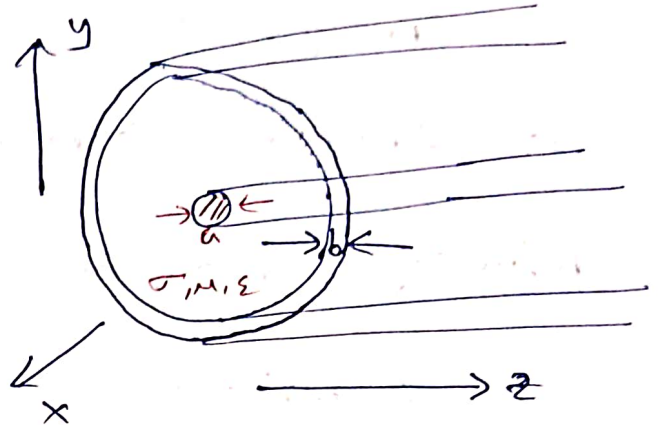


Unit - 6

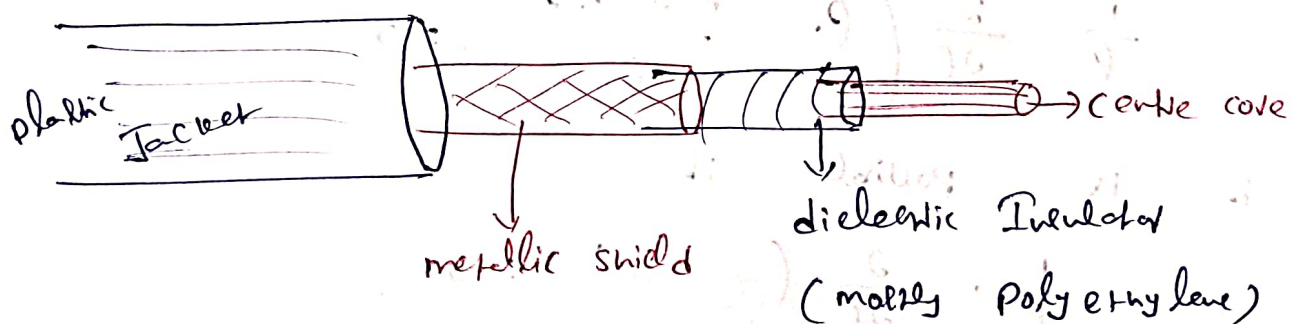
Co-axial Line

inner radius = a

outer " = b



- Two conductor structures
 - co-axial line
 - SHIP line
 - microstrip line
- For all two conductor structures TEM mode is possible
- co-axial line is not suitable to handle high powers
- To handle high power - waveguide is good
- co-axial - Both conductors are placed in a same geometrical axis (align with each other)



TEM mode:-

no longitudinal components
(Propagation)

$$\bar{H}_{zs} = \bar{E}_{zs} = 0$$

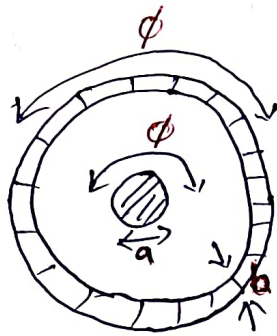
only transverse components

$$\bar{H}_{\phi s} \neq 0, \bar{E}_{\phi s} \neq 0, \bar{H}_{\rho s} \neq 0, \bar{E}_{\rho s} \neq 0$$

Assuming propagation along z-direction

$$\nabla^2 \phi(\rho, \phi) = 0$$

$$\frac{1}{\rho} \cdot \frac{\partial}{\partial \rho} \left(\rho \cdot \frac{\partial \phi(\rho, \phi)}{\partial \rho} \right) + \frac{1}{\rho^2} \cdot \frac{\partial^2 \phi(\rho, \phi)}{\partial \phi^2} = 0$$



inner conductor potential = V_0
outer " " = 0

Assumption:-

- potential will not vary in the direction of ϕ .

Then equation will be

$$\frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \cdot \frac{\partial \phi(\rho, \phi)}{\partial \rho} \right) = 0$$

it is possible if

$$\frac{1}{\rho} \cdot \frac{\partial}{\partial \rho} (\text{const}(c)) = 0$$

then $c = \rho \cdot \frac{\partial \phi(\rho)}{\partial \rho}$

$$\frac{\partial \phi(r)}{\partial r} = c/r$$

$$\partial \phi(r) = \frac{c}{r} \cdot \partial r$$

$$\int \partial \phi(r) = \int \frac{c}{r} \partial r$$

$$\phi(r) = c \ln(r) + D$$

upto $r=a$ potential $\phi(r=a) = V_0 = c \ln(a) + D$

at $r=b$ " $\phi(r=b) = 0 = c \ln(b) + D$

from the above $D = -c \ln(b)$

$$\phi(r=a) = c \ln(a) - c \ln(b) = V_0$$

$$c = \frac{V_0}{\ln(a/b)}$$

Then $\phi(r) = \frac{V_0}{\ln(a/b)} \ln(r) - c \cdot \ln(b)$

$$= \frac{V_0}{\ln(a/b)} \ln(r) - \frac{V_0}{\ln(a/b)} \ln(b)$$

$$= \frac{V_0}{\ln(a/b)} \ln(r/b)$$

$$\phi(r) = V_0 \ln(r/b) / \ln(a/b)$$

Electric field component

$$E = -\nabla \phi(r) = -\nabla V \quad \text{potential function}$$

$$= \text{gradient (scalar)}$$

↓

$$\frac{\partial}{\partial \rho} \phi(r) \alpha_\rho + \frac{1}{\rho} \cdot \frac{\partial}{\partial \phi} \phi(r) \alpha_\phi + \frac{\partial}{\partial z} \phi(r) \alpha_z$$

$\downarrow$
0

$$\bar{E}(r) = - \frac{V_0}{\ln(b/a)} \cdot \frac{\partial}{\partial \rho} [\ln(\rho/b)]$$

$$\bar{E}(r) = \frac{V_0}{\ln(b/a)} \cdot \frac{1}{\rho} \cdot b$$

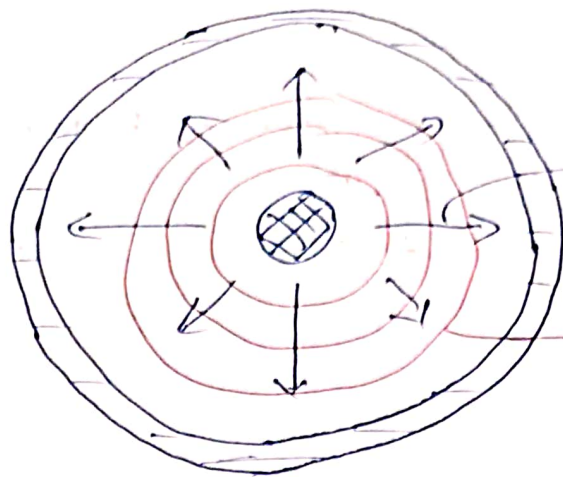
$$= \frac{V_0}{\ln(b/a)} \cdot \frac{1}{\rho}$$

$$\bar{E}_\rho(\rho, z) = \frac{V_0}{\ln(b/a)} \cdot \frac{1}{\rho} \cdot e^{-j\beta z} \hat{a}_\rho$$

$$\bar{H}_\phi = \frac{1}{\eta} (\hat{z} \times \hat{E}) = \frac{1}{\eta} \left(\hat{z} \times \frac{V_0}{\ln(b/a)} \cdot \frac{1}{\rho} \cdot e^{-j\beta z} \hat{a}_\rho \right)$$

Highal order
mag.

$$= \frac{1}{\eta} \cdot \frac{V_0}{\ln(b/a)} \cdot \frac{1}{\rho} \cdot e^{-j\beta z} \hat{a}_\phi$$



Characteristic Impedance:-

$$Z_0 = \frac{V_0}{I_0}$$

$$I = \oint \vec{H} \cdot d\vec{l}$$

in ϕ direction

$$\text{then } d\vec{l} = \rho d\phi$$

$$= \int_0^{2\pi} \left(\frac{1}{\eta} \cdot \frac{V_0}{\ln(b/a)} \cdot \frac{1}{\rho} \right) \cdot \rho d\phi$$

$$= \frac{V_0}{\eta \ln(b/a)} \cdot 2\pi = \frac{2\pi V_0}{\eta \ln(b/a)} e^{-j\beta z}$$

$$Z = \frac{V_0}{I_0} = \frac{V_0}{\left(\frac{2\pi V_0}{\eta \ln(b/a)} \right) e^{-j\beta z}} = \frac{\eta}{2\pi} \ln(b/a) e^{+j\beta z}$$

Dominant mode:-

- TEM mode is the dominant mode
- for TEM mode $f_c = 0$: i.e. it will allow all low signals

$$- \quad \bar{E}_\rho = \frac{V_0}{\ln(b/a)} \cdot \frac{1}{\rho} \cdot e^{-j\beta z} \hat{a}_\rho$$

$$- \quad \bar{H}_\phi = \frac{1}{\eta} \cdot \frac{V_0}{\ln(b/a)} \cdot \frac{1}{\rho} \cdot e^{-j\beta z} \hat{a}_\phi$$

∴ In the above equation we don't have any frequency component (Independent of frequency)

- if we want observe with respect to time then

$$- \quad \bar{E}_\rho(r, \phi, z, t) = \bar{E}_\rho \cdot e^{j\omega t}$$

Higher order modes:-

- TE & TM modes $f_c > f_c(\text{TEM mode})$

- Whatever the frequency is allowed by TE & TM modes, same frequency can be allowed by TEM mode.

Maximum power Handling Capacity:-

$$\vec{E}_\rho = \frac{V_0}{\ln(b/a)} \cdot \frac{1}{\rho} \cdot e^{-j\beta z} \hat{a}_\rho$$

$$|\vec{E}_\rho| = \frac{V_0}{\ln(b/a)} \cdot \frac{1}{\rho}$$

maximum when $\rho = a$,

Then maximum break down voltage V_0 at $\rho = a$

$$V_{\max} (\text{break down}) = |E_d| \cdot a \cdot \ln(b/a)$$

$$\text{Maximum power handling } P_{\max} = \frac{V_{\max}^2}{2 Z_0}$$

$$Z_0 = \frac{\eta}{2\pi} \ln(b/a),$$

$$\begin{aligned} \text{Then } P_{\max} &= \frac{|E_d|^2 \cdot a^2 \cdot \ln^2(b/a)}{2 \cdot \frac{\eta}{2\pi} \cdot \ln(b/a)} \\ &= \frac{\pi a^2 E_d^2 \ln(b/a)}{\eta} \end{aligned}$$

E_d = max field strength for dielectric to break down.

for air filled $E_d = 3 \times 10^6 \text{ V/m}$,

for $a = 1 \text{ mm}$ & $b/a = 2$

for $a = 5 \text{ mm}$ $b/a = 2$

$$P_{\max} = \frac{\pi (10^{-3})^2 (\ln(2)) \cdot (3 \times 10^6)^2}{120 \pi}$$

$$= 51.98 \text{ kW}$$

$$P_{\max} = \frac{\pi (10^{-3})^2 (\ln(2)) (3 \times 10^6)^2}{120 \pi}$$

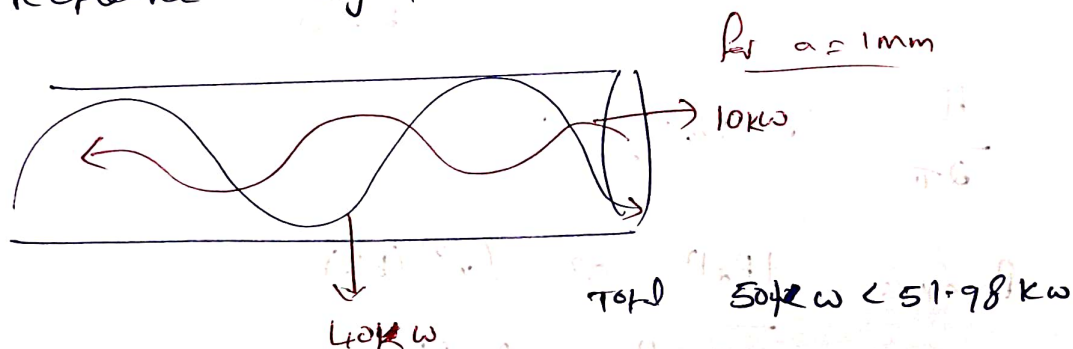
$$= 1299.5 \text{ kW}$$

$$V_{\max} = (3 \times 10^6) (10^{-3}) \ln(2)$$

$$= 2.07 \text{ kV}$$

$$V_{\max} = 10.35 \text{ kV}$$

- This $P_{\max}$ is for both Transmitted signal and Reflected signal



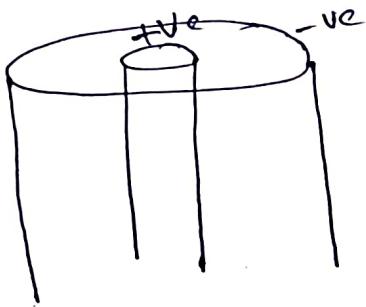
- We need to evaluate the Transmitted signal and Reflected signal, then we need to pick the suitable cable.

