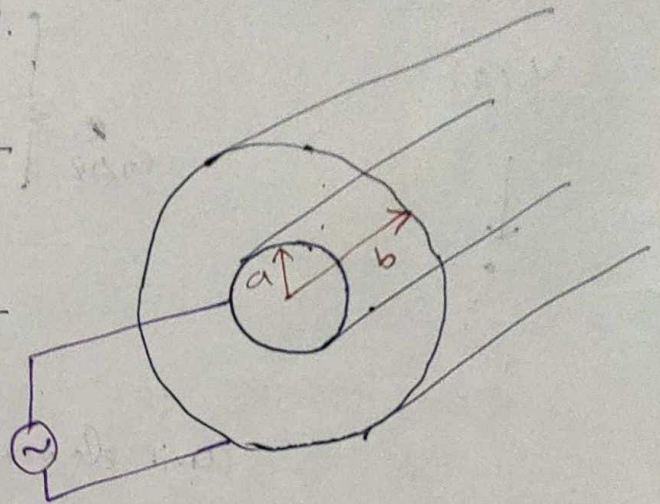
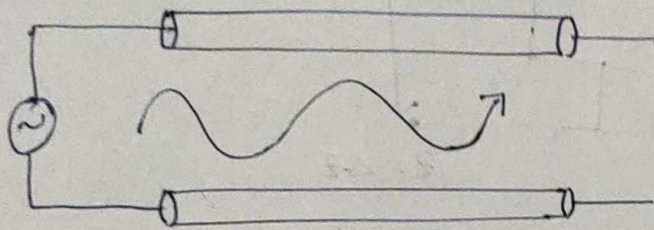
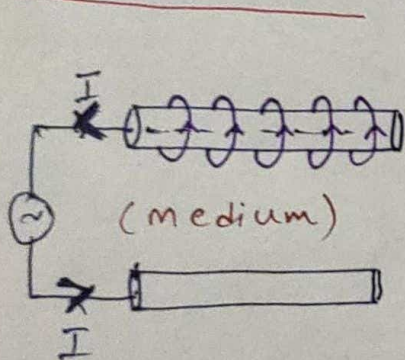


Transmission line:-

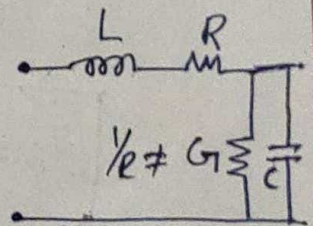
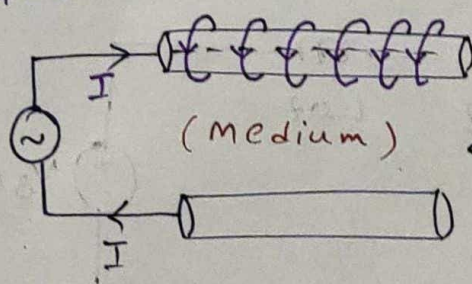
It is a two conducting system which provides medium for transmission of energy or power.



Equivalent ckt. of Txion line:-

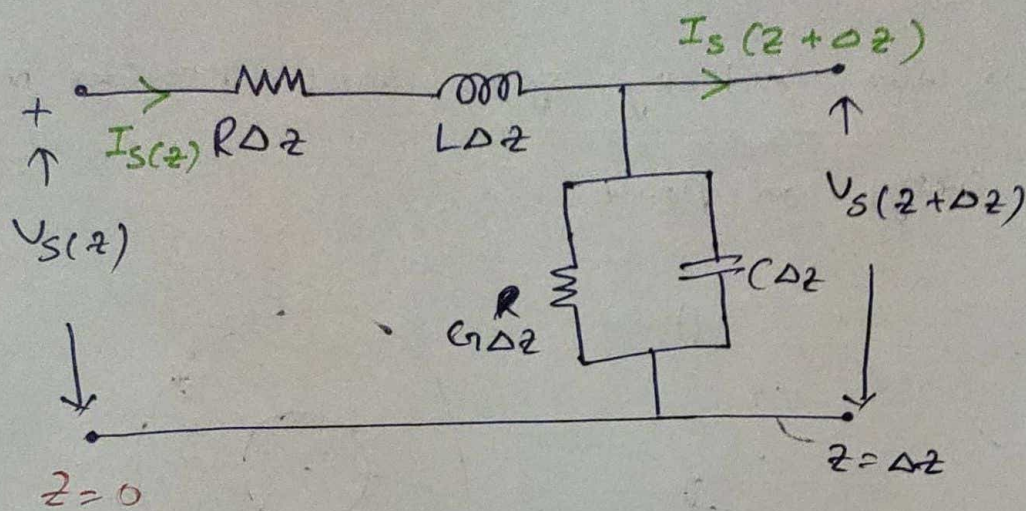


(Biot-Savart law)



Primary constant of Txion line	$R =$ Resistance per unit length (Ω/m)
	$L =$ Inductance " " " (H/m)
	$G =$ Conductance " " " (Ω^{-1}/m or S/m)
	$C =$ Capacitance " " " (F/m)

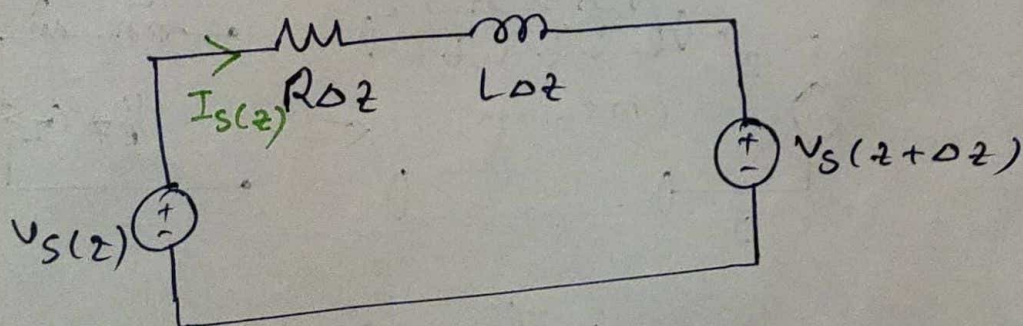
Transmission line equation



$$\begin{aligned} V_S(z) &\neq V_S(z+\Delta z) \\ I_S(z) &\neq I_S(z+\Delta z) \end{aligned}$$

$l = \Delta z$
(unit element)

KVL in the outer loop



$$V_S(z) - I_S(z) R \Delta z - j\omega L I_S(z) \Delta z - V_S(z + \Delta z) = 0$$

$$-V_S(z + \Delta z) + V_S(z) = I_S(z) \Delta z [R + j\omega L]$$

$$\frac{V_S(z + \Delta z) - V_S(z)}{\Delta z} = -I_S(z) (R + j\omega L)$$

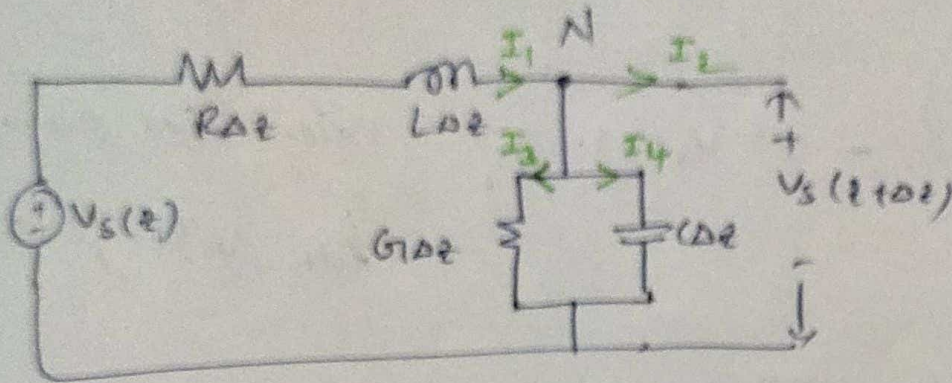
$$\lim_{\Delta z \rightarrow 0} \frac{V_S(z + \Delta z) - V_S(z)}{\Delta z} = -I_S(z) (R + j\omega L)$$

$$\frac{dV_S(z)}{dz} = -I_S(z) (R + j\omega L) \rightarrow (1)$$

(2)

$$\frac{d^2 V_S(z)}{dz^2} = - \frac{dI_S(z)}{dz} (R + j\omega L) \rightarrow (2)$$

Apply KCL at Node



$$I_1 = I_2 + I_3 + I_4$$

$$I_S(z) = I_S(z + \Delta z) + (V_S(z + \Delta z)) \cdot G\Delta z$$

$$+ V_S(z + \Delta z) j\omega C \Delta z = 0.$$

$$I_S(z + \Delta z) - I_S(z) = -V_S(z + \Delta z) \Delta z [G + j\omega C]$$

$$\lim_{\Delta z \rightarrow 0} \frac{dI_S(z)}{dz} = \lim_{\Delta z \rightarrow 0} -V_S(z + \Delta z) (G + j\omega C)$$

$$\frac{dI_S(z)}{dz} = -V_S(z) \cdot (G + j\omega C) \rightarrow (3)$$

$$\frac{d^2 I_S(z)}{dz^2} = - \frac{dV_S(z)}{dz} (G + j\omega C) \rightarrow (4)$$

$$\frac{d^2 V_S(z)}{dz^2} = - \frac{dI_S(z)}{dz} (R + j\omega L) \rightarrow (2)$$

$$\frac{d^2 V_s(z)}{dz^2} = (R+j\omega L)(G+j\omega C) V_s(z)$$

$$\frac{d^2 V_s(z)}{dz^2} - (R+j\omega L)(G+j\omega C) V_s(z) = 0$$

$$\boxed{\frac{d^2 V_s(z)}{dz^2} - \gamma^2 V_s(z) = 0}$$

Transmission line eqn

for voltage wave

In a similar way

$$\boxed{\frac{d^2 I_s(z)}{dz^2} - \gamma^2 I_s(z) = 0}$$

γ = Propagation constant

$$\gamma = \alpha + j\beta = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$(D^2 - \gamma^2) V_s = 0$$

$$V_s(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

$$(D^2 - \gamma^2) I_s = 0$$

$$D = \pm \gamma$$

$$I_s(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$$

forward
travelling

Backward
travelling

Char's Impedance:

it is defined as ratio of forward travelling voltage wave to the forward travelling current wave at any point of transmission line.

$$Z_0 = \frac{V_0^+}{I_0^+} = \frac{-V_0^-}{I_0^-} = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$$

Lossless Transmission line:

$$V_S(z) = V_0^+ e^{-\gamma z}, \quad I_S(z) = I_0^+ e^{-\gamma z}$$

$$Z_0 = \frac{V_S(z)}{I_S(z)} \quad \left\{ \begin{array}{l} V_S(z) = V_0^+ e^{-\gamma z} \\ \frac{dV_S(z)}{dz} = V_0^+ e^{-\gamma z} (-\gamma) = -\gamma V_S(z) \end{array} \right.$$

