

6.3 weak law of Large Numbers

Let $\{X_n\}$ be a sequence of r.v's & let $\bar{X}_n = \frac{1}{n} \sum_{i=1}^n X_i$ be the mean of first n r.v's.

The weak laws deal with limits of probabilities involving $\bar{X}_n$.

The strong laws deal with probabilities involving limits of $\bar{X}_n$.

Definition of weak law of Large numbers

A sequence $\{X_n\}$ of r.v's is said to satisfy the weak law of Large numbers (WLLN) if

$$\lim_{n \rightarrow \infty} P \left[\left| \frac{S_n}{n} - E\left(\frac{S_n}{n}\right) \right| < \epsilon \right] = 1$$

for any $\epsilon > 0$, where $S_n = \sum_{i=1}^n X_i$ i.e., $\frac{S_n}{n} \xrightarrow{P} E\left(\frac{S_n}{n}\right)$

Theorem 1: Let $\{X_n\}$ be a sequence of r.v's & let $S_n = X_1 + X_2 + \dots + X_n$ with $B_n = V(S_n) < \infty$. If $\frac{B_n}{n^2} \rightarrow 0$ as $n \rightarrow \infty$, then

$$\text{for any } \epsilon > 0, \lim_{n \rightarrow \infty} P \left[\left| \frac{S_n}{n} - E\left(\frac{S_n}{n}\right) \right| < \epsilon \right] = 1$$

i.e., $\{X_n\}$ satisfies WLLN

Pf: On applying Chebychev's inequality to the variables $\frac{S_n}{n}$,

we have

$$P \left[\left| \frac{S_n}{n} - E\left(\frac{S_n}{n}\right) \right| \geq \epsilon \right] \leq \frac{V\left(\frac{S_n}{n}\right)}{\epsilon^2} = \frac{V(S_n)}{n^2 \epsilon^2} = \frac{B_n}{n^2 \epsilon^2} \rightarrow 0$$

as $n \rightarrow \infty$. Thus

$$\lim_{n \rightarrow \infty} P \left[\left| \frac{S_n}{n} - E\left(\frac{S_n}{n}\right) \right| \geq \epsilon \right] = 0$$

$$\Rightarrow \lim_{n \rightarrow \infty} P \left[\left| \frac{S_n}{n} - E\left(\frac{S_n}{n}\right) \right| < \epsilon \right] = 1$$

$\Rightarrow \{X_n\}$ satisfies WLLN.

Corollary 1: let $\{x_n\}$ be a sequence of r.v.'s, $\bar{X}_n = \frac{S_n}{n}$ &
 $\mu = E\left(\frac{S_n}{n}\right)$. If $\frac{B_n}{n^2} \rightarrow 0$ as $n \rightarrow \infty$ then

$$\lim_{n \rightarrow \infty} P[\bar{X}_n \leq k] = \begin{cases} 0, & \text{if } k < \mu \\ 1, & \text{if } k > \mu \end{cases}$$

Pf: Since WLLN holds for $\{x_n\}$, we have

$$\lim_{n \rightarrow \infty} P\left[\left|\frac{S_n}{n} - E\left(\frac{S_n}{n}\right)\right| < \epsilon\right] = 1$$

$$\Rightarrow \lim_{n \rightarrow \infty} P[|\bar{X}_n - \mu| < \epsilon] = 1$$

$$\Rightarrow \lim_{n \rightarrow \infty} P[|\bar{X}_n - \mu| \geq \epsilon] = 0$$

Since $\{\bar{X}_n \leq \mu - \epsilon\} \subseteq \{|\bar{X}_n - \mu| \geq \epsilon\}$, we have

$$P(\bar{X}_n \leq \mu - \epsilon) \leq P(|\bar{X}_n - \mu| \geq \epsilon)$$

$$\Rightarrow \lim_{n \rightarrow \infty} P(\bar{X}_n \leq \mu - \epsilon) \leq \lim_{n \rightarrow \infty} P(|\bar{X}_n - \mu| \geq \epsilon)$$

$$\Rightarrow \lim_{n \rightarrow \infty} P(\bar{X}_n \leq \mu - \epsilon) = 0$$

$$\Rightarrow \lim_{n \rightarrow \infty} P(\bar{X}_n \leq k) = 0 \text{ where } k = \mu - \epsilon \text{ i.e., } k < \mu \text{ since } \epsilon > 0$$

$$\Rightarrow \lim_{n \rightarrow \infty} P(\bar{X}_n \leq k) = 0 \text{ if } k < \mu$$

