

① Line Integrals :- $\int \vec{F} \cdot d\vec{r}$

Cartesian $(x, y, z) \rightarrow dx \hat{x} + dy \hat{y} + dz \hat{z}$

Cylindrical $(\rho, \phi, z) \rightarrow d\rho \hat{\rho} + d\phi \hat{\phi} + dz \hat{z}$

Spherical $(r, \theta, \phi) \rightarrow dr \hat{r} + r d\theta \hat{\theta} + r \sin\theta d\phi \hat{\phi}$

② Surface Integrals :- $\iint \vec{F} \cdot d\vec{S}$

Cartesian :- $xy - dx dy dz$

$yz - dy dz dx$

$zx - dz dx dy$

Cylindrical :- $\rho \phi - d\rho \cdot \rho d\phi \cdot dz$

$\phi z - \rho d\phi \cdot dz \cdot \rho$

$\rho z - d\rho \cdot dz \cdot \rho$

Spherical :- $\theta \phi - r d\theta r \sin\theta d\phi \cdot r$

$\phi r - r \sin\theta d\phi \cdot dr \cdot r \sin\theta$

$r \theta - dr r d\theta \cdot r \sin\theta$

③ Volume Integral :- $\iiint dv$

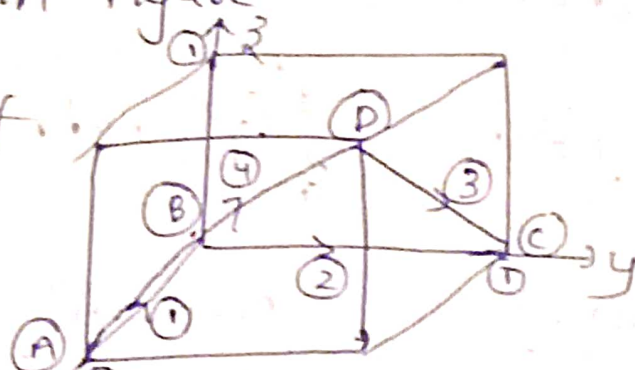
Cartesian $dx dy dz$

Cylindrical $d\rho \rho d\phi dz$

Spherical $dr r d\theta r \sin\theta d\phi$

① Given that $\vec{F} = x^2 \vec{a}_x - xz \vec{a}_y - y^2 \vec{a}_z$
calculate the circulation of $\vec{F}$ around the
(closed) path shown in figure.

sol:- The circulation of
 $\vec{F}$ around path L is
given by



$$\oint_L \vec{F} \cdot d\vec{l} = \left(\int_1 + \int_2 + \int_3 + \int_4 \right) \vec{F} \cdot d\vec{l}$$

line-①:- $A \rightarrow B$
 $(x, y, z) \rightarrow (x, y, 0)$
 $(1, 0, 0) \rightarrow (0, 0, 0)$

$$d\vec{l} = dx \vec{a}_x \quad \vec{F} = x^2 \vec{a}_x$$

$$\int_1^0 x^2 \vec{a}_x \cdot dx \vec{a}_x \rightarrow \int_1^0 x^2 dx = \left[\frac{x^3}{3} \right]_1^0 = -\frac{1}{3}$$

line-②:- $B \rightarrow C$
 $(0, 0, 0) \rightarrow (0, 1, 0)$

$$d\vec{l} = dy \vec{a}_y \quad \vec{F} = -xz \vec{a}_y$$

$$\int_0^1 (-xz) dy = 0$$

line-③:- $C \rightarrow D$
 $(0, 1, 0) \rightarrow (1, 1, 1)$
 $y=1$

$$d\vec{l} = dx \vec{a}_x + dz \vec{a}_z$$

$$\vec{F} = x^2 \vec{a}_x - xz \vec{a}_y - y^2 \vec{a}_z$$

$$\vec{F} = x^2 \vec{a}_x - xz \vec{a}_y - \vec{a}_z$$

$$\int \vec{F} \cdot d\vec{r} = \int (x^2 dx - dz)$$

$$dx = dz$$

$$\int \vec{F} \cdot d\vec{r} = \int_0^1 (x^2 - 1) dx = \left[\frac{x^3}{3} - x \right]_0^1 = -\frac{2}{3}$$

line - ④ :- $D \rightarrow A$
 $(1, 1, 1) \rightarrow (1, 0, 0)$
 $\searrow x=1$

$$d\vec{r} = dy \vec{a}_y + dz \vec{a}_z$$

$$\vec{F} = x^2 \vec{a}_x - xz \vec{a}_y - y^2 \vec{a}_z$$

$$\vec{F} = x^2 \vec{a}_x - z \vec{a}_y - y^2 \vec{a}_z$$

$$\int \vec{F} \cdot d\vec{r} = \int (-z dy - y^2 dz)$$

$$dz = dy \rightarrow z = y$$

$$\int \vec{F} \cdot d\vec{r} = \int_1^0 (-y - y^2) dy = \left[-\frac{y^2}{2} - \frac{y^3}{3} \right]_1^0 = \frac{5}{6}$$

By putting all these together, we obtain

$$\oint_L \vec{F} \cdot d\vec{r} = -\frac{1}{3} + 0 - \frac{2}{3} + \frac{5}{6} = -\frac{1}{6}$$

4. Gradient of a Scalar

The gradient of a scalar field V is a vector that represents both the magnitude and the direction of the maximum space rate of increase of V .

$$dV = \frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} dy + \frac{\partial V}{\partial z} dz$$

$$= \left(\frac{\partial V}{\partial x} a_x + \frac{\partial V}{\partial y} a_y + \frac{\partial V}{\partial z} a_z \right) (dx a_x + dy a_y + dz a_z)$$

$$\boxed{\vec{G} = \frac{\partial V}{\partial x} a_x + \frac{\partial V}{\partial y} a_y + \frac{\partial V}{\partial z} a_z}$$

Then

$$dV = \vec{G} \cdot d\vec{l} = G \cos \theta dl$$

$$(\cos) \frac{dV}{dl} = G \cos \theta$$

cartesian :-

$$\nabla V = \frac{\partial V}{\partial x} a_x + \frac{\partial V}{\partial y} a_y + \frac{\partial V}{\partial z} a_z$$

cylindrical :-

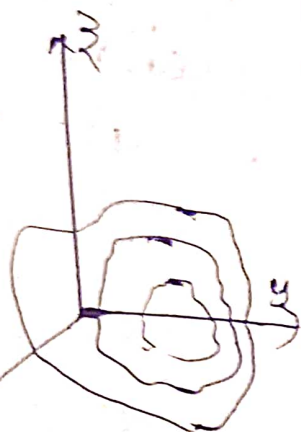
$$\nabla V = \frac{\partial V}{\partial \rho} a_\rho + \frac{1}{\rho} \frac{\partial V}{\partial \phi} a_\phi + \frac{\partial V}{\partial z} a_z$$

spherical :-

$$\nabla V = \frac{\partial V}{\partial r} a_r + \frac{1}{r} \frac{\partial V}{\partial \theta} a_\theta + \frac{1}{r \sin \theta} \frac{\partial V}{\partial \phi} a_\phi$$

properties :-

- 1) The magnitude of ∇V equals the maximum rate of change in V per unit distance.
- 2) ∇V points in the direction of the maximum rate of change in V .



3) ∇V at any point is perpendicular to the constant V surface that passes through that point.

4) The projection of ∇V in the direction of a unit vector $\bar{a}$ is $\nabla V \cdot \bar{a}$ and is called the directional derivative of V along $\bar{a}$.

5) If $\bar{A} = \nabla V$, V is said to be the scalar potential of $\bar{A}$.

problems:-

① find the gradient of the following scalar fields:-

a) $V = e^{-z} \sin 2x \cosh y$

b) $U = \rho^2 z \cos 2\phi$

c) $W = 10r \sin^2 \theta \cos \phi$

d) $V = x^2 y z$; e) $V = e^{-2x} \cos y \sin z$.

