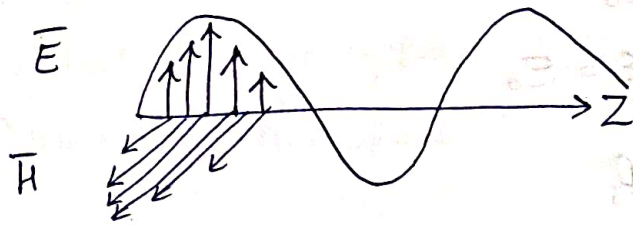


UNIT-2

Plane-Wave equation :-



Time-varying:-

$$\textcircled{1} \nabla \times \bar{E} = - \frac{\partial \bar{B}}{\partial t}$$

$$\textcircled{2} \nabla \times \bar{H} = \bar{J} + \bar{J}_d = \sigma \bar{E} + \frac{\partial \bar{D}}{\partial t}$$

$$\bar{B} = \mu \bar{H} = \mu \bar{H}_0 e^{j\omega t}$$

$$\bar{D} = \epsilon \bar{E} = \epsilon \bar{E}_0 e^{j\omega t}$$

$$\begin{aligned} \nabla \times \bar{E} &= -\mu j\omega \bar{H} \\ \nabla \times \bar{H} &= \sigma \bar{E} + j\omega \epsilon \bar{E} \end{aligned}$$

$$\nabla \times (\nabla \times \bar{E}) = -j\omega \mu (\nabla \times \bar{H})$$

$$\nabla \times (\nabla \times \bar{E}) = -j\omega \mu (\sigma + j\omega \epsilon) \bar{E}$$

$$\nabla \times (\nabla \times \bar{A}) = \nabla(\nabla \cdot \bar{A}) - \nabla^2 \bar{A}$$

$$\nabla(\nabla \cdot \bar{E}) - \nabla^2 \bar{E} = -\nabla^2 \bar{E}$$

$$\nabla^2 \bar{E} = \gamma^2 \bar{E}$$

$$\nabla \cdot \bar{E} = \text{Div. electric field}$$

$$\downarrow = 0$$

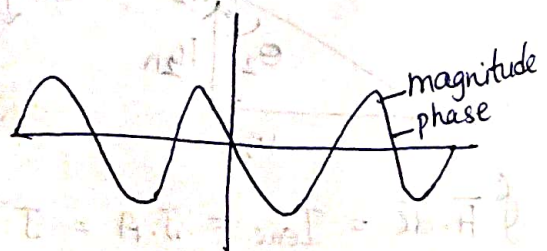
Source

$$\nabla^2 \bar{E} - \gamma^2 \bar{E} = 0$$

$$(D^2 - \gamma^2) \bar{E} = 0$$

$$D = \pm \gamma$$

$$\bar{E} = \frac{E_0^+ e^{-\gamma z}}{\downarrow \text{+ve}(z)} + \frac{E_0^- e^{+\gamma z}}{\downarrow \text{-ve}(z)}$$



$$E(z) = E_0^+ e^{-\gamma z}$$

- one direction
- vary w.r.t $\odot$

$$E(z, t) = E_0^+ e^{-\gamma z} e^{j\omega t}$$

$$\gamma = \alpha + j\beta$$

$$E(z, t) = E_0^+ e^{-\alpha z} e^{j\omega t}$$

$$E(z, t) = E_0^+ e^{-\alpha z} \cos(\omega t - \beta z) A_E$$

$E_0^+ \rightarrow$ magnitude

$\alpha \rightarrow$ attenuation constant

$z \rightarrow$ direction of propagation

$\omega \rightarrow$ frequency

$t \rightarrow$ time

$\beta \rightarrow$ phase constant

$A_E \rightarrow$ direction of $\vec{E}$ field

$$A_k = A_E \times A_H$$

direction of propagation

$$\vec{H} = H_0^+ e^{-\alpha z} \cos(\omega t - \beta z) A_H$$

$$t = 0, \pi/4, \pi/2, 3\pi/4, \pi$$

$$z = 0, \lambda/4, \lambda/2, 3\lambda/4, \lambda$$

Case (i): $t = 0$,

$$z = \underline{\hspace{2cm}}$$

Case (ii): $t = \pi/4$

$$z = \underline{\hspace{2cm}}$$

$\alpha =$ attenuation constant

$\beta =$ phase constant

$$\underline{\alpha, \beta, V_p, \eta, \theta_n}$$

$$\gamma = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$$

$$\alpha^2 - \beta^2 + 2j\alpha\beta = j\omega\mu\sigma - \omega^2\mu\epsilon$$

$$\boxed{\alpha^2 - \beta^2 = -\omega^2\mu\epsilon} \rightarrow \textcircled{1}$$

$$\sqrt{\alpha^2 + \beta^2} = \left(\sqrt{\omega^4\mu^2\epsilon^2 + \omega^2\mu^2\sigma^2} \right)^{1/2}$$

