

ANGLE MODULATION & DEMODULATION

- In angle Modulation the frequency (or) phase of the carrier signal is varied according to the message signal.
- In this method the amplitude of the carrier is kept constant.
- Angle modulation is defined as the process in which the total angle of the carrier signal i.e., $\theta(t)$ is varies w.r.to, message signal amplitude variations.

Mathematical representation :-

consider the un-modulated carrier signal is

$$c(t) = A_c \cos[2\pi f_c t + \phi] \rightarrow \textcircled{1}$$

where $A_c \rightarrow$ Amplitude of carrier signal

$\phi \rightarrow$ Some phase angle

Let us consider the total angle

$$\theta(t) = 2\pi f_c t + \phi \rightarrow \textcircled{2}$$

$$\therefore c(t) = A_c \cos[\theta(t)] \rightarrow \textcircled{3}$$

consider $c(t)$ as real part of rotating phasor $A \cdot e^{j\phi}$.
let us denote rotating phasor by C . so that

$$C = A \cdot e^{j\phi} \rightarrow \textcircled{4}$$

$$c(t) = \text{Re}(A \cdot e^{j\phi})$$

$$c(t) = A \cdot \text{Re}(e^{j\phi}) = A \cdot \cos\phi \rightarrow \textcircled{5}$$

Types of Angle Modulation :-

The phase angle ϕ can be varied in two ways thus there are two types of angle modulation techniques.

1. Frequency Modulation
2. Phase Modulation

Frequency Modulation :- If the angle modulation occurs due to dependence of carrier frequency on message signal is called as frequency modulation.

Phase Modulation :- If the angle modulation occurs due to dependence of phase angle of carrier frequency on message signal is called as phase modulation.

Advantages :-

- Noise reduction
- Improved system fidelity.
- More efficient use of power.

Disadvantages :-

- Increased Bandwidth.
- More complex circuitry.

Applications :-

- Radio Broadcasting
- Two way mobile Radio
- Microwave communication
- T.V. Sound transmission
- Cellular radio
- Satellite Communication.

Frequency Modulation :-

- In FM the instantaneous frequency is varied linearly with a message signal $m(t)$.
- let us consider
The frequency of carrier signal before modulation = f_c
freq. of carrier signal after freq. modulation = f_i
where f_i is instantaneous freq.

$$\therefore \boxed{f_i = f_c + k_f \cdot m(t)}$$

k_f is freq. sensitivity of freq. modulator.

— for $m(t) = 0$, then there is no modulation.

$$\therefore \boxed{f_i = f_c}$$

- k_f specifies the amount of freq. change in the carrier signal per 1V change in message signal.

NOTE :-

— for $m(t) = 0 \rightarrow f_c = f_i$

$m(t) = +ve \rightarrow f_c > f_i$

$m(t) = -ve \rightarrow f_c < f_i$

- In frequency modulation message signal voltage variations are converted as carrier signal freq. variations; so, it is called as voltage-to-frequency conversion.

- for $m(t) = A_m \cos(2\pi f_m t)$

$$\text{— } f_i = f_c + (k_f \cdot m(t)) \rightarrow \text{freq. deviation}$$

Maximum possible frequency of resulting FM signal

$$f_{\max} = f_c + k_f \cdot A_m$$

Minimum possible freq. of resulting FM signal

$$f_{\min} = f_c - k_f \cdot A_m \quad f_{\min} = f_c + k_f (-A_m)$$

$$\text{Max. frequency deviation} = \text{Max}[k_f \cdot m(t)] = \Delta f.$$

$$\therefore \boxed{\Delta f = k_f \cdot A_m}$$

$$\therefore f_{\max} = f_c + \Delta f$$

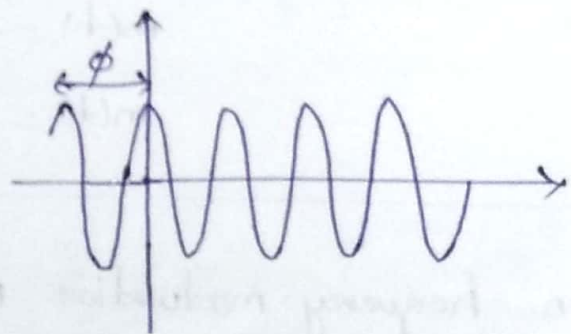
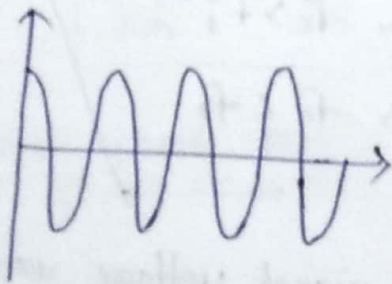
$$f_{\min} = f_c - \Delta f$$

$$\therefore \text{total freq. swing of FM signal} = f_{\max} - f_{\min} = 2\Delta f.$$

General expression of FM signal :-

let us consider

$$\text{the carrier signal } c(t) = A_c \cos[2\pi f_c t + \phi]$$



$$\text{WKT } c(t) = A_c \cos \theta(t)$$

$$\theta(t) = 2\pi f_c t + \phi$$

derivative w.r.t. t

$$\frac{d\theta(t)}{dt} = 2\pi f_c$$

General expression for resulting FM signal

(3)

$$S_{FM}(t) = A_c \cos[\theta_i(t)]$$

$$f_i = f_c + k_f \cdot m(t)$$

where $\theta_i(t) = 2\pi f_i t + \phi$

$$\frac{d}{dt} \theta(t) = 2\pi f_i$$

~~taking integration o.B.S.~~

$$\therefore f_i = \frac{1}{2\pi} \frac{d}{dt} \theta(t)$$

~~taking integration o.B.S.~~

$$\theta_i(t) = 2\pi \int f_i dt$$

$$= 2\pi \int (f_c + k_f m(t)) dt$$

$$\theta_i(t) = 2\pi f_c t + 2\pi k_f \int m(t) dt$$

$$\therefore S_{FM}(t) = A_c \cos \left[2\pi f_c t + 2\pi k_f \int m(t) dt \right]$$

SINGLE TONE FM :-

W.K.T, $S_{FM}(t) = A_c \cos \left[2\pi f_c t + 2\pi k_f \int m(t) dt \right]$

Assume $m(t) = A_m \cos(2\pi f_m t)$

$$\therefore S_{FM}(t) = A_c \cos \left[2\pi f_c t + 2\pi k_f \int A_m \cos(2\pi f_m t) dt \right]$$

$$= A_c \cos \left[2\pi f_c t + 2\pi k_f A_m \cdot \frac{\sin 2\pi f_m t}{2\pi f_m} \right]$$

$$S_{FM}(t) = A_c \cos \left[2\pi f_c t + \frac{k_f \cdot A_m}{f_m} \sin(2\pi f_m t) \right]$$

$$= A_c \cos \left[2\pi f_c t + \frac{\Delta f}{f_m} \sin(2\pi f_m t) \right]$$

$$S_{FM}(t) = A_c \cos \left[2\pi f_c t + \beta \sin(2\pi f_m t) \right]$$

NOTE:- $k_f \cdot A_m = \Delta f$

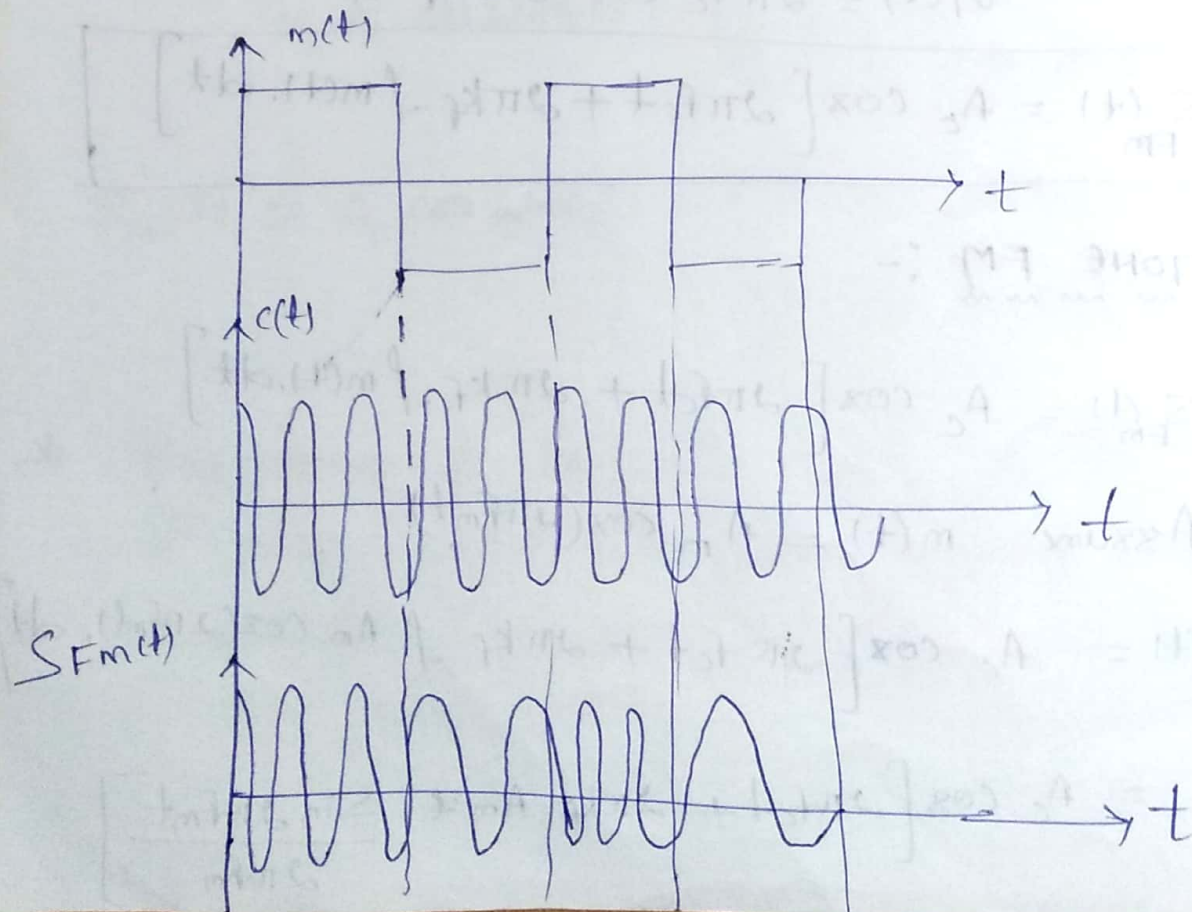
Where $\beta = \frac{\Delta f}{f_m} \Rightarrow$ Modulation index of FM.

