

Electromagnetic Theory

chapter-3 Problems

- Q1) In a free space ($z \leq 0$) a plane wave with
 $\vec{H} = 10 \cos(10^8 t - \beta z) \hat{a}_x \text{ mA/m}$
 is incident normally on a lossless medium ($\epsilon = 2\epsilon_0$ and $\mu = 8\mu_0$) in region $z \geq 0$. Determine the reflected wave H_r, E_r and the transmitted wave H_t, E_t .

Soln:

$$E_r = -\frac{10}{3} \eta_0 \cos(10^8 t + \frac{1}{3} z) \hat{a}_y \text{ mV/m}$$

$$H_r = -\frac{10}{3} \cos(10^8 t + \frac{1}{3} z) \hat{a}_x \text{ mA/m}$$

$$E_t = -\frac{40}{3} \eta_0 \cos(10^8 t - \frac{4}{3} z) \hat{a}_y \text{ mV/m}$$

$$H_t = \frac{20}{3} \cos(10^8 t - \frac{4}{3} z) \hat{a}_x \text{ mA/m}$$

- Q2) A 5-GHz uniform plane wave $E_i = 10 e^{-j\beta z} \hat{a}_x \text{ V/m}$ in free space is incident normally on large plane, lossless dielectric slab ($z > 0$) having $\epsilon = 4\epsilon_0$ $\mu = \mu_0$. Find the reflected wave E_r and the transmitted wave E_t .

Soln:

$$-3.333 \exp(j\beta_1 z) \hat{a}_x \text{ V/m}, \quad 6.667 \exp(-j\beta_2 z) \hat{a}_x \text{ V/m}$$

$$\text{where } \beta_2 = 2\beta_1 = 200\pi/3$$

- Q3) Given a uniform plane wave in air as
 $E_i = 40 \cos(\omega t - \beta z) \hat{a}_x + 30 \sin(\omega t - \beta z) \hat{a}_y \text{ V/m}$.

a) Find H_i

b) If the wave encounters a perfectly conducting plate normal to the z axis at $z=0$ find the reflected wave $\vec{E}_r$ and $\vec{H}_r$.

c) what are the total $\vec{E}$ and $\vec{H}$ fields for $z \leq 0$?

d) calculate the time-average pointing vectors for $z \leq 0$ and $z \geq 0$

Soln: $\vec{H}_i = -\frac{1}{4\pi} \sin(\omega t - \beta z) \hat{a}_x + \frac{1}{3\pi} \cos(\omega t - \beta z) \hat{a}_y \text{ mA/m}$

$$\vec{E}_r = -40 \cos(\omega t + \beta z) \hat{a}_x - 30 \sin(\omega t + \beta z) \hat{a}_y \text{ V/m}$$

$$\vec{H}_r = \frac{1}{3\pi} \cos(\omega t + \beta z) \hat{a}_y - \frac{1}{4\pi} \sin(\omega t + \beta z) \hat{a}_x \text{ A/m}$$

$$E_1 = 0 \quad E_2 = 0$$

$$P_{1, \text{avg}} = 0 \quad P_{2, \text{avg}} = 0$$

94) The plane wave $\vec{E} = 50 \sin(\omega t - 5x) \hat{a}_y$ V/m in a lossless medium ($\mu = 4\mu_0$, $\epsilon = \epsilon_0$) encounters a lossy medium ($\mu = \mu_0$, $\epsilon = 4\epsilon_0$, $\sigma = 0.1$ mhos/m) normal to x-axis at $x=0$. Find

- a) Γ , τ , S b) E_x and H_x
c) E_t and H_t d) the time average Poynting vectors in both regions.

soln:

a) $0.8186 \angle 171.1^\circ$, $0.2295 \angle 33.56^\circ$, 10.025

b) $40.93 \sin(\omega t + 5x + 171.9^\circ)$ V/m,
 $-54.3 \sin(\omega t + 5x + 171.9^\circ) \hat{a}_z$ mA/m

c) $11.47 e^{-6.021x} \sin(\omega t - 7.826x + 33.56^\circ) \hat{a}_y$ V/m,
 $120.2 e^{-6.021x} \sin(\omega t - 7.826x - 4.01^\circ) \hat{a}_z$ mA/m

d) $0.5469 \hat{a}_x$ W/m², $0.5469 \exp(-12.04x) \hat{a}_x$ W/m²

95) An EM wave travels in free space with electric field component

$$E_s = 100 e^{j(0.866y + 0.5z)} \hat{a}_x \text{ V/m}.$$

Determine

- a) ω and λ
b) The magnetic field component
c) The time average power in the wave.

soln:

a) $\omega = 3 \times 10^8$ rad/sec, $\lambda = 6.283$ m

b) $H_s = (1.33 \hat{a}_y - 2.3 \hat{a}_z) e^{j(0.866y + 0.5z)}$ mA/m

c) $11.49 \hat{a}_y + 6.631 \hat{a}_z$ W/m²

96) Repeat 95 for

$$\vec{E} = (10 \hat{a}_y + 5 \hat{a}_z) \cos(\omega t + 2y - 4z) \text{ V/m in free space}$$

soln:

a) $\omega = 1.342 \times 10^9$ rad/sec, $\lambda = 1.405$ m

b) $-29.66 \cos(1.342 \times 10^9 t + 2y - 4z) \hat{a}_x$ mA/m

c) $-0.07415 \hat{a}_y + 0.1489 \hat{a}_z$ W/m²

Q7) A uniform plane wave in air with $\vec{E} = 8 \cos(\omega t - 4x - 3z) \hat{a}_y$ V/m is incident on a dielectric slab ($z \geq 0$) with $\mu_r = 1$, $\epsilon_r = 2.5$, $\sigma = 0$. Find

- The polarisation of wave
- The angle of incidence
- The reflected $\vec{E}$ field
- The transmitted $\vec{H}$ field.

Soln:

- perpendicular polarization
- $\theta_i = 53.13^\circ$
- $\vec{E}_r = -3.112 \cos(15 \times 10^8 t - 4x + 3z) \hat{a}_y$ V/m
- $\vec{H}_t = (-17.69 \hat{a}_x + 10.37 \hat{a}_z) \cos(15 \times 10^8 t - 4x - 6.819z)$ mA/m

Q8) If the plane wave of given in Q6 is incident on a dielectric medium having $\sigma = 0$, $\epsilon = 4\epsilon_0$, $\mu = \mu_0$ and occupying $z \geq 0$ calculate

- The angle of incidence, reflection and transmission
- The reflection and transmission coefficients.
- The total $\vec{E}$ field in free space
- The total $\vec{E}$ field in the dielectric
- The Brewster angle.

Soln:

- $26.56^\circ, 26.56^\circ, 12.92^\circ$
- $-0.295, 0.647$
- $(10 \hat{a}_y + 5 \hat{a}_z) \cos(\omega t + 2y - 4z) + (-2.946 \hat{a}_y + 1.473 \hat{a}_z) \cos(\omega t + 2y + 4z)$ V/m
- $(7.055 \hat{a}_y + 1.618 \hat{a}_z) \cos(\omega t + 2y - 8.718z)$ V/m
- 63.43°

Q9) a) For a normal incidence upon the dielectric dielectric interface for which $\mu_1 = \mu_2 = \mu_0$, we define R and T as reflection and transmission coefficients for average power i.e., $P_{\text{ave}} = R P_{\text{ave}}$ and

$P_{t,ave} = T P_{i,ave}$. Prove that

a) $R = \left(\frac{n_1 - n_2}{n_1 + n_2} \right)^2$ and $T = \left(\frac{4n_1 n_2}{n_1 + n_2} \right)^2$

b) Determine the ratio n_1/n_2 so that the reflected and transmitted waves have the same average power where n_1 and n_2 are the refractive indices of the media

Q10) The plane wave $\vec{E} = 30 \cos(\omega t - z) \hat{a}_x$ V/m in air normally hits a lossless medium ($\mu = \mu_0$, $\epsilon = 4\epsilon_0$) at $z=0$.

a) Find Γ , τ and S

b) calculate the reflected electric & magnetic fields.

Soln:

a) $\Gamma = -1/3$, $\tau = 2/3$, $S = 2$

b) $E_x = -10 \cos(\omega t + z) \hat{a}_x$ V/m

$H_x = 26.53 \cos(\omega t + z) \hat{a}_y$ mA/m

Q11) A uniform plane wave in air with

$\vec{H} = 4 \sin(\omega t - 5x) \hat{a}_y$ A/m

is uniformly incident on a plastic region with the parameters $\mu = \mu_0$, $\epsilon = 4\epsilon_0$ and $\sigma = 0$.

a) obtain the total electric field in air

b) calculate the time-averaging power density in the plastic region.

c) Find the standing wave ratio.

Soln

a) $\vec{E} = -1.508 \sin(\omega t - 5x) \hat{a}_z + 0.503 \sin(\omega t + 5x) \hat{a}_z$ kV/m

b) $2.68 \hat{a}_x$ kW/m²

c) $S = 2$

Q12) A plane wave in free space with $\vec{E} = 3.6 \cos(\omega t - 3x) \hat{a}_y$ V/m is incident normally on an interface at $x=0$. If a lossless medium with $\sigma=0$, $\epsilon_r = 12.5$ exists for $x \geq 0$

and the reflected wave has $H_x = -1.2 \cos(\omega t + 3\pi) \hat{a}_x$ mA/m, find μ_2 .

soln:

$$\mu_2 = 2.604 \times 10^{-5} \text{ H/m}$$

✓Q13)

Region 1 is a lossless medium for which $y \geq 0$ $\mu = \mu_0$, $\epsilon = 4\epsilon_0$ where as region 2 is free space $y \leq 0$. If a plane wave $E = 5 \cos(10^8 t + \beta y) \hat{a}_z$ V/m exist in region 1. Find:

- total electric field component of wave in region 2.
- the time-average poynting vector in region 1
- the time-average poynting vector in region 2

✓Q14)

A plane wave having the electric field component $E_i = 24 \cos(3 \times 10^8 t - \beta y) \hat{a}_z$ and travelling in free space is incident normally on a lossless medium with $\mu = \mu_0$ and $\epsilon = 9\epsilon_0$ which occupies the region $y \geq 0$. The reflected magnetic field component is given by —

soln: $-\frac{1}{10\pi} \cos(3 \times 10^8 t + y) \hat{a}_x$ A/m

✓Q15)

The magnetic field component of a plane wave in a lossless dielectric is

$$H = 30 \sin(2\pi \times 10^8 - 5x) \hat{a}_z \text{ mA/m}$$

- If $\mu_r = 1$ and find ϵ_r
- calculate wavelength & wave velocity.
- calculate the wave impedance.
- Determine the polarisation of the wave
- Find the corresponding electric field component
- Find the displacement current density

✓Q16)

In nonmagnetic medium

$$E = 50 \cos(10^9 t - 8x) \hat{a}_y + 40 \sin(10^9 t - 8x) \hat{a}_z \text{ V/m}$$

find the dielectric constant ϵ_r , and the corresponding H .

✓ Q17) In a certain medium $\vec{E} = 10 \cos(2\pi \times 10^7 t - \beta x)(\hat{a}_y + \hat{a}_z) \text{ V/m}$
If $\mu = 50\mu_0$ and $\epsilon = \epsilon_0 \epsilon_r$ $\sigma = 0$. Find β and H .

✓ Q18) Which of the following media may be treated as conducting at 8 MHz? (MCQ)

- A) Wet marshy soil ($\epsilon = 15\epsilon_0$ $\mu = \mu_0$ $\sigma = 10^{-2} \text{ S/m}$)
- B) Intrinsic germanium ($\epsilon = 16\epsilon_0$ $\mu = \mu_0$ $\sigma = 0.025 \text{ S/m}$)
- C) sea water ($\epsilon = 81\epsilon_0$ $\mu = \mu_0$ $\sigma = 25 \text{ S/m}$)

Q19) calculate the skin depth and velocity of propagation for uniform plane wave at frequency 6 MHz in polyvinylchloride ($\mu_r = 1$, $\epsilon_r = 4$, $\tan \delta = 7 \times 10^{-2}$)

✓ Q20) An uniform plane wave in a lossy medium has a phase constant of 1.6 rad/m at 10^7 Hz and its magnitude reduced by 60% for every 2 m travelled. Find the skin depth and speed of the wave

✓ Q21) The distance (in meters) a wave has to propagate in a medium having a skin depth of 0.1 m so that the amplitude of wave attenuates by 20 dB is _____ (MCQ)

- A) 0.12 B) 0.23 C) 0.46 D) 2.3

✓ Q22) A uniform plane wave in air with $\vec{H} = 4 \sin(\omega t - 5x) \hat{a}_y \text{ A/m}$ is incident normally on a plastic region with parameters $\mu = \mu_0$ $\epsilon = 4\epsilon_0$ and $\sigma = 0$

- a) obtain the total electric field in air
- b) Find the standing wave ratio

✓ Q23) A plane wave in free space ($z \leq 0$) is incident normally on a large block of material with $\epsilon_r = 12$

$\mu_r = 3$ and $\sigma = 0$ which occupies $z \geq 0$. If the incident electric field is

$$E = 30 \cos(\omega t - z) \hat{a}_y \text{ V/m}$$

Find:

- ω
- the standing wave ratio
- the reflected magnetic field
- the average power density of transmitted wave.

✓ Q24) A 30 MHz uniform plane wave with

$$\vec{H} = 10 \sin(\omega t + \beta x) \hat{a}_z \text{ mA/m}$$

exists in region $x \geq 0$ having $\sigma = 0$, $\epsilon = 9\epsilon_0$, $\mu = 4\mu_0$.

At $x = 0$, the wave encounters free space. Determine

- The polarization of the wave
- the phase constant β
- the displacement current density in region $x \geq 0$.
- The reflection & transmitted magnetic fields.
- The average power density in each region.

✓ Q25) A uniform plane wave in air is normally incident

on a infinite lossless dielectric material having

$\epsilon = 3\epsilon_0$ and $\mu = \mu_0$. If the incident wave is $E_i =$

$$10 \cos(\omega t - 5) \hat{a}_y \text{ V/m. Find}$$

- λ and ω of the wave in air and the transmitted wave in dielectric field.
- The incident H_i field
- Γ and T
- The total electric field and time average power in both regions.