Ans:- a) $V = e^{-z} \sin 2x \cosh y$

$$\nabla V = \frac{\partial V}{\partial x} \bar{a}_x + \frac{\partial V}{\partial y} \bar{a}_y + \frac{\partial V}{\partial z} \bar{a}_z$$

$$= 2e^{-z} \cos 2x \cosh y \bar{a}_x + e^{-z} \sin 2x \sinh y \bar{a}_y$$

$$- e^{-z} \sin 2x \cosh y \bar{a}_z$$

b) $\nabla U = \frac{\partial U}{\partial \rho} \bar{a}_\rho + \frac{1}{\rho} \frac{\partial U}{\partial \phi} \bar{a}_\phi + \frac{\partial U}{\partial z} \bar{a}_z$

$$= 2\rho z \cos 2\phi \bar{a}_\rho - 2\rho z \sin 2\phi \bar{a}_\phi + \rho^2 \cos 2\phi \bar{a}_z$$

$$c) \nabla \omega = \frac{\partial \omega}{\partial r} a_r + \frac{1}{r} \frac{\partial \omega}{\partial \theta} a_\theta + \frac{1}{r \sin \theta} \frac{\partial \omega}{\partial \phi} a_\phi$$

$$= 10 \sin^2 \theta \cos \phi a_r + 10 \sin 2\theta \cos \phi a_\theta - 10 \sin \theta \sin \phi a_\phi$$

$$d) \nabla V = \frac{\partial V}{\partial x} a_x + \frac{\partial V}{\partial y} a_y + \frac{\partial V}{\partial z} a_z$$

$$= 2xy a_x + x^2 a_y + x^2 y a_z$$

$$e) \nabla V = \frac{\partial V}{\partial x} a_x + \frac{\partial V}{\partial y} a_y + \frac{\partial V}{\partial z} a_z$$

$$= e^{-2x} \cos y \sin z a_x + 2e^{-2x} \sin y \sin z a_y$$

$$e^{-2x} \cos y \cos z a_z$$

$$② a) U = x^2 y + xy z$$

$$b) V = \rho z \sin \phi + z^2 \cos^2 \phi + \rho^2 \quad c) f = \cos \theta \sin \phi \ln r + r^2$$

$$\text{Ans:- } a) \frac{\partial U}{\partial x} a_x + \frac{\partial U}{\partial y} a_y + \frac{\partial U}{\partial z} a_z$$

$$= y(2x + z) a_x + x(x + z) a_y + xy a_z$$

$$b) (z \sin \phi + \rho) a_\rho + (z \cos \phi - \frac{z^2}{\rho} \sin \phi) a_\phi +$$

$$(p \sin \phi + 2z \cos^2 \phi) a_z$$

$$c) \left(\frac{\cos \theta \sin \phi}{r} + 2r\phi \right) a_r - \frac{\sin \theta \sin \phi}{r} \ln r a_\theta +$$

$$\left(\frac{\cot \theta}{r} \cos \phi \ln r + r \csc \theta \right) a_\phi$$

③ Given $\omega = x^2 y^2 + xy z$, compute $\nabla \omega$ and the direction derivative $d\omega/dl$ in the direction $3a_x + 4a_y + 12a_z$ at $(2, 1, 0)$.

Sol:- $\Delta w = \frac{\partial w}{\partial x} a_1 + \frac{\partial w}{\partial y} a_2 + \frac{\partial w}{\partial z} a_3$

$= (2xy + yz)a_1 + (2x^2y + xz)a_2 + 12y$

at $(2, -1, 0)$:- $\nabla w = 4a_1 - 8a_2 - 2a_3$

$\frac{dw}{dt} = \nabla w \cdot a_1 = (4, -8, -2) \cdot \frac{(3, 4, 12)}{13} = \frac{4}{13}$

* Divergence theorem :-

The divergence of $\vec{A}$ at a given point P is the outward flux per unit volume as the volume shrinks about P .

$\text{div } \vec{A} = \nabla \cdot \vec{A} = \lim_{\Delta V \rightarrow 0} \frac{\oint_S \vec{A} \cdot d\vec{S}}{\Delta V}$

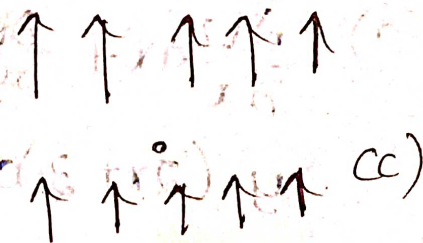
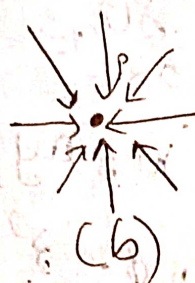
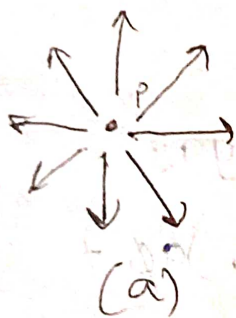


Illustration of the divergence of a vector field at P a) positive b) negative c) zero

$\oint_S \vec{A} \cdot d\vec{S} = \int_V \nabla \cdot \vec{A} dV$ This is called the divergence theorem.

Cartesian :- $\nabla \cdot \vec{A} = \frac{\partial A_1}{\partial x} + \frac{\partial A_2}{\partial y} + \frac{\partial A_3}{\partial z}$

cylindrical :- $\nabla \cdot \vec{A} = \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho A_\rho) + \frac{1}{\rho} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$

spherical: $\nabla \bar{A} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 A_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (A_\theta \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial A_\phi}{\partial \phi}$;

properties:- 1) It produces a scalar field
2) The divergence of a scalar V , $\text{div } V$, makes no sense.

3) $\nabla \cdot (\bar{A} + \bar{B}) = \nabla \bar{A} + \nabla \cdot \bar{B}$

4) $\nabla \cdot (\bar{V} \bar{A}) = \bar{V} \Delta \cdot \bar{A} + \bar{A} \cdot \Delta \bar{V}$

Divergence theorem states that the total outward flux of a vector field $\bar{A}$ through the closed surface S is the same as the volume integral of the divergence of $\bar{A}$.

$$\oint_S \bar{A} \cdot d\bar{S} = \int_V \nabla \cdot \bar{A} dV$$

* Curl of vector:- The curl of $\bar{A}$ is an axial vector whose magnitude is the maximum circulation of $\bar{A}$ per unit area as the area tends to zero and whose direction is the normal direction of the area when the area is oriented so as to make the circulation maximum.

$$\text{Curl}(\bar{A}) = \nabla \times \bar{A} = \left(\lim_{\Delta S \rightarrow 0} \frac{\oint_L \bar{A} \cdot d\bar{l}}{\Delta S} \right) \text{an max}$$

Cartesian:-

$$\nabla \times \vec{A} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$\nabla \times \vec{A} = \left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right] \hat{a}_x + \left[\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right] \hat{a}_y + \left[\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right] \hat{a}_z$$

cylindrical:-

$$\nabla \times \vec{A} = \frac{1}{\rho} \begin{vmatrix} \hat{a}_\rho & \rho \hat{a}_\phi & \hat{a}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$$

$$\nabla \times \vec{A} = \left[\frac{1}{\rho} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right] \hat{a}_\rho + \left[\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho} \right] \hat{a}_\phi + \frac{1}{\rho} \left[\frac{\partial (\rho A_\phi)}{\partial \rho} - \frac{\partial A_\rho}{\partial \phi} \right] \hat{a}_z$$

spherical:-

$$\nabla \times \vec{A} = \frac{1}{r^2 \sin \theta} \begin{vmatrix} \hat{a}_r & r \hat{a}_\theta & r \sin \theta \hat{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & r A_\theta & r \sin \theta A_\phi \end{vmatrix}$$

$$\nabla \times \vec{A} = \frac{1}{r \sin \theta} \left[\frac{\partial (A_\phi \sin \theta)}{\partial \theta} - \frac{\partial A_\theta}{\partial \phi} \right] \hat{a}_r + \frac{1}{r} \left[\frac{1}{\sin \theta} \frac{\partial A_r}{\partial \phi} - \frac{\partial (r A_\phi)}{\partial \theta} \right] \hat{a}_\theta + \frac{1}{r} \left[\frac{\partial (r A_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta} \right] \hat{a}_\phi$$

properties:-

- 1) The curl of a vector field is another vector field.
- 2) The curl of a scalar field V , $\nabla \times V$, makes no sense.

$$3) \nabla \times (\vec{A} + \vec{B}) = \nabla \times \vec{A} + \nabla \times \vec{B}$$

$$4) \nabla \times (\vec{A} \times \vec{B}) = \vec{A} (\nabla \cdot \vec{B}) - \vec{B} (\nabla \cdot \vec{A}) + (\vec{B} \cdot \nabla) \vec{A} - (\vec{A} \cdot \nabla) \vec{B}$$

5) The divergence of the curl of a vector field vanishes, i.e., $\nabla \cdot (\nabla \times \vec{A}) = 0$.