$$\alpha^2 + \beta^2 = \left(\sqrt{\omega^4\mu^2\epsilon^2 + \omega^2\mu^2\sigma^2} \right)$$

$$\alpha^2 + \beta^2 = \omega^2\mu\epsilon \sqrt{1 + \frac{\omega^2\mu^2\sigma^2}{\omega^4\mu^2\epsilon^2}}$$

$$\boxed{\alpha^2 + \beta^2 = \omega^2\mu\epsilon \sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2}} \rightarrow \textcircled{2}$$

$$\frac{\sigma}{\omega\epsilon} \rightarrow \text{loss tangent}$$

$$\textcircled{1} + \textcircled{2} \Rightarrow 2\alpha^2 = \omega^2\mu\epsilon \left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1 \right]$$

$$\alpha = \omega \left[\left(\frac{\mu\epsilon}{2} \right) \left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1 \right] \right]^{1/2}$$

$$\textcircled{2} - \textcircled{1} \Rightarrow \beta = \omega \left[\left(\frac{\mu\epsilon}{2} \right) \left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1 \right] \right]^{1/2}$$

Phase velocity: $\left(\frac{dz}{dt} \right)$

$$\frac{d}{dt}(\omega t - \beta z) = \omega - \beta \frac{dz}{dt}$$

$$\boxed{V_p = \frac{dz}{dt} = \frac{\omega}{\beta}}$$

$$\eta \text{ (Intrinsic impedance)} := E/H$$

Cause for EF and MF relation

$$V = IR, \quad R = \frac{V}{I} = \frac{\oint \vec{E} \cdot d\vec{l}}{\oint \vec{H} \cdot d\vec{l}} = \eta$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{E} = -j\omega \mu \vec{H}$$

$$= \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x(z) & E_y(z) & E_z(z) \end{vmatrix}$$

$\neq 0 \quad = 0 \quad = 0$

$$= + a_y \left(\frac{\partial E_x(z)}{\partial x} \right)$$

$$E(z) = E_0 e^{-\gamma z}$$

$$= -\gamma \cdot E(z) a_y = \nabla \times \vec{E}$$

$$-\gamma \vec{E} = -j\omega \mu \vec{H}$$

$$\frac{\vec{E}}{\vec{H}} = \eta = \frac{j\omega \mu}{\gamma} = \frac{j\omega \mu}{\sqrt{j\omega \mu (\sigma + j\omega \epsilon)}}$$

$$\eta = \sqrt{\frac{(\mu/\epsilon)}{(1 - j \cdot \frac{\sigma}{j\omega})}} = \sqrt{\frac{j\omega \mu}{\sigma + j\omega \epsilon}}$$

$[j\omega \epsilon]$

$$\underline{\theta_n} := \frac{\angle \theta_1}{\angle \theta_2} = \frac{a_1 + jb_1}{a_2 + jb_2}$$

$$\theta_n = \frac{1}{2} \left[\tan^{-1} \left(\frac{\theta}{(\mu/\epsilon)} \right) - \tan^{-1} \left(\frac{-\sigma/\omega \epsilon}{1} \right) \right]$$

$$\theta_n = \frac{1}{2} \left[0 - (-) \tan^{-1} \left(\frac{\sigma}{\omega \epsilon} \right) \right]$$

$$\theta_n = \frac{1}{2} \tan^{-1} \left(\frac{\sigma}{\omega \epsilon} \right)$$

$$\tan(2\theta_n) = \frac{\sigma}{\omega \epsilon}$$

remaining all
are zero

$$Z = \vec{E} \times \vec{y}$$

electric
field

Relationship between α and β :- (complex permittivity)

$$\frac{\alpha}{\beta} = \frac{\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1}{\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1} = \frac{\sqrt{1 + \tan^2(2\theta_n)} - 1}{\sqrt{1 + \tan^2(2\theta_n)} + 1}$$

$$\boxed{\frac{\alpha}{\beta} = \tan(\theta_n)} = \frac{\sin \theta_n}{\cos \theta_n}$$

- | | | |
|---|---|------------------------------------|
| ① Lossy
② Lossy less
③ Good conductor
④ free space | } | σ, ϵ, μ
mediums |
|---|---|------------------------------------|

① Lossy dielectric $\sigma \approx \omega\epsilon$

Conductivity (σ) $\neq 0$
 $\neq \infty$

$\epsilon = \epsilon_r \epsilon_0$
 $\neq 1$

$\mu = \mu_r \mu_0$
 $\neq 1$

② Loss less $\left(\frac{\sigma}{\omega\epsilon} \ll 1\right)$ (or) $(\sigma \ll \omega\epsilon)$

$\sigma = 0$

$\epsilon = \epsilon_r \epsilon_0$

$\mu = \mu_r \mu_0$

③ Good conductor $\left(\frac{\sigma}{\omega\epsilon} \gg 1\right)$ (or) $(\sigma \gg \omega\epsilon)$

$\sigma \approx \infty$

(very high)

