

wave guides:-

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①

Wave guide is a structure that guides waves such as EM waves or sound with min loss of energy by reflecting the transmission energy to one direction.

wave guides are capable of directing power precisely to where it is needed, can handle large amounts of power and it acts as a HPF.

wave guides are often used at microwave frequencies ($> 300 \text{ MHz}$)

wave guides are commonly used in microwave communications and radar installations.

Need to know about waveguide:-

- ① it is a hollow metallic pipe
- ② wave theory is conducted in waveguide analysis
- ③ power handling capacity is high
- ④ operating freq is 3 GHz to 100 GHz in waveguide
- ⑤ operating modes are TE or TM modes (cannot support TEM mode)
- ⑥ large surface area of waveguide reduces copper losses
- ⑦ dielectric losses are less in waveguide
- ⑧ Low signal attenuation at high frequencies than transmission line.

⑨ it acts as a high pass filter

⑩ wave Impedance (Char's Imp) is a function of frequency.

Definition

it is a hollow metallic pipe design | Travelling wave line is a conductor or wire which carries wave energy

Structure

Metal waveguides are typically one enclosed conductor filled with an

Traveling wave

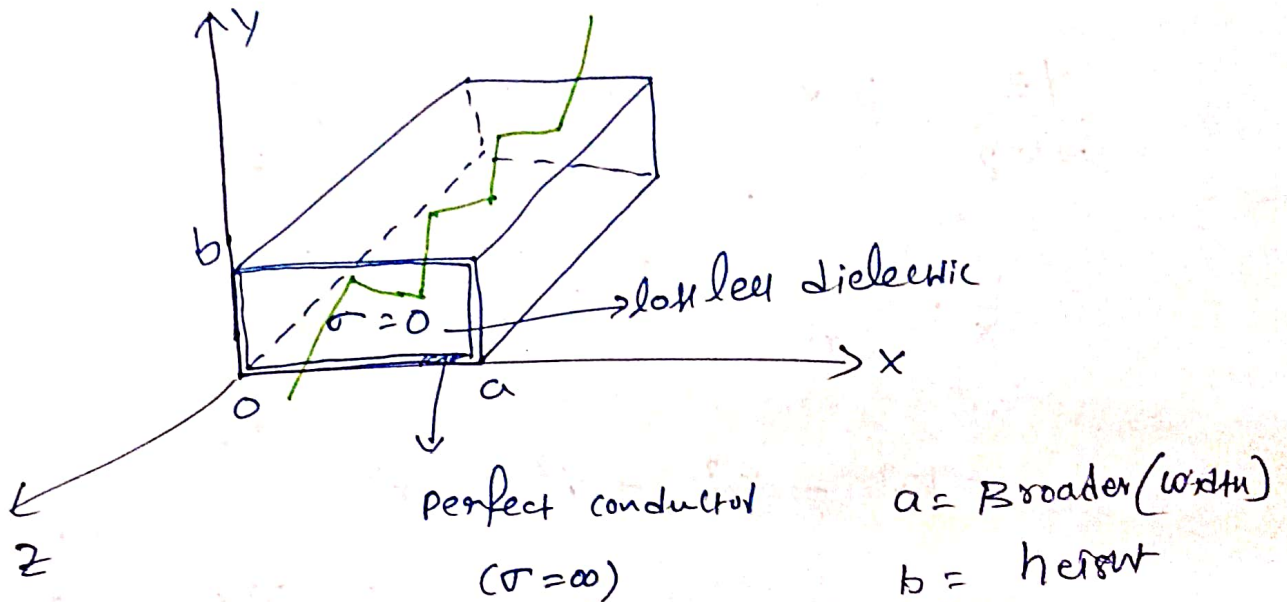
Waveguide

Travelling wave

wave guide equation:-

Previously we had derived

$$\nabla^2 E_s - \gamma^2 E_s = 0 \quad \gamma^2 = j\omega\mu(\sigma + j\omega\epsilon)$$



$$\text{Aspect ratio} = \frac{\omega}{h} = a/b$$

for a standard rectangular waveguide $a \geq 2b$

$$\gamma^2 = j\omega^2\mu\epsilon \quad [\text{wave travelling medium } \sigma = 0]$$

$$\gamma^2 = -\omega^2\mu\epsilon = -k^2$$

$$\boxed{\nabla^2 E_s + k^2 E_s = 0} \quad k = \omega \sqrt{\mu\epsilon}$$

for magnetic field $\nabla^2 H_s + k^2 H_s = 0$

$$\vec{E}_s (E_{xs}, E_{ys}, E_{zs}), \quad \vec{H}_s (H_{xs}, H_{ys}, H_{zs}).$$

for z -component of electric field

$$\frac{\partial^2 E_{zs}}{\partial x^2} + \frac{\partial^2 E_{zs}}{\partial y^2} + \frac{\partial^2 E_{zs}}{\partial z^2} + k^2 E_{zs} = 0$$

To find the above partial differential eq'n.

$$E_{25} = X(x), Y(y), Z(z).$$

$$\text{Then } \frac{\partial^2 XYZ}{\partial x^2} + \frac{\partial^2 XYZ}{\partial y^2} + \frac{\partial^2 XYZ}{\partial z^2} + k^2 XYZ = 0$$

$$\frac{YZ}{XYZ} \cdot \frac{\partial^2 X}{\partial x^2} + \frac{XZ}{XYZ} \cdot \frac{\partial^2 Y}{\partial y^2} + \frac{XY}{XYZ} \cdot \frac{\partial^2 Z}{\partial z^2} + k^2 = 0$$

$$\frac{1}{X} \cdot \frac{\partial^2 X}{\partial x^2} + \frac{1}{Y} \cdot \frac{\partial^2 Y}{\partial y^2} + \frac{1}{Z} \cdot \frac{\partial^2 Z}{\partial z^2} + k^2 = 0$$

$$-k_x^2 - k_y^2 + v^2 = -k^2 \quad (-k_x^2, -k_y^2, v^2 \text{ are separation const.})$$

$$\frac{1}{X} \cdot \frac{\partial^2 X}{\partial x^2} = -k_x^2 \Rightarrow \frac{1}{X} \cdot \frac{\partial^2 X}{\partial x^2} + k_x^2 = 0$$

$$\frac{\partial^2 X}{\partial x^2} + k_x^2 X = 0 \Rightarrow D^2 + k_x^2 = 0$$
$$D = \pm j k_x$$

$$X(x) = (C_1 \cos k_x x + C_2 \sin k_x x)$$

$$\text{Similarly } Y(y) = (C_3 \cos k_y y + C_4 \sin k_y y)$$

$$Z(z) = (C_5 e^{-vz} + C_6 e^{vz})$$

$$E_{25} = X(x) Y(y) Z(z)$$

$$= (C_1 \cos k_x x + C_2 \sin k_x x) (C_3 \cos k_y y + C_4 \sin k_y y) (C_5 e^{-vz} + C_6 e^{vz})$$

if wave is travelling along +ve z-direction

(3)

$$E_{zs} = (C_1 \cos k_x x + C_2 \sin k_x x) (C_3 \cos k_y y + C_4 \sin k_y y) e^{-\gamma z}$$

$$E_{zs} = (A \cos k_x x + B \sin k_x x) (C \cos k_y y + D \sin k_y y) e^{-\gamma z} \cdot e^{j\omega t}$$

similarly $\longrightarrow$ (1)

$$H_{zs} = (P \cos k_x x + Q \sin k_x x) (R \cos k_y y + S \sin k_y y) e^{-\gamma z} \cdot e^{j\omega t}$$

$\longrightarrow$ (2)

To find x, y component in E & H.

We know $\nabla \times E = -\frac{\partial B}{\partial t}$

$$\nabla \times E = -j\omega \mu H_s$$

$$-j\omega \mu (H_{xs} \hat{a}_x + H_{ys} \hat{a}_y + H_{zs} \hat{a}_z) = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_{xs} & E_{ys} & E_{zs} \end{vmatrix}$$

$$-j\omega \mu H_{xs} = \left[\frac{\partial E_{zs}}{\partial y} - \frac{\partial E_{ys}}{\partial z} \right] \longrightarrow (3)$$

$$-j\omega \mu H_{ys} = - \left[\frac{\partial E_{zs}}{\partial x} - \frac{\partial E_{xs}}{\partial z} \right] \longrightarrow (4)$$

$$-j\omega \mu H_{zs} = \left[\frac{\partial E_{ys}}{\partial x} - \frac{\partial E_{xs}}{\partial y} \right] \longrightarrow (5)$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \mathbf{J}_d$$

$$\nabla \times \mathbf{H} = (\sigma + j\omega\epsilon) \mathbf{E}_s \quad (\sigma = 0) \rightarrow \text{Inside the wave guide}$$

$$j\omega\epsilon \left[E_{xs} \hat{a}_x + E_{ys} \hat{a}_y + E_{zs} \hat{a}_z \right]$$

$$= \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_{xs} & H_{ys} & H_{zs} \end{vmatrix}$$

$$j\omega\epsilon E_{xs} = \left[\frac{\partial H_{zs}}{\partial y} - \frac{\partial H_{ys}}{\partial z} \right] \rightarrow (6)$$

$$j\omega\epsilon E_{ys} = - \left[\frac{\partial H_{zs}}{\partial x} - \frac{\partial H_{xs}}{\partial z} \right] \rightarrow (7)$$

$$j\omega\epsilon E_{zs} = \left[\frac{\partial H_{ys}}{\partial x} - \frac{\partial H_{xs}}{\partial y} \right] \rightarrow (8)$$

from (3), (4), (5) & (6), (7), (8)

$$j\omega\epsilon E_{xs} = \frac{\partial H_{zs}}{\partial y} - \frac{\partial}{\partial z} \left[\frac{1}{j\omega\mu} \left(\frac{\partial E_{zs}}{\partial x} - \frac{\partial E_{xs}}{\partial z} \right) \right]$$

$$j\omega\epsilon E_{xs} = \frac{\partial H_{zs}}{\partial y} - \frac{1}{j\omega\mu} \cdot \frac{\partial^2 E_{zs}}{\partial x \partial z} + \frac{1}{j\omega\mu} \cdot \frac{\partial^2 E_{xs}}{\partial z^2}$$

$$E_{zs} = E_{z0} \cdot e^{-\gamma z}, \quad E_{xs} = E_{x0} \cdot e^{-\gamma z}$$

$$\frac{\partial E_{xs}}{\partial z} = -\gamma E_{x0} \cdot e^{-\gamma z}$$

$$\frac{\partial^2 E_{xs}}{\partial z^2} = \sqrt{2} E_{x0} e^{-\sqrt{2}z} = \sqrt{2} E_{xs}.$$

(4)

$$\frac{\partial E_{zs}}{\partial z} = -\sqrt{2} E_{z0} e^{-\sqrt{2}z}$$

$$= -\sqrt{2} E_{zs}.$$

$$j\omega \epsilon E_{xs} = \frac{\partial H_{zs}}{\partial y} - \frac{1}{j\omega \mu} \cdot \frac{\partial}{\partial x} E_{zs} (-\sqrt{2}) + \frac{1}{j\omega \mu} \sqrt{2} E_{xs}$$

$$\left(\frac{-\omega^2 \mu \epsilon E_{xs} - \sqrt{2} E_{xs}}{j\omega \mu} \right) = \frac{\sqrt{2}}{j\omega \mu} \cdot \frac{\partial E_{zs}}{\partial x} + \frac{\partial H_{zs}}{\partial y}$$

$$(\omega^2 \mu \epsilon + \sqrt{2}) E_{xs} = -\sqrt{2} \cdot \frac{\partial E_{zs}}{\partial x} - j\omega \mu \frac{\partial H_{zs}}{\partial y}$$

$$\boxed{E_{xs} = -\frac{\sqrt{2}}{h^2} \cdot \frac{\partial E_{zs}}{\partial x} - \frac{j\omega \mu}{h^2} \cdot \frac{\partial H_{zs}}{\partial y}} \rightarrow (9)$$

$$h^2 = \sqrt{2} + \omega^2 \mu \epsilon = k_x^2 + k_y^2 = \text{Transverse Propagation constant.}$$

Similarly

$$E_{xs} = -\frac{\sqrt{2}}{h^2} \cdot \frac{\partial E_{zs}}{\partial x} - \frac{j\omega \mu}{h^2} \cdot \frac{\partial H_{zs}}{\partial y}$$

$$E_{ys} = -\frac{\sqrt{2}}{h^2} \cdot \frac{\partial E_{zs}}{\partial y} + \frac{j\omega \mu}{h^2} \cdot \frac{\partial H_{zs}}{\partial x}$$

$$H_{xs} = -\frac{\sqrt{2}}{h^2} \cdot \frac{\partial H_{zs}}{\partial x} - \frac{j\omega \epsilon}{h^2} \cdot \frac{\partial E_{zs}}{\partial y}$$

$$H_{ys} = -\frac{\sqrt{2}}{h^2} \cdot \frac{\partial H_{zs}}{\partial y} - \frac{j\omega \epsilon}{h^2} \cdot \frac{\partial E_{zs}}{\partial x}$$

any configuration of E -field & H -field that exist inside the waveguides must be a solution of Maxwell's equation.