Condition $R=0, G=0$

$$\gamma = \alpha + j\beta = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$\alpha + j\beta = \sqrt{j^2 \omega^2 LC} = j\omega \sqrt{LC}$$

$$\boxed{\beta = \omega \sqrt{LC}, \quad \alpha = 0}$$

phase velocity $v_p = \omega/\beta = \frac{\omega}{\omega \sqrt{LC}} = \frac{1}{\sqrt{LC}}$

$$\boxed{v_p = \frac{1}{\sqrt{LC}}}$$

Char's Imp $Z = \sqrt{L/C} = \sqrt{\frac{R+j\omega L}{G+j\omega C}}$

$$\begin{aligned} \frac{V_S(z)}{I_S(z)} &= \frac{R+j\omega L}{\sqrt{(R+j\omega L)(G+j\omega C)}} \\ &= \sqrt{\frac{R+j\omega L}{G+j\omega C}} = Z_0 \end{aligned}$$

Distortionless Transmission Line:-

$$\text{Condition : } \frac{R}{L} = \frac{G}{C}$$

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$= \sqrt{RG \left(1 + j\omega \frac{L}{R}\right) \left(1 + j\omega \frac{C}{G}\right)}$$

$$= \sqrt{RG \left(1 + \frac{j\omega C}{G}\right)^2}$$

$$= \sqrt{RG} \left(1 + j\omega \frac{C}{G}\right) = \sqrt{RG} + j\omega \sqrt{\frac{R \cdot C^2}{G}}$$

$$\boxed{\alpha = \sqrt{RG}}$$

$$\boxed{\beta = \omega \sqrt{\frac{R \cdot C^2}{G}}}$$

$$\frac{R}{G} = \frac{L}{C} \Rightarrow \boxed{\beta = \omega \sqrt{LC}}$$

$$\text{Char's Imp } Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{R}{G} \frac{(1 + j\omega L/R)}{(1 + j\omega C/G)}}$$

$$\boxed{Z_0 = \sqrt{R/G} = \sqrt{L/C}}$$

$$\text{Phase velocity } \textcircled{V_P} = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

① Lossless Transmission line $\alpha = 0$, $\beta = \omega \sqrt{LC}$,

$$V_p = \frac{1}{\sqrt{LC}}, \quad Z_0 = \sqrt{L/C}$$

② Distortionless Transmission line

$$\alpha = \sqrt{RG}, \quad \beta = \omega \sqrt{LC}, \quad V_p = \frac{1}{\sqrt{LC}}, \quad Z_0 = \sqrt{R/G}, \text{ or } \sqrt{L/C}$$

Low Loss Transmission line:

Condition:- $R \ll j\omega L, \quad G \ll j\omega C$

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} = \sqrt{j\omega L \cdot j\omega C \left(1 + \frac{R}{j\omega L}\right) \left(1 + \frac{G}{j\omega C}\right)}$$

$$= j\omega \sqrt{LC} \left(1 - j \frac{R}{\omega L}\right)^{1/2} \left(1 - \frac{jG}{\omega C}\right)^{1/2}$$

$$= j\omega \sqrt{LC} \left(1 + j \frac{R}{2\omega L}\right) \left(1 + j \frac{G}{2\omega C}\right)$$

$$= j\omega \sqrt{LC} \left(1 + j \frac{R}{2\omega L} + j \frac{G}{2\omega C} - \frac{RG}{4\omega^2 LC}\right)$$

$$= j\omega \sqrt{LC} + \omega \sqrt{LC} \cdot \frac{R}{2\omega L} + \omega \sqrt{LC} \cdot \frac{G}{2\omega C}$$

$$\alpha + j\beta = j\omega \sqrt{LC} + \omega \sqrt{LC} \cdot \frac{R}{2\omega L} + \omega \sqrt{LC} \cdot \frac{G}{2\omega C}$$

$$\beta = \omega \sqrt{LC}$$

$$\alpha = \frac{R}{2} \sqrt{\frac{C}{L}} + \frac{G}{2} \sqrt{\frac{L}{C}}$$

$$V_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{j\omega L}{j\omega C} \left(1 + \frac{R}{j\omega L}\right) \left(1 + \frac{G}{j\omega C}\right)^{-1}}$$

$$Z_0 = \sqrt{\frac{L}{C}} \left(1 - j \frac{R}{\omega L}\right)^{1/2} \left(1 - j \left(\frac{G}{\omega C}\right)\right)^{-1/2}$$

$$Z_0 = \sqrt{\frac{L}{C}} \left(1 - j \frac{R}{2\omega L}\right) \left(1 + j \frac{G}{2\omega C}\right)$$

$$Z_0 = \sqrt{\frac{L}{C}} \left[1 - j \cdot \frac{R}{2\omega L} + j \cdot \frac{G}{2\omega C} + \frac{RG}{2\omega^2 LC}\right]$$

$$Z_0 = \sqrt{\frac{L}{C}} \left[1 - j \cdot \frac{R}{2\omega L} + j \cdot \frac{G}{2\omega C}\right]$$

To make Z_0 is real, $\text{Imag}(Z_0) = 0$.

$$\text{SO } j \cdot \frac{G}{2\omega C} = j \cdot \frac{R}{2\omega L} \Rightarrow \boxed{\frac{R}{L} = \frac{G}{C}}$$

⑤ An air line has char Imp of 70Ω and phase const of 3 rad/m at 100 MHz , calculate Inductance per meter and capacitance per meter of the line.

① Air filled b/w two conductors.

loss less :- $R = G = 0$.

$$Z_0 = 70 = \sqrt{L/C} \rightarrow (1)$$

$$\beta = 3 = \omega \sqrt{LC} \rightarrow (2)$$

$$Z_0 \cdot \beta = (70)(3) = \sqrt{\frac{L}{C}} \cdot \omega \cdot \sqrt{LC}$$

$$210 = \omega \cdot L$$

$$L = \frac{210}{\omega} = \frac{210}{2\pi \times 100 \times 10^6} = 334.2 \frac{\text{nH}}{\text{m}}$$

$$70 = \sqrt{L/C} \Rightarrow 70 = \sqrt{\frac{\text{known}}{C}}$$

$$C = 68.2 \text{ PF/m}$$

② A transmission line operating at 500 MHz, has $Z_0 = 80 \Omega$

③ $\alpha = 0.04 \text{ NP/m}$, $\beta = 1.5 \text{ rad/m}$. Find line parameters R , L , G and C .

④ From given data $Z_0 = 80 \Omega$ (Real)

and

$$\alpha = 0.04 \text{ NP/m}$$

$\alpha \neq 0$

distortion will happen in distortion less line only.

$$\alpha = 0.04 = \sqrt{RG} \rightarrow (1)$$

$$\beta = 1.5 = \omega \sqrt{LC} \rightarrow (2)$$

$$Z_0 = 80 = \sqrt{R/G} = \sqrt{L/C} \rightarrow (3)$$

$$\alpha Z_0 = R, \quad \boxed{R = 3.2 \text{ } \Omega/\text{m}}$$

$$\frac{\alpha}{Z_0} = G \Rightarrow G = \frac{0.04}{80} = \frac{4 \times 10^{-2} \times 10^{-1}}{80} = 0.5 \times 10^{-3}$$

$$\boxed{G = 5 \times 10^{-4} \text{ } \Omega/\text{m}}$$

$$\beta Z_0 = \omega L$$

$$\boxed{L = \frac{\beta Z_0}{\omega} = \frac{1.5 \times 80}{2\pi \times 500 \times 10^6} = 38.1 \text{ nH/m}}$$

$$\boxed{C = \frac{\beta}{Z_0 \omega} = \frac{1.5}{80 \times 2\pi \times 500 \times 10^6} = 5.9 \text{ pF/m}}$$

⑥

④ A distortionless line has $z_0 = 60 \Omega$, $\alpha = 20 \text{ mNP/m}$

⑤ $u = 0.6c$, where c is the speed of light in vacuum.

Find R, L, G, C and λ at 100 MHz

①

$$\alpha = \sqrt{RG} = 20 \times 10^{-3} \rightarrow (1)$$

$$v_p = 0.6 \times 3 \times 10^8 = \frac{1}{\sqrt{LC}} \rightarrow (2) \quad \text{where } v_p = \omega/\beta$$

$$z_0 = 60 = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}} \rightarrow (3)$$

$$R = \alpha z_0 = 60 \times 20 \times 10^{-3} = 1.2 \Omega/\text{m}$$

$$G = \frac{\alpha}{z_0} = \frac{20 \times 10^{-3}}{60} = 0.33 \times 10^{-3} \text{ S/m}$$

$$v_p z_0 = \frac{1}{C} \Rightarrow C = \frac{1}{3 \times 10^8 \times 0.6 \times 60} = 92.59 \text{ pF/m}$$

$$\frac{z_0}{v_p} = \sqrt{\frac{L}{C}} \times \frac{\sqrt{LC}}{1} = L$$

$$L = \frac{60}{1.8 \times 10^8} = 0.33 \mu\text{H/m}$$

④ A telephone line has $R = 30 \text{ } \Omega/\text{km}$, $L = 100 \text{ mH/km}$,

$G = 0$ and $C = 20 \text{ nF/km}$, at $f = 1 \text{ kHz}$, obtain

(a) Char's Imp of the line

(b) Propagation const

(c) Phase velocity

$\Rightarrow$ Imping will occur in low loss line.

allow: distortion less.

$$\alpha = \sqrt{RG} = 0, \quad \beta = \omega \sqrt{LC} = 2\pi \times 10^3 \sqrt{100 \times 10^{-3} \times 20 \times 10^{-9}}$$

$$\boxed{\beta = 8.85 \text{ rad/km}}$$

$$\boxed{Z_0 = \sqrt{\frac{L}{C}} = 70.7 \Omega}$$

$$\boxed{V_p = \frac{\omega}{\beta} = \frac{2\pi \times 10^3}{8.88} \text{ km/s}}$$

(7)

(5) A telephone line has the following parameters

(14) $R = 40 \Omega/m$, $G = 400 \mu S/m$, $L = 0.2 \text{ mH/m}$, $C = 0.5 \text{ nF/m}$

(a) if the line operates at 10 MHz , calculate the char'l Imp Z_0 and velocity u .

(b) After how many meters will the voltage drop by 30 dB in the line?