$\epsilon = j\epsilon_0$
 $\epsilon_r = 1$

$\mu = \mu_r \mu_0$

④ Free space

$\sigma = 0$

$\epsilon = \epsilon_0$

$\mu = \mu_0$

Complex permittivity :

$\nabla \times \vec{H} = \vec{J} + \vec{J}_d$

$= j\omega \left(\epsilon \left(1 - j \frac{\sigma}{\omega\epsilon} \right) \right) \vec{E}$

$= \sigma \vec{E} + j\omega\epsilon \vec{E}$

$= j\omega\epsilon \vec{E}$

$= j\omega\epsilon \vec{E} \left(1 + \frac{\sigma}{j\omega\epsilon} \right)$

$= j\omega\epsilon \vec{E} \left(1 - j \left(\frac{\sigma}{\omega\epsilon} \right) \right)$

$\epsilon = \epsilon' - j\epsilon'' = \epsilon - j \frac{\sigma}{\omega}$

$\epsilon' = \epsilon, \quad \epsilon'' = \frac{\sigma}{\omega}$

$\frac{\epsilon''}{\epsilon'} = \text{loss tangent}$

Lossy dielectric	Lossy less dielectric	Good conductor	Free space
$\sigma \neq 0, \mu = \mu_r \mu_0$ $\epsilon = \epsilon_r \epsilon_0$	$\sigma = 0, \mu = \mu_r \mu_0$ $\epsilon = \epsilon_r \epsilon_0$	$\sigma \approx \infty, \mu = \mu_r \mu_0$ $\epsilon = \epsilon_0$	$\sigma = 0, \mu = \mu_0 = 4\pi \times 10^{-7}$ $\epsilon = \epsilon_0 = \frac{10^{-9}}{36\pi} \text{ F/m}$
$\alpha = \frac{\omega}{2} \sqrt{\frac{\sigma}{\epsilon}} \sqrt{1 + \left(\frac{\sigma}{\omega \epsilon}\right)^2}$ $\beta = \frac{\omega}{2} \sqrt{\frac{\sigma}{\epsilon}} \sqrt{1 + \left(\frac{\sigma}{\omega \epsilon}\right)^2}$ $\eta = \frac{\omega \mu}{2} \sqrt{\frac{\epsilon}{\sigma}} \sqrt{1 + \left(\frac{\sigma}{\omega \epsilon}\right)^2}$ $V_p = \frac{\omega}{\beta}$ $\theta_n = \tan^{-1} \left(\frac{\sigma}{\omega \epsilon} \right)$	$0 = \frac{\omega}{2} \sqrt{\frac{\sigma}{\epsilon}} \sqrt{1 + \left(\frac{\sigma}{\omega \epsilon}\right)^2}$ $\alpha = \frac{\omega}{2} \sqrt{\frac{\sigma}{\epsilon}} \sqrt{1 + \left(\frac{\sigma}{\omega \epsilon}\right)^2}$ $\beta = \frac{\omega}{2} \sqrt{\frac{\sigma}{\epsilon}} \sqrt{1 + \left(\frac{\sigma}{\omega \epsilon}\right)^2}$ $\eta = \frac{\omega \mu}{2} \sqrt{\frac{\epsilon}{\sigma}} \sqrt{1 + \left(\frac{\sigma}{\omega \epsilon}\right)^2}$ $V_p = \frac{\omega}{\beta}$ $\theta_n = \tan^{-1} \left(\frac{\sigma}{\omega \epsilon} \right)$	$\alpha = 0$ $\beta = \frac{\omega}{2} \sqrt{\frac{\sigma}{\epsilon}} \sqrt{1 + \left(\frac{\sigma}{\omega \epsilon}\right)^2}$ $\eta = \frac{\omega \mu}{2} \sqrt{\frac{\epsilon}{\sigma}} \sqrt{1 + \left(\frac{\sigma}{\omega \epsilon}\right)^2}$ $V_p = \frac{\omega}{\beta}$ $\theta_n = \tan^{-1} \left(\frac{\sigma}{\omega \epsilon} \right)$	$\alpha = 0$ $\beta = \frac{\omega}{2} \sqrt{\frac{\sigma}{\epsilon}} \sqrt{1 + \left(\frac{\sigma}{\omega \epsilon}\right)^2}$ $\eta = \frac{\omega \mu}{2} \sqrt{\frac{\epsilon}{\sigma}} \sqrt{1 + \left(\frac{\sigma}{\omega \epsilon}\right)^2}$ $V_p = \frac{\omega}{\beta}$ $\theta_n = \tan^{-1} \left(\frac{\sigma}{\omega \epsilon} \right)$

P) A 50 MHz plane wave travelling in a lossless material with permeability $(\mu) = 2.25$, $\epsilon = 1.25$ then find $\alpha, \beta, \eta, V_p, \theta_n$.