Besides this field must also satisfy the boundary condition.

There are different field configurations that satisfy these requirements and accordingly each such configuration is called "mode".

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Types of modes:-

① Transverse Electromagnetic mode (TEM-mode):- In this mode electric field, magnetic field and direction of propagation are $\perp$ to each other.

In which direction wave propagation will occur, in that direction E -field and H -field will be zero.

i.e. if propagation in z -direction then

$$\boxed{E_z = H_z = 0}$$

$$E_x = -\frac{1}{h^2} \frac{\partial E_z}{\partial x} - \frac{j\omega\mu}{h^2} \frac{\partial H_z}{\partial y}$$

(5)

$$E_{ys} = -\frac{1}{h^2} \cdot \frac{\partial E_{zs}}{\partial y} + \frac{j\omega\mu}{h^2} \cdot \frac{\partial H_{zs}}{\partial x}$$

$$H_{xs} = -\frac{1}{h^2} \cdot \frac{\partial H_{zs}}{\partial x} - \frac{j\omega\epsilon}{h^2} \cdot \frac{\partial E_{zs}}{\partial y}$$

$$H_{ys} = -\frac{1}{h^2} \cdot \frac{\partial H_{zs}}{\partial y} - \frac{j\omega\epsilon}{h^2} \cdot \frac{\partial E_{zs}}{\partial x}$$

if $E_{zs} = H_{zs} = 0$, then

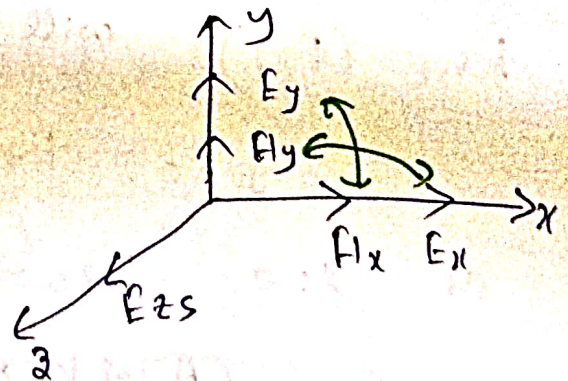
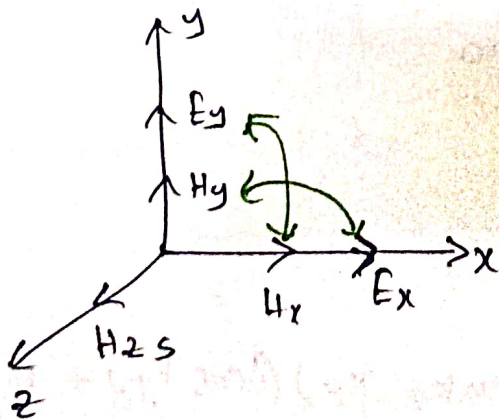
$$\boxed{E_{xs} = E_{ys} = H_{xs} = H_{ys} = 0} \quad \text{All fields vanish, so}$$

that's why TEM mode doesn't exist.

② Transverse electric mode (TE_{mn} mode):-

Electric field will be zero in the direction of propagation.

$$\boxed{E_{zs} = 0, H_{zs} \neq 0} \quad (\text{H-mode})$$



③ Transverse magnetic mode (TM_{mn} mode):-

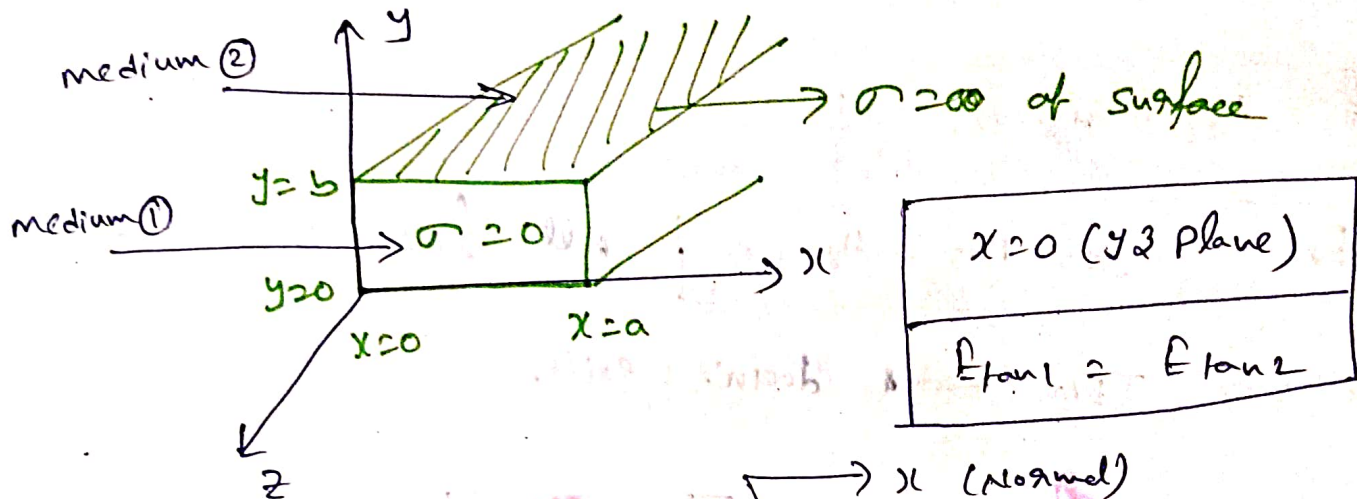
magnetic field will be zero in the direction of propagation

$$\boxed{H_{zs} = 0, E_{zs} \neq 0} \quad (\text{E-mode})$$

Transverse magnetic mode:-

Condition :- $H_{zs} = 0, E_{zs} \neq 0$

$$E_{zs} = (A \cos k_x x + B \sin k_x x) (C \cos k_y y + D \sin k_y y) e^{-\gamma z}$$



$x = \text{const}$ } x (Normal)
 y (Tangential).

$$E_{y1} = E_{y2}, E_{z1} = E_{z2}$$

Since medium ② is perfect conductor E_{tan2} (E_{y2} & E_{z2}) are zero.
 E_{n2} (E_{x2}) will be zero.

$$E_{y1} = 0, E_{z1} = 0.$$

$$\text{at } x=0, E_{zs} = 0.$$

$$0 = (A \cos k_x x_0 + B \sin k_x x_0) (C \cos k_y y + D \sin k_y y) e^{-\gamma z}$$

$$0 = A (C \cos k_y y + D \sin k_y y), \quad \boxed{A=0}$$

at $y=0$, $E_{zs} = 0$ (xz plane) } y normal (y)
 x tangential (xz)

$$\Rightarrow E_{x2} = 0, E_{z2} = 0$$

(6)

$$0 = B \sin k_x x [(\cos k_y x_0 + 0 \sin k_y x_0)] e^{-\sqrt{2} z}$$

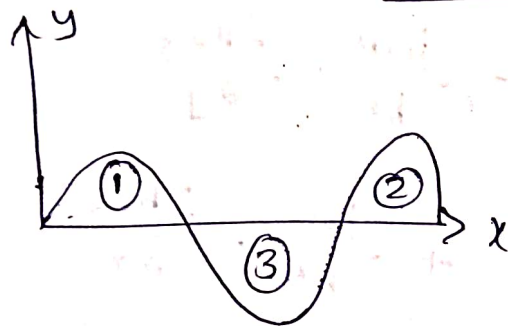
$$\boxed{C = 0}$$

$$E_{zs} = B \sin k_x x \cdot 0 \sin k_y y$$

at $x=a$, $E_{zs} = 0$

$$0 = B \sin(k_x a) \cdot 0 \sin(k_y y)$$

$$k_x a = m\pi, \quad \boxed{k_x = \frac{m\pi}{a}} \quad m = \text{no. of half cycle variation in } x\text{-direction}$$



at $y=b$, $E_{zs} = 0$,

$$0 = B \sin\left(\frac{m\pi}{a}x\right) \cdot 0 \sin(k_y b) \cdot e^{-\sqrt{2} z}$$

$$k_y b = n\pi, \quad k_y = \frac{n\pi}{b}, \quad n = \text{no. of half cycle variation in } y\text{-direction.}$$

$$\boxed{E_{zs} = E_0 \sin\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) e^{-\sqrt{2} z}}$$

E_0 = Peak amplitude of electric field.

m and n can not be zero.

$m \geq 1$ & $n \geq 1$ i.e. TM_{11} mode is min mode exist in the TM_{mn} mode.

Transverse Electric mode (TE_{mn} mode)

Condition :- $E_{zs} = 0$, $H_{zs} \neq 0$

$$H_{zs} = (P \cos k_x x + Q \sin k_x x) (R \cos k_y y + S \sin k_y y) e^{-\gamma z}$$

Boundary condition :- $x=0$, $E_{zs} = 0$, $E_{ys} = 0$ (yz plane) $\Downarrow$
 $\frac{\partial H_{zs}}{\partial x} = 0$

$$E_{xs} = -\frac{1}{h^2} \frac{\partial E_{zs}}{\partial x} - \frac{j\omega\mu}{h^2} \frac{\partial H_{zs}}{\partial y}$$

$$E_{ys} = -\frac{1}{h^2} \frac{\partial E_{zs}}{\partial y} + \frac{j\omega\mu}{h^2} \frac{\partial H_{zs}}{\partial x}$$

$$H_{xs} = -\frac{1}{h^2} \frac{\partial H_{zs}}{\partial x} - \frac{j\omega\epsilon}{h^2} \frac{\partial E_{zs}}{\partial y}$$

$$H_{ys} = -\frac{1}{h^2} \frac{\partial H_{zs}}{\partial y} - \frac{j\omega\epsilon}{h^2} \frac{\partial E_{zs}}{\partial x}$$

$$x=a, \quad E_{zs} = 0 \quad \frac{\partial H_{zs}}{\partial x} = 0$$

$(E_{ys} = 0) \Rightarrow$

$$y=0, \quad y=b$$

xz plane

$$(E_{xs} = 0) \Rightarrow \frac{\partial H_{zs}}{\partial y} = 0$$

$(E_{ys} = 0) \Rightarrow$

at $x=0$, $\frac{\partial H_{zs}}{\partial x} = 0$,

$$\frac{\partial H_{zs}}{\partial x} = \left[-P \sin k_x \cdot x + Q k_x \cos k_x \cdot x \right]$$

$$\left[R \cos k_y y + S \sin k_y y \right] e^{-\gamma z}$$

$$0 = Q k_x \left[R \cos k_y y + S \sin k_y y \right] e^{-\gamma z}$$

$Q=0$

$$H_{zs} = P \cos k_x \cdot x \left[R \cos k_y y + S \sin k_y y \right] e^{-\gamma z}$$

at $y=0$, $\frac{\partial H_{zs}}{\partial y} = 0$.

$$0 = P \cos k_x \cdot x \left[-R k_y \sin k_y y + S k_y \cos k_y y \right] e^{-\gamma z}$$

$$0 = P \cos k_x \cdot x \cdot S k_y$$

$S=0$

$$H_{zs} = P \cos (k_x \cdot x) R \cos (k_y y) e^{-\gamma z}$$

at $x=a$, $\frac{\partial H_{zs}}{\partial x} = 0$

$$0 = -P \sin (k_x \cdot a) \cdot R \cos (k_y y) e^{-\gamma z}$$

$P \neq 0$, $k_x a = m\pi$

$$\boxed{k_x = \frac{m\pi}{a}}$$

$$H_{zs} = P \cos\left(\frac{m\pi}{a} x\right) R \cos(k_y y) e^{-\gamma z}$$

at $y=b$, $\frac{\partial H_{zs}}{\partial y} = 0$

$$\frac{\partial H_{zs}}{\partial y} = -P \cos\left(\frac{m\pi}{a} x\right) R \sin(k_y y) k_y e^{-\gamma z}$$

$$0 = -P \cos\left(\frac{m\pi}{a} x\right) \cdot R \sin(k_y b) k_y \cdot e^{-\gamma z}$$

$$k_y b = n\pi$$

$$k_y = \frac{n\pi}{b}$$

$$H_{zs} = H_0 \cos\left(\frac{m\pi}{a} x\right) \cos\left(\frac{n\pi}{b} y\right) e^{-\gamma z}$$

↑
peak value of m & n field, m & n can not be

zero simultaneously.

