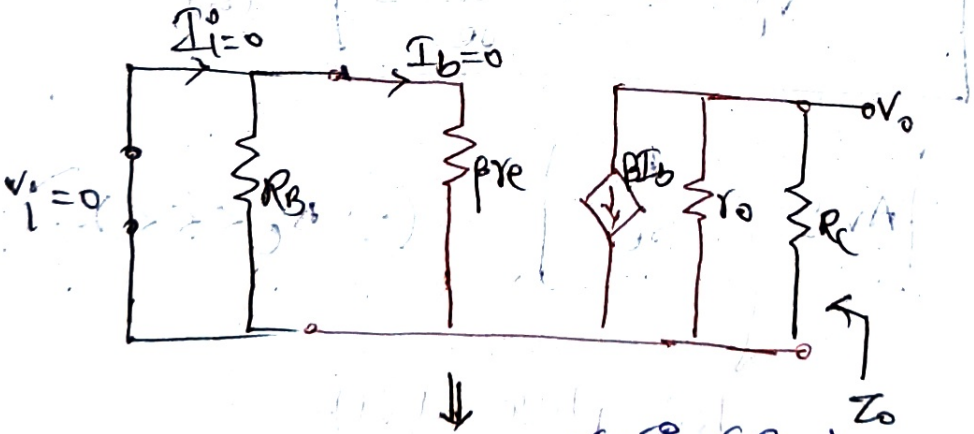
(OR)

* 'Z_o' is a Thevenin's equivalent impedance hence to calculate 'Z_o', make all independent sources in-active i.e

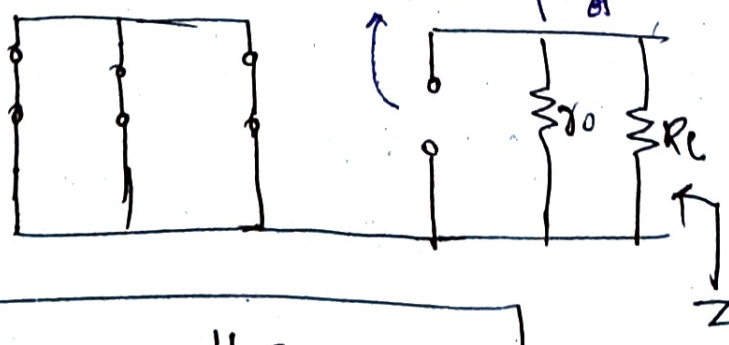
↳ voltage sources ⇒ shorted

↳ current sources ⇒ opened.

i.e. from circuit



$\because I_b = 0$ ⇒ Supply opened



$$\therefore \boxed{Z_o = r_o \| R_c \cong R_c}$$

* Calculating 'Av' :-

we have $A_V = \frac{V_o}{V_i}$ — (1)

But $V_o = I_L (R_C \| r_o) = I_o (R_C \| r_o)$

$V_o = (-\beta I_b)(R_C \| r_o)$ $(\because I_o = -I_c)$ — (2)

but $I_b = \frac{V_i}{\beta r_e}$ $(\because V_i = I_b \cdot \beta r_e)$ — (3)

substitute (3) in (2),

$\therefore V_o = -\beta \cdot (R_C \| r_o)\left(\frac{V_i}{\beta r_e}\right)$

$\therefore \boxed{A_V = \frac{V_o}{V_i} = \frac{-R_C \| r_o}{r_e}}$ (or)

$\boxed{A_V \cong \frac{-R_C}{r_e}}$ $(\because R_C << r_o)$

180° phase shift

* Calculating A_I :-

We have $A_I = \frac{I_o}{I_i}$ ——— (1)

But from Circuit above

$I_o = -I_c = -\beta I_b$ ——— (2)

but $I_b = I_i \times \frac{R_B}{(R_B + \beta r_e)}$ ——— (3)

∴ substitute (3) in (2).

∴ $I_o = (-\beta) \cdot I_i \cdot \left(\frac{R_B}{R_B + \beta r_e}\right)$

$$\therefore \boxed{A_I = \frac{I_o}{I_i} = \frac{-\beta \cdot R_B}{R_B + \beta r_e}} \quad \text{(or)}$$

$$\boxed{A_I \simeq -\beta}$$

(change current direction).

Hence assume current direction, opposite unlike shown in above fig, we get

$$\therefore \boxed{A_I \simeq \beta}$$

it is clear that the direction should be opposite for I_o (in above fig).

Hence:

$$I_o = I_c \left(\frac{r_o}{r_o + R_c}\right) \quad (\because \text{current division})$$

$$\therefore I_o = \frac{(\beta I_b) \cdot r_o}{(r_o + R_c)} \quad \text{——— (1)}$$

but $I_b = I_i \times \frac{R_B}{R_B + \beta r_e}$ ——— (2)

$\therefore$ from (1) and (2)

$$I_o = \left(\frac{\beta r_o}{r_o + R_c}\right) \left\{ I_i \cdot \frac{R_B}{R_B + \beta r_e} \right\}$$

$$\therefore \boxed{A_I = \frac{I_o}{I_i} = \frac{\beta \cdot r_o \cdot R_B}{(r_o + R_c)(R_B + \beta r_e)}} \quad \text{(or)}$$

but, $r_o \geq 10 R_c$ & $R_B \geq 10 (\beta r_e)$

$$\therefore \boxed{A_I \cong \beta}$$

(Choose 'β' large for high gains)

(OR)

$$A_I = \frac{I_o}{I_i}$$

Directly $\uparrow$ $\quad -V_o$

But $I_i = \frac{V_i}{Z_i}$ and $I_o = \frac{-V_o}{R_c}$

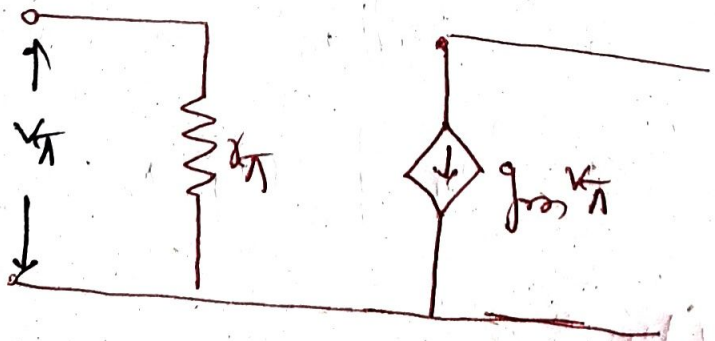
$$\therefore A_I = \frac{-V_o/R_c}{(V_i/Z_i)}$$

$$= \left(-\frac{V_o}{V_i}\right)\left(\frac{Z_i}{R_c}\right)$$

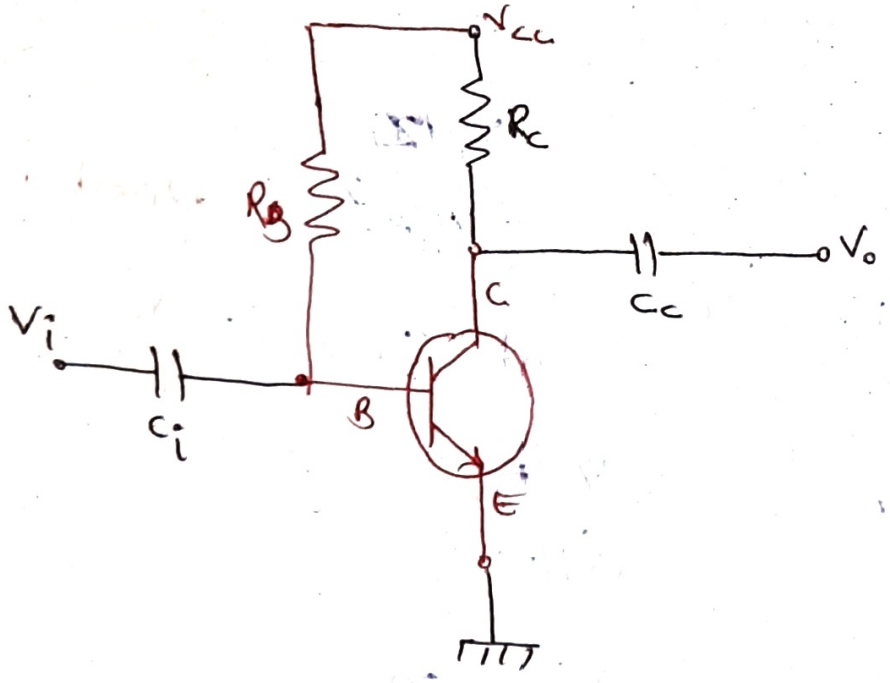
$$\therefore \boxed{A_I = -A_v \cdot \frac{Z_i}{R_c}}$$

*** Fixed Bias Analysis using "γ_π" model

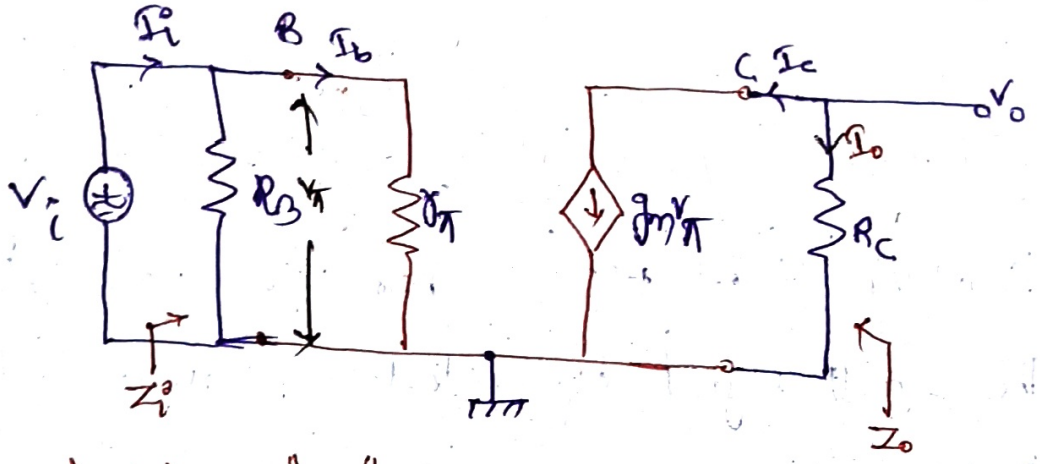
Consider ideal BJT where "$\gamma_o = \infty$", hence the small signal equivalent circuit will be given as



applying above to fixed bias circuit will be given as follows.



The r_π - Equivalent model for above fixed bias circuit is given as follows)



* Calculating "Z_i"

$$Z_i = R_B \parallel r_\pi \quad (or)$$

$$\boxed{Z_i \cong r_\pi} \quad \left(\text{if } R_B >>> r_\pi\right)$$

* Calculating "Z_o" :-

from above ckt :-

$$\boxed{Z_o = R_C}$$

* Calculating "Av":

$$A_V = \frac{V_o}{V_i} \quad \text{— (1)}$$

but $V_o = I_o R_c = -I_c \cdot R_c$

$$= -g_m V_\pi \cdot R_c \quad \text{— (2)}$$

substitute (2) in (1)

$$\therefore A_V = \frac{-g_m V_\pi \cdot R_c}{V_i}$$

but $V_i = V_\pi$ (from ckt)

$$\therefore A_V = \frac{-g_m V_\pi R_c}{V_\pi}$$

$$\therefore \boxed{A_V = -g_m R_c}$$

* Calculating A_I:

$$A_I = \frac{I_o}{I_i} = \frac{-g_m V_\pi}{I_b} = \frac{-g_m r_\pi}{I_i \cdot \left(\frac{R_B}{R_B + r_\pi}\right)}$$

$$\therefore \boxed{A_I = \frac{-g_m \cdot Z_i}{\left(\frac{R_B}{R_B + r_\pi}\right)} \simeq -g_m Z_i \simeq -g_m r_\pi}$$

$$\therefore \boxed{A_I = -g_m \cdot r_\pi}$$

Note if `ro` is considered (practical BJT)

↳ $Z_i = R_B \| r_\pi \cong r_\pi$

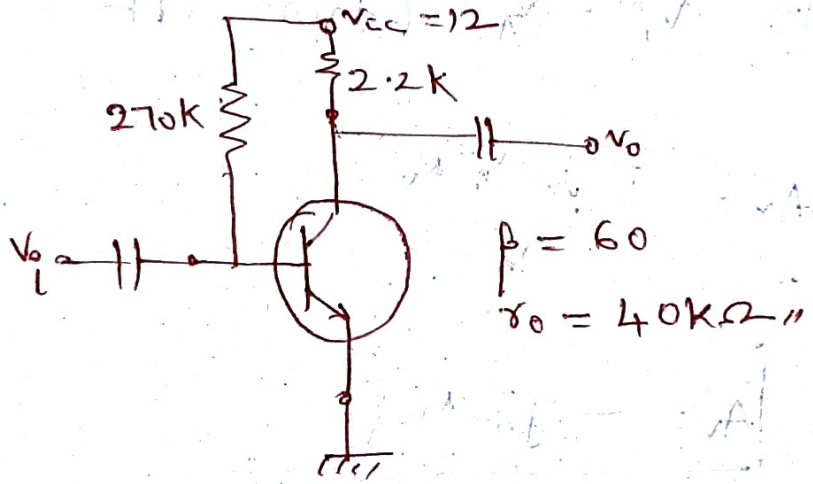
↳ $Z_o = R_c \| r_o$

↳ $A_V = -g_m (R_c \| r_o) \Rightarrow -G_m \times Z_o$

↳ $A_I = -g_m r_\pi \Rightarrow -G_m \times Z_i$

Example Problem:

* Determin, Z_i, Z_o, A_V for ckt shown



Sol: we have, $\boxed{r_e = \frac{26mV}{I_e}}$

$\therefore r_e = \frac{\eta V_T}{I_e} \cong \frac{V_T}{I_e}$

DC-Analysis:

$I_B = \frac{12 - 0.7}{270k} = 41.85\mu A$

$\therefore I_e = I_B + I_c = I_B + \beta I_B$

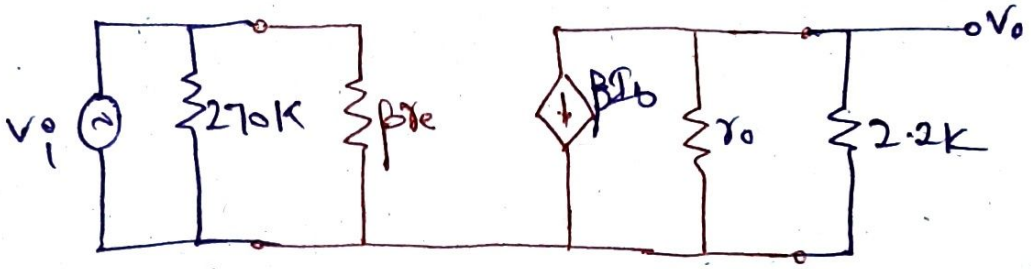
$= (1+\beta) I_B$

$\therefore I_e = (60+1) \cdot I_B = 61 \times 45.85 \times 10^{-6}$

$= 2.55\ mA$

$\therefore r_e = \frac{26\ mV}{2.55\ mA} = 10.18\ \Omega$ //

AC - Analysis



① $Z_i = (270k) \parallel (60 \times 10.18)$

R_B ; β ; r_e

$= 619.5\Omega$ $\left(\simeq \beta r_e = 621\Omega\right)$

② $Z_o = r_o \parallel 2.2k$

$= 40k \parallel 2.2k = 2.085k\Omega$ (or)

③ $Z_o \simeq 2.2k = R_C$

④ $A_V = \frac{-R_C}{r_e} = \frac{-2.2k}{10.18\Omega}$

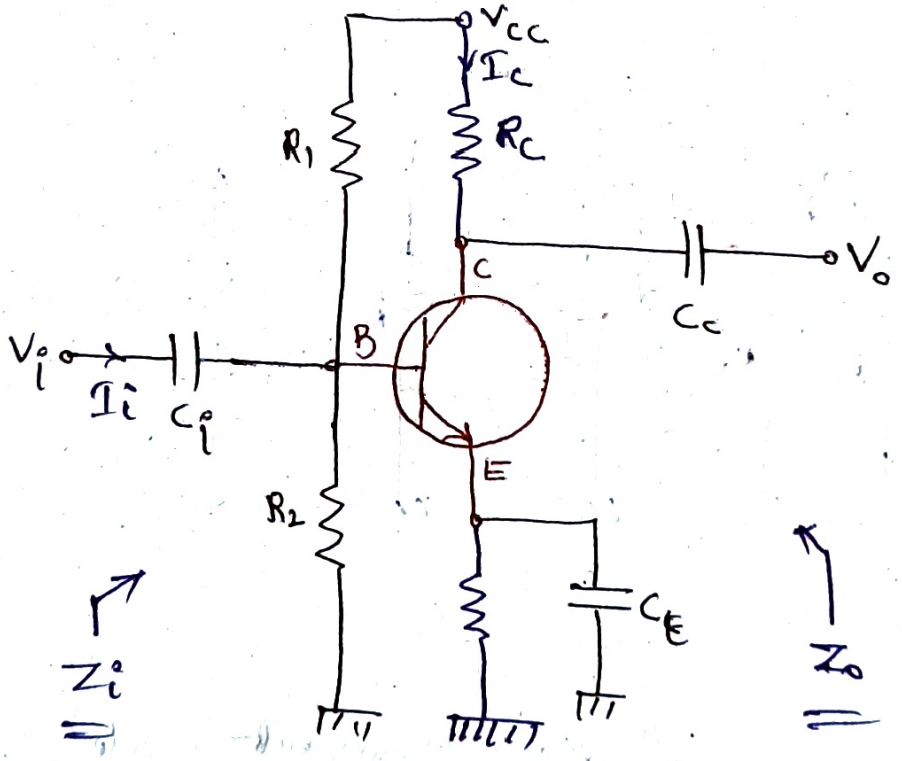
$A_V = -216.1$ //

(2) Small-signal - Low frequency - RC coupling - Single stage - Voltage-Diver Bias - CE Amplifier } with Bypass capacitors

(OR)

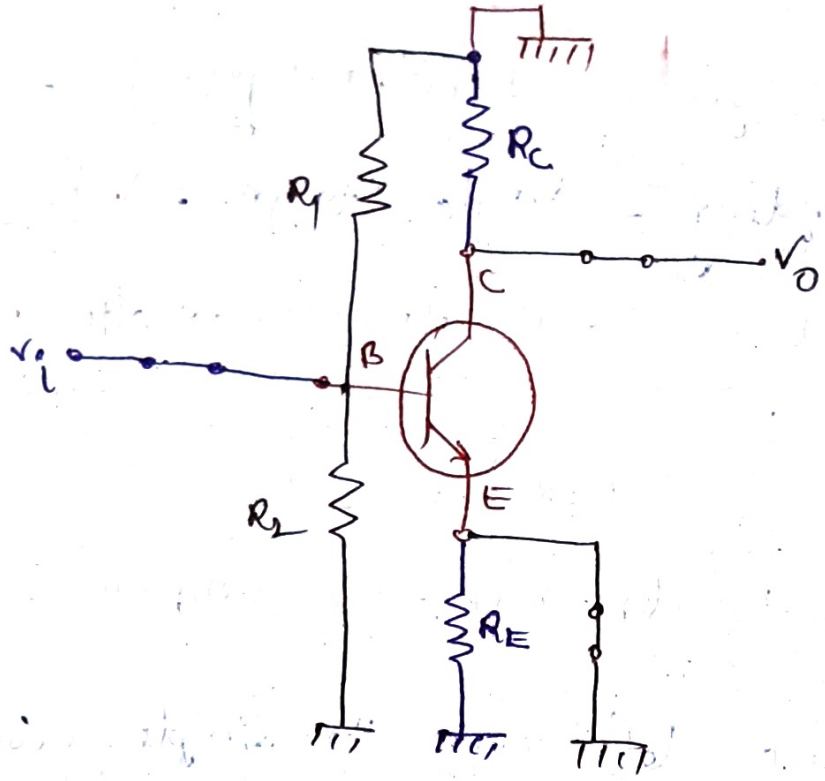
Voltage Divider Bias - CE Amplifier

* At first let's see the simple circuit without R_s & R_L. Later we extend it

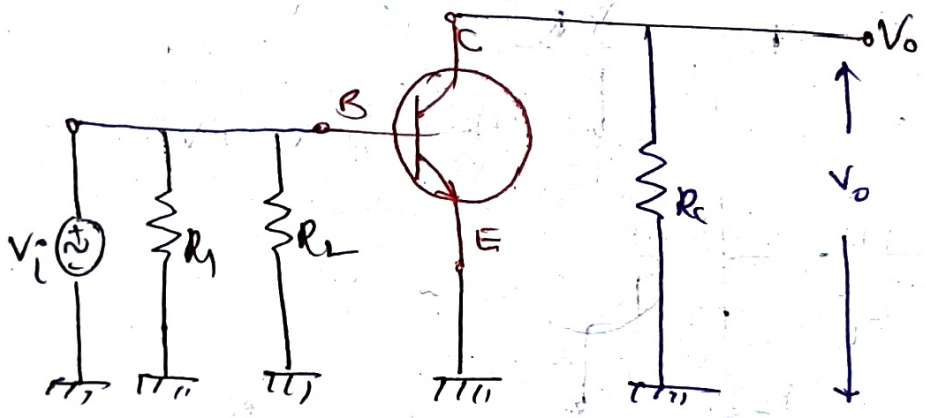


* Above circuit has a bypass capacitor "C_E"

* Now, the AC-equivalent circuit is given as follows.

