Modulation:-

It is the process of varying the characteristics of high frequency carrier in accordance with instantaneous values of modulating or message or baseband signal.

(or)

It is a frequency translation technique which converts baseband or low frequency signal to bandpass or high frequency signal.

Need for modulation:-

Modulation technique is used for the communication of base band signal. The need for modulation (or) advantages of modulation is given below:

1. Reduces the height of antenna.
2. Avoids mixing of signal.
3. Increases the range of communication
4. Allows multiplexing of signals
5. Allows adjustments (or) changes in bandwidth
6. Improves quality of reception.

Signals in the Modulation Process

1. Message or modulating signal

The signal which contains a message to be transmitted is called as a message signal. It is a baseband signal, which has to undergo the process of modulation, to get transmitted. Hence, it is also called as the modulating signal.

2. Carrier Signal :-

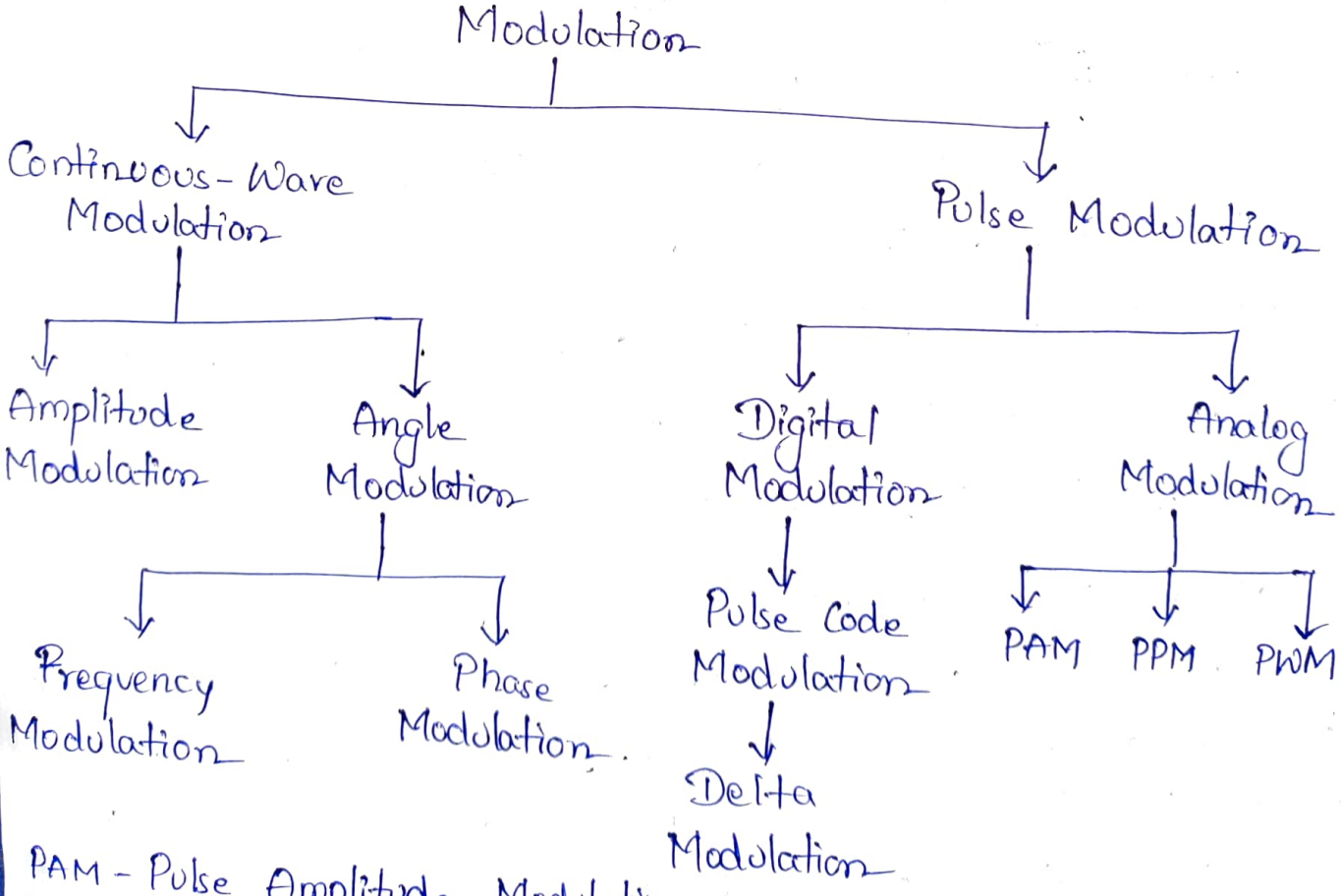
The high frequency signal which has a certain phase, frequency and amplitude but contains no information, is called a carrier signal. It is an empty signal. It is used to carry the signal to the receiver after modulation.

3. Modulated Signal

The resultant signal after the process of modulation, is called as the modulated signal. This signal is a combination of the modulating signal & the carrier signal

Types of Modulation

Depending upon the modulation technique used, modulation is classified as shown below.

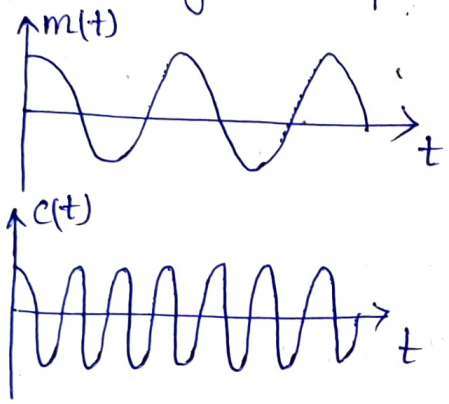


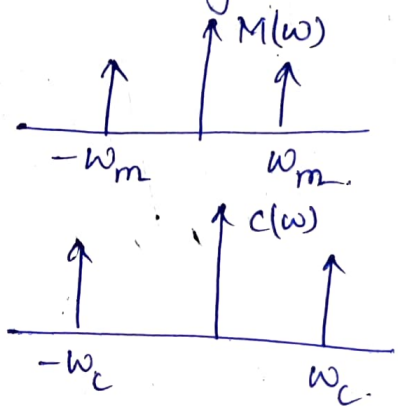
PAM - Pulse Amplitude Modulation
PPM - Pulse Position Modulation
PWM - Pulse Width Modulation.

Amplitude Modulation

In Amplitude modulation, the amplitude of a carrier signal is varied in accordance with the instantaneous values of modulating signal.

Consider a modulating voltage $m(t) = V_m(t) = V_m \cos \omega_m t$ & the high frequency carrier signal be $c(t) = V_c(t) = V_c \cos \omega_c t$





From the definition of amplitude modulation the amplitude of the carrier is changed according to the instantaneous values of amplitude of message/modulating signal.

$$V_{AM} = V_c + m(t).$$

$$= V_c + V_m \cos \omega_m t$$

$$= V_c \left[1 + \frac{V_m}{V_c} \cos \omega_m t\right].$$

$$= V_c \left[1 + \mu \cos \omega_m t\right]$$

$$= V_c \left[1 + k_a m(t)\right].$$

where $\frac{V_m}{V_c} = \mu$ is called modulation index
• depth of modulation
modulation factor

where $k_a = \frac{1}{V_c}$ = Amplitude sensitivity.

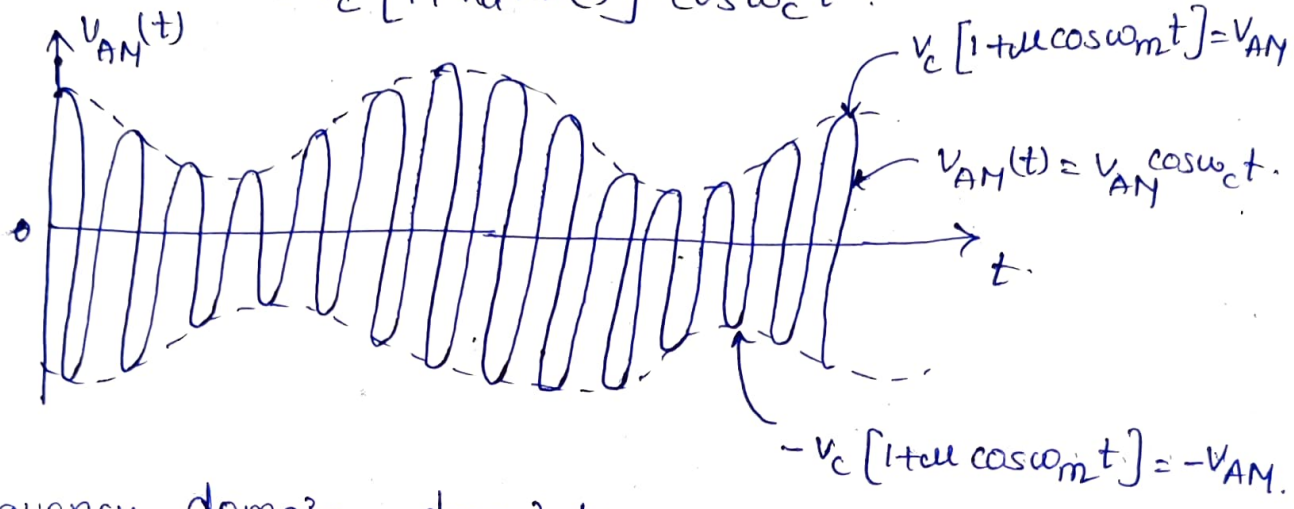
∴ The instantaneous amplitude of modulated signal is given by

$$v_{AM}(t) = V_{AM}\cos\omega_c t.$$

$$= V_c\left[1+\mu\cos\omega_m t\right]\cos\omega_c t$$

(or).

$$= V_c\left[1+k_a m(t)\right]\cos\omega_c t.$$



Frequency domain description

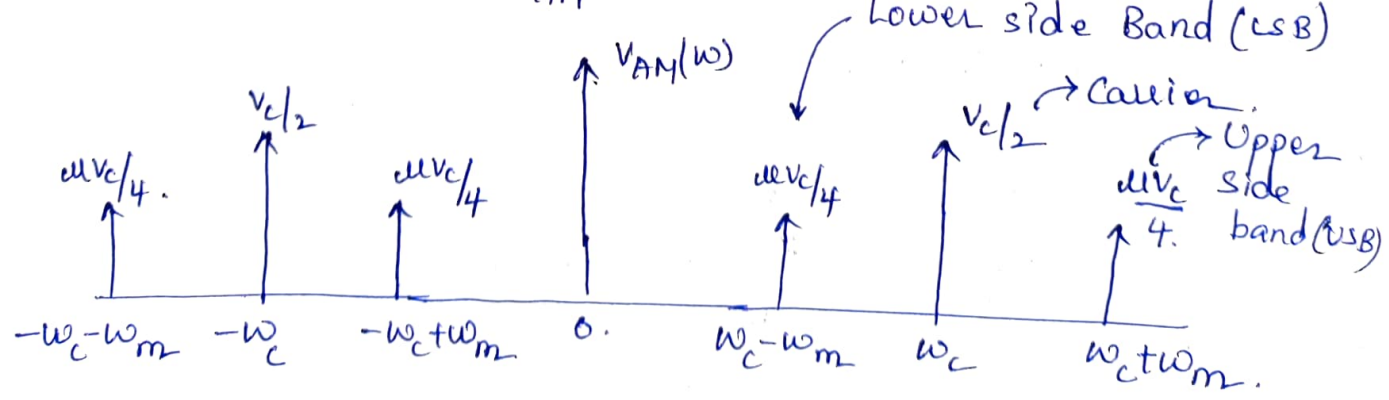
$$v_{AM}(t) = V_c\left[1+\mu\cos\omega_m t\right]\cos\omega_c t.$$

$$= V_c\cos\omega_c t + \mu V_c\cos\omega_m t\cos\omega_c t. \times\frac{2}{2}$$

$$= V_c\cos\omega_c t + \frac{\mu V_c}{2}\left[\cos(\omega_c+\omega_m)t + \cos(\omega_c-\omega_m)t\right]$$

$$= V_c\cos\omega_c t + \frac{\mu V_c}{2}\cos(\omega_c+\omega_m)t + \frac{\mu V_c}{2}\cos(\omega_c-\omega_m)t.$$

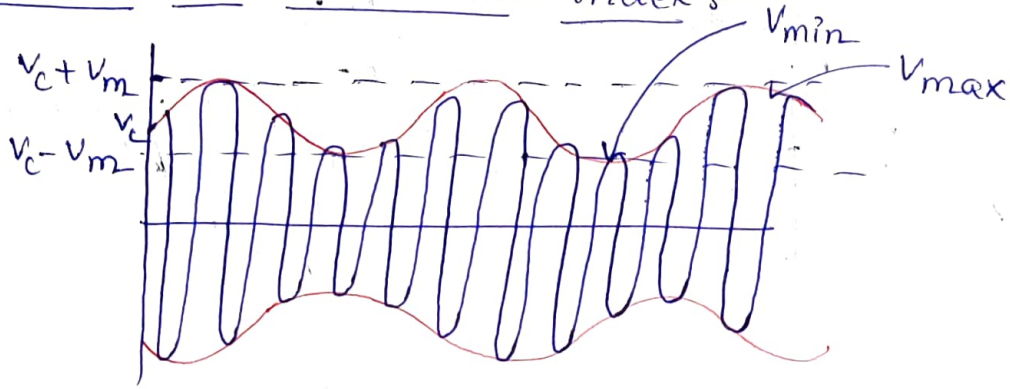
$v_{AM}(t) \longleftrightarrow V_{AM}(\omega)$.



Multiplication with cos signal shifts the spectrum of the signal to both sides & makes magnitude half.

$$B.W = \omega_c + \omega_m - (\omega_c - \omega_m)$$
$$= \omega_c + \omega_m - \omega_c + \omega_m$$
$$\boxed{B.W = 2\omega_m}$$

Calculation of Modulation Index :-



$$V_{max} - V_{min} = V_c + V_m - (V_c - V_m)$$
$$= 2V_m.$$
$$V_m = \frac{V_{max} - V_{min}}{2}$$

As $V_{max} = V_c + V_m$.

$$V_c = V_{max} - V_m.$$
$$= V_{max} - \frac{V_{max} - V_{min}}{2}.$$
$$= \frac{2V_{max} - V_{max} + V_{min}}{2}$$
$$= \frac{V_{max} + V_{min}}{2}.$$

II Method.

$$m = \frac{V_m}{V_c}.$$

$$V_{max} = V_c[1 + m]$$
$$V_{min} = V_c[1 - m].$$

$V_{max} = V_c + V_c m \rightarrow (1)$

$V_{min} = V_c - V_c m \rightarrow (2)$

Solving equations (1) & (2).

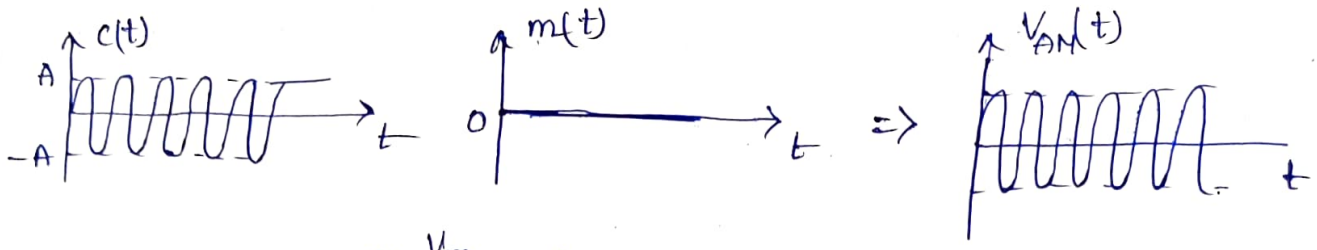
$V_{max} - V_{min} = 2V_c m.$

$V_{max} + V_{min} = 2V_c.$

$$\boxed{m = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}}$$

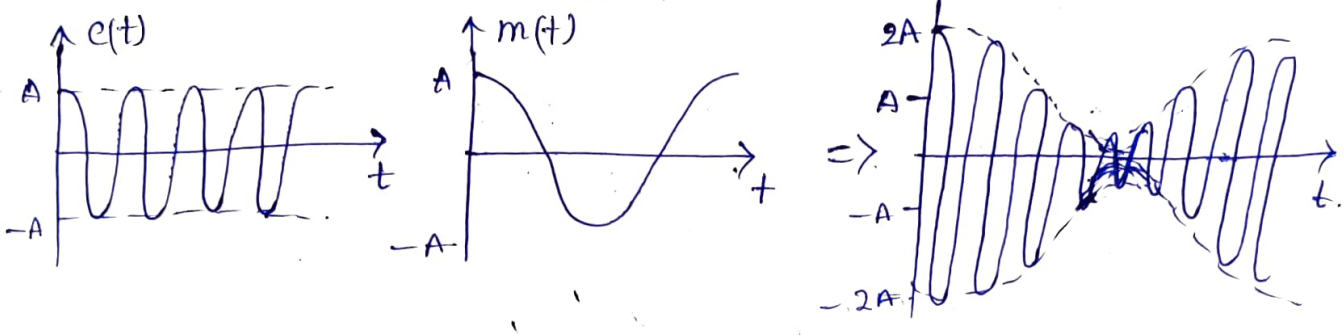
Modulation Index $m = \frac{V_m}{V_c} = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}$.

Case(i):- No modulation



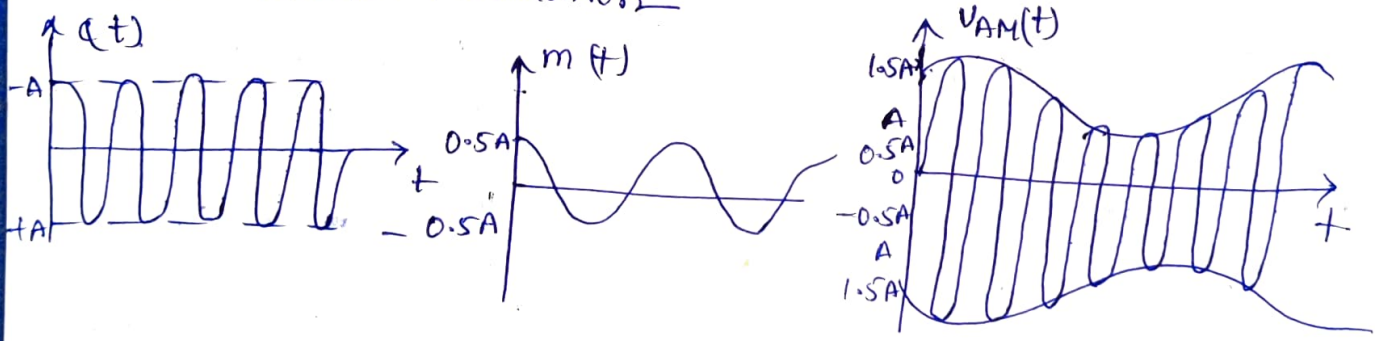
$$\mu = \frac{V_m}{V_c} = \frac{0}{A} = 0.$$

Case(ii):- Perfect Modulation.



$$\mu = \frac{V_m}{V_c} = \frac{A}{A} = 1 \simeq 100\% \simeq \frac{V_{max} - V_{min}}{V_{max} + V_{min}} = \frac{2A - 0}{2A + 0} = 1 \simeq 100\%.$$

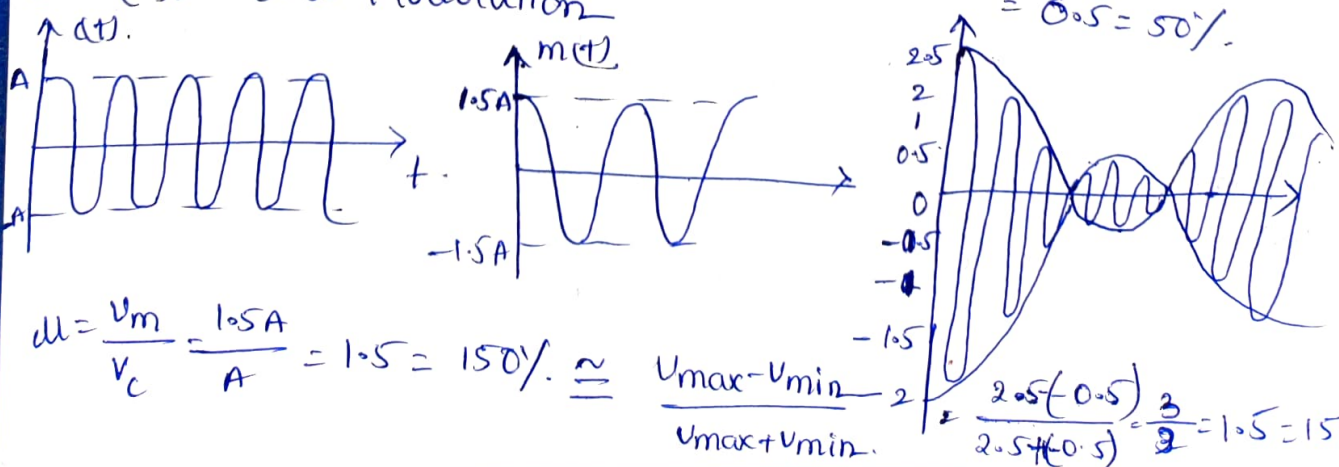
Case(iii):- Under Modulation



$$\mu = \frac{V_m}{V_c} = \frac{0.5A}{A} = 0.5 \simeq 50\%. \simeq \frac{V_{max} - V_{min}}{V_{max} + V_{min}} = \frac{1.5A - 0.5A}{1.5A + 0.5A} = \frac{1A}{2A}$$

$$= 0.5 = 50\%.$$

Case(iv):- Over Modulation



$$\mu = \frac{V_m}{V_c} = \frac{1.5A}{A} = 1.5 = 150\%. \simeq \frac{V_{max} - V_{min}}{V_{max} + V_{min}}$$

$$\frac{2.5 - (-0.5)}{2.5 + (-0.5)} = \frac{3}{2} = 1.5 = 150\%.$$

$\mu = \frac{V_m}{V_c} \leq 1$ i.e as long as the % modulation is less than unity, there exists a one to one relationship between message signal & envelope of the modulated signal.

Power relations in AM:-

$$m(t) = V_m(t) = V_m \cos\omega_m t$$
$$c(t) = V_c(t) = V_c \cos\omega_c t.$$

$$V_{AM}(t) = \underbrace{V_c \cos\omega_c t}_{\text{Carrier}} + \underbrace{\frac{\mu V_c}{2} \cos(\omega_c + \omega_m)t}_{\text{USB}} + \underbrace{\frac{\mu V_c}{2} \cos(\omega_c - \omega_m)t}_{\text{LSB}}.$$

Carrier component of the modulated wave has the same amplitude as the unmodulated carrier i.e amplitude of the carrier is unchanged.

∴ energy is added or subtracted depending on the other components. The modulated wave contains extra energy in the two sidebands.

It is clear from the above equation that the amplitude of the sidebands depends on the modulation index $\mu \left(= \frac{V_m}{V_c}\right)$. The total power in the modulated wave will also depend on the modulation index.

The total power in the modulated wave is given by

$$P_t = \frac{V_{carrier}^2}{R} + \frac{V_{LSB}^2}{R} + \frac{V_{USB}^2}{R}.$$

$$= P_c + P_{LSB} + P_{USB}.$$

where all the 3 voltages are RMS values & R

is resistance of antenna through which power is distributed.

$$P_c = \frac{V_{carrier}^2}{R} = \frac{(V_c/\sqrt{2})^2}{R} = \frac{V_c^2}{2R}.$$

$$P_{LSB} = \frac{V_{LSB}^2}{R} = \frac{\left(\frac{mV_c}{2}/\sqrt{2}\right)^2}{R} = \frac{m^2V_c^2}{8R}.$$

$$P_{USB} = \frac{V_{USB}^2}{R} = \frac{\left(\frac{mV_c}{2}/\sqrt{2}\right)^2}{R} = \frac{m^2V_c^2}{8R}.$$

Total power in AM modulated wave is given by

$$P_t = P_c + P_{LSB} + P_{USB}$$

$$= \frac{V_c^2}{2R} + \frac{m^2V_c^2}{8R} + \frac{m^2V_c^2}{8R}.$$

$$= \frac{V_c^2}{2R} + \frac{m^2V^2}{4R} = \frac{V_c^2}{2R}\left[1 + \frac{m^2}{2}\right].$$

$$\boxed{P_t = P_c\left[1 + \frac{m^2}{2}\right]}$$

for $m = 1$ i.e 100% modulation

$$P_t = \frac{V_c^2}{2R}\left[1 + \frac{m^2}{2}\right].$$

$$\boxed{P_t = 1.5\frac{V_c^2}{2R} = 1.5P_c}$$

Power in sidebands $P_{LSB} = P_{USB} = \frac{m^2V_c^2}{8R} = \frac{V_c^2}{2R}\left[\frac{m^2}{4}\right].$

$$\boxed{P_{LSB} = P_{USB} = P_c\frac{m^2}{4}.}$$

Current relations in AM.

$$P_t = P_c\left[1 + \frac{m^2}{2}\right].$$

$$I_t^2R = I_c^2R\left[1 + \frac{m^2}{2}\right].$$

$$\boxed{I_t = I_c\sqrt{\left[1 + \frac{m^2}{2}\right]}}$$

Efficiency :-

It is defined as the ratio of power in sidebands to total power, because the sidebands are containing the actual information

$$\% \eta = \frac{\text{Power in sidebands}}{\text{total power}}$$

$$= \frac{P_{LSB} + P_{USB}}{P_t}$$

$$= \frac{\frac{m^2 V_c^2}{8R} + \frac{m^2 V_c^2}{8R}}{\frac{V_c^2}{2R}\left[1 + \frac{m^2}{2}\right]} = \frac{\frac{2m^2 V_c^2}{8R}}{\frac{V_c^2}{2R}\left[1 + \frac{m^2}{2}\right]}$$

$$= \frac{m^2/2}{\frac{2+m^2}{2}} = \frac{m^2}{2+m^2}$$

$$\boxed{\eta = \frac{m^2}{2+m^2}}$$

for $m = 1$ i.e 100% modulation

$$\eta = \frac{1}{2+1} = \frac{1}{3} \Rightarrow \eta\% = 33.3\%.$$

i.e only 33.3% of power is utilized & the remaining power is wasted by the transmission of carrier along with sidebands.

Problems :-

(1) The amplitude of a sinusoidal carrier is modulated by a single sinusoid to obtain the amplitude modulated signal $s(t) = 5\cos 1600\pi t + 20\cos 1800\pi t + 5\cos(2000\pi t)$ what is the value of modulation index.

Sol :- The AM modulated wave can be expressed as

$$s(t) = A_c \cos\omega_c t + \frac{\mu A_c}{2}\cos(\omega_c+\omega_m)t + \frac{\mu A_c}{2}\cos(\omega_c-\omega_m)t.$$

On comparing the above equation with

$$s(t) = 5\cos 1600\pi t + 20\cos 1800\pi t + 5\cos(2000\pi t)$$

$A_c = 20.$

$\frac{\mu A_c}{2} = 5.$ $\Rightarrow \mu A_c = 40.$

$\mu A_c = 10$ $\Rightarrow \mu = \frac{10}{20} = 0.5.$

(2) The time domain equation for the AM signal for single tone modulation is

$$s(t) = A_c[1+\mu\cos 2\pi f_m t]\cos 2\pi f_c t \text{ (General Equation)}$$

$$s(t) = (20+4\cos 500\pi t)\cdot\cos(2\pi\times 10^5 t)\text{v}.$$

Calculate the modulation index & total power of the AM wave.

Sol :-

$s(t) = (20+4\cos 500\pi t)\cdot\cos(2\pi\times 10^5 t)\text{v}$

On comparing the above equation

$$s(t) = V_c[1+\mu\cos\omega_m t]\cos\omega_c t.$$

From the given equation

$$s(t) = 20\left[1+\frac{4}{20}\cos 500\pi t\right]\cos(2\pi\times 10^5 t)\text{v}.$$

$m = \frac{4}{20} = \frac{1}{5} = 0.2$ $\boxed{m = 0.2}$

Total power of AM signal

$P_t = P_c\left[1 + \frac{m^2}{2}\right]$ $P_c = \frac{V_c^2}{2R} = \frac{(20)^2}{2} = 200\,W.$

$= 200\left[1 + \frac{(0.2)^2}{2}\right]$

$\boxed{P_t = 204\,W}$

(3) A broadcast radio transmitter radiates 5kw power when the modulation index percentage is 60%. How much is the carrier power.

Sol:- $P_t = P_c\left[1 + \frac{m^2}{2}\right]$.

$P_c = \frac{P_t}{\left[1 + \frac{m^2}{2}\right]} = \frac{5\times10^3}{1 + \frac{(0.6)^2}{2}} = \frac{5000}{1 + \frac{0.36}{2}} = \frac{5000}{\frac{2.36}{2}} = 4237.28$

$P_c = 4.237\,kw$

(4) To obtain a modulation index of 75%, what should be the amplitude of message signal if the amplitude of carrier is 12V?

$m = 0.75$ $m = \frac{V_m}{V_c} \Rightarrow V_m = V_c \cdot m = 0.75 \times 12$

$= 9V$

(5) The amplitude modulated wave is with maximum amplitude 12V & the minimum amplitude 8V. Find the modulation index.

$m = \frac{V_{max} - V_{min}}{V_{max} + V_{min}} = \frac{12-8}{12+8} = \frac{4}{20} = \frac{1}{5} = 0.2 \Rightarrow 20\%.$

(6) If the frequency of carrier signal is 2MHz & it is modulated by a message signal of 2kHz, then what are the frequency components present in amplitude.

modulated wave?

Sol:-

$$s(t) = V_c \cos\omega_c t + \frac{\mu V_c}{2}\cos(\omega_c+\omega_m)t + \frac{\mu V_c}{2}\cos(\omega_c-\omega_m t$$

Frequency components of AM wave.

f_c, $f_c - f_m$, $f_c + f_m$.

$f_c = 2MHz$, $f_{LSB} = 2\times10^6 - 2\times10^3 = 2\times10^6 - 0.002\times10^6$

$= 1.998\ MHz$.

$f_{USB} = 2\times10^6 + 2\times10^3 = 2\times10^6 + 0.002\times10^6$

$= 2.002\ MHz$

(7) For the amplitude modulated wave $s(t) = 100(1+0.6\sin 6280t)\sin(2\pi\times10^6 t)$, determine.

frequency of message signal — Modulation index.

frequency of carrier signal — Frequency of upper & lower side bands

Sol:-

On comparing the given equation with

$s(t) = V_m(1+\mu\cdot\sin\omega_m t)\sin\omega_c t$.

$V_m = 100V$.

$\mu = 0.6$.

$\omega_m = 6280$

$\omega_c = 2\pi\times10^6$

$2\pi f_c = 2\pi\times10^6$

$f_c = 10^6 = 1MHz$.

$2\pi f_m = 6280$

$f_m = \frac{6280}{2\pi} = 999.49$

$= 1000Hz = 1kHz$

Frequency of message signal $f_m = 1kHz$.