Further, $P(\bar{X}_n \leq \mu + \epsilon) + P(|\bar{X}_n - \mu| > \epsilon) \geq 1$, since the region is larger than sample space covered.

$$\Rightarrow \lim_{n \rightarrow \infty} P(\bar{X}_n \leq \mu + \epsilon) \geq 1 \quad (\because \lim_{n \rightarrow \infty} P(|\bar{X}_n - \mu| > \epsilon) = 0)$$

$$\Rightarrow \lim_{n \rightarrow \infty} P(\bar{X}_n \leq \mu + \epsilon) = 1$$

$$\Rightarrow \lim_{n \rightarrow \infty} P(\bar{X}_n \leq k) = 1 \text{ where } k = \mu + \epsilon \text{ i.e., } k > \mu \because \epsilon > 0$$

$$\Rightarrow \lim_{n \rightarrow \infty} P(\bar{X}_n \leq k) = 1 \text{ if } k > \mu$$

$$\therefore \lim_{n \rightarrow \infty} P(\bar{X}_n \leq k) = \begin{cases} 0, & k < \mu \\ 1, & k > \mu \end{cases}$$

Variations of WLLN:

Some special cases of theorem 1.

Theorem 2: (Bernoulli's WLLN)

Let $\{X_n\}$ be a sequence of Bernoulli trials with probab. of success equal to p . If S_n is the no. of successes in n trials, then

$$\lim_{n \rightarrow \infty} P \left[\left| \frac{S_n - np}{n} \right| < \epsilon \right] = 1, \forall \epsilon > 0$$

Theorem 3: (Khinchine's WLLN)

Let $\{X_n\}$ be a sequence of i.i.d.r.v's with $E(X_i) = \mu < \infty$, $i=1, 2, \dots$ then the WLLN holds i.e.,

$$\lim_{n \rightarrow \infty} P \left[\left| \frac{S_n}{n} - \mu \right| > \epsilon \right] = 0$$

Theorem 4: (Bernstein's WLLN)

Let $\{X_n\}$ be a sequence of r.v's for which $\text{Var}(X_n) = \sigma_n^2 \leq k$, $\forall i$, where k is independent of n . If $\sigma_{ij} = \text{cov}(X_i, X_j) \rightarrow 0$ as $|i-j| \rightarrow \infty$ (Asymptotic uncorrelatedness) then the WLLN holds.

Examples: 1) Let $\{X_n\}$ be i.i.d.r.v's with mean μ & variance σ^2 , if
$$\frac{X_1^2 + X_2^2 + \dots + X_n^2}{n} \xrightarrow{P} C \text{ as } n \rightarrow \infty$$

for some constant C ($0 \leq C < \infty$), then find C .

Sol: Here $E(X_i) = \mu$ & $V(X_i) = \sigma^2 \forall i$

Let $S_n = X_1^2 + X_2^2 + \dots + X_n^2$. Then

$$E(S_n) = E(X_1^2 + X_2^2 + \dots + X_n^2)$$

$$= n E(X_1^2)$$

$$= n [V(X_1) + (E(X_1))^2]$$

($\because$ X 's are i.i.d.r.v's)

$$\begin{aligned} \because V(X_1) &= \sigma^2 \\ E(X_1^2) - (E(X_1))^2 &= \sigma^2 \\ E(X_1^2) &= \sigma^2 + \mu^2 \end{aligned}$$

$$\Rightarrow E(S_n) = n(\sigma^2 + \mu^2) \quad \therefore E(X^2) = V(X) + E(X)^2 = \sigma^2 + \mu^2 \text{ each for each } X^2 \text{ in } S_n$$

$$\Rightarrow E\left(\frac{S_n}{n}\right) = \frac{1}{n} E(S_n) = \frac{1}{n} [n(\sigma^2 + \mu^2)]$$

$$E\left(\frac{S_n}{n}\right) = \sigma^2 + \mu^2 \quad \text{--- (3)}$$

(By Khinchine's WLLN), we have

$$\left(\frac{S_n}{n}\right) \xrightarrow{P} E\left(\frac{S_n}{n}\right) \text{ as } n \rightarrow \infty \quad \text{--- (4)}$$

from (3) & (4)

$$E\left(\frac{S_n}{n}\right) = C = \sigma^2 + \mu^2$$

2) If the i.i.d.r.v's X_r ($r=1, 2, \dots$) assume the value $2^{r-2\ln r}$ with probab. $\frac{1}{2^r}$, examine if the WLLN holds for the sequence $\{X_k\}$

