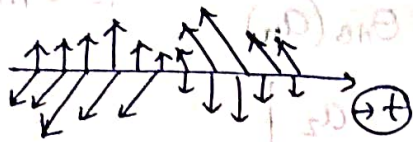


Vector  
mag direction



$$\vec{A} = A_x \vec{a}_x + A_y \vec{a}_y + A_z \vec{a}_z$$

$\downarrow$  mag       $\nearrow$  direction  
 $A_x$        $\vec{a}_x$

- unit vector  $\vec{a}_A = \frac{A_x}{\sqrt{A_x^2 + A_y^2 + A_z^2}} \vec{a}_x + \dots$

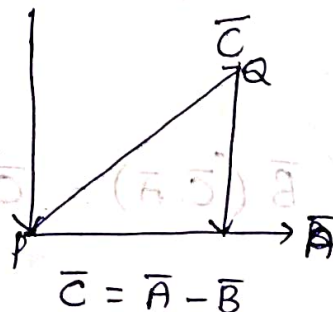
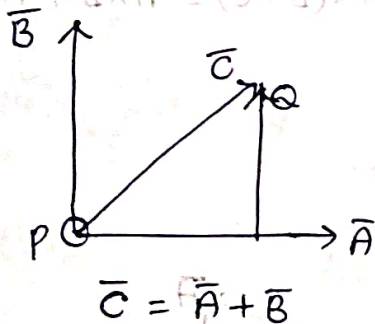
$|\vec{a}_A| = 1$  (to find only direction)

- Addition

$$\vec{A} = A_x \vec{a}_x + A_y \vec{a}_y + A_z \vec{a}_z$$

$$\vec{B} = B_x \vec{a}_x + B_y \vec{a}_y + B_z \vec{a}_z$$

$$\vec{A} + \vec{B} = (A_x + B_x) \vec{a}_x + (A_y + B_y) \vec{a}_y + (A_z + B_z) \vec{a}_z$$



- multiplication

② scalar vector

③ scalar vector

Scalar (dot) :-

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta_{AB} \quad \theta = 0^\circ = \max$$

$$= (A_x \cdot B_x) (\vec{a}_x \cdot \vec{a}_x) + (A_y \cdot B_y) (\vec{a}_y \cdot \vec{a}_y) + (A_z \cdot B_z) (\vec{a}_z \cdot \vec{a}_z)$$

$$A_x \cdot A_x = 1$$

$$A_x \cdot A_y = 0$$

$$A_x \cdot A_z = 0$$

$$= 6 + 2 + 3 = 11$$

Vector (cross):

$$\vec{A} \times \vec{B} = |\vec{A}| |\vec{B}| \sin \theta_{AB} (\hat{n}) \quad 90^\circ = \max$$

$$= \begin{vmatrix} a_x & a_y & a_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= a_x [A_y B_z - B_y A_z] + a_y [A_x B_z - B_x A_z] + a_z [A_x B_y - B_x A_y]$$

Scalar triple :

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = \vec{B} \cdot (\vec{C} \times \vec{A}) = \vec{C} \cdot (\vec{A} \times \vec{B})$$

Commutative law  $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$   $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

Associative law

Distributive law  $\vec{A}(\vec{B} + \vec{C}) = \vec{A}\vec{B} + \vec{A}\vec{C}$   $\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$

Vector triple :

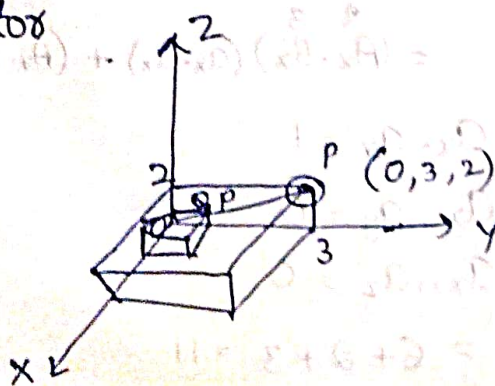
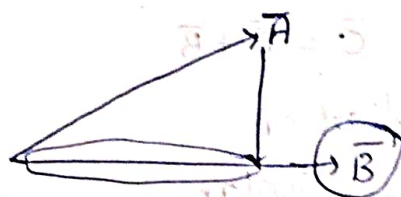
$$\vec{A} (\underbrace{\vec{B} \cdot \vec{C}}_{\text{constant}}) = \vec{B} (\vec{C} \cdot \vec{A}) = \vec{C} (\vec{A} \cdot \vec{B})$$

- Components of vectors :-

( $\vec{A}$  influence on  $\vec{B}$  without considering  $\vec{B}$ )

③  $A_B = (\vec{A} \cdot \hat{a}_B)$

④  $\vec{A}_B = (\underbrace{\vec{A} \cdot \hat{a}_B}_{\text{scalar}}) \hat{a}_B = \text{vector}$



Position vector

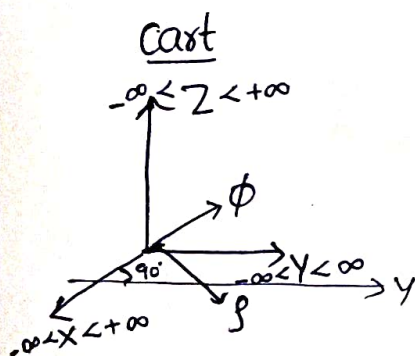
$$\vec{OP} = \vec{P} - \vec{O} = (0-0)a_x + (3-0)a_y + (2-0)a_z \\ = (3a_y + 2a_z)$$

Distance vector

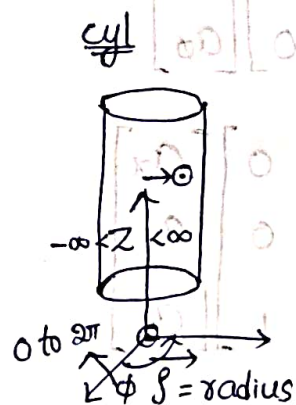
$$\vec{QP} = \vec{P} - \vec{Q} = (0-0)a_x + (3-1)a_y + (2-1)a_z \\ = |2a_y + a_z| \\ = \sqrt{4+1} = \sqrt{5}$$

Coordinate System :-

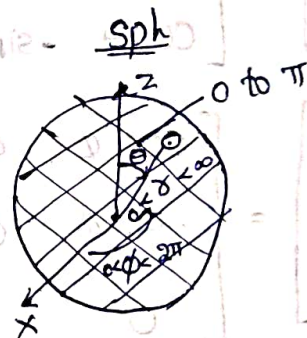
- Cartesian (rectangular)
  - cylindrical
  - Spherical
- } orthogonal



$(x, y, z)$



$(\rho, \phi, z)$



$(r, \theta, \phi) = f(\rho, \phi, z)$   
 $= f(x, y, z)$

$$(x, y, z) = f(\rho, \phi, z) \\ = f(r, \theta, \phi)$$

$$(\rho, \phi, z) = f(x, y, z) \\ = f(r, \theta, \phi)$$

$$\cos \phi = \frac{x}{\rho}$$

$$\tan \phi = y/x$$

$$x = \rho \cos \phi$$

$$\phi = \tan^{-1}(y/x)$$

$$y = \rho \sin \phi$$

$$z = z$$

$$z = z$$

$$\rho = \sqrt{x^2 + y^2}$$

$$x = r \sin \theta \cos \phi$$

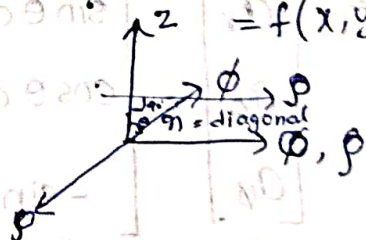
$$\rho = r \sin \theta$$

$$y = r \sin \theta \sin \phi$$

$$z = r \cos \theta$$

$$z = r \cos \theta$$

$$\phi = \phi$$



$$\tan \theta = \frac{\rho}{z} \left\{ \begin{array}{l} \sin \theta = \frac{\rho}{r} \\ \theta = \tan^{-1}(\frac{\rho}{z}) \\ \rho = r \sin \theta \\ r = \sqrt{\rho^2 + z^2} \\ z = r \cos \theta \\ \phi = \phi \end{array} \right.$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{x^2 + y^2}}{z}\right)$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$z = \tan^{-1}(y/x)$$