6) The curl of the gradient of a scalar field vanishes, i.e., $\nabla \times \nabla V = 0$.

$$\oint_L \vec{A} \cdot d\vec{l} = \int_S (\nabla \times \vec{A}) \cdot d\vec{s}$$

* Stokes' theorem:-

Stokes' theorem states that the circulation of a vector field $\vec{A}$ around a path L is equal to the surface integral of the curl of $\vec{A}$ over the open surface S bounded by L . provided that $\vec{A}$ and $\nabla \times \vec{A}$ are continuous on S .

$$\oint_L \vec{A} \cdot d\vec{l} = \int_S (\nabla \times \vec{A}) \cdot d\vec{s}$$

$$\oint_L \vec{E} \cdot d\vec{l} \quad (\text{Cor}) = \iint_S (\nabla \times \vec{E}) \cdot d\vec{s}$$

* Laplacian of scalar :-

The Laplacian of a scalar field V , written as $\nabla^2 V$, is the divergence of the gradient of V .

$$\text{Laplacian } V = \nabla \cdot \nabla V = \nabla^2 V$$

Cartesian :-

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

Cylindrical :-

$$\nabla^2 V = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial V}{\partial \rho} \right) + \frac{1}{\rho^2} \left(\frac{\partial^2 V}{\partial \phi^2} \right) +$$

Spherical :-

$$\nabla^2 V = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial V}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial V}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \left(\frac{\partial^2 V}{\partial \phi^2} \right)$$

A scalar field V is said to be harmonic in a given region if its Laplacian vanishes in that region. In other words, if $\nabla^2 V = 0$ is satisfied in the region.

Maxwell's Equations:-

⇒ (1) Gauss's law:-

statement:- states that the area integral of the electric field over any closed surface is equal to the net charge enclosed in the surface.

$$\oint \vec{E} \cdot d\vec{s} = \iiint_V \rho_v dv.$$

⇒ (2) Non-existence of magnetic monopoles

The divergence of magnetic field density is zero, i.e., magnetic field density is divergence-less.

$$\oint_S \vec{B} \cdot d\vec{s} = 0$$

⇒ (3) Ampere's Law:-

statement:- It states that the line integral of magnetic field intensity around any closed path is equal to total current enclosed by the closed path.

$$\nabla \times \vec{H} = \vec{J}$$

$$\oint \vec{H} \cdot d\vec{l} = \iint_S \vec{J} \cdot d\vec{s}$$

④ Faraday's law:-

Statement:- It states that induced emf is equal to time rate of change of magnetic flux linkage.

$$\nabla \times \vec{E} = - \mu \frac{\partial \vec{B}}{\partial t}$$

Lenz law; $\nabla \times \vec{E} = - \mu \frac{\partial \vec{H}}{\partial t}$

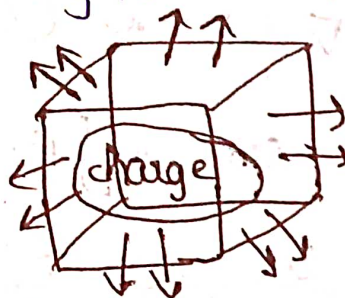
① Divergence — $\left[\begin{array}{l} \text{electric field} \rightarrow \nabla \cdot \vec{D} = \rho_v \\ \text{magnetic field} \rightarrow \nabla \cdot \vec{B} = 0 \end{array} \right.$

electric field $\rightarrow \oint \vec{D} \cdot d\vec{s} = \iiint_V \rho_v dV \rightarrow$ Gauss law

magnetic field $\rightarrow \oint \vec{B} \cdot d\vec{s} = 0 \rightarrow$ non-existence of magnetic monopole

① $\nabla \cdot \vec{D} = \rho_v$

The divergence of flux density is equal to charge enclosed by the closed surface.



$$\oint_{1-6} \vec{D} \cdot d\vec{s} = \iiint_V \rho_v dV$$

surface
integral

volume integral

$$\Rightarrow \iiint_V (\nabla \cdot \vec{D}) dV = \iiint_V \rho_v dV$$

(i) $\nabla \cdot \vec{B} = 0$

Divergence of magnetic field is equal to constant field closed surface.



$\Rightarrow \oint \vec{B} \cdot d\vec{l} = 0$

$\iiint_V (\nabla \cdot \vec{B}) dV = 0$

$\Rightarrow$ (2) curl $\left\{ \begin{array}{l} \text{Electric field} \rightarrow \nabla \times \vec{E} = 0 \\ \text{magnetic field} \rightarrow \nabla \times \vec{H} = \vec{I} \end{array} \right.$

Electric field $\rightarrow \oint \vec{E} \cdot d\vec{l} = 0 \rightarrow$ conservative nature
magnetic field $\rightarrow \oint \vec{H} \cdot d\vec{l} = \iint_S \vec{I} \cdot d\vec{s} \rightarrow$ Ampere's law

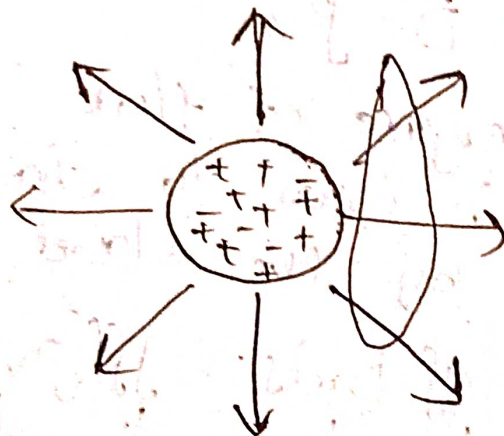
(i) $\nabla \times \vec{E} = 0$ (conservative nature)

$w = - \int \vec{F} \cdot d\vec{l}$

$\frac{w}{q} = - \int \vec{E} \cdot d\vec{l}$

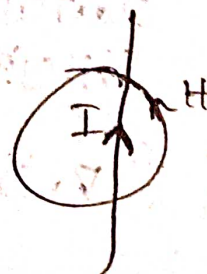
$V_{AB} = - \int \vec{E} \cdot d\vec{l}$

$V_{BA} = \int \vec{E} \cdot d\vec{l}$



(ii) $\nabla \times \vec{H} = \vec{I}$ (Ampere's law)

$\oint \vec{H} \cdot d\vec{l} = I_{enc}$
 $= \int A$
 $= \iint_S d\vec{s}$



$$\iint (\nabla \times \vec{H}) d\vec{S} = \iint \vec{I} d\vec{S}$$

Time-varying field:-

$$(1) \quad \nabla \cdot \vec{D} = \rho_v$$

$$(2) \quad \nabla \cdot \vec{B} = 0$$

$$(3) \quad \nabla \times \vec{E} = - \frac{\partial \vec{B}}{\partial t} \rightarrow \text{Faraday's law}$$

$$(4) \quad \nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \Rightarrow \text{modified Ampere's law.}$$

wave propagation :-

Electromagnetic wave propagation :-

In general, waves are means of transporting energy or information.

*. Electric field :- wave equation :-

$$\nabla \times \bar{E} = -\mu(j\omega)H$$

$$D = \epsilon E$$

$$\nabla \times \bar{E} = -\mu \frac{\partial H}{\partial t} \quad (\because H = H_0 e^{j\omega t})$$

$$\nabla \times \bar{E} = -\mu(j\omega)H$$

$$\nabla \times (\nabla \times \bar{E}) = -j\omega\mu(\nabla \times \bar{H})$$

$$(\because \nabla \times \bar{H} = j + \frac{\partial D}{\partial t})$$

$$\Rightarrow D = \epsilon E$$

$$\epsilon = \epsilon_0 j\omega t$$

$$\nabla \times \bar{H} = J + j\omega\epsilon E \quad (\because J = \sigma E)$$

$$\nabla \times \bar{H} = \sigma E + j\omega\epsilon E$$

$$\nabla \times \bar{H} = (\sigma + j\omega\epsilon)E$$

$$\nabla \times \nabla \times \bar{E} = -j\omega\mu(\sigma + j\omega\epsilon)E$$

$$(\because \nabla \times \nabla \times \bar{A} = \nabla(\nabla \cdot \bar{A}) - \nabla^2 \bar{A})$$

$$\nabla(\nabla \cdot \bar{E}) - \nabla^2 \bar{A} = -j\omega\mu(\sigma + j\omega\epsilon)E$$

$$(\because \nabla \cdot \mathbf{E} = 0 \Rightarrow \nabla \cdot \mathbf{E} = \rho / \epsilon \rightarrow \text{source free})$$