Frequency of carrier signal $f_c = 1MHz$.

Modulation index $= \mu = 0.6$.

Frequency of upper side band $\Rightarrow f_c + f_m = 1.001\ MHz$.

Frequency of lower side band $\Rightarrow f_c - f_m = 0.999\ MHz$

(8) The total power content of AM signal is 1000 watts. Determine the power being transmitted at the carrier frequency & each of the sideband when the % modulation is 100.

Sol:- $m\% = 100\%$.

$P_t = 1.5 P_c$

$1000 = 1.5 P_c$

$P_c = \frac{1000}{1.5} = 666.67$ watts

$P_{SB} = P_t - P_c$

$= 333.3\omega$

$P_{LSB} = P_{USB} = \frac{P_{SB}}{2}$

$= 166.67.$

(9) The antenna current of an AM transmitter is 10A, when it is modulated to a depth of 30% by an audio signal. It increases to 11A when another signal modulates the carrier signal. what will be the total modulation index & the modulation index due to second signal.

$I_t = I_c \sqrt{1 + \frac{m^2}{2}}$

$10 = I_c \sqrt{1 + \frac{(0.3)^2}{2}}$

$10 = I_c (1.022)$

$I_c = 9.784A$

$I_{t_2} = I_c \sqrt{1 + \frac{m_t^2}{2}}$

$11 = 9.784 \sqrt{1 + \frac{m_t^2}{2}}$

$1 + \frac{m_t^2}{2} = 1.263$

$m_t^2 = 0.527$

$m_t = 0.725$

$m_t^2 = m_1^2 + m_2^2$

$(0.725)^2 = (0.3)^2 + m_2^2$

$m_2^2 = 0.435$

$m_2 = 0.65$

$= 65\%$

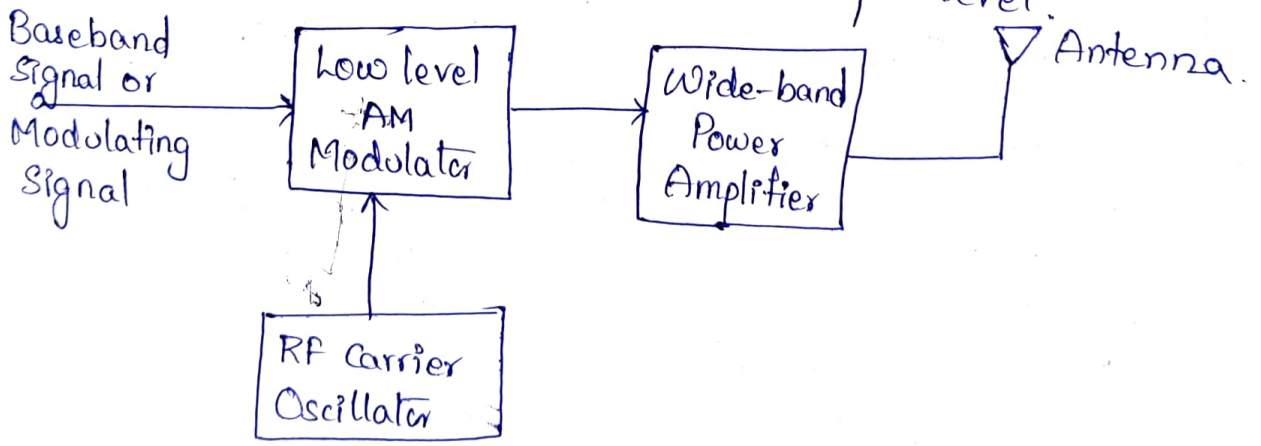
Generation of AM waves :-

The device which is used to generate an Amplitude modulated wave is known as Amplitude Modulator.

The methods of AM Generation are broadly classified as (i) Low-level Amplitude Modulation
(ii) High-level Amplitude Modulation

Low level Amplitude Modulation

In a low-level amplitude modulation system, the modulation is done at low power level. At low power levels, a very small power is associated with the carrier signal & the modulating signal. so output power of modulation is low. Therefore, the power amplifiers are required to boost the amplitude modulated signals upto desired output level.

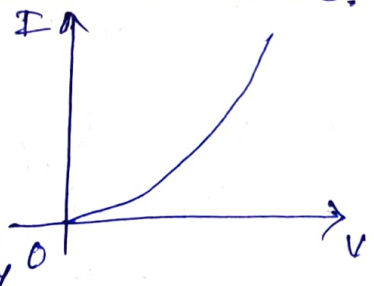


A wide band power amplitude is used just to preserve the sidebands of the modulated signal.
Amplitude modulated systems, employing modulation at low power levels are also called low-level amplitude modulation transmitters.

Square-law diode modulation & switching modulation are examples of low-level modulation.

Square-law diode modulation :- (or) **Square law modulator**

Square law diode modulation circuit make use of nonlinear current-voltage characteristics of diode. This method is suited at low-voltage levels because of the fact that current-voltage characteristic of a diode is highly non linear particularly in the low voltage region.

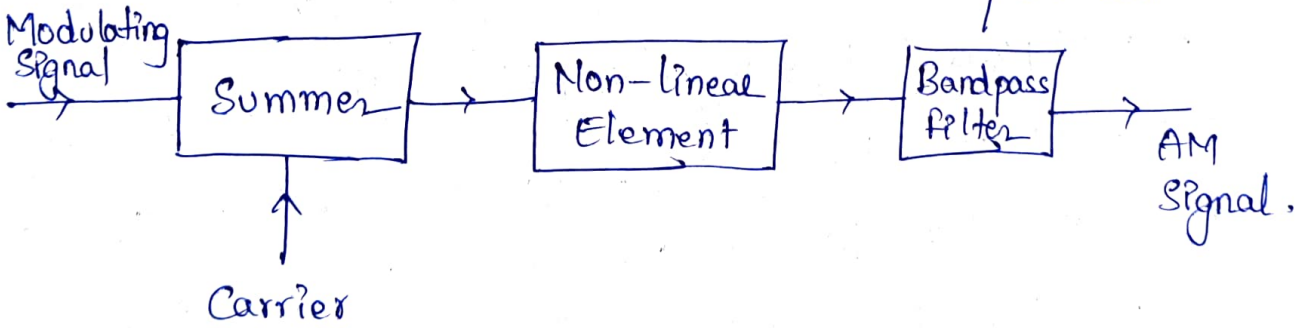


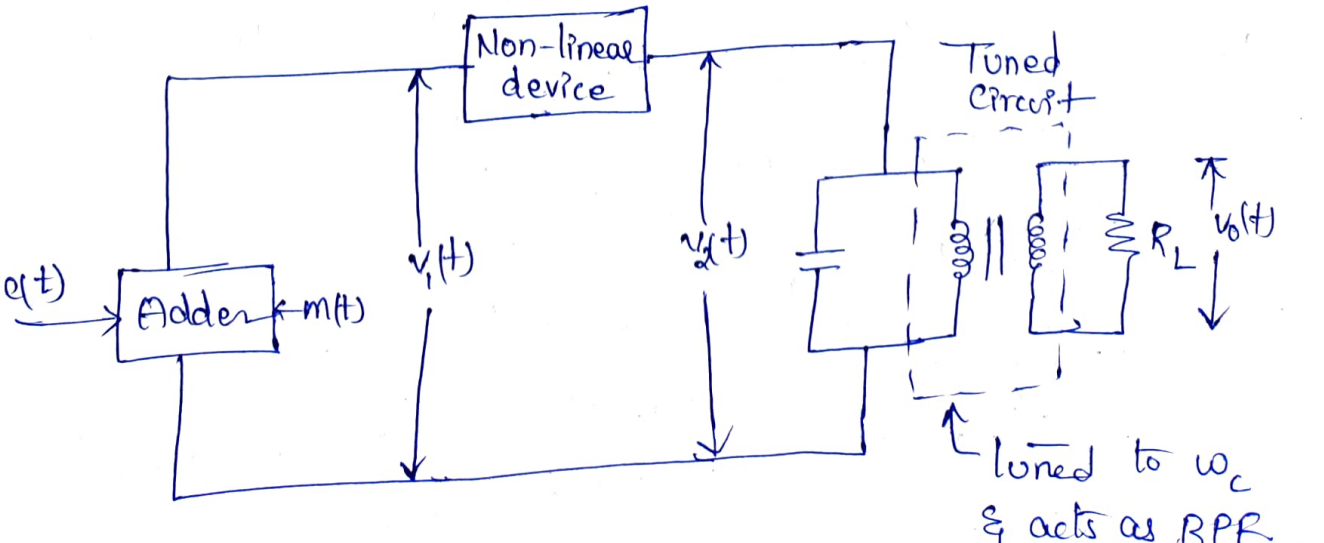
It contains 3 elements

(i) Summer : For summing carrier & modulating signal

(ii) Non-linear element (diode) : For implementing square law modulator

(iii) Bandpass filter (BPF) : For extracting the desired modulation products.





The output & input relationship of non-linear devices is as follows

$$V_0 = a_1 V_i + a_2 V_i^2 + a_3 V_i^3 + \ldots$$

If V_i is small, first two terms are significant & remaining terms can be neglected

$$V_0 = a_1 V_i + a_2 V_i^2$$

As the modulating signal $m(t)$ & carrier signal $c(t)$ is given as input to adder, the output of adder is

$$V_1(t) = m(t) + c(t)$$

$$= m(t) + V_c \cos \omega_c t.$$

When the above signal $V_1(t)$ is passed through a non-linear device, the output $V_2(t)$ is

$$V_2(t) = a_1 V_1(t) + a_2 V_1^2(t)$$

$$= a_1 [m(t) + V_c \cos \omega_c t] + a_2 [m(t) + V_c \cos \omega_c t]^2.$$

$$V_2(t) = a_1 [m(t) + V_c \cos \omega_c t] + a_2 [m^2(t) + V_c^2 \cos^2 \omega_c t + 2 m(t) V_c \cos \omega_c t]$$

$$= a_1 m(t) + a_1 V_c \cos \omega_c t + a_2 m^2(t) + a_2 V_c^2 \cos^2 \omega_c t + 2 a_2 m(t) V_c \cos \omega_c t.$$

The above signal is passed through a bandpass filter. It passes the frequencies that are centered at ω_c.

$$V_0(t) = a_1 V_c \cos \omega_c t + 2 a_2 m(t) V_c \cos \omega_c t.$$

$$= a_1 V_c \cos \omega_c t \left[1 + \frac{2a_2}{a_1} m(t)\right]$$

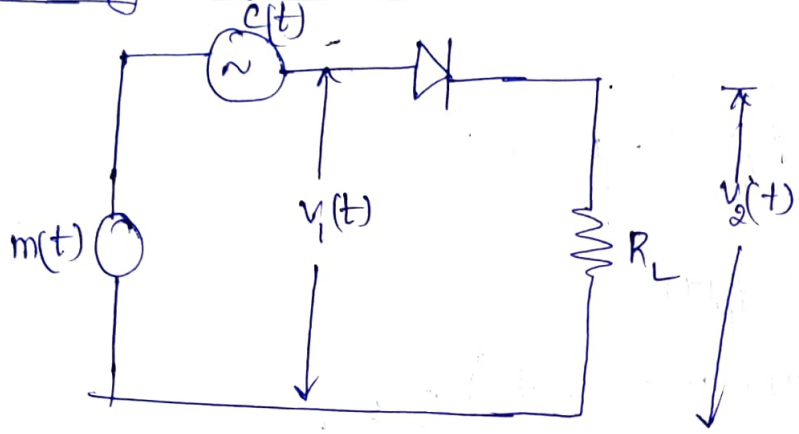
$$= V_c a_1 \left[1 + \frac{2a_2}{a_1} V_m \cos \omega_m t\right] \cos \omega_c t.$$

On comparing the above equation with the equation

$$V_{AM}(t) = V_c [1 + u \cos \omega_m t] \cos \omega_c t.$$

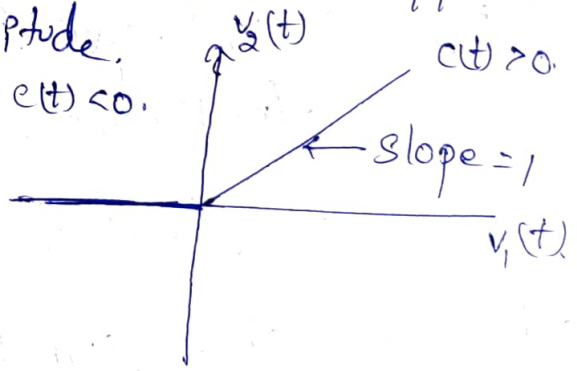
$$u = \frac{2a_2}{a_1} V_m.$$

Switching Modulator :-



Here, the diode is assumed to act as ideal switch i.e it presents zero impedence when it is forward biased & infinite impedence when it is reverse biased.

The diode switch is controlled by carrier wave c(t) & assume that the carrier voltage c(t) applied to diode is large is amplitude.



The i/p voltage

$$v_1(t) = m(t) + c(t)$$
$$= m(t) + V_c \cos \omega_c t.$$

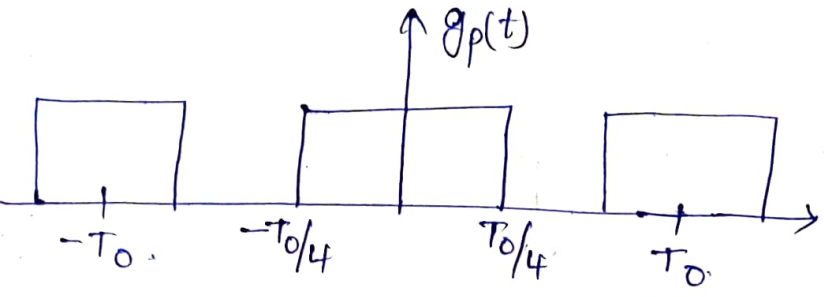
The resulting load voltage

$$v_2(t) = v_1(t) \quad \text{if } c(t) > 0$$
$$= 0 \quad \text{if } c(t) < 0.$$

$v_2(t)$ is changing in between $v_1(t)$ & 0 periodically. That is w.r.t to fixed period output is changing with fixed pulse train & is based on $v_1(t)$ periodically.

The Output $v_2(t) = v_1(t) g_p(t)$

where $g_p(t)$ is a periodic pulse train of duty cycle equal to half period $T_0/2$.



$$V_2(t) \cong V_1(t) \cdot g_p(t).$$

$$= (m(t) + V_c \cos\omega_c t)\, g_p(t);$$

Using Fourier series, $g_p(t)$ can be written as

$$g_p(t) = \frac{1}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos\left[\omega_c t (2n-1)\right].$$

$$= \frac{1}{2} + \frac{2}{\pi} \cos\omega_c t + \underbrace{\text{odd harmonics.}}$$

have higher frequencies so neglect it.

So $V_2(t) = V_1(t) \cdot g_p(t)$.

$$= (m(t) + V_c \cos\omega_c t)\left(\frac{1}{2} + \frac{2}{\pi}\cos\omega_c t\right)$$

$$= \frac{m(t)}{2} + \frac{V_c}{2}\cos\omega_c t + \frac{2}{\pi} m(t)\cos\omega_c t + \frac{2}{\pi} V_c \cos^2\omega_c t.$$

When the above signal is passed through BPF.

$$Y(t) = \frac{V_c}{2}\cos\omega_c t + \frac{2}{\pi} m(t)\cos\omega_c t.$$

$$= \frac{V_c}{2}\cos\omega_c t \left[1 + \frac{4}{\pi V_c} m(t)\right].$$

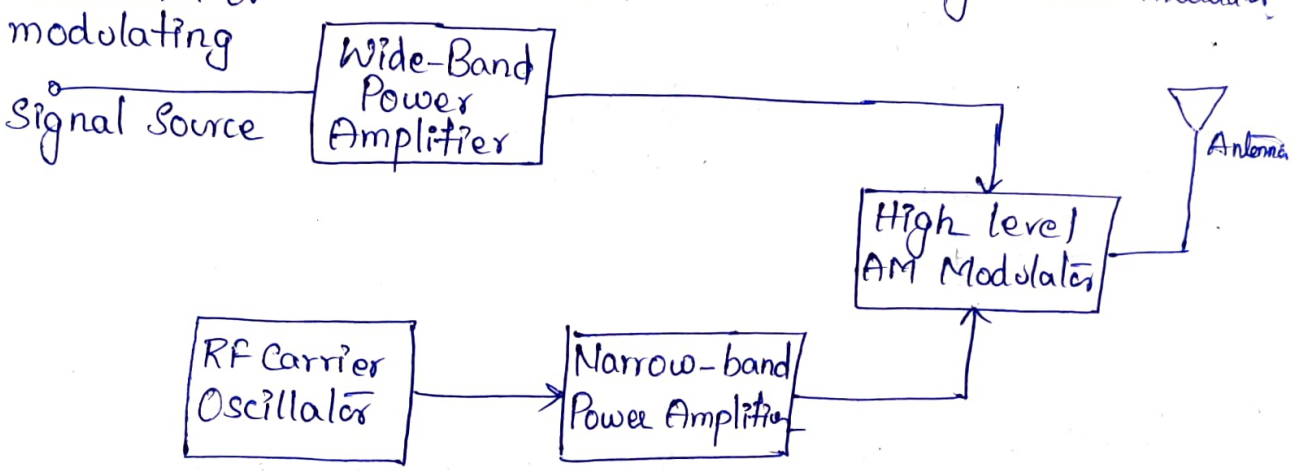
$$= \frac{V_c}{2}\left[1 + \frac{4}{\pi V_c} m(t)\right]\cos\omega_c t.$$

On comparing the above equation with standard equation, Amplitude sensitivity $k_a = \frac{4}{\pi V_c}$

High Level Amplitude Modulation:-

In a high-level amplitude modulation system, the modulation is done at high power level. so, the baseband & carrier signal must be at high power levels.

The modulating signal & carrier signal are first power amplified & then applied to AM high level modulator.



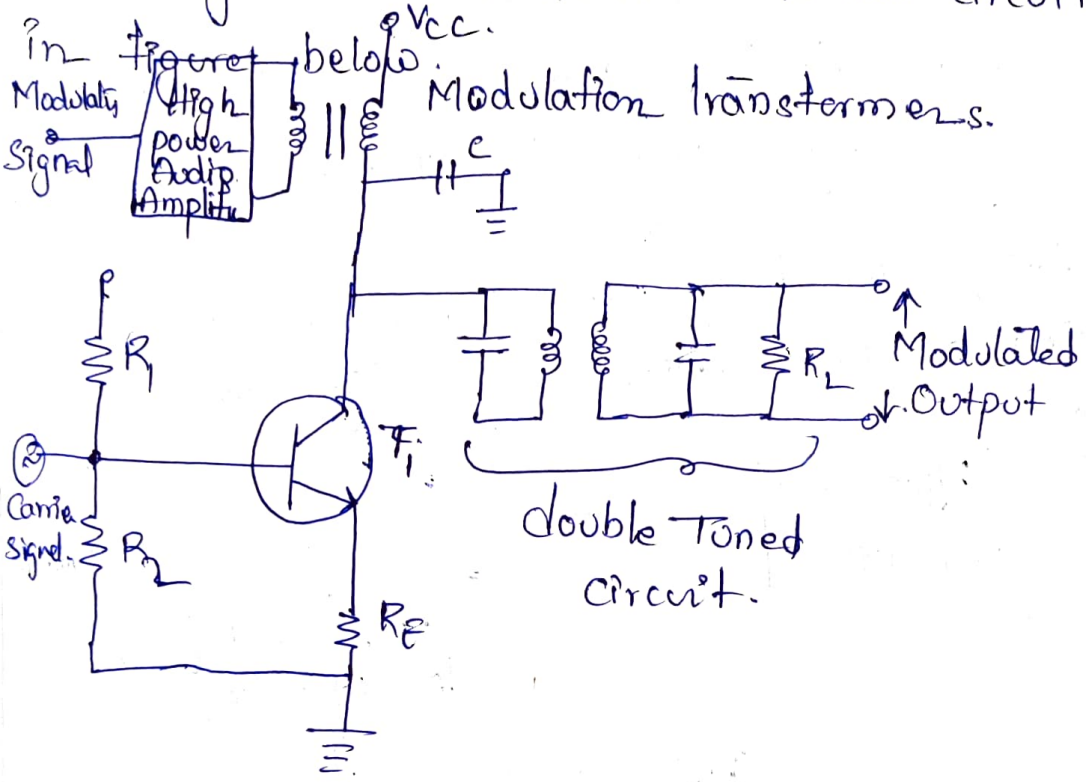
For modulating signal, the wide band power amplifier is required just to preserve all the frequency components present in modulating signal.

For carrier signal, the narrow band power amplifier is required because it is a fixed-frequency signal.

The Collector modulation method is the example of high-level modulation.

Collector Modulation Method

A high level collector modulator circuit is shown in figure below. Modulation transformers.



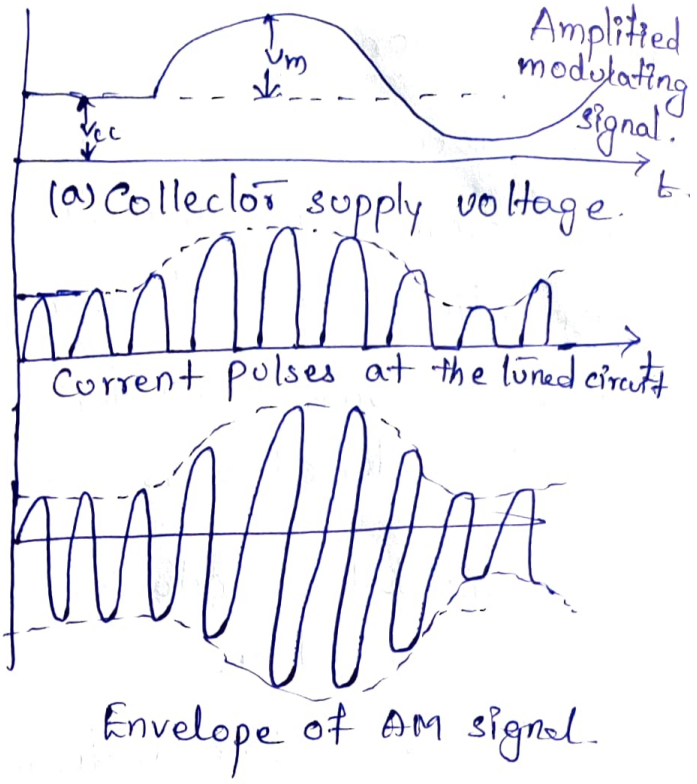
The transistor T_1 makes a radio frequency (RF) class-c amplifier. The class c amplifier conducts only a portion of the positive cycles of the carrier signal applied at the base of transistor Q_1.

The output of the transistor is connected to double tuned circuit for better performance.

With the help of modulation transformer, the modulating signal is applied at the collector of T_1.

The high power audio amplifier amplifies the modulating signal to a high power level. The secondary winding of the modulating transformer T_1 is connected in series with the dc supply voltage V_{cc}. Therefore the modulating voltage will add to or subtract from V_{cc}.

The varying supply voltage is then applied to the class C amplifier. The amplitude of collector current pulses will vary in accordance with the modulating signal. These current pulses pass through the tuned circuit. The AM wave is produced at the output of the tuned circuit.



The slowly changing carrier supply voltage V_c is expressed as

$$V_c = V_{cc} + V_m(t)$$
$$= V_{cc} + V_m \cos\omega_m t.$$

The modulating index m is give by $m = \frac{V_m}{V_{cc}}$

$$V_c = V_{cc} + mV_{cc} \cdot \cos\omega_m t.$$
$$V_c = V_{cc}(1 + m\cos\omega_m t). \rightarrow (1)$$

The modulated output voltage will be

$$V_o = V_c \cos\omega_c t. \rightarrow (2).$$

Substituting (1) in (2)

$$V_o = V_{cc}(1 + m\cos\omega_m t)\cos\omega_c t.$$

which is the required expression for AM wave.

Demodulation of AM waves :-

The process of extracting a modulating or baseband signal from the modulated signal is called demodulation or detection.

The devices used for demodulation or detection are called demodulators or detectors.

For amplitude modulation, detectors or demodulators are categorized as

(i) Square - Law detectors
(ii) Envelope detectors

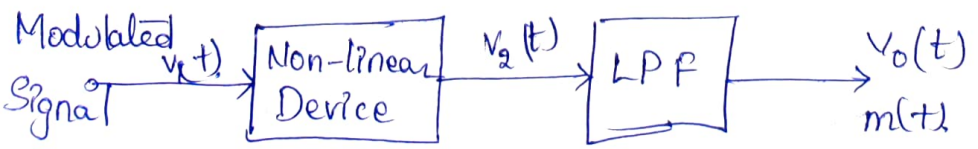
AM signal with large carrier are detected using the envelope detector.

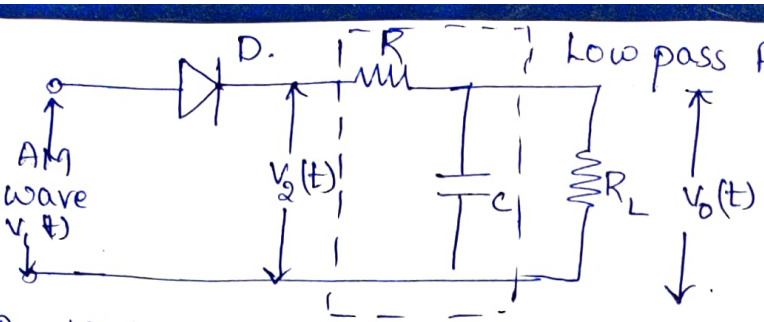
A low level amplitude modulated signal can only be detected by using a square-law detectors in which a device operating in the non-linear region is used to detect the modulating signal.

Square Law Detector

A square law detector circuit is used for detecting modulated signal of small amplitude (i.e below 1 volt) so that the operating region may be restricted to the non-linear portion of the V-I characteristics.

The circuit diagram of a square law detector is shown below.





A diode has a non-lineal relationship between input & Output. $V_2(t) = a_1 v_1(t) + b v_1^2(t)$ ⟶ (1).

The AM wave expression can be given as.

$$V_1(t) = V_c(1 + m\cos\omega_m t)\cos\omega_c t.$$
$$= V_c(1 + k_a m(t))\cos\omega_c t \longrightarrow (2)$$

Substituting (2) in (1)

$$V_2(t) = a[V_c(1+k_a m(t))\cos\omega_c t] + b[V_c^2(1+k_a m(t))^2\cos^2\omega_c t]$$

$$= aV_c\cos\omega_c t + aV_c k_a m(t)\cos\omega_c t + b\left[V_c^2\cos^2\omega_c t(1 + k_a^2 m^2(t) + 2k_a m(t))\right].$$

$$= \underbrace{aV_c\cos\omega_c t}_{\text{carrier}} + \underbrace{aV_c k_a m(t)\cos\omega_c t}_{\text{modulated}} + \underbrace{bV_c^2\cos^2\omega_c t}_{\text{squared carrier}} + \underbrace{bV_c^2 k_a^2 m^2(t)\cos^2\omega_c t}_{\text{squared modulated}} \cdot \left[\frac{1+\cos 2\omega_c t}{2}\right]$$
$$+ 2b k_a m(t) V_c^2\left[1 + \frac{\cos 2\omega_c t}{2}\right].$$

$$= aV_c\cos\omega_c t + aV_c k_a m(t)\cos\omega_c t + bV_c^2\cos^2\omega_c t + \frac{b}{2}V_c^2 k_a^2 m^2(t)$$
$$+ \frac{b}{2}V_c^2 k_a^2 m^2(t)\cos 2\omega_c t + 2b k_a m(t) V_c^2 + b k_a m(t) V_c^2(\cos 2\omega_c t)$$

The above equation is passed through a low pass filter. The low pass filter is used to extract the desired message signal.

$$\boxed{V_0(t) = bV_c^2 k_a m(t)}.$$

But the LPF passes the term $\frac{b}{2}V_c^2 k_a^2 m^2(t)$ in addition to the desired signal.

The ratio of desired signal to undesired signal is.

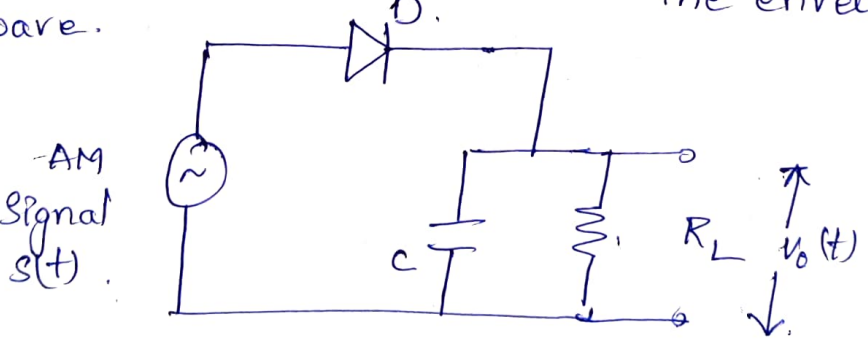
$$\frac{bV_c^2 k_a m(t)}{\frac{b}{2}V_c^2 k_a^2 m^2(t)} = \frac{2}{k_a m(t)}.$$

In order to minimize the distortion, the ratio should be maximized by keeping $|k_a m(t)| \simeq 1$ for all t.

Envelope Detector :-

The Envelope detector is a linear detector which produces an output signal (original signal) that follows the envelope of the AM wave.

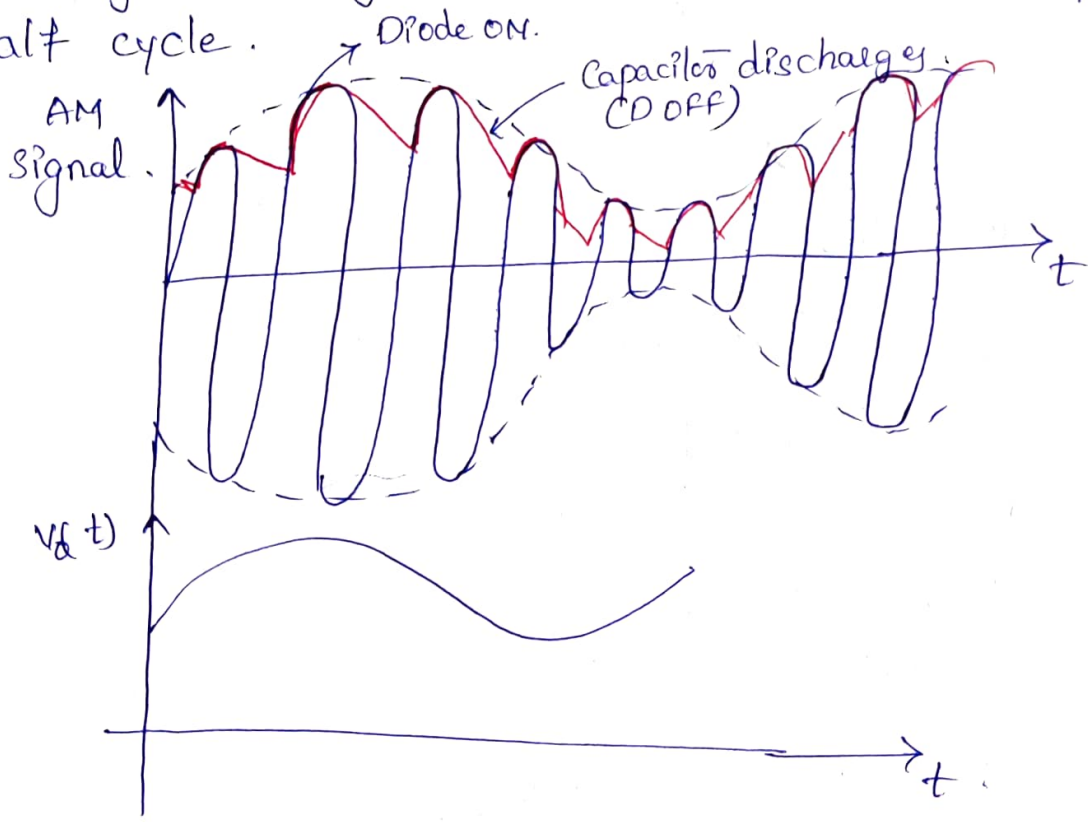
In this detector, a diode operates in a linear region of its characteristics to extract the envelope of an AM wave.



