ENGINEERING SECOND YEAR: SEMESTER-2

20EC2204	Electromagnetic waves and Guided media	PCC	3L: 1T: 0P	4 credits

Course Learning Objective

1. Introduce the fundamental importance of electromagnetic theory and wave propagation phenomena for an electronics and communicationengineer.
2. Understanding guided media role for efficient power transmission in communication systems,between microwave subsystems, optical fiber systems.
3. Introduce to the higher order modes of propagation in guidingmedia.

Course Content

Unit-I **(6 hours)**

Introduction

Application, Review of vector algebra (dot product, cross product, scalar and vector components of vector), coordinate systems (rectangular, cylindrical, spherical coordinate systems), vector calculus(gradient, curl, divergence)

Review of Electrostatics, Magnetostatics, electrodynamics and Maxwell equations and boundary conditions.

Unit-II **(8 hours)**

Wave Propagation

Wave solution to Maxwell equations, Uniform plane wave solution, propagation constant , Propagation of uniform plane waves in perfect dielectric and in lossy medium (conductor, lossy dielectric), Wave polarization, Power and Poynting vector.

Unit-III **(8 hours)**

Wave propagation at interfaces

Reflection, refraction, Normal Incidence, Oblique Incidence ,effects of wave polarization in reflection and refractions, total internal reflection, Brewster angle, phase and group velocities.

Unit-IV **(8 hours)**

Transmission Lines: Parameters

Transmission Lines- Equations of Voltage and Current on TX line, Propagation constant , characteristic impedance, reflection coefficient and VSWR, Impedance Transformation, Power transfer on TX line, Smith chart.

Unit-V **(6hours)**

Waveguides-I

General solution of TEM, TE, TM waves, parallel plate waveguide, rectangular waveguide, circularwaveguide.

Unit-VI **(7 hours)**

Waveguides-II

Coaxial line, power handling capacity, strip line, microstrip, wave velocity and dispersion, RF connectors, excitation of waveguide.

Learning Resources

Text books

1. Matthew N.O.Sadiku, *'Elements of Electromagnetics'*, Oxford University Press , 6th edition,2014.
2. William H. Hayt Jr. and John A. Buck, *'Engineering Electromagnetics'*, 7th edition, 2006, TMH.

Reference books

1. I.E.C.JordanandK.G.Balmain , *'Electromagnetic Waves and Radiating Systems'*, PHI, 2nd Edition,2000.
2. John Kraus and Daniel fleisch, *'Electromagnetics with applications'* ,McGraw-hill international edition ,5th edition ,1999.

Web Resource

1. Prof David Staeling,MIT- Opencourseware, 'Electromagnetics and Applications', URL:https://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-013- electromagnetics-and-applications-spring-2009/index.htm
2. Prof R K Shivgaonkar, NPTEL-IIT Bombay, *'Transmission Lines and EMWaves'* URL: http://nptel.ac.in/courses/117101056/
3. Prof Harish shankar Ramachandra, NPTEL-IIT Madras, *'ElectromagneticFields'* URL: http://nptel.ac.in/courses/108106073/

Course Outcomes

At the end of the course, the student will be able to

CO 1	Apply vector calculus to static electric-magnetic fields in different engineering situations.
CO 2	Analyze Maxwell's equation in different forms: differential and integral and

Video-2

Maxwell's equation:-

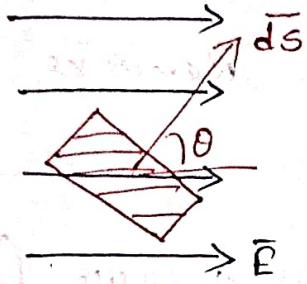
Static fields:-

Differential (or) Point form	Integral form.	Remarks.
$\nabla \cdot D = \rho_v$	$\oint_S D \cdot ds = \int_V \rho_v \, dv$	Gauss's law.
$\nabla \cdot B = 0$	$\oint_S B \cdot ds = 0$	Nonexistence of m·c monopole.
$\nabla \times E = 0$	$\oint_l E \cdot dl = 0$	conservative nature of e·c field.
$\nabla \times H = J$	$\oint_l H \cdot dl = \int_S J \cdot ds$	Ampere's law.

Time varying fields:-

$\nabla \cdot D = \rho_v$	$\oint_S D \cdot ds = \int_V \rho_v \, dv$	Gauss's law
$\nabla \cdot B = 0$	$\oint_S B \cdot ds = 0$	Non existence of m·c monopole.
$\nabla \times E = -\partial B / \partial t$	$\oint_L E \cdot dl = \frac{-\partial}{\partial t} \int_S B \cdot ds$	Faraday's law.
$\nabla \times H = J + \frac{\partial D}{\partial t}$	$\oint_L H \cdot dl = \int_S (J + \partial D/\partial t) \cdot ds$	modified ampere's circuit law

Flux:- it is measure of how much electric field lines (69) m²c. passes through any surface



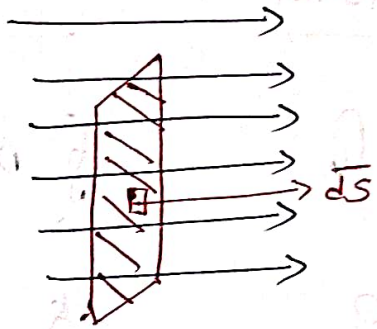
$$d\psi = \bar{E} \cdot d\bar{s}$$

$$\psi = \int_S \bar{E} \cdot d\bar{s}$$

$$\bar{E} \cdot d\bar{s} = E\, ds \cos\theta.$$

$$\psi = \int_S E\, ds \cos\theta$$

max flux (flow through when $\theta = 0$.



here

$$\psi = \int_S E\, ds$$

$$\boxed{\psi = E \times \text{area}}$$

Gauss law:- it states that total e·c flux passing thro any closed surface is equal to charge enclosed by that closed surface.

$$\oint \bar{D} \cdot d\bar{s} = Q_{enc.}$$

$\bar{D}$ = electric flux density. (field lines by considering the medium properties)

$= \varepsilon \bar{E} = (C/m^2)$ → coulomb

ρ_v = charge density of volume. (coulomb/m^3)

↑ volume

$\bar{D}$ = coulomb/m^2 → surface

$$\Phi = \int_v \rho_v \, dv.$$

$$\oint_S \bar{D} \cdot d\bar{s} = \int_v \rho_v \, dv.$$

as per statement of divergence theorem.

$$\oint_S (\bar{D} \cdot d\bar{s}) = \int_v (\nabla \cdot \bar{D}) \, dv = \int_v \rho_v \, dv$$

$$\boxed{\nabla \cdot \bar{D} = \rho_v}$$

for source free region $\rho_v = 0$ so $\nabla \cdot \bar{D} = 0$.

it means field was divergence less.

Maxwell's second equation (electro static field).

Total work done to move a point charge around the closed loop in presence of e.s. field is always equal to zero.

(also refer Sadiku text book P.NO : 132).

$$\oint_l E \cdot dl = 0.$$

as per the stoke's theorem

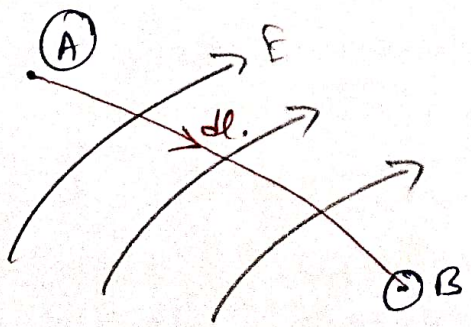
$$\oint_l E \cdot dl = \int_S (\nabla \times E) \cdot dS = 0.$$

i.e. $\boxed{\nabla \times E = 0}$

as per coulomb's law

Force b/w charges $F = \frac{K Q_1 Q_2}{R^2}$

Electric field Intensity $E = \frac{F}{Q}$ (or) $F = QE$.



work done to move a point charge from point A to B.

$$dW = -F \cdot dl$$

$$W = -Q \int_A^B E \cdot dl.$$

$$V = \frac{W}{Q} = -\int_A^B E \cdot dl.$$

Then if we move a point charge in a closed loop then $U = V_{AB} + V_{BA} = 0 = \oint_l E \cdot dl$.

$$\oint_l E \cdot dl = 0.$$

From Stokes theorem $\oint_l E \cdot dl = \int (\nabla \times E) \cdot ds = 0$

$$\boxed{\nabla \times E = 0}$$

- conservative nature of e^{-}c field.

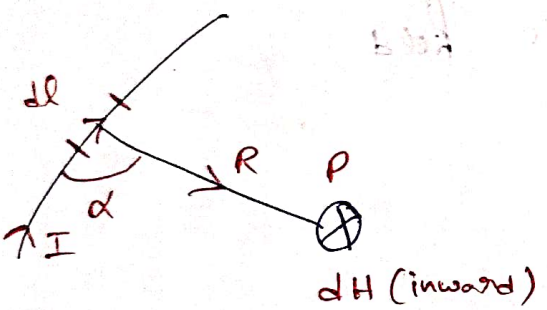
Maxwell's eq'n on Magneto Static field

Faraday's law :- it states that induced emf in a closed circuit is equal to time rate of change in flux linkage with the circuit

$$V_{emf} = -N\frac{d\phi}{dt}$$

[also read Sadiku text book P.NO : 275]

as per Biot - Savart's law



$$dH = \frac{K I\, dl \sin\alpha}{4\pi R^2}$$

as per ampere's circuit law.

$$\oint_L H \cdot dl = I_{enc}$$

from Stoke's theorem $\int (\nabla \times H) \cdot ds = I_{enc}$

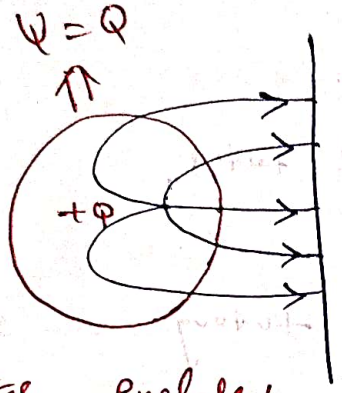
we know $I_{enc.} = J \cdot A$

$$= \int J \cdot ds$$

So we may write $\boxed{\nabla \times H = J}$

Maxwell's eqn on magneto static field

$\nabla \cdot B = 0$ → Isolated magnetic charge does not exist.



Q - Charge enclosed

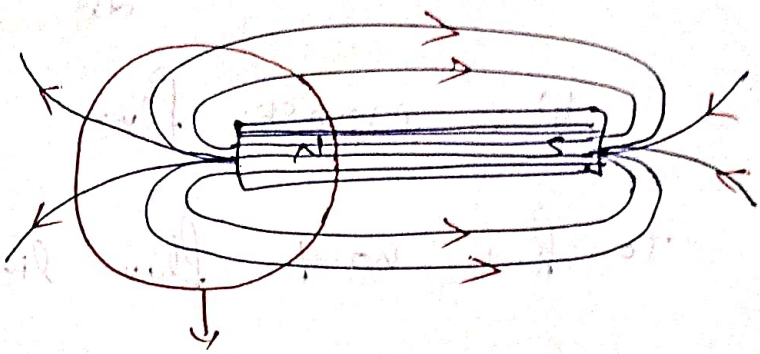
Ψ = flux leaving surface

from the gauss law.

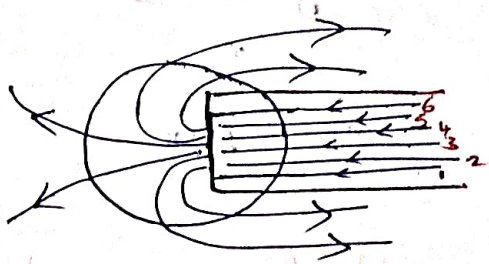
$$\oint D \cdot ds = Q.$$

$$\Downarrow$$

$$\Psi = Q.$$



$\Psi = 0.$



from gauss divergence theorem

$$\oint B \cdot ds = \int (\nabla \cdot B)\, dv = 0.$$

$$\boxed{\nabla \cdot B = 0}$$

Maxwell's eqn for time varying electric field :-

we know from faraday's law

$$V_{emf} = -N \cdot \frac{d\Psi}{dt}$$

Ψ = magnetic flux, N = no. of turns.

To find no. of flux lines rolling through a surface

$$\Psi = \oint D \cdot ds = \int B \cdot ds$$

↑ elec. flux ↑ mag flux.

so $$V_{emf} = -\frac{d}{dt} \int B \cdot ds.$$

while we are deriving this eqn to define the conservative nature of elec field.

$$dw = -F \cdot dl$$

$$w = -Q \int E \cdot dl$$

$$V = \frac{w}{Q} = -\int E \cdot dl$$

for a closed loop

$$V_{emf} = +\oint E \cdot dl = -\frac{d}{dt} \int B \cdot ds.$$

$$V_{emf} = \oint E \cdot dl = \int -\frac{\partial B}{\partial t} \cdot ds.$$

From Stoke's theorem.

$$\int (\nabla \times E) \cdot ds = \int -\frac{\partial B}{\partial t} \cdot ds.$$

$$\nabla \times E = -\partial B / \partial t$$

Maxwell's eqn for time varying magnetic field

$$= \nabla \times H = J + J_d = J + \frac{\partial D}{\partial t}$$

$$= D = \varepsilon_0 E$$

From static case we know $\nabla \times H = J$.

From the properties of vector calculus.

$$\nabla \cdot (\nabla \times \text{vector}) = 0.$$

i.e $\nabla \cdot (\nabla \times H) = \nabla \cdot J = 0.$

but from continuity eqn $\nabla \cdot J = -\frac{\partial \rho_v}{\partial t}$

i.e $I = \oint J \cdot ds = -\frac{dQ_{in}}{dt}$

net outward flux from the surface will be equal to

charge decreasing inside the surface.

$$\oint J \cdot dS = -\frac{d Q_{in}}{dt}$$

we know closed surface = volume.

charge inside the volume $Q = \int \rho_v \, dv$.

from divergence theorem,

$$\oint J \cdot dS = \int_V (\nabla \cdot J) \, dV = -\frac{d}{dt} \int \rho_v \, dv$$

$$= -\int \frac{\partial \rho_v}{\partial t} \, dv.$$

$$\text{SO} \quad \nabla \cdot J = -\frac{\partial \rho_v}{\partial t}$$ - continuity eqn.

SO $\nabla \times H = J$ - will not be suitable for time varying condition, we need to modify it.

$$\nabla \times H = J + J_d.$$

$$\nabla \cdot (\nabla \times H) = 0 = \nabla \cdot J + \nabla \cdot J_d$$

$$\nabla \cdot J_d = -\nabla \cdot J = \frac{\partial \rho_v}{\partial t} = \frac{\partial}{\partial t} (\nabla \cdot D) = \nabla \cdot \frac{\partial D}{\partial t}$$

$$J_d = \frac{\partial D}{\partial t}$$

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

Unit - 1 Summary

- Scalar – A
- vector – $\bar{A} = A_x a_x + A_y a_y + A_z a_z$
- unit vector $a_A = \frac{\bar{A}}{|A|} = \frac{\bar{A}}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$
- vector addition & substraction.

$C = \bar{A} + \bar{B}$ $\quad C = \bar{A} - \bar{B}$

- Position vector

$r_{op} = r_p - r_o = x a_x + y a_y + z a_z$

- distance vector

$r_{pq} = r_q - r_p$

$= (x_q - x_p) a_x + (y_q - y_p) a_y + (z_q - z_p) a_z$

Vector multiplication

① Scalar

$\bar{A} \cdot \bar{B} = |A||B| \cos\theta_{AB}$.

$= A_x B_x + A_y B_y + A_z B_z$.

- satisfy commutative & distributive law.

② vector.

$\bar{A} \times \bar{B} = |A||B| \sin\theta_{AB}\, a_n$

$$= \begin{vmatrix} a_x & a_y & a_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

- satisfy only distributive law.

$a_x \times a_y = a_z$, $a_y \times a_z = a_x$.

③ Scalar Triple product

$A \cdot (B \times C) = B \cdot (C \times A) = C \cdot (A \times B)$

④ vector triple product

Cartesian coordinate system

$\bar{A} = A_x a_x + A_y a_y + A_z a_z$

$-\infty < x < \infty$, $-\infty < y < \infty$

$-\infty < z < \infty$.

circular cylindrical system

$\bar{A} = A_\rho a_\rho + A_\phi a_\phi + A_z a_z$.

$0 \le \rho < \infty$, $0 \le \phi < 2\pi$, $-\infty < z < \infty$

Spherical co-ordinate system

$\bar{A} = A_r a_r + A_\theta a_\theta + A_\phi a_\phi$.

$0 \le r < \infty$, $0 \le \theta < \pi$, $0 \le \phi < 2\pi$

distance

$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2$

$d^2 = \rho_2^2 + \rho_1^2 - 2\rho_1\rho_2 \cos(\phi_2 - \phi_1) + (z_2 - z_1)^2$

$x = \rho\cos\phi$

$y = \rho\sin\phi$

$d^2 = r_2^2 + r_1^2 - 2r_1 r_2 \cos\theta_2 \cos\theta_1 - 2r_1 r_2 \sin\theta_2 \sin\theta_1 \cos(\phi_2 - \phi_1)$.

$x = r\sin\theta\cos\phi$

$y = r\sin\theta\sin\phi$

Co-ordinate conversion:

Cartesian .→

(x, y, z)

$\rho = \sqrt{x^2+y^2}$, $\phi = \tan^{-1}(y/x)$, $z = z$

$r = \sqrt{x^2+y^2+z^2}$, $\phi = \tan^{-1}(y/x)$.

$\theta = \tan^{-1}\dfrac{\sqrt{x^2+y^2}}{z}$.

Cylindrical

(ρ, ϕ, z)

$x = \rho\cos\phi$, $y = \rho\sin\phi$, $z = z$.

$r = \sqrt{\rho^2+z^2}$, $\theta = \tan^{-1}\rho/z$, $\phi = \tan^{-1}\phi$.

Spherical.

r, θ, ϕ

$x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$

$z = r\cos\theta$.

Vector conversion

Cartesian.

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

Cylindrical.

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

Spherical

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta & 0 & \cos\theta \\ \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

Differential length :-

Cartesian :-

$$d\bar{l} = dx\,a_x + dy\,a_y + dz\,a_z$$

cylindrical :-

$$d\bar{l} = d\rho\,a_\rho + \rho\,d\phi\,a_\phi + dz\,a_z$$

Spherical :-

$$d\bar{l} = dr\,a_r + r\,d\theta\,a_\theta + r\sin\theta\,d\phi\,a_\phi$$

Differential surface :-

Cartesian :-

$$d\bar{s} = dy\,dz\,a_x, \quad = dx\,dz\,a_y \quad = dx\,dy\,a_z$$

cylindrical :-

$$= \rho\,d\phi\,dz\,a_\rho \quad = d\rho\,dz\,a_\phi \quad = \rho\,d\rho\,d\phi\,a_z$$

Spherical

$$d\bar{s} = r^2\sin\theta\,d\theta\,d\phi\,a_r$$

$$= r\sin\theta\,dr\,d\phi\,a_\theta$$

$$= r\,dr\,d\theta\,a_\phi.$$

Volume :-

$$dv = dx\,dy\,dz \qquad dv = \rho\,d\rho\,d\phi\,dz.$$

Line Integral :-

$$\oint \bar{A}\cdot d\bar{l} = \int_a^b |A|\cos\theta\,dl$$

surface Integral :-

$$\psi = \int_S \bar{A}\cdot a_n\,ds$$

$$= \int_S \bar{A}\cdot d\bar{s}$$

volume Integral :-

$$= \int \rho_v\,dv.$$

Del operator :-

$$\nabla = \partial/\partial x\,a_x + \partial/\partial y\,a_y + \partial/\partial z\,a_z.$$

$$= a_\rho\frac{\partial}{\partial\rho} + a_\phi\frac{\partial}{\partial\phi}\frac{1}{\rho} + a_z\cdot\partial/\partial z$$

$$= a_r\,\partial/\partial r + a_\theta\cdot\frac{1}{r}\cdot\frac{\partial}{\partial\theta} + \frac{1}{r\sin\theta}\cdot\frac{\partial}{\partial\phi}\,a_\phi.$$

gradient of scalar :-

$$\nabla V = \frac{\partial V}{\partial l}a_\rho + \frac{1}{\rho}\cdot\frac{\partial V}{\partial\phi}a_\phi + \frac{\partial V}{\partial z}a_z,$$

$$= \frac{\partial V}{\partial r}a_r + \frac{1}{r}\cdot\frac{\partial V}{\partial\theta}a_\theta + \frac{1}{r\sin\theta}\cdot\frac{\partial V}{\partial\phi}a_\phi$$

$$= \frac{\partial V}{\partial x}a_x + \frac{\partial V}{\partial y}a_y + \frac{\partial V}{\partial z}a_z.$$

Divergence of vector :-

$$\nabla\cdot\bar{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla\cdot\bar{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2A_r) + \frac{1}{r\sin\theta}\cdot\frac{\partial}{\partial\theta}(A_\theta\sin\theta) + \frac{1}{r\sin\theta}\cdot\frac{\partial A_\phi}{\partial\phi}.$$

Curl of vector :-

$$\nabla\times\bar{A} = \begin{vmatrix} a_x & a_y & a_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & A_y & A_z \end{vmatrix} \qquad \nabla\times\bar{A} = \frac{1}{\rho}\begin{vmatrix} a_\rho & \rho a_\phi & a_z \\ \partial/\partial\rho & \partial/\partial\phi & \partial/\partial z \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$$

$$\nabla\times A = \frac{1}{r^2\sin\theta}\begin{vmatrix} a_r & r a_\theta & r\sin\theta\,a_\phi \\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\phi. \\ A_r & rA_\theta & r\sin\theta A_\phi \end{vmatrix}$$

Divergence theorem :-

$$\oint \bar{A}\cdot d\bar{s} = \int_V (\nabla\cdot A)\,dv$$

Stoke's theorem :-

$$\oint \bar{A}\cdot d\bar{l} = \int_S (\nabla\times A)\cdot d\bar{s}$$

Summary: $\gamma^2 = \sigma + j\omega\varepsilon$; $\boxed{\gamma = \sqrt{\sigma + j\omega\varepsilon} = \alpha + j\beta}$, $V_p = \omega/\beta$ → (6) | E = V/M, H = A/M.

(1) $\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]}$ | (2) $\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1\right]}$

(3) $\eta = \sqrt{\frac{j\omega\mu}{j\omega\varepsilon\left(1+\frac{\sigma}{j\omega\varepsilon}\right)}}$ | (5) $|\eta| = \frac{\sqrt{\mu/\varepsilon}}{\left[1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{1/4}}$, | (4) $\theta_\eta = \frac{1}{2}\tan^{-1}\left(\frac{\sigma}{\omega\varepsilon}\right)$

	lossy dielectric	loss less dielectric	free space	good conductor
	$(\sigma \neq 0,\ \varepsilon = \varepsilon_r\varepsilon_0,\ \mu = \mu_r\mu_0)$ $\sigma \simeq \omega\varepsilon$	$\sigma \simeq 0,\ \varepsilon = \varepsilon_r\varepsilon_0,\ \mu = \mu_r\mu_0$ $\sigma << \omega\varepsilon$	$\sigma = 0,\ \mu = \mu_0,$ $\varepsilon = \varepsilon_0.$ $\mu_r = 1$ $\varepsilon_r = 1$	good conductor $\sigma \simeq \infty,\ \mu = \mu_0\mu_r$ $\varepsilon = \varepsilon_0.\ \sigma >> \omega\varepsilon.$
α Np/m	$\omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]}$	$\varepsilon_0 = 10^{-9}/36\pi$ $\alpha = 0,\ \mu_0 = 4\pi\times10^{-7}$	$\alpha = 0$	$\alpha = \sqrt{\pi f\mu\sigma}$
β rad/m	$\omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1\right]}$	$\frac{\omega}{c}\sqrt{\mu_r\varepsilon_r}$	$\beta = \frac{\omega}{c}$ $\mu_r = 1$ $\varepsilon_r = 1$	$\beta = \sqrt{\pi f\mu\sigma}.$
η Ω	$\sqrt{\frac{j\omega\mu}{j\omega\varepsilon\,(1+\sigma/j\omega\varepsilon)}}$	$\eta = \sqrt{\mu/\varepsilon}$ $\lvert\eta\rvert = 120\pi.$ if $\mu_r = \varepsilon_r = 1$	$\lvert\eta\rvert = 120\pi$	$\eta = \sqrt{\frac{j\omega\mu}{\sigma}}$ $\lvert\eta\rvert = \sqrt{\frac{\mu\omega}{\sigma}}$
θ_η deg	$\frac{1}{2}\tan^{-1}\left(\frac{\sigma}{\omega\varepsilon}\right)$			$\theta_\eta = \pi/4.$

video - 3

Wave eqn :-

as from maxwell's eqn $\nabla \cdot \bar{D} = \rho_v$

$$\nabla \cdot \varepsilon \bar{E} = \rho_v$$

for source free region $\rho_v = 0$.

if medium is Homogenous - Then ε is Independent of the space (x, y, z)

$$\varepsilon (\nabla \cdot \bar{E}) = 0$$

$$\boxed{\nabla \cdot \bar{E} = 0 \cdot} \rightarrow (1)$$

$$\nabla \cdot \bar{B} = 0 \Rightarrow \boxed{\nabla \cdot \bar{H} = 0} \rightarrow (2)$$

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

$$\nabla \times \bar{E} = -\frac{\partial}{\partial t}(\mu \bar{H})$$

$$\boxed{\nabla \times \bar{E} = -j\omega \mu \bar{H}} \rightarrow (3)$$

with time factor

$\boxed{\bar{H} = H_0 e^{j\omega t}}$ $\quad \frac{\partial \bar{H}}{\partial t} = j\omega \cdot \bar{H}$

$\boxed{\bar{E} = E_0 e^{j\omega t}}$

$$\nabla \times \bar{H} = \bar{J}_c + \bar{J}_D$$

$\bar{J}_c$ ↓ conduction current

$\bar{J}_D$ → displacement current

$$\boxed{\nabla \times \bar{H} = \sigma \bar{E} + j\omega \varepsilon \bar{E}} \rightarrow (4)$$

$$\nabla \times \nabla \times \bar{E} = -j\omega\mu (\nabla \times \bar{H})$$

$$\nabla \times \nabla \times \bar{E} = -j\omega\mu (\sigma + j\omega\varepsilon)\bar{E}$$

as per vector identity principle.

$$\nabla \times \nabla \times \bar{A} = \nabla(\nabla \cdot \bar{A}) - \nabla^2 \bar{A}$$

$$\nabla \times \nabla \times \bar{E} = -j\omega\mu (\sigma + j\omega\varepsilon)\bar{E} = \nabla(\nabla \cdot \bar{E}) - \nabla^2 \bar{E}$$

In the source free region $\boxed{(\nabla \cdot \bar{E}) = \rho_v/\varepsilon}$

No source $\rho_v = 0$.

$$\nabla \cdot \bar{E} = 0 \quad \text{so}$$

$$\nabla^2 \bar{E} = j\omega\mu (\sigma + j\omega\varepsilon).\bar{E}$$

$$\text{assume } \gamma = \sqrt{j\omega\mu (\sigma + j\omega\varepsilon)}$$

$$= \text{Propagation const. (unit } m^{-1})$$

$$= \alpha + j\beta.$$

α = attenuation constant (NP/m)

1NP = 8.686 dB.

α - defines change in amplitude as wave travels through the medium.

β = Phase constant (rad/m) = $\frac{2\pi}{\lambda}$

= Phase shift per unit distance.

$$\boxed{\nabla^2 \bar{E} - \gamma^2 \bar{E} = 0}$$ - plane wave eqn for e-c field.

Plane wave eqn for mc field

$$\nabla \times \nabla \times \bar{H} = (\sigma + j\omega\varepsilon)(\nabla \times \bar{E}).$$

$$\boxed{\nabla^2 \bar{H} - \gamma^2 \bar{H} = 0}$$ - plane wave eqn for mc field.

(Video - 4)

Solution of plane wave :-

$$\nabla^2 E_s - \gamma^2 E_s = 0.$$

$(D^2 - \gamma^2) E_s = 0$ above wave is travelling in z-direction

$D^2 - \gamma^2 = 0$

$D = \pm\gamma \Rightarrow$ 2 diff roots.

Solution $E_s(z) = E_0^+ e^{-\gamma z} + E_0^- e^{+\gamma z}$.

Same way solution for plane wave eqn (M.C).

$H_s(z) = H_0^+ e^{-\gamma z} + H_0^- e^{+\gamma z}$.

$H_0^+ e^{-\gamma z}$: travelling in $+z$ direction

$H_0^- e^{+\gamma z}$: wave travelling in $-z$ direction

$E_0^+ e^{-\gamma z} \rightarrow z = \infty \quad E_0^+ e^{-\infty} = \frac{E_0^+}{e^{\infty}} = 0$ ↓ finite

$E_0^- e^{+\gamma z} \Rightarrow z = -\infty \quad E_0^- e^{-\infty} = 0$ ↓ finite

Assume wave is travelling +ve z-direction

$\overline{E_{s(z)}} = E_0 e^{-\gamma z} \bar{a}_E \rightarrow$ it's a vector.

$\overline{E_{s(z)}} = E_0 e^{-(\alpha + j\beta) z} \bar{a}_E$

$\overline{E_{s(z)}} = E_0 e^{-\alpha z} \cdot e^{-j\beta z} \bar{a}_E$

if we consider the time factor.

$\bar{E}(z,t) = E_0 e^{-\alpha z} e^{-j\beta z} e^{j\omega t}$

$$\boxed{\bar{E}(z,t) = E_0 e^{-\alpha z} e^{+j(\omega t - \beta z)} \bar{a}_E}$$

$$E(z,t) = Re\left[E_0 e^{-\alpha z} e^{j(\omega t - \beta z)} \hat{a}_E\right]$$

$$E(z,t) = \underbrace{E_0 e^{-\alpha z}}_{\text{magnitude}} \cos(\omega t - \beta z)\, \hat{a}_E$$

- $E(z,t)$ → unit → $\frac{V}{m}$
- E_0: Peak amplitude of e-c field
- α: attn const
- ω: frequency
- β: Phase Constant
- z: direction of Propagation (-ve → +z, +ve → -z)
- $\hat{a}_E$: direction of field

Solution of plane wave eqn:

if we assume $\alpha = 0$ (i.e No attenuation).

$$\bar{E}(z,t) = E_0 \cos(\omega t - \beta z)\, \bar{a}_E$$

$$\beta z = \frac{2\pi}{\lambda} \times z.$$

Case -1:- $t = 0$ $\quad E(z,t) = E_0 \cos(-\beta z)\, \hat{a}_E$

z	βz	E
0	0	E_0
$\lambda/4$	$\pi/2$	0
$\lambda/2$	π	$-E_0$
$3\lambda/4$	$3\pi/2$	0
λ	2π	E_0

Case 2:- $E(z,t) = E_0 \cos(\omega t - \beta z)$

$t = T/4 \quad E(z,t) = E_0 \cos\left(\frac{2\pi}{T} \cdot \frac{T}{4} - \beta z\right)$

$= E_0 \cos\left(\frac{\pi}{2} - \beta z\right) = E_0 \sin \beta z$

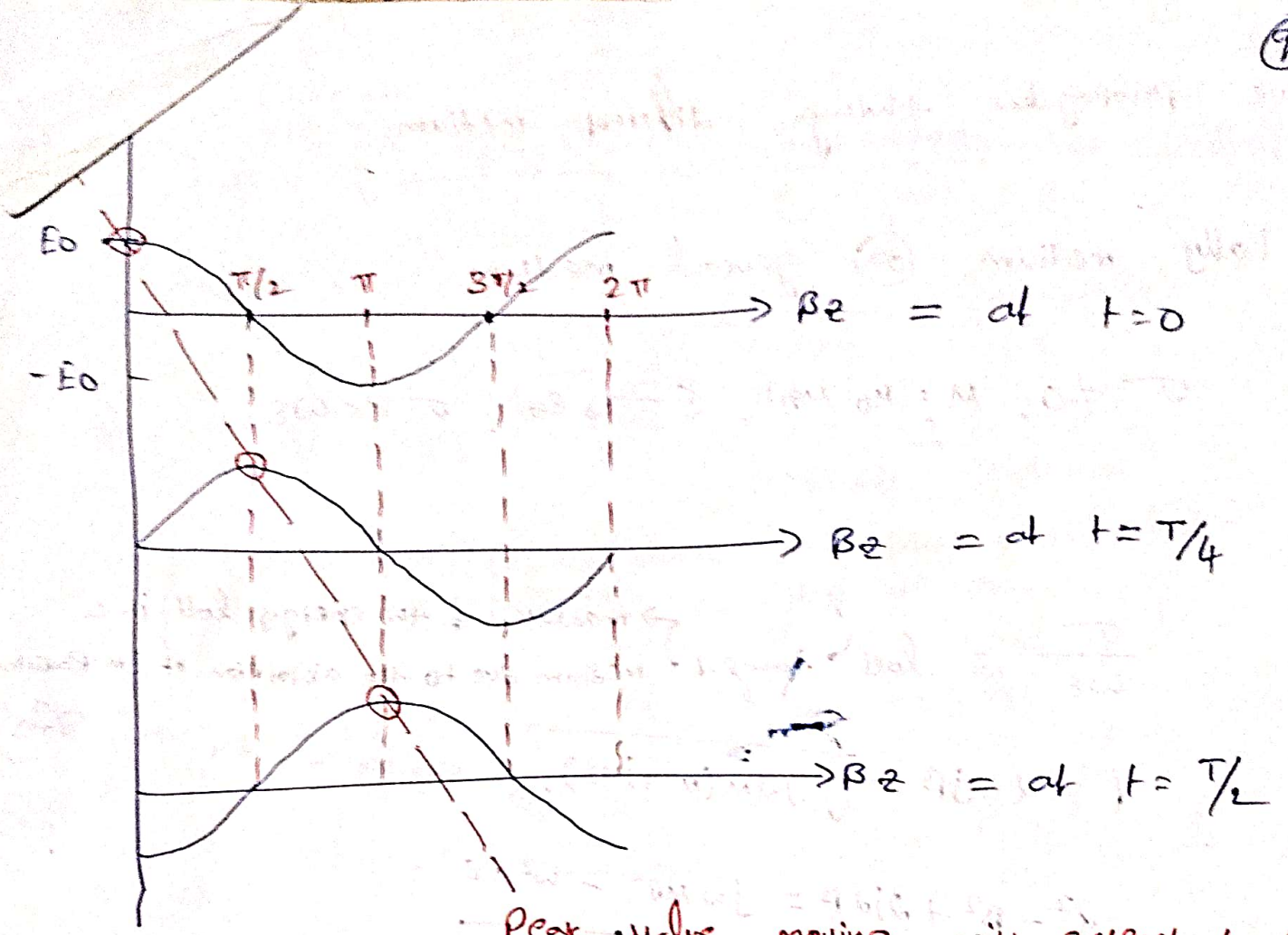
z	βz	E
0	0	0
$\lambda/8$	$\pi/2$	E_0
$\lambda/2$	π	0
$3\lambda/8$	$3\pi/2$	$-E_0$
λ	2π	0

Case 3:- $E(z,t) = E_0 \cos(\omega t - \beta z)$

$t = T/2, \quad E(z,t) = E_0 \cos\left(\frac{2\pi}{T} \times \frac{T}{2} - \beta z\right)$

$= E_0 \cos(\pi - \beta z) = -E_0 \cos(\beta z)$

z	βz	E
0	0	$-E_0$
$\lambda/4$	$\pi/2$	0
$\lambda/2$	π	$+E_0$
$3\lambda/4$	$3\pi/2$	0
λ	2π	$-E_0$



So we can say that wave is travelling.

Different media:-

- Write here about diff media (Param Sudiksha text book)

$\mu_0 = 4\pi \times 10^{-7}$ H/m
$\varepsilon_0 = 10^{-9}/36\pi$ F/m

① Lossy dielectric medium
$\sigma \neq 0$, $\mu = \mu_0\mu_r$, $\varepsilon = \varepsilon_0\varepsilon_r$ ($\sigma \lll \omega\varepsilon$) ($\sigma \approx \omega\varepsilon$)

② Loss less dielectric medium
$\sigma = 0$, $\varepsilon = \varepsilon_r\varepsilon_0$, $\mu = \mu_r\mu_0$ ($\sigma << \omega\varepsilon$).

③ good conductor (due to polarization negligible situation $\varepsilon_r = 1$)
$\sigma \approx \infty$, $\varepsilon = \varepsilon_0$, $\mu = \mu_r\mu_0$ ($\sigma >> \omega\varepsilon$)

④ free space
$\sigma = 0$, $\varepsilon = \varepsilon_0$, $\mu = \mu_0$

(Video - 5)

Wave Propagation through different medium

(applicable in all mediums)

① Lossy medium ② general medium.