(A) (a) $Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{40 + j 2\pi \times 10 \times 10^6 \times 0.2 \times 10^{-6}}{400 \times 10^{-6} + j 2\pi \times 10 \times 10^6 \times 0.5 \times 10^{-9}}}$

$$Z_0 = 10.76 - j 7.86 \Omega$$

$$= 13.34 \angle -36.24^\circ$$

$$\text{velocity} = \omega / \beta = v_p$$

to find β . $\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$

$$= \sqrt{41.93 \angle 17.44^\circ \times 31.42 \times 10^{-2} \angle 89.89^\circ}$$

$$= 2.15 + j 2.925 =$$

$$\beta = 2.925 \text{ rad/m.}$$

$$\text{velocity} = \omega / \beta = \frac{2\pi \times 10 \times 10^6}{2.925} = 2.148 \times 10^7 \text{ m/s}$$

(b)

α = atten const

Verify it once

$$\alpha = 2.15 \text{ NP/m}$$

$$= 2.15 \times 8.686 \text{ dB/m}$$

$$= 18.675 \text{ dB/m}$$

$$1 \text{ NP} = 8.686 \text{ dB}$$

$$\alpha l = 30$$

$$l = \frac{30}{18.675} = 1.606 \text{ m}$$

$$V_S(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

if wave travelling in forward direction

$$V_S(z) = V_0 e^{-(\alpha + j\beta)z}$$

$$V_S(z) = V_0 e^{-\alpha z}$$

(8)

Q A distortion less line operating at 120 MHz, has

5 $R = 20 \Omega/m$, $L = 0.3 \mu H/m$ and $C = 63 \text{ pF/m}$.

(a) determine γ , $\vec{u}$ ^{velocity} γ 20

(b) How far will a voltage wave travel before it is reduced to 20% of its Initial magnitude

(c) How far will it travel to suffer a 45° phase shift.

(A) line is a distortion less line. so

$$\frac{R}{G} = \frac{L}{C} \Rightarrow G = \frac{RC}{L} = \frac{20 \times 63 \times 10^{-12}}{0.3 \times 10^{-6}}$$

$$G = 4.2 \times 10^{-3} \text{ S/m}$$

$$\alpha = \sqrt{RG} = \sqrt{20 \times 4.2 \times 10^{-3}} = 0.2898 \text{ NP/m.}$$

$$\beta = \omega \sqrt{LC} = 2\pi \times 120 \times 10^6 \sqrt{0.3 \times 10^{-6} \times 63 \times 10^{-12}}$$

$$\beta = 3.278 \text{ rad/m.}$$

$$\gamma = \alpha + j\beta = 0.2898 + j 3.278$$

velocity $V_p = \omega/\beta = \frac{2\pi \times 120 \times 10^6}{3.278} = 2.3 \times 10^8 \text{ m/sec}$

$$Z_0 = \sqrt{L/C} = \sqrt{\frac{0.3 \times 10^{-6}}{63 \times 10^{-12}}} = 69 \Omega$$

- (b) wave travel before it is reduced to 20% of its Initial magnitude.

$$V(z) = V_0 \cdot e^{-\alpha z}$$

$$0.20 V_0 = V_0 e^{-0.2898 \times z}$$

$$\ln(0.2) = -0.2898 z$$

$$z = 5.54 \text{ m.}$$

(c) phase constant = $\frac{\text{phase shift}}{\text{distance.}}$

$$\frac{45^\circ}{\beta} = \text{distance}$$

$\beta \rightarrow \text{phase const}$

$$z = \frac{45\pi}{180^\circ \times 3.278} = 0.2396 \text{ m}$$

- (Q) A lossless transmission line operating at 4.5 GHz has $L = 2.4 \text{ nH/m}$ and $Z_0 = 85 \Omega$. Calculate the phase constant β & phase velocity v_p .

(A) $\beta = \omega \sqrt{LC} = 2\pi \times 4.5 \times 10^9 \sqrt{2.4 \times 10^{-6} \times C}$

$$Z_0 = \sqrt{L/C} \quad \sqrt{C} = \frac{\sqrt{L}}{Z_0} = 2.3 \times 10^{12}$$

$$\beta = 798.33 \text{ rad/m}$$

Velocity $v_p = \omega/\beta = \frac{2\pi \times 4.5 \times 10^9}{798.33} = 3.542 \times 10^7 \text{ m/sec}$

Q A lossless transmission line uses a dielectric insulating material with $\epsilon_r = 4$ if its line capacitance is $C = 10 \text{ pF/m}$, find

(a) Phase velocity (b) line Inductance

(c) char'l Impedance

$$(a) \quad v_p = \frac{c}{\sqrt{\epsilon_r}} = \frac{3 \times 10^8}{\sqrt{4}} = 1.5 \times 10^8 \text{ m/sec}$$

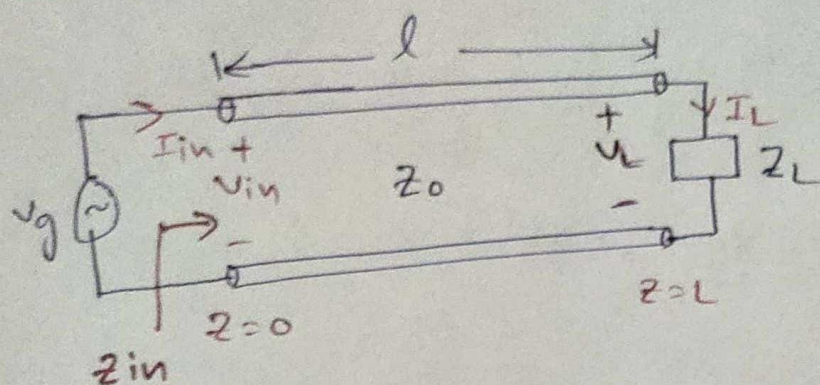
$$(b) \quad v_p = \frac{1}{\sqrt{LC}} \Rightarrow L = \frac{1}{C v_p^2} = \frac{1}{10 \times 10^{-12} \times (1.5 \times 10^8)^2}$$

$$L = 4.45 \text{ nH/m}$$

$$(c) \quad Z_0 = \sqrt{L/C} = \sqrt{\frac{4.45 \times 10^{-6}}{10 \times 10^{-12}}} = 667.1 \text{ } \Omega$$

(9)

Input Imp of Traction Line



$$V_s(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

$$I_s(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$$

$$Z_0 = \frac{V_0^+}{I_0^+} = \text{Char Imp}$$

$$= -\frac{V_0^-}{I_0^-}$$

$$I_s(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{\gamma z} \rightarrow (2)$$

at $z=0$, $V_s(z=0) = [V_0^+ + V_0^- = V_{in}] \rightarrow (3)$

at $z=0$ $I_s(z=0) = \frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0} = I_{in} \rightarrow (4)$

$$[V_0^+ - V_0^- = I_{in} Z_0] \rightarrow (4)$$

$$V_0^+ = V_{in} - V_0^-$$

$$\text{so} \rightarrow [V_{in} - V_0^-] - V_0^- = I_{in} Z_0$$

$$V_{in} - 2V_0^- = I_{in} Z_0$$

$$V_0^- = \frac{V_{in} - I_{in} Z_0}{2} \quad \text{similarly.}$$

$$V_0^+ = \frac{V_{in} + I_{in} Z_0}{2}$$

at $z=l$ $V_s(z=l) = [V_0^+ e^{-\gamma l} + V_0^- e^{\gamma l} = V_L] \rightarrow (5)$

$$I_s(z=l) = \frac{V_0^+}{Z_0} e^{-\gamma l} - \frac{V_0^-}{Z_0} e^{\gamma l} = I_L$$

$$[V_0^+ e^{-\gamma l} - V_0^- e^{\gamma l} = I_L Z_0] \rightarrow (6)$$

from (5) & (6)

$$V_0^+ = \left(\frac{V_L + I_L Z_0}{2} \right) e^{+\gamma l} = \left(\frac{I_L Z_L + Z_0 I_L}{2} \right) e^{+\gamma l}$$

$$V_0^- = \left(\frac{I_L Z_L - I_L Z_0}{2} \right) e^{-\gamma l}$$

Input Impedance $Z_{in} = \frac{V_{in}}{I_{in}} = \frac{V_0^+ + V_0^-}{\frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0}} = Z_0 \left[\frac{V_0^+ + V_0^-}{V_0^+ - V_0^-} \right]$ NIS

Char's Imp = Z_0 = entire line Impedance
= Txion line "

$$Z_{in} = Z_0 \left[\frac{\left(\frac{I_L Z_L + I_L Z_0}{2} \right) e^{+\gamma l} + \left(\frac{I_L Z_L - I_L Z_0}{2} \right) e^{-\gamma l}}{\left(\frac{I_L Z_L + I_L Z_0}{2} \right) e^{+\gamma l} - \left(\frac{I_L Z_L - I_L Z_0}{2} \right) e^{-\gamma l}} \right]$$

$$= Z_0 \left[\frac{Z_L \left(\frac{e^{+\gamma l} + e^{-\gamma l}}{2} \right) + Z_0 \left(\frac{e^{+\gamma l} - e^{-\gamma l}}{2} \right)}{Z_0 \left(\frac{e^{+\gamma l} + e^{-\gamma l}}{2} \right) + Z_L \left(\frac{e^{+\gamma l} - e^{-\gamma l}}{2} \right)} \right]$$

$$= Z_0 \left[\frac{Z_L \cosh \gamma l + Z_0 \sinh \gamma l}{Z_0 \cosh \gamma l + Z_L \sinh \gamma l} \right]$$

$$Z_{in} = Z_0 \left[\frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l} \right]$$

for lossy transmission line $\gamma = \alpha + j\beta$.

for lossless transmission line $\alpha = 0$, $\gamma = j\beta$.

$$\tanh(\gamma l) = \frac{e^{\gamma l} - e^{-\gamma l}}{e^{\gamma l} + e^{-\gamma l}} = \frac{j(e^{j\beta l} - e^{-j\beta l})/2j}{(e^{j\beta l} + e^{-j\beta l})/2}$$

$$= j \frac{\sin \beta l}{\cos \beta l} = j \tan \beta l.$$

then

$$Z_{in} = Z_0 \left[\frac{Z_L + j Z_0 \tan \beta l}{Z_0 + j Z_L \tan \beta l} \right]$$

↳ for lossless transmission line.

βl = electrical length (rad)

Input Imp for different length of transmission line

(11)

$$Z_{in} = Z_0 \left[\frac{Z_L + j Z_0 \tan \beta l}{Z_0 + j Z_L \tan \beta l} \right]$$

Case 1:- $l = \lambda/8$ $\beta l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{8} = \pi/4$

$$Z_{in} = Z_0 \left[\frac{Z_L + j Z_0 \tan(\pi/4)}{Z_0 + j Z_L \tan(\pi/4)} \right]$$

$$Z_{in} = Z_0 \left[\frac{Z_L + j Z_0}{Z_0 + j Z_L} \right]$$

$$|Z_{in}| = Z_0 \frac{\sqrt{Z_L^2 + Z_0^2}}{\sqrt{Z_0^2 + Z_L^2}}$$

$$|Z_{in}| = Z_0$$

Case 2:- $l = (2n-1)\lambda/4$ $n=1, 2, 3, \dots$

$$n=1, \quad l = \lambda/4, \quad \beta l = \frac{2\pi}{\lambda} \times \lambda/4 = \pi/2$$

$$Z_{in} = Z_0 \left[\frac{Z_L + j \tan \beta l \cdot Z_0}{Z_0 + j \tan \beta l \cdot Z_L} \right] = Z_0 \left[\frac{\frac{Z_L}{\tan \beta l} + j Z_0}{\frac{Z_0}{\tan \beta l} + j Z_L} \right]$$

$$Z_{in} = \frac{Z_0 \times j Z_0}{j Z_L} = \frac{Z_0^2}{Z_L}$$

if $Z_L = \text{open ckt}, \quad Z_{in} = \text{short ckt}$

$Z_L = \text{short ckt}, \quad Z_{in} = \text{open ckt}$

$Z_L = \text{capacitive}$

$Z_{in} = \text{Inductive}$

$Z_L = \text{Inductive}$

$Z_{in} = \text{capacitive}$

that is why $\lambda/4$ length transmission line is called as Impedance Inversion device.