Sol $\alpha = 0$

$$\beta = \omega \sqrt{\mu \epsilon} = 50 \times 10^6 \sqrt{2.25 \times 1.25} = 83852549.1$$

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{2.25}{1.25}} = 1.3416$$

$$V_p = \frac{1}{\sqrt{\mu \epsilon}} = \frac{1}{\sqrt{2.25 \times 1.25}} = 0.7453$$

$$\tan(2\theta_n) = 0 \Rightarrow 2\theta_n = 0 \Rightarrow \theta_n = 0.$$

P) A lossy non magnetic material with $\sigma = 10^{-4}$ mho/m and $\epsilon_r = 1.35$, $\mu_r = 1.65$ operating at a frequency $\omega = 2.5$ MHz then find loss tangent of that material α, β, η , and V_p .

Sol Loss tangent $= \frac{\sigma}{\omega \epsilon} = \frac{10^{-4}}{2.5 \times 10^6 \times 1.9366 \times 10^{-11}}$

$$= \frac{10^{-4} \times 10^5}{2.5 \times 1.9366} = 3.35103.$$

Poynting Theorem:

$$\nabla(\bar{E} \times \bar{H}) = -\frac{\partial}{\partial t} (E' \text{ field})$$

$$= -\frac{\partial}{\partial t} (M \text{ field})$$

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

$$\nabla \times \bar{H} = \sigma \bar{E} + \frac{\partial \bar{B}}{\partial t}$$

$$\nabla \cdot (\bar{A} \times \bar{B}) = \bar{B}(\nabla \times \bar{A}) - \bar{A}(\nabla \times \bar{B})$$

$$\nabla \cdot (\bar{E} \times \bar{H}) = \bar{H}(\nabla \times \bar{E}) - \bar{E}(\nabla \times \bar{H}) \rightarrow \textcircled{3}$$

$$\int_V \nabla \cdot (\bar{E} \times \bar{H}) = -\frac{\partial}{\partial t} (\text{Energy decaying of electric field})$$

$$= -\frac{\partial}{\partial t} (\text{Energy decaying of magnetic field})$$

$$\bar{H}(\nabla \times \bar{E}) :$$

$$\nabla \times \bar{E} = -\mu \frac{\partial \bar{H}}{\partial t}$$

$$\bar{H}(\nabla \times \bar{E}) = -\mu \left(\bar{H} \frac{\partial \bar{H}}{\partial t} \right)$$

$$\frac{\partial}{\partial t} (\bar{A} \cdot \bar{B}) = \bar{A} \frac{\partial \bar{B}}{\partial t} + \bar{B} \frac{\partial \bar{A}}{\partial t}$$

$$\frac{\partial}{\partial t} (\bar{A} \cdot \bar{A}) = \bar{A} \cdot \frac{\partial \bar{A}}{\partial t} + \bar{A} \frac{\partial \bar{A}}{\partial t}$$

$$\frac{\partial}{\partial t} (\bar{A}^2) = 2 \left(\bar{A} \frac{\partial \bar{A}}{\partial t} \right)$$

$$\bar{A} \cdot \frac{\partial \bar{A}}{\partial t} = \frac{1}{2} \frac{\partial}{\partial t} \bar{A}^2$$

$$\bar{H} \cdot \frac{\partial \bar{H}}{\partial t} = \frac{1}{2} \frac{\partial}{\partial t} \bar{H}^2$$

$$W_c = \frac{1}{2} C V^2 = \frac{1}{2} \frac{\epsilon A}{d} \oint \bar{E} dL$$

$$= \frac{1}{2} \frac{\epsilon A}{d} \bar{E}^2 d^2$$

$$= \frac{1}{2} \epsilon \bar{E}^2 (\text{volume})$$

$$= \frac{1}{2} \mu \bar{H}^2$$

$$\bar{H}(\nabla \times \bar{E}) = -\frac{\partial}{\partial t} \frac{1}{2} \mu \bar{H}^2 \rightarrow \textcircled{1}$$

$$\nabla \times \bar{H} = \sigma \bar{E} + \epsilon \frac{\partial \bar{E}}{\partial t}$$

$$\bar{E} \cdot (\nabla \times \bar{H}) = \sigma \bar{E}^2 + \frac{\partial}{\partial t} \frac{1}{2} \epsilon \bar{E}^2 \rightarrow \textcircled{2}$$

eq ① and ② in ③

$$\nabla \cdot (\bar{E} \times \bar{H}) = -\frac{\partial}{\partial t} \left(\frac{1}{2} \mu \bar{H}^2 \right) - \sigma \bar{E}^2 - \frac{\partial}{\partial t} \left(\frac{1}{2} \epsilon \bar{E}^2 \right)$$

$$\int \nabla \cdot (\bar{E} \times \bar{H}) dV = -\frac{\partial}{\partial t} \int \frac{1}{2} \mu \bar{H}^2 dV - \int \sigma \bar{E}^2 dV \rightarrow \text{conductivity}$$