Coaxial connectors:-

Prof. Amitabha Bhattacharya
JIT, Kharagpur, (Lect - 10)

- cables to circuit connection establishment
 - Energy Transmission from one end to another end
- Char's:-

- Low VSWR
- High repeatability after a connect disconnect cycle
- mechanical strength



- the circumference $= 1 \lambda$

- To limit circumference ① to reduce multimodal propagation

② reduce power loss

③ reduce signal distortion

- height $h \approx \lambda/4$
of connector

① APC 3.5

- Amphenol precision connector - 3.5 mm
- good for repeatable connections
- very low of VSWR
- $Z_0 = 50 \Omega$ suitable for good power handling and min attenuation
- up to 34 GHz

② APC - 7

- Amphenol precision connector - 7mm
- good for repeatable connecting
- $Z_0 = 50\Omega$ (actual - is complete 3.5mm)
- extreme low VSWR
- operates 1.02 to 18 GHz

③ BNC

- Bayonet Navy connectors
- frequency about 4 GHz
- $Z_0 = 50\Omega$ to 75Ω

④ SMA

- sub-miniature A
- upto 24 GHz
- good for higher order modes of operation.

⑤ SMC

- sub-miniature C
- $Z_0 = 50\Omega$
- diameter upto 3.17mm
- upto 7 GHz

⑥ TNC

- Threaded Navy connector
- upto 12 GHz.

⑦ TYPE N → Navy

- 1 - 18 GHz
- $Z_0 = 50 - 75 \Omega$
- VSWR Extremely Low.

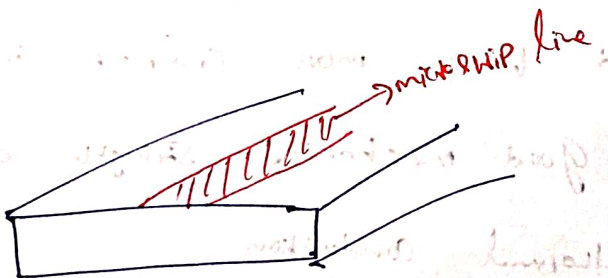
SHIP line:-

- To overcome the disadvantages of waveguide and coaxial line, SHIP line concept had developed

- waveguide is not flexible

- Coaxial line not able handle more power

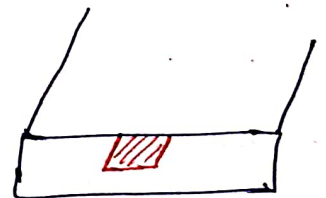
and the connections in between line is difficult



micro strip

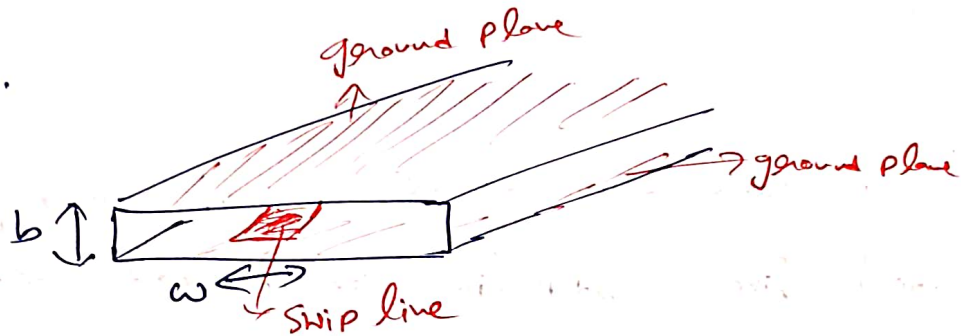


coplanar waveguide



SHIP lines

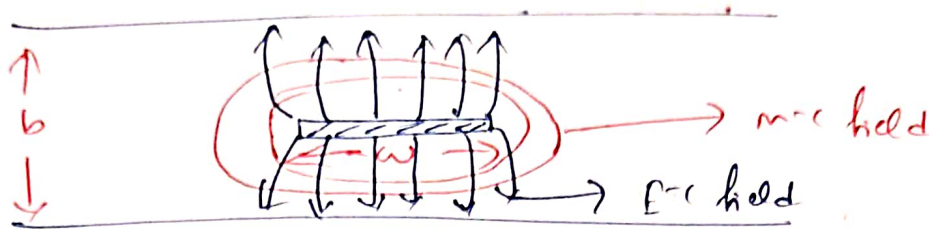
- This line consists of a strip placed between two parallel plates - but function as ground planes
- The strip line was often used in microwave filters and couplers besides its fabrication difficulty till other forms of planar transmission lines became popular
- Because of its natural shielding against incoming spurious signals and minimized emission, strip line became a viable candidate for multilayer PCB circuits at high frequencies.



- Substrate material :
 - small loss tangent
 - A large dielectric constant results in a short propagation wavelength and hence more compact circuits
 - good mechanical strength and thermal conduction

- Fig 4 - 4.4 Constant

- Strip line has 2 conductors and a homogeneous dielectric. It can support a TEM mode.



Phase velocity of a TEM mode

$$V_p = \frac{1}{\sqrt{\mu_0 \epsilon_0 \epsilon_r \mu_r}} \quad \text{for dielectric medium}$$

$$\beta = \frac{\omega}{V_p} = \text{propagation constant}$$

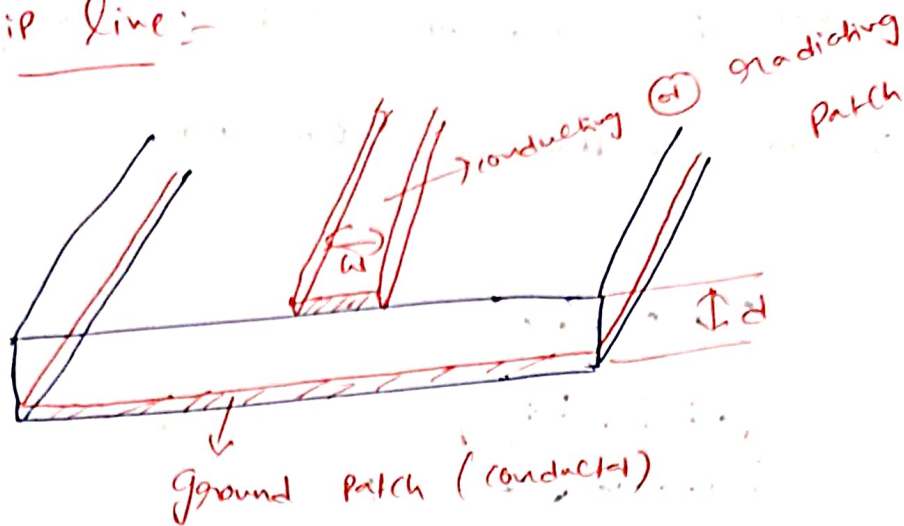
$$Z_0 = \sqrt{\frac{L}{C}} = \text{Characteristic Impedance of a transmission line}$$

- for practical computation

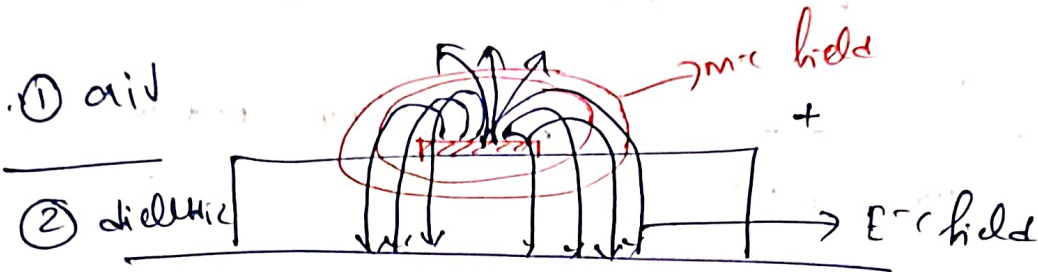
$$Z_0 = \frac{30 \pi}{\sqrt{\epsilon_r}} \cdot \frac{b}{w_e + 0.441b} \quad \text{where } \frac{w_e}{b} = \frac{w}{b} \begin{cases} 1 & \text{for } w/b > 0.35 \\ (0.35 - w/b)^2 & \text{for } w/b < 0.35 \end{cases}$$

$$\frac{w}{b} = \begin{cases} x & \text{for } \sqrt{\epsilon_r} Z_0 > 120 \\ 0.85 - \sqrt{0.6 - x} & \text{for } \sqrt{\epsilon_r} Z_0 < 120 \end{cases} \quad \text{where } x = \frac{30 \pi}{\sqrt{\epsilon_r} Z_0}$$

Microstrip Line:



- Pure TEM wave cannot propagate



- in TEM mode the phase velocity should be same in the entire direction

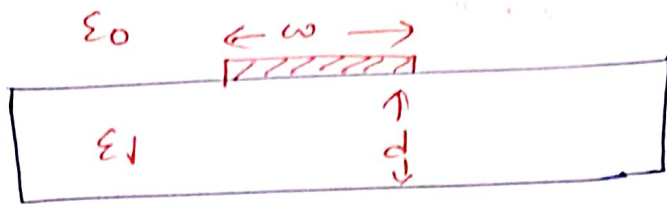
- But here wave first travels in air v_{p1} and then enters into dielectric medium v_{p2}

$$v_{p1} \neq v_{p2}$$

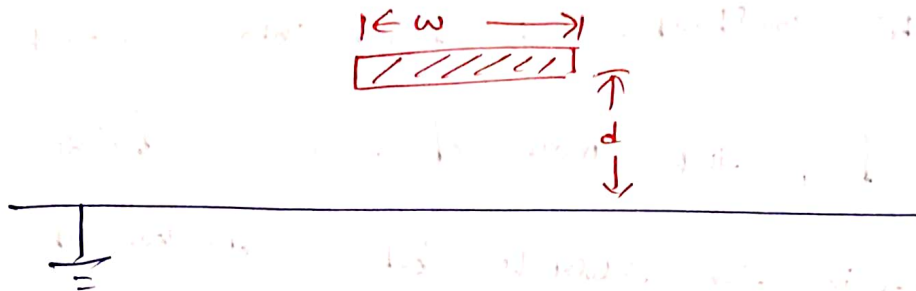
- Quasi TEM mode possible

- it is possible because $d \ll \lambda$.