TE_{10} or TE_{01} will be the minimum mode that
can exist in the TE_{mn} mode

Dispersion Relation in waveguide (waveguide parameters)

$$h^2 = \beta^2 + \omega^2 \mu \epsilon = k_x^2 + k_y^2$$

$$\beta = \sqrt{k_x^2 + k_y^2 - \omega^2 \mu \epsilon}$$

$$\beta = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2 \mu \epsilon}$$

At cut off freq $\omega = \omega_c$, $\beta = 0$.

$$0 = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega_c^2 \mu \epsilon}$$

$$\omega_c^2 \mu \epsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$(2\pi f_c)^2 \mu \epsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$f_c = \frac{1}{2\sqrt{\mu \epsilon}} \left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 \right]^{1/2}$$

$$f_c = \frac{v_p}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

for air filled waveguide

$$v_p = c = 3 \times 10^8 \text{ m/s}$$

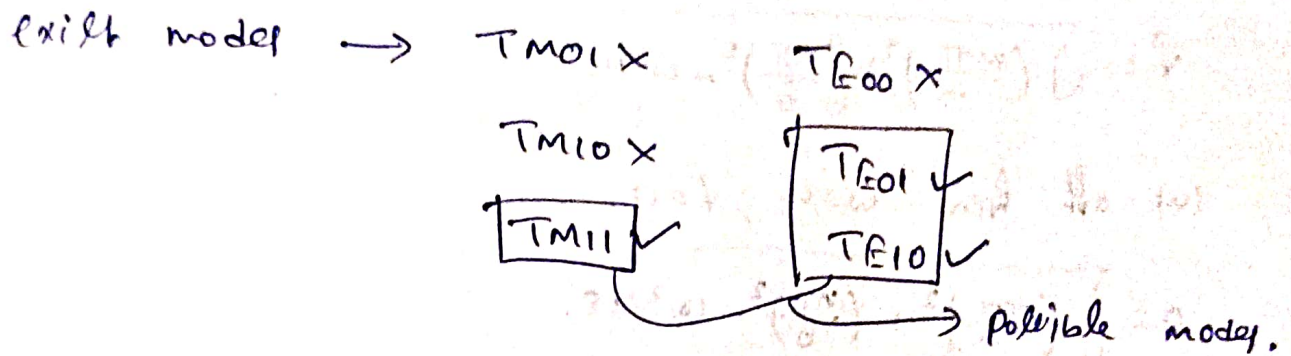
Cut off frequency:- The min frequency above which wave propagates and below which wave attenuates

Cutoff wavelength:- Maximum wavelength above which wave attenuates and below which wave propagates.

$$\lambda_c = \frac{v_p}{f_c} = \frac{v_p}{\frac{v_p}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}} = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$

Dominant mode :-

The mode which gives min cut off freq and maximum cut off wavelength.



TE_{10} mode $\rightarrow m=1, n=0$

$$(f_c)_{TE_{10}} = \frac{v_p}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} = \frac{v_p}{2} \sqrt{\left(\frac{1}{a}\right)^2}$$

$$(f_c)_{TE_{10}} = \left(\frac{v_p}{2a}\right)$$

TE_{01} mode $\rightarrow m=0, n=1$

$$(f_c)_{TE_{01}} = \frac{v_p}{2} \sqrt{\left(\frac{1}{b}\right)^2} = \frac{v_p}{2b}$$

for a standard rectangular wave guide

$$a \geq 2b.$$

Hence TE_{10} mode will give minimum cut off

frequency.

SO TE_{10} mode is dominant mode

Cut-off wavelength for TE₁₀ mode

$$\lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}} = 2a \text{ dominant mode}$$

Now

$$\gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2 \mu \epsilon}$$

$$\gamma = \sqrt{\omega_c^2 \mu \epsilon - \omega^2 \mu \epsilon} = \alpha + j\beta$$

	γ	α	β	Wave propagation
$\omega = \omega_c$	0	0	0	non-propagation mode
$\omega < \omega_c$	real	$\neq 0$	0	non-propagation
$\omega > \omega_c$	Imag	0	$\neq 0$	propagating mode

guided - phase constant:-

$$\gamma = j\beta_g = \sqrt{\omega_c^2 \mu \epsilon - \omega^2 \mu \epsilon}$$

$$j\beta_g = \sqrt{(-1) [\omega^2 \mu \epsilon - \omega_c^2 \mu \epsilon]}$$

$$\beta_g = \sqrt{\omega^2 \mu \epsilon \left(1 - \frac{\omega_c^2 \mu \epsilon}{\omega^2 \mu \epsilon}\right)}$$

$$\beta_g = \omega \sqrt{\mu \epsilon} \sqrt{1 - (f_c/f)^2}$$

$$\boxed{\beta_g = \beta \sqrt{1 - (f_c/f)^2}}$$

Guide wavelength:-

$$\lambda_g = \frac{2\pi}{\beta_g}$$

$$\lambda_g = \frac{2\pi}{\beta \sqrt{1 - (f_c/f)^2}} = \frac{\lambda}{\sqrt{1 - (f_c/f)^2}} = \frac{\lambda}{\sqrt{1 - (\lambda/\lambda_c)^2}}$$

$$\lambda_g^2 = \frac{\lambda^2}{1 - (\lambda/\lambda_c)^2}, \quad 1 - \frac{\lambda^2}{\lambda_c^2} = \frac{\lambda^2}{\lambda_g^2}$$

$$\frac{1}{\lambda^2} - \frac{1}{\lambda_c^2} = \frac{1}{\lambda_g^2} \Rightarrow \frac{1}{\lambda^2} = \frac{1}{\lambda_c^2} + \frac{1}{\lambda_g^2}$$

Guided Phase velocity:-

$$v_{pz} = \omega / \beta_g = \frac{\omega}{\beta \sqrt{1 - (f_c/f)^2}}$$

$$v_{pz} = \frac{v_p}{\sqrt{1 - (f_c/f)^2}}$$

guided group velocity

$$v_{gz} = \frac{\partial \omega}{\partial \beta_g}$$

$$\beta_g = \beta \sqrt{1 - (f_c/f)^2} = \sqrt{\omega^2 \mu \epsilon - \omega_c^2 \mu \epsilon}$$

$$\frac{\partial \beta_g}{\partial \omega} = \frac{1}{2 \sqrt{\omega^2 \mu \epsilon - \omega_c^2 \mu \epsilon}} (2\omega (\mu \epsilon)) = \frac{1}{\omega \sqrt{\mu \epsilon} \sqrt{1 - (\omega_c/\omega)^2}}$$

$\times \omega \mu$

$$\frac{\partial \beta_g}{\partial \omega} = \frac{\sqrt{\mu \epsilon}}{\sqrt{1 - (f_c/f)^2}} = \frac{1}{v_p \sqrt{1 - (f_c/f)^2}} \quad (10)$$

$$v_{g2} = \frac{\partial \omega}{\partial \beta_g} = v_p \sqrt{1 - (f_c/f)^2}$$

$$v_{p2} = \frac{v_p}{\sqrt{1 - (f_c/f)^2}}$$

$$v_p^2 = v_{p2} v_{g2}$$

$$v_p = \sqrt{v_{p2} \cdot v_{g2}}$$

④ A K-band waveguide (1.067 cm x 0.533 cm) is filled by dielectric material with $\epsilon_r = 6.8$, if it operates in the dominant TE_{10} mode at 6 GHz, determine the

① f_c ② v_p ③ λ (waveguide wavelength)

$$a) (f_c)_{TE_{10}} = \frac{v_p}{2a} = \frac{c}{\sqrt{\epsilon_r} \cdot 2a} = \frac{3 \times 10^{10}}{\sqrt{6.8} (2 \times 1.067)}$$

$$= 5.391 \text{ GHz}$$

$$b) v_{p2} = \frac{v_p}{\sqrt{1 - (f_c/f)^2}} = \frac{3 \times 10^8 / \sqrt{6.8}}{\sqrt{1 - (5.391/6)^2}}$$

$$= 2.62 \times 10^8 \text{ m/sec}$$

$$\lambda_g = \frac{\lambda}{\sqrt{1 - (f_c/f)^2}} = \frac{v_p/f}{\sqrt{1 - (5.391/6)^2}} = \frac{c/\sqrt{\epsilon_r} \times 6 \times 10^8}{\sqrt{1 - (5.391/6)^2}}$$

$$\lambda_g = 4.368 \text{ cm}$$

Intrinsic Impedance for TEM mode

$$E_{zs} = 0, \quad H_{zs} \neq 0.$$

$$\eta_{TE} = \frac{E_x}{H_y} = -\frac{E_y}{H_x}$$

$$H_{zs} = H_0 \cos\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right) e^{-\gamma z}$$

$$E_{xs} = -\frac{\sqrt{\epsilon}}{h^2} \frac{\partial E_{zs}}{\partial x} - \frac{j\omega\mu}{h^2} \frac{\partial H_{zs}}{\partial y}$$

$$= -\frac{j\omega\mu}{h^2} \frac{\partial}{\partial y} \left[H_0 \cos\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right) e^{-\gamma z} \right]$$

$$= -\frac{j\omega\mu}{h^2} \left(\frac{n\pi}{b}\right) H_0 \cos\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) e^{-\gamma z}$$

$$= +\frac{j\omega\mu}{h^2} \left(\frac{n\pi}{b}\right) H_0 \cos\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) e^{-\gamma z}$$

$$H_{ys} = -\frac{\sqrt{\epsilon}}{h^2} \frac{\partial H_{zs}}{\partial y} - \frac{j\omega\epsilon}{h^2} \frac{\partial E_{zs}}{\partial x}$$

$$= -\frac{\sqrt{\epsilon}}{h^2} \frac{\partial}{\partial y} \left[H_0 \cos\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) e^{-\gamma z} \right]$$

$$H_{ys} = \frac{\sqrt{\epsilon}}{h^2} H_0 \left(\frac{n\pi}{b}\right) \cos\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) e^{-\gamma z}$$

$$\eta_{TE} = \frac{E_{xs}}{H_{ys}} = \frac{\frac{j\omega\mu}{h^2} \left(\frac{n\pi}{b}\right) H_0 \cos\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) e^{-\gamma z}}{\frac{\sqrt{\epsilon}}{h^2} H_0 \left(\frac{n\pi}{b}\right) \cos\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) e^{-\gamma z}}$$

$$\eta_{TE} = \frac{j\omega M}{j\beta_g} = \frac{\omega M}{\beta_g} = \frac{\omega M}{\beta \sqrt{1 - (f_c/f)^2}}$$

$$\frac{\omega M}{\beta} = \frac{\omega M}{\omega \sqrt{\mu \epsilon}} = \sqrt{\frac{\mu}{\epsilon}} = \eta$$

$$\boxed{\eta_{TE} = \frac{\eta}{\sqrt{1 - (f_c/f)^2}}}$$

Intrinsic Impedance for TM_{mn} mode ($E_{zs} \neq 0, H_{zs} = 0$)

$$\boxed{\eta_{TM} = \frac{E_{xs}}{H_{ys}}}$$

$$E_{zs} = E_0 \sin\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) e^{-\gamma z}$$

$$E_{xs} = -\frac{1}{h^2} \cdot \frac{\partial E_{zs}}{\partial x} - \frac{j\omega M}{h^2} \cdot \frac{\partial H_{zs}}{\partial y} \xrightarrow{=0}$$

$$E_{xs} = -\frac{1}{h^2} \cdot E_0 \left(\frac{m\pi}{a}\right) \cos\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) e^{-\gamma z}$$

$$H_{ys} = -\frac{1}{h^2} \cdot \frac{\partial H_{zs}}{\partial y} - \frac{j\omega \epsilon}{h^2} \cdot \frac{\partial E_{zs}}{\partial x}$$

$$= -\frac{j\omega \epsilon}{h^2} E_0 \left(\frac{m\pi}{a}\right) \cos\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) e^{-\gamma z}$$