Case 3:- $l = \frac{n\lambda}{2}$, $n = 1, 2, 3, \dots$

$n=1$, $l = \frac{\lambda}{2}$ (Half-wavelength), $\beta l = \frac{2\pi}{\lambda} \times \frac{\lambda}{2} = \pi$.

$$Z_{in} = Z_0 \left[\frac{Z_L + j Z_0 \tan \beta l}{Z_0 + j Z_L \tan \beta l} \right] = Z_0 \left[\frac{Z_L + j Z_0 \tan \pi}{Z_0 + j Z_L \tan \pi} \right]$$

$$Z_{in} = Z_L$$

if length of transmission line is Integer multiples of $\lambda/2$.
load Impedance appears as input Impedance without any change

I/P Imp for diff load conditions

Case 1:- $Z_L = Z_0$ $\rightarrow$ Character Impedance (matched condition)

$$Z_{in} = Z_0 \left[\frac{Z_L + j Z_0 \tan \beta l}{Z_0 + j Z_L \tan \beta l} \right]$$

$$Z_{in} = Z_0 \left[\frac{Z_0 + j Z_0 \tan \beta l}{Z_0 + j Z_0 \tan \beta l} \right]$$

$$\boxed{Z_{in} = Z_0}$$

Case 2:-

$z_L = 0$ (Short circuited)

$$Z_{in} = z_0 \left[\frac{z_L + j z_0 \tan \beta l}{z_0 + j z_L \tan \beta l} \right]$$

$$Z_{in} = z_0 \left[\frac{0 + j z_0 \tan \beta l}{z_0 + 0} \right]$$

$$Z_{in} = j z_0 \tan \beta l = Z_{sc}$$

i/p Impedance of Transmission line when load is short circuited is denoted by (Z_{sc})

(1) $0 < l < \lambda/4$ then $0 < \beta l < \pi/2$ $\rightarrow$ First Quadrant

$$Z_{sc} = j z_0 \tan(\beta l)$$

$$\boxed{IWL = X_L}$$

$$\beta l \Rightarrow 0$$

$$Z_{sc} = 0$$

$$\beta l \Rightarrow \pi/2$$

$$Z_{sc} = \infty$$

$\rightarrow$ purely Inductive

Second

(2) $\lambda/4 < l < \lambda/2$ then $\pi/2 < \beta l < \pi$

$$Z_{sc} = j z_0 \tan(\beta l)$$

$$\boxed{X_C = \frac{1}{j\omega C}}$$

$$\beta l \rightarrow \frac{2\pi}{\lambda}, \lambda/4 \Rightarrow \pi/2, Z_{sc} = \infty$$

$$\beta l \rightarrow \frac{2\pi}{\lambda}, \lambda/2 \Rightarrow \pi, Z_{sc} = 0$$

$\rightarrow$ purely Capacitive

③ $\frac{\pi}{2} < \beta l < \frac{3\pi}{2} \Rightarrow \pi < \beta l < \frac{3\pi}{2}$ (3rd quadrant)

$$Z_{sc} = j Z_0 \tan \beta l = j Z_0 \cdot 0 \rightarrow \text{for } \beta l = \pi$$

$$\text{for } \beta l = \frac{3\pi}{2} \quad Z_{sc} = j Z_0 \tan \beta l = -\infty.$$

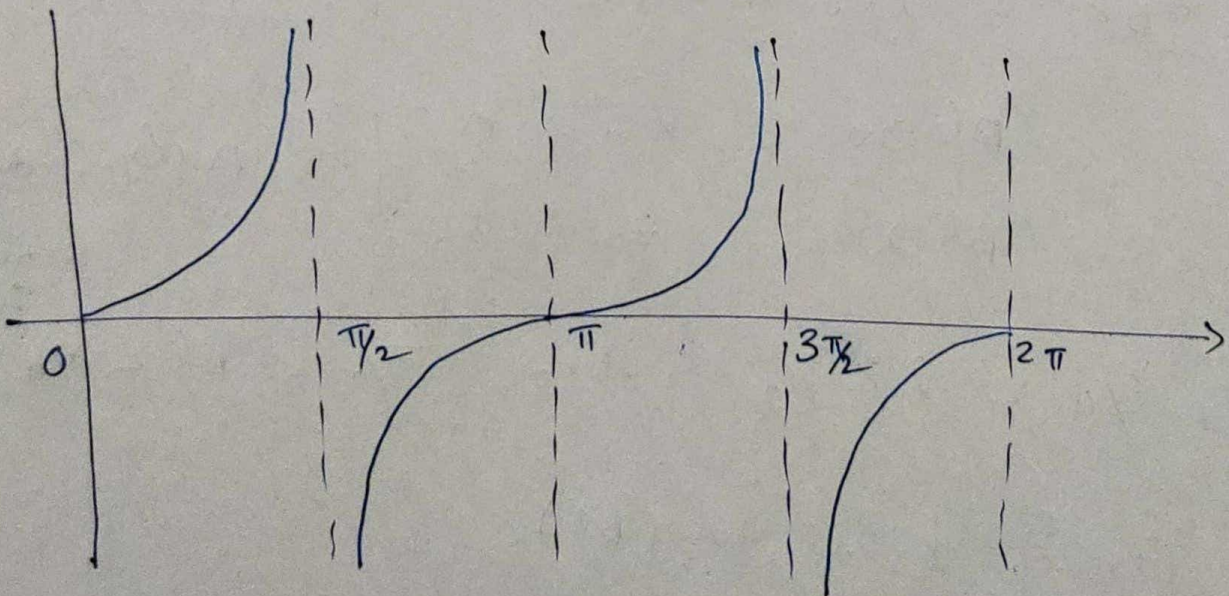
$$Z_{sc} \Rightarrow 0 \rightarrow +\infty$$

(Purely Inductive)

④ $\frac{3\pi}{4} < \beta l < \pi, \quad \frac{3\pi}{2} < \beta l < 2\pi$ (4th quadrant)

$$Z_{sc} \Rightarrow -\infty \rightarrow 0.$$

(Purely capacitive)



Case 3: $z_L = \infty$ (open circuited)

$$z_{in} = z_0 \left[\frac{z_L + j z_0 \tan \beta l}{z_0 + j z_L \tan \beta l} \right]$$

$$= z_0 \left[\frac{1 + \frac{j z_0}{z_L} \tan \beta l}{\frac{z_0}{z_L} + j \tan \beta l} \right] = z_0 \left[\frac{1 + 0}{0 + j \tan \beta l} \right]$$

$$z_{in} = -j z_0 \cot \beta l = z_{oc}$$

i/p Impedance of Transmission line when load is open circuited

(1) $0 < l < \lambda/4, \quad 0 < \beta l < \pi/2$

$$\beta l \Rightarrow 0 \quad \cot \beta l = \frac{\cos \beta l}{\sin \beta l} = \frac{\cos(0)}{\sin(0)} = \frac{1}{0} = \infty$$

$$\beta l \Rightarrow \pi/2 \quad \text{capacitive} \quad = \frac{\cos(\pi/2)}{\sin(\pi/2)} = \frac{0}{1} = 0$$

$$z_{oc} = -j z_0 \cot \beta l = -\infty \text{ to } -0$$

(2) $\lambda/4 < l < \lambda/2, \quad \pi/2 < \beta l < \pi$

$$\beta l \Rightarrow \pi/2 \quad \cot \beta l = \frac{\cos(\pi/2)}{\sin(\pi/2)} = \frac{0}{1} = 0$$

$$\beta l \Rightarrow \pi \quad = \frac{\cos(\pi)}{\sin(\pi)} = \frac{-1}{0} = -\infty$$

Inductive

$$z_{oc} = -j z_0 \cot \beta l = -0 \text{ to } \infty$$

③ $\lambda/2 < l < 3\lambda/4 \Rightarrow \pi < \beta l < 3\pi/2$

$\beta l = \pi \quad \cot \beta l = \frac{\cos(\pi)}{\sin(\pi)} = \frac{-1}{0} = -\infty$

$\beta l = 3\pi/2 \quad \cot \beta l = \frac{\cos(3\pi/2)}{\sin(3\pi/2)} = \frac{0}{-1} = -0.$
Capacitive

$Z_{oc} = -j Z_0 \quad \cot \beta l = \infty \text{ to } 0$

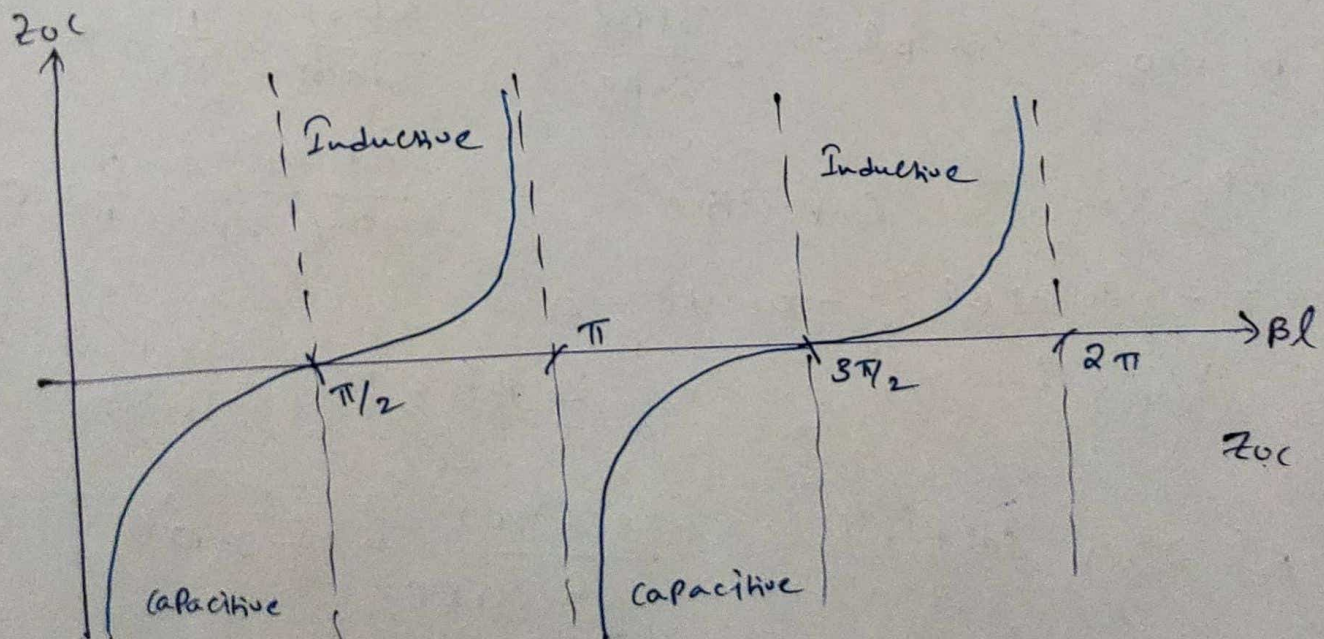
④ $3\lambda/4 < l < \lambda \Rightarrow 3\pi/2 < \beta l < 2\pi$