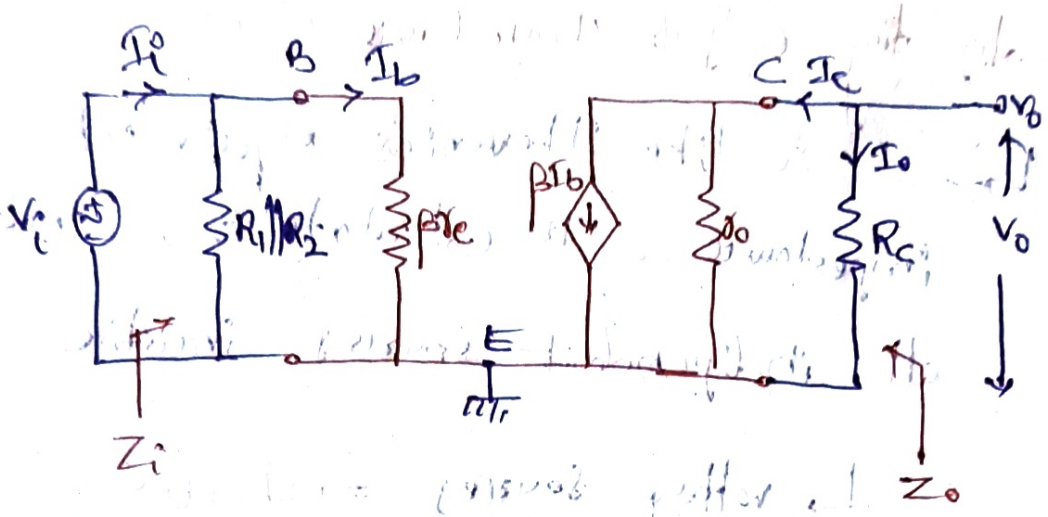


* The above circuit can be re-drawn as follows



* Now,

Draw small signal equivalent model of BJT ; i.e



* Calculating input Impedance (Z_i) :—

from Cruit above,

$$\boxed{Z_i = (R_1 \| R_2) \,//\, \beta r_e}$$

OR

$$Z_i = \frac{V_i}{I_i} \quad \text{——— (1)}$$

but $V_i = (I_b) \cdot \beta r_e$ ——→ (2) and

$$I_b = I_i \times \frac{(R_1 \| R_2)}{(R_1 \| R_2 + \beta r_e)} \quad \text{——— (3)}$$

substitute (3) in (2)

$$V_i = \left\{ I_i \times \frac{R_1 \| R_2}{(R_1 \| R_2 + \beta r_e)} \right\} \cdot \beta r_e$$

$$\therefore \boxed{Z_i = \frac{V_i}{I_i} = \frac{(R_1 \| R_2) \cdot \beta r_e}{(R_1 \| R_2) + \beta r_e} \simeq (R_1 \| R_2) \,//\, \beta r_e}$$

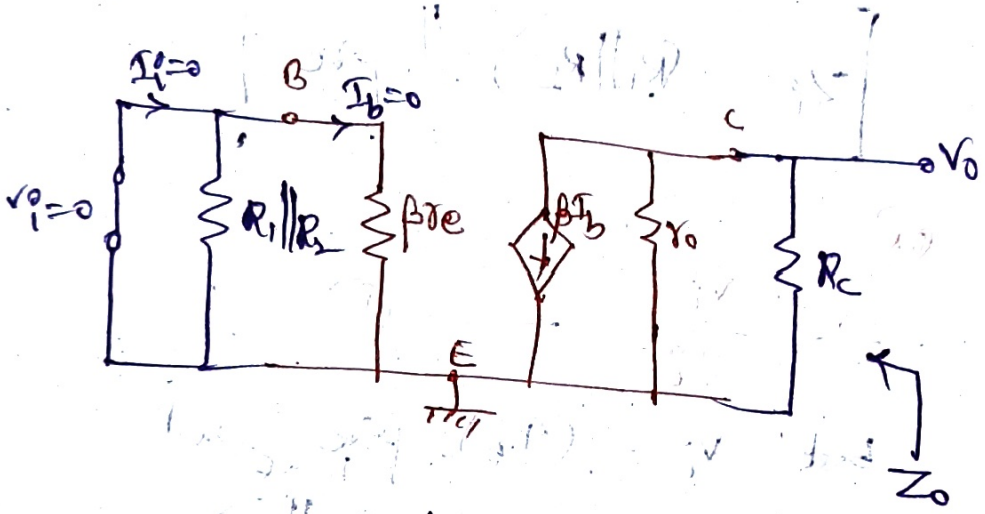
* Calculating Output Impedance (Z_o)

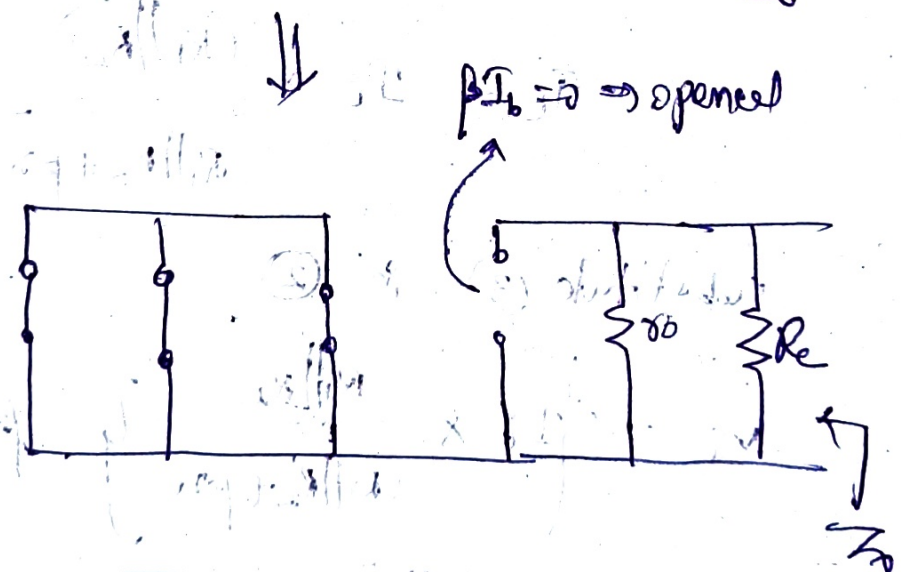
↳ 'Z_o' is like Thevenin's equivalent impedance. To calculate 'Z_o' make all independent sources inactive i.e.

↳ voltage sources ⇒ shorted

↳ Current sources ⇒ opened.

Hence, the circuit is





$$\therefore \boxed{Z_o = R_c \| r_o \cong R_c}$$

* Calculating Voltage Gain (A_V) :-

we have, $A_V = \frac{V_o}{V_i}$ — (1)

But, $V_o = (-\beta I_b) . R_C$ — (2)

but $I_b = \frac{V_i}{\beta r_e}$ $\left(\because V_i = (\beta r_e) . I_b\right)$

$\therefore V_o = -\beta \left(\frac{V_i}{\beta r_e}\right) . R_C$

$$\therefore \boxed{\frac{V_o}{V_i} = A_V = \frac{-R_C}{r_e}}$$

* Calculating Current Gain (A_I) :-

we have, $A_I = \frac{I_o}{I_i}$ — (1)

but $I_o = -I_C \times \left(\frac{r_o}{r_o + R_C}\right)$

$$I_o = \frac{-\beta . I_b . r_o}{r_o + R_C} \quad \text{— (2)}$$

but $I_b = I_i \times \frac{R_1 \| R_2}{(R_1 \| R_2 + \beta r_e)}$ — (3)

from (2) and (3)

$$I_o = \left(\frac{-\beta r_o}{r_o + R_C}\right) . \left\{ I_i \left(\frac{R_1 \| R_2}{R_1 \| R_2 + \beta r_e}\right) \right\}$$

$$\therefore \boxed{A_I = \frac{I_o}{I_i} = \left(\frac{-\beta r_o}{r_o + R_c}\right) \cdot \left\{\frac{R_1 \| R_2}{R_1 \| R_2 + \beta r_e}\right\}}$$

but $r_o >>> R_c$ & $R_1 \| R_2 >> \beta r_e$

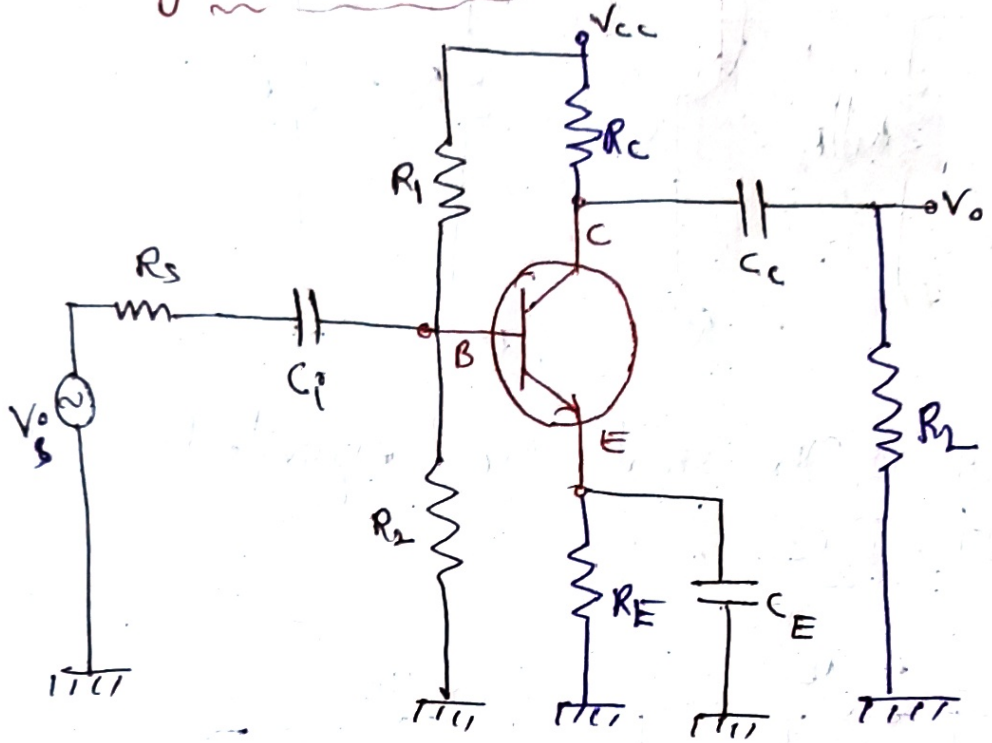
$$\therefore \boxed{A_I \simeq \beta}$$ (← indicates the direction of I_o - need to be considered opposite) ⇒

OR

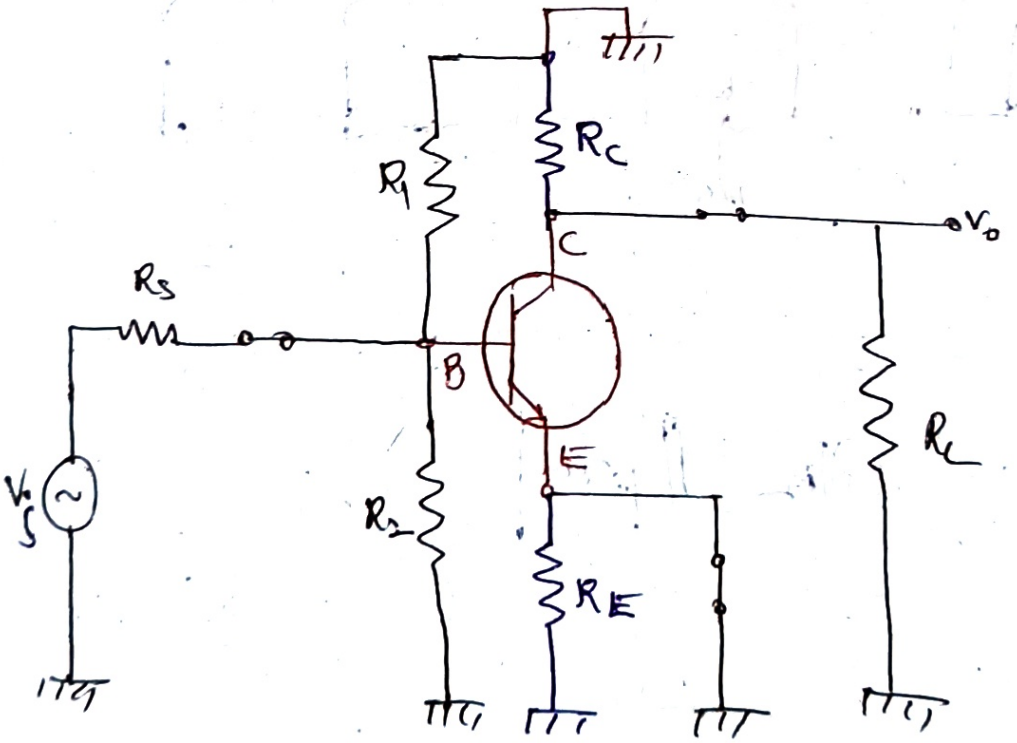
$$A_I = \frac{I_o}{I_i} = \frac{-V_o / R_c}{(V_i / Z_i)}$$

$$\therefore \boxed{A_I = -A_V \cdot \frac{Z_i}{R_c}}$$ //

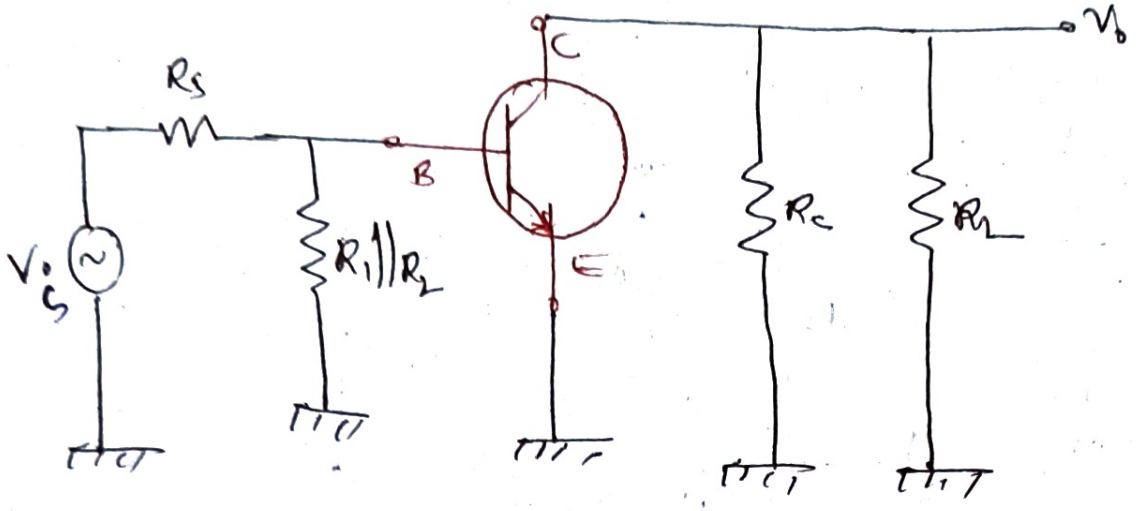
* Voltage divider - Bias with R_s, R_L and a Bypass capacitor :—



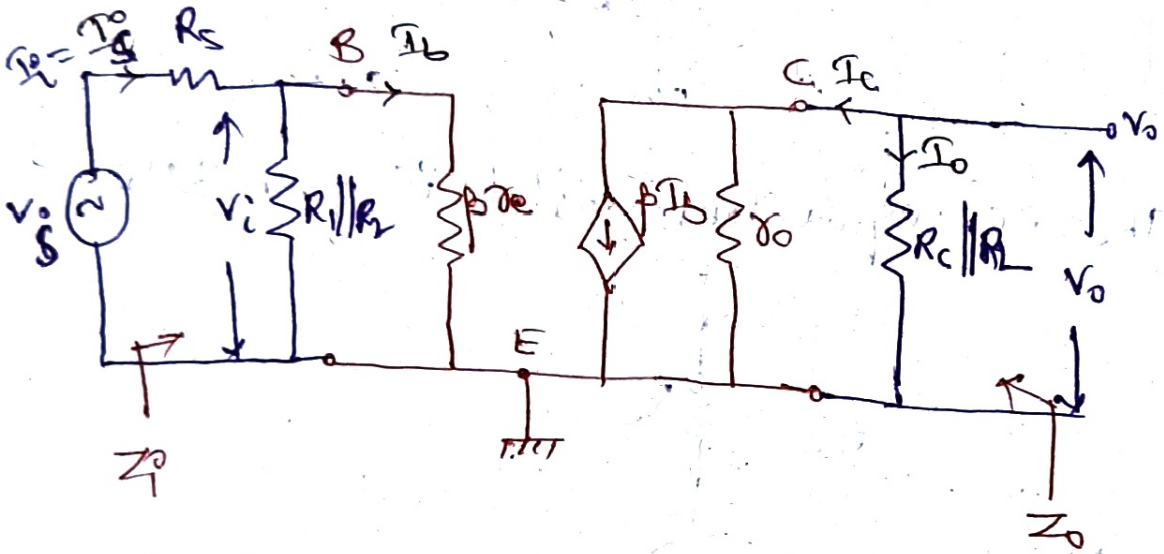
* The AC equivalent model is given below.



* The Circuit can be re-drawn as follows.



Now; draw the small signal equivalent model of BJT; i.e



* Calculating " Z_i "

$$Z_i = (R_1 \| R_2) \| \beta r_e$$

(OR)

$$Z_i = \frac{V_i}{I_i} \quad \text{---} (1)$$

but $V_i = (I_b) \cdot (\beta r_e)$ ---(2)

but $I_b = I_i \dfrac{(R_1 \| R_2)}{R_1 \| R_2 + \beta r_e}$. ---(3)

$$\therefore V_i = I_i \left(\frac{(R_1 \| R_2)}{R_1 \| R_2 + \beta r_e} \right) (\beta r_e)$$

$$\therefore Z_i = \frac{V_i}{I_i} = \frac{(R_1 \| R_2)(\beta r_e)}{(R_1 \| R_2) + (\beta r_e)}$$

$$\therefore \boxed{Z_i = (R_1 \| R_2) \| \beta r_e}$$

* Calculating "Z_o":

↳ Z_o is like thevenins equivalent impedance

to calculate 'Z_o' make all indipendent sources in-active' i.e.

voltage sources ⇒ shorted

current sources ⇒ opened

and $R_L = \infty$ ⇒ opened //

Hence, if we do so, we get

$$\boxed{Z_o = r_o \| R_C}$$ but

practically

$$\boxed{Z_o = r_o \| R_C \| R_2}$$ //

* Calculating 'Av' :-

we have $A_V = \frac{V_o}{V_i}$ — (1)

but $V_o = (-\beta I_b) \left\{ \frac{r_o \| R_c}{(r_o \| R_c) + R_L} \right\} \cdot R_L$

I_o — (2)

but $I_b = \frac{V_i}{\beta r_e}$ — (3)

∴ substitute (3) in (2)

$$V_o = -\beta \left(\frac{V_i}{\beta r_e}\right) \left\{ \frac{r_o \| R_c}{(r_o \| R_c) + R_L} \right\} \cdot R_L$$

$$\therefore \frac{V_o}{V_i} = A_V = -\left(\frac{r_o \| R_c}{r_o \| R_c + R_L}\right) \cdot \frac{1}{r_e} \cdot R_L$$

but $r_o >> 10 R_c$

$$\therefore A_V = \left(\frac{-R_c}{R_c + R_L}\right)\left(\frac{1}{r_e}\right) \cdot R_L$$

$$\boxed{A_V \cong \frac{-R_c}{r_e}} \quad (\because R_L >> R_c)$$

∴ overall voltage Gain (A_{VS}) is given by

$$\boxed{A_{VS} = A_V \cdot \frac{Z_i}{Z_i + R_s}}$$

* Calculating A_I :-

$$A_I = \frac{I_o}{I_i} \quad \text{---} \quad (1)$$

but $I_o = \dfrac{I_c \cdot (r_o \,||\, R_c)}{(r_o \,||\, R_c) + R_L}$

$$\therefore I_o = (\beta \cdot I_b) \frac{(r_o \,||\, R_c)}{(r_o \,||\, R_c) + R_L} \quad \text{---} \quad (2)$$

but $I_b = I_i \cdot \dfrac{(R_1 \,||\, R_2)}{(R_1 \,||\, R_2) + \beta r_e}$ ---(3)

from (2) and (3)

$$I_o = \beta \left(\frac{r_o \,||\, R_c}{r_o \,||\, R_c + R_L} \right) \cdot \left\{ I_i \frac{(R_1 \,||\, R_2)}{(R_1 \,||\, R_2) + \beta r_e} \right\}$$

$$\therefore \boxed{A_I = \frac{I_o}{I_i} = \beta \left(\frac{r_o \,||\, R_c}{r_o \,||\, R_c + R_L} \right) \left(\frac{R_1 \,||\, R_2}{R_1 \,||\, R_2 + \beta r_e} \right)}$$

but $r_o >> 10 R_c$; $r_o \,||\, R_c = R_c$ and

$R_1 \,||\, R_2 >> \beta r_e$

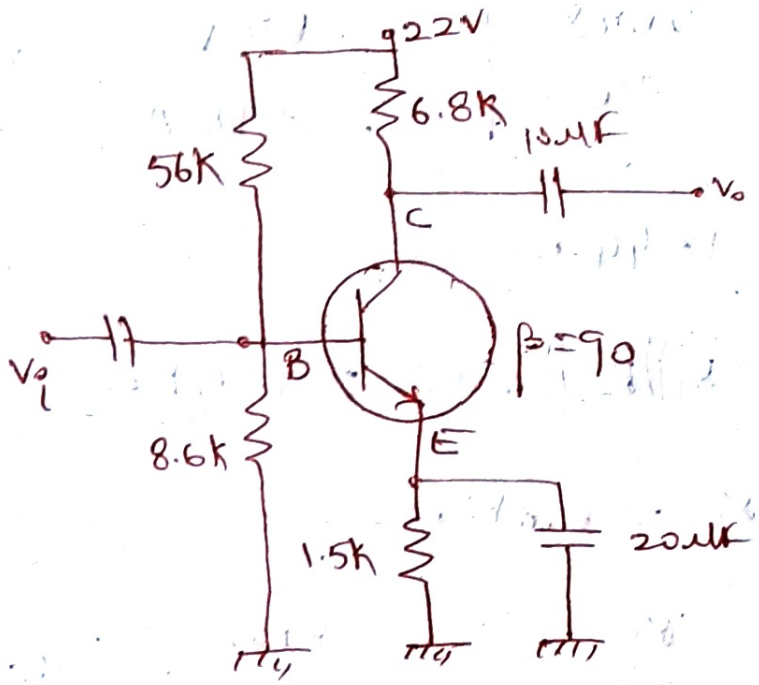
$\therefore$

$$\boxed{A_I \cong \beta}$$

$\therefore$ over all current Gain $\boxed{A_{Is} = A_I \cdot \dfrac{R_s}{R_s + Z_i}}$