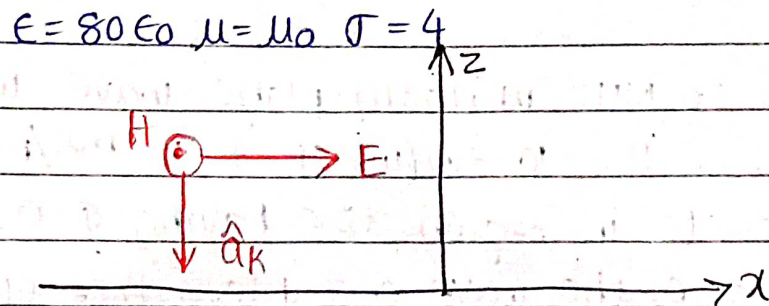
✓ Q26) A signal in air ($z \geq 0$) with electric field component

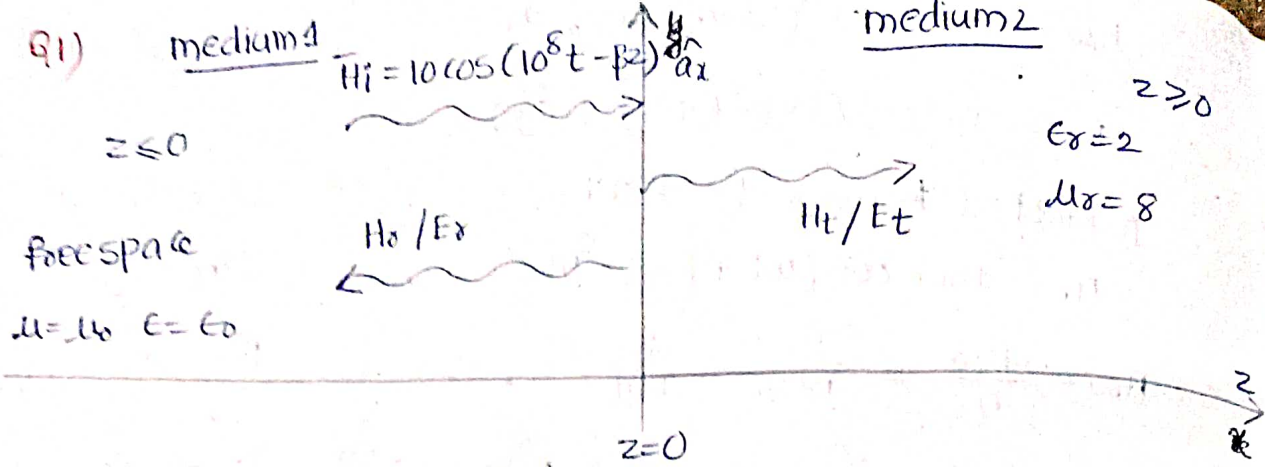
$$E = 10 \sin(\omega t + 3z) \hat{a}_x \text{ V/m}$$

hits normally the ocean surface at $z = 0$ as

shown in figure below. Assuming that the ocean surface is smooth and that $\epsilon = 80\epsilon_0$, $\mu = \mu_0$ and $\sigma = 4 \text{ mhos/m}$ in ocean. Determine:-

- ω
- The ~~air~~ wavelength of signal in air.
- The loss tangent and intrinsic impedance of ocean.
- The reflected & transmitted E field.





$$\beta_1 = \frac{\omega}{c} = \frac{10^8}{3 \times 10^8} = \frac{1}{3}$$

$$\eta_1 = 120\pi \Omega \quad \eta_2 = 120\pi \sqrt{\frac{\mu_r}{\epsilon_r}} = 120\pi \times \sqrt{\frac{8}{2}} = 240\pi$$

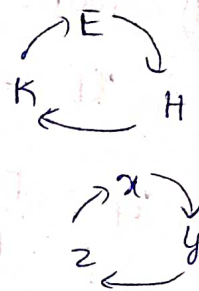
$$E_i = E_o \cos(\omega t - \beta_1 z) \hat{a}_{E_i} \text{ V/m}$$

$$\omega = 10^8 \text{ rad/sec} \quad \beta = \frac{1}{3} \text{ rad/m}$$

$$\hat{a}_{E_i} = \hat{a}_{H_i} \times \hat{a}_{k_i} = \hat{a}_x \times \hat{a}_z = -\hat{a}_y$$

$$E_{i0} = H_{i0} \eta_1 = 10 \times 120\pi \text{ mV/m}$$

$$= 1.2\pi \text{ mV/m}$$



$$\Gamma_E = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{240\pi - 120\pi}{240\pi + 120\pi} = \frac{120\pi}{360\pi} = \frac{1}{3}$$

$$\Gamma_E = \frac{E_{r0}}{E_{i0}} = \frac{1}{3}$$

$$E_{r0} = E_{i0} \times \frac{1}{3} = \frac{1.2 \times \pi}{3} = 0.4\pi$$

$$\vec{E}_r = E_{r0} \cos(\omega t + \beta z) (-\hat{a}_y)$$

$$E_r = 0.4\pi \cos(10^8 t + \frac{z}{3}) (-\hat{a}_y) \text{ V/m}$$

$$\vec{H}_r = \frac{E_{r0}}{\eta_1} \cos(10^8 t + \frac{z}{3}) \hat{a}_{H_r}$$

$$\hat{a}_{H_r} = \hat{a}_{k_r} \times \hat{a}_{E_r} = -\hat{a}_z \times -\hat{a}_y = -\hat{a}_x$$

$$\frac{H_{r0}}{\eta_1} H_{r0} = \frac{E_{r0}}{\eta_1} = \frac{0.4\pi}{120\pi} = \frac{0.4}{120} = \frac{1}{300} = \frac{10}{3} \times 10^{-3} = \frac{10}{3} \text{ mA/m}$$

$$\vec{H}_r = \frac{10}{3} \cos(10^8 t + \frac{z}{3}) (-\hat{a}_x) \text{ mA/m}$$

$$\text{Transmission coefficient } \tau_E = \frac{2\eta_2}{\eta_2 + \eta_1} = \frac{2 \times 240\pi}{240\pi + 120\pi}$$

$$\tau_E = \frac{480\pi}{360\pi} = \frac{4}{3}$$

$$\tau_E = \frac{E_{t0}}{E_{i0}} = \frac{4}{3}$$

$$E_{t0} = \frac{4}{3} \times E_{i0} = \frac{4}{3} \times 1.2\pi = 1.6\pi$$

$$\bar{E}_t = E_{t0} \cos(\omega t - \beta_2 z) (-\hat{a}_y) \text{ V/m}$$

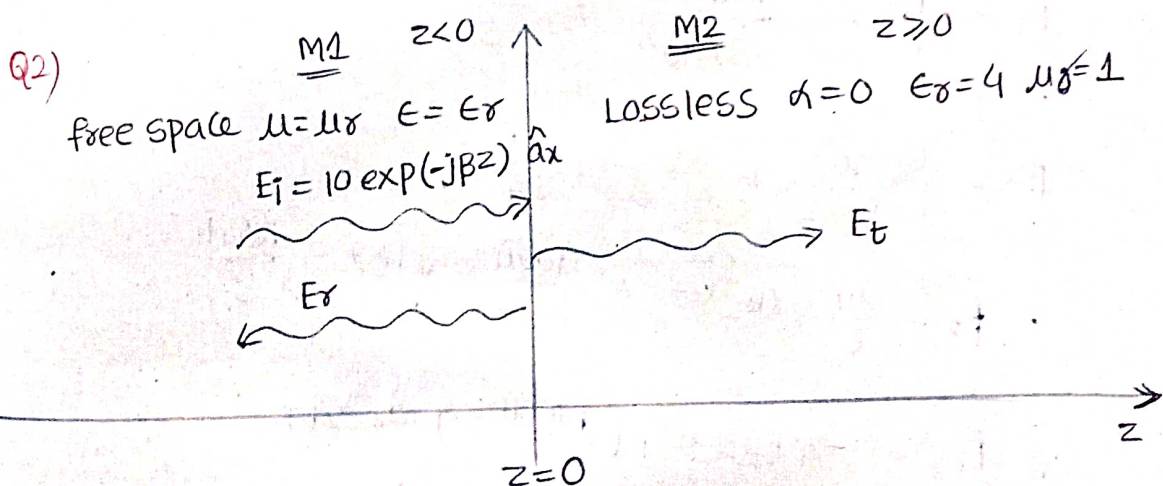
$$\beta_2 = \frac{\omega}{c} \sqrt{\mu_{r2} \epsilon_{r2}} = \frac{10^8}{3 \times 10^8} \times \sqrt{2 \times 8} = \frac{4}{3}$$

$$E_t = 1.6\pi \cos(10^8 t - \frac{4}{3} z) (-\hat{a}_y) \text{ V/m}$$

$$H_t = H_{t0} \cos(10^8 t - \frac{4}{3} z) \hat{a}_x$$

$$H_{t0} = \frac{E_{t0}}{\eta_2} = \frac{1.6\pi}{240\pi} = \frac{16}{2400} = \frac{2}{300} = \frac{20}{3} \times 10^{-3}$$

$$H_t = \frac{20}{3} \cos(10^8 t - \frac{4}{3} z) \text{ mA/m } \hat{a}_x$$



$$f = 5 \text{ GHz}$$

$$\eta_1 = 120\pi \quad \eta_2 = 120\pi \sqrt{\frac{\mu_{r2}}{\epsilon_{r2}}} = 120\pi \times \sqrt{\frac{1}{4}} = 60\pi$$

$$\text{Reflection coefficient } \Gamma_E = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{60\pi - 120\pi}{60\pi + 120\pi} = \frac{-60\pi}{180\pi} = -\frac{1}{3}$$

$$\Gamma_E = \frac{E_{r0}}{E_{i0}} = -\frac{1}{3}$$

$$\Rightarrow E_{r0} = -\frac{1}{3} \times E_{i0} = -\frac{1}{3} \times 10 = -\frac{10}{3} \text{ V/m}$$

$$\vec{E}_{os} = E_{o0} \exp(+j\beta_1 z) \hat{a}_x \text{ V/m}$$

$$= -\frac{10}{3} \exp(j\beta_1 z) \hat{a}_x \text{ V/m}$$

$$\vec{E}_{os} = -3.333 \exp(j\beta_1 z) \hat{a}_x \text{ V/m}$$

Transmission coefficient $\tau_E = \frac{2\eta_2}{\eta_2 + \eta_1} = \frac{2 \times 60\pi}{120\pi + 60\pi}$

$$\tau_E = \frac{120\pi}{180\pi} = \frac{2}{3}$$

$$\tau_E = \frac{E_{t0}}{E_{i0}} = \frac{2}{3}$$

$$\Rightarrow E_{t0} = \frac{2}{3} \times E_{i0} = \frac{2}{3} \times 10 = \frac{20}{3}$$

$$\vec{E}_{ts} = E_{t0} \exp(-j\beta_2 z) \hat{a}_x \text{ V/m}$$

$$\vec{E}_{ts} = \frac{20}{3} \exp(-j\beta_2 z) \hat{a}_x \text{ V/m}$$

$$\vec{E}_{ts} = 6.667 \exp(-j\beta_2 z) \hat{a}_x \text{ V/m}$$

$$\omega = 2\pi f = 2\pi \times 5 \times 10^9 \text{ rad/sec} = 10\pi \times 10^9 \text{ rad/sec}$$

$$\beta_1 = \frac{\omega}{c} = \frac{10\pi \times 10^9}{3 \times 10^8} = \frac{100\pi}{3}$$

$$\beta_2 = \frac{\omega}{c} \sqrt{\frac{\mu_{r2}}{\epsilon_{r2}}} = \frac{100\pi}{3} \times \sqrt{\frac{1}{4}}$$

$$\beta_2 = \frac{\omega}{c} \sqrt{\mu_{r2} \epsilon_r} = \frac{100\pi}{3} \times \sqrt{1 \times 4} = \frac{200\pi}{3}$$

$$\therefore \vec{E}_{os} = -3.333 \exp(j \frac{100\pi}{3} z) \hat{a}_x \text{ V/m}$$

$$\vec{E}_{ts} = 6.667 \exp(-j \frac{200\pi}{3} z) \hat{a}_x \text{ V/m}$$

Q3) $\vec{E}_i = \underbrace{40}_{E_{i01}} \cos(\omega t - \beta z) \hat{a}_x + \underbrace{30}_{E_{i02}} \sin(\omega t - \beta z) \hat{a}_y$ V/m

$\eta_1 = 120\pi \Omega$ (free space/air)

a) $H_{i01} = \frac{E_{i01}}{120\pi} = \frac{40}{120\pi} = \frac{1}{3\pi}$

$H_{i02} = \frac{E_{i02}}{120\pi} = \frac{30}{120\pi} = \frac{1}{4\pi}$

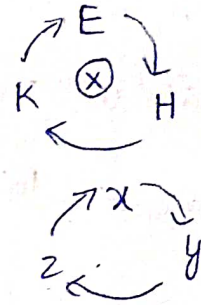
$\hat{a}_{ki} = \hat{a}_z$

$\hat{a}_{Hi1} = \hat{a}_{ki} \times \hat{a}_{Ei1} = \hat{a}_z \times \hat{a}_x = \hat{a}_y$

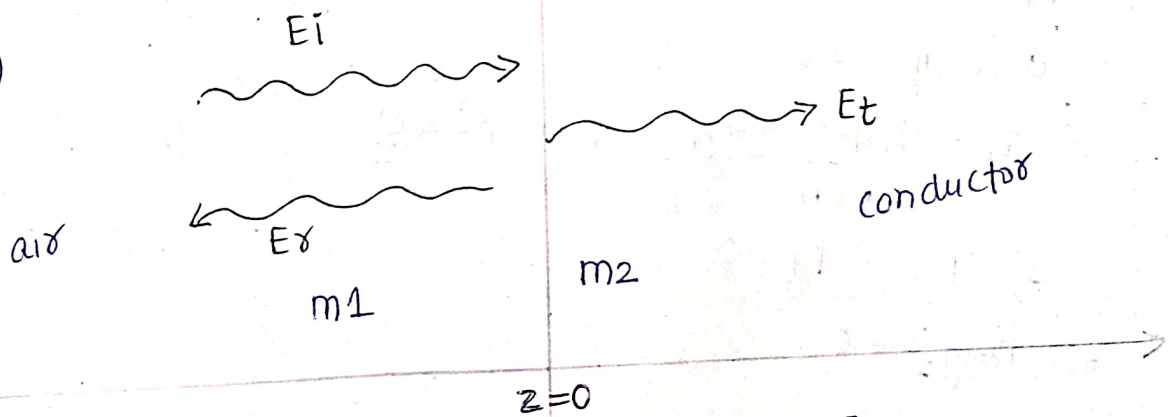
$\hat{a}_{Hi2} = \hat{a}_{ki} \times \hat{a}_{Ei2} = \hat{a}_z \times \hat{a}_y = -\hat{a}_x$

$\vec{H}_i = \frac{1}{3\pi} \cos(\omega t - \beta z) \hat{a}_y + \frac{1}{4\pi} \sin(\omega t - \beta z) (-\hat{a}_x)$

$\vec{H}_i = \frac{1}{4\pi} \sin(\omega t - \beta z) (-\hat{a}_x) + \frac{1}{3\pi} \cos(\omega t - \beta z) \hat{a}_y$



b)



medium 2 perfect conductor $\Rightarrow \frac{\sqrt{2}}{\omega \epsilon_2} \gg 1$

$\eta_2 \rightarrow 0$

$\Gamma_E = \frac{E_{r0}}{E_{i0}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = -1$

$E_{r0} = -E_{i0}$

$\Rightarrow \vec{E}_r = -40 \cos(\omega t + \beta z) \hat{a}_x - 30 \sin(\omega t + \beta z) \hat{a}_y$ V/m

$H_r = \frac{E_r}{\eta_1} = \frac{E_{r0}}{120\pi}$ $H_{r01} = \frac{-40}{120\pi} = \frac{-1}{3\pi}$ $H_{r02} = \frac{-30}{120\pi} = \frac{-1}{4\pi}$