$$\nabla^2 \mathbf{A} = \mu (\sigma + j\omega\epsilon) \mathbf{E}$$

$$\nabla^2 \mathbf{A} = \underbrace{j\omega\mu (\sigma + j\omega\epsilon)}_{\gamma^2} \mathbf{E}$$

$$\nabla^2 \mathbf{A} = \gamma^2 \mathbf{E} = 0$$

$$\nabla^2 \mathbf{A} - \gamma^2 \mathbf{E} = 0$$

$$\gamma = \sqrt{j\omega\mu (\sigma + j\omega\epsilon)} = \alpha + j\beta$$

where α is attenuation constant

β is phase constant

$$(\nabla^2 - \gamma^2) \mathbf{E} = 0$$

$$\nabla^2 - \gamma^2 = 0$$

$$\nabla^2 = \gamma^2$$

$$\nabla = \pm \gamma$$

$$E_0 e^{\gamma z} + E_0 e^{-\gamma z}$$

$+z$ direction $z \rightarrow 0$ to ∞

$-z$ direction $z \rightarrow -\infty$ to 0

$$\mathbf{E} = E_0 e^{-\gamma z} e^{j\omega t}$$

$$\mathbf{E} = E_0 e^{-(\alpha + j\beta)z} e^{j\omega t}$$

$$E = E_0 e^{-\alpha z} e^{j(\omega t - \beta z)}$$

$$E(\omega, t) = \text{Re}(E)$$

$$\boxed{\bar{E} = E_0 e^{-\alpha z} \cos(\omega t - \beta z) \bar{a}_E}$$

where E_0 is magnitude

α is a attenuation constant

z is a direction of propagation

ω is angular frequency

β is phase constant

E is a direction of electric field

lly magnetic field wave equation is

$$\boxed{\bar{H} = H_0 e^{-\alpha z} \cos(\omega t - \beta z) \bar{a}_H}$$

at $t=0$

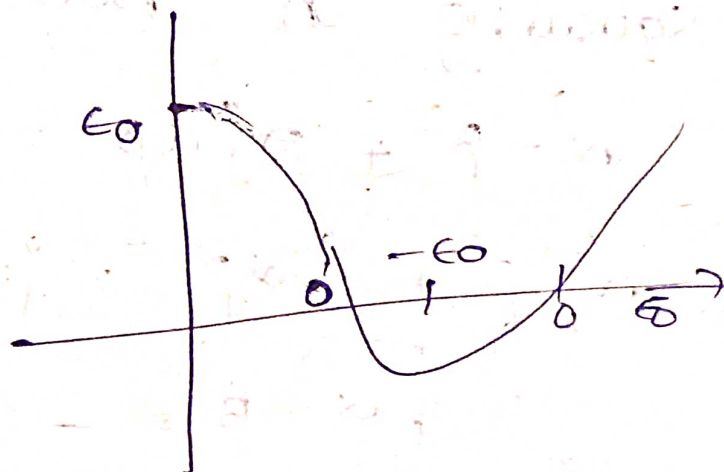
$$\bar{E} = E_0 \cos(\omega t - \beta z) \bar{a}_E$$

$$t=0, T/4, T/2, 3T/4, T$$

at $t=0$

$$\bar{E} = E_0 \cos(-\beta z) \bar{a}_E$$

z	βz	$\bar{E}$
0	0	E_0
$1/4$	$\frac{2\pi \cdot 1}{4} = \frac{\pi}{2}$	0
$1/2$	$\frac{2\pi \cdot 1}{2} = \pi$	$-E_0$
$3/4$	$\frac{2\pi \cdot 1}{4} = \frac{3\pi}{2}$	0
1	2π	E_0



Medium

where; σ is conductivity
 μ is permeability ($\mu_0 \mu_r$)
 ϵ is permittivity ($\epsilon_0 \epsilon_r$)

- ① Free Space ($\sigma=0, \mu=\mu_0, \epsilon=\epsilon_0$)
- ② Lossless dielectric ($\sigma=0, \mu=\mu_0 \mu_r, \epsilon=\epsilon_0 \epsilon_r$)
- ③ Lossy dielectric ($\sigma \neq 0, \mu=\mu_0 \mu_r, \epsilon=\epsilon_0 \epsilon_r$)
- ④ Good conductor ($\sigma \approx \infty, \epsilon=\epsilon_0, \mu=\mu_0 \mu_r$)

parameters: $\gamma = \alpha + j\beta, \eta, \theta_\eta, v_p$

Lossy dielectric:
 $\Rightarrow$ ① $\gamma = \alpha + j\beta$:

A Lossy dielectric is a medium in which an EM wave loses power as it propagates due to poor conduction.

$$\gamma = \alpha + j\beta$$

$$\sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = \alpha + j\beta$$

Squaring on both sides

$$\alpha^2 - \beta^2 + 2j\alpha\beta = j\omega\mu(\sigma + j\omega\epsilon)$$

$$\alpha^2 - \beta^2 + 2j\alpha\beta = j\omega\mu\sigma - \omega^2\mu\epsilon$$

$$\boxed{\alpha^2 - \beta^2 = -\omega^2\mu\epsilon} \rightarrow \text{①}$$

$$\sqrt{\alpha^2 + \beta^2} = \sqrt{(\mu\omega\sigma)^2 + (\omega^2\mu\epsilon)^2}$$

squaring on Both sides

$$\alpha^2 + \beta^2 = \sigma^2 \omega^2 \mu^2 + \omega^4 \mu^2 \epsilon^2$$

$$\alpha^2 + \beta^2 = \sqrt{\omega^4 \mu^2 \epsilon^2 \left(\frac{\sigma^2}{\omega^2 \epsilon^2} + 1 \right)}$$

$$\boxed{\alpha^2 + \beta^2 = \omega^2 \mu \epsilon \left(\sqrt{1 + \left(\frac{\sigma}{\omega \epsilon} \right)^2} \right)} \rightarrow (2)$$

adding (1) and (2) we get

$$\alpha^2 = \omega^2 \mu \epsilon \left(\sqrt{1 + \left(\frac{\sigma}{\omega \epsilon} \right)^2} - 1 \right)$$

$$\alpha = \frac{\omega^2 \mu \epsilon}{2} \left(\sqrt{1 + \left(\frac{\sigma}{\omega \epsilon} \right)^2} - 1 \right)$$

$$\therefore \boxed{\alpha = \omega \sqrt{\frac{\mu \epsilon}{2}} \sqrt{1 + \left(\frac{\sigma}{\omega \epsilon} \right)^2} - 1}$$

$$\text{||ly} \boxed{\beta = \omega \sqrt{\frac{\mu \epsilon}{2}} \sqrt{1 + \left(\frac{\sigma}{\omega \epsilon} \right)^2} + 1}$$

$$\Rightarrow (2) \quad \eta = \frac{E}{H} = \frac{\sqrt{j\omega\mu}}{\sigma + j\omega\epsilon}$$

$$\nabla \times \bar{E} = - \frac{\partial \bar{B}}{\partial t} = -\mu \frac{\partial \bar{H}}{\partial t}$$

$$\bar{H} = H_0 e^{-\gamma z} e^{j\omega t}$$

$$\bar{H} = H_0 e^{j\omega t}$$

$$\frac{\partial H}{\partial t} = j\omega \bar{H}$$

$$\nabla \times \bar{E} = -j\omega \mu \bar{H}$$

$$\bar{H} = \frac{-1}{j\omega \mu} (\nabla \times \bar{E})$$

$\swarrow$ y-axis $\searrow$ x-axis

$$\nabla \times \bar{E} = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix}$$

$\underbrace{\quad \quad \quad}_{\text{non-zeroes}}$

$$\nabla \times \bar{E} = a_y \frac{\partial}{\partial z} (E_x)$$

$$E = E_0 e^{-\gamma z} \cdot e^{j\omega t} \quad \rightarrow t=0$$

$$\boxed{\frac{\partial E_x}{\partial z} = -\gamma E}$$

$$\bar{H} = \frac{-1}{j\omega \mu} (-\gamma E)$$

$$\bar{H} = \frac{\gamma}{j\omega \mu} E$$

$$\frac{\bar{H}}{E} = \frac{\gamma}{j\omega \mu}$$

$$\frac{\bar{H}}{E} = \frac{j\omega \mu}{\gamma} \Rightarrow \frac{j\omega \mu}{\sqrt{j\omega \mu (\sigma + j\omega \epsilon)}}$$

$$\frac{\bar{E}}{H} = \frac{\sqrt{j\omega\mu} \cdot \sqrt{j\omega\mu}}{\sqrt{j\omega\mu} \cdot \sqrt{\sigma + j\omega\epsilon}}$$

$$\boxed{\frac{\bar{E}}{H} = \frac{\sqrt{j\omega\mu}}{\sqrt{\sigma + j\omega\epsilon}}}$$

$$\boxed{\eta = \frac{\bar{E}}{H} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}}$$