Eg:- For given BJT design find

r_e, Z_i, Z_o, A_v, A_I; (if $r_o = \infty$)



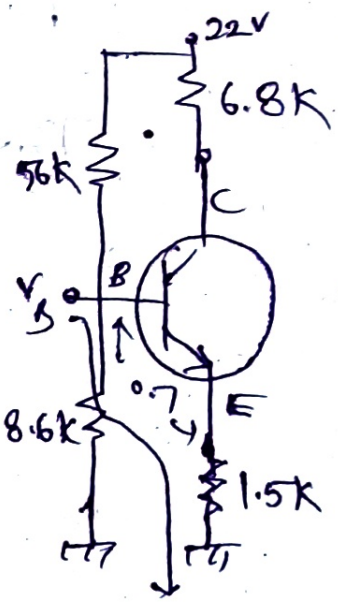
Sol:-

* DC Analysis:

$$\therefore V_B = V_{CC}\left(\frac{R_2}{R_1+R_2}\right)$$

$$= 22.\left(\frac{8.6K}{8.6K+56K}\right)$$

$$\boxed{V_B = 2.81V}$$



* apply KVL @ i/p Loop

$$V_B = V_{BE} + V_E \Rightarrow V_E = V_B - V_{BE}$$

$$= 2.81 - 0.7$$

$$\boxed{V_E = 2.11V}$$

$$\therefore \boxed{I_E = \frac{V_E}{R_E}} = \frac{2.11}{1.5K} = 1.41\,mA$$

$$\therefore \boxed{r_e = \frac{26mV}{I_E}} = \frac{26mV}{1.41mA} = 18.44\Omega$$

$\therefore$ (a) $r_e = 18.44\Omega$.

(b) $Z_i = R_1 \| R_2 \| \beta r_e = 1.35K\Omega$

(c) $Z_o \cong R_C = 6.8K\Omega$

(d) $A_V = \frac{-R_C}{r_e} = \frac{-6.8K\Omega}{18.44\Omega} = -368.76$

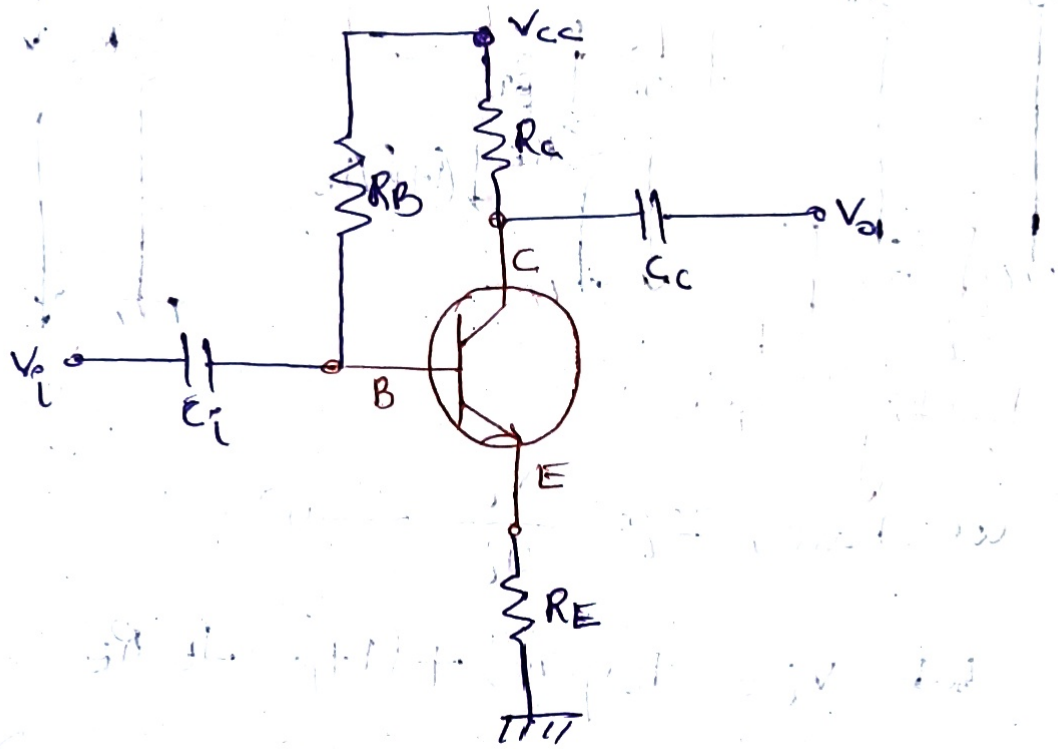
(e) $A_I = \beta \cdot \left(\frac{R_1 \| R_2}{R_1 \| R_2 + \beta r_e}\right)$ $(\because \beta r_e < R_1 \| R_2)$

$= 73.04$

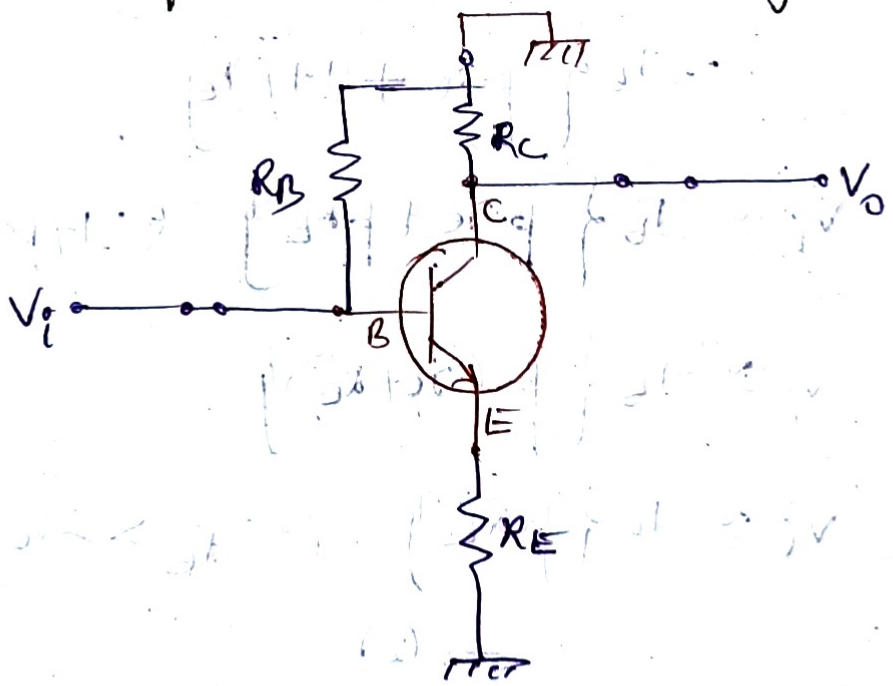
* Small signal - Low frequency - RC coupling single stage - Emitter Bias - CE Amplifier

(OR)

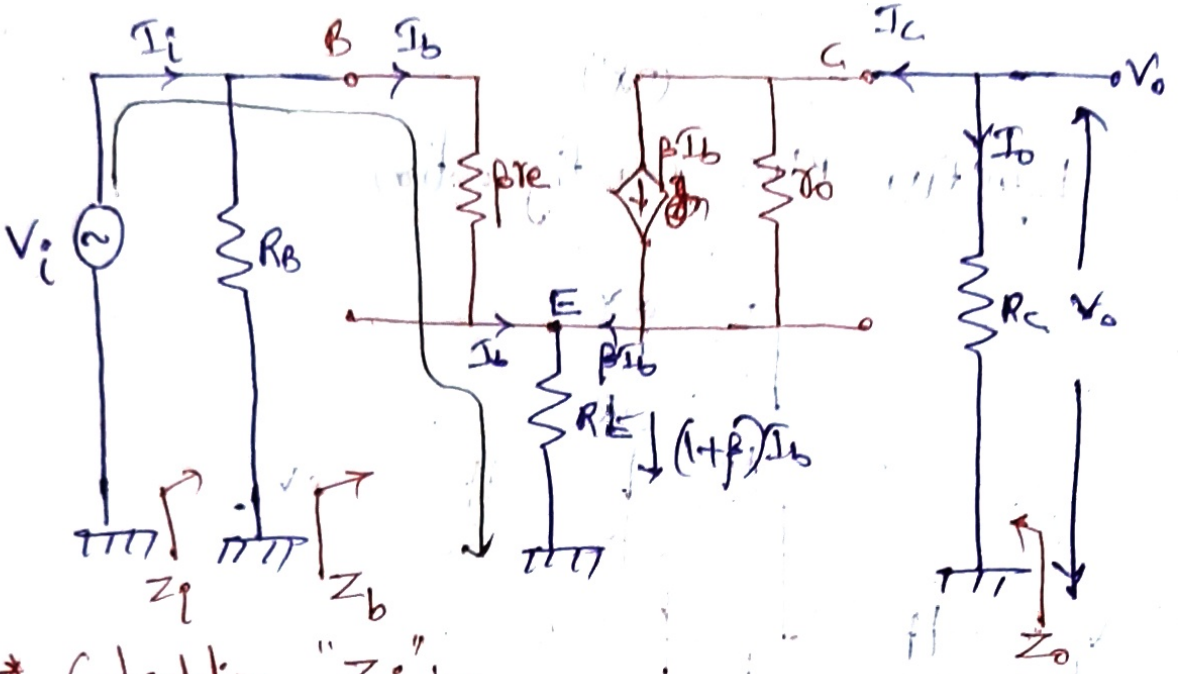
Emitter Bias configuration



The AC-equivalent circuit is given as.



Now, The small signal Equivalent circuit of BJT is given, as.



* Calculating "Z_i" :-

we have, $Z_i = \frac{V_i}{I_i}$ — ①

but $V_i = I_b(\beta r_e) + (1+\beta) I_b . R_E$

$(\because V_E = (1+\beta) I_b)$

$= I_b \{ \beta r_e + (1+\beta) R_E \}$

$\therefore V_i \simeq I_b \{ \beta r_e + \beta R_E \}$ $(\because 1+\beta \simeq \beta)$

$V_i \simeq I_b \{ \beta (r_e + R_E) \}$

$V_i \simeq I_b \{ \beta R_E \}$ $(\because R_E >> r_e)$

— ②

$$\therefore Z_b = \frac{V_b}{I_b} = \beta R_E$$

$$\therefore \boxed{Z_i = R_B \parallel Z_b} \quad (\because \text{from fig}).$$

i.e $$\boxed{Z_i = R_B \parallel \beta R_E}$$

(6P)

from (2), $V_i = I_b \{(\beta R_E)\}$

but $I_b = I_i \left\{ \frac{R_B}{R_B + \beta R_E} \right\}$ → overall impedance

$$\therefore V_i = I_i \left(\frac{R_B}{R_B + \beta R_E} \right) \beta R_E$$

$$\therefore Z_i = \frac{V_i}{I_i} = \frac{(R_B) \cdot (\beta R_E)}{(R_B) + (\beta R_E)}$$

$$\boxed{Z_i = R_B \parallel \beta R_E}$$

* Calculating Z_o'

$$Z_o = r_o \parallel R_c \quad (\because \text{Thevenins Equivalent})$$

$$\boxed{Z_o \cong R_c}$$

* Calculating 'Av':

we have $A_v = \frac{V_o}{V_i} \quad \text{—— (1)}$

but $V_o = (-\beta I_b) \cdot R_C \quad \text{—— (2)}$

but $I_b = \frac{V_i}{Z_b}$

$\therefore V_o = (-\beta)\left(\frac{V_i}{Z_b}\right) \cdot R_C$

$\therefore \boxed{\frac{V_o}{V_i} = A_v = \frac{-\beta R_C}{Z_b}}$

$\therefore A_v = \frac{-\beta R_C}{\beta R_E}$

$\therefore \boxed{A_v = \frac{-R_C}{R_E}}$

* Calculating "A_I" :—

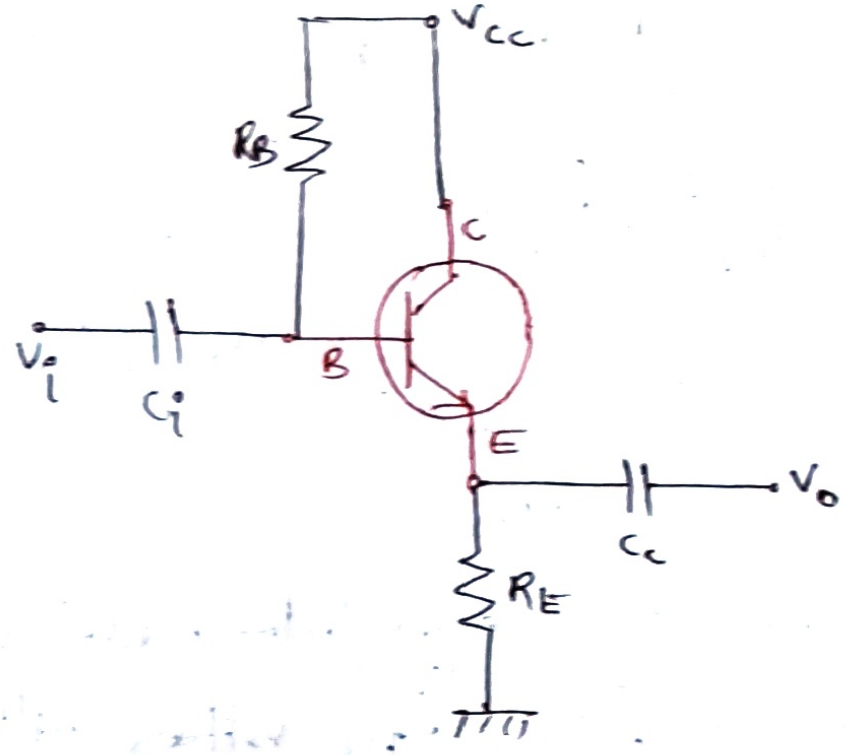
we have $A_I = \frac{I_o}{I_i} \quad \text{—— (1)}$

$\therefore A_I = \frac{-\beta I_b}{I_i}$

$= \frac{-\beta \left\{ I_i \left(\frac{R_B}{R_B + Z_b} \right) \right\}}{I_i}$

$\therefore \boxed{A_I = \left| \frac{\beta R_B}{R_B + Z_b} \right| \cong -A_v \frac{Z_i}{R_C}}$

* Small Signal Analysis of Emitter-Follower (CC- configuration) :-



* DC- Analysis -

$V_i = 0.7 + V_b$ (or)

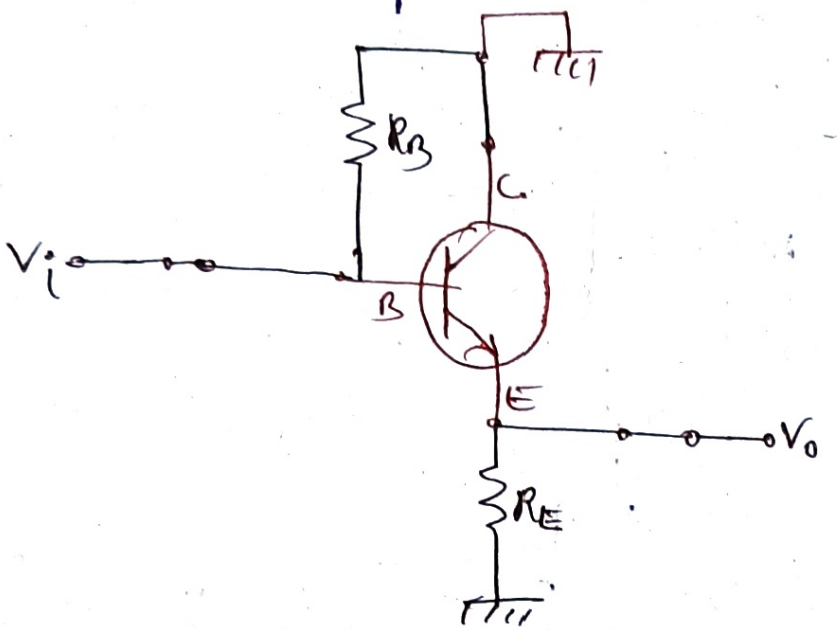
$V_o = V_i - 0.7$ (or)

$$\boxed{V_o \cong V_i} \Rightarrow \boxed{\frac{V_o}{V_i} \cong 1}$$

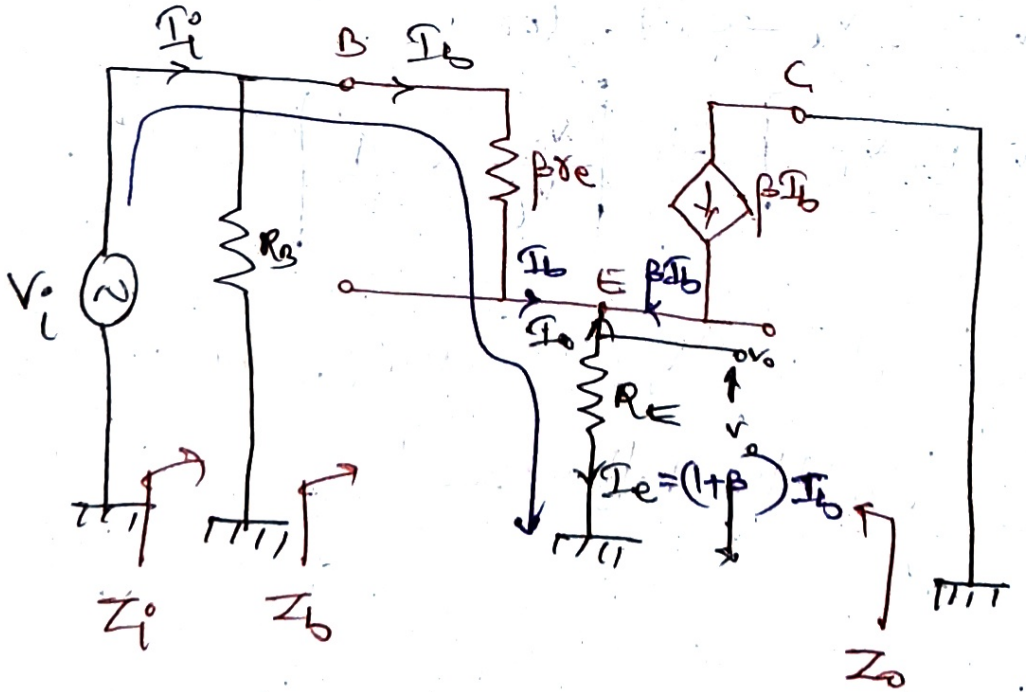
'V_o' follows "V_i" hence name given as "Emitter Follower"

AC - Analysis :-

↳ The AC equivalent model is given as



↳ The small signal equivalent model is given as follows {for voltage follower Z_i = High & Z_o = very low ⇒ we can neglect "r_o"}.



Calculating "Z_i":-

we have, $Z_i = \frac{V_i}{I_i}$

apply KVL @ i/p.

$$V_i = I_b(\beta r_e) + (1+\beta).I_b.R_E$$

$$= I_b\{\beta r_e + (1+\beta)R_E\}$$

$$\simeq I_b\{\beta r_e + \beta R_E\} \quad (\because 1+\beta \simeq \beta)$$

$$\therefore \boxed{Z_b = \frac{V_i}{I_b} = \beta(r_e + R_E)}$$

$$\boxed{Z_b \simeq \beta R_E} \quad (\because R_E >> r_e)$$

Now

$$\boxed{Z_i = R_B \parallel Z_b} \quad \text{i.e.}$$

$$\boxed{Z_i = R_B \parallel \beta R_E}$$

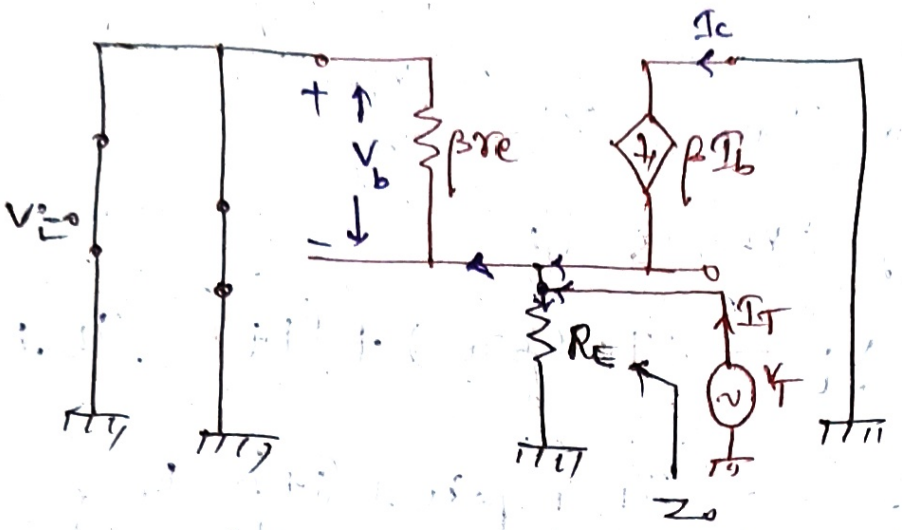
* Calculating 'Z_o'.