- ~~TEM~~ - v_{p1} , z_0 , $E \propto H$ are mildly frequency dependent



- Char's Imp for microstrip line $Z_0 = \sqrt{\frac{L}{C}}$



- Removed dielectric medium

Phase velocity $V_p = c = \frac{1}{\sqrt{L C_{air}}}$ $\Rightarrow L = \frac{1}{C_0^2 C_{air}}$

(from TXION line concept)

$\downarrow$
velocity of light

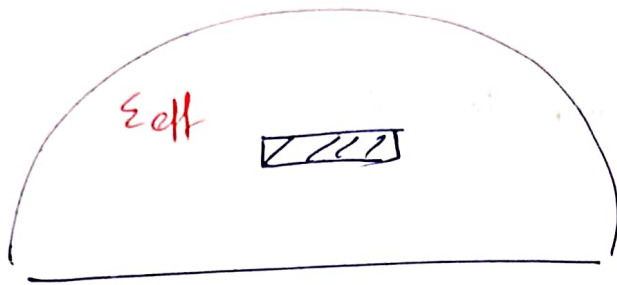
- L will not affect due to dielectric change

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{1}{C_0^2 C_{air} \cdot C}} = \frac{1}{C_0} \cdot \frac{1}{\sqrt{C \cdot C_{air}}}$$

$$V_p = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{C_0^2 C_{air}} \cdot C}} = C_0 \cdot \sqrt{\frac{C_{air}}{C}}$$

$$V_p = c_0 \sqrt{\frac{c_{air}}{c}} = \frac{c_0}{\sqrt{c/c_{air}}} = \frac{c_0}{\sqrt{\epsilon_r}}$$

$$\epsilon_r = \frac{c}{c_{air}}$$



Equivalent geometry of
Quasi-TEM microstrip line

Effective dielectric constant: it takes into account by considering the fact that most of the electric fields are contained within the substrate, but a fraction of the total energy exists within the air above the board.

$$\epsilon_{eff} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \cdot \frac{1}{\sqrt{1 + 12(d/w)}}$$

approximate formula

$$1 < \epsilon_{eff} < \epsilon_r \quad V_p = \frac{c_0}{\sqrt{\epsilon_{eff}}} \quad \beta = k \sqrt{\epsilon_{eff}}$$

Characteristic Impedance

$$Z_0 = \begin{cases} \frac{60}{\sqrt{\epsilon_{eff}}} \ln \left(\frac{8d}{w} + \frac{w}{4d} \right) & \text{for } w/d \leq 1 \\ \frac{120\pi}{\sqrt{\epsilon_r} \left(w/d + 1.39 + 0.66 \ln(w/d + 1.44) \right)} & \text{for } w/d \geq 1 \end{cases}$$

Q:- calculate the width and length of microstrip line for 50Ω characteristic Impedance and a 90° phase shift at 2.5GHz , The substrate thickness $d = 0.127\text{cm}$ with $\epsilon_r = 2.20$.

$$\textcircled{A} \quad \frac{w}{d} = \begin{cases} \frac{8e^A}{2^{2A}-2} & \text{for } w/d < 2 \\ \frac{2}{\pi} \left[B-1 - \ln(2B-1) + \frac{\epsilon_r-1}{2\epsilon_r} \left\{ \ln(B-1) + 0.39 - \frac{0.61}{\epsilon_r} \right\} \right] & \text{for } w/d \geq 2 \end{cases}$$

$$A = \frac{Z_0}{60} \sqrt{\frac{\epsilon_r+1}{2}} + \frac{\epsilon_r-1}{\epsilon_r+1} \left(0.23 + \frac{0.11}{\epsilon_r} \right)$$

$$B = \frac{377\pi}{2Z_0\sqrt{\epsilon_r}}$$

$$B = 7.985 \quad \frac{w}{d} = 3.081$$

$$\epsilon_{eff} = 1.87 \quad (\text{use } \epsilon_{eff} \text{ formula})$$

The line length l for a 90° phase shift

$$\phi = 90^\circ = \beta l = \sqrt{\epsilon_{eff}} k_0 l$$

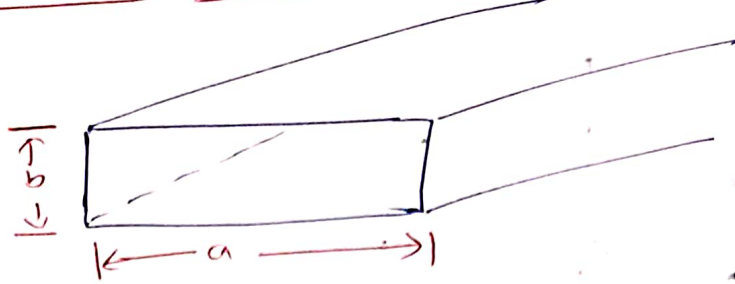
$$k_0 = \frac{2\pi f}{c} = 52.35/\text{m}$$

$$l = \frac{90^\circ (\pi/180^\circ)}{\sqrt{\epsilon_{eff}} k_0} = 2.19\text{cm}.$$

<u>Char't</u>	<u>Coax</u>	<u>waveguide</u>	<u>Strip line</u>	<u>microstrip</u>
① model:- Prefitted other	TEM TM, TE	TE ₁₀ TM, TE	TEM TM, TE	Quasi TEM Hybrid TE, TM
② Dispersion	None	medium	None	Low
③ Bandwidth	High	Low	High	High
④ Loss	medium	Low	High	High
⑤ Power capacity	medium	High	Low	Low
⑥ physical size	Large	Large	medium	Small
⑦ ease of fabrication	medium	medium	Early	Early
⑧ Integration with other components	Hard	Hard	Easy	Easy

Excitation of waveguide

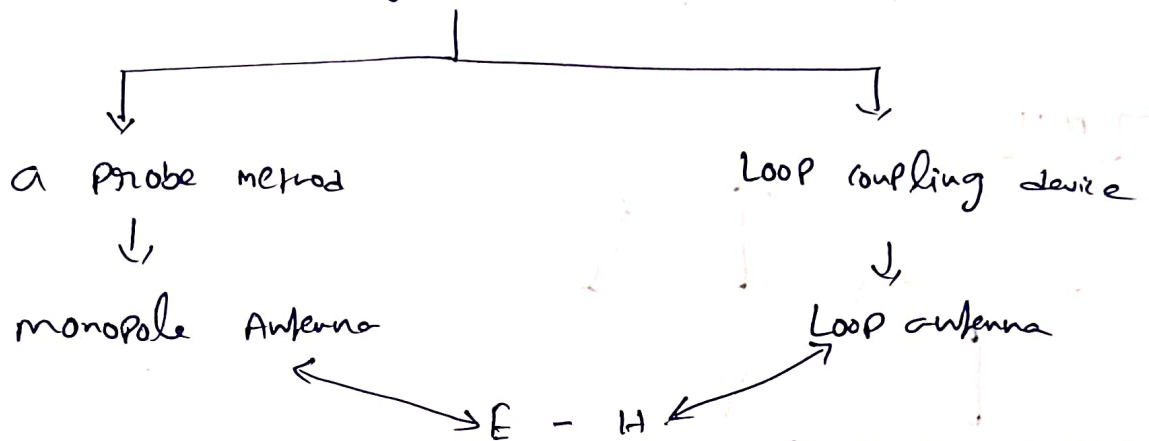
- In depth analysis available in Sergio PO-207 test book



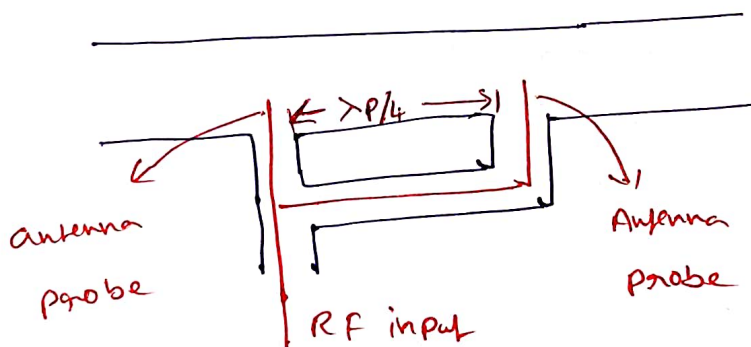
unit - 4

- possible modes - TE_{mn} & TM_{mn}
 - $\downarrow$ TE_{10} (dominant mode)
 - $\downarrow$ TM_{11} (dominant mode)

- excite the waveguide with a particular mode



TE₁₀:-

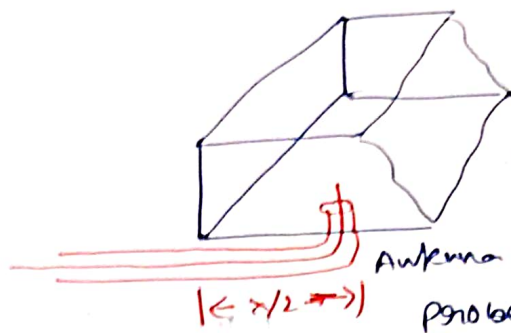


① Probe excitation

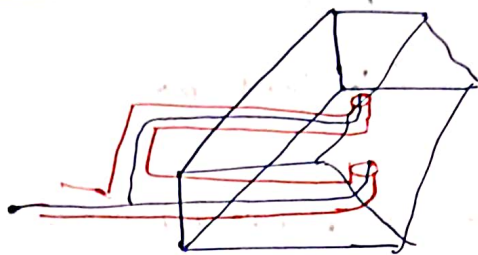
② Aperture "