Here C - filter capacitance, R_L - load resistance

During the positive half cycle of the AM signal, the diode is forward biased & acts as closed switch. The capacitor charges to the peak value of the input signal.

During the negative half cycle, the diode is reverse biased & act as open circuit. Hence the capacitor discharges through the R_L till the next positive half cycle.



The capacitor C charges when the diode is ON & the 'c' discharges when diode is OFF.

The charging time constant is short. $RC << \frac{1}{f_c}$.

The discharging time constant is long $Rc >> \frac{1}{f_m}$

The spikes are introduced due to charging & discharging of the capacitor.

In order to minimize spikes, the time constant 'Rc' should be very high $\frac{1}{f_c} << Rc << \frac{1}{f_m}$.

But the time constant cannot be increased beyond a certain limit. Therefore, the time-constant is an important consideration for envelope detector.

Advantages of Amplitude Modulation:-

1. AM transmitters are less complex
2. AM receivers are simple, detection is easy.
3. AM receivers are cost efficient.
4. AM waves can travel a longer distance
5. Low bandwidth.

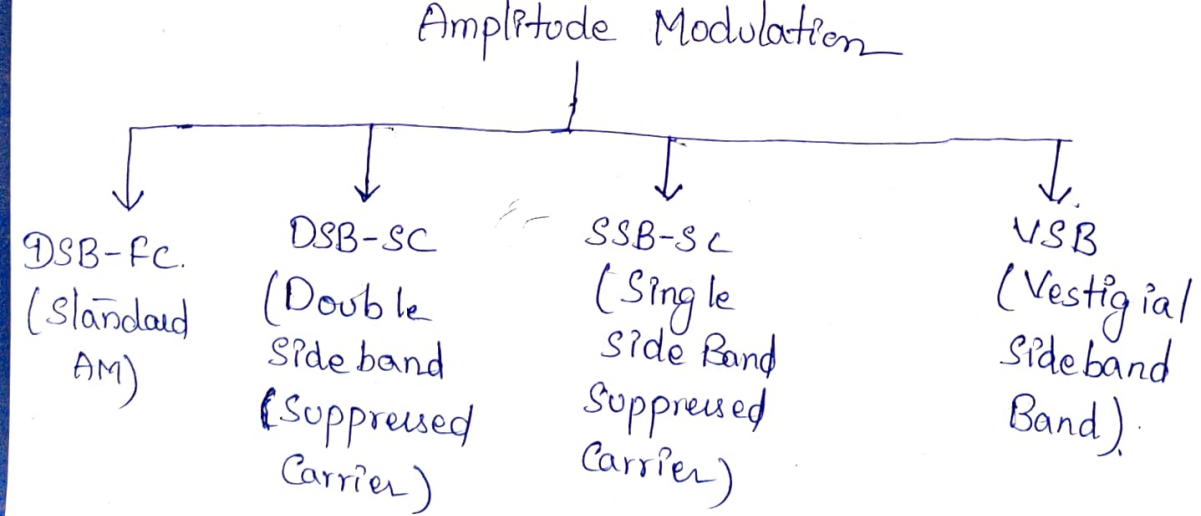
Disadvantages of Amplitude Modulation.

1. AM waves gets affected due to Noise.
2. Data loss is also more due to noise.
3. Power wastage takes place.
4. It is a bandwidth inefficient system.

Applications of Amplitude Modulation

1. Radio broadcasting
2. Picture transmission in TV system.

Types of Amplitude Modulation

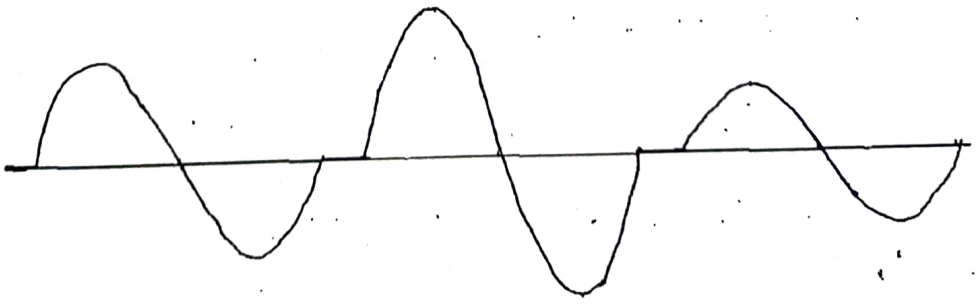


DSB-Modulation

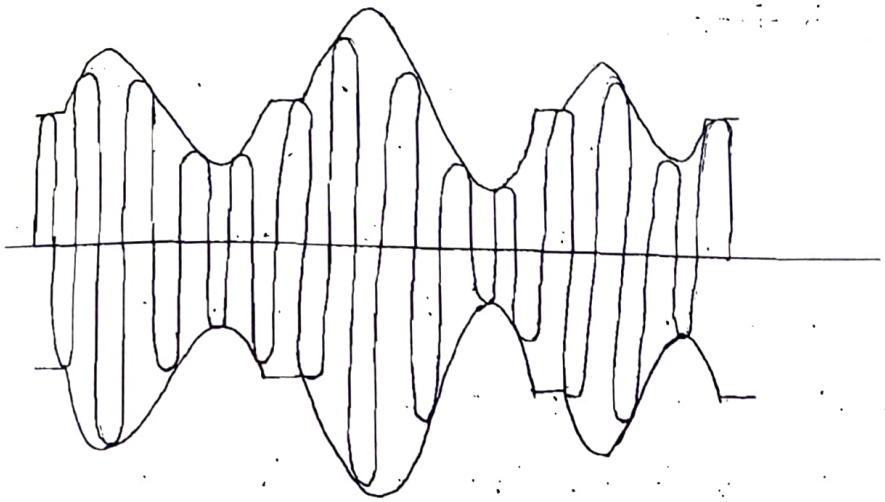
Introduction :=

When the carrier is amplitude modulated by a single sine wave the resulting signal consists of three frequencies i.e... Original carrier and two side bands. This system is known as Double side-band full carrier (DSBFc).

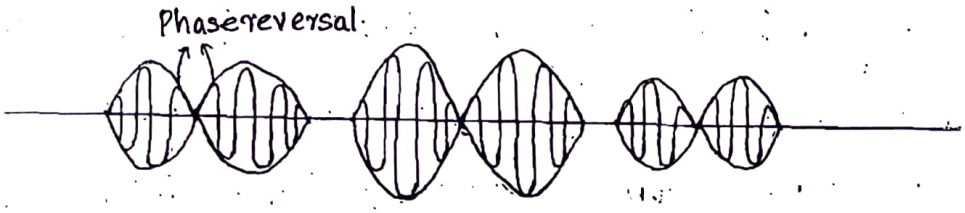
Generally we know that the carrier signal does not convey any information. The real information is conveyed by two side bands. But two third of power is wasted to transmit the carrier signal. So we can supress the carrier and remaining signal contains only lower and upper side bands. Then the signal is called Double side band supressed carrier. (DSBSc). Now the power wasted for carrier can be put in to the side bands, to transmit for longer distances.



(a) modulating signal



(b) AM wave



(c) Supressed carrier wave.

Time domain description :=

When the carrier is supressed during the modulation process, the resulting is only the algebraic sum of upper and lower side bands. The equation of DSBSC AM signal is given as

$$s(t) = m(t)\, c(t)$$

$$= A_m \cos 2\pi f_m t \cdot A_c \cos 2\pi f_c t$$

$$= \frac{A_m A_c}{2}\left[\cos 2\pi (f_c + f_m)t + \cos 2\pi (f_c - f_m)t\right]$$

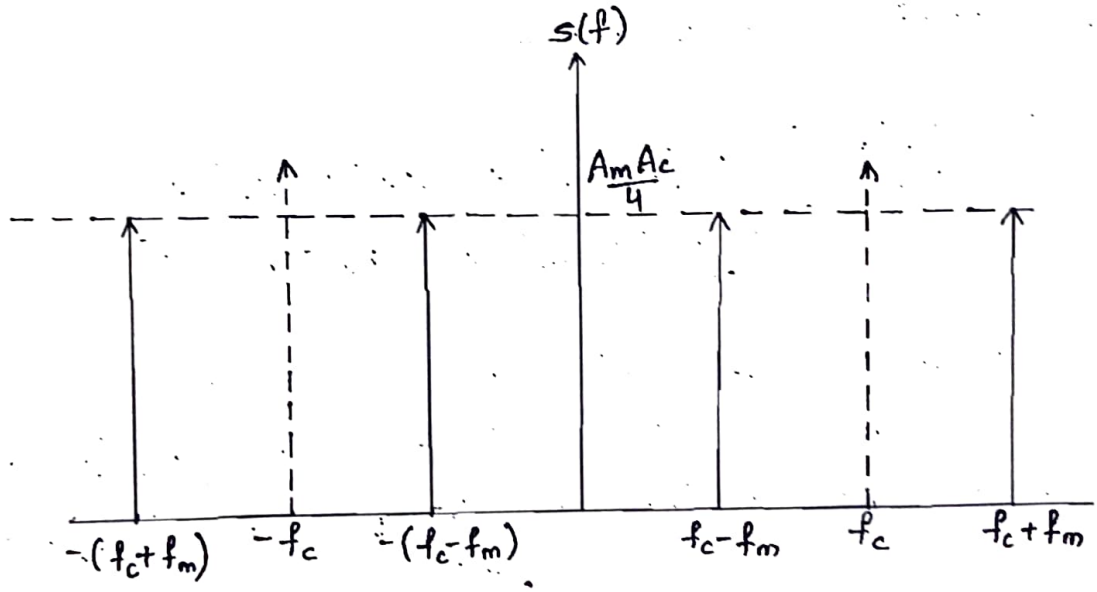
$f_c + f_m$ is upperside band; $f_c - f_m$ is lower side band.

Frequency domain description :=

In DSBSC, as the carrier is supressed it is denoted by dotted line. The spectrum space occupied by DSB is $2f_m$ or $2w$, which is same as that for a conventional AM signal.

From time domain description we have

$$s(t) = \frac{A_c A_m}{4}\left[\delta(f-(f_c+f_m)) + \delta(f+(f_c+f_m)) + \delta(f-(f_c-f_m)) + \delta(f+(f_c-f_m))\right]$$



Spectrum of DSBSC modulated wave

Band width :=

The spectrum space occupied by DSB-SC is $2f_m$, so its band width is $2f_m$.

Single tone modulation :-

If the modulating signal contains only one frequency then it is called single tone modulation.

The sinusoidal modulating signal is given by

$$m(t) = A_m \cos 2\pi f_m t.$$

Therefore, DSBSC modulated wave is given by

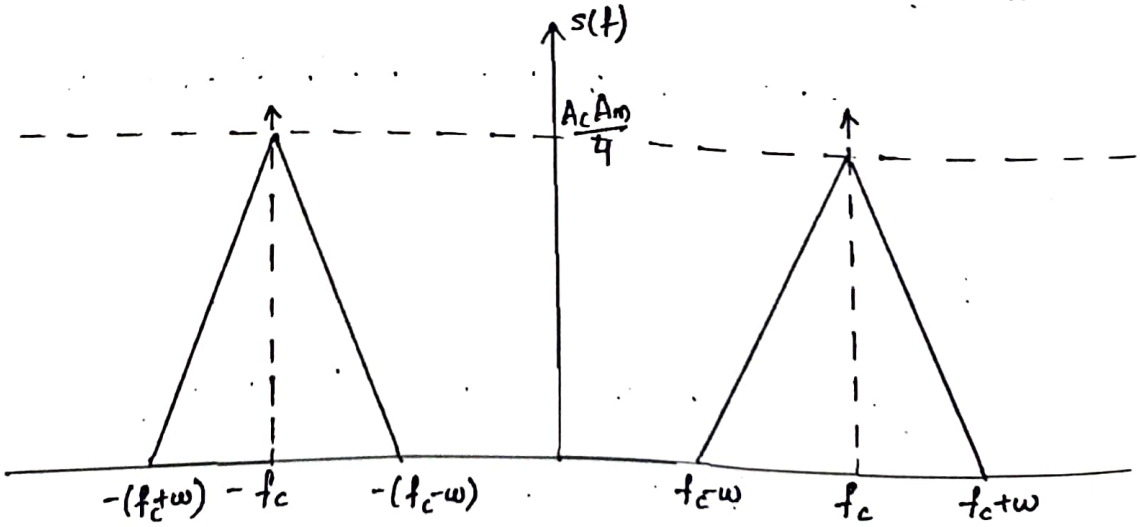
$$s(t) = A_c A_m \cos 2\pi f_c t \cos 2\pi f_m t$$

$$= \frac{1}{2} A_c A_m \left[\cos 2\pi (f_c + f_m)t + \cos 2\pi (f_c - f_m)t\right]$$

By applying fourier transform

$$s(f) = \frac{A_c A_m}{4} \left[\delta(f-(f_c+f_m) + \delta(f+(f_c+f_m) + \right.$$
$$\left. \delta(f-(f_c-f_m) + \delta(f+[f_c-f_m)\right]$$

We can observe that the spectrum of DSBSC modulated wave for the case of a sinusoidal modulating wave consists of delta functions located at $f_c \pm f_m$ & $-f_c \pm f_m$



spectrum of single tone AM-SC

Power :=

As the carrier is absent the total power is only due to side bands.

$$P_{DSBFC} = P_c + P_{sc} = P_c + P_c \frac{\mu^2}{2}$$

Carrier is supressed so

$$P_{DSBSC} = P_c \frac{\mu^2}{2}$$

$$\% \text{ power saving in DSBSC} = \frac{P_{DSBFC} - P_{DSBSC}}{P_{DSBFC}} \times 100$$

$$= \frac{P_c\left(1 + \frac{\mu^2}{2}\right) - P_c \frac{\mu^2}{2}}{P_c\left(1 + \frac{\mu^2}{2}\right)} \times 100$$

$$= \frac{2}{2+\mu^2} \times 100.$$

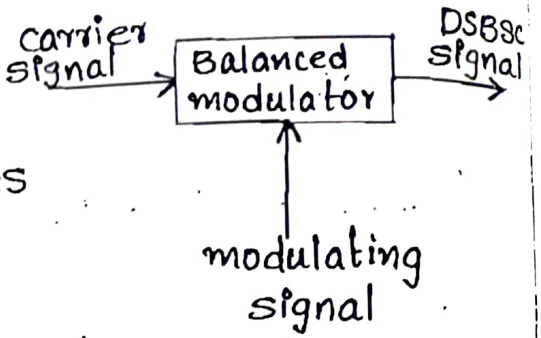
If $\mu = 1$ then % power saving in DSBSC is 66.6%

Generation of DSBSC wave :=

For generating a DSBSC signal we have seen that m(t) and c(t) should get multiplied with each other. So this type of can be obtained by using a product modulator or balanced modulator.

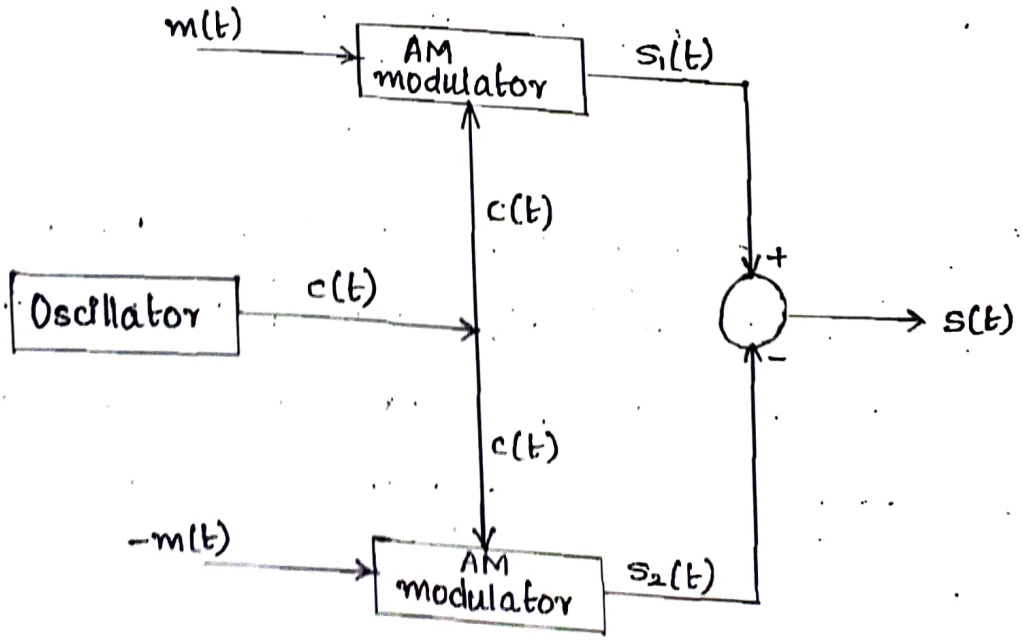
Balanced Modulator :=

The balanced modulator is used to supress the carrier from the AM signal. The inputs are carrier and modulating signal. The out puts are upper and lower side bands.



Principle used in balanced modulator :=

When two signals at different frequencies are passed through a non linear resistance, the AM signal is generated with supressed carrier. The non linear devices such as diode, JFET and transistor are used in balanced modulator.



The balanced modulator circuit consists of two standard amplitude modulators which are arranged in balanced configuration, to supress the carrier. The two modulators are identical except for the sign reversal of the modulating wave applied to the input of one of them.

The outputs of modulators can be given as

$$s_1(t) = A_c(1 + k_a m(t)) \cos 2\pi f_c t$$

$$s_2(t) = A_c(1 - k_a m(t)) \cos 2\pi f_c t$$

$$s(t) = s_1(t) - s_2(t)$$

$$= A_c \cos 2\pi f_c t \left[1 + k_a m(t) - 1 + k_a m(t)\right]$$

$$= A_c \cos 2\pi f_c t \left(2 k_a m(t)\right)$$

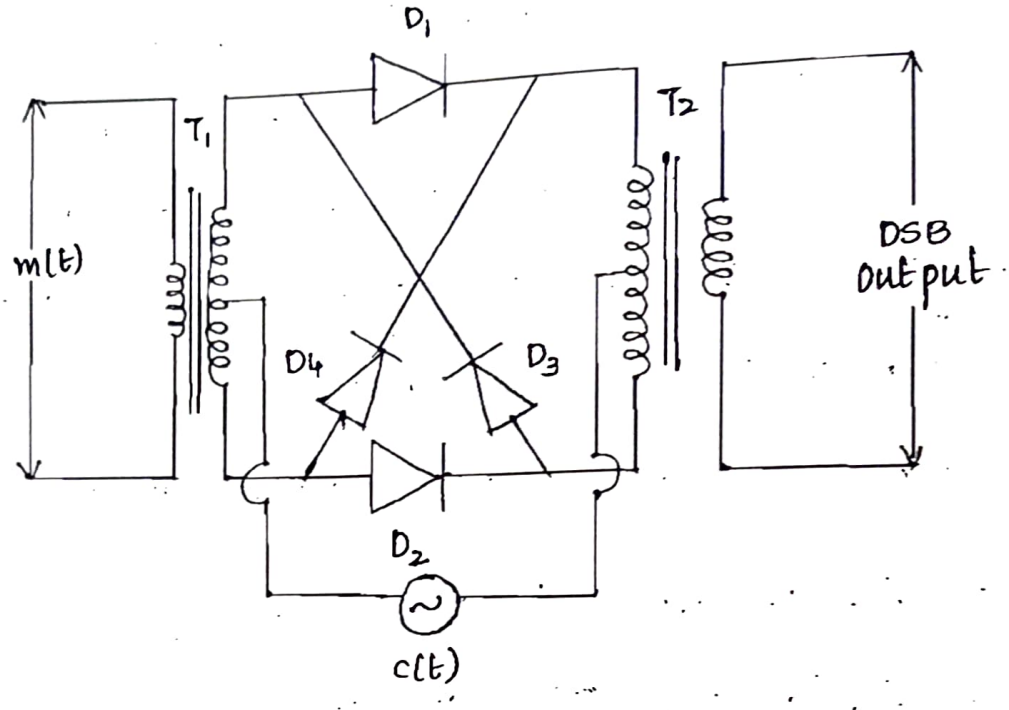
$$= 2 k_a m(t) c(t).$$

The balanced modulator output is equal to the product of modulating and carrier.

There are two types of balanced modulators.

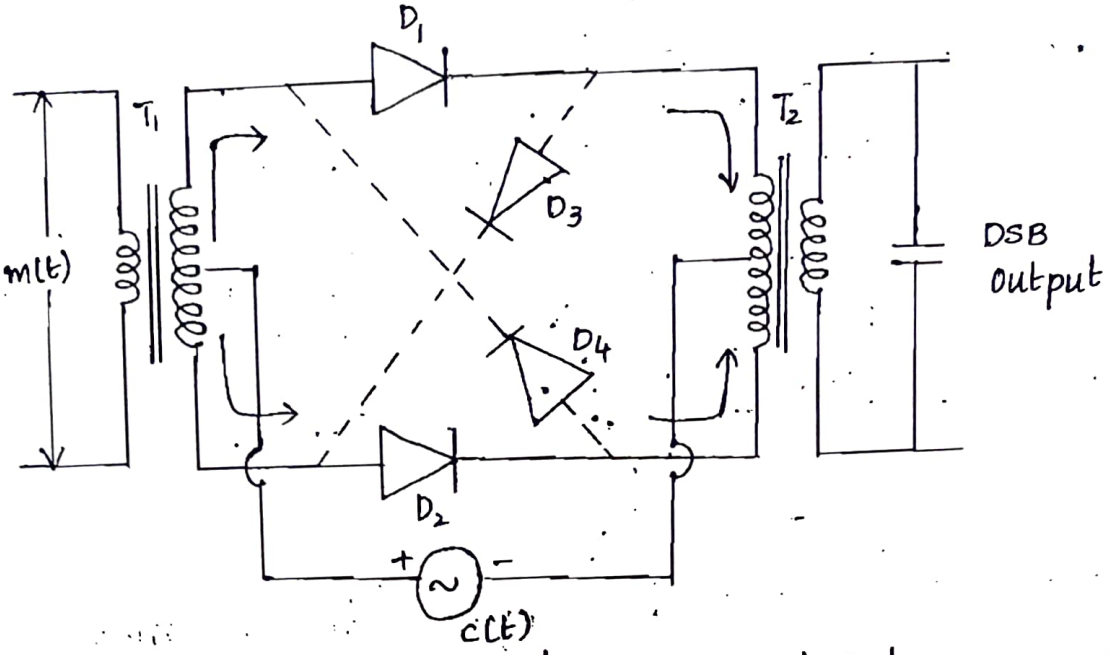
* Diode or Ring or Lattice modulator.
* JFET modulator.

Diode Modulator :=



Diode modulator consists of an input transformer T_1 and an output transformer T_2 and four diodes connected in a bridge circuit. In this the carrier signal is applied to center taps of input and output transformers and modulating signal is applied to input transformer T_1. The output appears across secondary of the transformer. The diodes connected in the bridge acts like switches and are controlled by carrier signal because it is higher in frequency and amplitude than the modulating signal.

Positive half cycle of carrier signal:-

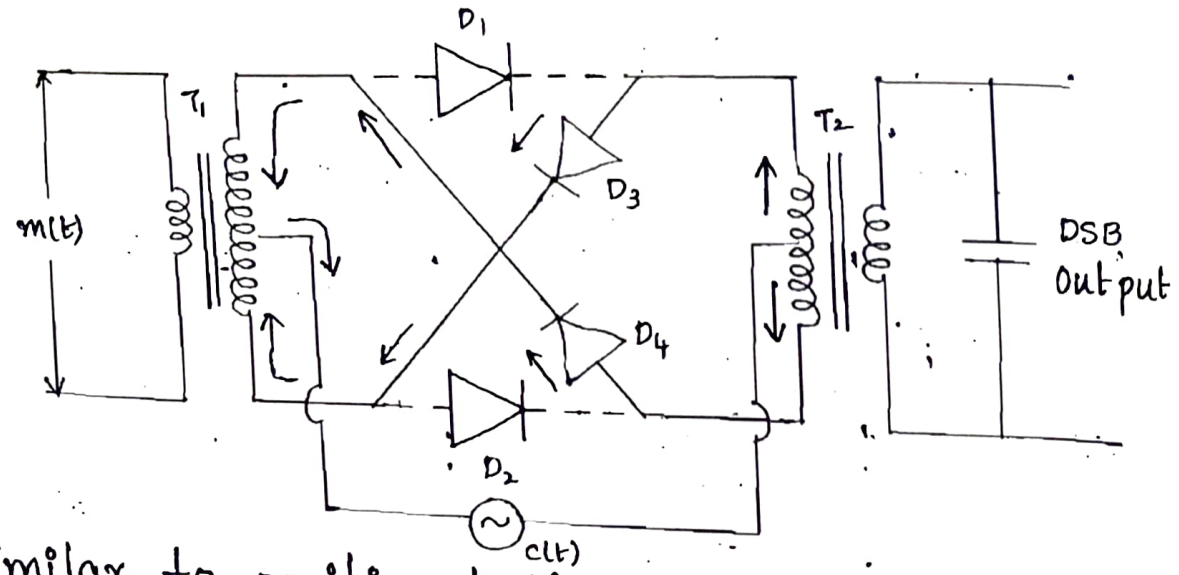


In this we assume that the modulating input is zero. In positive half cycle of carrier signal, the diodes D_1 and D_2 are forward biased and D_3 and D_4 are reverse biased. The current divides equally in upper and lower portions of primary winding of T_2. The current produce magnetic field which is equal and opposite in upper and lower part of the winding. Hence they cancel each other and produce no output at secondary of T_2. Thus the carrier is supressed.

Negative half of carrier signal:-

In negative half of carrier signal the diodes D_1 and D_2 are reverse biased and D_3 and D_4 are

forward biased.



Similar to positive half cycle, the magnetic fields in primary winding of T_2 are equal and opposite canceling each other. Therefore output at T_2 secondary is zero.

The supression of carrier in both the half cycles depends on diode characteristics and exactness of center tap of transformer to give exactly equal upper and lower currents and magnetic fields.

With modulating signal:-

Before we assumed that the modulating input is zero. But now consider a low frequency sine wave is applied to primary of T_1 as modulating signal. This will appears across T_1 secondary. The diodes D_1 and D_2 are forward biased and they connect secondary

of T_1 to primary of T_2. As a result the modulating signal is applied to primary of T_2. In negative half cycle, the diodes D_3 and D_4 are forward biased. ∴ they connect secondary of T_1 to primary of T_2 with reverse connections which results in 180° phase shift in modulated signal.