$\sigma \neq 0, \quad \mu = \mu_0 \mu_r, \quad \varepsilon = \varepsilon_0 \varepsilon_r, \quad \sigma << \omega\varepsilon$

(- loss electromagnetic waves around you by Solar SIV)

$\frac{\sigma}{\omega\varepsilon}$ = loss tangent → measure of the energy loss in a medium due to the absorption of EM Radiation

$\gamma = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$ → squaring both the sides

$\alpha^2 - \beta^2 + 2j\alpha\beta = j\omega\mu\sigma - \omega^2\mu\varepsilon.$

$$\boxed{\alpha^2 - \beta^2 = -\omega^2\mu\varepsilon. \rightarrow (1)}$$

$2\alpha\beta = \omega\mu\sigma.$

$\sqrt{\alpha^2 + \beta^2} = \sqrt{\sqrt{(\sigma\omega\mu)^2 + (\omega^2\mu\varepsilon)^2}}$ — modulus

$\alpha^2 + \beta^2 = \sqrt{\sigma^2\mu^2\varepsilon^2 + \omega^4\mu^2\varepsilon^2}$

$\alpha^2 + \beta^2 = \sqrt{\omega^4\mu^2\varepsilon^2\left(1 + \frac{\omega^2\mu^2\sigma^2}{\omega^4\mu^2\varepsilon^2}\right)}$ → common

$$\boxed{\alpha^2 + \beta^2 = \omega^2\mu\varepsilon\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} \rightarrow (2)}$$ → Add eq 1 & eq 2

$2\alpha^2 = \omega^2\mu\varepsilon\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} - \omega^2\mu\varepsilon.$

$$\alpha^2 = \frac{\omega^2\mu\varepsilon}{2}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]$$

- The loss tangent represents the fraction of the EM energy that is lost as heat in the medium.

$$\alpha = \omega \sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1\right]} \Rightarrow \text{attenuation constant}$$

Same way

$$\beta = \omega \sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right]}$$

$\Rightarrow$ Phase constant.

if wave is travelling in $+z$ direction.

$$\bar{E}(z,t) = E_0 e^{-\alpha z} \cos(\omega t - \beta z)\, \hat{a}_E$$

if $\omega t - \beta z = \text{const}$ (phase)

$$\frac{d}{dt}(\omega t - \beta z) = \text{const}$$

$$\omega - \beta \cdot \frac{dz}{dt} = 0$$

$$\frac{dz}{dt} = \omega/\beta = v_p = \text{phase velocity}$$

$\nabla \times E = \frac{\partial E_x(z)}{\partial z} a_y$

$\bar{H} = -\frac{1}{j\omega\mu} a_y \cdot \frac{\partial E_x(z)}{\partial z}$

$E_x(z) = E_0 e^{-\gamma z}$

$\bar{H} = \pm \frac{1}{j\omega\mu}(+\gamma) E_x(z) a_y$

$H = \frac{\gamma}{j\omega\mu} E_x(z)$

$\eta = E/H = \frac{j\omega\mu}{\gamma}$

for lossy medium $\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$

$\sigma << \omega\varepsilon$

Intrinsic Impedance :-

$$\eta = \frac{E_0}{H_0} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$$

(How?)

$$\eta = \sqrt{\frac{j\omega\mu}{j\omega\varepsilon\left(1+\frac{\sigma}{j\omega\varepsilon}\right)}}$$

$$|\eta| = \frac{\sqrt{\mu/\varepsilon}}{\left[1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{1/4}}$$

$\nabla \times E$

$\nabla \times E = -\partial B/\partial t$

$\nabla \times E = -j\omega\mu \bar{H}$

$\bar{H} = -\frac{1}{j\omega\mu}(\nabla \times E)$

$E \rightarrow x$ direction

$H \rightarrow y$ "

propagation $\rightarrow z$ direction

$$\begin{vmatrix} a_x & a_y & a_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x & E_y & E_z \end{vmatrix}$$

$$\theta_\eta = 0 - \frac{1}{2}\tan^{-1}\left(\frac{-\sigma/\omega\varepsilon}{1}\right)$$

$$\theta_\eta = \frac{1}{2}\tan^{-1}\left(\frac{\sigma}{\omega\varepsilon}\right)$$

$$\tan 2\theta_\eta = \frac{\sigma}{\omega\varepsilon}$$

5 - Imp Parameter — $\alpha, \beta, \eta, \theta_\eta, v_p$

Relation ship b/w α & β

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1\right]}$$

$$\frac{\alpha}{\beta} = \sqrt{\frac{\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1}{\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1}}$$

$$\frac{\alpha}{\beta} = \sqrt{\frac{\sqrt{1+(\tan 2\theta)} - 1}{\sqrt{1+\tan^2 2\theta} + 1}}$$

$$\frac{\alpha}{\beta} = \sqrt{\frac{\sec\theta - 1}{\sec\theta + 1}} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \sqrt{\frac{2\sin^2\theta/2}{2\cos^2\theta/2}}$$

$$\boxed{\frac{\alpha}{\beta} = \tan\theta/2} \qquad \boxed{\theta_\eta = \theta/2}$$

Electric field eqn wave along +z direction.

$$\bar{E} = E_0 e^{-\alpha z} \cos(\omega t - \beta z)\, \bar{a}_E$$

$$\bar{H} = H_0 e^{-\alpha z} \cos(\omega t - \beta z)\, \bar{a}_H$$

$$\frac{E_0}{H_0} = \eta \rightarrow \text{Impedance (Intrinsic)}$$

$$H_0 = E_0/\eta = \frac{E_0}{|\eta| \angle \theta_\eta}$$

$$\bar{H} = \frac{E_0}{|\eta| \angle \theta_\eta} e^{-\alpha z} \cos(\omega t - \beta z)\, \bar{a}_H$$

$$= \frac{E_0}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - \theta_\eta)\, \bar{a}_H$$

$$\hat{a}_k = \hat{a}_E \times \hat{a}_H$$

$\hat{a}_k$ ← direction of propagation.

$\hat{a}_E$ ↓ e.m. field direction

$\hat{a}_H$ → m.f. field direction

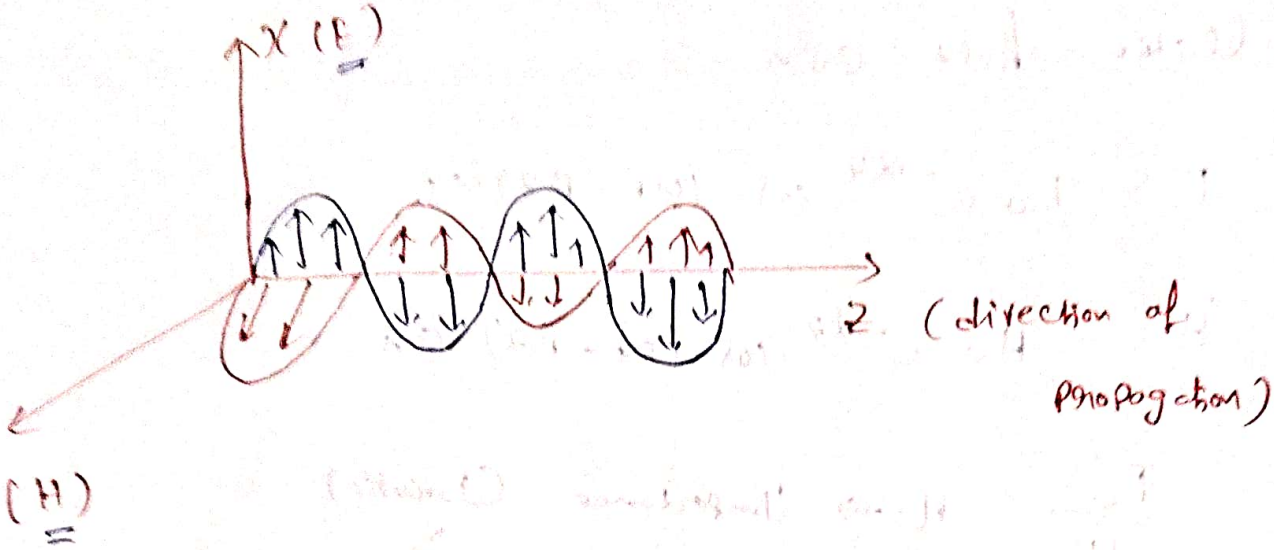
- if wave is travelling in the z-direction & e.m. field is having x-component, what will be the m.f. field direction

$$\hat{a}_z = \hat{a}_x \times \hat{a}_y$$

$\hat{a}_z$ ← direction

$\hat{a}_x$ ↓ e.m. field

$\hat{a}_y$ → m.f. field



Transverse Electromagnetic field wave

Lec-6

Wave propagation through lossless medium

In the previous section we had derived

$$\alpha = \omega \sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]}$$

In the lossless medium, No attenuation so

$\alpha = 0$, $\sigma = 0$, $\mu = \mu_0 \mu_r$, & $\varepsilon = \varepsilon_0 \varepsilon_r$.

↓ (conductivity)

$$\alpha = \omega \sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+(0)^2} - 1\right]} = 0.$$

$$\boxed{\alpha = 0}$$

Phase constant

$$\beta = \omega \sqrt{\frac{\mu\varepsilon}{2}\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1}$$

$$= \omega \sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1}+1\right]}$$

$$\boxed{\sigma = 0.}$$

$$= \omega \sqrt{[1+1]\frac{\mu\varepsilon}{2}} \qquad = \omega\sqrt{\mu\varepsilon}.$$

$$\beta = \omega\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r} \qquad = \omega\sqrt{4\pi\times10^{-7}\times\frac{10^{-9}}{36\pi}\cdot\mu_r\varepsilon_r}$$

$$= \omega\sqrt{\frac{10^{-16}}{9}\mu_r\varepsilon_r}$$

$$\beta = \frac{\omega}{3\times10^8}\sqrt{\mu_r\varepsilon_r} \qquad = \frac{\omega}{c}\sqrt{\mu_r\varepsilon_r}.$$

if $\mu_r = 1$, $\boxed{\beta = \frac{\omega}{c}\sqrt{\mu_r}}$ → Non-magnetic medium $\mu_r = 1$

most of practical dielectric media are lossless in nature.

Intrinsic Impedance :-

intrinsic Imp $\eta = \sqrt{\dfrac{j\omega\mu}{\sigma+j\omega\varepsilon}}$.

$$|\eta| = \sqrt{\frac{\sqrt{\sigma^2+(\omega\mu)^2}}{\sqrt{\sigma^2+(\omega\varepsilon)^2}}} = \frac{\sqrt{\omega\mu}}{(\sigma^2+(\omega\varepsilon)^2)^{1/4}}$$

$$|\eta| = \frac{\sqrt{\mu/\varepsilon}}{\left[1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{1/4}}.$$

Same way

$$\boxed{\tan 2\theta_\eta = \frac{\sigma}{\omega\varepsilon}}$$

For lossless medium $\sigma = 0$

$$|\eta| = \sqrt{\mu/\varepsilon} = \sqrt{\frac{\mu_0 \mu_r}{\varepsilon_0 \varepsilon_r}} \quad (\theta_\eta = 0)$$

$$|\eta| = 120\pi \sqrt{\frac{\mu_r}{\varepsilon_r}}$$

$\mu_0 = 4\pi \times 10^{-7}$

$\varepsilon_0 = 10^{-9}/36\pi$

Lossless medium

if $\mu_r = \varepsilon_r = 1$ then

$$\boxed{|\eta| = 120\pi}$$

we know that phase velocity

$$\phi = \omega t - \beta z.$$

$$\frac{d\phi}{dt} = \omega - \frac{\partial z}{\partial t} \cdot \beta.$$

$$\frac{\partial z}{\partial t} = v_p = \frac{\partial \phi}{\partial t} = \frac{\omega}{\beta}$$

For lossless medium $\beta = \omega \sqrt{\mu_r \varepsilon_r} = \frac{\omega}{c} \sqrt{\mu_r \varepsilon_r}$.

Then $v_p = \dfrac{\omega}{\frac{\omega}{c}\sqrt{\mu_r \varepsilon_r}} = \dfrac{c}{\sqrt{\mu_r \varepsilon_r}} = v_p$

Wave Propagation Through free space (air)

$\sigma = 0$, $\mu = \mu_0$, $\varepsilon = \varepsilon_0$

if $\sigma = 0$.

$$\alpha = \omega \sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1\right]}$$

So $\alpha = 0$.

$$\beta = \omega \sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right]}$$

So $\beta = \frac{\omega}{c}\sqrt{\mu_r \varepsilon_r}$. $\eta = 120\pi \sqrt{\frac{\mu_r}{\varepsilon_r}}$ → Low Loss.

↓ Low Loss

$\beta = \frac{\omega}{c}$ $\eta = 120\pi = \eta_0$

$$v_p = \frac{\omega}{\beta} = c$$

low loss medium.

$\alpha = 0$, $\beta = \omega\sqrt{\mu\varepsilon}$. =

Wave Propagation Through Good Conductor

$\sigma \simeq \infty$, $\mu = \mu_0 \mu_r$, $\varepsilon = \varepsilon_0$, $\sigma >> \omega\varepsilon$

$$\alpha = \omega \sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]}, \quad \beta = \omega \sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1\right]}$$

$$\alpha = \omega \sqrt{\frac{\mu\varepsilon}{2} \cdot \frac{\sigma}{\omega\varepsilon}}$$

$$= \sqrt{\frac{\omega^2 \mu\varepsilon\sigma}{2\omega}} = \sqrt{\frac{\sigma\mu\omega}{2}}$$

$$\boxed{\alpha = \sqrt{\pi f \mu \sigma} = \beta}$$

Intrinsic Impedance

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = \sqrt{\frac{j\omega\mu}{\sigma(1 + j\omega\varepsilon/\sigma)}}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma}}$$

$\sigma \simeq \infty$. $\frac{j\omega\varepsilon}{\sigma}$ very low and negligible.

$$|\eta| = \sqrt{\frac{\omega\mu}{\sigma}}$$

$$\theta_\eta = \frac{1}{2} \tan^{-1}\left(\frac{\omega\mu}{\frac{\sigma}{0}}\right) = \frac{1}{2}\tan^{-1}(\infty)$$

$$\theta_\eta = \frac{1}{2} \times \pi/2 = \pi/4$$

Then $\eta = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ$

$= \sqrt{\frac{\pi f \mu}{\sigma}} (1+j)$. $\Rightarrow$ impedance is complex & inductive in nature.

$$\boxed{V_p = \frac{\omega}{\beta}}$$

Skin depth :-

- it is a measure of how much a plane wave penetrate through medium

- it is the distance at which electric field becomes $1/e$ times of peak amplitude of e-c field.

$$E(z,t) = E_0 e^{-\alpha z} \cos(\omega t - \beta z)$$

$\frac{E_0}{e} = E_0 e^{-\alpha \delta}$ $\hookrightarrow$ wave travelling along z direction

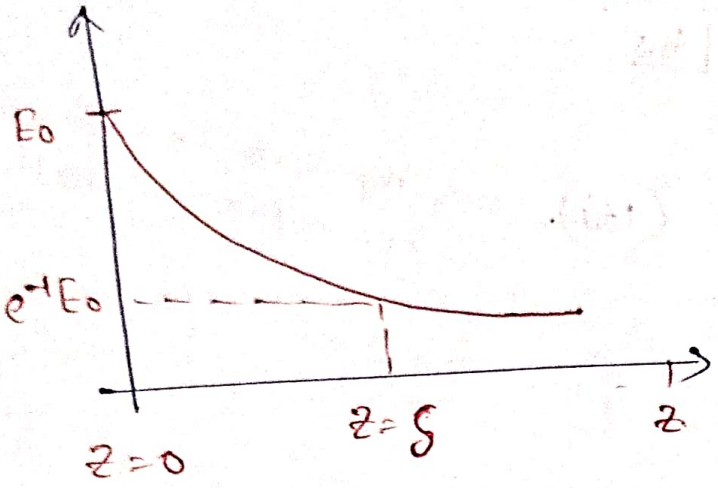
and wave should move δ distance to reduce its peak amplitude to E_0/e.

$$e^{-1} = e^{-\alpha \delta}.$$

$$\alpha \delta = 1$$

$$\boxed{\delta = 1/\alpha}$$

- valid for lossy medium
- " " good conductor



Complex Permittivity :-

$$\nabla \times \bar{H} = J + J_D = J + \frac{\partial D}{\partial t}$$

$$= J + \varepsilon \cdot \frac{\partial E}{\partial t} = J + J_c$$

$$J = \sigma E, \quad E = E_0 e^{j\omega t}$$

$$\frac{\partial E}{\partial t} = j\omega \cdot E$$

so $$\nabla \times \bar{H} = (\sigma + j\omega\varepsilon)\, E$$

$$= j\omega\varepsilon \left(1 + \frac{\sigma}{j\omega\varepsilon}\right) E$$

$$\nabla \times \bar{H} = j\omega \left[\varepsilon \left(1 - j\frac{\sigma}{\omega\varepsilon}\right)\right] E$$

$$\varepsilon = \varepsilon' - j\varepsilon''$$

Permittivity $= \varepsilon$

$$\varepsilon'' = \sigma/\omega$$
$$\varepsilon = \varepsilon'$$

ε_c = complex Permittivity $= \varepsilon \left(1 - j\frac{\sigma}{\omega\varepsilon}\right)$

$$\epsilon_c = \epsilon - j\frac{\sigma}{\omega}$$

$$\epsilon_c = \epsilon' - j\epsilon''$$

$$\frac{\text{imaginary part}}{\text{real part}} \quad \frac{\epsilon''}{\epsilon'} = \frac{\sigma}{\omega\epsilon} = \text{loss tangent.}$$

Good conductor $\eta_c = \sqrt{\frac{\omega\mu}{\sigma}}$ — σ high.

Perfect conductor $\sigma = \infty$, $\eta = 0$

(5) A 50 MHz uniform plane wave is propagating in a material with relative permeability and permittivity of 2.25 and 1 respectively. The material is assumed to be loss less find the phase constant of the wave propagation

(1) $f = 50$ MHz, $\mu = 2.25$, $\epsilon = 1$.

material = loss less.

$$V_p = \frac{\omega}{\beta} = \frac{2\pi f}{\beta} = \text{Phase velocity} \quad \therefore \frac{2\pi \times 50}{\pi/2} = 200$$

for loss less $\beta = \frac{\omega}{c}\sqrt{\mu_r \epsilon_r}$ = phase constant

$$= \frac{2\pi \times 50 \times 10^6}{3 \times 10^8}\sqrt{(2.25)(1)}$$

$$= 10^8 \times \pi / 3 \times 10^8 \cdot (1.5) = \pi/2 \text{ rad/m}$$

$\alpha = 0$ for lossless medium

$\downarrow$

$\sigma = 0$

$$\alpha = \omega \sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]}$$

$$\eta = 120\pi \sqrt{\frac{\mu_r}{\varepsilon_r}} \quad \text{— for lossless medium}$$

$$\eta = 120\pi \sqrt{\frac{2.25}{1}} = 180\pi.$$

(2) A non-magnetic lossy dielectric material with relative permittivity $\varepsilon_r = 2.25$ and conductivity $= 10^{-4}$ mho is applied with EM wave of 2.5 MHz, then what is the loss tangent.

(A) Material = lossy dielectric

for lossy dielectric $= \frac{\sigma}{\omega\varepsilon}$ = lossy tangent

$$= \frac{10^{-4}}{2\pi \times 2.5 \times 10^6 \times 2.25 \times \frac{10^{-9}}{36\pi}} = 0.32$$

$\frac{\sigma}{\omega\varepsilon} << 1 \rightarrow$ lossy medium.

$$\alpha = \omega \sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]}$$

$$= 2.5\times10^6 \sqrt{\frac{\mu_r \cdot \mu_0 \cdot \varepsilon_r \cdot \varepsilon_0}{2}\left[\sqrt{1+(0.32)^2} - 1\right]}$$

$$= 1.975\times10^{-3}$$

$$\beta = \omega \sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1\right]}$$

$$\eta = \frac{\sqrt{\mu/\varepsilon}\ \angle}{\left[1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{1/4}}$$

$$\theta_\eta = \frac{1}{2}\tan^{-1}\left(\frac{\sigma}{\omega\varepsilon}\right).$$

Lec-7

(Q) An EM wave propagating in a certain medium is described by $E = 25 \sin(2\pi \times 10^6 t - 6x)\, a_z$ V/m

(a) Determine the direction of wave propagation

(b) Compute period T, wavelength λ, velocity u.

(A) given $\bar{E} = 25 \sin(2\pi \times 10^6 t - 6x)\, a_z$.

$E_0 = 25$, $\omega = 2\pi \times 10^6$, $\beta = 6$ rad/m.

$E = E_0 \sin(\omega t - \beta z)\, \hat{a}_x$

$\hat{a}_x$ → direction of E

↓

direction of propagation (wave)

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{2\pi \times 10^6} = 1\ \mu sec$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{6} = \frac{\pi}{3}\ m.$$

Direction of wave propagation = x.

" " E-wave " = z.

→ in the question not given any information about μ_r & ε_r

so we can take $\varepsilon_r = \mu_r = 1$.

$$v_p = \omega/\beta = \frac{8\pi \times 10^6}{6} = 1.047 \times 10^6 \text{ m/sec.}$$

Q :- At 50 MHz, a lossy dielectric material is characterized by $\varepsilon = 3.6\,\varepsilon_0$, $\mu = 2.1\,\mu_0$, $\sigma = 0.08$ S/m. if $\bar{E}_x = 6e^{-\gamma x}\,\bar{a}_z$ V/m

(a) γ (b) λ, (c) u (d) η (e) H.

Ans:- material :- lossy, $\sigma \neq 0$, $\varepsilon = \varepsilon_r \varepsilon_0$,
dielectric $\mu = \mu_r \mu_0$.

and $f = 50$ MHz,

$\varepsilon_r = 3.6$, $\mu_r = 2.1$, $\sigma = 0.08$ S/m.

$\bar{a}_E = \hat{a}_z \rightarrow$ direction of E-wave propagation.

(a) $\gamma = \alpha + j\beta$.

free space	$\sigma = 0$,	$\varepsilon = \varepsilon_0$, $\mu = \mu_0$
Loss less dielectric	$\sigma \simeq 0$,	$\varepsilon = \varepsilon_r \varepsilon_0$, $\mu = \mu_r \mu_0$. ($\sigma \ll \omega\varepsilon$)
Lossy "	$\sigma \neq 0$,	$\varepsilon = \varepsilon_r \varepsilon_0$, $\mu = \mu_r \mu_0$ ($\sigma \simeq \omega\varepsilon$)
good conductor	$\sigma \simeq \infty$,	$\varepsilon = \varepsilon_0$, $\mu = \mu_r \mu_0$ ($\sigma \gg \omega\varepsilon$)

From the above discussion,

if $\sigma \neq 0 \rightarrow$ there two possible cases

(1) lossy dielectric ($\sigma \simeq \omega\varepsilon$)

(2) good conductor ($\sigma > \omega\varepsilon$).

To find (1) first calculate $\frac{\sigma}{\omega\varepsilon}$.

$$= \frac{0.08 \rightarrow \sigma}{2\pi \times 50 \times 10^6 \times 3.6 \times 10^{-9}/36\pi} = 8.$$

(f; $\varepsilon = \varepsilon_r \cdot \varepsilon_0$)

$\frac{\sigma}{\omega\varepsilon} > 1 \rightarrow$ good conductor.

$\alpha = \beta = \sqrt{\pi f \mu \sigma}$ = for good conductor.

You can go with $\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right]}$ and $\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1\right]}$

$= 5.758$ (But answers will differ)

$\gamma = \alpha + j\beta = 5.758 + j\,5.578 \text{ /m}.$

(b) $\lambda = 2\pi/\beta = \frac{2\pi}{5.758} = 1.091 \text{ m}.$

(c) velocity $v = \lambda f = 1.091 \times 50 \times 10^6$

$= 54.55 \times 10^6 \text{ m/sec}$

(D) $|\eta| = \sqrt{\frac{\mu\omega}{\sigma}} = \sqrt{\frac{2\pi f \times \mu_0 \cdot \mu_{r1}}{\sigma}}$

$|\eta| = 101.8\ \Omega \qquad \theta_\eta = \pi/4$

(E) Expression of MI field.

$\bar{H} = H_0 e^{-\sqrt{\cdot}x} \cdot \bar{a}_H$

$H_0 = E_0/\eta = \frac{6}{(101.8)\angle 45^\circ}$

$= 58.93 \angle -45^\circ$ mA/m.

$\bar{a}_H = \bar{a}_{IC} \times \bar{a}_E = \hat{a}_x \times \hat{a}_z = -\hat{a}_y$.

$\bar{H} = 58.93 \angle -45^\circ \; e^{-(5.7+j5.7)x}(-\hat{a}_y)$ mA/m.

$= 58.93\ e^{-\sqrt{\cdot}x}\ e^{-j\pi/4}\cdot(-\hat{a}_y)$ mA/m.

(a) A lossy material has $\mu = 5\mu_0$, $\varepsilon = 2\varepsilon_0$, at 5 MHz, the phase constant is 10 rad/m. Calculate.

(A) loss tangent (B) ε (complex permittivity)

(b) σ (d) α (attenuation constant)

(e) intrinsic Impedance (η).

Ans

(A) loss tangent $= \frac{\sigma}{\omega\varepsilon}$

given $\mu_r = 5$, $\varepsilon_r = 2$, $f = 5$ MHz, $\beta = 10$ rad/m

$$\beta = \frac{\omega}{c}\sqrt{\frac{\mu_r\varepsilon_r}{2}\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1}$$

$\frac{\sigma}{\omega\varepsilon} = 1823$.

$\frac{\sigma}{\omega\varepsilon} >> 1 \rightarrow$ good conductor

(B) $\sigma = 1823 \times \omega\varepsilon$. $\hookrightarrow \varepsilon_0\varepsilon_r$.

$= 1.013$ S/m.

You can use α, β, γ formulas of good conductor

(C) Complex permittivity

$\varepsilon_c = \varepsilon' - j\varepsilon''$

$\varepsilon'' = \sigma/\varepsilon$, $= 3.22 \times 10^{-8}$

$\varepsilon' = \varepsilon = 2\varepsilon_0 = 2 \times 10^{-9}/36\pi = 17.7 \times 10^{-12} - j\,3.22 \times 10^{-8}$

(d) $$\alpha = \frac{\omega}{c}\sqrt{\frac{\mu_r \varepsilon_r}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1\right]}$$

$$= 9.995 \quad \text{NP/M.}$$

(e) η (Intrinsic Impedance)

$$\eta = \sqrt{\frac{j\omega\mu}{j\omega\varepsilon\left(1+\frac{\sigma}{\omega\varepsilon}\right)}} = \text{general formula.}$$

$$\eta = \sqrt{\frac{\omega\mu}{\sigma}} = \text{for good conductor}$$

if $\sigma >> \omega\varepsilon$.

$$\eta = 13.95 \angle 45^\circ$$

(f) a non magnetic medium has $\eta = 240 \angle 30^\circ\ \Omega$ Then find (a) The loss tangent (b) Dielectric const (c) Complex permitivity (d) α at 1 MHz

(A) $\eta = 240\Omega, \quad \theta_\eta = 30^\circ.$

(a) $\theta_\eta = \frac{1}{2}\tan^{-1}\left(\frac{\sigma}{\omega\varepsilon}\right). \quad \frac{\sigma}{\omega\varepsilon} = \sqrt{3}.$

(b) $\varepsilon_r = 12. \quad \eta = \frac{120\pi/\sqrt{\varepsilon_r}}{\left[1+(\sqrt{3})^2\right]^{1/4}}$

$\varepsilon_r = \left[2.46/2 = 1.23\right]$

(c) $\varepsilon_c = \varepsilon' - j\varepsilon''$

$= \varepsilon - j\frac{\sigma}{\omega} = \varepsilon\left(1 - j\frac{\sigma}{\omega\varepsilon}\right)$.

$= \varepsilon_0\varepsilon_r \left(1 - j\sqrt{3}\right) = 1.23\,\varepsilon_0 \left(1 - j\sqrt{3}\right)$

(d) $\alpha = \frac{\omega}{c}\sqrt{\frac{\mu_r\varepsilon_r}{2}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]}$

$\alpha = 0.0164$ Np/m

$= \frac{2\pi \times 10^6}{3\times 10^8}\sqrt{\frac{1\times 1.23}{2}\left[\sqrt{1+(\sqrt{3})^2} - 1\right]}$

(Q) Alkine 'sea water' $\sigma = 4$ S/m, $\varepsilon_r = 80$, $\mu_r = 1$.

& $f = 100$ MHz, calculate.

(a) v_p (b) λ (c) δ (d) η.

(a) $v_p = \frac{\omega}{\beta}$ $\qquad \frac{\sigma}{\omega\varepsilon} = \frac{4}{2\pi\times 10^8 \times 80\times 10^{-9}/36\pi}$

$\frac{\sigma}{\omega\varepsilon} = 9$

$\sigma > \omega\varepsilon$ → So good conducted.

For good conducted

$$\beta = \alpha = \sqrt{\pi f \mu \sigma}$$

You can use this one also

$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right]}$

$$= \sqrt{\pi \times 108 \times \mu_0 \times \mu_r \times 4}$$

$\mu_0 = 4\pi \times 10^{-7}$

$\alpha = \beta = 39.73$

$$VP = \frac{2\pi \times 108}{39.73} = 15.81 \times 10^6 \text{ m/sec}$$

(b) $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{39.73} = 0.158 \text{ m}$.

This will hold for free space

$f = c/\lambda$ $\lambda = \frac{3\times10^8}{100\times10^6} = 3\text{m}$

(c) $\delta = 1/\alpha = \frac{1}{39.73} = 25.16 \text{ mm}$

(d) $\eta = \sqrt{\frac{\mu\omega}{\sigma}} = \sqrt{\frac{\mu_0 \times \mu_r \times 2\pi f}{\sigma}}$

$\mu_r = 1$, $\mu_0 = 4\pi \times 10^{-7}$ $f = 108$, $\sigma = 4$

$\eta = 14.05 \angle 45^\circ$

Lec-8

Q The amplitude of a wave travelling through a lossy non magnetic medium reduces by 18% every meter. if wave operates at 10 MHz, and the e.f field leads M.f field by 24° then

(a) Propagation constant (γ).

(b) The wavelength (λ).

(c) The skin depth (δ)

(d) Conductivity of the medium (σ).

A

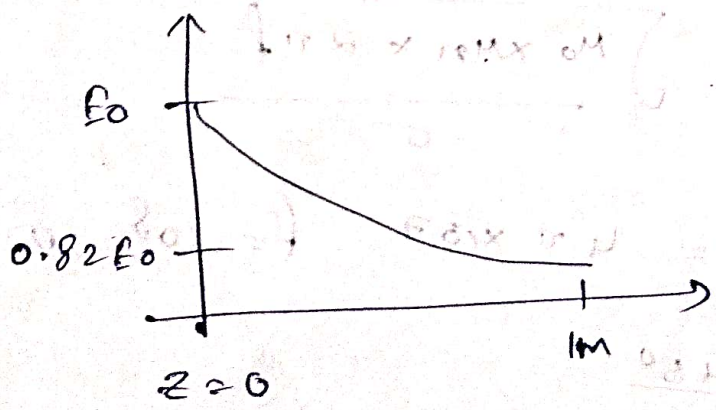
we know that

Phase

$$E(z,t) = E_0 e^{-\alpha z} \cos(\omega t - \beta z)\, \hat{a}_E$$

$E_0 e^{-\alpha z}$: amplitude; ω : freq; βz : direction of propagation; $\hat{a}_E$: direction of e- field.

So from the given data



amplitude $= E_0 e^{-\alpha z}$

at $z = 1$ m, amp $= 0.82 E_0$

$0.82 E_0 = E_0 e^{-\alpha}$

$\alpha = 0.198$

To find propagation constant

$$\gamma = \alpha + j\beta.$$

$\alpha = 0.198$. To find β:

we know $\alpha/\beta = \tan\theta/2 = \tan\theta_n$

$$\beta = \frac{\alpha}{\tan\theta_n} = \frac{0.198}{\tan(24^0)} = 0.445 \text{ rad/m}$$

$$\gamma = \alpha + j\beta = (0.198 + j0.445)\ m^{-1}$$

(B) $$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.445} = 14.09 \text{ m.}$$

(C) Skin depth $\delta = 1/\alpha$

$$\delta = \frac{1}{0.198} = 5.05 \text{ m.}$$

(D) conductivity (σ)

$$\alpha = \frac{\omega}{c}\sqrt{\frac{\mu_r \varepsilon_r}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]}$$

$$\tan\theta_n = \frac{\sigma}{\omega\varepsilon}. \qquad \theta_n = \theta/2$$

$\theta = 2\theta_n \hookrightarrow 24^\circ$

$$\frac{\sigma}{\omega\varepsilon} = \tan(48^\circ)$$

$$\sigma = 2\pi \times 10 \times 10^6 \times 3.63 \times 10^{-9}/36\pi \times \tan 48^\circ$$

$$\boxed{\sigma = 2.24 \times 10^{-3}}$$

(Q) in a certain medium with $\mu = \mu_0$ $\varepsilon = 4\varepsilon_0$

$$H = 12\, e^{-0.1y} \sin(\pi \times 10^8 t - \beta y)\, a_x \text{ A/m.}$$

Then find.

(a) wave period T (b) wavelength (c) electric field

(d) phase diff b/w E & H.

Ans:-

(a) $\frac{2\pi}{\omega} = T = \frac{2\pi}{2\pi \times f} = \frac{2\pi}{\pi \times 10^8} = 20\ \text{nsec}$

(b) $\lambda = \frac{2\pi}{\beta}$. you can use $\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right]}$

$$\beta = \frac{\omega}{c}\sqrt{\mu_r \varepsilon_r} = \frac{\pi \times 10^8}{3 \times 10^8}(\sqrt{4}) = 2\pi/3$$

$$\lambda = 2\pi/(2\pi/3) = 3\text{m}.$$

(c) Electric field E.

$$\hat{E} = E_0 e^{-\alpha y} \cdot \sin(3\pi \times 10^8 t - \beta y)\, \hat{a}_E$$

$$\alpha = \frac{\omega}{c} \sqrt{\frac{\mu_r \varepsilon_r}{\sqrt{2}} \sqrt{1 + \left(\frac{\sigma}{\omega \varepsilon}\right)^2}}$$

we know α from

$$H = 12\, e^{0.1y} \sin(3\pi \times 10^8 t - \beta y)\, \hat{a}_x \text{ A/m.}$$

$$\alpha = 0.1, \quad \beta = \frac{2\pi}{3}.$$

$$\alpha/\beta = \tan\theta_n$$

$$\theta_n = \tan^{-1}\left(\frac{0.3}{2\pi}\right) = 2.73^\circ$$

$$\hat{a}_E = \hat{a}_H \times \hat{a}_{ic}$$

$$= \hat{a}_y \times \hat{a}_y = \hat{a}_z = \hat{a}_E$$

we know $\theta_n = \theta/2$ → angle of load tangent

↓

phase of Impedance

$\tan(2\theta) = \alpha/\beta$

$\tan(2\theta) = 2\alpha/\beta$

$\theta = 5.46^\circ$

to find E_0, we need η

$|\eta| = |E_0|/|H_0|$

$|H_0| = 12, \quad |\eta| = \dfrac{\sqrt{\mu/\varepsilon}}{\left(1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2\right)^{1/4}}$

$|\eta| = 188.06\,\Omega$

$|E_0| = |\eta|\,|H_0| = (188.06)(12) = 2256.8$

$\bar{E} = (2256.8)\, e^{-0.1y} \sin\left(\pi \times 10^8 t - \frac{2\pi}{3} y + 2.730\right) \hat{a}_z$

Q) in a medium.

$$\bar{E} = 16 e^{-0.05x} \sin(2\times10^8 t - 2x)\, a_z \ \text{V/m}.$$

($2\pi f$, β)

Find

(a) Propagation constant

(b) The wavelength

(c) Speed of the wave

(D) Skin depth.

Ans

(a) $\gamma = \alpha + j\beta = (0.05 + j2)\,m^{-1}$

(b) $\boxed{\lambda = 2\pi/\beta} = 2\pi/2 = \pi\ m.$

(c) $v_p = \omega/\beta = \dfrac{2\pi f}{\beta} = \dfrac{2\times10^8}{2}$

(d) $\alpha = 1/\delta \Rightarrow \delta = 1/\alpha = \dfrac{1}{0.05} = 2m.$

(Q) A uniform wave in air has.

$$E = 10 \cos(2\pi 10^6 t - \beta z)\, \hat{a}_y$$

(a) Calculate β

(b) Find H.

(Ans) Given $\alpha = 0$ = free space. ($\sigma = 0$).

$E_0 = 10$, $f = 10^6$ Hz.

wave Travelling along z-direction

E-c field along y-direction.

medium = air = free space So $\alpha = 0$

(a) So $\beta = \frac{\omega}{c} = \frac{2\pi \times 10^6}{3 \times 10^8} = 0.02$ rad/m

(b) $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.02} = 3.114 \times 100$

$= 314$ m.