$\hat{a}_{Hr1} = \hat{a}_{kr} \times \hat{a}_{Er1} = -\hat{a}_z \times \hat{a}_x = -\hat{a}_y$

$\hat{a}_{Hr2} = \hat{a}_{kr} \times \hat{a}_{Er2} = -\hat{a}_z \times \hat{a}_y = \hat{a}_x$

$\vec{H}_r = \left\{ \frac{-1}{3\pi} \cos(\omega t + \beta z) (-\hat{a}_y) + \left(\frac{-1}{4\pi} \right) \sin(\omega t + \beta z) \hat{a}_x \right\}$ A/m

c) We observe that

$$E_i = -E_r \quad \& \quad H_i = -H_r$$

$$\Rightarrow \left. \begin{aligned} E_i + E_r &= 0 \\ H_i + H_r &= 0 \end{aligned} \right\} \text{ for } z \leq 0$$

d) case (i) $z \leq 0$:

$$P_{avg_1} = \frac{E_{i0}^2}{2|\eta_1|} e^{-2\alpha z} \cos \theta \hat{a}_z + \frac{E_{r0}^2}{2|\eta_1|} e^{+2\alpha z} (-\hat{a}_z)$$

$$\alpha = 0 \quad \theta = 0$$

$$P_{avg_1} = \frac{1}{2 \times 120\pi} \left\{ (40^2 + 30^2) \hat{a}_z \right\} + \frac{1}{2 \times 120\pi} \left\{ [(-40)^2 + (-30)^2] (-\hat{a}_z) \right\}$$

$$P_{avg_1} = 0$$

case (ii) $z \geq 0$

$$\tau_E = \frac{2\eta_2}{\eta_2 + \eta_1} = 0 \quad \text{since } \eta_2 = 0$$

$$\Rightarrow E_{to} = E_t = 0$$

$$\Rightarrow P_{avg_2} = 0$$

Q4) Given $\vec{E} = 50 \sin(\omega t - 5x) \hat{y}$ V/m

medium 1 (lossless)

$$\mu_{r1} = 4$$

$$\epsilon_{r1} = 1$$

$$\beta_1 = 5 \text{ rad/m}$$

$$\eta_1 = 120\pi \sqrt{\frac{\mu_{r1}}{\epsilon_{r1}}}$$

$$\eta_1 = 120\pi \times \sqrt{\frac{4}{1}} = 240\pi \Omega$$

$$\beta_1 = \frac{\omega}{c} \sqrt{\mu_r \epsilon_r}$$

$$\Rightarrow 5 = \frac{\omega}{3 \times 10^8} \times \sqrt{4 \times 1}$$

$$\Rightarrow \omega = \frac{3 \times 5 \times 10^8}{2} = 7.5 \times 10^8 \text{ rad/sec}$$

medium 2 (lossy)

$$\mu_{r2} = 1$$

$$\epsilon_{r2} = 4$$

$$\sigma = 0.1 \text{ mhos/m}$$

$$x=0$$

$$\beta_2 = ?$$

$$\frac{\sigma}{\omega \epsilon} = \tan \theta = \frac{0.1 \times 36\pi}{7.5 \times 10^8 \times 10^{-9} \times 4}$$

$$= \frac{6}{5} \pi = 1.2\pi$$

$$\eta_2 = 120\pi \sqrt{\frac{\mu_{r2}}{\epsilon_{r2}}}$$

$$\sqrt{\sec \theta}$$

$$\sec^2 \theta = 1 + \tan^2 \theta$$

$$\sec \theta = \sqrt{1 + 1.44}$$

$$\eta_2 = 120\pi \sqrt{\frac{1}{4}} = \frac{60\pi}{(\sqrt{1 + 1.44\pi^2})^{1/2} (1 + 1.44\pi^2)^{1/4}}$$

$$\eta_2 = 95.445$$

$$\tan 2\theta_{\eta_2} = \tan \theta = 1.2\pi$$

$$\tan 2\theta_{\eta_2} = 1.2\pi$$

$$\theta_{\eta_2} = \frac{1}{2} \tan^{-1}(2\pi) = 37.57^\circ$$

$$\eta_2 = |\eta_2| \angle \theta_{\eta_2} = 95.445 \angle 37.57^\circ$$

$$a) \Gamma_E = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{95.445 \angle 37.57^\circ - 240\pi}{95.445 \angle 37.57^\circ + 240\pi} = 0.8186 \angle 171.08^\circ$$

$$\tau_E = 1 + |\Gamma_E| = 1 + 0.8186 \angle 171.08^\circ = 0.2295 \angle 33.56^\circ$$

$$S = \frac{1 + |\Gamma_E|}{1 - |\Gamma_E|} = \frac{1 + 0.8186}{1 - 0.8186} = 10.025$$

since medium 2 is lossy we need to find α & β .

$$\alpha = \frac{\omega}{c} \sqrt{\frac{\mu_{r2} \epsilon_{r2}}{2} \{ \sec \theta - 1 \}}$$

$$\alpha = \frac{7.5 \times 10^8}{3 \times 10^8} \times \sqrt{\frac{4}{2} \{ \sqrt{1 + 1.44\pi^2} - 1 \}}$$

$$\alpha = 6.021 \text{ NP/m}$$

$$\beta = \frac{\omega}{c} \sqrt{\frac{\mu_{r2} \epsilon_{r2}}{2} \{ \sec \theta + 1 \}}$$

$$\beta = \frac{7.5 \times 10^8}{3 \times 10^8} \times \sqrt{\frac{4}{2} \{ \sqrt{1 + 1.44\pi^2} + 1 \}}$$

$$\beta = 7.826 \text{ rad/m}$$

$$b) E_x = E_{x0} \cos(\omega t + 5x) \hat{a}_y = \operatorname{Re} \{ E_{x0} e^{j(\omega t + 5x)} \} \hat{a}_y$$

$$\frac{E_{x0}}{E_{i0}} = |\Gamma_E| \neq \Gamma_E$$

$$E_{x0} = E_{i0} |\Gamma_E| \neq \Gamma_E = 50 \times 0.8186 \neq 171.08$$

$$E_{x0} = 40.93 \neq 171.08 = 40.93 e^{j171.08}$$

$$E_{x0} = \operatorname{Re} \{ 40.93 e^{j171.08} e^{j(\omega t + 5x)} \} \hat{a}_y$$

$$E_{x1} = \operatorname{Re} \{ 40.93 e^{j(\omega t + 5x + 171.08)} \} \hat{a}_y$$

$$E_{x1} = 40.93 \cos(\omega t + 5x + 171.08) \hat{a}_y \text{ V/m}$$

$$H_x = \frac{E_{x0}}{|n_1|} \cos(\omega t + 5x + 171.08^\circ) \hat{a}_{H_{x1}} \quad K \quad E \quad H$$

$$\hat{a}_{H_{x1}} = \hat{a}_{K_{x1}} \times \hat{a}_{E_{x1}} = -\hat{a}_x \times \hat{a}_y = -\hat{a}_z$$

$$H_{x0} = \frac{E_{x0}}{|n_1|} = \frac{40.93}{240\pi} = -54.28 \text{ mA/m}$$

$$H_{x1} = -54.28 \cos(\omega t + 5x + 171.08^\circ) \hat{a}_z \text{ mA/m}$$

$$c) \frac{E_{t0}}{E_{i0}} = |\Gamma_E| \neq \Gamma_E$$

$$E_{t0}' = E_{i0} \times \{ |\Gamma_E| \neq \Gamma_E \}$$

$$E_{t0}' = 50 \times 0.2295 \neq 33.56 = 11.475 \neq 33.56$$

But 2nd medium is lossy so

$$E_t = E_{t0}' e^{-\alpha x} = 11.475 e^{-6.021x} \neq 33.56$$

$$E_t = \operatorname{Re} \{ E_{t0}' e^{j(\omega t - 5x + 33.56^\circ)} \} \hat{a}_y$$

$$= \operatorname{Re} \{ 11.475 e^{-6.021x} e^{j(33.56 + \omega t - 5x)} \}$$

$$E_t = 11.475 e^{-6.021x} \cos(\omega t - 5x + 33.56^\circ) \text{ V/m } \hat{a}_y$$

$$H_t = \frac{E_{t0}}{|n_2| \times n_2} \cos(\omega t - 5x + 33.56^\circ) \hat{a}_{Ht}$$

$$\hat{a}_{Ht} = \hat{a}_{Kt} \times \hat{a}_{Et} = \hat{a}_x \times \hat{a}_y = \hat{a}_z$$

$$H_{t0} = \frac{11.475}{95.445} \angle -37.57^\circ = 120.22 \angle -37.57^\circ \text{ mA/m}$$

$$H_t = 120.22 \cos(\omega t - 5x + 33.56 - 37.57) \hat{a}_z \text{ mA/m}$$

$$H_t = 120.22 \cos(\omega t - 5x - 4.01^\circ) \hat{a}_z \text{ mA/m}$$

Q5) given $\vec{E}_s = 100 e^{j(0.866y + 0.5z)} \hat{a}_x$
 We know that $\beta = \frac{\omega}{c}$

$\Rightarrow \omega = \beta c$

a) General expression for E field $\vec{E} = E_0 e^{j(\omega t - \vec{k} \cdot \vec{r})} \hat{a}_E$
 In phasor form $\vec{E}_s = E_0 e^{-j(\vec{k} \cdot \vec{r})} \hat{a}_E$

$\Rightarrow \vec{k} \cdot \vec{r} = xk_x + yk_y + zk_z = -0.866y - 0.5z$

$\Rightarrow \vec{k} = -0.866 \hat{a}_y - 0.5 \hat{a}_z$

$\beta = |\vec{k}| = \sqrt{k_x^2 + k_y^2 + k_z^2} = \sqrt{(-0.866)^2 + (-0.5)^2} = 0.9999 \approx 1 \text{ rad/m}$

$\omega = \beta c = 1 \times 3 \times 10^8 = 3 \times 10^8 \text{ rad/sec}$

$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{1} = 2\pi \text{ m}$

b) From Maxwell's eqⁿ:

$\nabla \times \vec{E} = -j\omega\mu\vec{H}$

Replace ∇ by $\vec{k}$ and j by -1

$\Rightarrow \vec{k} \times \vec{E} = -\omega\mu\vec{H}$

$\Rightarrow \vec{H} = \frac{1}{\omega\mu} \{\vec{k} \times \vec{E}\}$

$= \frac{1}{3 \times 10^8 \times \mu_0} \times \left\{ [-0.866 \hat{a}_y - 0.5 \hat{a}_z] \times [100 e^{j(0.866y + 0.5z)} \hat{a}_x] \right\}$

$= \frac{1}{3 \times 10^8 \times 4\pi \times 10^{-7}} \times \left\{ 86.6 e^{j(0.866y + 0.5z)} \hat{a}_z - 50 e^{j(0.866y + 0.5z)} \hat{a}_y \right\}$

$= 0.23 e^{j(0.866y + 0.5z)} \hat{a}_z - 0.132 e^{j(0.866y + 0.5z)} \hat{a}_y$

c) Total time average power $P_{avg} = \frac{E_0^2 \hat{a}_k}{2\eta} = \frac{100^2}{2 \times 120\pi} \frac{\vec{k}}{|\vec{k}|}$

$P_{avg} = \frac{10000}{240\pi} \{ 0.866 \hat{a}_y + 0.5 \hat{a}_z \}$

$P_{avg} = - [11.485 \hat{a}_y + 6.631 \hat{a}_z] \text{ W/m}^2$

$$Q6) \quad \vec{E} = (10\hat{a}_y + 5\hat{a}_z) \cos(\omega t + 2y - 4z) \text{ V/m}$$

$$\vec{E} = E_0(x, y, z) \cos(\omega t - \vec{k} \cdot \vec{r})$$

$$\Rightarrow -\vec{k} \cdot \vec{r} = 2y - 4z$$

$$\vec{k} = -2\hat{a}_y + 4\hat{a}_z$$

$$|\vec{k}| = \sqrt{k_x^2 + k_y^2 + k_z^2} = \sqrt{4 + 16} = \sqrt{20}$$

$$a) \quad \beta = \sqrt{20} \text{ rad/m}$$

$$\omega = \beta c = \sqrt{20} \times 3 \times 10^8 = 1.342 \times 10^9 \text{ rad/sec}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\sqrt{20}} = 1.4049 \approx 1.405 \text{ m}$$

b) Again from Maxwell's eqⁿ

$$\nabla \times \vec{E} = -j\omega\mu\vec{H}$$

Replace ∇ by $\vec{k}$ and j by -1

$$\vec{k} \times \vec{E} = \omega\mu\vec{H}$$

$$\Rightarrow \vec{H} = \frac{1}{\omega\mu} \{ \vec{k} \times \vec{E} \}$$

$$\Rightarrow \vec{H} = \frac{1}{\omega\mu_0} \{ [-2\hat{a}_y + 4\hat{a}_z] \times [10\hat{a}_y \cos(\omega t + 2y - 4z) + 5\hat{a}_z \cos(\omega t + 2y + 4z)] \}$$

$$\vec{H} = \frac{1}{\omega\mu_0} \{ -2 \times 5 \cos(\omega t + 2y - 4z) \hat{a}_x - 4 \times 10 \cos(\omega t - 2y - 4z) \hat{a}_x \}$$

$$\vec{H} = \frac{-(10 + 40)}{\omega\mu_0} \{ \cos(1.342 \times 10^9 t + 2y - 4z) \hat{a}_x \}$$

$$\vec{H} = \frac{-50}{1.342 \times 10^9 \times 4\pi \times 10^{-7}} \cos(1.342 \times 10^9 t + 2y - 4z) \hat{a}_x$$

$$\vec{H} = -29.65 \cos(1.342 \times 10^9 t + 2y - 4z) \hat{a}_x \text{ mA/m}$$

$$c) \quad P_{avg} = \frac{E_0^2}{2\eta} \hat{a}_k = \left\{ \frac{(\sqrt{10^2 + 5^2})^2}{2 \times 120\pi} \right\} \left\{ \frac{-2y + 4z}{\sqrt{4 + 16}} \right\}$$

$$= \frac{10^2}{2 \times 120\pi} \hat{a}_k = -0.0741 \hat{a}_y + 0.1482 \hat{a}_z$$

Q7) $E_i = 8 \cos(\omega t - 4x - 3z) \hat{a}_y$ V/m $\rightarrow$ Medium 1 (air)

$$\vec{E}_i = E_{i0} \cos(\omega t - \vec{k}_i \cdot \vec{r}) \hat{a}_E \quad \Downarrow$$