'Z_o' is thevenins equivalent impedance.

hence to calculate 'Z_o' make independent sources inactive i.e

↳ voltage source ⇒ shorted

↳ Current source ⇒ opened



Connect a test source as shown $\left[\because Z_o = \frac{V_T}{I_T}\right]$

$$\therefore I_T + i_c = \frac{V_T}{R_E} + \frac{V_b}{\beta r_e}$$

$$\therefore I_T + \beta I_b = \frac{V_T}{R_E} + \frac{V_b}{\beta r_e}$$

but $V_T = -V_b$ and $I_b = \frac{V_b}{\beta r_e}$

$$\therefore I_T + \beta \cdot \left(\frac{V_b}{\beta r_e}\right) = \frac{V_T}{R_E} + \frac{V_b}{\beta r_e}$$

$$\therefore I_T = \frac{V_T}{R_E} - \frac{V_T}{\beta r_e} + \frac{V_T}{r_e}$$

$$I_T = V_T \left[\frac{1}{R_E} + \frac{1}{r_e} - \frac{1}{\beta r_e}\right]$$

but $\frac{1}{\beta r_e}$ is small value, neglect it

$$\therefore \frac{V_T}{I_T} = Z_o = \frac{1}{\frac{1}{R_E} + \frac{1}{r_e}} \simeq r_e \,||\, R_E$$

$$\therefore \boxed{Z_o = r_e \,||\, R_E \simeq r_e}$$

* Calculating Av:

Applying voltage division rule.

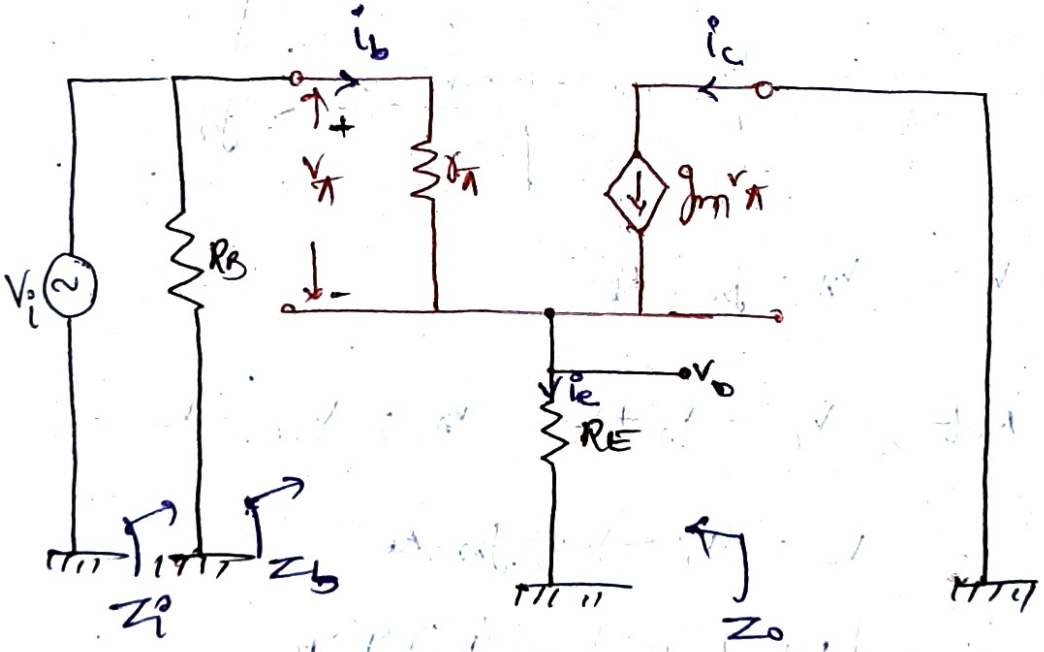
$$V_o = V_i \times \frac{R_E}{R_E + r_e\beta}$$

but $R_E >> \beta r_e$.

$$\therefore \boxed{V_o \cong V_i}$$

* Emitter follower Analysis [2nd method]

Consider 'r_π' & '$g_m v_\pi$' as shown for small signal Analysis



* Voltage Gain: (Av)

$$V_o = V_i \times \frac{R_E}{R_E + r_\pi} \cong V_i \quad (\because R_E >> r_\pi)$$

$$\therefore \boxed{\frac{V_o}{V_i} \cong 1}$$

(22)

$$V_o = i_e \cdot R_E$$

$$= (i_b + i_c) R_E$$

$$= \left(\frac{v_\pi}{r_\pi} + g_m v_\pi\right) R_E$$

$$V_o = v_\pi \left[\frac{1}{r_\pi} + g_m\right] R_E$$

↳ neglected

But,

$$g_m v_\pi = \beta I_b \quad \text{i.e.}$$

$$g_m v_\pi = \beta \left(\frac{v_\pi}{r_\pi}\right)$$

$$\therefore g_m = \frac{\beta}{r_\pi} \quad (\text{or}) \quad r_\pi = \frac{\beta}{g_m}$$

$$\therefore \frac{1}{r_\pi} = \frac{g_m}{\beta} \simeq \frac{26\,mV}{\beta} << g_m$$

$$\therefore V_o = v_\pi \cdot g_m \cdot R_E$$

But, $V_i = v_\pi + V_o \Rightarrow v_\pi = V_{in} - V_o$

$$\therefore V_o = (V_i - V_o)\, g_m R_E$$

$$\therefore V_o \left[1 + g_m R_E\right] = (V_i)\, g_m R_E$$

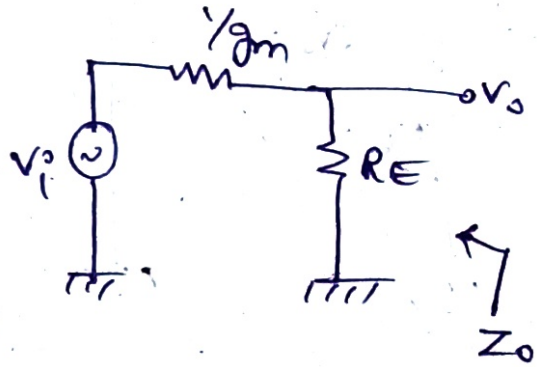
$$\therefore A_V = \frac{V_o}{V_i} = \frac{g_m R_E}{1 + g_m R_E} \simeq \frac{R_E}{\frac{1}{g_m} + R_E}$$

But usually, $\frac{1}{g_m} \ll R_E$

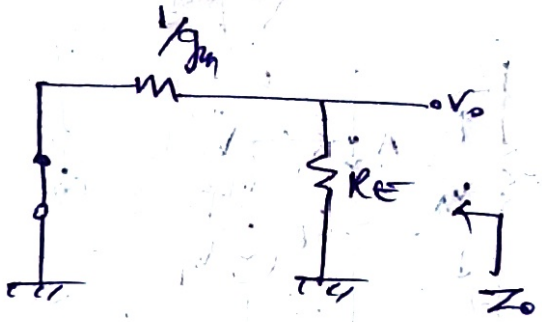
$\therefore \boxed{A_V \cong 1}$

Note:

$$A_V = \frac{V_o}{V_i} = \frac{R_E}{R_E + \frac{1}{g_m}}$$ ⇒ voltage divider expression



∴ Z_o is thevenin equivalent impedance, to calculate Z_o, make "$V_i \Rightarrow 0$".



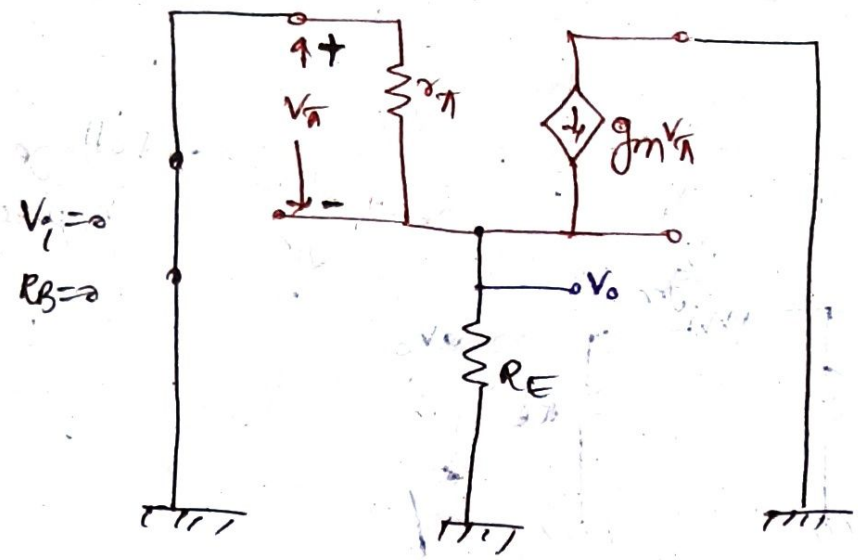
$$\therefore Z_o = \frac{1}{g_m} \parallel R_E$$

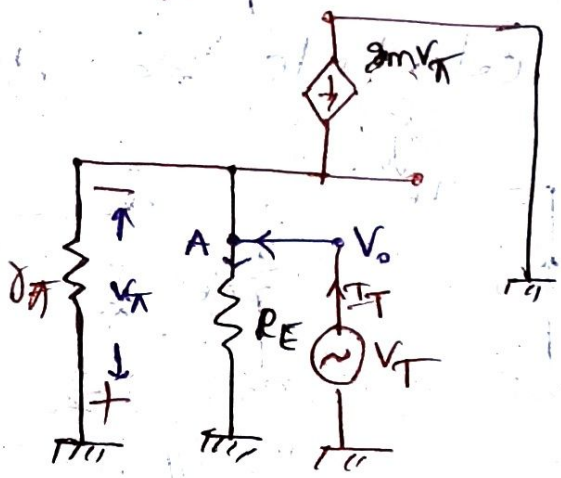
[∵ but $\frac{1}{g_m} = r_e \Rightarrow Z_o = r_e \parallel R_E \cong r_e$]

it can be also calculated conventionally by following method

Calculating 'Zo':

make all independent source = 0.





It is clear that $V_o = -V_\pi$

Now, connect a test source and apply KCL at node 'A' shown $\left[Z_o = \frac{V_T}{I_T} \right]$

$$\therefore I_T + i_c = \frac{V_T}{R_E} + \frac{V_T}{r_\pi}$$

$$\therefore I_T + g_m v_\pi = V_T \left\{ \frac{1}{R_E} + \frac{1}{r_\pi} \right\}$$

but $v_\pi = -v_T$

$$\therefore I_T - g_m v_T = v_T \left[\frac{1}{R_E} + \frac{1}{r_\pi} \right]$$

$$\therefore I_T = v_T \left[g_m + \frac{1}{R_E} + \frac{1}{r_\pi} \right]$$

$$\therefore Z_o = \frac{v_T}{I_T} = \frac{1}{\frac{1}{R_E} + \left(\frac{1}{r_\pi} + g_m \right)}$$

↓ neglected (as given above)

$$\therefore Z_o = \frac{1}{\frac{1}{R_E} + g_m}$$

$$= \frac{1}{\frac{1}{R_E} + \frac{1}{1/g_m}}$$

$$= \boxed{R_E \,//\, \frac{1}{g_m} = Z_o}$$

Input impedance "Z_i":—

KVL @ i/p

$$v_i = i_b \cdot r_\pi + i_e R_E$$

$$= i_b r_\pi + ((1+\beta)\, i_b) \cdot R_E$$

$$v_i = i_b \left[r_\pi + (1+\beta) R_E \right]$$

$$\therefore Z_b = \frac{v_i}{i_b} = r_\pi + (1+\beta) R_E \cong \beta R_E$$

$$\therefore Z_i = Z_b \,//\, R_B$$

For above if 'r_o' is given, then

* $Z_{in} = R_B \| Z_b$

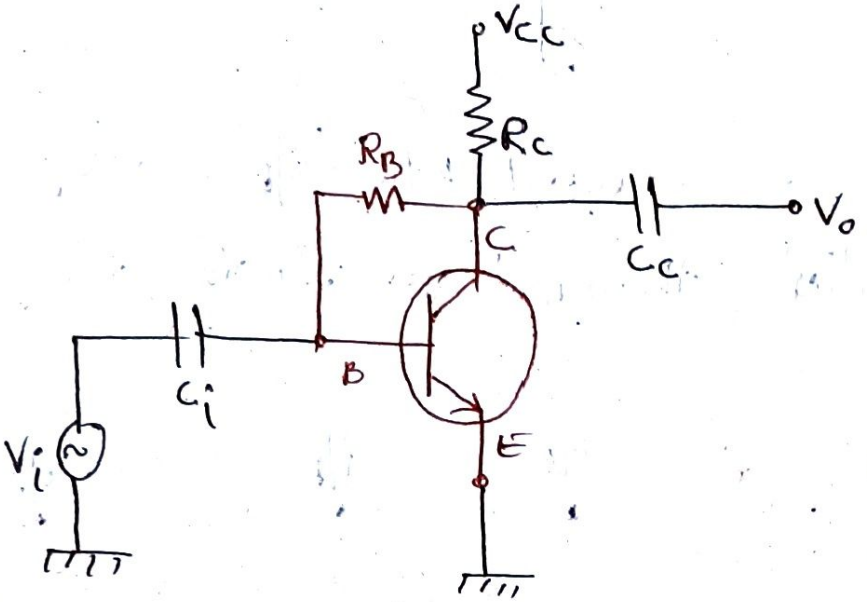
(where $Z_b = r_\pi + (1+\beta)(R_E \| r_o)$)

* $Z_o = (1/g_m) \| (R_E \| r_o)$

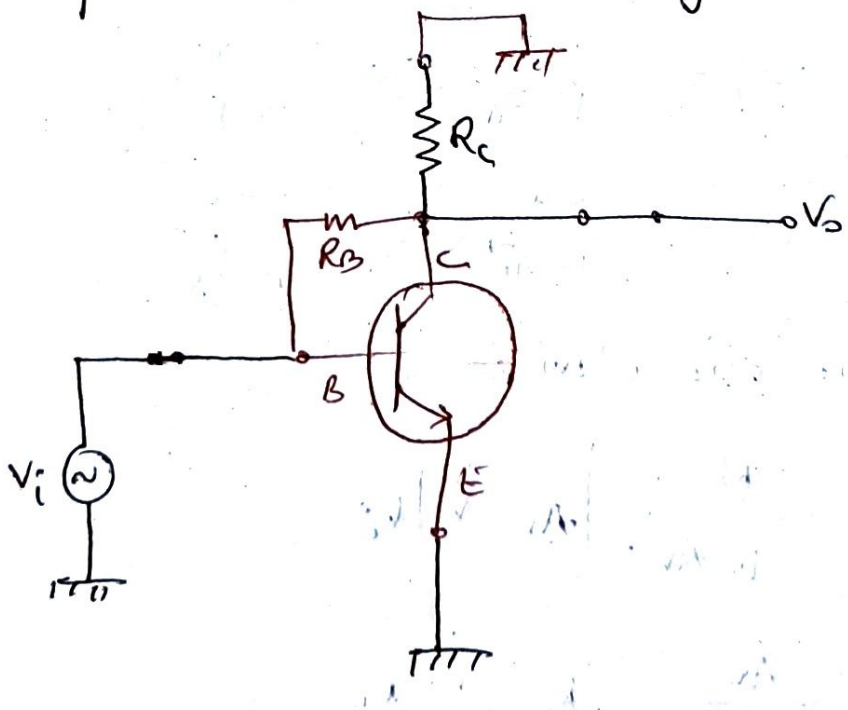
* $A_V = \dfrac{R_E \| r_o}{1/g_m + (R_E \| r_o)} \simeq 1$ //

* Single Stage - Low frequency - RC coupling small signal Analysis of Collector-feedback configuration :-

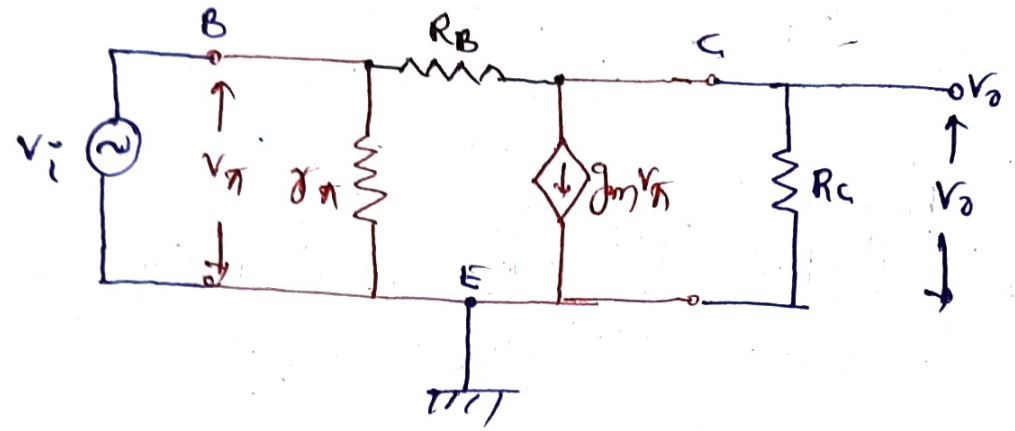
Base Bias with collector feedback configuration :-



AC - equivalent model is given as follows.

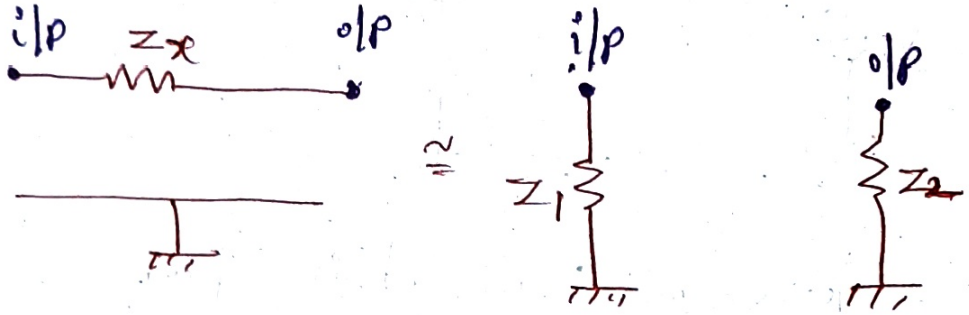


the small signal equivalent model of BJT is given as.



As 'R_B' is connected b/w 'i/p' and 'o/p', use 'Miller's theorem' for simple analysis.

i.e:



where, $Z_1 = \dfrac{Z_x}{1-A_V} \simeq \dfrac{Z_x}{1-k}$

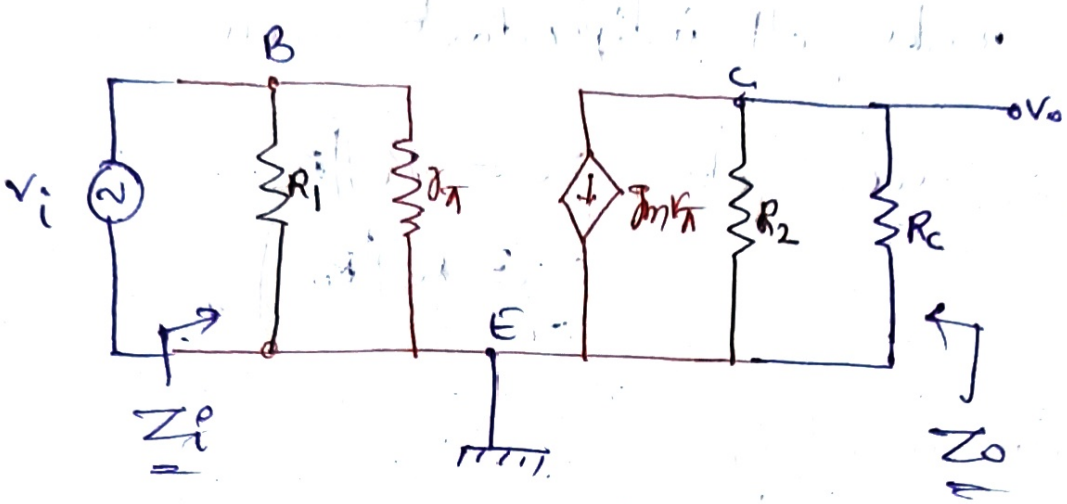
$Z_2 = \dfrac{Z_x}{1-A_V^{-1}} \simeq \dfrac{Z_x}{1-1/k}$

Hence, for our circuit-

$$R_1 = \frac{R_B}{1-A_V} \Big|_{A_V = V_o/V_i}$$

$$R_2 = \left(\frac{A_V}{A_V-1}\right) . R_B \cong R_B \;//$$

Hence, the circuit can be redrawn as.