Sol: G.T $\{X_r\}$ is i.i.d.r.v's. It is enough to S.T $E(X_r)$ exists $\forall r$ by Khinchine's WLLN

$$E(X_r) = \sum_{r=1}^{\infty} X_r P(X_r) = \sum_{r=1}^{\infty} 2^{r-2\ln r} \cdot \frac{1}{2^r}$$

$$= \sum_{r=1}^{\infty} (2^{-2})^{\ln r}$$

$$= \sum_{r=1}^{\infty} \left(\frac{1}{4}\right)^{\ln r}$$

$$= \sum_{r=1}^{\infty} r^{\ln\left(\frac{1}{4}\right)}$$

$$= \sum_{r=1}^{\infty} \left(\frac{1}{r}\right)^{\ln 4}$$

$$[\because a^{\ln n} = n^{\ln a}]$$

$\sum_{n=0}^{\infty} r^n$ converges $r < 1$
diverges $r > 1$

P-series $\sum_{x=1}^{\infty} \frac{1}{x^p}$ converges $p > 1$
diverges $p \leq 1$

converges since $\ln 4 = 1.386 > 1$

$\therefore E(X_r)$ exists $\Rightarrow E(X_r) < \infty$

$\therefore$ WLLN holds for the sequence $\{X_r\}$ by Khinchine's theorem

3) let $\{X_n\}$ be a sequence of i.i.d $U(0,1)$ r.v's. for the geometric mean $G_n = (X_1 \cdot X_2 \cdots X_n)^{1/n}$, s.t $G_n \xrightarrow{P} c$ where c is some constant, find c .

Sol: Let $Y = -\ln X$ where $X \sim U(0,1)$ & $f(x) = \frac{1}{1-0} = 1, 0 < x < 1$
 $\Rightarrow$ The cdf of Y is

$$\begin{aligned} F_Y(y) &= P(Y \leq y) = P(-\ln X \leq y) \\ &= P(\ln X \geq -y) \\ &= P(X \geq e^{-y}) \\ &= \int_{e^{-y}}^1 1 dx = [x]_{e^{-y}}^1 = 1 - e^{-y} \end{aligned}$$

The pdf of Y is

$$f_Y(y) = \frac{d}{dy} [F_Y(y)] = 0 - (-e^{-y}) = e^{-y} \text{ for } y > 0$$

$$\begin{aligned} \text{Then } E(Y) &= \int_0^{\infty} y e^{-y} dy = [y(-e^{-y})]_0^{\infty} - \int_0^{\infty} -e^{-y} dy \\ &= (-e^{-y})_0^{\infty} = -[0-1] = 1 \end{aligned}$$

$$\begin{aligned} E(Y^2) &= \int_0^{\infty} y^2 e^{-y} dy = [y^2(-e^{-y})]_0^{\infty} + \int_0^{\infty} 2y e^{-y} dy \\ &= 2 \end{aligned}$$

$$V(Y) = E(Y^2) - (E(Y))^2 = 2 - 1 = 1$$

Thus, the sequence $\{Y_n\}$ is i.i.d with finite mean $E(Y_n) = 1$

Hence by Khinchine's WLLN

$$\sum_{i=1}^n \frac{Y_i}{n} \xrightarrow{P} E(Y) = 1 \quad \text{--- (1)}$$

$$\text{But } \ln G_n = \ln (X_1 X_2 \cdots X_n)^{1/n}$$

$$= \frac{1}{n} [\ln X_1 + \ln X_2 + \cdots + \ln X_n]$$

$$= \sum_{i=1}^n \ln \frac{X_i}{n}$$

$$= - \sum_{i=1}^n \frac{Y_i}{n} \Rightarrow \sum_{i=1}^n \frac{Y_i}{n} = -\ln G_n \quad \text{--- (2)}$$