Cart

$$\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} \cos \phi & -\sin \phi & 0 \\ \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_\rho \\ a_\phi \\ a_z \end{bmatrix}$$

$$\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} \sin \theta \cos \phi & \cos \theta \cos \phi & -\sin \phi \\ \sin \theta \sin \phi & \cos \theta \sin \phi & \cos \phi \\ \cos \theta & -\sin \theta & 0 \end{bmatrix} \begin{bmatrix} a_\rho \\ a_\theta \\ a_\phi \end{bmatrix}$$

cyl

$$\begin{bmatrix} a_\rho \\ a_\phi \\ a_z \end{bmatrix} = \begin{bmatrix} \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \\ \cos \theta & -\sin \theta & 0 \end{bmatrix} \begin{bmatrix} a_r \\ a_\theta \\ a_\phi \end{bmatrix}$$

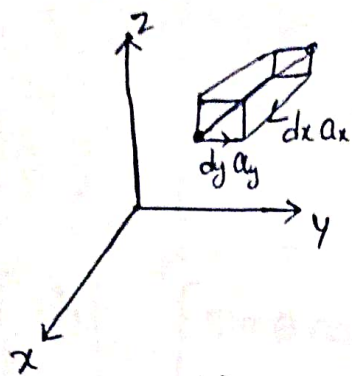
$$\begin{bmatrix} a_\rho \\ a_\phi \\ a_z \end{bmatrix} = \begin{bmatrix} \cos \phi & \sin \phi & 0 \\ -\sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$$

sph

$$\begin{bmatrix} a_r \\ a_\theta \\ a_\phi \end{bmatrix} = \begin{bmatrix} \sin \theta \cos \phi & \sin \theta \sin \phi & \cos \theta \\ \cos \theta \cos \phi & \cos \theta \sin \phi & -\sin \theta \\ -\sin \phi & \cos \phi & 0 \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$$

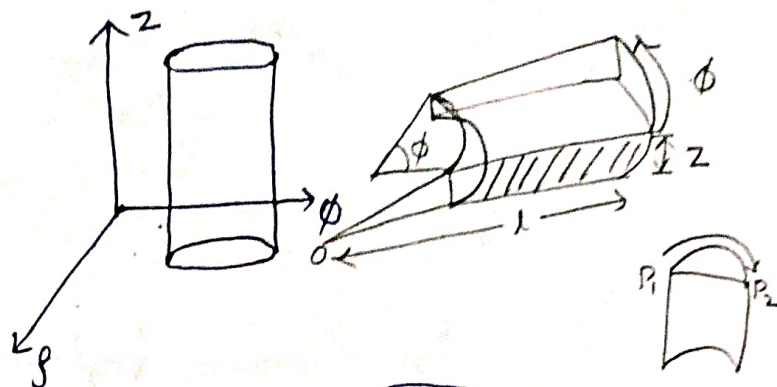
$$\begin{bmatrix} a_r \\ a_\theta \\ a_\phi \end{bmatrix} = \begin{bmatrix} \sin \theta & 0 & \cos \theta \\ \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} a_\rho \\ a_\theta \\ a_\phi \end{bmatrix}$$

Cart



diff region  
 $\int \vec{A} \cdot d\vec{l} = dx a_x + dy a_y + dz a_z$   
 field

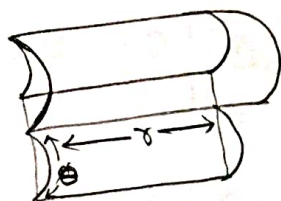
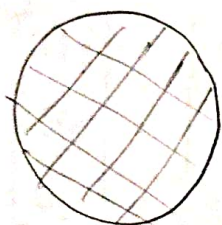
cyl



$$d\vec{l} = d\rho a_\rho + (\rho d\phi) a_\phi + dz a_z$$

radius      angle  
length

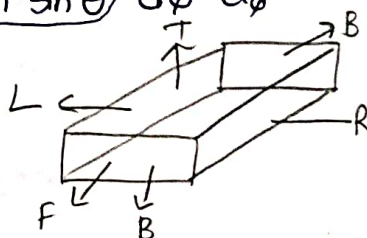
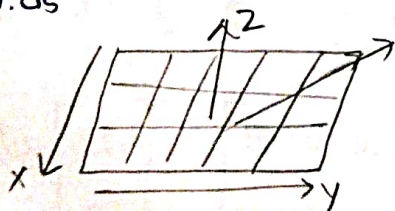
Sph



$$\begin{aligned} d\vec{s} &= dr (r d\theta) a_\theta \\ &= dr (r \sin\theta d\phi) a_\phi \\ &= (r d\theta)(r \sin\theta d\phi) a_r \end{aligned}$$

$$d\vec{l} = dr a_r + r d\theta a_\theta + (r \sin\theta) d\phi a_\phi$$

$\int \vec{A} \cdot d\vec{s}$



T  $\rightarrow d\vec{s} = dx dy (a_z)$

B  $\rightarrow d\vec{s} = dx dy (-a_z)$

R  $\rightarrow d\vec{s} = dx dz (a_y)$

cyl

$$d\vec{s} = d\rho (\rho d\phi) a_z$$

$$= d\rho dz (a_\rho)$$

$$= (\rho d\phi) (dz) (a_\phi)$$

Del Operator ( $\nabla$ ) :-

$$\begin{aligned}\nabla &= \frac{\partial}{\partial x} a_x + \frac{\partial}{\partial y} a_y + \frac{\partial}{\partial z} a_z \\&= \frac{\partial}{\partial \rho} a_\rho + \frac{1}{\rho} \frac{\partial}{\partial \phi} a_\phi + \frac{\partial}{\partial z} a_z \\&= \frac{\partial}{\partial r} a_r + \frac{1}{r} \frac{\partial}{\partial \theta} a_\theta + \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi} a_\phi\end{aligned}$$

- gradient

$$\nabla V = \frac{\partial V}{\partial x} a_x + \frac{\partial V}{\partial y} a_y + \frac{\partial V}{\partial z} a_z \quad \left( V = xy + \left( \frac{x^2+z}{y} \right) + x^3 \right)$$

$$\begin{aligned}dV &= \frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} dy + \frac{\partial V}{\partial z} dz \\&= \left( \frac{\partial V}{\partial x} a_x + \frac{\partial V}{\partial y} a_y + \frac{\partial V}{\partial z} a_z \right) (dx a_x + dy a_y + dz a_z) \\&= \vec{G} \cdot d\vec{r} = |\vec{G}| |d\vec{r}| \cos \theta_{\vec{G}, d\vec{r}}\end{aligned}$$