→ Depending on modulation index of FM, the FM wave can be divided into two types.

(i) Narrow Band FM (NBFM) ($\beta \leq 1$)

(ii) Wide Band FM (WBFM) ($\beta > 1$)

Wave form representation of FM :-



Narrowband Frequency Modulation (NBFM) :-

- * for NBFM the Modulation index β is less than 1.
- * the general FM wave is

$$S_{FM}(t) = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t]$$

$$S_{FM}(t) = A_c \cos(2\pi f_c t) \cos(\beta \sin 2\pi f_m t) - A_c \sin(2\pi f_c t) \sin(\beta \sin 2\pi f_m t)$$

Assume $\theta = \beta \sin 2\pi f_m t$.

for NBFM, $\beta \leq 1$ (small)

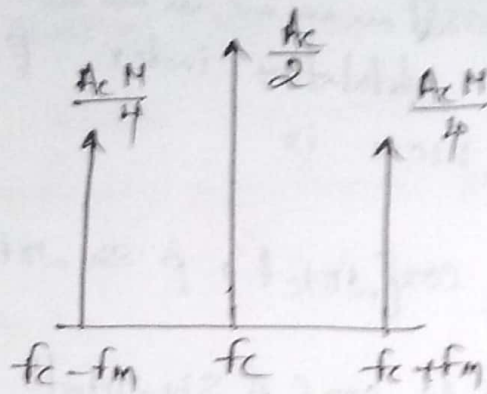
for small θ ; $\cos \theta \approx 1$
 $\sin \theta \approx \theta$

$$\begin{aligned} \therefore S_{FM}(t) &\approx A_c \cos(2\pi f_c t) \cdot 1 - A_c \sin(2\pi f_c t) \cdot \beta \sin(2\pi f_m t) \\ &\approx A_c \cos(2\pi f_c t) - A_c \beta \sin(2\pi f_c t) \sin(2\pi f_m t) \\ &\approx A_c \cos(2\pi f_c t) - \frac{A_c \beta}{2} \left[\cos 2\pi(f_c - f_m)t - \cos 2\pi(f_c + f_m)t \right] \end{aligned}$$

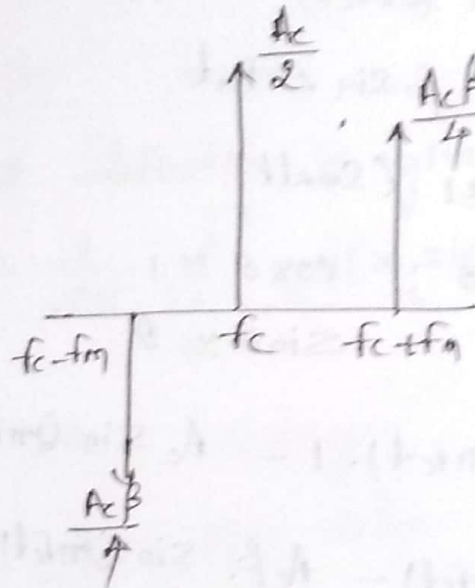
$$S_{FM}(t) \approx A_c \cos(2\pi f_c t) - \frac{A_c \beta}{2} \cos 2\pi(f_c - f_m)t + \frac{A_c \beta}{2} \cos 2\pi(f_c + f_m)t$$

- * Expressions of AM and NBFM will be same except 180° phase shift at LSB frequency component.
- * Bandwidth of NBFM is $2f_m$, it is similar to AM wave.

$S_{AM}(f) \leftrightarrow$



$S_{NBFM}(f) \leftrightarrow$



Power of NBFM :-

$$P_t = P_c + P_{USB} + P_{LSB}$$

$$P_c = \frac{A_c^2}{2R} \quad P_{USB} = \frac{A_c^2 \beta^2}{8R} = P_{LSB}$$

$$\therefore P_t = \frac{A_c^2}{2R} + \frac{A_c^2 \beta^2}{8R} + \frac{A_c^2 \beta^2}{8R}$$

$$= \frac{A_c^2}{2R} + \frac{A_c^2 \beta^2}{4R}$$

$$P_t = \frac{A_c^2}{2R} \left[1 + \beta^2 \right]$$

$$\therefore \boxed{P_t = P_c (1 + \beta^2/2)}$$

- * The channel B.W and Transmission loss are same as AM. (5)
- Requirements of NBFM will be almost same as AM.
- * Because of its much similarities with AM, NBFM is given least practical significance compared to WBFM.

WIDE BAND FM :-

- * In wide band FM we are using "Bessel function".

Bessel function :-

The standard definition is given as

$$J_n(x) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(x \sin \theta - n\theta)} d\theta$$

where n - Integer

x - real value.

Properties of Bessel function :-

- 1) " $J_n(x)$ " decreases as ' n ' value increases.

$$J_0(x) > J_1(x) > J_2(x) > \dots$$

$$2) J_{-n}(x) = (-1)^n J_n(x)$$

$$J_n(x) = \begin{cases} J_n(x) ; n, \text{ even} \\ -J_n(x) ; n, \text{ odd} \end{cases}$$

$$3) \sum_{n=-\infty}^{\infty} J_n^2(x) = 1$$

4) $J_n(x)$ always result in real value.

General expression of WBFM :-

General expression for FM is

$$S_{FM}(t) = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t]$$

Now $\cos \theta = \operatorname{Re}[e^{j\theta}]$

$$\therefore S_{FM}(t) = A_c \cdot \operatorname{Re} \left[e^{j(2\pi f_c t + \beta \sin 2\pi f_m t)} \right]$$

$$S_{FM}(t) = A_c \cdot \operatorname{Re} \left\{ e^{j2\pi f_c t} \cdot e^{j\beta \sin 2\pi f_m t} \right\} \rightarrow \textcircled{1}$$

* Now, $e^{j\beta \sin 2\pi f_m t}$ is a continuous periodic function with period $T = 1/f_m$.

Above signal is periodic, so $x(t) = x(t+T)$

$$\therefore e^{j\beta \sin 2\pi f_m t} = e^{j\beta \sin 2\pi f_m (t + 1/f_m)} \rightarrow \textcircled{2}$$

* Fourier series is used to find the frequency analysis of continuous periodic signal. So, the exponential F-S is given as

$$F(t) = \sum_{n=-\infty}^{\infty} C_n \cdot e^{j2\pi f_0 t n} ; f_0 = \frac{1}{T} \rightarrow \textcircled{3}$$

$$\text{Where } C_n = \frac{1}{T} \int_{-T/2}^{T/2} e^{-jn2\pi f_0 t} \cdot dt \cdot f(t)$$

$$\text{So, } e^{j\beta \sin 2\pi f_m t} = \frac{1}{1/f_m} \int_{-1/2 f_m}^{1/2 f_m} e^{j\beta \sin 2\pi f_m t} \cdot e^{-jn2\pi f_m t} \cdot dt$$

$$C_n = f_m \int_{-1/2f_m}^{1/2f_m} e^{j(\beta \sin 2\pi f_m t - n 2\pi f_m t)} dt$$

Let us consider

$$2\pi f_m t = \theta, \quad \text{for } t = -1/2f_m \rightarrow \theta = -\pi$$

$$2\pi f_m dt = d\theta \quad t = 1/2f_m \rightarrow \theta = +\pi$$

$$\therefore C_n = f_m \int_{-\pi}^{\pi} e^{j(\beta \sin \theta - n\theta)} \frac{d\theta}{2\pi f_m}$$

$$C_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(\beta \sin \theta - n\theta)} d\theta.$$