$$= -\frac{\partial}{\partial t} \int \frac{1}{2} \epsilon \bar{E}^2 dV$$

Ohmic loss

Poynting vector :-

$$\vec{P} = \vec{E} \times \vec{H}$$

$$A_k = A_E \times A_H \begin{matrix} \downarrow & \downarrow & \searrow \text{in y-axis} \\ z & \text{in x-axis} & H_y \neq 0 \\ E_x \neq 0 & & \end{matrix}$$

$$\begin{vmatrix} a_x & a_y & a_z \\ E_x & E_y & E_z \\ H_x & H_y & H_z \end{vmatrix}$$

$$\Rightarrow \vec{P} = (E_x H_y) a_z$$

$$E_x = E_0 e^{-\alpha z} \cos(\omega t - \beta z) a_x \rightarrow \textcircled{1}$$

$$H_y = H_0 e^{-\alpha z} \cos(\omega t - \beta z) a_y$$

direction of propagation

$$V = IR \Rightarrow R = \frac{V}{I}$$

$$\eta = \frac{E_0}{H_0} \Rightarrow H_0 = \frac{E_0}{\eta}$$

$$\eta = |n| \sqrt{\frac{\mu}{\epsilon}} = \frac{\sqrt{\mu/\epsilon}}{\sqrt{1-j(\frac{\sigma}{\omega\epsilon})}}$$

$$H_y = \frac{E_0}{|n|} e^{-\alpha z} \cos(\omega t - \beta z - \theta_n) a_y \rightarrow \textcircled{2}$$

$$= \frac{E_0}{|n|} e^{-\alpha z} e^{-j\beta z} e^{j\omega t} e^{-j\theta_n}$$

$$\vec{P} = [E_x H_y] a_z$$

$$= \frac{E_0^2}{2|n|} e^{-2\alpha z} [\cos(2\omega t - 2\beta z - \theta_n) + \cos \theta_n] a_z$$

$$P_{avg} = \frac{1}{T} \int_0^T \vec{P} dt$$

$$P_{avg} = \frac{E_0^2}{2|n|} e^{-2\alpha z} \cos \theta_n a_z$$

Polarization :- $\Rightarrow$ path or tip of electric field

Electric field direction

- linear
- circular
- elliptical

Polarization:

$$E = E_{0x} \cos(\omega t - \beta z) e^{-\alpha z} a_x + E_{0y} \cos(\omega t - \beta z) e^{-\alpha z} a_y e^{j\delta}$$

Linear:- $\delta = (2n)\pi$ $\left| \begin{array}{l} E_{0x} = E_{0y} = E_0 \\ \delta = (2n+1)\pi \end{array} \right.$

Circular:- $\delta = (2n+1)\pi/2$ $\left| \begin{array}{l} E_{0x} = E_{0y} = E_0 \end{array} \right.$

Elliptical:- $\delta = (2n+1)\pi/2$ $\left| \begin{array}{l} E_{0x} \neq E_{0y} \end{array} \right.$

$n = 0, 1, \dots, \infty$

$n = 1, \delta = 3\pi/2$

$$e^{j\frac{3\pi}{2}} = \cos\left(\frac{3\pi}{2}\right) + j\sin\left(\frac{3\pi}{2}\right)$$

$$= 0 - j = -j$$

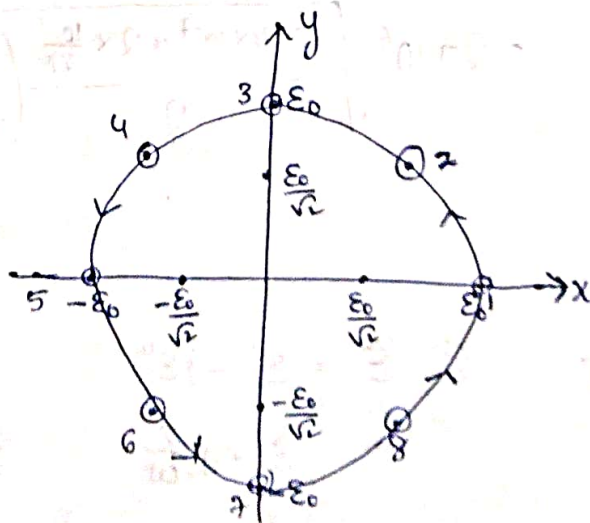
$$E = E_{0x} \cos(\omega t - \beta z) e^{-\alpha z} a_x$$

$$+ E_{0x} \cos\left(\frac{3\pi}{2} + \omega t - \beta z\right) e^{-\alpha z} a_y$$

$$= E_0 \cos(\omega t - \beta z) e^{-\alpha z} a_x + E_0 \sin(\omega t - \beta z) e^{-\alpha z} a_y$$

$z=0$ $\vec{E} = \frac{E_0 \cos(\omega t)}{E_y} a_x \rightarrow E_x$
 $+ \frac{E_0 \sin(\omega t)}{E_y} a_y$