\* Max rate of change

- Divergence (dot) (div)

$$\begin{aligned}\nabla \cdot \vec{A} &= \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \\&= \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho A_\rho) + \frac{1}{\rho} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z} \\&= \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 A_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta A_\theta) + \frac{1}{r \sin \theta} \frac{\partial A_\phi}{\partial \phi}\end{aligned}$$

$$\textcircled{1} \quad \nabla(u+v) = \nabla u + \nabla v$$

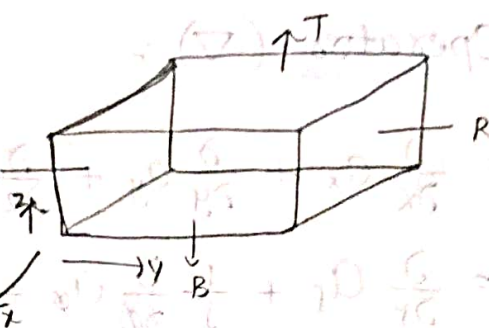
$$\textcircled{2} \quad \nabla(uv) = u \cdot \nabla v + v \cdot \nabla u$$

$$\nabla \cdot (\vec{A} + \vec{B}) = \nabla \cdot \vec{A} + \nabla \cdot \vec{B}$$

$$\nabla \cdot (v \vec{A}) = v (\nabla \cdot \vec{A}) + \vec{A} \cdot \nabla v$$

- Divergence Theorem :-

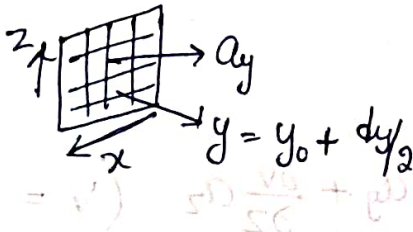
$$\oint \vec{A} \cdot \vec{ds} = \iiint_V (\nabla \cdot \vec{A}) dv$$



$$= \left( \iint_B + \iint_T + \iint_F + \iint_{\text{Back}} + \iint_R + \iint_L \right) \vec{A} \cdot \vec{ds}$$

Right

$$= \int_x \int_z \vec{A} \cdot \vec{ds}$$



$$A = A(x_0, y_0, z_0) + (x-x_0) \frac{\partial A}{\partial x} + (y-y_0) \frac{\partial A}{\partial y} + (z-z_0) \frac{\partial A}{\partial z} + \dots$$

$$\vec{A} = A(x_0, y_0, z_0) + \left( y_0 + \frac{dy}{2} - y_0 \right) \frac{\partial A}{\partial y}$$

$$= A(x_0, y_0, z_0) + \left( \frac{dy}{2} \right) \frac{\partial A}{\partial y}$$

$$= \int_x \int_z \left[ A(x_0, y_0, z_0) + \left( \frac{dy}{2} \right) \frac{\partial A}{\partial y} \right] dx dz$$

$$= \int_x \int_z A(x_0, y_0, z_0) dx dz + \int_x \int_z \frac{dy}{2} \frac{\partial A}{\partial y} dx dz$$

left  $y = y_0 - \frac{dy}{2}$

$$\vec{A} = A(x_0, y_0, z_0) - \frac{dy}{2} \frac{\partial A}{\partial y}$$

$$= - \int_x \int_z A(x_0, y_0, z_0) dx dz + \int_x \int_z \frac{dy}{2} \frac{\partial A}{\partial y} dx dz$$

$$\text{Right} + \text{left} = \int_x \int_z \frac{dy}{2} \frac{\partial A}{\partial y} dx dz + \int_x \int_z \frac{dy}{2} \frac{\partial A}{\partial y} dx dz$$

$$= \int_x \int_z \frac{\partial A}{\partial y} dx dy dz$$

$$\text{Top} + \text{Bottom} = \int_x \int_y \frac{\partial A}{\partial z} dx dy dz$$

$$\text{Front} + \text{Back} = \int \int_z \frac{\partial A_x}{\partial x} dx dy dz$$

$$= \int \int \int \left( \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \right) dx dy dz$$

$$= \int \int \int (\nabla \cdot \bar{A}) dx dy dz = \oint \bar{A} \cdot d\bar{s}$$

Curl (Stoke's Theorem) (Cross) :-

$$\nabla \times \bar{A}$$

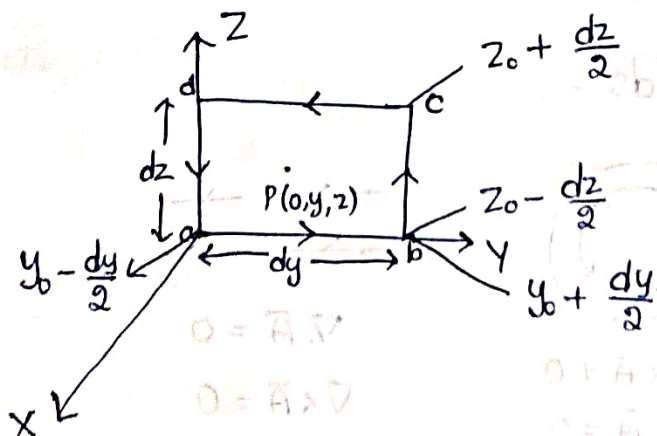
$$\nabla \times \bar{A} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} \quad d\bar{l} = dx \hat{a}_x + dy \hat{a}_y + dz \hat{a}_z$$

$$\nabla \times \bar{A} = \frac{1}{\rho} \begin{vmatrix} \hat{a}_\rho & \rho \hat{a}_\phi & \hat{a}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix} \quad d\bar{l} = d\rho \hat{a}_\rho + \rho d\phi \hat{a}_\phi + dz \hat{a}_z$$

$$\nabla \times \bar{A} = \frac{1}{r^2 \sin \theta} \begin{vmatrix} \hat{a}_r & r \hat{a}_\theta & r \sin \theta \hat{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & r A_\theta & r \sin \theta A_\phi \end{vmatrix} \quad d\bar{l} = dr \hat{a}_r + r d\theta \hat{a}_\theta + r \sin \theta d\phi \hat{a}_\phi$$

$$\oint \bar{A} \cdot d\bar{s} = \int (\nabla \cdot \bar{A}) dv$$

$$\oint \bar{A} \cdot d\bar{l} = \int (\nabla \times \bar{A}) \cdot d\bar{s}$$



$$\oint \vec{A} \cdot d\vec{l} = \left( \int_{ab} + \int_{bc} + \int_{cd} + \int_{da} \right) \vec{A} \cdot d\vec{l}$$

Qb

$$\begin{aligned} \int_{ab} \vec{A} \cdot d\vec{l} &= \int_{ab} \left[ A(x_0, y_0, z_0) + \left( z_0 - \frac{dz}{2} + z_0 \right) \frac{\partial A_z}{\partial y} \right] dy \\ &= \int_{ab} A(x_0, y_0, z_0) dy + \int_{ab} \frac{dz}{2} \frac{\partial A_z}{\partial y} dy \end{aligned}$$

Cd

$$= \int_{cd} \left[ A(x_0, y_0, z_0) + \left( z_0 + \frac{dz}{2} - z_0 \right) \frac{\partial A_z}{\partial y} \cdot dy \right] (-1)$$

$$= - \int_{cd} A(x_0, y_0, z_0) (-1) dy - \int_{cd} \frac{dz}{2} \frac{\partial A_z}{\partial y} dy$$

$$= \iint \left( - \frac{\partial A_z}{\partial y} \right) dy dz = ab + cd$$

$$= \iint \left( \frac{\partial A_y}{\partial z} \right) dy dz = bc + da$$

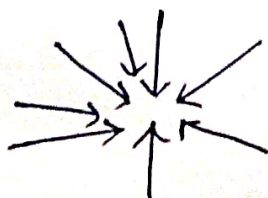
$$= \iint \left( \frac{\partial A_y}{\partial z} - \frac{\partial A_z}{\partial y} \right) dy dz$$