Now $\vec{k}_i \cdot \vec{r} = 4x + 3z$

$$\vec{k}_i = 4\hat{a}_x + 3\hat{a}_z$$

$$\vec{k}_i = |\vec{k}_i| \cos \theta_i \hat{a}_z + |\vec{k}_i| \sin \theta_i \hat{a}_x$$

$$|\vec{k}_i| = \sqrt{4^2 + 3^2} = \sqrt{16 + 9} = 5 \quad \Rightarrow \beta_1 = 5 \text{ rad/m}$$

$$\vec{k}_i = 5 \cos \theta_i \hat{a}_z + 5 \sin \theta_i \hat{a}_x$$

$$5 \cos \theta_i = 3$$

$$5 \sin \theta_i = 4$$

$$\beta_2 = \frac{\omega}{c} \sqrt{\mu \epsilon_2}$$

$$\Rightarrow \frac{5 \sin \theta_i}{5 \cos \theta_i} = \frac{4}{3} \quad |\vec{k}_t| = \beta_2 = 5 \times \sqrt{1 \times 2.5} = 5\sqrt{2.5}$$

$$\Rightarrow \tan \theta_i = \frac{4}{3}$$

$$b) \theta_i = \tan^{-1}(4/3) = 53.13^\circ$$

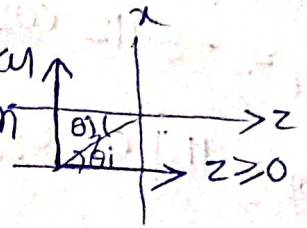
$$c) \eta_1 = 120\pi \, \Omega$$

$$\eta_2 = 120\pi \sqrt{\frac{\mu \epsilon_2}{\epsilon_0}} = 120\pi \times \sqrt{\frac{1}{2.5}} = 238.43 \, \Omega$$

$$\beta_1 \sin \theta_i = \beta_2 \sin \theta_t \quad (\text{Snell's law})$$

$$\Rightarrow \theta_t = \sin^{-1} \left\{ \frac{\beta_1}{\beta_2} \times \sin \theta_i \right\} = \sin^{-1} \left\{ \frac{5}{5\sqrt{2.5}} \times \sin(53.13) \right\}$$

$$\theta_t = 30.39^\circ$$



$$\Gamma_{\perp} = \frac{n_2 \cos \theta_i - n_1 \cos \theta_t}{n_2 \cos \theta_i + n_1 \cos \theta_t}$$

$$\Gamma_{\perp} = \frac{238.43 \cos(53.13) - 120\pi \cos(30.39)}{238.43 \cos(53.13) + 120\pi \cos(30.39)} = -0.389$$

$$\tau_{\perp} = 1 + \Gamma_{\perp} = 1 - 0.389 = 0.611$$

$$\Gamma_{\perp} = \frac{E_{r0}}{E_{i0}} = -0.389$$

$$\Rightarrow E_{r0} = -0.389 E_{i0} = -0.389 \times 8 = -3.112 \text{ V/m}$$

$$\vec{E}_r = -3.112 \cos(\omega t - \vec{k}_r \cdot \vec{r}) \hat{a}_y \text{ V/m}$$

$$\vec{k}_r = |k_r| \cos \theta_{r1} (-\hat{a}_z) + |k_r| \cos \theta_{r2} (\hat{a}_x)$$

$$\vec{k}_r = 5 \cos(53.13) \hat{a}_x - 5 \cos(30.39) \hat{a}_z$$

$$\vec{k}_r = 4 \hat{a}_x - 3 \hat{a}_z$$

$$\vec{k}_r \cdot \vec{r} = (4 \hat{a}_x - 3 \hat{a}_z) \cdot (x \hat{a}_x + y \hat{a}_y + z \hat{a}_z) = 4x - 3z$$

$$\Rightarrow \vec{E}_r = -3.112 \cos(\omega t - (4x - 3z)) \hat{a}_y \text{ V/m}$$

$$\Rightarrow \vec{E}_r = -3.112 \cos(\omega t - 4x + 3z) \hat{a}_y \text{ V/m}$$

Transmitted magnetic field:- $H_t = ?$

we have

$$\vec{H}_t = \frac{\vec{k}_t \times \vec{E}_t}{\omega \mu_2} \quad (\text{OR})$$

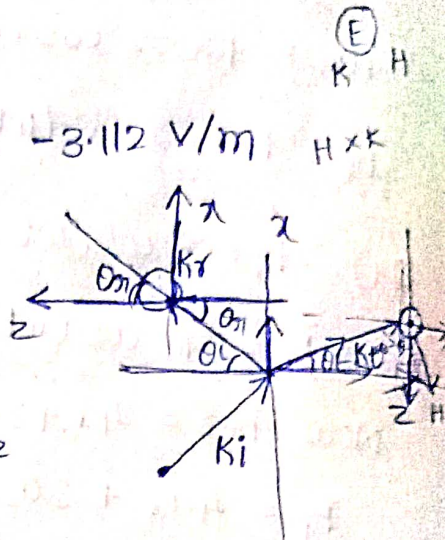
$$\vec{H}_t = \{ H_{t0} \cos \theta_t (-\hat{a}_x) + H_{t0} \sin \theta_t \hat{a}_z \} \cos(\omega t - \vec{k}_t \cdot \vec{r})$$

$$\vec{k}_t \cdot \vec{r} = ?$$

$$\vec{k}_t = |k_t| \cos \theta_t \hat{a}_z + |k_t| \sin \theta_t \hat{a}_x$$

$$\vec{k}_t = 5 \sqrt{2} \cdot 5 \cos(30.39) \hat{a}_z + 5 \sqrt{2} \cdot 5 \sin(30.39) \hat{a}_x$$

$$\vec{k}_t = 6.819 \hat{a}_z + 4 \hat{a}_x$$



$$\vec{K}_t \cdot \vec{r} = (4\hat{a}_x + 6.819\hat{a}_z) \cdot (x\hat{a}_x + y\hat{a}_y + z\hat{a}_z)$$

$$\vec{K}_t \cdot \vec{r} = 4x + 6.819z$$

$$\beta_1 = \frac{\omega}{c} \Rightarrow \omega = \beta_1 c = 5 \times 3 \times 10^8 = 15 \times 10^8 \text{ rad/sec}$$

$$\vec{E}_t = E_{t0} \cos(\omega t - \vec{K}_t \cdot \vec{r}) \hat{a}_y$$

$$= T_1 E_{t0} \cos(\omega t - (4x + 6.819z)) \hat{a}_y$$

$$= 0.611 \times 8 \cos(\omega t - 4x - 6.819z) \hat{a}_y$$

$$\vec{E}_t = 4.888 \cos(15 \times 10^8 t - 4x - 6.819z) \hat{a}_y$$

$$\vec{H}_t = \frac{\vec{K}_t \times \vec{E}_t}{\omega \mu_2}$$

$$= \frac{(4\hat{a}_x + 6.819\hat{a}_z) \cdot \{4.888 \cos(15 \times 10^8 t - 4x - 6.819z)\} \hat{a}_y}{15 \times 10^8 \times 1 \times 4\pi \times 10^{-7}}$$

$$= 10.37 \cos(15 \times 10^8 t - 4x - 6.819z) \hat{a}_z - 17.68 \cos(15 \times 10^8 t - 4x - 6.819z) \hat{a}_x$$

$$\vec{H}_t = \{-17.68 \hat{a}_x + 10.37 \hat{a}_z\} \cos(15 \times 10^8 t - 4x - 6.819z) \text{ mA/m}$$

Q8) $\vec{E} = (10\hat{a}_y + 5\hat{a}_z) \cos(\omega t + 2y - 4z) \text{ V/m}$

$$\vec{E}_i = E_0 \cos(\omega t - \vec{k}_i \cdot \vec{r}) \text{ V/m}$$

$$\vec{E}_i = (10\hat{a}_y + 5\hat{a}_z) \cos(\omega t - (4z - 2y)) \text{ V/m}$$

$$\vec{k}_i \cdot \vec{r} = -2y + 4z$$

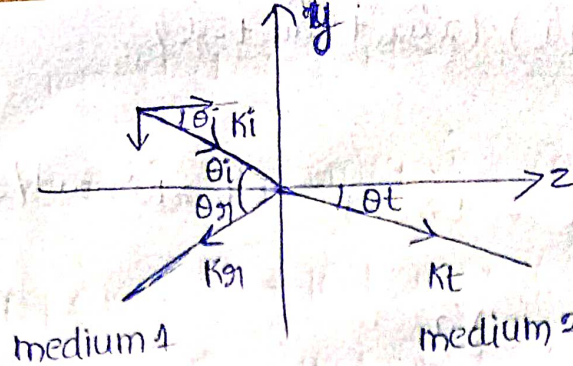
$$\vec{k}_i = -2\hat{a}_y + 4\hat{a}_z$$

$$\beta_1 = |\vec{k}_i| = \sqrt{4 + 16} = \sqrt{20} = 2\sqrt{5} \text{ rad/m}$$

$$\beta_1 = \frac{\omega}{c}$$

$$\Rightarrow \omega = \beta_1 c = 2\sqrt{5} \times 3 \times 10^8 = 6\sqrt{5} \times 10^8 \text{ rad/sec}$$

$$\beta_2 = \frac{\omega}{c} \sqrt{\mu\epsilon_2} = \frac{6\sqrt{5} \times 10^8 \times \sqrt{4 \times 1}}{3 \times 10^8} = 4\sqrt{5} \text{ rad/m}$$



$$k_i = \underbrace{-2\hat{a}_y}_{\text{-ve } y} + \underbrace{4\hat{a}_z}_{\text{+ve } z}$$

$$\bar{k}_i = |k_i| \cos \theta_i \hat{a}_z + |k_i| \sin \theta_i (-\hat{a}_y)$$

$$\bar{k}_i = |k_i| \cos \theta_i \hat{a}_z - |k_i| \sin \theta_i \hat{a}_y$$

$$\bar{k}_i = 2\sqrt{5} \cos \theta_i \hat{a}_z - 2\sqrt{5} \sin \theta_i \hat{a}_y$$

$$\bar{k}_i = 4\hat{a}_z - 2\hat{a}_y$$

$$\Rightarrow \begin{aligned} 2\sqrt{5} \cos \theta_i &= 4 \\ -2\sqrt{5} \sin \theta_i &= -2 \end{aligned}$$

$$\Rightarrow \frac{\sin \theta_i}{\cos \theta_i} = \frac{2}{4}$$

$$\Rightarrow \tan \theta_i = \frac{1}{2}$$

$$\Rightarrow \theta_i = \tan^{-1}(0.5) = 26.56^\circ$$

$$\theta_i = \theta_r = 26.56^\circ$$

$$\Rightarrow \beta_1 \sin \theta_i = \beta_2 \sin \theta_t$$

$$\beta_2 = \frac{\omega}{c} \sqrt{\mu_0 \epsilon_0} = 4\sqrt{5} \text{ rad/m}$$

$$\Rightarrow 2\sqrt{5} \sin 26.56 = 4\sqrt{5} \sin \theta_t$$

$$\Rightarrow \theta_t = \sin^{-1} \left\{ \frac{2\sqrt{5}}{4\sqrt{5}} \sin 26.56 \right\} = \sin^{-1} \left\{ \frac{\sin 26.56}{2} \right\}$$

$$\theta_t = 12.918 = 12.92^\circ$$

a) $\theta_i = 26.56^\circ$

$\theta_r = 26.56^\circ$

$\theta_t = 12.92^\circ$

Transmission electric field =

$$E_t = \{E_{t0} \cos \theta_t \hat{a}_y + E_{t0} \sin \theta_t \hat{a}_z\} \cos(\omega t - \vec{k}_t \cdot \vec{r})$$

$$\vec{k}_t = |k_t| \cos \theta_t \hat{a}_z + |k_t| \sin \theta_t (-\hat{a}_y)$$

$$\vec{k}_t = |k_t| \cos \theta_t \hat{a}_z - |k_t| \sin \theta_t \hat{a}_y$$

$$\vec{k}_t = 4\sqrt{5} \cos(12.92) \hat{a}_z - 4\sqrt{5} \sin(12.92) \hat{a}_y$$

$$\vec{k}_t = 8.718 \hat{a}_z - 2 \hat{a}_y$$

$$E_{t0} = \tau_{11} E_{i0} = 0.705 \times \sqrt{10^2 + 5^2} = 7.882 \text{ V/m}$$

$$E_t = 7.882 \{ \cos(12.92) \hat{a}_y + \sin(12.92) \hat{a}_z \} \cos(\omega t + 2y - 8.718z)$$

$$E_t = \{7.68 \hat{a}_y + 1.762 \hat{a}_z\} \cos(\omega t + 2y - 8.718z)$$

c) Total E field in free space:

$$\vec{E}_1 = \{ (10 \hat{a}_y + 5 \hat{a}_z) \cos(\omega t + 2y - 4z) \} + \{ (-2.94 \hat{a}_y + 1.469 \hat{a}_z) \cos(\omega t + 2y + 4z) \} \text{ V/m}$$

d) Total E field in dielectric:

$$\vec{E}_2 = (7.68 \hat{a}_y + 1.762 \hat{a}_z) \cos(\omega t + 2y - 8.718z)$$

e) Brewster angle =

$$\theta_{B11} = \tan^{-1} \left\{ \sqrt{\frac{\epsilon_{r2}}{\epsilon_{r1}}} \right\} = \tan^{-1}(\sqrt{4})$$

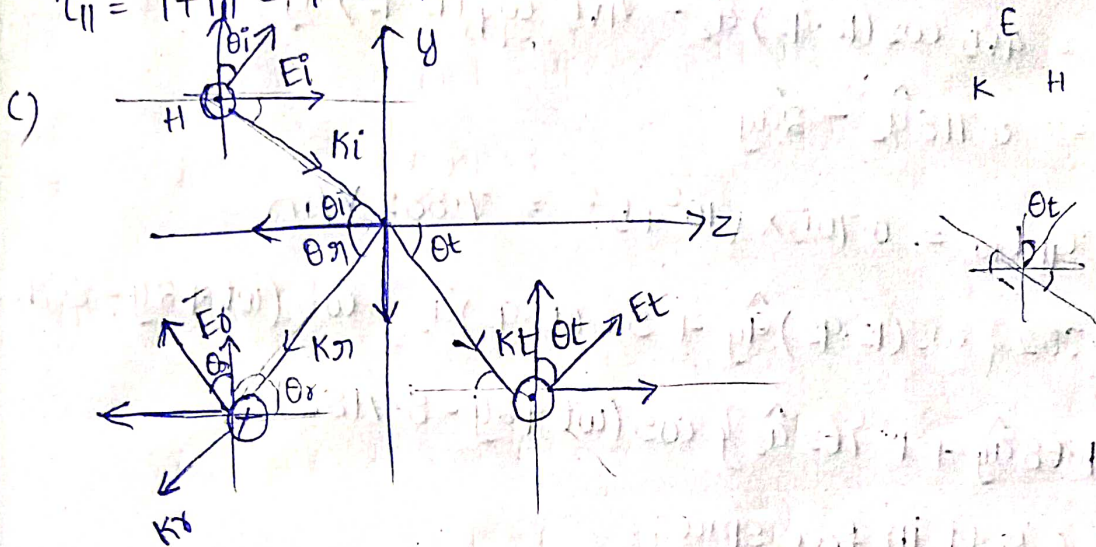
$$\theta_{B11} = \tan^{-1}(\sqrt{4}) = 63.43^\circ$$

$$b) \Gamma_{11} = \frac{n_2 \cos \theta_t - n_1 \cos \theta_i}{n_2 \cos \theta_t + n_1 \cos \theta_i}$$

$$n_1 = 120\pi \quad n_2 = 120\pi \sqrt{\frac{\mu_0 \epsilon_2}{\epsilon_1}} = 120\pi \sqrt{\frac{1}{4}} = 60\pi$$

$$\Gamma_{11} = \frac{60\pi \cos(12.92^\circ) - 120\pi \cos(26.56^\circ)}{60\pi \cos(12.92^\circ) + 120\pi \cos(26.56^\circ)} = \frac{-0.2946}{1.2946} = -0.295$$

$$\tau_{11} = 1 + \Gamma_{11} = 1 - 0.295 = 0.705$$



Reflected electric field:-

$$\vec{E}_r = (E_{r0} \cos \theta_{r1} \hat{a}_y - E_{r0} \sin \theta_{r1} \hat{a}_z) \cos(\omega t - (\vec{k}_r \cdot \vec{r}))$$

$$E_{r0} = E_{i0} \Gamma_{11} = \sqrt{10^2 + 5^2} (-0.294) = -3.287 \text{ V/m}$$

$$\begin{aligned} \vec{k}_r &= |k_{r1}| \cos \theta_{r1} (-\hat{a}_z) + |k_{r1}| \sin \theta_{r1} \hat{a}_y \\ &= (2\sqrt{5} \cos(26.56) \hat{a}_z + 2\sqrt{5} \sin(26.56) \hat{a}_y) \\ &= -[4\hat{a}_z + 2\hat{a}_y] \end{aligned}$$

$$\vec{k}_r \cdot \vec{r} = \{-(4\hat{a}_z + 2\hat{a}_y)\} \cdot \{x\hat{a}_x + y\hat{a}_y + z\hat{a}_z\}$$

$$= -4z - 2y$$

$$\vec{E}_r = -3.287 \{ \cos(26.56) \hat{a}_y - \sin(26.56) \hat{a}_z \} \cos(\omega t + 4z + 2y) \text{ V/m}$$

$$\vec{E}_r = (-2.94 \hat{a}_y + 1.469 \hat{a}_z) \cos(\omega t + 4z + 2y) \text{ V/m}$$