Probe mode

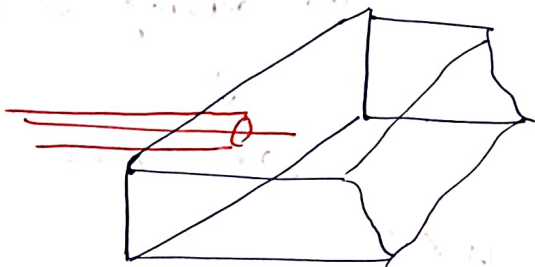
TE₁₀



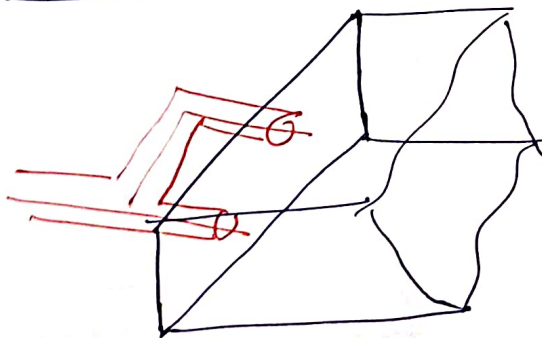
TE₂₀



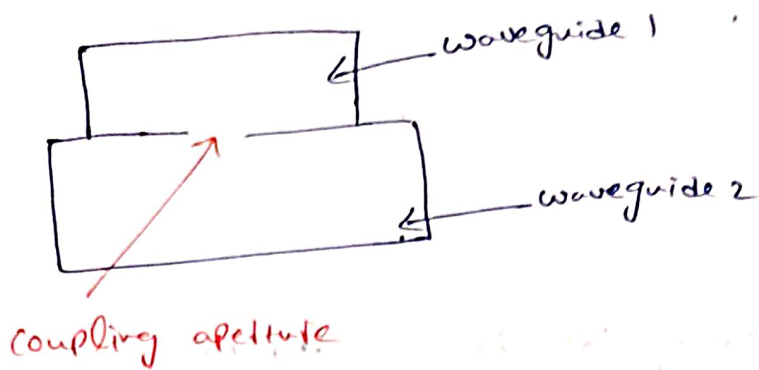
TM₁₁



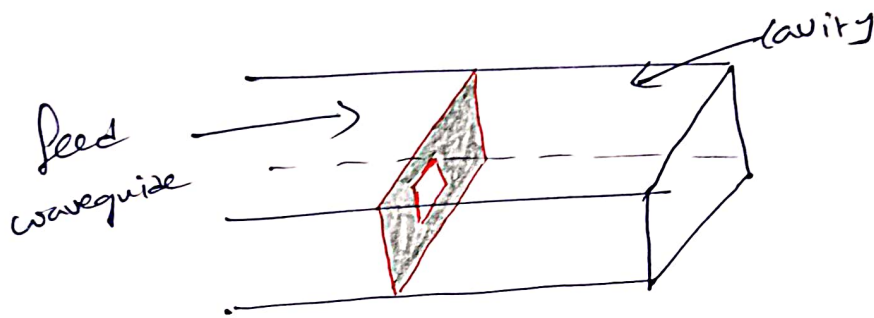
TM₂₁



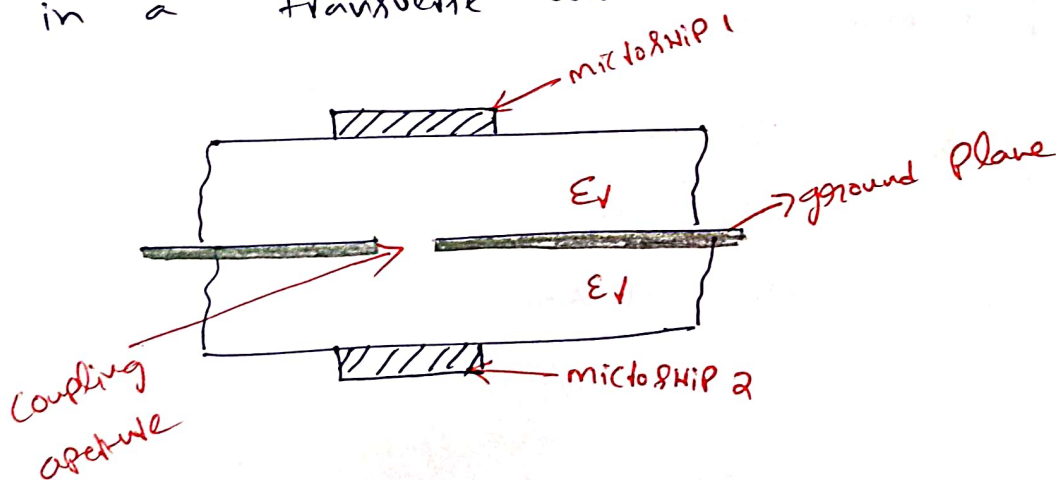
Aperture coupling:-



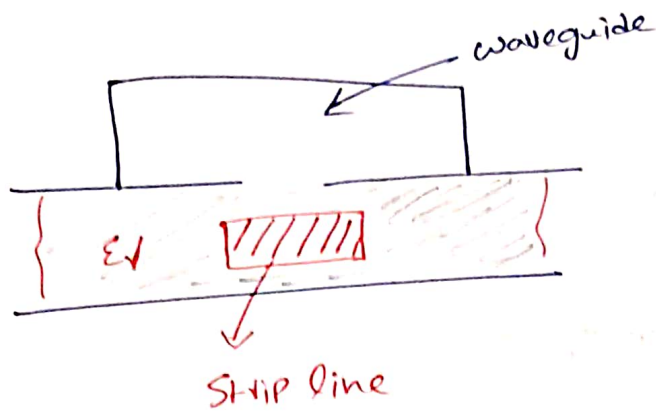
(a) Coupling between two waveguides via an aperture in the common broad wall



(b) Coupling to a waveguide cavity via an aperture in a transverse wall.



(c) Coupling between two microstrip lines via an aperture in the common ground plane



(d) coupling from a waveguide to a stripline via an aperture

attenuation constant for coaxial line

$$\alpha_c = \frac{P_{\text{loss}}}{2 \cdot P_{\text{Txed}}}$$

Wave propagation $E(z) = E_0 e^{-\alpha z} e^{j\beta z}$

Power density $P(z) = P_0 e^{-2\alpha z}$

- power at $z = P(z)$

- " " $(z+dz) = P(z) e^{-2\alpha dz}$

The differential power loss is

$$dP = P(z) - P(z+dz) = P(z) (1 - e^{-2\alpha dz})$$

if dz is small then $1 - e^{-2\alpha dz} \approx -2\alpha dz$

$$\text{Then } dP = P(z) - P(z+dz) \approx -2\alpha dz P(z)$$

$$\frac{dP}{dz} = -2\alpha P(z)$$

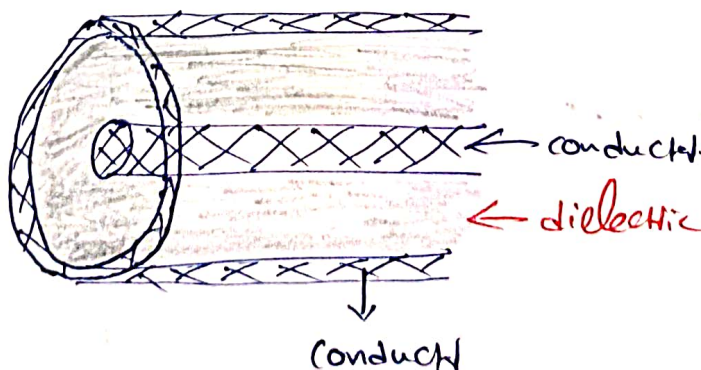
- power decays exponentially with a factor of 2α .

So $\alpha_c = \frac{P_{\text{loss}}(c)}{2 \cdot P_{\text{Txed}}}$

$c \rightarrow$ conductor loss.

$$\alpha = \frac{P_{\text{loss}}(c) + P_{\text{loss}}(d)}{2 \cdot P_{\text{Txed}}}$$

$d \rightarrow$ dielectric loss.



Power calculations from Poynting theorem

$$P = \frac{1}{2} \operatorname{Re} [E \times H^*] \Rightarrow \text{it is phasor representation}$$

in general $P = E \times H^*$

in phasor form $P(t) = \operatorname{Re} [E e^{j\omega t}] \times \operatorname{Re} [H e^{j\omega t}]$

From Identity Principle

$$P(t) = \operatorname{Re} [E e^{j\omega t}] \times \operatorname{Re} [H e^{j\omega t}] = \frac{1}{2} \operatorname{Re} [E \times H^*] + \frac{1}{2} \operatorname{Re} [E \times H^* e^{j2\omega t}]$$

Real power $P_R = \frac{1}{2} \operatorname{Re} [E \times H^*]$

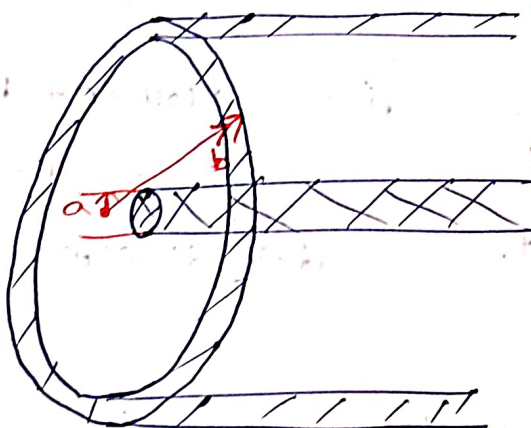
$$= \frac{1}{2\eta} |E|^2 \quad \text{or} \quad \frac{\eta}{2} |H|^2$$

Power Txed P_T

— Power delivered via dielectric

media i.e. $\rho = a$ to b

$\phi = 0$ to 2π



Then $P_{Txed} = \frac{1}{2\eta} \int_{\rho=a}^b \int_{\phi=0}^{2\pi} |E|^2 d\phi ds$

$$= \frac{1}{2\eta} \int_{\rho=a}^b \int_{\phi=0}^{2\pi} \left[\frac{V_0}{\ln(b/a)} \cdot \frac{1}{\rho} \cdot e^{-j\beta z} a_\phi \right]^2 \cdot d\ell \cdot (\rho d\phi).$$

$$= \frac{V_0^2}{2\eta (\ln(b/a))^2} \cdot \int_{\rho=a}^b \int_{\phi=0}^{2\pi} \frac{1}{\rho^2} \cdot \rho \cdot d\ell \cdot d\phi.$$

at $z=0$.

$$= \frac{V_0^2}{2\eta (\ln(b/a))^2} \left[\ln(\rho) \right]_a^b [\cancel{2\pi}]$$

$$= \frac{V_0^2 \pi}{\eta \ln(b/a)^2} [\ln(b/a)] = \frac{V_0^2 \pi}{\eta \ln(b/a)}$$

$$\boxed{P_{\text{Txed}} = \frac{V_0^2 \pi}{\eta \ln(b/a)}}$$

Power loss (P_L):- per unit length $z=0$ to 1m .

$$P_L = \iint_S \frac{\eta}{2} |H|^2 ds = \frac{R_s}{2} \int_{z=0}^1 \int_{\phi=0}^{2\pi} \left[|H(\rho=a)|^2 + |H(\rho=b)|^2 \right] dz$$

$R_s \Rightarrow$ Surface Resistance

$$P_L = \frac{R_s}{2} \left[\int_{z=0}^1 \int_{\phi=0}^{2\pi} \left[\frac{1}{\eta} \cdot \frac{V_0}{\ln(b/a)} \cdot \frac{1}{\rho} \cdot e^{-j\beta(\hat{\phi})} \right]_{z=0}^2 dz (\rho d\phi) \right.$$

$$\left. + \int_{z=0}^1 \int_{\phi=0}^{2\pi} \left[\frac{1}{\eta} \cdot \frac{V_0}{\ln(b/a)} \cdot \frac{1}{\rho} e^{-j\beta(\phi)} \right]^2 \cdot dz \cdot (\rho d\phi) \right]$$

$$P_L = \frac{R_s}{2} \cdot \frac{1}{\eta^2} \cdot \frac{V_0^2}{(\ln(b/a))^2} \left[\int_{z=0}^1 \int_{\phi=0}^{2\pi} \frac{1}{\rho^2} \cdot \rho \cdot dz d\phi \right. \\ \left. + \int_{z=0}^1 \int_{\phi=0}^{2\pi} \frac{1}{\rho^2} \cdot \rho \cdot dz d\phi \right]$$

$$P_L = \frac{R_s}{2} \cdot \frac{1}{\eta^2} \cdot \frac{V_0^2}{(\ln(b/a))^2} \cdot \left(\frac{1}{a} \cdot (z)_0^1 (\phi)_0^{2\pi} \right. \\ \left. + \frac{1}{b} (z)_0^1 (\phi)_0^{2\pi} \right)$$

$$P_L = \frac{R_s}{2} \cdot \frac{1}{\eta^2} \cdot \frac{V_0^2}{(\ln(b/a))^2} \cdot \left(\frac{2\pi}{a} + \frac{2\pi}{b} \right)$$

$$P_L = \frac{R_s \pi \cdot 2}{2 \eta^2} \cdot \frac{V_0^2}{(\ln(b/a))^2} \cdot \left(\frac{1}{a} + \frac{1}{b} \right)$$

$$P_L = \frac{R_s \cdot \pi \cdot V_0^2}{\eta^2 (\ln(b/a))^2} \cdot \left(\frac{1}{a} + \frac{1}{b} \right)$$

$$d_L = \frac{P_L}{2 \cdot P_T} = \frac{1}{2} \left[\frac{R_s \cdot \pi \cdot \overset{P_L}{V_0^2} \left(\frac{1}{a} + \frac{1}{b} \right)}{2 \eta^2 (\ln(b/a))^2} \right] \cdot \frac{\cancel{\eta} \cancel{\ln(b/a)}}{\frac{V_0^2 \cdot \cancel{\pi}}{1/P_T}}$$

$$d_L = \frac{1}{2} \cdot \left[\frac{R_s}{\eta} \cdot \frac{1}{\ln(b/a)} \cdot \left(\frac{1}{a} + \frac{1}{b} \right) \right]$$

To find maximum power handling capacity - "a" value

$$P_{\max} = \frac{\pi E_d^2}{\eta} \cdot a^2 \cdot \ln(b/a)$$

$$= c \cdot a^2 \cdot \ln(b/a) \quad \text{const } c = \frac{\pi E_d^2}{\eta}$$

- we will try to find optimal "a" value for maximum power handling capacity.

$$\frac{dP_{\max}}{da} = -c \left[2a \ln(b/a) - a^2 \cdot \frac{1}{a/b} \cdot \frac{1}{b} \right] = 0$$

$$= -c [2a \ln(b/a) - a] = 0$$

$$2 \ln(b/a) - 1 = 0$$

$$\ln(b/a) = 1/2 \quad \ln(x) = 1/2$$

$$x = 1.65 = b/a$$

$$\text{for this } b/a \quad Z_0 = \frac{\eta}{2\pi} \ln(b/a)$$

$$= \frac{377}{2\pi} \ln(1.65) = 30 \Omega$$

⑦ if $b/a = 1/3$ then what will be the change of ~~the~~
if a varied from 3 cm to 6 cm. Assume all the
Remaining parameters are same & ~~the~~ ~~same~~