— This is Bessel function

$$\text{So, } \boxed{C_n = J_n(\beta)}$$

Substitute C_n value in eqⁿ ②

$$e^{j\beta \sin 2\pi f_m t} = \sum_{n=-\infty}^{\infty} J_n(\beta) \cdot e^{jn 2\pi f_m t} \rightarrow \textcircled{4}$$

From eqⁿ ④ and eqⁿ ①

$$S_{FM}(t) = A_c \cdot \text{Re} \left\{ e^{j2\pi f_c t} \cdot \sum_{n=-\infty}^{\infty} J_n(\beta) \cdot e^{jn 2\pi f_m t} \right\}$$

$$= A_c \cdot \text{Re} \left\{ \sum_{n=-\infty}^{\infty} J_n(\beta) \cdot e^{j2\pi (f_c + n f_m) t} \right\}$$

$$S_{FM}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos[2\pi(f_c + n f_m)t]$$

$$\therefore S_{WBFM}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos[2\pi(f_c + n f_m)t] ; \beta > 1$$

Spectrum analysis of WBFM :-

$$S_{WBFM}(t) = A_c \sum_{n=-\infty}^{\infty} \cos 2\pi(f_c + n f_m)t \cdot J_n(\beta)$$

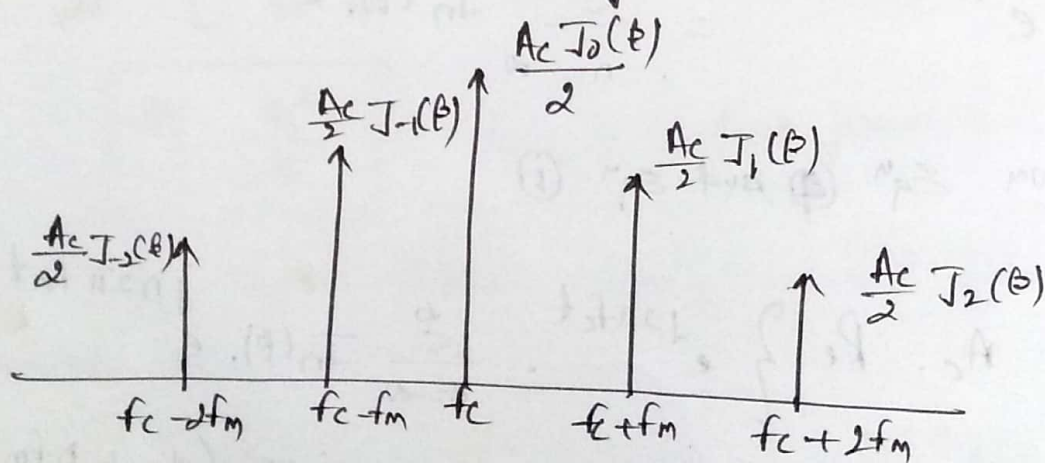
$$= A_c J_0(\beta) \cos 2\pi(f_c)t + A_c J_1(\beta) \cos[2\pi(f_c + f_m)t] + A_c J_{-1}(\beta) \cos[2\pi(f_c - f_m)t] + A_c J_2(\beta) \cos[2\pi(f_c + 2f_m)t] + A_c J_{-2}(\beta) \cos[2\pi(f_c - 2f_m)t]$$

* for β is small

$$J_0(\beta) \approx 1 \quad J_1(\beta) \approx \beta/2 \quad \boxed{J_0(0) = 1}$$

$$J_{-1}(\beta) \approx -\beta/2$$

* for small β values we get NBFM exp.



* NBFM contains carrier freq. component and infinite no. of LSB's and USB's.

- The actual B.W of WBFM is infinite.
- for WBFM, most of the strength will be contained by lower order side bands and are said to be significant, higher order side bands and are said to be insignificant.

Power of WBFM :-

$$P_t = \dots P_{LSB2} + P_{LSB1} + P_c + P_{USB1} + P_{USB2} + \dots$$

$$P_c = \frac{A_c^2 J_0^2(\beta)}{2R} \quad P_{USB1} = \frac{A_c^2 J_1^2(\beta)}{2R}, \quad P_{LSB1} = \frac{A_c^2 J_{-1}^2(\beta)}{2R}$$

$$P_{USB2} = \frac{A_c^2 J_2^2(\beta)}{2R}, \quad P_{LSB2} = \frac{A_c^2 J_{-2}^2(\beta)}{2R}$$

$$\therefore P_t = \dots \frac{A_c^2 J_{-2}^2(\beta)}{2R} + \frac{A_c^2 J_{-1}^2(\beta)}{2R} + \frac{A_c^2 J_0^2(\beta)}{2R} + \frac{A_c^2 J_1^2(\beta)}{2R} + \frac{A_c^2 J_2^2(\beta)}{2R} + \dots$$

$$= \frac{A_c^2}{2R} \left[\dots J_{-2}^2(\beta) + J_{-1}^2(\beta) + J_0^2(\beta) + J_1^2(\beta) + J_2^2(\beta) + \dots \right]$$

$$= \frac{A_c^2}{2R} \left[\sum_{n=-\infty}^{\infty} J_n^2(\beta) \right]$$

$$\therefore \boxed{P_t = \frac{A_c^2}{2R}} \quad \text{this is actual power of FM}$$

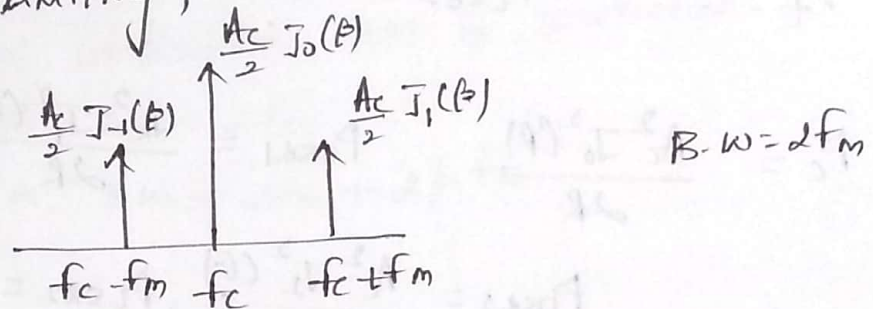
→ Power of signal before modulation and after modulation remains same.

→ for FM, the power of carrier signal before and after modulation will be same.

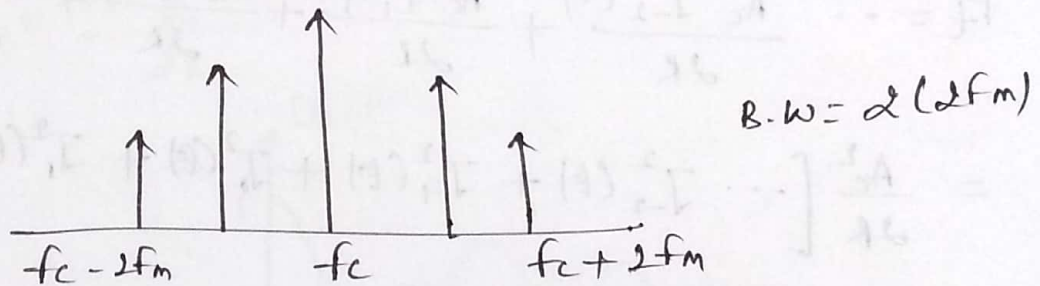
Practical Bandwidth of FM (CARSONS RULE) :-

- The actual B.W of WBFM is infinity. For transmission of signal it should be band limited by retaining or significant sidebands and eliminating insignificant sideband.
- Assume FM signal contains significant sidebands up to 1st order.