(calculating voltage Gain (Av) :-

$$V_o = -I_c (R_c // R_2)$$

$$\simeq -I_c (R_c // R_B)$$

$$v_o = -g_m v_\pi (R_c // R_B)$$

But $v_\pi = v_i$ (from above fig)

$$\therefore v_o = v_i \left[-g_m (R_c // R_B) \right]$$

$$\therefore \boxed{A_v = \frac{v_o}{v_i} = -g_m (R_c // R_B)}$$

But $R_B \gg R_c$

$$\therefore \boxed{A_v \simeq -g_m R_c}$$

o/p impedance.

make all indipendant sources = 0

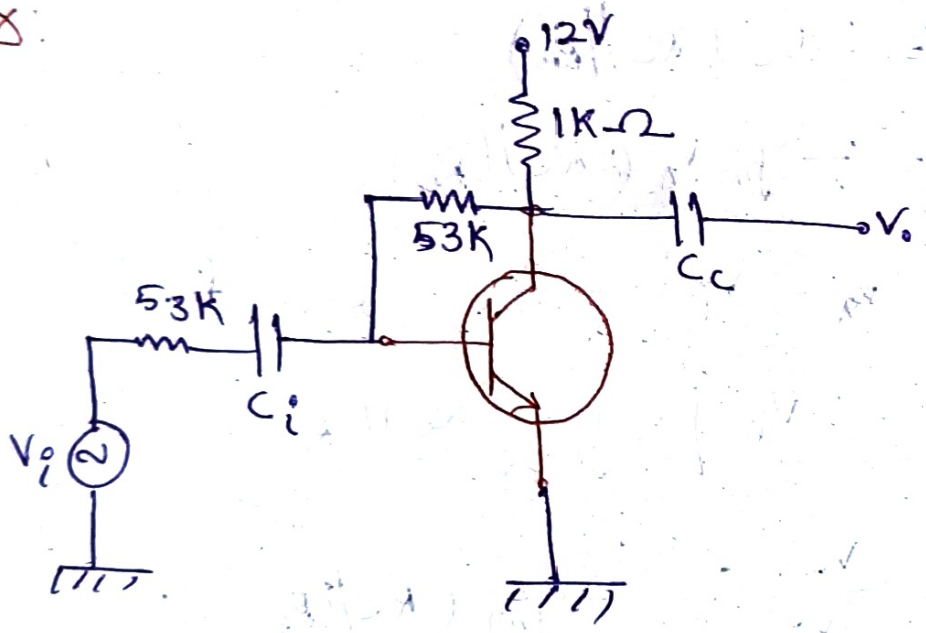
∴ we get, $Z_o = R_c \| R_2$

$$\boxed{Z_o \cong R_c \| R_3}$$ //

i/p impedance.

$$\boxed{Z_i = r_1 \| \beta r_\pi}$$ (from fig).

Ex.



Find $\frac{V_o}{V_i}$. Assume $V_{BE} = 0.7$; $\beta_F = 60$.

and $V_T = 26\,mV$ //

sol:-

* DC - Analysis:

we have, $I_B = \dfrac{V_{CC} - V_{BE}}{R_B + (1+\beta)R_C}$

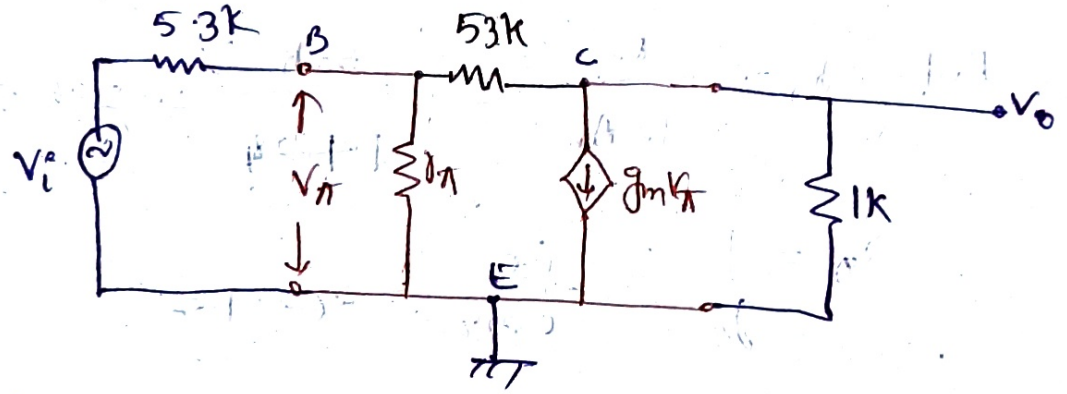
and $I_C = \beta I_B$

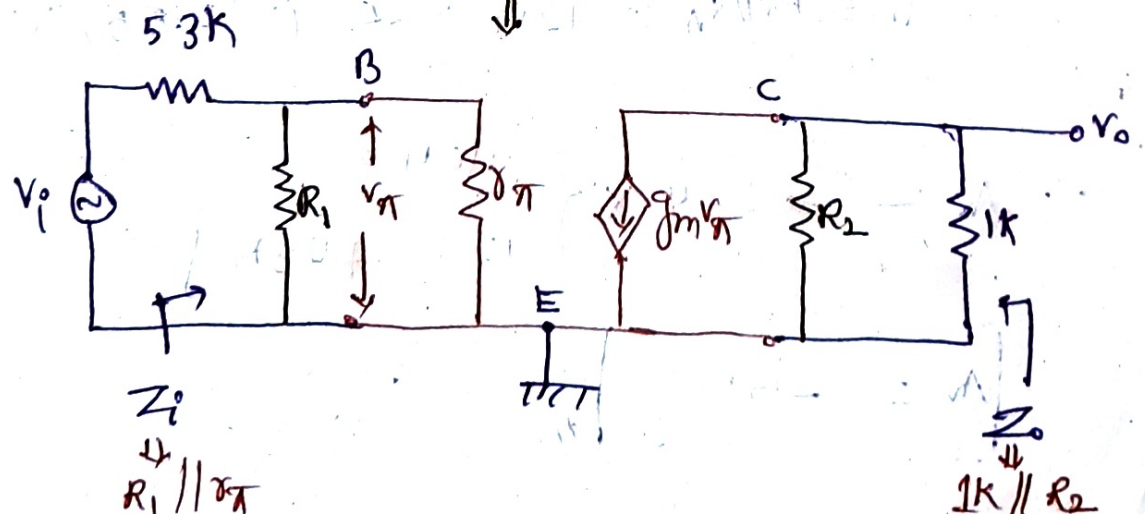
$\therefore I_B = \dfrac{12 - 0.7}{53K + (61 \times 1K)} = 99.12\,\mu A$

$\therefore I_C = \beta I_B = 60 \times 99.12\,\mu A = 5.94\,mA$ //

$\therefore g_m = \dfrac{I_C}{V_T} = \dfrac{5.94\,mA}{26\,mV} = 0.228$ siemen

* AC - Analysis:





where, $R_1 = \left.\frac{R_B}{1-A_V}\right|_{A_V = \frac{V_o}{V_\pi}}$

$$R_2 \cong R_B = 53K$$

* Voltage Gain (A_V)

$$A_V = -g_m (R_C // R_2) = -g_m (R_C // R_2)$$

$$= -0.228 [1k // 53k]$$

$$= -224$$

Now:

$$A_{VS} = A_V \left(\frac{Z_i}{Z_i + R_S}\right)$$

$$= -224 \left\{\frac{R_1 // r_\pi}{R_1 // r_\pi + R_S}\right\}$$

But $R_1 = \frac{R_B}{1-A_V} = \frac{53k}{1+224} = 235.5\Omega$

$$r_\pi = \frac{\beta}{g_m} = \frac{60}{0.228} = 263.1\Omega$$

$$\therefore Z_{in} = R_1 // r_\pi = 124.3\Omega \quad (\because 235.5 // 263)$$

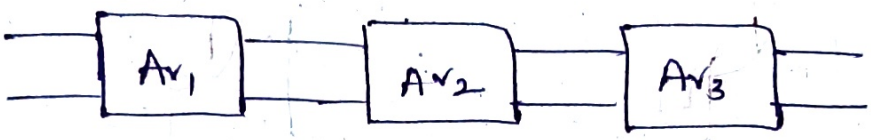
$$\therefore A_V = -224 \times \left\{\frac{124.3}{124.3 + 53k}\right\}$$

$$\boxed{A_V = -5.13}$$

BJT- multistage Amplifiers :-

↳ Single stage voltage Gain may not sufficient for driving a 'Load'. in such cases multiple stages are cascaded / Cascoded known as "multistage Amplifiers"

*



∴ then overall $A_V = (A_{V_1}) * (A_{V_2}) * (A_{V_3})$

* We have already seen, how to couple different stages of amplifiers and advantages, disadv in unit-II.

* Here in this particular topic, we try to analyse different multi stage amplifiers using small signal analysis

* Amplifiers & Classification :-

Def of an Amplifier :

The electronic device which increases the level of given AC i/p signal by transfering the input through a Dynamic Resistive device of different resistances is an "Amplifier"

* If the amplitude of i/p signal is lesser than o/p (where o/p is Level shifted quantity of i/p) then it is called as "Small signal Ampl"

Ex: voltage, Current, Transconductance and Transresistance amplrs.

* If o/p amplitude = i/p amplitude but if 'o/p' is strengthed signal of i/p then it is "Large signal Ampl"

Ex: Power Ampl.

* if single transistor amplr is not suitable to drive the load in terms of 'phase' and/or "Frequency" and/or "Amplitude" and/or "Gain" then - "Multi-Stage Amplrs" are needed.

↳ In multi-stage if all the transistors are of same type & configuration then it is "Cascading"

↳ In multistage if all the amplrs of different type then it is "Cascoding"

* If o/p of one stage is not suitable to drive 'second stage' or if o/p is not suitable to drive a load then to over come this limitation 'Coupling' is used.

↳ if Capacitors only or Inductors only are used for coupling

then it is Capacitive coupling or Inductive coupling like "Tuned Amplifiers", or oscillators etc....

↳ if 'Capacitors' with support of Resistors are used for coupling then it is "RC-coupled Amplr"

Ex: small signal Amplrs

↳ if 'Transformers' are used for coupling then it is "Transformer Coupled Amplr" which is a Power Amplification system where "Impedance match" is mandatory.

↳ If there is no need of coupling then they are "Direct coupled Amplrs"

Ex: Darlington Pair, Op-Amps...

* Since

Amplr must need 'Transistor' where transistor is made up of 'Diodes' and diode has a 'Capacitance' which

is frequency variable component implies Amplifier can't perform same operations for all frequencies

1. if ampl can amplify upto 20kHz then it is "Af Amplifier"
   - Here 'C.E' ampl is best due to high 'Gains' and "FET" is more preferred due to high stability
   - * Upto High freq (Hf), CE is preferred where as under 'V.H.F or U.H.F' "C.B" is preferred under B.J.T.
   - * Highest 'Bandwidth ampl' is 'MOSFET'. it is has highest cutoff frequency
   - * 'Op-Amp' (Operational Amplifier) is a "DC-amplifier" i.e. it can amplify extreme low level frequencies even in the order of "Hz"

* MULTISTAGE - AMPLIFIERS :-

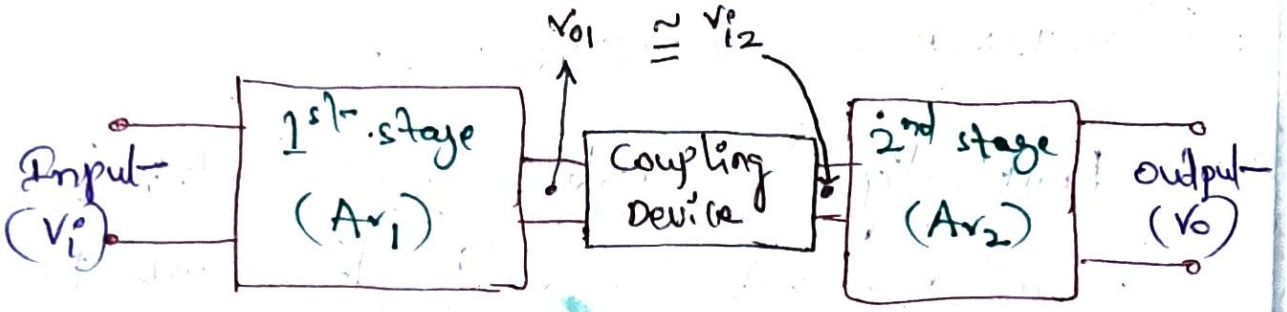
↳ Practically a 'Single-stage' amplifier's gain may not be sufficient enough to drive the "Load".

Hence 'Output' of one stage is connected as 'Input' for another stage and so on untill to get required output usually known as "Multistage amplifiers" {as it contains multiple stages}

* Multistage amplrs obtained by connecting several single stage amplifiers in series or cascaded form.

↳ Here in multistage amplrs, the output is 'Coupled' to the input of next stage using a "Coupling Device". This process is knaonces "Coupling mechanism" or simply "Coupling".

Ex: The following fig. shows a 2-stage amplifier connected in cascade form.



where, A_{V_1} = voltage Gain of stage-1

A_{V_2} = " " " " -2

Hence, here, the over all voltage Gain is given as

$$A_V = \frac{V_o}{V_i} = \left(\frac{V_o}{V_2}\right) \times \left(\frac{V_2}{V_i}\right)$$

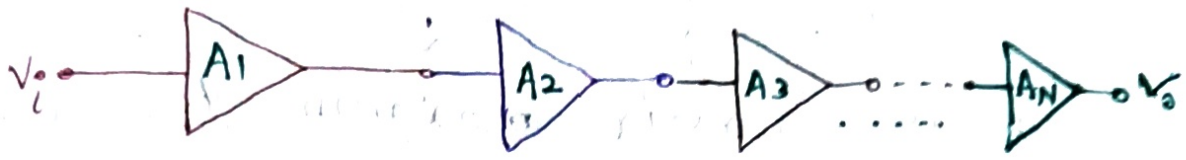
$$\boxed{A_V = A_{V_1} \times A_{V_2}}$$

$$A_{V_2} = \frac{V_o}{V_2} \Rightarrow \frac{V_o}{V_{i2}}$$

$$A_{V_1} = \frac{V_2}{V_i} = \frac{V_{i2}}{V_i}$$

i.e. The over all 'gain' in a multistage amplifiers is the multiplication of individual stage gains. (if No - Loading Effect)

it is also true for 'N'-stage amplifiers as shown below



$$\therefore \boxed{(A_V)_{overall} = A_1 \cdot A_2 \cdot A_3 \cdots A_N}$$

Here $A_1, A_2 \ldots A_N$ are calculated by considering 'Loading Effect'.

* This kind of "Cascading" generally used for

① To increase the 'Gain' which is sufficient to drive the Load

② To achieve desired input and o/p Resistances i.e. for good impedance matching

More About Coupling:—

'Coupling' achieves proper impedance match within stages of 'multistage amplifiers'. Usually coupling mechanism is made using some coupling devices like Capacitors, Resistors and/or Transformers

The purpose of coupling devices are

* To transfer maximum 'Ac' from o/p of one stage to the i/p of next stage
* to isolate 'DC' from o/p i.e. it blocks 'DC' and allows required 'Ac' to next stage

Types of Coupling:-

Usually there are 4 types of coupling mechanisms

① Impedance Coupling / Tuning:

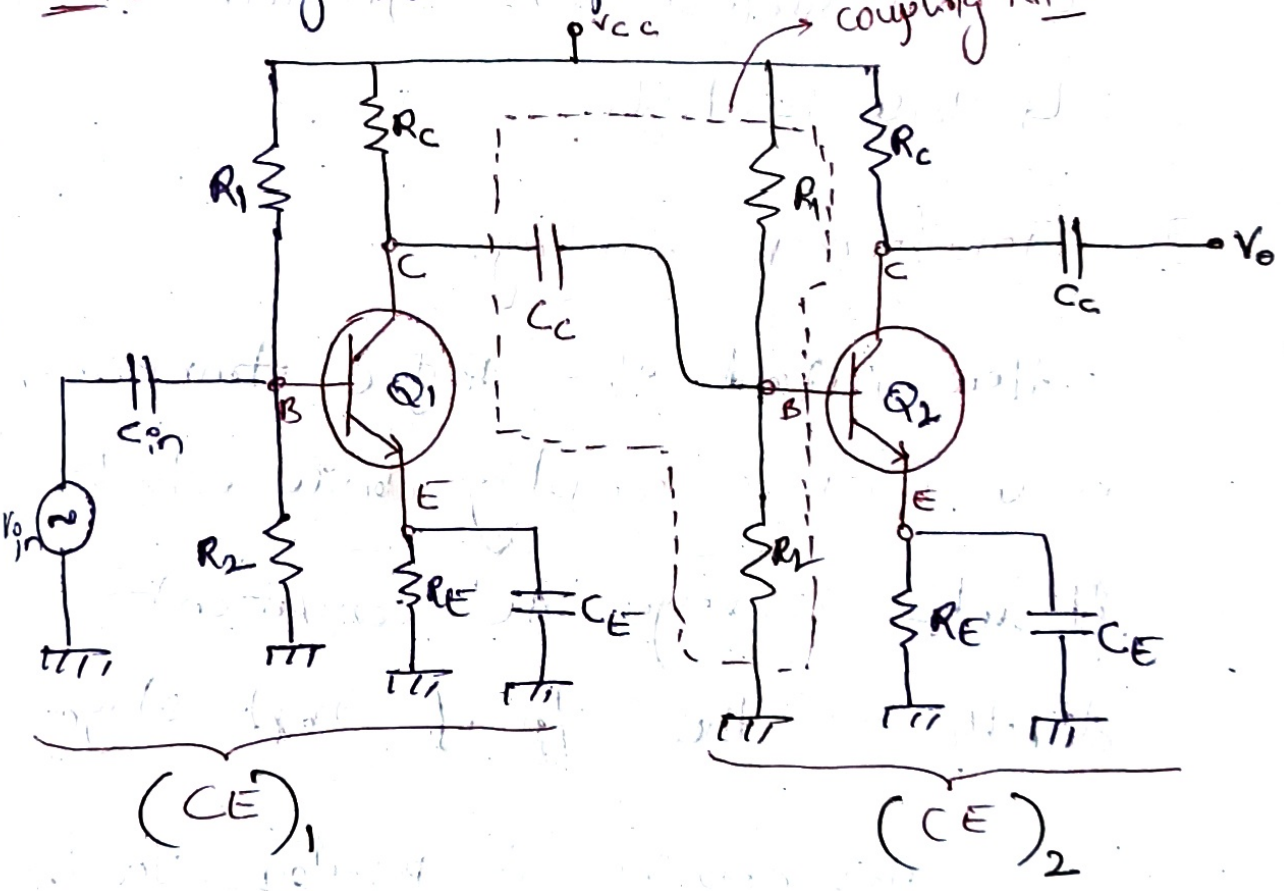
The coupling network that uses Inductance & Capacitance as coupling elements & Like Tuned Amplifier &

This is not so popular and rarely used.

(2) R-C { Resistance - Capacitance } Coupling

↳ it is mostly used technique which uses simple connection of 'R' & 'C' combination. The capacitor which allows 'AC' and blocks 'DC' is the main coupling element used here.

Ex:- Using 'BJT' Amplifs



let in above fig. — Q_1 & Q_2 are identical. Hence it uses mostly same "R's" and "C's" in amplf design.

* Advantages of RC coupling

↳ it Best suitable for Audio - Frequency' Range

↳ simple, cost effective.

↳ Gain is increased highly

* Dis-advantages & drawbacks

↳ Bandwidth Reduced.