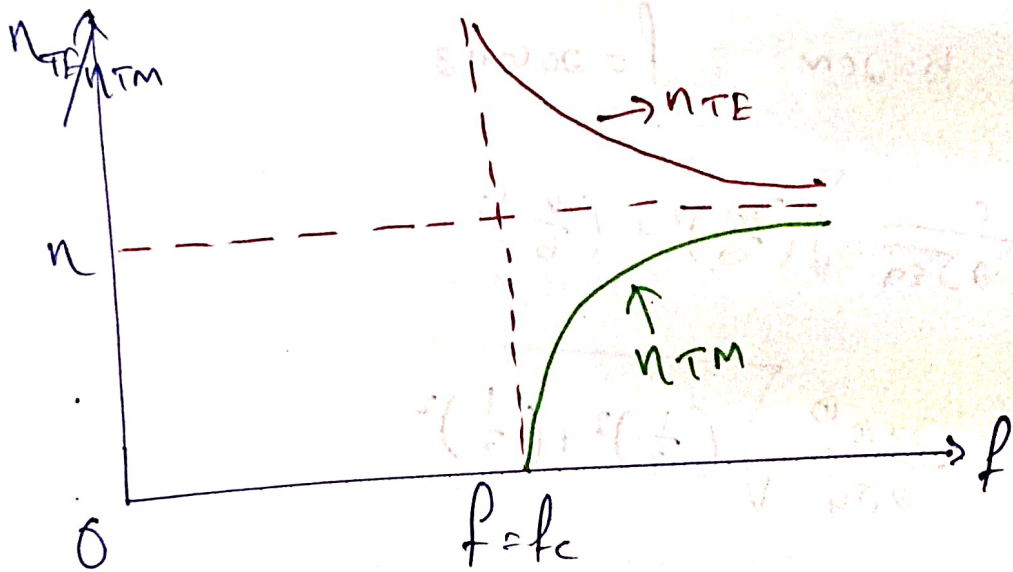
$$\eta_{TM} = \frac{E_{xs}}{H_{ys}} = \frac{1}{j\omega \epsilon} = \frac{j\beta_g}{j\omega \epsilon}$$

$$= \frac{\beta \sqrt{1 - (f_c/f)^2}}{\omega \epsilon} = \frac{\omega \sqrt{\mu \epsilon}}{\omega \epsilon} \sqrt{1 - (f_c/f)^2}$$

$$\eta_{TM} = \eta \sqrt{1 - (f_c/f)^2}$$

$$\eta_{TE} \times \eta_{TM} = \frac{\eta}{\sqrt{1 - (f_c/f)^2}} \times \eta \sqrt{1 - (f_c/f)^2}$$

$$\boxed{\eta = \sqrt{\eta_{TE} \times \eta_{TM}}}$$



$$\eta = \sqrt{\mu/\epsilon} = 120\pi \sqrt{\mu_r/\epsilon_r}$$

Air-filled wave guide $\eta = \eta_0 = 120\pi$

Q A 2cm by 3cm waveguide is filled with a dielectric material with $\epsilon_r = 4$, if the waveguide operates at 20 GHz

with TE_{11} mode, find

(a) cut off freq (b) The phase constant

(c) the phase velocity

(a) $a = 3\text{cm}$, $b = 2\text{cm}$, $f = 20\text{GHz}$.

$$(f_c)_{TM} = \frac{c}{2\sqrt{\epsilon_r}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$= \frac{3 \times 10^{10}}{2\sqrt{4}} \sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{1}{2}\right)^2}$$

$$(f_c)_{TM} = 4.51\text{ GHz}$$

$$(b) \beta_g = \beta \sqrt{1 - (f_c/f)^2}$$

$$= \frac{\omega}{v_p} \sqrt{1 - (f_c/f)^2} = 816.18\text{ rad/m}$$

$$(c) v_{pz} = \frac{v_p}{\sqrt{1 - (f_c/f)^2}} = 1.53 \times 10^8$$

(P1 - 59)

1) A tunnel is modeled as an air-filled metallic rectangular waveguide with dimensions $a = 8\text{m}$ & $b = 16\text{m}$. Determine whether the tunnel will pass.

- (a) A 1.5 MHz broadcast signal
(b) A 120-MHz broadcast signal.

(A) $(f_c)_{TE_{10}} = \frac{c}{2a} = \frac{3 \times 10^8}{2 \times 8} = 18.75\text{ MHz}$

$(f_c)_{TE_{10}} > f$ (tunnel will not pass 1.5 MHz AM)

(B) since $f > (f_c)_{TE_{10}}$ Tunnel will pass 120 MHz FM

2) in an air-filled rectangular waveguide, the cutoff freq of TE_{10} mode is 5 GHz whereas that of TE_{01} mode is 12 GHz, calculate

(a) Dimension of the guide

(b) cutoff freq of the next three higher TE modes

(i) since $f > (f_c)_{TE_{10}}$ tunnel will pass 120 MHz FM

the cutoff freq for TE_{11} mode if the guide is

filled with a lossless material having $\epsilon_{r0} = 2.25$ and

$\mu_{r0} = 1$.

$$\textcircled{A} \quad (f_c)_{TE_{10}} = \frac{c}{2a} \quad 5 \times 10^9 = \frac{3 \times 10^{10}}{2a}$$

$$\boxed{a = 3 \text{ cm}} \quad (f_c)_{TE_{01}} = \frac{c}{2b}$$

$$12 \times 10^9 = \frac{3 \times 10^{10}}{2b}$$

$$\boxed{b = 1.25 \text{ cm}}$$

$$(f_c)_{TE_{11}} = \frac{c}{2} \sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{1}{1.25}\right)^2}$$

$$(f_c)_{TE_{20}} = \frac{c}{a} = \frac{3 \times 10^{10}}{3} = 10 \text{ GHz}$$

$$(f_c)_{TE_{02}} = \frac{c}{b} = \frac{3 \times 10^{10}}{1.25} = 24 \text{ GHz}$$

next three higher modes, TE_{10} , TE_{01} , TE_{20}

$\textcircled{C}$ if dielectric is filled $\epsilon_r = 2.25$

$$(f_c)_{TE_{11}} = \frac{13}{\sqrt{2.25}} = 8.66 \text{ GHz}$$

$$(f_c)_{TM_{11}} = \frac{c}{2} \sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{3.75}\right)^2}$$

$$12 \times 10^9 = \frac{3 \times 10^{10}}{2} \sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{3.75}\right)^2}$$

$$\boxed{a = 1.32 \text{ cm}}$$

$$\boxed{b > a} \quad (f_c)_{TE_{01}} = \frac{c}{2b} = \frac{3 \times 10^{10}}{2 \times 3.75} = 40 \text{ GHz}$$

Q) An air-filled rectangular waveguide operates at 40 GHz. if the cutoff freq of the TE_{12} mode is 25 GHz, calculate the wavelength, phase constant, phase velocity and intrinsic Impedance of this mode

a) Guided wavelength $\lambda_g = \frac{\lambda}{\sqrt{1 - (f_c/f)^2}}$

$$= \frac{(3 \times 10^8 / 40 \times 10^3)}{\sqrt{1 - (25/40)^2}}$$

b) Guided phase const $\beta_g = \frac{2\pi}{\lambda_g} =$

c) Guided phase velocity $v_{p2} = \frac{c}{\sqrt{1 - (f_c/f)^2}}$

$$= \frac{3 \times 10^8}{\sqrt{1 - (25/40)^2}} =$$

Q) An air-filled hollow rectangular waveguide is 1.50 m long and is capped at the end with a metal plate. If a short pulse of freq 7.2 GHz is introduced into the i/p end of the guide. how long does it take the pulse to return to the i/p end? Assume that the cut-off freq of the guide is 6.5

$$t_{gr} = \frac{2l}{v_{g2}}$$

$$v_{g2} = c \sqrt{1 - (f_c/c)^2} = 3 \times 10^8 \sqrt{1 - \left(\frac{6.5}{7.2}\right)^2}$$

$$t_{gr} = 2.325 \mu\text{sec} = 1.29 \times 10^{-8} \text{ s}$$

4) Calculate the dimensions of air-filled rectangular waveguide for which the cut-off freqs for TM_{11} and TE_{03} modes are both equal to 12 GHz. at 8 GHz, determine which

the dominant mode will propagate or evanesce in the waveguide.

$$(f_c)_{TE_{03}} = \frac{c}{2} \times \frac{3}{b}$$

$$10^9 \times 12 = \frac{3 \times 3 \times 10^{10}}{2b}$$

$$b = \frac{90}{24} = 3.75 \text{ cm}$$

④ Intrinsic Impedance of TE_{12}

$$\eta_{TE} = \frac{\eta_0}{\sqrt{1 - (f_c/f)^2}} = \frac{120\pi}{\sqrt{1 - (25/48)^2}} = 482.9 \Omega$$

⑤ An air-filled rect waveguide is to be constructed to operate at 4 GHz in the dominant mode, specify the guide dimensions a and b such that the operating freq is 12% higher than the cut off freq for the dominant mode (TE_{10}) and 15% below the cut off freq of the next highest mode

(A) $f \geq 1.12 (f_c)_{TE_{10}}$

$$4 \times 10^9 \geq 1.12 \times \frac{c}{2a}$$

$$a = \frac{1.12 \times 3 \times 10^{10}}{4 \times 10^9}$$

$$a = 4.2 \text{ cm}$$

$$f \leq c/2b$$

$$b \leq \frac{3 \times 10^{10}}{2 \times 4 \times 10^9} \times 0.85$$

$$b = 3.1875 \text{ cm}$$

Q An $6\text{cm} \times 4\text{cm}$ air-filled waveguide is given. What modes are possible if $f = 6\text{GHz}$ is to be propagated down the guide?

$(f_c)_{\text{TE}_{10}} = \frac{c}{2a} = \frac{3 \times 10^{10}}{2 \times 6} = 2.5\text{GHz} \quad (f > (f_c)_{\text{TE}_{10}})$
 $(f_c)_{\text{TE}_{01}} = \frac{c}{2b} = \frac{3 \times 10^{10}}{2 \times 4} = 3.75\text{GHz} \quad (f > (f_c)_{\text{TE}_{01}})$
 $(f_c)_{\text{TE}_{20}} = \frac{c}{a} = \frac{3 \times 10^{10}}{6} = 5\text{GHz} \quad (f > (f_c)_{\text{TE}_{20}})$
 $\times (f_c)_{\text{TE}_{02}} = \frac{c}{b} = \frac{3 \times 10^{10}}{4} = 7.5\text{GHz} \quad (f < (f_c)_{\text{TE}_{02}})$
 $(f_c)_{\text{TE}_{11}/\text{TM}_{11}} = \frac{c}{2} \sqrt{\left(\frac{1}{6}\right)^2 + \left(\frac{1}{4}\right)^2} = 4.51\text{GHz} \quad (f > (f_c)_{\text{TE}_{11}})$

Q Design a rectangular waveguide with an aspect ratio of 3 to 1 for use in the K band ($18-26.5\text{GHz}$). Assume that the guide is air filled.