"a" value for minimum attenuation constant

$$\alpha_c = \frac{1}{2} \left[\frac{R_s}{\eta} \cdot \frac{1}{\ln(b/a)} \cdot \left(\frac{1}{a} + \frac{1}{b} \right) \right]$$

$$= \frac{1}{2} \left[\frac{R_s}{\eta} \cdot \frac{1}{\ln(b/a)} \cdot \frac{1}{a} \left(1 + \frac{1}{b} \cdot a \right) \right]$$

$$= \frac{1}{2} \left[\frac{R_s}{\eta} \cdot \frac{1}{\ln(b/a)} \cdot \frac{1}{a} \left(1 + a/b \right) \right]$$

$$= \frac{1}{2} \left[\frac{R_s}{\eta} \cdot \frac{1}{\ln(b/a)} \cdot \frac{1}{a} \cdot \frac{a}{b} \left(\frac{b}{a} + 1 \right) \right]$$

$$= \frac{1}{2} \left[\frac{R_s}{\eta} \cdot \frac{1}{\ln(b/a)} \cdot \frac{1}{a} \cdot \frac{(1+b/a)}{(b/a)} \right]$$

$$\frac{\partial \alpha_c}{\partial a} = \frac{\partial}{\partial a} \left[\frac{1}{2} \cdot \frac{R_s}{\eta} \cdot \frac{1}{\ln(b/a)} \left(\frac{1}{a} + \frac{1}{b} \right) \right]$$

$$= \frac{R}{2\eta} \left[\frac{1}{a} \cdot \left(\frac{1}{\ln(b/a)} \right)^2 \left(\frac{1}{a} + \frac{1}{b} \right) + \frac{1}{\ln(b/a)} \cdot \left(-\frac{1}{a^2} \right) \right] = 0$$

$$\frac{d}{dx} \left[\frac{1}{\ln(1/x)} \right] = \frac{1}{x \cdot \ln(x)^2}$$

$$\frac{d}{dx} \left[\frac{1}{\ln(1/x)} \right] \cdot \frac{d}{dx} \left[\frac{1}{f} \right] = \frac{-\frac{d}{dx} f}{f^2} = -\frac{\frac{d}{dx} \ln(1/x)}{\ln(1/x)^2}$$

$$= \frac{-d\left(\frac{1}{x}\right) \cdot \frac{d}{dx}\left(\frac{1}{x}\right)}{\ln\left(\frac{1}{x}\right)^2} = \frac{-x \left(-\frac{1}{x^2}\right)}{\ln\left(\frac{1}{x}\right)^2} = \frac{1}{x \cdot \ln(x)^2}$$

$$\frac{d}{da} \left(\frac{1}{\ln(b/a)} \right) = \frac{1}{a \cdot (\ln(b/a))^2}$$

$$\frac{\partial \alpha}{\partial a} = \frac{R}{2\pi} \cdot \frac{1}{a} \cdot \frac{1}{\ln(b/a)} \left[\frac{1}{\ln(b/a)} \cdot \left(\frac{1}{a} + \frac{1}{b} \right) - \frac{1}{a} \right] = 0$$

$$\frac{1}{\ln(b/a)} \left(\frac{1}{a} + \frac{1}{b} \right) - \frac{1}{a} = 0$$

$$\frac{a}{\ln(b/a)} \left(\frac{1}{a} + \frac{1}{b} \right) - 1 = 0$$

$$\frac{a}{\ln(b/a)} \left(\frac{1}{a} + \frac{1}{b} \right) = 1$$

$$a \left(\frac{1}{a} + \frac{1}{b} \right) = \ln(b/a)$$

$$\left(1 + \frac{a}{b} \right) = \ln(b/a) \Rightarrow \ln(b/a) = \left(1 + \frac{1}{b/a} \right)$$

$$\ln(x) = \left(1 + \frac{1}{x} \right) \quad (iv)$$

$$\frac{a}{b} \left(\frac{b}{a} + 1 \right) = \ln(b/a)$$

$$\left(1 + \frac{b}{a} \right) = \frac{b}{a} \ln(b/a)$$

$$(1+x) = x \ln(x) \Rightarrow x = 3.59 = b/a$$

for this $Z_0 = \frac{377}{2\pi} \ln(3.59) = 77\Omega$

- max power handling
Capacity $b/a = 1.65$

- Left attenuation
at $b/a = 3.59$

- Choose optimal always
based on requirement

Q if $b/a = 1/4$, $V_0 = 5 \text{ V/m}$
and line is filled with loss
less material then find

(a) Electric field at $\rho = 3$,

(b) mag field at $\rho = 3$.

(A) $E(\rho) = \frac{V_0}{\ln(b/a)} \cdot \frac{1}{\rho} \cdot e^{-j\beta z} a_\rho$
at $z=0$

$$E(3) = \frac{5}{\ln(4)} \cdot \frac{1}{3} \cdot a_\rho$$

$$H_\phi = \frac{1}{\eta} \cdot \frac{V_0}{\ln(b/a)} \cdot \frac{1}{\rho} \cdot e^{-j\beta z} a_\phi$$

$$= \frac{1}{377} \cdot \frac{5}{\ln(4)} \cdot \frac{1}{3} \cdot a_\phi$$

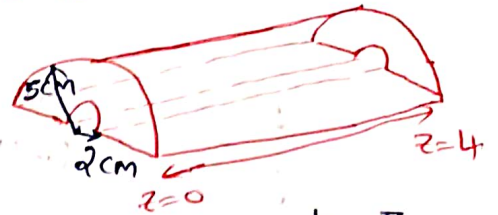
Q The Impedance of co-axial line
is $Z_0 = 78 \Omega$, it is filled
with air, then find the value
of "a". To $P_{\max} = 95 \text{ kW}$.

$$Z_0 = \frac{\eta}{2\pi} \ln(b/a) = 78 \Omega$$

$$P_{\max} = \frac{\pi a^2 |E_d|^2 \ln(b/a)}{\eta}$$

air filled $|E_d| = 3 \times 10^6 \text{ V/m}$
 $\eta = 377 \Omega$

Q find the Power Txed for
the following cell section
if $V_0 = 10 \text{ V}$



(A) Power Txed $= \frac{1}{2\eta} \int_{\rho=a}^b \int_{\phi=0}^{\pi} |E|^2 ds$

$$= \frac{1}{2\eta} \int_{\rho=2\text{cm}}^{5\text{cm}} \int_{\phi=0}^{\pi} \left[\frac{V_0}{\ln(b/a)} \cdot \frac{1}{\rho} \right]^2 \rho \cdot d\phi \cdot dz$$

$$= \frac{V_0^2 \cdot \pi}{\eta \cdot \ln(b/a)}$$

$$= \frac{(10)^2 \cdot (180^\circ)}{377 \cdot \ln(5\text{cm}/2\text{cm})}$$

Q find the power loss for the
above mention cell.

$$P_L = \frac{R_s}{2} \int_{z=0}^4 \int_{\phi=0}^{\pi} \left[(H(\rho=2\text{cm}))^2 + (H(\rho=5\text{cm}))^2 \right] ds$$

$$P_L = \frac{R_s}{2} \cdot 4 \cdot \frac{V_0^2 \cdot \pi}{(\ln(5\text{cm}/2\text{cm}))^2} \cdot \left(\frac{1}{2\text{cm}} + \frac{1}{5\text{cm}} \right)$$

Q find the attenuation constant

$$\alpha = \frac{P_L}{2 \cdot P_T}$$

Q An air filled Rectangular waveguide R_1 is operating at the frequency 2 GHz and another air filled Rectangular waveguide R_2 is operating at 4 GHz, The guide wavelengths of these waveguides at their respective frequencies are equal. If the cut-off frequency of waveguide R_1 is 1 GHz, what is the cut-off frequency of the waveguide R_2 in GHz?

Q we know $\lambda_g = \frac{\lambda}{\sqrt{1 - (f_c/f)^2}}$ $\lambda = \frac{2\pi}{\beta}$

$$\beta = \omega \sqrt{\mu \epsilon} = 2\pi f \sqrt{\mu \epsilon}$$

$$\lambda_g = \frac{2\pi}{2\pi f \sqrt{\mu \epsilon} \sqrt{1 - (f_c/f)^2}}$$

we know $\lambda_{g1} = \lambda_{g2}$

$$\frac{1}{f_1 \sqrt{\mu_1 \epsilon_1} \sqrt{1 - (f_{c1}/f_1)^2}} = \frac{1}{f_2 \sqrt{\mu_2 \epsilon_2} \sqrt{1 - (f_{c2}/f_2)^2}}$$

$$\mu_1 = \mu_2 \quad \& \quad \epsilon_1 = \epsilon_2$$

$$\frac{1}{\sqrt{f_1^2 - f_{c1}^2}} = \frac{1}{\sqrt{f_2^2 - f_{c2}^2}}$$

$$f_2^2 - f_{c2}^2 = f_1^2 - f_{c1}^2$$

$$(4 \text{ GHz})^2 - f_{c2}^2 = (2 \text{ GHz})^2 - (1 \text{ GHz})^2$$

$$\boxed{f_{c2}^2 = 13 \text{ GHz}^2}$$

Q A square waveguide carries TE_{11} mode whose axial magnetic field is given by

$$H_z = H_0 \cos\left(\frac{\pi x}{\sqrt{8}}\right) \cos\left(\frac{\pi y}{\sqrt{8}}\right) \text{ A/m, where waveguide}$$

dimensions are in cm. what is the cut-off frequency of

(A)

$$f_c = \frac{1}{2\pi \sqrt{\mu_0 \epsilon_0}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$f_c = \frac{1}{2 \sqrt{\mu_0 \epsilon_0}} \left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 \right]^{1/2}$$

$$= \frac{1}{2 \sqrt{\mu_0 \epsilon_0}} \sqrt{\left(\frac{1}{\sqrt{8}}\right)^2 + \left(\frac{1}{\sqrt{8}}\right)^2}$$

$$H_z = H_0 \cos\left(\frac{m\pi}{a} x\right) \cos\left(\frac{n\pi}{b} y\right) \text{ A/m.}$$

$$\frac{m\pi}{a} = \frac{\pi}{\sqrt{8}}$$

$$\frac{n\pi}{b} = \frac{\pi}{\sqrt{8}}$$

$$f_c = 7.5 \text{ GHz}$$

Q An air filled rectangular waveguide with dimensions $a = 75 \text{ mm}$, $b = 37.5 \text{ mm}$ has same guide wavelength at frequencies f_1 and f_2 when operated at TE_{10} and TE_{20} modes respectively. if the frequency f_1 is 13 GHz , what is frequency $f_2 = ?$

(A)

$$f_1 \rightarrow TE_{10} \quad f_2 \rightarrow TE_{20}$$

$$(\lambda_g)_1 = (\lambda_g)_2$$

$$\frac{2\pi}{\beta_1 \sqrt{1 - (f_c/f_1)^2}} = \frac{2\pi}{\beta_2 \sqrt{1 - (f_c/f_2)^2}}$$

$$\frac{2\pi}{\sqrt{\mu_1 \epsilon_1} \sqrt{f_1^2 - f_{c1}^2}} = \frac{2\pi}{\sqrt{\mu_2 \epsilon_2} \sqrt{f_2^2 - f_{c2}^2}}$$

$$\mu_1 = \mu_2, \quad \epsilon_1 = \epsilon_2$$

$$f_1^2 - f_{c1}^2 = f_2^2 - f_{c2}^2$$

$$f_{c1} = \frac{1}{2\sqrt{\mu\epsilon}} \left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 \right]^{1/2}$$

$$= \frac{c}{2} \left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 \right]^{1/2}$$

$$TE_{10} \Rightarrow \frac{c}{2} \left[\left(\frac{1}{a}\right)^2 + 0 \right]^{1/2} = \frac{c}{2a} = \frac{3 \times 10^8}{2 \times 75 \times 10^{-3}}$$

$$TE_{20} \Rightarrow \frac{c}{2} \left[\left(\frac{2}{a}\right)^2 + 0 \right]^{1/2} = \frac{c \cdot 2}{2 \cdot a} = \frac{c}{a}$$

$$\frac{1}{2} f_{c1} = f_{c2} \Rightarrow f_{c1} = \frac{1}{2} f_{c2} \quad f_{c2} = \frac{c}{a} = \frac{3 \times 10^8}{75 \times 10^{-3}}$$

$$\textcircled{1} \quad f_{c1} = \underline{2 \text{ GHz}}$$

$$f_{c2} = \underline{4 \text{ GHz}}$$

$$= \frac{1}{25} \times 10^{12} = 0.04 \times 10^{12} = 4 \text{ GHz}$$

$$(13 \text{ GHz})^2 - (2 \text{ GHz})^2 = f_2^2 - (4 \text{ GHz})^2$$

$$169 - 4 + 16 = f_2^2 \quad 169 + 12 = f_2^2$$

$$f_2 = 2\sqrt{13} \text{ GHz}$$

$$f_2^2 = 181 \text{ GHz}^2$$

$$= \sqrt{181} \text{ GHz}$$

$$13 \times 4 = 52$$

④ A cylindrical waveguide with radius of 3.5 cm has waves travelling in TM_{12} mode. The value of 1st zero of 2nd order Bessel function is 7. Find the cut-off wavelength for this mode?