$$\oint \vec{A} \cdot d\vec{l} = \iint (\nabla \times \vec{A}) \cdot d\vec{s}$$

Laplacian of the scalar :-

$$\nabla \cdot (\nabla V) = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

Classification of fields :-



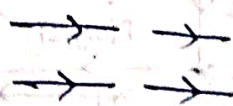
$$\nabla \cdot \vec{A} \neq 0$$

$$\nabla \times \vec{A} = 0$$



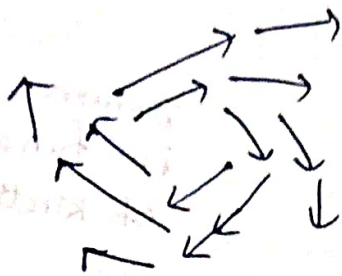
$$\nabla \times \vec{A} \neq 0$$

$$\nabla \cdot \vec{A} = 0$$



$$\nabla \cdot \vec{A} = 0$$

$$\nabla \times \vec{A} = 0$$



$$\nabla \cdot \vec{A} \neq 0$$

$$\nabla \times \vec{A} \neq 0$$

$$\textcircled{1} \nabla \times (\vec{A} + \vec{B}) = \nabla \times \vec{A} + \nabla \times \vec{B}$$

$$\textcircled{2} \nabla \times (\vec{A} \times \vec{B}) = \vec{A}(\nabla \cdot \vec{B}) - \vec{B}(\nabla \cdot \vec{A}) + (\vec{B} \cdot \nabla)\vec{A} - (\vec{A} \cdot \nabla)\vec{B}$$

$$\textcircled{3} \nabla \times (\nabla A) = \nabla(\nabla \times \vec{A}) + (\nabla \cdot \nabla \times \vec{A})$$

$$\Rightarrow \text{curl}(\text{gradient } A) = 0$$

$$\Rightarrow \text{curl}(\text{gradient } A) = 0$$

Maxwell's equations:-

Static - fixed

Dynamic - time varying

$$\textcircled{1} \nabla \cdot \vec{D} = \rho_v \quad \oint \vec{D} \cdot d\vec{s} = \int \rho_v dv \rightarrow \text{Gauss law}$$

$$\textcircled{2} \nabla \cdot \vec{B} = 0 \quad \oint \vec{B} \cdot d\vec{s} = 0 \rightarrow \text{Non-existence of magnetic monopole}$$

$$\textcircled{3} \nabla \times \vec{E} = 0 \quad \oint \vec{E} \cdot d\vec{l} = 0 \rightarrow \text{Conservative nature of electric field}$$

$$\textcircled{4} \nabla \times \vec{H} = \vec{J} \quad \oint \vec{H} \cdot d\vec{l} = \iint \vec{J} \cdot d\vec{s} \rightarrow \text{Ampere's law}$$

$$\oint \vec{D} \cdot d\vec{s} = \int_V (\nabla \cdot \vec{D}) dV = \int_V \rho_v dV$$

$\downarrow$   
 $\epsilon \cdot \vec{E}$

$\downarrow$   
 volume  
 charge density

The net flux inside is equal flux coming out of the surface.

$$\oint \vec{B} \cdot d\vec{s} = 0$$

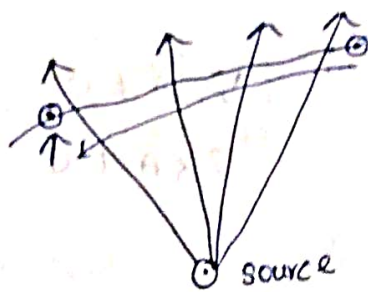
$\downarrow$   
 $\mu \vec{H}$

incoming flux = outgoing flux

We can't separate north & south pole.

$$V_{emf} = \frac{P}{Q}$$

$$dW = - \vec{F} \cdot d\vec{l}$$



return  
in form  
of kinetic energy

Coulomb's:  $F \propto \frac{Q_1 Q_2}{4\pi r^2}$

$$\vec{E} = \vec{F}/Q$$

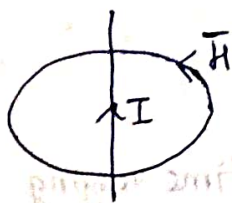
$$V_{emf} = \frac{W}{Q} = - \int \vec{E} \cdot d\vec{l}$$

$$dW = - \vec{E} Q \cdot d\vec{l} \quad V_{emf} = A - B - A = - \oint \vec{E} \cdot d\vec{l} = 0$$

$$V = \frac{dW}{Q} = - \vec{E} \cdot d\vec{l} \quad \iint_S (\nabla \times \vec{E}) \cdot d\vec{s} = 0$$

from this we can say, electric field has conservative nature.

Ampere's law:- deals with current along with magnetic field.



$e^-$  move with constant velocity have  
constant current

$$\oint \vec{H} \cdot d\vec{l} = I_{enc}$$

$= J(A) \rightarrow \text{Area}$

$$\iint_S (\nabla \times \vec{H}) \cdot d\vec{s} = \iint_S \vec{J} \cdot d\vec{s}$$



Time-varying fields:-

$$\textcircled{1} \nabla \cdot \vec{D} = \rho_v \quad \textcircled{2} \nabla \cdot \vec{B} = 0$$

$$\textcircled{3} \nabla \times \vec{E} = - \frac{\partial \vec{B}}{\partial t} \rightarrow \text{faraday's law}$$

$$\textcircled{4} \nabla \times \vec{H} = \vec{J} + \vec{J}_d = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

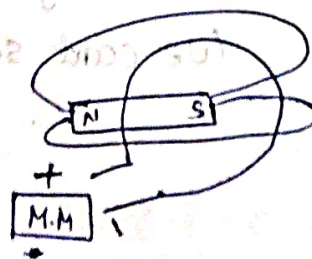
$\hookrightarrow$  modified ampere's law

$$V_{emf} = - N \frac{d\phi}{dt}$$

$$V_{emf} = - \frac{d\phi}{dt}$$

$$\Psi \rightarrow \text{flux} = \iint \vec{B} \cdot d\vec{s}$$

magnetic flux



$$V_{emf} = \iint \left[ -\frac{\partial B}{\partial t} \right] d\vec{s}$$

$$\oint \vec{E} \cdot d\vec{l} = \iint \left( -\frac{\partial B}{\partial t} \right) d\vec{s}$$

$$\iint (\nabla \times \vec{E}) d\vec{s} = \iint \left( -\frac{\partial B}{\partial t} \right) d\vec{s}$$

$$\nabla \times \vec{E} = -\frac{\partial B}{\partial t}$$

(4)  $\Rightarrow$

$$= \nabla \cdot (\nabla \times \vec{H}) = 0 \neq \nabla \cdot \vec{J}$$

$\downarrow$   
div

$\downarrow$   
curl

$$\nabla \cdot \vec{J} = -\frac{\partial \rho_v}{\partial t}$$

$$\oint \vec{J} \cdot d\vec{s} = -\frac{dQ_{in}}{dt} \rightarrow \text{equation of continuity}$$

$$= Q = \int \rho_v dv$$

$$\oint \vec{J} \cdot d\vec{s} = -\frac{d}{dt} \left( \int \rho_v dv \right)$$

$$\oint \vec{J} \cdot d\vec{s} = \int -\frac{\partial \rho_v}{\partial t} dv$$

$$\nabla \cdot \vec{J} = -\frac{\partial \rho_v}{\partial t} \neq 0$$

$$\nabla \times \vec{H} = \vec{J} + \vec{J}_d$$

$$\nabla \cdot (\nabla \times \vec{H}) = \nabla \cdot \vec{J} + \nabla \cdot \vec{J}_d = 0$$

$$\nabla \cdot \vec{J} \neq \nabla \cdot \vec{J}_d = \nabla \cdot \left( \frac{\partial \rho_v}{\partial t} \right)$$

$$\nabla \cdot \vec{J}_d = \frac{\partial \rho_v}{\partial t}$$

$$= \frac{\partial}{\partial t} (\nabla \cdot \vec{D})$$

$$\nabla \cdot \vec{J}_d = \nabla \cdot \frac{\partial \vec{D}}{\partial t}$$

$$\vec{J}_d = \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \times \vec{H} = \vec{J} + \vec{J}_d = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\vec{D} = \epsilon \vec{E}$$