Aspect ratio $= \frac{a}{b} = 3 \Rightarrow a = 3b$

$(f_c)_{\text{TE}_{10}} = \frac{c}{2a} \quad b = a/3 = 0.833/3$

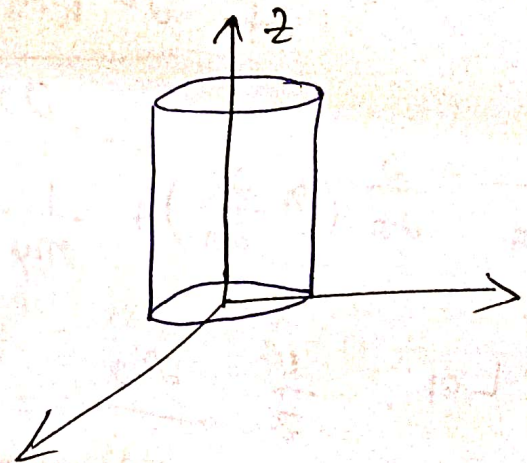
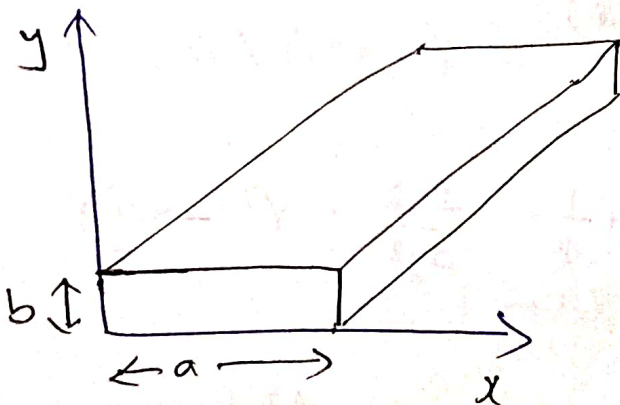
$18 \times 10^9 = \frac{3 \times 10^{10}}{2 \times a} \quad b = 0.277\text{cm}$

$a = \frac{30}{36} = 0.833\text{cm}$

(1)

Circular wave guide

- it is a hollow metallic tube of circular cross section
- CWG are used in rotational coupling
- Dominant mode in RWR is TE_{10}
 ξ CWG is TE_{11}
- A hollow metallic tube is made up of brass or copper



Sol'n of wave eq'n in cylindrical coordinate

The e-c field and m-c field eq'n are given by

$$\nabla^2 E = \gamma^2 E$$

$$\nabla^2 H = \gamma^2 H$$

$$\text{Where } \gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = \alpha + j\beta$$

The circular or cylindrical components of e-c field or m-c field satisfy the complex scalar wave equation or Helmholtz eq'n.

$$\nabla^2 \psi = \gamma^2 \psi$$

The scalar Helmholtz eqn in cylindrical coordinates

$$\text{is } \left[\frac{1}{r} \cdot \frac{\partial}{\partial r} \left(r \cdot \frac{\partial \psi}{\partial r} \right) + \frac{1}{r^2} \left(\frac{\partial^2 \psi}{\partial \phi^2} \right) + \frac{\partial^2 \psi}{\partial z^2} = \gamma^2 \psi \right]$$

Using method of separation of variables

$$\psi = R(r) \phi(\phi), z(z)$$

$$\frac{1}{r} \left(\frac{\partial}{\partial r} \cdot r \cdot \frac{\partial R(r) \phi(\phi) z(z)}{\partial r} \right) + \frac{1}{r^2} \cdot \frac{\partial^2 R(r) \phi(\phi) z(z)}{\partial \phi^2}$$

$$+ \frac{\partial^2 R(r) \phi(\phi) z(z)}{\partial z^2} = \gamma^2 R(r) \phi(\phi)$$

$$\frac{1}{Rr} \frac{\partial}{\partial r} \left(r \frac{\partial R}{\partial r} \right) + \frac{1}{r^2 \phi} \cdot \frac{\partial^2 \phi}{\partial \phi^2} + \frac{1}{z} \cdot \frac{\partial^2 z}{\partial z^2} = \gamma^2 \rightarrow \textcircled{1}$$

$$\text{Let } \frac{1}{z} \cdot \frac{\partial^2 z}{\partial z^2} = \gamma^2 \Rightarrow \frac{\partial^2 z}{\partial z^2} = \gamma^2 z.$$

$$\frac{\partial^2 z}{\partial z^2} - \gamma^2 z = 0.$$

$$(D^2 - \gamma^2) z = 0.$$

$$D = \pm \gamma$$

$$z = A e^{-j \gamma z} + B e^{j \gamma z}$$

$\gamma =$ propagation const in the waveguide

(2)

$$\frac{1}{rR} \frac{\partial}{\partial r} \left(r \cdot \frac{\partial R}{\partial r} \right) + \frac{1}{r^2 \phi} \cdot \frac{\partial^2 \phi}{\partial \phi^2} + (v_g^2 - v^2) = 0$$

$$\frac{r}{R} \frac{\partial}{\partial r} \left(r \cdot \frac{\partial R}{\partial r} \right) + \frac{1}{\phi} \cdot \frac{\partial^2 \phi}{\partial \phi^2} + (v_g^2 - v^2) r^2 = 0$$

$$\text{Let } \frac{1}{\phi} \cdot \frac{\partial^2 \phi}{\partial \phi^2} = -n^2$$

$$\frac{\partial^2 \phi}{\partial \phi^2} + n^2 \phi = 0$$

$\phi \rightarrow \text{gammma}$

$r \rightarrow \text{radius}$

$$(D^2 + n^2) \phi = 0, \quad D = \pm nj$$

$$\frac{r}{R} \frac{\partial}{\partial r} \left(r \cdot \frac{\partial R}{\partial r} \right) - n^2 + (v_g^2 - v^2) r^2 = 0$$

$$r \frac{\partial}{\partial r} \left(r \cdot \frac{\partial R}{\partial r} \right) - n^2 R + (v_g^2 - v^2) r^2 R = 0$$

$$r \frac{\partial}{\partial r} \left(r \cdot \frac{\partial R}{\partial r} \right) - n^2 R + k_c^2 r^2 R = 0$$

$$r \frac{\partial}{\partial r} \left(r \cdot \frac{\partial R}{\partial r} \right) + (k_c^2 r^2 - n^2) R = 0$$

This is Bessel equation of order n

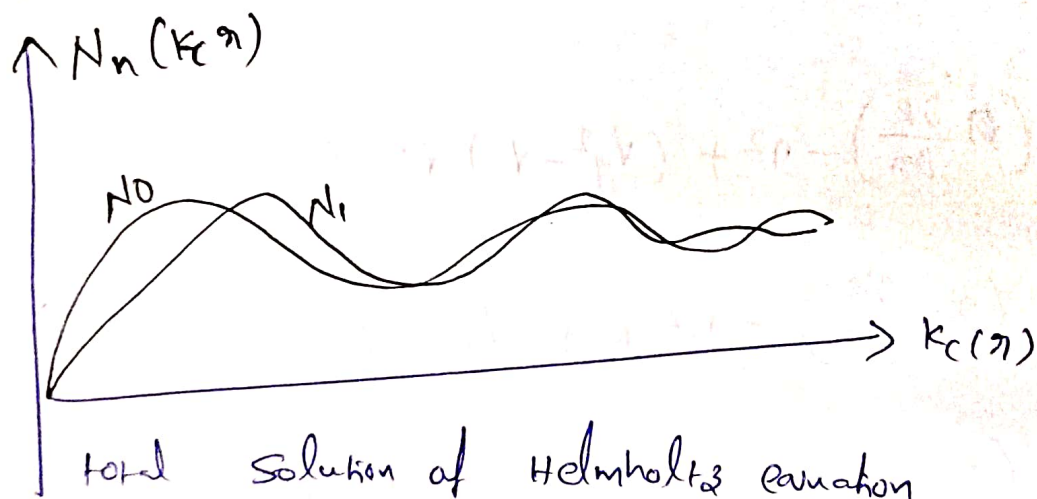
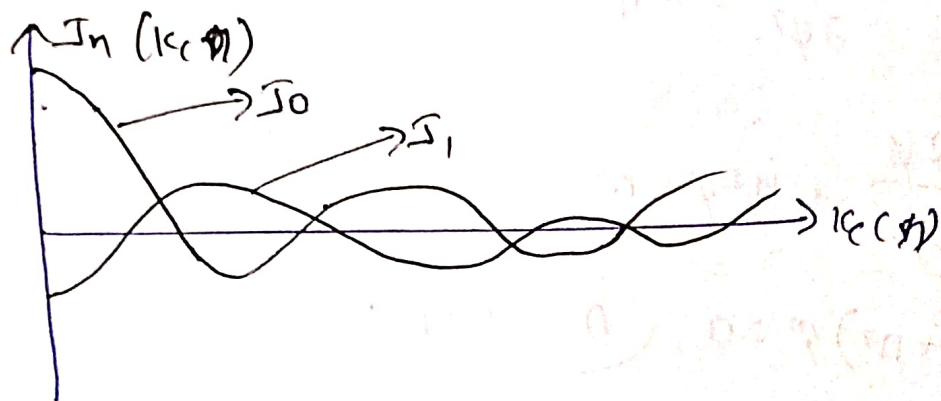
$$\text{where } k_c^2 = v_g^2 - v^2$$

Solution of the Bessel eq'n is

$$R = P J_n(k_c r) + Q N_n(k_c r)$$

Where $J_n(k_c a) =$ Bessel's function of first kind, which represent standing wave of $\cos(k_c a)$ for $a < a$

$N_n(k_c r) =$ Bessel's function of second kind, which represent standing wave of $\sin(k_c r)$ for $a > a$



Total solution of Helmholtz equation

$$[P J_n(k_c r) + Q N_n(k_c r)] [C \cos(n\phi) + D \sin(n\phi)] [e^{j\gamma_g z} + e^{-j\gamma_g z}]$$

At $r=0$, $N_n \rightarrow \infty$, $D \rightarrow 0$ $\rightarrow$ +z direction

$$\Psi = P J_n(k_c r) [C \cos(n\phi) + D \sin(n\phi)] e^{-j\gamma_g z}$$

for lossless medium $\alpha=0$, $\gamma_g = -j\beta_g$

$$\Psi = P J_n(k_c r) \sqrt{c^2 + D^2} \cos(n\phi - \tan^{-1}(D/C)) e^{-j\beta_g z}$$

(3)

$$\boxed{\Psi = \Psi_0 J_n(k_r r) \cos(n\phi) e^{-j\beta_g z}}$$

$$k_r^2 = \gamma_g^2 - \gamma^2$$

$$k_r^2 = (j\beta_g)^2 - (j\beta)^2$$

$$k_r^2 = -\beta_g^2 + \beta^2$$

$$k_r^2 = -\beta_g^2 + (\omega\sqrt{\mu\epsilon})^2$$

$$\boxed{k_r^2 = -\beta_g^2 + \omega^2\mu\epsilon}$$

using method of separation of variables

$$\Psi = R(r) \phi(\phi) z(z)$$

$$\Psi = \Psi_0 J_n(k_r r) \cos(n\phi) e^{-j\beta_g z}$$

where $J_n(k_r r)$ = n^{th} order Bessel function of first kind
representing a standing of $\cos(k_r r)$ for $r < a$

$$k_r^2 + \gamma^2 = \gamma_g^2$$

$$k_r^2 = \omega^2\mu\epsilon - \beta_g^2$$

TE modes in circular waveguide.

$$E_{zs} = 0, \quad H_{zs} \neq 0$$

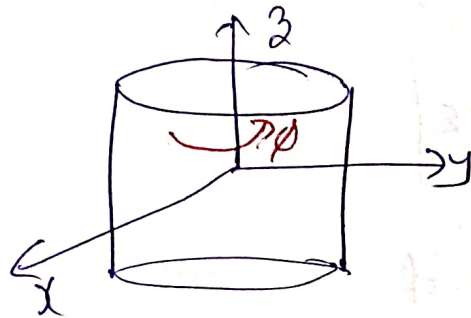
$$H_{zs} = H_0 J_n(k_r r) \cos(n\phi) e^{-j\beta_g z}$$

Boundary condition:- the tangential component of electric field must be zero at inner surface of CWG at $r=a$

At $r=a$, $E_\phi = 0$ or $\frac{\partial H_{z0}}{\partial r} = 0$.

$$\frac{\partial H_z}{\partial r} = H_0 J_n(k_r a) \cos(n\phi) e^{-j\beta_g z} = 0$$

hence $J_n(k_r a) = 0$



$$E_\phi = \frac{j\omega\mu}{k_r^2} \cdot \frac{\partial H_z}{\partial r}$$

$$E_r = -\frac{j\omega\mu}{k_r^2 r} \cdot \frac{\partial H_z}{\partial \phi}$$

Let zero of $J_n(k_r a)$ for TE_{np} modes.