$$\textcircled{A} \quad n=1, \quad m=2, \quad \left(f_c\right)_{TM_{12}} = \frac{x_{np} \cdot v_p}{2\pi a}$$

①

$$k_c a = x_{np}$$

$$\omega_c \sqrt{\mu\epsilon} \cdot a = x_{np}$$

$$\omega_c = \frac{x_{np}}{\sqrt{\mu\epsilon} \cdot a}$$

$$f_c = \frac{\chi_{np}}{2\pi\sqrt{\mu\epsilon} \cdot a}$$

$$(f_c) = \frac{7}{2\pi\sqrt{\mu_0\epsilon_0} \cdot a} = \frac{7}{2\pi\sqrt{\mu_0\epsilon_0} \cdot 3.5} = \frac{7 \cdot c}{2\pi \cdot 3.5}$$

$$\lambda_c = \frac{2\pi}{f_c} \quad f = \frac{c}{\lambda} \quad \lambda = \frac{c}{f} = \frac{2c}{2\pi} = \frac{c}{\pi}$$

$$\lambda_c = \frac{c}{c/\pi} = \pi \text{ cm.}$$

④ A hollow air filled rectangular waveguide has internal dimensions of $a \text{ cm} \times b \text{ cm}$. if $a = 2b$ and the cut-off frequency of the TE_{02} mode is 12 GHz , the cut-off frequency of the dominant mode is

$$\textcircled{A} \quad f_c = \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

air filled $\mu = \mu_0 \quad \epsilon = \epsilon_0$

$$\text{then } f_c = \frac{c}{2} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$TE_{02} \quad f_c = \frac{c}{2} \sqrt{\left(\frac{2 \cdot \pi}{b}\right)^2} = 12 \text{ GHz}$$

$$\frac{3 \times 10^8}{2} \cdot \frac{2 \cdot \pi}{b} = 12 \times 10^9$$

$$\frac{3}{2} \cdot \pi \cdot \frac{\pi}{b} = 12 \times 10^9$$

$$b = \frac{\pi}{40} =$$

$$a = 2b = \frac{\pi}{20}$$

$$(f_c)_{TE_{10}} = \frac{c}{2} \sqrt{\left(\frac{\pi}{a}\right)^2}$$

$$(f_c)_{TE_{20}} = \frac{c}{2} \sqrt{\left(\frac{2\pi}{b}\right)^2}$$

$$b = a/2 \quad (f_c)_{TE_{20}} = \frac{c}{2} \sqrt{\left(\frac{4\pi}{a}\right)^2}$$

$$= 4 \frac{c}{2} \sqrt{\left(\frac{\pi}{a}\right)^2}$$

$$(f_c)_{TE_{20}} = 4 (f_c)_{TE_{10}}$$

$$(f_c)_{TE_{10}} = \frac{(f_c)_{TE_{20}}}{4} = \frac{12 \text{ GHz}}{4}$$

$$= 3 \text{ GHz}$$

Q) Two rectangular waveguides, one air-filled with internal dimensions $2 \text{ cm} \times 1 \text{ cm}$ and the other dielectric filled with internal dimensions $1 \text{ cm} \times 0.5 \text{ cm}$, have the same cut-off frequency for the dominant mode. Then find ϵ_r for dielectric material.

A)

$$f_{c1} = f_{c2}$$

$$\downarrow \quad \quad \downarrow$$

$$TE_{10} \quad \quad TE_{10}$$

$$\frac{1}{\sqrt{\mu\epsilon}} \cdot \left(\frac{1}{a_1}\right) = \frac{1}{\sqrt{\mu\epsilon}} \cdot \left(\frac{1}{a_2}\right)$$

$$\frac{1}{\sqrt{\mu_0 \epsilon_0} \sqrt{\mu_{r1} \epsilon_{r1}}} \cdot \left(\frac{1}{2 \text{ cm}}\right) = \frac{1}{\sqrt{\mu_0 \epsilon_0} \sqrt{\mu_{r2} \epsilon_{r2}}} \cdot \left(\frac{1}{1 \text{ cm}}\right)$$

$$\frac{1}{2} \cdot \frac{1}{\sqrt{\epsilon_{r1}}} = \frac{1}{\sqrt{\epsilon_{r2}}} \cdot \frac{1}{1}$$

$$\boxed{\epsilon_{r2} = 4}$$

Q Consider an air filled rectangular waveguide with a cross section of $5\text{ cm} \times 3\text{ cm}$. For this waveguide, the cut-off frequency (in MHz) of TE_{21} mode is

A

$$f_c = \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$= \frac{3 \times 10^8}{2} \sqrt{\left(\frac{2}{5 \times 10^{-2}}\right)^2 + \left(\frac{1}{3 \times 10^{-2}}\right)^2}$$

$$= 7810 \text{ MHz}$$

Q TEM propagation is NOT possible in
- air filled cylindrical waveguide

Q The interior of a $\frac{20}{3}\text{ cm} \times \frac{20}{4}\text{ cm}$ rectangular waveguide is completely filled with a dielectric of $\epsilon_r = 4$. Then kind wave of free space wavelength shorter than _____ can be propagated in the TE_{11} mode

A

$$\lambda_c = c/f_c$$

$$(f_c)_{TE_{11}} = \frac{1}{2\sqrt{\mu_0 \mu_r \cdot \epsilon_0 \epsilon_r}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$= \frac{1}{2\sqrt{\mu_0 \cdot \epsilon_0 \cdot 4}} \sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2}$$

$$= \frac{3 \times 10^8}{4} \sqrt{\left(\frac{3}{20}\right)^2 + \left(\frac{4}{20}\right)^2}$$

$$= \frac{3 \times 10^8}{4} \cdot \frac{1}{20 \times 10^{-2}} \sqrt{9 + 16}$$

$$= \frac{3 \times 10^8}{4} \cdot \frac{1}{204} \cdot 5$$

$$= \frac{3}{16} \cdot 10^8 \times 10^2$$

$$\lambda_c = \frac{c}{\frac{c}{4} \sqrt{\left(\frac{3}{20 \times 10^{-2}}\right)^2 + \left(\frac{4}{20 \times 10^{-2}}\right)^2}}$$

$$\lambda_c = \frac{4 \times 20^4 \times 10^{-2}}{5} =$$

Q) A rectangular waveguide of width w and height h has cut-off frequencies for TE_{10} and TE_{11} modes in the ratio 1:2. The aspect ratio w/h rounded off to two decimal places is _____

$$\frac{(f_c)_{TE_{10}}}{(f_c)_{TE_{11}}} = \frac{1}{2}$$

$$\frac{\frac{1}{2\pi\mu\epsilon} \sqrt{\left(\frac{1}{w}\right)^2 + \left(\frac{0}{h}\right)^2}}{\frac{1}{2\pi\mu\epsilon} \sqrt{\left(\frac{1}{w}\right)^2 + \left(\frac{1}{h}\right)^2}} = \frac{1}{2}$$

$$\frac{1/w}{1/w \sqrt{1 + (w/h)^2}} = \frac{1}{2}$$

$$2 = \sqrt{1 + (w/h)^2}$$

$$4 = 1 + (\omega/\omega_c)^2 \quad \boxed{\omega/\omega_c = \sqrt{3}}$$

Q) The cut off frequency of TE_{01} mode of an air filled rectangular waveguide having inner dimensions $a \text{ cm} \times b \text{ cm}$ ($a > b$) is twice that of the dominant TE_{10} mode. When the waveguide is operated at a frequency which is 25% higher than the cut off frequency of the dominant mode, the guide wavelength is found to be 4 cm. the value of b (in cm). is _____

(A) $(f_c)_{TE_{01}} = 2(f_c)_{TE_{10}} \Rightarrow a = 2b$

$$f = 1.25 (f_c)_{TE_{10}} \Rightarrow \lambda_g = 4 \text{ cm} \quad \underline{\underline{b = 2}}$$

$$a = 2b$$

$$f = 1.25 (f_c)_{TE_{10}}$$

$$\lambda/\lambda_c = 1.25 \cdot \lambda_c/2a$$

$$\frac{1}{\lambda_c^2} + \frac{1}{\lambda_g^2} = \frac{1}{\lambda^2}$$

$$\left(\frac{1}{2a}\right)^2 + \left(\frac{1}{4 \times 10^{-2}}\right)^2 = \left(\frac{1.25}{2a}\right)^2$$

$$a = 6 \text{ cm}$$

$$b = \frac{a}{2} = 3 \text{ cm.}$$

③ Consider an air filled rectangular waveguide with dimensions $a = 2.286 \text{ cm}$ and $b = 1.016 \text{ cm}$. at 10 GHz operating frequency, the value of propagation constant (per meter) of the corresponding propagating mode. (β_g)

①

$$\beta_g = \beta \sqrt{1 - (f_c/f)^2}$$

$$= 2\pi f \sqrt{\mu\epsilon} \sqrt{1 - (f_c/f)^2}$$

$$= 2\pi (10 \times 10^9) \sqrt{\mu_0 \epsilon_0} \sqrt{1 - \frac{f_c}{10 \times 10^9}}$$

$$f_c = \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2} \quad \text{TE}_{10}$$

$$= 6.5 \text{ GHz}$$

$$\beta_g = 158 / \text{m}$$

④ The longitudinal component of the magnetic field inside an air filled rectangular waveguide made of a perfect electric conductor is give by

$$H(x, y, z, t) = 0.1 \cos(25\pi x) \cos(30.5\pi y) \cos(12\pi \times 10^6 t - \beta z) \text{ A/m}$$

The cross-sectional dimensions of the waveguide are given as $a = 0.08 \text{ m}$ and $b = 0.033 \text{ m}$. the mode of

Propagation in side tee waveguide is

- $E_{zs} = 0$, $H_{zs} \neq 0$

So it's TE mode

Then $\frac{m\pi}{a} = 25\pi$, $\frac{n\pi}{b} = 30.3\pi$

$$m = 25a$$

$$= 25 \times \frac{82}{1004}$$

$$m = 2$$

$$n = \frac{30.3 \times 33}{1000}$$

$n = 1$	$m = 2$
---------	---------

So TE ₂₁ mode

Q An air-filled rectangular waveguide of internal dimensions a cm $\times$ b cm ($a > b$) has a cutoff frequency of 6 GHz for the dominant TE₁₀ mode, for the same waveguide, if the cutoff frequency of the TM₁₁ mode is 15 GHz, the cutoff frequency of the TE₀₁ mode in GHz is _____

A $(f_c)_{TE10} = \frac{c}{2a} = 6 \text{ GHz}$

$$(f_c)_{TM11} = \frac{c}{2} \sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2} = 15 \text{ GHz}$$

$$(f_c)_{TE01} = \frac{c}{2b}$$

⑦ for a rectangular waveguide of internal dimensions $a \times b$ ($a > b$) and

$$(f_c)_{TE_{11}} = \frac{(f_c)_{TE_{10}} + (f_c)_{TE_{20}}}{2}$$

- if $a = 5 \text{ cm}$ find value of b if.