This balanced modulator works perfectly when the carrier signal is square wave

The expression can be given as

$$c(t) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos 2\pi f_c t(2n-1).$$

$$s(t) = c(t) \cdot m(t)$$

$$= \left(\frac{4}{\pi} \cos 2\pi f_c t - \frac{4}{3\pi} \cos(3 \times 2\pi f_c t) + \cdots\right) m(t)$$

$$= \frac{4}{\pi} \cos 2\pi f_c t \; m(t) - \frac{4}{3\pi} \cos(3 \times 2\pi f_c t) m(t) + \cdots$$

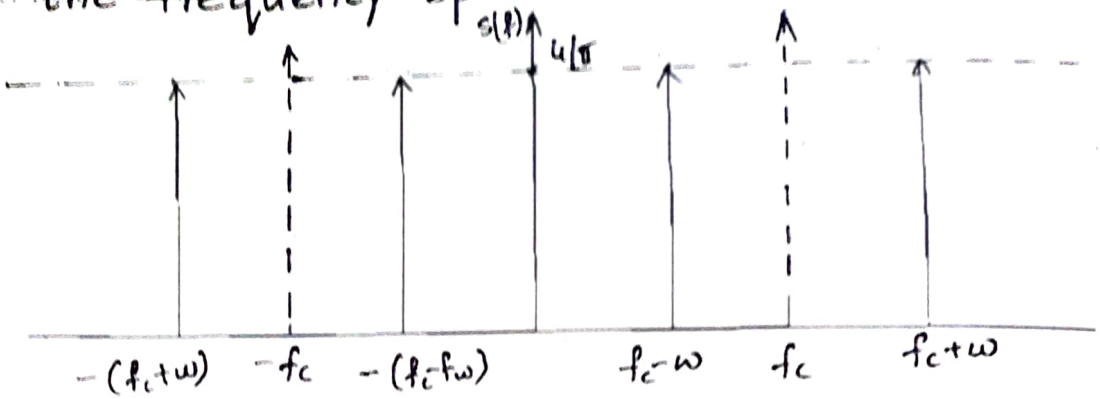
Apply fourier transforms

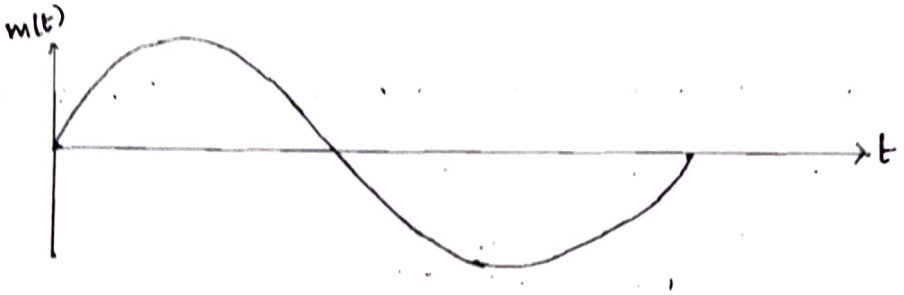
$$s(f) = \frac{4}{\pi}\left[M(f-f_c) + M(f+f_c)\right] - \frac{4}{3\pi}\left[M(f-3f_c) + M(f+3f_c)\right] + \cdots$$

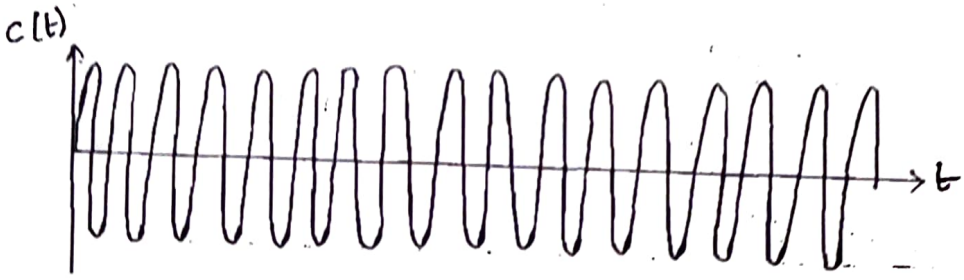
Eliminate high frequency terms then we get

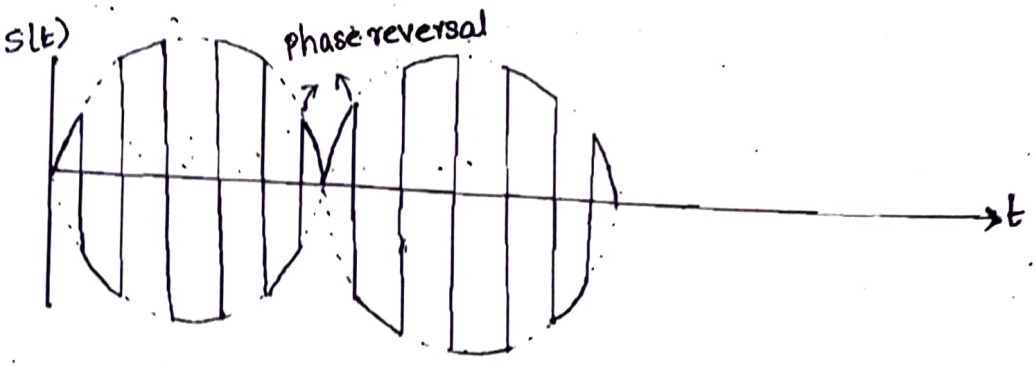
$$s(f) = \frac{4}{\pi}\left[M(f-f_c) + M(f+f_c)\right].$$

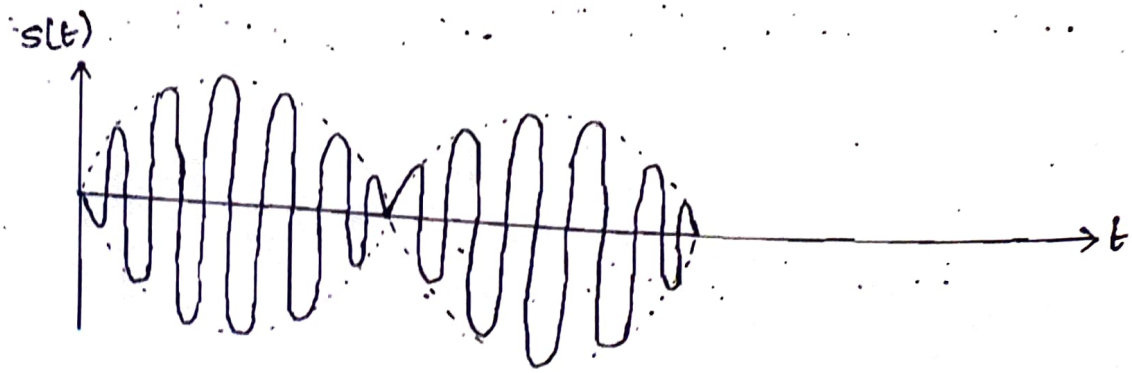
Then the frequency spectrum is



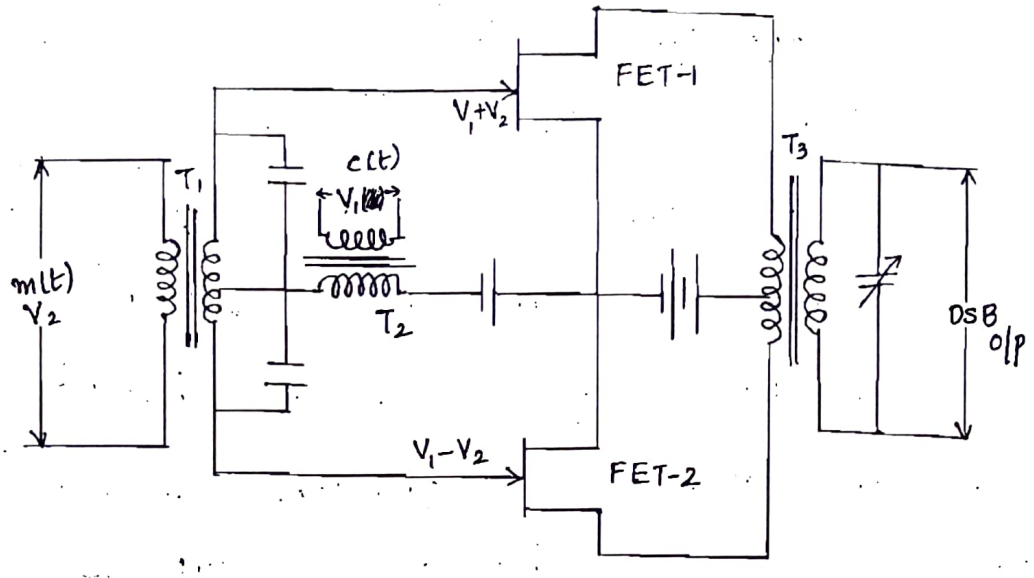








JFET balanced Modulator :-



This modulator consists of input transformers T_1 and T_2, an output transformer T_3 and two FETs. Here the carrier signal is applied to centre taps of input T_1 and output T_3 through transformer T_2. The modulating signal is applied to input transformer T_1. The carrier voltage is applied to two tag gates in phase where as modulating voltage appears 180° out of phase, at the gates, since they are at the opposite ends of center tapped transformer.

In the absence of modulating signal, the FET currents due to carrier signal are equal in amplitude but opposite in direction and cancel each each other resulting in no output of secondary of Transformer. Thus carrier is supressed.

When modulating signal is applied, the FET currents due are due to carrier as well as the modulating signal. The FET currents due to carrier are equal and opposite and cancel eachother. As modulating signal is applied 180° out of phase at the gate, the FET currents are equal but not opposite Hence they do not cancel each other. Therefore the out put is two side bands with supressed carrier

The supression of carrier depends on symmetry of circuit components.

Mathematical Expression for balanced modulator :=

The transfer characteristics of FET (I_D, V_{GS}) can be given as $I_D = I_0 + aV_{gs} + bV_{gs}^2 + \dots$

Here I_0 is maximum drain current when $V_{GS} = 0$

a and b are constants.

From this $I_{D_1} = I_0 + aV_{gs_1} + bV_{gs_1}^2$

$$I_{D_2} = I_0 + aV_{gs_2} + bV_{gs_2}^2$$

As the drain currents are in opposite direction th current through primary of coil of T_3 can be given as $I_P = I_{D_1} - I_{D_2}$

$$I_p = a(V_{gs_1} - V_{gs_2}) + b(V_{gs_1}^2 - V_{gs_2}^2)$$

Here $V_{gs_1} = V_1 + V_2$ and $V_{gs_2} = V_1 - V_2$.

$$I_p = a(2V_2) + b(4V_1V_2).$$

We know that $V_1 = c(t) = A_c \cos\omega_c t$

$V_2 = m(t) = A_m \cos\omega_m t$

$$I_p = 2aA_m \cos\omega_m t + 4bA_cA_m \cos\omega_c t \cos\omega_m t.$$

$$= 2aA_m \cos\omega_m t + 2bA_cA_m[\cos(\omega_c - \omega_m)t + \cos(\omega_c + \omega_m)t]$$

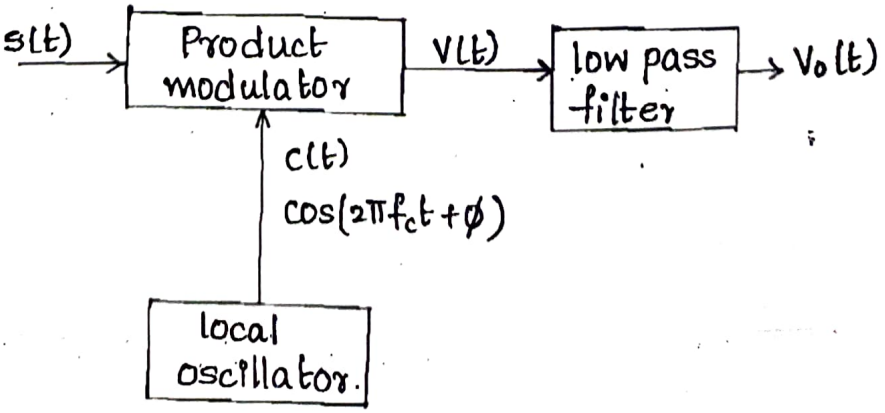
$2a\,A_m \cos\omega_m t$ is message signal.

$\omega_c - \omega_m$ is lower side band

$\omega_c + \omega_m$ is upper side band.

From the above expression we can say that the carrier is supressed.

Coherent detection of DSBSC modulated wave:=



The modulating signal $m(t)$ is recovered from a DSBSC wave $s(t)$; by multiplying $s(t)$ with a locally generated sinusoidal wave and then passing through a low pass filter.

For faith ful recovery of message signal $m(t)$ the output of local oscillator should be exactly coherent or synchronized in both frequency and phase with carrier wave $c(t)$ which is used to generate $s(t)$. This method of demodulation is called coherent or synchronous detection.

The output of product modulator is given as

$$V(t) = s(t) \cos(2\pi f_c t + \phi)$$

$$= A_c \cos 2\pi f_c t \cdot \cos(2\pi f_c t + \phi)\, m(t).$$

$$= \frac{1}{2} A_c \cos\phi\, m(t) + \frac{1}{2} A_c \cos(4\pi f_c t + \phi)\, m(t).$$

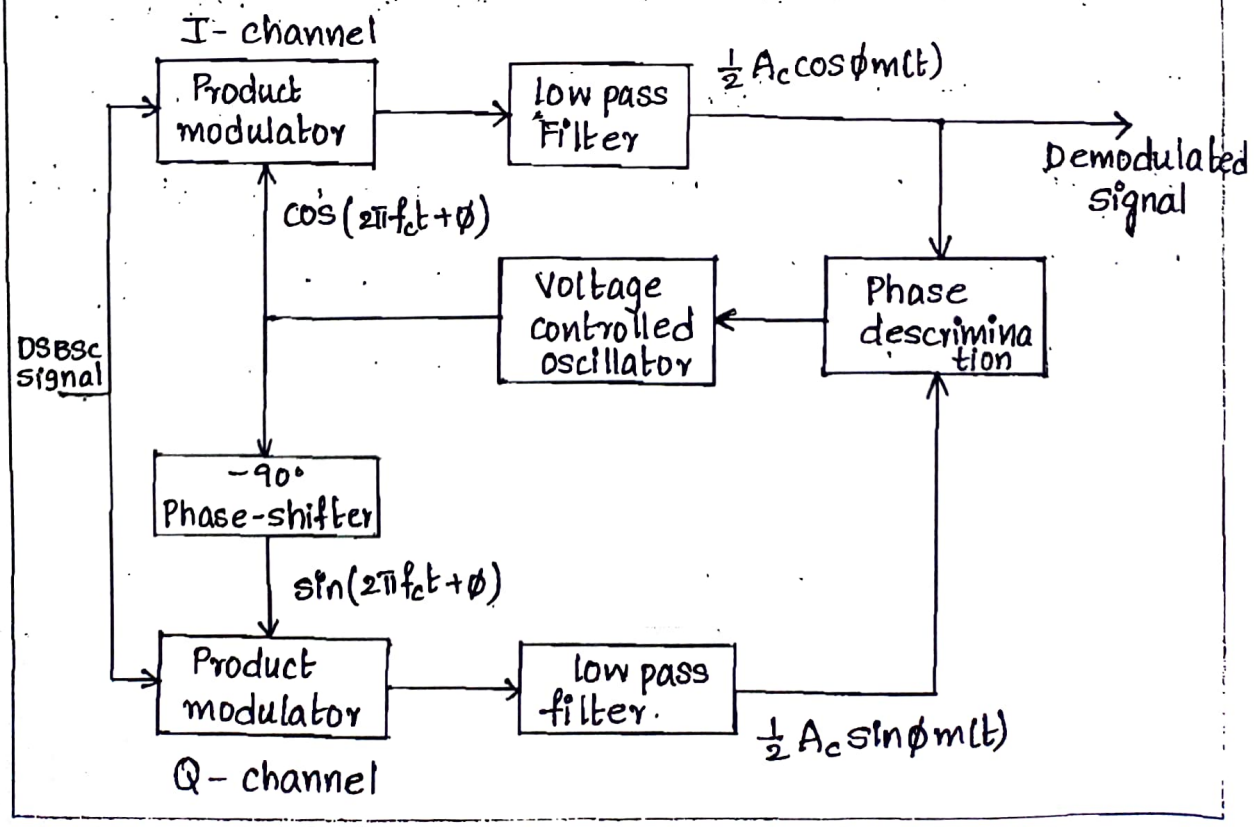
The out put consists of two terms. They are scaled version of message signal and unwanted term. The unwanted term is removed by passing through low pass filter.

The output is $V_o(t) = \frac{1}{2} A_c \cos\phi\, m(t)$.

When phase error ϕ is constant, the demodulated signal $V_o(t)$ is proportional to $m(t)$. It is maximum when $\phi = 0$ and minimum when $\phi = \pm \pi/2$. The zero demodulated signal which occurs when $\phi = \pm \pi/2$ is called quadrature null effect.

Because of phase error ϕ the detector output is attenuated by a factor $\cos\phi$. If $\cos\phi$ is constant the detector output gives undistorted modulating signal. But practically $\cos\phi$ is not constant. So to maintain the local oscillator in perfect synchronism we use costas loop.

Costas loop :=



The coastas loop is nothing but the negative feedback system designed to main t maintain the local-oscillator synchronous with carrier wave.

It consists of two coherent product modulators supplied with same input signal. The local oscillator signal supplied to productmodulators are 90° out of phase. The frequency of local oscillator is adjusted to be same as carrier frequency f_c. The product modulator in the upper path is called "in phase coherent detector or I-channel" where as, which is in lower path is called "quadrature phase detector or Q-channel". The outputs of these two channels are given to phase descriminator which consists of a multiplier followed by a low pass filter, produces a dc control signal proportional to phase error ϕ. This dc control signal is used to correct the phase error in local oscillator.

SSB Modulation

Introduction:=

When the carrier is amplitude modulated by a single sine wave, the resulting signal consists of three frequencies i.e. Original carrier and two side bands ($f_c \pm f_m$). In normal AM system both the side bands and carrier are transmitted. This is known as DSBFC. But we know that the carrier signal doesnot convey any information. so we can supress the carrier and we get DSBSC.

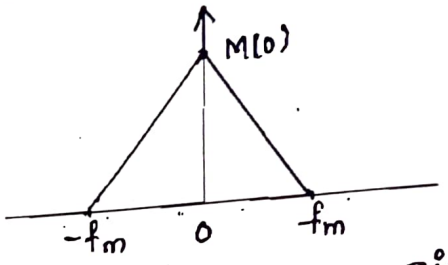
As the two side bands carry same information there is no need to transmit both the side bands to convey information. One side band may be supressed. The resulting signal is single side band known as single side band supressed carrier (SSB).

Definition :- "A modulation process in which the modulated signal contains no carrier component and has only one side band is called single side band modulation or SSB modulation"

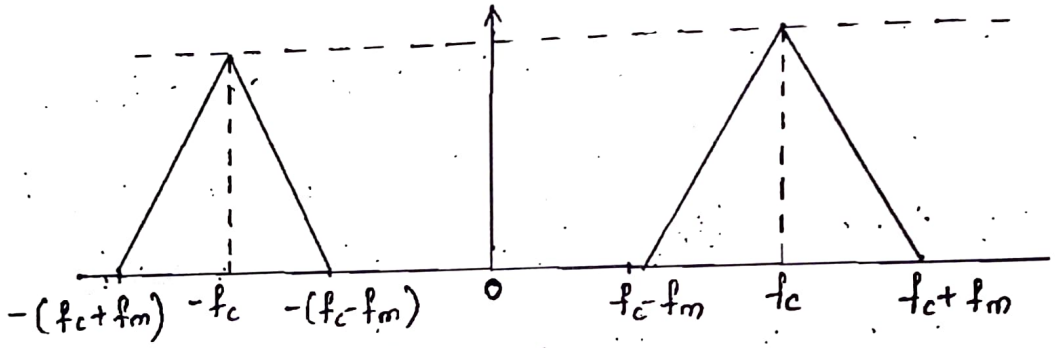
So the power which is wasted to transmit the carrier signal and a side band, can be put in to a single side band for stronger signals over longer distances.

Frequency domain description :=

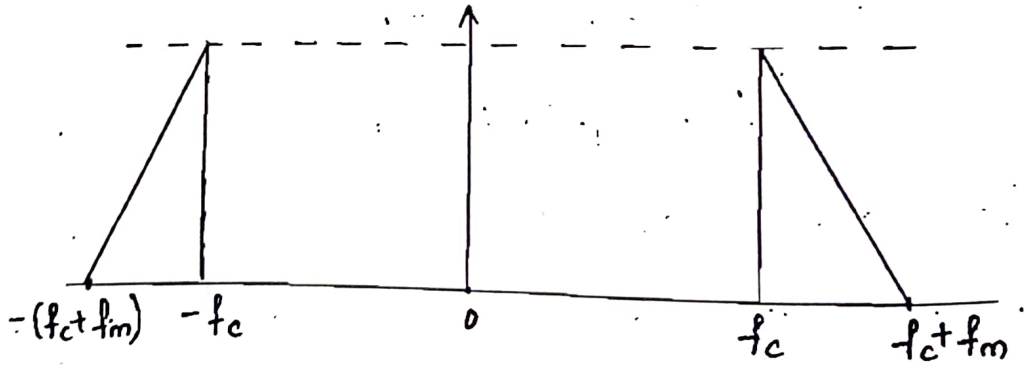
The frequency domain display of single side band modulated wave depends on the side band which is transmitted. The spectrum of DSBSC modulated wave can be obtained by multiplying m(t) and c(t).



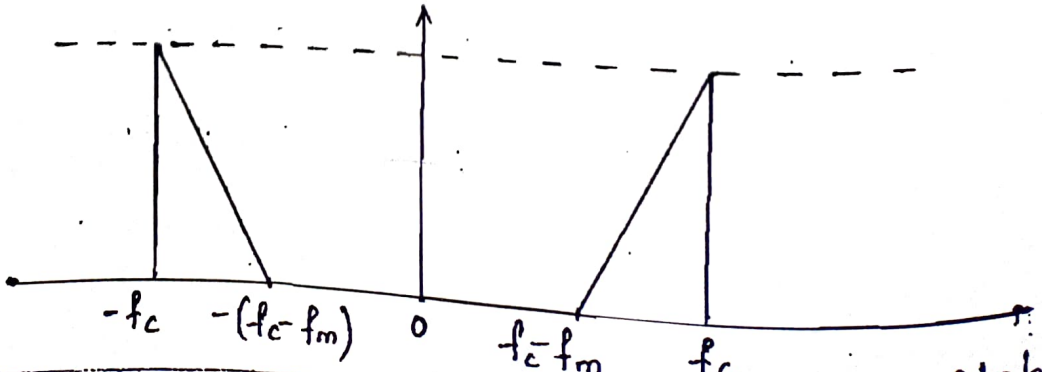
Spectrum of message signal



Spectrum of DSBSC wave



Spectrum of SSB with upper sideband



lower sideband

Advantages of SSB :=

* The spectrum space occupied by SSB signal is f_m which is only half that of AM and DSB. This reduction in frequency or band width allows more signals to transmit in same frequency range.
* Due to supression of carrier and one side band, power is saved.
* SSB signal has less band width than an AM or DSB so there will be less noise on it.
* Fading does not occur in SSB.

Dis advantages:=

* The generation and reception of SSB signal is a complex process.
* SSB is not used for transmission of high frequency signals such as music signals.

Power relations in SSB wave:=

% power saving w.r.to DSBFC.

We know that total power transmitted in AM is

$$P_t = P_c + P_{USB} + P_{LSB}$$

$$= \frac{A_c^2}{2R} + \frac{\mu^2 A_c^2}{8R} + \frac{\mu^2 A_c^2}{8R}$$

$$= \frac{A_c^2}{R}\left[1 + \frac{\mu^2}{2}\right] = P_c\left(1 + \frac{\mu^2}{2}\right).$$

If the carrier and one side band is supressed, then the total power transmitted in SSB-SC is

$$P_{SSB} = P_{LSB} = P_{USB}$$

$$P_{SSB} = \frac{\mu^2 A_c^2}{8R} = P_c\left(\frac{\mu^2}{4}\right).$$

Power saved due to transmitting only one side band component is given by

$$\% \text{ power saving} = \frac{P_t - P_{SSB}}{P_t} \times 100$$

$$= \frac{P_c\left(1+\frac{\mu^2}{2}\right) - \frac{\mu^2}{4}P_c}{P_c\left(1+\frac{\mu^2}{2}\right)} \times 100$$

$$= \frac{1+\frac{\mu^2}{2} - \frac{\mu^2}{4}}{1+\frac{\mu^2}{2}} \times 100$$

$$= \frac{4+\mu^2}{4+2\mu^2} \times 100.$$

If $\mu = 1$ then

$$\% \text{ power saving} = \frac{5}{6} \times 100 = 83.33\%.$$

Thus 83.33% power is saved due to supression of carrier wave and one side band.

% power saving w.r.to DSB-sc system :=

We know that the total power transmitted in DSB-SC is

$$P_{DSBSC} = P_{LSB} + P_{USB}$$

$$= \frac{\mu^2 A_c^2}{8R} + \frac{\mu^2 A_c^2}{8R} = P_c \frac{\mu^2}{2}$$

total power transmitted in SSB-sc is

$$P_{SSB} = P_{LSB} = P_{USB}$$

$$P_{SSB} = \frac{A_c^2 \mu^2}{8R} = P_c \frac{\mu^2}{4}$$

$$\% \text{ power saving} = \frac{P_{DSBSC} - P_{SSB}}{P_{DSBSC}} \times 100$$

$$= \frac{P_c \frac{\mu^2}{2} - P_c \frac{\mu^2}{4}}{P_c \frac{\mu^2}{2}} \times 100$$

$$= \frac{\mu^2}{2} \times 100$$

if $\mu = 1$ then

$$\% \text{ power saving} = \frac{1}{2} \times 100 = 50\%$$

Thus 50% of power is saved due to supression of one side band from DSB-sc signal.

Time domain description :=

The SSB signal is generated by passing a DSBSC modulated wave through a band pass filter having a transfer function $H_u(f)$.

We know that $S_{DSBSC}(t) = A_c m(t) \cos(2\pi f_c t)$.

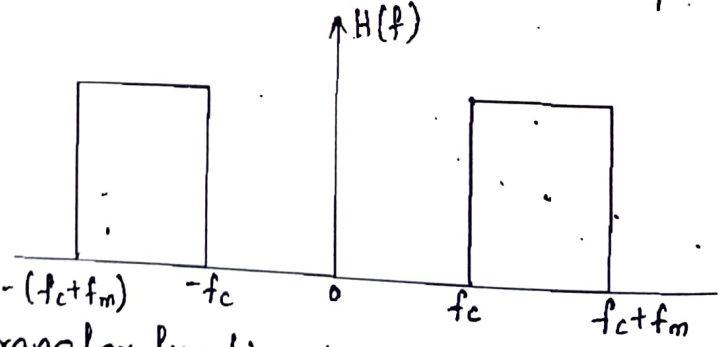
The low pass complex envelope of DSBSC modulated wave is expressed as

$$\tilde{s}_{DSBSC}(t) = A_c m(t).$$

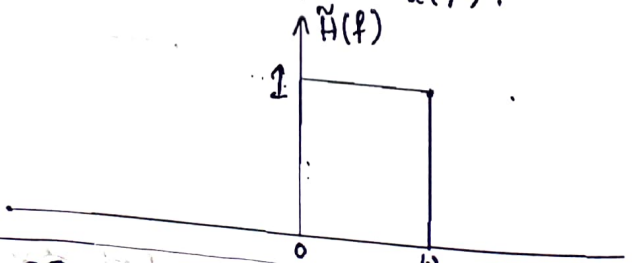
Let $s_u(t)$ be the SSB wave which has only upper-side band, and $\tilde{s}_u(t)$ be the complex envelope of $s_u(t)$.

$$s_u(t) = Re\left[\tilde{s}_u(t)\, e^{j2\pi f_c t}\right]$$

The frequency response of ideal band pass filter is



Replace transfer function $H_u(f)$ by an equivalent low-pass filter transfer function $\tilde{H}_u(f)$.



Where $\tilde{H}_u(f) = \frac{1}{2}(1 + sgn(f)) \quad 0 < f < \omega$

$= 0 \quad$ otherwise.

the spectrum of complex envelope of DSBSC modulated wave $\tilde{s}_{DSBSc}(t)$ is

$$\tilde{S}_{DSBSc}(f) = A_c M(f).$$

Now $\tilde{S}_u(f) = \tilde{H}(f)\, \tilde{S}_{DSBSc}(f)$

$$= \frac{1}{2}(1 + sgn(f))\, A_c M(f).$$

$$= \frac{A_c}{2}[M(f) + M(f)\, sgn(f)].$$

from the definition of Hilbert transform we have $\hat{m}(t) = -j\, sgn(f)\, M(f).$

$$\therefore \tilde{S}_u(f) = \frac{A_c}{2}[M(f) + j\hat{m}(t)]$$

$$\tilde{S}_u(t) = \frac{A_c}{2}[m(t) + j\hat{m}(t)].$$

We have $S_u(t) = Re[\tilde{S}_u(t)\, e^{j2\pi f_c t}].$

$$S_u(t) = Re\left[\frac{A_c}{2}(m(t) + j\hat{m}(t))(\cos 2\pi f_c t + j \sin 2\pi f_c t\right]$$

$$\therefore S_u(t) = \frac{A_c}{2}[m(t)\cos 2\pi f_c t - \hat{m}(t)\sin 2\pi f_c t].$$

This is time domain representation of a wave containing only an upper side band.

The time domain representation of SSB modulated wave containing only lower side band is given by

$$S_L(t) = \frac{A_c}{2}\left[m(t)\cos 2\pi f_c t + \hat{m}(t)\sin 2\pi f_c t\right].$$

$\frac{A_c}{2}$ = scaling factor ; $m(t)$ = in phase component,

$\hat{m}(t)$ = out of phase component.

Single tone modulation :=

If the modulating signal contains only one frequency, then it is single tone modulation.

Consider the sinusoidal modulating signal

$$m(t) = A_m \cos 2\pi f_m t.$$

By changing phase of input signal by 90° with out changing its amplitude we get hilbert transform $\hat{m}(t)$.

$$\hat{m}(t) = A_m \cos(2\pi f_m t - 90°)$$

$$= A_m \sin 2\pi f_m t.$$

We know that

$$S_u(t) = \frac{A_c}{2}\left[m(t)\cos 2\pi f_c t - \hat{m}(t)\sin 2\pi f_c t\right].$$

Substitute $m(t)$ and $\hat{m}(t)$ in $S_u(t)$

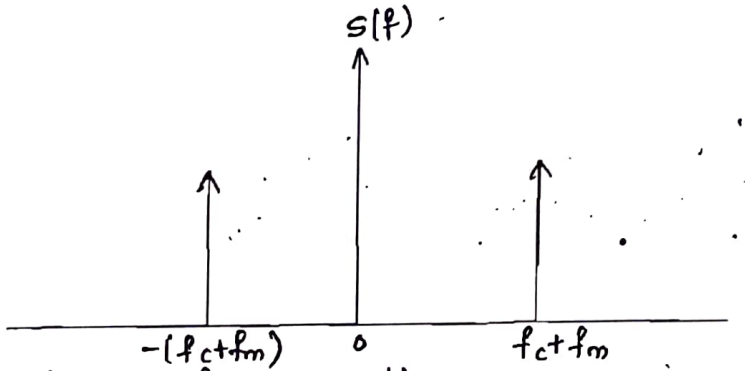
$$S_u(t) = \frac{A_c}{2}\left[A_m \cos 2\pi f_m t \cos 2\pi f_c t - A_m \sin 2\pi f_m t \sin 2\pi f_c t\right].$$

$$= \frac{A_c A_m}{2} \cos(f_c + f_m) 2\pi t$$

$$\therefore S_u(t) = \frac{A_c A_m}{2} \cos\left[2\pi (f_c + f_m) t\right].$$

This is SSB wave obtained by transmitting only upper side band.

Now



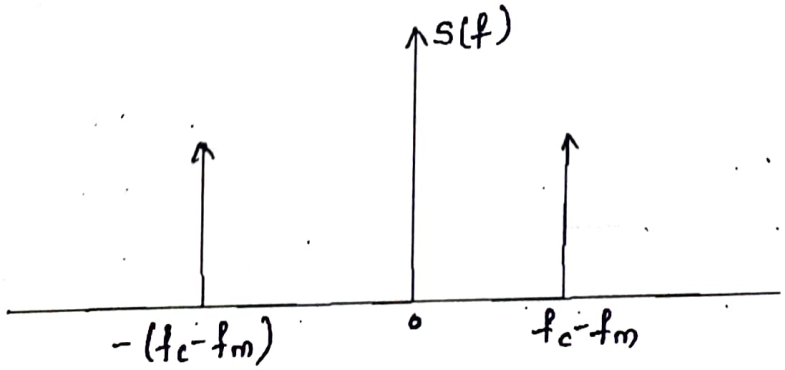
Spectrum of SSB with USB

Now for lower side band we have

$$S_L(t) = \frac{A_c}{2}\left[m(t) \cos 2\pi f_c t + \hat{m}(t) \sin 2\pi f_c t\right]$$

$$= \frac{A_c}{2}\left[A_m \cos 2\pi f_m t \cos 2\pi f_c t + m A_m \sin 2\pi f_m t \sin 2\pi f_c t\right].$$

$$= \frac{A_c A_m}{2}\left[\cos 2\pi (f_c - f_m) t\right].$$



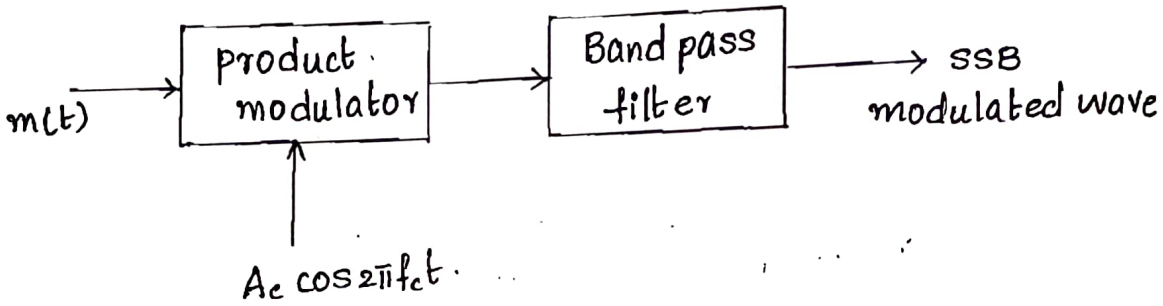
Spectrum of SSB with LSB

Generation of SSB signals :=

There are 3 different methods for generation of SSB signal.