P	n=0	n=1	n=2	n=3	n=4
1	3.8	1.8	3.0	4.2	5.3
2	7.0	5.3	6.7	8.0	9.2
3	10.1	8.5	9.9	11.3	12.6
4	13.3	11.7	13.1		

$$k_r a = x'_{np}$$

n = number of feeds in ϕ direction $0 < \phi < \pi$

p = no. of feed points in radial direction

$$J_0'(3.832) = 0$$

$$J_2(6.706) = 0$$

Waveguide parameters:-

$$k_c^2 = -\beta_g^2 + \omega^2 \mu \epsilon \quad \cancel{k_c^2} h^2$$

at $\omega = \omega_c$, $\beta_g = 0$

$$k_c^2 = \omega_c^2 \mu \epsilon$$

$$\left(\frac{x'_{np}}{a} \right)^2 = (2\pi f_c)^2 \mu \epsilon,$$

$$f_c = \frac{x'_{np}}{2\pi a \sqrt{\mu \epsilon}}$$

$$\boxed{f_c = \frac{x'_{np} v_p}{2\pi a}} \text{ cut off freq.}$$

for air filled, $v_p = c$.

$$f_c = \frac{x'_{np} c}{2\pi a} = \frac{x'_{np} (3 \times 10^{10})}{2\pi (a)_{cm}}$$

$$f_c =$$

The minimum value of f_c denotes dominant mode

$$(f_c)_{TE_{11}} = \frac{1.841 \times 3 \times 10^{10}}{2\pi (a)_{cm}} = \left(\frac{8.79}{(a)_{cm}} \right) \text{GHz}$$

Cut off wavelength $\lambda_c = \frac{v_p}{f_c} = \frac{2\pi a}{\chi'_{np}}$

for air-filled ξ dominant mode TE_{11} .

$$\lambda_c = \frac{2\pi}{1.841} a_{cm}$$

$$\lambda_c = 3.419 \text{ cm}$$

Phase velocity

$$v_{pz} = \frac{\omega}{\beta_g} = \frac{v_p}{\sqrt{1 - (\beta_c/\beta)^2}}$$

Guided wavelength $\lambda_g = \frac{2\pi}{\beta_g} = \frac{\lambda}{\sqrt{1 - (\beta_c/\beta)^2}}$

Intrinsic Impedance $\eta_{TE} = \frac{\eta}{\sqrt{1 - (\beta_c/\beta)^2}}$

TM mode in circular waveguide

$$E_{zs} \neq 0, H_{zs} = 0$$

$$E_{zs} = E_0 J_n(k_r r) \cos(n\phi) e^{-j\beta_g z}$$

Boundary condition:- at $r=a$ $E_{zs} = 0$

$$E_0 J_n(k_r a) \cos(n\phi) e^{-j\beta_g z} = 0$$

hence

$$J_n(k_r a) = 0$$

$$k_r a = x_{np}$$

Table

p	$n=0$	$n=1$	$n=2$	$n=3$
1	2.4	3.8	5.1	6.3
2	5.5	7.1	8.4	9.7
3	8.6	10.1	11.6	13.0
4	11.7	13.3	14.7	

p^{th} root of n^{th} order Bessel Harmonics.

$$k_c a = x_{np}$$

Cut-off freq for TM_{np} mode

$$(f_c)_{TM_{np}} = \frac{x_{np} \cdot V_p}{2\pi a}$$

The minimum mode is TM_{np} is TM_{01}

Intrinsic Impedance

$$\eta_{TM} = \eta \sqrt{1 - (f_c/f)^2}$$

dominant mode TE_{11}

Degenerate mode:- The modes having same cut off frequency.

if $X'_{np} = X_{np}$ we get same cut off frequency.

$$TE_{0p} = TM_{1p}$$

ex:-

$$TE_{01} = TM_{10}$$

$$TE_{02} = TM_{20}$$

$$TE_{03} = TM_{30}$$

Evanescent mode:- mode that will not be supported by

waveguide $TE_{no} \& TM_{no}$.

⑥ A TE_{11} dominant mode is propagating through a coax. The radius of the guide is $a = 5\text{cm}$ and guide contains an air dielectric ($\epsilon_r = 1$).

① Determine cut off frequency

② The wavelength in the guide for a operating freq of 3GHz

③ Determine the wave impedance in the guide

$$\textcircled{a} (f_c)_{TE_{11}} = \frac{8.79}{(a)_{\text{cm}}} \text{ GHz} = \frac{8.79}{5} = 1.758 \text{ GHz}$$

$$\begin{aligned} \textcircled{b} \text{ guided wavelength } \lambda_g &= \frac{\lambda}{\sqrt{1 - (f_c/f)^2}} \\ &= \frac{\left(\frac{3 \times 10^8}{3 \times 10^9} \right)}{\sqrt{1 - \left(\frac{1.758}{3} \right)^2}} = 12.3 \text{ cm} \end{aligned}$$

$$\textcircled{c} Z_{TE} = \frac{\eta}{\sqrt{1 - (f_c/f)^2}} = \frac{120\pi}{\sqrt{1 - \left(\frac{1.758}{3} \right)^2}} = 465.25 \Omega$$

Note:- if in any question $v_p = 0.6c$ @ $0.4c$ instead of mention the μ_r & ϵ_r value, he will give $v_p = 0.6c$ i.e. wherever $\sqrt{\mu\epsilon} \Rightarrow$ need to replace with $1/0.6c$

Reason:- $\sqrt{\mu_0 \epsilon_0} = 1/c = 1/v_p$
if we alter the μ_r & ϵ_r then only velocity change

④ An air filled coax has a radius of 2cm and is to carry energy at a frequency of 10GHz. Find all TE_{np} of TM_{np} for which energy transmission is possible

for the wave propagation $f > f_c$

for TE_{np} mode, $f > \frac{x'_{np} \times c}{2\pi a}$

$$f > \frac{x'_{np} \times 3 \times 10^{10}}{2\pi \times 2}$$

$$x_{np} < \frac{10 \times 10^9 \times 4\pi}{3 \times 10^{10}}$$

TE₁₁, TE₀₁, TE₂₁

$$\boxed{x'_{np} < 4.188}$$

⑤ An air filled circular waveguide is to be operated at a frequency of 6 GHz and is to have dimensions such that $f_c = 0.8f$ for the dominant mode.

Determine. (a) the diameter of the guide

(b) The wavelength λ_g and phase velocity

of the guide

$$(a) \quad (f_c)_{TE_{11}} = \frac{1.841 \times c}{2\pi a}$$

$$d = \frac{1.841 \times 3 \times 10^{10}}{\pi \times 0.8 \times 6 \times 10^9} = 3.66$$

(b) $\gamma_g = \frac{\lambda}{\sqrt{1 - (f_c/f)^2}} = \frac{8 \cdot (3 \times 10^8 / 6 \times 10^9)}{\sqrt{1 - (0.8)^2}} = 8.33 \text{ cm}$ (7)

(c) $v_{p2} = \frac{v_p}{\sqrt{1 - (f_c/f)^2}} = \frac{3 \times 10^8}{\sqrt{1 - (0.8)^2}} = 5 \times 10^8 \text{ m/sec}$

(Q) An air-filled circular waveguide of 2cm inside radius is operated in the TE_{01} mode.

(a) Compute the cut off frequency.

(b) if the guide is to be filled with a dielectric material of $\epsilon_r = 2.25$, to what value must its radius be changed in order to maintain the cut off frequency at its original value?

(a) $(f_c)_{TE_{01}} = \frac{x'_{np} \times c}{2\pi a} = \frac{x'_{01} \times 3 \times 10^{10}}{2\pi \times 2}$
 $= \frac{3.832 \times 3 \times 10^{10}}{4\pi}$

$(f_c)_{TE_{01}} = 8.07 \text{ GHz}$

(b) $8.07 \times 10^9 = \frac{x'_{np} \times v_p}{2\pi a} \quad a = \frac{3.832 \times c}{8 \times 10^9 \sqrt{2.25}} \times \frac{1}{2\pi}$
 $a = 1.33 \text{ cm.}$

① A TE_{11} wave is propagating through a circular waveguide. The diameter of the guide is 10 cm, and the guide is air filled.

(a) Find the cut off freq

(b) Find the wavelength λ_g in the guide for a frequency of 3 GHz

(c) Determine the wave Impedance in the guide

① $a = 10 \text{ cm} = 10 \times 10^{-2} \text{ m}$

② $(f_c)_{TE_{11}} = \frac{\chi'_{np}}{2\pi a \sqrt{\mu\epsilon}}$ air filled $\mu_r = \epsilon_r = 1$

χ'_{np} for $TE_{11} \Rightarrow 1.81$

Then $(f_c)_{TE_{11}} = \frac{1.81 \times 3 \times 10^8}{2\pi a} = \frac{1.81 \times 3 \times 10^8}{2 \times 3.14 \times 10^{-1}} =$

③ $\lambda_g = \frac{\lambda}{\sqrt{1 - (f_c/f)^2}} = \frac{c}{f \sqrt{1 - (f_c/f)^2}} = \frac{c}{\sqrt{f^2 - f_c^2}}$

$= \frac{3 \times 10^8}{\sqrt{(3 \times 10^9)^2 - (1.81 \times 10^9)^2}}$

④ $\eta_{TE} = \frac{E_x}{H_y} = \frac{\omega \mu}{\beta_g} = \frac{\eta}{\sqrt{1 - (f_c/f)^2}}$

Helmholtz equation for TEM mode (RF design Basics)

in general Helmholtz equation

$$(\nabla^2 - \gamma^2) \vec{E}_s = 0 \quad \text{or} \quad (\nabla^2 - \gamma^2) \vec{H}_s = 0$$

$$\vec{E}_s \Rightarrow E_{xs}, E_{ys}, E_{zs}$$

$$\vec{H}_s \Rightarrow H_{xs}, H_{ys}, H_{zs}$$

To find $\vec{E}_s$ & $\vec{H}_s$ first we derive E_{zs} and H_{zs}

Then with help of $\nabla \times \vec{E} = -j\omega\mu \vec{H} \rightarrow (1)$

$$\nabla \times \vec{H} = \sigma \vec{E} + j\omega\epsilon \vec{E}$$

$\sigma = 0$ for lossless line i.e. dielectric

$$\nabla \times \vec{H} = j\omega\epsilon \vec{E} \rightarrow (2)$$

from (1)

$$\begin{aligned} -j\omega\mu H_{xs} &= \frac{\partial E_{zs}}{\partial y} - \frac{\partial E_{ys}}{\partial z} \\ -j\omega\mu H_{ys} &= \frac{\partial E_{xs}}{\partial z} - \frac{\partial E_{zs}}{\partial x} \\ -j\omega\mu H_{zs} &= \frac{\partial E_{ys}}{\partial x} - \frac{\partial E_{xs}}{\partial y} \end{aligned} \quad \left| \quad \begin{aligned} j\omega\epsilon E_{xs} &= \frac{\partial H_{zs}}{\partial y} - \frac{\partial H_{ys}}{\partial z} \\ j\omega\epsilon E_{ys} &= \frac{\partial H_{xs}}{\partial z} - \frac{\partial H_{zs}}{\partial x} \\ j\omega\epsilon E_{zs} &= \frac{\partial H_{ys}}{\partial x} - \frac{\partial H_{xs}}{\partial y} \end{aligned} \right.$$

from these equations we had concluded that

$$E_{xs} = -\frac{\gamma}{h^2} \cdot \frac{\partial E_{zs}}{\partial x} - \frac{j\omega\mu}{h^2} \cdot \frac{\partial H_{zs}}{\partial y}$$

$$E_{ys} = -\frac{\gamma}{h^2} \cdot \frac{\partial E_{zs}}{\partial y} + \frac{j\omega\mu}{h^2} \cdot \frac{\partial H_{zs}}{\partial x}$$

$$H_{xs} = -\frac{\gamma}{h^2} \cdot \frac{\partial H_{zs}}{\partial x} + \frac{j\omega\epsilon}{h^2} \cdot \frac{\partial E_{zs}}{\partial y}$$

$$H_{ys} = -\frac{\gamma}{h^2} \cdot \frac{\partial H_{zs}}{\partial y} - \frac{j\omega\epsilon}{h^2} \cdot \frac{\partial E_{zs}}{\partial x}$$

But this equations are not suitable for TEM mode analysis. Why because

In TEM mode $E_z = H_z = 0$

Then $E_{yx} = H_{yx} = E_{xy} = H_{xy} = 0$

Then we need to add the equation which work sustain with out E_z & H_z .