Q10) medium 1 : air $\mu_1 = \mu_0$ $\epsilon_2 = \epsilon_0$

medium 2 : lossless medium : $\mu_1 = \mu_0$ $\epsilon_2 = 4\epsilon_0$

$$\vec{E}_i = 30 \cos(\omega t - z) \hat{a}_x \text{ V/m}$$

$$\eta_1 = 120\pi \Omega \quad \eta_2 = 120\pi \times \sqrt{\frac{\mu_{r2}}{\epsilon_{r2}}} = 120\pi \times \sqrt{\frac{1}{4}} = 60\pi \Omega$$

$$a) \quad \Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{60\pi - 120\pi}{60\pi + 120\pi} = -\frac{1}{3}$$

$$\tau = \frac{2\eta_2}{\eta_2 + \eta_1} = \frac{2 \times 60\pi}{60\pi + 120\pi} = \frac{2}{3}$$

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 1/3}{1 - 1/3} = 2$$

$$b) \quad \Gamma = \frac{E_{\text{ro}}}{E_{i0}} = -\frac{1}{3}$$

$$\Rightarrow E_{\text{ro}} = -\frac{1}{3} \times E_{i0} = -\frac{1}{3} \times 30 = -10$$

$$\vec{E}_r = -10 \cos(\omega t + z) \hat{a}_x \text{ V/m}$$

$$\vec{H}_r = H_{r0} \cos(\omega t + z) \hat{a}_{Hr}$$

$$H_{r0} = \frac{E_{r0}}{\eta_2} = \frac{-10}{60\pi} = -26.525 \text{ mA/m}$$

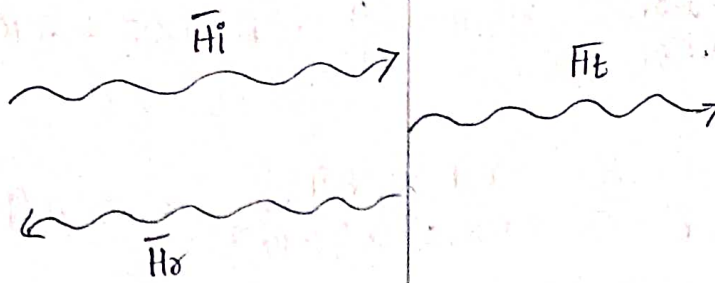
$$\hat{a}_{Hr} = \hat{a}_{kr} \times \hat{a}_{Er} = -\hat{a}_z \times \hat{a}_x = -\hat{a}_y$$

$$\vec{H}_r = -26.525 \cos(\omega t + z) (-\hat{a}_y)$$

$$\vec{H}_r = 26.525 \cos(\omega t + z) \hat{a}_y \text{ mA/m}$$

Q11)

m_1
 $a_1 \delta$
 $\mu = \mu_0$
 $\epsilon = \epsilon_0$



m_2
 Plastic
 $\mu \sigma_2 = 1$
 $\epsilon \sigma_2 = 4$

Given $\bar{H}_i = 4 \sin(\omega t - 5x) \text{ A/m} \hat{y}$ $H_{i0} = 4 \text{ A/m}$

a) $\beta_1 = 5 \text{ rad/m}$

$$\beta_1 = \frac{\omega}{c}$$

$$\omega = \beta_1 c = 5 \times 3 \times 10^8 = 15 \times 10^8 \text{ rad/sec}$$

$$\eta_1 = 120\pi \Omega$$

$$E_i = E_{i0} \sin(\omega t - 5x) \hat{y} \text{ V/m}$$

$$E_{i0} = H_{i0} \times 120\pi = 4 \times 120\pi = 1507.96 = 1.508 \text{ kV/m}$$

$$\hat{a}_{Ei} = \hat{a}_{Hi} \times \hat{a}_{ki} = \hat{a}_y \times \hat{a}_x = -\hat{a}_z$$

$$E_i = 1.508 \sin(\omega t - 5x) (-\hat{a}_z) \text{ kV/m}$$

$$\eta_2 = 120\pi \sqrt{\frac{\mu \sigma_2}{\epsilon \sigma_2}} = 120\pi \times \sqrt{\frac{1}{4}} = 60\pi$$

$$\Gamma_E = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{60\pi - 120\pi}{60\pi + 120\pi} = \frac{-60\pi}{180\pi} = -\frac{1}{3}$$

$$\Gamma_E = \frac{E_{r0}}{E_{i0}} = -\frac{1}{3}$$

$$E_{r0} = -\frac{1}{3} \times E_{i0} = -\frac{1}{3} \times 1.508 = -0.503 \text{ kV/m}$$

$$E_r = -0.503 \sin(\omega t - 5x) (-\hat{a}_z) \text{ kV/m}$$

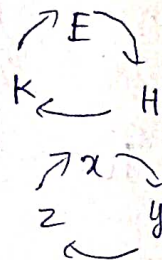
$$E_t = E_i + E_r$$

$$E_t = \{-1.508 \sin(\omega t - 5x) \hat{a}_z + 0.503 \sin(\omega t - 5x) \hat{a}_z\} \text{ kV/m}$$

b) $P_{avg} = \frac{E_0^2}{2|\eta|} e^{-2\alpha x} \hat{a}_x$

$$E_0 = E_{t0}$$

$$\tau_E = \frac{2\eta_2}{\eta_2 + \eta_1} = \frac{2 \times 60\pi}{120\pi + 60\pi} = \frac{120\pi}{180\pi} = \frac{2}{3}$$



$$\tau_E = \frac{E_{to}}{E_{io}} = \frac{2}{3}$$

$$\Rightarrow E_{to} = \frac{1.508 \times 2}{3} = 1.005 \text{ kV/m}$$

$$P_{avg} = \frac{E_{to}^2}{2|n|} = \frac{(1.005 \times 10^3)^2}{2 \times 120\pi} = 2.679 \text{ kW/m}^2$$

$$S = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+\frac{1}{3}}{1-\frac{1}{3}} = \frac{\frac{4}{3}}{\frac{2}{3}} = 2$$

Q12)

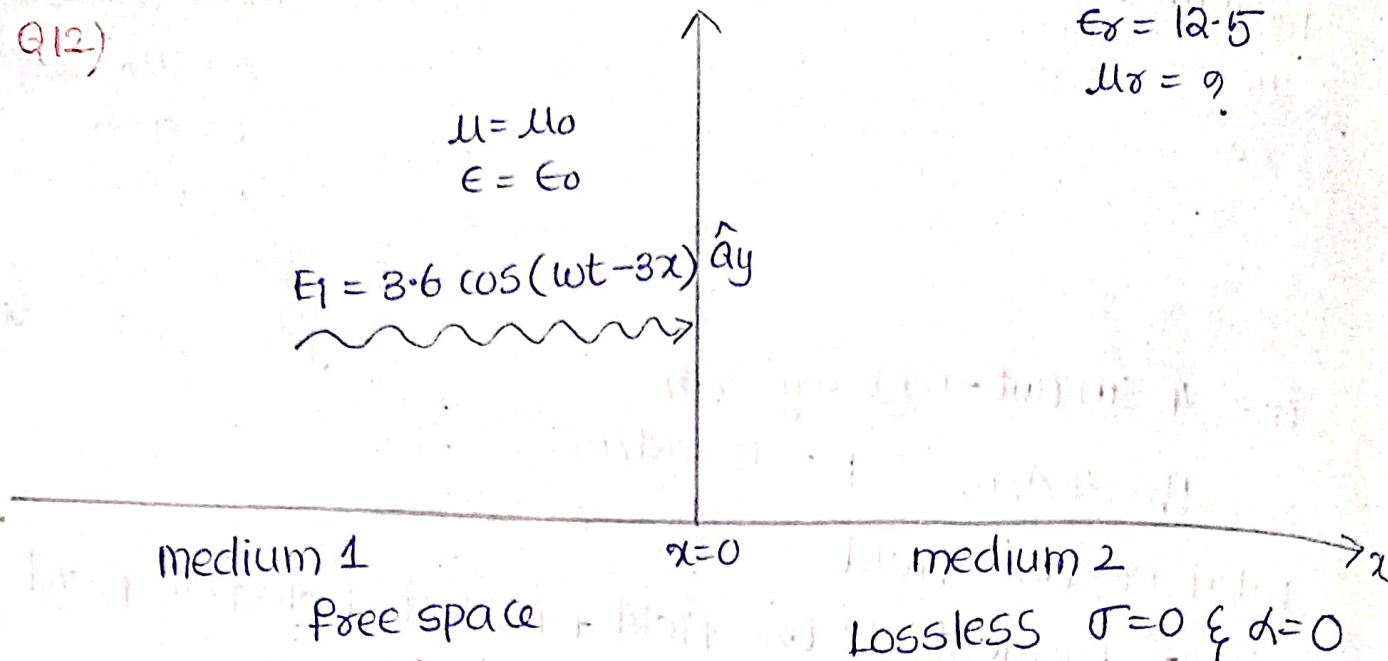
$$\epsilon_r = 12.5$$

$$\mu_r = 9$$

$$\mu = \mu_0$$

$$\epsilon = \epsilon_0$$

$$E_1 = 3.6 \cos(\omega t - 3x) \hat{a}_y$$



$$\eta_2 = 120\pi \sqrt{\frac{\mu_{r2}}{\epsilon_{r2}}} = 120\pi \times \sqrt{\frac{\mu_r}{12.5}}$$

$$\vec{H}_2 = 1.2 \cos(\omega t + 3x) (-\hat{a}_z)$$

$$H_{x0} = 1.2 \text{ mA/m}$$

$$\eta_1 = 120\pi \text{ (Free space)}$$

$$\frac{E_{x0}}{\eta_1} = H_{x0}$$

$$E_{x0} = \eta_1 H_{x0} = 120\pi \times 1.2 = 0.144\pi \text{ V/m}$$

$$\Gamma_E = \frac{E_{x0}}{E_{i0}} = \frac{0.144\pi}{3.6} = 0.04\pi = 0.125$$

$$\Gamma_E = \frac{\eta_2 - \eta_1}{\eta_1 + \eta_2}$$

$$\Gamma_E = \frac{120\pi \sqrt{\frac{\mu_r}{12.5}} - 120\pi}{120\pi \sqrt{\frac{\mu_r}{12.5}} + 120\pi} = 0.125$$

$$\Gamma_E = \frac{\sqrt{\mu_r} - \sqrt{12.5}}{\sqrt{\mu_r} + \sqrt{12.5}} = 0.125$$

$$\Rightarrow \frac{\frac{\sqrt{\mu_0}}{\sqrt{12.5}} - 1}{\frac{\sqrt{\mu_0}}{12.5} + 1} = 0.125$$

$$\Rightarrow \frac{\sqrt{\mu_0}}{\sqrt{12.5}} - 1 = 0.125 \frac{\sqrt{\mu_0}}{\sqrt{12.5}} + 0.125$$

$$\Rightarrow \frac{\sqrt{\mu_0}}{\sqrt{12.5}} (1 - 0.125) = 1.125$$

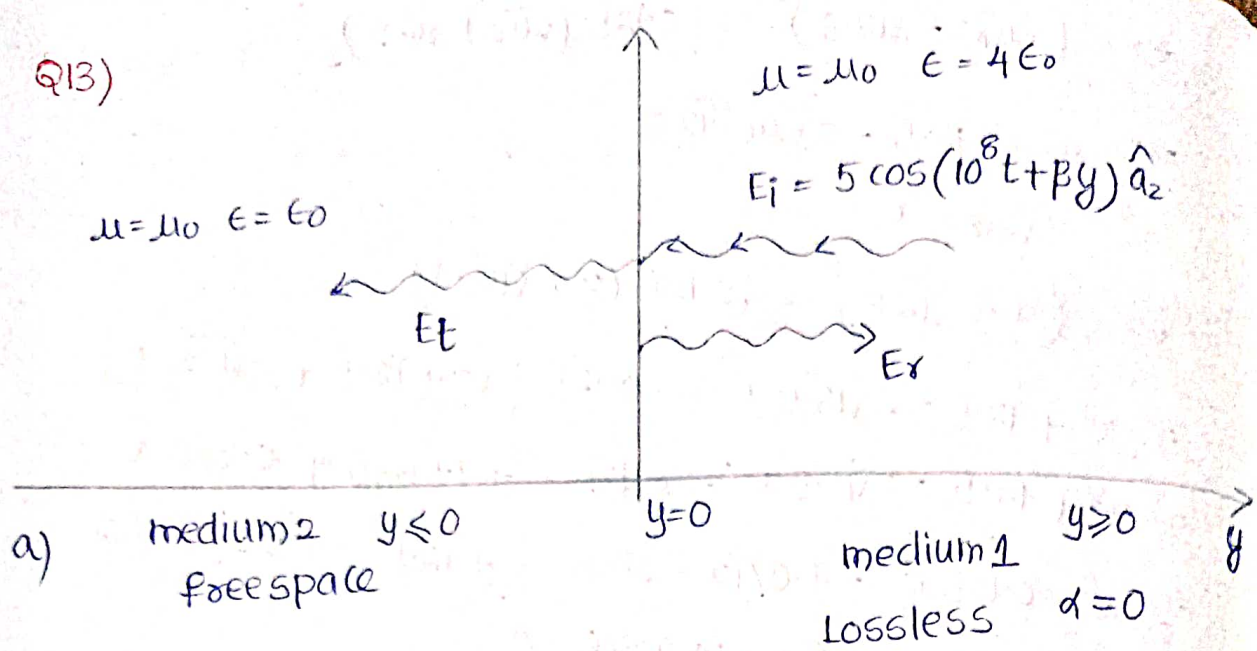
$$\Rightarrow \sqrt{\mu_0} = \frac{1.125 \times \sqrt{12.5}}{0.875}$$

$$\mu_0 = \left(\frac{1.125 \times \sqrt{12.5}}{0.875} \right)^2 = 20.663$$

$$\mu_2 = \mu_{r2} \mu_0 = 4\pi \times 10^{-7} \times 20.663 = ~~2.5~~ 259.66 \times 10^{-7} \text{ H/m}$$

$$\mu_2 = 2.59 \times 10^{-5} \text{ H/m}$$

Q13)



medium 1:

$$E_{i0} = 5 \text{ V/m}$$

$$\omega = 10^8 \text{ rad/sec}$$

$$\beta_1 = \frac{\omega}{c} \sqrt{\mu_0 \epsilon_0}$$

for medium 2:

Transmitted electric field:

$$E_t = E_{t0} \cos(\omega t + \beta_2 y) \hat{a}_z$$

Transmission coefficient $\tau_E = \frac{2\eta_2}{\eta_2 + \eta_1}$

$$\eta_2 = 120\pi \text{ (free space)}$$

$$\eta_1 = 120\pi \sqrt{\frac{\mu_0 \epsilon_0}{\epsilon_0}} = 120\pi \times \sqrt{\frac{1}{4}} = 60\pi$$

$$\tau_E = \frac{2 \times 120\pi}{120\pi + 60\pi} = \frac{240\pi}{180\pi} = \frac{4}{3}$$

$$\tau_E = \frac{E_{t0}}{E_{i0}} = \frac{4}{3}$$

$$E_{t0} = E_{i0} \times \frac{4}{3} = 5 \times \frac{4}{3} = \frac{20}{3} \text{ V/m}$$

$$\beta_2 = \frac{\omega}{c} = \frac{10^8}{3 \times 10^8} = \frac{1}{3}$$

$$E_t = \frac{20}{3} \cos(10^8 t + \frac{1}{3} y) \hat{a}_z$$

b) Average Poynting vector in region 1:

$$P_{avg,1} = \frac{E_o^2}{2|n|} e^{+2dy} \cos \theta_\eta (-\hat{a}_y)$$

for medium 1 we have 2 waves
 $\left\{ \begin{array}{l} \text{Incident wave} \\ \text{Reflected wave} \end{array} \right.$

$$E_x = E_{x0} \cos(\omega t - \beta y) \hat{a}_z$$

$$\Gamma_E = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{120\pi - 60\pi}{120\pi + 60\pi} = \frac{60\pi}{180\pi} = \frac{1}{3}$$

$$\Gamma_E = \frac{E_{x0}}{E_{i0}} = \frac{1}{3} \Rightarrow E_{x0} = \frac{1}{3} E_{i0}$$

$$\Rightarrow E_{x0} = \frac{1}{3} \times 5 = \frac{5}{3}$$

$$\beta_1 = \frac{\omega}{c} \sqrt{\mu_0 \epsilon_0}$$

$$= \frac{10^8}{3 \times 10^8} \sqrt{1 \times 4}$$

$$= \frac{2}{3}$$

$$E_x = \frac{5}{3} \cos(10^8 t - \beta y) \hat{a}_z$$

$$E_x = \frac{5}{3} \cos(10^8 t - \frac{2}{3} y) \hat{a}_z$$

$$E_1 = E_i + E_r$$

$$E_i = \left\{ 5 \cos(10^8 t + \frac{2}{3} y) \hat{a}_z + \frac{5}{3} \cos(10^8 t - \frac{2}{3} y) \hat{a}_z \right\} \text{ V/m}$$

$$\begin{aligned} \beta &= 0 & |n| &= 60\pi & E_{i0} &= 5 \\ \theta_\eta &= 0 & & & E_{x0} &= \frac{5}{3} \end{aligned}$$

$$P_{avg,1} = \frac{E_{i0}^2}{2|n|} e^{+2dy} \cos \theta_\eta (-\hat{a}_y) + \frac{E_{x0}^2}{2|n|} e^{-2dy} \cos \theta_\eta \hat{a}_y$$

$$= \frac{25}{2 \times 60\pi} (-\hat{a}_y) + \frac{25}{9 \times 2 \times 60\pi} \hat{a}_y$$

$$= \frac{25}{2 \times 60\pi} \left\{ -1 + \frac{1}{9} \right\} \hat{a}_y$$

$$= \frac{25}{2 \times 60\pi} \left\{ \frac{-9+1}{9} \right\} \hat{a}_y$$

$$= \frac{-25 \times 8}{120\pi \times 9} \hat{a}_y \text{ W/m}^2$$

$$P_{avg,1} = -0.0589 \hat{a}_y \text{ W/m}^2$$

$$c) P_{avg_2} = \frac{E_0^2}{2|n|} e^{+2\alpha y} \cos \theta_n (-\hat{a}_y)$$

$$\Rightarrow P_{avg_2} = \frac{20^2}{3^2 \times 2 \times 120\pi} (-\hat{a}_y)$$

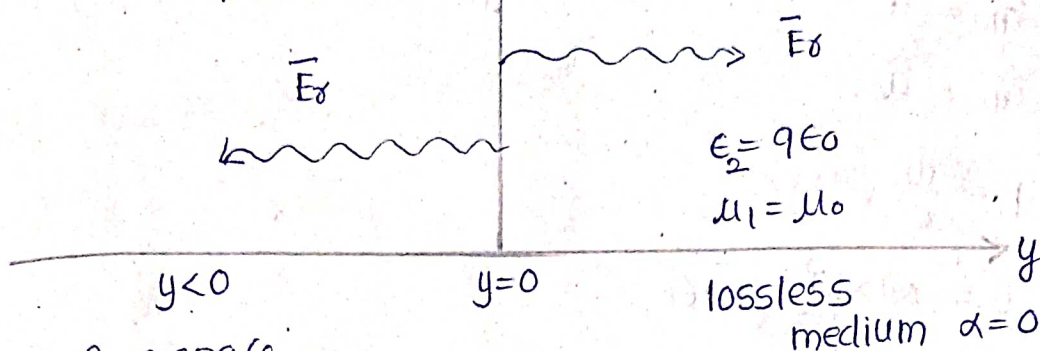
$$= \frac{-400}{9 \times 240\pi} \hat{a}_y$$

$$P_{avg} = -0.0589 \hat{a}_y \text{ W/m}^2$$

914)

$$E_i = 24 \cos(\underbrace{3 \times 10^8 t}_{\omega} - \beta y) \hat{a}_z$$

$$E_i = 24 \cos(3 \times 10^8 t - \beta y) \hat{a}_z$$



free space

$$\mu_1 = \mu_0 \quad \epsilon_1 = \epsilon_0$$

$$\beta_1 = \frac{\omega}{c} = \frac{3 \times 10^8}{3 \times 10^8} = 1 \text{ rad/m}$$

$$\beta_2 = \frac{\omega}{c} \sqrt{\epsilon_2 \mu_2} = \frac{3 \times 10^8}{3 \times 10^8} \times \sqrt{9 \times 1} = 3 \text{ rad/m}$$

$$\eta_1 = 120\pi$$

$$\eta_2 = 120\pi \sqrt{\frac{\mu_2}{\epsilon_2}} = 120\pi \times \sqrt{\frac{1}{9}} = \frac{120\pi}{3} = 40\pi$$

$$\Gamma_E = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{40\pi - 120\pi}{40\pi + 120\pi} = \frac{-80\pi}{160\pi} = -\frac{1}{2}$$

$$\Gamma_E = \frac{E_{r0}}{E_i} = -\frac{1}{2}$$

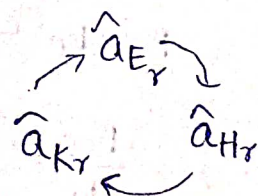
$$E_{r0} = -\frac{1}{2} E_i = -\frac{1}{2} \times 24 = -12$$

$$\bar{H}_r = H_{r0} \cos(\omega t + \beta y) \hat{a}_{Hr}$$

$$\bar{H}_r = \frac{E_{r0}}{\eta_1} \cos(3 \times 10^8 t + y) \hat{a}_{Hr}$$

$$\bar{H}_r = \frac{-12}{120\pi} \cos(3 \times 10^8 t + y) (-\hat{a}_x)$$

$$\bar{H}_r = \frac{1}{10\pi} \cos(3 \times 10^8 t + y) \hat{a}_x$$



$$a_{Er} = \hat{a}_z$$

$$\hat{a}_{kr} = -\hat{a}_y$$

$$\Rightarrow \hat{a}_{kr} \times \hat{a}_{Er} = \hat{a}_{Hr}$$

$$\Rightarrow \hat{a}_{Hr} = -\hat{a}_y \times \hat{a}_z$$

$$\hat{a}_{Hr} = -\hat{a}_x$$

915) $\vec{H} = 30 \sin(2\pi \times 10^8 t - 5x) \hat{a}_z \text{ mA/m}$

$H_0 = 30 \text{ mA/m}$

$\omega = 2\pi \times 10^8 \text{ rad/sec}$ lossless medium

$\alpha = 0$

$\beta = 5 \text{ rad/m}$

$\mu_r = 1 \quad \epsilon_r = 9$

$\hat{a}_k = \hat{a}_x$

$\hat{a}_{H1} = \hat{a}_z$

a) $\beta = \frac{\omega}{c} \sqrt{\mu_r \epsilon_r}$

$5 = \frac{2\pi \times 10^8}{3 \times 10^8} \sqrt{1 \times \epsilon_r}$

$\frac{5 \times 3}{2\pi} = \sqrt{\epsilon_r}$

$\epsilon_r = \left(\frac{5 \times 3}{2\pi}\right)^2 = 5.69$

b) $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{5} \text{ m}$

$v_p = \frac{\omega}{\beta} = \frac{2\pi \times 10^8}{5} = \frac{2}{5} \pi \times 10^8 \text{ m/sec}$

c) $\eta = 120\pi \sqrt{\frac{\mu_r}{\epsilon_r}} = 120\pi \times \sqrt{\frac{1}{5.69}} = 158.04 \Omega$

d) polarisation = direction of electric field $\hat{a}_E$

$\hat{a}_E = \hat{a}_H \times \hat{a}_k$

$\hat{a}_E = \hat{a}_z \times \hat{a}_x = \hat{a}_y$

$\hat{a}_E = \hat{a}_z \times \hat{a}_x = \hat{a}_y$

$\hat{a}_x \times \hat{a}_y = \hat{a}_z$

e) $E = E_0 \sin(2\pi \times 10^8 t - \beta x) \hat{a}_E$

$\frac{E_0}{H_0} = \eta \Rightarrow E_0 = \eta H_0 = 158.04 \times 30 = 4.741 \text{ V/m}$

$\vec{E} = 4.741 \sin(2\pi \times 10^8 t - 5x) \hat{a}_y \text{ V/m}$

f) $J_D = \epsilon \frac{\partial E}{\partial t}$

$= \epsilon_0 \epsilon_r \frac{\partial}{\partial t} \{4.741 \sin(2\pi \times 10^8 t - 5x) \hat{a}_y\}$

$= \frac{2\pi \times 10^8 \times 5.69 \epsilon_0 \times 4.741 \cos(2\pi \times 10^8 t - 5x) \hat{a}_y}{2\pi \times 10^8}$

$$J_D = \frac{5.69 \times 4.741}{3\pi \times 10^8}$$

$$J_D = \frac{5.69 \times 4.741 \times 3\pi \times 10^8 \times 10^{-9}}{36\pi} \cos(3\pi \times 10^8 t - 5x) \hat{a}_y$$

$$J_D = 0.149 \cos(3\pi \times 10^8 t - 5x) \hat{a}_y$$

916) for non magnetic medium $\mu_r = 1$

$$\beta = \frac{\omega}{c} \sqrt{\mu_r \epsilon_r}$$

$$\omega = 10^9 \text{ rad/sec}$$

$$\beta = 8 \text{ rad/m}$$

$$E_{01} = 50 \hat{a}_{E1} = \hat{a}_y$$

$$E_{02} = 40 \hat{a}_{E2} = \hat{a}_z$$

$$\Rightarrow 8 = \frac{10^9}{3 \times 10^8} \sqrt{\epsilon_r}$$

$$\hat{a}_k = \hat{a}_x$$

$$\Rightarrow \sqrt{\epsilon_r} = \frac{8 \times 3 \times 10^8}{10^9} = 2.4$$

$$\eta = 120\pi \sqrt{\frac{1}{\epsilon_r}}$$

$$\Rightarrow \epsilon_r = (2.4)^2 = 5.76$$

$$\eta = 120\pi \sqrt{\frac{1}{5.76}} = \frac{120\pi}{2.4}$$

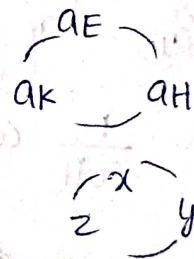
$$\eta = 50\pi$$

magnetic field $H = H_1 + H_2$

$$\bar{H}_1 = \frac{E_{01}}{\eta} \cos(3\pi \times 10^8 t - 8x) \hat{a}_{H1}$$

$$\hat{a}_{H1} = \hat{a}_k \times \hat{a}_{E1} = \hat{a}_x \times \hat{a}_y = \hat{a}_z$$

$$H_{01} = \frac{E_{01}}{\eta} = \frac{50}{50\pi} = \frac{1}{\pi}$$



$$\bar{H}_1 = \frac{1}{\pi} \cos(3\pi \times 10^8 t - 8x) \hat{a}_z$$

$$\bar{H}_2 = \frac{E_{02}}{\eta} \sin(10^9 t - 8x) \hat{a}_{H2}$$

$$H_{02} = \frac{E_{02}}{\eta} = \frac{40}{50\pi} = \frac{4}{5\pi}$$

$$\hat{a}_{H2} = \hat{a}_k \times \hat{a}_{E2}$$

$$= \hat{a}_x \times \hat{a}_z$$

$$= -\hat{a}_y$$

$$\bar{H}_2 = \frac{4}{5\pi} \sin(10^9 t - 8x) (-\hat{a}_y)$$

$$\bar{H} = H_1 + H_2$$

$$H = -\frac{4}{5\pi} \sin(10^9 t - 8x) \hat{a}_y + \frac{1}{\pi} \cos(10^9 t - 8x) \hat{a}_z$$

A/m

Q17) $\vec{E} = 10 \cos(2\pi \times 10^7 t - \beta x) (\hat{a}_y + \hat{a}_z) \text{ V/m}$