* Filter or frequency descrimination method.
* Phase shift method.
* Weavers or third method.

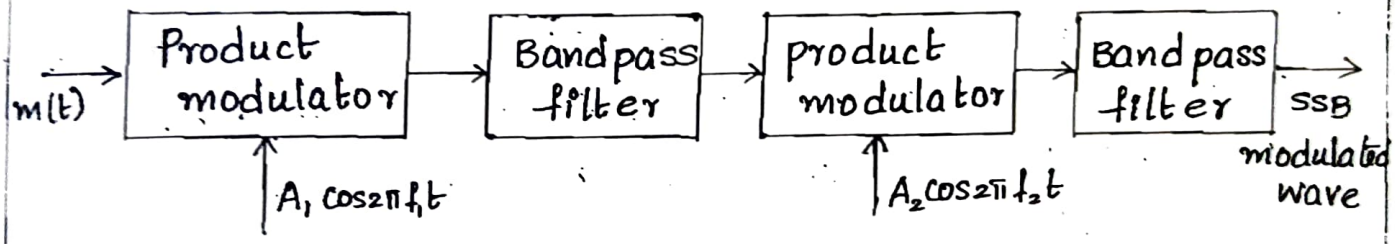
Filter or frequency descrimination method :=



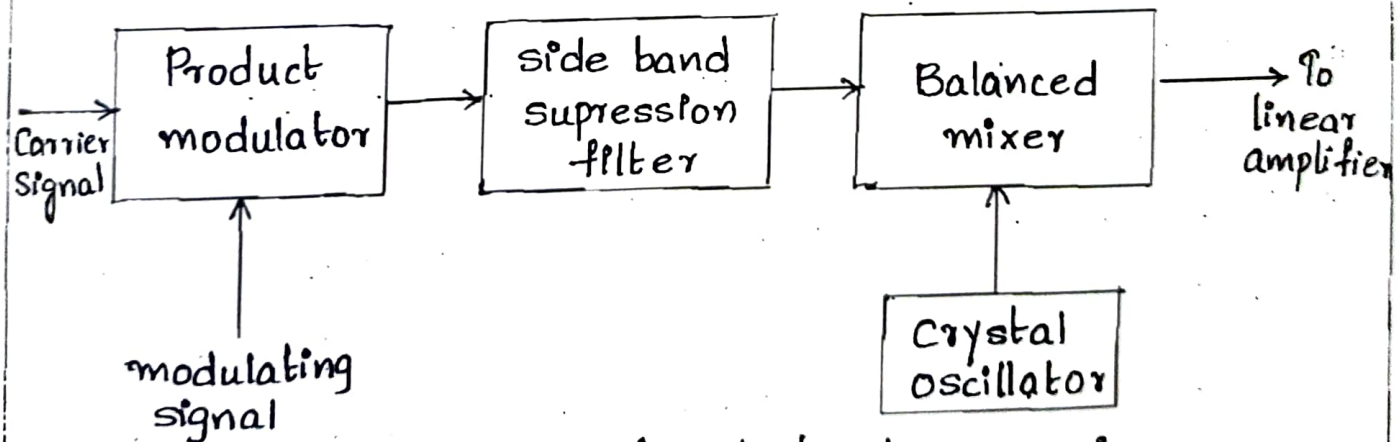
The filter is used to remove unwanted side band. The filter must have flat pass band and extremely high attenuation out side the pass band. In order to have above response the quality factor (Q) of the tuned circuits must be very high. Q-factor increases as the difference between modulating frequency f_m and carrier frequency f_c increases. Carrier frequency is usually same as transmitter frequency.

To generate an SSB modulated wave occupying a frequency that is higher than message signal, it is very difficult to design a filter, that pass desired

side band and reject the other, using single stage modulation. In double stage modulation the SSB wave at the first filter is used as modulating wave for second product modulator. It produce a DSBSC wave with a spectrum that is symmetrically spaced about second carrier frequency f_2. The frequency seperation between side bands of DSBSC modulated wave is effectively twice the first carrier frequency f_1. ∴ It is easy to remove unwanted side band.



Block diagram of two stage SSB modulator.



Filter method of side band supression

For transmitting higher frequency, the Q value should be very high which can not be achieved practically. So initial modulation is carried out. The balanced modulator supress the carrier and

filter supress the side band. The frequency of SSB is very low when compared to transmitter frequency. This frequency is boosted up to transmitter frequency by balanced mixer and crystal oscillator. This process is called "up conversion". The side band signal is then amplified by linear amplifier.

Advantages:=

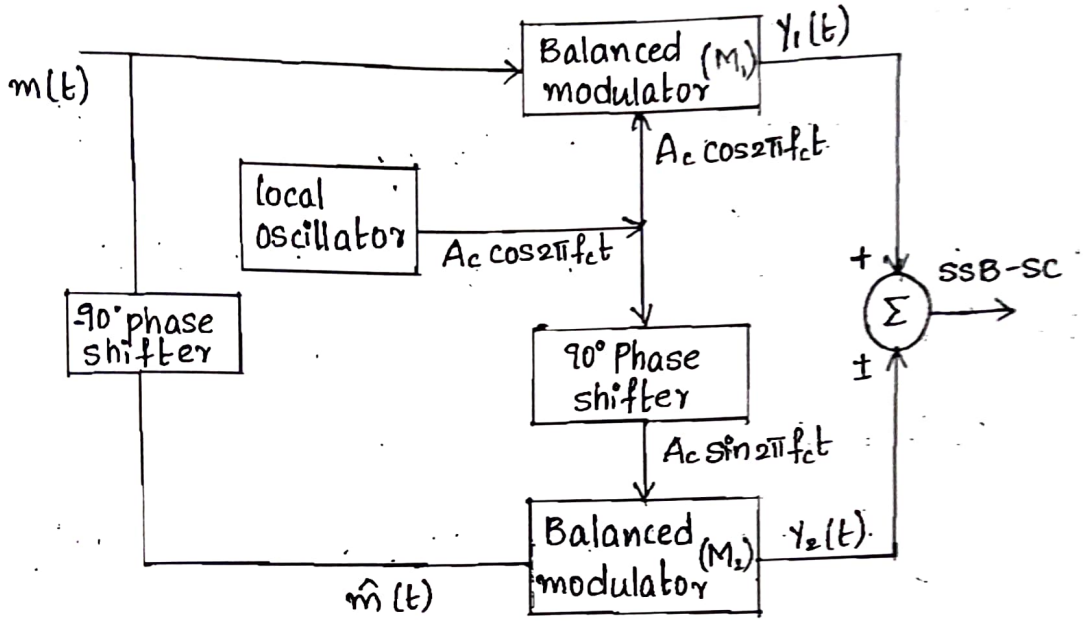
* The filter method gives side band supression up to 50 dB which is quite adequate.
* The side band filters also helps to attenuate carrier if present in output of balanced modulator.
* Band width is sufficiently flat and wide.

Dis advantages:=

* They are bulky.
* At lower audio frequencies expensive filters are required.
* As modulation takes place at lower carrier frequency repeated mixing is required in conjunction with extremely stable oscillators to generate SSB at high radio frequency.

Phase descrimination method :-

In this method we use two balanced modulators and two phase networks to obtain an SSB wave



The modulating signal m(t) and carrier signal are applied to balanced modulator (M_1) produce DSBSC wave that translates the spectrum of m(t) symmetrically spaced about the carrier frequency f_c

The modulating signal m(t) and carrier signal are phase shifted by 90° is applied to (M_2) which produce DSBSC wave with side bands of same amplitude but different phase spectra. Then we get $\hat{m}(t)$ which is hilbert transform of m(t).

The outputs of balanced modulator are identical in amplitude spectra, but phase spectra may be vector addition or subtraction.

Consider $c(t) = A_c \cos 2\pi f_c t$

$m(t) = A_m \cos 2\pi f_m t$

The output of balanced modulator M_1 is $Y_1(t)$

$$Y_1(t) = A_c A_m \cos 2\pi f_c t \cdot \cos 2\pi f_m t$$

$$= \frac{A_c A_m}{2}\left(\cos 2\pi (f_c - f_m)t + \cos 2\pi (f_c + f_m)t\right).$$

The input of balanced modulator M_2 is $\hat{m}(t)$.

The output is $Y_2(t)$.

$$Y_2(t) = A_c A_m \sin 2\pi f_c t \cdot \sin 2\pi f_m t$$

$$= \frac{A_c A_m}{2}\left(\cos 2\pi (f_c - f_m)t - \cos 2\pi (f_c + f_m)t\right).$$

Expression for lower side band is

$$Y_1(t) + Y_2(t) = A_c A_m \cos\left(2\pi (f_c - f_m)t\right).$$

Expression for upper side band is

$$Y_1(t) - Y_2(t) = A_c A_m \cos 2\pi (f_c + f_m)t.$$

Advantages:=

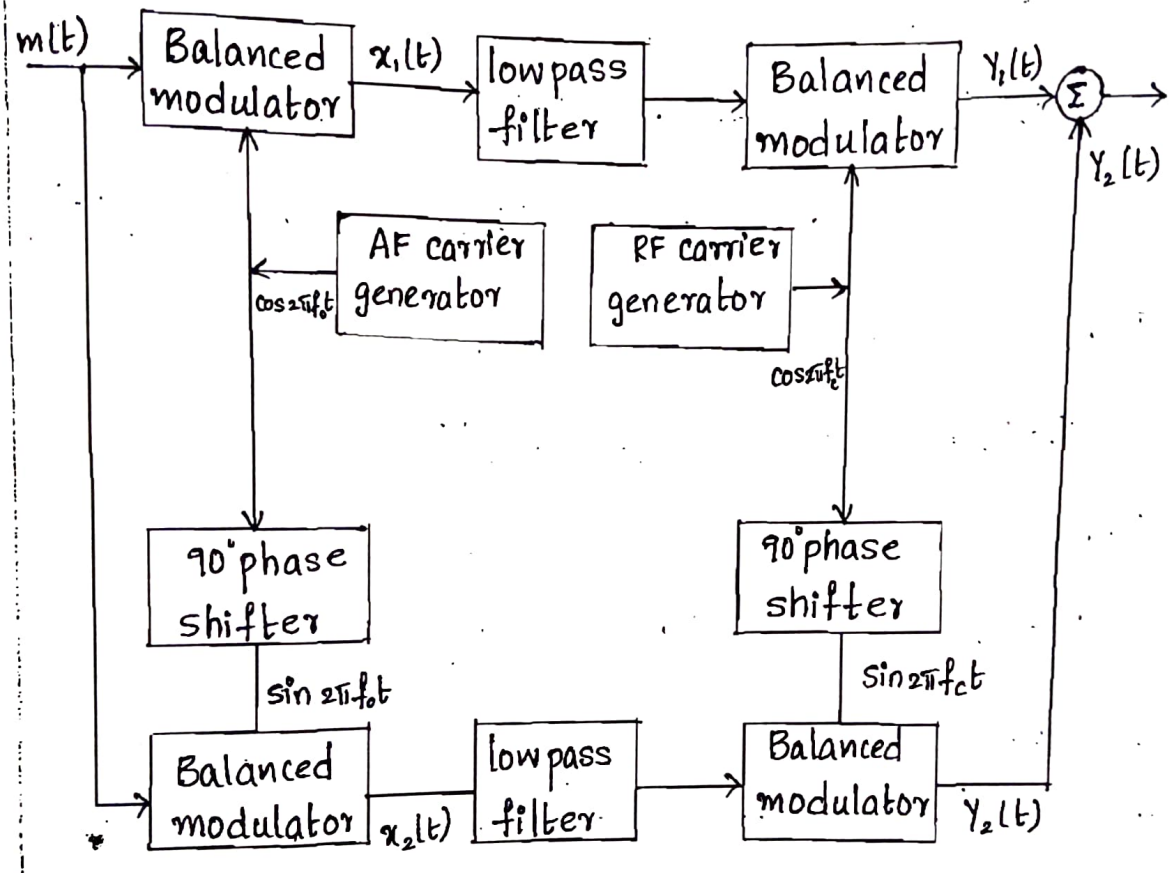
* One side band along with carrier are supressed.
* There is no need of very sharp cut off filter.
* Choice of addition or subtraction of modulator out put gives upper or lower side band.

Dis advantages:=

* The circuit is complex and expensive.
* The phase shifter should produce exact phase shift of 90°. In practise it is very difficult to design.

Weavers or third method :=

Third method of generating SSB-SC wave was invented by D.K. Weaver in 1950. This method eliminates the limitations of using sharp bandpass filter. This method consists of four balanced modulators and two lowpass filters.



The input of low pass filter -1 is

$$x_1(t) = \cos(2\pi f_m t) \cdot \cos(2\pi f_0 t).$$

$$= \frac{1}{2}\left(\cos 2\pi (f_0 + f_m)t + \cos 2\pi (f_0 - f_m)t\right).$$

As it is lowpass filter it allows only low frequency i.e... $\frac{1}{2}\cos 2\pi (f_0 - f_m)t$. This is multiplied with $\cos 2\pi f_c t$ to yield $Y_1(t)$.

$$Y_1(t) = \frac{1}{2}\cos 2\pi (f_0 - f_m)t \cdot \cos 2\pi f_c t.$$

$$Y_1(t) = \frac{1}{4}\left(\cos 2\pi(f_c + f_o - f_m)t + \cos 2\pi(f_c - f_o + f_m)t\right).$$

Similarly, the input of lowpass filter -2 is

$$x_2(t) = \cos 2\pi f_m t \sin 2\pi f_o t$$

$$= \frac{1}{2}\left(\sin 2\pi(f_o + f_m)t + \sin 2\pi(f_o - f_m)t\right).$$

The out put is $\frac{1}{2}\sin 2\pi(f_o - f_m)t$. This is multiplied by $\sin 2\pi f_c t$ to get $Y_2(t)$.

$$Y_2(t) = \frac{1}{2}\sin\left(2\pi(f_o - f_m)t\right).\sin 2\pi f_c t.$$

$$= \frac{1}{4}\left(\cos 2\pi(f_c - f_o + f_m)t - \cos 2\pi(f_c + f_o - f_m)t\right).$$

Expression for lower side band.

$$Y_1 - Y_2 = \frac{1}{2}\cos 2\pi\left(f_o + f_c - f_m\right)t$$

Expression for upper side band.

$$Y_1 + Y_2 = \frac{1}{2}\cos 2\pi(f_c - f_o + f_m)t.$$

Advantages :=

* It does not require a side band supression filter nor a wide band audio phase shift network.
* Critical parts or adjustments are not required.
* low audio frequencies can be easily transmitted.
* Side bands also be switched easily.

Disadvantages :-

* The system is more complex than other two.
* DC coupling may be needed to avoid loss of signal components close to audio carrier frequency.

Comparison between SSB generation methods.

Sl no	Parameter	Filter method	Phase shift method	Third method
1	Method used	Filter is used to remove the unwanted side band.	Phase shifting technique is used to remove unwanted side band	Similar to phase shift method. But carrier is phase shifted by 90°
2	90° phase shift	not required	Requires complex phase shift network	Phase shift network is simple RC ckt
3	Possible frequency range of SSB	not possible to generate SSB at any frequency	possible to generate SSB at any frequency	possible to generate at any frequency
4	Need for up-conversion	required	not required	not required
5	Complexity	less	medium	high.
6	Design aspects	Q of tuned ckt, filter type, it size, weight and upper frequency limit	Design of 90° phase shifter for entire modulating frequency range	Symmetry of balanced modulator.
7	Bulkiness	Yes	No	No
8	Switching ability	Not possible with existing circuit Extra filter and switching network is necessary	Easily possible	Easily possible. But extra crystal is required.

Demodulation of SSB waves :=

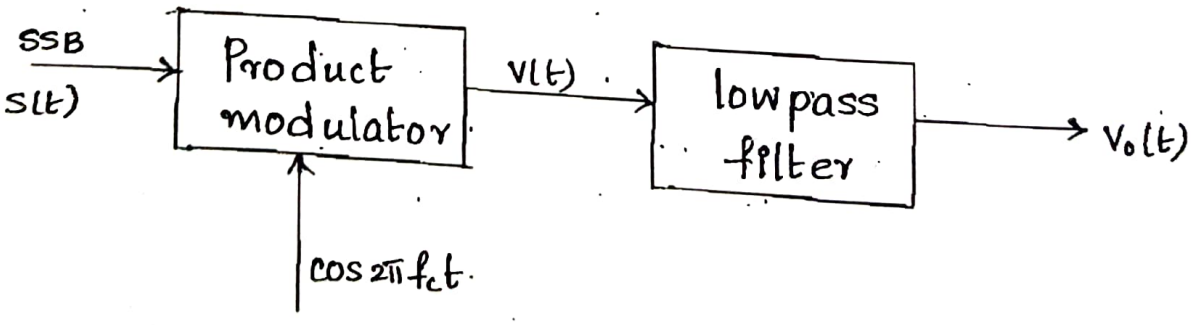
There are two types of demodulation techniques

* Coherent detection method
* Envelope detector method.

Coherent detection method :=

We can recover the message signal $m(t)$ by coherent detection method by following procedure

* Consider SSB wave either $s_u(t)$ or $s_l(t)$.
* Apply it to product modulator.
* Apply locally generated carrier $\cos 2\pi f_c t$ as second input to product modulator
* Apply out put to lowpass filter. The out put of low pass filter is recovered signal $m(t)$.



The output of product modulator is $v(t)$ which is product of SSB and carrier wave.

$$s(t) = \frac{A_c}{2}\left(m(t)\cos 2\pi f_c t \pm \hat{m}(t)\sin 2\pi f_c t\right).$$

$V(t) = s(t) \cos 2\pi f_c t$

$$V(t) = \frac{A_c}{2} \cos 2\pi f_c t \left(m(t) \cos 2\pi f_c t \pm \hat{m}(t) \sin 2\pi f_c t \right)$$

$$= \frac{A_c}{2} m(t) \cos^2 2\pi f_c t \pm \frac{A_c}{2} \hat{m}(t) \cos 2\pi f_c t \sin 2\pi f_c t$$

$$= \frac{A_c}{2} m(t) \left(\frac{1 + \cos 4\pi f_c t}{2} \right) \pm \frac{A_c}{2} \hat{m}(t) \left(\frac{\sin 4\pi f_c t}{2} \right)$$

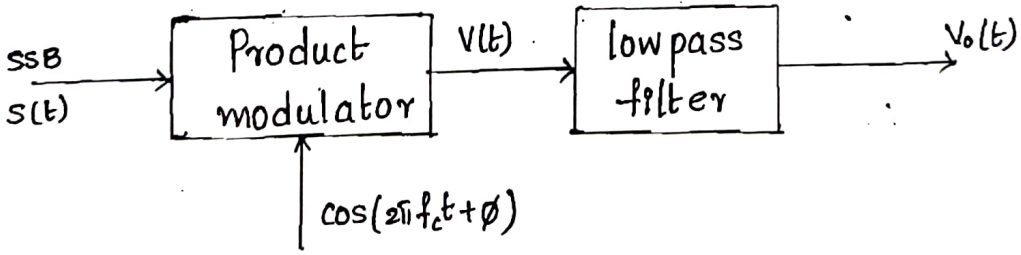
$$= \underbrace{\frac{A_c m(t)}{4}}_{\text{wanted signal}} + \underbrace{\frac{A_c}{4} \left(m(t) \cos 4\pi f_c t \pm \hat{m}(t) \sin 4\pi f_c t \right)}_{\text{unwanted signal}}$$

If $V(t)$ is passed through low pass filter then we get

$$V_0(t) = \frac{A_c m(t)}{4}$$

This is message signal with scaling factor $A_c/4$.

But in practise there in phase error ϕ in locally generated carrier wave.



Now $s(t) = \frac{A_c}{2} \left(m(t) \cos 2\pi f_c t \pm \hat{m}(t) \sin 2\pi f_c t \right)$

$$V(t) = s(t) \cos(2\pi f_c t + \phi)$$

$$= \frac{A_c}{2} \left(m(t) \cos 2\pi f_c t \cos(2\pi f_c t + \phi) \pm \hat{m}(t) \sin 2\pi f_c t \cdot \cos(2\pi f_c t + \phi) \right)$$

$$= \frac{A_c}{4}\left[m(t)\left(\cos(4\pi f_c t+\phi)+\cos\phi\right) \pm \hat{m}(t)\left(\sin(4\pi f_c t+\phi)+\sin\phi\right)\right].$$

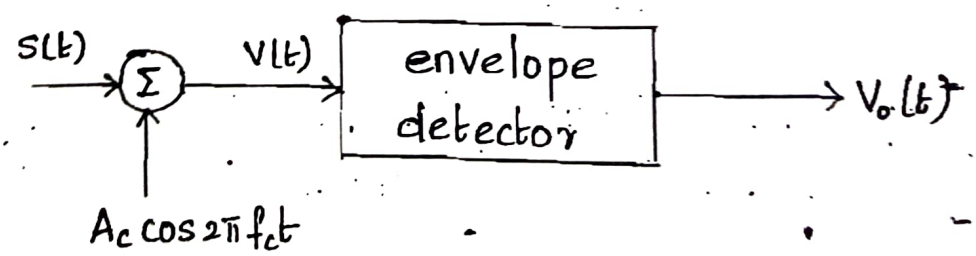
$$= \frac{A_c}{4}\left[m(t)\cos\phi \pm \hat{m}(t)\sin\phi\right] + \frac{A_c}{4}\left(m(t)\cos(4\pi f_c t+\phi) \pm \hat{m}(t)\sin(4\pi f_c t+\phi)\right]$$

$\therefore$ the desired signal

$$V_o(t) = \frac{A_c}{4}\left[m(t)\cos\phi \pm \hat{m}(t)\sin\phi\right].$$

Envelope detector method :=

In this method the SSB signal is demodulated by adding local carrier to it at the reciever and then passing the resultant wave through an envelope detector.



Consider SSB-SC with upper side band as

$$s(t) = \left(m(t)\cos 2\pi f_c t - \hat{m}(t)\sin 2\pi f_c t\right)$$

The local carrier is given by

$$c(t) = A_c \cos 2\pi f_c t$$

The resulting signal is given by

$$V(t) = S(t) + A_c \cos 2\pi f_c t$$

$$= (m(t) \cos 2\pi f_c t - \hat{m}(t) \sin 2\pi f_c t) + A_c \cos 2\pi f_c t$$

$$= \cos 2\pi f_c t \,(m(t) + A_c) - \hat{m}(t) \sin 2\pi f_c t.$$

The output of envelope detector is given by

$$V_o(t) = \sqrt{(\text{inphase component})^2 + (\text{Out of phase component})^2}$$

$$= \sqrt{(A_c + m(t))^2 + (\hat{m}(t))^2}$$

$$= \sqrt{A_c^2 + m^2(t) + 2A_c m(t) + \hat{m}^2(t)}$$

Assuming that $m^2(t) << A_c \Rightarrow \hat{m}^2(t) << A_c$

$$V_o(t) = \sqrt{A_c^2 + 2A_c m(t)}.$$

$$= \sqrt{A_c^2 \left(1 + \frac{2m(t)}{A_c}\right)}$$

$$= A_c \sqrt{1 + \frac{2m(t)}{A_c}}.$$

Since $|m(t)| << A_c$

$$V_o(t) = A_c \left(1 + \frac{2m(t)}{2A_c}\right) \qquad \left[\because (1+x)^{1/2} = 1 + \frac{1}{2}x\right]$$

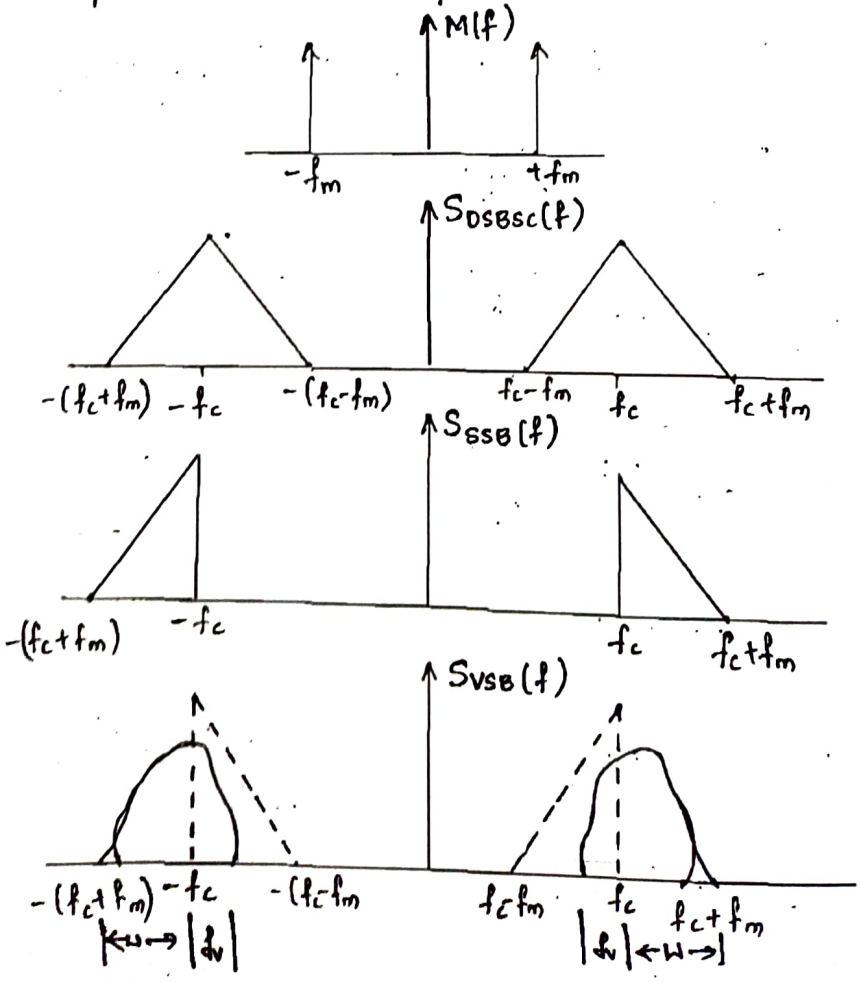
$$V_o(t) = A_c + m(t).$$

In this method local carrier frequency should match with original carrier. If not distortion occurs.

Vestigial side band modulation:=

When the message signal contains extremely low frequencies SSB modulation is not appropriate. Because the upper and lower side bands meet at carrier frequency and it becomes difficult to isolate one the side band. To over come this (VSB) Vestigidal side band modulation is used. In this modulation one side band is passed completely along with a trace or vestige of other side band. This technique is mostly used in television transmission.

Frequency domain description:=

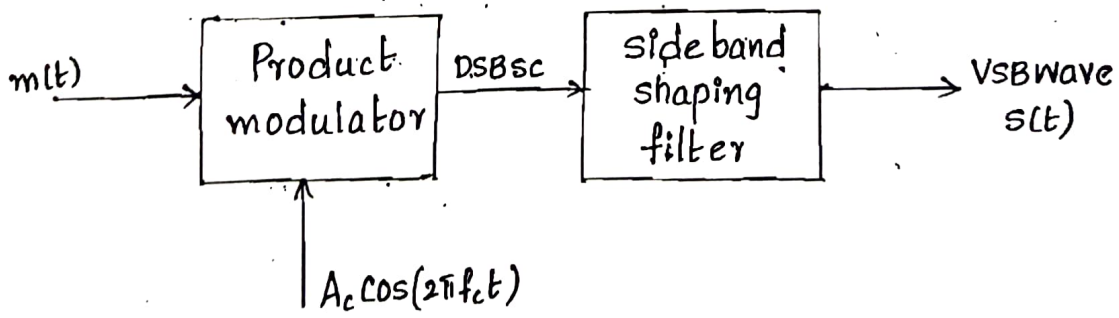


Here the lower side band is modified into vestigial side band. The transmitted vestige of LSB compensate for the removed part of USB. The transmission band width required by VSB modulated wave is given by band width = $W + f_v$.

Where W is band width of message signal and f_v is band width of VSB. The VSB requires band width almost equal to SSB transmission. It retains the excellent low frequency base band characteristics of double side band modulation.

Generation of VSB modulated wave:-

We can generate the VSB modulated wave by passing DSBSC modulated wave through a side band shaping filter. The filter is designed to provide desired spectrum of VSB modulated wave.



The relation between the transfer function H(f) of the filter and spectrum S(f) of VSB modulated wave s(t) defined by

$$S(f) = \frac{A_c}{2}\left[M(f-f_c) + M(f+f_c)\right] H(f).$$

Demodulation of VSB modulated wave:=

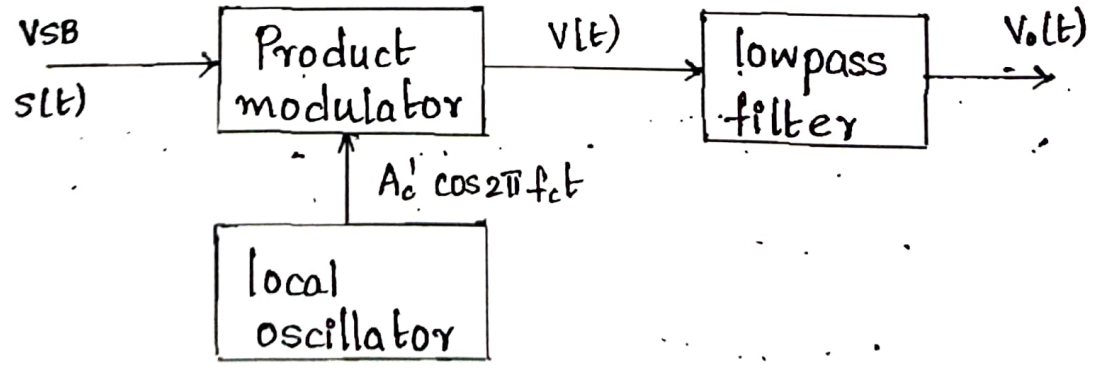
There are two methods of demodulation

* Coherent detection
* Envelope detection.