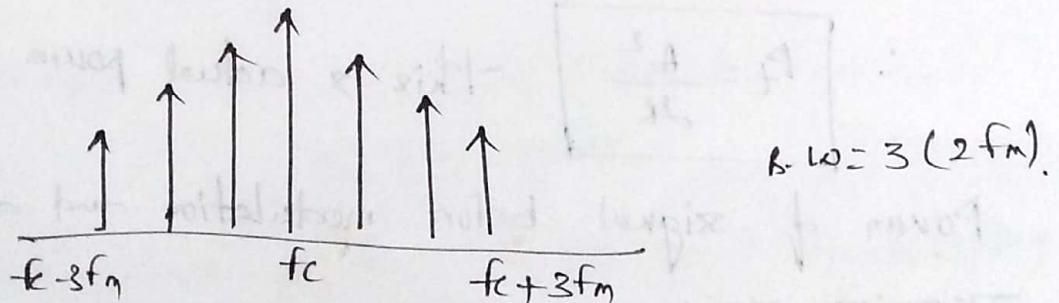
After band limiting,



- for SB's up to 2nd order



- for SB's up to 3rd order



- According to Carson, FM signal contains significant sidebands up to $(\beta + 1)$ order.

after Bandlimiting, $B.W = (\beta + 1) 2f_m$

W.K.T. $\beta = \frac{\Delta f}{f_m}$

$\therefore B.W = \left(\frac{\Delta f}{f_m} + 1\right) 2f_m$

$$B.W = (\Delta f + f_m) 2$$

→ for continuous band of frequency signal

$B.W = (\beta + 1) 2W$

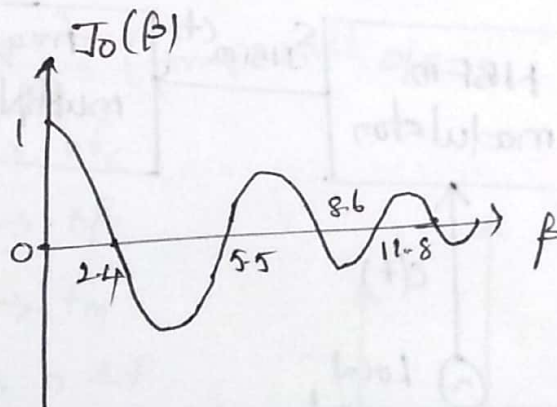
→ for effective B.W utilization AM is preferred.

AM $B.W = 2f_m$

FM $B.W = 2f_m + 2\beta f_m$

Modulation Efficiency (η) :-

W.K.T $P_c = \frac{A_c^2 J_0^2(\beta)}{2R}$



$J_0(\beta) = 0$, $\beta = 2.4, 5.5, 8.6, 11.8, \dots$
Eigen values.

→ for above values of β , $P_c = 0$ and modulation efficiency of WBFM corresponds to 100%.

Generation of FM signal :-

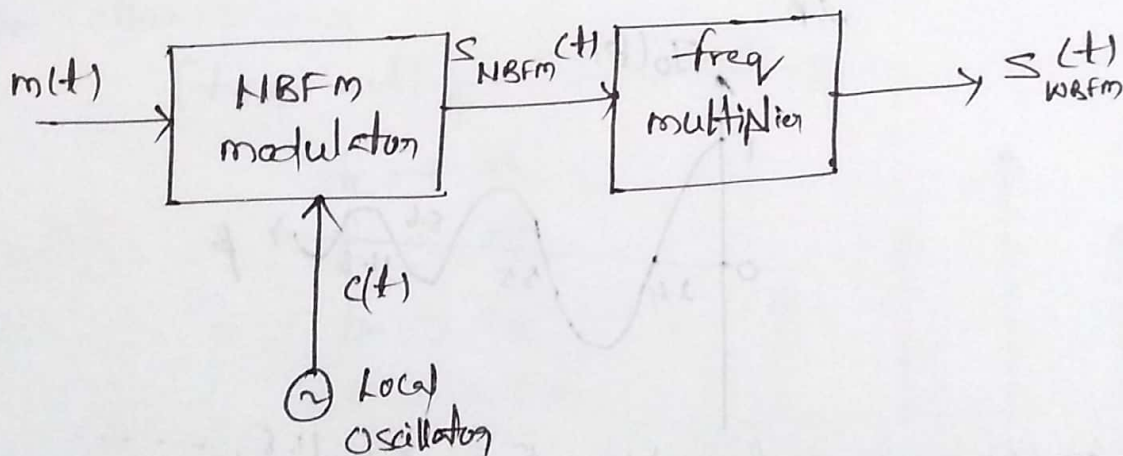
there are two methods of generating FM signal.

(i) Indirect Method

(ii) Direct Method.

Indirect Method :-

- * This method is also called as "Armstrong method".
- * In this method a narrow band FM is first produced by Modulating the carrier by the baseband S/L.
- * The resulting signal is then frequency multiplied to have the desired frequency deviation.
- * In this method NBFM signal is generated indirectly from NBFM signal.

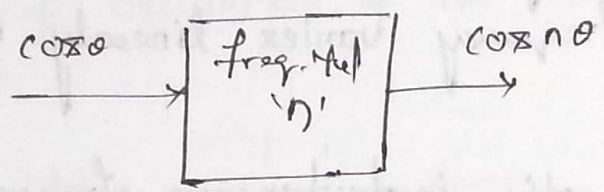
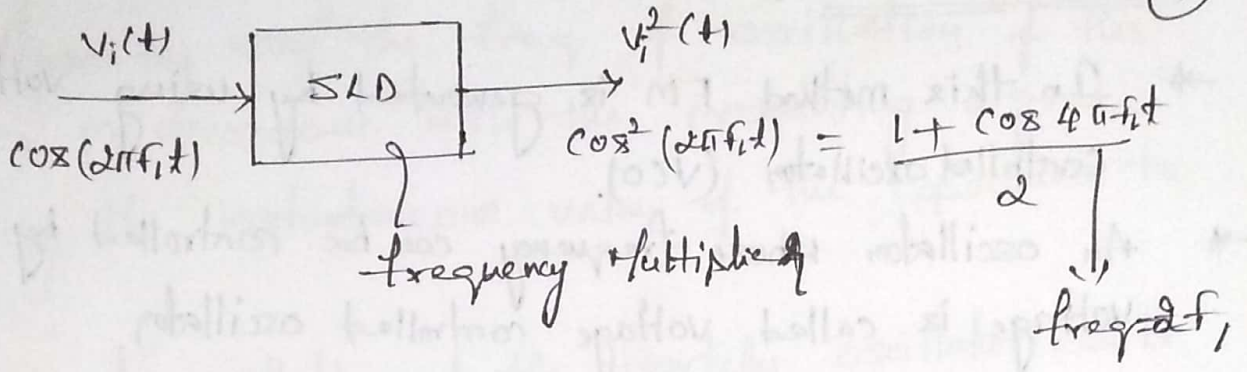


- * frequency multiplier is nothing but square law device followed by proper BPF.

WKT

$$v_i(t) \rightarrow \boxed{\text{SLD}} \rightarrow a_1 v_i(t) + a_2 v_i^2(t) + a_3 v_i^3(t) + \dots$$

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W.K.T. $S_{NRFM}(t) = A_c \cos \left[2\pi f_c t + \beta \sin(2\pi f_m t) \right] ; \beta \leq 1$

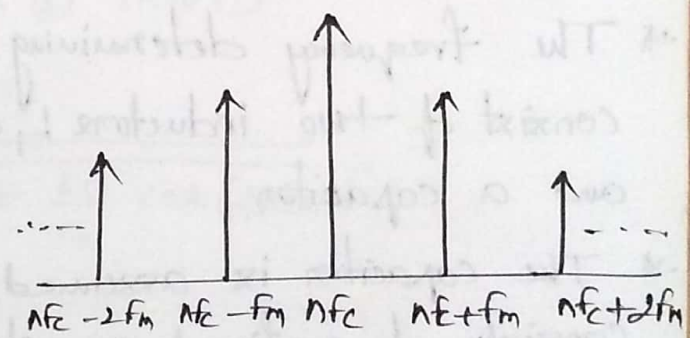
$(\text{freq. mul})_{o/p} = S_{NRFM}(t) = A_c \cos \left[2\pi n f_c t + n\beta \sin(2\pi f_m t) \right]$

'n' should be such that $n\beta > 1$

for this

$(\text{freq. mul})_{i/p} = (\text{freq. mul})_{o/p}$

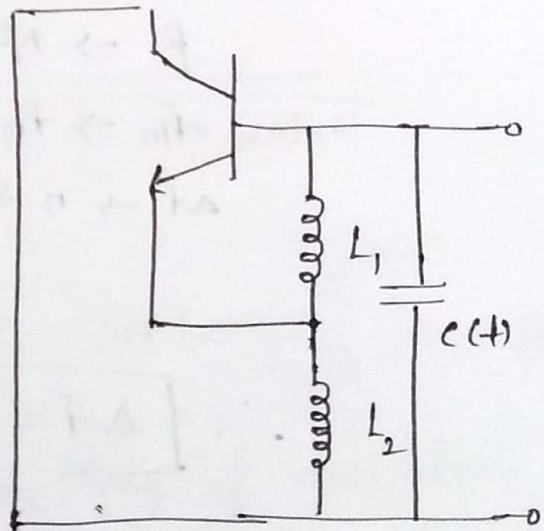
- $f_c \rightarrow n f_c$
- $\beta \rightarrow n\beta$
- $f_m \rightarrow f_m$
- $\Delta f \rightarrow n \Delta f$



$\therefore \Delta f = \beta f_m$

Direct Method :-

- * In this method FM is generated by using voltage controlled oscillator (VCO).
- * An oscillator whose frequency can be controlled by an external voltage is called voltage controlled oscillator.
- * The oscillation of frequency varies linearly with control voltage in VCO.
- * Thus in direct FM, the instantaneous frequency of carrier wave is made to vary in accordance with the baseband signal by using VCO.
- * Simple method for implementing is to use a sinusoidal oscillator having a relatively high value of 'quality factor' Q of frequency determining network.
- * To control the oscillation frequency by symmetrical incremental variation of the relative components.
- * The frequency determining N/W consist of two inductors L_1 and L_2 and a capacitor.
- * The capacitor is assumed to consist of a fixed capacitor shunted by a voltage variable capacitor such as varactor.
- * The capacitance of varactor diode varies by the applied voltage.
- * Applying the modulating signal across this capacitor.
- * It is possible to vary the capacitance in the



Frequency determining N/w. ~~and~~

(10)

- * And then by vary the freq of oscillation of the oscillator in accordance with the modulating signal.
- * Assuming the instantaneous value of the capacitance to be $c(t)$.
- * The freq. of oscillation of the Hartely oscillator can be written as

$$f_i(t) = \frac{1}{2\pi \sqrt{(L_1 + L_2) \cdot c(t)}} \rightarrow (1)$$

- * for a sinusoidal modulating signal $m(t)$ of freq. f_m .
- * the capacitance $c(t)$ can be expressed as

$$c(t) = c_0 + \Delta c \cos(2\pi f_m t) \rightarrow (2)$$

where c_0 is the total capacitance in the absence of modulation.