$E_0 = 10$

$\omega = 2\pi \times 10^7 \text{ rad/m}$

$\mu_r = 50$

$\sigma = 0$

$\epsilon_r = 2$

$d = 0 \text{ (lossless)}$

$\hat{a}_{E1} = \hat{a}_y \quad \hat{a}_{E2} = \hat{a}_z$

$\beta = \frac{\omega}{c} \sqrt{\mu_r \epsilon_r} = \frac{2\pi \times 10^7}{3 \times 10^8} \times \sqrt{50 \times 2} = \frac{2\pi \times 10^7 \times 10}{3 \times 10^8} = \frac{2\pi}{3}$

$\beta = \frac{2\pi}{3} \text{ rad/m}$

$\eta = 120\pi \sqrt{\frac{\mu_r}{\epsilon_r}} = 120\pi \times \sqrt{\frac{50}{2}} = 120\pi \times \sqrt{25} = 5 \times 120\pi$

$\eta = 600\pi \Omega$

$\vec{H} = H_0 \cos(2\pi \times 10^7 t - \frac{2\pi}{3} x) (\hat{a}_{H1} + \hat{a}_{H2}) \text{ A/m}$

$H_0 = \frac{E_0}{\eta} = \frac{10}{600\pi} = \frac{1}{60\pi}$

$\hat{a}_{H1} = \hat{a}_k \times \hat{a}_{E1} = \hat{a}_x \times \hat{a}_y = \hat{a}_z$

$\hat{a}_{H2} = \hat{a}_k \times \hat{a}_{E2} = \hat{a}_x \times \hat{a}_z = -\hat{a}_y$

$\vec{H} = \frac{1}{60\pi} \cos(2\pi \times 10^7 t - \frac{2\pi}{3} x) (\hat{a}_z - \hat{a}_y) \text{ A/m}$

Q18) for medium is said to be conducting $\frac{\sigma}{\omega\epsilon} \gg 1$ at $f = 8 \text{ MHz}$

A) Wet marshy soil :-

$\frac{\sigma}{\omega\epsilon} = \frac{10^{-2}}{2\pi \times 8 \times 10^6 \times 15 \epsilon_0} = \frac{10^{-2} \times 36\pi}{2\pi \times 8 \times 10^6 \times 15 \times 10^{-9}} = 1.5$

B) Intrinsic germanium :-

$\frac{\sigma}{\omega\epsilon} = \frac{0.025}{2\pi \times 8 \times 10^6 \times 16 \epsilon_0} = \frac{0.025 \times 36\pi}{2\pi \times 8 \times 10^6 \times 16 \times 10^{-9}} = 3.515$

C) sea water :-

$\frac{\sigma}{\omega\epsilon} = \frac{25}{2\pi \times 8 \times 10^6 \times 81 \epsilon_0} = \frac{25 \times 36\pi}{2\pi \times 8 \times 10^6 \times 81 \times 10^{-9}} = 694.44$

$\frac{\sigma}{\omega\epsilon} = 694.44 \gg 1$

$\Rightarrow$ sea water is good conductor at $f = 8 \text{ MHz}$

Q20) phase constant $\beta = 1.6 \text{ rad/m}$
 $f = 10^7 \text{ Hz}$ $\omega = 2\pi \times 10^7 \text{ rad/sec}$

$$E = E_0 e^{-\alpha z} \cos(\omega t - \beta z) \hat{a}_E$$

Magnitude reduced by 60% for every am

$$\Rightarrow \text{when } z=0 \text{ magnitude } E(z=0) = E_0 e^{-\alpha(0)} = E_0$$

$$\Rightarrow \text{when } z=m \text{ magnitude } E(z=am) = E_0 e^{-\alpha z}$$

$$\Rightarrow 0.4 E_0 = E_0 e^{-2\alpha}$$

$$\Rightarrow 0.4 = e^{-2\alpha}$$

$$\Rightarrow \ln 0.4 = -2\alpha$$

$$\Rightarrow \alpha = \frac{\ln 0.4}{-2} = 0.458 \text{ NP/m}$$

$$\delta = \frac{1}{\alpha} = \frac{1}{0.458} = 2.183 \text{ m}$$

$$v_p = \frac{\omega}{\beta} = \frac{2\pi \times 10^7}{1.6} = 1.25\pi \times 10^7 \text{ m/sec}$$

Q21) Skin depth $\delta = \frac{1}{\alpha}$

$$\Rightarrow \alpha = \frac{1}{\delta} = \frac{1}{0.1} = 10$$

$$(\alpha)_{\text{dB}} = 20 \log \alpha$$

$$20 = 20 \log \alpha$$

$$\Rightarrow \log \alpha = 1$$

$$\boxed{\alpha = 10}$$

$$E = E_0 e^{-\alpha z}$$

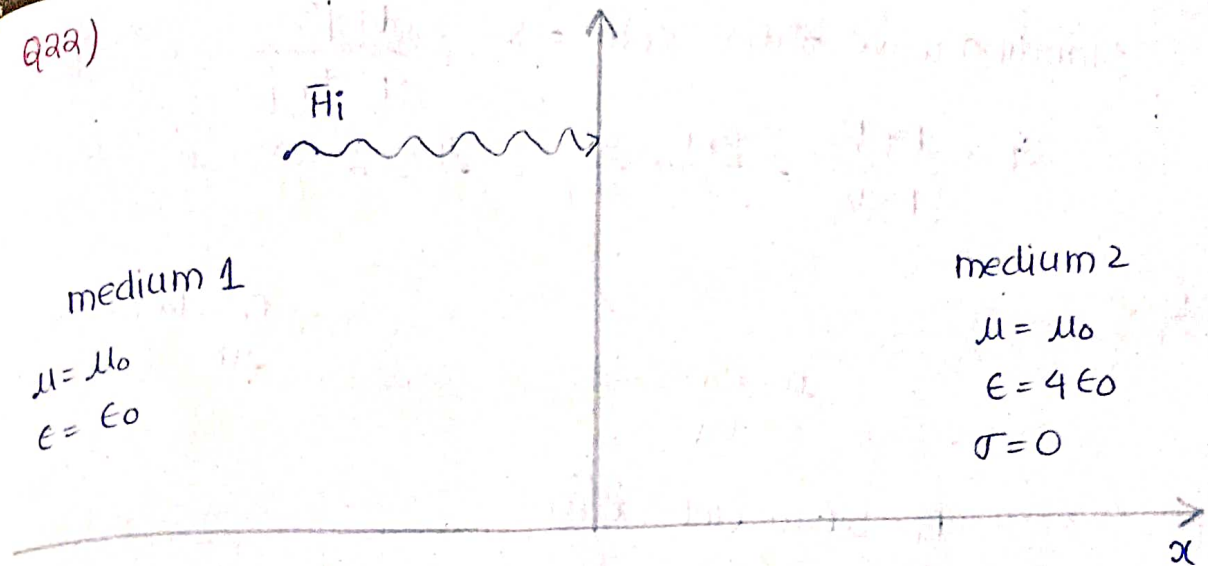
$$\frac{E_0}{10} = E_0 e^{-\alpha z}$$

$$0.1 = e^{-10z}$$

$$\ln(0.1) = -10z$$

$$z = \frac{\ln(0.1)}{-10} = 0.23 \text{ m}$$

Q22)



$$\vec{H} = 4 \sin(\omega t - 5x) \hat{a}_y \text{ A/m}$$

$$H_0 = 4 \text{ A/m} \quad \beta = 5 \text{ rad/m}$$

Total electric field

= Incident electric field + Reflected electric field

$$E_i = E_{i0} \sin(\omega t - 5x) \hat{a}_E$$

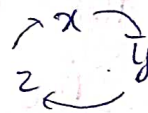
$$\beta_1 = \frac{\omega_1}{c} \Rightarrow \omega_1 = \beta_1 c = 5 \times 3 \times 10^8 = 15 \times 10^8 \text{ rad/sec}$$

$$\hat{a}_{E_i} = \hat{a}_H \times \hat{a}_k = \hat{a}_y \times \hat{a}_x = -\hat{a}_z$$



$$E_{i0} = H_0 \eta_1 = 4 \times 120 \pi = 480 \pi$$

$$E_i = 480 \pi \sin(15 \times 10^8 t - 5x) (-\hat{a}_z) \text{ V/m}$$



$$\eta_1 = 120 \pi \quad \eta_2 = 120 \pi \sqrt{\frac{\mu_0 \epsilon_2}{\epsilon_0 \epsilon_2}} = 120 \pi \times \sqrt{\frac{1}{4}} = 60 \pi$$

$$\Gamma_E = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{60 \pi - 120 \pi}{60 \pi + 120 \pi} = \frac{-60 \pi}{180 \pi} = -\frac{1}{3}$$

$$\Gamma_E = \frac{E_{r0}}{E_{i0}} = -\frac{1}{3}$$

$$E_{r0} = -\frac{1}{3} \times E_{i0} = -\frac{1}{3} \times 480 \pi = -160 \pi$$

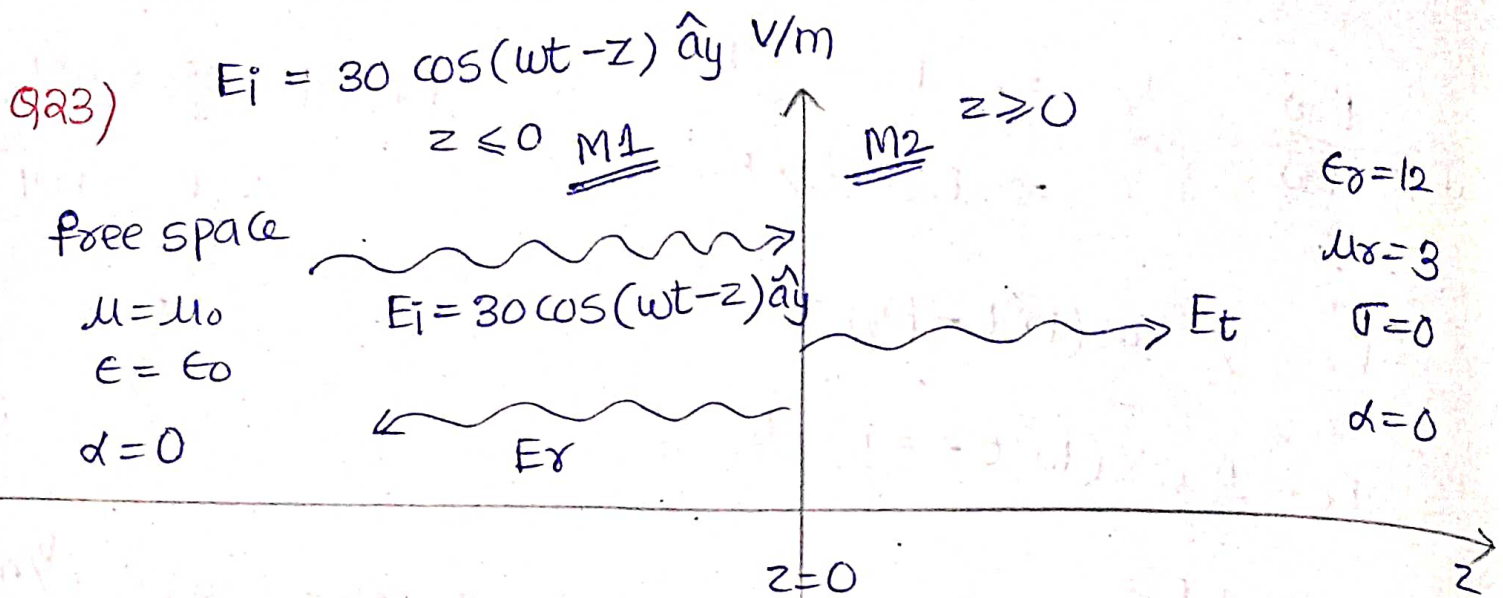
$$\hat{a}_{E_r} \neq \hat{a}_H$$

$$E_r = -160 \pi \sin(15 \times 10^8 t + 5x) (-\hat{a}_z) \text{ V/m}$$

$$\vec{E} = E_i + E_r = \{-480 \pi \sin(15 \times 10^8 t - 5x) \hat{a}_x + 160 \pi \sin(15 \times 10^8 t + 5x) \hat{a}_z\} \text{ V/m}$$

standing wave ratio $SWR = S_E = \frac{1 + |\Gamma_E|}{1 - |\Gamma_E|}$

$$S_E = \frac{1 + 1/3}{1 - 1/3} = \frac{3+1}{3} \times \frac{3}{3-1} = \frac{4}{2} = 2$$



medium 1:

$$E_{i0} = 30$$

$$\beta_1 = 1$$

$$\omega = ?$$

$$\beta_1 = \frac{\omega}{c} \Rightarrow \omega = \beta_1 c$$

$$\omega = 1 \times 3 \times 10^8$$

a) $\omega = 3 \times 10^8$ rad/sec

$$\Gamma_E = \frac{n_2 - n_1}{n_2 + n_1}$$

$$n_1 = 120\pi$$

$$n_2 = 120\pi \sqrt{\frac{\mu_r}{\epsilon_r}} = 120\pi \times \sqrt{\frac{3}{12}} = 60\pi$$

$$\Gamma_E = \frac{60\pi - 120\pi}{60\pi + 120\pi} = \frac{-60\pi}{180\pi} = -\frac{1}{3}$$

b) standing wave ratio $SWR = \frac{1 + |\Gamma_E|}{1 - |\Gamma_E|} = \frac{1 + \frac{1}{3}}{1 - \frac{1}{3}} = \frac{\frac{4}{3}}{\frac{2}{3}} = 2$

c) Reflected electric field $E_r = E_{r0} \cos(\omega t + \beta z) \hat{a}_y$

$$\omega = 3 \times 10^8 \text{ rad/sec}$$

$$\beta = 1 \text{ rad/m}$$

$$\Gamma_E = \frac{E_{r0}}{E_{i0}} = -\frac{1}{3}$$

$$E_{r0} = E_{i0} \left(-\frac{1}{3}\right) = 30 \times \left(-\frac{1}{3}\right) = \underline{\underline{-10}}$$

$$E_{z0} = -10 \cos(3 \times 10^8 t + z) \hat{a}_y$$

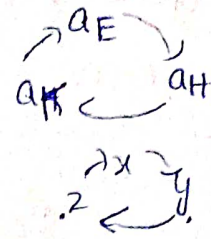
$$E_{z0} = 10 \cos(3 \times 10^8 t + z) (-\hat{a}_y)$$