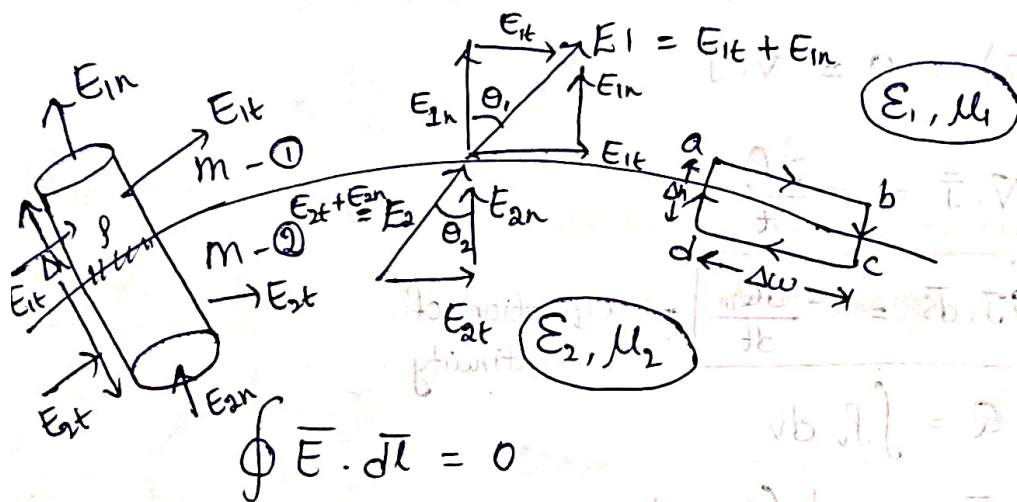
$$\vec{B} = \mu \vec{H}$$

# Boundary Conditions :

- ① Dielectric - Dielectric
- ② Conductor - Dielectric
- ③ Conductor - free space

## ① Dielectric - Dielectric

$\epsilon$  - ability to store



$$\oint \vec{E} \cdot d\vec{l} = 0$$

$$E_{it} \Delta w - \frac{\Delta h}{2} E_{1n} - \frac{\Delta h}{2} E_{2n} - E_{2t} \Delta w + \frac{\Delta h}{2} E_{2n} + \frac{\Delta h}{2} E_{1n} = 0$$

$\downarrow$   $\downarrow$   $\downarrow$   $\downarrow$   
 $ab$   $bc$   $cd$   $da$

$$E_{it} \Delta w - E_{2t} \Delta w = 0$$

$$E_{it} = E_{2t}$$

$$\frac{D_{it}}{\epsilon} = \frac{D_{2t}}{\epsilon}$$

$$\vec{D} = \epsilon \vec{E} + \vec{P} = \epsilon_0 \chi \vec{E}$$

$$\oint \vec{D} \cdot d\vec{S} = Q_s$$

$$- \frac{\Delta h}{2} D_{2n} + \frac{\Delta h}{2} D_{1n} = \left( \frac{Q_s}{\epsilon_0} \right) \Delta w = 0$$

$$- D_{2n} + D_{1n} = 0$$

$$D_{1n} = D_{2n}$$

Q) Let  $\vec{A} = 3\vec{a}_y + 4\vec{a}_z$  and  $\vec{B} = 4\vec{a}_x - 10\vec{a}_y + 5\vec{a}_z$  then find

- ① a unit vector  $\vec{a}_c$  i.e parallel to  $3\vec{A} + 2\vec{B}$  with magnitude-15
- ② find angle b/w  $\vec{A}$  &  $\vec{B}$
- ③ find vector component  $\vec{A} \rightarrow \vec{B}$
- ④ find vector component i.e.,  $\perp$  to both  $\vec{A}$  &  $\vec{B}$

Sol:  $15 \vec{a}_c = (\vec{A}_{(3\vec{A} + 2\vec{B})}) 15$

$$\begin{aligned} 3\vec{A} + 2\vec{B} &= 3(3\vec{a}_y + 4\vec{a}_z) + 2(4\vec{a}_x - 10\vec{a}_y + 5\vec{a}_z) \\ 3\vec{A} + 2\vec{B} &= 9\vec{a}_y + 12\vec{a}_z + 8\vec{a}_x - 20\vec{a}_y + 10\vec{a}_z \end{aligned}$$

$$3\vec{A} + 2\vec{B} = 8\vec{a}_x - 11\vec{a}_y + 22\vec{a}_z$$

$$= 15 \left[ \frac{8}{\sqrt{64+121+484}} \vec{a}_x - \frac{11}{\sqrt{64+121+484}} \vec{a}_y + \frac{22}{\sqrt{64+121+484}} \vec{a}_z \right]$$

②  $\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta_{AB}$

$$|\vec{A}| = \sqrt{3^2 + 4^2} = \sqrt{9+16} = 5$$

$$|\vec{B}| = \sqrt{16+100+25} = \sqrt{141}$$

$$\vec{A} \cdot \vec{B} = (3\vec{a}_y + 4\vec{a}_z) \cdot (4\vec{a}_x - 10\vec{a}_y + 5\vec{a}_z)$$

$$= (0 - 30 + 20) = -10$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= 55\vec{a}_x - 16\vec{a}_y - 12\vec{a}_z$$

$$= \sqrt{(55)^2 + (16)^2 + (12)^2}$$

$$= \vec{a}_x [A_y B_z - A_z B_y] + \vec{a}_y [A_z B_x - A_x B_z] + \vec{a}_z [A_x B_y - A_y B_x]$$

$$= \vec{a}_x (15+40) + \vec{a}_y (16-0) + \vec{a}_z (0-12)$$

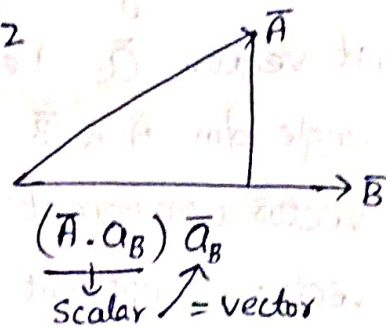
$$\vec{A} \times \vec{B} = |\vec{A}| |\vec{B}| \sin \theta_{AB}$$

$$\theta_{AB} = \sin^{-1} \frac{(\vec{A} \times \vec{B})}{|\vec{A}| |\vec{B}|}$$

$$= \sin^{-1}$$

$$③ \quad a_B = \frac{4}{\sqrt{16+100+25}} a_x - \frac{10}{\sqrt{141}} a_y + \frac{5}{\sqrt{141}} a_z$$

$$(\vec{A} \cdot \vec{a}_B) = \text{scalar} \cdot \vec{a}_B = \underline{\hspace{2cm}}$$



④ Cross product b/w  $\vec{A}$  &  $\vec{B}$ .

Q) Consider a straight line in a  $xy$ -plane i.e.,  $3x+2y=6$  then find a vector from origin to the straight line which makes  $90^\circ$  with it.