(c) $H = H_0 \cos(\omega t - \beta z)\, \hat{a}_H$

$\hookrightarrow = \hat{a}_k \times \hat{a}_E$

$= \hat{a}_z \times \hat{a}_y$

$= -\hat{a}_x$

$H_0 = E_0/\eta_0 = \frac{10}{120\pi} = \frac{1}{12\pi}$

$= \frac{.1}{12\pi}$

(24)

(Q) The magnetic field component of an EM wave propagating through a non magnetic medium ($\mu = \mu_0$) is

$$\bar{H} = 25 \sin (2\times10^8 t + 6x)\, \hat{a}_y \text{ mA/m}.$$

Then calculate

(a) direction of wave propagation

(b) ε

(c) electric field intensity

Ans $H = H_0 e^{-\alpha x} \sin(\omega t - \beta z)\, \hat{a}_H$ → wave prop-g th-

Given $H_0 = 25$ mA/m.

$\omega = 2\times10^8$ rad/sec

m.c field direction $\hat{a}_H = \hat{a}_y$

(a) wave propagation $\hat{a}_k = -\hat{a}_x$

(b) $\varepsilon = \varepsilon_0 (\varepsilon_r = 81)\, \varepsilon_0$

$$\beta = \frac{\omega}{c} \sqrt{\mu_r \varepsilon_r}$$

$\varepsilon_r = 81$,

$\varepsilon = 7.17 \times 10^{-10}$ F/m.

Electric field Intensity equation for lossless medium

i.e. $\alpha = 0$ & $\sigma = 0$

$$\bar{E} = E_0 \sin(\omega t + \beta z)\, \hat{a}_E$$

$$\eta = E_0 / H_0 \qquad E_0 = \eta H_0.$$

$$= 120\pi \cdot (25 \times 10^{-3}).$$

↓

lossless medium

$$E_0 = \eta \cdot H_0 = 1.047 \text{ V/m}.$$

$$\hat{a}_E = \hat{a}_H \times \hat{a}_{ic}$$

$$= \hat{a}_y \times (-\hat{a}_H) = \hat{a}_z$$

$$\bar{E} = 1.047 \sin(2 \times 10^8 t + 6x)\, \hat{a}_z \text{ V/m}.$$

Q) if $\bar{H} = 10 \sin(\omega t - 4z)\, \hat{a}_y$ mA/m in a material for which $\sigma = 0$, $\mu = \mu_0$, $\varepsilon = 4\varepsilon_0$. find

(a) ω (b) λ & (c) I_d

Ans:-

$H_0 = 10$ mA/m, $\hat{a}_k = \hat{a}_z$, $\hat{a}_H = \hat{a}_y$

$\beta = 4$, $\varepsilon_r = 4$ (a) $\beta = \frac{\omega}{c}\sqrt{\mu_r \varepsilon_r} = \frac{\omega}{3\times10^8}\sqrt{4}$

$\omega = 6\times10^8$ rad/sec

(b) $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{4} = \pi/2$ m.

(c) $\bar{J}_d = \frac{\partial \bar{D}}{\partial t} = \varepsilon \cdot \frac{\partial \bar{E}}{\partial t}$

$\bar{E} = E_0 \sin(\omega t - 4z)$

$\hat{a}_E = \hat{a}_H \times \hat{a}_k = \hat{a}_y \times \hat{a}_z = -\hat{a}_x$.

$\eta = 120\pi/\sqrt{\varepsilon_r} = 60\pi$ $\eta = E_0/H_0 = 60\pi$

$\bar{E} = 600\pi \times 10^{-3} \sin(\omega t - 4z)(-\hat{a}_x)$

Lect-9

(Q) a manufactured produced a feasible material. with $\mu = 750\,\mu_0$, $\varepsilon = 5\varepsilon_0$ and $\sigma = 10^{-6}$ S/m at 10 MHz.

(a) would you classify the material as lossless, lossy or conducting?

(b) calculate β and λ?

(c) Determine the phase difference b/w two points seperated by 2m.

(d) find η.

(Ans)

(a) $$\frac{\sigma}{\omega\varepsilon} = \frac{10^{-6}}{2\pi \times 10^7 \times 5 \times \varepsilon_0} = 3.59 \times 10^{-4} \ll$$

$\boxed{\sigma \ll \omega\varepsilon}$ so it's low loss dielectric.

(b) $$\beta = \omega \sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1\right]}.$$

in low loss dielectric.

$$\beta = \frac{\omega}{c}\sqrt{\mu_r \varepsilon_r} = \frac{2\pi \times 10^7}{3 \times 10^8}\sqrt{750 \times 5}$$

$= 12.81$ rad/m

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{12.81} = 0.49 m$$

Phase shift ϕ = β x distance

$$\phi = 12.81 \times 2 = 25.62 \text{ rad.}$$

$$\eta = \frac{120\pi \sqrt{\mu_r/\epsilon_r}}{\left[1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2\right]^{1/4}} = \frac{\sqrt{\mu/\epsilon}}{\left[1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2\right]^{1/4}}$$

$\eta = \sqrt{\mu/\epsilon}$ => for lossy dielectric.

$$\eta = 4.62 \text{ k}\Omega$$

5) A lossy dielectric has an $\eta = 200 \angle 30°\ \Omega$ at a particular radian frequency ω, if at that frequency the plane wave propagating through the dielectric has the magnetic field component is given by

$$\bar{H} = 10\, e^{-\alpha x} \cos(\omega t - \gamma_2 x)\, \hat{a}_y \text{ A/m}$$

Calculate attenuation constant, skin depth, electric field and direction of polarization.

(Aw)

$$\eta = 200 \angle 30^\circ, \quad H_0 = 10, \quad \beta = 1/2,$$

$$a_k = a_x, \quad a_H = a_y$$

$$\alpha/\beta = \tan \theta/2 \pm \beta \tan \theta_\eta$$

(a) $$\alpha = \beta \tan \theta_\eta = \frac{1}{2} \tan(30^\circ) = \frac{1}{2\sqrt{3}} \text{ NP/m.}$$

(b) Skin depth $$\delta = 1/\alpha = 2\sqrt{3} \text{ m.}$$

(c) electric field for lossy medium

$$E = E_0 e^{-\alpha x} \cos(\omega t - \beta a_k) \, a_E$$

$$a_k \downarrow a_x$$

$$a_H = a_y \qquad a_E = a_H \times a_k$$

$$= -(a_x \times a_y) = -a_z$$

$$|E| = |H_0||\eta|$$

$$H_0 = 10, \quad |\eta| = 200$$

$$E_0 = 2000 \angle 30^\circ$$

$$\bar{E} = 2000 \angle 30^\circ \, e^{-\alpha x} \cos(\omega t - x/2)(-a_z).$$

$$= -2 e^{-\alpha x} \cos(\omega t - \frac{x}{2} + 30^\circ)(a_z). \text{ KV/m.}$$

5) The Intrinsic impedance of certain medium is $28.1 \angle 45^0\ \Omega$ at 1 MHz and the skin depth is 2m. find the σ, Vp & λ.

Ans

given $\eta = 28.1 \angle 45^0$

$\theta_\eta = 45^0 \quad |\eta_0| = 28.1\ \Omega$

$\tan\theta_\eta = \frac{\sigma}{\omega\varepsilon} = \sigma > \omega\varepsilon$ → good conductor

$\delta = 1/\alpha = 1/\beta$

$\beta = 1/\delta = 0.5$ rad/m

$\lambda = 2\pi/\beta = \frac{2\pi}{0.5} = 4\pi$ m.

$V_p = \frac{\omega}{\beta} = \frac{2\pi \times 10^6}{0.5} = 4\pi \times 10^6$ m/sec

for good conductor $\eta = \sqrt{\frac{\omega\mu}{\sigma}}$

$\alpha = \sqrt{\pi f \mu \sigma} = 1/\delta$

$\delta = 1/\alpha = \frac{1}{\sqrt{\pi f \mu \sigma}}$

$\eta \cdot \delta = \sqrt{\frac{2\pi f \mu}{\sigma}} \cdot \frac{1}{\sqrt{\pi f \mu \sigma}} \Rightarrow 28.1 \times 2 = \sqrt{2}/\sigma$

$\sigma = 25.16 \times 10^{-3}$ S/m

(Q) a plane wave propagating in a dielectric medi

with $\varepsilon_r = 8$, $\mu_r = 2$ hai.

$$\bar{E} = 0.5\, e^{-z/3} \sin(10^8 t - \beta z)\, a_x$$

find the value of phase constant, skin depth, n

Vp & $\bar{H}$

Ans

$$\bar{E} = E_0\, e^{-\alpha z} \sin(\omega t - \beta a_k)\, a_E .$$

$$\text{So } \alpha = 1/3 = \frac{\omega}{c}\sqrt{\frac{\mu_r \varepsilon_r}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]}$$

$$\frac{1}{3} = \frac{10^8}{3\times10^8}\sqrt{\frac{8\times2}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]}$$

$$1+(8)^2 = \sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}$$

$$\frac{\sigma}{\omega\varepsilon} = 0.515 \ll 1.$$

$\sigma \ll \omega\varepsilon$ → lossy medium.

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1\right]}$$

$$\beta = \frac{10^8}{3\times10^8}\sqrt{\frac{8\times2}{2}\left[\sqrt{1+(0.515)^2}+1\right]} = 1.374 \text{ rad/m}$$

For lossy dielectric

$$\eta = \omega\sqrt{\mu/\varepsilon} = 120\pi\sqrt{\mu_r/\varepsilon_r} = 177.73\,\Omega.$$

$$\theta_\eta = \frac{1}{2}\tan^{-1}\left(\frac{\sigma}{\omega\varepsilon}\right)$$

$$\boxed{\theta_\eta = 13.62^\circ}$$

Magnetic field $\vec{H} = H_0 e^{-\alpha(\text{dielectric wave})} \cdot \cos(\omega t - \beta z)\hat{a}_y$

$$\hat{a}_E = \hat{a}_H \times \hat{a}_k = \hat{a}_x = (\hat{a}_{xy}) \times \hat{a}_z$$

$$\hat{a}_{xy} \rightarrow \hat{a}_y$$

$$H_0 = E_0/\eta = \frac{0.5}{177.73\,\angle 13.62^\circ} = 2.81\,\angle -13.62^\circ \text{ mA/m}.$$

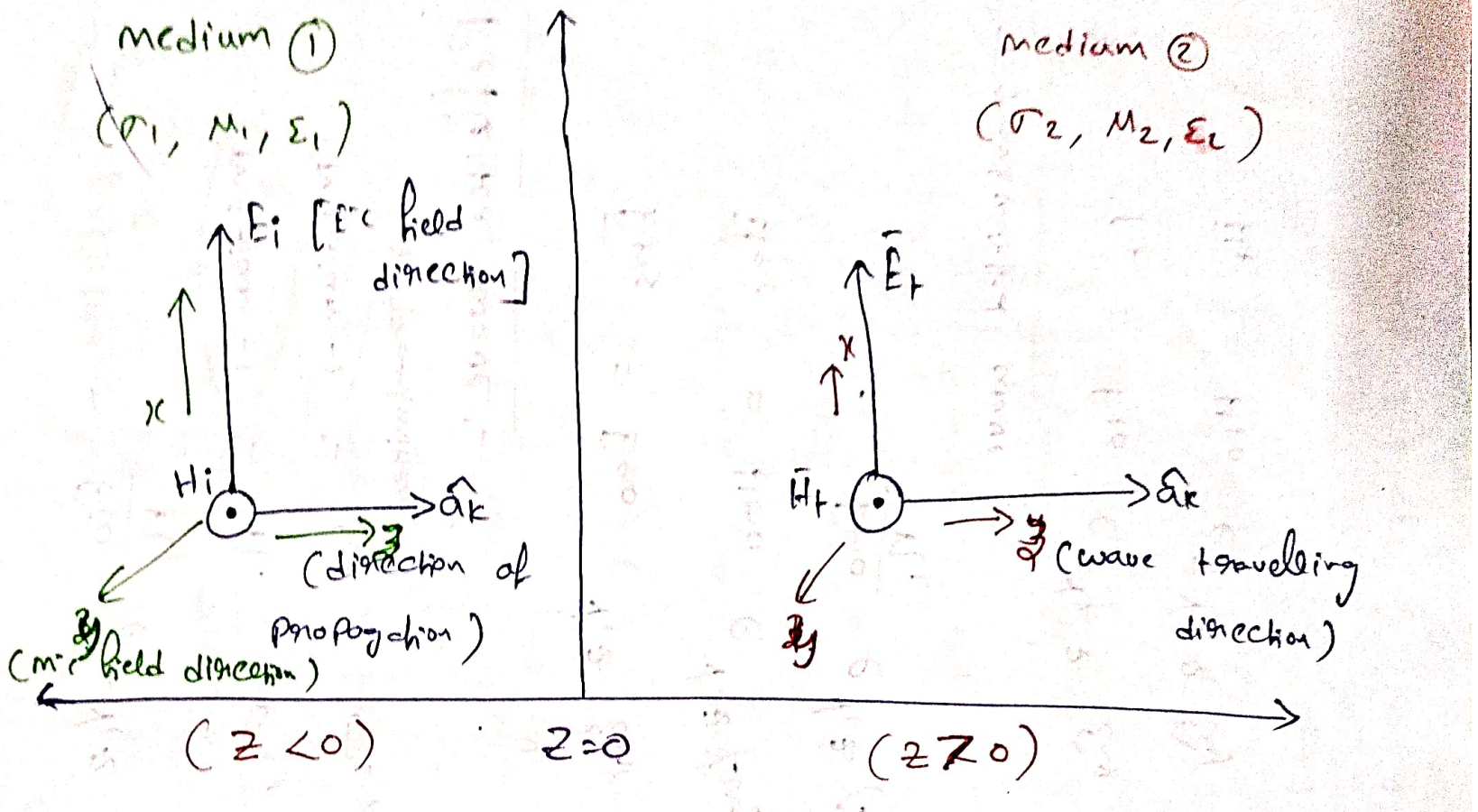
$$H = 2.81\,\angle -13.62^\circ\; e^{-\alpha z}\cos(10^8 t - \beta z)\hat{a}_y \text{ mA/m}$$

Lect - 10

Plane wave travel from one medium to another

Reflection of plane wave at (normal) Incidence

Need to remember



i → incident

r → reflected

t → transmitted.

+ ŷ ⊙ – out side the page

– ŷ ⊗ – Inside the page



Incident wave :-

(1) — $E_{is} = E_{io}\, e^{-\gamma_1 z}\, \hat{a}_x$ (direction of wave; $\hat{a}_x$ → direction of e.c field.)

(2) — $\bar{H}_{is} = H_{io}\, e^{-\gamma_1 z}\, \hat{a}_y$ (→ direction of m.c field)

Incident, Reflected = $\frac{E_{io}}{\eta_1} e^{-\gamma_1 z}\, \hat{a}_y$

Medium-1 :- $\eta_1, \sigma_1, \mu_1, \varepsilon_1$

Medium-2 :- $\eta_2, \sigma_2, \mu_2, \varepsilon_2$ → Transmitted

Reflected wave :-

$\bar{E}_{rs} = E_{ro}\, e^{\gamma_1 z}\, \hat{a}_x \rightarrow (3)$ ($\rightarrow$ -ve z direction)

$\bar{H}_{rs} = H_{ro}\, e^{\gamma_1 z}(-\hat{a}_y)$

$\therefore = \frac{E_{ro}}{\eta_1} e^{\gamma_1 z}(-\hat{a}_y) \rightarrow (4)$

Transmitted wave :-

$\bar{E}_t = E_{to}\, e^{-\gamma_2 z}\, \hat{a}_x \longrightarrow (5)$

$\bar{H}_t = H_{to}\, e^{-\gamma_2 z}\, \hat{a}_y = \frac{E_{to}}{\eta_2} e^{-\gamma_2 z}\, \hat{a}_y \rightarrow (6)$

Total electric field in medium ①

$$\bar{E}_{1s}(z) = \bar{E}_{is}(z) + \bar{E}_{rs}(z)$$

Total electric field in medium ②

$$E_{2s} = \bar{E}_{ts}(z)$$

Electric Boundary Condition :- $E_{1t} = E_{2t}$; $D_{1n} - D_{2n} = \rho_s$

(derive it) - Page no. 187

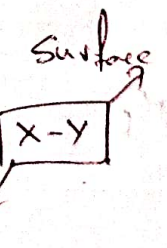
E(tangential component in medium-1) Sadiku book

= E (tangential component in medium-2).

In our scenario wave travelling along z-direction and boundary located at $z=0$ so.

z - component is normal

x, y - " is tangent



made with

$$E_1(z=0) = E_{is}(z=0) + E_{rs}(z=0)$$

$$E_{tan1} = E_{i0} + E_{r0}$$

$$E_2(z=0) = E_{ts}(z=0), \quad E_{tan2} = E_{ts}(z=0)$$

$$E_{tan2} = E_{t0}$$

From Boundary condition

$$E_{io} + E_{ro} = E_{to}$$

in Normal Incidence case

$$\frac{E_{io}}{E_{io}} + \frac{E_{ro}}{E_{io}} = \frac{E_{to}}{E_{io}}$$

$$1 + \Gamma_E = \tau_E \rightarrow (7)$$

Reflection coefficient :-

$$= \frac{\text{Peak value of reflected electric field}}{\text{" " " Incident " "}}$$

$$\Gamma_E = \frac{E_{ro}}{E_{io}} \rightarrow (8)$$

Transmission coefficient (τ_E) :-

$$= \frac{\text{Peak value of transmitted electric field}}{\text{" " " Incident " "}}$$

$$\tau_E = E_{to} / E_{io} \rightarrow (9)$$

Magnetic Boundary Condition :- $B_{1n} = B_{2n}$; $H_{1t} - H_{2t} = \rho_L$

H (tangential Component at medium 1) - Page no. 340 (Sadiku text book)

= H (tangential Component at medium 2).

Total magnetic field in medium 1

$$H_1 = H_{is}(z) + H_{as}(z)$$

Total magnetic field in medium 2

$$H_2 = H_t(z)$$

$$H_{io}(z=0) + H_{ro}(z=0) = H_{to}(z=0).$$

$$\frac{E_{io}}{\eta_1} - \frac{E_{ro}}{\eta_1} = \frac{E_{to}}{\eta_2}$$

$$1 - \frac{E_{ro}}{E_{io}} = \frac{\eta_1}{\eta_2} \frac{E_{to}}{E_{io}}$$

$$1 - \tau_E = \frac{\eta_1}{\eta_2} \tau_E \quad \rightarrow (10)$$

adding eq'n (9) & (10)

$$1 + \Gamma_E = \tau' \quad \rightarrow (9)$$

$$1 - \Gamma_E = \frac{\eta_1}{\eta_2}\tau' \quad \rightarrow (10)$$

$$2 = \left[1 + \frac{\eta_1}{\eta_2}\right]\tau_E$$

$$\boxed{\tau_E = \frac{2\eta_2}{(\eta_1 + \eta_2)}} \quad \rightarrow (11)$$

from eq'n (9) $\quad 1 + \Gamma_E = \tau_E$

from eq'n (11) $\quad \tau_E = \dfrac{2\eta_2}{(\eta_1 + \eta_2)}$

so $\quad 1 + \Gamma_E = \dfrac{2\eta_2}{(\eta_1 + \eta_2)}$

$$\boxed{\Gamma_E = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}} \quad \rightarrow (12)$$

if both the mediums are loss less & non magnetic.

↓

if $\mu_r = 1$, ε_r

$\eta_1 = 120\pi/\sqrt{\varepsilon_{r_1}}$, $\quad \eta_2 = 120\pi/\sqrt{\varepsilon_{r_2}}$

$$\Gamma_E = \frac{\frac{120\pi}{\sqrt{\varepsilon_{r2}}} - \frac{120\pi}{\sqrt{\varepsilon_{r1}}}}{\frac{120\pi}{\sqrt{\varepsilon_{r2}}} + \frac{120\pi}{\sqrt{\varepsilon_{r1}}}} = \frac{\sqrt{\varepsilon_{r1}} - \sqrt{\varepsilon_{r2}}}{\sqrt{\varepsilon_{r1}} + \sqrt{\varepsilon_{r2}}}$$

$$\Gamma_E = \frac{\sqrt{\varepsilon_{r1}} - \sqrt{\varepsilon_{r2}}}{\sqrt{\varepsilon_{r1}} + \sqrt{\varepsilon_{r2}}} \Rightarrow \text{for lossless medium}$$

Refractive index = $\frac{\text{velocity of wave at vaccum}}{\text{velocity of wave in medium}}$

↓ if it is lossless $\mu_r = 1, \varepsilon_r$

$$n = \frac{c}{v_p} = \frac{c}{c/\sqrt{\varepsilon_r}} = \sqrt{\varepsilon_r}$$

$$n = \sqrt{\varepsilon_r} \Rightarrow \text{for lossless medium}$$

refractive index = n $\quad \eta = n$

intrinsic Impedance = η — To make this happen both mediums have $\mu_{r1} = \mu_{r2} = 1$

Reflection coefficient in terms of refractive index

$$\Gamma_E = \frac{n_2 - n_1}{n_1 + n_2} \Rightarrow \text{Refractive Index.}$$

$$\Gamma_E = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \Rightarrow \text{Intrinsic Impedance}$$

Reflection coefficient for magnetic field :-

We consider, magnetic field direction will same.

Incident wave :-

$$\bar{E}_{is} = E_{io}\, e^{-\gamma_1 z}\, \hat{a}_x$$

$$= \eta_1 H_{io}\, e^{-\gamma_1 z}\, \hat{a}_x \rightarrow (1)$$

we are taging to find reflection coefficient for magnetic field.

$$\bar{H}_{is} = H_{io}\, e^{-\gamma_1 z}\, \hat{a}_y \rightarrow (2)$$

Reflected wave :-

$$\bar{E}_{rs} = E_{ro}\, e^{+\gamma_1 z}\, (-\hat{a}_x)$$

$$= \eta_1 H_{ro}\, e^{+\gamma_1 z}\, (-\hat{a}_x) \rightarrow (3)$$

$$\bar{H}_{rs} = H_{ro}\, e^{\gamma_1 z}\, \hat{a}_y \rightarrow (4)$$

Transmitted wave :-

$$\bar{E}_{ts} = E_{to}\, e^{-\gamma_2 z}\, \hat{a}_x$$

$$= \eta_2 H_{to}\, e^{-\gamma_2 z}\, \hat{a}_x \rightarrow (5)$$

$$\bar{H}_{ts} = H_{to}\, e^{-\gamma_2 z}\, \hat{a}_y \rightarrow (6)$$

From Boundary condition, [Electric field]

$$E_{tan1} = E_{tan2} \quad \text{at} \quad z=0.$$

$$E_{is} + E_{rs} = E_{ts} \Rightarrow E_{io} + E_{ro} = E_{to}$$

$$\eta_1 H_{io} - \eta_2 H_{ro} = \eta_2 H_{to}$$

$$1 - \frac{H_{ro}}{H_{io}} = \frac{H_{to}}{H_{io}} \cdot \frac{\eta_2}{\eta_1}$$

$$\boxed{1 - \Gamma_H = \tau_H \cdot \frac{\eta_2}{\eta_1}} \rightarrow (7)$$

Boundary condition for magnetic field :-

$$H_{tan1} = H_{tan2} \quad \text{at} \quad z=0$$

$$H_{io} + H_{ro} = H_{to}$$

$$1 + \frac{H_{ro}}{H_{io}} = \frac{H_{to}}{H_{io}}$$

$$\boxed{1 + \Gamma_H = \tau_H} \longrightarrow (8)$$

$(7) + (8) \Rightarrow$

$$2 = \left(1 + \frac{\eta_2}{\eta_1}\right) \tau_H$$

$$\boxed{\tau_H = \frac{2\eta_1}{\eta_1 + \eta_2}} \rightarrow (9)$$

$$1 + \Gamma_H = \frac{2\eta_2}{\eta_2 + \eta_1}$$

$$\Gamma_H = \frac{2\eta_1}{\eta_2 + \eta_1} - 1$$

$$\boxed{\Gamma_H = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2}} \rightarrow (10)$$

$$\boxed{\begin{array}{l} \Gamma_H = -\Gamma_E \\ \tau_H = 1 + \Gamma_H \\ \tau_E = 1 + \Gamma_E \end{array}} \rightarrow (11)$$

Case 1:- if medium 2 is perfect conductor

for perfect conductor

$\sigma >> \omega\varepsilon \qquad \sigma = \infty.$ $\quad \mu = \mu_0 \mu_r, \quad \varepsilon = \varepsilon_0.$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma}} \rightarrow 0, \Rightarrow \boxed{\eta_2 = 0.} \quad \boxed{\Gamma_E = -1}$$

Case 2:- if medium 1 is perfect conducted.

$$\eta_1 = 0.$$

$$\Gamma_E = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = +1$$

Case 3:- $n_2 = n_1$

$$\Gamma_E = \frac{n_2 - n_1}{n_2 + n_1} = 0$$

VSWR = $\frac{V_{max}}{V_{min}} = \frac{V_{inc} + V_{ref}}{V_{inc} - V_{ref}}$ (Standing Wave Ratio)

destructive Interference

$= \frac{1 + \Gamma_E}{1 - \Gamma_E}$

Range of Reflection coefficient

$-1 < \Gamma < +1$

Lect-12

Standing wave Pattern (or) Ratio:-

$$S_E = \left| \frac{E_{max}}{E_{min}} \right| = \frac{1 + |\Gamma_E|}{1 - |\Gamma_E|} \Rightarrow \text{for electric field.}$$

$$S_H = \left| \frac{H_{max}}{H_{min}} \right| = \frac{1 + |\Gamma_H|}{1 - |\Gamma_H|}$$

We know that $\Gamma_H = -\Gamma_E$

$$\therefore S_H = \frac{1 + |-\Gamma_E|}{1 - |-\Gamma_E|} = \frac{1 + |\Gamma_E|}{1 - |\Gamma_E|} = S_E$$

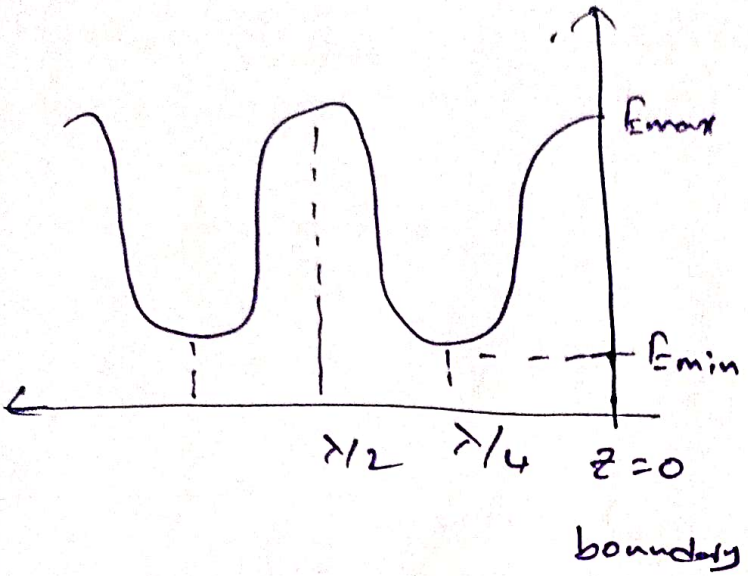
$$\boxed{S_H = S_E}$$

$$\boxed{-1 \leq \Gamma_E \leq 1}$$

$$\boxed{1 \leq S_E \leq \infty}$$

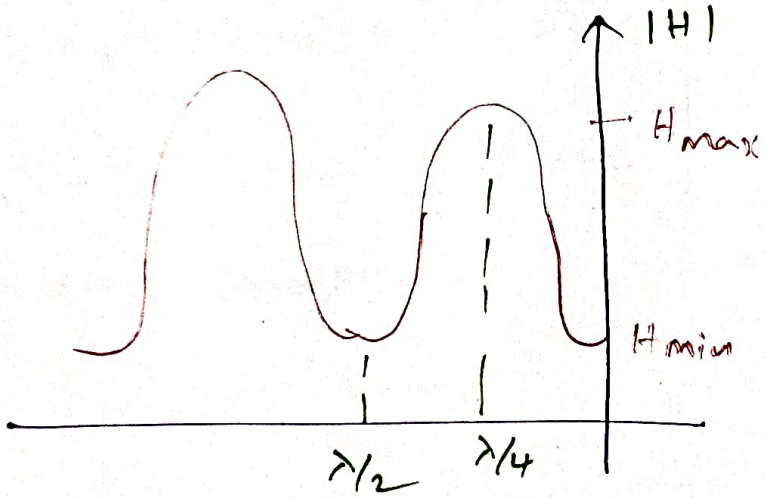
Important

Case 1:- $n_2 > n_1$



$$\Gamma_E = \frac{n_2 - n_1}{n_2 + n_1}$$

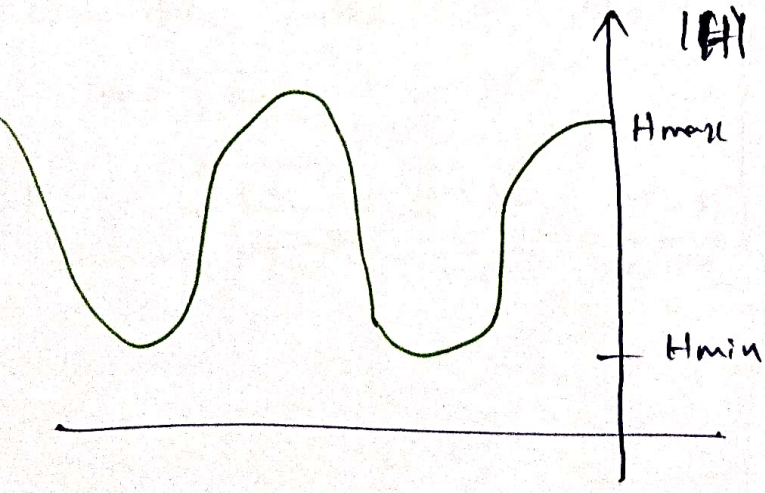
So Γ_E = +ve

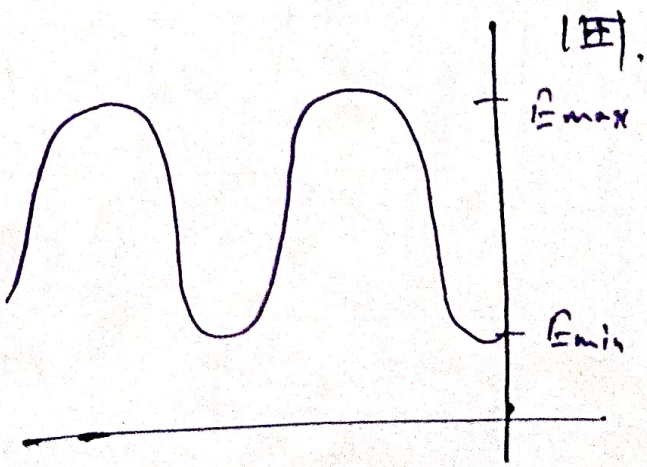


$$\Gamma_H = \frac{n_1 - n_2}{n_2 + n_1}$$

So Γ_H = -ve

Case 2:-





Summary

Normal Incidence & Electric field case

$\Rightarrow E_{io} + E_{ro} = E_{to}$,

$\Rightarrow \Gamma_E = E_{ro}/E_{io}$.

$\Rightarrow 1 + \Gamma_E = \tau_E$

$\Rightarrow \tau_E = E_{to}/E_{io}$

$E_{is} = E_{io}\, e^{-\gamma_1 z}\, \hat{a}_x$

$E_{rs} = E_{ro}\, e^{\gamma_1 z}\, \hat{a}_x$

$E_t = E_{to}\, e^{-\gamma_2 z}\, \hat{a}_x$

$H_{is} = H_{io}\, e^{-\gamma_1 z}\, \hat{a}_y$

$H_{rs} = H_{ro}\, e^{\gamma_1 z}\, (-\hat{a}_y)$
$= -H_{ro}\, e^{\gamma_1 z}\, (\hat{a}_y)$

$H_t = H_{to}\, e^{-\gamma_2 z}\, (\hat{a}_y)$.

Boundary condition

$E_{tan1} = E_{tan2}$

$\downarrow$

$E_i + E_r = E_t$

$H_{tan1} = H_{tan2}$,

$\downarrow \qquad \downarrow$

$H_i + (+H_r) = H_t$

$\downarrow \qquad \downarrow$

$\frac{E_{io}}{\eta_1} - E_{ro}/\eta_1 = \frac{E_{to}}{\eta_2}$

Refractive Index $n = c/v_p$

$n = \sqrt{\varepsilon_r}$.

$\Gamma_E = \frac{\sqrt{\varepsilon_{r1}} - \sqrt{\varepsilon_{r2}}}{\sqrt{\varepsilon_{r1}} + \sqrt{\varepsilon_{r2}}} = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2}$

$1 - \Gamma_E = \frac{\eta_1}{\eta_2}\, \tau_E$

$\tau_E = 1 + \Gamma_E$.

$\tau_E = \frac{2\eta_2}{\eta_1 + \eta_2}$

$\Gamma_E = \frac{\eta_2 - \eta_1}{\eta_1 + \eta_2}$

Normal Incidence and magnetic field wave Reflection coefficient for magnetic field

$E_{is} = E_{io} e^{-\gamma_1 z} \hat{a}_x$

$E_{rs} = -\eta_1 H_{ro} e^{\gamma_1 z} \hat{a}_x$

$= -E_{ro} e^{\gamma_1 z} \hat{a}_x$

$E_{ts} = E_{to} e^{-\gamma_2 z} \hat{a}_x$

$H_{is} = H_{io} e^{-\gamma_1 z} \hat{a}_y$

$H_{rs} = H_{ro} e^{\gamma_1 z} \hat{a}_y$

$H_{ts} = H_{to} e^{-\gamma_2 z} \hat{a}_y$

$E_{tan1} = E_{tan2}$

$H_{tan1} = H_{tan2}$

$E_{is} + E_{rs} = E_{ts}$

$H_{io} + H_{ro} = H_{to}$

$\eta_1 H_{io} - \eta_2 H_{ro} = \eta_2 H_{to}$

$1 - \frac{H_{ro}}{H_{io}} = \frac{H_{to}}{H_{io}} \cdot \frac{\eta_2}{\eta_1}$

$1 - \frac{H_{ro}}{H_{io}} = \frac{H_{to}}{H_{io}}$

$1 - \Gamma_H = \tau_H \cdot \frac{\eta_2}{\eta_1}$

$1 + \Gamma_H = \tau_H$

$\tau_H = \frac{2\eta_1}{\eta_1 + \eta_2}$

$\Gamma_H = \frac{\eta_1 - \eta_2}{\eta_1 + \eta_2}$

finally.

$\Gamma_H = -\Gamma_E$

$\tau_H = 1 + \Gamma_H$

$\tau_E = 1 + \Gamma_E$

$S_E = 1 + |\Gamma_E| / 1 - |\Gamma_E|$

$S_H = 1 + |\Gamma_H| / 1 - |\Gamma_H|.$

$-1 \leq \Gamma_E \leq 1$

$1 \leq S_E \leq \infty$

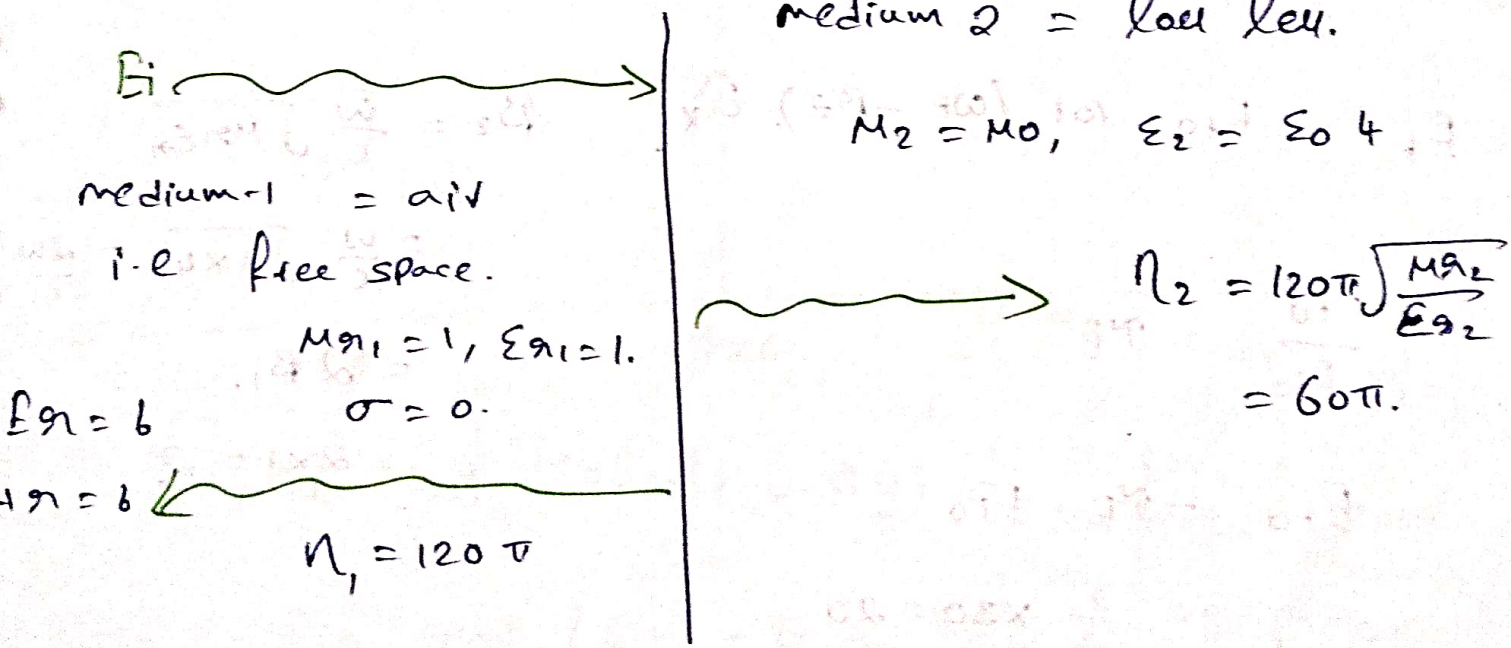
(P) The plane wave $\bar{E} = 30 \cos(\omega t - z)\, \hat{a}_x$ V/m in air normally hits a lossless medium ($\mu = \mu_0$, $\varepsilon = 4\varepsilon_0$) at $z = 0$

(A) Find Γ, τ and S

(B) Calculate the reflected electric and mr field.