From (1) & (2)

$$-\ln G_n \xrightarrow{P} 1 \quad \text{i.e., } G_n \xrightarrow{P} e^{-1}$$

$$\therefore C = \frac{1}{e}$$

4) let X_i can have only two values i^α & $-i^\alpha$ with equal probabilities. If $\{X_i\}$ is a sequence of i.i.v's, then S.T WLLN holds if $\alpha < \frac{1}{2}$

Sol: $E(X_i) = i^\alpha \frac{1}{2} - i^\alpha \frac{1}{2} = 0$ &
 $V(X_i) = E(X_i^2) = i^{2\alpha} \frac{1}{2} + i^{2\alpha} \frac{1}{2} = i^{2\alpha}$

let $S_n = \sum_{k=1}^n X_k$. Then

$$B_n = V(S_n) = \sum_{i=1}^n V(X_i)$$

($\because X_i$'s are Independent)

$$= \sum_{i=1}^n i^{2\alpha}$$

$$= 1^{2\alpha} + 2^{2\alpha} + \dots + n^{2\alpha}$$

$$= \int_0^n x^{2\alpha} dx \quad [\text{Euler-Maclaurin formula}]$$

$$B_n = \left[\frac{x^{2\alpha+1}}{2\alpha+1} \right]_0^n = \frac{n^{2\alpha+1}}{2\alpha+1}$$

$$\frac{B_n}{n^2} = \frac{1}{n^2} \left[\frac{n^{2\alpha+1}}{2\alpha+1} \right] \rightarrow 0 \text{ as } n \rightarrow \infty \text{ if } \alpha < \frac{1}{2}$$

Thus $\frac{B_n}{n^2} \rightarrow 0$ as $n \rightarrow \infty$ when $\alpha < \frac{1}{2}$

$\therefore \{X_n\}$ holds WLLN when $\alpha < \frac{1}{2}$

Conditions of WLLN: when the sequence $\{X_n\}$ is independent

- 1) $E(X_i)$ exists $\forall i$
- 2) $B_n = \text{Var}(X_1 + X_2 + \dots + X_n) < \infty$
- 3) $\lim_{n \rightarrow \infty} \frac{B_n}{n^2} = 0$

5) Examine whether WLLN holds for the sequence $\{X_k\}$ of I.R.V's defined by $P(X_k = \pm 2^k) = 2^{-(2k+1)}$

$$P(X_k = 0) = 1 - 2^{-2k}$$

Sol: Given sequence $\{X_k\}$ are I.R.V's

$$\begin{array}{ccc} X_k & 0 & 2^k \quad -2^k \\ P(X_k) & 1 - 2^{-2k} & 2^{-(2k+1)} \quad 2^{-(2k+1)} \end{array}$$

$$\begin{aligned} (i) \quad E(X_k) &= \sum x_k P(X_k) \\ &= 0(1 - 2^{-2k}) + 2^k(2^{-(2k+1)}) - 2^k(2^{-(2k+1)}) \\ &= 0 \quad \forall k \text{ exists} \end{aligned}$$

$$\begin{aligned} (ii) \quad B_n &= \text{Var}(X_1 + X_2 + \dots + X_n) \\ &= V(X_1) + V(X_2) + \dots + V(X_n) \quad (\because \{X_n\} \text{ are I.R.V}) \end{aligned}$$

$$\begin{aligned} \text{Var}(X_k) &= E(X_k^2) - (E(X_k))^2 \\ &= \sum x_k^2 P(X_k) - 0 \\ &= 0(1 - 2^{-2k}) + (2^k)^2(2^{-(2k+1)}) + (-2^k)^2(2^{-(2k+1)}) \\ &= 2^{2k-2k-1} + 2^{2k-2k-1} \\ &= 2^{-1} + 2^{-1} \end{aligned}$$

$$V(X_k) = 1 \quad \forall k$$

$$\begin{aligned} B_n &= V(X_1) + V(X_2) + \dots + V(X_n) \\ &= 1 + 1 + \dots + 1 \\ &= n \end{aligned}$$

$B_n < \infty$ is satisfied

$$(iii) \quad \lim_{n \rightarrow \infty} \frac{B_n}{n^2} = \lim_{n \rightarrow \infty} \frac{n}{n^2} = 0$$

By above three conditions we conclude that given sequence $\{X_k\}$ satisfies WLLN.