$$\underline{Ans} \quad \vec{K} \cdot \vec{L} = 0$$

$$(\vec{OP}) \cdot (\vec{AB}) = 0$$

$$(x a_x + y a_y) \cdot (-2 a_x + 3 a_y) = 0$$

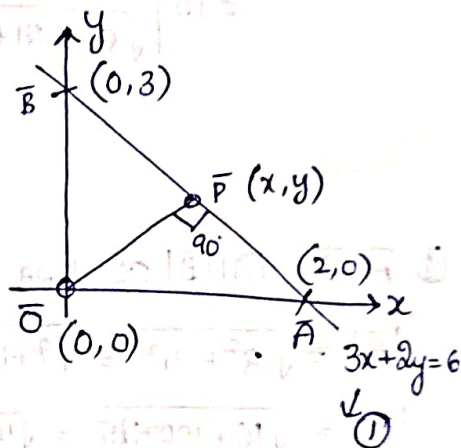
$$-2x + 3y = 0 \rightarrow ②$$

$$3x + 2y = 6 \rightarrow ①$$

$$x = \frac{18}{13}, \quad y = \frac{12}{13}$$

$$\vec{OP} = x a_x + y a_y$$

$$\vec{OP} = \frac{18}{13} a_x + \frac{12}{13} a_y$$



Q)  $P(2, 3, -5) \rightarrow \text{cyl, sph}$

$Q(3, 180^\circ, 2) \rightarrow \text{sph, cart}$

$R(4, 60^\circ, 210^\circ) \rightarrow \text{cyl, cart}$

①  $P(2, 3, -5) \quad x=2, y=3, z=-5$

$$\underline{Cyl}: \quad \rho = \sqrt{x^2 + y^2} = \sqrt{4+9} = \sqrt{13}$$

$$\phi = \tan^{-1}(y/x) = \tan^{-1}(3/2)$$

$$z = z = -5$$

Sph:  $r = \sqrt{x^2 + y^2 + z^2}$   
 $\phi = \tan^{-1}(y/x)$   
 $\theta = \tan^{-1}\left(\frac{\sqrt{x^2 + y^2}}{z}\right)$

Q  $\Rightarrow r = \sqrt{\rho^2 + z^2}$

$\phi = \phi$

$\theta = \tan^{-1}(\rho/z)$

$x = \rho \cos \phi$

$y = \rho \sin \phi$

$z = z$

R  $\Rightarrow x = r \sin \theta \cos \phi$

$y = r \sin \theta \sin \phi$

$z = r \cos \theta$

$\phi = \phi$

$\rho = r \sin \theta$

$z = r \cos \theta$

Q  $\frac{\rho}{\sqrt{\rho^2 + z^2}} (\cos \phi a_\rho - \sin \phi a_\phi - z \cos \phi a_z)$   
 $A_\rho a_\rho + A_\phi a_\phi + A_z a_z$

@ spherical @ cartesian

A:  $\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin \theta & \cos \theta & 0 \\ \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$   
 $A_\rho = \frac{\rho \cos \theta}{\sqrt{\rho^2 + z^2}}$   
 $A_\phi = \frac{-\rho \sin \phi}{\sqrt{\rho^2 + z^2}}$   
 $A_z = \frac{-\rho z \cos \phi}{\sqrt{\rho^2 + z^2}}$

$A_r = \frac{\rho \sin \theta \cos \phi}{\sqrt{\rho^2 + z^2} \rightarrow r \cos \theta} + 0 + \frac{(-\rho z \cos \phi \cos \theta)}{\sqrt{\rho^2 + z^2}}$

$A_r = \frac{\rho r \sin^2 \theta \cos \phi}{\sqrt{r^2 \cos^2 \theta + r^2 \sin^2 \theta}} + 0 + \frac{(-r \sin \theta r \cos \theta \cos \theta \cos \phi)}{\sqrt{r^2 \cos^2 \theta + r^2 \sin^2 \theta}}$

$A_r = \sin^2 \theta \cos \phi + (-r \sin \theta \cos^2 \theta \cos \phi)$

$A_\theta = \frac{\rho \cos \theta \cos \phi}{\sqrt{\rho^2 + z^2}} + 0 + \frac{\rho z \sin \theta \cos \phi}{\sqrt{\rho^2 + z^2}}$

$= \frac{r \sin \theta \cos \theta \cos \phi}{\sqrt{r^2 \sin^2 \theta + r^2 \cos^2 \theta}} + 0 + \frac{r \sin \theta r \cos \theta \sin \theta \cos \phi}{\sqrt{r^2 \cos^2 \theta + r^2 \sin^2 \theta}}$

$A_\theta = \sin \theta \cos \theta \cos \phi + r \sin^2 \theta \cos \theta \cos \phi$

$$A_\phi = \frac{-\rho \sin \phi}{\sqrt{\rho^2 + z^2}} = \frac{-\gamma \sin \theta \cancel{\cos \theta} \sin \phi}{\sqrt{\gamma^2 \sin^2 \theta + \gamma^2 \cos^2 \theta}}$$

$$A_\phi = -\sin \theta \sin \phi$$

(b) Cartesian

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos \phi & -\sin \phi & 0 \\ \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}$$

$$A_x = \cos \phi \left[ \frac{\rho \cos \phi}{\sqrt{\rho^2 + z^2}} \right] - \sin \phi \left[ \frac{-\rho \sin \phi}{\sqrt{\rho^2 + z^2}} \right]$$

$$A_y = \sin \phi \left[ \frac{\rho \cos \phi}{\sqrt{\rho^2 + z^2}} \right] + \cos \phi \left[ \frac{-\rho \sin \phi}{\sqrt{\rho^2 + z^2}} \right]$$

$$A_z = A_z$$

$$A_x = \frac{\rho \cos^2 \phi}{\sqrt{\rho^2 + z^2}} + \frac{\rho \sin^2 \phi}{\sqrt{\rho^2 + z^2}} = \frac{\rho}{\sqrt{\rho^2 + z^2}} = \frac{\sqrt{x^2 + y^2}}{\sqrt{x^2 + y^2 + z^2}}$$

$$A_y = \frac{\rho \cos \phi \sin \phi}{\sqrt{\rho^2 + z^2}} - \frac{\rho \sin \phi \cos \phi}{\sqrt{\rho^2 + z^2}} = 0$$

$$A_z = A_z$$

Q) The surfaces  $\rho = 3$  &  $5$ ,  $z = 3$  &  $4.5$ ,  $\phi = 100^\circ$  to  $130^\circ$

(a) for the given values identify a closed surface

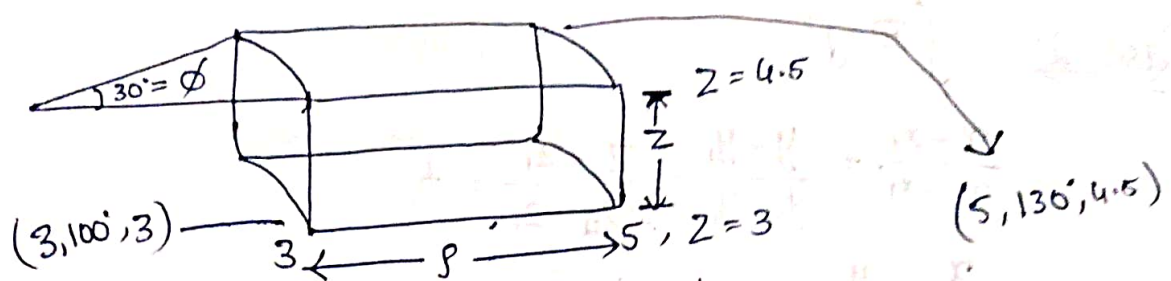
(b) find the volume

(c) find the surface area

(d) find the length of the lines

(e) find the length of the longest line lies inside that volume.

a)

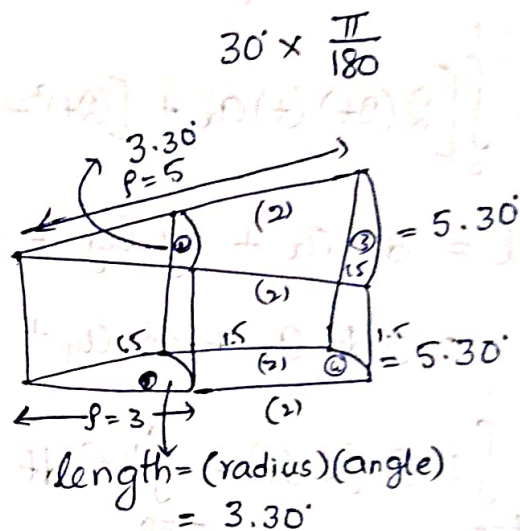
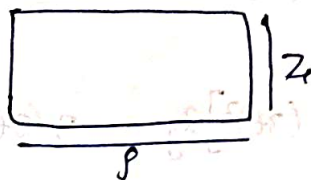


b)

$$= \int_{\rho=3}^5 \int_{z=3}^{4.5} \int_{\phi=100}^{130} dV \begin{cases} dx dy dz \\ d\rho \rho d\phi dz \\ d\rho \rho d\theta \rho \sin\theta d\phi \end{cases}$$

c) (S-1) = (P)

$$\int_{\rho=3}^5 \int_{z=3}^{4.5} dz d\rho$$



a) find  $\oint \vec{F} \cdot d\vec{r}$ ,  $\vec{F} = 2xy \vec{a}_x + (x^2 - z^2) \vec{a}_y - 3xz^2 \vec{a}_z$