(Ans) Relationship b/w Γ and S.

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} \qquad \Gamma_E = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$



$$\Gamma_E = \frac{60\pi - 120\pi}{60\pi + 120\pi} = -1/3, \qquad \tau_E = 1 + \Gamma_E = 1 - 1/3 = 2/3$$

$$S = \frac{1 + |\Gamma_E|}{1 - |\Gamma_E|} = \frac{1 + 1/3}{1 - 1/3} = 2$$

$\overline{E_R} = E_{R0} \cos(\omega t + z)\hat{a}_x$

$\hat{a}_E = \hat{a}_H \times \hat{a}_K$

$\hat{a}_x = (-a_y \times -a_z)$

$\hat{a}_H$

$\Gamma_E = \frac{E_{R0}}{E_{i0}}$ $\quad E_{R0} = E_{i0} \cdot \Gamma_E$

$= 30 \times (-1/3) = -10$

$\overline{E_R} = -10 \cos(\omega t + z)\hat{a}_x$.

$\overline{H_R} = H_{R0} \cos(\omega t + z)\hat{a}_{HR}$

$H_{R0} = \frac{E_{R0}}{\eta}$

$= \frac{-10}{120\pi} = -\frac{1}{12\pi}$.

$= \frac{-1}{12\pi} \cos(\omega t + z)(-\hat{a}_y)$

$= +\frac{1}{12\pi} \cos(\omega t + z)\hat{a}_y$

$E_t = E_{t0} \cos(\omega t - \beta z)\hat{a}_x$

$\beta_2 = \frac{\omega}{c}\sqrt{\mu_R \varepsilon_R}$

$= \frac{\omega}{c}\sqrt{1 \times 4} = \frac{2\omega}{c}$

$= 2\beta_1$.

$= 2 \times 1 = 2$.

$\frac{E_{t0}}{E_{i0}} = \tau_E$

$E_{t0} = \tau_E \cdot E_{i0}$

$= \frac{2}{3} \times 30 = 20$.

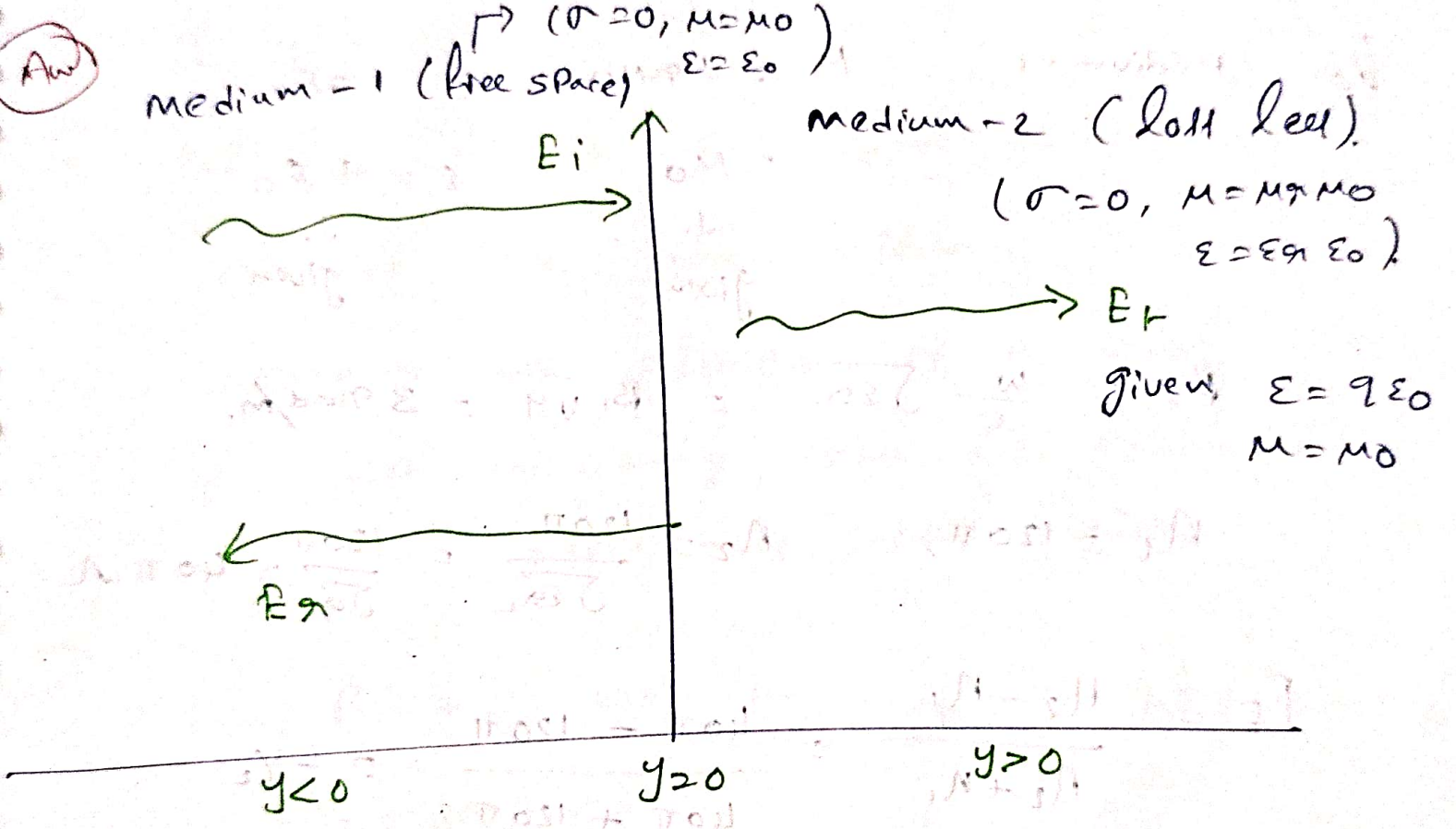
$\overline{E_t} = 20 \cos(\omega t - 2z)\hat{a}_x$

$H_t = H_{t0} \cos(\omega t - \beta_2 z) = \frac{E_{t0}}{\eta_2} \cos(\omega t - 2z)\hat{a}_y$

$= \frac{20}{60\pi} \cos(\omega t - 2z)\hat{a}_y$

(Q) :- A plane wave having the electric field component $E_i = 24 \cos(3\times10^8 t - \beta y)\,\hat{a}_z$ and travelling in free space is incident normally on a lossless medium with $\mu = \mu_0$ & $\varepsilon = 9\varepsilon_0$, which occupies the region $y \geq 0$. The reflected wave field component ?

(Ans)



$E_i = 24 \cos(3\times10^8 t - \beta y)\,\hat{a}_z$

$E_i = E_{i0} \cos(\omega t - \beta a_k)\,\hat{a}_E$

$\omega = 3\times10^8$

$E_{i0} = 24$

wave direction = $\bar{a}_k = \hat{a}_y$

E-C field direction = $\bar{a}_E = \hat{a}_z$

$$\beta = \omega \sqrt{\frac{\mu\varepsilon}{2}\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1}$$

For medium-1, $\mu = \mu_0$, $\varepsilon = \varepsilon_0$, $\sigma = 0$

So $\beta_1 = \omega\sqrt{\mu_{r1}\cdot\varepsilon_{r1}} = \omega/c = \frac{3\times10^8 \rightarrow \text{given}}{3\times10^8} = 1$

For medium-2, $\mu = \mu_r\mu_0$, $\varepsilon = \varepsilon_{r2}\varepsilon_0$

$= \mu_0$ (↓ given) $\quad \varepsilon = 9\varepsilon_0$ (↓ given)

$$\beta_2 = \frac{\omega}{c}\sqrt{\varepsilon_{r2}} = \beta_1\sqrt{9} = 3 \text{ rad/m}.$$

$$\eta_1 = 120\pi, \quad \eta_2 = \frac{120\pi}{\sqrt{\varepsilon_{r2}}} = \frac{120\pi}{\sqrt{9}} = 40\pi\ \Omega.$$

$$\Gamma_E = \frac{\eta_2-\eta_1}{\eta_2+\eta_1} = \frac{40\pi-120\pi}{40\pi+120\pi} = -\gamma_2$$

$$\Gamma_E = \frac{E_{r0}}{E_{i0}} = -\gamma_2 \Rightarrow \boxed{E_{r0} = 24\times(-1/2) = -12.}$$

$\bar{H}_r = H_{r0}\cos(\omega t - \beta_1 a_k)\, a_H$ $\quad a_k = a_H \times a_E$ (↓ ↓) $\quad a_2 = -a_y \times -a_y$

$\bar{H}_r = H_{r0}\cos(\omega t + 1(+a_y))\, a_H$

$= \frac{E_{r0}}{\eta_1}\cos(3\times10^8 t + y)(-a_x)$

$$\bar{H}_{2} = \frac{1}{10\pi} \cos(3\times10^{8}t + \beta y)\,(\hat{a}_x)$$

(Q) The magnetic field component of a plane wave in a lossless dielectric is

$$\bar{H} = 30 \sin(2\pi\times10^{8}t - 5x)\,\hat{a}_z \ \text{mA/m}.$$

(a) If $\mu_r = 1$, find ε_r

(b) Calculate the β & η Wave velocity

(c) Determine the wave impedance

(d) Determine the polarization of the wave

(e) find the corresponding electric field component

(f) " " displacement current density

(Ans) $\beta = +5$, $a_k = a_x$, $a_{E} = a_z$, $\omega = 2\pi\times10^{8}$

$H_{i0} = 30\times10^{-3}$.

(A) we know $\beta = \frac{\omega}{c}\sqrt{\mu_r\varepsilon_r}$ for lossless medium.

$$5 = \frac{2\pi\times10^{8}}{3\times10^{8}}\sqrt{\varepsilon_r}$$

$$\boxed{\varepsilon_r = 5.69}$$

(B) $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{5}$ m.

$v_p = \frac{\omega}{\beta} = \frac{2\pi \times 10^8}{5} = \frac{2\pi}{5} \times 10^8$ m/sec

(C) $\eta = \frac{120\pi}{\sqrt{\varepsilon_R}} = \frac{120\pi}{15/2\pi} = 157.91\ \Omega$

(D) $\hat{a}_E = \hat{a}_H \times \hat{a}_k$

$= \hat{a}_z \times \hat{a}_x$

$\hat{a}_E = \hat{a}_y$

Polarization will be decided by electric field.

(E) $\bar{E} = E_0 \sin(2\pi \times 10^8 t - \beta x)\, \hat{a}_E$

$\eta = E_0/H_0 \qquad E_0 = \eta_0 H_0 = 4.73$ V/m

$\bar{E} = 4.73 \sin(2\pi \times 10^8 t - 5x)\, \hat{a}_y$

(F) $J_D = \frac{\partial \bar{D}}{\partial t} = \varepsilon \cdot \frac{\partial \bar{E}}{\partial t}$

$= 5.69\, \varepsilon_0 \frac{\partial}{\partial t} [4.73 \sin(\omega t - 5x)\, \hat{a}_y]$

$\bar{J}_D = 5.69\, \varepsilon_0 \times 4.73\, \omega \cos(\omega t - 5x)\, \hat{a}_y$

$\omega \rightarrow 2\pi \times 10^8$

$= 0.149 \cos(\omega t - 5x)\, \hat{a}_y$ A/m

(9) in non magnetic medium,

$$\bar{E} = 50 \cos(10^9 t - 8x)\,\hat{a}_y + 40 \sin(10^9 t - 8x)\,\hat{a}_z \text{ V/m}.$$

find dielectric const (ε) and $\bar{H}$

(A) non magnetic - loss less. $\mu_r \to 1$ (not mentioned)

$$\beta = \omega/c\sqrt{\varepsilon_r}$$

$$\sqrt{\varepsilon_r} = \frac{8 \times 3 \times 10^8}{10^9} = 5.76$$

given $\bar{E} = E_1 + E_2$

$$\bar{H}_1 = H_{o1} \cos(\omega t - \beta x)\,\hat{a}_{H_1}$$

$$= \frac{50}{50\pi} \cos(\omega t - 8x)\,\hat{a}_z$$

$$\hat{a}_{ic} = \hat{a}_E \times \hat{a}_H$$

$\downarrow$ $\downarrow$ $\downarrow$

x y (3) need to find

$$H_{o1} = E_{o1}/\eta.$$

$E_a = 50$

$$\eta = \frac{120\pi}{\sqrt{\varepsilon_r}}$$

$$= \frac{120\pi}{\sqrt{5.6}} = 50\pi$$

$$\bar{H}_1 = \frac{1}{\pi} \cos(10^9 t - 8x)\,\hat{a}_z$$

$$H_{o2} = \frac{E_{o2}}{\eta}$$

$$= \frac{40}{50\pi}$$

$$\bar{H}_2 = H_{o2} \cos(\omega t - \beta x)\,\hat{a}_{H_2}$$

$$= \frac{40}{50\pi} \cos(10^9 t - 8x)(-\hat{a}_y)$$

$$\hat{a}_{ic} = \hat{a}_E \times \hat{a}_H$$

$\downarrow_x$ $\downarrow_z$ (y) $\rightarrow$ it shold be

(Q) in a certain medium $\bar{E} = 10 \cos(2\pi \times 10^7 t - \beta x)(\hat{a}_y + \hat{a}_z)$

if $\mu = 50\mu_0$, $\varepsilon = 2\varepsilon_0$, and $\sigma = 0$, find β & η

(A) given $\mu_R = 50$, $\varepsilon_R = 2$.

$\sigma = 0$ → loss less dielectric (or) free space

$$\beta = \omega \sqrt{\frac{\mu\varepsilon}{2}\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1} \qquad \sigma = 0$$

$$\beta = \omega\sqrt{\mu\varepsilon} = 2\pi \times 10^7 \sqrt{50 \times \mu_0 \times 2 \times \varepsilon_0}$$

$$= \frac{2\pi}{3}$$

$$\bar{H} = H_0 \cos(\omega t - \beta x)(a_{H_1} + a_{H_2})$$

$$H_0 = E_0/\eta \qquad \eta = \sqrt{\mu/\varepsilon} = 600\pi$$

$$\bar{H} = \frac{10}{600\pi} \cos(2\pi \times 10^7 t - \beta x)(\hat{a}_z - \hat{a}_y)$$

(4) which of the following media may be treated as conducting at 8 MHz

(a) wet marshy soil ($\varepsilon = 15\varepsilon_0, \mu = \mu_0, \sigma = 10^{-2}$ S/m)

(b) Intrinsic germanium ($\varepsilon = 16\varepsilon_0, \mu = \mu_0, \sigma = 0.025$ S/m)

(c) Sea water ($\varepsilon = 18\varepsilon_0, \mu = \mu_0, \sigma = 25$ S/m)

(a) $\frac{\sigma}{\omega\varepsilon} = \frac{10^{-2}}{2\pi \times 8 \times 10^6 \times 15\varepsilon_0} = 1.5.$ ↳ it's not a conductor

for good conductor $\frac{\sigma}{\omega\varepsilon} \gg 1.$

(b) $\frac{\sigma}{\omega\varepsilon} = \frac{0.025}{2\pi \times 8 \times 10^6 \times 16\varepsilon_0} = 3.515$ ↳ not a conductor.

(c) $\frac{\sigma}{\omega\varepsilon} = \frac{25}{2\pi \times 8 \times 10^6 \times 81\varepsilon_0} = 694 \gg$ (good conductor)

) calculate the skin depth and the velocity of propagation for a uniform plane wave at frequency 6 MHz travelling in polyvinylchloride ($\mu_R = 1$, $\varepsilon_R = 4$, $\tan\theta_h = 7 \times 10^{-2}$)

) $\delta = 1/\alpha =$

$$\alpha = \omega \sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]}$$

we know $\tan\theta = \frac{\sigma}{\omega\varepsilon}$

$\theta\eta = \tan^{-1}(7\times10^{-2}) = 4.004°$

$\theta_n = \theta/2 = 8°$

$\tan(8°) =$

$$\alpha = \frac{2\pi\times6\times10^{6}}{3\times10^{8}}\sqrt{\frac{1\times4}{2}\left[\sqrt{1+\tan^2 80} - 1\right.}$$

$\alpha = 8.79\times10^{-3}$ | $\delta = 1/\alpha = 113.76$

(Q) A uniform plane wave in a lossy medium has a phase constant of 1.6 rad/m at 10^7 Hz and it's magnitude is reduced by 60% for every 2 m travelled, find the skin depth and speed of the wave.

(A) $\bar{E} = E_0 e^{-\alpha z}\cos(\omega t - \beta z)\hat{a}_E$

$0.4E_0 = E_0 e^{-\alpha\times2}$

$\ln(0.4) = -2\alpha.$

$\boxed{\alpha = 0.458 \text{ NP/m}}$

$\delta = 1/\alpha = \frac{1}{0.458}$

$= 2.813$ m.

$V_P = \frac{\omega}{\beta} = \frac{2\pi\times10^7}{1.6} = 3.92\times10^7.$

Q) The distance (in meters) a wave has to propagate in a medium having a skin depth of 0.1m so that the amplitude of the wave attenuates by 20 dB, is.

A)

$\delta = 0.1m$,

$E = E_0 e^{-\alpha z}$

$\frac{E_0}{10} = E_0 e^{-\frac{1}{\delta} \times z}$.

$\delta = 1/\alpha$.

$0.1 = e^{-10z}$.

$\ln(0.1) = -10z$.

$-2.3 = -10z$.

$\boxed{z = 0.23}$

$(x)_{dB} = 20 \log x$.

↳ voltage
current
electric
magnetic

$20 = 20 \log x$

$\log x = 1$

$\boxed{x = 10}$

Lecture - 16

Poynting Theorem:-

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \rightarrow (1)$$

$$\nabla \times \bar{H} = -\mu \cdot \frac{\partial \bar{H}}{\partial t}$$

$$\boxed{\bar{H} \cdot (\nabla \times \bar{E}) = -\mu \bar{H} \cdot \frac{\partial \bar{H}}{\partial t} = -\frac{\mu}{2} \cdot \frac{\partial H^2}{\partial t}} \rightarrow (1)$$

$$-\mu \cdot \frac{\partial H^2}{\partial t} = -H\mu \cdot \frac{\partial H}{\partial t} \quad (2) \Rightarrow -\mu H \frac{\partial H}{\partial t} = -\frac{\mu}{2} \frac{\partial H^2}{\partial t}$$

$$\nabla \times \bar{H} = J + J_d = \sigma \bar{E} + \frac{\partial \bar{D}}{\partial t} = \sigma \bar{E} + \varepsilon \cdot \frac{\partial \bar{E}}{\partial t}$$

$$\bar{E} \cdot (\nabla \times \bar{H}) = \sigma (\bar{E} \cdot \bar{E}) + \varepsilon \bar{E} \cdot \frac{\partial \bar{E}}{\partial t}$$

$$\boxed{\bar{E} \cdot (\nabla \times \bar{H}) = \sigma E^2 + \frac{1}{2} \varepsilon \cdot \frac{\partial E^2}{\partial t}} \rightarrow (2)$$

as per vector Identity.

$$\nabla \cdot (\bar{A} \times \bar{B}) = \bar{B} \cdot (\nabla \times \bar{A}) - \bar{A} \cdot (\nabla \times \bar{B})$$

$\bar{A} \rightarrow \bar{E}, \quad \bar{B} \rightarrow \bar{H}$

$$\nabla \cdot (\bar{E} \times \bar{H}) = \bar{H} \cdot (\nabla \times \bar{E}) - \bar{E} \cdot (\nabla \times \bar{H})$$

$$\nabla \cdot (\bar{E} \times \bar{H}) = -\frac{1}{2} \cdot \mu \cdot \frac{\partial H^2}{\partial t} - \sigma E^2 - \frac{1}{2} \varepsilon \cdot \frac{\partial E^2}{\partial t}$$

$$\nabla \cdot (\bar{E} \times \bar{H}) = -\sigma E^2 - \frac{\partial}{\partial t} \left(\frac{1}{2} \varepsilon E^2\right) - \frac{\partial}{\partial t} \left(\frac{1}{2} \mu H^2\right)$$

Energy stored in electric field

$$W_E = \int_V \frac{1}{2} \bar{D} \cdot \bar{E} \, dV$$

$$= \int_V \frac{1}{2} (\varepsilon \bar{E} \cdot \bar{E}) \, dV = \int_V \frac{1}{2} \varepsilon E^2 \, dV.$$

$P = V \cdot I$

$W = \int P \, dt$

$= \int V \cdot I \, dt$

$= \int V \cdot C \frac{dV}{dt} \, dt$

$W = \frac{1}{2} CV^2$

$w = \frac{1}{2} \cdot \frac{\varepsilon A}{d} E^2 \cdot d$

$= \frac{1}{2} \cdot \varepsilon \cdot E^2 \cdot A \cdot d$

Stored per volume

$w_v = \frac{1}{2} \varepsilon E^2$

Energy stored in mc field.

$$W_H = \int_V \frac{1}{2} (\bar{B} \cdot \bar{H}) \, dV = \int_V \frac{1}{2} \mu H^2 \, dV$$

$$\nabla \cdot (\bar{E} \times \bar{H}) = -\sigma E^2 - \frac{\partial}{\partial t} \left(\frac{1}{2} \varepsilon E^2 \right) - \frac{\partial}{\partial t} \left(\frac{1}{2} \mu H^2 \right)$$

$$\int_V \nabla \cdot (\bar{E} \times \bar{H}) \, dV = -\int \sigma E^2 \, dV - \frac{\partial}{\partial t} \int_V \frac{1}{2} \varepsilon E^2 \, dV$$

$$- \partial/\partial t \int \frac{1}{2} \mu H^2 \, dV.$$

$$\int_V \nabla \cdot (\bar{E} \times \bar{H}) \, dV = -\int_V \sigma E^2 \, dV - \frac{\partial}{\partial t} \int_V \frac{1}{2} \varepsilon E^2 \, dV$$

$$- \partial/\partial t \int \frac{1}{2} \mu H^2 \, dV.$$

- $\int_V \nabla \cdot (\bar{E} \times \bar{H}) \, dV$ → total power leaving volume
- $\int_V \sigma E^2 \, dV$ → Ohmic power dissipated
- $\frac{\partial}{\partial t} \int_V \frac{1}{2} \varepsilon E^2 \, dV$ → decrease in rate of energy stored in e-c field
- $\partial/\partial t \int \frac{1}{2} \mu H^2 \, dV$ → decrease in rate of energy stored in mc field.

$$\oint (\bar{E} \times \bar{H}) \cdot ds = \int_V \sigma E^2 \, dV - \frac{\partial}{\partial t} \int_V \frac{1}{2} \varepsilon E^2 \, dV - \frac{\partial}{\partial t} \int_V \frac{1}{2} \mu H^2 \, dV.$$

Poynting vector $\bar{P} = \bar{E} \times \bar{H}$ (Power density).

LECT - 17

Suppose wave → +z direction

E field → +x " ($\hat{a}_E = \hat{a}_x$)

M field → +y "

In a lossy medium.

$$\bar{E}(z,t) = E_0 e^{-\alpha z} \cos(\omega t - \beta z)\, a_x$$

$$\bar{H}(z,t) = \frac{E_0}{\eta} e^{-\alpha z} \cos(\omega t - \beta z)\, a_y$$

$$\bar{H}(z,t) = \frac{E_0}{|\eta| \angle \theta_\eta} e^{-\alpha z} \cos(\omega t - \beta z - \theta_\eta)\, \hat{a}_y$$

Poynting vector $\bar{P} = \bar{E} \times \bar{H}$

$$\bar{P} = \begin{vmatrix} a_x & a_y & a_z \\ E_x & E_y = 0 & E_z = 0 \\ H_x = 0 & H_y & H_z = 0 \end{vmatrix} = a_z [E_x \cdot H_y]$$

$$= E_0 e^{-\alpha z} \cos(\omega t - \beta z)\, \hat{a}_x \times \frac{E_0}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - \theta_\eta)$$

$$= \frac{E_0^2}{|\eta|} e^{-2\alpha z} \cos(\omega t - \beta z) \cos(\omega t - \beta z - \theta_\eta)\, \hat{a}_z$$

$$\bar{P} = \frac{E_0^2}{2|\eta|} e^{-2\alpha z} \left[\cos(2\omega t - 2\beta z - \theta_\eta) + \cos(\theta_\eta)\right] \hat{a}_z$$

Time average $\bar{P}_{avg} = \frac{1}{T} \int_T \bar{P}\, dt$

$$\bar{P}_{avg} = \frac{1}{T} \int_T \frac{E_0^2}{2|\eta|} e^{-2\alpha z} \left[\cos(2\omega t - 2\beta z - \theta_\eta) + \cos\theta_\eta\right] \hat{a}_z\, dt$$

$$= \frac{1}{T} \int_T \frac{E_0^2}{2|\eta|} e^{-2\alpha z} \cos(2\omega t - 2\beta z - \theta_\eta)\, dt\, \hat{a}_z + \frac{1}{T} \int_T \frac{E_0^2}{2|\eta|} e^{-2\alpha z} \cos\theta_\eta\, dt\, \hat{a}_z$$

$$= \frac{1}{T} \int_T \frac{E_0^2}{2(\eta)} e^{-2\alpha z} \cos\theta_\eta \, dt \, \hat{a}_z = \frac{E_0^2}{2(\eta)} e^{-2\alpha z} \cos\theta_\eta \frac{1}{T} \int_0^T dt$$

$$P_{avg} = \frac{E_0^2}{2(\eta)} e^{-2\alpha z} \cos\theta_\eta \, \hat{a}_z .$$

General representation

$$\bar{P}_{avg} = \frac{E_0^2}{2(\eta)} e^{-2\alpha k} \cos\theta_\eta \, \hat{a}_{ic}$$

for loseless medium. $\sigma = 0$, $\alpha = 0$, $\gamma = j\beta$, $\theta_\eta = 0$

$$\bar{P}_{avg} = \frac{E_0^2}{2(\eta)} \hat{a}_{ic}$$

↳ direction of propagation.

Power

$$P = \int_S \bar{P}_{avg} \cdot d\bar{s}$$

Lec 4 - 18

Power density in two different medium

$$P_i = \frac{E_0^2}{2\eta_1}, \quad P_R = \frac{E_{R0}^2}{2\eta_1}, \quad P_t = \frac{E_{t0}^2}{2\eta_2}$$

$$\frac{\text{Reflected power}}{\text{Incident power}} = \frac{\frac{E_{R0}^2}{2\eta_1} \times \text{Area}}{\frac{E_{i0}^2}{2\eta_1} \times \text{Area}} = \left| \frac{E_{R0}}{E_{i0}} \right|^2 = |\Gamma_1|^2$$

How much power is reflected back $|\Gamma|^2$.

$$\frac{\text{Transmitted power}}{\text{Incident power}} = \frac{\frac{E_{to}^2}{2\eta_2} \times \text{Area}}{\frac{E_{io}^2}{2\eta_1} \times \text{Area}} = \left|\frac{E_{to}}{E_{io}}\right|^2 \times \frac{\eta_1}{\eta_2} = |\tau_E|^2 \times \frac{\eta_1}{\eta_2}$$

$$\frac{\text{Transmitted power}}{\text{Incident power}} = \frac{\text{Incident power} - \text{reflected power}}{\text{Incident power}}$$

$$= 1 - \frac{\text{reflected power}}{\text{Incident power}}$$

$$\frac{\text{Transmitted power}}{\text{Incident power}} = 1 - |\Gamma_E|^2$$

Lect - 21

Polarization :-

Polarization is tracing of tip of electric field as a function of time (in a plane ⊥ to the direction of propagation) at a given point in space.

① linear Polarization :-

$$E(z,t) = E_{ox} \cos(\omega t - \beta z)\,\hat{a}_x + E_{oy} \cos(\omega t - \beta z)\, e^{j\delta}\,\hat{a}_y$$

For a wave to be linearly polarized $\delta = n\pi$.

$n = 0, 1, 2 \ldots$

if $E_{ox} = E_{oy} = E_0$

$$E(z,t) = E_0 \cos(\omega t - \beta z)\,a_x + E_0 \cos(\omega t - \beta z)\,a_y \cdot e^{j\delta}$$

Phase difference $\delta = \theta_y - \theta_x$.

Case - 1 :- $\delta = 2n\pi \quad n = 0, 1, 2 \ldots$

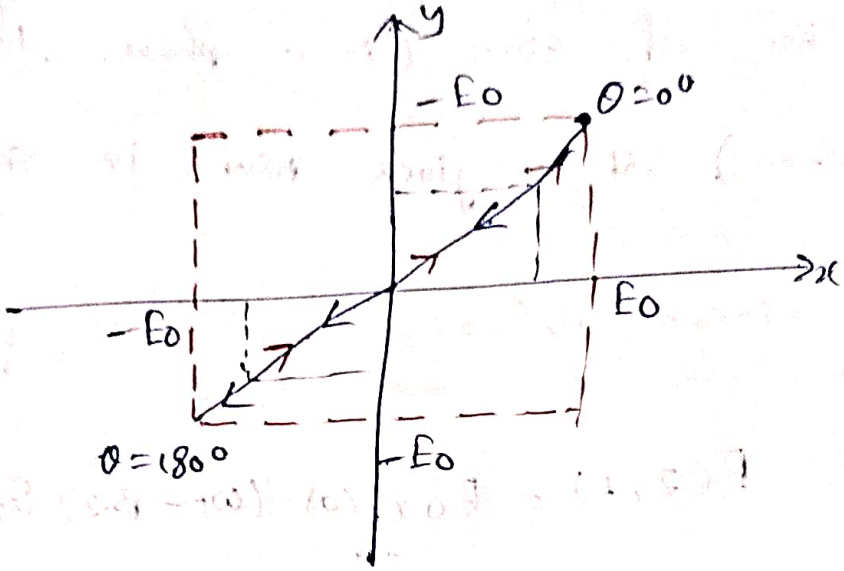
$$e^{j\delta} = e^{j2n\pi} = (\cos 2n\pi + j \sin 2n\pi) = 1.$$

$$E(z,t) = E_0 \cos(\omega t - \beta z)\hat{a}_x + E_0 \cos(\omega t - \beta z)\,a_y$$

at a given point $(z = 0)$

$$E(0,t) = E_0 \cos(\omega t)\,\hat{a}_x + E_0 \cos(\omega t)\,\hat{a}_y$$

θ rad	E_{ox}	E_{oy}
0	E_0	E_0
45°	$E_0/\sqrt{2}$	$E_0/\sqrt{2}$
90°	0	0
135°	$-E_0/\sqrt{2}$	$-E_0/\sqrt{2}$
180°	$-E_0$	$-E_0$
225°	$-E_0/\sqrt{2}$	$-E_0/\sqrt{2}$
270°	0	0
315°	$E_0/\sqrt{2}$	$E_0/\sqrt{2}$
360°	E_0	E_0



Case - 2 :- $\delta = (2n+1)\pi \quad n = 0, 1, 2 \ldots$

$$e^{j\delta} = e^{j(2n+1)\pi} = \cos(2n+1)\pi + j\sin(2n+1)\pi = -1.$$

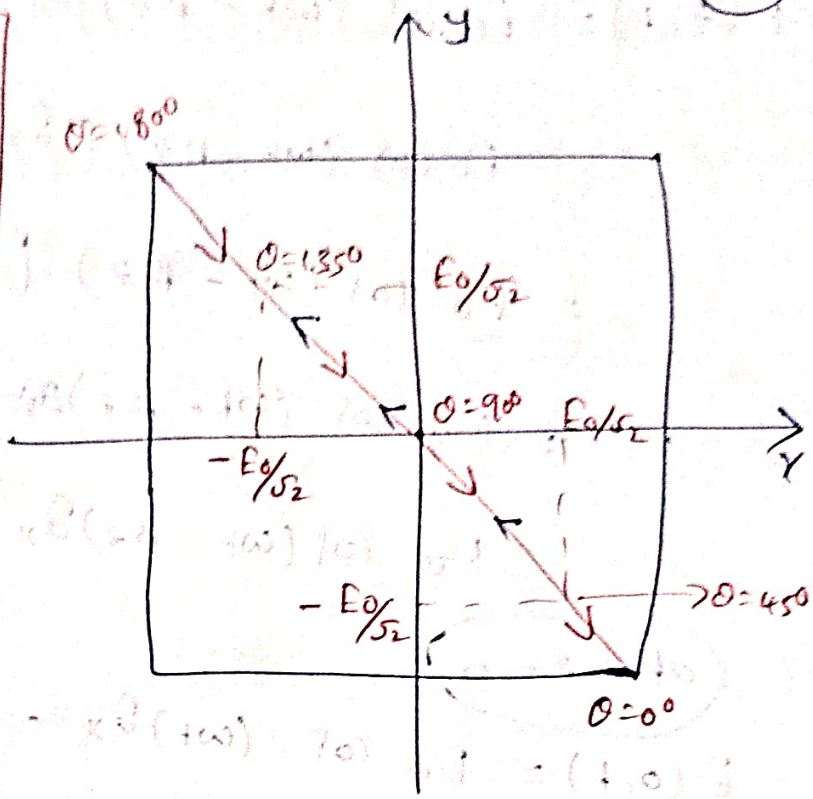
$$E(z,t) = E_0 \cos(\omega t - \beta z)\,\hat{a}_x + e^{j\delta} E_0 \cos(\omega t - \beta z)\,\hat{a}_y$$

$$= E_0 \cos(\omega t - \beta z)\,\hat{a}_x - E_0 \cos(\omega t - \beta z)\,\hat{a}_y$$

at a point in space, $(z = 0)$

$$E(z,t) = E_0 \cos \omega t\,\hat{a}_x - E_0 \cos(\omega t)\,\hat{a}_y$$

θ	E_{ox}	E_{oy}
0	E_0	$-E_0$
45°	$E_0/\sqrt{2}$	$-E_0/\sqrt{2}$
90°	0	0
135°	$-E_0/\sqrt{2}$	$E_0/\sqrt{2}$
180°	$-E_0$	E_0
225°	$-E_0/\sqrt{2}$	$E_0/\sqrt{2}$
270°	0	0
315°	$E_0/\sqrt{2}$	$-E_0/\sqrt{2}$
360°	E_0 "	$-E_0$ "



Linear Polarization

Lect : 22

Circular Polarization

$$E(z,t) = E_{ox}\cos(\omega t - \beta z)\hat{a}_x + e^{j\delta} E_{oy}\cos(\omega t - \beta z)a_y$$

Condition for circular polarization,

$E_{ox} = E_{oy}$ & phase difference

$$\delta = (2n+1)\pi/2$$

$$E(z,t) = E_0\cos(\omega t - \beta z)\hat{a}_x + e^{j\delta} E_0\cos(\omega t - \beta z)a_y$$