ωt	E_x	E_y
0°	E_0	0
45°	$\frac{E_0}{\sqrt{2}}$	$\frac{E_0}{\sqrt{2}}$
90°	0	E_0
135°	$-\frac{E_0}{\sqrt{2}}$	$\frac{E_0}{\sqrt{2}}$
180°	$-E_0$	0
225°	$-\frac{E_0}{\sqrt{2}}$	$-\frac{E_0}{\sqrt{2}}$
270°	0	$-E_0$
315°	$\frac{E_0}{\sqrt{2}}$	$-\frac{E_0}{\sqrt{2}}$
360°	E_0	0



P) An electromagnetic wave is travelling in a free space with electric field $\vec{E} = \frac{E_0}{25} e^{-\alpha z} \cos(\omega t - \beta z) \hat{a}_z$ V/m then find

- a) α b) β c) Direction of propagation d) $\vec{H}$ e) η

Soln a) for free space $\alpha = 0$

b) $\beta = \omega \sqrt{\mu_0 \epsilon_0} = 6$

c) Direction of propagation = y-axis

d) $\vec{H} = H_0 e^{-\alpha y} \cos(\omega t - \beta y) \hat{a}_y$ A/m

$\hat{a}_k = \hat{a}_E \times \hat{a}_H$

$y = 2 \times \boxed{x}$

① H_0 ③ $\hat{a}_k = \hat{a}_y$ ⑤ $\beta = 6$

② $\alpha = 0$ ④ $\omega = 2\pi \times 10^6$ $\eta = \frac{E_0}{H_0} \Rightarrow H_0 = \frac{E_0}{\eta}$

① A lossy material has $\mu = 5\mu_0$, $\epsilon = 2\epsilon_0$, $\beta = 10$ rad/m, $f = 1$ MHz then find a) conductivity (σ) b) $\frac{\sigma}{\omega\epsilon}$

c) complex permittivity d) α, β, η

$\omega = 2\pi f$
 $= 2\pi \times 10^6$

Soln a) $\beta = \omega \sqrt{\frac{\mu\epsilon}{2} \left(\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon} \right)^2} + 1 \right)}$ ($\sigma = 5.09 \times 10^{20}$)

$= 2\pi \times 10^6 \sqrt{\frac{(5\mu_0)(2\epsilon_0)}{2} \left(\sqrt{1 + \left(\frac{\sigma^2}{(2\pi \times 10^6)^2 (2\epsilon_0)^2} \right)} + 1 \right)}$

$= 2\pi \times 10^6 \sqrt{\frac{5 \times 4 \times 10^{-7} \times 2 \times \frac{10^{-9}}{36\pi}}{2} \left(\sqrt{1 + \frac{\sigma^2}{(2\pi \times 10^6)^2 (2 \times \frac{10^{-9}}{36\pi})^2}} + 1 \right)}$

b) $\frac{\sigma}{\omega\epsilon}$

c) $\epsilon = \epsilon' - j\epsilon''$

$= \epsilon - j \frac{\sigma}{\omega}$

p) A particular medium intrinsic impedance $\eta = 240$ at an angle of 30° and $\mu_r = 1$ then find @ loss tangent $f = 1 \text{ MHz}$

⑥ Complex permittivity ⑦ α at 1 MHz ⑧ β at 1 MHz

Sol: $\eta = \frac{\sqrt{\mu/\epsilon}}{\sqrt{1 - j\left(\frac{\sigma}{\omega\epsilon}\right)}} = |\eta| \angle \theta_n$

⑥ $\epsilon = \epsilon - j \frac{\sigma}{\omega}$

$|\eta| = \frac{\sqrt{\mu/\epsilon}}{\left(\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2}\right)^{1/2}} = 240$

$\theta_n = \frac{1}{2} \tan^{-1}\left(\frac{\sigma}{\omega\epsilon}\right) = 30^\circ$

$\theta_n = \frac{\sigma}{\omega\epsilon} = \tan(60^\circ) = \sqrt{3}$

$|\eta| = \frac{\sqrt{\frac{1 \times 4\pi \times 10^{-7}}{\epsilon}}}{\left(\sqrt{1 + (\sqrt{3})^2}\right)^{1/2}} = 240$

$= \frac{\sqrt{\frac{1 \times 4\pi \times 10^{-7}}{\epsilon}}}{\sqrt{2}} = 240$

$|\eta| = \sqrt{\frac{1 \times 4\pi \times 10^{-7}}{\epsilon}} = \sqrt{2} \times 240$

$\Rightarrow \sqrt{\epsilon} = \frac{\sqrt{1 \times 4\pi \times 10^{-7}}}{\sqrt{2} \times 240}$

$\epsilon = 1.09 \times 10^{-11} ; \sigma = 1.186 \times 10^{-4}$

p) Consider a medium $\sigma = 6 \text{ S/m}$ and $\mu_r = 6$ and $\epsilon_r = 40$, operating frequency $= 10 \text{ MHz}$ then find