③ θ_η

$$\eta = \sqrt{\frac{j\omega\mu}{j\omega\epsilon(1 + \frac{\sigma}{j\omega\epsilon})}} \Rightarrow \sqrt{\frac{\mu}{\epsilon(1 + \frac{\sigma}{j\omega\epsilon})}}$$

$$\eta = \frac{\sqrt{\mu/\epsilon}}{\sqrt{1 - j\frac{\sigma}{\omega\epsilon}}}$$

$$\eta = |\eta| \theta_\eta$$

$$\theta_\eta = \frac{\eta}{|\eta|}$$

$$|\eta| = \frac{\sqrt{\mu/\epsilon}}{\sqrt{\sqrt{1 + (\frac{\sigma}{\omega\epsilon})^2}}} \quad (\text{or})$$

$$|\eta| = \frac{\sqrt{\mu/\epsilon}}{\left(\sqrt{1 + (\frac{\sigma}{\omega\epsilon})^2}\right)^{1/4}}$$

$$\theta_l = \frac{\tan^{-1}\left(\frac{0}{\sqrt{\mu/\epsilon}}\right)}{\frac{1}{2}\tan^{-1}\left(\frac{\sigma/\omega\epsilon}{1}\right)}$$

$$\theta_l = 0 - \frac{1}{2}\tan^{-1}\left(-\frac{\sigma}{\omega\epsilon}\right)$$

$$\theta_l = -\frac{1}{2}\tan^{-1}\left(-\frac{\sigma}{\omega\epsilon}\right)$$

$$\boxed{\theta_l = \frac{1}{2}\tan^{-1}\left(\frac{\sigma}{\omega\epsilon}\right)}$$

$$\tan(2\theta_l) = \sigma/\omega\epsilon \Rightarrow \tan\theta_l = \sigma/\omega\epsilon$$

$$\therefore \boxed{\theta_l = 2\theta_l}$$

$$\Rightarrow \textcircled{4} \quad v_p = \text{Phase velocity}$$

$$v_p = \frac{d\theta}{dt}$$

$$E = E_0 e^{-\alpha z} \underbrace{\cos(\omega t - \beta z)}_{\cos(\theta)} \vec{a}_E$$

$$v_p = \frac{d}{dz}(\theta)$$

$$\omega - \beta \frac{dz}{dt} = 0$$

$$\omega = \beta \frac{dz}{dt}$$

$$\therefore \boxed{\frac{dz}{dt} = \frac{\omega}{\beta} = v_p}$$

Free Space :-

$\sigma = 0$, $\epsilon = \epsilon_0$, $\mu = \mu_0$

lossy dielectric

$$1) \alpha = \omega \sqrt{\frac{\mu \epsilon}{2}} \sqrt{\left(1 + \left(\frac{\sigma}{\omega \epsilon}\right)^2\right) - 1} \rightarrow (1)$$

$$2) \beta = \omega \sqrt{\frac{\mu \epsilon}{2}} \sqrt{\left(1 + \left(\frac{\sigma}{\omega \epsilon}\right)^2\right) + 1} \rightarrow (2)$$

$$3) \eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} \rightarrow (3)$$

$$4) \theta \eta = \frac{1}{2} \tan^{-1} \left(\frac{\sigma}{\omega \epsilon} \right) \rightarrow (4)$$

$$5) v_p = \frac{\omega}{\beta} \rightarrow (5)$$

$\sigma = 0$ in eq - (1)
 $\epsilon = \epsilon_0$
 $\mu = \mu_0$

$$(1) \alpha = \omega \sqrt{\frac{\mu \epsilon}{2}} \sqrt{\left(1 + \frac{0}{\omega \epsilon}\right)^2 - 1}$$

$$\boxed{\alpha = 0}$$

$$(2) \beta = \omega \sqrt{\frac{\mu \epsilon}{2}} \sqrt{(1+0) - 1}$$

$$\beta = \omega \sqrt{\frac{\mu \epsilon}{2}}$$

$$\beta = \omega \sqrt{\frac{\mu_0 \epsilon_0}{2}}$$

$$\boxed{\beta = \omega \sqrt{\mu_0 \epsilon_0}}$$

$$\left(\because \mu_0 = 4\pi \times 10^{-7} \right. \\ \left. \epsilon_0 = 10^{-9} / 36\pi \right)$$

$$(3) \eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} \Rightarrow \sqrt{\frac{\mu}{-\epsilon}} = \sqrt{\frac{\mu_0}{-\epsilon_0}}$$

$$\eta = \sqrt{\frac{4\pi \times 10^{-7}}{10^{-9} / 36\pi}}$$

$$\eta = 2\pi (10)(6) = 120\pi$$

$$\boxed{\eta = 377 \Omega}$$

$$(4) \theta\eta = \frac{1}{2} \tan^{-1} \left(\frac{\sigma}{\omega\epsilon} \right)$$

$$\boxed{\theta\eta = 0}$$

$$(5) v_p = \frac{\omega}{\beta} \Rightarrow \frac{\cancel{\omega}}{\cancel{\omega} \sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu\epsilon}}$$

$$\boxed{v_p = \frac{1}{\sqrt{\mu_0 \epsilon_0}}}$$

Lossless Dielectric

$$\sigma = 0, \mu = \mu_0 \mu_r, \epsilon = \epsilon_0 \epsilon_r$$

$$1) \boxed{\alpha = 0}$$

$$2) \beta = \omega \sqrt{\mu\epsilon}$$

$$\boxed{\beta = \omega \sqrt{\mu_0 \mu_r \epsilon_0 \epsilon_r}}$$

$$3) \eta = \sqrt{\frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r}} = \sqrt{\frac{\mu_0}{\epsilon_0}} \cdot \sqrt{\frac{\mu_r}{\epsilon_r}}$$

$$\boxed{\eta = 120\pi \cdot \sqrt{\frac{\mu_r}{\epsilon_r}}}$$

$$4) \theta\eta = \frac{1}{2} \tan^{-1} \left(\frac{0}{\omega\epsilon} \right)$$

$$= \frac{1}{2} \tan^{-1}(0)$$

$$\boxed{\theta\eta = 0}$$

$$5) v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\epsilon}}$$

$$\boxed{v_p = \frac{1}{\sqrt{\mu_0 \mu_r \epsilon_0 \epsilon_r}}}$$

Good conductor:-

$$\sigma \rightarrow \infty, \mu = \mu_0, \epsilon = \epsilon_0$$

$$1) \alpha = \omega \sqrt{\frac{\mu \epsilon}{2}} \cdot \frac{\sigma}{\omega \epsilon}$$

$$\alpha = \omega \sqrt{\frac{\mu}{2}} \cdot \frac{\sigma}{\omega}$$

$$\boxed{\alpha = \sqrt{\frac{\omega \mu \sigma}{2}}}$$

$$2) \beta = \sqrt{\frac{\omega \mu \sigma}{2}}$$

$$3) \eta' = \sqrt{\frac{\omega \mu}{\sigma}} \angle 45^\circ$$

$$4) \theta \eta = \frac{1}{2} \tan^{-1} \left(\frac{\infty}{\omega \epsilon} \right)$$

$$\boxed{\theta \eta = 0}$$

$$5) v_p = \frac{\omega}{\beta} = \sqrt{\frac{2\omega}{\mu \sigma}}$$

$$\boxed{v_p = \sqrt{\frac{2\omega}{\mu \sigma}}}$$

Relation between:- α and β

$$\Rightarrow \frac{\alpha}{\beta} = \frac{\omega \sqrt{\frac{\mu \epsilon}{2}} \sqrt{(1 + (\frac{\sigma}{\omega \epsilon})^2) - 1}}{\omega \sqrt{\frac{\mu \epsilon}{2}} \sqrt{(1 + (\frac{\sigma}{\omega \epsilon})^2) + 1}}$$

$$\frac{\alpha}{\beta} = \frac{\sqrt{(1 + (\frac{\sigma}{\omega \epsilon})^2) - 1}}{\sqrt{(1 + (\frac{\sigma}{\omega \epsilon})^2) + 1}}$$