$\beta l = 3\pi/2 \quad \cot \beta l = -0.$

$\beta l = 2\pi \quad \cot \beta l = \frac{\cos(2\pi)}{\sin(2\pi)} = \frac{1}{0} = \infty.$

Inductive

$Z_{oc} = -j Z_0 \quad \cot \beta l = 0 \text{ to } -\infty.$



(3)

② A certain transmission line operating at $\omega = 10^6$ rad/s has
 ① $\alpha = 8$ dB/m, $\beta = 1$ rad/m, and $Z_0 = 60 + j40 \Omega$, and
 is 2 m long, if the line is connected to a source of
 10 V, $Z_g = 40 \Omega$ and terminated by a load of $20 + j50 \Omega$.
 Then find

(a) i/p Impedance.

(b) Sending-end current

(c) current at the middle of the line

(A) $\alpha \rightarrow$ measured in NP/m

but here $\alpha = 8$ dB/m.

$$1 \text{ NP} = 8.686 \text{ dB.}$$

$$1 \text{ dB} = \frac{1}{8.686} \text{ NP}$$

$$8 \text{ dB} = \frac{1}{8.686} \times 8 \text{ NP/m}$$

$$= 0.921 \text{ NP/m}$$

$$Z_{in} = Z_0 \left[\frac{Z_L + Z_0 \tanh(\gamma l)}{Z_0 + Z_L \tanh(\gamma l)} \right]$$

$$\gamma = \alpha + j\beta$$

$$\gamma = 0.921 + j(1)$$

$$\gamma l = (0.921 + j(1)) 2$$

$$= 1.84 + j2$$

$$Z_{in} = (60 + j40) \left[\frac{(20 + j50) + (60 + j40) (\tan(1.84 + j2))}{(60 + j40) + (20 + j50) (\tan(1.84 + j2))} \right]$$

$$\tan(x + jy) = \frac{\sinh 2x}{\cosh 2x + \cosh 2y} + j \frac{\sin 2y}{\cosh 2x + \cosh 2y}$$

$$\tan(1.84 + j2) = \frac{\sinh(2 \times 1.84)}{\cosh(2 \times 1.84) + \cosh(2 \times 2)}$$

$$+ j \frac{\sin(2 \times 2)}{\cosh(2 \times 1.84) + \cosh(2 \times 2)}$$

$$= 1.033 - j 0.0392$$

$$Z_{in} = (60.25 + j 38.79) \Omega$$

(b) Sending-end current

$$I_{in} = \frac{V_g}{Z_{in} + Z_g} = \frac{10 \angle 0^\circ}{(60.25 + j(38.79)) + 40}$$

$$= 93.03 \angle -21.15^\circ \text{ mA}$$

(c) $V_S(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$

$$I_S(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{+\gamma z}$$

at $z = l/2$, $V_S(z = l/2) = V_0^+ e^{-\gamma l/2} + V_0^- e^{+\gamma l/2}$

$$I_S(z = l/2) = \frac{V_0^+}{Z_0} e^{-\gamma l/2} - \frac{V_0^-}{Z_0} e^{+\gamma l/2}$$

(4)

$$V_0^+ = \frac{1}{2} (V_{in} + I_{in} Z_0)$$

$$V_0^- = \frac{1}{2} (V_{in} - I_{in} Z_0)$$

$$V_0^+ = \frac{1}{2} \left[10 \angle 0^\circ + 93.03 \angle -21.15^\circ \times (60 + j40) \right]$$

$$= 6.687 \angle 12.08^\circ$$

$$V_0^- = \frac{1}{2} \left[10 \angle 0^\circ - 93.03 \angle -21.15^\circ \times (60 - j40) \right]$$

$$= 0.0518 \angle 266^\circ$$

$$I_s(z=l/2) = \frac{6.687 \angle 12.08^\circ e^{-\alpha} e^{-j\beta} - 0.0518 \angle 266^\circ e^{\alpha} e^{j\beta}}{(60 + j40)}$$

$$= 35.1 \angle 281^\circ \text{ mA}$$

④ A 40 m long transmission line shown in figure has

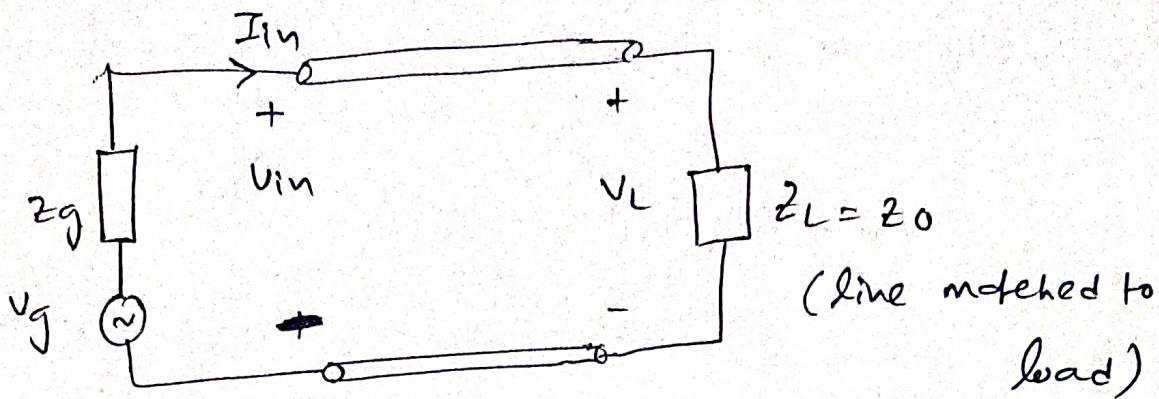
② $V_g = 15 \angle 0^\circ \text{ V rms}$, $Z_g = 30 + 60j \Omega$, $V_L = 5 \angle -48^\circ \text{ V rms}$

if the line is matched to the load, calculate

① V_P I_{imp} Z_{in}

② Sending end current I_{in} & voltage V_{in}

(A)



(a)

$$\dot{V}_p \quad I_{imp} \quad \text{Char's } I_{imp}$$

$$Z_{in} = Z_o = 30 + j60 = Z_g \rightarrow \text{source } I_{imp}$$

(b)

$$I_{in} = \frac{V_g}{Z_{in} + Z_g} = \frac{15 \angle 0^\circ \text{ rms}}{Z_o + Z_g}$$

$$= \frac{15}{2 Z_g} \text{ rms}$$

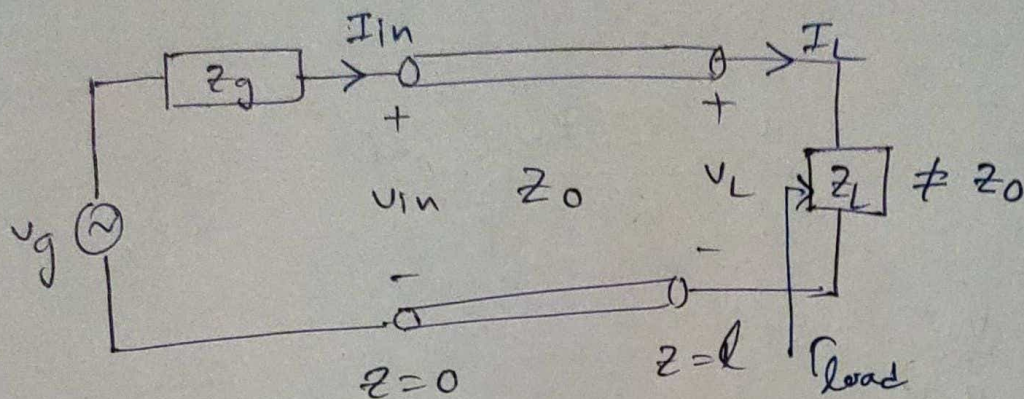
$$= \frac{7.5}{30 + j60} \text{ rms}$$

$$= 0.05 \angle -63.4^\circ \text{ rms}$$

(c)

$$V_{in} = \frac{V_g \cdot Z_{in}}{Z_{in} + Z_o} = \frac{V_g \cdot Z_o}{2 Z_o} = V_g$$

Reflection coefficient at load.