$\alpha, \beta, V_p, \eta, \sigma/\omega\epsilon$

$\epsilon = \epsilon_r \epsilon_0$
 $= 40 \times \frac{10^{-9}}{36\pi}$

Sol: $\frac{\sigma}{\omega\epsilon} = \frac{6}{2\pi \times 10^6 \times 3.5367 \times 10^{-10}}$

$= 3.5367 \times 10^{-10}$

$= 2700.058412$

$V_p = \frac{\omega}{\beta}$

$$\alpha = \omega \sqrt{\frac{\mu \epsilon}{2} \left(\sqrt{1 + \left(\frac{\sigma}{\omega \epsilon} \right)^2} - 1 \right)}$$

$$\mu = \mu_r \mu_0$$

$$= 6 \times 4\pi \times 10^{-7}$$

$$= 7.5398 \times 10^{-6}$$

$$\frac{\mu \epsilon}{2} = \frac{7.5398 \times 10^{-6} \times 3.5367 \times 10^{-10}}{2}$$

P) In a certain medium with $\mu = \mu_0$ and $\epsilon = 4\epsilon_0$ and $\bar{H} = 12e^{-0.1y} \sin(\pi 10^8 t - \beta y) \hat{a}_x$ A/m then find electric field, phase difference b/w the EF and MF and also β ?

$$\bar{H} = 12e^{-0.1y} \sin(\pi 10^8 t - \beta y) \hat{a}_x$$

$$\bar{H} = H_0 e^{-\alpha z} \cos(\omega t - \beta z) \hat{a}_H$$

$$\textcircled{1} \bar{E} = E_0 e^{-\alpha y} \sin(\omega t - \beta y) \hat{a}_H \text{ V/m}$$

$$\eta = \frac{E_0}{H_0} \Rightarrow E_0 = \eta H_0$$

$$\eta = \frac{\sqrt{\mu/\epsilon}}{\sqrt{1 - j\left(\frac{\sigma}{\omega \epsilon}\right)}}$$

$$a_k = a_E \times a_H$$

$$y = \textcircled{a_z} \times \hat{a}_x$$

$$\alpha = \omega \sqrt{\frac{\mu \epsilon}{2} \left(\sqrt{1 + \left(\frac{\sigma}{\omega \epsilon} \right)^2} - 1 \right)}$$

$$\begin{matrix} x & y & z & x \\ | & | & | & | \\ z & x & y & \end{matrix}$$

$$\beta = \omega \sqrt{\frac{\mu \epsilon}{2} \left(\sqrt{1 + \left(\frac{\sigma}{\omega \epsilon} \right)^2} + 1 \right)}$$

P) If electric field was represented by

$$\bar{E} = 10 \cos(2\pi 10^8 t - \beta z) \hat{a}_x \text{ V/m then find}$$

Ⓐ β Ⓑ λ Ⓒ $\bar{H}$

$$\alpha = 0 \rightarrow \sigma = 0 \rightarrow \begin{cases} \text{free space} \rightarrow \mu_r = \epsilon_r = 1 \\ \text{loss less} \rightarrow \mu_r = \mu_0 \mu_r \\ \epsilon_r = \epsilon_0 \epsilon_r \end{cases}$$

$$\vec{E} = E_0 e^{-\alpha z} \cos(\omega t - \beta z) \hat{a}_E \text{ V/m}$$

$$\left\{ \begin{array}{l} d, \beta, V_p, \\ \eta, \frac{\sigma}{\omega \epsilon}, \lambda, \\ \theta_n, \vec{H}, \vec{E} \end{array} \right.$$

$$\beta = \omega \sqrt{\mu_0 \epsilon_0} = 2\pi 10^6 \sqrt{4\pi \times 10^{-7} \times \frac{10^{-9}}{36\pi}} \\ = \omega/c = \frac{1}{150} \pi = 0.02$$

$$\beta = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{\beta} = 100\pi = 314.1592654$$

$$\vec{H} = H_0 e^{-\alpha z} \cos(\omega t - \beta z) \hat{a}_H \text{ A/m}$$

$$a_k = a_E \times a_H$$

$$\alpha = 0, \omega = 2\pi 10^6, \beta = 0.02 \quad \eta = \sqrt{\frac{\mu}{\epsilon}}$$

$$\eta = \frac{E_0}{H_0} \Rightarrow H_0 = \frac{E_0}{\eta}$$

$$H_0 = \frac{10}{120\pi} = 0.0265 \quad \eta = 120\pi = 376.99 = 377 \Omega$$

$$\vec{H} = 0.0265 \cos(2\pi 10^6 t - 0.02 z) \hat{a}_y$$

2) The MF component of an EM wave is propagating through a non-magnetic medium $\mu = \mu_0$, $\vec{H} = 25 \sin(2\pi 10^8 t + 6x) \hat{a}_y \text{ A/m}$ then calculate (a) ϵ (b) $\vec{E}$ and (c) direction of propagation.