Case 1:- n = even (Let us n = 0)

$$\delta = (2n+1)\pi/2 = \pi/2$$

$$e^{j\delta} = e^{j\pi/2} = \cos(\pi/2 + j\sin\pi/2) = j$$

$$E(z,t) = E_0 \cos(\omega t - \beta z)\hat{a}_x + j E_0 \cos(\omega t - \beta z)\hat{a}_y$$

$$= E_0 \cos(\omega t - \beta z)[\hat{a}_x + j\hat{a}_y], \quad \delta = \phi_y - \phi_x$$

$$= E_0 \cos(\omega t - \beta z)[\hat{a}_x + e^{j\pi/2}\hat{a}_y].$$

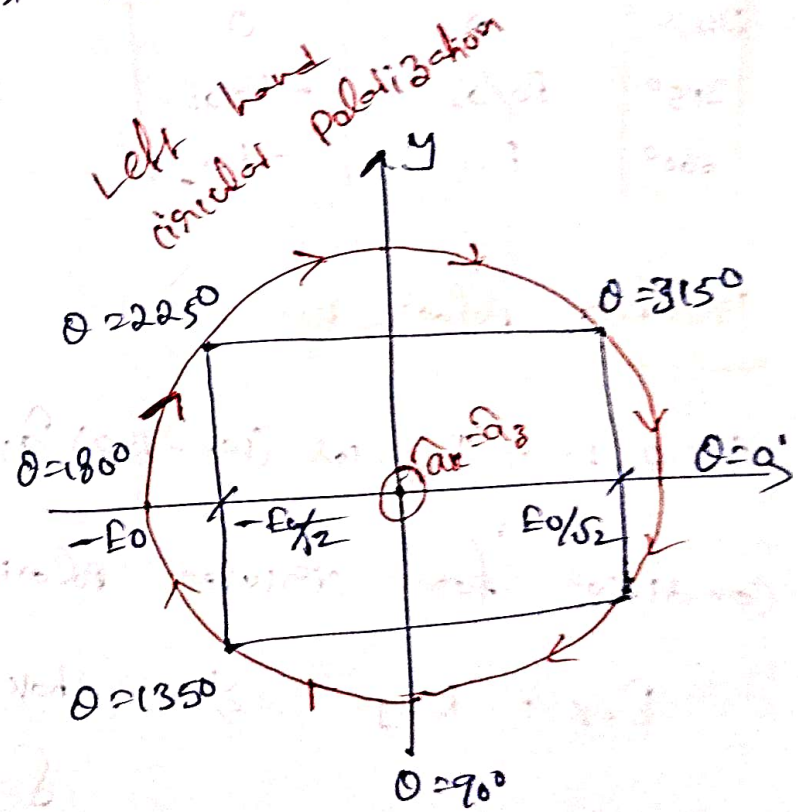
$$= E_0 \cos(\omega t - \beta z)\hat{a}_x + E_0 \cos(\omega t - \beta z + 90^\circ)\hat{a}_y.$$

$$= E_0 \cos(\omega t - \beta z)\hat{a}_x - E_0 \sin(\omega t - \beta z)\hat{a}_y$$

$\omega t - z = 0$

$$E(0,t) = E_0 \cos(\omega t)\hat{a}_x - E_0 \sin \omega t \hat{a}_y$$

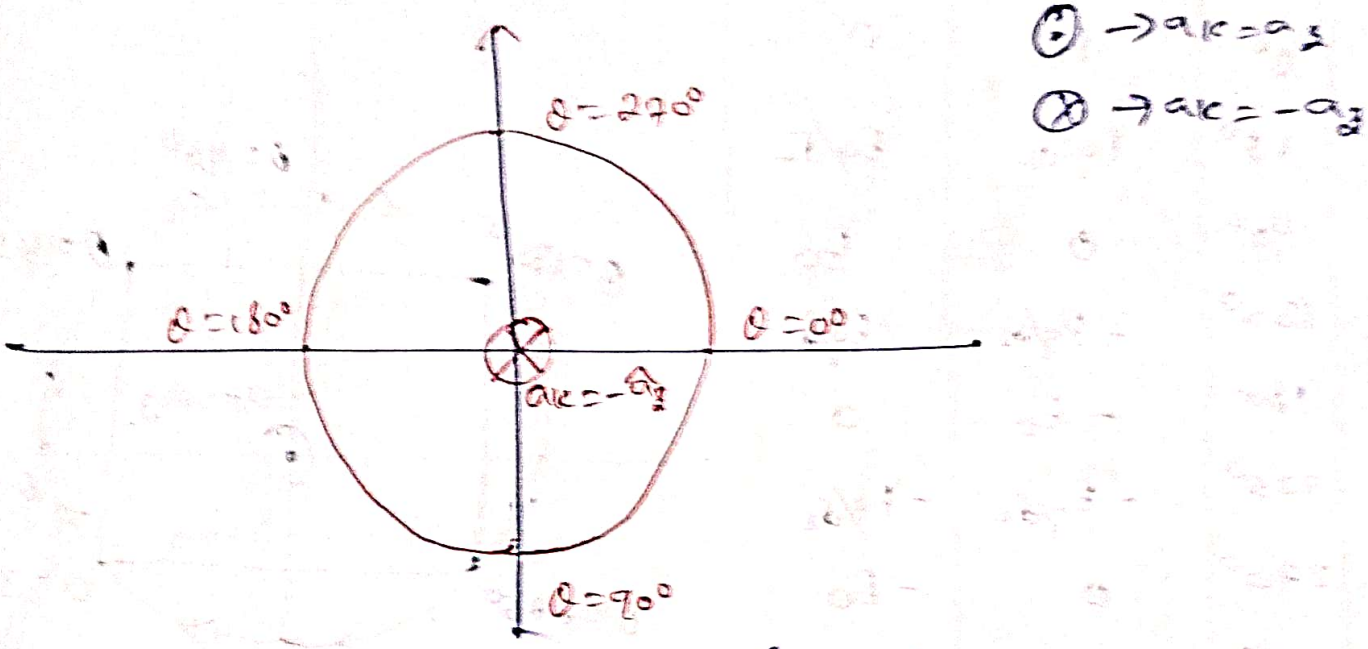
$\theta = \omega t$	E_{ox}	E_{oy}
0	E_0	0
45°	$E_0/\sqrt{2}$	$-E_0/\sqrt{2}$
90°	0	$-E_0$
135°	$-E_0/\sqrt{2}$	$-E_0/\sqrt{2}$
180°	$-E_0$	0
225°	$-E_0/\sqrt{2}$	$E_0/\sqrt{2}$
270°	0	E_0
315°	$E_0/\sqrt{2}$	$-E_0/\sqrt{2}$
360°	E_0	0



direction of propagation - z.

if $a_{rc} = \hat{a}_z$ — left hand circular polarization

if $a_{rc} = -\hat{a}_z$ — polarization converted to Right hand circular polarization.



Right hand circular polarization

Case 2: n = odd (n = 1)

$$\delta = (2n+1)\pi/2 = \frac{3\pi}{2}$$

$$e^{j\delta} = e^{j3\pi/2} = \cos\frac{3\pi}{2} + j\sin\frac{3\pi}{2} = -j$$

$$\bar{E}(z,t) = E_0 \cos(\omega t - \beta z)\,\hat{a}_x + e^{j\delta} E_0 \cos(\omega t - \beta z)\,\hat{a}_y$$

$$= E_0 \cos(\omega t - \beta z)\,\hat{a}_x - j E_0 \cos(\omega t - \beta z)\,\hat{a}_y$$

$$= E_0 \cos(\omega t - \beta z)\,(\hat{a}_x - j\hat{a}_y)$$

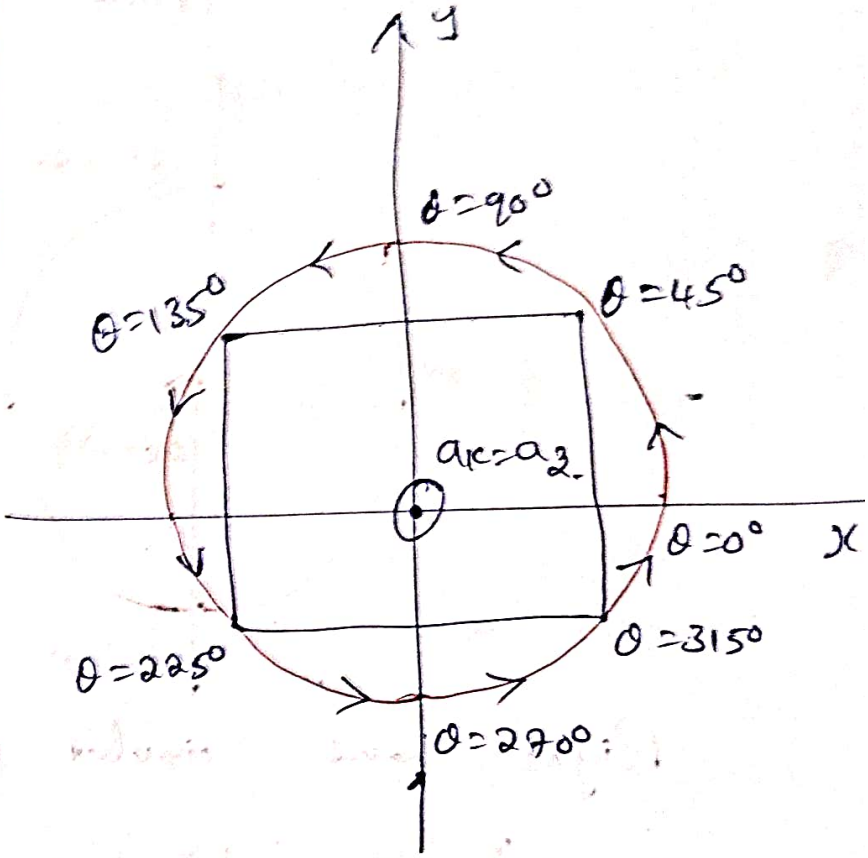
$$= E_0 \cos(\omega t - \beta z)\,\hat{a}_x + E_0 \cos(\omega t - \beta z - 90^\circ)\,\hat{a}_y$$

$$E = E_0 \cos(\omega t - \beta z)\,\hat{a}_x + E_0 \sin(\omega t - \beta z)\,\hat{a}_y$$

At $z=0$

$$\vec{E}(0,t) = E_0 \cos(\omega t)\hat{a}_x + E_0 \sin(\omega t)\hat{a}_y$$

θ	E_{ox}	E_{oy}
0°	E_0	0
45°	$E_0/\sqrt{2}$	$E_0/\sqrt{2}$
90°	0	E_0
135°	$-E_0/\sqrt{2}$	$E_0/\sqrt{2}$
180°	$-E_0$	0
225°	$-E_0/\sqrt{2}$	$-E_0/\sqrt{2}$
270°	0	$-E_0$
315°	$E_0/\sqrt{2}$	$-E_0/\sqrt{2}$
360°	E_0	0

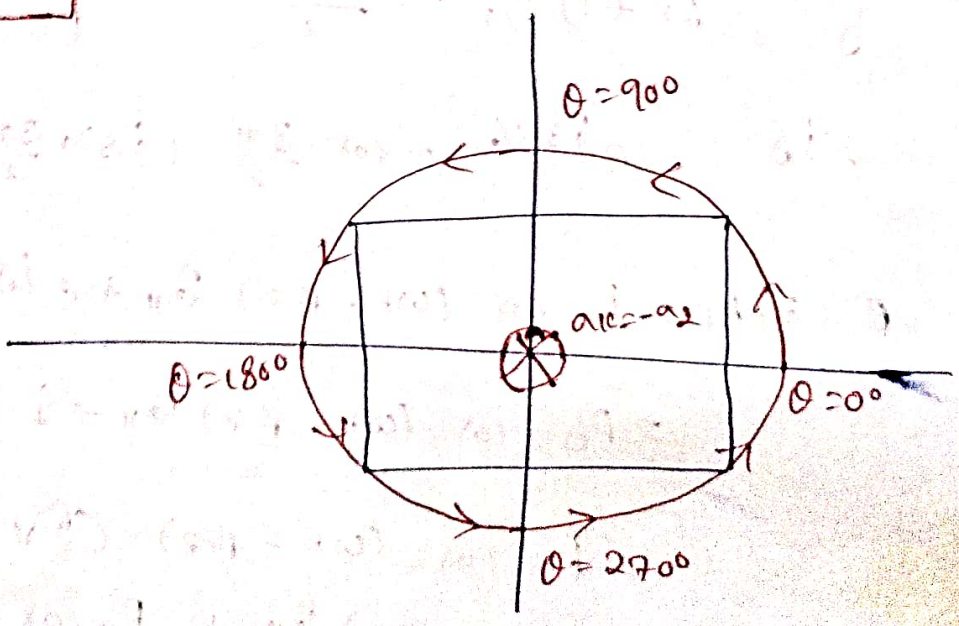


Right hand circularly polarized

if $a_{ic} = -\hat{a}_z$

⊗ → Inside

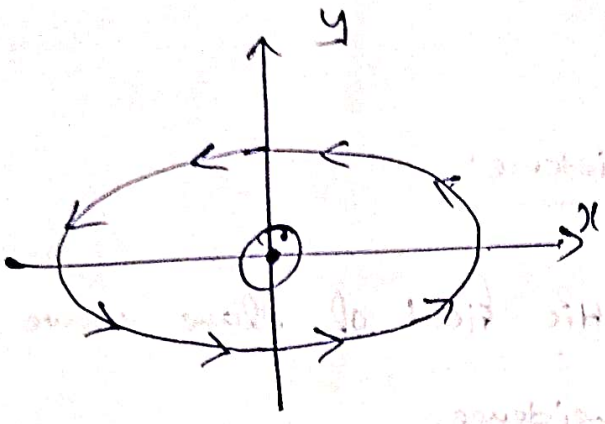
⊙ → out side



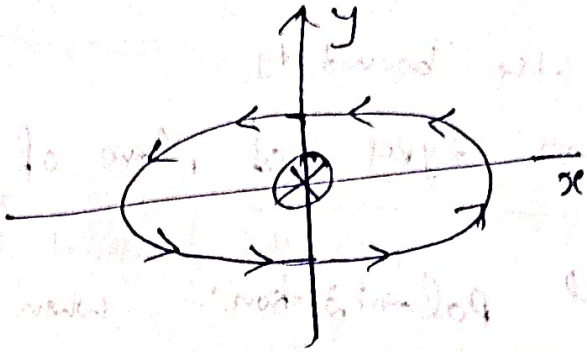
Elliptically Polarization

For elliptical polarization

$E_{ox} \neq E_{oy}$ and $\delta = (2n+1)\pi/2$

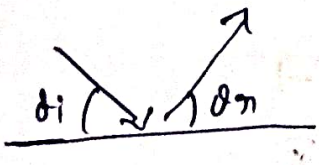


Right hand



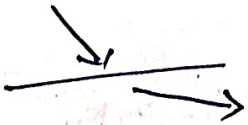
left hand

Reflection:-



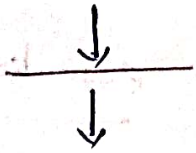
- mostly $\theta_i = \theta_r$

Refraction:-



- occur when wave pass from one medium to other and change direction
- it can be calculated from snell's law

Transmission:-



Scattering:-



Reflection of Plane wave at oblique Incidence :- (Lect-23)

Oblique incidence :- it is also preferred to as plane of incidence. plane of incidence is defined as plane containing incident wave and ⊥ unit normal vector to the boundary

two types of plane of Incidence :-

(a) **⊥ly polarization :-** when electric field of plane wave is parallel to the plane of Incidence. ← vertical polarization

(b) **⊥lr polarization :-** when electric field is ⊥lr to the plane of Incidence.

Maxwell's equation :- $\nabla \to k, \quad j \to -1$

$$\nabla \times \bar{E} = -j\omega\mu\bar{H} \to (1), \quad \nabla \times \bar{H} = (\sigma + j\omega\varepsilon)\bar{E}$$

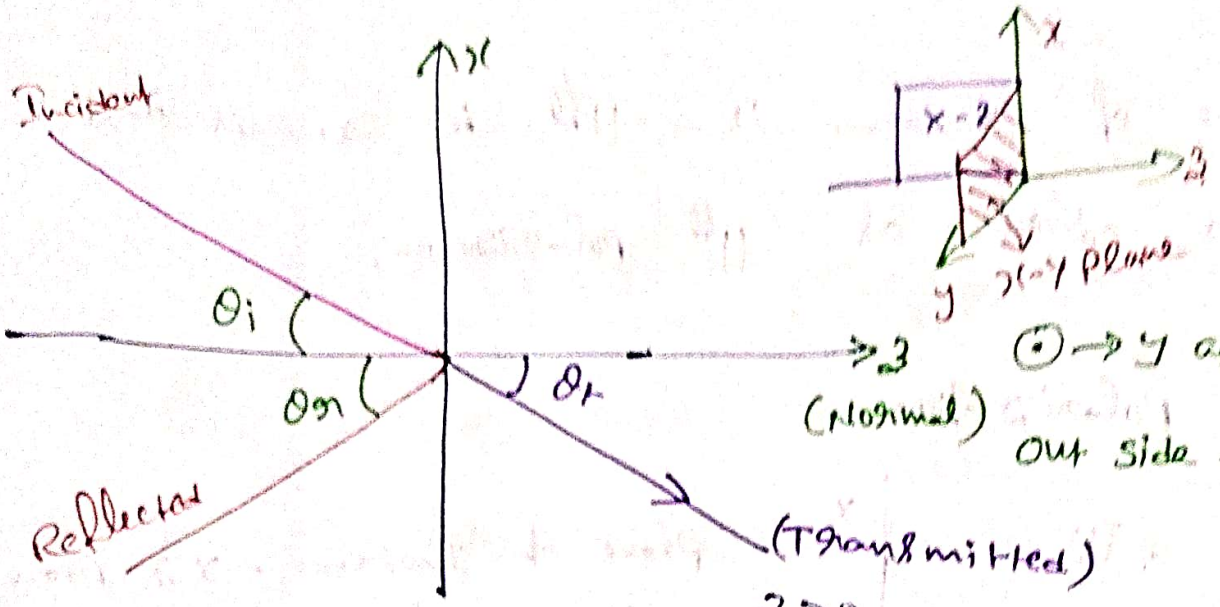
For lossless medium, $\sigma = 0$.

$$\nabla \times \bar{H} = j\omega\varepsilon\bar{E} \to (2), \quad \nabla \cdot \bar{E} = 0 \to (3)$$

$$\nabla \cdot \bar{H} = 0 \to (4)$$

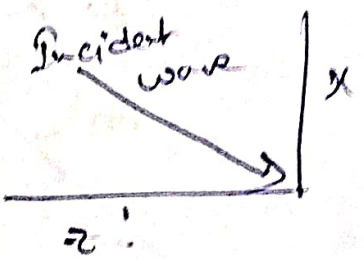
$$\bar{k} \times \bar{E} = \omega\mu\bar{H} \to (1), \quad \bar{k} \times \bar{H} = -\omega\varepsilon\bar{E} \to (2)$$

$$\bar{k} \cdot \bar{E} = 0 \to (3) \quad \bar{k} \cdot \bar{H} = 0 \to (4)$$



⊙ → y axis

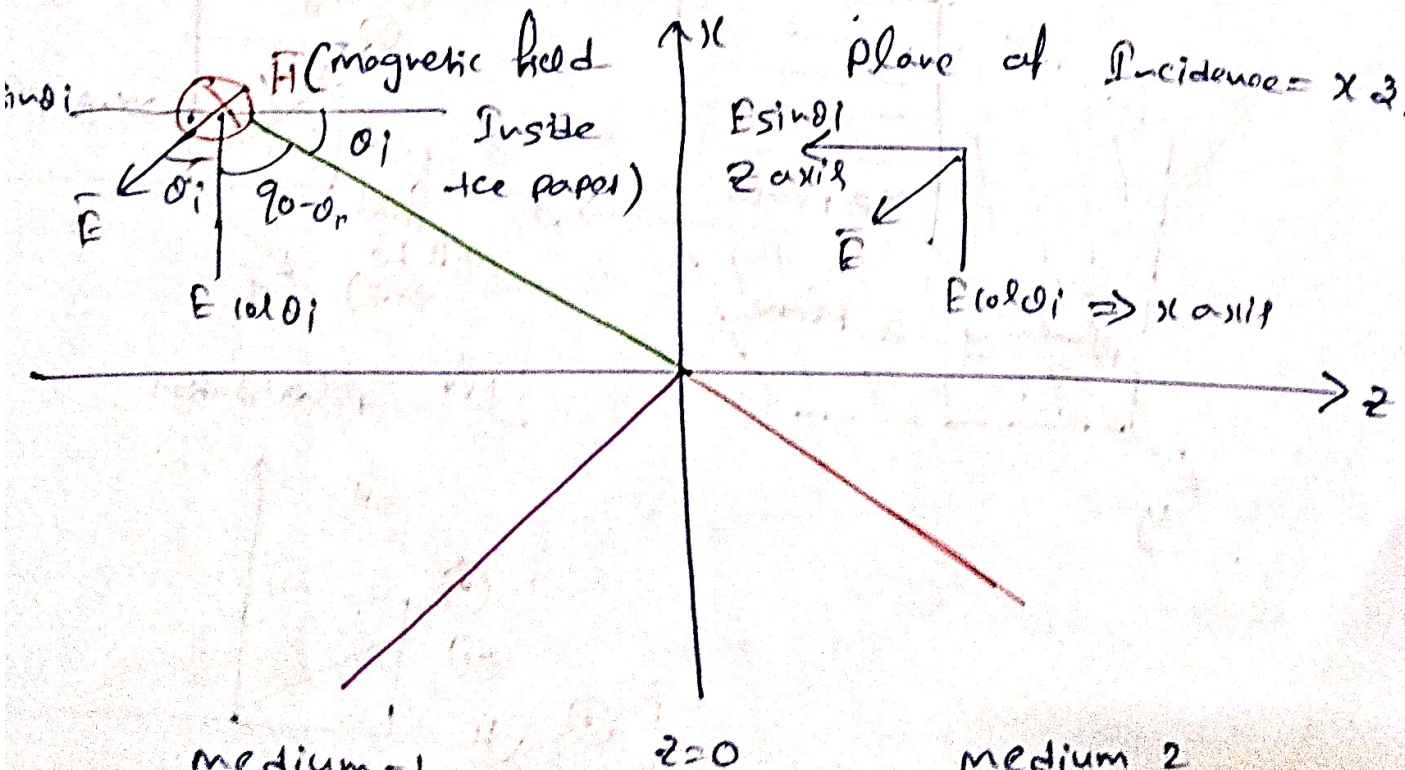
Out side the paper



Plane of Incidence → x z plane

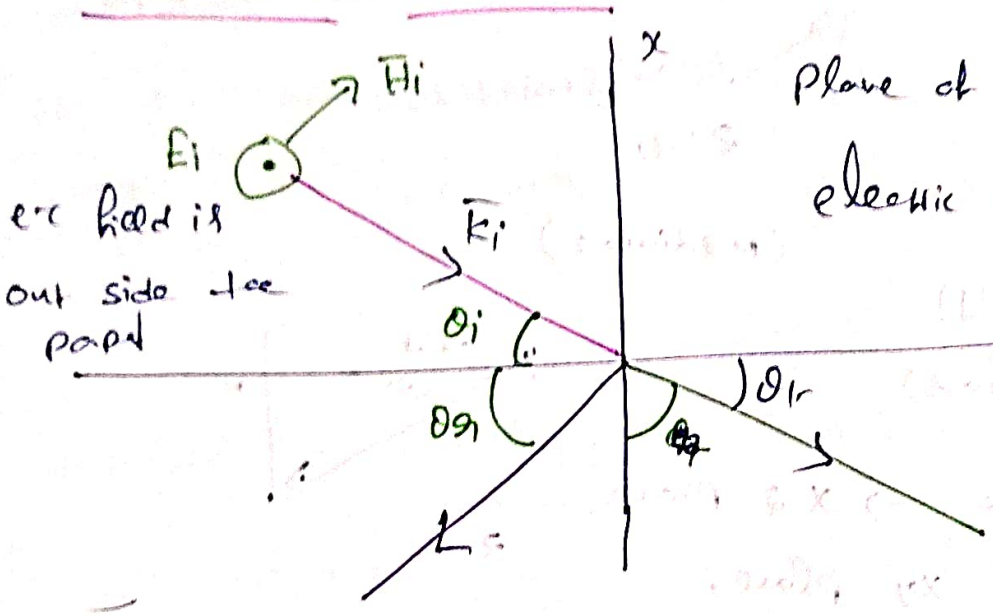
Boundary → xy plane.

- angle of Incidence, reflection or transmission is measured from normal to the boundary



- here plane of Incidence is ⊥ll to x-axis and z-ax
- so it is called as ⊥ll polarization.

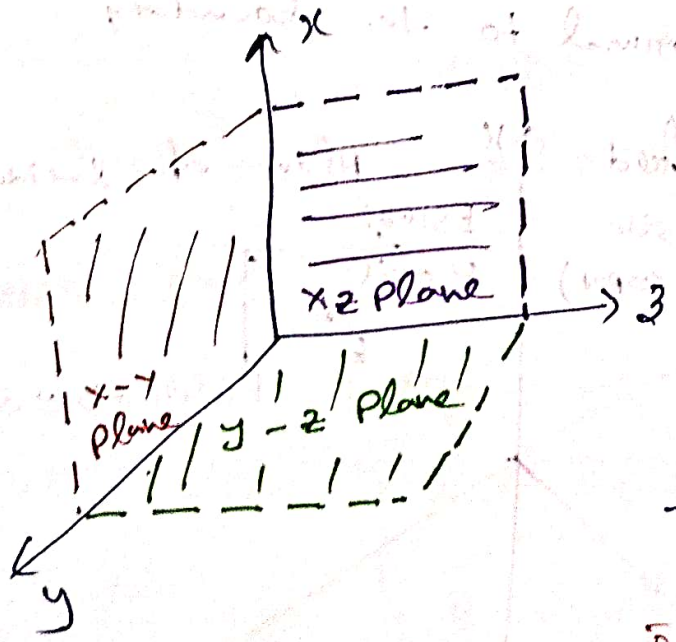
⊥ll Polarization

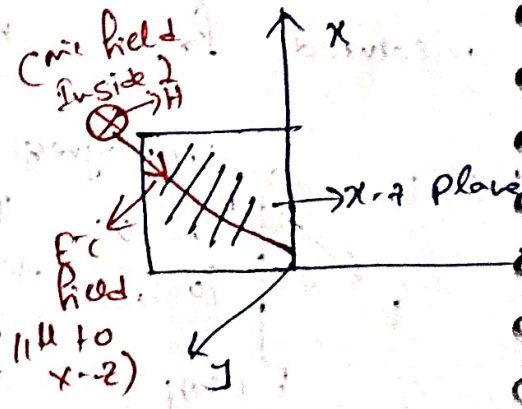


Plane of Incidence - x z plane
electric field → y-direction.
e-c field is out side the paper

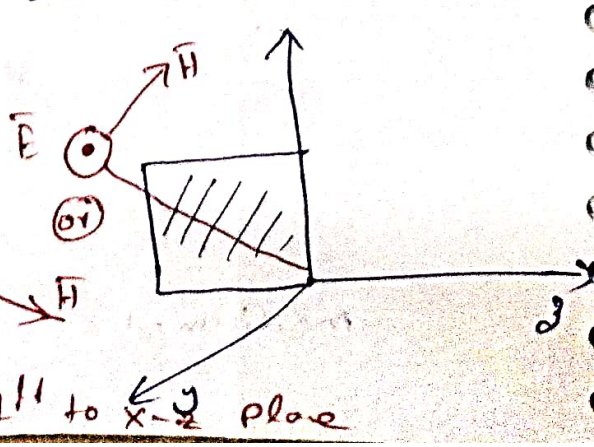
⊙ → out side the paper

||ll Polarization





⊥lr Polarization



Lect - 20

Representation of wave

$$\bar{E}(x,y,z,t) = (E_{ox}\,a_x + E_{oy}\,a_y + E_{oz}\,a_z)\, e^{j(\omega t - \bar{k}\cdot\bar{r})}$$

$\bar{k}$ = propagation vector

$$\bar{k} = k_x a_x + k_y a_y + k_z a_y$$

(Indicate wave incidence direction)

$$\beta = |\bar{k}| = \sqrt{k_x^2 + k_y^2 + k_z^2}$$

$\bar{r} = \bar{r}$ = Position vector

$= x a_x + y a_y + z a_z$

$\cos(\omega t - \beta z)$ ⋯

↓

if wave propagate along z-direction only.

$\bar{E}\cdot\bar{r} = x k_x + y k_y + z k_z$

- if $\bar{E} = E_{ox}\,\bar{a}_E\, e^{j(\omega t - 3x + 4z)}$

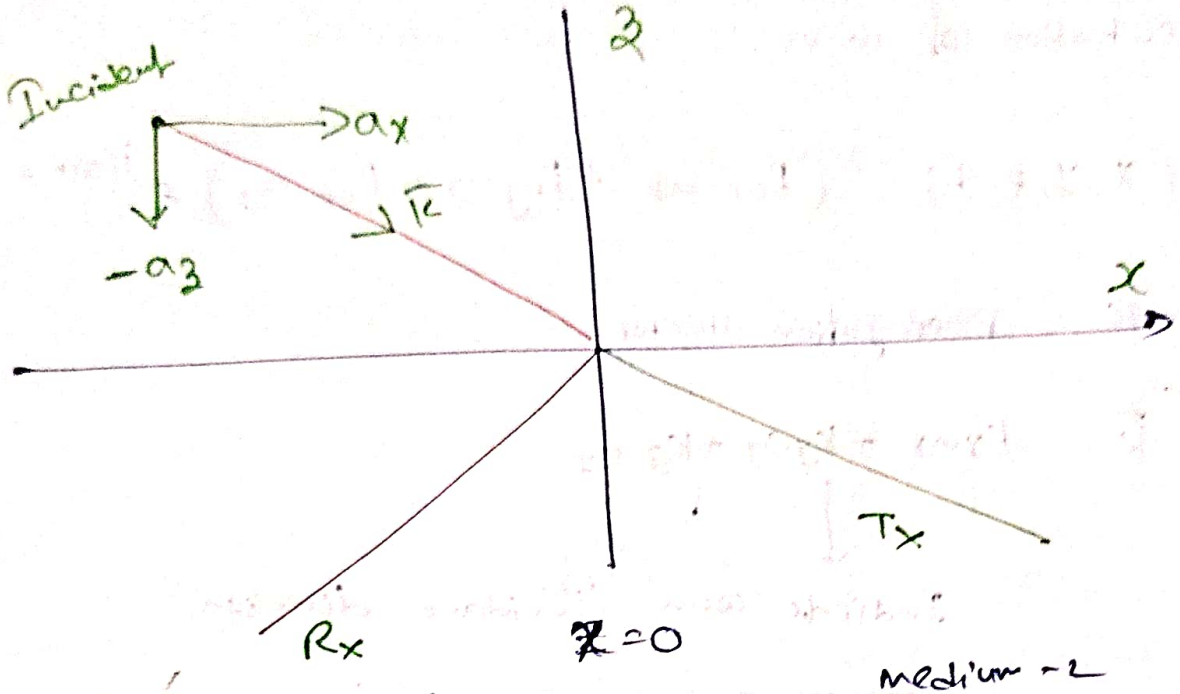
$= E_{ox}\,\bar{a}_E\, e^{j(\omega t - (3x - 4z))}$

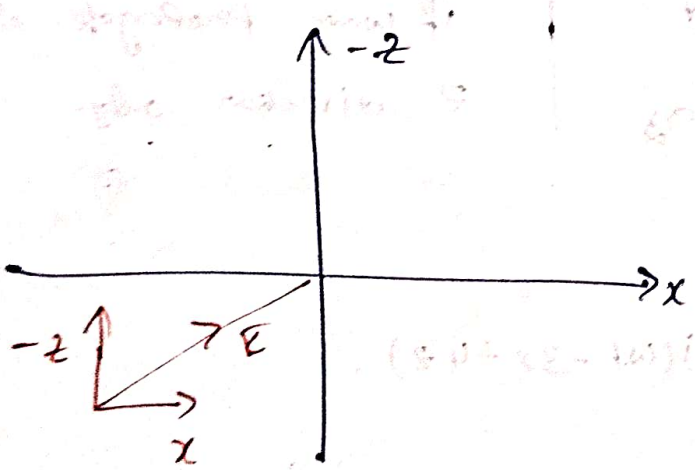
$\bar{k}\cdot\bar{r} = 3x - 4z$

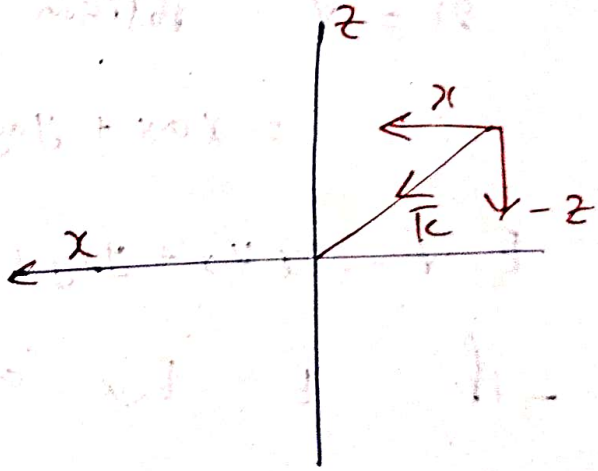
$\bar{r} = x\bar{a}_x + y a_y + z a_z$.

$k = k_x a_x + k_y a_y + k_z a_y = 3a_x - 4a_z$

- if $\bar{E}$ is incident wave the wave was travelling along $3a_x + 4(-a_z)$ i.e a_x & $-a_z$ plane

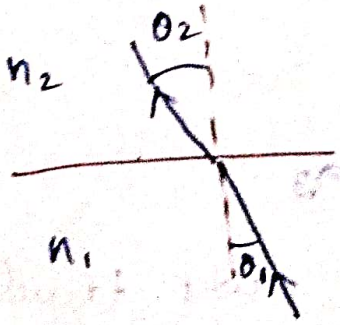






Snell's law:- law of refraction

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$



$n_1 = n_2$ = refractive Index

$$= \frac{\text{Velocity of light in a vacuum}}{\text{" " " " " medium}}$$

$$= \frac{c}{v_p = v_p}$$

(how fast phase change with that velocity wave travels)

Boundary conditions that must be satisfied for all x & y

(1) $\omega_i = \omega_{ref} = \omega_t = \omega$.
↳ freq matching condition

(2) $k_{ix} = k_{refx} = k_{tx} = k_x$
↓
Incidence propagation vector x-component

(3) $k_{iy} = k_{refy} = k_{ty} = k_y$
↳ phase matching condition.