Δc is the maximum change in the capacitance value. substituting $c(t)$ from (2) into (1)

$$f_i(t) = \frac{1}{2\pi \sqrt{(L_1 + L_2) c_0}} \cdot \frac{1}{\sqrt{1 + \frac{\Delta c}{c_0} \cos(2\pi f_m t)}}$$

$$\therefore f_i(t) = f_0 \cdot \left(1 + \frac{\Delta c}{c_0} \cos(2\pi f_m t)\right)^{-1/2} \rightarrow (3)$$

where f_0 is unmodulated freq. of oscillation

$$\text{i.e., } f_0 = \frac{1}{2\pi \sqrt{(L_1 + L_2) c_0}} \rightarrow (4)$$

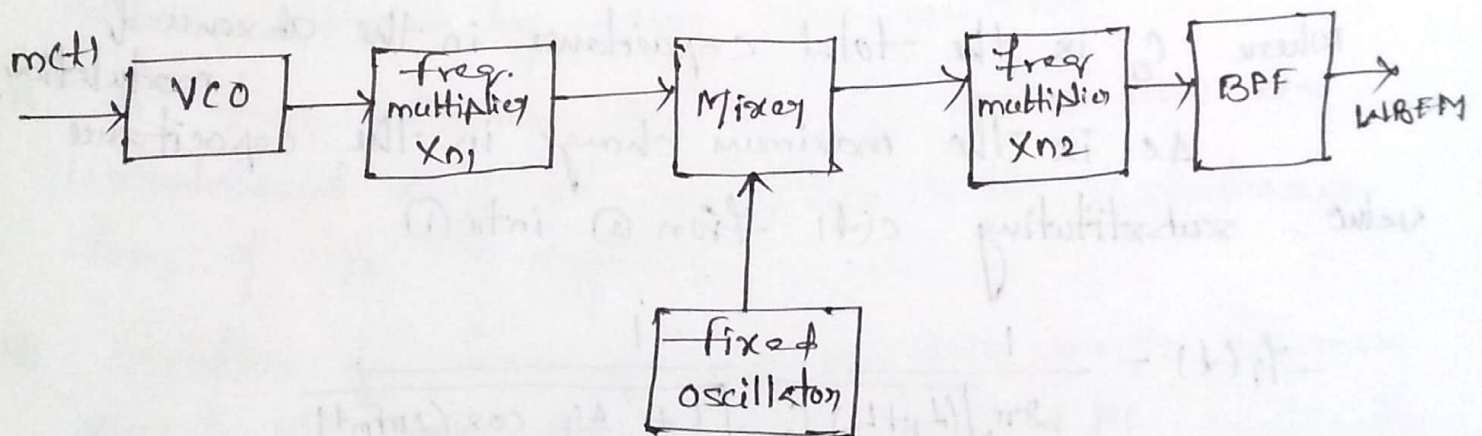
- * If the maximum change in capacitance ΔC is small compared with unmodulated capacitance C_0 .

$$f_i(t) = f_0 \cdot \left[1 - \frac{\Delta C}{2C_0} \cdot \cos(2\pi f_m t) \right]$$

Assuming $\frac{\Delta C}{2C_0} = -\frac{\Delta f}{f_0}$

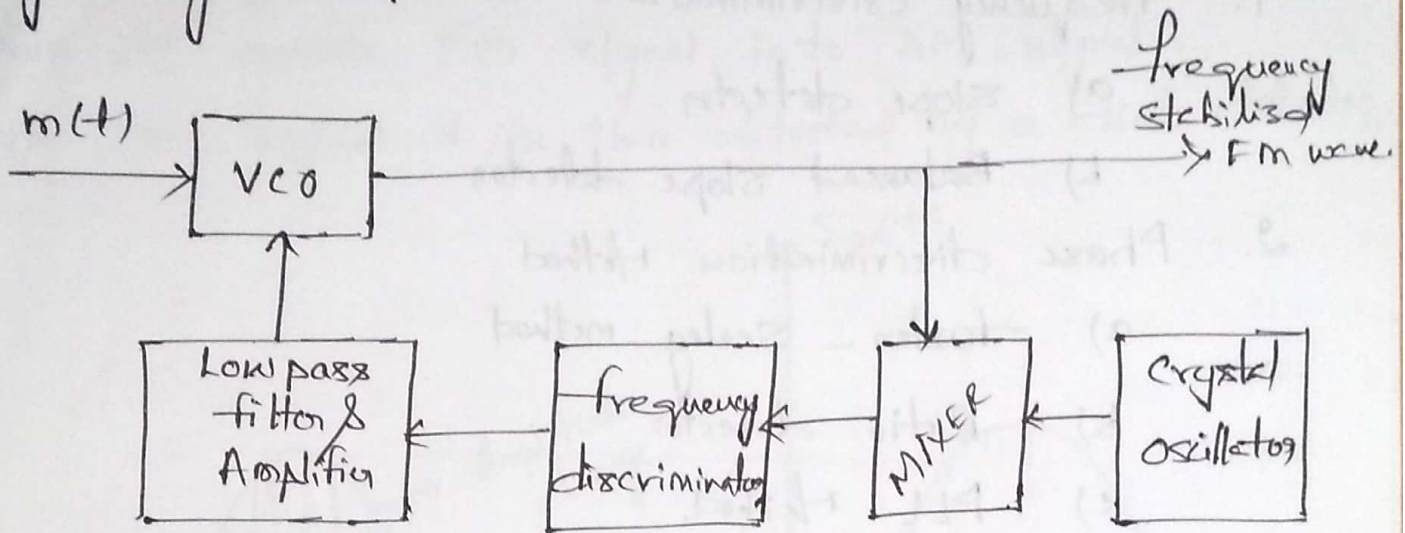
$$\therefore f_i(t) = f_0 + \Delta f \cos(2\pi f_m t)$$

- * The wave generated by this method is not wideband.
- * In order to obtain WBFM with required frequency deviation we may use the following.



- * In comparison with indirectly generated FM, the VCO generated FM,
- * the VCO generated FM produces large deviation so little multiplication is need to convert into WBFM.
- * this method has poor frequency stability. the reason is that in this method the basic oscillator is not stable oscillator like crystal oscillator.

- * So, feedback method is used to stabilize the signal generated. (ii)



- * The o/p of FM generator is applied to a mixer together with o/p from crystal oscillator, and difference is extracted.
- * The mixer o/p is applied to freq. discriminator and then filtered by using Low pass filter.
- * A freq. discriminator, the o/p voltage has an instantaneous amplitude proportional to the instantaneous freq. of i/p FM signal.
- * Deviation of freq. of generated FM w.r. to reference signal will cause freq. discrimination - LPF combination to develop a dc. voltage with a polarity determined by sense of freq. drift of modulator.
- * After amplification the dc voltage is fed back to VCO of the freq. modulator.
- * The freq. modulator will modify its freq. in a direction that tends to restore the carrier frequency to its required value.