$$\theta_n = 45^\circ \quad \alpha = 0 \rightarrow \sigma = 0 \rightarrow \begin{cases} \text{loss less} (\epsilon = \epsilon_0 \epsilon_r, \mu = \mu_0 \mu_r) \\ \text{free space} (\epsilon = \epsilon_0, \mu = \mu_0) \end{cases}$$

$$\beta = \omega \sqrt{\mu \epsilon} \\ \downarrow \quad \downarrow \quad \downarrow \\ -6 \quad 2\pi f \quad \mu_0$$

$$-\beta = 2\pi 10^8 \sqrt{4\pi \times 10^{-7} \times \epsilon}$$

$$\beta = \text{magnitude} +ve$$

$$\left(\frac{-6}{2\pi 10^8}\right)^2 = 4\pi \times 10^{-7} \times \epsilon$$

$$\epsilon = \left(\frac{-6}{2\pi 10^8}\right)^2 \frac{1}{4\pi \times 10^{-7}}$$

$$\vec{E} = E_0 \sin(\omega t - \beta(-x)) \hat{a}_E \text{ V/m}$$

$$a_E$$

$$x \frac{y}{z} \frac{1}{x} = 1$$

$$\eta = E_0/H_0$$

$$\eta = \frac{\sqrt{\mu/\epsilon}}{\sqrt{1-j(\frac{\sigma}{\omega \epsilon})}} \rightarrow 0$$

$$a_k = a_E \times a_H$$

$$(-x) = a_z \times a_y$$

$$E_0 = \eta H_0$$

Q) If $\vec{H} = 10 \sin(\omega t - 4z) \hat{a}_z$ A/m and $\mu = \mu_0$, $\epsilon = 4\epsilon_0$,

$\sigma = 0$ then find. (a) ω (b) $\frac{\partial \vec{D}}{\partial t}$ (c) λ

α β η

$\vec{z} = (-y)\hat{x} + x\hat{y}$

$$\beta = \omega \sqrt{\mu \epsilon}$$

$$\downarrow$$

$$4 = \omega \sqrt{4 \times 10^{-7} \times 4 \times \frac{10^{-9}}{36\pi}}$$

$$\omega = \frac{4}{\sqrt{4 \times \frac{10^{-16}}{9}}}$$

$$= 3.6 \times 10^8$$

$$\omega = 600\,000\,000$$

$$\vec{E} = E_0 e^{-\alpha z} \sin(\omega t - \beta z) \hat{a}_E \text{ V/m}$$

$$= \sqrt{\frac{\mu}{\epsilon}} \mu_0 \sin(\omega t - 4z) (-\hat{a}_y) \text{ V/m}$$

$$\omega = 2\pi f = 2\pi \frac{c}{\lambda} = 3 \times 10^8$$

$$\left(\beta = \frac{2\pi}{\lambda}\right) \rightarrow \text{only for free space}$$

Q) Manufacturer produced a material with $\mu = 750\mu_0$ and $\epsilon = 5\epsilon_0$ and $\sigma = 10^{-6}$ S/m and $f = 10$ MHz then find type of material, α , β , η , phase difference b/w two points separated by 6m

$$\boxed{\sigma \ll \omega \epsilon}$$

loss tangent $\frac{\sigma}{\omega \epsilon} \ll 1 \Rightarrow$ loss less (ideal $\sigma = 0$)
(practical $\sigma \approx 0$)

free space

$$\mu_r \approx \epsilon_r \approx 1$$

$$\frac{\sigma}{\omega \epsilon} \gg 1 \Rightarrow \text{good conductor}$$

$$\omega = 2\pi f = 2 \times \pi \times 10 \times 10^6 = 2\pi \times 10^7 =$$

$$\epsilon = 5\epsilon_0 = 5 \times \frac{10^{-9}}{36\pi} =$$

$$\frac{\sigma}{\omega \epsilon} = \frac{10^{-6}}{2\pi \times 10^7 \times 5 \times \frac{10^{-9}}{36\pi}} = \frac{10^{-6}}{5 \times \frac{10^{-2}}{18}} = \frac{10^{-4}}{5/18} = 3.6 \times 10^{-4}$$

phase variation tell by β

phase difference = $\beta \times \text{distance}$

$$\beta = \omega \sqrt{\frac{\mu \epsilon}{2} \left(\sqrt{1 + \left(\frac{\sigma}{\omega \epsilon} \right)^2} + 1 \right)}$$

$$\beta = \omega \sqrt{\mu \epsilon}$$

$$\alpha = 0$$

$$= 2\pi \times 10^7 \times \sqrt{750 \times 4\pi \times 10^{-7} \times 5 \times \frac{10^{-9}}{36\pi}}$$

$$= 2\pi \times 10^7 \sqrt{750 \times 5 \times \frac{10^{-16}}{9}} = 12.825$$

$$\text{phase difference} = 76.952$$

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{750 \times 4\pi \times 10^{-7}}{5 \times \frac{10^{-9}}{36\pi}}} = 4617.179$$