↳ Not suitable for impedance matching

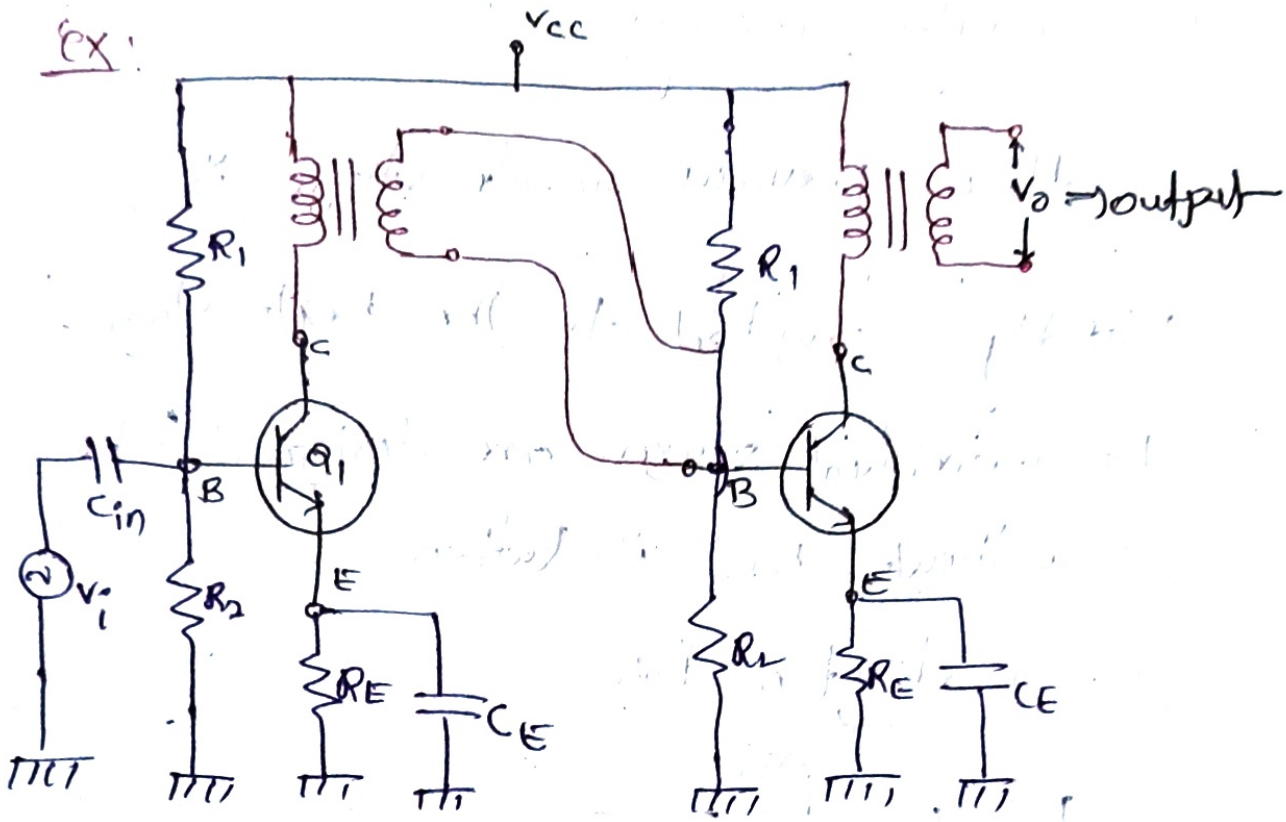
↳ Noise is high.

(3) Transformer Coupling.

Here instead of — 'R & C' transformer is used as a coupling device which it self- conveys 'Ac' component directly to the i/p of next stage.

This coupling is popular for its efficiency and its impedance matching.

Ex:



* Advantages:

↳ Good impedance matching

↳ High Efficiency & Gain

* Dis-advantage:

↳ Due to transformer, circuit become bulky & heavy

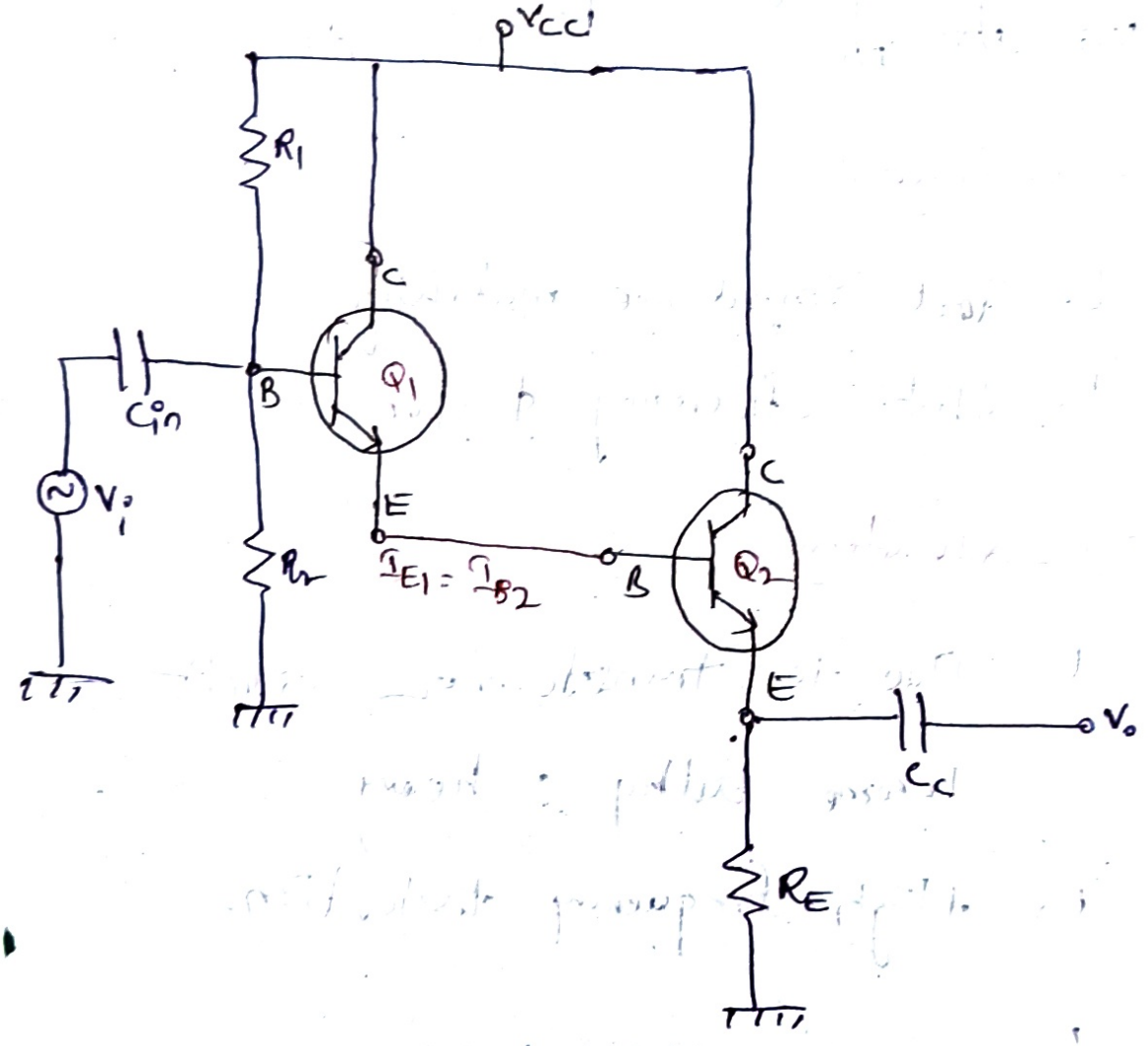
↳ High frequency distortion.

↳ occupies more space.

(4) Direct Coupling:-

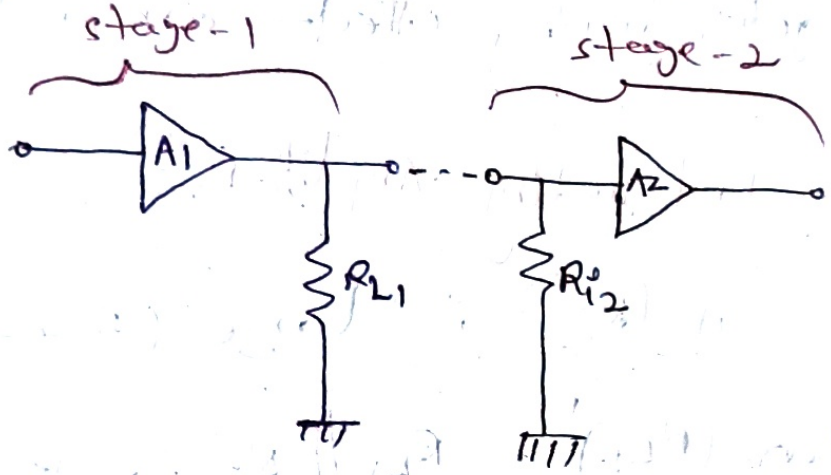
Here previous amplr stage is 'Directly' coupled to the next stage. The individual stages are designed like so without 'DC' isolation.

Ex: Darlington Pair



* Loading Effect in multistage Amplifier

↳ Loading effect refers to decrease / alter in voltage Gain of one amplifier stage due to input impedance of next stage.



Hence, from above, the Effective load for 'A1' (i.e stage 1) is

$$(R_{L_1})' = R_{L_1} \parallel R_{i_2}'$$

↳ Hence, due to change in Load impedance of 'A1' following possible cases may arise

Case (i): if R_{i_2}' is 'medium or low'

Then $(R_{L_1})'$ decreases due to which voltage Gain 'A1' decreases

↳ This decrement in Gain due to i/p resistance / impedance of 2nd stage is known as "Loading effect"

↳ This kind of amplifiers where we can get 'loading effect' are called "Interacting Amplifiers"

Case (ii): if (R_{i_2}) is Large $(\infty \Omega)$

then $(R_{L_1})' = R_{L_1} \,//\, R'_{i_2} \cong R_{L_1}$

∴ 'Ai' doesn't changes as impedance not changed.

↳ This kind of amplrs where loading effect doesnot occur are called as "Non-Interacting Amplrs".

* multistage Amplrs
  - Interacting Amplrs
  - Non Interacting Amplrs

* Amplifier - Frequency Response :-

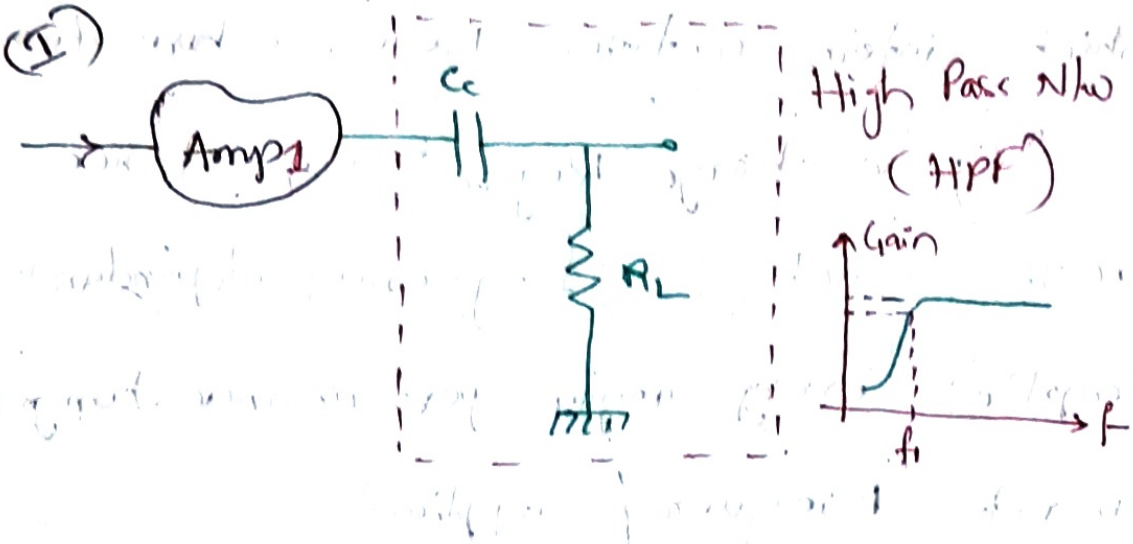
Since, every amplr made up of Transistors which intern contains 'Diodes' where due to "Space charge Region" capacitance exists which is frequency dependant- implies every amplr performance changes w.r.t- 'Frequency' applied

- Hence, in the frequency range of amplification any Noise/Distortions / Non-Linearity exists then amplification becomes unstable which is not- advisible.

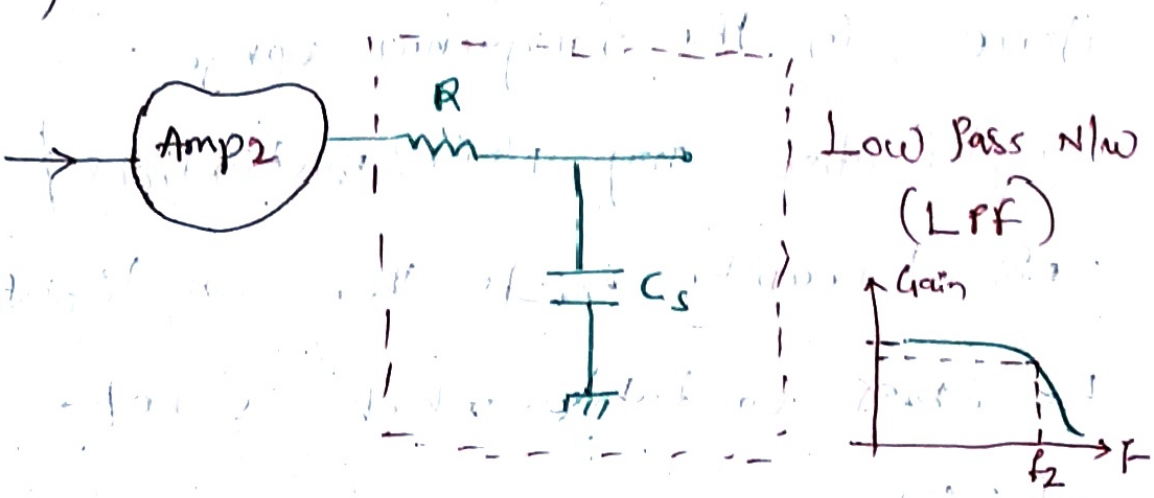
Hence to make 'Ac stability it is must need to find the Bandwidth i.e set of frequencies where faithful amplification is possible and to take care 'Noise' should never present-

Note: under Amplr design, following connections are observed.

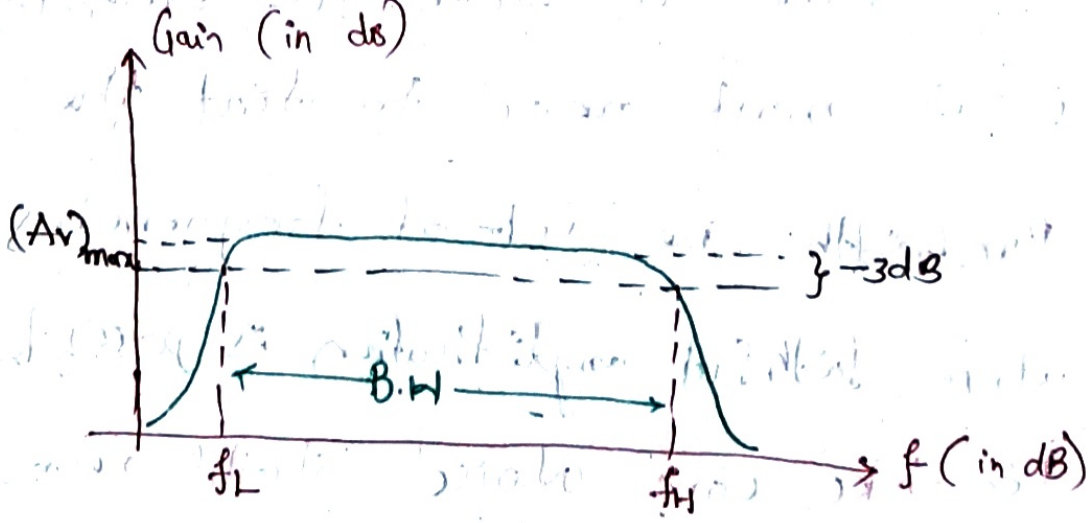
(I)



(II)

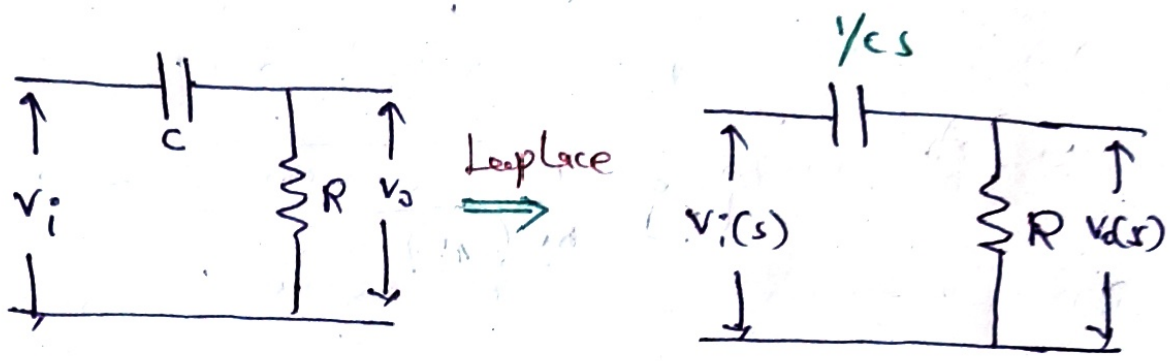


Together, Amplr response is given as **Bandpass**



Note: High Frequency Response

Consider following n/w & its Laplace equivalent ckt ←



Hence,

$$V_o(s) = V_i(s)\left(\frac{R}{R+\frac{1}{Cs}}\right)$$

$$\therefore A_v = \frac{V_o(s)}{V_i(s)} = \frac{R}{R+\frac{1}{Cs}}$$

$$= \frac{RCs}{1+RCs} \quad (\text{or}) \quad \boxed{\frac{\tau s}{1+\tau s}}$$

$$A_v = \frac{1}{1+\frac{1}{RCs}}$$

Now, put $s = j\omega$ in above equation.

$$\therefore A_v = \frac{1}{1+\frac{1}{RC(j\omega)}}$$

$$\therefore A_V = \frac{1}{1 - j\left(\frac{1}{RC\omega}\right)}$$

But $\omega = 2\pi f$,

$$\therefore A_V = \frac{1}{1 - j\,\frac{1}{RC(2\pi f)}}$$

$$= \frac{1}{1 - j\left\{\frac{1/2\pi RC}{f}\right\}}$$

Now let $\boxed{f_1 = \frac{1}{2\pi RC}}$

$$\therefore \boxed{A_V = \frac{1}{1 - j\left(\frac{f_1}{f}\right)}}$$ and, magnitude of gain is given as

$$\boxed{|A_V| = \frac{1}{\sqrt{1 + \left(\frac{f_1}{f}\right)^2}}} \quad ***$$

Now

Case (i): if $(f << f_1)$

then $\left(\frac{f_1}{f}\right) >> 1$ which tends to '∞' and $|A_V|$ tends to '0' (observe above expression)

Case (ii): if $(f >>> f_1)$

then $\left(\frac{f_1}{f}\right) << 1$ tends to '0' and

$$|A_V| = |A_{V_{max}}| = 1$$

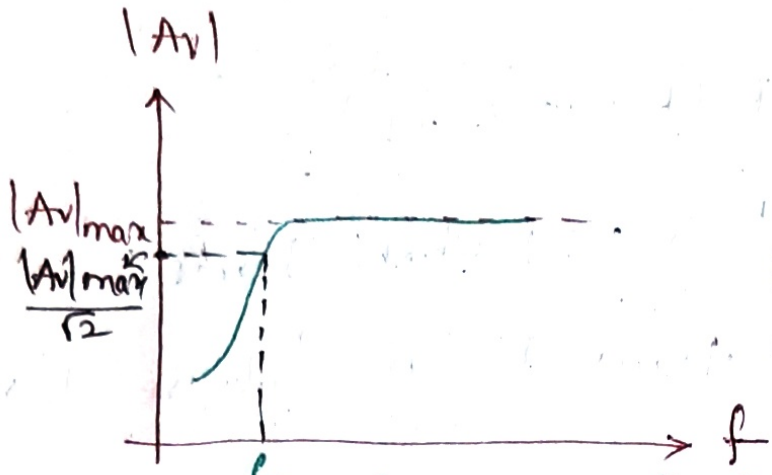
Case (iii): if $f = f_1$

then $$|A_V| = \frac{1}{\sqrt{1+1}} = \frac{1}{\sqrt{2}}$$

$$\therefore |A_V| = \frac{1}{\sqrt{2}}$$

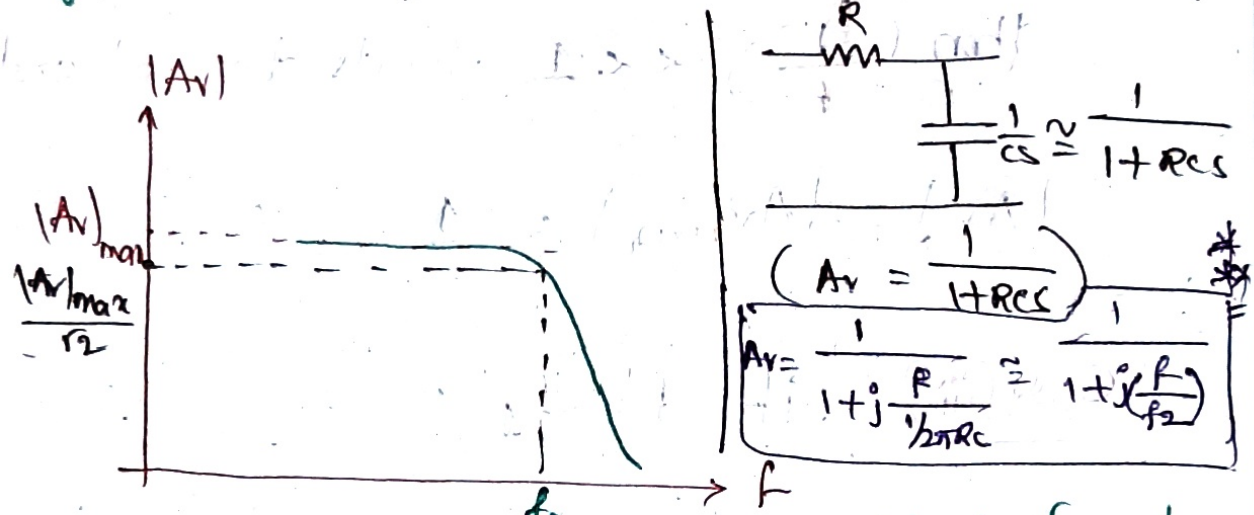
$$\boxed{|A_V| = \frac{|A_V|_{max}}{\sqrt{2}}}$$

Try to plot above characteristics as given below:



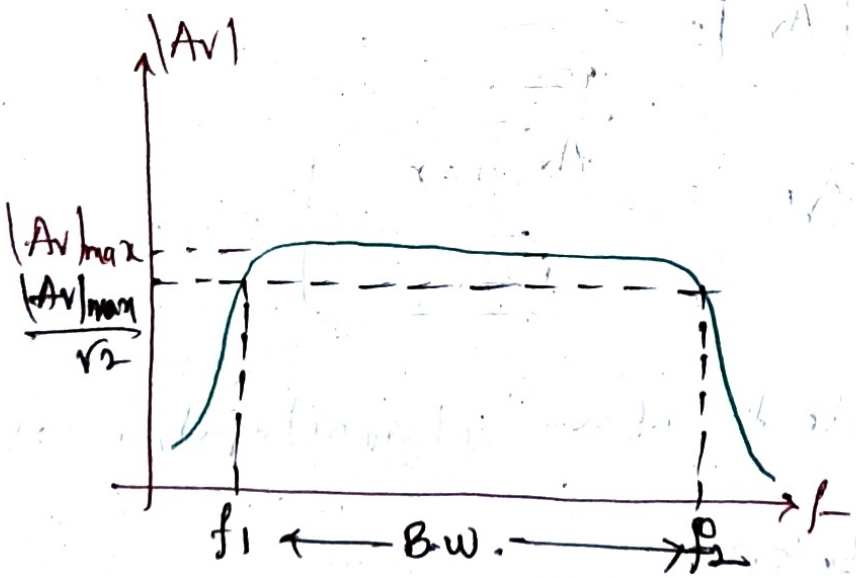
f_1 ⇒ Lower Cut off freq. / Corner freq.