FM Demodulation :-

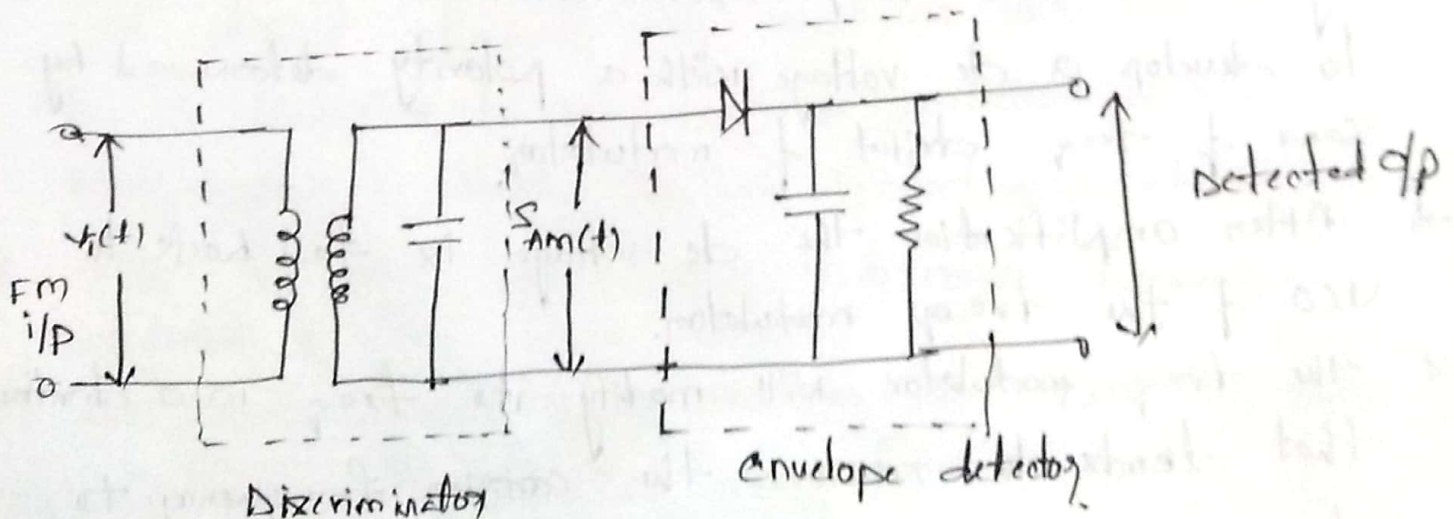
1. frequency discrimination Method
 - a) Slope detector
 - b) Balanced slope detector
2. Phase discrimination Method
 - a) Foster - Seeley method
 - b) Ratio detector
 - c) PLL Method.

→ Except PLL all other demodulators need transformers, for their construction, so that PLL demodulation is only ~~preferred~~ preferred for FM demodulation.

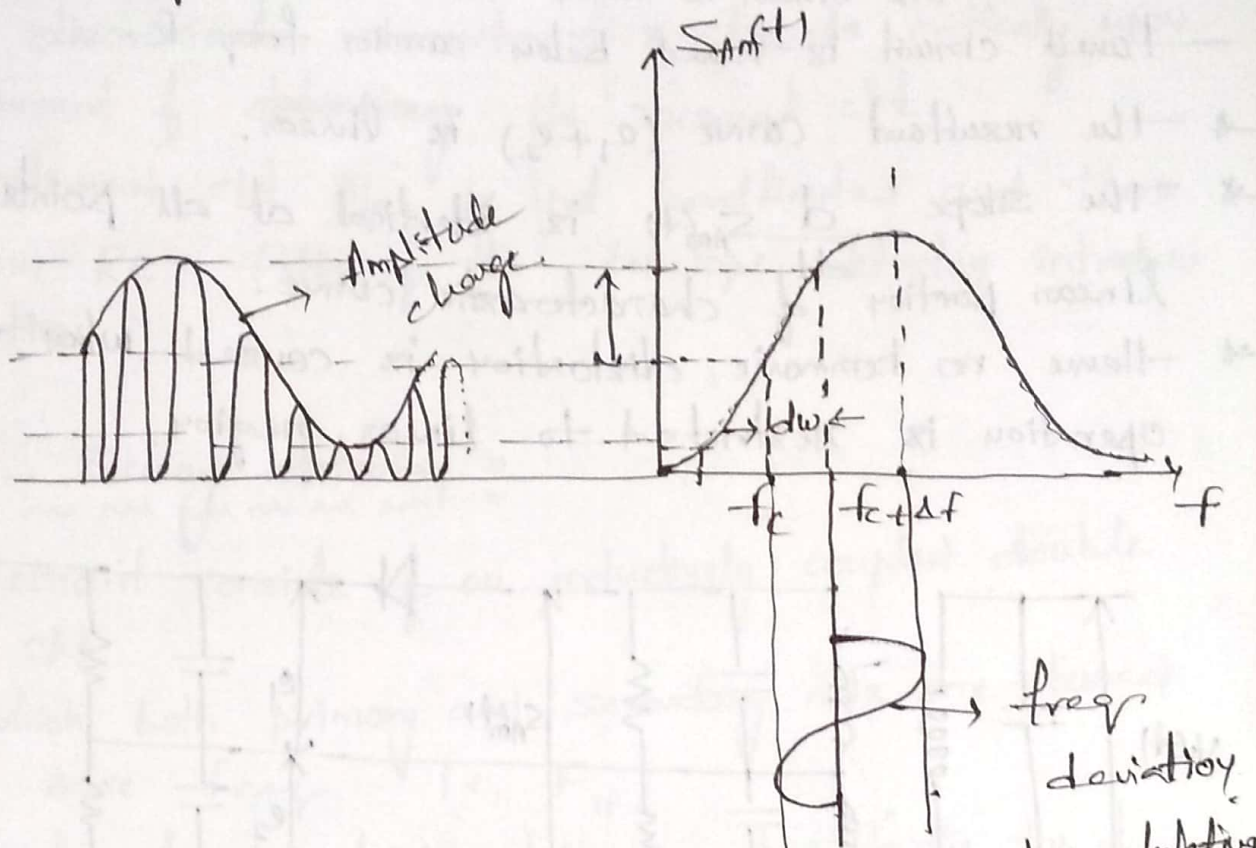
Slope detector :-

The principle of slope detector depends on the slope of the freq response characteristics of freq. selective network.

Simple tuned detector (Single-tuned ckt) :-



- * The circuit consists of a tuned ckt which is slightly tuned below carrier freq. f_c . (12)
- * this ckt converts FM signal into AM signal.
- * the AM signal is then detected by a diode detector.



- * At the output of envelope detector the original modulating signal is obtained.
- * this ckt is simple and inexpensive.
- * It does not eliminate the amplitude variations and the o/p is sensitive to any amplitude variation in the i/p FM signal, which causes distortion in demodulated signal.
- * this ckt has non-linear ch's cause harmonic distortion.
- * the non-linearity is due to fact that the ~~the~~ slope is not same at every point.

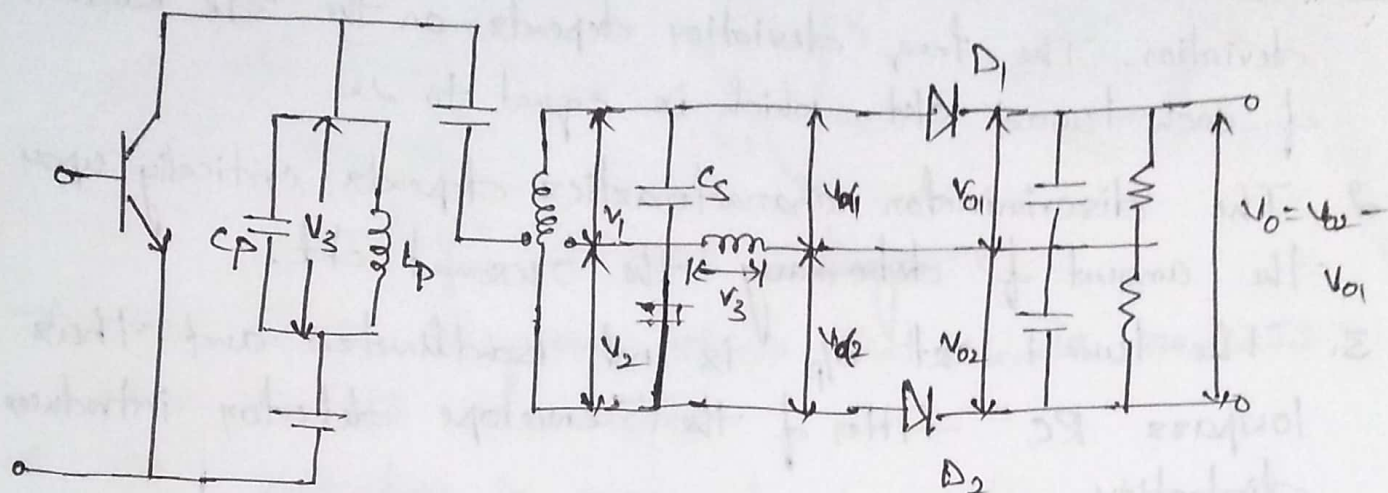
Disadvantages :-

(13)

1. The linear characteristic is limited to a small freq deviation. The freq deviation depends on the 3dB bandwidth of each-tuned ckt which is equal to 2ω .
2. The discriminator characteristics depends critically upon the amount of ~~determining~~ the resonant ckt.
3. The tuned ckt o/p is not bandlimited and thus lowpass RC filter of the envelope detector introduces distortion.

Foster - Seely detector :-

- * this circuit consists of an inductively coupled double-tuned ckt.
- * in which both primary and secondary coils are tuned to the same freq. i.e., F_{if} .
- * The center of secondary coil is connected to the top of primary through a capacitor C .
- * The capacitor blocks dc from primary to secondary and it couples the signal freq from primary to center-tapping of the secondary.
- * ~~Entire~~ entire voltage appears across L except a small drop across C .
- * By suitable choice of C and L drop across C can be negligible.
- * The center-tapping of secondary coil has an equal and opposite voltage across each half winding.