@  $(0,0,0) \xrightarrow{(1)} (0,1,0) \xrightarrow{(2)} (2,1,0) \xrightarrow{(3)} (2,1,3)$

⑥  $(0,0,0) \rightarrow (2,1,3)$

a)

$$\textcircled{1} \Rightarrow \int_{y=0}^1 \vec{F} \cdot d\vec{r} = 0$$

$(x=0, z=0)$

$$\vec{F} = 2xy \vec{a}_x + (x^2 - z^2) \vec{a}_y - 3xz^2 \vec{a}_z = 0$$

$$d\vec{r} = dy \vec{a}_y$$

$$\textcircled{2} \int_{x=0}^2 \vec{F} \cdot d\vec{r} = \int_{x=0}^2 2x dx = 2 \cdot \frac{x^2}{2} \Big|_0^2$$

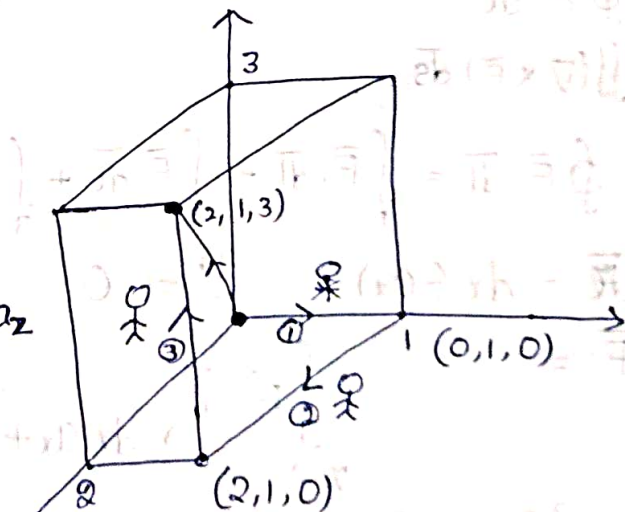
$(y=1, z=0)$

$$= 2 \cdot \frac{4}{2} = \textcircled{4}$$

$$\vec{F} = 2x(1) \vec{a}_x + (x^2 - 0^2) \vec{a}_y - 3x(0)^2 \vec{a}_z$$

$$= 2x \vec{a}_x + x^2 \vec{a}_y$$

$$d\vec{r} = dx \vec{a}_x$$



$$\vec{F} \cdot d\vec{r} = (2x \vec{a}_x + x^2 \vec{a}_y) \cdot dx \vec{a}_x = 2x dx$$

Case-2:-  $\int \vec{F} \cdot d\vec{l}$

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1} = t$$

$$\frac{x}{2} = \frac{y}{1} = \frac{z}{3} = t$$

$$x = 2t \quad y = t \quad z = 3t$$

$$dx = 2dt \quad dy = dt \quad dz = 3dt$$

$$t = 0 \text{ to } 1$$

$$= \int_0^1 [2(2t)(t)a_x + [(2t)^2 - (3t)^2]a_y - 3(2t)(3t)^2 a_z] d\vec{l}$$

$$d\vec{l} = dx a_x + dy a_y + dz a_z$$

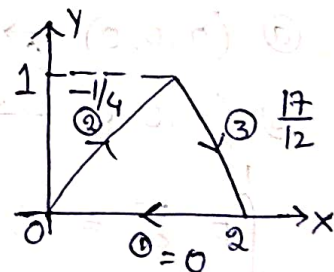
$$= 2dt a_x + dt a_y + 3dt a_z$$

$$= \int_{t=0}^1 4t^2 (2dt) + \int_{t=0}^1 4t^2 dt - \int_{t=0}^1 9t^2 dt - 18 \int_{t=0}^1 t^3 dt$$

P) Given that  $\vec{F} = x^2y a_x - y a_y$  then find

①  $\oint \vec{F} \cdot d\vec{l}$

②  $\iint (\nabla \times \vec{F}) \cdot d\vec{S}$



$$\oint \vec{F} \cdot d\vec{l} = \int_1 \vec{F} \cdot d\vec{l} + \int_2 \vec{F} \cdot d\vec{l} + \int_3 \vec{F} \cdot d\vec{l}$$

①  $\Rightarrow \oint \vec{F} \cdot d\vec{l} = \int_1 \vec{F} \cdot d\vec{l} + \int_2 \vec{F} \cdot d\vec{l} + \int_3 \vec{F} \cdot d\vec{l}$

$$\vec{F} = x^2y a_x - y a_y$$

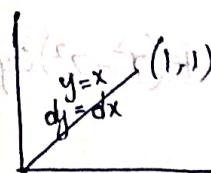
$$\vec{F} = x^3 a_x - x a_y$$

$$d\vec{l} = dx a_x + dy a_y$$

$$\int \vec{F} \cdot d\vec{l} = \int x^3 dx - x dx$$

$$= \left[ \frac{x^4}{4} - \frac{x^2}{2} \right]_0^1$$

$$= \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$$



③/6

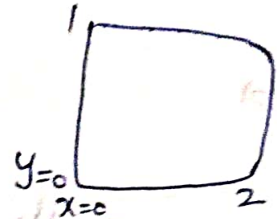
$$\nabla \times \vec{F} = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 y & -y & 0 \end{vmatrix}$$

$$= a_x [0 - 0] - a_y [0] + a_z [0 - x^2]$$

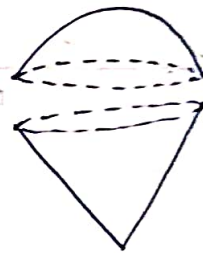
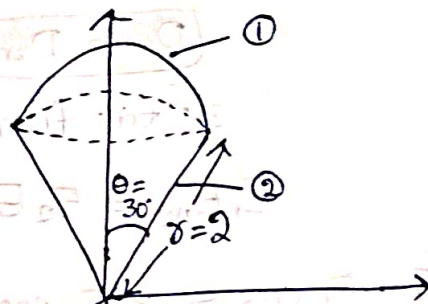
$$= -x^2 a_x + (-x^2) a_z = -x^2 a_z$$

$$= \iint (-x^2 a_z) \vec{ds} \rightarrow dx dy (-a_z)$$

$$= \int_{x=0}^1 \int_{y=0}^x x^2 dx dy + \int_{x=1}^2 \int_{y=0}^{2-x} x^2 dx dy = 7/6$$



P)  $\vec{Q} = r \sin \theta \vec{a}_r + r \cos \theta \vec{a}_\theta + r \vec{a}_\phi$



①  $\oint \vec{Q} \cdot d\vec{l} \rightarrow \int d\phi \vec{a}_\phi$  (4π)

$$\vec{Q} \cdot d\vec{l} = r^2 \sin \theta d\phi$$

$$\oint \vec{Q} \cdot d\vec{l} = \int_{\phi=0}^{2\pi} 4 \frac{1}{2} d\phi = 2 [\phi]_0^{2\pi} = 4\pi$$