Ig, Low frequency Response may given as

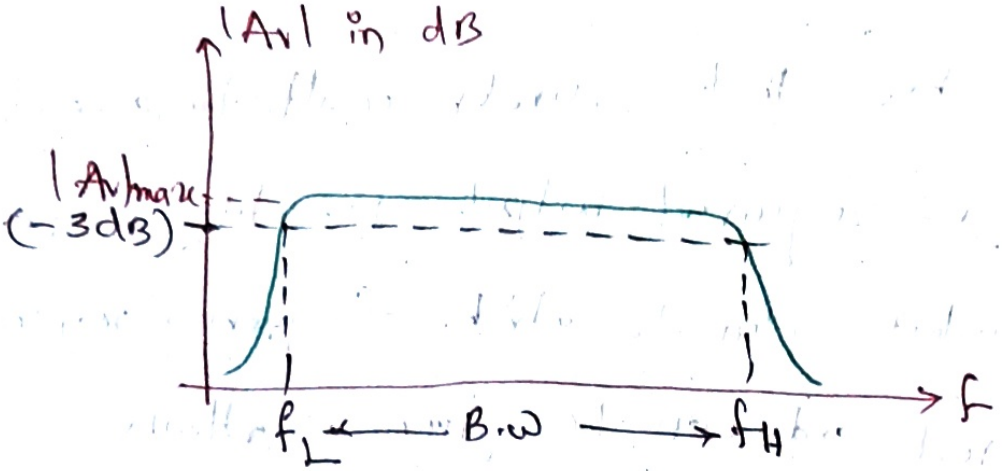


f_2 ⇒ Upper Cut off freq / corner freq.

Hence if combined



(OR) if we consider gain in 'dB'



Where, B.W = Bandwidth = $f_2 - f_1$ (or)

$= f_H - f_L$

f_L = Lower-Cut-off frequency

f_H = Upper/Higher " "

The above frequency is for only single stage amplr. for multi stage the frequency response varies according to the Number of stages cascaded which is given below.

* Frequency Response of Multistage Amplrs:

We know that, under multistage amplrs to get required output N-stages are cascaded due to which frequency response altered and can be given as follows

Effect of Cascading on Cut-off frequencies

↳ Consider a cascade of N-identical Non-Interacting amplrs. Assume that each ampl has cut off frequencies f_{L1} & f_{H1}. Then

Case(i): Effect on Lower Cut-off frequency

we have, Low frequency voltage Gain for each amplr is

$$A_{VL_1} = \frac{A_{VM_1}}{\sqrt{1+\left(\frac{f_{L_1}}{f}\right)^2}}$$

if 'N' such amplrs are cascaded. Then

$$A_{V_L} = A_{VL_1} \times A_{VL_2} \times \ldots\ldots A_{VLN}$$
$$= (A_{VL_1})^N //$$

$$\therefore A_{VL} = (A_{VL_1})^N = \frac{(A_{Vm_1})^N}{\sqrt{\left[1+\left(\frac{f_{L_1}}{f}\right)^2\right]^N}}$$

$$= \frac{(A_{Vm_1})^N}{\left\{1+\left(\frac{f_{L_1}}{f}\right)^2\right\}^{N/2}}$$

$$A_{V_L} = \frac{A_{Vm}}{\left\{1+\left(\frac{f_{L_1}}{f}\right)^2\right\}^{N/2}} \quad \text{---(1)}$$

where $A_{Vm} = (A_{Vm_1})^N$. But

at $f = f_L \Rightarrow A_{V_L} = \frac{A_{Vm}}{\sqrt{2}}$

$\therefore$ from (1),

$$\frac{A_{Vm}}{\sqrt{2}} = \frac{A_{Vm}}{\left[1+\left(\frac{f_{L_1}}{f_L}\right)^2\right]^{N/2}}$$

squaring on both sides

$$2 = \left[1+\frac{f_{L_1}}{f_L}\right]^N$$

$$\therefore 2^{1/N} - 1 = \left(\frac{f_{L_1}}{f_L}\right)$$

$$\therefore \frac{f_{L1}}{f_L} = \sqrt{2^{1/N} - 1}$$

$$\therefore \boxed{f_L = \frac{f_{L1}}{\sqrt{2^{1/N} - 1}}}$$

i.e. for $N \geq 2$, $\sqrt{2^{1/N} - 1} < 1$

$$\therefore \boxed{f_L > f_{L1}}$$

Hence Cascading increases Lower Cutoff frequency (moves towards right side)

Case(ii): Effect on Higher-Cutoff frequency

we have, for single stage amplr

$$\boxed{A_{VH1} = \frac{A_{VM1}}{\sqrt{1 + \left(\frac{f}{f_{H1}}\right)^2}}}$$

hence, for N-stage amplrs

$$A_{VH} = \frac{(A_{VM1})^N}{\left[1 + (f/f_{H1})^2\right]^{N/2}} = \frac{A_{VM}}{\left[1 + (f/f_{H1})^2\right]^{N/2}}$$

But @ $f = f_{H1}$, $A_{V_H} = A_{vm}/\sqrt{2}$

$$\therefore \frac{A_{vm}}{\sqrt{2}} = \frac{A_{vm}}{\left[1 + \left(\frac{f}{f_H}\right)^2\right]^{N/2}}$$

Squaring on both sides

$$2 = \left[1 + \left(\frac{f_H}{f_{H1}}\right)^2\right]^N$$

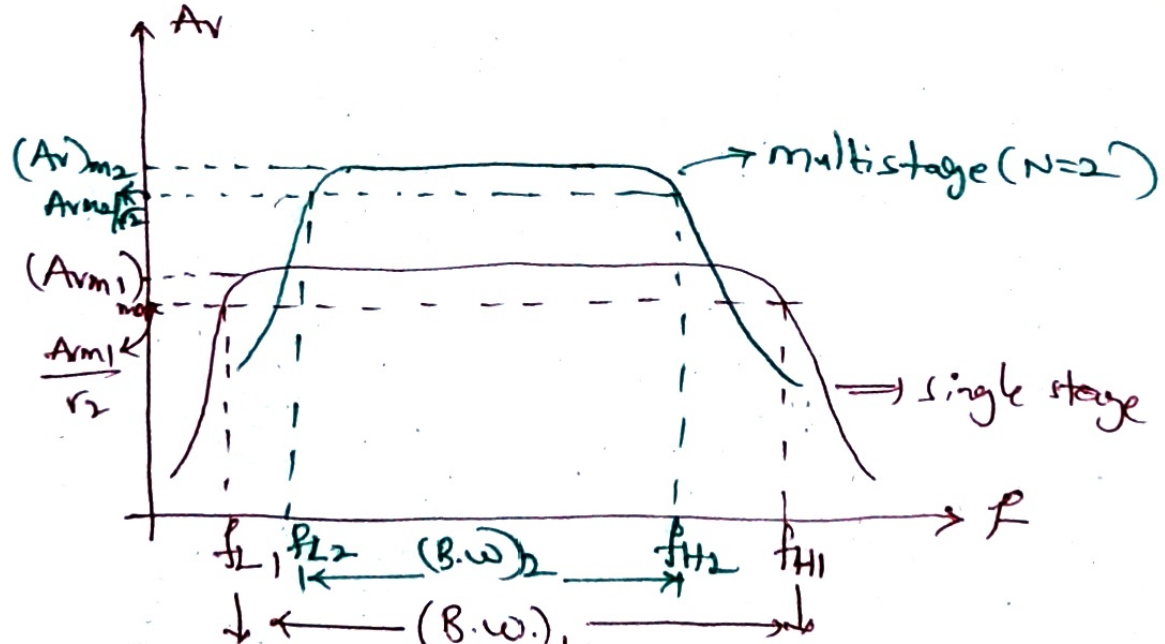
$$\therefore \boxed{f_H = f_{H1} \times \sqrt{2^{1/N} - 1}}$$

for $N \geq 2$, $\sqrt{2^{1/N} - 1} < 1$, i.e.

$$\boxed{f_H < f_{H1}}$$

Hence Cascading decreases upper cutoff frequencies (moves towards Left side)

The overall response is given below (for N=2)

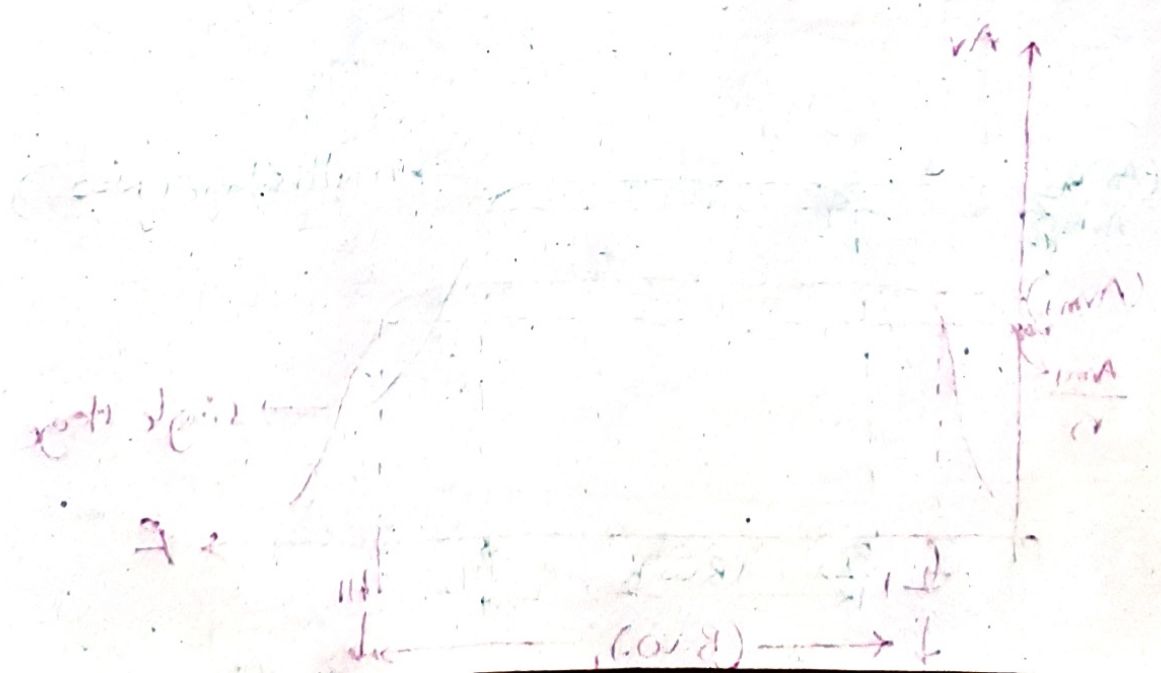


Note:

If 'N' identical amplifiers having B.W. as $(B.W)_1$ are cascaded then overall Bandwidth is

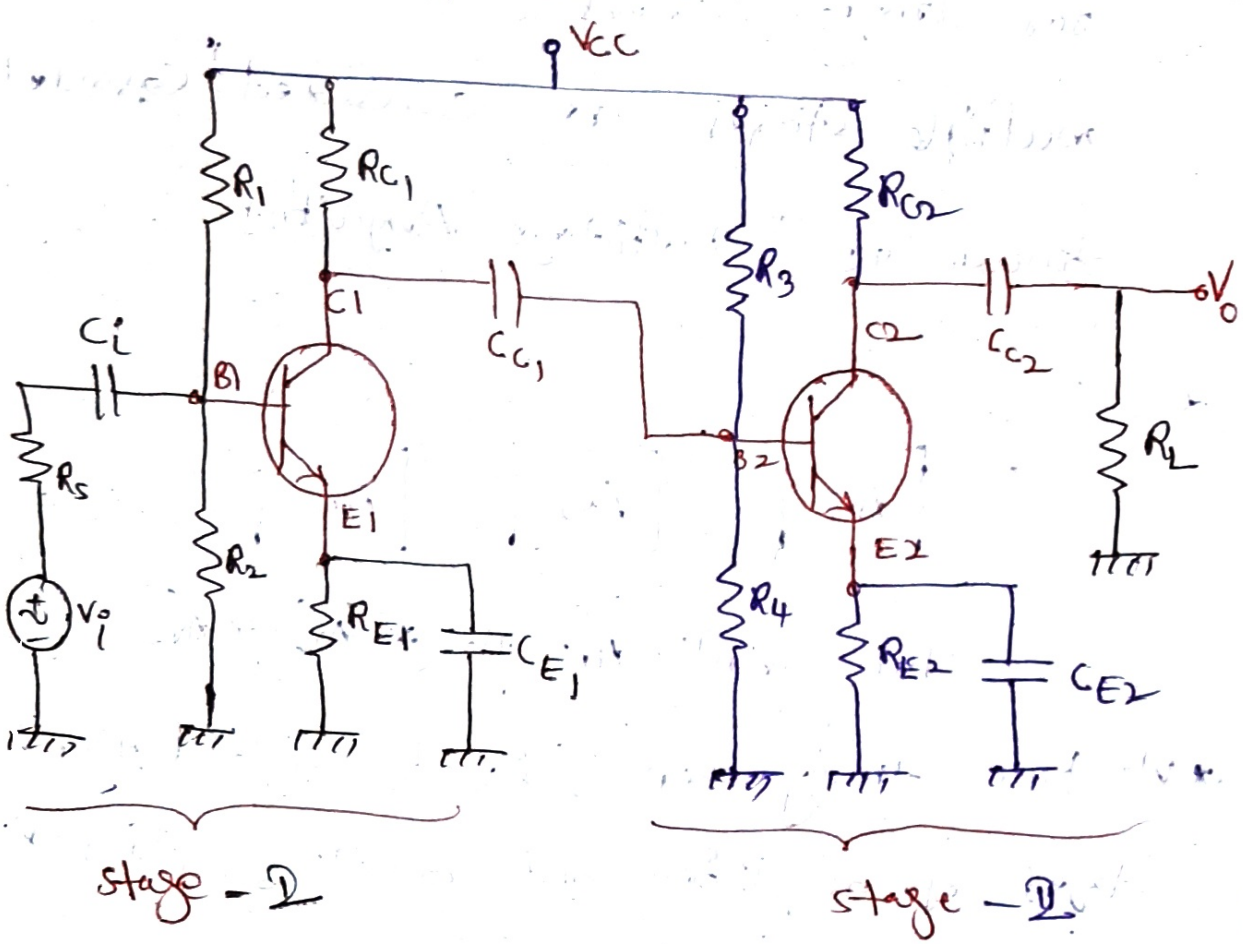
$$B.W = (B.W)_1 \times \sqrt{2^{1/N} - 1}$$

i.e. $(B.W)$ decreases as Number of stages increased.



① RC-Coupled Amplifier {Cascaded} Analysis

↳ it is designed using 'CE-CE' stages as shown below



Note: DC-analysis is also important to analyse AC-analysis effectively.

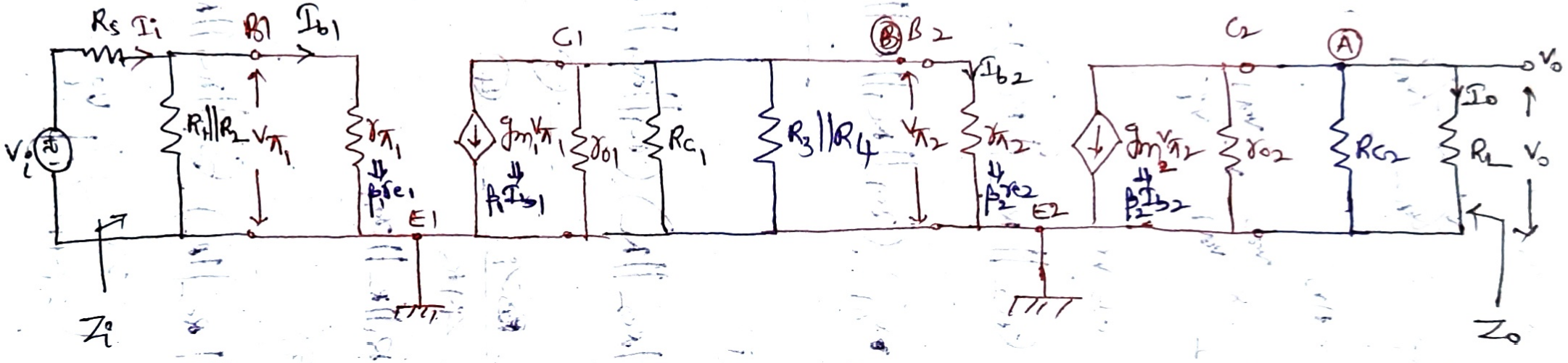
* The AC-equivalent model is given below for two stage RC-coupled amplifier.

Here under AC-analysis

↳ all DC-sources ⇒ Grounded

↳ Capacitors ⇒ shorted

The Ac- small signal equivalent circuit is given as



The calculations case A_v, Z_i, Z_o, etc.