* V_1 and V_2 are equal in magnitude but opposite in phase.

* The radio freq voltages $V_{\alpha 1}$ and $V_{\alpha 2}$ applied to the diodes D_1 & D_2 are given as

$$V_{\alpha 1} = V_3 + V_1, \quad V_{\alpha 2} = V_3 - V_2$$

* $V_{\alpha 1}$ and $V_{\alpha 2}$ depends on phasor relationship b/w V_1, V_2 & V_3 .

* The phase portion of V_1 & V_2 relative to V_3 depend upon
— tuned secondary coil at resonance (or) off the resonance.

At resonance :- When i/p voltage has a freq equal to the resonant freq. If of tuned secondary V_3 is inphase quadrature with V_1 and V_2 . The magnitude of V_1 & V_2 are equal.

Off resonance :- When i/p signal freq is above the resonant freq. the phase difference b/w V_3 & V_1 is 145° . The phase difference b/w V_3 & V_2 is 135° .

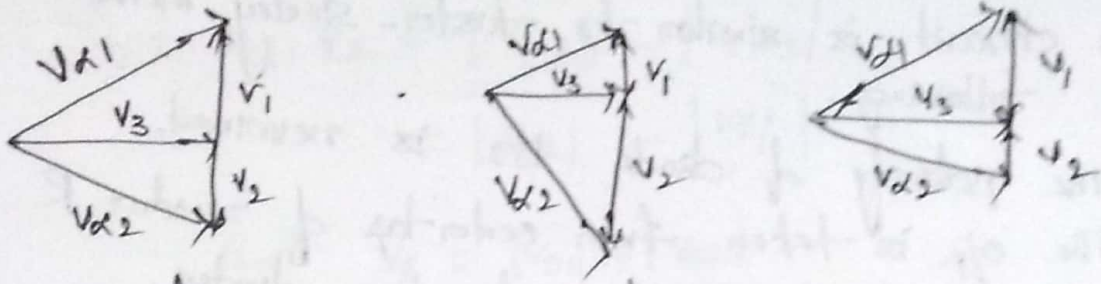


Fig: phasor diagrams

- * The phase diagrams shows that V_{d1} is reduced where V_{d2} is increased.
- * The situation is reversed when voltage has freq below. Fig.
- * Hence the amplitude of voltage V_{d1}, V_{d2} will vary with instantaneous frequency.

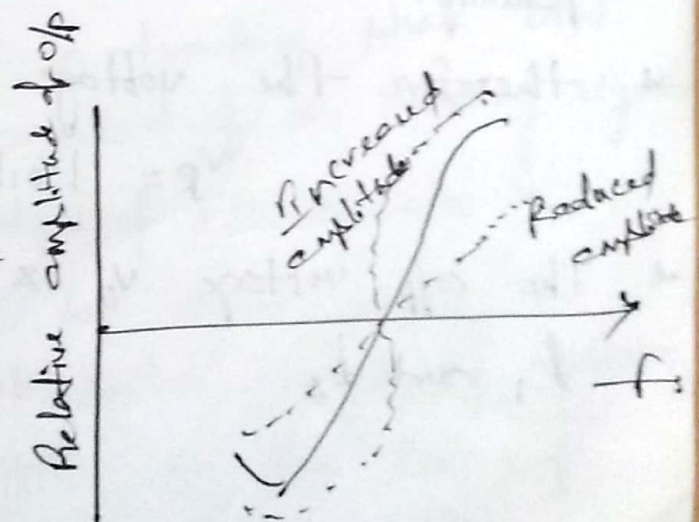
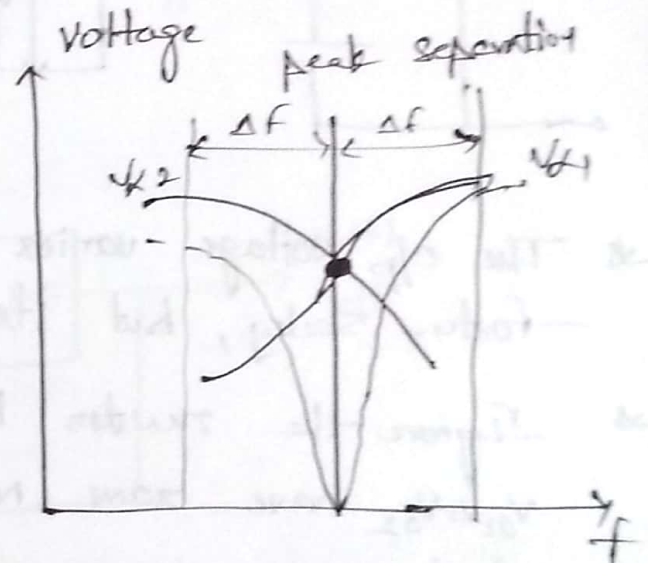
* The RF voltage V_{d1} and V_{d2} are rectified by the diodes D_1 & D_2 to produce voltage V_{o1} & V_{o2} .

* The RF components are passed by capacitor leaving only modulating freq components and a dc term.

* The o/p voltage V_o is

$$V_o = |V_{o2}| - |V_{o1}|$$

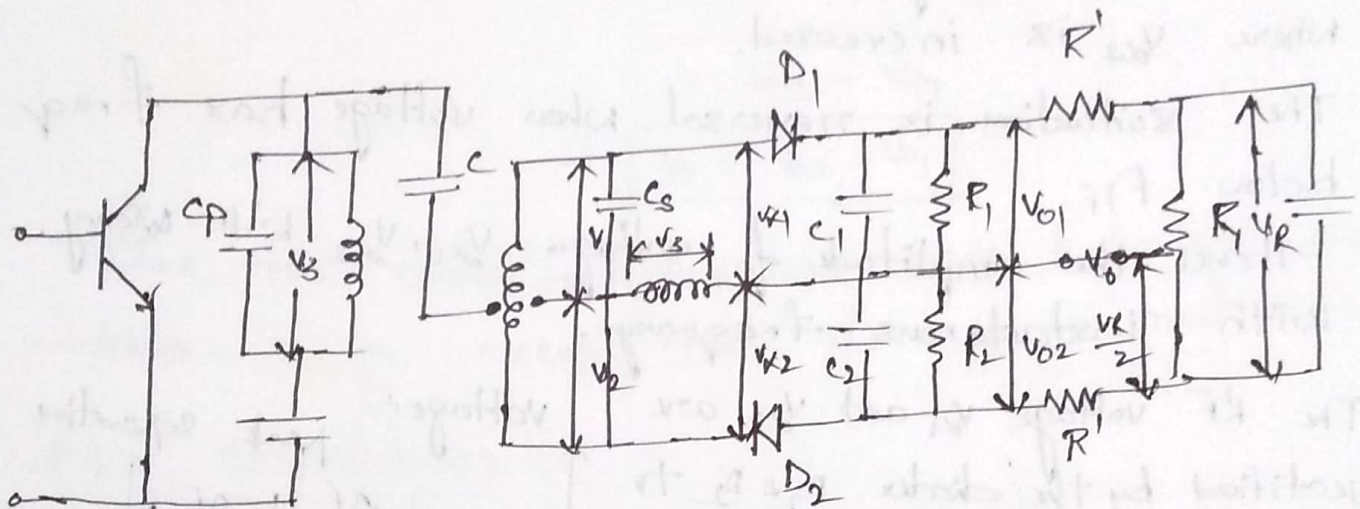
Hence V_o will vary with instantaneous freq. in accordance with difference $|V_{d2}| - |V_{d1}|$.



Ratio detector :-

* This circuit is similar to Foster-Seeley method except the following.

- The polarity of diode D_2 is reversed.
- The o/p is taken from center tap of resistor R which shunts the load impedance of two diodes.