Reflected magnetic field:

$$H_z = H_{z0} \cos(\omega t + \beta z) \hat{a}_H$$

$$H_{z0} = \frac{E_{z0}}{\eta} = \frac{10}{120\pi} = \frac{1}{12\pi}$$

$$\hat{a}_{H_z} = \hat{a}_k \times \hat{a}_E = -\hat{a}_z \times -\hat{a}_y = -\hat{a}_x$$



$$H_z = \frac{1}{12\pi} \cos(3 \times 10^8 t + z) (-\hat{a}_x)$$

d) Transmission coefficient $\tau_E = \frac{2\eta_2}{\eta_2 + \eta_1} = \frac{2 \times 60\pi}{120\pi + 60\pi}$

$$\tau_E = \frac{120\pi}{180\pi} = \frac{2}{3}$$

$$\tau_E = \frac{E_{t0}}{E_{i0}} = \frac{2}{3}$$

$$E_{t0} = E_{i0} \frac{2}{3} = 30 \times \frac{2}{3} = 20$$

d) Average power density of transmitted wave

$$P_{avg}|_t = \frac{E_{t0}^2}{2|\eta|} e^{-2\alpha z} \cos \theta \eta \hat{a}_z$$

$$= \frac{20^2}{2 \times 60\pi} \hat{a}_z \text{ W/m}^2$$

$$P_{avg}|_t = 1.061 \text{ W/m}^2 \hat{a}_z$$

Q24) $\vec{H}_i = 10 \sin(\omega t + \beta x) \hat{a}_z \text{ mA/m}$

Medium 1: ($x \geq 0$) $\sigma = 0$ $\epsilon = 9\epsilon_0$ $\mu = 4\mu_0$

Medium 2: ($x < 0$) free space

$$\eta_1 = 120\pi \sqrt{\frac{\mu_1}{\epsilon_1}} = 120\pi \sqrt{\frac{4}{9}} = \frac{120 \times 2}{3} \pi = 80\pi \Omega$$

$$\eta_2 = 120\pi \Omega$$

$$H_{i0} = 10 \text{ mA/m} \quad E_{i0} = H_{i0} \times \eta_1 = 10 \times 80\pi = 800\pi \text{ V/m}$$

$$E_i = 0.8\pi \sin(\omega t + \beta x) \hat{a}_{E_i} \text{ V/m}$$

$$\hat{a}_{E_i} = \hat{a}_{H_i} \times \hat{a}_{k_i} = \hat{a}_z \times (-\hat{a}_x) = -\hat{a}_y$$

$$E_i = 0.8\pi \sin(\omega t + \beta x) (-\hat{a}_y) \text{ V/m}$$

a) $\therefore$ The wave is polarised in $-\hat{a}_y$ direction

$$b) \beta_1 = \frac{\omega}{c} \sqrt{\epsilon_1 \mu_1} = \frac{2\pi \times 30 \times 10^6}{3 \times 10^8} \times \sqrt{9 \times 4} = 3.769 \approx 3.77 \text{ rad/m}$$

$$c) \vec{J}_D = \frac{\partial \vec{D}}{\partial t} = \epsilon \frac{\partial \vec{E}}{\partial t} = 9\epsilon_0 \frac{\partial}{\partial t} \left\{ \frac{\partial}{\partial t} (-0.8\pi \sin(\omega t + \beta x) \hat{a}_y) \right\}$$

$$= -(9\epsilon_0)(0.8\pi)(\omega) \cos(\omega t + \beta x) \hat{a}_y$$

$$\vec{J}_D = -9\epsilon_0 \times 0.8\pi \times 60\pi \times 10^6 \cos(6\pi \times 10^6 t + 3.77x) \hat{a}_y$$

$$J_D = 37.6 \cos(6\pi \times 10^6 t + 3.77x) (-\hat{a}_y) \text{ mA/m}$$

$$d) \Gamma_E = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{120\pi - 80\pi}{120\pi + 80\pi} = \frac{40\pi}{200\pi} = \frac{1}{5}$$

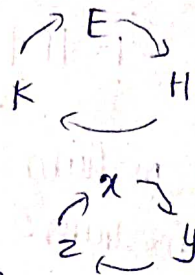
$$\Gamma_E = \frac{E_{r0}}{E_{i0}} = \frac{1}{5}$$

$$E_{r0} = \frac{1}{5} \times E_{i0} = \frac{1}{5} \times 0.8\pi =$$

$$\tau_E = \frac{2\eta_2}{\eta_2 + \eta_1} = \frac{2 \times 120\pi}{120\pi + 80\pi} = \frac{240\pi}{200\pi} = \frac{6}{5}$$

$$\tau_E = \frac{E_{t0}}{E_{i0}} = \frac{6}{5}$$

$$E_{t0} = \frac{6}{5} \times E_{i0} = \frac{6}{5} \times 0.8\pi$$



$$\bar{E}_x = E_{x0} \sin(\omega t - \beta x) (-\hat{a}_y)$$

$$\bar{E}_x = \frac{1}{5} \times 0.8\pi \sin(\omega t - \beta x) (-\hat{a}_y)$$

$$\bar{H}_x = \frac{E_{x0}}{\eta_1} \sin(\omega t - \beta x) (\hat{a}_{Hx})$$

$$H_{x0} = \frac{E_{x0}}{\eta_1} = \frac{\frac{1}{5} \times 0.8\pi}{80\pi} = 2 \text{ mA/m}$$

$$\hat{a}_{Hx} = \hat{a}_{kx} \times \hat{a}_E = \hat{a}_x \times -\hat{a}_y = -\hat{a}_z$$

$$\Rightarrow \bar{H}_x = 2 \sin(\omega t - \beta x) (-\hat{a}_z) = -2 \sin(\omega t - \beta x) \hat{a}_z \text{ mA/m}$$

$$\bar{E}_t = E_{t0} \sin(\omega t + \beta x) (-\hat{a}_y)$$

$$\bar{E}_t = \frac{6}{5} \times 0.8\pi \sin(\omega t + \beta x) (-\hat{a}_y)$$

$$\bar{H}_t = \frac{E_{t0}}{\eta_2} \sin(\omega t + \beta x) \hat{a}_{Ht}$$

$$H_{t0} = \frac{E_{t0}}{\eta_2} = \frac{\frac{6}{5} \times 0.8\pi}{120\pi} = 8 \text{ mA/m}$$

$$\hat{a}_{Ht} = \hat{a}_{kt} \times \hat{a}_{Et} = -\hat{a}_x \times -\hat{a}_y = \hat{a}_z$$

$$\bar{H}_t = 8 \sin(\omega t + \beta x) \hat{a}_z \text{ mA/m}$$

e) Region 1:

$$P_{avg1} = \frac{E_{t0}^2}{2|\eta_1|} e^{2\alpha x} \cos \theta_{\eta_1} (-\hat{a}_x) + \frac{E_{x0}^2}{2|\eta_1|} e^{-2\alpha x} \cos \theta_{\eta_1} \hat{a}_x$$

$$= \frac{(0.8\pi)^2}{2 \times 80\pi} (-\hat{a}_x) + \frac{(\frac{1}{5} \times 0.8\pi)^2}{2 \times 80\pi} \hat{a}_x$$

$$= (-12.5 \hat{a}_x + 0.502 \hat{a}_x) \text{ mW/m}^2$$

$$= -11.998 \text{ mW/m}^2 \approx 12$$

$$= -0.012 \text{ mW/m}^2$$

Region 2:

$$P_{avg2} = \frac{E_{t0}^2}{2|\eta_2|} e^{2\alpha x} \cos \theta_{\eta_1} (-\hat{a}_x)$$

$$= \frac{(\frac{6}{5} \times 0.8\pi)^2}{2 \times 120\pi} (-\hat{a}_x) = 0.01206 \text{ W/m}^2$$

Q25) $E_i = 10 \cos(\omega t - z) \hat{a}_y \text{ V/m}$

$\beta_1 = 1 \text{ rad/m}$

$\beta_1 = \frac{\omega}{c}$

a) $\Rightarrow \omega = \beta_1 c = 1 \times 3 \times 10^8 = 3 \times 10^8 \text{ rad/sec}$

$\beta_2 = \frac{\omega}{c} \sqrt{\mu \epsilon_2} = \frac{3 \times 10^8}{3 \times 10^8} \times \sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}} \text{ rad/m}$

$\beta_2 = \frac{\omega}{c} \sqrt{\mu \epsilon_2} = \frac{3 \times 10^8}{3 \times 10^8} \times \sqrt{1 \times 3} = \sqrt{3} \text{ rad/m}$

$\lambda_1 = \frac{2\pi}{\beta_1} = 2\pi \text{ m}$ $\left| \begin{array}{l} \eta_1 = 120\pi \\ \eta_2 = 120\pi \sqrt{\frac{\mu \epsilon_2}{\epsilon_1}} \\ = 120\pi \times \sqrt{\frac{1}{3}} = \frac{120\pi}{\sqrt{3}} \end{array} \right.$

$\lambda_2 = \frac{2\pi}{\beta_2} = \frac{2\pi}{\sqrt{3}} = 3.6275 \text{ m}$

b) $H_i = H_{i0} \cos(\omega t - z) \hat{a}_{Hi}$

$H_{i0} = \frac{E_{i0}}{\eta_1} = \frac{10}{120\pi} = \frac{1}{12\pi} = 26.525 \text{ mA/m}$

$\hat{a}_{Hi} = \hat{a}_{ki} \times \hat{a}_{Ei} = \hat{a}_z \times \hat{a}_y = -\hat{a}_x$

$H_i = -26.525 \cos(3 \times 10^8 t - z) \hat{a}_x \text{ mA/m}$

c) $\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\frac{120\pi}{\sqrt{3}} - 120\pi}{\frac{120\pi}{\sqrt{3}} + 120\pi} = \frac{120\pi(1 - \sqrt{3})}{120\pi(1 + \sqrt{3})} = -0.2679 = -0.268$

$\tau = \frac{2\eta_2}{\eta_2 + \eta_1} = \frac{2 \times \frac{120\pi}{\sqrt{3}}}{\frac{120\pi}{\sqrt{3}} + 120\pi} = \frac{2}{1 + \sqrt{3}} = 0.732$

d) Region 1:

$E_1 = E_i + E_o$

$\frac{E_{o0}}{E_{i0}} = \Gamma = -0.268$

$E_o = E_{o0} \cos(\omega t + z) \hat{a}_y$

$E_{o0} = 10 \times -0.268 = -2.68$

$E_o = -2.68 \cos(\omega t + z) \hat{a}_y \text{ V/m}$

$\bar{E}_1 = E_i + E_o$

$= 10 \cos(\omega t - z) \hat{a}_y - 2.68 \cos(\omega t + z) \hat{a}_y \text{ V/m}$

$$P_{avg1} = \frac{E_{i0}^2}{2|n_1|} e^{-2\alpha z} \cos \theta_{\eta_1} \hat{a}_z + \frac{E_{r0}^2}{2|n_1|} e^{+2\alpha z} \cos \theta_{\eta_1} (-\hat{a}_z)$$

$$P_{avg1} = \left(\frac{10^2}{2 \times 120\pi} - \frac{(-2.68)^2}{2 \times 120\pi} \right) \hat{a}_z = 0.1231 \text{ W/m}^2 \hat{a}_z$$

Region 2:

$$E_2 = E_t = E_{t0} \cos(\omega t - \frac{z}{\beta_2}) \hat{a}_y \text{ V/m}$$

$$\Gamma = \frac{E_{r0}}{E_{i0}} = 0.732 \Rightarrow E_{t0} = E_{i0} \times 0.732 = 10 \times 0.732 = 7.32$$

$$E_2 = E_t = 7.32 \cos(\omega t - \frac{z}{\beta_2}) \hat{a}_y \text{ V/m}$$

$$P_{avg2} = \frac{E_{t0}^2}{2|n_2|} e^{-2\alpha z} \cos \theta_{\eta_2} \hat{a}_z = \frac{7.32^2}{2 \times 120\pi} \times \sqrt{3} = 0.1231 \hat{a}_z \text{ W/m}^2$$

Q26) $\vec{E} = 10 \sin(\omega t + 3z) \hat{a}_x$

$$\beta_1 = 3 \text{ rad/m}$$

$$\beta_1 = \frac{\omega}{c} \text{ (air)}$$

a) $\omega = \beta_1 c = 3 \times 3 \times 10^8 = 9 \times 10^8 \text{ rad/sec}$

b) $\lambda_1 = \frac{2\pi}{\beta_1} = \frac{2\pi}{3} = 2.094 \text{ m}$

c) Loss tangent $\tan \theta = \frac{\sigma}{\omega \epsilon} = \frac{4 \times 36\pi}{9 \times 10^8 \times 80 \times 10^{-9}} = 2\pi = 6.283$

~~tan theta~~ $\tan \theta = \tan 2\theta_{\eta}$

$$\theta_{\eta} = \frac{1}{2} \tan^{-1}(\tan \theta) = \frac{1}{2} \times \tan^{-1}(2\pi) = 40.47^\circ$$

$$\theta_{\eta} = 40.47^\circ$$

$$|n_2| = 120\pi \sqrt{\frac{\mu_{r2}/\epsilon_{r2}}{\sqrt{1 + \tan^2 \theta}}} = 120\pi \times \sqrt{\frac{1}{80}} \times \frac{1}{(\sqrt{1 + (2\pi)^2})^{1/2}} = 16.71$$

$$n_2 = 16.71 \angle 40.71^\circ \quad n_1 = 120\pi$$

d) $\Gamma_E = \frac{n_2 - n_1}{n_2 + n_1} = \frac{16.71 \angle 40.47^\circ - 120\pi}{16.71 \angle 40.47^\circ + 120\pi} = 0.935 \angle 176.68^\circ$

$$\epsilon \cdot \tau_E = \frac{2n_2}{n_2 + n_1} = \frac{2 \times 16.71 \angle 40.47^\circ}{16.71 \angle 40.47^\circ + 120\pi} = 0.0857 \angle 38.87^\circ$$

$$\alpha_2 = \frac{\omega}{c} \sqrt{\frac{\mu_{r2} \epsilon_{r2}}{2} \left\{ \sqrt{1 + \left(\frac{\sigma}{\omega \epsilon} \right)^2} - 1 \right\}}$$

$$\alpha_2 = \frac{9 \times 10^8}{3 \times 10^8} \times \sqrt{\frac{80}{2} \left\{ \sqrt{1 + 6.283^2} - 1 \right\}} = 43.936 \text{ NP/m}$$

$$\beta_2 = \frac{\omega}{c} \sqrt{\frac{\mu_{r2} \epsilon_{r2}}{2} \left\{ \sqrt{1 + \left(\frac{\sigma}{\omega \epsilon} \right)^2} + 1 \right\}}$$

$$\beta_2 = \frac{9 \times 10^8}{3 \times 10^8} \times \sqrt{\frac{80}{2} \left\{ \sqrt{1 + (2\pi)^2} + 1 \right\}} = 51.48 \text{ rad/m}$$

$$\Gamma = \frac{E_{r0}}{E_{i0}} = 0.935 \angle 176.7^\circ$$

$$E_{r0} = E_{i0} \times 0.935 \angle 176.7^\circ$$

$$E_{r0} = 9.35 \angle 176.7^\circ$$

$$\bar{E}_x = 9.35 \sin(\omega t - 3z + 176.7^\circ) \hat{a}_y \text{ V/m}$$

$$\tau = \frac{E_{t0}}{E_{i0}} = 0.0857 \angle 38.89^\circ$$

$$E_{t0} = E_{i0} \times 0.0857 \angle 38.89^\circ$$

$$E_{t0} = 0.857 \angle 38.89^\circ$$

$$\bar{E}_t = 0.857 e^{+43.936z} \sin(\omega t + 51.48z + 38.89^\circ) \text{ V/m}$$