Coherent detection:=

In this method the VSB wave s(t) is passed through a coherent detector and determining the necessary condition for detector output to provide undistorted message signal m(t). In this s(t) is multiplied with locally generated wave $A_c' \cos 2\pi f_c t$ which is synchronous with carrier wave in both frequency and phase.

$$\therefore V(t) = s(t) \cdot A_c' \cos 2\pi f_c t.$$



$$V(t) = s(t)\, A_c' \cos 2\pi f_c t$$

converting it in to frequency domain

$$V(f) = \frac{A_c'}{2}\left(S(f-f_c) + S(f+f_c)\right)$$

We have $S(f) = \frac{A_c}{2}\left(M(f-f_c) + M(f+f_c)\right) H(f)$

Substitute $s(f)$ in $V(f)$.

$$V(f) = \frac{A_c A_c'}{4}\Big[M(f-f_c-f_c) + M(f-f_c+f_c) + M(f+f_c-f_c) + M(f+f_c+f_c)\Big]\left(H(f-f_c) + H(f+f_c)\right).$$

$$= \frac{A_c A_c'}{4}\Big[M(f-2f_c) + 2M(f) + M(f+2f_c)\Big]\left(H(f-f_c) + H(f+f_c)\right).$$

By passing it through a low pass filter

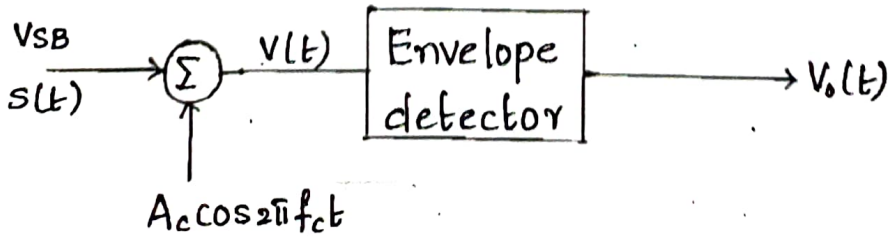
$$V_0(f) = \frac{A_c A_c'}{2} M(f)\left[H(f-f_c) + H(f+f_c)\right]$$

Condition for getting original message signal with a scaling factor and with out any distortion is given as $H(f-f_c) + H(f+f_c) = 2H(f)$.

So $V_0(f) = \frac{A_c A_c'}{2} M(f) \cdot 2H(f)$.

$$V_0(f) = A_c A_c' M(f) H(f).$$

Envelope detection method :=



To make demodulation of VSB wave an envelope detector is used at the receiving end and it is necessary to transmit a carrier along with the modulated wave. The scaled expression of VSB wave by factor k_a with carrier component $A_c \cos 2\pi f_c t$ can be given by

$$V(t) = s(t) + A_c \cos 2\pi f_c t$$

$$= A_c \cos 2\pi f_c t + \frac{A_c}{2} k_a m(t) \cos 2\pi f_c t - \frac{A_c}{2} k_a m_q(t) \sin 2\pi f_c t$$

$$= A_c \left(1 + \frac{k_a m(t)}{2}\right) \cos 2\pi f_c t - \frac{A_c}{2} k_a m_q(t) \sin 2\pi f_c t$$

$$V_o(t) = \sqrt{(\text{inphase component})^2 + (\text{out of phase component})^2}$$

$$= \sqrt{\left(A_c \left(1 + \frac{k_a}{2} m(t)\right)\right)^2 + \left(\frac{A_c}{2} k_a m_q(t)\right)^2}$$

$$= \sqrt{A_c^2 \left(1 + \frac{k_a}{2} m(t)\right)^2 + A_c^2 \left(\frac{k_a m_q(t)}{2}\right)^2}$$

$$= A_c \left(1 + \frac{k_a}{2} m(t)\right) \left[1 + \frac{\frac{k_a}{2} m_q(t)}{1 + \frac{k_a}{2} m(t)}\right]^{1/2}$$

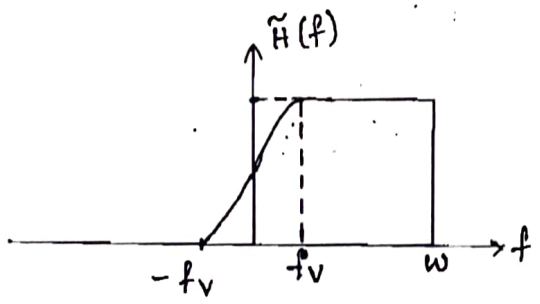
The detector output is distorted by quadrature component $m_q(t)$. To reduce the distortion we can decrease the modulation index k_a or we can decrease the quadrature phase component $m_q(t)$.

Time domain description :=

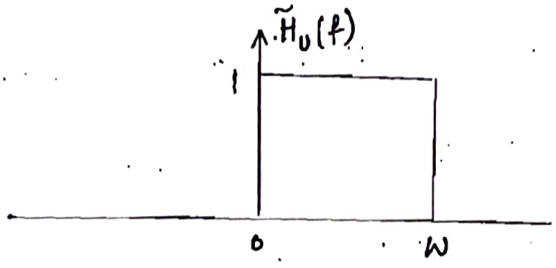
let s(t) be VSB modulated wave containing a vestige of lower side band, and it is an output of side band shaping filter in response to DSBSC modulated wave

If H(f) is transfer function of side band shaping filter then $\tilde{H}(f)$ can be given as

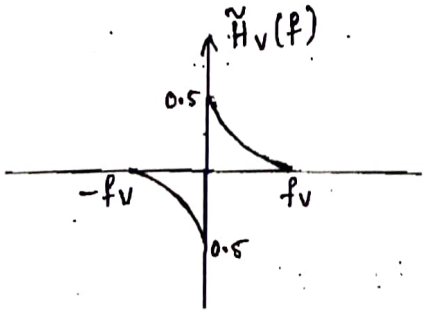
$$\tilde{H}(f) = \tilde{H}_u(f) - \tilde{H}_v(f).$$



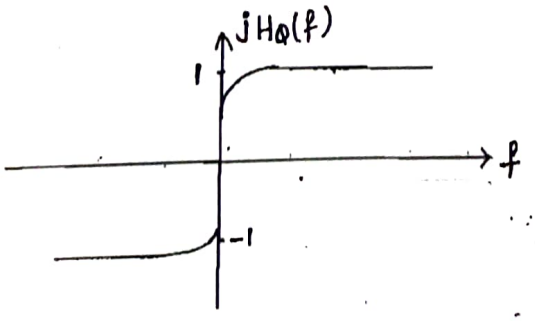
Response of side band shaping filter



first component. $\tilde{H}_u(f)$



second component $\tilde{H}_v(f)$



Response of $jH_Q(f)$

Where

(i) $\tilde{H}_0(f)$ is for complex low pass filter which is equal to the band pass filter designed to reject lower side band

(ii) $\tilde{H}_v(f)$ is for both generation of vestige of lower side band and for removal of some portion in upper side band.

We know that $\tilde{H}_0(f) = \frac{1}{2}(1 + sgn(f)) \quad 0 < f < \omega$

$= 0 \quad$ elsewhere.

$\therefore \tilde{H}(f) = \frac{1}{2}(1 + sgn(f)) - \tilde{H}_v(f) \quad f_v < f < \omega$

$= 0 \quad$ else where.

As signum function and $\tilde{H}_v(f)$ are odd functions, they contain imaginary values inverse fourier transforms There exist a new expression

$$H_Q(f) = \frac{1}{j}\left(sgn(f) - 2\tilde{H}_v(f)\right).$$

$\tilde{H}(f)$ can be written as

$\tilde{H}(f) = \frac{1}{2}(1 + j H_Q(f)) \quad f_v < f < \omega$

$= 0 \quad$, else where.

The VSB modulated wave $s(t)$ is given as

$$s(t) = Re\left(\tilde{s}(t)\, e^{j2\pi f_c t}\right).$$

$$\tilde{S}(f) = \tilde{H}(f)\, \tilde{S}_{DSBSC}(f)$$

$\tilde{S}_{DSBSC}(f) = A_c M(f)$

$\tilde{S}(f) = \frac{1}{2} A_c \left(1 + j H_Q(f)\right) M(f)$.

Converting it in to time domain

$\tilde{s}(t) = \frac{1}{2} A_c \left(m(t) + j m_Q(t)\right)$

$s(t) = Re\left(\tilde{s}(t) e^{j2\pi f_c t}\right)$.

$= Re\left(\frac{1}{2} A_c (m(t) + j m_Q(t))\right) \cdot \left(\cos 2\pi f_c t + j \sin 2\pi f_c t\right)$

$= Re\left(\frac{A_c}{2} m(t) \cos 2\pi f_c t + j \frac{A_c}{2} m(t) \sin 2\pi f_c t + j \frac{A_c}{2} m_Q(t) \cos 2\pi f_c t - \frac{A_c}{2} m_Q(t) \sin 2\pi f_c t\right)$.

$= \frac{A_c}{2} m(t) \cos 2\pi f_c t - \frac{A_c}{2} m_Q(t) \sin 2\pi f_c t$.

$\therefore s(t) = \frac{A_c}{2}\left(m(t) \cos 2\pi f_c t - m_Q(t) \sin 2\pi f_c t\right)$.

This is expression for USB with a vestige of LSB.

Similarly the expression for LSB with a vestige of USB is given as

$s(t) = \frac{A_c}{2}\left(m(t) \cos 2\pi f_c t + m_Q(t) \sin 2\pi f_c t\right)$.

Advantages of VSB :=

* Low frequencies near f_c are transmitted with out any attenuation
* Band width is reduced compared to DSB.
* VSB signals are relatively easy to generate.
* VSB inherits the advantages of DSB and SSB but avoid their disadvantages at a small cost.

Comparison of amplitude modulation techniques:=

Sl no	Parameter	standard AM	SSB	DSBSC	VSB
1	Power	High	less	medium	less than DSBSC but greater than SSB
2	Bandwidth	$2f_m$	f_m	$2f_m$	$f_m < BW < 2f_m$
3	Carrier Supression	NO	Yes	Yes	NO
4	Receiver Complexity	simple	complex	complex	simple.
5	Modulation type	Non linear	linear	linear	linear
6	Sideband Supression	NO	One sided Completely	NO	One sideband supressed partly
7	Transmission efficiency	minimum	maximum	moderate	moderate
8	Application	Radio communication	Point to point communication Preffered for long distances	Point to point communication	Television broad casting

Angle Modulation

Angle modulation is defined as the process in which the total phase angle of a carrier wave is varied in accordance with the instantaneous value of the modulating signal while keeping the amplitude of the carrier constant.

Mathematical Representation:-

An unmodulated carrier signal is expressed as

$$c(t) = A\cos(\omega_c t + \theta_0)$$

where A = Amplitude of carrier
ω_c = Carrier frequency
θ_0 = phase angle.

Substituting $\omega_c t + \theta_0 = \phi$,

$$c(t) = A\cos\phi$$

The total phase angle $\phi = \omega_c t + \theta_0$

Differentiating on both sides w.r.t 't'

$$d\phi = \omega_c dt$$

$$\omega_c = \frac{d\phi}{dt}$$ where $\frac{d\phi}{dt}$ may vary with time.

The time dependent angular velocity or angular frequency is known as instantaneous angular velocity or instantaneous angular frequency.

$$\frac{d\phi}{dt} = \omega_i \Rightarrow \phi = \int \omega_i$$

If the angle ϕ is varied according to the instantaneous value of the message signal, the carrier signal is then said to be angle modulated.

Angle Modulation

Frequency Modulation (FM)

Frequency of the carrier is varied according to the message signal.

Phase Modulation (PM)

Phase angle of the carrier is varied according to the message signal.

Applications :-

(i) Radio broadcasting
(ii) Two way mobile radio
(iii) Microwave communication
(iv) TV sound transmission
(v) Cellular radio
(vi) Satellite communication

Phase Modulation :-

Phase Modulation (PM) is that type of angle modulation in which the phase angle ϕ is varied lineaely with a baseband or modulating signal $x(t)$ about an unmodulated phase angle $\omega_c t + \theta_0$.

In Phase Modulation, the instantaneous value of the phase angle is equal to the phase angle of the unmodulated carrier $(\omega_c t + \theta_0)$ plus a time-varying component which is proportional to modulating signal $x(t)$.

The unmodulated carrier signal is given as

$$c(t) = A\cos(\omega_c t + \theta_0)$$

$$c(t) = A\cos\phi$$

where $\phi = \omega_c t + \theta_0$

Neglecting θ_0, $\phi = \omega_c t$

According to phase Modulation, this phase angle ϕ is varied linearly with a baseband or modulating signal $x(t)$. Let the instantaneous value of phase angle be denoted by ϕ_i. Therefore,

$$\phi_i = \omega_c t + k_p . x(t)$$

where k_p is the proportionality constant & is known as phase sensitivity of the modulator

The expression for phase modulated wave will be

$$s(t) = A\cos\phi_i$$

$$\boxed{s(t) = A\cos[\omega_c t + k_p \cdot x(t)]}$$

Frequency Modulation:-

Frequency Modulation is that type of angle modulation in which the instantaneous frequency ω_i is varied linearly with a message or baseband signal $x(t)$ about an unmodulated carrier frequency ω_c.

This means that the instantaneous value of the angular frequency ω_i will be equal to the carrier frequency ω_c plus a time-varying component proportional to the baseband signal $x(t)$.

The instantaneous frequency is given by

$$\omega_i = \omega_c + k_f \cdot x(t)$$

where k_f is proportionality constant & is known as frequency sensitivity of the modulator.

The expression for instantaneous phase angle ϕ_i is

$$\phi_i = \int \omega_i \, dt.$$

$$= \int [\omega_c + k_f x(t)] \, dt.$$

$$= \omega_c t + k_f \int x(t) \, dt.$$

The expression for frequency modulated wave is

$$s(t) = A\cos \phi_i$$

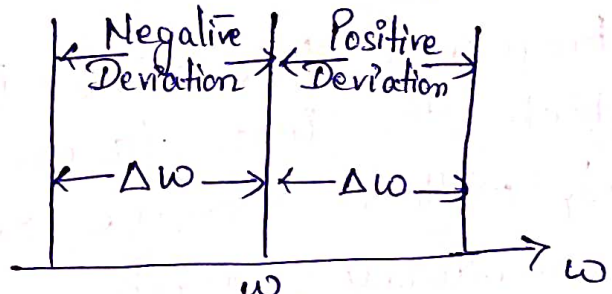
$$= A\cos [\omega_c t + k_f \int x(t) \, dt].$$

$$\boxed{s(t) = A \cos \left[\omega_c t + k_f \int_0^t x(t) \, dt\right]}$$

Frequency Deviation:-

The maximum change in instantaneous frequency from the average frequency ω_c is called frequency deviation.

$$\boxed{\Delta\omega = |k_f \cdot x(t)|_{max}}$$



Relationship between Phase Modulation (PM) and Frequency Modulation:-

The expression for phase modulated (PM) wave is

$$s(t) = A \cos [\omega_c t + k_p x(t)].$$

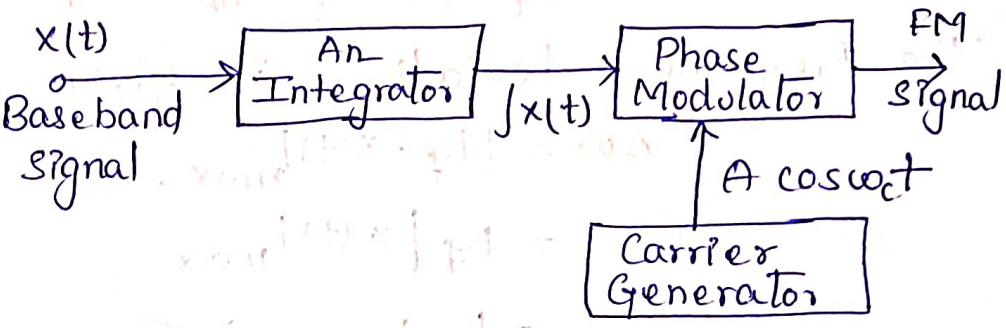
The expression for frequency modulated (PM) wave is

$$s(t) = A \cos \left[\omega_c t + k_f \int_0^t x(t) \, dt\right].$$

The above two equations are closely related to each other because in both the cases there is a variation in the total phase angle.

FM using PM :-

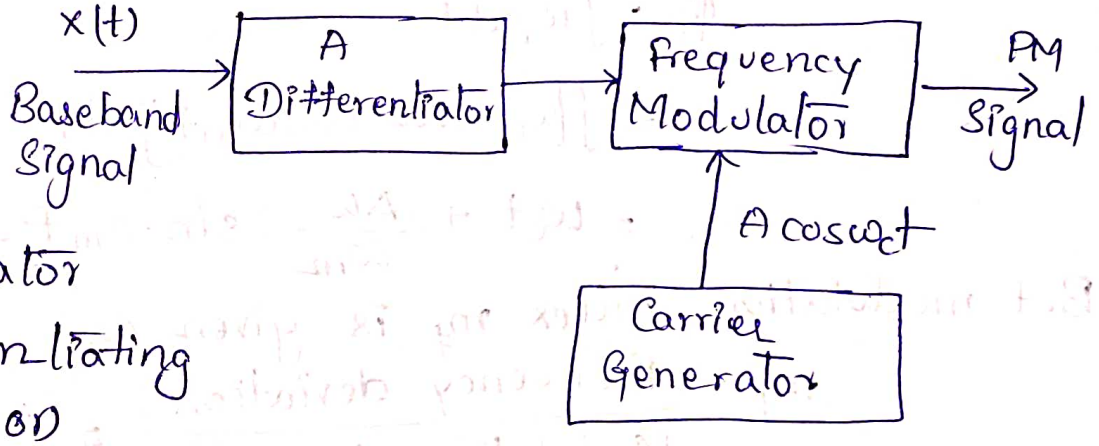
To get FM using PM, first integrate the baseband signal & then apply to the phase Modulator as shown in fig.



Generation of FM using Phase Modulator.

PM using FM :-

A PM wave can be generated by using frequency modulator by first differentiating the modulating (or) baseband signal x(t) & then applying to the frequency modulator as shown in fig.



Generation of PM using Frequency Modulator

Mathematical Expression for Single Tone Frequency Modulation

Let the expression for a carrier signal be

$$c(t) = A \cos \omega_c t.$$

Let the modulating signal be

$$x(t) = V_m \cos \omega_m t.$$

The instantaneous frequency of the modulated signal is given as

$$\omega_i = \omega_c + k_f \cdot x(t)$$

$$\omega_i = \omega_c + k_f \cdot V_m \cos\omega_m t \rightarrow (1) \quad [\because x(t) = V_m \cos\omega_m t]$$

The Frequency deviation is given as

$$\Delta\omega = |k_f \cdot x(t)|_{max}.$$

$$= k_f |x(t)|_{max}$$

$$= k_f \cdot V_m \rightarrow (2)$$

Substituting (2) in (1).

$$\omega_i = \omega_c + \Delta\omega \cdot \cos\omega_m t.$$

The total phase angle of modulated wave is

$$\phi_i = \int \omega_i \, dt.$$

$$= \int [\omega_c + \Delta\omega \cos\omega_m t] \, dt.$$

$$= \omega_c t + \frac{\Delta\omega}{\omega_m} \cdot \sin\omega_m t \rightarrow (3)$$

But modulation index m_f is given as

$$m_f = \frac{\text{Frequency deviation}}{\text{Modulating frequency}} = \frac{\Delta\omega}{\omega_m} \rightarrow (4)$$

Substituting (4) in (3) $\quad \phi_i = \omega_c t + m_f \cdot \sin\omega_m t.$

The expression of FM wave is given as

$$s(t) = A\cos\phi_i.$$

$$\boxed{s(t) = A\cos[\omega_c t + m_f \sin\omega_m t]}$$

↳ This is the required mathematical expression for single tone FM wave.

Problems :-

(1) A single tone FM is represented by the voltage equation as $v(t) = 12\cos(6\times10^8 t + 5\sin 1250t)$

Determine the following

(i) Carrier frequency (iii) the modulation index

(ii) Modulating frequency. (iv) Maximum deviation

(v) What power will this FM wave dissipate in 10Ω resistors.

Sol:- $V(t) = 12\cos(6\times10^8 t + 5\sin 1250t)$.

$v(t) = A\cos(\omega_c t + m_f \sin\omega_m t)$.

(i) Carrier frequency

$\omega_c = 6\times10^8 \Rightarrow f_c = \frac{6\times10^8}{2\pi} = 95.5\,MHz$

(ii) Modulating frequency

$\omega_m = 1250 \Rightarrow f_m = \frac{1250}{2\pi} = 199\,Hz$.

(iii) Modulation index $m_f = 5$.

(iv) Max. deviation $m_f = \frac{\Delta\omega}{\omega_m} = \frac{\Delta f}{f_m} \Rightarrow \Delta f = m_f \cdot f_m$

$= 5\times199 = 995\,Hz$

(v) Power dissipated $= P = \frac{V_{rms}^2}{R} = \frac{(12/\sqrt{2})^2}{10} = \frac{72}{10} = 7.2W$.

(2) What is the modulation index of an FM signal having a carrier swing of 100 kHz when the modulating signal has a frequency of 8 kHz.

Sol:- Carrier swing $= 2\times\Delta f \Rightarrow \Delta f = \frac{100}{2} = 50\,kHz$.

modulation index $m_f = \frac{\Delta f}{f_m} = \frac{50}{8} = 6.25$.

(3) A 107.6 MHz carrier signal is frequency modulated by a 7 kHz sine wave. The resultant FM signal has a frequency deviation of 50 kHz. Determine the following

(i) The carrier swing of the FM signal.

(ii) The highest & lowest frequencies attained by the modulated signal.

(iii) The Modulation index of FM wave.

Sol:- $f_c = 107.6\,MHz$ $f_m = 7\,kHz$. $\Delta f = 50\,kHz$.

(i) Carrier swing $= 2 \times \Delta f = 100\,kHz$.

(ii) $f_H = f_c + \Delta f$ $f_L = f_c - \Delta f$.

$= 107.65\,MHz$ $= 107.55\,MHz$.

(iii) Modulation index $m_f = \frac{\Delta f}{f_m} = \frac{50 \times 10^3}{7 \times 10^3} = 7.143$.

(4) Determine the frequency deviation & carrier swing for a frequency modulated (FM) signal which has a resting frequency of 105.00 MHz & whose upper frequency is 105.007 MHz when modulated by a particular wave. Find the lowest frequency reached by the FM wave.

Sol:- $f_c = 105.00\,MHz$ $f_H = 105.007$.

$f_H = f_c + \Delta f$

$\Delta f = f_H - f_c$

$= 0.007\,MHz$

$= 7\,kHz$.

Carrier swing $= 2 \times \Delta f$

$= 14\,kHz$

$f_L = f_c - \Delta f$

$= 105 - 0.007$

$= 104.993\,MHz$

Types of Frequency Modulation (FM) :-

Depending upon the value of frequency sensitivity k_f, FM can be divided as

(i) Narrow Band FM : In this case, k_f is small & hence the bandwidth of FM is narrow.

(ii) Wideband FM :- In this case, k_f is large & hence the FM signal has a wide bandwidth

Narrow Band FM :—

It is used in voice communication & radio broadcasting

It is used to improve spectrum efficiency.

Modulating index for NBFM is less than 1. $m_f << 1$.

The Frequency modulated wave is given as

$$s(t) = A\cos(\omega_c t + m_f \sin\omega_m t)$$

$$s(t) = A\left[\cos\omega_c t \cos(m_f \sin\omega_m t) - \sin\omega_c t \sin(m_f \sin\omega_m t)\right]$$

If m_f is very small, $\cos(m_f \sin\omega_m t) \cong 1$

$$\sin(m_f \sin\omega_m t) \cong m \sin\omega_m t.$$

$$s(t) = A\left[\cos\omega_c t - m_f \sin\omega_m t \sin\omega_c t\right].$$

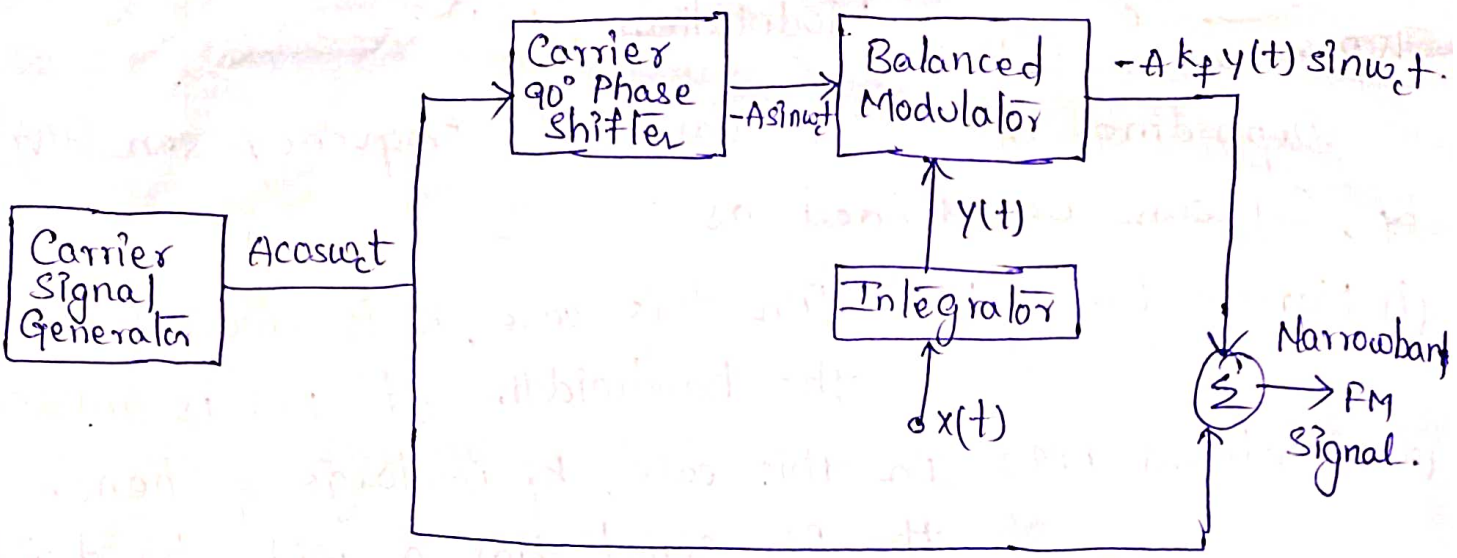
$$\boxed{s(t) = A\left[\cos\omega_c t - m_f\, y(t) \sin\omega_c t\right] \quad \text{where } y(t) = \int x(t)}$$

$$\rightarrow s(t) = A\left[\cos\omega_c t - \frac{\Delta\omega}{\omega_m}\sin\omega_m t \sin\omega_c t\right] \qquad \left[m_f = \frac{\Delta\omega}{\omega_m}\right.$$

$$= A\left[\cos\omega_c t - \frac{k_f (x(t))_{max}}{\omega_m}\sin\omega_m t \cdot \sin\omega_c t\right] \qquad \Delta\omega = k_f x(t)$$

$$= A\left[\cos\omega_c t - \frac{k_f V_m}{\omega_m}\sin\omega_m t \cdot \sin\omega_c t\right] \qquad x(t) = V_m \cos\omega_m t$$

$$\boxed{s(t) = A\left[\cos\omega_c t - k_f\, y(t) \sin\omega_c t\right]} \qquad y(t) = \int x(t) = \frac{V_m}{\omega_m}\sin\omega_m t$$



Wide Band Frequency Modulation :-

When the value of modulation index 'm_f' is very large, then the FM signal has a wide bandwidth.

$$m_f >> 1$$

Consider a single tone FM wave,

$$s(t) = A_c \cos(\omega_c t + m_f \sin\omega_m t) \longrightarrow (1)$$

This wave is produced by a sinusoidal modulating wave $x(t)$ which is a periodic function.

It can be obtained by multiplying the NBFM signal with suitable frequency multipliers.

$$s_{FM}(t) = A_c e^{j(\omega_c t + \beta \sin\omega_m t)} \quad \text{where } \beta = m_f = \text{modulating index.}$$

$$= Re\left[A_c e^{j(\omega_c t + \beta \sin\omega_m t)}\right].$$

$$= Re\left[A_c e^{j\omega_c t} \cdot e^{j\beta \sin\omega_m t}\right].$$

$$= Re\left[\tilde{s}(t)\, e^{j\omega_c t}\right] \longrightarrow (2)$$

where $\tilde{s}(t) = A_c e^{j\beta \sin\omega_m t} \longrightarrow (3)$

↳ Complex envelope of the FM signal.

↳ periodic function of time with 'f_m'

To expand the $\tilde{s}(t)$ using fourier series

$$\tilde{s}(t) = \sum_{n=-\infty}^{\infty} c_n \exp(+jn\omega_m t) \longrightarrow (4)$$

where

$$c_n = f_m \int_{-1/2 f_m}^{1/2 f_m} \tilde{s}(t)\, e^{-jn\omega_m t}\, dt \longrightarrow (5)$$

Substitute eq. (3) in (5).

$$c_n = f_m \int_{-1/2 f_m}^{1/2 f_m} A_c\, e^{j\beta \sin n\omega_m t}\, e^{-jn\omega_m t}\, dt.$$

$$c_n = \frac{A_c}{2\pi} \int_{-\pi}^{+\pi} e^{[j\beta \sin nx - jnx]}\, dx$$

Let
$x = \omega_m t$
$= 2\pi f_m t.$
$\frac{dx}{dt} = 2\pi f_m$
$dx = 2\pi f_m dt$
$t = \infty$

$$\boxed{c_n = A_c J_n(\beta)} \longrightarrow (6).$$

where

$$J_n(\beta) = \frac{1}{2\pi} \int_{-\pi}^{+\pi} e^{[j\beta \sin nx - jnx]}\, dx.$$

$J_n(\beta)$ is n^{th} order Bessel function of the first kind.

Substitute (6) in (4).

$$\tilde{s}(t) = \sum_{n=-\infty}^{\infty} A_c J_n(\beta) \exp(j\omega_m n t) \longrightarrow (7).$$

To get wideband FM wave, substitute (7) in (2).

$$s_{FM}(t) = Re[\tilde{s}(t) e^{j\omega_c t}]$$

$$= Re\left[\sum_{n=-\infty}^{\infty} A_c J_n(\beta)\, e^{jn\omega_m t}\, e^{j\omega_c t}\right]$$

$$= Re\left[\sum_{n=-\infty}^{\infty} A_c J_n(\beta)\, e^{j[\omega_c t + n\omega_m t]}\right].$$

$$\boxed{s_{WBFM}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(\omega_c t + n\omega_m t)}$$

Fourier series time-domain representation of WBFM signal is

$$s_{WBFM}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(2\pi (f_c + nf_m)t)$$

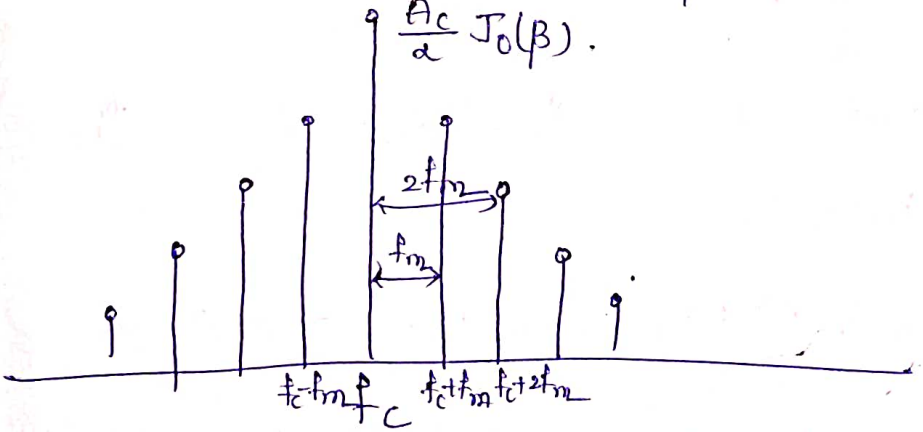
Frequency domain Representation :-

Fourier transform of $s_{WBFM}(t)$ is given as

$$S_{WB}(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta) [\delta(f - f_c - nf_m) + \delta(f + f_c + nf_m)]$$

WBFM signal contains the carrier with an amplitude $\frac{A_c}{2} J_0(\beta)$. It contains an infinite no. of sidebands spaced symmetrically on either sides of the carrier with $\pm f_m, \pm 2f_m, \pm 3f_m \ldots$.