Reflection coefficient = $\frac{\text{Reflected voltage wave}}{\text{Incident voltage wave}}$

$$V_S(z) = \underbrace{V_0^+ e^{-\gamma z}}_{\substack{\uparrow \\ \text{Incident} \\ \text{wave}}} + \underbrace{V_0^- e^{+\gamma z}}_{\substack{\uparrow \\ \text{Reflected} \\ \text{wave}}}$$

$$\Gamma_V = \Gamma_{\text{load}} = \frac{V_0^- e^{+\gamma l}}{V_0^+ e^{-\gamma l}} = \frac{V_0^-}{V_0^+} e^{+2\gamma l} \rightarrow (1)$$

$$V_S(l) = V_0^+ e^{-\gamma l} + V_0^- e^{+\gamma l} = V_L \rightarrow (2)$$

$$I_S(l) = \frac{V_0^+}{z_0} e^{-\gamma l} - \frac{V_0^-}{z_0} e^{+\gamma l} = I_L$$

$$V_0^+ e^{-\gamma l} - V_0^- e^{+\gamma l} = I_L z_0 \rightarrow (3)$$

$$(2) + (3) \quad 2 V_0^+ e^{-\gamma l} = V_L + I_L z_0$$

$$V_0^+ = \frac{V_L + I_L z_0}{2 e^{-\gamma l}} = \frac{I_L z_L + I_L z_0}{2 e^{-\gamma l}}$$

$$V_0^+ = \frac{I_L (Z_L + Z_0) e^{-\gamma l}}{2}$$

② - ③

$$V_0^- = \frac{I_L (Z_L - Z_0)}{2} \cdot e^{-\gamma l}$$

$$\Gamma_{\text{Load}} = \frac{V_0^-}{V_0^+} e^{2\gamma l}$$

$$= \frac{\frac{I_L}{2} (Z_L - Z_0) e^{-\gamma l}}{\frac{I_L}{2} (Z_L + Z_0) e^{\gamma l}} \cdot e^{2\gamma l}$$

$$= \left(\frac{Z_L - Z_0}{Z_L + Z_0} \right) e^{-2\gamma l} \cdot e^{2\gamma l}$$

$$\Gamma_{\text{Load}} = \frac{Z_L - Z_0}{Z_L + Z_0} \Rightarrow \text{Reflection coefficient at load}$$

magnetic reflection coefficient $\Gamma_H = -\Gamma_E$

current reflected coefficient

$$\Gamma_I = \frac{\text{Reflected current wave}}{\text{Incident current wave}}$$

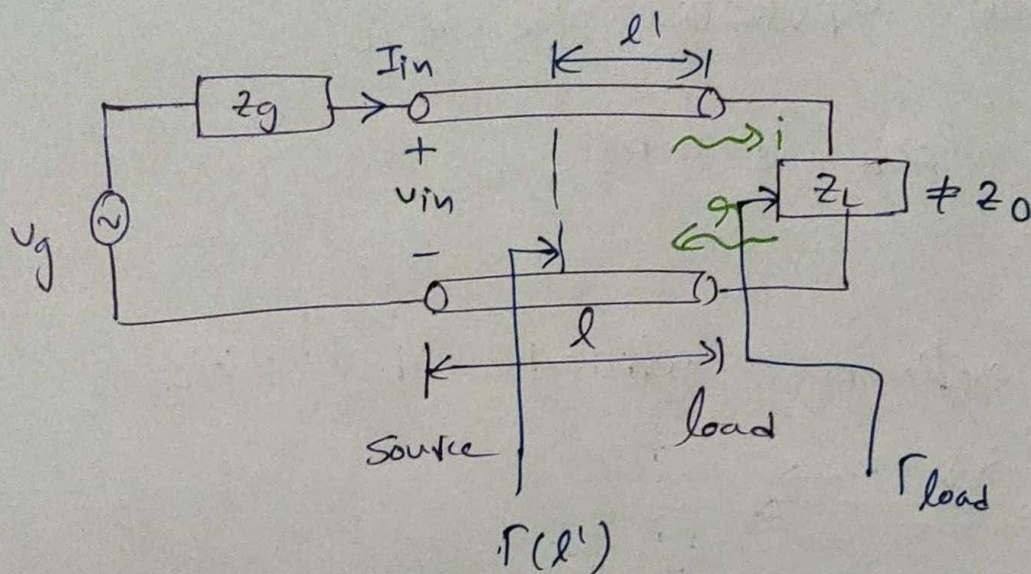
$$\Gamma_I = \frac{-V_0^- / Z_0 \cdot e^{\gamma l}}{V_0^+ / Z_0 \cdot e^{-\gamma l}} = - \left(\frac{V_0^-}{V_0^+} \right) e^{2\gamma l}$$

$$\Gamma_I = - \left(\frac{Z_L - Z_0}{Z_L + Z_0} \right)$$

for matched condition $z_L = z_0$

$$\boxed{\Gamma_V = \Gamma_I = 0}$$

Reflection coefficient at any distance l' from load



$$V_S(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

$$V_S(z = l - l') = V_0^+ e^{-\gamma(l-l')} + V_0^- e^{\gamma(l-l')}$$

$$= \underbrace{V_0^+ e^{-\gamma l} \cdot e^{\gamma l'}}_{\text{forward}} + \underbrace{V_0^- e^{\gamma l} \cdot e^{-\gamma l'}}_{\text{Backward}}$$

$$\Gamma(l') = \frac{V_0^- e^{\gamma l} e^{-\gamma l'}}{V_0^+ e^{-\gamma l} e^{\gamma l'}} = \left[\frac{V_0^-}{V_0^+} e^{2\gamma l} \right] e^{-2\gamma l'}$$

$$\boxed{\Gamma(l') = \Gamma_{load} e^{-2\gamma l'}} \quad \Gamma_{load} = \frac{z_L - z_0}{z_L + z_0}$$

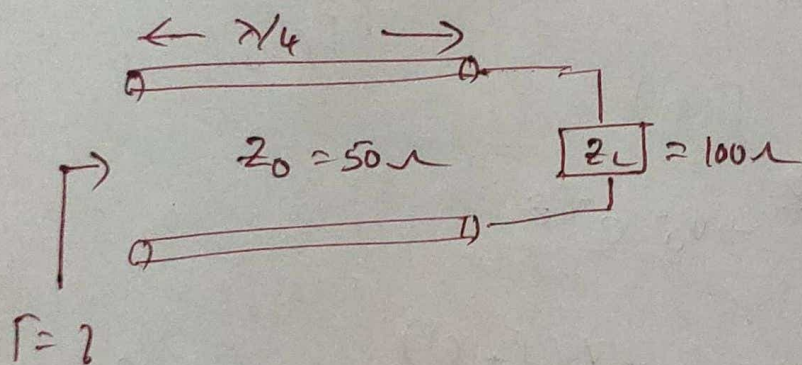
$$\Gamma(l') = \Gamma_{\text{load}} e^{-2(\alpha + j\beta)l'}$$

$$= \underbrace{\Gamma_{\text{load}} e^{-2\alpha l'}}_{\text{magnitude}} \cdot \underbrace{e^{-j2\beta l'}}_{\text{phase}}$$

For lossless transmission line $\alpha = 0$.

$$\Gamma(l') = \Gamma_{\text{load}} \cdot e^{-j2\beta l'}$$

Q3) Find reflection coefficient at $l' = \lambda/4$



Q4)

$$\Gamma_{\text{load}} = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{100 - 50}{100 + 50} = \frac{1}{3}$$

$$\Gamma(l' = \lambda/4) = \Gamma_{\text{load}} \cdot e^{-j2\beta l'}$$

$$= \frac{1}{3} \cdot e^{-j2\beta \cdot \lambda/4} = \frac{1}{3} (\cos \pi - j \sin \pi)$$

$$\beta = \frac{2\pi}{\lambda}$$

$$= -\frac{1}{3} = -\frac{1}{3}$$

Reflection coefficient for complex load:- (5)

$$Z_L = R + jX$$

$$\Gamma_{\text{load}} = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{R + jX - Z_0}{R + jX + Z_0} = \frac{(R - Z_0) + jX}{(R + Z_0) + jX}$$

$$\Gamma_{\text{load}} = \sqrt{\frac{(R - Z_0)^2 + X^2}{(R + Z_0)^2 + X^2}} \left[\tan^{-1}\left(\frac{X}{R - Z_0}\right) - \tan^{-1}\left(\frac{X}{R + Z_0}\right) \right]$$

$$\Gamma_{\text{load}} = |\Gamma_{\text{load}}| e^{j\phi}$$

at any distance l' from load

$$\Gamma(l') = |\Gamma_{\text{load}}| e^{j\phi} \cdot e^{-2\alpha l'}$$

$$= |\Gamma_{\text{load}}| e^{j\phi} \cdot e^{-2\alpha l'} \cdot e^{-j2\beta l'}$$

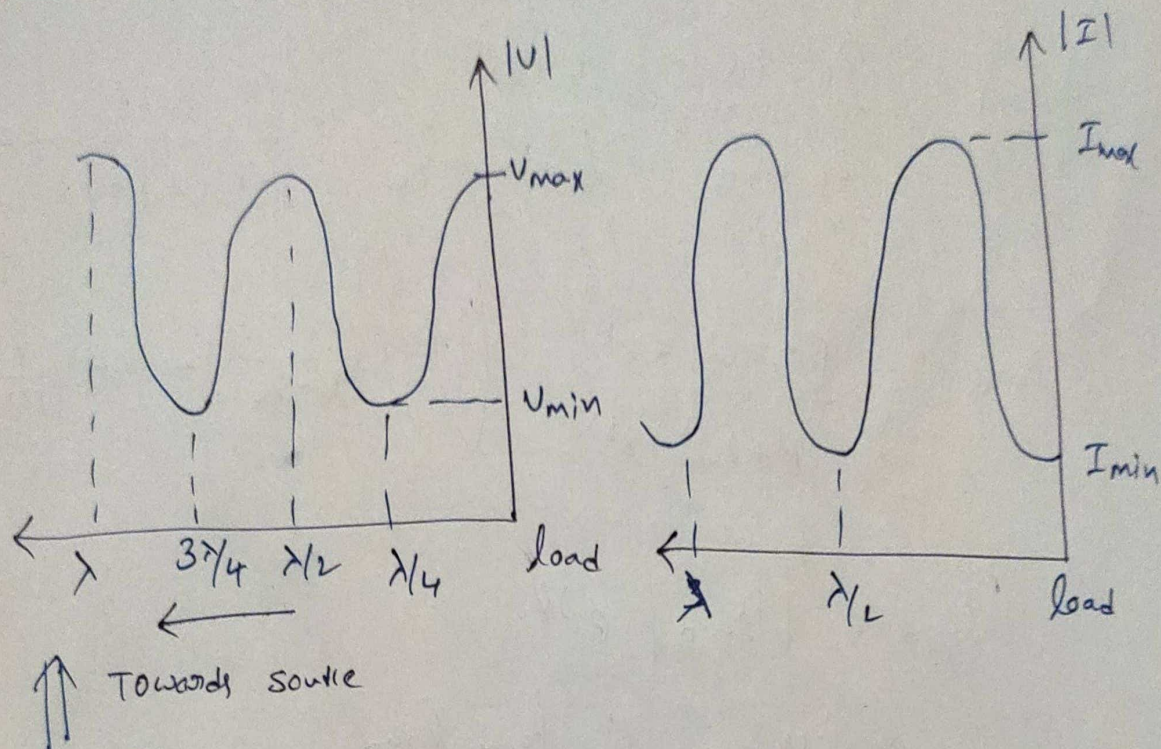
$$\Gamma(l') = |\Gamma_{\text{load}}| \cdot e^{-2\alpha l'} \cdot e^{j(\phi - 2\beta l')}$$

for loss less $\alpha = 0$ $\Gamma(l') = |\Gamma_{\text{load}}| \cdot e^{j(\phi - 2\beta l')}$

Standing wave Ratio:-

$$V_{\text{SWR}} = \left| \frac{V_{\text{max}}}{V_{\text{min}}} \right| = \left| \frac{I_{\text{max}}}{I_{\text{min}}} \right| \left\{ \begin{array}{l} \text{VSWR at load} \\ \text{VSWR at } l' \end{array} \right.$$
$$= \left[\frac{1 + |\Gamma_v|}{1 - |\Gamma_v|} \right] \left\{ \begin{array}{l} \text{VSWR at load} \\ \text{VSWR at } l' \end{array} \right.$$
$$V_{\text{SWR}} = \frac{1 + |\Gamma_{\text{load}}|}{1 - |\Gamma_{\text{load}}|}$$
$$V_{\text{SWR}} = \frac{1 + |\Gamma(l')|}{1 - |\Gamma(l')|}$$