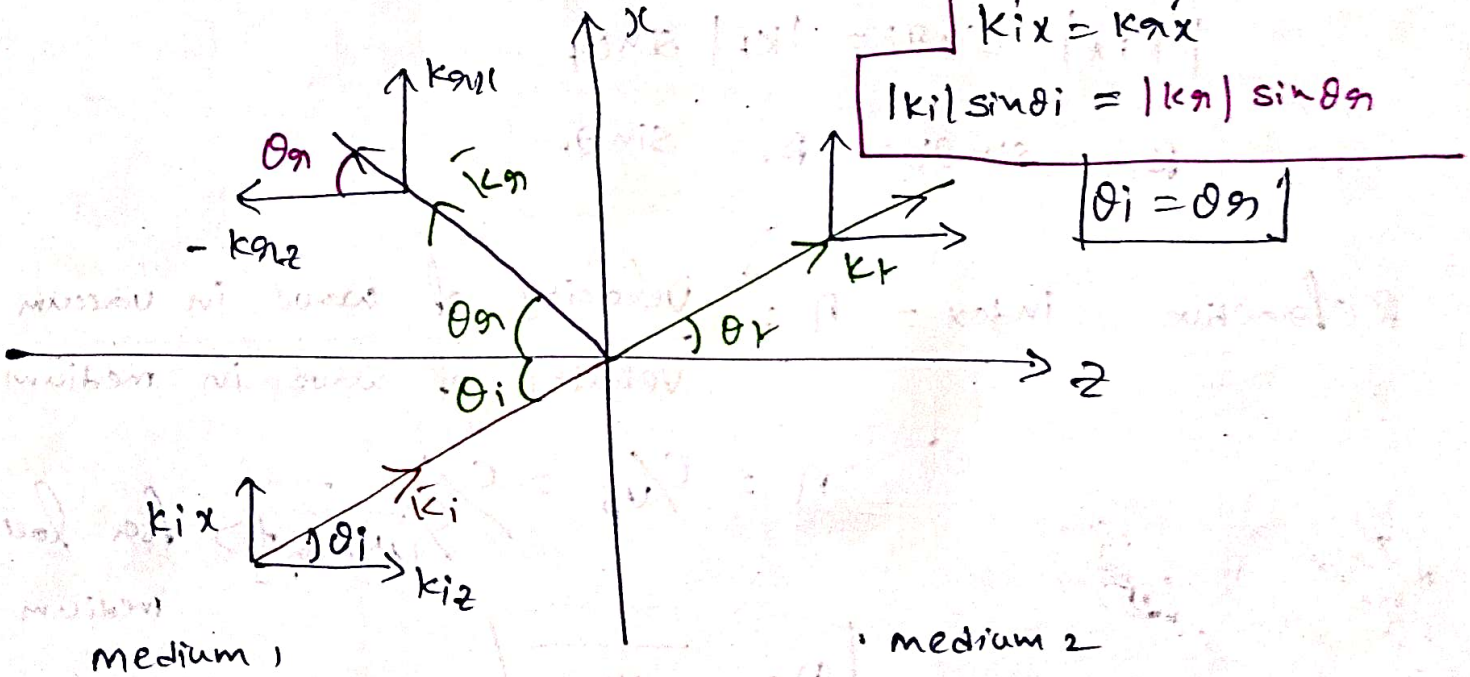
This condition only for k_i, k_{ref}, k_t
↓ $(k_{ix}+k_{iy}+k_{iz})$
↓ $(k_{rx}+k_{ry}+k_{rz})$
$(k_{ix}+k_{iy}+k_{iz})(k_{rx}+k_{ry}+k_{rz})$

one part of k_i — one part of k_{ref}

$k_{ix} = k_{refx}$

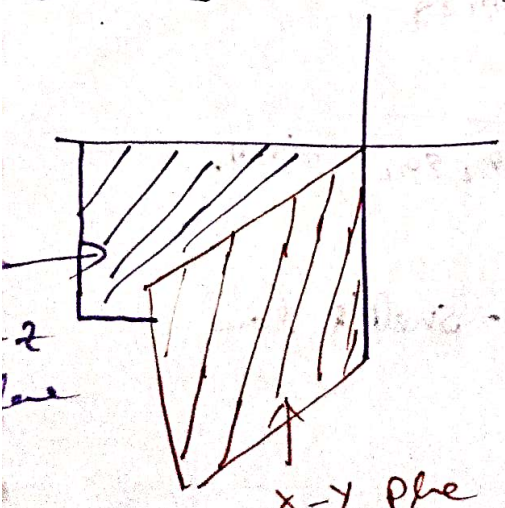
$|k_i| \sin\theta_i = |k_{ref}| \sin\theta_{ref}$

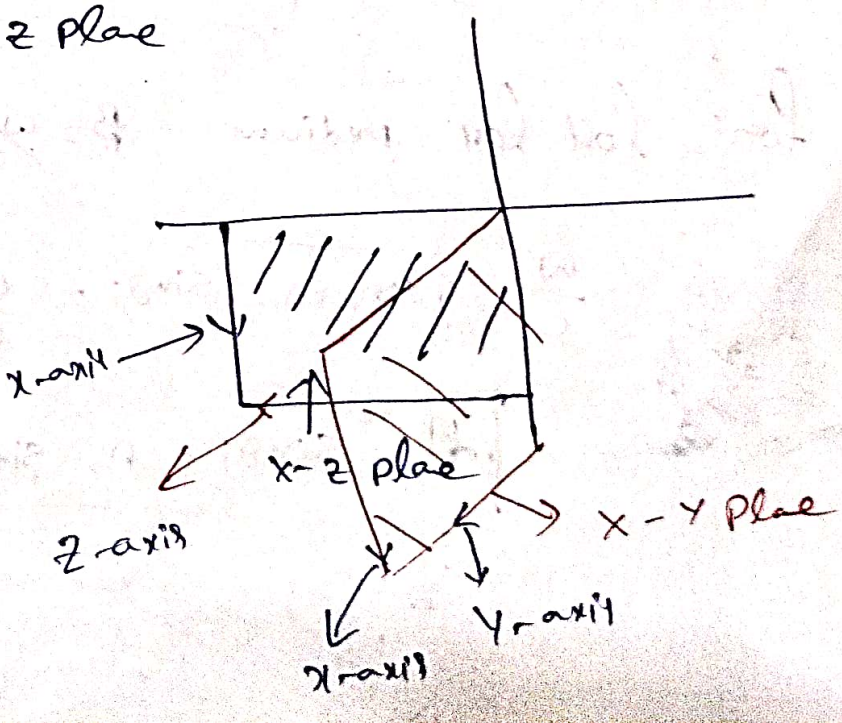
$\theta_i = \theta_{ref}$



Plane of Incidence = x z plane

Boundary = x-y plane.





(A) $k_{ix} = k_{gx}$

$|k_i| \sin\theta_i = |k_g| \sin\theta_g$

$\theta_i = \theta_g$

angle of Incidence = angle of reflectance.

(B) $k_{ix} = k_{tx}$ (Snell's law not applicable for same medium Trion) i.e not for reflection

$|k_{ix}| \sin\theta_i = |k_t| \sin\theta_t$ ⇒ is law of refraction

$$\boxed{\beta_1 \sin\theta_i = \beta_2 \sin\theta_t}$$

Refractive index = η = $\frac{\text{velocity of wave in vaccum}}{\text{velocity of wave in medium.}}$

$$\boxed{v_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu\cdot\varepsilon}} = \frac{c}{\sqrt{\mu_r\varepsilon_r}}} \quad \eta = c/v_p = c/\left[c/\sqrt{\mu_r\varepsilon_r}\right] \rightarrow \text{for low loss medium}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right]} \qquad \boxed{\eta = \sqrt{\mu_r\varepsilon_r}}$$

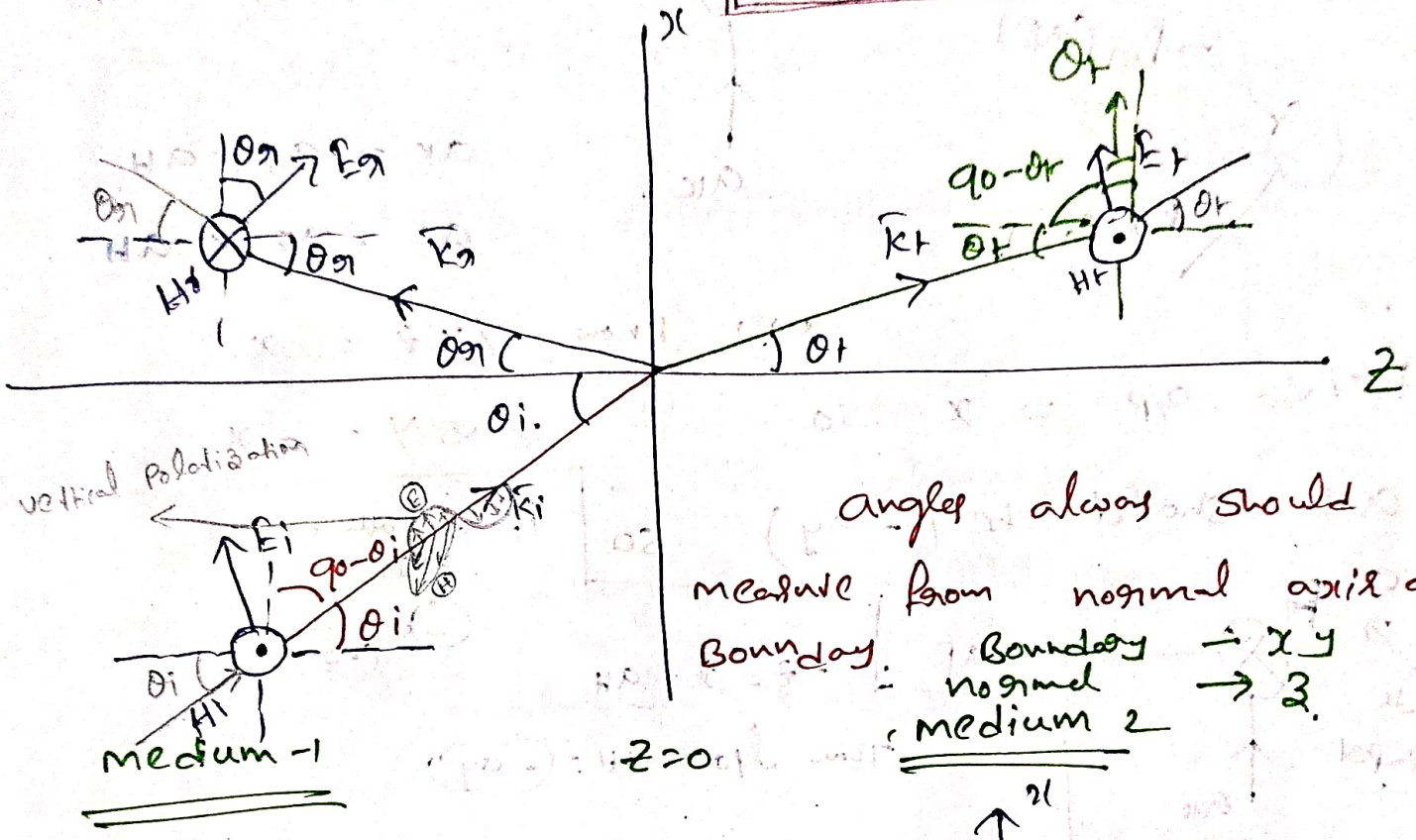
For low loss medium $\beta = \omega/c\sqrt{\mu_r\varepsilon_r}$

$$\frac{\omega}{c}\sqrt{\mu_{r_1}\varepsilon_{r_1}}\sin\theta_i = \frac{\omega}{c}\sqrt{\mu_{r_2}\varepsilon_{r_2}}\sin\theta_t$$

$$\boxed{n_1 \sin\theta_i = n_2 \sin\theta_t} \text{ - Snell's law.}$$

Parallel Polarization :-

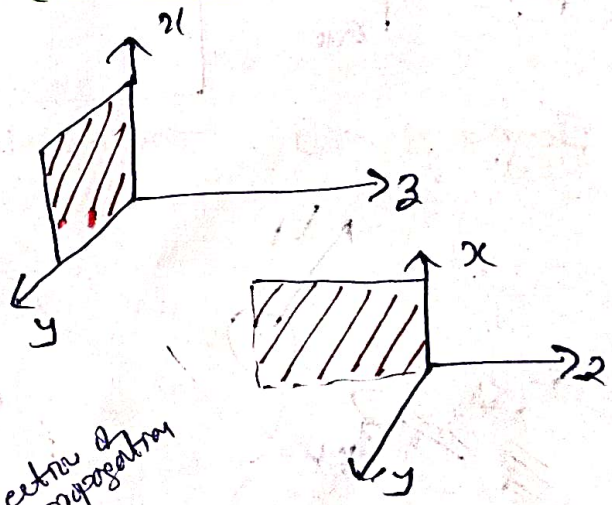
E-field direction should be same i.e +x axis.

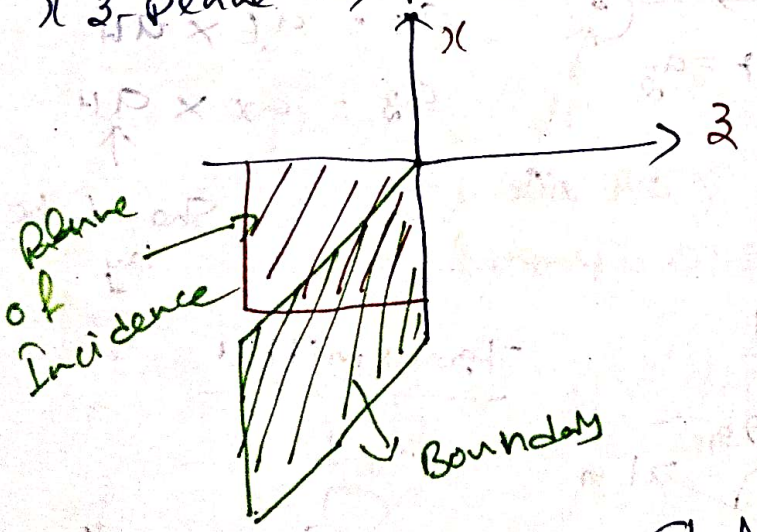


angles always should measure from normal axis of Boundary. Boundary → xy
normal → z.

xy-plane → Boundary

xz-plane → Plane of Incidence



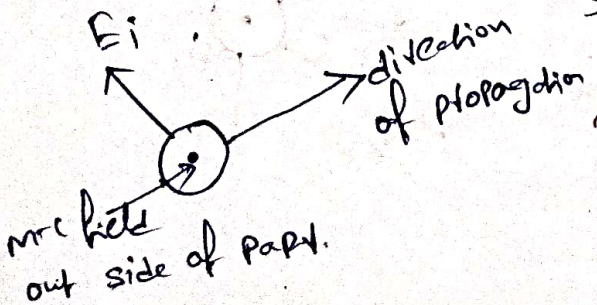


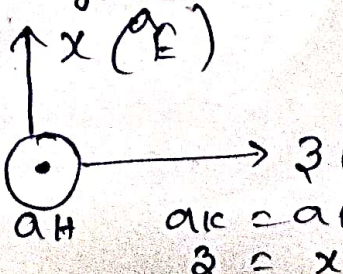
direction of propagation

$$a_{IC} = \bar{a}_E \times \bar{a}_H$$

Remember this representation

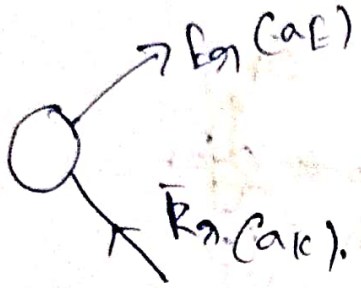
Should always satisfy Incidence

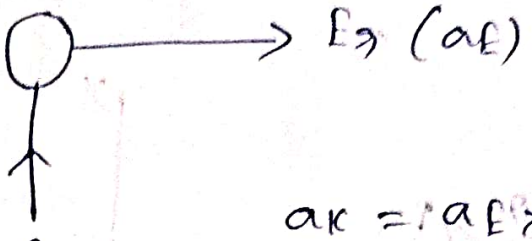




$a_{IC} = a_E \times a_H$
z = x × y.

Reflected plane :-





$a_K = a_E \times a_H$

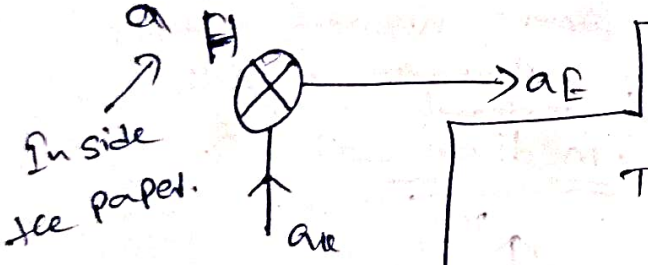
$a_x = a_z \times a_H$

we know $y \times z = x$.

here o/p = x so.

$z \times y = -x$.

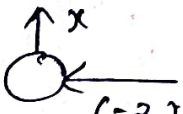
a_H should be $(-a_y)$ so



$-z = x \times a_H$

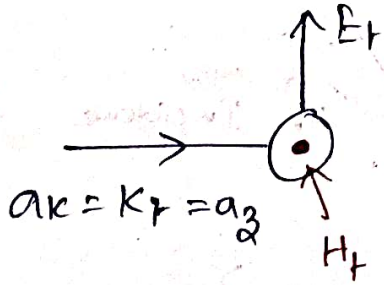
Then $a_H = (-a_y)$

if we allure



Same way Transmitted plane :-





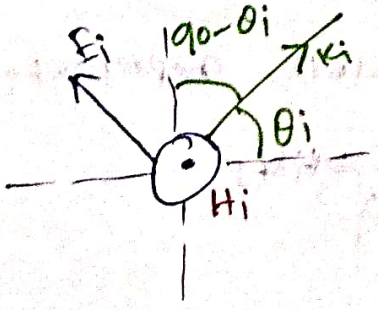
$a_K = a_E \times a_H$

$a_z = a_x \times a_H$

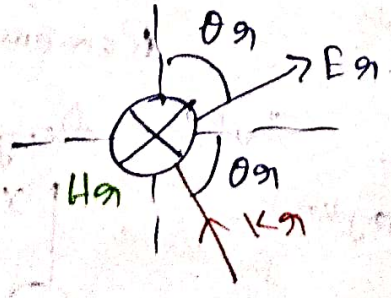
(out side the paper)

Should be a_y

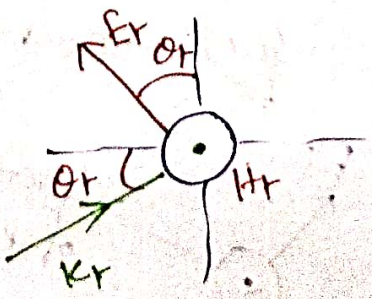
Incidence



Reflected



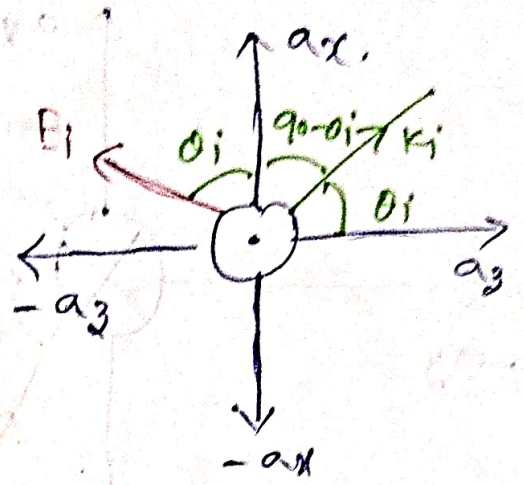
Transmitted



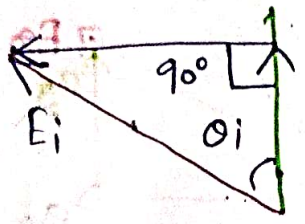
Incidence Plane

$$\bar{E}_i = (E_{ox}\,a_x + E_{oy}\,a_y + E_{oz}\,a_z)\, e^{j(\omega t - \bar{k}\cdot\bar{r})}$$

$$\bar{r} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$$

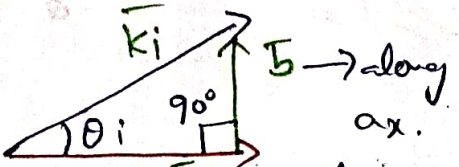


$$\bar{k}_i = |\bar{k}_i| \cos\theta_i\, \hat{a}_z + |\bar{k}_i| \sin\theta_i\, a_x$$



$$\sin\theta_i = \frac{b}{\bar{k}_i}$$

$$\bar{b} = |\bar{k}_i| \sin\theta_i\, a_x$$



$$\bar{a} = |\bar{k}_i| \cos\theta_i\, a_z$$

$$\bar{k} = \bar{a} + \bar{b}$$

$$\bar{k}_i = \beta_1 \cos\theta_i\, a_z + \beta_1 \sin\theta_i\, a_x$$

β_1 → medium 1

> β = magnitude of the propagation vector
>
> $k = k_x a_x + k_y a_y + k_z a_z$
>
> $\therefore |k| = \beta$

$$\bar{k}\cdot\bar{r} = \beta_1 \cos\theta_i\, z + \beta_1 \sin\theta_i\, x$$

$$\bar{E}_i = \left[E_{io}\cos\theta_i\, a_x + E_{io}\sin\theta_i\,(-\hat{a}_z)\right] e^{j(\omega t - \bar{k}_i\cdot\bar{r})}$$

$$\bar{E}_{is} = \left[E_{io}\cos\theta_i\, a_x + E_{io}\sin\theta_i\,(-a_z)\right] e^{-j(\beta_1 z\cos\theta_i + \beta_1 x\sin\theta_i) - \theta}$$

let Time factor suppressed.

$$\bar{H}_i = \left[\frac{E_{io}}{\eta_1}\hat{a}_y\, e^{-j(\beta_1 z\cos\theta_i + \beta_1 x\sin\theta_i)}\right] = \frac{E_{io}}{\eta_1} e^{-j(\beta_1 z\cos\theta_i - \beta_1 x\sin\theta_i)}\hat{a}_y \quad -②$$

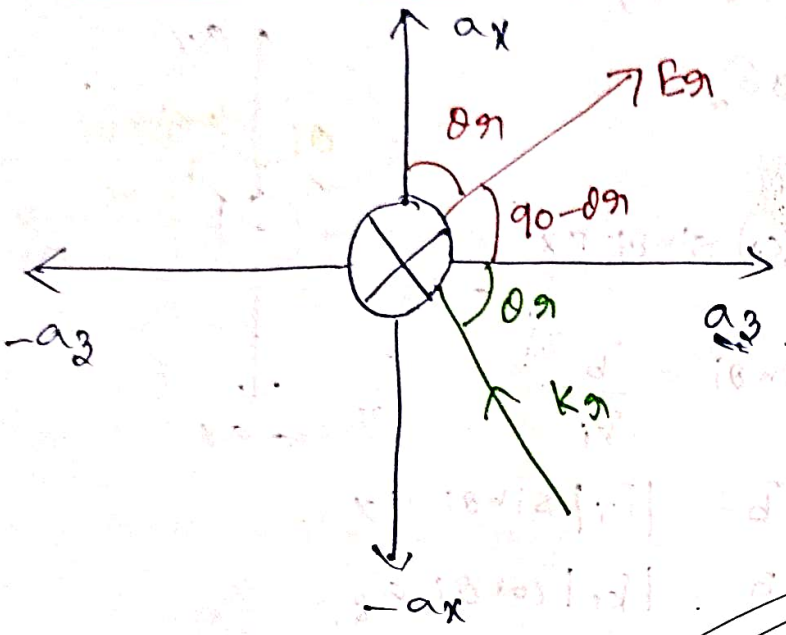
$\hat{a}_y$ → m-c field

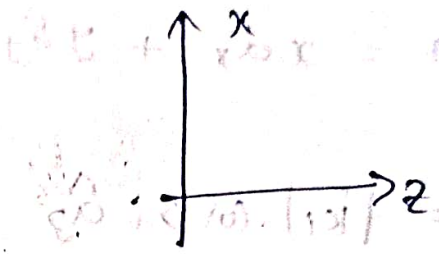
$\beta_1 x\sin\theta_i$ → wave propagation direction

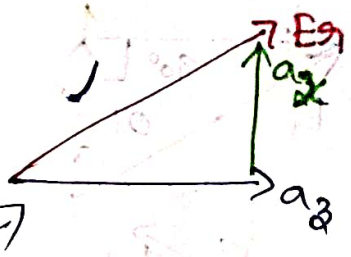
E-c field direction — plane of Incidence — x-z plane

m-c " " — y direction.

Reflected plane (or) wave

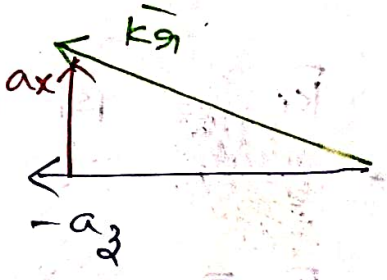






$$\bar{E}_{rs} = (E_{ro} \cot\theta_r \, a_x + E_{ro} \sin\theta_r \, \hat{a}_z)$$

$$e^{-j(\bar{k}\cdot\bar{r})}.$$



$$\bar{k} = (\bar{k}_r) \cot\theta_r (-a_z) + \bar{k}_r \sin\theta_r \, \bar{a}_x$$

$$\bar{k} = -\beta_1 \cot\theta_r a_z + \beta_1 \sin\theta_r a_x$$

(medium 1.)

$$\bar{k}\cdot\bar{r} = -\beta_1 \cot\theta_r \, z + \beta_1 \sin\theta_r \, x$$

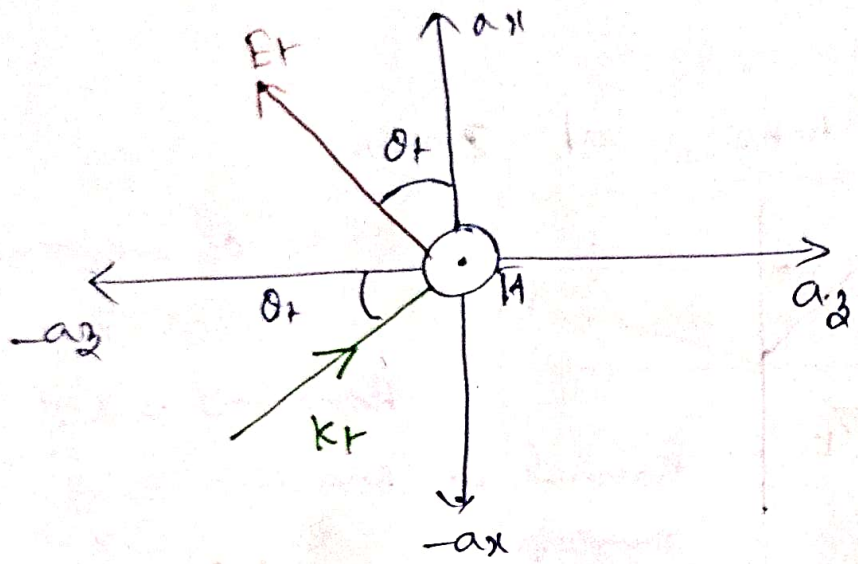
$\bar{r} = x a_x + y a_y + z a_z \rightarrow$ Position vector

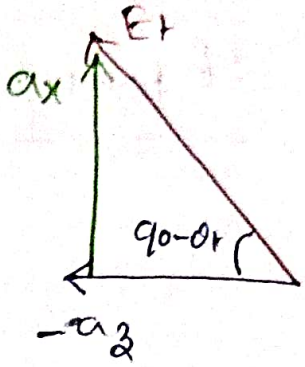
$$\vec{E}_{rs} = (E_{ro} \cos\theta_r \hat{a}_x + E_{ro} \sin\theta_r \hat{a}_z) e^{j(-\beta_1 z \cos\theta_r + \beta_1 x \sin\theta_r)} \quad \text{—③}$$

$$\bar{H}_{rs} = H_{ro}(-a_y) e^{-j(-\beta_1 z \cot\theta_r + \beta_1 x \sin\theta_r)}.$$

$$= \frac{E_{ro}}{\eta_1}(-\hat{a}_y) e^{-j(-\beta_1 z \cot\theta_r + \beta_1 x \sin\theta_r)}. \rightarrow ④$$

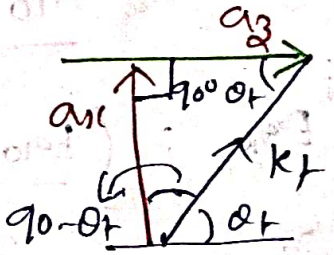
Transmitted wave (or) Plane





$$\bar{E}_t = \left[E_{to} \cos\theta_t \hat{a}_x + E_{to} \sin\theta_t (-\hat{a}_3)\right] e^{-j[\bar{k} \cdot \bar{r}]}$$

$$\bar{r} = x a_x + y a_y + z a_3.$$



$$\bar{k} = |k_t| \cos\theta_t a_3 + |k_t| \sin\theta_t a_x$$

($|k_t| \downarrow \beta_2$; $|k_t| \sin\theta_t \downarrow \beta_2$)

$$\bar{k} \cdot \bar{r} = \beta_2 \cos\theta_t z + \beta_2 \sin\theta_t x$$

$$\bar{E}_t = \left[E_{to} \cos\theta_t \hat{a}_x + E_{to} \sin\theta_t (-\hat{a}_3)\right] \quad \text{— (5)}$$

$$e^{-j(\beta_2 \cos\theta_t z + \beta_2 \sin\theta_t x)}$$

$$H_t = \frac{E_{to}}{\eta_2} a_y \, e^{-j(\beta_2 \cos\theta_t z + \beta_2 \sin\theta_t x)} \quad \text{— (6)}$$

$\frac{E_{to}}{\eta_2} a_y$ → mr field direction

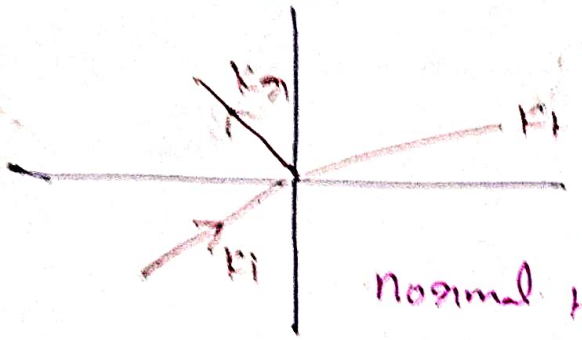
$\beta_2 \cos\theta_t z + \beta_2 \sin\theta_t x$ → field propagation

tangential components — x & y direction (for Boundary)

normal components — z direction (for Boundary)

Boundary condition

$E_{tan1} = E_{tan2}$ at $z = 0$



Plane of Incidence = x - z

Boundary = x-y plane

Normal to Boundary = z - direction

Tangential " " = x - y "

$$E_i = \left[\underbrace{E_{i0}\cos\theta_i\, a_x}_{T} - \underbrace{E_{i0}\sin\theta\, a_z}_{N}\right] e^{-j\left(\underbrace{\beta_1 z\cos\theta_i}_{N} + \underbrace{\beta_1 x \sin}_{T}\right.}$$

$$H_i = \underbrace{H_{i0}\, a_y}_{T}\, e^{-j\left(\underbrace{\beta_1 z\cos\theta_i}_{N} + \underbrace{\beta_1 x \sin\theta_i}_{T}\right)}$$

$$E_r = \left(\underbrace{E_{r0}\cos\theta_r a_x}_{T} + \underbrace{E_{r0}\sin\theta_r a_z}_{N}\right) e^{-j\left(\underbrace{-\beta_1 \cos\theta_r a_z}_{N} + \underbrace{\beta_1 \sin\theta}_{T}\right.}$$

$$H_r = \underbrace{H_{r0}\,(-a_y)}_{T}\, e^{-j\left(-\beta_1 z\cos\theta_r + \beta_1 x \sin\theta_r\right)}$$

$$E_t = \left[\underbrace{E_{t0}\cos\theta_t\, a_x}_{T} + \underbrace{E_{t0}\sin\theta_t\,(-a_z)}_{N}\right] e^{-j\left(\underbrace{\beta_2 z\cos\theta_t}_{N} + \underbrace{\beta_2 \sin\theta_t x}_{T}\right)}$$

$$H = \underbrace{H_{t0}\, a_y}_{T}\, e^{-j\left(\underbrace{\beta_2 z\cos\theta_t}_{N} + \underbrace{\beta_2 \sin\theta_t x}_{T}\right)}$$

For electric field $E_{tan1} = E_{tan2}$.

at medium ① incidence tangential component.

$E_{i0}\cos\theta_i\, e^{-j\beta_1 x \sin\theta_i}$

reflected tangential component.

$E_{r0}\cos\theta_r\, e^{-j\beta_1 x \sin\theta_r}$.

Medium ② tangential component → Boundary condition

$= E_{to} \cos\theta_t \, e^{-j\beta_2 \sin\theta_t}$

$E_{tan1} = E_{tan2}$

$E_{io} \cos\theta_i \, e^{-j\beta_1 x \sin\theta_i} + E_{ro} \cos\theta_r \, e^{-j\beta_1 x \sin\theta_r} = E_{to} \cos\theta_t \, e^{-j\beta_2 \sin\theta_t x}$

we know that $\theta_i = \theta_r$ & $\beta_1 \sin\theta_i = \beta_2 \sin\theta_t$

↳ Snell's law.

So $E_{io} \cos\theta_i \, e^{j\beta_1 x \sin\theta_i} + E_{ro} \cos\theta_i \, e^{j\beta_1 x \sin\theta_i} = E_{to} \cos\theta_t \, e^{-j\beta_2 \sin\theta_t}$

$E_{io} \cos\theta_i + E_{ro} \cos\theta_i = E_{to} \cos\theta_t$

$(E_{io} \cos\theta_i)\left(1 + \frac{E_{ro}}{E_{io}}\right) = E_{to} \cos\theta_t$ / $E_{io} \cos\theta_i$

$1 + \frac{E_{ro}}{E_{io}} = \frac{E_{to}}{E_{io}} \cdot \frac{\cos\theta_t}{\cos\theta_i}$

$$1 + \Gamma_{11} = \cdot \tau_{11} \frac{\cos\theta_t}{\cos\theta_i} \quad \text{— (7)}$$

Γ_{11} → ll^R Polarization

τ_{11} → Transmission coefficient

Boundary condition for M-F fields

at $z = 0$

$H_{tan1} = H_{tan2}$

$H_i + H_r = H_t$

tangential ↔ tangential

$H_{io} - H_{ro} = H_{to} \Rightarrow \frac{E_{io}}{\eta_1} - \frac{E_{ro}}{\eta_1} = E_{to}/\eta_2$

$$\frac{1}{\eta_1}\left[E_{i0} - E_{r0}\right] = E_{t0}/\eta_2$$

$$1 - E_{r0}/E_{i0} = \eta_1/\eta_2 \; \frac{E_{t0}}{E_{i0}}$$

$$1 - \Gamma_{11} = \frac{\eta_1}{\eta_2}\,\tau_{11} \quad \text{— (6)}$$

$$1 + \Gamma_{11} = \tau_{11}\,\frac{\cot\theta_t}{\cot\theta_i}$$

Add Equation 7 & 8

$$2 = \tau_{11}\left[\frac{\eta_1}{\eta_2} + \frac{\cot\theta_2}{\cot\theta_i}\right] = \tau_{11} = \frac{2\eta_2 \cot\theta_i}{\eta_2\cot\theta_t + \eta_1\cot\theta_i}$$

$$1 - \Gamma_{11} = \frac{\eta_1}{\eta_2} \cdot \frac{2\eta_2\cot\theta_i}{\eta_2\cot\theta_t + \eta_1\cot\theta_i}$$

$$\Gamma_{11} = 1 - \left(\frac{2\eta_1\cot\theta_i}{\eta_2\cot\theta_t + \eta_1\cot\theta_i}\right) = \frac{\eta_2\cot\theta_t - \eta_1\cot\theta}{\eta_2\cot\theta_t + \eta_2\cot}$$

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$$\Gamma_{11} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$

Brewster angle :-

The angle of incidence for which reflection coefficient become zero is called Brewster angle.

$\Gamma_{11} = 0$ if $(\theta_i + \theta_t) = \pi/2$

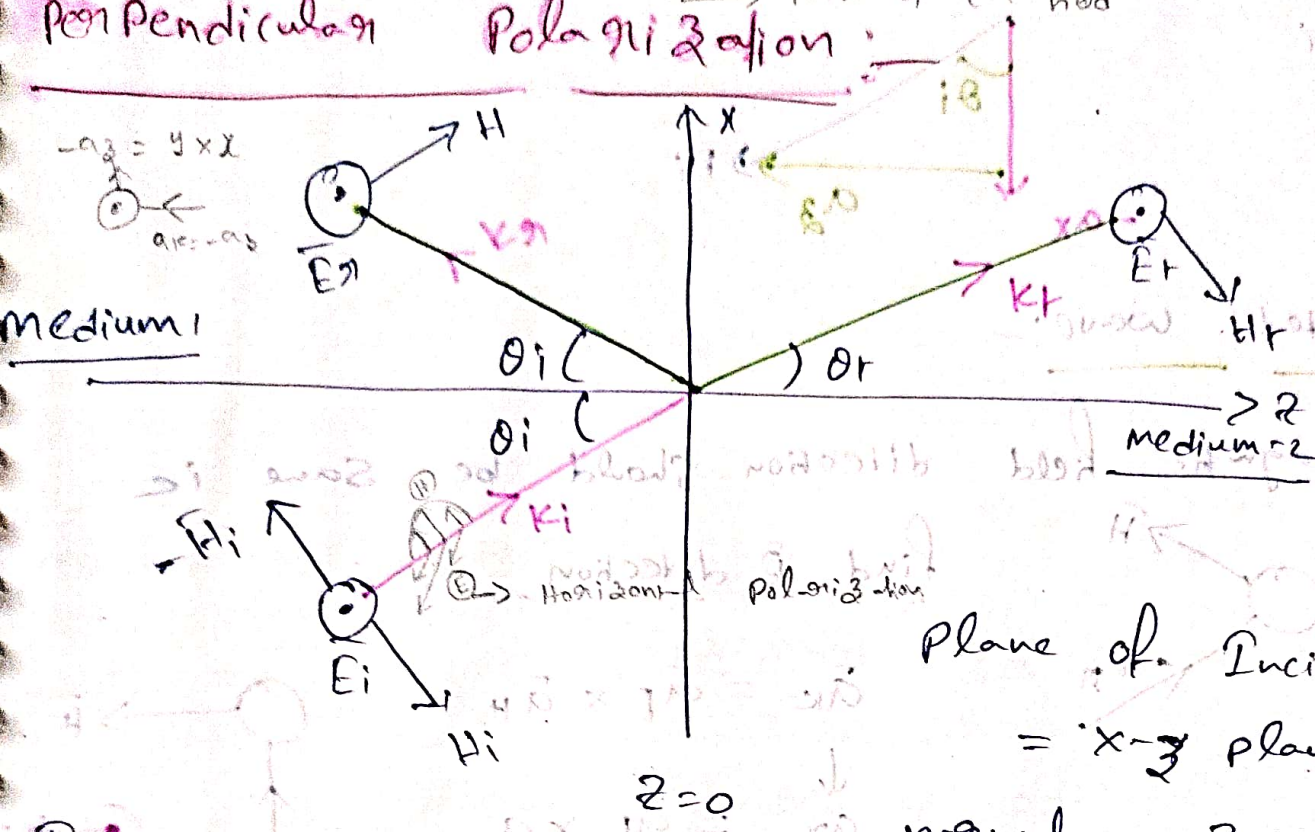
$$\Gamma_{11} = 0, \quad = \frac{\eta_2\cot\theta_t - \eta_1\cot\theta_i}{\eta_2\cot\theta_t + \eta_1\cot\theta_i}$$

$$\eta_2\cot\theta_t = \eta_1\cot\theta_B$$

if both mediums are non-magnetic lossless dielectric

$$\theta_{B11} = \tan^{-1}\left(\sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}}}\right)$$

Perpendicular Polarization :



Plane of Incidence
= x-z plane

normal = z-axis.