- * The o/p voltage varies with i/p signal frequency as in Foster-Seeley, but its magnitude is reduced to half.
- * Ignore the resistor R' and capacitor C , the voltage V_{01} & V_{02} have same magnitude but V_{02} is of reverse polarity.
- * therefore the voltage V_R is sum of $V_{01} + V_{02}$

$$V_R = |V_{01}| + |V_{02}|$$
- * The o/p voltage V_0 is taken across the terminals t_1 and t_2 .

for the ckt diagram

(15)

$$V_0 = V_{E1, t2} = |V_{E, t1}| - |V_{E, t2}|$$

$$= |V_{O2}| - |V_{R/2}|$$

But $V_R = |V_{O1}| + |V_{O2}|$

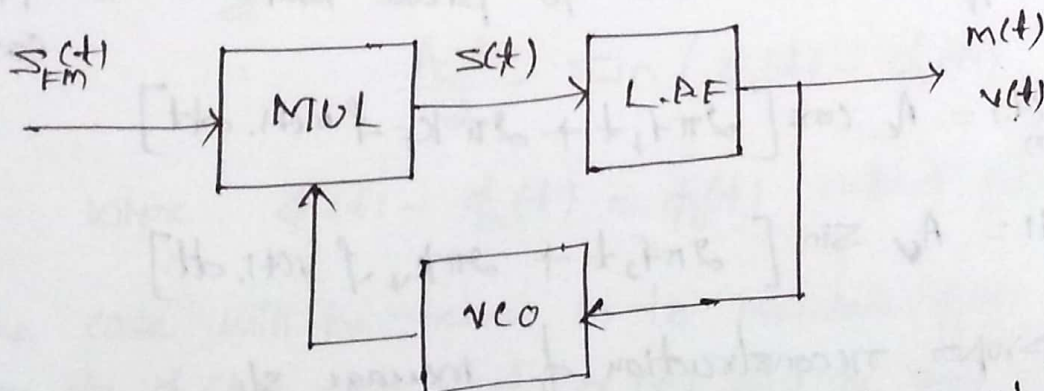
$$\therefore V_0 = V_{O2} - \left(\frac{V_{O1} + V_{O2}}{2} \right)$$

$$= V_{O2} - V_{O1}$$

$$\therefore \boxed{V_0 = V_{O2} - V_{O1}}$$

Thus, the ratio detector has same behaviour as Foster-Seeley except that its o/p is reduced.

PLL Demodulator :-



- * "Phase locked loop" is used in tracking phase and frequency of the carrier component of an incoming signal.
- * PLL is useful in demodulation of FM signals in presence of large noise and low signal power.
- * PLL is a negative feedback system consists of Multiplier, LPF and VCO connected together in the form of a feed back loop.

- for FM demodulation by synchronous detection, the local oscillator has to be replaced by VCO.
- The input of VCO will be message signal which is taken as feedback from output.
- As msg signal amplitude is varied, correspondingly freq. of VCO o/p will be varied.
- So, that freq synchronisation can be maintained, onto FM S/L to be demodulated.

$$k.f.T \quad S_{FM}(t) = A_c \cos \left[2\pi f_c t + 2\pi k_f \int m(t) dt \right]$$

$$(VCO)_{o/p} \rightarrow S_v(t) = A_v \cos \left[2\pi f_2 t + 2\pi k_v \int v(t) dt \right]$$

$$\text{Let } f_c = f_1, \quad 2\pi f_1 t + 2\pi k_f \int m(t) dt = \phi_1(t)$$

$$2\pi f_2 t + 2\pi k_v \int v(t) dt = \phi_2(t)$$

- VCO o/p should have 90° phase shift w.r.to Transmitter carrier

$$\therefore S_{FM}(t) = A_c \cos \left[2\pi f_1 t + 2\pi k_f \int m(t) dt \right]$$

$$S_v(t) = A_v \sin \left[2\pi f_2 t + 2\pi k_v \int v(t) dt \right]$$

- for proper reconstruction of message S/L
 1. f_1 should be made equal to f_2 then PLL is said to be in lock mode.
 2. $\phi_1(t)$ should be made equal to $\phi_2(t)$ then PLL is said to be in capture mode.
- Due to input of negative feedback formed by VCO, within no time f_1 is made is equal to f_2 and PLL will go to lock mode.

$$\text{i.e., } f_1 = f_2 = f_c \quad (16)$$

$$\therefore S_{\text{Tx}}(t) = A_c \cos [2\pi f_c t + \phi_1(t)]$$

$$S_{\text{V}}(t) = A_v \sin [2\pi f_c t + \phi_2(t)]$$

→ the multiplication o/p is

$$(\text{mul})_{\text{o/p}} = S_{\text{Tx}}(t) \cdot S_{\text{V}}(t)$$

$$= A_c A_v \cos (2\pi f_c t + \phi_1(t)) \cdot \sin [2\pi f_c t + \phi_2(t)]$$

$$\text{W.K.T } \boxed{2 \cos A \sin B = \sin(A+B) - \sin(A-B)}$$

$$\therefore (\text{mul})_{\text{o/p}} = \frac{A_c A_v}{2} \left[\sin (2\pi f_c t + 2\pi f_c t + \phi_1(t) + \phi_2(t)) - \frac{A_c A_v}{2} \left[\sin (2\pi f_c t - 2\pi f_c t + \phi_1(t) - \phi_2(t)) \right] \right]$$

$$S(t) = \frac{A_c A_v}{2} \sin (4\pi f_c t + \phi_1(t) + \phi_2(t)) - \frac{A_c A_v}{2} \sin (\phi_1(t) - \phi_2(t))$$

where $\phi_1(t) - \phi_2(t) = \phi_e(t)$ called phase error.

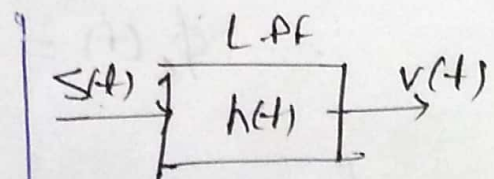
→ The case will be taken in to maintain $\phi_1(t)$ is very close to $\phi_2(t)$ so that $\phi_e(t)$ will be very small.

$$\therefore S(t) = \frac{A_c A_v}{2} \sin (4\pi f_c t + \phi_1(t) + \phi_2(t)) - \frac{A_c A_v}{2} \sin [\phi_e(t)]$$

$$\text{where } \phi_e(t) = \text{small}$$

$$\sin \phi_e(t) = \phi_e(t)$$

$$V(t) = \phi_e(t) \otimes h(t)$$



$$V(t) = S(t) \otimes h(t)$$

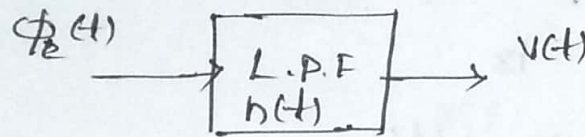
$$= (1 + 2) \otimes h(t)$$

$$V(t) = 1 \otimes h(t) + 2 \otimes h(t)$$

↳ not allowed

→ 1st term of above eqⁿ contains very high freq and is not allowed by L.P.F.

Since, $\phi_e(t)$ is very small, $\sin \phi_e \approx \phi_e(t)$



$$v(t) = \phi_e(t) \otimes h(t)$$

$$V(f) = \phi_e(f) \cdot H(f) \longrightarrow \textcircled{1}$$

WKT $\phi_e(t) = \phi_1(t) - \phi_2(t)$

$$= \phi_1(t) - 2\pi k_v \int v(t) \cdot dt$$

$$\phi_e(t) = \phi_1(t) - 2\pi k_v \int \phi_e(t) \otimes h(t) \cdot dt$$

—taking derivative over both sides,

$$\frac{d}{dt} \phi_e(t) = \frac{d}{dt} \phi_1(t) - 2\pi k_v [\phi_e(t) \otimes h(t)]$$

—taking F-T on B.S.

$$j2\pi f \phi_e(f) = j2\pi f \phi_1(f) - 2\pi k_v [\phi_e(f) \cdot H(f)]$$

$$j + f \phi_e(f) = j + f \phi_1(f) - k_v [\phi_e(f) \cdot H(f)]$$

$$\phi_e(f) [jf + k_v H(f)] = jf \cdot \phi_1(f)$$

$$\therefore \phi_e(f) = \frac{jf \cdot \phi_1(f)}{jf + k_v H(f)}$$

if $H(f) = \infty$, $\phi_e(f) = 0$,

$$\phi_e(t) = 0 \Rightarrow \phi_1(t) = \phi_2(t)$$

$$\phi_2(F) = \frac{jF \phi_1(F)}{jF \left[1 + \frac{k_v}{jF} \cdot H(F) \right]}$$

$$\phi_2(F) = \frac{\phi_1(F)}{1 + \frac{k_v}{jF} \cdot H(F)}$$

Since $H(F)$ is high, $\Rightarrow 1 + \frac{k_v}{jF} \cdot H(F) \approx \frac{k_v}{jF} \cdot H(F)$

$$\therefore \phi_2(F) = \frac{\phi_1(F)}{\frac{k_v}{jF} \cdot H(F)}$$

From eqn (1)

$$V(F) = \frac{\phi_1(F)}{\frac{k_v \cdot H(F)}{jF}}$$

$$V(F) = \frac{jF \phi_1(F) \cdot H(F)}{k_v \cdot H(F)}$$

$$\Rightarrow V(F) = \frac{jF \phi_1(F)}{k_v} \times \frac{2\pi}{2\pi} \cdot H(F)$$

taking Inverse Fourier transform on both sides

$$v(t) = \frac{1}{2\pi k_v} \cdot \frac{d}{dt} \phi_1(t)$$

$$= \frac{1}{2\pi k_v} \cdot \frac{d}{dt} \left[2\pi k_f \cdot \int m(t) \cdot dt \right]$$

$$\boxed{v(t) = \frac{k_f}{k_v} \cdot m(t)}$$

- * frequency produced by VCO corresponds to lock mode of PLL.
- * LPF is responsible for capture mode of PLL.
- * PLL demodulation is essentially differentiation process.