Standing wave Pattern:

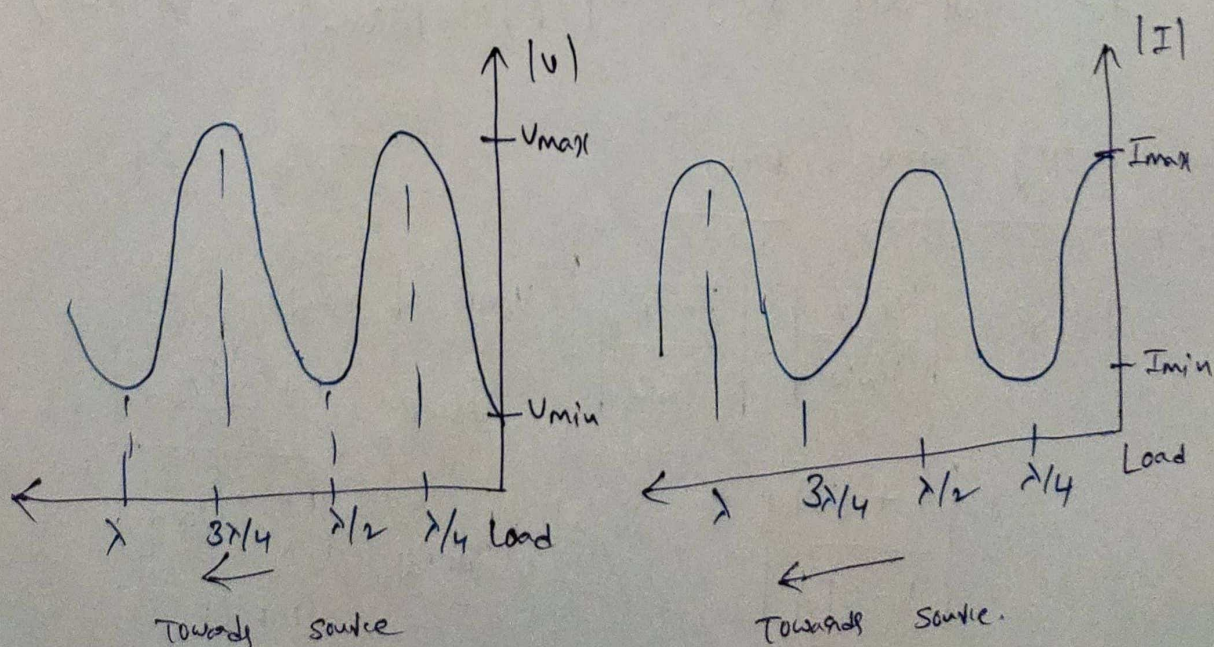


Case 1:- $Z_L > Z_0$ ($Z_L = \text{resistive}$)

① Distance b/w two successive maxima & min is $\lambda/2$.

② if $Z_L > Z_0$ $V_{\text{SWR}} = \frac{Z_L}{Z_0}$ $V_{\text{SWR}} = \frac{1 + \left| \frac{Z_L - Z_0}{Z_L + Z_0} \right|}{1 - \left| \frac{Z_L - Z_0}{Z_L + Z_0} \right|}$

Case 2:- $Z_0 > Z_L$ ($Z_L = \text{resistive}$).



Standing wave ratio, $z_0 > z_L$, $V_{SWR} = \frac{z_0}{z_L}$

Reflection coefficient & VSWR for different load

① Load is short circuited $z_L = 0$

$$\Gamma_{load} = \frac{z_L - z_0}{z_L + z_0} = -1, \quad SWR = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \infty$$

② Load is open circuited, $z_L = \infty$

$$\Gamma_{load} = \frac{1 - z_0/z_L}{1 + z_0/z_L} = 1, \quad SWR = \infty$$

③ Load is equal to Char's Impedance $z_L = z_0$

$$\Gamma_{load} = 0, \quad SWR = 1.$$

from above Range of $\Gamma \Rightarrow -1 \leq \Gamma \leq 1$

Range of $SWR \Rightarrow 1 \leq SWR \leq \infty$

Location of maxima & minima ($Z_L > Z_0$)

$$l_{\max}' = \frac{\phi + 2n\pi}{2\beta}, \quad \begin{array}{l} n=0, 1, 2, \dots \text{ if } \phi \geq 0 \\ n=1, 2, 3, \dots \text{ if } \phi < 0 \end{array}$$

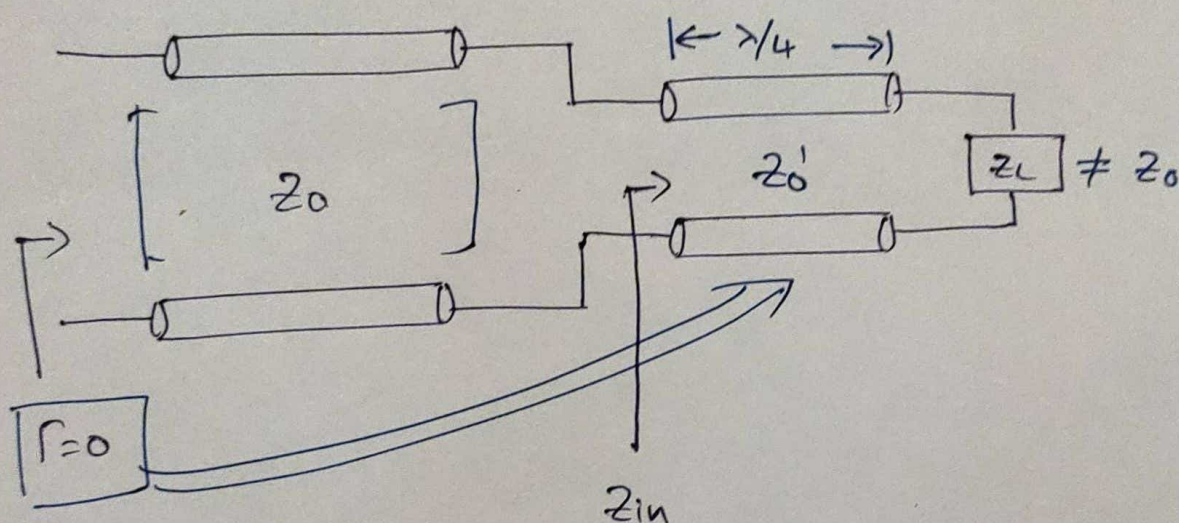
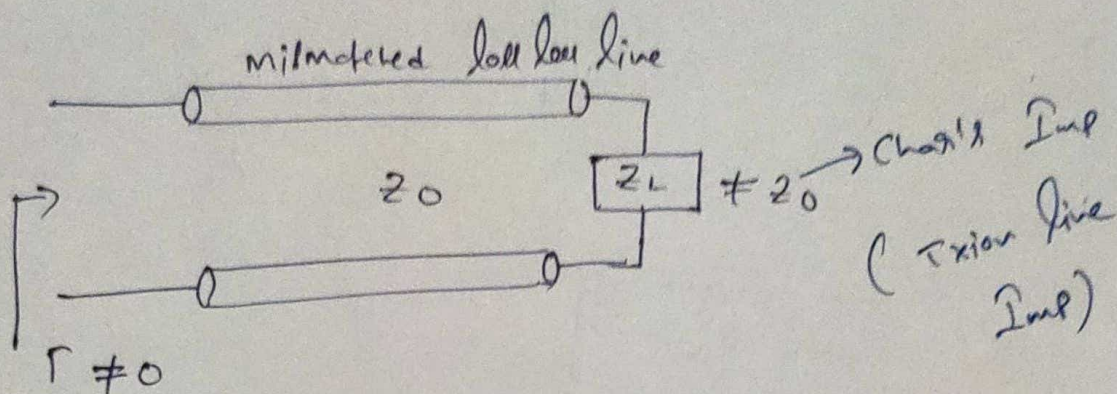
ϕ = phase of reflection coefficient (Z_L is complex)

$$\frac{2n\pi}{2\beta} = \frac{n\pi}{2\pi/\lambda} = \frac{n\lambda}{2}$$

$$n=0 \quad V_{\max} \rightarrow$$

$$n=1 \quad V_{\max}$$

Impedance matching using $\lambda/4$ Transformer



Now the situation is $\Rightarrow$ load Impedance of
lossless line will be input Impedance of $\lambda/4$ line

$$Z_L' = Z_{in}$$

$$Z_{in} = \frac{Z_0'^2}{Z_L} \rightarrow \textcircled{1}$$

$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right]$$

$$\beta l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2}$$

$$\tan\left(\frac{\pi}{2}\right) = \frac{1}{0} = \infty$$

$$= Z_0 \left[\frac{\cancel{Z_L} / \tan \beta l + jZ_0}{Z_0 / \tan \beta l + j\cancel{Z_L}} \right] = \frac{Z_0^2}{Z_L}$$

for $\Gamma = 0$, $z_{in} = z_0 \Rightarrow \textcircled{2}$

$$z_0 = \frac{z_0'^2}{z_L}$$

$\Downarrow$

but

$$z_{in} = \frac{z_0'^2}{z_L}$$

$$z_0' = \sqrt{z_0 \cdot z_L}$$

(8)

(D) Find i/p Imp of short-circuited co-axial transmission

(A) line if $z_0 = 65 + j38 \Omega$, $\gamma = 0.7 + j2.5/m$, $l = 0.8m$

(A) Short circuited

$$Z_{in} = -j z_0 \tan \beta l \quad (\text{lossless})$$

$$Z_{in} = z_0 \left[\frac{z_L + z_0 \tanh(\gamma l)}{z_0 + z_L \tanh(\gamma l)} \right] = z_0 \tanh(\gamma l)$$

$$\gamma l = (0.7 + j2.5)(0.8) = 0.56 + j2$$

$$\sinh(\gamma l) = \sinh(x + jy) = \sinh(x) \cosh(y) + j \cosh(x) \sinh(y)$$

$$\cosh(\gamma l) = \cosh(x + jy) = \cosh(x) \cosh(y) + j \sinh(x) \sinh(y)$$

$$\sinh(\gamma l) = -0.245 + j 0.0548$$

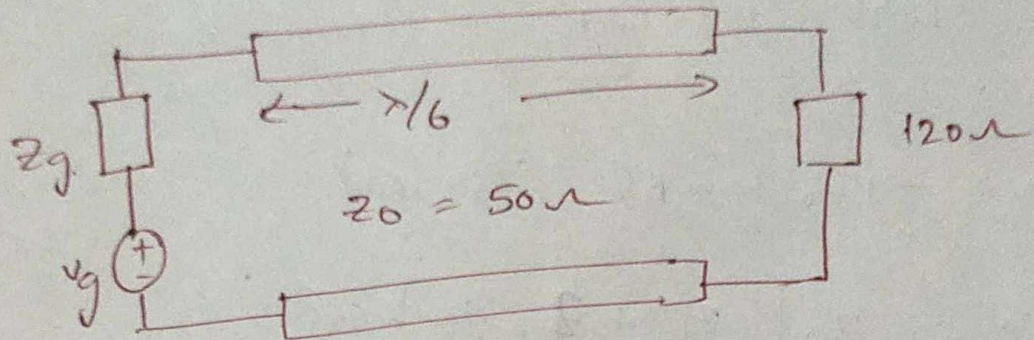
$$\cosh(\gamma l) = 0.483 + j 0.536$$

$$Z_{in} = \frac{(65 + j38)(-0.245 + j 0.0548)}{(-0.4831 + j 0.536)} = 113 + j 2.726 \Omega$$

Q Refer to the lossless transmission line shown in fig.