So consider the equations i.e. derived from

$$\nabla \times \vec{E} = -j\omega\mu\vec{H} \quad \nabla \times \vec{H} = j\omega\epsilon\vec{E}$$

$$-j\omega\mu H_x = \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}$$

$$-j\omega\mu H_y = \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}$$

$$-j\omega\mu H_z = \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}$$

$$j\omega\epsilon E_x = \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}$$

$$j\omega\epsilon E_y = \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}$$

$$j\omega\epsilon E_z = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}$$

if $E_z = H_z = 0$ in TEM mode

$$-j\omega\mu H_x = -\frac{\partial E_y}{\partial z}$$

$$-j\omega\mu H_y = \frac{\partial E_x}{\partial z} - \frac{\partial E_y}{\partial x}$$

$$H_z = 0$$

$$j\omega\epsilon E_x = -\frac{\partial H_y}{\partial z}$$

$$j\omega\epsilon E_y = \frac{\partial H_x}{\partial z} - \frac{\partial H_y}{\partial x}$$

$$E_z = 0$$

if wave propagate along z-direction then

$$E = E_0 e^{-\gamma z}$$

$$\gamma = \alpha + j\beta \quad \alpha = 0 \quad \text{if } \sigma = 0 \quad \text{then}$$

$$E = E_0 e^{-j\beta z}$$

$$\frac{\partial E_y}{\partial z} = -j\beta \cdot E_y, \quad \frac{\partial E_x}{\partial z} = -j\beta E_x$$

$$\frac{\partial H_y}{\partial z} = -j\beta H_y, \quad \frac{\partial H_x}{\partial z} = -j\beta H_x$$

then

$$\begin{aligned} -j\omega \mu H_x &= -(-j\beta E_y) \\ -j\omega \mu H_y &= -j\beta E_x \end{aligned} \quad \left| \quad \begin{aligned} j\omega \epsilon E_x &= -(-j\beta H_y) \\ j\omega \epsilon E_y &= -j\beta H_x \end{aligned} \right.$$

$$j\omega \epsilon E_x = j\beta H_y = j\beta \left[\frac{\beta}{\omega \mu} \right] E_x$$

$$\omega \epsilon E_x = \frac{\beta^2}{\omega \mu} E_x$$

$$\beta^2 = \omega^2 \mu \epsilon$$

$$\boxed{\beta^2 = k^2}$$

Then if we apply this condition for Helmholtz equation

$$(\nabla^2 - \gamma^2) \vec{E} = 0 \quad \text{or} \quad (\nabla^2 + k^2) \vec{E} = 0$$

$$\gamma^2 = j\omega \mu (\sigma + j\omega \epsilon) = -\omega^2 \mu \epsilon = -k^2$$

when $\sigma = 0$

Then
$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + k^2 \right) \bar{E} = 0$$

Suppose if we consider x-component

$$\left(\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} + k^2 E_x \right) = 0.$$

$$\frac{\partial^2}{\partial z^2} E_x = (-j\beta)(-j\beta) \cdot E_x$$

$$\left(\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} - \beta^2 E_x + k^2 E_x \right) = 0$$

$$\left(\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} \right) = 0 \quad \nabla^2 \bar{E} = 0$$

$$\nabla^2 \bar{E}(x, y) = 0$$

it is same for y component too.

Then we can define as (David M. Pozar
p. No : 88 & 89)

$$\nabla_{\perp}^2 \phi(x, y) \quad \nabla_{\perp}^2 \phi(x, y) = 0$$

$\Rightarrow$ Transverse components.

$$\left[\begin{matrix} E_x \\ E_y \end{matrix} \right] = \left[\begin{matrix} E_{x0} \\ E_{y0} \end{matrix} \right] e^{-j\beta z}$$

$$\nabla_{\perp}^2 \phi(x, y) = 0 \quad \phi(x, y) = \phi_0(x, y) e^{-j\beta z}$$

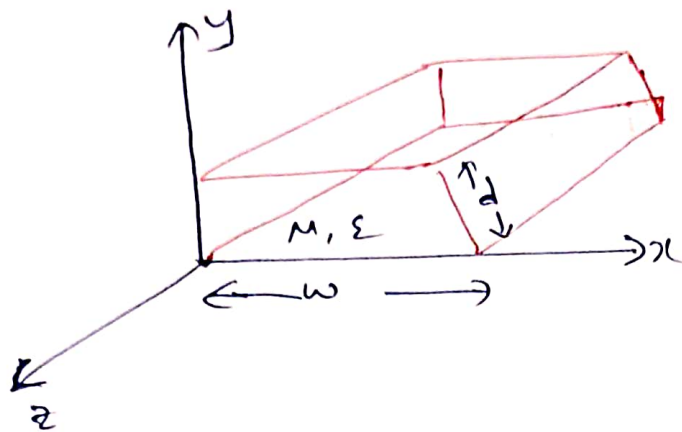
$$\nabla_{\perp}^2 \phi(x, y) = 0 \quad \phi(x, y) = \phi_0(x, y) e^{-j\beta z}$$

or we can write

For the transverse components

Parallel plate waveguide

(RTU wallah, EM possible)



TEM analysis

in general $(\nabla^2 - \mu\epsilon)\vec{E} = 0$ but for

general solution can TEM analysis $\nabla^2 \vec{E} = 0$ (or)

$\nabla^2 \phi(x,y) = 0 \rightarrow$ derived
topic

$$\nabla^2 V(x,y,z) = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

$$\textcircled{or} \quad \frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} = 0$$

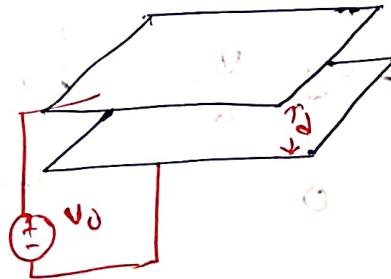
The parallel plate waveguide is uniform in the x and

z direction so the eq'n $\nabla^2 V = 0$ reduces to:

$$\frac{\partial^2 V}{\partial y^2} = 0$$

$$V(0) = 0, \quad V(d) = V_0$$

$$\frac{d^2 V}{dy^2} = 0 \quad 0 \leq y \leq d$$



$$\frac{d^2 V}{dy^2} = 0$$

$$\int \frac{d^2 V}{dy^2} = \int 0$$

$$\frac{dV}{dy} = A$$

$$\int \frac{dV}{dy} = \int A$$

$$V = Ay + B$$

$$V(y) = Ay + B$$

By Applying Boundary condition at $y=0$

$$V(0) = A \cdot (0) + B$$

$$B = 0$$

Apply the boundary condition at $y=d$

$$V(d) = V_0$$

$$A \cdot d + B = V(d)$$

$$A \cdot d = V_0$$

$$A = V_0/d$$

$$\text{So } V(y) = \frac{V_0}{d} y$$

The electric field $\vec{E} = -\nabla V$

$$= -a_x \frac{\partial V}{\partial x} - a_y \frac{\partial V}{\partial y} - a_z \frac{\partial V}{\partial z}$$

$$\vec{E} = -a_y \cdot \frac{\partial}{\partial y} \left(\frac{V_0}{d} y \right) - a_x \cdot \frac{\partial}{\partial x} \left(\frac{V_0}{d} y \right) - a_z \frac{\partial}{\partial z} \left(\frac{V_0}{d} y \right)$$

$$\vec{E} = -a_y \cdot \frac{V_0}{d}$$

$$\vec{E}(x, y) = \begin{cases} -a_y \frac{V_0}{d} & \text{for } 0 \leq x \leq w \text{ and } 0 \leq y \leq d \\ 0 & \text{otherwise} \end{cases}$$

$$\vec{E}(x, y, z) = \begin{cases} -a_y \cdot \frac{V_0}{d} \cdot e^{-j\beta z} & \text{for } 0 \leq x \leq w \text{ and } 0 \leq y \leq d \\ 0 & \text{otherwise} \end{cases}$$

if propagation along z-axis and $\vec{H}$ is along y-axis. Then

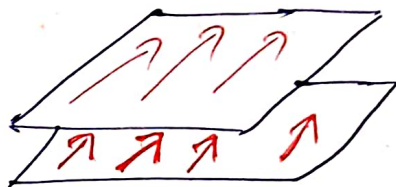
$$\vec{H}(x, y, z) = \frac{\vec{a}_z \times \vec{E}}{\eta} = \frac{\vec{a}_z \times \left(-\vec{a}_y \frac{V_0}{d} e^{-j\beta z} \right)}{\eta}$$

$$= \vec{a}_x \frac{V_0}{\eta d} e^{-j\beta z}$$

$$\vec{H}(x, y, z) = \begin{cases} \vec{a}_x \frac{V_0}{\eta d} e^{-j\beta z} & \text{for } 0 \leq x \leq w \text{ \& } 0 \leq y \leq d \\ 0 & \text{otherwise} \end{cases}$$

Impedance Z_{TEM} :-

$$Z_{TEM} = V_0 / I$$



we know $I = \oint \vec{H} \cdot d\vec{l}$

In our situation current is flowing in the direction $\vec{z}$ i.e. on the sheet.

$$\text{Then } I = \int_0^w (\vec{H}) \cdot (dx \vec{a}_x)$$

$$I = \int_0^w \left(\frac{V_0}{\eta d} \right) dx = \frac{V_0}{\eta d} \int_0^w dx = \frac{w}{d} \cdot \frac{V_0}{\eta}$$

$$Z_{TEM} = \frac{V_0}{I} = \frac{V_0}{\left(\frac{w}{d} \cdot \frac{V_0}{\eta} \right)} = \eta \cdot \frac{d}{w}$$

Propagation const:- (γ)

cut-off frequency:-

$$\gamma^2 = j\omega\mu(\sigma + j\omega\epsilon)$$

$$\sigma = 0 \quad \gamma^2 = -\omega^2\mu\epsilon$$

$$\gamma = \pm j\omega\sqrt{\mu\epsilon} = \alpha + j\beta$$

from this $\beta = \omega\sqrt{\mu\epsilon}$

$$f_c = 0$$

$\Rightarrow$ no cut off frequency
it allows all freq's

Q

$$\leftarrow w = 2.0 \text{ mm} \rightarrow$$

$$\epsilon_r = 2.3$$

$$\begin{array}{c} \uparrow \\ d = 0.5 \text{ mm} \\ \downarrow \end{array}$$

(a) What is Characteristic Impedance Z_{TEM} ?