⑧

$$\frac{c}{2} \sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2} = \frac{\frac{c}{2a} + \frac{c}{2a}}{2}$$

$$\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2} = \frac{\frac{1}{a} + \frac{2}{a}}{2}$$

$$a = 5 \text{ cm}, \quad b = 2 \text{ cm}$$

⑨ An air filled rectangular waveguide has inner dimensions of $3 \text{ cm} \times 2 \text{ cm}$. The wave impedance of the TE_{20} mode of propagation in the waveguide at a frequency of 30 GHz is (η for free space $= 377 \Omega$) 400 Ω

⑩

$$\eta_{TE_{20}} = \frac{\eta}{\sqrt{1 - (f_c/f)^2}} = \frac{\omega \mu}{\beta_g}$$

$$(f_c)_{TE_{20}} = \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{2}{a}\right)^2 + \left(\frac{1}{b}\right)^2} = \frac{c}{2} \cdot \frac{2}{a} = \frac{c}{a}$$

$$= \frac{3 \times 10^8}{3 \times 10^{-2}} = 10^{10} \text{ Hz}$$

$$\eta_{TE_{20}} = \frac{377}{\sqrt{1 - \left(\frac{c/a}{30 \times 10^9}\right)^2}} = \frac{377}{\sqrt{1 - \left(\frac{3 \times 10^8}{3 \times 10^2 \times 30 \times 10^9}\right)^2}} = \frac{377}{\sqrt{0.66}} = 400 \Omega$$

Q The $\vec{E}$ field in a rectangular waveguide of inner dimensions $a \times b$ is given by

$$\vec{E} = \frac{\omega \mu}{h^2} \left(\frac{\pi}{a} \right) H_0 \sin\left(\frac{2\pi x}{a}\right) \sin(\omega t - \beta z) \hat{y}$$

where H_0 is a constant, and a and b are the dimensions along the x -axis and y -axis respectively. The mode of propagation in the waveguide is

(A) $E_x = 0, \quad E_y \neq 0, \quad E_z = 0$

$$TE_{20}$$

Q A rectangular waveguide having TE_{10} mode as dominant mode is having a cutoff frequency of 18 GHz for the TE_{30} mode. Then find inner broad-wall dimension of the rectangular waveguide?

(A) $(f_c)_{TE_{mn}} = \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$

$$(f_c)_{TE_{30}} = \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{3}{a}\right)^2 + (0)}$$

$$= \frac{c}{2} \cdot \frac{3}{a} = 18 \times 10^9$$

$$\frac{3 \times 10^8}{2} \times \frac{3}{a} = 18 \times 10^9 \Rightarrow a = \frac{3}{40} = 5/2 \text{ cm}$$

① The cut-off wavelength λ_c for TE_{20} mode for a standard rectangular waveguide is

$$\textcircled{A} \quad (\lambda_c)_{TE_{20}} = \frac{c}{(f_c)_{TE_{20}}}$$

$$(f_c)_{TE_{20}} = \frac{c}{a} = \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$(\lambda_c)_{TE_{20}} = \frac{c}{f_c} = a.$$

② A rectangular waveguide measure $3 \times 4.5 \text{ cm}$ internally and has a 9 GHz signal propagated in it. The cut-off wavelength for TE_{10} mode is.

$$\lambda_c = \frac{\lambda}{\sqrt{1 - (f_c/f)^2}} = \frac{2\pi}{\omega\sqrt{\mu\epsilon} \sqrt{1 - (f_c/f)^2}}$$

$$(f_c) = \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

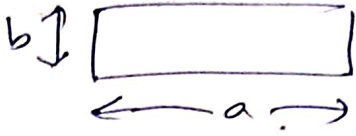
$$(f_c)_{TE_{10}} = \frac{1}{2\sqrt{\mu\epsilon}} \left(\frac{1}{a}\right) = \frac{c}{a} \times \frac{1}{3 \times 10^{-2}}$$

$$\lambda_c = \frac{c}{f \sqrt{1 - (f_c/f)^2}} = \frac{c}{\sqrt{f^2 - f_c^2}}$$

$$\lambda_c = \frac{3 \times 10^8}{\sqrt{9 \times 10^9 - \left(\frac{3 \times 10^8}{a} \times \frac{1}{3 \times 10^{-2}}\right)^2}} = 9 \text{ cm}$$

Q) if the height of the Rectangular waveguide is halved, the cut off wavelength of dominant mode

A) $(f_c)_{TE_{10}} = \frac{c}{2} \sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{0}{b}\right)^2}$

$$= \frac{c}{2a}$$


i.e. it depends on width not the height

So Remain unchanged

Q) The cut-off frequency of the dominant mode of a rectangular waveguide having aspect ratio more than 2 is 10 GHz , The inner broad wall dimension is given by.

A) dominant mode TE_{10}

So $(f_c)_{TE_{10}} = \frac{c}{2} \sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{0}{b}\right)^2}$

So there is no need to consider aspect Ratio.

$$10 \times 10^9 = \frac{3 \times 10^8}{2} \cdot \frac{1}{a}$$

$$\frac{200}{3} = \frac{1}{a} \quad \boxed{a = \frac{3}{200} = 1.5 \text{ cm.}}$$

Q) in an air-filled waveguide of dimensions $a \text{ cm} \times b \text{ cm}$, at a given frequency, the longitudinal component of electric field of TE_{32} mode is of the form

$E_z = 20 \sin(60\pi x) \sin(100\pi y)$, then which form would E_z have for the lowest order TM mode?

① Lowest mode in TM mode

$$(TM)_{11}$$

$$\begin{aligned} \text{Given } (TM)_{32} \Rightarrow E_z &= 20 \sin(60\pi x) \sin(100\pi y) \\ &= E_0 \sin\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) \end{aligned}$$

$$\begin{aligned} \frac{m\pi}{a} &= 60\pi & \frac{n\pi}{b} &= 100\pi \\ m &= 3, & n &= 2 \end{aligned}$$

$$\begin{aligned} 3 \frac{\pi}{a} &= 60\pi, & 2 \frac{\pi}{b} &= 100\pi \\ \frac{\pi}{a} &= 20\pi, & \frac{\pi}{b} &= 50\pi. \end{aligned}$$

$$\begin{aligned} \text{Then } (TM)_{11} \Rightarrow E_z &= 20 \sin\left(\frac{(1)\pi}{a}x\right) \sin\left(\frac{(1)\pi}{b}y\right) \\ &= 20 \sin(20\pi x) \sin(50\pi y). \end{aligned}$$

② For a rectangular waveguide of dimensions $a\sqrt{3}\text{ cm} \times a\text{ cm}$
the cut off frequency for the TE_{10} mode is 2 GHz .
What is the cut off frequency for TM_{11} mode in the
waveguide.

$$\begin{aligned} \textcircled{A} \quad (f_c) &= \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \\ &= \frac{c}{2} \sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2} = \frac{c}{2 \cdot a} = 2\text{ GHz} \\ (f_c)_{TE_{10}} &= \frac{c}{2 \cdot a\sqrt{3}} = 2 \times 10^9 \cdot \frac{3 \times 10^8}{2 \cdot a \cdot \sqrt{3}} = 2 \times 10^9 \end{aligned}$$

$$\frac{3}{2 \cdot a \cdot \sqrt{3}} = 200$$

$$a = \frac{3}{2 \cdot 200 \cdot \sqrt{3}} = \frac{3}{2 \times 1.73 \times 200} = 4.6 \text{ mm}$$

$$\begin{aligned} (f_c)_{TM_{11}} &= \frac{c}{2} \sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2} \\ &= \frac{c}{2} \sqrt{\left(\frac{1}{a\sqrt{3}}\right)^2 + \left(\frac{1}{a}\right)^2} = 4.6 \text{ GHz} \end{aligned}$$

⑦ A standard waveguide has inside wall dimensions of $a = 2.286 \text{ cm}$ and $b = 1.016 \text{ cm}$. What is the cut-off wavelength for TE_{01} mode?

$$\begin{aligned} \textcircled{7} \quad (f_c)_{TE_{01}} &= \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \\ &= \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{0}{a}\right)^2 + \left(\frac{1}{b}\right)^2} \\ &= \frac{c}{2} \cdot \frac{1}{b} = \frac{3 \times 10^8}{2} \cdot \frac{1}{1.016 \text{ cm}} \end{aligned}$$

$$\begin{aligned} (\lambda_c) &= c/f_c = c / \frac{c}{2 \cdot b} = 2 \cdot b \\ &= 2.032 \text{ cm} \end{aligned}$$

⑧ A standard air filled waveguide has inside wall dimensions of $a = 4.755 \text{ cm}$ and $b = 2.215 \text{ cm}$. Then find λ_c for dominant mode.

④ The cut-off frequency of a rectangular waveguide in dominant mode is 10 GHz. The width of the waveguide is.

⑤ A rectangular waveguide with inner dimensions $6\text{ cm} \times 3\text{ cm}$ has been designed for a single mode operation. Find the possible frequency range of operation such that the lowest frequency is 5% above the cut-off and the highest frequency is 5% below the cut-off of the next higher mode.

(A) $a = 6\text{ cm}$, $b = 3\text{ cm}$

Lower cut-off frequency TE_{10}

$$\begin{aligned} (f_c)_{TE_{10}} &= \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2} \\ &= \frac{c}{2} \sqrt{\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2} = \frac{c}{2} \cdot \frac{1}{a} = \frac{c}{2 \cdot a} \\ &= \frac{3 \times 10^8}{2 \times 6 \times 10^{-2}} = 0.25 \times 10^{10} = 2.5\text{ GHz} \end{aligned}$$

Next higher order mode

$$(f_c)_{TE_{20}} = \frac{c}{2} \sqrt{\left(\frac{2}{a}\right)^2} = \frac{c}{2} \cdot \frac{2}{a} = 5\text{ GHz.}$$

$$\text{Lower cut-off frequency } f_L = f_{c, TE_{10}} + 0.05 f_{c, TE_{10}} = 1.05 f_{c, TE_{10}}$$

$$= 1.05 f_{c TE10} = 2.625 \text{ GHz}$$

upper cut-off frequency $f_u = f_{c TE20} - 0.05 f_{c TE20}$

$$= 0.95 f_{c TE20}$$

$$= 4.75 \text{ GHz}$$

Q An air filled rectangular waveguide has (half-section) dimensions $a = 6 \text{ cm}$ and $b = 3 \text{ cm}$. given that

$$E_z = 5 \sin\left(\frac{2\pi x}{a}\right) \sin\left(\frac{3\pi y}{b}\right) \cos(10^{12}t - \beta z) \text{ V/m}$$

calculate the Intrinsic Impedance of this mode.

A $E_z \neq 0 \Rightarrow \eta_{TM} = \frac{\beta_y}{\omega \epsilon} \quad \beta_y = \beta \sqrt{1 - (f_c/f)^2}$

Dominant $(f_c)_{TE10} = \frac{c}{2a} \sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{0}{b}\right)^2} = \frac{c}{2a}$

$$\omega = 10^{12} = 2\pi f$$

$$f = \frac{10^{12}}{2\pi}$$

$$\eta_{TM} = \frac{\omega \sqrt{\mu \epsilon} \sqrt{1 - (f_c/f)^2}}{\omega \cdot \epsilon}$$

$$= \sqrt{\frac{\mu_0}{\epsilon_0}} \cdot \sqrt{1 - (f_c/f)^2}$$

$$= \sqrt{\frac{4\pi \times 10^{-7}}{10^{-9}/86\pi}} \sqrt{1 - \left(\frac{c/2a}{10^{12}/2\pi}\right)^2}$$

from the given data

$$\frac{m\pi}{a} = \frac{2\pi}{a}, \quad \frac{n\pi}{b} = \frac{3\pi}{b} \quad \text{for } m=2, n=3.$$

$$\eta_{TM_{23}} = \sqrt{\frac{\mu_0}{\epsilon_0}} \sqrt{1 - \left(\frac{(f_c)_{TM_{23}}}{f}\right)^2}$$

$$(f_c)_{TM_{23}} = \frac{f}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} = \frac{c}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{2}{a}\right)^2 + \left(\frac{3}{b}\right)^2}$$

$$= 15.81 \text{ GHz}$$

$$\eta_{TM_{23}} = 375.12 \Omega$$

Q When the dominant mode is propagated in an air filled rectangular waveguide, the guided wavelength for a frequency of 9000 MHz in 4cm. calculate breadth (a) of the guide?