6) A variable $\{X_k\}$ has the distribution $P(X_k=0) = 1 - \frac{2}{3^{2k+1}}$

$$P(X_k = \pm 3^k) = 3^{-(2k+2)}$$

If sequence $\{X_k\}$ is a sequence of IRV's then S.T the $\{X_k\}$ obeys WLLN.

6.4: Strong law of large numbers

Definition: A sequence of r.v's $\{X_n\}$ is said to satisfy the strong law of large numbers (SLLN) if

$$\left[\frac{S_n - E(S_n)}{n} \right] \xrightarrow{a.s.} 0 \text{ as } n \rightarrow \infty$$

Theorem 1: (Kolmogorov's SLLN)

This theorem is useful when the r.v's in the sequence are independent but not identically distributed.

Statement: Let $\{X_n\}$ be a sequence of i.i.v's with $E(X_i) = \mu$ & $V(X_i) = \sigma_i^2 < \infty$ for $i=1, 2, \dots$. If $\sum_{k=1}^{\infty} \frac{\sigma_k^2}{k^2} < \infty$, then the SLLN holds for the sequence $\{X_n\}$.

Theorem (2):

This theorem is helpful when the r.v's in the sequence are independent & identically distributed (i.i.d.)

Statement: The sequence $\{X_n\}$ of i.i.d. r.v's holds SLLN if $E(X_n)$ exists.

Theorem (3): (Borel's SLLN)

This theorem is helpful when the sequence consists of Bernoulli trials.

Statement: For a sequence of Bernoulli trials with constant probability of success, the SLLN holds.

Examples:

1) Let $\{X_n\}$ be a sequence of independent random variables with pmf given by $P(X_n = \pm 2^n) = \frac{1}{2^{(2n+1)}}$, $P(X_n = 0) = 1 - \frac{1}{2^{2n}}$

Does the SLLN hold for $\{X_n\}$?

Sol:

X_n	0	2^n	-2^n
$P(X_n)$	$1 - \frac{1}{2^{2n}}$	$\frac{1}{2^{(2n+1)}}$	$\frac{1}{2^{(2n+1)}}$

$$E(X_n) = \sum X_n P(X_n) = 0 \left(1 - \frac{1}{2^{2n}}\right) + 2^n \left(\frac{1}{2^{(2n+1)}}$$

$$- 2^n \left(\frac{1}{2^{(2n+1)}}\right) = 0$$

$$\sigma_n^2 = V(X_n) = E(X_n^2) - (E(X_n))^2 = \sum X_n^2 P(X_n) - 0$$

$$= 0^2 \left(1 - \frac{1}{2^{2n}}\right) + (2^n)^2 \cdot \frac{1}{2^{(2n+1)}} + (-2^n)^2 \cdot \frac{1}{2^{(2n+1)}}$$

$$= 2^{2n-2n-1} + 2^{2n-2n-1}$$

$$= 1 < \infty$$

$$\sum_{n=1}^{\infty} \frac{\sigma_n^2}{n^2} = \sum_{n=1}^{\infty} \frac{1}{n^2} = \sum_{n=1}^{\infty} \frac{1}{n^2} \text{ Converges } \left(\because \sum_{n=1}^{\infty} \frac{1}{n^p} \text{ converges if } p > 1 \right)$$

$\therefore$ the SLLN holds for $\{X_n\}$

2) for what value of α does the SLLN hold for sequence $P(X_k = \pm k^\alpha) = \frac{1}{2}$

Sol: we have $E(X_k) = k^\alpha \left(\frac{1}{2}\right) - k^\alpha \left(\frac{1}{2}\right) = 0$

$$V(X_k) = E(X_k^2) - (E(X_k))^2$$

$$= k^{2\alpha} \left(\frac{1}{2}\right) + k^{2\alpha} \left(\frac{1}{2}\right) - 0$$

$$= k^{2\alpha}$$

$$\sum_{k=1}^{\infty} \frac{\sigma_k^2}{k^2} = \sum_{k=1}^{\infty} \frac{k^{2\alpha}}{k^2} = \sum_{k=1}^{\infty} \frac{1}{k^{2-2\alpha}} \because \text{SLLN hold for sequence then}$$

$$\sum_{k=1}^{\infty} \frac{\sigma_k^2}{k^2} \text{ converges } \Rightarrow 2-2\alpha > 1$$

$$\Rightarrow 1 > 2\alpha$$

$$\Rightarrow \alpha < \frac{1}{2}$$

$\therefore$ SLLN holds if $\alpha < \frac{1}{2}$

3) let $\{X_n\}$ be a sequence of Irv's with pmf
 $P(X_n = \pm \frac{1}{n}) = \frac{1}{2}$ check whether SLLN holds
 for $\{X_n\}$ or not.