$$\left(\because \tan(2\theta \eta) = \sigma / \omega \epsilon \right)$$
$$\tan \theta \eta = \sigma / \omega$$

$$\frac{\alpha}{\beta} = \frac{\sqrt{(1 + \tan^2 \theta)} - 1}{\sqrt{(1 + \tan^2 \theta) + 1}}$$

$$= \frac{\sec \theta - 1}{\sec \theta + 1}$$

$$\frac{\alpha}{\beta} = \frac{\sqrt{1 - \cos \theta}}{\sqrt{1 + \cos \theta}}$$

$$\cos 2\theta = 2\cos^2\theta - 1$$

$$\cos \theta = 2\cos^2\left(\frac{\theta}{2}\right) - 1$$

$$\frac{\alpha}{\beta} = \frac{\sqrt{1 - 2\cos^2\left(\frac{\theta}{2}\right) + 1}}{\sqrt{1 + 2\cos^2\left(\frac{\theta}{2}\right) - 1}}$$

$$\frac{\alpha}{\beta} = \sqrt{\frac{1 - \cos^2\left(\frac{\theta}{2}\right)}{\cos^2\left(\frac{\theta}{2}\right)}}$$

$$\frac{\alpha}{\beta} = \tan\left(\frac{\theta}{2}\right)$$

$$\theta \rightarrow \theta\eta$$

$$\theta = 2\theta\eta$$

$$\frac{\theta}{2} = \theta\eta$$

$$\boxed{\tan(\theta\eta) = \frac{\alpha}{\beta}}$$

Where, $\theta\eta$ = angle of intrinsic impedance

α = attenuation constant

β = phase constant

*** Poynting theorem :-

statement:- states that the net power flowing out of a given volume V is equal to the time rate of decrease in the energy stored within V minus the conduction losses.

Proof:-

$$\oint_S (\vec{E} \times \vec{H}) \cdot d\vec{s} = \iiint_V \nabla \cdot (\vec{E} \times \vec{H}) dv$$

$$\therefore \nabla \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\nabla \times \vec{A}) - \vec{A} \cdot (\nabla \times \vec{B})$$

$$\vec{A} = \vec{E} \quad , \quad \vec{B} = \vec{H}$$

Electric
field

magnetic
field

$$\nabla \cdot (\vec{E} \times \vec{H}) = \vec{H} \cdot (\nabla \times \vec{E}) - \vec{E} \cdot (\nabla \times \vec{H})$$

$$\nabla \times \vec{E} = - \frac{\partial \vec{B}}{\partial t} = - \mu \frac{\partial \vec{H}}{\partial t}$$

$$\vec{H} \cdot (\nabla \times \vec{E}) = - \mu \vec{H} \cdot \frac{\partial \vec{H}}{\partial t}$$

in terms of square

$$\frac{\mu}{2} \frac{\partial H^2}{\partial t} = - \mu \cdot 2 \cdot H \cdot \frac{\partial H}{\partial t}$$

$$\boxed{\vec{H} \cdot (\nabla \times \vec{E}) = - \frac{\mu}{2} \frac{\partial H^2}{\partial t}} \rightarrow (1)$$

$$\Rightarrow \nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} = \sigma \vec{E} + \epsilon \frac{\partial \vec{E}}{\partial t}$$

$$\boxed{\vec{E} \cdot (\nabla \times \vec{H}) = \sigma (\vec{E})^2 + \frac{\epsilon}{2} \frac{\partial E^2}{\partial t}} \rightarrow (2)$$

substitute eq (1) and eq (2) in $\nabla \cdot (\vec{E} \times \vec{H})$

$$\nabla \cdot (\vec{E} \times \vec{H}) = - \frac{\mu}{2} \frac{\partial H^2}{\partial t} - \sigma \vec{E}^2 - \frac{\epsilon}{2} \frac{\partial E^2}{\partial t}$$

$$P(\vec{E}, \vec{H}) = \underbrace{-\frac{1}{2} \frac{\partial}{\partial t} \left(\frac{1}{2} \mu H^2 \right)}_{\text{ohmic losses}} - \underbrace{\frac{\partial}{\partial t} \left(\frac{1}{2} \epsilon E^2 \right)}_{\text{Gen of elect field}}$$

$\downarrow$ Total power leaving the volume (or) divergence of the electromagnetic field from the surface.

problems :-

- ① A 50 MHz uniform plane wave is propagating in a material with relative permeability and permittivity of 2.25 and respectively. The material is assumed to be lossless find the phase constant of the wave propagation.

Ans:- Given $f = 50 \text{ MHz} \Rightarrow 50 \times 10^6$
 $\mu_r = 2.25, \epsilon_r = 1$

lossless dielectric

$$\sigma = 0, \mu = \mu_r \mu_0, \epsilon = \epsilon_r \epsilon_0$$

$$(\because \mu_0 = 4\pi \times 10^{-7}, \epsilon_0 = 10^{-9}/36\pi)$$

$$\text{phase constant } (\beta) = \omega \sqrt{\frac{\mu \epsilon}{2 \sqrt{1 + \left(\frac{\sigma}{\omega \epsilon}\right)^2} + 1}}$$

$$\beta = \omega \sqrt{\mu \epsilon}$$

$$\beta = \omega \sqrt{\mu_0 \mu_r \epsilon_0 \epsilon_r}$$

$$(\because \omega = 2\pi f)$$

$$\beta = 2\pi \times 10^6 \sqrt{4\pi \times 10^{-7} (2.25) (10^{-9}/36\pi)(1)}$$

$$\beta = 2\pi \times 50 \times 10^6 \sqrt{4\pi \times 10^{-7} (2.25) (10^{-9}/36\pi)}$$

$$\beta = 3141.4 \sqrt{2.5 \times 10^{-7}}$$

$$\beta = 3141.4 \cdot \frac{1}{2000} = 1.5707 \text{ (or)} \frac{\pi}{2} \text{ rad/sec}$$

$$\boxed{\beta = \pi/2 \text{ rad/sec}}$$

$$\Rightarrow v_p = \frac{\omega}{\beta} = \frac{2\pi \times 50}{\pi/2} = 100 \times 2 = 200$$

$$\alpha = 0$$

$$\eta = 120\pi \sqrt{\frac{\mu_r}{\epsilon_r}} \Rightarrow 120\pi \sqrt{\frac{2.25}{1}}$$

$$\eta = 120\pi \sqrt{2.25}$$

$$\eta = 120\pi (1.5) = 180\pi$$

$$\boxed{\eta = 180\pi}$$

2) A non-magnetic lossy dielectric material with relative permittivity $\epsilon_r = 2.25$ and conductivity $(\sigma) = 10^{-4} \text{ mho/m}$ is applied with EM wave of 2.5 MHz then what is the loss tangent.

Sol:-

$$\text{lossy dielectric} = \frac{\sigma}{\omega \epsilon}$$
$$= 10^{-4}$$

$$\frac{2\pi \times 2.5 \times 10^6 \times 2.25 \times 10^{-9}}{36\pi} = 0.32$$

$$\frac{\sigma}{\omega \epsilon} \ll 1 \rightarrow \text{lossy medium}$$

$$\alpha = \omega \sqrt{\frac{\mu \epsilon}{2} \sqrt{(1 - (\frac{\sigma}{\omega \epsilon})^2) - 1}}$$

$$\alpha = 2\pi (2.5 \times 10^6) \sqrt{\frac{\mu_r \mu_0 \cdot \epsilon_r \epsilon_0}{2} \sqrt{(1 + (0.32)^2) - 1}}$$

$$\alpha = 1.975 \times 10^{-3}$$

$$\beta = \omega \sqrt{\frac{\mu \epsilon}{2} \sqrt{(1 - (\frac{\sigma}{\omega \epsilon})^2) + 1}}$$

$$\beta = 2\pi (2.5 \times 10^6) \sqrt{\frac{\mu_r \mu_0 \cdot \epsilon_r \epsilon_0}{2} \sqrt{(1 + (0.32)^2) + 1}}$$

$$\eta = \frac{\sqrt{\mu/\epsilon}}{[(1 + (\frac{\sigma}{\omega \epsilon})^2)]^{1/4}}$$

$$\theta \eta = \frac{1}{2} \tan^{-1} \left(\frac{\sigma}{\omega \epsilon} \right)$$

③ An EM wave propagation in a certain medium described by $\vec{E} = 25 \sin(2\pi \times 10^6 t - 6x) \hat{a}_z$ V/m. then find a) direction of wave propagation and b) period T c) wave length d) phase velocity V_p .