As modulation index increases & no. of sidebands increases



Advantages :-

1. Noise immunity is improved.
2. All the transmitted power is useful.
3. Covers a large area.
4. Adjacent channel interference is avoided due to guard bands.

Disadvantages:-

1. Very large bandwidth is required.
2. Equipments are complex.

Comparision of Wideband and Narrowband FM:-

S.NO	Parameter/ Characteristics	Wideband FM	Narrowband FM
1.	Modulation Index	Greater than 1	Less than 1
2.	Maximum deviation	75 kHz	5kHz
3.	Range of modulating frequency	30 Hz to 15kHz	30Hz to 3kHz
4.	~~Maximum~~ Bandwidth	Large, about 15 times higher & same as that of AM	Small, approximately.
5.	Applications	Entertainment broadcasting (can be used for high quality music transmission)	FM Mobile communication like police wireless, ambulance etc (This is used for speech transmission)

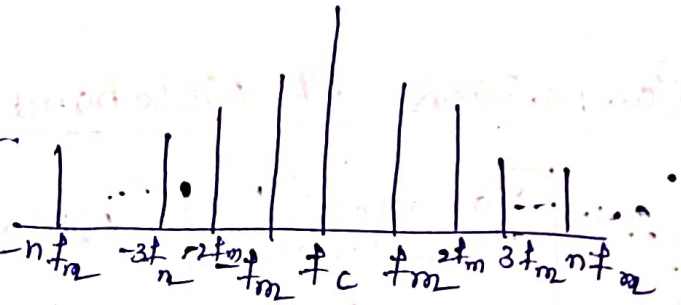
Transmission Bandwidth of FM Signal:-

Let us consider that in the spectrum of FM wave the no. of significant sidebands is n. Since the upper sidebands are separated by ω_m, therefore, they

form a frequency band of $n\omega_m$. A similar frequency band of $n\omega_m$ is formed by the lower sidebands. Thus, for a sidebands the bandwidth of FM wave is given by

$$B.W = 2n\omega_m \text{ radians/sec.}$$

$$B.W = 2nf_m \text{ Hz}$$



Carson's Rule or Practical Bandwidth :-

According to this rule, the FM bandwidth is given as twice the sum of the frequency deviation & the highest modulating frequency.

Mathematically

$$BW = 2(\Delta\omega + \omega_m)$$

$$\text{But } m_f = \frac{\Delta\omega}{\omega_m} \Rightarrow \Delta\omega = m_f\omega_m$$

$$BW = 2(m_f\omega_m + \omega_m)$$

$$\boxed{B.W = 2\omega_m(1+m_f)}$$

Case(i) If $\Delta\omega << \omega_m$ i.e $m_f << 1$ i.e. in case of narrowband FM

$$B.W = 2\omega_m(1+m_f)$$

$$\boxed{B.W = 2\omega_m}$$

Case(ii) If $\Delta\omega >> \omega_m$ i.e $m_f >> 1$ i.e in case of wideband FM

$$B.W = 2\omega_m(1+m_f)$$

$$= 2\omega_m m_f$$

$$\boxed{B.W = 2\Delta\omega}$$

Problems :-

(1) Find the bandwidth of a commercial FM transmission if frequency deviation $\Delta f = \pm 75$ kHz & modulating frequency $f_m = 15$ kHz

Sol :- $B.W = 2(\Delta f + f_m)$

$= 2(75+15) = 2 \times 90$

$= 180$ kHz.

(2) Determine the bandwidth of a narrowband FM signal which is generated by a 4kHz audio signal modulating a 125 MHz carrier.

$f_m = 4$ kHz $\quad f_c = 125$ MHz.

$B.W = 2f_m$

$= 2 \times 4 = 8$ kHz.

(3) The maximum deviation allowed in a FM broadcast system is ± 75 kHz. If the modulating signal is a single-tone sinusoid of 8kHz, determine the bandwidth of FM signal. what will be the bandwidth when modulating signal amplitude is doubled?

$\Delta f = \pm 75$ kHz. $\quad f_m = 8$ kHz.

$B.W = 2(\Delta f + f_m) = 2(75+8) = 166$ kHz.

Modulating signal amplitude doubled, deviation becomes $2 \times 75 = 150$ kHz.

$B.W = 2(\Delta f + f_m) = 2(150+8) = 316$ kHz.

Comparision of FM and PM Systems:-

S.No	FM System	PM System
1.	$s(t) = V_c \sin[\omega_c t + m_f \sin \omega_m t]$	$s(t) = V_c \sin[\omega_c t + m_p \sin \omega_m t]$
2.	Frequency deviation is proportional to modulating voltage.	Phase deviation is proportional to the modulating voltage.
3.	Associated with the change in f_c, there is some phase change.	Associated with the changes in phase, there is some change in f_c.
4.	m_f is proportional to the modulating voltage as well as the modulating frequency f_m.	m_p is proportional only to the modulating voltage.
5.	It is possible to receive FM on a PM receiver.	It is possible to receive PM on a FM receiver.
6.	Noise immunity is better than AM & PM.	Noise immunity is better than AM but worse than FM.
7.	Amplitude of the FM wave is constant.	Amplitude of the PM wave is constant
8.	Signal to Noise ratio is better than that of PM.	Signal to Noise ratio is inferior to that in FM.
9.	FM is widely used.	PM is used in some mobile systems.
10.	In FM, the frequency deviation is proportional to the modulating voltage only.	In PM, the frequency deviation is proportional to both the modulating voltage & modulating frequency.

Expression for frequency deviation for a PM wave:-

The total phase angle of a PM wave is given as

$$\phi_i = \omega_c t + k_p \cdot x(t) \quad \longrightarrow (1)$$

For a single tone modulating signal,

$$x(t) = V_m \cos \omega_m t \quad \longrightarrow (2)$$

Substituting (2) in (1)

$$\phi_i = \omega_c t + k_p V_m \cos \omega_m t.$$

In the above expression, the maximum departure in the phase is $k_p \cdot V_m$. This is called phase deviation denoted by θ_d.

Hence phase deviation, $\theta_d = k_p V_m$.

The expression for the PM wave becomes

$$S(t)_{PM} = A \cos \phi_i$$

$$= A \cos [\omega_c t + \theta_d \cos \omega_m t].$$

The instantaneous frequency is given as

$$\omega_i = \frac{d\phi_i}{dt} = \frac{d}{dt} [\omega_c t + \theta_d \cos \omega_m t].$$

$$= \frac{d}{dt} [\omega_c t + k_p V_m \cos \omega_m t].$$

$$= \omega_c - k_p V_m \omega_m \sin \omega_m t.$$

The maximum departure in frequency from ω_c is $k_p \cdot V_m \omega_m$

Hence, for PM, the frequency deviation is given by

$$\boxed{\Delta \omega_{PM} = k_p \cdot V_m \cdot \omega_m}$$

The frequency deviation for PM, $\Delta \omega_{PM} = k_f \cdot V_m$

For an equal bandwidth in FM & PM

$$k_f = k_p \cdot \omega_m .$$

Transmission Bandwidth of PM :—

Using Carson's rule, the transmission B.W of PM is

$$BW_{PM} = 2(\Delta\omega)$$
$$= 2 k_p v_m \omega_m .$$

For PM, the modulation index m_p will be same as the deviation θ_d.

$$m_p = k_p \cdot v_m = \theta_d .$$

Problems :—

(1) A baseband or modulating signal $x(t) = 5\cos(2\pi 15\times10^3 t)$ angle modulates a carrier signal $A\cos\omega_c t$. Determine the modulation index & bandwidth for

(i) (a) FM system (b) PM system.

(ii) Find the change in the bandwidth & modulation index for both FM & PM if modulating frequency f_m is reduced to 5 kHz.

Assume $k_f = k_p = 15$ kHz/volt.

Sol :— Given $v_m = 5$ $f_m = 15$ kHz.

(a) FM system :—

$$m_f = \frac{\Delta f}{f_m} \Rightarrow \Delta f = k_f \cdot v_m$$
$$= 15 \times 5 = 75 \text{ kHz}$$
$$= \frac{75}{15} = 5.$$

$$B.W = 2(\Delta f + f_m) = 2(75 + 15)$$
$$= 180 \text{ kHz}.$$

(b) PM system

$\Delta f = k_p \cdot V_m \cdot f_m$

$= 15k \times 5 \times 15 \times 10^3$

$= 1125\ MHz.$

$m_p = k_p \cdot V_m$

$= 15 \times 10^3 \times 5$

$= 75,000.$

$B.W = 2\Delta f = 2 \times 1125\ MHz.$

$= 2250\ MHz.$

(ii) Modulating frequency is reduced to 5kHz.

(a) FM System $f_m = 5kHz.$

$m_f = \frac{\Delta f}{f_m} = \frac{75}{5} = 15.$

$B.W = 2(\Delta f + f_m) = 2(75+5) = 2 \times 80 = 160kHz.$

(b) PM System

$m_p = k_p \cdot V_m$

$= 75,000.$

$\Delta f = k_p \cdot V_m \cdot f_m$

$= 15 \times 10^3 \times 5 \times 5 \times 10^3$

$= 375\ MHz.$

$B.W = 2 \cdot \Delta f$

$= 2 \times 375 = 750\ MHz.$

(2) In a PM system, the modulating frequency $f_m = 1kHz$, the modulating voltage $V_m = 2$ volt & the deviation is 6kHz. If the modulating voltage is raised to 4 volt then what will be the new deviation? If the modulating voltage is further increased to 8 volt & modulating frequency is reduced to 500Hz, what will be deviation?

Sol:- Given $f_m = 1kHz$, $V_m = 2$ $\Delta f = 6kHz$.

$$\Delta f = k_f \cdot V_m$$

$$k_f = \frac{\Delta f}{V_m} = \frac{6}{2} = 3\,kHz/V.$$

(a) $V_m = 4V$,

$$\Delta f = k_f \cdot V_m = 3 \times 4 = 12kHz$$

(b) $V_m = 8V$, $f_m = 500Hz$.

$$\Delta f = k_f \cdot V_m = 3 \times 8 = 24kHz.$$

(3) A 20MHz carrier is modulated by a 400Hz modulating signal. The carrier voltage is 5V & the maximum deviation is 10kHz. Write the mathematical expressi for the FM & PM waves. If the modulating frequency is increased to 2kHz keeping everything else constant, write the expressions for the PM & FM waves.

$f_c = 20MHz$ $\omega_c = 2\pi \times 20 \times 10^6 = 1.25 \times 10^8$ rad/sec.

$f_m = 400Hz$ $\omega_m = 2\pi \times 400 = 2513$ rad/sec. $A_c = 5$

$\Delta f = 10kHz$.

$$m_f = \frac{\Delta f}{f_m} = \frac{10k}{400} = 25.$$

Expression for FM wave & PM ware

$$S_{FM}(t) = A_c \cos(\omega_c t + m_f \sin \omega_m t)$$

$$= 5 \cos(1.25 \times 10^8 t + 25 \sin 2513t)$$

$$S_{PM}(t) = A_c \cos[\omega_c t + k_p \cdot x(t)]$$

$$= A_c \cos[\omega_c t + k_p \cdot V_m \cos \omega_m t]$$

$$= A_c \cos(\omega_c t + m_p \cos \omega_m t).$$

$$= 5 \cos(1.25 \times 10^8 t + 25 \cos 2513t].$$

(b) Modulating frequency is changed to $f_m = 2kHz$.

$$\omega_m = 2\pi \times 2 \times 1000 = 12566.3 \text{ rad/s}$$

$$m_f = \frac{\Delta f}{f_m} = \frac{10,000}{2,000} = 5.$$

But $m_p = 25$.

Expression of FM & PM wave.

$$s_{FM}(t) = A_c \cos(\omega_c t + m_f \sin \omega_m t)$$

$$= 5 \cos(1.25 \times 10^8 t + 5 \sin 12566.3t)$$

$$s_{PM}(t) = A_c \cos(\omega_c t + m_p \cos \omega_m t).$$

$$= 5 \cos(1.25 \times 10^8 t + 25 \cos 12566.3t)$$

(4) A carrier wave of amplitude 10V & frequency 100 MHz is frequency modulated by a sinusoidal voltage. The modulating voltage has an amplitude of 5V & frequency $f_m = 20kHz$. The frequency deviation constant is 2kHz/volt. Draw the frequency spectrum of FM wave.

[$J_0 = 0.94$ $J_1 = 0.24$ $J_2 = 0.03$].

Sol:- $A_c = 10V$ $f_c = 100MHz$. $f_m = 20kHz$. $\Delta f = 2kHz/volt$.

$$m_f = \frac{\Delta f}{f_m} = \frac{2}{20} = 0.5.$$

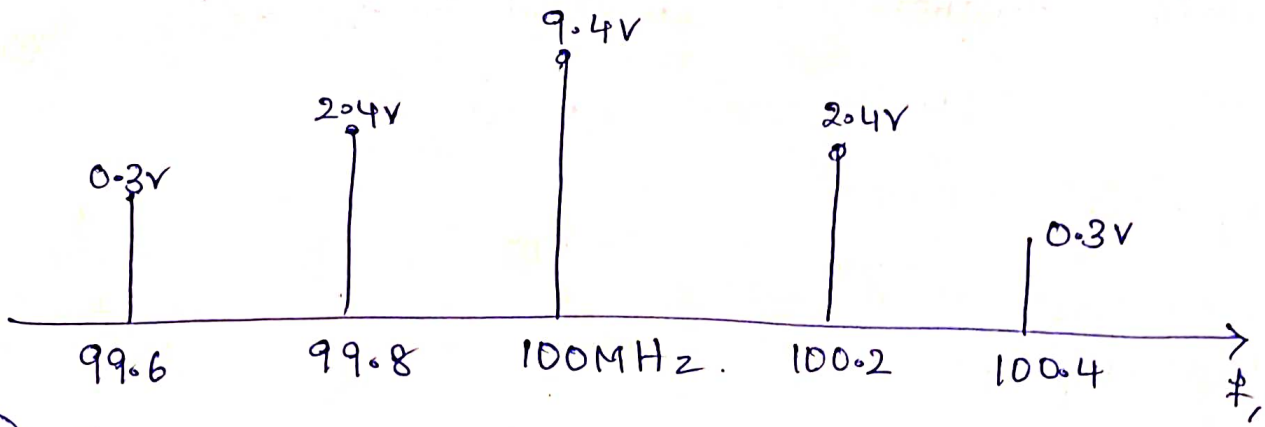
Carrier amplitude $= A_c \cdot J_0 = 10(0.94) = 9.4V$

Carrier amplitude for first pair of sidebands $= A_c \cdot J_1 = 10(0.24) = 2.4V$

second pair of sidebands $= A_c \cdot J_2 = 10(0.03) = 0.3V$.

$f_c \pm f_m = 100.2MHz$ (or) $99.8MHz$.

$f_c \pm 2f_m = 100.4MHz$ (or) $99.6MHz$.



(5). In a FM system, the audio frequency is 1kHz & audio voltage is 2V. The deviation is 4kHz. If the AF voltage is now increased to 8V & its frequency dropped to 500Hz, find the modulation index in each case & the corresponding bandwidth using carson's rule.

$f_{m_1} = 1kHz \quad V_{m_1} = 2V \rightarrow \Delta f = 4kHz \Rightarrow k_f \cdot V_m.$

$f_{m_2} = 500Hz \quad V_{m_2} = 8V \rightarrow \Delta f = k_f \cdot V_m.$

$\Rightarrow k_f = \frac{4\times10^3}{2} = 2\times10^3.$

$= 2\times10^3 \times 8.$

$= 16\times10^3$

$m_{f_1} = \frac{\Delta f}{f_{m_1}} = \frac{4}{1} = 4$

$m_{f_2} = \frac{\Delta f}{f_{m_2}} = \frac{16\times10^3}{500} = 32.$

$BW_1 = 2(\Delta f + f_{m_1}) = 2(4+1) = 10kHz.$

$BW_2 = 2(\Delta f + f_{m_2}) = 2(16+0.5) = 32kHz.$

FM Generation :-

The FM modulator circuit used for generating FM signals are divided as

(i) The direct method or parameter variation method.

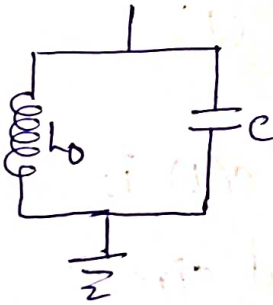
(ii) The Indirect method or the Armstrong method.

Methods of FM Generation

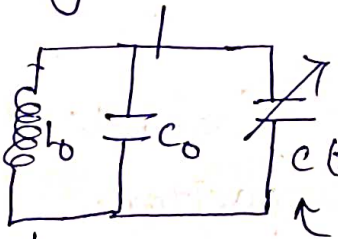
- Direct Method
  - Reactance Modulators
  - Varactor diode Modulators
- Indirect Methods
  - Armstrong method.

Direct Method or Parameter variation Method :-

In direct method, the baseband or modulating signal directly modulates the carrier. The carrier is generated with the help of an oscillator circuit which uses a parallel tuned LC circuit. The frequency of oscillation of the carrier generation is $\omega_c = \frac{1}{\sqrt{LC}}$.



$\Rightarrow f_0 = \frac{1}{2\pi\sqrt{L_0 C_0}}$.



$f' = \frac{1}{2\pi\sqrt{L_0(C_0 + C(t))}}$

An oscillator circuit whose frequency is controlled by a modulating voltage is called voltage controlled oscillator (VCO). The frequency of VCO is varied

according to the modulating signal simply by putting a shunt voltage variable capacitor with its tuned circuit.

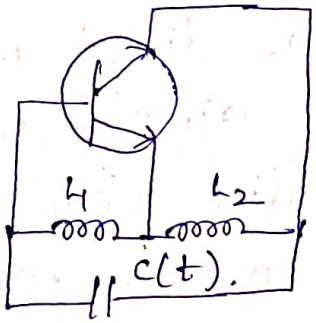
The various devices used to generate a variable capacitance are

(i) Varicap or Varactor diode (Reverse biased diode).
(ii) BJT or FET (Miller's effect).
(iii) Electron tube (Reactance Tube)
(iv) Klystron oscillator
(v) Multivibrator.

With the help of above devices voltage controlled oscillator is generated.

An example of direct FM is which uses a Hartley oscillator along with variable capacitance. i.e varactor diode.

The varactor diode is reverse biased. Its capacitance is dependent on the reverse voltage applied across it.



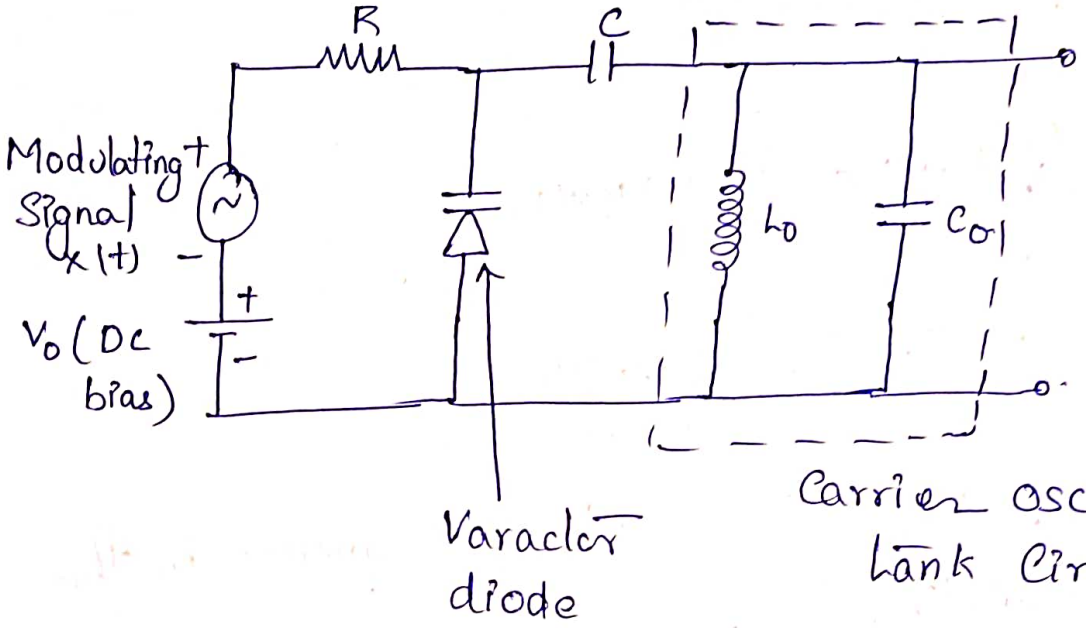
The frequency of oscillations of the Hartley oscillator is $f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)\,c(t)}}$ → (1)

where $c(t) = C + C_{varactor}$. This means that $c(t)$ is the effective capacitance of the fixed tuned circuit capacitance C & the varactor diode capacitance $C_{varactor}$.

Let the relation between the modulating voltage $x(t)$ & capacitance $c(t)$ be represented as.

Varactor Diode Method for FM Generation :-

The varactor diode is a semiconductor diode whose junction capacitance changes with dc bias voltage. This varactor diode is connected in shunt with the tuned circuit of the carrier oscillator.



In varactor diode FM generation arrangement, the capacitor C is made much smaller than the varactor diode capacitance C_d so that the radio frequency (RF) voltage from oscillator across the diode is small as compared to reverse bias d.c voltage across the varactor diode. In addition to this, the reactance of the capacitor C at the highest modulating frequency is made large enough compared to resistor R so that the shunting of the baseband or modulating signal through the tuned circuit may be checked.

$$c(t) = c - k_c x(t)$$

where k_c is the sensitivity of varactor capacitance.

Substituting $c(t)$ in equation (1).

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)(c-k_c x(t))}}$$

$$= \frac{1}{2\pi\left(\sqrt{(L_1+L_2)c - (L_1+L_2)k_c x(t)}\right.}$$

$$= \frac{1}{2\pi\sqrt{(L_1+L_2)c\left[1 - \frac{k_c x(t)}{c}\right]}}.$$

But $f_0 = \frac{1}{2\pi\sqrt{(L_1+L_2)c}}$ which is oscillator frequency in the absence of modulating signal.

$$f_i(t) = f_0\left[1 - \frac{k_c x(t)}{c}\right]$$

If the maximum change in the capacitance corresponding to the modulating wave is assumed to be small as compared to the unmodulated capacitance c then the above equation becomes.

$$f_i(t) = f_0\left[1 + \frac{k_c}{2c} x(t)\right].$$

$$= f_0 + \frac{f_0 k_c}{2c} \cdot x(t).$$

$$f_i(t) = f_0 + k_f \cdot x(t) \quad \left[\because k_f = \frac{f_0 k_c}{2c}\right].$$

where k_f is called as the frequency sensitivity of the modulator.

The capacitance C_d of the varactor diode is expressed as $C_d = \frac{k}{\sqrt{V_D}} = k V_D^{-1/2}$

Here v_d is the total instantaneous voltage across the varactor diode & it is written as

$$V_D = V_0 + x(t).$$

The frequency of oscillation of the carrier generation is given by $\omega_c = \frac{1}{\sqrt{LC}}$

Now, the total capacitance of the oscillator tank circuit will be $C_0 + C_d$ & thus the instantaneous frequency of oscillation ω_i is expressed as

$$\omega_i = \frac{1}{\sqrt{L_0(C_0 + C_d)}}$$

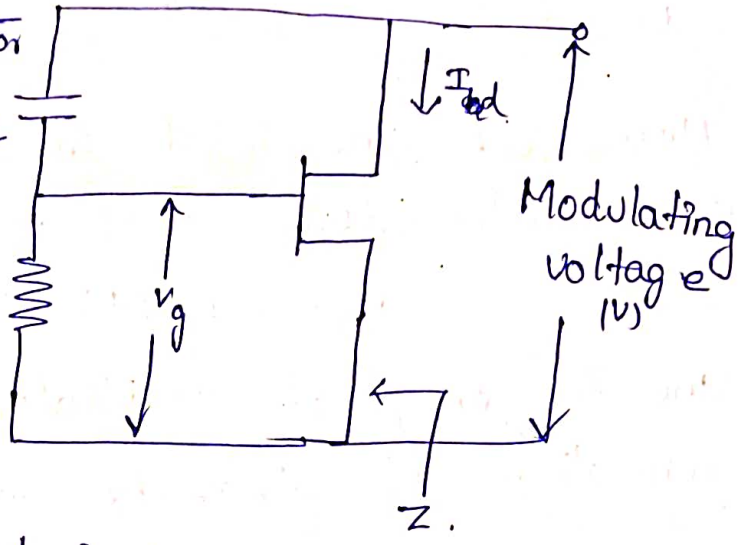
$$= \frac{1}{\sqrt{L_0\left(C_0 + k V_D^{-1/2}\right)}}$$

Thus the instantaneous frequency ω_i of FM signal depends upon V_D which in turn depends upon the value of the modulating signal $x(t)$.

Thus, the instantaneous oscillator frequency ω_i also depends upon the baseband or modulating signal $x(t)$ & hence frequency modulation is generated.

Basic FET Reactance Modulator :-

The basic reactance modulator using FET is shown in figure. The modulating voltage is applied between the terminals. The following assumptions are made



(i) Bias Network current i_b is negligible as compared to the drain current of FET, and

(ii) Drain to gate impedence (X_c) must be greater than the gate to source impedence (R) by more than 5:1.

Expression for Equivalent Capacitance

The gate voltage $V_g = i_b R$.

$$= \frac{VR}{R - jX_c}$$

The drain current of the FET is given by

$$i_D = g_m \times V_g .$$

Substituting $V_g \Rightarrow i_d = \frac{g_m \cdot R \cdot V}{R - jX_c}$.

Assuming that i_b is very small as compared to i_d, the impedence seen between terminals is given by

$$Z = \frac{V}{i_d} = \frac{V}{\frac{g_m RV}{R - jX_c}} = \frac{R - jX_c}{R g_m} = \frac{1}{g_m}\left[1 - j\frac{X_c}{R}\right]$$

Now if $X_C >> R$, then the above equation reduces to

$$Z = \frac{-jX_C}{g_m R}.$$

Z is reactive infact it is capacitive

$$Z = X_{eq}$$

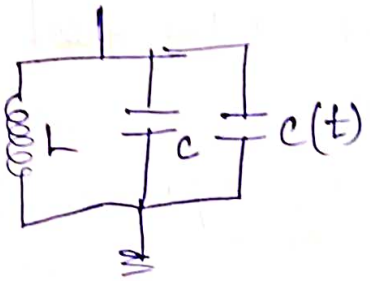
$$X_C = \frac{1}{2\pi f C}.$$

So $$Z = X_{eq} = \frac{X_C}{g_m R} = \frac{1}{2\pi f g_m R C}$$

$$Z = \frac{1}{2\pi f C_{eq}}.$$

where $C_{eq} = g_m R C$.

This expression shows that the FET is equivalent to a variable capacitance C_{eq}.



$$f = \frac{1}{2\pi\sqrt{L(C + C(t))}} \Rightarrow C(t) = C + C_{eq}.$$

where $C_{eq} = g_m R C$.

where the transconductance changes, C_{eq} changes, impedence changes, and C(t) will change which results in changing the frequency of oscillation.

Drawbacks of Direct Method for FM Generation:-

1. It is not easy to get a high order stability in carrier frequency. This is due to the fact that generation of carrier signal is directly affected by the baseband or modulating signal.
2. The non-linearity of the varactor diode produces a frequency variation due to harmonics of the modulating or baseband signal & therefore the FM signal is distorted.

Despite all above drawbacks, the direct method is utilized for high power FM generation in several applications.

Indirect Method (Armstrong Method) of FM Generation:-

In this method, the FM is obtained through the Phase modulation.

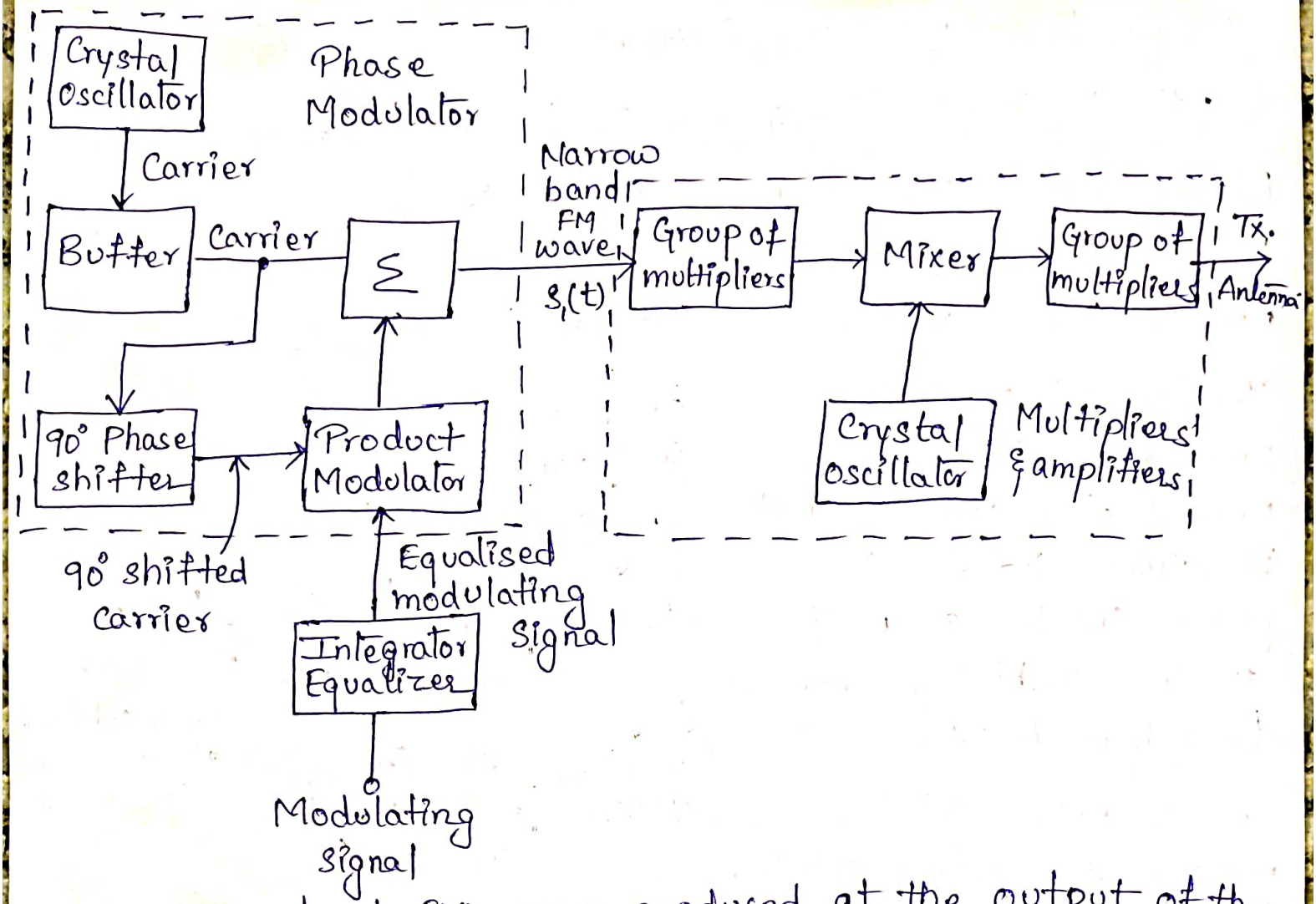
The working operation of this system can be understood by dividing it into two parts

Part I: Generate a narrow band FM wave using a phase modulator.