Boundary = x-y plane.

Incidence :-

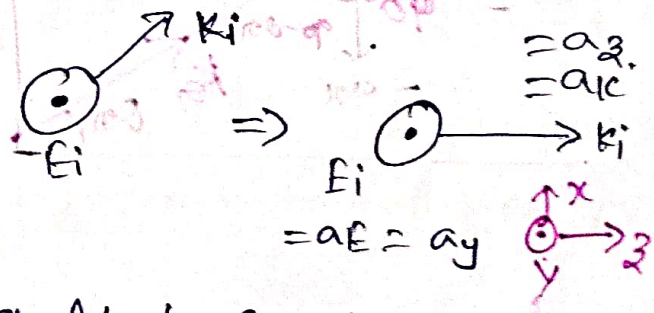
Here Ei - should be $\perp^{lr}$ to plane of Incidence.

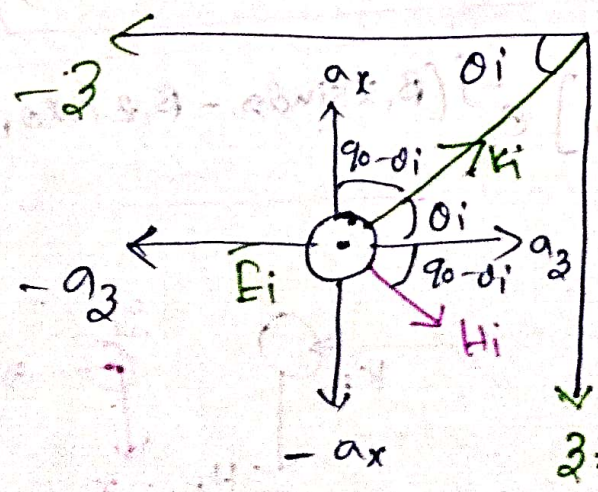
i.e Ei should be in y-direction (out of paper)

$a_{1c} = a_E \times a_H \rightarrow$

$a_3 = a_y \times (-a_x)$

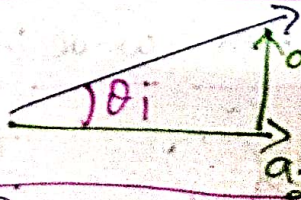
↳ a_H should be $(-a_x)$.





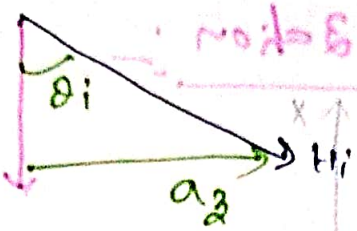
$$\bar{E}_{is} = E_{io}\,\hat{a}_y\, e^{-j(\beta_{1x}\sin\theta_i + \beta_{1z}\cos\theta_i)}$$

For $\bar{k}_i$ =



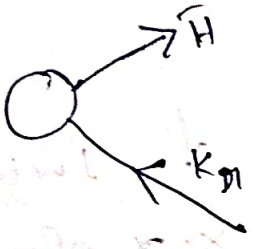
$$H_{is} = H_{io}\left[\sin\theta_i\, a_3 - \cos\theta_i\, a_x\right] e^{-j(\beta_{1x}\sin\theta_i + \beta_{1z}\cos\theta_i)}$$

For H_i



Reflected wave :-

magnetic field direction should be same ie

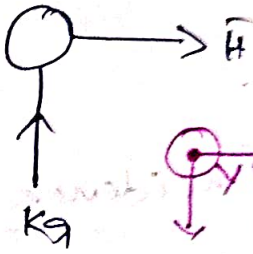


find $\bar{E}$ direction

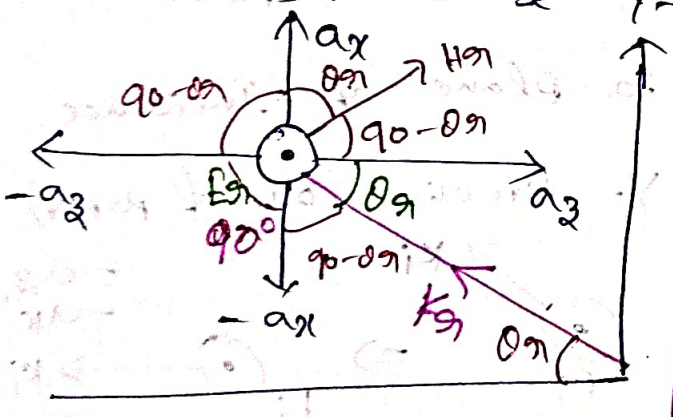
$$\bar{a}_K = \bar{a}_E \times \bar{a}_H$$

$$\downarrow$$

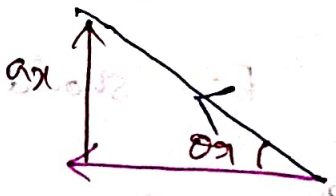
$$a_x = \bar{a}_E \times a_z$$

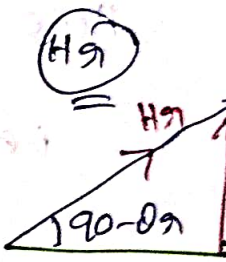


$\bar{a}_E$ should be y- direction i.e $\bar{a}_E = -a_y$





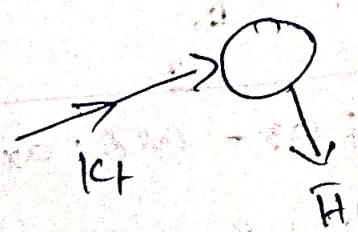




$$E_{rs} = E_{r0}\, \hat{a}_y\, e^{-j(\beta_1 x \sin\theta_r - \beta_1 z \cos\theta_i)}$$

$$H_{rs} = H_{r0}\left[\cos\theta_r \hat{a}_x + \sin\theta_r a_z\right] e^{-j(\beta_1 x \sin\theta_r - \beta_1 z \cos\theta_i)}$$

Transmitted wave :-

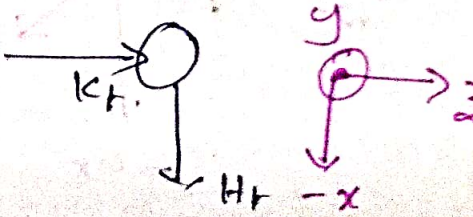


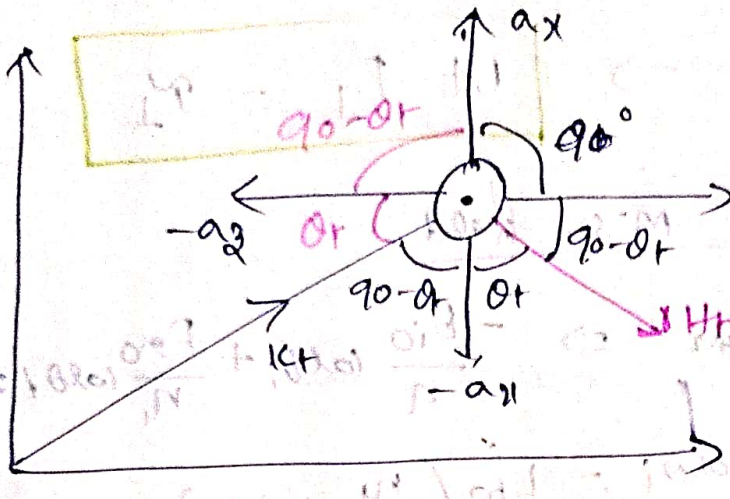
to find $\bar{a}_E$

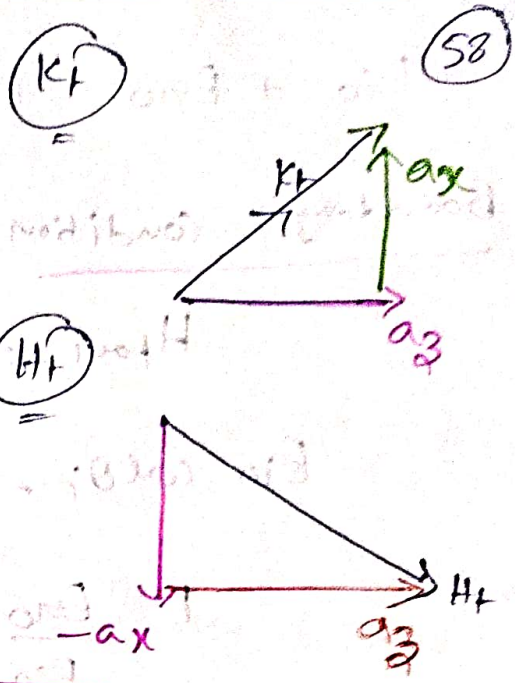
$$a_K = a_E \times a_H$$

$$\downarrow \quad = \quad \downarrow \quad \downarrow$$

$$a_z \quad = \quad a_y \times -a_x$$







Transmitted wave

$$E_t = \underbrace{E_{to}\,\hat{a}_y}_{T}\, e^{-j\left(\underbrace{\beta_2 x \sin\theta_t}_{T} + \underbrace{\beta_2 z \cos\theta_t}_{N}\right)}$$

$$H_t = H_{to}\left[\underbrace{\sin\theta_t\, a_z}_{N} - \underbrace{\cos\theta_t\, a_x}_{T}\right] e^{-j\left(\underbrace{\beta_2 x \sin\theta_t}_{T} + \underbrace{\beta_2 z \cos\theta_t}_{N}\right)}$$

Summary

$$E_{is} = \underbrace{E_{io}\,\hat{a}_y}_{T}\, e^{-j\left(\underbrace{\beta_1 x \sin\theta_i}_{T} + \underbrace{\beta_1 z \cos\theta_i}_{N}\right)}$$

$$H_{is} = H_{io}\left[\underbrace{\sin\theta_i\, a_z}_{N} - \underbrace{\cos\theta_i\, a_x}_{T}\right] e^{-j\left(\underbrace{\beta_1 x \sin\theta_i}_{T} + \underbrace{\beta_1 z \cos\theta_i}_{N}\right)}$$

$$E_{rs} = \underbrace{E_{ro}\,\hat{a}_y}_{T}\, e^{-j\left(\underbrace{\beta_1 x \sin\theta_r}_{T} - \underbrace{\beta_1 z \cos\theta_r}_{N}\right)}$$

$$H_{rs} = H_{ro}\left[\underbrace{\cos\theta_r\, a_x}_{T} + \underbrace{\sin\theta_r\, a_z}_{N}\right] e^{-j\left(\underbrace{\beta_1 x \sin\theta_r}_{T} - \underbrace{\beta_1 z \cos\theta_r}_{N}\right)}$$

Tangential components w.r.t Boundary — x - y direction (x-y plane)

normal components w.r.t Boundary — z - direction (x-y plane)

Electric field Boundary condition

$$E_{tan1} = E_{tan2}$$

$$E_{io}\, e^{-j\beta_1 x \sin\theta_i} + E_{ro}\, e^{-j\beta_1 x \sin\theta_r} = E_{to}\, e^{-j\beta_2 x \sin\theta_t}$$

$(\theta_i = \theta_r \;\&\; \beta_1 \sin\theta_i = \beta_2 \sin\theta_t) \Rightarrow$ Snell's law

$E_{io} + E_{ro} = E_{to} \Rightarrow \boxed{1 + \Gamma_\perp = \tau_\perp}$

Boundary condition for m.s. field

$H_{tan1} = H_{tan2} \Rightarrow -\frac{E_{io}}{\eta} \cos\theta_i + \frac{E_{ro}}{\eta_1} \cos\theta_r = -\frac{E_{to}}{\eta_2} \cos$

$E_{io} \cos\theta_i - E_{ro} \cos\theta_i = E_{to} \left(\frac{\eta_1}{\eta_2} \cos\theta_t\right)$

$1 - \frac{E_{ro}}{E_{io}} = \frac{E_{to}}{E_{io}} \left(\frac{\eta_1 \cos\theta_i}{\eta_2 \cos\theta_i}\right) \Rightarrow \boxed{1 - \Gamma_\perp = \tau_\perp \left(\frac{\eta_1 \cos\theta}{\eta_2 \cos\theta_i}\right.}$

From above equation.

$2 = \tau_\perp \left(1 + \frac{\eta_1 \cos\theta_t}{\eta_2 \cos\theta_i}\right) \Rightarrow \boxed{\tau_\perp = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}}$

From this we may write $\boxed{\Gamma_\perp = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}}$

Brewster angle does not exist for $\perp^{lr}$ polarization if both the medium are non-magnetic but have dielectric (Why?)

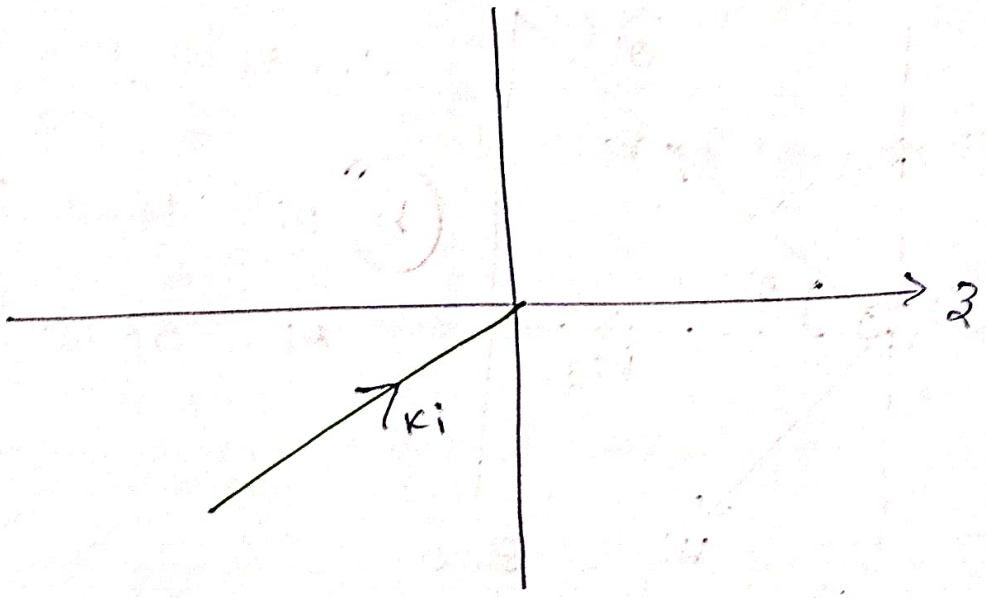
(1) A uniform plane wave in air with

$$E = 8 \cos(\omega t - 4x - 3z)\, a_y \text{ V/m}.$$

is incident on a dielectric slab ($z \geq 0$) with $\mu_r = 1.0$, $\varepsilon_r = 2.5$, $\sigma = 0$ mho/m.

(a) Polarization of the wave
(b) angle of Incidence.
(c) reflected E field.
(d) transmitted H field.

Ans



$z < 0$ [(medium - 1)] ($\mu_1 = \mu_0$, & $\varepsilon_1 = \varepsilon_0$)

$z = 0$

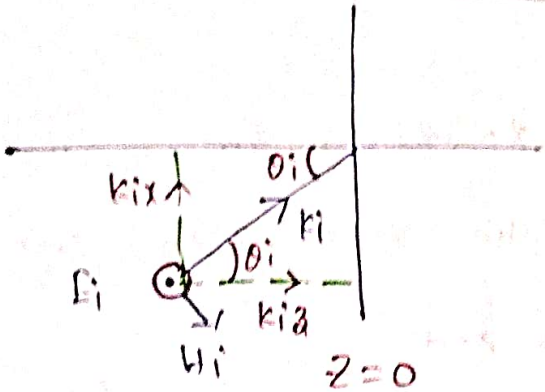
$z > 0$ (medium - 2) ($\mu_2 = \mu_0$ & $\varepsilon_2 = 2.5\varepsilon_0$)

$$E_i = 8 \cos[\omega t - (4x + 3z)]\, a_y.$$

$$E_i = E_0 \cos(\omega t - \bar{K}_i \cdot \bar{r})\, \bar{a}_{E_i}$$

(i) plane of incidence = x - z plane
direction of E-c field = $\hat{a}_y$
Type of polarization = $\perp^{lr}$.

$\bar{K}_i = 4a_x + 3a_z$ $\quad \beta_1 = |\bar{K}_i| = \sqrt{4^2 + 3^2} = 5$ rad/m



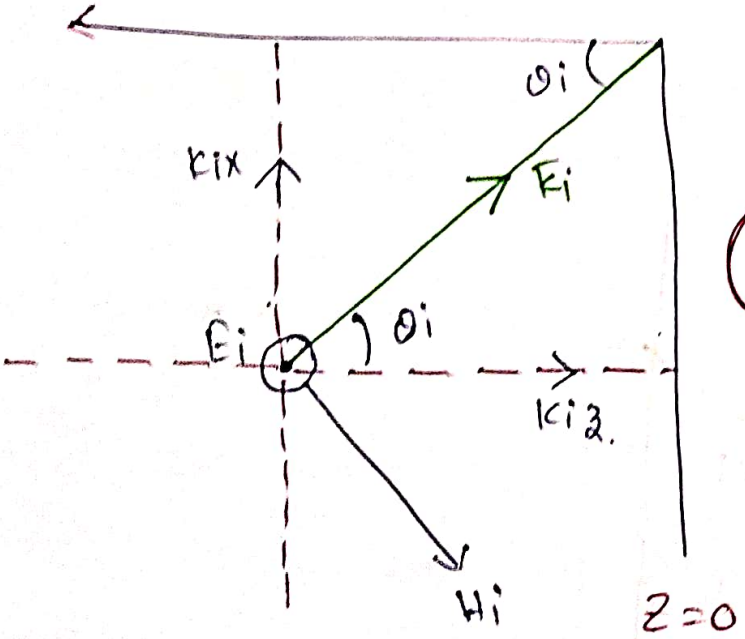
(xy plane)



$a_H = a_E \times a_H$

$a_z = a_y \times (-a_x)$

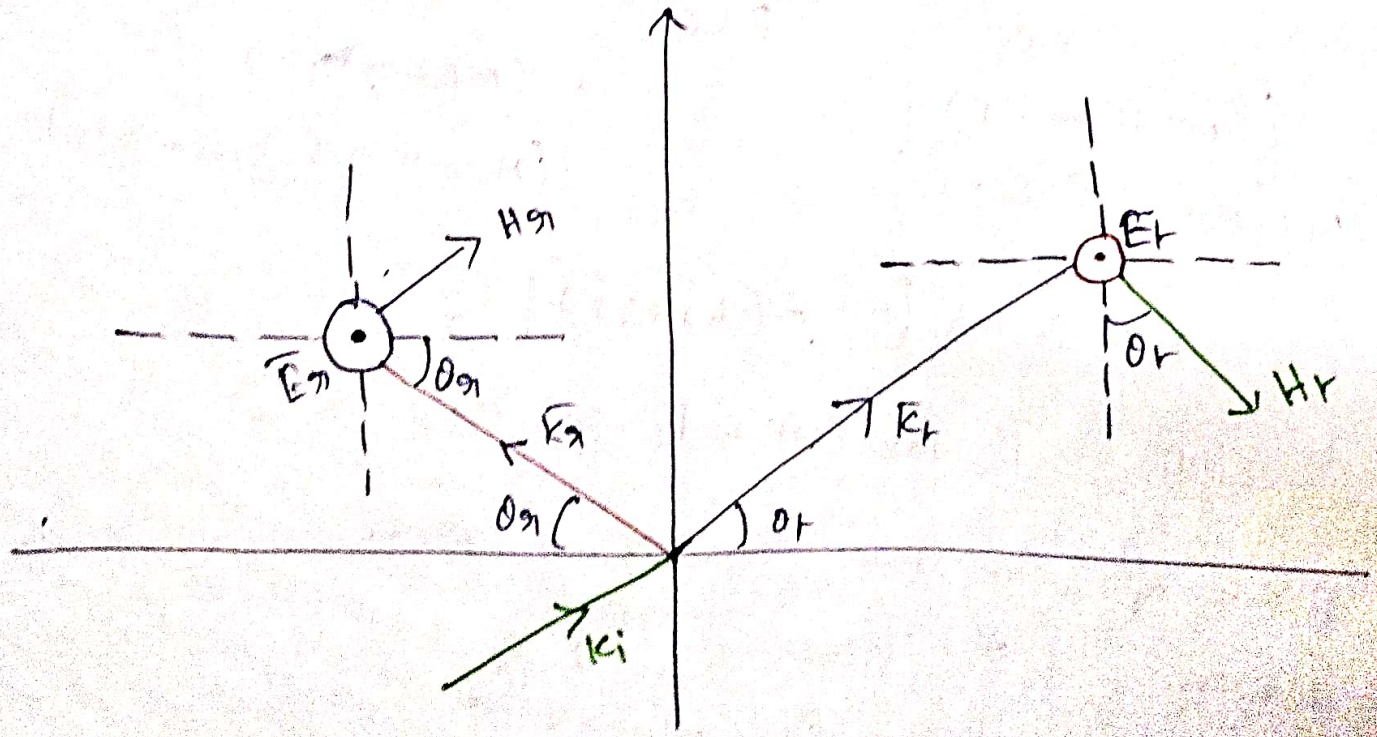
a_H should be $(-a_x)$



$\tan\theta_i = \frac{k_{ix}}{k_{iz}}$

(b) $\theta_i = \tan^{-1}(4/3)$

$\theta_i = 53.13^\circ$



we know $\theta_i = \theta_{91} = 53.13^\circ$

$$\beta_1 \sin\theta_i = \beta_2 \sin\theta_t$$

$$\beta = \omega \sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right]}$$

if $\mu_{91} = 1$, $\varepsilon_{91} = 1$

if $\sigma = 0$, $\beta = \omega\sqrt{\mu\varepsilon}$

$$\beta = \omega\sqrt{\mu_0 \times \varepsilon_0}$$

$$= \omega\sqrt{4\pi \times 10^{-7} \times \frac{36\pi \times 10^{-9}}{10^{-9} \times 36\pi}}$$

$$\beta = \omega\sqrt{\frac{1}{4} \times 36 \times \pi \times \frac{10^{-18}}{\pi}}$$

$$V_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{4\times 36\times \pi \cdot \frac{1}{9\times 10^{16}}}}$$

$$= \frac{\omega}{\beta} = 3\times 10^8 \text{ m/sec} = c.$$

$$\boxed{V_p = c = \omega/\beta}$$

if $\mu = \mu_0$ & $\varepsilon = \varepsilon_0$

So $\boxed{\beta_1 \sin\theta_i = \beta_2 \sin\theta_t}$

$$\frac{\omega}{c}\sin(53.13^\circ) = \frac{\omega}{c}\sqrt{2.5}\sin\theta_t$$

$$\boxed{\sin\theta_t = \frac{\sin(53.13^\circ)}{\sqrt{2.5}}} \quad \boxed{\theta_t = 30.39^\circ}$$

$$\Gamma_1 = \frac{\eta_2 \cot\theta_i - \eta_1 \cot\theta_t}{\eta_2 \cot\theta_i + \eta_1 \cot\theta_t}$$

$$\eta = \frac{\sqrt{\mu/\varepsilon}}{\sqrt{1 - j\left(\frac{\sigma}{\omega\varepsilon}\right)^2}} \quad \text{if } \sigma = 0$$

$\eta = \sqrt{\mu/\varepsilon} \Rightarrow$ medium 1 $\Rightarrow \sigma = 0, \ \varepsilon = \varepsilon_0, \mu = \mu_0$

So $\eta_1 = 120\pi$.

medium 2 $\Rightarrow \sigma = 0, \ \varepsilon_2 = 2.5\,\varepsilon_0, \ \mu_2 = \mu_0$

$$\eta_2 = \sqrt{\mu/\varepsilon} = 120\pi \cdot \frac{1}{\sqrt{\varepsilon_r}} = \frac{120\pi}{\sqrt{2.5}} = 238.43.$$

So $\eta_1 = 120\pi, \ \eta_2 = 238.43, \ \theta_i = \theta_r = 53.13^\circ$

$\theta_t = 30.39^\circ$

Then $$\Gamma_\perp = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} = -0.389$$

$$\tau_\perp = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} \quad \text{(or)} \quad \boxed{\tau_\perp = 1 + \Gamma_\perp}$$

$$\boxed{\tau_\perp = 1 - 0.389 = 0.611}$$

Reflected electric field

(c) $$\boxed{\bar{E}_r = E_{r0}\, a_y \cos(\omega t - \bar{k}_r \cdot \bar{r})}$$

$\bar{k}_r = |\bar{k}_r| \cos\theta_r (-\bar{a}_z) + |\bar{k}_r| \sin\theta_r (\bar{a}_x)$

$= B_1 \cos\theta_r (-a_z) + B_1 \sin\theta_r (a_x)$

$\overline{k_R} = -5 \cos 53.13^\circ a_z + 5 \sin 53.13^\circ a_x$

$= -3 a_z + 4 a_x = 4 a_x - 3 a_z.$

$\overline{k_R} \cdot \overline{r} = 4x - 3z.$

$\overline{E_R} = E_{R0} a_y \cos(\omega t - \overline{k_R} \cdot \overline{r})$

$E_{R0} = \Gamma_\perp E_{i0} = -0.389 \times 8 =$

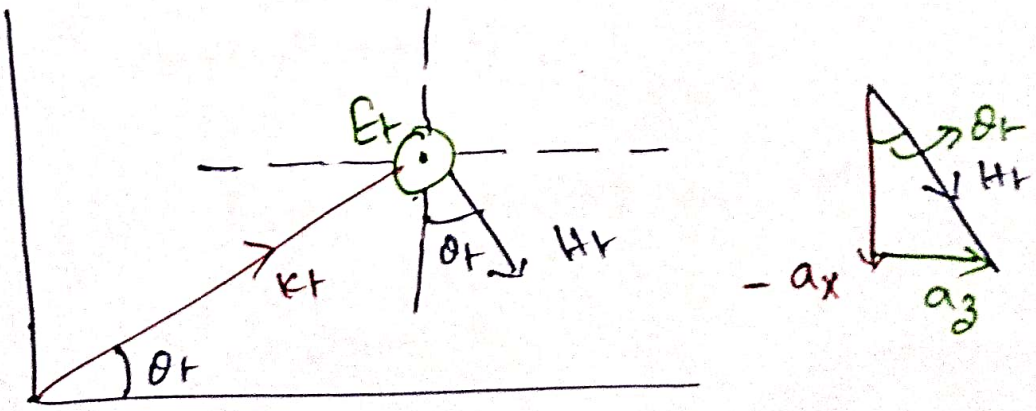
$E_{R0} = -3.112$

$B_1 = \omega / c \qquad \omega = B_1 \cdot c = 5 \times 3 \times 10^8 \text{ m/sec}$

$\omega = 15 \times 10^8 \text{ rad/sec.}$

$$\overline{E_R} = -3.112 a_y \cos[15 \times 10^8 t - (4x - 3z)] \text{ V/m}$$

Transmitted magnetic field.



$H_t = (H_{ox} a_x + H_{oy} a_y + H_{oz} a_z) \; e^{-j(\overline{k_t} \cdot \overline{r})}.$

$\overline{H_t} = [H_{to} \cos\theta_t (-a_x) + H_{to} \sin\theta_t (a_z)] \cos(\omega t - (\overline{k_t} \cdot \overline{r}))$

$$H_t = (-17.69 a_x + 10.37 a_z)$$

$$\cos(15 \times 10^8 t - 4x - 6.819 z) \quad mA/m$$

(6) An EM wave travels in free space with the electric field component

$$E_s = 100 e^{j(0.866y + 0.5z)} a_x \text{ V/m}$$

Then find (a) ω & λ
(b) magnetic field component
(c) time average power in the wave

Ans

$$\beta = \omega/c \Rightarrow \omega = \beta c$$

$$\bar{E} = 100 e^{j(\omega t - \bar{k}\cdot\bar{r})} \bar{a}_E$$

$$\bar{E}_s = 100 e^{-j(\bar{k}\cdot\bar{r})} \bar{a}_E$$

$$\bar{k}\cdot\bar{r} = -(0.866y + 0.5z)$$

$$\bar{k} = -(0.866 a_y + 0.5 a_z)$$

$$\beta = |k| = \sqrt{(0.866)^2 + (0.5)^2} = 1$$

$$\omega = \beta c = 1 \times 3\times10^8 \text{ rad/sec}$$

$$\lambda = 2\pi/\beta = \frac{2\pi}{1} = 2\pi m$$

Maxwell's Eq'n:-

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

$$\bar{k} \times \bar{E} = \omega\mu \bar{H}$$

$$\bar{H} = \frac{1}{\omega\mu}(\bar{k} \times \bar{E})$$

$$\bar{H} = \frac{1}{2\pi \times M_0}\left[-(0.866\bar{a}_y + 0.5\bar{a}_z) \times 100\, e^{j(0.866y + 0.5z)} a_x\right]$$

$$\bar{H} = -(1.33\bar{a}_y - 2.3\bar{a}_z)\, e^{-j(0.866y + 0.5z)} \text{ mA/m}$$

time average power density $\bar{P}_{avg} = \frac{E_0^2}{2\eta}$ a/c

$$\bar{P}_{avg} = \frac{(100)^2}{2 \times 120\pi}\left(\frac{-0.866\,\bar{a}_y - 0.5\bar{a}_z}{\sqrt{(0.866)^2 + (0.5)^2}}\right)$$

$$P_{avg} = -(11.49\,a_y + 6.63\,a_z) \text{ W/m}^2$$

Since direction of electric field is x-z plane of Incidence is yz plane.

So $\perp^{lr}$ polarization

(Q) $E = (10\,a_y + 5a_z)\cos(\omega t + 2y - 4z)$ V/m

in free space then find

(a) ω (b) magnetic field component

(c) time average power in the wave

(A)

$$\bar{E} = (10\,a_y + 5a_z)\cos[\omega t - (-2y + 4z)]$$

$$\bar{k} \cdot \bar{r} = -2y + 4z$$

$$\bar{k} = -2a_y + 4a_z$$

$$\beta = |\bar{k}| = \sqrt{(2)^2 + (4)^2} = \sqrt{20}$$

$$\beta = 4.472 \text{ rad/m}.$$

(a) $\omega = \beta c = 4.472 \times 3 \times 10^8$ m/sec

(b) $\lambda = 2\pi/\beta = \frac{2\pi}{4.472} = \frac{2 \times 3.14}{4.472} =$

(c) magnetic field component $\bar{H} = \frac{\bar{k} \times \bar{E}}{\omega\mu}$

$$\bar{H} = \frac{1}{1.3416 \times 10^9 \times \mu_0}[(-2a_y + 4a_z) \times (10\,a_y + 5a_z)\cos(\omega t - 2y + 4z)]$$

$$\bar{H} = \frac{(-10\,a_x - 40a_x)}{1.341 \times 10^9 \times 4\pi \times 10^{-7}}\cos(\omega t - 2y + 4z)$$

$$\bar{H} = -29.61\, a_x \cos(\omega t + 2y - 4z)\ \text{mA/m}.$$

Time average power $\bar{P}_{avg} = \frac{E_0^2}{2\eta}\, d_k$

$$P_{avg} = \left[\frac{\sqrt{10^2 + 5^2}}{2 \times 120\pi}\right]^2 * \frac{(-2a_y + 4a_z)}{\sqrt{2^2 + 4^2}}$$

$$P_{avg} = -0.074\, a_y + 0.1489\, a_z \quad \text{W/m}^2$$

Type of polarization :- 11ll polarization

Normal Incidence Numerically (Numericals)

A uniform plane wave in air with $\bar{H} = 4\sin(\omega t - 5x)\hat{a}_y$ A/m is normally incident on a plastic region with the parameters $\mu = \mu_0$, $\varepsilon = 4\varepsilon_0$ & $\sigma = 0$.

(a) Obtain the total electric field in air → medium (1)

(b) Find the standing wave ratio.

(A) medium 1 = air

medium 2 = ($\mu = \mu_0$, $\varepsilon_r = 4\varepsilon_0$ & $\sigma = 0$)

(a) total electric field = [Incident + Reflected]

$$E_i = E_{io}\sin(\omega t - 5x)\,\bar{a}_{E_i}$$

need to find ω, E_{io} & $\bar{a}_{E_i}$

we know $V_p = \omega/\beta_1 = c$.

$$\omega = c \cdot \beta_1 = 3\times10^8 \times 5.$$

$$\omega = 15\times10^8.$$

$\bar{a}_{E_i}$ ⇒ $a_{ic} = a_E \times a_H$

$a_x = a_E \times a_y$

a_E should be $-a_z$.

$E_{io} = H_{io} \times \eta$

$= 4 \times 120\pi$

$\bar{E}_i = 120\pi \times 2 \sin(15\times10^8 t - 5z)(-\bar{a}_y)$

$= 240\pi \sin(15\times10^8 t - 5z)(-\bar{a}_y)$ V/m

Reflection coefficient of electric field.

$$\Gamma_E = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\sqrt{\varepsilon_{r2}} - \sqrt{\varepsilon_{r1}}}{\sqrt{\varepsilon_{r1}} + \sqrt{\varepsilon_{r2}}} = \frac{1-\sqrt{4}}{1+\sqrt{4}} = -\frac{1}{3}$$

$$\Gamma_E = -\frac{1}{3} = \frac{E_{r0}}{E_{i0}}$$

$$-\frac{1}{3} = \frac{E_{r0}}{240\pi} \qquad E_{r0} = -\frac{240\pi}{3} = -80\pi$$

$$\bar{E}_r = -80\pi \sin(15\times10^8 t + 5z)(-\bar{a}_y)$$

$$= 80\pi \sin(15\times10^8 t + 5z)\,\bar{a}_y$$

Total electric field $\bar{E}_t = \bar{E}_i + \bar{E}_r$

(3) Standing wave ratio $S_E = \dfrac{1+|\Gamma_E|}{1-|\Gamma_E|}$

$$= \frac{1+\frac{1}{3}}{1-\frac{1}{3}} = \frac{4}{2} = 2.$$

$\boxed{S_E = 2}$

(2)

(Q) A plane wave in free space with $\bar{E} = 3.6 \cos(\omega t - 3x)\hat{a}_y$ V/m is incident normally an Interface at, $x=0$, if a lossless medium with $\sigma = 0$, $\varepsilon_{r1} = 12.5$ exists for $x \geq 0$ and the reflected wave has $\bar{H}_{r1} = -1.2 \cos(\omega t + 3x)\hat{a}_z$ mA/m find μ_2

(A)

$$\eta_2 = 120\pi\sqrt{\frac{\mu_{r2}}{\varepsilon_{r2}}}, \qquad \bar{H}_{r1} = 1.2 \cos(\omega t + 3x)$$

$H_{r10} = 1.2$, $\beta = +3$. Propagation $= -a_x$.