5 a) find Γ & S

b) det Z_{in} at the generator



A a)
$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{120 - 50}{120 + 50} = 0.411$$

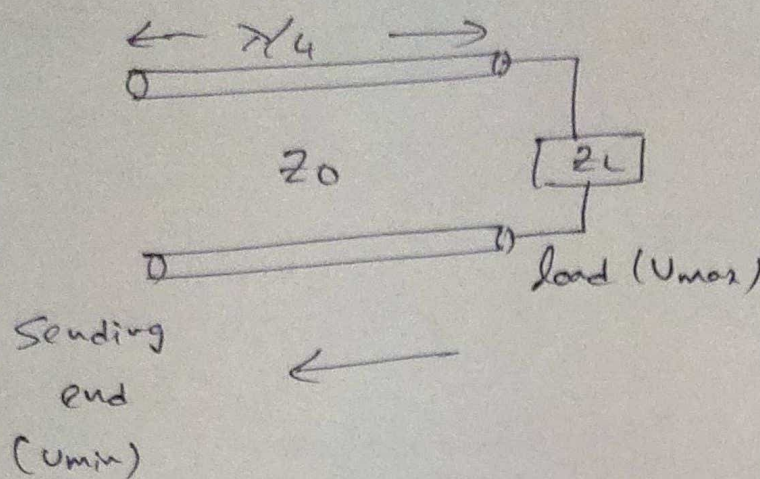
$$S = \frac{Z_L}{Z_0} = \frac{120}{50} = 2.4$$

b)
$$Z_{in} = Z_0 \left[\frac{Z_L + j Z_0 \tan(\beta l)}{Z_0 + j Z_L \tan(\beta l)} \right]$$

$$Z_{in} = 34.63 \angle -40.65^\circ$$

3 A quarter-wave lossless 100Ω line is terminated by a load $Z_L = 210\Omega$, if the voltage at the receiving end is $80V$, what is the voltage at the sending end.

A $l = \lambda/4$, $Z_0 = 100\Omega$, $Z_L = 210\Omega$.
if load is resistive, ($Z_L > Z_0$)



$$V_{SWR} = \frac{V_{max}}{V_{min}}$$

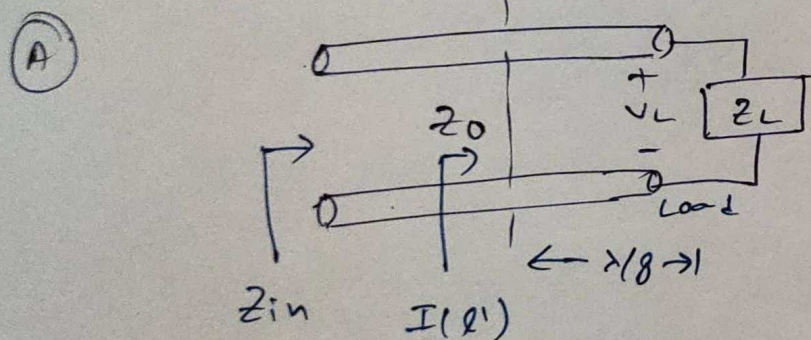
$$V_{min} = \frac{V_{max}}{V_{SWR}} = \frac{80}{2.1}$$

$$V_{min} = 38.11 \text{ V}$$

$$\text{for } Z_L > Z_0, \quad V_{SWR} = \frac{Z_L}{Z_0} = \frac{210}{100} = 2.1$$

⑥ A 500Ω load on a line has $V_L = 10 e^{j25^\circ}$

⑦ $Z_L = 50 e^{j30^\circ}$ find the current at $\lambda/8$ from the load



$$I_L = \frac{V_L}{Z_L} = \frac{10 e^{j25^\circ}}{50 e^{j30^\circ}}$$

$$I(l') = I_L e^{+V l'} = I_L e^{(\alpha + j\beta)l'}$$

$$= I_L e^{j\beta l'}$$

$$= I_L e^{j(\frac{2\pi}{\lambda} \times \lambda/8)}$$

$$= I_L e^{j45^\circ}$$

$$= 0.2 e^{j5^\circ} e^{j45^\circ}$$

$$= 0.2 e^{j40^\circ}$$

lossless $\alpha = 0$.

Q A coaxial line 5.6m long has distributed parameters

Q $R = 6.5 \Omega/m$, $L = 3.4 \text{ mH/m}$, $G = 8.4 \text{ mS/m}$, and

$C = 21.5 \text{ pF/m}$. if the line operates at 2 MHz,

calculate the char's Z_0 & end to end

propagation time delay.

(A)
$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$= 55.12 + j 45.85$$

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

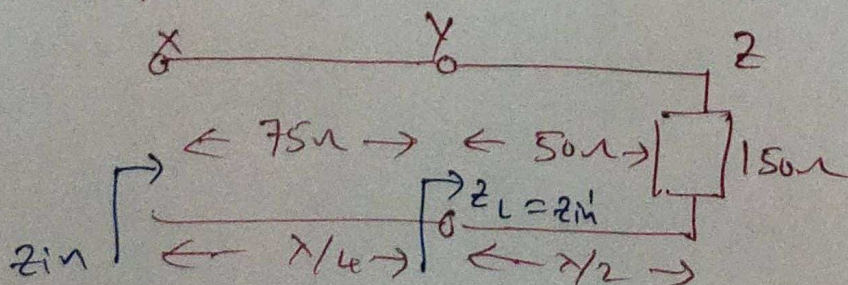
$$= 0.45 + j 0.39 \text{ m}^{-1}$$

$$t = \frac{l}{v_p} = \frac{l}{(\omega/\beta)} = \frac{\beta l}{\omega} = \frac{0.39 \times 5.6}{2\pi \times 2 \times 10^6} = 0.1738 \text{ } \mu\text{s}$$

Q Two lines are connected as shown in figure

Q determine (a) i/p Impedance (b) SWR for XY & YZ

(c) reflection coefficient at Z



(a) for $\lambda/2$ length T.L

$$Z_{in} = Z_L = 150 \Omega,$$

for $\lambda/4$ length T.L,

$$Z_{in} = \frac{Z_0^2}{Z_L} = \frac{(75)^2}{150} = 37.5$$

$$Z_{in} = Z_0 \left[\right]$$

(b) for $Y/2$ - section

$$SY_2 = \frac{Z_L}{Z_0} = \frac{150}{50} = 3$$

$$Z_L > Z_0$$

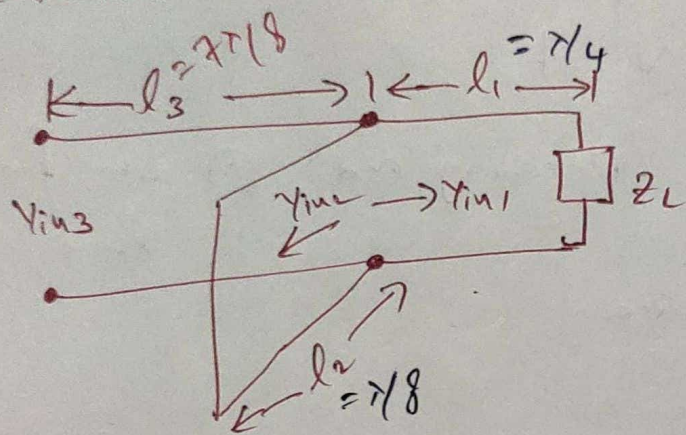
for XY - section $S_{XY} = \frac{150}{75} = 2$

(c) reflection coefficient at Z .

$$\Gamma_{load} = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{150 - 50}{150 + 50} = \frac{1}{2} = 0.5$$

(11)

Q A section of lossless transmission line is shunted across the main line as in figure if $l_1 = \lambda/4$, $l_2 = \lambda/8$ and $l_3 = 7\lambda/8$ find Y_{in1} , Y_{in2} , & Y_{in3} given that $Z_0 = 100 \Omega$, $Z_L = 200 + j150 \Omega$. Repeat the calculations as if the shunted section were open.



$$Z_{in1} = \frac{Z_0^2}{Z_L} = \frac{(100)^2}{200 + j150}$$

$$Y_{in1} = \frac{1}{Z_{in1}} = \frac{200 + j150}{(100)^2}$$

$$Y_{in1} = (20 + j15) \text{ mS}$$

$$Z_{in2} = Z_0 \left[\frac{Z_L + jZ_0}{Z_0 + jZ_L} \right] = jZ_0 = j100$$

$$Y_{in2} = \frac{1}{j100} = -j10 \text{ mS}$$

$$Y_L = Y_{in1} + Y_{in2} = 20 + j15 - j10 = 20 + j5 \text{ mS}$$

$$Z_L = \frac{1}{Y_L} = \frac{1}{(20 + j5) \times 10^{-3}} = 47.06 - j11.06 \Omega$$

$$\beta l = \frac{2\pi}{\lambda} \times \frac{7\lambda}{8} = \frac{7\pi}{4} \Rightarrow$$

$$\tan\left(\frac{7\pi}{4}\right) = -1$$

$$Z_{in3} = Z_0 \left[\frac{Z_L + jZ_0 \tan \beta l_3}{Z_0 + jZ_L \tan \beta l_3} \right] = Z_0 \left[\frac{47.06 - j11.76 + j100}{100 + j(-1)(47.06 - j11.76)} \right]$$

$$Y_{in3} = \frac{1}{Z_{in3}} = (6.408 + j 5.189) \text{ mS}$$

Q A 60- Ω lossless line is connected to a source

$V_g = 10 \angle 0^\circ \text{ V}_{\text{rms}}$, and $Z_s = 50 - j40 \Omega$ and terminated with a load $j40 \Omega$, if the line is 100 m long and $\beta = 0.25 \text{ rad/m}$

then calculate Z_{in} & V at

- (a) Sending end (b) Receiving end (c) 4 m from the load
- (d) 3 m from the source