(b) What value of w would make this Transmission line 50Ω ?

Ans

(a) $Z_{TEM} = \eta \cdot \frac{d}{w}$

$$\mu_r = 1$$

$$\epsilon_r = 2.3$$

$$\eta_0 = 377 \Omega$$

$$\eta = \eta_0 \sqrt{\frac{\mu_r}{\epsilon_r}} = 248.4 \Omega$$

$$Z_{TEM} = \frac{(248.4)(0.5)}{(2)} = 62.1 \Omega$$

(b)

$$w = \frac{\eta d}{Z_{TEM}}$$

$$= \frac{(248.4)(0.5)}{50} = 2.48 \text{ mm}$$

TE Analysis i.e. $E_z = 0$ (electric field is zero along

the direction of propagation) $H_z \neq 0$ (non

(Transverse Electric field mode)

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \frac{\partial^2 H_z}{\partial z^2} - \gamma^2 H_z = 0.$$

Since $H(x, y, z) = h_z(x, y) \cdot e^{-j\beta z}$ at $z = 0$

$= h_z(x, y)$. Then the equation will

be reduced to

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} - \gamma^2 H_z = 0$$

$$\frac{1}{x} \frac{\partial^2 H_z}{\partial x^2} + \frac{1}{y} \frac{\partial^2 H_z}{\partial y^2} + \frac{1}{z} \frac{\partial^2 H_z}{\partial z^2} - \gamma^2 = 0.$$

$$\frac{1}{x} \cdot \frac{\partial^2 x}{\partial x^2} + \frac{1}{y} \cdot \frac{\partial^2 y}{\partial y^2} + \frac{1}{z} \cdot \frac{\partial^2 z}{\partial z^2} - \gamma^2 = 0$$

$$\gamma^2 = \gamma_g^2 - k_c^2 = -k^2$$

$$\gamma_g = \alpha + j\beta$$

where $\alpha \geq 0$

$$\gamma_g^2 = -\beta^2$$

$$\frac{1}{x} \cdot \frac{\partial^2 x}{\partial x^2} + \frac{1}{y} \cdot \frac{\partial^2 y}{\partial y^2} - \beta^2 + k^2 = 0$$

$$\frac{1}{x} \cdot \frac{\partial^2 x}{\partial x^2} + \frac{1}{y} \cdot \frac{\partial^2 x}{\partial y^2} + k_c^2 = 0$$

$$k_c^2 = k^2 - \beta^2 \quad k_c = \sqrt{k^2 - \beta^2}$$

if $E_{zs} = 0$ then

$$H_x = -\frac{\sqrt{g}}{k^2} \cdot \frac{\partial H_{zs}}{\partial x}$$

$$E_x = -\frac{j\omega\mu}{h^2} \cdot \frac{\partial H_{zs}}{\partial y}$$

$$H_y = -\frac{\sqrt{g}}{h^2} \cdot \frac{\partial H_{zs}}{\partial y}$$

$$E_y = \frac{j\omega\mu}{h^2} \cdot \frac{\partial H_{zs}}{\partial x}$$

$$H_{zs} = H_{0z}$$

Assuming the waveguide is uniform in the direction of x

then there is no change then

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial x^2} = 0$$

$$\frac{1}{x} \cdot \frac{\partial^2 x}{\partial x^2} + \frac{1}{y} \cdot \frac{\partial^2 y}{\partial y^2} + k_c^2 = 0 \quad \text{reduced to}$$

$$\frac{1}{y} \cdot \frac{\partial^2 y}{\partial y^2} + k_c^2 = 0$$

$$\frac{\partial^2 y}{\partial y^2} + k_c^2 y = 0$$

$$\frac{\partial^2 H_{zs}}{\partial y^2} + k_c^2 H_{zs} = 0$$

$$H_{23} = A \sin(k_c y) + B \cos(k_c y)$$

To find the constants (A, B) and k_c we need to apply Boundary conditions.

In the Boundary conditions always we can apply for electric field so to get the equation in terms of electric field

$$E_x = -\frac{j\omega M}{h^2} \cdot \frac{\partial H_{23}}{\partial y}$$

$$= -\frac{j\omega M}{h^2} [A \cos(k_c y) \cdot k_c - B \sin(k_c y) \cdot k_c]$$

$$k_c^2 = h^2 = k^2 - \beta^2$$

at $y=0$ $E(x, 0) = 0$.

Then $E_x(x, 0) = -\frac{j\omega M}{h} [A \cos(0) - B \sin(0)] = 0$

$$A = 0$$

at $y=d$ $E(x, d) = 0$

$$E_x(x, d) = -\frac{j\omega M}{h} [-B \sin(k_c d)] = \frac{B j\omega M}{k_c} \sin k_c d$$

$$\sin k_c d = 0 \quad k_c d = m\pi$$

$$k_c = m\pi/d$$

$$H_{zs}(x, y, z) = B_m \cos\left(\frac{m\pi}{d} y\right) e^{-j\beta z}$$

from this we can derive

$$\underline{E_x, E_y, H_x \text{ \& } H_y}$$

No TE₀ mode

$$\begin{array}{lll} \text{if } m=0 & H_x = 0 & E_x = 0 \\ & H_y = 0 & E_y = 0 \\ & H_z = B e^{-j\beta z} & E_z = 0 \end{array}$$

So no TE₀ mode

Cut off frequency

$$k_c = \omega_c \sqrt{\mu \epsilon} = \frac{m\pi}{d} \Rightarrow \frac{m}{d} = \frac{k_c}{\pi}$$

$$f_c = \frac{m}{2d \sqrt{\mu \epsilon}} = \frac{k_c}{2\pi \sqrt{\mu \epsilon}}$$

Characteristic Impedance Z_{TE}

- TE_0 mode does not exist

- TE_1 is the lowest mode

$$Z_{TE} = \frac{E_{0x}}{H_{0y}} = - \frac{E_{0y}}{H_{0x}}$$

$$= \frac{\frac{j\omega\mu}{h} B \sin\left(\frac{m\pi y}{d}\right)}{\frac{j\beta}{h} \cdot B \sin\left(\frac{m\pi y}{d}\right)} = \frac{\omega\mu}{\beta}$$

$$\boxed{Z_{TE} = \frac{\omega\mu}{\beta}}$$

TM Analysis :-

$$H_{0,z} \text{ (or) } H_{25} = 0.$$

Laplace equation,

$$\frac{\partial^2 E_{zx}}{\partial x^2} + \frac{\partial^2 E_{zx}}{\partial y^2} + \frac{\partial^2 E_{zx}}{\partial z^2} - \gamma^2 E_{zx} = 0$$

$$\frac{1}{x} \cdot \frac{\partial^2 x}{\partial x^2} + \frac{1}{y} \cdot \frac{\partial^2 y}{\partial y^2} + \frac{1}{z} \cdot \frac{\partial^2 z}{\partial z^2} - \gamma^2 = 0$$

$$\gamma^2 = j\omega\mu (\sigma + j\omega\epsilon) \quad \text{if } \sigma = 0$$

$$\gamma^2 = -\omega^2\mu\epsilon = -k^2$$

$$\frac{1}{x} \cdot \frac{\partial^2 x}{\partial x^2} + \frac{1}{y} \cdot \frac{\partial^2 y}{\partial y^2} + \gamma^2 - \gamma^2 = 0$$

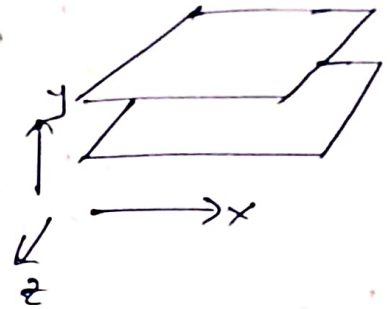
$$\gamma_g = (\alpha_g + j\beta_g)$$

$$\gamma_g^2 = -\beta_g^2 \quad \text{if } \alpha_g = 0$$

Then

$$\frac{1}{x} \cdot \frac{\partial^2 x}{\partial x^2} + \frac{1}{y} \cdot \frac{\partial^2 y}{\partial y^2} + k^2 - \beta^2 = 0$$

it is a parallel plate waveguide
along x-axis field is uniform



$$\text{Then } \frac{\partial}{\partial x} = \frac{\partial^2}{\partial x^2} = 0.$$

Then the equation should be

$$\frac{1}{y} \frac{\partial^2 y}{\partial y^2} + k_c^2 = 0$$

$$\text{we may write } \frac{\partial^2 E_{zs}}{\partial y^2} + k_c^2 E_{zs} = 0.$$

if $H_{zs} = 0$ from standard equations

$$H_x = \frac{j\omega\epsilon}{h^2} \frac{\partial E_{zs}}{\partial y}$$

$$E_x = -\frac{\gamma_g}{h^2} \frac{\partial E_{zs}}{\partial x}$$

$$H_y = -\frac{j\omega\epsilon}{h^2} \frac{\partial E_{zs}}{\partial x}$$

$$E_y = -\frac{\gamma_g}{h^2} \frac{\partial E_{zs}}{\partial y}$$

$$H_{zs} = 0, \quad E_{zs} = ? \text{ Need to derive}$$

$$\text{Solution for } \frac{\partial^2 E_{zs}}{\partial y^2} + k_c^2 E_{zs} = 0$$

$$E_{zs} = A \sin(k_y y) + B \cos(k_y y)$$

To find A & B need to apply Boundary conditions

$$\textcircled{1} \rightarrow \text{at } y=0 \quad E_{zs} \text{ must be } 0, \quad E_{zs}(x,0)=0$$

$$\textcircled{2} \rightarrow \text{at } y=d \quad " = 0 \quad E_{zs}(x,d)=0$$

$$E_{zs}(x,0) = A \sin(0) + B \cos(0) = 0$$

$$B=0$$

$$E_{zs}(x,d) = A \sin(k_y d) = 0$$

$$k_y d = m\pi \quad k_y = \frac{m\pi}{d} \quad m=0, 1, 2, \dots$$

$$E_{zs}(x,y,z) = A \sin\left(\frac{m\pi y}{d}\right) \cdot e^{-j\beta z} = A \sin\left(\frac{m\pi y}{d}\right) e^{-j\gamma_g z}$$

$$\gamma_g = \alpha_g + j\beta_g \quad \alpha_g = 0 \quad \text{Then } \gamma_g = j\beta_g$$

from this we can derive

$$H_x, H_y, E_x, E_y$$

$$E_z = A \sin\left(\frac{m\pi y}{d}\right) e^{-j\beta_g z}$$

$$H_z = \frac{1}{\eta} \frac{\partial E_z}{\partial y}$$

$$H_z = \frac{1}{\eta} \frac{\partial}{\partial y} \left(A \sin\left(\frac{m\pi y}{d}\right) e^{-j\beta_g z} \right)$$

TM₀ mode:

it exists when $m = 0$

$$H_x = \frac{j\omega\epsilon}{h} A e^{-j\beta z}$$

$$H_y = 0 \quad E_x = 0$$

$$H_z = 0$$

$$E_y = -\frac{\sqrt{g}}{h} A e^{-j\beta z}$$

$$E_z(x, y, z) = 0$$

Cut off frequency:-

$$k_c^2 = k^2 - \beta^2 = \omega_c^2 \mu\epsilon$$

$$k_c = \frac{m\pi}{d}$$

$$\omega_c = \frac{1}{\sqrt{\mu\epsilon}} \cdot \frac{m\pi}{d}$$

$$\boxed{f_c = \frac{m}{2d\sqrt{\mu\epsilon}}}$$

Characteristic Impedance Z_{TM} :-

$$Z_{TM} = \frac{E_x}{H_y} = -\frac{E_y}{H_x}$$

$$= \frac{-\frac{\sqrt{g}}{h} A \cos\left(\frac{m\pi y}{d}\right)}{\frac{j\omega\epsilon}{h} A \cos\left(\frac{m\pi y}{d}\right)} = \frac{\beta}{\omega\epsilon}$$