Let $R_1 \| R_2 = R_{B1}$

$R_3 \| R_4 = R_{B2}$

$Z_i = R_1 \| R_2 \| r_{\pi 1} = R_{B1} \| r_{\pi 1}$

$Z_o = r_{o2} \| R_{C_2} \| R_L$

* Calculating voltage Gain ("A_V") :—

we have $A_{VS} = \frac{V_o}{V_i} = \frac{V_o}{V_{\pi 2}} \times \frac{V_{\pi 1}}{V_{\pi 1}}$

$= A_{V_1} \times A_{V_2}$

$$A_{VS} = \frac{V_o}{V_{\pi 2}} \times \frac{V_{\pi 2}}{V_{\pi 1}} \times \frac{V_{\pi 1}}{V_i}$$

Now: from Circuit above.

$$V_o = (-g_{m_2} \cdot V_{\pi 2}) \cdot (r_{o2} \| R_{C_2} \| R_L) \quad \text{—①}$$

But, $V_{\pi 2} = (-g_{m_1} \cdot V_{\pi 1}) \cdot (r_{o1} \| R_{C_1} \| R_3 \| R_4)$ —②

Substitute ② in ①

$$V_o = \{g_{m_1} \cdot g_{m_2} V_{\pi 1} (r_{o2} \| R_{C2} \| R_L) (r_{o1} \| R_{C_1} \| R_3 \| R_4)$$ —③

but $V_{\pi 1} = V_i \times \frac{Z_i}{Z_i + R_S}$ —④

where $Z_i = R_1 \| R_2 \| r_{\pi 1} =$

Substitute ④ in ③

$$\therefore V_o = \{g_{m_1} \cdot g_{m_2} \cdot (r_{o2} \| R_{C2} \| R_L) (r_{o1} \| R_{C_1} \| R_3 \| R_4) \cdot \left(\frac{Z_i}{Z_i + R_S}\right)\} V_i$$

$$\therefore A_{Vs} = \frac{V_o}{V_i} = \left\{ g_{m_1} \cdot g_{m_2} \cdot (r_{o_2} \parallel R_{c_2} \parallel R_L) \cdot (r_{o_1} \parallel R_{c_1} \parallel R_3 \parallel R_4) \left(\frac{Z_i'}{Z_i' + R_s}\right) \right\}$$ //

Note:

* if identical BJT's are choosen, then

ideally,

$$A_V = (g_{m_1} R_{c_1}) \cdot (g_{m_2} R_{c_2})$$

$$\boxed{A_V \cong (g_m R_c)^2}$$

* Over all i/p impedance (R_i')

$$\boxed{R_i' = Z_i' + R_s}$$

* over all o/p impedance (R_o')

$$\boxed{R_o' \cong R_{c_2}} \quad // \quad (\text{if } R_L >>> R_c)$$

* Calculating Current Gain (A_{Is}):

$$A_{Is} = \frac{I_o}{I_i} = \frac{I_L}{I_i} = \frac{I_o}{V_{\pi 2}} \times \frac{V_{\pi 2}}{V_{\pi 1}} \times \frac{V_{\pi 1}}{I_{b1}} \times \frac{I_{b1}}{I_i'}$$

But, $I_o = (-g_{m_2} V_{\pi 2}) \left(\frac{r_{o_2} \parallel R_{c_2}}{r_{o_2} \parallel R_{c_2} + R_L} \right)$ ——— (1)

($\because$ Current divider)

Now apply, KCL at node (B) in the circuit

i.e. $(g_{m_1} \cdot V_{\pi 1}) + \frac{V_{\pi 2}}{r_{o1} \| R_{C1} \| R_{B2} \| r_{\pi 2}} = 0$

$\therefore V_{\pi 2} = V_{\pi 1}\left(-g_{m_1} \cdot (r_{o1} \| R_{C1} \| R_{B2} \| r_{\pi 2})\right)$ ——— (2)

Now, from input stage

$r_{\pi 1} = \frac{V_{\pi 1}}{I_{b1}}$ and

$I_{b1} = I_i \times \left(\frac{R_1 \| R_2}{R_1 \| R_2 + r_{\pi 1}}\right)$ ——— (3) $\rightarrow R_{B1}$

from all above

$$A_{I_S} = g_{m_2}\left(\frac{r_{o2} \| R_{C2}}{r_{o2} \| R_{C2} + R_L}\right) \times \left\{-g_{m_1}\left[r_{o1} \| R_{C1} \| R_{B2} \| r_{\pi 2}\right]\right\} \times r_{\pi 1} \times \left(\frac{R_{B1}}{R_{B1} + r_{\pi 1}}\right)$$

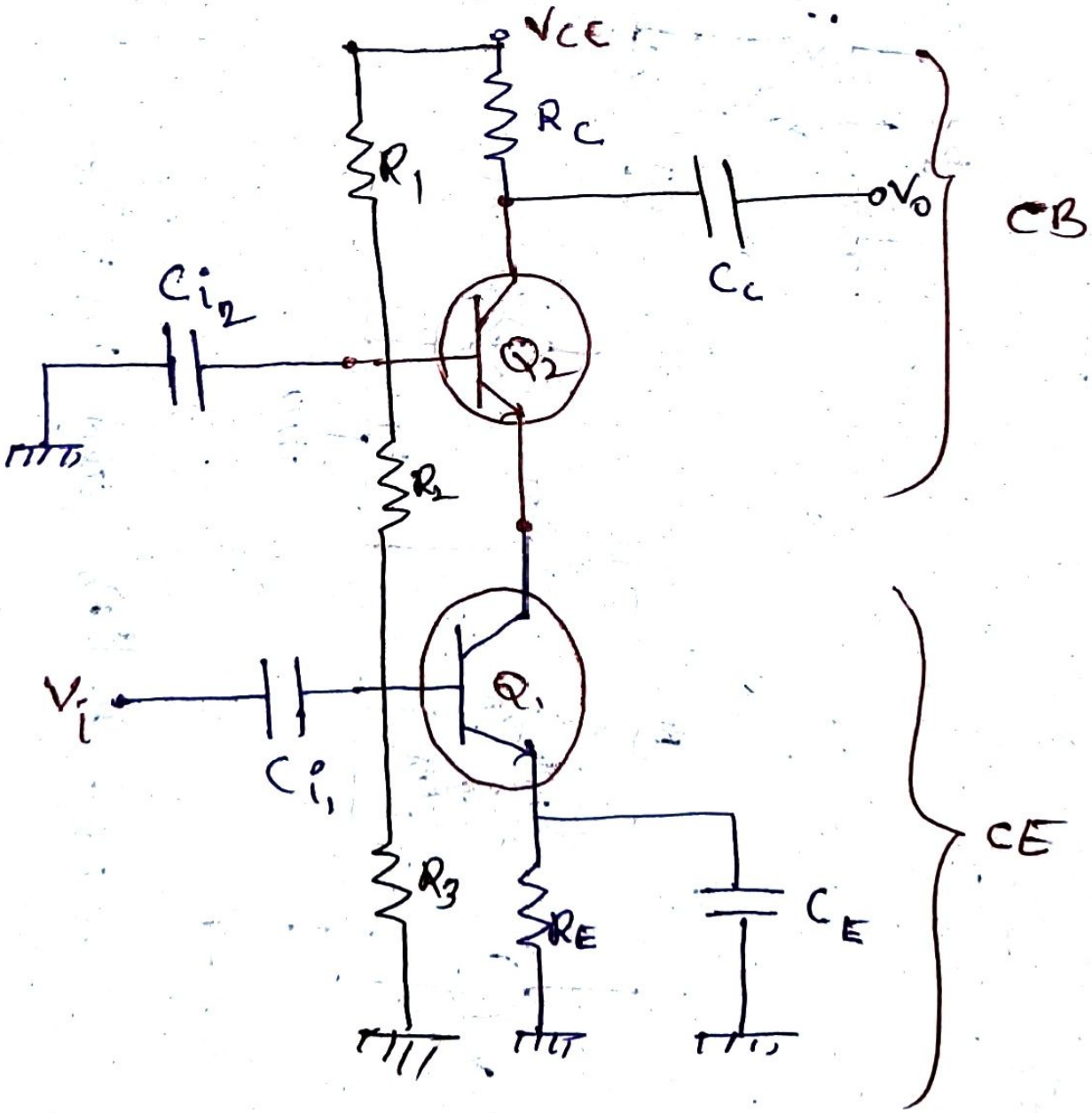
if $R_{C2} >> R_L$ & $R_{B1} >> r_{\pi 1}$, then

$$A_{I_S} = g_{m_1} \cdot g_{m_2}\left(R_C \| R_{B2} \| r_{\pi 2}\right) \cdot r_{\pi 1}$$

* Cascode Amplifier (CE-CB)

↳ It is a multistage amplifier of CE - CB connection

↳ As 'CE' has High Gain & 'CB' has high Bandwidth cascode contains advantages of both



The small signal model is given below

Calculating voltage Gain (A_V) :-

$$A_V = \frac{V_o}{V_i} \quad \text{---} \quad (1)$$

Now, apply KCL @ node (A) shown in above fig, i.e.,

$$g_{m_2} V_{\pi 2} + \left(\frac{V_o}{R_c}\right) = 0$$

$$\therefore g_{m_2} V_{\pi 2} = \frac{-V_o}{R_c} \quad \text{---} \quad (2)$$

Now, apply KCL @ node (B)

$$g_{m_2} V_{\pi 2} = \frac{-V_{\pi 2}}{r_{\pi 2}} + g_{m_1} V_{\pi 1}$$

$$\Rightarrow g_{m_2} V_{\pi 2} \cdot r_{\pi 2} = -V_{\pi 2} + g_{m_1} V_{\pi 1} \cdot r_{\pi 2}$$

$$\therefore V_{\pi 2} \{ g_{m_2} r_{\pi 2} + 1 \} = g_{m_1} V_{\pi 1} r_{\pi 2}$$

$$\therefore \frac{V_{\pi 2}}{V_{\pi 1}} = \frac{g_{m_1} r_{\pi 2}}{(g_{m_2} r_{\pi 2} + 1)} \quad \text{---} \quad (3)$$

But from fig,

$$V_{\pi 1} = V_i^o \Rightarrow \frac{V_{\pi 1}}{V_i^o} = 1 \text{ "}$$

$$\therefore \frac{V_o}{V_i^o} = \frac{V_o}{V_{\pi 2}} \cdot \frac{V_{\pi 2}}{V_{\pi 1}} \cdot \frac{V_{\pi 1}}{V_i^o}$$

$$\boxed{A_V = (-g_{m_2} R_C)\left(\frac{g_{m_1} \cdot r_{\pi 2}}{g_{m_2} r_{\pi 2} + 1}\right) \cdot 1}$$

(or)

$$g_{m_2} r_{\pi 2} + 1 \cong g_{m_2} r_{\pi 2}$$

$$\therefore A_V \cong -g_{m_2} \cdot R_C \cdot \frac{g_{m_1} r_{\pi 2}}{g_{m_2} r_{\pi 2}}$$

$$\therefore \boxed{A_V \cong -g_{m_1} R_C}$$

* Calculating "Z_i" & "Z_o"

from fig $Z_i = R_2 // R_3 // r_{\pi 1} = R_{31} // r_{\pi 1}$

$$Z_o = R_C$$

Calculating "A_I" :-

we have $A_I = I_o / I_i$

apply KCL @ node (A) :

$$I_o = -g_{m_2} V_{\pi 2}$$

$$\therefore \frac{I_o}{V_{\pi 2}} = -g_{m_2} \quad \text{---} \quad (1)$$

and, from previous analysis of voltage gain we already have

$$\frac{V_{\pi 2}}{V_{\pi 1}} = \frac{g_{m_1} r_{\pi 2}}{g_{m_2} r_{\pi 2} + 1} \quad \text{---} \quad (2)$$

and we have

$$\frac{V_{\pi 1}}{r_{\pi 1}} = i_{b1} \Rightarrow \frac{V_{\pi 1}}{i_{b1}} = r_{\pi 1} \quad \text{---} \quad (3)$$

and from fig.

$$I_{b1} = I_i \left\{ \frac{R_{B1}}{R_{B1} + r_{\pi 1}} \right\} \quad (\because \text{current division})$$

$$\therefore A_I = \frac{I_o}{I_i} = \frac{I_o}{V_{\pi 2}} \cdot \frac{V_{\pi 2}}{V_{\pi 1}} \cdot \frac{V_{\pi 1}}{I_{b1}} \cdot \frac{I_{b1}}{I_i}$$

$$\boxed{A_I = -g_{m_2} \left\{ \frac{g_{m_1} r_{\pi 2}}{g_{m_2} r_{\pi 2} + 1} \right\} \cdot \{ r_{\pi 1} \} \cdot \left\{ \frac{R_{B1}}{R_{B1} + r_{\pi 1}} \right\}}$$

(*R)

if, $g_{m_2} r_{\pi 2} + 1 \cong g_{m_2} r_{\pi 2}$

$$\therefore A_I = (-g_{m_2}) \left\{ \frac{g_{m_1} r_{\pi 2}}{g_{m_2} r_{\pi 2}} \right\} \{ r_{\pi 1} \} \left(\frac{R_{B1}}{R_{B1} + r_{\pi 1}} \right)$$

$$\therefore \boxed{A_I \cong (g_{m_1} r_{\pi 1}) \left\{ \frac{R_{B1}}{R_{B1} + r_{\pi 1}} \right\}}$$

* Darlington - Pair :- (CC - CC)

↳ it is named after its inventor sidney Darlington.

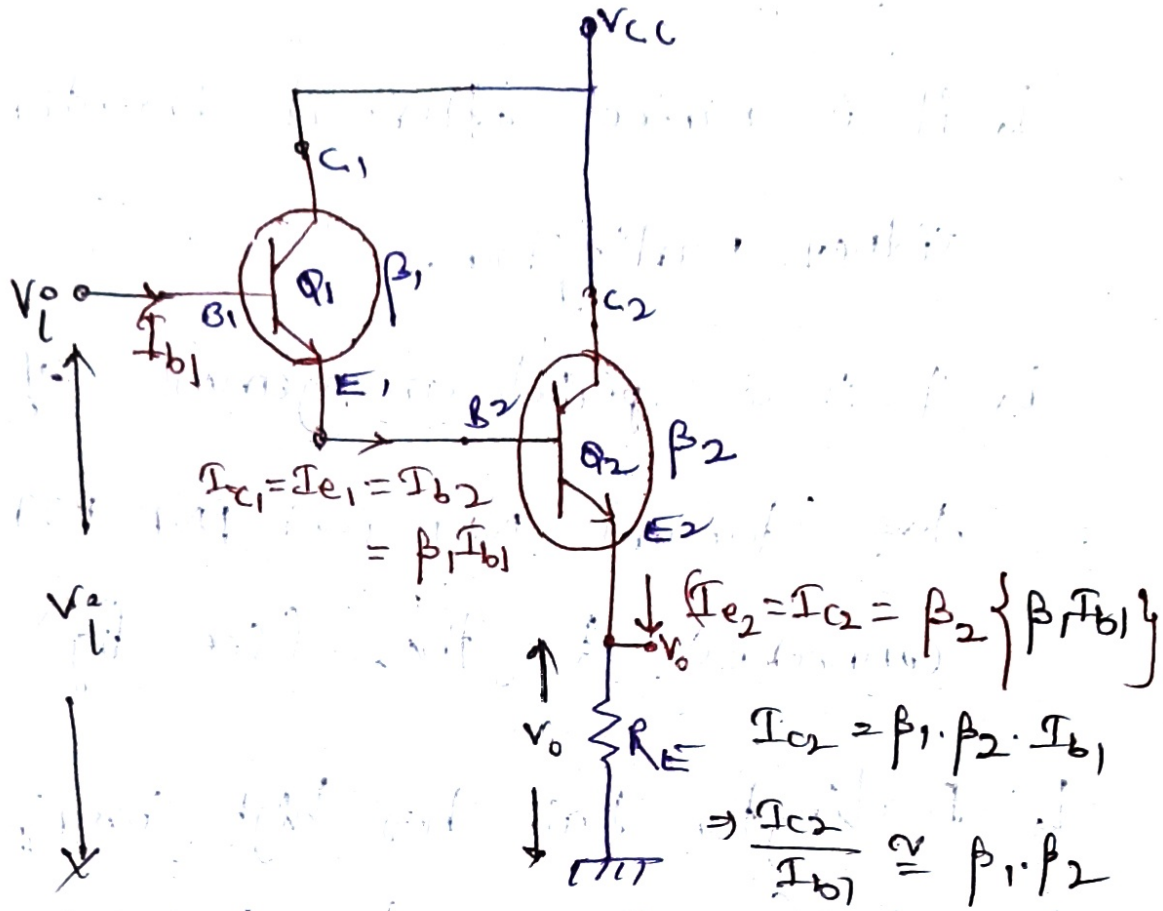
↳ it is a special arrangement of two standard NPN (or) PNP BJT's connected together (see fig).

↳ Darlington Pair has high current gains (as it multiplies the current gains of individual transistors)

↳ it is the combination of CC - CC configurations.

↳ mainly used in current amplification circuits & faster switching applications.

↳ A simple darlington pair is given below



operation:

As shown above,

For Q_1:-

$$I_{c_1} \simeq I_{e_1} = \beta I_{b1} \quad \text{—(1)}$$

for Q_2:-

It is clear that $I_{e_1} = I_{c_1}$ is given as i/p to 'Q_2'

$$\therefore I_{e_1} = I_{b_2} = \beta_1 I_{b1}$$

$$\therefore I_{e_2} = I_{c_2} = \beta_2 (I_{b_2}) = \beta_2 (\beta_1 I_{b1})$$

$$\therefore \boxed{I_{e_2} = I_o = \beta_1 \beta_2 \cdot I_{b1}}$$

Note:

ideally, i.e. if two transistors are perfectly matched, then $\beta_1 = \beta_2$

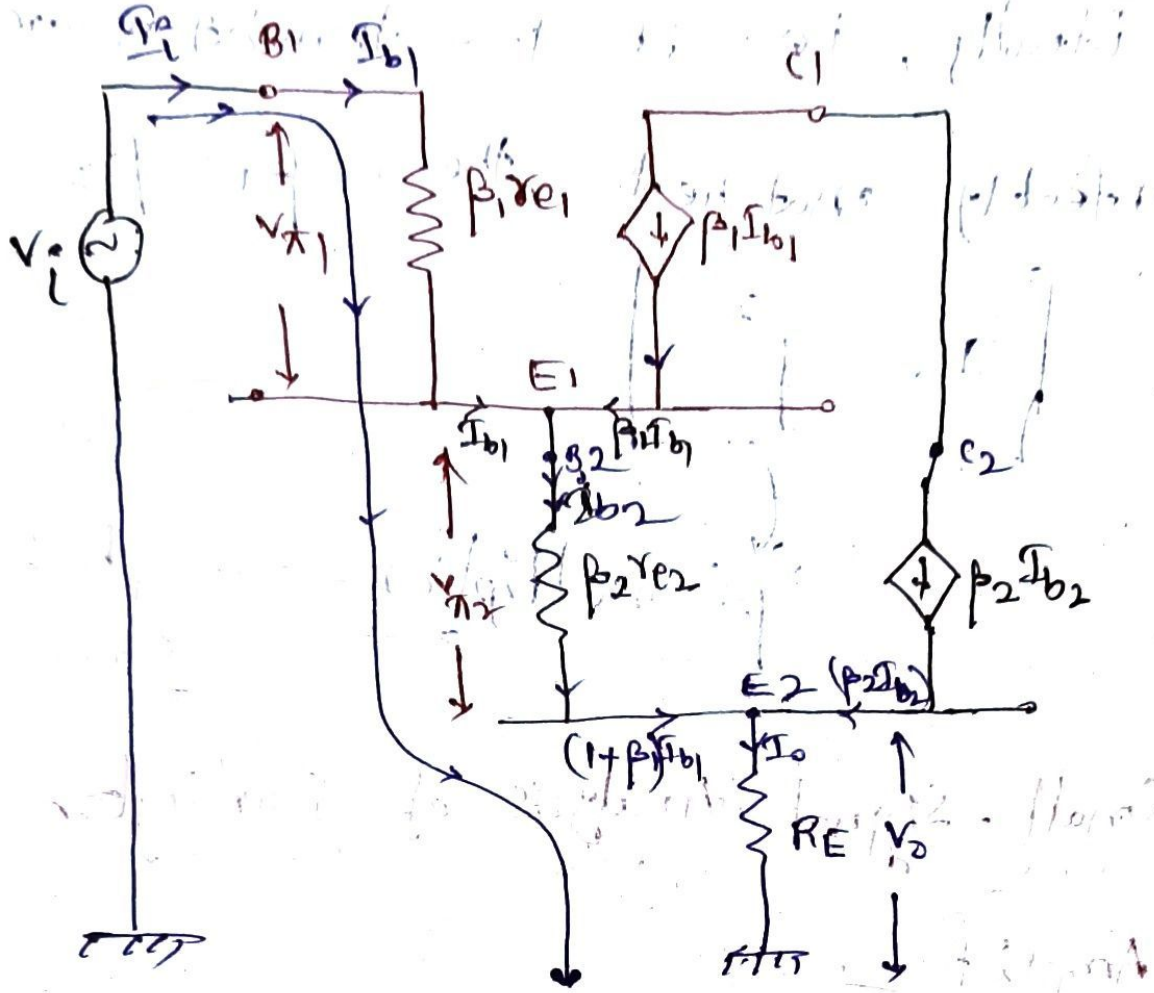
$$\therefore \boxed{I_o = \beta^2}$$

β^2 multiplier $\Rightarrow$ very high Gain

* Small-Signal Analysis of Darlington Amplifier:

Let β_1, r_{π_1} (β_{re_1}) are parameters of Q_1 & β_2, r_{π_2} are parameters of Q_2 shown in darlington pair shown above.

The small signal equivalent model is given below



Calculating Current Gain (A_I),

we have $A_I = \dfrac{I_o}{I_i}$

Sub $I_o = I_{b2} + \beta_2 I_{b2}$ — (1)

$= (1+\beta_2) I_{b2}$

and from fig,

$I_{b2} = I_{b1} + \beta_1 I_{b1}$

$= (1+\beta_1) I_{b1}$ — (2)

& $I_{b1} = I_i$

Substitute (2) in (1)

$$\therefore I_o = (1+\beta_2).(1+\beta_1).I_{b1}$$

$$I_o = (1+\beta_1+\beta_2+\beta_1\beta_2)\, I_i$$

$$\therefore \boxed{A_I = \frac{I_o}{I_i} = 1+\beta_1+\beta_2+\beta_1\beta_2}$$

Note:

if $\beta_1 = \beta_2 = \beta$, then

$$I_o = (1+\beta)(1+\beta).I_i$$

$$\therefore I_o = \beta^2 I_i \qquad (\because 1+\beta \simeq \beta)$$

$$\therefore \boxed{A_I = \frac{I_o}{I_i} = \beta^2}$$

* Darlington - Amplifier provides very high current gains (approximately β^2).

Calculating i/p impedance (Z_i)

we have, $Z_i = \frac{V_i}{I_i}$

apply KVL in path shown.

$$V_i = I_{b1}(\beta_1 r_{e1}) + I_{b2}(\beta_2 r_{e2}) + I_o R_E \quad \text{---(1)}$$

But $I_{b1} = I_i$,

$$I_{b2} = (1+\beta_1) I_{b1} = (1+\beta_1) I_i$$

$$I_o = \beta^2 I_i$$

∴ substitute all above in eq (1)

$$\therefore V_i = I_i \left\{ (\beta_1 r_{e1}) + (\beta_2 r_{e2})(1+\beta_1) + \beta^2 R_E \right\}$$

$$\therefore \boxed{Z_i = \frac{V_i}{I_i} = \beta_1 r_{e1} + \beta_1 \beta_2 r_{e2} + \beta^2 R_E}$$

(OR)

$$\boxed{Z_i \cong \beta^2 R_E}$$

(as r_{e1} & r_{e2} are very small)

* Calculating Z_o

$$\boxed{Z_o \cong R_E}$$

* Calculating voltage Gain (A_v) :-

more simply,

$$A_v = \frac{V_o}{V_i}$$

$$= \frac{I_o \cdot Z_o}{I_i \cdot Z_i}$$

$$= \frac{(\beta^2 I_i) R_E}{I_i (\beta^2 R_E)} \qquad \left(\because I_o = \beta^2 I_i,\ Z_i = \beta^2 R_E,\ Z_o = R_E \right)$$

$$\boxed{A_v \cong 1}$$