Sol:- Given $\vec{E} = 25 \sin(2\pi \times 10^6 t - 6x) \hat{a}_z$

$$E_0 = 25, \quad \omega = 2\pi \times 10^6 \quad \beta = 6 \text{ rad/m}$$

$$E = E_0 \sin(\omega t - \beta x) \hat{a}_z$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{2\pi \times 10^6} = 1 \text{ msec}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{6} = \frac{\pi}{3} \text{ m}$$

Direction of wave propagation = x

" " E-wave " = z

$$V_p = \frac{\omega}{\beta} = \frac{2\pi \times 10^6}{6} = 1.047 \times 10^6 \text{ m/sec}$$

④ At 50 MHz a lossy dielectric material is characterized by $\Sigma = 3.6 \Sigma_0$ and $\mu = 2.1 \mu_0$ and $\sigma = 0.08$ (or) 0.1 mho/m if $\vec{E}_x = 6e^{-\gamma x} \hat{a}_z$ V/m then find a) α b) V_p c) η and $\vec{H}$

Sol:-

$$\sigma \neq 0, \quad \Sigma = \Sigma_r \Sigma_0, \quad \mu = \mu_0 \mu_r$$

$$f = 50 \text{ MHz}$$

$$\Sigma_r = 3.6, \quad \mu_r = 2.1 \quad \sigma = 0.08 \text{ S/m}$$

$\vec{a}_E = \hat{a}_z \rightarrow$ direction of E-wave page

(a) $\gamma = \alpha + j\beta$

Calculate $\frac{\sigma}{\omega\epsilon} = \frac{0.8}{2\pi \times 50 \times 10^6 \times 3.6 \times 10^{-9} / 36\pi}$

$\frac{\sigma}{\omega\epsilon} > 1 \rightarrow$ good conductor

$\alpha = \beta = \sqrt{\pi f \mu \sigma} =$ for good conductor

$\beta = 5.758$

$\gamma = \alpha + j\beta = 5.758 + j5.758 / m$

(b) $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{5.758} = 1.091 \text{ m}$

(c) velocity $v = \lambda f = 1.091 \times 50 \times 10^6 = 54.55 \times 10^6 \text{ m/sec.}$

(d) $|\eta| = \sqrt{\frac{\mu\sigma}{\epsilon}} = \sqrt{\frac{2\pi f \times \mu_0 \mu_r}{\sigma}}$

$|\eta| = 101.8 \Omega \quad \theta_\eta = \pi/4$

(e) $\vec{H} = H_0 e^{-\gamma x} \vec{a}_H$

$H_0 = \frac{\epsilon_0}{\eta} = \frac{6}{(101.8)(450)}$

$= 58.93 \angle -45^\circ \text{ mA/m}$

$$\vec{a}_\pi = \vec{a}_k \times \vec{a}_\epsilon = \vec{a}_1 \times \vec{a}_3 = -\vec{a}_2$$

$$\vec{H} = 58.93 \angle 45^\circ e^{-(5.7 + j5.7)x} (-\vec{a}_2) \text{ mA/m}$$

$$\vec{H} = 58.93 e^{-\gamma x} e^{-j\pi/4} (-\vec{a}_2) \text{ mA/m}$$

5) A lossy material has $\mu = 5\mu_0$, $\epsilon' = 2\epsilon_0$, if at 5 MHz, the phase constant is 10 rad/m . calculate.

- a) loss tangent b) σ c) ϵ d) α
e) η .

Sol:- a) loss tangent $= \frac{\sigma}{\omega\epsilon}$

given $\mu_r = 5$, $\epsilon_r = 2$, $f = 5 \text{ MHz}$ $\beta = 10 \text{ rad/m}$

$$\beta = \frac{\omega}{c} \sqrt{\frac{\mu_r \epsilon_r}{2} \sqrt{(1 + (\frac{\sigma}{\omega\epsilon})^2) + 1}}$$

$$\frac{\sigma}{\omega\epsilon} = 1823$$

b) $\sigma = 1823 \times \omega\epsilon = 1.013 \text{ S/m}$

d) $\alpha = 9.995 \text{ Np/m}$

e) $\eta = \sqrt{\frac{\omega\mu}{\sigma}}$ = for good conductor if $\sigma \gg \omega\epsilon$

$$\eta = 13.95 \angle 45^\circ$$

c) complex permittivity

$$\epsilon_c = \epsilon' - j\epsilon''$$

$$\epsilon'' = \sigma/\epsilon = 3.22 \times 10^{-8}$$

$$\epsilon' = \epsilon = 2\epsilon_0 = 2 \times 10^{-9} / 36\pi = 17.7 \times 10^{-12} - j 3.22 \times 10^{-8}$$

(9) In a certain medium with $\mu = \mu_0$

$$\epsilon = 4\epsilon_0 \quad H = 12 e^{-0.1y} \sin(\pi \times 10^8 t - \beta y) \text{ a}_x$$

A/m. Then find a) wave period T
 b) wavelength c) electric field E , d) phase
 different b/w E & H

Sol: a) $T = \frac{2\pi}{\omega} = \frac{2\pi}{2\pi \times f} = 20 \text{ nsec}$

b) $\lambda = \frac{2\pi}{\beta} = \beta = 2\pi/3$

$$\lambda = \frac{2\pi}{\frac{2\pi}{3}} = \cancel{2\pi} \times \frac{3}{\cancel{2\pi}} = 3 \text{ m}$$

c) $E = E_0 e^{-\alpha y} \sin(\pi \times 10^8 t - \beta y) \hat{a}_E$

$$\alpha = \frac{\omega}{c} \sqrt{\frac{\mu_r \epsilon_r}{2} \sqrt{(1 + (\frac{\epsilon}{\omega^2})^2)} - 1}$$

$H = 12 e^{0.1y} \sin(\pi \times 10^8 t - \beta y) \text{ a}_x \text{ A/m}$

$\alpha = 0.1, \quad \beta = \frac{2\pi}{3}$

$\frac{\alpha}{\beta} = \tan \theta_\eta$

$\theta_\eta = \tan^{-1} \left(\frac{0.3}{2\pi} \right) = 2.73^\circ$

$\hat{a}_E = \hat{a}_H \times \hat{a}_k$

$= \hat{a}_x \times \hat{a}_y = \hat{a}_z = \hat{a}_E$

we know $\theta_\eta = \theta/2$

$\hookrightarrow$ phase of impedance

$\tan \theta_\eta = \alpha/\beta$

$$\tan \theta = \alpha \left(\frac{\alpha}{\beta} \right)$$

$$\theta = 5.46^\circ$$

to find ϵ_0 , we need η

$$|\eta| = \frac{|E_0|}{|H_0|}$$

$$|H_0| = 12, \quad |\eta| = \frac{\sqrt{\mu/\epsilon}}{(1 + (\frac{\epsilon}{\omega\epsilon})^2)^{1/4}}$$