A $f = 9000 \text{ MHz}, \quad \lambda_g = 4 \text{ cm},$

dominant mode $TE_{10} \quad (f_c)_{TE_{10}} = \frac{c}{2} \sqrt{\left(\frac{1}{a}\right)^2} = \frac{c}{2a}$

$$\lambda_g = 4 \text{ cm}, \quad \lambda_c = c/f_c$$

we know $k_c^2 = k^2 - \beta^2$

$$\lambda_c = c/f_c = 2a$$

$$k^2 = k_c^2 + \beta^2$$

$$\lambda = c/f$$

$$\left(\frac{2\pi}{\lambda}\right)^2 = \left(\frac{2\pi}{\lambda_c}\right)^2 + \left(\frac{2\pi}{\lambda_g}\right)^2$$

$$= \frac{3 \times 10^8}{9 \times 10^3 \times 10^6} = \frac{1}{30}$$

$$\left(\frac{2\pi}{2a}\right)^2 + \left(\frac{2\pi}{4 \times 10^{-2}}\right)^2 = \left(\frac{1}{30}\right)^2$$

$$\boxed{a = 3.015 \text{ cm}}$$

Q (a) Find the cut off frequency for dominant mode in rectangular waveguide with dimensions $a = 4\text{cm}$ and $b = 2\text{cm}$ filled with air

(b) Find the wave Impedance (η) for TE_{10} & TM_{11} modes in an air-filled rectangular waveguide with dimensions $a = 7\text{cm}$ and $b = 3.5\text{cm}$ operating at 6GHz .

A (a)
$$(f_c)_{TE_{10}} = \frac{c}{2a} = \frac{3 \times 10^8}{2 \times 4 \times 10^{-2}} = 3.75\text{GHz}$$

$$= \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

(b)
$$\eta_{TE} = \frac{\omega\mu}{\beta_g}$$

$$\beta_g = \beta \sqrt{1 - (f_c/f)^2}$$

$$\beta = \omega \sqrt{\mu\epsilon} \quad f = 6\text{GHz}$$

$$\eta_{TE_{10}} = \frac{\omega\mu}{\omega\sqrt{\mu\epsilon} \sqrt{1 - \left(\frac{f_c}{f}\right)_{TE_{10}}^2}}$$

$$= \frac{\sqrt{\mu/\epsilon}}{\sqrt{1 - \left(\frac{f_c}{f}\right)_{TE_{10}}^2}}$$

$$(f_c)_{TE_{10}} = \frac{c}{2a} = \frac{3 \times 10^8}{2 \times 7 \times 10^{-2}} = 2.142\text{GHz}$$

$$\eta_{TE10} = \frac{120\pi}{\sqrt{1 - \frac{2.142 \text{ GHz}^2}{6 \text{ GHz}^2}}} = \frac{120\pi}{\sqrt{1 - \frac{2.14}{6}}} = 403.56 \Omega$$

$$\eta_{TM11} = \frac{\beta_g}{\omega\epsilon} = \sqrt{\frac{\mu_0}{\epsilon_0}} \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

$$\left(\frac{f_c}{f}\right)_{TM11} = \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2} = 4.79 \text{ GHz}$$

$$\eta_{TM11} = 120\pi \sqrt{1 - \left(\frac{4.79 \text{ GHz}}{6 \text{ GHz}}\right)^2} = 226.94 \Omega$$

Q) In an air-filled rectangular waveguide, the vector electric field is given by.

$$\vec{E} = \cos(2\pi y) \exp\left[-j\left(\frac{40\pi}{3}\right)z + j\omega t\right] \hat{a}_x \text{ V/m.}$$

Find vector magnetic field and the phase velocity of the wave inside the waveguide.

A) $\vec{E} \Rightarrow x$ component, Wave propagates $\Rightarrow +z$ direction

$$\beta_g = \frac{40\pi}{3}, \quad \frac{m\pi}{a} = 0, \quad \frac{n\pi}{b} = 2\pi$$

From given data, $E_x \neq 0$, that means $E_z = 0$

so $H_z \neq 0$ so TM mode

to find ω

we know

$$k_c^2 = k^2 - \beta_g^2$$

$$k_c^2 = \omega_c^2 \mu \epsilon$$

$$(k_c)^2 = \frac{1}{25 \mu \epsilon} \cdot \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$2\pi f_c = \frac{2\pi}{25 \mu \epsilon} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$k_c^2 = \omega_c^2 \cdot \mu \epsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 = (0)^2 + (2\pi)^2$$

$$k_c^2 = (2\pi)^2 \Rightarrow \boxed{k_c = 2\pi}$$

$$k^2 = \beta_g^2 + k_c^2$$

$$\omega^2 \mu \epsilon = \left(\frac{40\pi}{3}\right)^2 + (2\pi)^2$$

$$\boxed{\omega = 12.7 \times 10^9 \text{ rad/sec}}$$

To find $\vec{H}$

$$\nabla \times \vec{E} = -j\omega \mu \vec{H} = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = -a_y \left[\frac{\partial E_x}{\partial z} \right] + a_z \left[\frac{\partial E_y}{\partial z} \right]$$

$$-j\omega \mu = \frac{\partial E_x}{\partial z} a_y - \frac{\partial E_y}{\partial z} a_z$$

$$\frac{\partial E_x}{\partial z} = \frac{\partial}{\partial z} \left[\cos(2\pi y) e^{np} \left(-j \frac{40\pi}{3} z + j\omega t \right) \right] a_x \text{ V/m}$$

$$= \left(-\frac{40\pi}{3} j \right) \cos(2\pi y) e^{(-j \frac{40\pi}{3} z + j\omega t)}$$

$$\frac{\partial E_x}{\partial z} = \sin(2\pi y) (2\pi) e^{(-j \frac{40\pi}{3} z + j\omega t)}$$

$$\vec{H} = -\frac{1}{j\omega\mu_0} \left[\int \frac{40\pi}{3} \cos 2\pi y \cdot e^{-j40\pi/3 z} dy \right] + \int 2\pi \sin(2\pi y) e^{-j40\pi/3 z} dy$$

To find v_{pg}

$$v_{pg} = \frac{\omega}{\beta_g} = \frac{12.7 \times 10^7}{(40\pi/3)} = 3.03 \times 10^8 \text{ m/sec}$$

② A Lossless, air-dielectric cylindrical waveguide, of inside diameter 3cm, is operated at 14 GHz; for the TM_{11} mode propagating in the +z direction, find the cutoff frequency, guide wavelength and wave impedance

n	P_{n1}	P_{n2}	P_{n3}	P_{n1}	P_{n2}	P_{n3}
0	3.8	7.0	10.1	2.4	5.5	8.6
1	1.8 TE_{01}	5.3	8.5	3.8 TM_{11}	7.0	10.1
2	3.0	6.7	9.9	5.1	8.4	11.6

③ Given data air-dielectric $\mu_r = \epsilon_r = 1$

$$a = 3\text{cm}, \quad f = 14 \text{ GHz}, \quad \text{mode} = TM_{11}$$

To find f_c

$$f_c = \frac{x'_{np}}{2\pi a \sqrt{\mu\epsilon}} = \frac{3.8}{2\pi \times 3 \times 10^{-2} \sqrt{\mu_0 \epsilon_0}} = \frac{3.8}{12.77} = 6.09 \text{ GHz}$$

To find β_g

$$\beta_g = \beta \sqrt{1 - (f_c/f)^2} = \omega \sqrt{\mu\epsilon} \sqrt{1 - (f_c/f)^2}$$

$$f_c = 6.09 \text{ GHz}, \quad f = 14 \text{ GHz},$$

$$\beta_g = 263.93$$

To find λ_g

$$\beta_g = \frac{2\pi}{\lambda_g} \quad \lambda_g = 2.38 \text{ cm}$$

$$\lambda_g = \frac{2\pi}{\beta_g} = \frac{2\pi}{263.93}$$

To find wave Impedance

$$\begin{aligned} \eta_{TM} &= \frac{j\omega\epsilon}{j\beta_g} = \frac{\omega\epsilon}{\beta_g} = \frac{\omega \times \epsilon_0}{\beta_g} \\ &= \frac{2\pi f \times (10^{-9}/36\pi)}{263.93} = 339.35 \Omega \end{aligned}$$

Q) Find the inside diameter of a hollow air-filled cylindrical waveguide so that a TE_{11} mode propagates at a frequency of 10 GHz, with the cut off wavelength of the mode being 1.3 times the operating wavelength.

A) given data air-filled $\epsilon_r = \mu_r = 1$

$$f = 10 \text{ GHz}, \quad \lambda_c = 1.3 \lambda, \quad \text{mode} \Rightarrow TE_{11}$$

$$(P_c) \frac{1}{TE_{11}} = \frac{x'_{np}}{2\pi a \sqrt{\mu\epsilon}} = \frac{1.8}{2\pi a \sqrt{\mu\epsilon}}$$

$$\lambda_c = c/f_c \quad f_c = c/\lambda_c = c/1.3\lambda$$

$$f_c = \frac{c}{1.3\lambda} = \frac{c}{1.3(c/f)} = \frac{f}{1.3} = \frac{10 \times 10^9}{1.3}$$

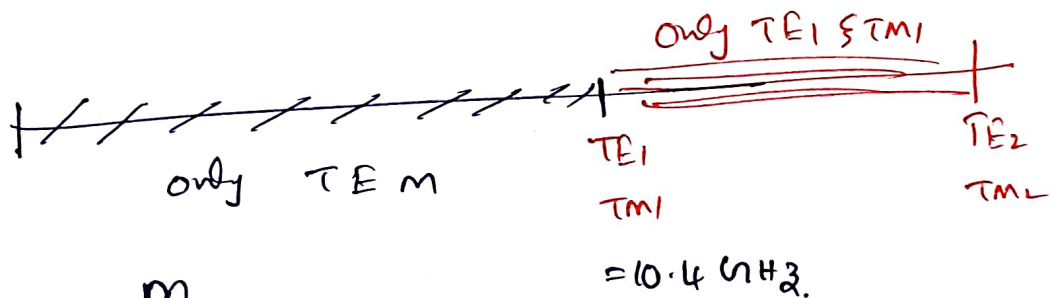
$$\begin{aligned}
 (f_c)_{TE_{11}} &= \frac{10 \times 10^9}{1.3} = \frac{1.8}{2\pi a \sqrt{\mu_0 \epsilon_0}} \\
 &= \frac{10^{10}}{1.3 \times 1.8} = \frac{1 \times 3 \times 10^8}{2\pi a}
 \end{aligned}$$

$$a = \frac{3 \times 10^8 \times 1.3 \times 1.8}{2\pi \times 10^{10}} = 1.142 \text{ cm}$$

(Q) A parallel-plate waveguide has plate separation $d = 1 \text{ cm}$ and is filled with teflon having dielectric constant $\epsilon_r = 2.1$. Determine the maximum operating frequency such that only the TEM mode will propagate. Also find the range of frequencies over which the TE_1 and TM_1 ($m=1$) modes, and no higher-order modes, will propagate.

(A) (Q) Find the maximum frequency can propagate only TEM mode

(Q) Find the Range of TE_1 & TM_1



$$f_c = \frac{m}{2d \sqrt{\mu \epsilon}}$$

$$\begin{aligned}
 TE_1 \text{ or } TM_1 \Rightarrow f_c &= \frac{1}{2 \times 1 \times 10^{-2} \times \sqrt{\mu_0 \times \epsilon_0 \times 2.1}} \\
 &= \frac{3 \times 10^8}{2 \times \sqrt{2.1} \times 10^{-2}} = 1.04 \times 10^{10} \\
 &= 10.4 \text{ GHz}
 \end{aligned}$$

$$TE_2 \text{ @ } TM_2 \quad f_c = \frac{2}{2\pi \sqrt{\mu_0 \times \epsilon_0 \times 2.1}}$$

$$f_c = 20.8 \text{ GHz}$$

so only TEM up to 10.4 GHz

only TE_1, TM_1 up to 20.8 GHz.