Part II: Use the frequency multipliers & mixer to obtain the required values of frequency deviation, carrier & modulation index.

Part I: Generation of a narrow band FM wave using a phase modulator.

The modulating signal $x(t)$ is passed through an integrator before applying it to the phase modulator as shown in figure



The narrow band FM wave produced at the output of the phase modulator be represented by

$$s_1(t) = V_{c_1} \cos(2\pi f_1 t + \phi_1(t))$$

where V_{c_1} is the amplitude & f_1 is the frequency of the carrier produced by the crystal oscillator.

The phase angle $\phi_1(t)$ of $s_1(t)$ is related to $x(t)$ as follows.

$$\phi_1(t) = 2\pi k_1 \int_0^t x(t)\, dt.$$

where k_1 represents the frequency sensitivity of the modulator.

If $\phi_1(t)$ is very small then $\cos[\phi_1(t)] = 1$ & $s_1(t)[\phi_1(t)] = \phi_1(t)$.

So $s_1(t) = V_{c_1} \cos[2\pi f_1 t + \phi_1(t)]$.

$$= V_{c_1}\{\cos[2\pi f_1 t] \cos\phi_1(t) - \sin(2\pi f_1 t) \sin\phi_1(t)\}$$

After approximation,

$$s_1(t) = V_{c_1}[\cos 2\pi f_1 t - \phi_1(t) \sin(2\pi f_1 t)]$$

$$= V_{c_1} \cos 2\pi f_1 t - V_{c_1} \phi_1(t) \sin 2\pi f_1 t.$$

Substituting $\phi_1(t) = 2\pi k_1 \int_0^t x(t)\,dt$.

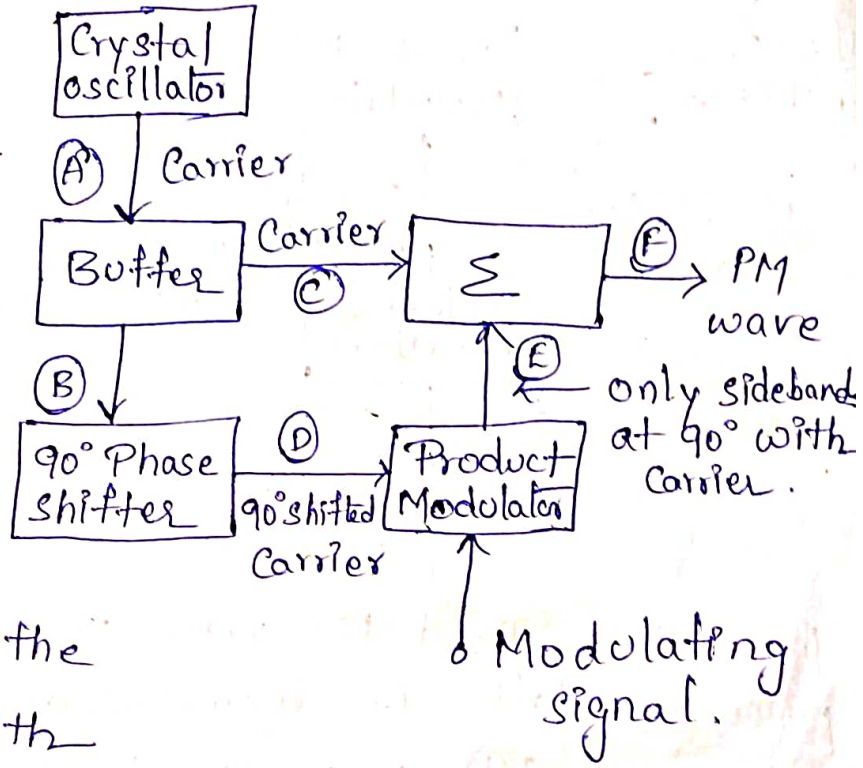
$$s_1(t) = V_{c_1} \cos(2\pi f_1 t) - 2\pi k_1 V_{c_1} \sin(2\pi f_1 t) \int_0^t x(t)\,dt$$

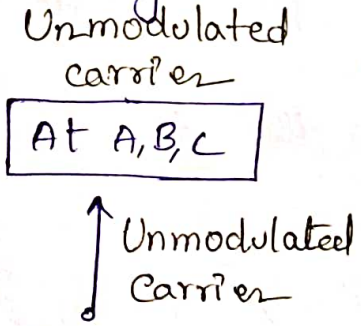
At the output of the phase modulator, a narrow band FM wave is obtained.

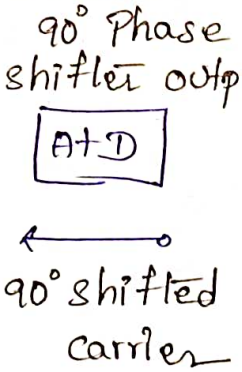
Part II: Implementation of the Phase Modulator

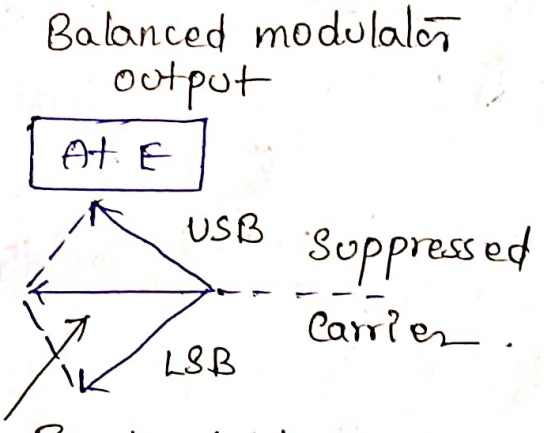
The crystal oscillator produces a stable unmodulated carrier which is applied to the 90° phase shifter as well as the combining network through a buffer.

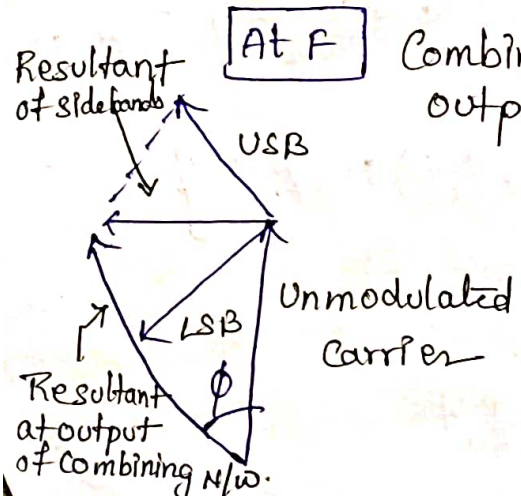
The 90° phase shifter produces a 90° phase shifted carrier. It is applied to the balanced modulator alongwith the modulating signal and a DSBSC signal is obtained.







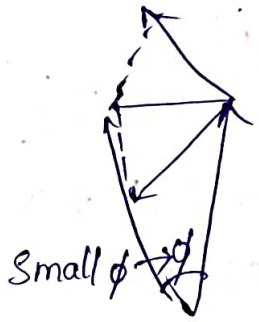




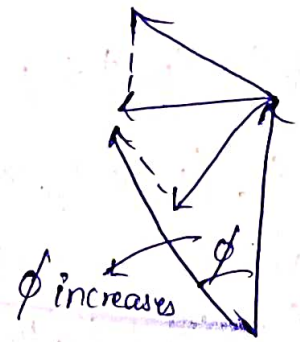
The DSBSC signal (two sidebands) & the original carrier without any phase shift are applied to a combining network (Σ).

As the modulation index is increased, the amplitude of sidebands will also increase. Hence the amplitude of their resultant increases. This will increase the angle ϕ made by the resultant with unmodulated carrier.

Low modulation index



High modulation index



Thus, the resultant at the output of the combining N/w is phase modulated.

Part III :- Use of Frequency Multipliers, Mixer & Amplifier.

The FM signal produced at the output of phase modulator has a low carrier frequency & low modulation index. They are increased to an adequately high value with the help of frequency multipliers & mixer.

Demodulation of FM waves :-

The message signal can be generated from FM wave with the below basic block diagram



An Envelope detector is used to generate message signal when AM signal is given as input to it. So an FM wave is passed through a differentiator &

an AM wave is generated.

Consider FM wave equation is

$$S_{FM}(t) = A_c \cos(\omega_c t + k_f \int x(t)\, dt)$$
$$= A_c \cos(\omega_c t + \phi(t)) \quad [\because \phi(t) = k_f \int x(t)\, dt]$$

when the above equation is passed through a differentiator

$$\frac{d}{dt}[S_{FM}(t)] = -A_c \sin(\omega_c t + \phi(t)) \left(\omega_c + \frac{d\phi(t)}{dt}\right)$$
$$= A_c \sin(\omega_c t + \phi(t) + \pi)\,(\omega_c + k_f x(t))$$
$$= A_c (\omega_c + k_f x(t)) \sin(\omega_c t + \phi(t) + \pi)$$

The above equation is similar to AM wave equation

$$= A_c (\omega_c + k_a m(t)) \cos\omega_c t.$$

When the differentiator output is passed through an Envelope detector, message signal $x(t)$ is generated.

Classification of FM demodulators :-

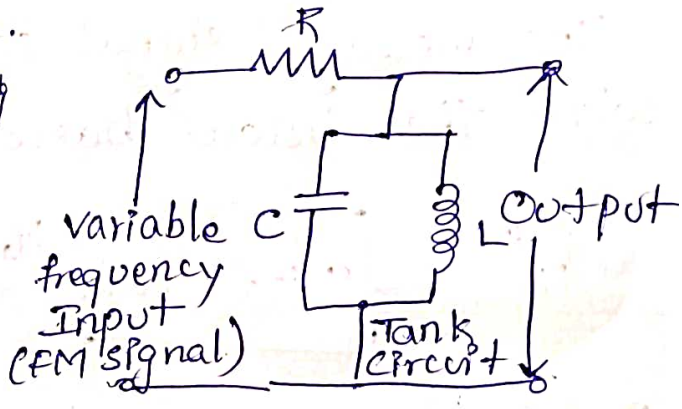
1. Balanced FM slope detector
2. Foster Seeley Discriminator

Balanced frequency Discriminator :-

The frequency discriminator operates on the principle of slope detection.

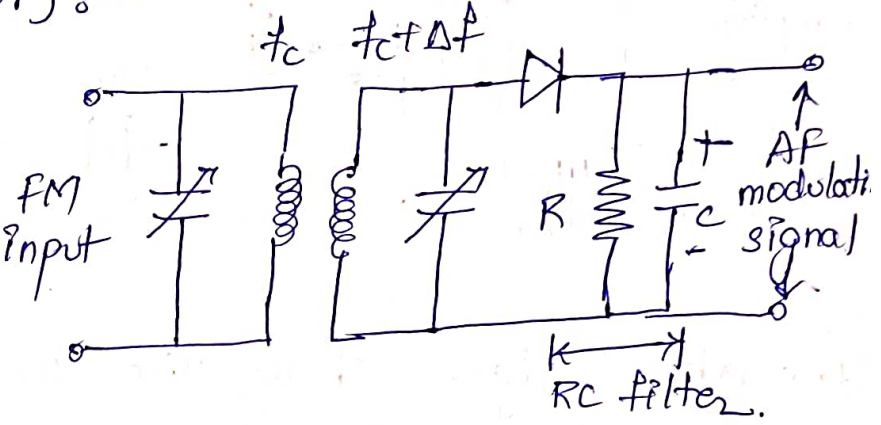
Principle of slope detection :-

Consider a tuned circuit as shown in figure. A frequency modulated signal is applied to the tuned circuit with frequency f_c & deviation Δf.



The resonant frequency of the tuned circuit is deliberately adjusted to $(f_c + \Delta f)$. The amplitude of the output voltage of the tank circuit depends on the frequency deviation of the input FM signal.

FM slope detector / simple FM slope detector (Frequency discriminator):-



The output voltage of the tank circuit is then applied to a simple diode detector of an RC load with proper time constant.

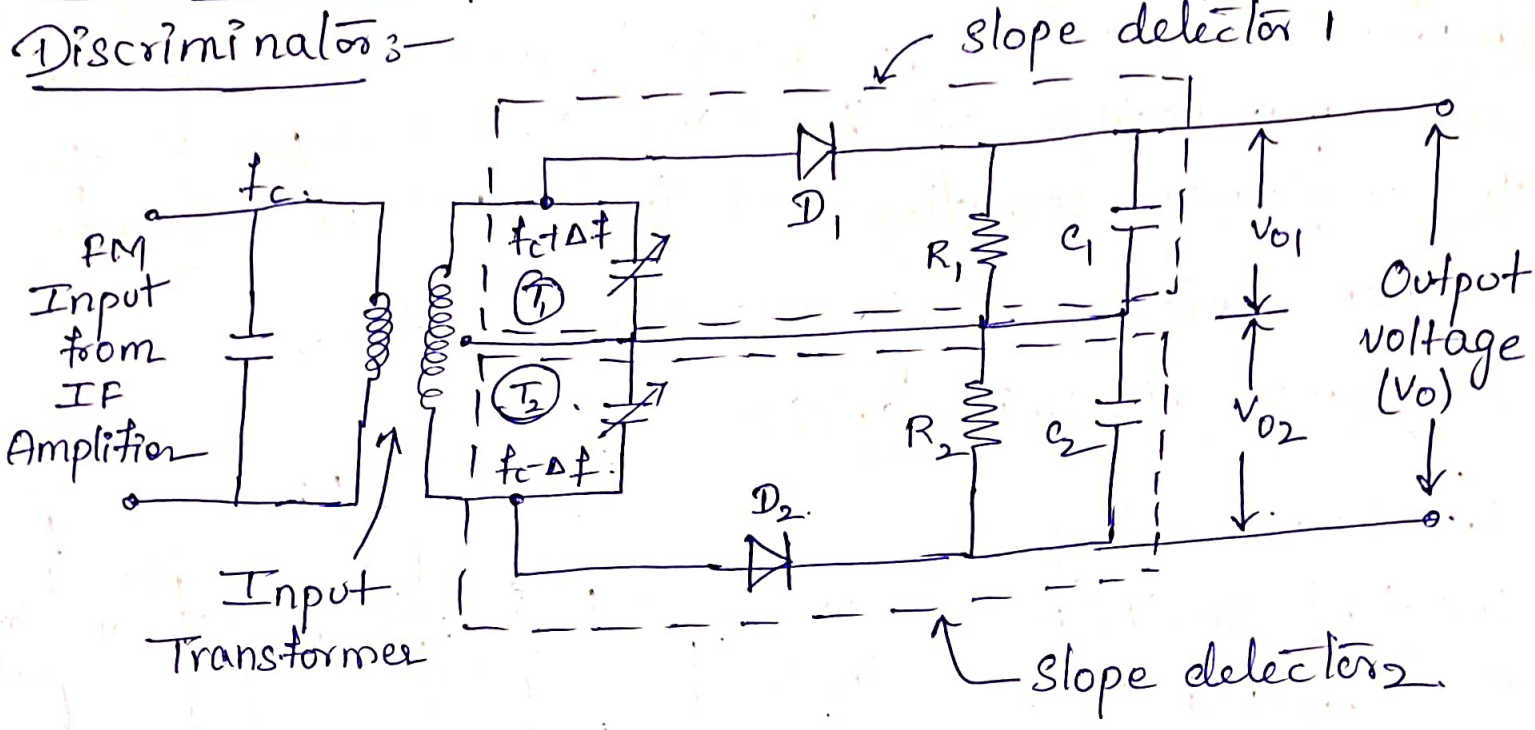
This detector is identical to the AM diode detector. Even though the slope detector circuit is simple it has the following drawbacks.

Drawbacks of slope detector

1. It is inefficient.
2. It is linear only over a limited frequency range.
3. It is difficult to adjust as the primary & secondary windings of the transformer must be tuned to slightly different frequencies.

The only advantage of basic slope detector circuit is its simplicity. To overcome the above drawbacks, a balanced slope detector is used.

Balanced FM slope Detector (Balanced Frequency Discriminator)



The above figure shows the circuit diagram of a balanced slope detector. It consists of two slope detector circuits. The input transformer is center tapped secondary. Hence the input voltages of the two transformers will be 180° out of phase. There are 3 tuned circuits, primary is tuned to f_c, upper tuned circuit of the secondary (T_1) is tuned above f_c by Δf i.e $f_c+\Delta f$ & lower tuned circuit of secondary is tuned below f_c by Δf i.e $f_c-\Delta f$.

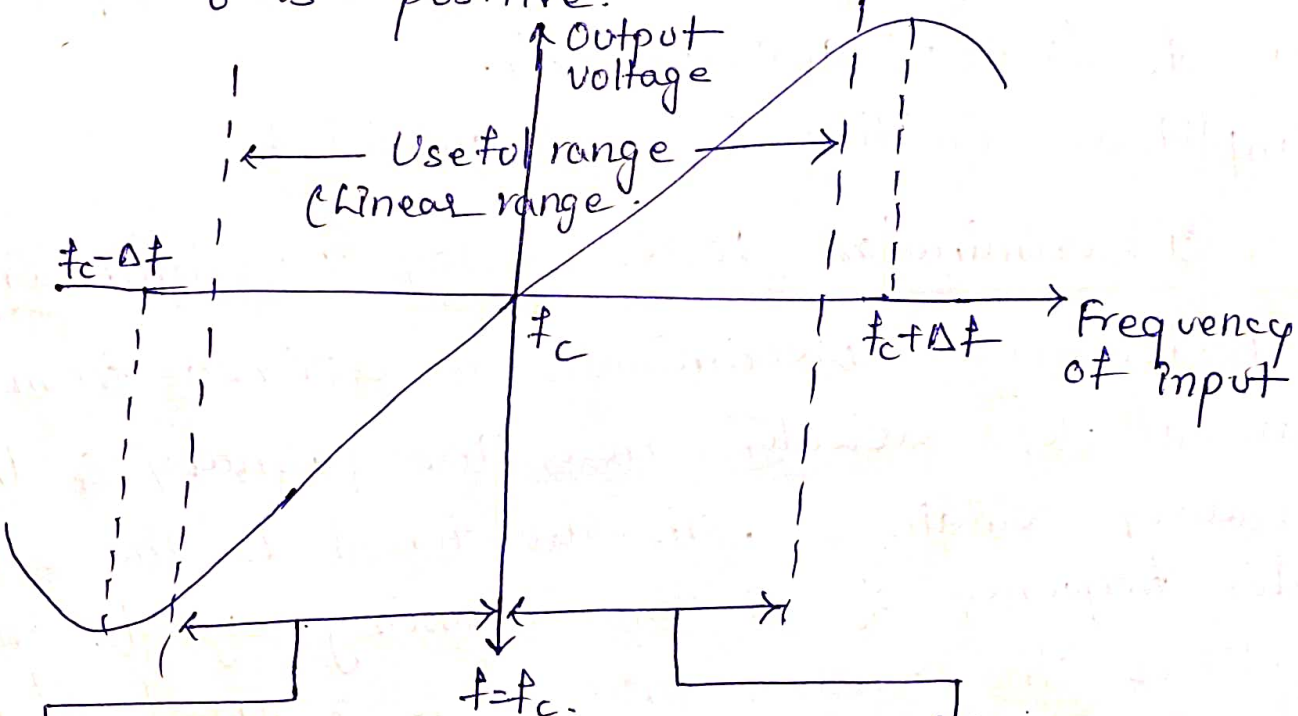
R_1C_1 & R_2C_2 are filters used to bypass the RF ripple. V_{01} & V_{02} are the output voltages of the two slope detectors. The final output voltage V_0 is obtained by taking the subtraction of the individual voltages V_{01} & V_{02} $\quad V_0 = V_{01} - V_{02}$.

Working Operation of the circuit :—

(i) $f_{in} = f_c$ ⇒ When the input frequency is equal to f_c, the induced voltage in the T_1 winding of secondary is exactly equal to that induced in the winding T_2.

The input voltages to both the diodes D_1 & D_2 will be same. Therefore, their dc voltages V_{o_1} & V_{o_2} will also be identical but they have opposite polarities. Hence the net output voltage $V_o = 0$.

(ii) $f_c < f_{in} < (f_c + \Delta f)$ ⇒ In this range of input frequency, the induced voltage in the winding T_1 is higher than that induced in T_2. Therefore input to diode D_1 is higher than D_2. So $V_{o_1} > V_{o_2}$. So V_o is positive.



$(f_c - \Delta f) < f < f_c$
Input to D_1 is less than input to D_2
↓
V_{o_1} is less than V_{o_2}
↓
V_o is negative.

$f = f_c$.
Input to D_1 and D_2 are equal
↓
V_{o_1} is equal to V_{o_2}
↓
$V_o = 0$

$f_c < f < (f_c + \Delta f)$
Input to D_1 is higher than input to D_2
↓
V_{o_1} is greater to V_{o_2}
↓
V_o is positive.

(iii) $(f_c - \Delta f) < f < f_c$ => In this range of input frequency, the induced voltage in the winding T_2 is higher than that induced in T_1. Therefore, the input to the diode D_2 is higher than D_1. So $V_{O_2} > V_{O_1}$. So V_O is negative.

Advantages of Balanced slope Detector :-

1. This circuit is more efficient than simple slope detector.
2. It has better linearity than simple slope detector.

Disadvantages of Balanced slope Detector :-

1. Even though linearity is good, it is not good enough.
2. This circuit is difficult to tune since three tuned circuits are to be tuned at different frequencies i.e f_c, $(f_c + \Delta f)$ and $(f_c - \Delta f)$.
3. Amplitude limiting is not provided.

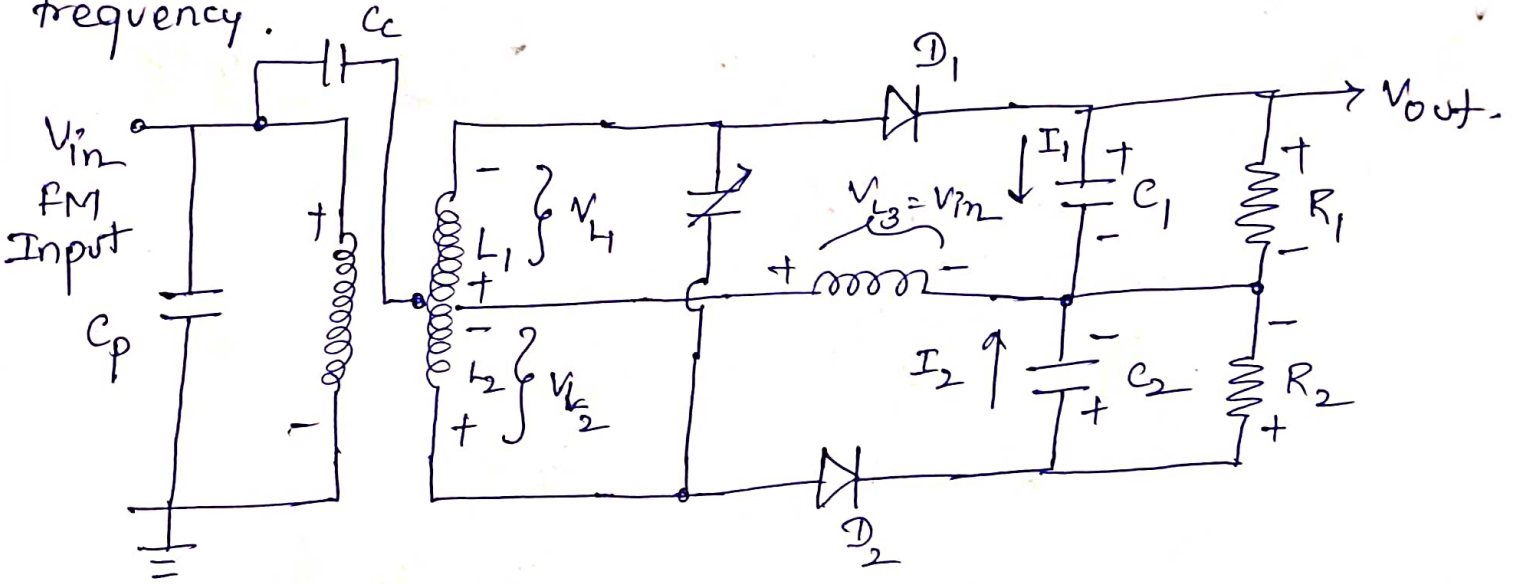
Phase Discriminator (Foster seeley Discriminator) :-

Foster seeley Discriminator is derived from the balanced slope detector. Here the primary & the secondary windings both are tuned to the same center frequency f_c of the incoming signal. This simplifies the tuning process to a great extent & it will yield better linearity than the balanced slope detector.

A capacitor C_c couples the primary voltage to secondary & blocks the dc component.

Principle of Operation :-

Even though Primary & secondary tuned circuits are toned to the same center frequency, the voltages applied to the diodes D_1 & D_2 are not constant. This is due to the change in phase shift between the primary & secondary windings depending on the input frequency.



(i) At $f_{in} = f_c$, the individual o/p voltage of the two diodes will be equal & opposite. & the phase b/w primary & secondary is 90°. So $V_o = 0$

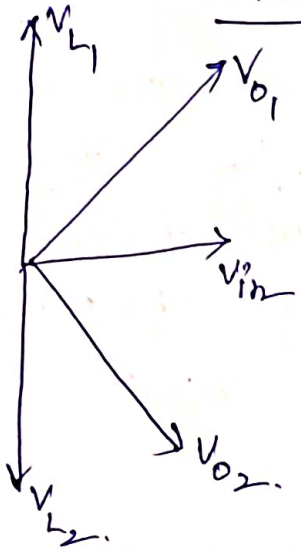
(ii) At $f_{in} > f_c$, the phase b/w primary & secondary windings is less than 90°

(iii) At $f_{in} < f_c$, the phase b/w primary & secondary is greater than 90°

Let Primary voltage be V_{in}.

& secondary voltage be V_{L_1}. As the secondary winding transformer is center tapped, V_{L_1} & V_{L_2} will be 180° out of phase.

When $f = f_c$.

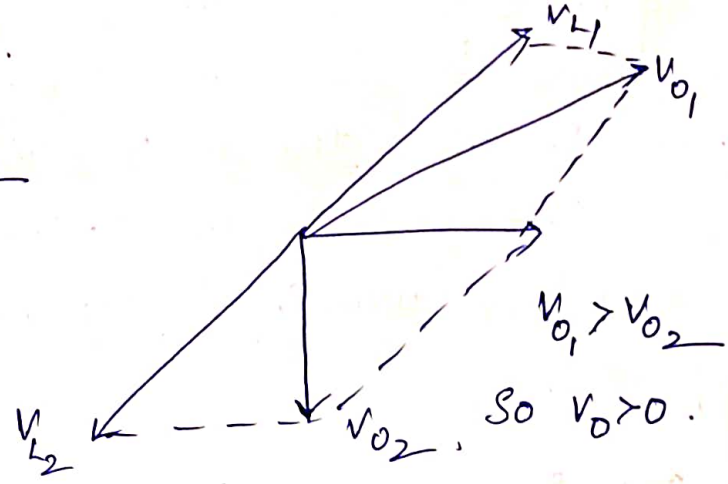


$V_{O_1} = V_{O_2}$.

$V_O = V_{O_1} - V_{O_2}$

$= 0$.

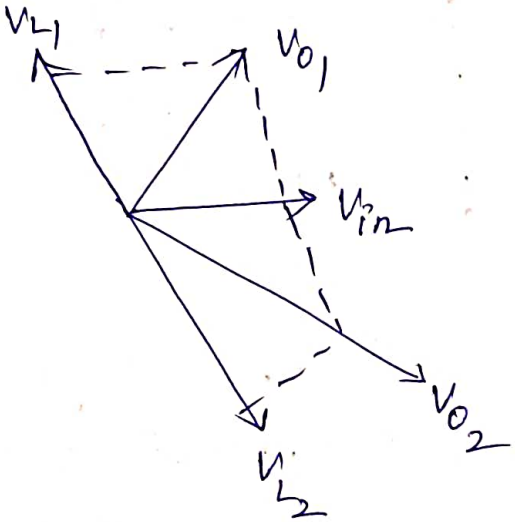
When $f_{in} > f_c$



$V_{O_1} > V_{O_2}$

So $V_O > 0$.

When $f_{in} < f_c$



$V_{O_2} > V_{O_1}$

$V_O = V_{O_1} - V_{O_2}$

So $V_O < 0$.

Advantages of Foster-Seeley FM discriminators:

1. It is more easy to align (tune) than balanced slope detector as there are only two tuned circuits & both are to be tuned at the same frequency f_c.
2. Linearity is better. This is because the operation of the circuit is dependent more on the primary to secondary phase relationship which is very much linear.

Disadvantages of Foster-Seeley FM discriminators:

It does not provide amplitude limiting. So in the presence of noise or any other spurious

amplitude variations, the demodulator output responds to them & produces errors.

Performance Comparision of FM & AM System:-

S.No	FM System	AM System
1.	Amplitude of FM wave is constant. It is independent of the modulation index.	Amplitude of AM wave will change with the modulating voltage.
2.	The transmitted power remains constant. It is independent of m_f.	Transmitted power is dependent on the modulation index.
3.	All the transmitted power is useful.	Carrier power & one sideband power are useless. useless.
4.	FM receivers are immune to Noise.	AM receivers are not immune to Noise
5.	It is possible to decrease noise further by increasing deviation.	This feature is absent in AM.
6.	Bandwidth = $2(\Delta f + f_m)$. The bandwidth depends on modulation index.	BW = $2f_m$. It is not dependent on the modulation index.
7.	BW is large. Hence, wide channel is required.	BW is much less than FM.
8	Space wave is used for propagation. So radius of transmission is limited to line of sight.	Ground wave & sky wave propagation is used. Therefore, larger area is covered than FM.

S.No	FM System	AM System
9.	It is possible to operate several transmitters on same frequency.	Not possible to operate more channels on the same frequency.
10.	FM transmission & reception equipment are more complex.	AM equipments are less complex.
11.	The number of sidebands having significant amplitudes depends on modulation index	Nomber of sidebands in AM will be constant and equal to 2.
12.	The information is contained in the frequency variation of the Carrier.	The information is contained in the amplitude variation of the Carrier.