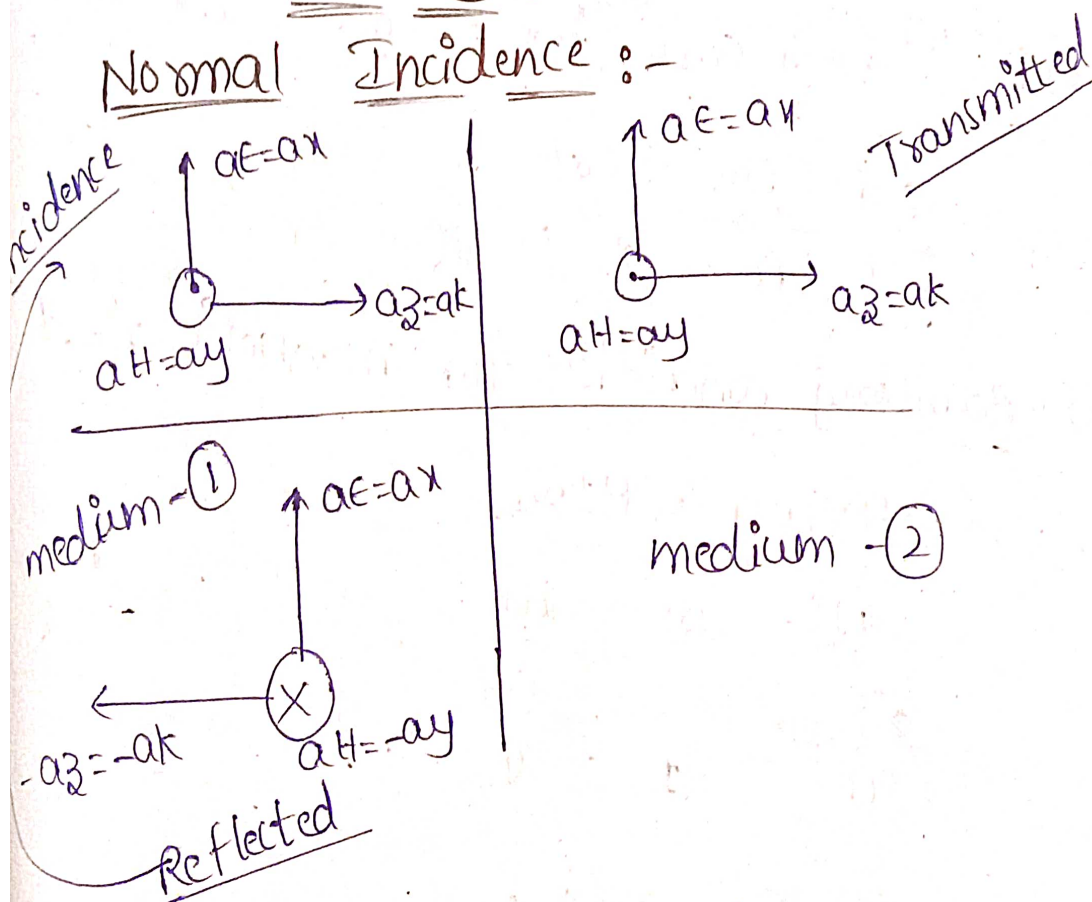
$$|\eta| = 188.06 \eta$$

$$|E_0| = |\eta| |H_0| = (188.06)(12) = 2256.8$$

$$\vec{E} = (2256.8) e^{-0.1y} \sin(\pi \times 10^8 t - \frac{2\pi}{3}y + 2.73^\circ) \hat{a}_z$$

Unit - (3) :-

Normal Incidence :-



Incidence - wave :- for electric and magnetic

$$E_i = E_{oi} e^{-\gamma_1 z} a_x, \quad H_i = H_{oi} e^{-\gamma_1 z} a_y$$

Reflected - wave for ef and mf

$$E_r = E_{or} e^{+\gamma_1 z} a_x, \quad H_r = H_{or} e^{+\gamma_1 z} (-a_y)$$

Transmitted - wave for ef and mf

$$E_t = E_{ot} e^{-\gamma_2 z} a_x, \quad H_t = H_{ot} e^{-\gamma_2 z} a_y$$

$\Rightarrow$ Boundary condition for electric field

$$E_{tan1} = E_{tan2}$$

$$E_i + E_r = E_t$$

$$E_{oi} e^{-\gamma_1 z} a_x + E_{or} e^{+\gamma_1 z} a_x = E_{ot} e^{-\gamma_2 z} a_x$$

$$E_{oi} + E_{or} = E_{ot}$$

$$\boxed{1 + \Gamma_E = \tau_E} \rightarrow \textcircled{1}$$

$\Rightarrow$ Boundary condition for magnetic field

$$H_{tan1} = H_{tan2}$$

$$H_{oi} - H_{or} = H_{ot}$$

$$\frac{E_{oi}}{\eta_1} - \frac{E_{or}}{\eta_1} = \frac{E_{ot}}{\eta_2}$$

$$E_{oi} - E_{or} = \frac{\eta_1}{\eta_2} \cdot E_{ot}$$

$$E_{oi} (1 - \Gamma_E) = \frac{\eta_1}{\eta_2} \cdot E_{ot}$$

$$1 - \frac{E_{or}}{E_{oi}} = \frac{n_1}{n_2} \cdot \frac{E_{ot}}{E_{oi}}$$

$$\boxed{1 - r_E = \frac{n_1}{n_2} \tau_E} \rightarrow (2)$$

from (1) and (2) equations we get

$$(1 + r_E) + (1 - r_E) = \tau_E \left(\frac{n_1}{n_2} \right) \cdot \tau_E$$

$$1 + \cancel{r_E} + 1 - \cancel{r_E} = \tau_E \left(\frac{n_1}{n_2} \right) \tau_E$$

$$2 = \tau_E \left(\frac{n_2 + n_1}{n_2} \right)$$

$$\therefore \boxed{\tau_E = \frac{2n_2}{n_1 + n_2}}$$

substitute τ_E in eq (1)

$$1 + r_E = \tau_E$$

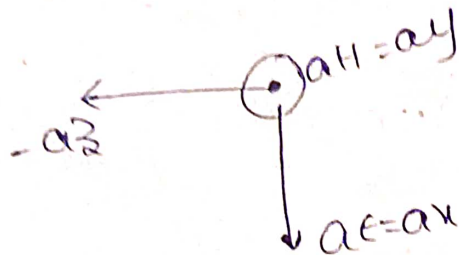
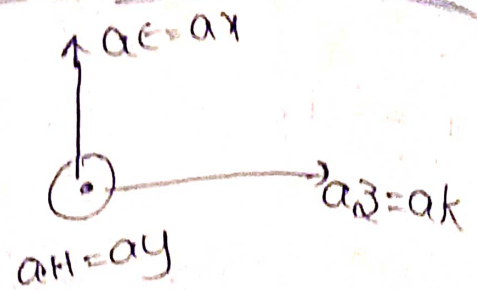
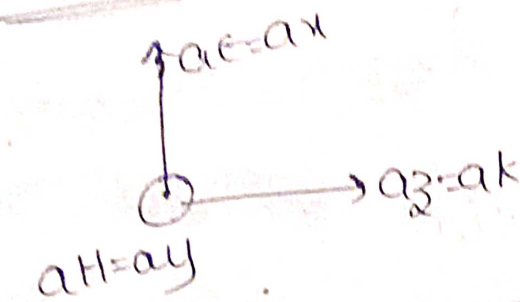
$$1 + r_E = \frac{2n_2}{n_1 + n_2}$$

$$r_E = \frac{2n_2}{n_1 + n_2} - 1$$

$$r_E = \frac{2n_2 - n_1 - n_2}{n_1 + n_2}$$

$$\therefore \boxed{r_E = \frac{n_2 - n_1}{n_1 + n_2}}$$

Normal Incidence for magnetic field:



$$\hat{a}_k = \hat{a}_e \times \hat{a}_H$$

$$-a_k = -a_x \times a_y$$

Boundary conditions for electric field:-
 x and y are (Tangential)
 z is normal

$$E_{oi} - E_{or} = E_{ot} \quad (E \text{ into } mf)$$

$$H_{oi} \eta_1 + H_{or} \eta_1 = H_{ot} \eta_2$$

$$H_{oi} + H_{or} = \frac{\eta_2}{\eta_1} H_{ot}$$

$$1 - \frac{H_{or}}{H_{oi}} = \frac{\eta_2}{\eta_1} \cdot \frac{H_{ot}}{H_{oi}}$$

$$\boxed{1 - \Gamma_H = \frac{\eta_2}{\eta_1} \cdot \tau_H} \rightarrow (1)$$

Boundary conditions for magnetic field:-

$$H_{tan1} = H_{tan2}$$

$$H_i + H_r = H_t$$

$$H_i + H_r = H_{ot}$$

$$1 + \frac{n_1 r_H}{n_1} = \frac{n_1}{n_1}$$

$$\boxed{1 + r_H = T_H} \rightarrow (2)$$

from (1) and (2) equations

$$1 - r_H = \frac{n_2}{n_1} (1 + r_H)$$

$$1 - r_H = \frac{n_2 (1 + r_H)}{n_1}$$

$$1 - r_H = \frac{n_2 + n_2 r_H}{n_1}$$

$$n_1 (1 - r_H) = n_2 + n_2 r_H$$

$$n_1 - n_1 r_H = n_2 + n_2 r_H$$

$$n_1 - n_2 = n_2 r_H + n_1 r_H$$

$$n_1 - n_2 = r_H (n_2 + n_1)$$

$$\therefore, \boxed{r_H = \frac{n_1 - n_2}{n_1 + n_2}}$$

substitute r_H in eq (2)

$$1 + \frac{n_1 - n_2}{n_1 + n_2} = T_H$$

$$\frac{n_1 + \cancel{n_2} + n_1 - \cancel{n_2}}{n_1 + n_2} = T_H$$

$$\boxed{T_H = \frac{2n_1}{n_1 + n_2}}$$