Sol: $E(X_n) = \frac{1}{n}(\frac{1}{2}) - \frac{1}{n}(\frac{1}{2}) = 0$

$$\begin{aligned}\sigma_n^2 &= E(X_n^2) - (E(X_n))^2 \\ &= \frac{1}{n^2}(\frac{1}{2}) + \frac{1}{n^2}(\frac{1}{2}) \\ &= \frac{1}{n^2}\end{aligned}$$

$$\sum_{n=1}^{\infty} \frac{\sigma_n^2}{n^2} = \sum_{n=1}^{\infty} \frac{1}{n^4} \quad (\because p=4>1)$$

is converges

$\therefore \{X_n\}$ obeys SLLN.

4) let $\{X_n\}$ be a sequence of Irv's with pmf
 $P(X_k = \pm 2^{-k}) = \frac{1}{2}$ check whether SLLN holds or
 not.

Sol: $E(X_k) = 2^{-k}(\frac{1}{2}) - 2^{-k}(\frac{1}{2}) = 0$

$$\sigma_k^2 = 2^{-2k} \frac{1}{2} + 2^{-2k} \frac{1}{2} = 2^{-2k}$$

$$\sum_{k=1}^{\infty} \frac{\sigma_k^2}{k^2} = \sum_{k=1}^{\infty} \frac{2^{-2k}}{k^2} = \sum_{k=1}^{\infty} \frac{1}{k^2 2^{2k}} \text{ converges.}$$

$\therefore \{X_n\}$ obeys SLLN.

5) let $\{X_n\}$ be i.i.d.r.v's with mean μ & variance σ^2 & as $n \rightarrow \infty$
 $\frac{X_1^2 + X_2^2 + \dots + X_n^2}{n} \xrightarrow{\text{a.s.}} c$ for some constant c
 $(0 \leq c < \infty)$ then find c .

Sol: $E(X_i) = \mu$ & $V(X_i) = \sigma^2 + i$

Let $S_n = X_1^2 + X_2^2 + \dots + X_n^2$ then

$$E(S_n) = E(X_1^2 + X_2^2 + \dots + X_n^2)$$

$$= n E(X_1^2)$$

$$= n [V(X_1) + (E(X_1))^2]$$

$$= n(\sigma^2 + \mu^2)$$

$$E\left(\frac{S_n}{n}\right) = \frac{1}{n} E(S_n) = \frac{1}{n} n(\sigma^2 + \mu^2)$$

$$= \sigma^2 + \mu^2$$

$$\therefore \frac{S_n}{n} \xrightarrow{a.s.} E\left(\frac{S_n}{n}\right) = \sigma^2 + \mu^2$$

$$\Rightarrow \frac{X_1^2 + \dots + X_n^2}{n} \xrightarrow{a.s.} C \text{ where } C = \sigma^2 + \mu^2$$

6) If the i.i.d.r.v's $\{X_n\}$ assume the value $2^{r-2\ln r}$ with probab $\frac{1}{2^r}$ for $r=1, 2, \dots$ Examine if SLLN holds for the sequence $\{X_n\}$

Sol: SLLN holds for i.i.d.r.v's $\{X_n\}$ if $E(X_n)$ exists.

$$E(X_r) = \sum_{r=1}^{\infty} X_r P(X_r) = \sum_{r=1}^{\infty} 2^{r-2\ln r} \cdot \frac{1}{2^r}$$

$$= \sum_{r=1}^{\infty} \left(\frac{1}{4}\right)^{\ln r}$$

$$[\because a^{\ln n} = n^{\ln a}]$$

$$= \sum_{r=1}^{\infty} r^{\ln\left(\frac{1}{4}\right)}$$

$$= \sum_{r=1}^{\infty} \frac{1}{r^{\ln 4}}$$

where $\ln 4 = 1.386 > 1$

which converges

Then $E(X_n)$ is finite hence SLLN holds for $\{X_n\}$