②  $\int_{\theta=0}^{30^\circ} \int_{\phi=0}^{2\pi} (\nabla \times \vec{Q}) \vec{ds} \rightarrow r d\theta r \sin \theta d\phi \vec{a}_r$  (S<sub>1</sub>)

$$(\cos \theta \vec{a}_r - 2\vec{a}_\theta + \cos \theta \vec{a}_\phi)$$

$$\nabla \times \vec{Q} = \frac{1}{r^2 \sin \theta} \begin{vmatrix} a_r & r a_\theta & r \sin \theta a_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & r A_\theta & r \sin \theta A_\phi \end{vmatrix}$$

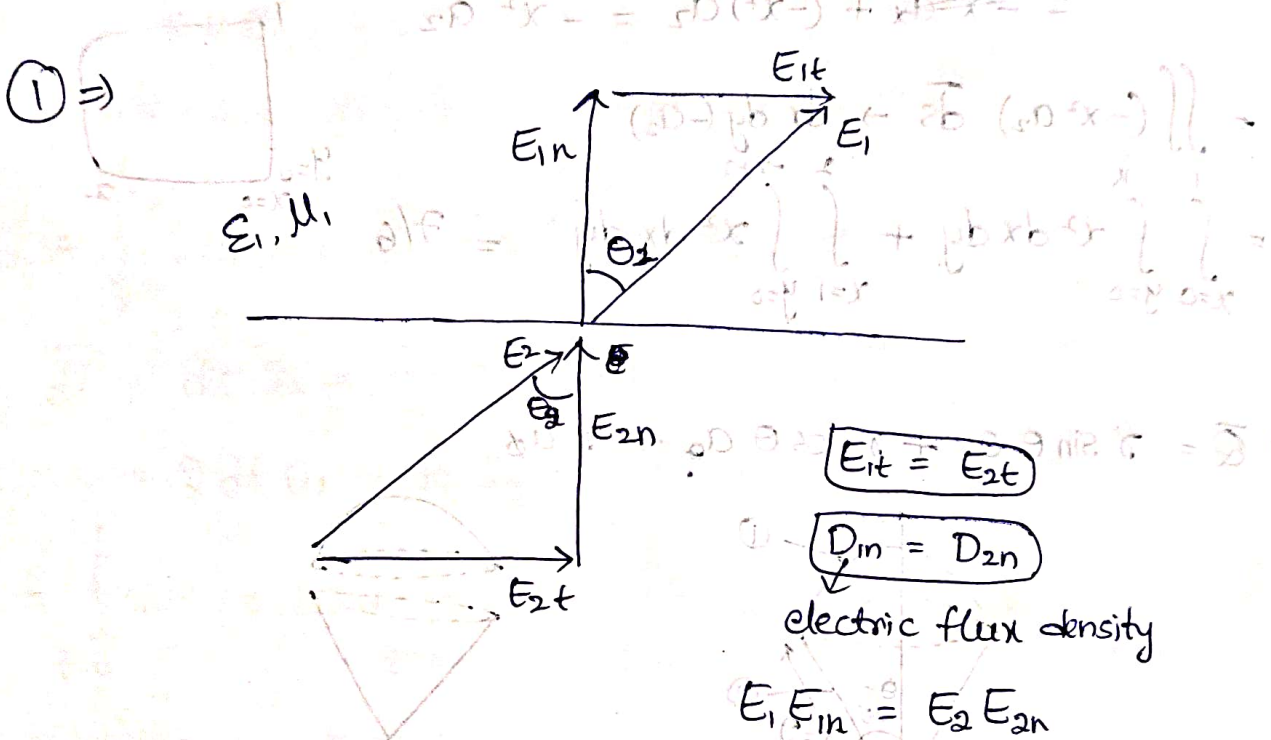
$$\begin{matrix} \downarrow & \downarrow & \downarrow \\ r \sin \theta & r \cos \theta & r \end{matrix}$$

(4π)

$$\int_0^{\pi} \int_0^{2\pi} (-2) d\eta \sin\theta d\phi = -4\pi$$

Electric field boundary conditions:-

- ① Dielectric - Dielectric
- ② Dielectric - conductor
- ③ Conductor - free space



①  $\leftarrow \sin \theta_1 E_1 = \sin \theta_2 E_2 \quad \& \quad E_1 \cos \theta_1 = E_2 \cos \theta_2 \rightarrow ②$

①  $\frac{\tan \theta_1}{\tan \theta_2 E_1} = \frac{\tan \theta_2}{E_2} \Rightarrow \frac{\tan \theta_1}{\tan \theta_2} = \frac{E_1}{E_2} = \frac{1}{1}$

②  $\Rightarrow$  potential difference = 0 because of free electrons

Dielectric - conductor

$E = 0$

$E_{1t} = E_{2t}$   
 $0 = E_{2t}$

$D_{1n} - D_{2n} = \rho_s$

if  $\rho_s = 0$

$D_{1n} = D_{2n}$

$0 = D_{2n}$

if  $\rho_s \neq 0$

$D_{2n} = \rho_s$

③ ⇒ Conductor - free space ( $\mu_r = \epsilon_r = 1$ )

$$\downarrow$$
  
 $E = 0$

$$\downarrow$$
  
 $\mu = \mu_0$

$$\epsilon = \epsilon_0$$

$$\mu = \mu_0, \epsilon = \epsilon_0$$

$$D_{2n} = \rho_s$$

$$E_2 D_{2n} = \rho_s$$

$$\epsilon_0 E_2 = \rho_s$$

④ Free space - Dielectric

$$E_{1t} = E_{2t}$$

$$D_{1n} = D_{2n}$$

$$\epsilon_0 E_1 = \epsilon_2 E_2$$

$$\frac{\tan \theta_1}{\tan \theta_2} = \frac{E_1}{E_2} = \frac{\epsilon_2}{\epsilon_1}$$

$$= \frac{\epsilon_2}{\epsilon_1}$$

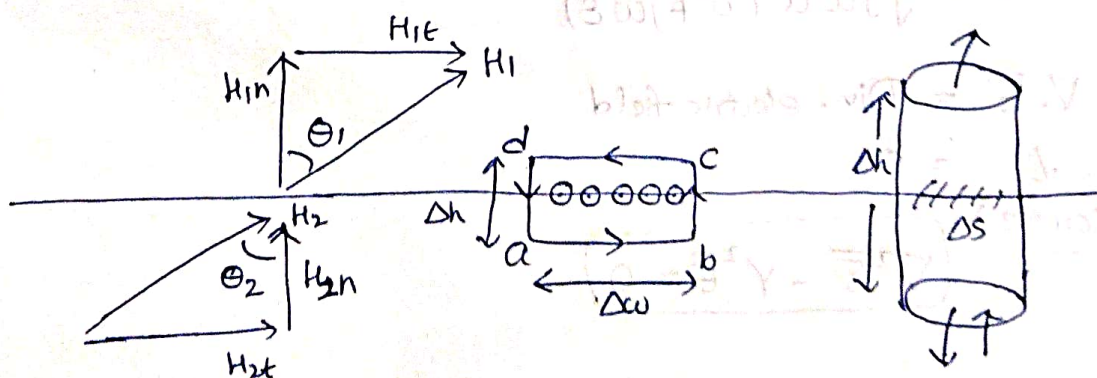
Magnetic field boundary conditions:

$$\oint \vec{E} \cdot d\vec{l} = 0$$

$$\oint \vec{D} \cdot d\vec{s} = \rho_s$$

$$\oint \vec{H} \cdot d\vec{l} = I_{enc}$$

$$\oint \vec{B} \cdot d\vec{s} = 0$$



$$\oint \vec{H} \cdot d\vec{l} = I_{enc} = J \cdot A = J \cdot \Delta h \cdot \Delta w$$

$$(H_{1t} - H_{2t}) \Delta w = J \cdot \Delta h \Delta w$$

$$\downarrow$$

$$H_{1t} = H_{2t}$$

$$B_{1n} = B_{2n}$$

Source is not inside the line  
So  $J = 0$