Medium 1 = free space. $\Rightarrow \eta_1 = 120\pi$

" 2 = lossless medium.

$$H_{r10} = 1.2 = E_{r10}/\eta_1 \Rightarrow E_{r10} = \eta_1 \times 1.2 \times 10^{-3}$$

$$= 120\pi \times 1.2 \times 10^{-3}$$

$$= 144\pi \times 10^{-3}$$

Reflection coefficient $\Gamma_E = \dfrac{E_{r10}}{E_{i0}} = \dfrac{144 \times 10^{-3}}{3.6} = 0.125$

$$\Gamma_E = \frac{120\pi\sqrt{\frac{\mu_{r2}}{\varepsilon_{r2}}} - 120\pi\sqrt{\frac{\mu_{r1}}{\varepsilon_{r1}}}}{120\pi\sqrt{\frac{\mu_{r2}}{\varepsilon_{r2}}} + 120\pi\sqrt{\frac{\mu_{r1}}{\varepsilon_{r1}}}} = 0.125$$

$$0.125 = \frac{\sqrt{\frac{\mu_{r2}}{12.5}} - 1}{\sqrt{\frac{\mu_{r2}}{12.5}} + 1}$$

$$\boxed{\mu_{r2} = 20.66}$$

$$\mu_2 = \mu_0 \mu_{r2} = 4\pi \times 10^{-7} \times 20.66$$

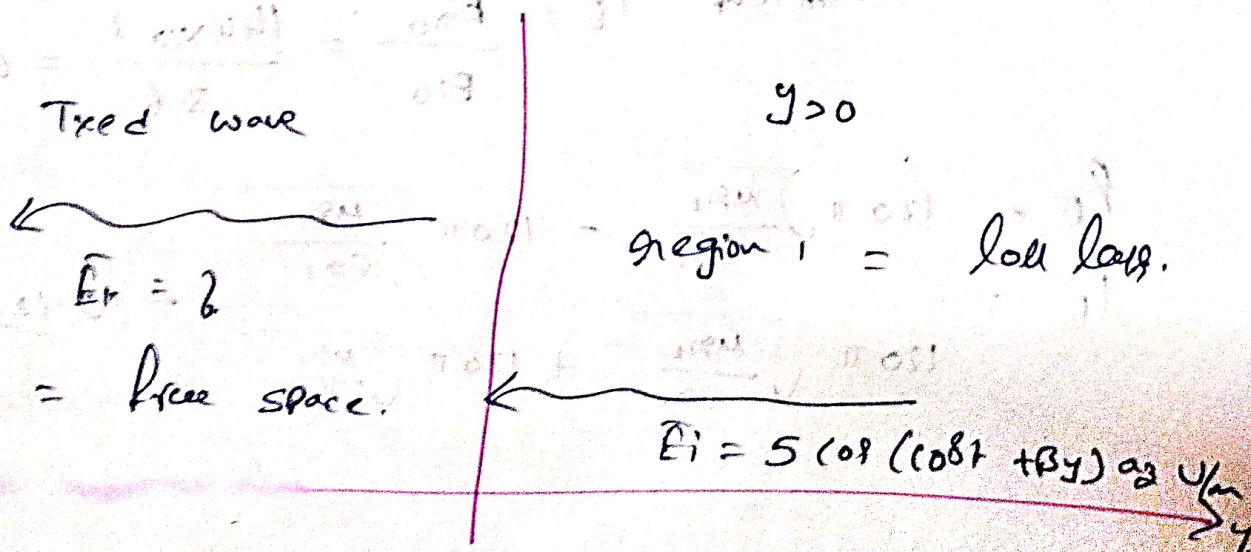
(Q) Region 1 is a lossless medium for which $y \geq 0$, $\mu = \mu_0$, $\varepsilon = 4\varepsilon_0$, whereas region 2 is free space. $y \leq 0$. if a plane wave $E_i = 5 \cos(10^8 t + \beta y)\, a_z$ V/m exists in region 1 Then find

(a) total E-c field component of the wave in region 2

(b) the time-average Poynting vector in region 1

(c) " " " " " region 2.

(A)



Transmitted electric field.

$$\bar{E}_t = E_{t0} \cos(\omega t + \beta_2 y)\, \hat{a}_3$$

$$\tau_E = \frac{2\eta_2}{\eta_2 + \eta_1} = \frac{2 \times 120\pi}{120\pi + \frac{120\pi}{\sqrt{\varepsilon_{r1}}}} \rightarrow (4)$$

$$= \frac{240\pi}{180\pi} = 4/3.$$

$$\frac{E_{t0}}{E_{i0}} = \tau_E \Rightarrow E_{t0} = 5 \times 4/3 = 20/3 \ \text{V/m}.$$

$$\beta_2 \omega/c = \frac{108}{3 \times 10^8} = 1/3.$$

$$\bar{E}_t = E_{t0} \cos(\omega t + \beta_2 y)\, \hat{a}_3$$

$$= 20/3 \cos(10^8 t + 1/3)\, \hat{a}_3$$

$$1 + \Gamma_E = \tau_E$$

$$\Gamma_E = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{120\pi - 60\pi}{120\pi + 60\pi} = 1/3. \ (\text{or})$$

(Q) A plane wave is free space ($z \le 0$) is incident normally on a lossy (perfect dielectric) of material with $\varepsilon_{r2} = 12$, $\mu_{r2} = 3$, $\sigma = 0$ which occupies $z > 0$, if the incident electric field is $\bar{E} = 30 \cos(\omega t - \beta z)\, a_y$ V/m

Then find (a) ω (b) S_E (Standing wave ratio) (c) reflected magnetic field.

(A) (a) $\beta = \omega/c \Rightarrow \omega = \beta_1 c = 1 \times 3 \times 10^8$ rad/sec

(b) $S_E =$

$$\Gamma_E = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{120\pi\sqrt{3/12} - 120\pi}{120\pi\sqrt{3/12} + 120\pi} = -1/3$$

$$S = \frac{1 + 1/3}{1 - 1/3} = 2.$$

(c) Reflected magnetic field (H_{r1})

$$\bar{H}_{r1} = H_{r10} \cos(\omega t - \beta z)\, a_{H_{r1}}.$$

$$\bar{E}_{r1} = E_{r10} \cos(\omega t + \beta z)\, a_y.$$

$$\Gamma = \frac{E_{r10}}{E_{i10}} \qquad E_{r10} = \Gamma E_{i10} = -1/3 \cdot 30 = -10$$

$$H_{r10} = E_{r10}/\eta = -10/120\pi =$$

$$\bar{H}_{1n} = \frac{-10}{120\pi} \cos\left(3\times10^8 t + \frac{1}{3}\right) \hat{a}_x$$

$$a_{H1} = \hat{a}_{k1} \times \hat{a}_{E1}$$

Lect - 15

A 30 MHz uniform plane wave with

$H = 10 \sin(\omega t + \beta x)\, a_z$ mA/m exists in region $x \geq 0$ having $\sigma = 0$, $\varepsilon = 9\varepsilon_0$, $\mu = 4\mu_0$ at $x = 0$. The wave encounters free space. Then find

(a) The polarization of the wave.

(b) The phase constant β.

(c) displacement current density in region $x \geq 0$

(d) reflected and fixed m.c. field.

(A) (a) $\hat{a}_k = \hat{a}_E \times \hat{a}_H$

$-\hat{a}_x = \hat{a}_E \times a_z$.

So $\hat{a}_E$ should be $-\hat{a}_y$

(b) $\beta = \frac{\omega}{c}\sqrt{\mu_r \varepsilon_r} = 2\pi \times 30 \times 10^6 / 3\times10^8 \sqrt{4 \times 9}$

$= 3.77$ rad/m

(c) $I_d = \partial D/\partial t = \varepsilon \partial E/\partial t$

$\bar{E}_t = E_{t0} \cos(\omega t + \beta x)(-\hat{a}_y)$

$\eta_1 = 120\pi \sqrt{\frac{\mu_1}{\varepsilon_1}} = 120\pi \sqrt{4/9} = 2/3 \times 120\pi$

$= 80\pi$

$\bar{E}_t = \eta_1 H_{i0} \cos(\omega t + \beta x)(-\hat{a}_y)$

$= -80\pi \times 10 \times 10^{-3} \cos(\omega t + 3.77x)(+\hat{a}_y)$

$= -2.51 \cos(\omega t + 3.77x)\, \hat{a}_y$

$\bar{I}_D = \frac{\partial \bar{D}}{\partial t} = \varepsilon \cdot \frac{\partial \bar{E}}{\partial t} = 9\varepsilon_0 \left[-2.51 (\omega) (-\sin(\omega t + 3.77x) \right.$

$\bar{I}_D = 9\varepsilon_0 \times 2.51 \times 2\pi \times 30 \times 10^6$

$\sin(\omega t + 3.77x)\, a_y$

(5) A uniform plane wave in air is normally incident on an infinite lossless dielectric material having $\varepsilon = 3\varepsilon_0$ and $\mu = \mu_0$. If the incident wave is $E_i = 10\cos(\omega t - 2)\hat{a}_y$ V/m

Find.

(a) λ & ω of the wave in air and the fixed wave in the dielectric medium

(b) Incident H_i field

(c) Γ & T (d) Total electric field in both region.

(A) (a) $\beta = \frac{2\pi}{\lambda}$ from the given data

$\beta = 1$, $\lambda = 2\pi m$

$\omega = \beta_1 c = 3 \times 10^8$ rad/sec.

(c) $\Gamma = \frac{\sqrt{\varepsilon_{r1}} - \sqrt{\varepsilon_{r2}}}{\sqrt{\varepsilon_{r1}} + \sqrt{\varepsilon_{r2}}} = \frac{1 - \sqrt{3}}{1 + \sqrt{3}}$

$\Gamma = -0.268$

$T = 1 + \Gamma = 1 - 0.268 = 0.732$

Lect - 19

Poynting theorem

(Q) in free space $H = 0.1 \cos(\omega t - \beta x)\, a_z$ A/m, the total power passing through a square plate of side 10 cm on plane $x + 2y = 1$ is approximately

(A)

$$P = P_{avg} \times \text{Area} \quad (\hat{a}_{\perp} \cdot \hat{a}_n)$$

$$P_{avg} = \frac{E_0^2}{2\eta} = \frac{\eta^2 H_0^2}{2\eta} = \frac{H_0^2 \eta}{2} = \frac{(0.1)^2 \times 120\pi}{2}$$

$$\text{Area} = 10 \times 10 \times 10^{-4}\ m^2$$

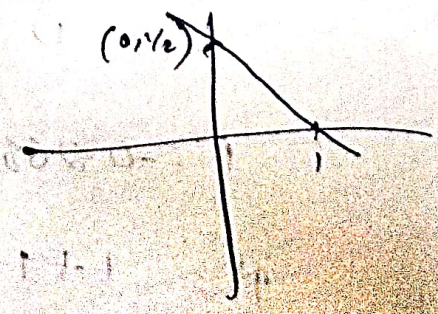
$$\hat{a}_{\perp} = \hat{a}_x$$

$$\phi = x + 2y - 1 \qquad \hat{a}_n = \frac{\nabla\phi}{|\nabla\phi|} = \frac{a_x + 2a_y}{\sqrt{1+4}}$$

$$P = \frac{(0.1)^2 \times 120\pi}{2} \times 100 \times 10^{-4} \left[\hat{a}_x \cdot \frac{a_x + 2a_y}{\sqrt{5}}\right]$$

$$P = \frac{(0.1)^2 \times 120\pi \times 10^{-2}}{2\sqrt{5}} = 8.425\ mw.$$

$x = 0,\ y = 1/2,\quad y = 0,\quad x = 1$



(6)

(5) The electric field intensity of a plane wave traveling in free space is given by the following expression

$$\bar{E}(x,t) = \bar{a}_y \, 24\pi \cos(\omega t - kox) \text{ V/m}.$$

in unit field. Consider a square area 10cm × 10cm on a plane $x + y = 1$.

The total time-averaged power (in mW) passing through the square area is

(A)

$$P = P_{avg} \times \text{Area} \times (\hat{a}_{ic} \cdot \hat{a}_n)$$

$$= \frac{(24\pi)^2}{2 \times 120\pi} \times 10 \times 10 \times 10^{-4} \times \left(\bar{a}_x \cdot \left(\frac{\bar{a}_x + \bar{a}_y}{\sqrt{2}}\right)\right)$$

$$= 53.3 \text{ mW}.$$

(2) In a non-magnetic medium ($\mu_R = 1$, $\sigma = 0$, $\varepsilon_R \neq 1$) $\bar{E} = 4 \sin(2\pi \times 10^7 t - 0.8x)\hat{a}_y$ V/m.

Then the intrinsic impedance is

the time average power carried by the wave is

the total power falling 100 cm² of plane $2x + y = 5$ is -

(a) $$\eta = \frac{120\pi}{\sqrt{\varepsilon_R}} = \frac{120\pi}{\sqrt{12/\pi}} = 10\pi^2 \, \Omega.$$

$$\beta = \frac{\omega}{c}\sqrt{\varepsilon_R}$$

$$\sqrt{\varepsilon_R} = \frac{\beta c}{\omega} = \frac{0.8 \times 3 \times 10^8}{2\pi \times 10^7} = 12/\pi$$

(b)

$$P_{avg} = \frac{E_0^2}{2\eta} a_{ic} = \frac{(4)^2}{2 \times 10\pi^2} a_x = \frac{8}{10\pi^2} a_x$$

(c)

$$\text{Power } P = \frac{8}{10\pi^2} \times 100 \times 10^{-4} [\hat{a}_x \cdot \hat{a}_n]$$

$$a_n = \frac{\nabla \phi}{|\nabla \phi|} \qquad \phi = 2x + y.$$

$$= \frac{2a_x + a_y}{\sqrt{5}}$$

$$P = \frac{8}{10\pi^2} \times 100 \times 10^{-4} \left[\hat{a}_x \cdot \frac{2\hat{a}_x + \hat{a}_y}{\sqrt{5}}\right]$$

$$= \frac{800 \times 10^{-4} \times 2}{10\pi^2\sqrt{5}} =$$

(c) $\bar{E} = (\bar{a}_x + j\bar{a}_y)\, e^{jkz - j\omega t}$

$\bar{H} = (k/\omega\mu)(\bar{a}_y + j\bar{a}_x)\, e^{jkz - j\omega t}$ → time-averaged

Poynting vector is

(a) Null vector (b) $(k/\omega\mu)\bar{a}_z$ (c) $(2k/\omega\mu)\bar{a}_z$

(d) $(k/2\omega\mu)\bar{a}_z$

(A)

$$\bar{P}_{avg} = \frac{1}{2} Re\left[\bar{E} \times \bar{H}^*\right]$$

$$= \frac{1}{2} Re\left[\bar{a}_x + j\bar{a}_y\right] e^{jkz - j\omega t} \times \frac{k}{\omega\mu}(\bar{a}_y - j\bar{a}_x)\, e^{-jkz + j\omega t}$$

$$= \frac{k}{2\omega\mu} Re\left[(\bar{a}_x + j\bar{a}_y) \times (\bar{a}_y - j\bar{a}_x)\right]$$

$$= \frac{k}{2\omega\mu} Re\left[\bar{a}_z - \bar{a}_z\right] = 0 = \text{Null vector}$$

(D) the time-averaged Poynting vector for a wave with

$\bar{E} = 24\, e^{j(\omega t + \beta z)}\, \bar{a}_y$ V/m in free space

(a) $-\frac{2.4}{\pi}\bar{a}_z$ (b) $\frac{2.4}{\pi}\bar{a}_z$ (c) $\frac{4.8}{\pi}\bar{a}_z$ (d) $-\frac{4.8}{\pi}\bar{a}_z$

(D)

$$\bar{P}_{avg} = \frac{E_0^2}{2\eta}\hat{a}_k = \frac{(24)^2}{2 \times 120\pi}(-\bar{a}_z) = \frac{-2.4}{\pi}(\bar{a}_z)\ W/m^2$$

Polarization

(Q) a plane electromagnetic wave along z-direction

$E_x = 2\cos(\omega t)$, $E_y = 2\cos(\omega t + 90^\circ)$.

(Ans)

$E_{ox} = E_{oy} = 2$

$\phi_y = 90^\circ$, $\phi_x = 0^\circ$

$\delta = \phi_y - \phi_x = 90^\circ$, दिशा = a_z

Left hand circularly Polarized.

(1) Linear :- $\delta = n\pi$ $E_{ox} = E_{oy}$

(2) circular $E_{ox} = E_{oy}$ & δ

a_z	$90^\circ / -270^\circ$	LHCP
$-a_z$	$90^\circ / -270^\circ$	RHCP
a_z	$-90^\circ / 270^\circ$	RHCP
$-a_z$	$-90^\circ / 270^\circ$	LHCP.

(1) Lect. no. 28, 29 -> Numericals on oblique incidence

Complex Permittivity :-

- it represents encompassing both energy storage and energy loss

$$\varepsilon = \varepsilon' - j\varepsilon''$$

ε' - Real Part - material's ability to store energy, also known as the dielectric constant

ε'' - Imag Part - material's energy loss, also known as the loss factor.

① Dielectric $\varepsilon' \gg \varepsilon''$

② Conductor $\varepsilon' < \varepsilon''$

③ Lossy dielectric $\varepsilon' \approx \varepsilon''$

$$\nabla \times \bar{H} = \bar{J} + \bar{J}_D = \bar{J} + \partial \bar{D}/\partial t$$

$$= \bar{J} + \varepsilon \cdot \frac{\partial \bar{E}}{\partial t}$$

$$= \sigma \bar{E} + j\omega\varepsilon \bar{E}$$

$$= j\omega\varepsilon \left(1 + \frac{\sigma}{j\omega\varepsilon}\right) \bar{E}$$

$$= j\omega \left[\varepsilon \left(1 - j \cdot \frac{\sigma}{\omega\varepsilon}\right)\right] \bar{E}$$

$$= j\omega\, \varepsilon_c \cdot \bar{E}$$

$$\varepsilon_c = \varepsilon' - j\varepsilon'' = \varepsilon - j \cdot \frac{\sigma}{\omega}$$

$$\varepsilon' = \varepsilon, \quad \varepsilon'' = \sigma/\omega.$$

Loss tangent :-

$$\tan\delta = \varepsilon''/\varepsilon' = \frac{\sigma}{\omega\varepsilon}$$

Conductivity $\sigma = \omega\varepsilon''$

Permittivity $\varepsilon'' = \sigma/\omega$

$$\tan\delta = \frac{\sigma}{\omega\varepsilon'} = \frac{\sigma}{\omega\varepsilon}.$$

Lossy dielectric ($\frac{\sigma}{\omega\varepsilon} \approx 1$) — Both conduction & Polarization are significant

Good dielectric - $\sigma/\omega\varepsilon \ll 1$ - Polarization dominates, Low conduction

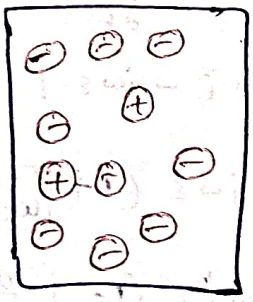
Good conductor - $\frac{\sigma}{\omega\varepsilon} \gg 1$ - conduction dominates, all energy lost as heat.

Conductors $\varepsilon_r = ?$

- conductors have high conductivity

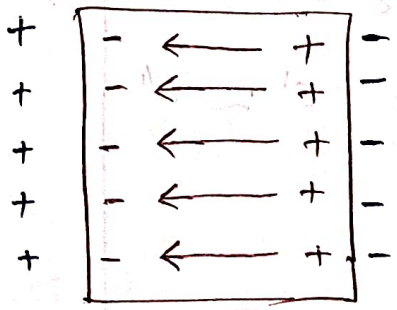
due to this Electric field inside a conductor is zero. $\boxed{\bar{E} = 0 \text{ in conductor}}$

Conductors have free charges (e⁻ns) that can move freely within the material. When the electric field is applied these free charges will accelerate and rearrange themselves to cancel out the e⁻c field. So $E_n = 0$.



(without applied)

① ⟹ Applied field (E_a)



② Induced field (E_i)

$E_n = E_a - E_i$

$E_n = \text{zero}$

ε_r = Ratio b/w the electric displacement field (D) and Electric field

$\bar{D} = \varepsilon \bar{E} \qquad \varepsilon = \bar{D}/\bar{E}$

- Conductors have low Polarizability, means e⁻c field does not Induce significant Polarization. as a result relative Permitivity is close to 1.

$\varepsilon_r = 1$ ⇒ due to (a) high conductivity
(b) negligible e^c field
(c) Low polarizability.

Brewster angle (or) polarizing angle :- Reference :- Sadiku Text book
P.No :- 469

↓
No Reflection (all energy will be transmitted into another medium)

- it is a angle of Incidence for which the reflection coefficient is zero.

Brewster angle for parallel polarization :-

$$\Gamma_{\parallel} = \frac{\eta_2 \cot\theta_t - \eta_1 \cot\theta_i}{\eta_2 \cot\theta_t + \eta_1 \cot\theta_i}$$

$$\Gamma_{\parallel} = 0 \Rightarrow \eta_2 \cot\theta_t - \eta_1 \cot\theta_i$$

$$\eta_2^2 (1 - \sin^2\theta_t) = \eta_1^2 (1 - \sin^2\theta_B)$$

from the Boundary conditions

$$k_{ix} = k_{tx}$$

$$|k_i| \sin\theta_i = |k_t| \sin\theta_t$$

$$\beta_1 \sin\theta_i = \beta_2 \sin\theta_t$$

if medium is lossless medium Then $\beta = \frac{\omega}{c}\sqrt{\mu \cdot \varepsilon}$

$$\frac{\omega}{c}\sqrt{\mu_{r1} \cdot \varepsilon_{r1}} \sin\theta_i = \frac{\omega}{c}\sqrt{\mu_{r2} \cdot \varepsilon_{r2}} \cdot \sin\theta_t$$

$$\boxed{\eta_1 \sin\theta_i = \eta_2 \sin\theta_t} \Rightarrow \boxed{\sin\theta_t = \frac{\eta_1}{\eta_2} \sin\theta_i = \frac{n_1}{n_2} \sin\theta_B}$$

↓
Refractive Index

$$\eta_2^2 \left(1 - \frac{\eta_1^2}{\eta_2^2} \sin^2\theta_{B_\parallel}\right) = \eta_1^2 (1 - \sin^2\theta_{B_\parallel})$$

if medium is lossless $\sigma = 0$ then

$$\frac{\mu_2}{\varepsilon_2} \left(1 - \frac{\varepsilon_1 \cdot \mu_1}{\varepsilon_2 \cdot \mu_2} \cdot \sin^2\theta_{B_\parallel}\right) = \frac{\mu_1}{\varepsilon_1} \left(1 - \sin^2\theta_{B_\parallel}\right)$$

$$\frac{\mu_2}{\varepsilon_2} - \frac{\mu_1}{\varepsilon_1} = \sin^2\theta_{B_\parallel} \left(\frac{\mu_2}{\varepsilon_2} \cdot \frac{\varepsilon_1 \cdot \mu_1}{\varepsilon_2 \cdot \mu_2} - \frac{\mu_1}{\varepsilon_1} \right)$$

$$\frac{(\varepsilon_1 \mu_2 - \mu_1 \varepsilon_2)}{\varepsilon_2 \varepsilon_1} = \sin^2\theta_{B_\parallel} \left(\frac{\mu_1 \cdot \varepsilon_1}{\varepsilon_2^2} - \frac{\mu_1}{\varepsilon_1} \right)$$

$$\frac{\mu_1 \varepsilon_2}{\varepsilon_2 \cdot \varepsilon_1} \left(-1 + \frac{\mu_2 \varepsilon_1}{\varepsilon_2 \mu_2}\right) = \sin^2\theta_{B_\parallel} \left(\frac{\mu_1 \cdot \varepsilon_1^2 - \mu_1 \varepsilon_2^2}{\varepsilon_2^2 \cdot \varepsilon_1} \right)$$

$$\left(\frac{\varepsilon_1 \mu_2}{\mu_1 \varepsilon_2} - 1 \right) = \sin^2\theta_{B_\parallel} \left(\left(\frac{\varepsilon_1}{\varepsilon_2}\right)^2 - 1 \right)$$

$$\sin^2\theta_{B_\parallel} = \frac{1 - \left(\frac{\mu_2 \varepsilon_1}{\mu_1 \varepsilon_2}\right)}{1 - (\varepsilon_1/\varepsilon_2)^2}$$

if medium was lossless and non-magnetic

with $\mu_1 = \mu_2 = \mu_0$ then

$$\sin^2\theta_{B_\parallel} = \frac{1 - \left(\frac{\varepsilon_1}{\varepsilon_2}\right)}{1 - (\varepsilon_1/\varepsilon_2)^2} = \frac{1}{1 + \varepsilon_1/\varepsilon_2}$$

$$\sin\theta_{B_\parallel} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1 + \varepsilon_2}} \Rightarrow \cot\theta_{B_\parallel} = \sqrt{\frac{\varepsilon_1}{\varepsilon_1 + \varepsilon_2}}$$

$$\cot^2\theta_{B_\parallel} = 1 - \sin^2\theta_B = \frac{\varepsilon_1}{\varepsilon_1 + \varepsilon_2}$$

$$\tan\theta_{B_{\parallel}} = \frac{\sin\theta_{B_{\parallel}}}{\cos\theta_{B_{\parallel}}} = \sqrt{\frac{\varepsilon_2}{\varepsilon_1+\varepsilon_2}} \Big/ \sqrt{\frac{\varepsilon_1}{\varepsilon_1+\varepsilon_2}}$$

$$= \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \tan\theta_B$$

$$\theta_{B_{\parallel}} = \tan^{-1}\left(\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\right)$$

if medium have $\mu_1 = \mu_2 = \mu_0$

Then $\eta = n$ → Refractive Index

↓

Impedance

otw wise if medium was only lossless

$$\eta = \sqrt{\mu/\varepsilon}, \quad n = \sqrt{\mu\varepsilon} = \frac{c}{v_p}$$

- Brewster angle will be possible for only Parallel Polarization i.e vertical Polarization
- not possible for Perpendicular Polarization i.e Horizontal Polarization.

Brewster angle for perpendicular Polarization:-

- For Perpendicular Polarization $\Gamma_{\perp} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$

$$\Gamma_{\perp} = 0 \quad = \quad \eta_2\cos\theta_i = \eta_1\cos\theta_t$$

$$\eta_2^2 (1 - \sin^2\theta_i) = \eta_1^2 (1 - \sin^2\theta_t)$$

$$(\theta_i = \theta_{B\perp})$$

and from Boundary conditions

$$\eta_1 \sin\theta_i = \eta_2 \sin\theta_t$$

$$\sin\theta_t = \frac{\eta_1}{\eta_2} \sin\theta_i = \frac{\eta_1}{\eta_2} \sin\theta_B$$

so

$$\eta_2^2 (1 - \sin^2\theta_{B\perp}) = \eta_1^2 \left(1 - \frac{\eta_1^2}{\eta_2^2} \sin^2\theta_{B\perp}\right)$$

if medium is lossless medium.

$$\frac{\mu_2}{\varepsilon_2} (1 - \sin^2\theta_{B\perp}) = \frac{\mu_1}{\varepsilon_1} \left(1 - \frac{\mu_1 \varepsilon_1}{\mu_2 \varepsilon_2} \cdot \sin^2\theta_{B\perp}\right)$$

$$\frac{\mu_2}{\varepsilon_2} - \frac{\mu_1}{\varepsilon_1} = \frac{\mu_2}{\varepsilon_2} \sin^2\theta_{B\perp} - \frac{\mu_1}{\varepsilon_1} \cdot \frac{\mu_1 \varepsilon_1}{\mu_2 \varepsilon_2} \sin^2\theta_{B\perp}$$

$$\frac{\varepsilon_1 \mu_2 - \mu_1 \varepsilon_2}{\varepsilon_1 \varepsilon_2} = \sin^2\theta_{B\perp} \left(\frac{\mu_2}{\varepsilon_2} - \frac{\mu_1^2}{\mu_2 \varepsilon_2}\right)$$

$$\frac{\mu_2 \varepsilon_1}{\varepsilon_1 \varepsilon_2} \left(1 - \frac{\mu_1 \varepsilon_2}{\varepsilon_1 \mu_2}\right) = \sin^2\theta_{B\perp} \left(\frac{\mu_2^2 - \mu_1^2}{\varepsilon_2 \cdot \mu_2}\right)$$

$$\mu_2 \left(1 - \frac{\mu_1 \varepsilon_2}{\varepsilon_1 \mu_2}\right) = \sin^2\theta_{B\perp} \cdot \left(\frac{\mu_2^2}{\mu_2} \left(1 - (\mu_1/\mu_2)^2\right)\right)$$

$$\sin^2\theta_{B\perp} = \frac{1 - \left(\frac{\mu_1 \varepsilon_2}{\varepsilon_1 \mu_2}\right)}{1 - (\mu_1/\mu_2)^2}$$

Note :- if medium is lossless and non magnetic

medium with $M_1 = M_2 = M_0$

Then $$\sin^2\theta_{B_\perp} = \frac{1-\left(\frac{M_0\varepsilon_2}{\varepsilon_1 M_0}\right)}{1-\left(M_0/M_0\right)^2}$$

$$\sin^2\theta_{B_\perp} = \frac{1-\left(\varepsilon_2/\varepsilon_1\right)}{1-(1)^2} = \frac{1-\left(\varepsilon_2/\varepsilon_1\right)}{0}$$

$$= \infty$$

Practically not possible.

- So Total Reflection (or) Brewster angle not possible in the case of perpendicular Polarization (or) Horizontal Polarization.

- if any medium will exist $\varepsilon_1 = \varepsilon_2 = \varepsilon_0$ & $M_1 \neq M_2$

Then $$\sin^2\theta_{B_\perp} = \frac{1-\left(\frac{M_1 \cdot \varepsilon_0}{\varepsilon_0 \cdot M_2}\right)}{1-\left(M_1/M_2\right)^2} = \frac{1}{1+\left(M_1/M_2\right)}$$

$$\sin\theta_{B_\perp} = \sqrt{\frac{M_2}{M_1+M_2}} \qquad \cos\theta_{B_\perp} = \sqrt{\frac{M_1}{M_1+M_2}}$$

$$\tan\theta_{B_\perp} = \sqrt{\frac{M_2}{M_1}}$$

$$\boxed{\theta_{B_\perp} = \tan^{-1}\left(\sqrt{\frac{M_2}{M_1}}\right)}$$

- But it is possible theoritically, not possible in practical.

Reference - Sadiku - P no - 610

Total Internal Reflection (or) Critical angle

As a light ray propagates from medium 1 to medium 2 Snell's law must be satisfied.

i.e $n_1 \sin\theta_1 = n_2 \sin\theta_2$

Where θ_1 is the incident angle in medium 1 and θ_2 is the transmission angle in medium 2.

The total internal Reflection occurs when

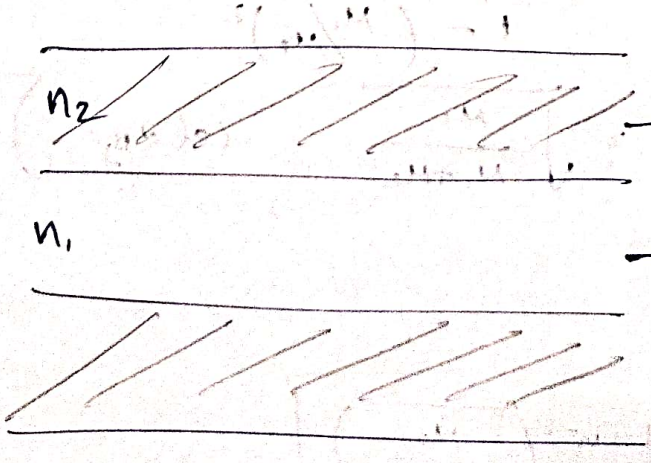
$$\theta_2 = 90^\circ$$

Then $n_1 \sin\theta_c = n_2$

$$\theta_c = \sin^{-1}\frac{n_2}{n_1}$$

it is valid only if $n_1 > n_2$

This concept mostly used in guided media



$n_1 = n_2 =$ Refractive Index $= \frac{\text{speed of wave in a vacuum}}{\text{speed of wave in medium.}}$

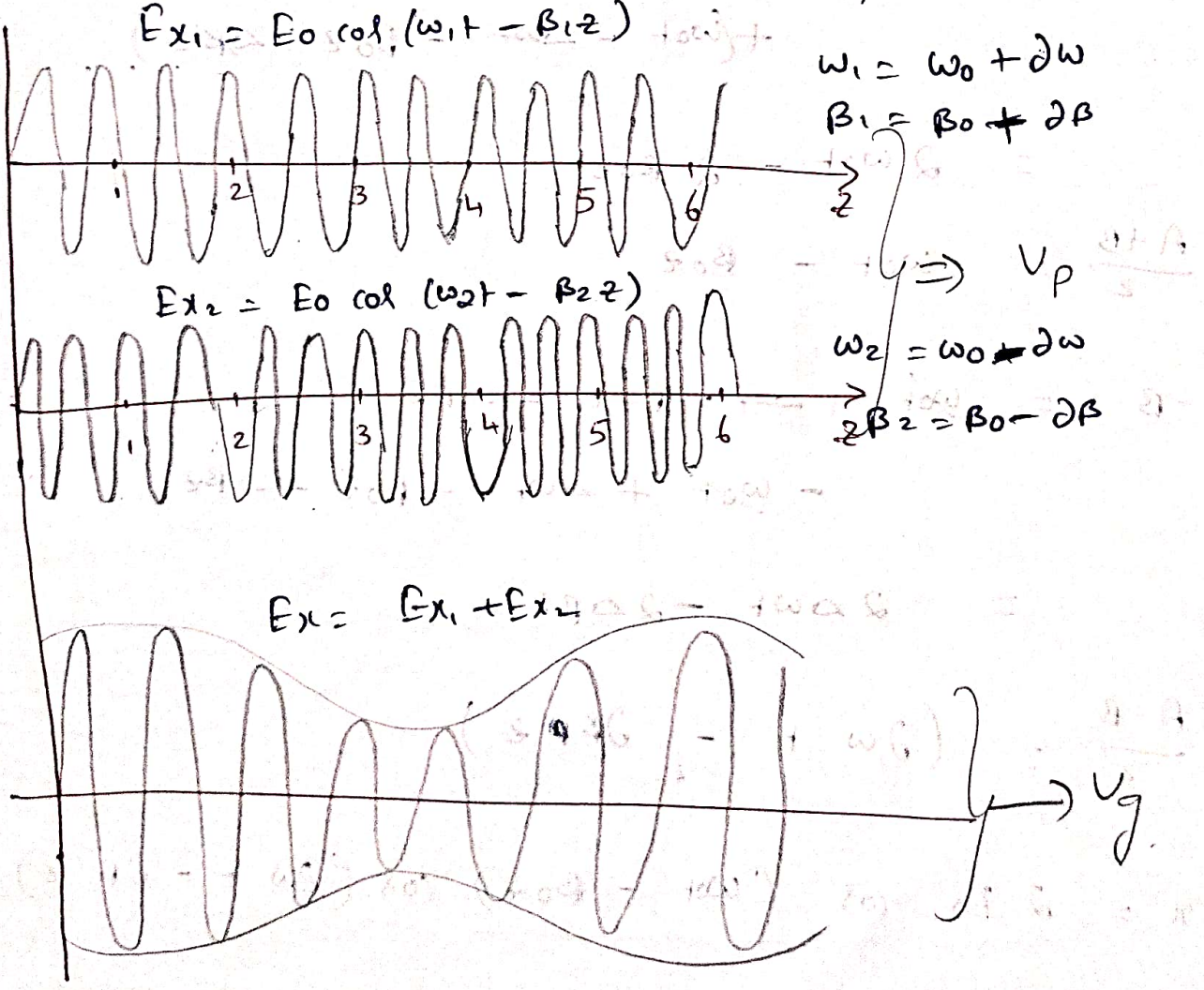
Phase velocity and group velocity :-

Phase velocity :-

it is the speed at which the phase of a wave oscillates. it can be expressed as $V_p = \omega/\beta$

group velocity :-

it is the speed at which the peak of envelope of wave pocket propagates. $V_g = \frac{d\omega}{d\beta}$



$$E_{x_1} = E_0 \cos((\omega_0 + d\omega)t - (\beta_0 + \Delta\beta)z)$$

$$E_{x_2} = E_0 \cos((\omega_0 - d\omega)t - (\beta_0 - \Delta\beta)z)$$

$$E_x = E_{x_1} + E_{x_2}$$

$$E_x = E_0 \cos\left[(\omega_0 + \Delta\omega)t - (\beta_0 + \Delta\beta)z\right]$$

$$+ E_0 \cos\left[(\omega_0 - \Delta\omega)t - (\beta_0 - \Delta\beta)z\right]$$

$$= E_0 \left[\cos A + \cos B\right]$$

$$= E_0 \cdot 2 \cdot \cos\left(\frac{A+B}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right)$$

$$A + B = (\omega_0 t + \Delta\omega t - \beta_0 z - \Delta\beta z) + (\omega_0 t - \Delta\omega t - \beta_0 z + \Delta\beta z)$$

$$= 2\omega_0 t - 2\beta_0 z$$

$$\frac{A+B}{2} = \omega_0 t - \beta_0 z$$

$$A - B = \omega_0 t + \Delta\omega t - \beta_0 z - \Delta\beta z - \omega_0 t + \Delta\omega t + \beta_0 z - \Delta\beta z$$

$$= 2\Delta\omega t - 2\Delta\beta z$$

$$\frac{A-B}{2} = (\partial\omega\, t - \partial\beta\, z)$$

$$E_x = \underbrace{2E_0 \cos(\omega_0 t - \beta_0 z)}_{\text{indicate wave}} \underbrace{\cos(\partial\omega\, t - \partial\beta\, z)}_{\text{Indicate envelope or Packet}}$$

$$V_p = \frac{dz}{dt} = \frac{\omega_0}{\beta_0}$$

$$\omega_0 \frac{dt}{dt} - \beta_0 \frac{dz}{dt} = 0 \qquad \boxed{\frac{dz}{dt} = \omega_0/\beta_0}$$

$$v_g = \frac{\partial \omega}{\partial \beta}$$

$$\partial \omega \cdot \frac{dt}{dt} - \partial \beta \cdot \frac{d\varepsilon}{dt} = 0$$

$$\boxed{\left(\frac{d\varepsilon}{dt}\right)_{envelope} = v_g = \frac{\partial \omega}{\partial \beta}}$$